The long-run output-inflation trade-off with menu costs

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Abstract

We examine the long-run output-inflation trade-off under the assumption that firms face menu costs and set prices in a state dependent fashion. We argue that these characteristics capture the idea that the long-run output-inflation trade-off is driven by (predictable) trend inflation, and the degree of price rigidity should be chosen optimally by firms in the long run, at least on average.

We find that state dependent pricing implies a non-trivial departure from long-run monetary neutrality in terms of output, and a larger one in terms of utility. This is because trend inflation substantially influences average mark-ups and relative price distortions. We find that price stability is optimal.

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1 Introduction

The output-inflation trade-off is determined by the nature of shocks that hit the economy, combined with nominal rigidities that prevent prices from adjusting in response to those shocks. One possible source of nominal rigidities is menu costs, which lie at the heart of many “New Keynesian” (for example, Clarida et al (1999)) and “New Neoclassical” (for example, Goodfriend and King (1997)) models. They were first introduced by Mankiw (1985) and Akerlof and Yellen (1985) as a means to offer microeconomic foundations to models with Keynesian characteristics, and generate inertia in nominal variables. They are implicitly assumed in many time dependent pricing models where firms choose a fixed price over the life of a contract (for example, Calvo (1983)) or explicitly modeled in state dependent pricing models as a source of nominal rigidity (for example, Dotsey et al (1999)).

Aside from menu costs, other forms of nominal rigidity have been suggested that may influence the output-inflation trade-off, including information gathering and processing costs (Mankiw and Reis (2002)), imperfect credibility (Erceg and Levin (2003)), and imperfect common knowledge (Woodford (2001)). While all these sources of nominal rigidity may potentially be important in determining the short-run output-inflation trade-off, menu costs are likely to be dominant in determining the long-run output-inflation trade-off. The reason for this is that the short-run trade-off is driven in large part by unpredictable shocks, about which agents and the central bank are likely to have incomplete and asymmetric information, leading to important
roles for many sources of nominal rigidity. But these unpredictable shocks are by definition equal to zero on average in the long-run, implying that they play little role in determining the long-run output-inflation trade-off.

In contrast, trend inflation continues to play a role in determining the output-inflation trade-off in the long run. And in the long run, trend inflation is known by agents, implying little role for information gathering and processing costs, imperfect credibility, and imperfect common knowledge on trend inflation's output effects. But if menu costs exist, they remain important in determining the long-run output-inflation trade-off. We therefore turn off all mean-zero shocks in the model, and then ask what the real effects of (predictable) trend inflation are in the presence of menu costs.\(^1\) In the long run, maximizing agents may be expected to learn about trend inflation and determine their optimal pricing response to trend inflation, both in terms of the price that they set and the average contract length for which that price holds, consistent with state dependent pricing.

Microeconomic evidence of the importance of menu costs includes Levy and Young (2004) who analyze coca cola, whose nominal price remained fixed over a 70 year period, Cecchetti (1986) who showed that magazine prices change more often when inflation is higher, consistent with menu costs, and Levy et al (2002) who find that orange juice prices are more flexible in response to shocks that are larger, more persistent, and on which markets have more information.

\(^1\)One random shock remains in the model. The menu costs that firms must pay to adjust their price each period are identically and independently distributed across firms, as in Dotsey et al (1997, 1999). This assumption ensures tractability by reducing the dimensionality of the state space.
consistent with menu cost models with state contingent pricing. Evidence across broader classes of goods includes Kashyap (1995) who examines catalog data and finds that prices are updated infrequently, but more frequently when the inflation rate is higher, and Levy et al (1997) who directly measure menu costs at supermarkets, and find these are non-trivial. They also find that a supermarket chain that is required by law to place a separate tag on every item for sale experiences menu costs almost three times as high, and changes prices much less frequently, than other supermarket chains. In contrast, survey results (Blinder (1994), Blinder et al (1998) and Hall et al (2000)) suggest that firms do not consider menu costs as a very important consideration when deciding whether to set prices. However, as Blanchard (1994) comments, a central theme of the sticky price literature is that costs of price change that may be trivial to, and have only minor implications for profits of, the individual price setter can still have large macroeconomic effects (Mankiw (1985)).

State Contingent pricing models are based on the principle that firms face menu costs and adjust their prices whenever it is in their best interests to do so (that is, the benefits from adjusting price outweigh the menu costs of doing so). Elsewhere, such models have had mixed success explaining real world data, since they imply that the degree of price stickiness (or average contract length) responds to shocks in a way that reduces the persistent effects of shocks. Given that micro-founded macroeconomic models generally fail to generate empirically plausible persistence (see, for example, Holden and Driscoll (2003), Fuhrer and Moore (1995), and the discussion in Mankiw (2001)), state dependent pricing is unlikely to provide realistic short-run dynamics.
However, our objective here is not to generate realistic dynamics to unpredictable shocks, but instead to ask what the real effects of (predictable) trend inflation are. We answer this question in a state dependent pricing model based on Dotsey et al (1997, 1999).

Earlier, Danziger (1988) and Benabou and Koniczyny (1994) investigate the output-inflation trade-off in a state dependent pricing model incorporating (s,S) pricing, as in Caplin and Leahy (1991). We demonstrate some important differences between the two frameworks, and find that price stability maximizes welfare in our model across a range of parameter values.

The next section outlines our theoretical model, and section 3 contains a study of the long-run output-inflation trade-off implied by the theoretical model. Section 4 concludes.

2 Theoretical Model

Here we develop a theoretical model in the spirit of Dotsey et al (1999) to investigate the long-run relationship between output and inflation. As in their model, the fixed cost of price change is a random variable that firms draw each period so that only a portion of firms will generally change prices in any given period, but all firms that do so choose the same price, since the expected cost of future price changes is the same for all firms. Note that it is a highly non-linear model, but because we are interested in possible multiple equilibria in the model, it
is solved numerically without linearizing around the steady state, as in Burstein (2006).\textsuperscript{2,3} The building-blocks of the model are outlined below.

\subsection{Production}

Firms hire labor and produce a differentiated good. They optimally choose when and by how much to adjust their price subject to random menu costs that are drawn from the same distribution for all firms.

Each firm (indexed by $i$) produces good $i$. In order to produce this, it hires labor ($N_{it}$) from consumers, which the firm then utilizes via linear production technology

\[ Y_{it} = A_t N_{it}, \]  

where $A_t$ represents productivity. The labor market is perfectly competitive, so workers are paid their marginal product

\[ \frac{W_{it}}{P_t} = \frac{Y_{it}}{A_t}. \]  

\textsuperscript{2}See also John and Wolman (2004), who solve a similar model analytically, but only in the case where the maximum contract length is two periods. The focus of their analysis is the existence of multiple equilibria.

\textsuperscript{3}One difference between our model and Burstein (2006) is the assumption here of a distorted steady state. That is, the absence of a lump-sum tax financed production subsidy to offset the distortion introduced by monopolistic competition. As King and Wolman (1999) point out, the effect of this is likely to be small.
2.2 Price Setting

Firms set prices consistent with profit maximization. Real profits for a price set in period $t$, applying to period $t+h$, are given by

$$\pi_{it,t+h} = \frac{\Pi_{it,t+h}}{P_{t+h}} = \frac{P_{it} - MC_{it,t+h}}{P_{t+h}} C_{it,t+h},$$

(3)

implying a price setting decision that satisfies

$$\max E_t \left[ \sum_{h=0}^{J-1} \beta^h \eta_{t,t+h} \left( \frac{P_{it} - MC_{it,t+h}}{P_{t+h}} \right) C_{it,t+h} \right],$$

(4)

where $\eta_{t,t+h}$ is the probability that the price applies in period $t+h$, and $J$ is the maximum number of periods for which firms choose to fix prices.\footnote{$J$ is optimally determined by the price setting decisions of firms, and depends critically on the distribution of menu costs together with the level of trend inflation. With non-zero trend inflation and finite maximum menu costs, all firms will eventually decide to adjust their prices, ensuring finite $J$.} As we will show later, demand by consumer $j$ is given by

$$C_{it}(j) = \left( \frac{P_{it}}{P_t} \right)^{-\epsilon} C_t(j).$$

(5)

Thus

$$P_{it} = \frac{\epsilon}{\epsilon - 1} E_t \left[ \sum_{h=0}^{J-1} \beta^h \eta_{t,t+h} C_{t+h}P_{t+h}^{\epsilon-1}MC_{it,t+h} \right].$$

(6)
2.3 Probability of Changing Price

Since we are interested in the long-run output-inflation trade-off, we will focus on the steady state of our model. Suppose that \( \alpha_i \) is the portion of firms that change prices after \( i \) periods, and \( \alpha_J = 1 \) where \( J \) is the maximum length for which any firms choose to keep prices fixed. Then \( \eta_0 = 1 \) (when prices are reset, they apply with probability 1 in the period in which they are reset), \( \eta_J = 0 \) (prices are all reset within \( J \) periods), and

\[
\eta_{i+1} = \eta_i (1 - \alpha_{i+1}) \quad \forall 1 \leq i < J. \tag{7}
\]

Defining \( \theta_i \) as the portion of firms who last set prices \( i \) periods ago,

\[
\theta_i = \frac{\eta_i}{\sum_{j=0}^{J} \eta_j}. \tag{8}
\]

Suppose that changing prices requires the firm to pay a random cost given by \( \omega_t(\alpha_{it}) \), independently drawn from a distribution that is bounded from above, multiplied by the real wage.\(^5\)\(^6\) Then the \textit{ex ante} expected cost of adjustment in any given period is given by

\[
\Omega_t(\alpha_{it}) = \frac{W_t}{P_t} \left[ \int_{0}^{\alpha_{it}} \omega_t(\alpha) d\alpha \right] / \alpha_{it}. \tag{9}
\]

We follow the case considered in John and Wolman (2004) where \( \omega \) is distributed according to the \( \beta \) distribution on \([0, B]\) where we set \( B = 0.07 \). We consider the special case where \( \omega \) is

\(^5\) With random menu costs, \( \alpha_i \) is both the portion of firms that adjust prices after \( i \) periods and the \textit{ex ante} probability of each individual firm adjusting its price after \( i \) periods.

\(^6\) Rotemberg (2005) provides one rationale for random price adjustment costs. He argues that the cost to a firm of changing its price depends on how consumers will respond to that price change, which is a function of whether they perceive the price change to be “fair” or not. Since consumer perceptions of fairness are likely to be stochastic and time-varying, the cost of changing prices in any given period may also be stochastic.
uniform-distributed.7

Defining $V_h(P^*_t)$ as the value function for a firm that set its price in period $t$ with the price applying in period $t + h$,

$$V_h(P^*_t) = \pi_h(P^*_t) + \beta E_t[\alpha_{h+1}(V_0(P^*_{t+h+1}) - \Omega(\alpha_{h+1})) + (1 - \alpha_{h+1})V_{h+1}(P^*_t)],$$

(10)

where $P^*_t = P^*_t$ is the optimal price set by all firms setting prices in period $t$. Firms will choose whether to change their price or not in any given period so as to maximize this value function. Given the form of $\omega(\alpha)$ above, there will exist some $h$ such that

$$V_0(P^*_{t+h}) - \omega(1) > V_J(P^*_t),$$

(11)

implying that all firms who have not already done so will find it optimal to change prices in period $t + J$. Thus for some $h = J - 1$,

$$V_{J-1}(P^*_t) = \pi_{J-1}(P^*_t) + \beta E_t[(V_0(P^*_{t+h}) - \Omega(1))].$$

(12)

For all other periods, the ex ante probability with which a given firm changes its price in period $h + 1$ (given by $\alpha_{h+1}$) maximizes the firm’s value function, $V_h(P^*_t)$.8

Real profits are given by

$$\pi_{t,t+h} = \Pi_{t,t+h} = \frac{(P^*_t)^{-\epsilon}[P^*_t - \phi C^\sigma_{t+h}(N_{t+h})^\psi P_{t+h}/A_{t+h}]}{P^\epsilon_{t+h}} C_{t+h},$$

(13)

7The results are qualitatively robust to alternative parameter choices here. For the base parameterization considered below, this level of menu cost implies an average contract length of 2.5 periods (6.5 periods) with trend inflation of 5% (2%) in the absence of shocks.

8A Nash equilibrium obtains when the optimal ex ante probability is equal to the ex post proportion of firms changing price in all periods, at all horizons.
where the aggregate price index in period $t$ is defined as

$$P_t = \left[ \sum_{h=1}^{J} \alpha_h \theta_{h-1} P_{it-1}^{1-\epsilon} + \sum_{h=1}^{J} (1 - \alpha_h) \theta_{h-1} P_{it-h}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}. \quad (14)$$

### 2.4 Consumption Decision

Consumers (indexed by $j$) have a utility function given by

$$U_t(j) = \frac{C_t(j)^{1-\sigma}}{1-\sigma} - \frac{\phi}{1+\psi} N_t(j)^{1+\psi}, \quad (15)$$

where

$$C_t(j) = \left[ \int_0^1 C_{it}(j) \frac{\epsilon}{1+\epsilon} \frac{dt}{\Psi_0} \right]^{\frac{1}{1-\epsilon}} \quad (16)$$

is a composite commodity made up of all the different types of goods produced in the economy, $\epsilon > 1$, and $N_t(j)$ is labor supplied by consumer $j$. Each consumer is also assumed to own an equal share of all firms.

The consumer’s consumption decision satisfies

$$\min \quad \int_0^1 C_{it}(j) P_{it}(j) \quad (17)$$

$$s.t. \quad C_t(j) = \left[ \int_0^1 C_{it}(j) \frac{1}{1+\epsilon} \frac{dt}{\Psi_0} \right]^{\frac{1}{1-\epsilon}}, \quad (18)$$

implying demand for each individual good of

$$C_{it}(j) = \left( \frac{P_{it}}{P_t} \right)^{-\epsilon} C_t(j), \quad (19)$$
where the price of the composite commodity is given by

\[ P_t = [\int_0^1 P_t(j)^{1-\epsilon} \, dj]^{1/\epsilon}. \]  

(20)

The consumer’s aggregate demand is then governed by the quantity equation,\(^9\)

\[ C_t(j) \leq \frac{M_t}{P_t}. \]  

(21)

### 2.5 Labor Supply Decision

Since the labor market is perfectly competitive, the consumer’s labor supply (or equivalently wage-setting) decision satisfies

\[
\max U_t(j) = \frac{C_t(j)^{1-\sigma}}{1-\sigma} - \frac{\phi}{1+\psi} N_t(j)^{1+\psi}
\]

\[ s.t. \quad P_t C_t(j) = W_t(j) N_t(j) + \Pi_t, \]  

(22)

(23)

where workers earn income from wages and profits, implying that

\[ W_t(j) = W_t = \phi C_t^\sigma N_t^{\psi} P_t, \]  

\[ W_t(j) = W_t = \phi C_t^\sigma N_t^{\psi} P_t, \]  

(24)

where \( N_t(j) = N_t; \quad C_t(j) = C_t. \)

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\(^9\)John and Wolman (2004) show that the results with this assumption are very similar to assuming a cash-in-advance constraint. This is not surprising, since we are considering a steady-state model without shocks, and in equilibrium households will always hold exactly the correct amount of cash for their desired transactions. We also examine a version of the model in which money enters directly into the utility function, and obtain results qualitatively similar to those reported here (results available from the authors).
2.6 Market Clearing

Market-clearing in the goods market implies that demand equals consumption for each good,

\[ C_{it} = Y_{it}, \]

while aggregate production is given by

\[ Y_t = \int_0^1 Y_{it} di = A_t N_t \]

\[ = \left[ \sum_{h=1}^J \alpha_h \theta_{h-1} P_{it}^{-\epsilon} + \sum_{h=1}^J (1 - \alpha_h) \theta_{h-1} P_{it-h}^{-\epsilon} \right] P_t^\epsilon C_t. \]  

In the labor market, labor demand for an individual firm \( i \) is given by

\[ N_{it} = \frac{Y_{it}}{A_t}, \]

Aggregating over all firms, we have

\[ N_t = \frac{Y_t}{A_t}, \]

which may then be combined with the solution to the consumer’s problem above to determine equilibrium wages and labor.

2.7 Solving the Steady State Model

As in Burstein (2006) and John and Wolman (2004), we assume that the money supply will grow at some rate \( \mu \) each period in steady state, implying that all nominal variables grow at the
same rate (that is, \( P_t = \tau_1 M_t \) and \( P_t^* = \tau_2 M_t \) for constants \( \tau_1, \tau_2 \)).

To solve the model, we first choose arbitrary initial values of \( J \) and the \( \alpha \)'s, and update all other variables consistent with these, until they converge. We then update \( J \) and the \( \alpha \)'s using

\[
\alpha_{h+1} = \omega^{-1}[V_0 - V_{h+1}],
\]

(30)

which is a necessary condition for optimality from the firm’s value function. Using these new values of \( J \) and \( \alpha \)'s, we again update all the other variables, and so on, until all variables converge.

However, as John and Wolman (2004) point out, at this point we do not have sufficiency for equilibrium. In particular, the \( \alpha \)'s we have obtained need not represent the best response of an individual firm, given the \( \alpha \)'s chosen by all other firms. To confirm that we do indeed have an equilibrium, we than take each candidate that satisfies equation (30) and examine the optimal \( \alpha_1 \) and \( \alpha_2 \) of an individual firm, taking the \( \alpha \)'s of other firms (and therefore \( Y, C, N \), etc) as given, using a grid search. If the optimal \( \alpha \)'s of an individual firm coincide with those assumed of other firms, we have verified that this is indeed an equilibrium.\(^{10}\)

In principle, it is possible that there may be more than one equilibria, or indeed no equilibrium at all due to strategic complementarity in the price setting decisions of firms. To ensure that we have considered all such possibilities, we consider a range of possible initial values for the \( \alpha \)'s when we solve the model, (including two extreme assumptions: \( \alpha_h = 1\forall h \) and \( \alpha_h = 0\forall h < 60;\)

\(^{10}\)Our use of grid searches is necessarily limited to searching over \((\alpha_1, \alpha_2)\) due to finite computing capacity, since computing power required increases exponentially with \( J \).
\( \alpha_{60} = 1 \). When combined with our test for sufficiency outlined above, we were left with either zero or one equilibrium in all cases.\(^{11}\)

### 2.8 Parameter Values

We take our parameter values from Gali, Gertler and Lopez-Salido (2007): \( \beta = 0.99; \phi \rightarrow 1 \); \( \sigma \in \{1, 5\}; \psi \in \{1, 5\} \); and \( \epsilon \in \{6, 7.5\} \) (some strategic complementarity across goods, implying an average price mark-up over marginal cost of 15\% to 20\% at zero percent trend inflation).

### 3 Results

Figures 1a and 1b display the long-run output-inflation trade-off implied by our model, across our different calibrations. There are two main results to note. First in the case where firms have relatively little market power (\( \epsilon = 7.5 \), implying approximately 15\% markup), there are regions over which no equilibrium exists. This is because in these regions, which coincide with the transition from a maximum contract length of three periods to two periods, firms’ price setting decisions are “strategic substitutes.” If the aggregate \( \alpha \) is high, then an individual firm’s optimal response is to choose a low probability of price change, and vice versa. This is due in part to the discrete nature of the decision regarding maximum contract length. Indeed, a mixed-

\(^{11}\)By no equilibrium, we mean that there is no “pure strategy” equilibrium. It is likely that a “mixed strategy” equilibrium still exists; see the discussion in John and Wolman (2004).
strategy equilibrium that allowed firms to randomize over one and two period contracts is likely
to eliminate the no-equilibrium range. But as market power increases, implying larger average
mark-ups and less dependence of an individual firm’s price setting decision on the decisions of
others, the range of inflation over which no equilibrium exists shrinks; with $\epsilon = 6$, we find no
evidence of a region with no equilibrium using a step size for trend inflation of 0.01%.

Second, with a coefficient of relative risk aversion of $\sigma = 1$ (implying log utility in con-
sumption), output is maximised at $0^+$, the lowest possible positive rate of trend inflation. This is
consistent with the earlier literature using $(s,S)$ pricing models such as Danziger (1988) and Ben-
abou and Koniczny (1994), and in that framework is due to the role of discounting at low rates
of inflation (see Danziger (1988) for a discussion). Here the mechanism is different. Consumers
respond to increased relative price volatility (which is increasing in trend inflation, as we will
discuss later) by increasing their consumption of goods from non-adjusting firms by more than
they decrease their consumption of goods from adjusting firms, implying that output increases.
But as inflation increases further, the marginal returns to substituting cheaper goods for more
expensive goods falls, and so output declines. Note, however, that the vertical axis is measured
in percent deviations from zero trend inflation; with $\sigma = 1$, money is approximately neutral in
terms of output.

With $\sigma = 5$, in contrast, output is increasing in trend inflation except at those regions

\footnote{A similar no-equilibrium range exists at the transition from a maximum contract length of two periods to one
period. We could find no evidence of a similar “no equilibrium” range during the transition between a maximum
contract length of three and four periods using a step-size for inflation of 0.01%.}
where the maximum contract length falls. The mechanism is the same as for \( \sigma = 1 \), but here the marginal disutility to risk averse agents from decreased consumption in the face of price volatility is amplified, and as a result the effect of inflation on output does not disappear as inflation rises.\(^{13,14}\)

So how do changes in inflation affect utility? King and Wolman (1999) develop a model similar to ours, in which there are two sources of distortion: relative price distortion, and mark-up distortion. Mark-up distortion is due to deviations in the marginal product of labor from the marginal rate of substitution between labor and leisure, which is inversely related to the markup of price over social marginal cost.\(^{15}\) Relative price distortion is due to deviations in relative prices driving consumers’ consumption basket away from its preferred composition (in this case, equal quantities of each good in the economy). King and Wolman (1999) show that an increase in inflation has two offsetting effects on the average mark-up. Firms choose a higher mark-up when they adjust prices, but the average markup deteriorates more rapidly due to the higher trend inflation. In their model, the former effect dominates except at low levels of trend inflation. In contrast, relative price distortion is eliminated at zero inflation, so that welfare is maximized somewhere between zero and the level that minimizes average mark-up. However, they find that the welfare effects of average mark-up are very small, and as a result, optimal inflation is very close to zero.\(^{16}\)

\(^{13}\)Note that at high rates of trend inflation, full monetary neutrality returns. However, this is a function of the discrete time nature of the model: in a continuous time model, money would remain non-supernatural.

\(^{14}\)Empirical evidence on the relationship between output and inflation is mixed: see Bullard and Keating (1995) and Rappach (2003), for example.

\(^{15}\)See also Gali et al (2007).

\(^{16}\)See Woodford (2003) for a more detailed analysis of the desirability of price stability in sticky price models with time dependent pricing, and also Khan et al (2003) who incorporate the Friedman principle that deviations
While both relative price distortion and mark-up distortion are present here, one key difference between our model and King and Wolman (1999) is the price setting mechanism. They use Taylor (1980) contracts, so that the degree of price stickiness is independent of the level of trend inflation, while here state-dependent pricing implies that the degree of price stickiness is itself endogenous.

Figure 2 plots both of these sources of disutility, for $\sigma = 1$.\textsuperscript{17} Relative price distortion, measured as $\sum_{j=1}^{J} \theta_j (P_{it-j} - P_t)^2$, is increasing with trend inflation, but so too are average mark-ups (measured as $\sum_{j=1}^{J} \theta_j (P_{it-j}/MC_t)$), in contrast to King and Wolman (1999).\textsuperscript{18} As inflation increases from zero, the average contract length decreases dramatically (see Figure 3). This reduced average contract length implies that the average price charged by competing firms in future periods increases quickly with trend inflation. Thus a profit maximizing firm that sets its price today with the expectation that it will remain fixed into future periods will mark-up their price over marginal cost by more today than they would at zero trend inflation. These increases are non-monotonic, however, due to the discrete nature of the maximum contract length, with a drop in both average mark-ups and price volatility each point that the maximum contract length reduces by one period.\textsuperscript{19}

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\textsuperscript{17} Qualitatively similar graphs result for $\sigma = 5$.

\textsuperscript{18} In a version of our model incorporating time dependent pricing (in the form of Calvo contracts), average mark-ups do not increase with trend inflation close to zero percent trend inflation, consistent with King and Wolman (1999).

\textsuperscript{19} Indeed, at high levels of inflation, all firms change price every period, so that price volatility disappears and average mark-ups return to zero trend inflation levels.
Figures 4a and 4b combines the different sources of disutility, and show that welfare is decreasing in inflation for all our calibrations, implying that price stability is optimal in our model. Note too that welfare is more sensitive to inflation than output is, with 10% inflation resulting in a welfare loss of between 0.8% and 5%, depending on the calibration.

Our results stand in contrast to the earlier results Danziger (1988) obtained in a state dependent pricing model based on \((s,S)\) pricing, due to differences in the utility function. In Danziger (1988), the marginal utility derived from consuming a good is independent of the remainder of the consumer’s consumption bundle. In contrast, our Dixit-Stiglitz aggregator over consumption goods (16) implies that the marginal utility derived from each good depends on all other goods in the consumer’s consumption bundle. Thus our model incorporates strategic complementarity in the price setting decisions of firms, while Danziger’s (1988) model does not. An increase in trend inflation has a direct effect in raising the prices set by firms, but a further indirect one, in response to the price increases by competing firms. This strategic complementarity in turn results in average mark-ups increasing in trend inflation in our model, ensuring that welfare is maximized at zero trend inflation.

In this model, output is assumed to be perfectly flexible. If output changes were also subject to rigidities or adjustment costs as in Danziger (2007), this would generate two effects. First consumers’ ability to respond to relative price volatility by adjusting quantities in their consumption bundle would be diminished; and second firms would respond to output rigidities by adjusting prices more readily, reducing the underlying level of relative price volatility. Given
that relative price volatility is a major driver of welfare in the model, the first of these effects
would strengthen our conclusion that price stability is optimal, while the second effect would at
least partly offset this. We leave the determination of the net effect for future study.

4 Conclusions

We examine the long-run output-inflation trade-off under the assumption that firms face
menu costs and set prices in a state dependent fashion. We argue that these characteristics
capture the idea that the long-run output-inflation trade-off is driven by (predictable) trend
inflation, and the degree of price rigidity should be chosen optimally by firms in the long run, at
least on average.

We find that state dependent pricing implies a non-trivial departure from long-run monetary
neutrality in terms of output, and a larger one in terms of utility. This is because trend inflation
substantially influences average mark-ups and relative price distortions. The optimal level of
trend inflation is zero.

5 References

   with Wage and Price Inertia, Quarterly Journal of Economics 100(Supplement), 823-38


Figure 1a. Output-Inflation Trade-off; $\sigma = 1$

$\epsilon = 6, \psi = 1$ (---); $\epsilon = 7.5, \psi = 1$ (--); $\epsilon = 6, \psi = 5$ (· · ·); $\epsilon = 7.5, \psi = 5$ (− − −);
Figure 1b. Output-Inflation Trade-off; $\sigma = 5$

$\epsilon = 6, \psi = 1$; $\epsilon = 7.5, \psi = 1$; $\epsilon = 6, \psi = 5$; $\epsilon = 7.5, \psi = 5$;
Figure 2. Sources of disutility $\sigma = 1$

$\epsilon = 6, \psi = 1$ (---); $\epsilon = 7.5, \psi = 1$ (---); $\epsilon = 6, \psi = 5$ (---); $\epsilon = 7.5, \psi = 5$ (---);

Average Markup

Price Volatility
Figure 3. Average Contract Length $\sigma = 1$

$\epsilon = 6, \psi = 1$ (---); $\epsilon = 7.5, \psi = 1$ (---); $\epsilon = 6, \psi = 5$ (\ldots); $\epsilon = 7.5, \psi = 5$ (\ldots);
Figure 4a. Welfare-Inflation Trade-off; $\sigma = 1$

$\epsilon = 6, \psi = 1$ (---); $\epsilon = 7.5, \psi = 1$ (---); $\epsilon = 6, \psi = 5$ (---); $\epsilon = 7.5, \psi = 5$ (---);
Figure 4b. Welfare-Inflation Trade-off; $\sigma = 5$

$\epsilon = 6, \psi = 1$ (---); $\epsilon = 7.5, \psi = 1$ (---); $\epsilon = 6, \psi = 5$ (---); $\epsilon = 7.5, \psi = 5$ (---);