Producing forecasts with DSGE models: Issues and solutions

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DSGE forecast production at RBNZ

- Raw database
- Monitoring quarters
- External projections
- Judgment
- Main risks

Model-equivalent variables
- SA & smoothing
- MV filter
- Judgment

Historical simulations
- 1st pass forecasts
- 2nd pass forecasts

In-sample database
- Understand init.cond.
- Understand 1st pass
- Understand 2nd pass

Alternative scenarios
- Understand alternatives

Understand alternatives

Supplementary information:
- Historical simulations
- MV filter
- Model-equivalent variables
- Judgment
- In-sample database
- Understand init.cond.
- Understand 1st pass
- Understand 2nd pass
- Alternative scenarios
- Understand alternatives

Note: The diagram illustrates the workflow for producing DSGE forecasts at RBNZ, with key stages including raw database processing, model-equivalent variable generation, historical simulations, and judgment to produce forecasts and assess alternative scenarios.
Some of the issues (and solutions)

1. Treating stochastic trends
2. Adding judgment to DSGE forecasts
3. Communicating DSGE forecasts
1. Stochastic trends

- Data typically suggest many more than one common real trend
- How we treat stochastic trends determines how we set up initial conditions for forecasts
- Some of them important for monetary policy, e.g. relative price of tradables / non-tradables
1. Stochastic trends: Two types of solutions

<table>
<thead>
<tr>
<th>Outside</th>
<th>Inside</th>
</tr>
</thead>
<tbody>
<tr>
<td>build a stationary model</td>
<td>introduce a number of unit roots in the model</td>
</tr>
<tr>
<td>pre-filter data accordingly</td>
<td>use sector-specific productivities et c.</td>
</tr>
<tr>
<td>choose univariate or multivariate methods</td>
<td>Kalman filter with diffuse initial condition</td>
</tr>
</tbody>
</table>
1. Stochastic trends: Inside solution

- BGP models with multiple stochastic trends
- No need to stationarise model: first-order accurate solution valid with unit roots retained
- Taylor (log-)expansion around a snapshot of BGP at an arbitrary date; Taylor coefficients independent of that date
- Preferable form

\[
\begin{bmatrix}
  a_t \\
  x_t^f \\
\end{bmatrix} =
\begin{bmatrix}
  T & 0 \\
  F & 0 \\
\end{bmatrix}
\begin{bmatrix}
  a_{t-1} \\
  x_{t-1}^f \\
\end{bmatrix} + R e_t
\]

\[ x_t^b = U a_t \]

\[ T := \begin{bmatrix}
  I & T_1 \\
  0 & T_2 \\
\end{bmatrix} \]
1. Stochastic trends: Preferences & technology

• Stochastic trends in relative prices and real quantities
  – sector-specific productivities
  – sector-specific iceberg costs, etc.

• Nominal expenditure ratios must stable in the long run (for the approximate solution to hold)

• Implications for elasticities of substitution:
  – unitary in long run
  – reduced in short run by deep habit, adjustment costs, etc.
1. Stochastic trends: Example of utility function

- Tradables and non-tradables driven by distinct unit-root productivity processes (not cointegrated)
- Utility with consumption of tradables and non-tradables

\[
\begin{align*}
\max \ E_0 \sum \beta^t \left( \log \Gamma_t + \cdots \right) \\
\Gamma_t := \left( C_t^\tau - \chi C_{t-1}^\tau \right)^\omega \left( C_t^n - \chi C_{t-1}^n \right)^{1-\omega}
\end{align*}
\]
1. Stochastic trends: Curse of dimensionality

- KITT has more than 20 observables, very few combinations pass co-integration tests
- Demand for pre-filtering (MPC & governor): previous model’s legacy
- No unique/universal trend-cycle decomposition within DSGEs
- Pragmatic solution?
  - include only some (=policy important) trends
  - pre-filter remaining trends
  - use MV filters with some consistency imposed
1. Stochastic trends: Designing MV filters

- MV filter with I(2) (HP-like) trends
- NIPA identities and other definitions – e.g. long-run sustainability of current account
- Would be identical to univariate filters...
- ...unless we start imposing tunes/judgment:

<table>
<thead>
<tr>
<th>Automatic</th>
<th>Forecast-specific</th>
</tr>
</thead>
<tbody>
<tr>
<td>stabilising trends in e.g.</td>
<td>tuning e.g.</td>
</tr>
<tr>
<td>• nominal ratios</td>
<td>• gaps in GDP components</td>
</tr>
<tr>
<td>• growth and interest rates in pre-sample and post-sample</td>
<td>• growth rates in relative prices</td>
</tr>
</tbody>
</table>
1. Stochastic trends: Example of MV filter

- Nominal consumption/GDP
- NT inflation
- World oil price/World PPI
- Consumption gap
- Export gap
- Output gap
- BOP and investment income rate
- Consumption/GDP deflator
- Consumption deflator/CPI
- Consumption deflator/CPI
2. Adding judgment: Why?

• Core (DSGE) projection models must have good forecasting properties to inform policy decisions
• ...but staff & policymakers
  – process information in many other different ways
  – have access to off-model information
• Baseline needs to be staff projection combining many pieces of information rather than model simulation
• Governor signs the baseline and the forecasts need to reflect his beliefs
## 2. Adding judgment: What kind?

<table>
<thead>
<tr>
<th></th>
<th><strong>Structural judgment</strong></th>
<th><strong>Reduced-form judgment</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conditional on distribution of particular structural shocks</td>
<td>Conditional on distributions of particular endogenous variables</td>
</tr>
<tr>
<td><strong>Unanticipated mode</strong></td>
<td>Future judgment not foreseen</td>
<td>Future judgment foreseen</td>
</tr>
<tr>
<td><strong>Hard conditions</strong></td>
<td>Judgment imposed as a single point</td>
<td>Judgment imposed as a distribution</td>
</tr>
<tr>
<td><strong>Density conditions</strong></td>
<td></td>
<td></td>
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</tbody>
</table>

**Reduced-form judgment:**

<table>
<thead>
<tr>
<th></th>
<th><strong>Exactly determined</strong></th>
<th><strong>Underdetermined</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of judgment conditions = number of shocks backed out</td>
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</table>
2. Adding judgment: How?

- Structural & unanticipated: Rather trivial...
  \[ x_t = T x_{t-1} + R e_t \]

- Structural & anticipated: Solution needs to be expanded forward
  \[ x_t = T x_{t-1} + R_0 e_t + R_1 E_t[e_{t+1}] + \cdots + R_k E_t[e_{t+k}] \]

- Reduced-form & unanticipated: Equivalent to Kalman filter
  \[ x_t = T x_{t-1} + R e_t \]
  \[ C_t x_t = y_t + \omega_t \]
2. Adding judgment: How?

- Reduced-form & anticipated, point forecast: Extension of Waggoner-Zha for RE and uncertainty in initial conditions
  \[ x_t = T x_{t-1} + R_0 e_t + R_1 E_t[e_{t+1}] + \cdots + R_k E_t[e_{t+k}] \]
  \[ C_t x_t = y_t + \omega_t \]

- Find likelihood-maximising paths for
  \[ z = [x'_{t-1} \quad e'_t \cdots e'_{t+k} \quad \omega'_t \cdots \omega'_{t+k}]' \]

- Solve
  \[ \min z' \Omega^+ z \quad \text{s.t.} \quad C x = y + \omega \Rightarrow D z = d \]

- Exactly determined solution independent of \( \Omega \)
3. Communicating forecasts

• Unravel story for policymakers who are non-modellers but understand models
• Maximise structural interpretation
• Many possible perspectives, e.g.
  – contributions of individual historical shocks from Kalman filter
  – effects of individual data outturns on differences between this and last forecast rounds
• Here: focus on reconciliation trees
3. Communicating forecasts: Reconciliation trees

• Create a tree of equations following a certain logic, choose LHS variables
• Quantify first-order effects of RHS variables
• Do not use $t+1$ terms; expand expectations forward instead
• Forward expansions lead to Beveridge-Nelson decomposition
3. Communicating forecasts: Example of reconciliation tree
3. Communicating forecasts: Example of one reconciliation node

![Graph showing RMC in non-tradables, weighted contributions, and productivity shocks over the years 2005:1 to 2011:1.](Image)
3. Communicating forecasts: Beveridge-Nelson

- Asymptotic forecast of any (unit-root) variable adjusted for deterministic drift
  \[ \tilde{x}_t = E_t[x_{t+\infty}] - (d_{t+\infty} - d_t) \]

- Extremely easy to evaluate for triangular forms
  \[
  \begin{bmatrix}
  a^1_t \\
  a^2_t
  \end{bmatrix} = \begin{bmatrix}
  I & T_1 \\
  0 & T_2
  \end{bmatrix}
  \begin{bmatrix}
  a^1_{t-1} \\
  a^2_{t-1}
  \end{bmatrix} + \ldots
  \tilde{a}^1_t = a^1_t + T_1 (I - T_2)^{-1} a^2_t
  \tilde{x}_t^b = U_1 \tilde{a}^1_t
  \]

- Easy to evaluate contributions of individual shocks and judgment
3. Communicating forecasts: Example of Beveridge-Nelson

- Iterate UIP forward

\[ s_t = E_t \left[ s_{t+\infty} - \sum (i_{t+k} - i_{t+k}^*) + \cdots \right] \]

- Explain BN trends

\[ E_t [s_{t+\infty}] = E_t \left[ p_{t+\infty} - p_{t+\infty}^* - (a_{t+\infty} - a_{t+\infty}^*) \right] \]
3. Communicating forecasts: Example of Beveridge-Nelson

Nominal exchange rate and its Beveridge-Nelson
Lessons learned from NZ experience

• Projection models need to be exposed to a large variety of (simple) techniques for modellers to:
  – understand DSGE-based forecasts
  – communicate the forecasts with non-modellers
  – maximise the structural DSGE interpretation

• Devices to translate model outcomes into language understandable to other staff and end-users

• Everything needs to be applied effectively and seamlessly (that’s what you get when using IRIS...)