Allocating systemic risk across institutions: Methodology and Policy Applications

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Focus on the system

- Key lesson from crisis:
  - Emphasis on the system
  - Policy objective to mitigate systemic risk
  - “Macroprudential” approach

- Many prudential tools are institution-specific

- Instruments need to be calibrated on the basis of individual firm’s contribution to system-wide risk
Contributions of this paper

- Propose an allocation procedure of systemic risk to individual institutions based on the “Shapley Value”
  - Efficient, fair, general and robust

- Use the procedure to illustrate the relative importance of different drivers of system-wide risk
  - Size, individual risk and interconnectedness

- Use it to demonstrate how policy tools can be designed to deal with the externalities of systemic importance
  - Macroprudential tools
Allocating systemic risk: Shapley value

- The Shapley value methodology has one requirement:
  - a characteristic function, which …
  - … maps any subgroup of institutions into a measure of risk

- The Shapley value of an institution = its average contribution to the risk of all subgroups of institutions in the system.

\[
ShV_i(\Sigma) = \frac{1}{n} \sum_{n_S=1}^{n} \frac{1}{c(n_s)} \sum_{S \subseteq i \atop |S|=n_S} \left[ g(S) - g(S - \{i\}) \right]
\]

- Degree of systemic importance = Shapley value
Simple example with the Shapley value

- Three players: A, B and C

<table>
<thead>
<tr>
<th>Subgroup</th>
<th>Subgroup output</th>
<th>Marginal contribution of A</th>
<th>Marginal contribution of B</th>
<th>Marginal contribution of C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>4</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>.</td>
<td>4</td>
<td>.</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>.</td>
<td>.</td>
<td>4</td>
</tr>
<tr>
<td>A, B</td>
<td>9</td>
<td>5</td>
<td>5</td>
<td>.</td>
</tr>
<tr>
<td>A, C</td>
<td>10</td>
<td>6</td>
<td>.</td>
<td>6</td>
</tr>
<tr>
<td>B, C</td>
<td>11</td>
<td>.</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>A, B, C</td>
<td>15</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Shapley value</td>
<td>.</td>
<td>4.5</td>
<td>5</td>
<td>5.5</td>
</tr>
</tbody>
</table>
Why Shapley value?

- **Efficient**: allocates total quantity of risk exactly
- **Fair**: allocates risk according to contributions
  - Includes all bilateral links
- **Flexible**: can be applied to any portfolio measure of system-wide risk
- **Robust to model uncertainty**: allocations corresponding to different models can be combined in a straightforward (linear) way to produce robust estimate of systemic contribution
Application using Expected Shortfall

- Define system-wide risk as the credit risk on the combined portfolio of liabilities of “banks” in the system
  - Think of the deposit insurer’s problem
- Expected Shortfall as the risk metric
  - Expected loss in the tail
- Used single-factor default mode model
  - A bank pays back or defaults and pays 1-LGD
- Use two different value functions (1) constant conditioning event [Acharya et al (2009) and Huang, Zhao, Zhu (2009)] (2) conditioning event dependent on coalition
Different drivers of systemic importance

- Drivers considered: size, PD, exposure to common factor

A system of large internationally active banks

- No single driver explains satisfactorily systemic importance ...
The impact of PD and common-factor exposure

- Intuitive results
- An increase in the PD raises systemic importance
- Higher exposure to the common factor …
  - … implies that the bank is more likely to fail with others
  - raises systemic importance
Interaction between different drivers

- Changes in PD have a greater impact on the systemic importance of institutions that are more exposed to the common factor …
Impact of size

- Ceteris paribus systemic importance increases at least proportionately with size of the institution
Size: a convex impact on systemic importance
Impact of size

- Ceteris paribus systemic importance increases at least proportionately with size of the institution

- **Theorem:**
  - Two banks \{B,S\} that are identical except for size
  - B is larger than S
  - \( \frac{ShV(B)}{ShV(S)} > \frac{\text{size of } (B)}{\text{size of } (S)} \)

- Intuition: larger banks appear more often in tail events
Policy intervention: “macro” vs “micro”

- **Objective** of the intervention
  - Attain a given level of systemic risk
  - Equalise systemic importance across institutions, controlling for institutions’ sizes

- Stylised system (mechanical application)
  - Higher capital $\rightarrow$ lower PD
Policy intervention: concrete example

<table>
<thead>
<tr>
<th>0. Initial system</th>
<th>1. Attain target level of systemic risk (ES = 10) with equal PDs</th>
<th>2. Equalise contributions to systemic risk (keeping ES = 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share in total ES</td>
<td>PD (Capital)</td>
<td>Share in total ES</td>
</tr>
<tr>
<td>34%</td>
<td>0.31% (4.0%)</td>
<td>37%</td>
</tr>
<tr>
<td>66%</td>
<td>0.31% (4.0%)</td>
<td>63%</td>
</tr>
</tbody>
</table>

Five banks with a **low** exposure to the common factor ($\rho_{low} = 0.30$)

Five banks with a **high** exposure to the common factor ($\rho_{high} = 0.70$)

*Memo:*

**Total ES and capital**

12.5 (100%) 10 (100%) 10 (100%)

- “Efficiency” result: greater loading on systematic risk implies that a given change in capital (ie PD) has a greater impact on systemic importance
- Opposite outcome also possible, if there are more interactions …
Banks that differ only in size

- Capital charge combinations that result in target level of system-wide risk
- Equal capital charges to both institutions
- Capital charges that equate contributions to system-wide risk
Banks that differ in size and correlation

Equal capital charges to both institutions

Capital charges that equate contributions to system-wide risk
Conclusions

- Shapley methodology provides a neat way to allocate risk
  - Flexibility and robustness

- Attribution of risk needs to look at all drivers and interactions
  - Importance of models
  - Size has a non-linear effect

- Macroprudential policy can lead to re-allocation of capital
Thank you!

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