New Zealand Zero-Coupon Yield Curves: A Principal-Components Analysis

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Abstract

This paper describes the statistical properties of a new set of zero-coupon bond yields constructed from New Zealand Government Bond market data. These yields are constructed following the method of Nelson and Siegel (1987) and the extension by Svensson (1994). Trends in the shape of the zero-coupon curve since 1993 are documented. Despite several notable differences between the shapes of the yield curves derived for New Zealand and those for other OECD economies, results in this paper generally confirm that movements in the returns on New Zealand government bonds can be decomposed into the same *level*, *slope*, and *curvature* principal-components that a large literature has documented for various other economies. However, we do find some evidence that a fourth principal-component may also be important in explaining bond return variation.

*This research was conducted while the author was on leave from the Reserve Bank of New Zealand. The views expressed in this paper are those of the author and do not necessarily reflect those of the Bank for International Settlements or the Reserve Bank of New Zealand.

1 Introduction

This paper describes the statistical properties of a new data-set of zero-coupon bond yields constructed from New Zealand Government Bond market data. Such yields are a cornerstone of fixed income analysis, where they are used to price a variety of derivative instruments, and are also an essential input in the estimation of structural models of yield curve dynamics. Though zero-coupon yields can be derived from interest rates on corporate debt or from swap rates, these rates are subject to credit risk and cannot be directly used to price other securities or in arbitrage-free models of the yield curve. In contrast, zero-coupon curves derived from yields to maturity on government bonds are risk-free by definition – insofar as there is negligible risk of sovereign default – and so provide reference rates ideally suited to the uses noted above. However, the estimation of these curves is complicated by the fact that there are typically a large number of government bonds on issue, each differing in maturity and the coupon offered, and which do not always trade in sufficiently liquid secondary markets. Finally, in the absence of a complete set of government debt securities for all maturities, zero-coupon curves must instead be estimated by fitting to the available yield data.

Though there are several alternative approaches to estimating zero-coupon yields, two types of methods have become especially popular. Function-based methods – including the particular form proposed by Nelson and Siegel (1987) and extended by Svensson (1994) – rely on fitting a single mathematical model to yield data on bonds of different maturities. Though the Nelson and Siegel specification and its extensions are the most commonly used functional forms for deriving zero-coupon yields, there is, in principle, no restriction on the choice of function. The key requirement is that that various parameterisations of the function must permit representations of the various shapes of Treasury yield curves over the business cycle.

The other major class of methods relies on fitting piecewise polynomials – usually quadratic, cubic or exponential – to bond yield data, with parameters governing the degree of smoothness of the fitted curve. The choice between these two methods will often depend on whether the researcher is interested in obtaining a higher degree of smoothness in the estimated curve or in better fit to the raw yields.[1]

[1] For a selection of alternative methods used see Bolder et al. (2004) (Canada), Schich (1997) (Germany), Ricart and Siscic (1995) (France), Svensson (1994) (Sweden), Anderson

This paper focuses on the evolution of the New Zealand yield curve since 1993. The approach is similar to that of Bolder et al. (2004) who study the Canadian yield curve. We describe the changing distributional properties of the levels of short- and long-term New Zealand interest rates and of their daily changes. We also consider how measures of the slope and curvature of the yield curve have changed over time, and perform a principal-components analysis to determine the common factors that influence the shape of the yield curve. The chief motivation for this paper is that this data-set should prove useful input in more structural investigations of the New Zealand yield curve, with the conclusions of the principal-components analysis in section 4 providing a statistical benchmark for evaluation of results from these investigations.

2 Estimating zero-coupon curves for New Zealand

We use the method of Nelson and Siegel (1987) and the extension by Svensson (1994) to construct zero-coupon curves. Given the ubiquity of the method, only a brief sketch of the method is presented here.[2] Following Svensson, we assume that the instantaneous forward rate at maturity m is given by the function:

$$f_m = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \left(\frac{m}{\tau_1}\right) \exp\left(-\frac{m}{\tau_1}\right) + \beta_3 \left(\frac{m}{\tau_2}\right) \exp\left(-\frac{m}{\tau_2}\right) \quad (1)$$

where $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1$ and τ_2 are parameters to be estimated. Continuous compounding of the instantaneous forward rate to maturity results in the zero-coupon (or spot) interest rate. Consequently, the latter is defined as the

et al. (1999) (United Kingdom) and Fama and Bliss (1987) (United States). See BIS (2005) for a discussion of the relative merit of different techniques in use at various central banks.

[2] In surveying central banks on the technical details of their derivations of zero-coupon curves, BIS (2005) notes that the majority of reporting banks have adopted variants of the Nelson and Siegel method.

integral of (1) from 0 to m:

$$
\begin{aligned}
s_m &= -\frac{1}{m}\int_0^m f_m(s)\mathrm{ds} \\
&= \beta_0 + \beta_1\left(1-\exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{-m}{\tau_1}\right)^{-1} \\
&\quad +\beta_2\left(\left(1-\exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{m}{\tau_1}\right)^{-1}-\exp\left(\frac{-m}{\tau_1}\right)\right) \\
&\quad +\beta_3\left(\left(1-\exp\left(-\frac{m}{\tau_2}\right)\right)\left(\frac{m}{\tau_2}\right)^{-1}-\exp\left(-\frac{m}{\tau_2}\right)\right) \qquad (2)
\end{aligned}
$$

The original Nelson and Siegel specification is achieved by setting $\beta_3 = 0$. In this case, the infinite-maturity spot rate is β_0, and the starting point of the curve, i.e. as $m \to 0$, is given by $(\beta_0 + \beta_1)$. The spot rate can take on a range of common yield curve shapes including those that are monotonically increasing or decreasing, and convex or concave shapes where the location, size and direction of the hump is determined by the sign and magnitude of β_2 and τ_1. (These facts point to one additional advantage in our use of the Nelson and Siegel method. The three parameters – β_0, β_1 and β_2 can be directly interpreted in terms of terms of the level, slope and curvature principal-components derived in section 4.) The additional parameters β_3 and τ_2 introduced by Svensson allow for the possibility of a second hump, a feature that turns out to be especially important in estimating the New Zealand term structure.

The discount function – that is, the price of a dollar delivered at maturity m – is given by:

$$d_m = \exp\left(\frac{-ms_m}{100}\right), \qquad (3)$$

and the theoretical price and yield of a bond can then be written in terms of this discount function, the coupon rate and a given set of parameters for the spot rate.[3] Given observations on the prices and yields on bonds of different maturities, estimating the zero-coupon curve, s_m, for a particular trade date is a matter of determining the set of parameters that minimises the sum of squared differences between the theoretical and observed price, or between the theoretical and observed yield to maturity.

[3]See Svensson (1994) for algebraic expressions.

2.1 Data

We use maximum likelihood to fit the model to yield data on government bonds and bills made available by the Reserve Bank of New Zealand. We minimise yield errors following the oft-reported observation that the alternative of minimising price errors can result in loss of fit (as measured by yield errors) for instruments with very short maturities. Svensson also makes the argument that minimising yield rather than price errors is more natural for monetary policy analysis.

The data used for estimation consists of 3724 daily observations on secondary market yields from 01 March 1993 to 08 November 2007 on 1-, 2-, 3-, 5- and 10-year benchmark government bonds that pay variable coupon semi-annually.[4] We also use yields on Treasury bills with 1-, 2- and 3- and 6-months to maturity that pay no coupon, and the interest rate on overnight inter-bank transactions. The short end of the estimated zero-coupon curve on any given day is restricted to equal the prevailing overnight rate.[5]

Figure 1 provides a composite picture of New Zealand Government bond yield curves over the sample period. Dividing the sample into halves, government bond yield levels are generally lower and much less volatile at all maturities in the period 2000-07 than between 1993-99 (see table 1). We continue to contrast between these two subsamples in the analysis below.

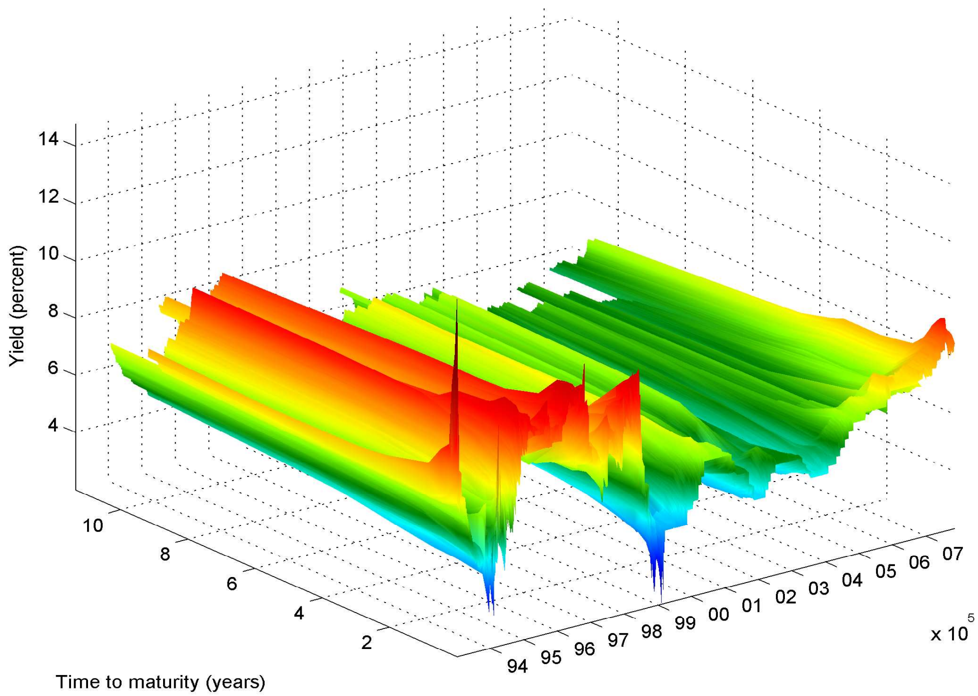
2.2 Estimation process

The procedure we adopt for estimating zero-coupon yields from government bond data is simple. For every trading date in our sample we fit the original Nelson and Siegel model to the yields on bonds for the maturities noted above. We use the estimated parameters on date $t-1$ as the starting values for the maximum likelihood estimation of the term structure on date t. For some dates in our sample, the original specification results in yield errors that are very high in comparison to those for neighbouring dates or

[4] A formal 3-year benchmark for New Zealand government bonds was discontinued in November 2006. To determine the appropriate benchmark from November 2006 to November 2007 we use the bond with the shortest maturity beyond three years.

[5] Though the maturity profile in our estimations is skewed to short-term maturities, this can be justified by the fact that much of the variation in the yield curve shapes actually occurs at these shorter maturities.

Figure 1: New Zealand Government bond yield curves: 1993–2007.



in atypical curve shapes. These errors and atypical shapes are likely to be a consequence either of errors in our estimation data or due to unusual prevailing market conditions. Given that is impossible to distinguish between the two possibilities *ex post* we do not censor any of the estimation results in constructing the data-set, leaving subsequent research projects to filter the results as necessary.

However, there are also some years in our sample (such as the 2003-07 period) when the yield errors are systematically large for a prolonged period of time suggesting that the Nelson and Siegel specification is not a good fit to the observed yield for that period. Consequently, for these years we estimate the model using the extended Svensson specification instead. Table 2 provides a summary of the fit of the model.

Bolder et al. (2004) find a near-secular improvement in the fit of an exponential spline model to Canadian data over the period 1986-2003. The authors attribute this to improvements in bond issuance and trading procedures which allow for more efficient pricing across all maturities. No such

Table 1: Summary of New Zealand Government bond yields (percent)

		O/N^a	1m	2m	3m	6m	1y	2y	3y	5y	10y
1993-1999	Mean	7.07	7.27	7.30	7.31	7.30	7.14	7.09	7.10	7.12	7.15
	SD	1.98	1.85	1.79	1.73	1.60	1.45	1.18	1.07	0.97	0.87
2000-2007	Mean	6.37	6.53	6.54	6.55	6.56	6.06	6.00	6.05	6.08	6.11
	SD	0.99	1.03	1.04	1.05	1.04	0.76	0.61	0.53	0.43	0.36
1993-2007	Mean	6.74	6.93	6.94	6.96	6.96	6.63	6.58	6.61	6.63	6.66
	SD	1.64	1.57	1.54	1.50	1.42	1.29	1.10	1.00	0.93	0.85

[a]Overnight interbank rate

Table 2: Model fit: Root-mean square yield errors (censored, annual means)

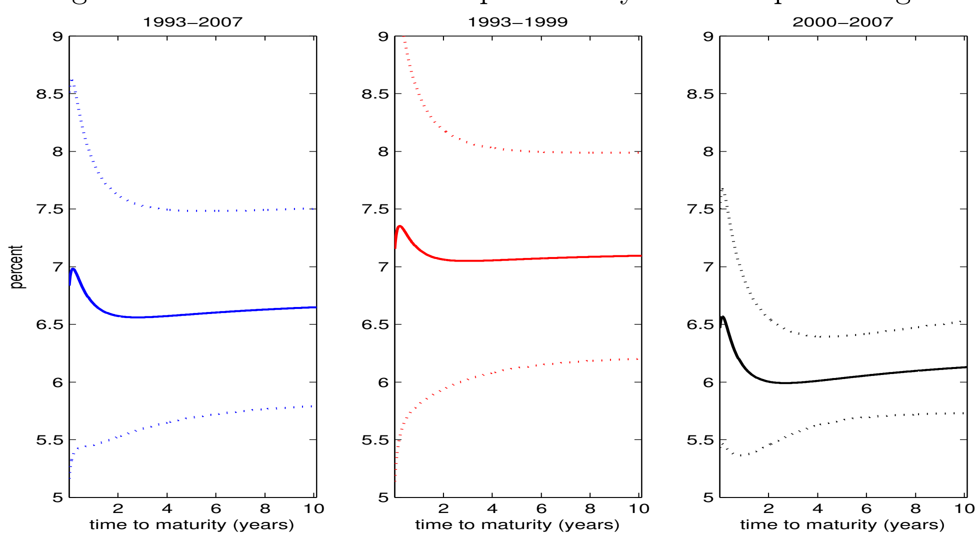
Year	RMSYE (bps)	Year	RMSYE (bps)
1993	10.25	2000	4.43
1994	12.31	2001	9.63
1995	6.46	2002	15.82
1996	10.42	2003	8.55
1997	7.18	2004	8.25
1998	11.77	2005	8.75
1999	14.67	2006	13.58
		2007	23.35
1993-99	10.44	2000-07	11.54

trend is apparent from table 2 for New Zealand data, where the smallest errors are in 2000, and the largest in 2007.[6] With respect to the rationale noted above, this may well be because the New Zealand Debt Management Office had already completed an overhaul of bond issuance procedures by 1993, moving away from a program of ad-hoc issuance of bonds with irregular maturities that traded in illiquid markets to a program developed in consultation with financial markets participants featuring regular issuance of benchmark bonds that are actively traded in secondary markets.

3 Summary statistics

In summarising the statistical properties of the estimated zero-coupon yields over the sample period, we focus on the 1-year and 10-year yields as representative short- and long-term interest rates, and define the slope of the curve as the difference between these two yields. We also consider a measure of the degree of curvature of the yield curve, defining this (in terms of a duration-neutral portfolio choice) as the difference between the 5-year rate and the average of the 1- and 10-year rates.

Figure 2: New Zealand zero-coupon bond yields: sample averages



[6]It is not unreasonable that the errors are largest in 2007, a year that featured several liquidity crunches, especially in the second half, causing short-term spreads to spike well-above historical averages.

Figure 2(a) shows the mean yield curve over the full sample with one standard deviation bounds, and the adjacent panels shows the same data for the periods 1993-1999 and 2000-2007. The average zero curve over the full sample features interest rates that rise above the overnight rate in the very short-term, which then decline for maturities from about three months to about two years, before beginning to rise again (marginally) for maturities through to 10 years. Standard errors around the estimated yield curves are large in the full sample: the average 3-month rate is 7.0 percent over the full sample, and the one standard deviation band is large and ranges from 5.3 to 8.5 percent. The bands do become narrower for maturities up to about 3 years, before stabilising to encompass a range of about 1.75 percent.

The dispersion about the mean zero curve is much larger in the 1990s than in the 2000s for all maturities. The other major difference between the sub-samples is that the short-term hump is dated at an average maturity of 9 months in the 1990s and at only 2 months in 2000s; a likely consequence of the Reserve Bank's move to use the OCR as the monetary policy instrument in 1999 giving it greater influence on short term interest rates.

Figure 3 and 4 show the slope and curvature of the zero curve (as defined above) over the full sample and we see that both of these yield curve measures become markedly less volatile in the 2000s. Though the volatilty decreases, the properties of the levels are much the same: the zero curve was negatively sloped on 55 percent of all trading dates between 1993 and 1999, while the corresponding figure for 2000-2007 is 57 percent. In contrast to the experience of many other countries, yield curve inversions do not appear to have become less common with time. The middle-maturity curvature (by our definition) of the yield curve is generally negative, suggesting a convex hump in the curve is the most likely out-turn, which is corroborated by the average curves shown in figure 2.

Table 3 provides a summary of the descriptive statistics on the five measures of the yield curve considered in this section, including Jarque-Berra probabilities for normality of the measures in the final column. The downward level shift and the reduction in volatility of yields are apparent, especially at the long end of the curve. We reject the normality hypothesis (on the basis of the Jarque-Berra statistics at the 5% level) for all measures of the yield curve in both sub-periods.

Table 3: Summary of New Zealand zero-coupon yields–levels

Variable	Mean	Max	Min	Std Dev	Skew	Kurtosis	JB Prob
1. 1993-2007							
3-month yield	7.00%	10.67%	3.93%	1.55%	0.22	2.03	0.000
1-year yield	6.83%	10.50%	4.49%	1.30%	0.40	2.37	0.000
10-year yield	6.62%	9.44%	4.99%	0.87%	0.82	3.22	0.000
Slope	-0.21%	2.31%	-3.76%	0.95%	-0.00	2.67	0.000
Curvature	-0.16& 0.61%	-1.65%	0.32%	-0.41	3.01	0.000	
2. 1993-1999							
3-month yield	7.34%	10.67%	3.93%	1.76%	-0.13	1.78	0.000
1-year yield	7.28%	10.50%	4.49%	1.44%	-0.11	2.04	0.000
10-year yield	7.08%	9.44%	4.99%	0.90%	0.17	2.78	0.003
Slope	-0.21%	2.31%	-3.76%	0.92%	-0.14	3.24	0.000
Curvature	-0.13%	0.61%	-1.65%	0.34%	-0.73	3.50	0.000
3. 2000-2007							
3-month yield	6.59%	9.54%	4.77%	1.13%	0.32	2.07	0.000
1-year yield	6.30%	8.40%	4.78%	0.86%	0.30	2.26	0.000
10-year yield	6.09%	7.12%	5.04%	0.41%	0.51	2.42	0.000
Slope	-0.21%	2.2%	-2.1%	0.98%	0.15	2.16	0.000
Curvature	0.19%	0.56%	-0.85%	0.30%	0.03	2.43	0.000

Figure 3: 3m-10y slope of the New Zealand zero coupon curve

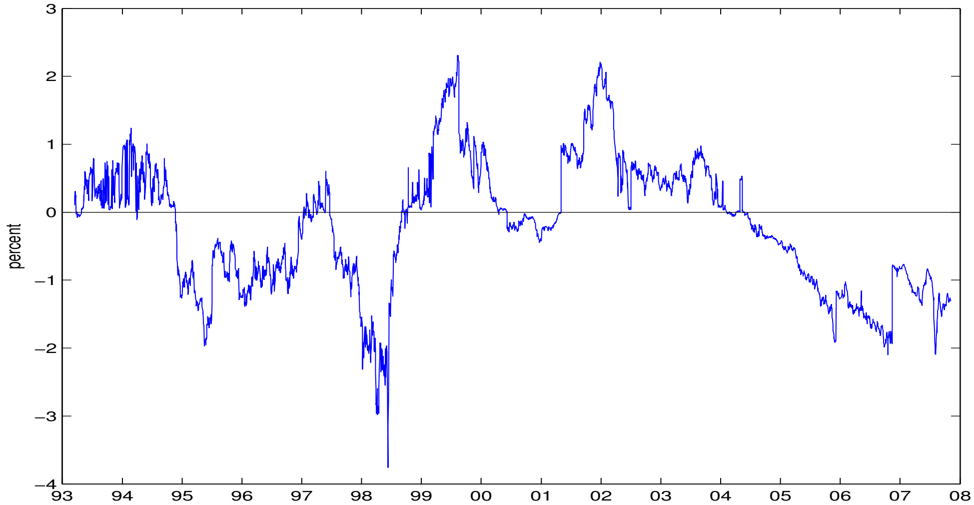
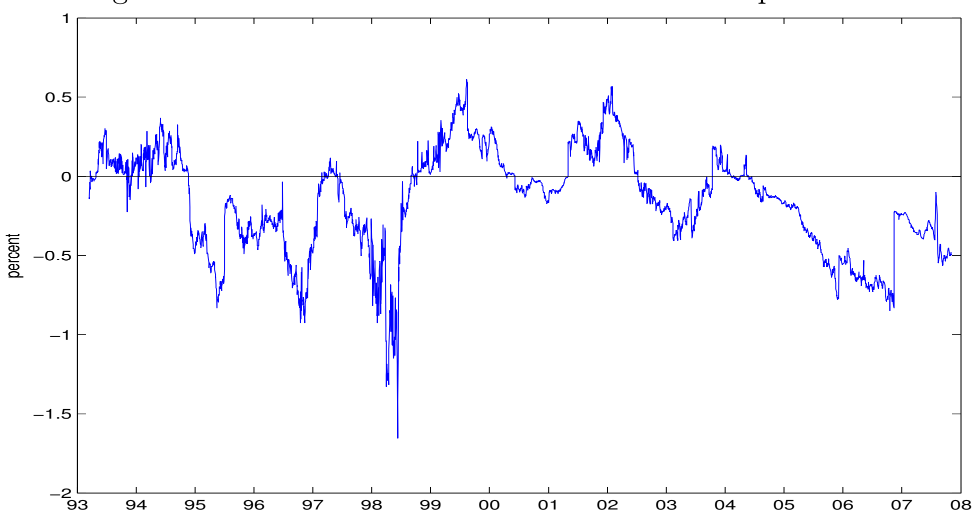


Figure 4: Curvature of the New Zealand zero coupon curve



3.1 Yield curve measures–first differences

We have already noted that zero-coupon yields derived from government bond yields are useful for pricing other securities, and have used this fact to motivate the analysis. As Bolder et al. (2004) explain, the first difference (or daily change) of the yield curve is considered by many market analysts and traders to be synonymous with the short-term risk and return on government bonds. This is because a zero-coupon bond carries no interest payment and over a very short period of time, the contribution to the price of the bond from its accretion to par value is negligible, leaving yield changes as the only fundamental driver of price changes. Bolder et al. go on to note that most security-pricing and risk-management models assume that these returns are normally distributed and show that this assumption is rejected on the basis of the zero-coupon yields they determine from Canadian data, concluding that "models that make the assumption of normality could be producing results that provide inaccurate prices or risk measures".

Table 4 contains summary statistics for the first differences of the five measures of the yield curve we consider, and again the hypothesis of normality is rejected for all yield curve measures for all periods. More interestingly, we find that the results for New Zealand echo those for Canada where the yield differences are highly leptokurtic and subject to extreme outliers (evidenced by the fact that the standard deviations are several orders of magnitude larger than the means.) The implication of these results for asset- and risk-pricing models is that assumptions of normality are likely to result in underestimating the probabilities of both very small and very large changes in yields, unless appropriate hedging strategies are in place.[7]

4 Principal-components analysis

In the previous section, we noted that hedging instruments are often necessary to hedge against the risk that pricing models underestimate very low and very high returns to holding bonds. A common hedging strategy is based on the idea of Macaulay duration which assumes that yield changes on bonds of all maturities are very similar. However, several authors have shown that though bond yield changes do often occur in parallel, they have done so by different amounts for different maturities at various times in history, implying that slope and curvature, and perhaps other, considerations are also likely to be important in designing interest rate derivatives and hedges.

In this section, we use a common non-parametric statistical technique – principal-components analysis (PCA) – to explore the dynamic behaviour of bond yields in New Zealand.

[7] See Bliss (1997), Bolder et al. (2004), and Anderson et al. (1999).

Table 4: Summary of New Zealand zero-coupon yields–first differences

Variable	Mean	Max	Min	Std Dev	Skew	Kurtosis	JB Prob
1. 1993-2007							
3-month yield	0.00%	1.35%	-0.77%	0.10%	0.92	27.88	0.00
1-year yield	0.00%	1.03%	-1.00%	0.09%	0.03	27.28	0.00
10-year yield	-0.00%	0.71%	-0.61%	0.08%	0.59	15.53	0.00
Slope	-0.00%	1.05%	-0.91%	0.11%	0.88	29.06	0.00
Curvature	-0.00%	0.59%	-0.44%	0.04%	0.90	34.52	0.00
2. 1993-1999							
3-month yield	-0.00%	1.35%	-0.77%	0.13%	0.76	16.65	0.00
1-year yield	-0.00%	1.03%	-0.96%	0.12%	0.48	16.37	0.00
10-year yield	-0.00%	0.71%	-0.61%	0.09%	0.40	12.67	0.00
Slope	-0.00%	1.05%	-0.91%	0.14%	0.32	19.07	0.00
Curvature	-0.00%	0.59%	-0.44%	0.06%	0.29	21.68	0.00
3. 2000-2007							
3-month yield	0.00%	0.32%	-0.38%	0.03%	-0.80	36.54	0.00
1-year yield	0.00%	0.31%	-1.00%	0.05%	-6.01	112.5	0.00
10-year yield	0.00%	0.69%	-0.28%	0.06%	1.32	16.61	0.00
Slope	-0.00%	1.02%	-0.46%	0.07%	5.00	74.48	0.00
Curvature	0.0%	0.56%	-0.21%	0.03%	5.49	106.5	0.00

4.1 An overview of the method

Following Litterman and Scheinkman (1991), changes in bond yields of different maturities over time have often been assessed by extracting principal-components (or factors) of the data. That is, though bond yield dynamics reflect a multitude of macroeconomic and financial markets developments, they are correlated across different maturities. Consequently, principal-components analysis aims to decompose these dynamics in terms of a new, smaller set of linearly independent random variables.[8]

Again, given the wealth of literature detailing the use of principal-components for examining bond yield dynamics, we provide only the essential elements of the technique.[9]

Let X be an $n \times T$ matrix of standardised bond yield changes where n is the number of instruments under consideration (ten in this case) and T is the number of observation dates. We seek an orthonormal matrix M which yields a transformation $MX = Y$ such that the covariance of Y is a diagonal matrix. (This transformation allows us to translate a matrix of correlated bond yield data into an uncorrelated matrix.) The principal components of X are given by the rows of M.

The matrix M is easily constructed using an eigenvector decomposition. To see this, note that $E(YY') = E(MXX'M') = M\Omega M'$ where E is the expectations operator and Ω is the covariance matrix of X and will be positive definite so long as none of the yield changes are an exact linear combination of the others. We can diagonalise Ω using the factorisation, $\Omega = FDF'$ where F is the orthogonal matrix of eigenvectors of Ω and D consists of the corresponding (strictly positive) eigenvalues, arranged in decreasing order on the diagonal. Then, making the substitution $F' = M$, we have $\Omega = M'DM$ and $Cov(Y) = D$.

In summary, the principal-components of X are given by the rows of M and the i^{th} diagonal element of D is the variance of X along the i^{th} principal-component. Given that we have ordered the eigenvalues of D, the largest proportion of covariance in X is explained by the first row of M – that is, by

[8] PCA simply involves restating the data in terms of a new basis to help reveal the underlying dependencies and structure. From this perspective, PCA is closely linked to singular value decompositions of the data.

[9] See Bliss (1997) for a different approach to understanding the common drivers of bond yields using factor analysis.

Table 5: Zero coupon yield changes: percentage variation explained by principal-components (full and sub-sample)

Period	1st PC	2nd PC	3rd PC	4th PC	Total
1993-1999	55.8	21.3	12.7	6.0	95.9
2000-2007	44.2	29.5	12.9	8.5	95.2
1993-2007	54.2	23.0	12.4	6.1	95.8

the first principal-component – and so on. The first k principal-components explain much of the correlation in the rows of X if the remaining $n - k$ eigenvalues are relatively small.

The original data can be retrieved from the principal-components by $X = M^{-1}Y = M'Y$ since M is orthogonal. Therefore, the i^{th} zero-coupon bond return can be recovered as:

$$X_i = \mu_i + \sum_{j=1}^{n} \sigma_i m'_{i,j} Y_i \tag{4}$$

where μ_i is the sample mean of the i^{th} series of bond returns and σ_i its sample standard deviation.[10] If only the first k principal-components are found to explain much of the variation in bond returns then the above summation need only be over $j = 1, 2, ..., k$ elements of i^{th} row of M.

4.2 Results

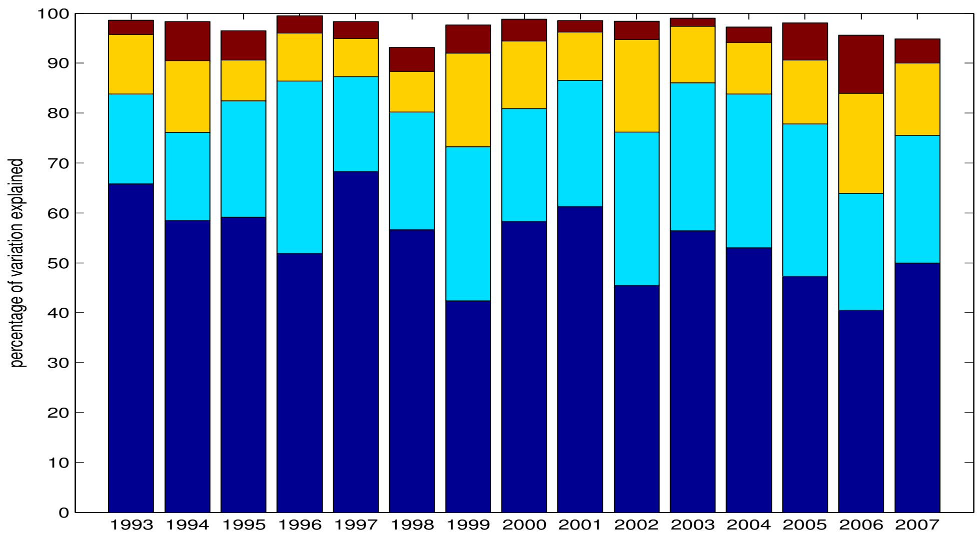
We extract principal-components from standardised zero-coupon daily yield differences for each year in our sample, and then again for the full sample. We calculate the individual and cumulative variation explained by the principal-components (as measured by the ratio of each eigenvalue of Ω to the sum of all eigenvalues), and figure 5 contains a year-by-year plot of the results, while table 5 contains a summary of full- and sub-sample results.

Similar analyses for other economies commonly report that the first three principal-components explain more than 95 percent of the variation in bond returns.[11] To arrive at this threshold for our full sample of New Zealand

[10] See Bolder et al. (2004) for this expansion.

[11] See, for example, Litterman and Scheinkman (1991), Bliss (1997), and Diebold and Li (2004).

Figure 5: Zero-coupon yield changes: variation explained by principal-components (annual)



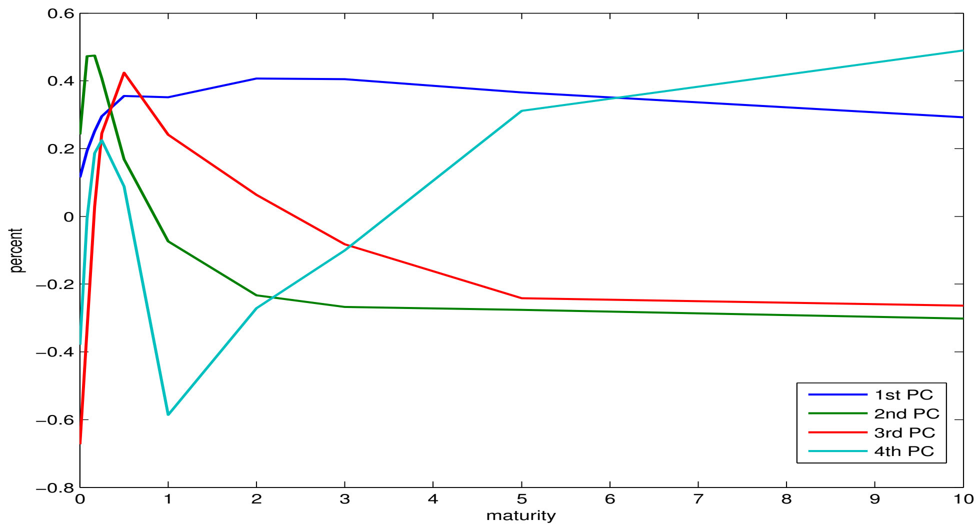
bond yields, we need four principal-components, a fact also true of both subsamples. Turning to the year-by-year evidence in figure 5, the evidence is less clear cut with three principal-components appearing to suffice to explain 95 percent of variation in some years, and four components being necessary in others – 1994, 95, 98, 2005, 06, and 07.

Between the two subsamples, it appears that the first PC explains much less bond return variation in the 2000s than in the 1990s, and the second and fourth PCs explain more. The year-by-year results provide finer detail: the first PC has tended to explain less of the variation over time from a high of 68 percent in 1997 to a low of 39 percent in 2006. The second PC has become more important, especially in recent years. It is interesting to note that the first principal-component contributes much less to the explanation of variation in bond returns than the results in Bolder et al. (2004) for Canada or Bliss (1997) for the US, where it is typically found to explain around 90 percent of the variation by itself. In contrast to these studies, the higher order PCs are significantly more important.

In summary, the first three principal-components explain over 90 percent of the correlation between New Zealand zero-coupon yields (though four are necessary to unequivocally explain more than 95 percent), and make more equal contributions to explained variances than found in results for other countries where the first component dominates.

Figure 6 shows how interest rates at each maturity respond to a change in a given principal-component for the full sample while figure 7 contains the same information on a year-by-year basis. Assessing the full-sample results first, we see that an increase in the first PC results in zero-coupon bond returns across all maturities: a level shift in the terminology of Litterman and Scheinkman (1991).

Figure 6: Response of zero-coupon rates to changes in first four PCs: full sample



An increase in the second PC causes a decay across the term-structure and has been interpreted as a slope factor in that an increase in this PC causes short-maturity returns to rise and long-maturity returns to fall.[12] Conversely, a decrease in this PC can be expected to flatten the curve. This component appears to become much more important in the second sub-sample, when it explains almost two-thirds of the variation in bond returns explained by the first PC. This result of a decay across the term-structure is in contrast to the findings of Bliss (1997) and Bolder et al. (2004) and others who find the second PC to result in an increase in the term-structure with maturity.

The third PC conforms with what has been termed the "curvature factor" in the literature: an increase in this PC causes the yield curve to fall at the short- and long-ends while pushing up the curve at maturities between one

[12]The statement on the uniform decay of bond yield changes in response to changes in the second PC ignores the initial increase in very short term returns which may be due to the fact that we restrict the short end of the curve to the overnight interbank rate.

and three years. The importance of this PC has stayed relatively constant between the two sub-samples.

Finally, the fourth PC – which has been considered unimportant in describing movements in the term-structure of interest rates in other studies – appears to shift the curve down in the very short term, then up for maturities between two- and six-months, down again for maturities upto three years, before again positively influencing the longer end of the curve. This PC has also become a more important influence on the New Zealand curve in recent years.[13]

The characteristic level, slope, curvature and 'double-hump' effects of the four principal-components noted above are generally observed in the year-by-year results too in figure 7, though there are discrepancies with the findings above. For example, in some years the first PC can no longer be interpreted as a level factor (such as in 1999), while in others it changes sign (as in 1993 or 2003). The slope factor also changes sign, but has generally tended to follow the full-sample profile in later years.

5 Conclusion

In this paper, we have described the properties of a new data-set of zero-coupon yield curves. The curves are constructed by fitting a function to yields on coupon-bearing New Zealand government and Treasury bills. Our preliminary statistical analysis suggests that the model of Nelson and Siegel (1987) fits well to New Zealand data in the period since 1993. We further confirm that the assumptions of normality of bond yield levels or in bond returns (as measured by the first difference of the yields) do not hold true of the data. In the case of the latter the distribution is found to exhibit high kurtosis and to be subject to extreme outliers. This implies that portfolios constructed to manage bond risk are unlikely to perform adequately if they only account for changes in the level of returns across the New Zealand yield curve.

Using principal-components analysis, we further show that four principal-components are necessary to explain more than 95 percent of the variation in the New Zealand term structure of interest rates. This is in contrast to studies

[13] This may prove useful in examining the suggestion that international financial conditions and monetary policy have had a much larger influence on the longer end of the New Zealand yield curve, as compared to domestic policy, in recent years.

on other sovereign yield curves that find evidence for the importance of only three factors in explaining the variation. The first principal-component – the level – explains much less of this variation in the second half of our sample than in the first half, while the second principal component – the slope – becomes increasingly important.

The statistical results in this paper should prove useful as a benchmark for more analytical investigations of yield curve dynamics that employ this new set of zero-coupon yields.

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Figure 7: Response of zero-coupon rates to changes in first four PCs: year-by-year