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International Risk Sharing and Wealth Allocation with Higher Order Cumulants

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Abstract

We study international risk sharing across countries differing in size, openness, and productivity distributions, emphasizing fat tails. In a canonical IRBC model, safer economies benefit through asset and terms-of-trade revaluations, while riskier ones smooth consumption at the cost of lower wealth. Calibrated to non-Gaussian shocks, country size and openness, the model predicts welfare gains between 0.09% and 3.0% of permanent consumption (median 0.34%). Assuming Gaussian shocks leaves the median gains virtually unchanged but reduces them for the most tail-exposed economies. In contrast, imposing equal country size and eliminating home bias sharply compresses the distribution of gains and lowers their overall magnitude. Clustering economies by openness, size, and higher moments aligns well with the cross-country distribution of gains.

Key words: Asymmetries in Risk, Openness, Country size, Tail Risk, Gains from Risk Sharing, Consumption Smoothing, Terms of Trade, Wealth Transfers.
JEL classification: F15, F41, G15.

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1 Introduction

The Global Financial Crisis, the euro-area sovereign debt crisis, the early impacts of climate change, and, more recently, the COVID-19 pandemic have all highlighted the global economy's vulnerability to disruptive shocks. Policymakers face an environment characterized by heightened macroeconomic and financial risks, in which both idiosyncratic and global disturbances propagate asymmetrically across borders, deepening cross-country disparities in wealth and welfare. As shown in Table 1, countries differ substantially in the cyclical properties of log per-capita real GDP: across all moments reported in the table, values in the top five percent of the distribution are several times larger than those in the bottom five percent. This evidence motivates a re-examination of the potential benefits of cross-border insurance, placing key dimensions of heterogeneity—including the marked differences in country size and openness also highlighted in the table— at the forefront of the analysis.

This paper studies the welfare gains from sharing production risk across countries that differ in size, openness, and underlying risk, under non-Gaussian productivity shocks. Using a canonical international real business cycle (IRBC) framework, we provide both analytical insight into the mechanisms through which heterogeneous economies benefit from cross-border insurance and a quantitative evaluation of how heterogeneity shapes potential welfare gains.

Our contribution is threefold. First, we map the gains from risk sharing onto the equilibrium adjustment of asset prices, wealth, consumption, and terms of trade, distinguishing between level effects (LE) on unconditional consumption, and smoothing effects (SE) on the variability of the stochastic discount factor. Second, using perturbation methods we derive an analytical decomposition of macroeconomic and financial effects of risk sharing by cumulants (equivalently moments), enabling a detailed assessment of how asymmetries in risk—up to the fourth order—affect welfare differences across countries of unequal size and openness.[1] Finally, we complement our analytical results with numerical simulations based

[1] Cumulants are tightly related to moments. In Section 3 we provide a proper definition of cumulants

Table 1: Distribution of higher moments of cyclical GDP, size, and openness

	Standard deviation	**Skewness**	**Kurtosis**	**Population size**†	**Consumption import share**‡
Min	0.02	-5.89	3.05	0.39	11.81
5%	0.03	-5.22	3.90	4.99	14.05
50%	0.04	-0.97	6.33	39.30	24.84
95%	0.09	0.14	46.95	451.39	35.66
Max	0.10	0.70	55.64	1438.07	52.38

† In millions. ‡ In percentage points.
Source: Detrended (Hodrick–Prescott filter, $\lambda = 100$) log real GDP per capita from the Barro–Ursúa macroeconomic dataset Barro and Ursúa (2008) spliced with PWT 11.0; 1901–2023; the sample includes the 39 economies used in the quantitative analysis; extreme observations are retained. The consumption import share is the foreign value-added content of final domestic demand from the OECD Inter-Country Input–Output tables, averaged over 2016–2022.

on global methods, calibrating the model to data on size, openness, and output distributions for 39 economies.

Based on our theoretical decomposition, we show analytically and numerically that, all else equal, countries more exposed to extreme events—manifested in larger variance, more negative skewness, or thicker tails in the distribution of income (endogenous to production uncertainty)—derive the greatest benefits in terms of consumption smoothing in the transition from financial autarky to complete markets. Safer countries, instead, benefit from the macroeconomic analog of an insurance premium driven by the equilibrium re-pricing of assets and goods. This market-based implicit wealth transfer tends to improve their terms of trade. Remarkably, we find that safety commands both a welfare and a financial premium: theory predicts a positive relationship between welfare and asset prices under perfect risk sharing. This is in sharp contrast with financial autarky, under which non-traded risk causes this relationship to be uniformly negative.

We characterize how the interactions between elasticities, country size, openness,

and explain that cumulants are useful in solving our model via perturbation methods.

and output variability crucially weigh on the endogenous status of safe and risky countries and the welfare relevance of macroeconomic insurance. The integration of smaller economies into complete global markets has negligible effects on state-contingent prices (i.e., world asset prices). As a result, smaller economies face a larger budget set, since the prices of the states in which they buy or sell insurance do not move against them. For given size, in turn, gains from risk sharing vary with openness, i.e., home bias in demand. While openness tends to reduce a country's exposure to own output shocks, it also makes it easier to stabilize utility by substituting domestic with imported goods. We show that the trade elasticity plays a comparably important role: the larger this elasticity, the more efficient the substitution, but also the greater the relative exposure of domestic income to domestic, as opposed to foreign, productivity shocks. Analytically, complementing Martin (2008), we specifically characterize how the welfare gains from sharing output tail risk depend jointly on trade elasticities and risk aversion, both governing the mapping of productivity shocks onto income and consumption risk.

Overall, our findings show that an assessment of the potential gains from risk sharing requires a proper calibration of heterogeneity in economic fundamentals. In our empirical application, the standard IRBC model—calibrated with non-Gaussian output shock estimates, country size (governed by population size), and openness data—predicts welfare gains from sharing productivity risk that range between approximately 0.09% and 3.0% of permanent consumption, with a median of about 0.34%.[2] In the upper tail of the distribution, the magnitude of these gains is relatively high compared to most leading contributions on the subject. For instance, Coeurdacier et al. (2020) find that financial integration under complete markets yields gains around 0.5% of permanent consumption, up to 1% in some extreme specifications—somewhat comparable to the welfare gains from eliminating the business cycle in Lucas (2003).[3] By contrast, departing from log-normality, Martin proposes a

[2]Labor substitution plays a large role in self-insurance. These higher numbers are generated with Greenwood et al. (1988) (GHH) preferences. Separable CRRA preferences generate gains half as large. Epstein and Zin (1989) preferences are known to further magnify these gains.

[3]Lucas (2003) estimates that fully eliminating business-cycle uncertainty would raise permanent consumption by less than 1% (possibly as little as 0.01%).

striking revision of Lucas' estimates, with gains up to about 14%, "largely attributable to higher cumulants" (Martin, 2008, p. 75). Most of these differences are due to assumptions concerning technology and preferences. Endowment economies tend to generate larger gains. Models with more margins for self-insurance generate a smaller range of gains.

In spite of its stylized nature, our model allows countries to differ along five dimensions—three moments, size, and openness—all shaping the gains from trade through complex interactions. Indeed, in our evidence-based assessment, the ranking of gains closely aligns with a statistical clustering of countries by size, openness, and moments. A key takeaway from our analysis is that output uncertainty in general, and non-Gaussianity ("fat tails") in particular, play a significant role in generating large gains from risk sharing, provided that economies are modeled allowing for the heterogeneity in size and openness observed in the data.

Methodologically, we adopt a two-pronged approach. For economic insight, Section 4 solves the model analytically using standard higher-order perturbation methods.[4] Under suitable simplifying restrictions, these methods yield transparent, tractable expressions, in line with a large body of macroeconomic and financial research where they are the dominant technique. As a byproduct, we present an algorithm for deriving time-0 relative marginal utility (consumption) under complete markets—naturally interpreted as "Negishi weights"—for a general class of DSGE models. To facilitate comparison with existing work, in the analytical section we adopt a two-country specification of the model.

For the quantitative analysis we use global solution methods and conduct two sets of exercises. First, we perform a controlled experiment with generated data to reassess the insights from the analytical solution in more general settings, through numerical exercises that relax simplifying assumptions while retaining the two-country version of the model. Second, we calibrate the full three-country version to data spanning 120 years. The calibration proceeds country by country: each country in turn serves as the home economy, with the two foreign blocks, given by Advanced Economies (AE) and Emerging Market Economies

[4]See, among others, Holmes (1995), Judd (1998), Schmitt-Grohé and Uribe (2004), Lombardo and Sutherland (2007), Kim et al. (2008), Andreasen et al. (2018), and Lombardo and Uhlig (2018).

(EME). This specification allows for heterogeneity in size, openness and business-cycle moments in the "rest of the world", and — going beyond the two-country structure — balances cross-country heterogeneity against computational tractability. The AE/EME partition is a conventional choice, made for concreteness rather than as the unique mapping of the model to the data.

Literature. Our work draws on three main strands. First is the extension of portfolio and asset-pricing theory beyond mean-variance. Early contributions, such as Samuelson (1970), warned against limiting analysis to first and second moments. Higher moments jointly determine asset values and the degree of hedging they provide.[5] Second, from a macro-finance perspective, following Rietz (1988), Barro (2006) and Barro and Ursúa (2008) document the importance of non-Gaussian shocks for both equity prices and welfare. A large literature has since estimated the frequency and severity of rare events using macroeconomic data (e.g., Martin, 2013a; Nakamura et al., 2013) or financial data (e.g., Martin, 2008; Backus et al., 2011). Backus et al. (2011) highlight cumulants as both a source of intuition and a practical tool.[6]

Bridging these two strands of literature, our analysis is closely related to Martin's (2008; 2013a; 2013b) integration of cumulants into asset-pricing theory (see also Kyle and Todorov, 2022). We apply this approach to the welfare and asset-price implications of sharing production risk among heterogeneous countries, highlighting the role of trade elasticities and openness in mapping productivity shocks into income risk.

Third, we build on the large literature on cross-border risk sharing (e.g., van Win-

[5]Ingersoll (1975) shows that skewness influences efficient portfolio frontiers similarly to second moments. Kraus and Litzenberger (1976) extend the CAPM to include the third moment, improving empirical fit. Harvey and Siddique (2000) and Smith (2007) argue that co-skewness helps explain the cross-section of returns. Bekaert and Engstrom (2017) build a model generating skewness in consumption growth and its observed correlation with option-implied volatility. Extending to the fourth moment, Fang and Lai (1997) find that investors require compensation for higher variance and kurtosis, but accept lower returns for higher skewness.

[6]Guvenen et al. (2018) show that idiosyncratic income fluctuations display non-Gaussian features, with skewness and kurtosis affecting the welfare cost of incomplete insurance.

coop, 1999; Athanasoulis and van Wincoop, 2000; Lewis and Liu, 2015). In line with recent work (e.g., Gourinchas and Jeanne, 2006; Coeurdacier et al., 2020), we stress that cross-border insurance can reduce precautionary saving and induce reallocation of production factors. We emphasize the role of higher-order cumulants and level effects, and show how the resulting equilibrium allocation of demand and labor shapes the terms of trade. This novel result—linking the "market-based transfers" from risk sharing to the Keynes-Ohlin debate on the "transfer problem" (Keynes, 1929; Ohlin, 1929)—follows from the fact that risk sharing entails a permanent shift in relative wealth, which can raise the relative price of a country's goods if either consumption is home-biased, or labor supply falls as households become richer, or both.

Finally, our results speak to the policy debate, where gains from risk sharing are typically gauged solely by consumption smoothing—volatility and cross-country consumption correlations (e.g., Viñals, 2015; Constâncio, 2016; as well as Obstfeld (1994) and Kose et al. (2009)). Yet smoothing is only part of the total gain: relatively safe countries also benefit from implicit wealth transfers through asset revaluation and terms-of-trade adjustments (See e.g. Ljungqvist and Sargent, 2012; Engel, 2016; Coeurdacier et al., 2020).

The paper is structured as follows. Section 2 presents the model. Section 3 outlines the solution methods. Section 4 analytically decomposes the level and smoothing effects of risk sharing by cumulants. Section 5 discusses numerical results. In particular, Section 5.2 applies the analysis to 39 economies. Section 6 concludes.

2 Model

As a general framework for our analysis, we specify a model economy consisting of three economies, labeled A, B and C, identical except for their size, degree of openness and the probability distribution of total factor productivity (TFP). Country size (n_A, n_B and n_C) is normalized to sum up to 1. In each period the state of the economy consists of a realization of the stochastic TFP processes and a distribution of financial assets. See Online Appendix A for a compact summary of the equations needed to solve the model.

By setting $n_C = 0$ in the analytical section 4.1 and part of the quantitative section 5, we will be able to compare and contrast our contribution to the large body of work on international risk sharing relying on two-country models. Furthermore, by setting labor share in the production function to zero and eliminating home bias in consumption the model simplifies to the Lucas (1978) orchard model (Martin, 2013b), which we use in the analytical section. We will use the full model at the end of the paper in an empirical application.

2.1 Consumer Problem

The representative agent in country A consumes a bundle of domestic and foreign goods $C_{A,t}$. Domestically she trades in units of capital at price $P_{A,K,t}$, and supplies labor $L_{A,t}$ at a wage $w_{A,t}$ and capital $K_{A,t}$ at the rate $r_{A,K,t}$ to firms. Internationally, she trades in Arrow-Debreu securities with residents in countries B and C, at the price $\Lambda_{A,t+1|t}$, that pay one unit of consumption in period $t+1$. We denote by $S_{A,t} = S_{A,B,t} + S_{A,C,t}$ the net-trade of securities with the other two countries. Capital must be purchased one period in advance and is assumed to be constant at the aggregate level. All prices are in units of the consumption basket.

Denoting households' welfare at time 0 as $\mathcal{W}_{A,0}$, the representative country A household solves the following problem:

$$\max_{C_{A,t},S_{A,t+1},K_{A,t+1},L_{A,t}} \mathcal{W}_{A,0} := E_0 \sum_{t=0}^{\infty} \delta^t U\left(C_{A,t}, L_{A,t}\right) \tag{2.1}$$

subject to the individual budget constraint:

$$C_{A,t} + E_t\left(\Lambda_{A,t+1|t} S_{A,t+1}\right) + P_{A,K,t} K_{A,t+1} = r_{A,K,t} K_{A,t} + w_{A,t} L_{A,t} + S_{A,t} + P_{A,K,t} K_{A,t}. \tag{2.2}$$

We find it instructive to entertain two alternative specifications of preferences:

$$U\left(C_{A,t}, L_{A,t}\right) := \begin{cases} \dfrac{C_{A,t}^{1-\rho} - 1}{1-\rho} - \chi \dfrac{L_{A,t}^{1+\varphi}}{1+\varphi} & \text{[CRRA]} \\ \\ \dfrac{\left(C_{A,t} - \chi \frac{L_{A,t}^{1+\varphi}}{1+\varphi}\right)^{1-\rho}}{1-\rho} & \text{[GHH],} \end{cases} \tag{2.3}$$

with $\rho > 0$, the degree of risk aversion, $\varphi > 0$, the inverse Frisch elasticity of labor supply. Both specifications share the same constant-relative-risk-aversion (power) felicity with identical curvature ρ; they differ only in separability. Throughout, "CRRA" is shorthand for the *separable* case (first line), in which the labor-disutility term is additive, and "GHH" for the *non-separable* case (second line), in which the same CRRA aggregator is applied to the composite $C_{A,t} - \chi L_{A,t}^{1+\varphi}/(1+\varphi)$.[7]

Contrasting the model's implications under the two alternative preference specifications highlights the role of the labor adjustment margin for international risk sharing. In the production economy studied in Section 5, shocks can be partially smoothed through adjustments in hours worked—a mechanism emphasized by Swanson (2012). This self-insurance channel is operative under our first preference specification, which allows for wealth effects on labor supply, but is absent under the second.

Total domestic consumption in each country is a constant elasticity of substitution

[7] In addition we solve the model under Epstein and Zin (1989) (EZ) preferences—codes available upon request. Solving the model under (EZ) preferences requires additional state variables, strongly affecting the speed of convergence. The main benefit from using the more computationally demanding EZ preferences is the additional insight from positing a higher degree of risk aversion, which implies larger welfare effects of risk sharing.

(CES) function of domestic and foreign goods, e.g., for country A:

$$C_{A,t} = \left(\nu_A^{\frac{1}{\theta}} c_{A,A,t}^{\frac{\theta-1}{\theta}} + (1-\nu_A)^{\frac{1}{\theta}} C_{A,t}^{*\,\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}; \tag{2.4}$$

$$C_{A,t}^{*} = \left(\varsigma_A^{\frac{1}{\theta}} c_{A,B,t}^{\frac{\theta-1}{\theta}} + (1-\varsigma_A)^{\frac{1}{\theta}} c_{A,C,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}. \tag{2.5}$$

$C_{A,t}$ is total consumption of goods in country A, $C_{A,t}^{*}$ is total foreign goods consumed in country A, small letters $c_{A,B,t}$ and $c_{A,C,t}$ denote country B and C goods consumed by country A, $\theta > 0$ is the trade elasticity and $\nu_A \in (0,1)$ is a function of relative size of countries, and the degree of home bias (or equivalently degree of openness), $\lambda_A \in (0,1]$, such that $1-\nu_A = (1-n_A)\lambda_A$ is the import consumption share. Finally, $1-\varsigma_A = \frac{n_C}{1-n_A}$ represents the share of country C's goods within country A's foreign goods basket.

The relative prices associated with the above preferences obey the following:

$$1 = \nu_A p_{A,t}^{1-\theta} + (1-\nu_A) P_{A,t}^{*\,1-\theta}; \qquad P_{A,t}^{*\,1-\theta} = \varsigma_A p_{B,t}^{1-\theta} + (1-\varsigma_A) p_{C,t}^{1-\theta}, \tag{2.6}$$

where $p_{i,t}$ is the price of output of country i in terms of country A's consumption basket. Define $Q_{i,j}$ as the real, bilateral exchange rate between country i and j and $ToT_{i,j}$ as terms of trade between country i and j. For example, $ToT_{A,B,t} = \frac{p_{B,t}}{p_{A,t}}$. As the law of one price holds in our economy, we have $p_{B,t} = Q_{A,B,t} p_{B,t}^{*}$ and $p_{C,t} = Q_{A,C,t} p_{C,t}^{*}$.

Demands for country A specific goods by domestic and foreign residents are

$$c_{A,B,t} = \varsigma_A \left(\frac{p_{B,t}}{P_{A,t}^{*}} \right)^{-\theta} C_{A,t}^{*}; \qquad c_{A,C,t} = (1-\varsigma_A) \left(\frac{p_{C,t}}{P_{A,t}^{*}} \right)^{-\theta} C_{A,t}^{*}, \tag{2.7a}$$

$$c_{A,A,t} = \nu_A \left(p_{A,t} \right)^{-\theta} C_{A,t}; \qquad C_{A,t}^{*} = (1-\nu_A) \left(P_{A,t}^{*} \right)^{-\theta} C_{A,t}. \tag{2.7b}$$

Total demand for home goods is:

$$Y_t = \left(c_{A,A,t} + \frac{n_B}{n_A} c_{B,A,t} + \frac{n_C}{n_A} c_{C,A,t} \right). \tag{2.8}$$

The first order conditions of the consumer problem are:

$$C_{A,t} : U_C(C_{A,t}, L_{A,t}) - \zeta_{A,t} = 0, \tag{2.9a}$$

$$L_{A,t} : U_L(C_{A,t}, L_{A,t}) + w_{A,t}\zeta_{A,t} = 0, \tag{2.9b}$$

$$S_{A,t+1} : \delta\zeta_{A,t} - \zeta_{A,t-1}\Lambda_{A,t|t-1} = 0, \tag{2.9c}$$

$$K_{A,t+1} : P_{A,K,t} = E_t\delta\frac{\zeta_{A,t+1}}{\zeta_{A,t}}\left(r_{A,k,t+1} + P_{A,K,t+1}\right), \tag{2.9d}$$

where $\zeta_{A,t}$ is the Lagrange multiplier on the budget constraint, so that $\Lambda_{A,t|t-1}$ is the stochastic discount factor (SDF).

2.2 Risk-sharing and the cross-border distribution of wealth

By combining equations (2.9a) and (2.9c) (and the foreign counterparts) and iterating backward, we obtain risk-sharing conditions:

$$\zeta_{B,t} = Q_{A,B,t}\zeta_{A,t}\kappa_{A,B}^{-\rho} \qquad \zeta_{C,t} = Q_{A,C,t}\zeta_{A,t}\kappa_{A,C}^{-\rho}. \tag{2.10}$$

where

$$\kappa_{A,B} := \frac{X_{B,0}}{X_{A,0}}Q_{A,B,0}^{\frac{1}{\rho}}, \qquad \kappa_{A,C} := \frac{X_{C,0}}{X_{A,0}}Q_{A,C,0}^{\frac{1}{\rho}}. \tag{2.11}$$

and $\kappa_{B,C}$ can be obtained by taking the ratio of the above.[8] In the above expressions, $X_{j,0} = C_{j,0}$ with CRRA preferences while $X_{j,0} = C_{j,0} - \chi\frac{L^{1+\varphi}}{1+\varphi}$ with GHH preferences. The variable κ^ρ, typically referred to either as "risk-sharing constant" or "Negishi weight", is the ratio of the marginal utilities of consumption of residents in two countries, evaluated in the risk sharing equilibrium at the time in which the complete markets regime starts (the

[8] When not ambiguous we drop the subscripts of $\kappa_{A,j}$, $j = \{A, B, C\}$, when referring to the generic risk-sharing constant.

initial "time zero").[9] It reflects the initial cross-border distribution of wealth assessed at the equilibrium prices of real and financial assets owned by residents in each country.[10] As is well known (e.g., Ljungqvist and Sargent, 2012), κ is pinned down by the "time-0" asset allocation conditions. Complete markets prevent relative marginal utilities from diverging and opening inefficient wealth gaps.[11] With two weights to be determined, we need two conditions, i.e., we impose zero net foreign assets at time 0:

$$S_{j,0} = E_0 \sum_{t=0}^{\infty} \Lambda_{j,t|0} \left(C_{j,t} - p_{j,t} Y_{j,t}\right) = 0;\; j = \{A, B\}. \tag{2.12a}$$

2.3 Firms

Each country produces a differentiated good. In each country, an infinite number of firms operate in perfectly competitive markets using a Cobb-Douglas technology in capital ($K_{A,t}$) and labor ($L_{A,t}$), subject to an exogenous stochastic process for TFP (possibly correlated across countries) $D_{A,t}$. The aggregate production function is:

$$Y_{A,t} = D_{A,t} K_{A,t}^{\alpha} L_{A,t}^{1-\alpha}. \tag{2.13}$$

Goods markets are competitive. For tractability and to highlight the macroeconomic implications of wealth effects on relative demand and labor supply, throughout the analysis we posit that the aggregate capital stock is constant, and conveniently abstract from labor in the analytical section—we treat labor as an endogenous variable in our quantitative section, as it is potentially important in determining relative price movements and welfare.

In each period the representative firm rents capital at rate $r_{A,K,t}$ and hires workers at the real wage $w_{A,t}$ (in units of the consumption basket) from households to solve the

[9] In our text, we will use interchangeably "risk-sharing constant" and Negishi weight.

[10] Negishi (1960) devised an algorithm to find equilibrium allocations under complete markets. Under CRRA, via a simple transformation weights can be referred to relative consumption.

[11] For a definition and discussion of wealth gap, see Corsetti et al. (2010).

following problem:

$$\min_{K_{A,t},L_{A,t}} r_{A,K,t}K_{A,t} + w_{A,t}L_{A,t} \tag{2.14}$$

subject to the production function (2.13). The associated factor demands satisfy the standard conditions: $K_{A,t} = p_{A,t}\alpha\frac{Y_{A,t}}{r_{A,K,t}}$ and $L_{A,t} = p_{A,t}(1-\alpha)\frac{Y_{A,t}}{w_{A,t}}$. For future reference, it is useful to define global income (in country A consumption units) as:

$$I_{w,t} := n_A p_{A,t} Y_{A,t} + n_B p_{B,t} Y_{B,t} + n_C p_{C,t} Y_{C,t}. \tag{2.15}$$

2.4 Output and Productivity

In line with the literature we posit a stochastic process driving TFP specific to each country j ($j = \{A, B, C\}$), assumed to follow an AR(1) process in logs:

$$\ln D_{j,t} = (1-\varphi_j)\ln \bar{D}_j + \varphi_j \ln D_{j,t-1} + \omega\varepsilon_{j,t}; \tag{2.16}$$

where the parameter $\varphi_j \in (0,1)$ measures the persistence of the TFP process, $\varepsilon_{j,t}$ is a serially-uncorrelated exogenous innovation, ω is the perturbation parameter (identical across countries) such that if $\omega = 0$ the model is deterministic, and a bar over variables indicates the deterministic steady state value. The process is analogous for the other two countries. In the global solution of the model we assume that $D_{j,t}$ is generated by mixed-Gaussian distributions (to match the skewness and excess kurtosis observed in the data) as described in the Online Appendix G. To derive our analytical solution, we restrict the probability distribution of $\varepsilon_{j,t}$ as follows:

Assumption 2.1 (Probability distribution for the analytical solution). *The probability distribution of the innovation $\varepsilon_{j,t}$, $j = \{A, B, C\}$, is characterized as follows:*

Mean	*Cross-Moment*	*Variance*	*Skewness*	*Kurtosis*
$E(\varepsilon_{j,t}) = 0$	$E\left(\varepsilon^{r}_{A,t}\varepsilon^{l}_{B,t}\varepsilon^{s}_{C,t}\right) = \chi^{\dagger}_{r,l,s}$	$E\left(\varepsilon^{2}_{j,t}\right) = \gamma_j$	$E\left(\varepsilon^{3}_{j,t}\right) = \phi_j$	$E\left(\varepsilon^{4}_{j,t}\right) = \eta_j$

$^{\dagger}r, l, s \in \{1, 2, 3, 4\}$

In the perturbation analysis we compute solutions up to the fourth order of accuracy. For analytical transparency and feasibility of our numerical applications, in the main text we restrict $\chi_{r,l,s} = 0$. One key takeaway from our analysis is that, even in symmetric economies, gains from risk sharing are generated by asymmetries in the *intensity* and transmission of shocks at country level. In the Online Appendix F, we show that, because of these asymmetries, gains are positive and can be substantial even if shocks are perfectly correlated across borders.

Importantly, we model the distribution of the log of TFP, as opposed to its level. For the sake of illustration, note that, setting $\varphi_A = 0$, a series expansion of the log of $D_{A,t}$ around $\omega = 0$ (see below for a broader discussion) yields $E\left(D^{\tau}_{A,t}\right) = 1+\frac{\tau^2}{2}\gamma_A+\frac{\tau^3}{6}\phi_A+\frac{\tau^4}{24}\eta_A+\mathcal{O}\left(\omega^5\right)$, where $\tau \in \mathbb{R}$ is an arbitrary constant. This shows that the distribution of the level of TFP is not mean invariant: even with $\tau = 1$ the mean of TFP depends on higher moments.[12]

3 Solution Methods

In our analysis, we rely on both higher-order perturbation methods and global solution methods—the former to derive tractable analytical expressions providing economic insight, the latter to corroborate and generalize our analytical insight with numerical analysis, as well as to carry out quantitative exercises into the assessment of the potential gains from

[12]We do not adjust the process to rule out this dependence. First, it would be computationally cumbersome. Second, and more importantly, there is no empirical reason to believe that the underlying process has a mean-invariant property. Third, it is precisely through the effect on mean consumption and labor that higher moments can have a particularly strong impact on welfare.

risk sharing.[13]

Higher-Order Perturbation. We generate an order-decomposition of the variables in the model by applying higher-order series expansions with respect to ω around the riskless equilibrium ($\omega = 0$)—that allows us to characterize the higher-order state-space solution of the model. In our model, the state space solution is a function of exogenous state variables only, hence the series expansion amounts to an order-decomposition of the variables in terms of higher-order terms of the exogenous shocks. Integrating over the probability distribution of these shocks this decomposition will collapse to a decomposition in moments.

The perturbation method we use in our analysis is standard—it is described by Holmes (1995) in general terms, and by Judd (1998) and Lombardo and Uhlig (2018) for DSGE models in particular.[14] Our approach begins with the recognition that, in solving the model, all variables—including endogenous states—must be expressed as functions of a scaling parameter that embeds the stochastic process into the economy. In our notation this scaling parameter is denoted by ω, loading the innovation in the TFP AR(1) process (2.16). So, for example, we write $C_t := C(t; \omega)$, which highlights the dependence of consumption on the scale parameter ω. Then the m-order log-series-expansion of $C(t; \omega)$ around $\omega = 0$ is

$$\ln C\left(t, \omega\right) \approx \sum_{i=0}^{m} C_t^{(i)} \frac{1}{i!} \omega^i, \text{ where } C_t^{(i)} := \left. \frac{\partial^i \ln C(t; \omega)}{\partial \omega^i} \right|_{\omega=0}. \tag{3.1}$$

For our purposes, it is worth stressing that standard perturbation approaches to the solution of DSGE models are closely related to the derivation of cumulants discussed by

[13] As is well understood, perturbation methods (series expansions) are an ideal tool for digging deep into the analytical solution in terms of cumulants, although not (necessarily) accurate in numerical applications—e.g., with significant departures from the steady state in strongly non-linear systems, they may yield poor quantitative results. We address this issue by comparing analytical results with numerical global solution in Appendix I.

[14] See also Lombardo and Sutherland (2007) and in particular Devereux and Sutherland (2011) in which perturbation methods are used to solve for portfolio choices in open economies.

Martin (2008, 2013a) and Backus et al. (2011).[15] To wit, following Backus et al. (2011), given the *moment-generating* function for a random variable x and a real variable ω, $h(\omega, x) = E(e^{\omega x})$, then $f(\omega, x) = \ln h(\omega, x)$ defines the *cumulant-generating* function. Provided that $h(\omega, x)$ is analytic, we can write its series expansion as

$$f(\omega, x) = \sum_{m=1}^{\infty} f_m(x) \frac{\omega^m}{m!}, \tag{3.2}$$

where $f_m := \left.\frac{\partial^m f(\omega, x)}{\partial \omega^m}\right|_{\omega=0}$ and $f_m(x)$ is the m-order cumulant. Cumulants are tightly related to moments: in line with Assumption 2.1 one can show that $f_{2,j} = \gamma_j$, $f_{3,j} = \phi_j$, $f_{4,j} = \eta_j - 3\gamma_j^2$, with $j = \{A, B, C\}$. Technical details of our approach are presented in the Online Appendix B, which also offers a practical algorithm specifically designed for numerical solutions of DSGE models of any size, e.g., using Dynare (Juillard, 1996).[16]

Global methods. To solve the model with global methods, we adopt a standard approach (e.g., Judd, 1998). Since for each value of κ allocations and prices can be determined from a static problem, the solution proceeds in two steps. In the first step, we compute policy functions for consumption, labor, and prices—that is, the variables that enter the value functions and the budget constraint. In the second step, which requires evaluating expectations, we solve for the policy functions of the value functions and Arrow securities using a fixed-point iteration. Further details are provided in the Online Appendix C.

[15] It is important to note that our solution algorithm is distinct from the "risky-steady-state" approaches discussed in the literature (e.g., Coeurdacier et al., 2011). In the case of the Negishi weights (κ) in the class of models we are considering—i.e., with power utility also of the Epstein and Zin (1989) type—the recursivity of the solution is a natural outcome of the functional form of the model combined with the time-invariance of κ. No ad-hoc deviation from standard perturbation methods is required.

[16] To solve our model, one can also use the Wolfram Mathematica code, available from the authors.

4 Analytics of Risk Sharing

For comparison with the literature, throughout the section we will rely on a two-country version of our model. For analytical transparency, we further impose a set of educated assumptions (relaxed in Section 5.1).

Assumption 4.1 (Model in the analytical section)**.** *Two country model ($n_C = 0$), no labor in production ($\alpha = 1$), fixed aggregate capital ($K_t = 1$); identical steady state endowments $\bar{D}_A = \bar{D}_B$, purchasing power parity (PPP) meaning that $\lambda_A = \lambda_B = \lambda = 1$.*

We solve our model up to 4th order approximation, the minimum order that can capture kurtosis. We will use superscripts "*cm*" and "*au*" respectively, to denote variables under complete markets and financial autarky.

4.1 Level vs. smoothing effects (LE vs. SE) of risk sharing

The point of departure of our analysis is the observation that, per equations (2.11), (2.9a) (and the foreign counterparts) and the resource constraint, κ determines the consumption share of a country in global income. Under Assumption 4.1, this share is

$$\frac{C_{A,t}}{I_{w,t}} = \mu_A := \frac{E_0 \sum_{t=0}^{\infty} \Lambda_{t|0} p_{A,t} Y_{A,t}}{E_0 \sum_{t=0}^{\infty} \Lambda_{t|0} I_{w,t}} = \frac{1}{n_A + n_B \kappa_{A,B}}, \tag{4.1}$$

where $\Lambda_{t|0} = \Lambda_{A,t|0} = \Lambda_{B,t|0}$ and $I_{w,t} = n_A p_{A,t} Y_{A,t} + n_B p_{B,t} Y_{B,t}$, consistent with (2.15). Note that if $\kappa_{A,B} = 1$, then $C_{A,t} = I_{w,t}$.

By using the foreign counterpart of equation (4.1), $\kappa_{A,B}$ can be written as the ratio of the relative present discounted value of the flow of output in each country—assessed at equilibrium prices:[17]

$$\kappa_{A,B} = \frac{E_0 \sum_{t=0}^{\infty} \Lambda_{t|0} p_{B,t} Y_{B,t}}{E_0 \sum_{t=0}^{\infty} \Lambda_{t|0} p_{A,t} Y_{A,t}} = \frac{p_{B,0} Y_{B,0} + P_{B,K,0}}{p_{A,0} Y_{A,0} + P_{A,K,0}}, \tag{4.2}$$

with $P_{j,K,t} = E_t \sum_{i=1}^{\infty} \delta^i \left(\frac{I_{w,t+i}}{I_{w,t}} \right)^{-\rho} p_{j,t+i} Y_{j,t+i}$ where $j = A, B$.[18] Equation (4.2) establishes a tight relationship between relative consumption shares, asset prices and the dynamics of relative output and relative good prices, i.e., the terms of trade. As further discussed below, through the equilibrium adjustment in asset prices and the terms of trade, transitioning from financial autarky to perfect risk sharing results in an implicit (market-based) wealth transfer between economies that are asymmetric in risk.[19]

In light of the above, it is apparent that risk sharing arrangements impinge on the equilibrium allocation through two key channels. The first one works through the repricing of a country's assets and output at the time when the new financial regime comes into effect (time zero). The equilibrium distribution of wealth and consumption across borders changes when the human and physical asset endowment of a country is re-valued at the new equilibrium prices, reflecting the adjustment in the agents' stochastic discount factor. The second one works via equilibrium consumption smoothing under efficient risk sharing, reflecting the new equilibrium process for consumption and labor. Throughout the paper, we will structure our analysis around these two channels, distinguishing between Level Effects and Smoothing Effects of risk sharing, defined as follows:

Definition 4.1. *Level Effect (LE) of risk sharing is the time-zero equilibrium adjustment in consumption and leisure, captured by κ.*

[17]To derive analogous expressions (as (4.1)) for the Foreign country, make use of the fact that $C_{B,t} = \frac{1 - n_A \mu_A}{1 - n_A} I_{w,t}$. Equation (4.2), directly follows from taking the ratio of equation (4.1) and its foreign counterpart.

[18]$P_{j,K,t}$ is derived from equations (2.9d) using Assumption 4.1.

[19]Observe that this result is in line with the textbook example in Obstfeld and Rogoff (1996, Ch 5, equation (45) where $n_A = n_B = 1/2$). As shown by these authors, with CRRA preferences and PPP, the share of a country's consumption in world output, denoted μ_j, coincides with the portfolio shares of a country j in the world optimal portfolio of claims to countries' current and future output. Thus μ_j captures the optimal exposure to country risk from the vantage point of investors.

Definition 4.2. *Smoothing Effect (SE) of risk sharing is the equilibrium adjustment in the (mean) response of consumption and leisure to shocks.*

The gains from risk sharing stem from the combination of these two effects, which do not need to go in the same direction. To fix ideas, anticipating our results below, consider two countries of equal size with asymmetric income risk. The relatively safe country will be an equilibrium provider of insurance—hence it will benefit mainly from an appreciation of its assets. Its residents will enjoy higher consumption (of goods and leisure) on average. Conversely, the relatively risky country will gain from smoothing its residents' consumption, at the cost of a drop in their average consumption. Most of the applied work on international risk sharing assumes ex-ante symmetric economies, in which case the LE is irrelevant. But any heterogeneity and asymmetry across borders, in technology, taste or size, weighs on the endogenous determination of relative wealth, as well as of relative risk.

For future reference, it is useful to define the counterpart of κ in financial autarky, as the ratio of the multipliers of the budget constraint (see chapter 22 in Azzimonti et al., forthcoming). In financial autarky, this ratio varies period by period in response to shocks. Yet, one can obtain a measure of relative wealth comparable with κ by taking unconditional means. For our purposes, one important feature of such a measure is that, holding Assumption 4.1, unconditionally consumption is identical across borders. Hence, in the rest of the analytical sections, we can interpret log-$\kappa_{A,B}$ under complete markets in terms of cross-border wealth adjustment relative to autarky.[20]

[20] Under Assumption 4.1, we have seen that $\kappa_{A,B} = \dfrac{C_{B,0}}{C_{A,0}}$. Under autarky, relative consumption is identical to relative income. From the goods-demand system (equations (2.7)) we can express relative income, and thus relative consumption, as a function of the terms of trade, i.e.,

$$\frac{C_{B,t}}{C_{A,t}} = \frac{p_{B,t}Y_{B,t}}{p_{A,t}Y_{A,t}} = \left(\frac{p_{A,t}}{p_{B,t}}\right)^{\theta-1}, \tag{4.3}$$

where, in our endowment economy, relative prices (the terms of trade) are identical to relative TFP. This implies that the average log-ratio of consumption is equal to the average log-ratio of TFP, which is zero by Assumption 2.1. Thus the implicit log-$\kappa_{A,B}$ under autarky—defined as average log-consumption-ratio—is zero.

4.2 Decomposition of LE and SE by cumulants

Perturbation methods allow us to offer a tight and insightful expression of the contribution of level and smoothing effects to a country's gains from risk sharing by cumulants.[21] In particular, solving our model under Assumption 4.1 to fourth order of approximation, the gains from risk sharing for country A can be written as:

$$\mathcal{W}^{cm}_{A,0} - \mathcal{W}^{au}_{A,0} = \underbrace{-\left(a_{\kappa,2}\kappa^{(2)}_{A,B} + a_{\kappa,2^2}\kappa^{(2)}_{A,B}{}^2 + a_{\kappa,3}\kappa^{(3)}_{A,B} + a_{\kappa,4}\kappa^{(4)}_{A,B} + a_{\kappa,\Gamma}\kappa^{(2)}_{A,B}\Gamma\right)}_{\text{level effect LE}}$$
$$+ \underbrace{\left(a_{\gamma,A}\gamma_A - a_{\gamma,B}\gamma_B\right) + \left(a_{\phi,B}\phi_B - a_{\phi,A}\phi_A\right) + \left(a_{\eta,A}\eta_A - a_{\eta,B}\eta_B\right)}_{\text{smoothing effect SE}} + \mathcal{O}\left(\omega^5\right), \tag{4.4}$$

where $\Gamma = \gamma_A + \gamma_B$ is total variance, and $\kappa^{(m)}_{A,B}$ $(m = 2, 3, 4)$ are the m-order terms of the ln-series-expansion of κ around $\omega = 0$:

$$\kappa^{(2)}_{A,B} = b_\gamma\left(\gamma_A - \gamma_B\right); \quad \kappa^{(3)}_{A,B} = b_\phi\left(\phi_B - \phi_A\right), \tag{4.5}$$
$$\kappa^{(4)}_{A,B} = -b_{\gamma_A^2}\gamma_A^2 + b_{\gamma_B^2}\gamma_B^2 + b_{\gamma_B\gamma_A}\gamma_B\gamma_A + b_\eta\left(\eta_A - \eta_B\right). \tag{4.6}$$

with $a_\bullet$ and $b_\bullet$ complex convolutions of deep parameters (ρ, θ, δ and n_A). While the $a_\bullet$ and $b_\bullet$ may have either sign, they are nonnegative in our model calibration as well as in most standard calibrations adopted by the literature (see Online Appendix D).[22]

The above expressions detail the analytics of our definitions of LE and SE. The Level Effect of risk sharing includes only terms in $\kappa_{A,B}$, determined at time 0 and time invariant.[23] As opposed to LE, SE reflects the equilibrium response of consumption (of goods and leisure) to shocks over time, and is thus written as a convolution of moments (pinned down by the

[21]Since the only state variables in our model are the exogenous shocks, cumulants depend only on shocks' moments.

[22]In Online Appendix D, it is easy to verify that under symmetry in size and moments $\kappa^{(4)}_{A,B} = 0$.

[23]In our stylized model, under autarky, there are no forward looking equations: consumption and leisure have no "level shift" reflecting precautionary saving. Under complete markets the level shift reflects the time-0 solution of asset allocations.

time-0 expectations of the convolutions of shocks). Notably, LE and SE interact through the last term in the first line of (4.4). Unless, unconditionally, consumption is identical across countries (i.e., unless $\kappa^{(2)}_{A,B}=0$), this interaction term suggests that welfare gains of a country vary with total variance Γ.

The expression (4.4) provides a framework to study the marginal contribution of smoothing and wealth effects to the welfare gains from risk sharing reflecting cross-country asymmetries. Provided $a_{\bullet}>0$ and $b_{\bullet}>0$, $\kappa_{A,B}$ is increasing in the relative variance $(\gamma_A-\gamma_B)$, relative (negative) skewness $(\phi_B-\phi_A)$ and relative kurtosis $(\eta_A-\eta_B)$ of the productivity distribution in country A. As country A becomes riskier, country B—now safer—gets a compensation for providing insurance.

In general, cumulants and deep parameters interact in complex ways in determining the gains from risk sharing, as discussed in the three subsections below.

4.2.1 Risk aversion and trade elasticities

We start by demonstrating that assessing the gains from risk sharing requires a careful mapping of output quantity fluctuations (at second and higher moments), onto income and consumption risk—a mapping that is a complex function of the economy's structure. The following proposition elaborates on this point, under some (admittedly strong) assumptions that enhance economic interpretation:

Proposition 4.3 (Gains from risk sharing)**.** *Define $\Gamma:=\gamma_A+\gamma_B$, $\Phi:=\phi_A+\phi_B$ and $H:=\eta_A+\eta_B$. Under Assumption 4.1 and countries with equal size ($n_A=n_B=\frac{1}{2}$):*

$$\begin{aligned}\mathcal{W}^{cm}_{A,0}-\mathcal{W}^{au}_{A,0}=&\frac{\delta(\theta-1)^2\rho}{24(1-\delta)\theta^3}\left[-\frac{((\rho+1)+2\theta(\rho-2))}{2}\Phi-(\rho+1)(\theta-1)\phi_A\right]+\\&+\frac{\delta(\theta-1)^2\rho}{48(1-\delta)\theta^3}\left[\frac{a_H}{8\theta}H+(\theta-1)(\rho-1)(\rho+1)\eta_A+6\delta\theta(\rho-1)^2\left(\gamma_B\gamma_A-\frac{1}{4}\Gamma^2\right)\right]\\&+\frac{\delta(\theta-1)^2\rho}{8(1-\delta)\theta^2}\Gamma+\mathcal{O}\left(\omega^5\right),\end{aligned}\tag{4.7}$$

where $a_H = 3\rho + \rho^2 - 6 + 2\theta(1+\rho^2) + \theta^2(10 - 15\rho + 3\rho^2)$.

Proof. The proof follows from direct calculation. □

Proposition 4.3 offers a decomposition of welfare gains in cumulants, where subsequent rows relate to increasing orders. Under our simplifying assumptions, the second order cumulant contributes to welfare gains symmetrically across countries. In other words, the variance does not affect the *relative* gains from risk sharing: equally-sized countries gain the same, irrespective of the ratio γ_A/γ_B. While special, this case illustrates the relevance of the SE and LE decomposition: varying the ratio γ_A/γ_B, in relative terms, any gains in smoothing are exactly offset by losses in wealth level. However, in (4.7), there is no perfect offset between the LE and SE effects of the third and fourth moments—total and country-specific moments can increase or decrease the gains depending on parameter values, especially on trade elasticity θ and risk aversion ρ. Observe that, as in Cole and Obstfeld (1991), if $\theta = 1$ (under PPP) there are no gains from moving from financial autarky to complete markets: terms of trade movements perfectly insure risk emanating from output fluctuations, by ensuring that relative output prices move in proportion to relative quantities so as to keep relative income constant.[24]

Away from the Cole and Obstfeld case ($\theta \neq 1$), trade elasticity and risk aversion jointly shape the gains from risk sharing. Intuitively, θ regulates the income exposure of a country's residents to domestic vs. foreign output shocks. To show this most clearly, we write the first order approximation of country A's income under Assumption 4.1 and equal-size countries as follows:

$$p_{A,t}^{(1)\,cm} + Y_{A,t}^{(1)} = (2\theta)^{-1}\left(\ln D_{B,t} - \ln D_{A,t}\right) + \ln D_{A,t}. \tag{4.8}$$

If $\theta > 1$, implying that relative prices move less than proportionally to relative output, home

[24] However, a unit trade elasticity does not grant efficient sharing of productivity risk if demand is home biased, except in the special case of symmetric openness and log utility, see Corsetti and Pesenti (2005).

income responds more to home productivity shocks than to foreign shocks (in the limit case $\theta \to \infty$ domestic income responds only to domestic TFP); vice versa if $\theta < 1$. In reference to the analytical results in 4.2.4 below and the quantitative analysis in the next section, it is worth noting here that also home bias impinges on the income risk from productivity disturbances. In particular, the trade elasticity threshold at which home income responds more to home productivity shocks than to foreign ones becomes $\theta > 2 - \frac{1}{\lambda}$— see Figure 1 in the Online Appendix E.[25] With some conceptual latitude, one can regard trade elasticities and home bias as co-determinants of the "quantity" of domestic income risk.

Following up on this interpretation, risk aversion, ρ, modulates the (consumption) "price" of risk. In the solution of the model, ρ enters only through $\kappa_{A,B}$.[26] Observe that, for equal-sized countries, $n_A = n_B = \frac{1}{2}$, log utility ($\rho = 1$) rules out LE from risk sharing (i.e., κ's are zero). When $\rho = 1$ welfare gains are independent of cross-border differences in even moments (eliminating η_A from (4.7)), but not independent of differences in odd moments ((4.7) still a function of ϕ_A).

4.2.2 Elasticities and non-Gaussianity

The following corollary details how trade elasticity (θ) and risk aversion (ρ) jointly drive the contribution of deviations from Gaussianity to welfare:

Corollary 4.4 (Non-Gaussianity and Welfare Gains)**.** *Under Assumption 4.1 and countries with equal size ($n_A = n_B = \frac{1}{2}$) the contribution of non-Gaussian tails to the welfare gains from risk sharing is regulated by the following coefficients:*

[25] With home bias ($\lambda \neq 1$), home income can be written as $p^{(1)\,cm}_{A,t} + Y^{(1)}_{A,t} = (\ln D_{A,t}((2\theta - 3)\lambda + 2) + \ln D_{B,t}\lambda)\,(2(\theta - 1)\lambda + 2)^{-1}$.

[26] To wit, the second order term of the $\kappa_{A,B}$ expansion under PPP is $\kappa^{(2)}_{A,B} = \frac{\delta(\theta-1)}{\theta^2}\left\{\Gamma\left[n_B - n_A + \theta(1 - 2\rho n_B)\right] + 2\gamma_A\theta(\rho - 1)\right\}$.

$$
\begin{aligned}
\phi_A &\Rightarrow -\tfrac{\delta(\theta-1)^3\rho(\rho+1)}{24(1-\delta)\theta^3}; && \Phi \Rightarrow -\tfrac{\delta(\theta-1)^2\rho(2\theta(\rho-2)+\rho+1)}{48(1-\delta)\theta^3};\\
\xi_A &\Rightarrow \tfrac{\delta(\theta-1)^3\rho\left(\rho^2-1\right)}{48(1-\delta)\theta^3}; && \Xi \Rightarrow \tfrac{\delta(\theta-1)^2\rho a_H}{384(1-\delta)\theta^4},
\end{aligned}
$$

where $\xi_A := \eta_A - 3\gamma_A^2$ denotes excess kurtosis, $\Xi := \xi_A + \xi_B$ and a_H is defined in Proposition 4.3.

Proof. Under Gaussianity $\phi_j = \xi_j = 0$ for $j = A, B$. The terms in equation (4.7) associated with individual or aggregate "skewness" (ϕ_j and Φ respectively) measure the contribution of asymmetric tails relative to Gaussianity, without further calculation. In line with definition of cumulants in (3.2), the coefficients for individual excess kurtosis and aggregate excess kurtosis are derived by replacing η_j with $f_{4,j} + 3\gamma_j^2$ in (4.7) and collecting terms. □

It is well known that, unless risk aversion is sufficiently high, risk premia remain moderate (Campbell, 2017). Backus et al. (2011) specifically characterize how premia respond to tail risk, showing how the relevance of third and fourth-order cumulants rises with risk aversion. Our results highlight once more that in an open economy risk aversion systematically interacts with trade elasticity in determining premia and thus allocations and welfare gains.[27]

4.2.3 Country size

Gains from risk sharing can vary significantly with the relative size of a country, since size determines the incidence of the demand for and supply of a country's assets and goods in the global equilibrium. Specifically, the integration into world financial markets of a large (i.e., with non-infinitesimal size) country affects state-contingent prices. This means that

[27]To fix ideas, using the parameter values discussed in Section 5 below, i.e., $\theta = 1.5$, $\delta = 0.98$ and $\rho = 4$, the coefficients displayed in Corollary 4.4 amount to $\phi_A \approx -1.51$, $\Phi \approx -3.33$, $\xi_A \approx 2.27$ and $\Xi \approx 1.73$ (for comparison $\Gamma \approx 2.72$). It should be noted that, while the magnitudes of the coefficients are comparable, those of the moments are *typically* not—recall that these are raw moments, i.e., not scaled.

the price of states of the world in which the large country seeks insurance will rise. The price of the state of the world in which it provides insurance will fall. It follows that, upon joining globally efficient capital markets, the budget set faced by a country will tend to be smaller, the larger its size. By the same token, a country with infinitesimal size will enjoy a "privilege": relative to the pre-insurance status quo, the price of the insurance it seeks in global markets will not be rising when the country joins the world financial market. For a country with a small size n_A, global income is largely determined by country B's income—which in turn implies a low $P_{B,K,t}$. Everything else equal, $\kappa_{A,B}$ is decreasing in n_A.

That said, when considering the overall gains from risk sharing, the impact of country size depends on its interactions with other parameters of the model. As we show in Section 5, this interaction generates a rich set of results, suggesting welfare gains may or may not be monotonic in size. The following proposition illustrates a case in point, stating a necessary condition for smaller countries to benefit more from risk sharing—relative to larger ones—highlighting the relevance of the TFP distribution in addition to intra- and intertemporal elasticities. For clarity, the proposition is derived under cross-country symmetry in all parameters except size.

Assumption 4.2 (Simplified model with size difference)**.** *Countries have identical TFP probability distributions, i.e.,* $\gamma_A = \gamma_B = \frac{\Gamma}{2}$, $\phi_A = \phi_B = \frac{\Phi}{2}$, $\eta_A = \eta_B = \frac{H}{2}$, *but possibly different size, i.e.,* $n_A \neq n_B$. *PPP holds, i.e.,* $\lambda_A = \lambda_B = 1$.

Proposition 4.5 (Size and Welfare Gains from Risk Sharing)**.** *Under Assumption 4.2 and using a fourth-order approximation,*

$$\left.\frac{\partial\left(\mathcal{W}^{cm}_{A,0} - \mathcal{W}^{au}_{A,0}\right)}{\partial n_A}\right|_{n_A = n_B} \begin{cases} = 0 \quad if & \theta = 1 \\ < 0 \quad iff & \theta \neq 1 \,\&\, \Gamma > \frac{1}{2}(\rho - 1)\Phi - \frac{1}{24\theta^2}P(\rho,\theta)H, \end{cases} \tag{4.10}$$

where $P(\rho, \theta) = -5 + \rho^2 + \theta(6 + 3\rho - \rho^2) + \theta^2(2 - 9\rho + 3\rho^2)$.

Proof. The proof follows from direct calculation (see Appendix K). □

We will comment on this proposition below, comparing it with its analog derived for openness. For now, we note a specific implication of the interaction of size and cumulants, discussed in the Appendix L: if a country is sufficiently small, smoothing and level effects may have the same sign. Infinitesimal-size countries may gain both in smoothing and wealth.[28]

4.2.4 Country Openness

While openness is negatively related, if only weakly, to country size in the data, it has a distinct and significant effect on risk sharing. For any given size, a country with a high degree of home bias in consumption tends to be more exposed to own income risk and less able to stabilize marginal utility by substituting domestic goods with imports. As income risk and the efficiency of substituting domestic with foreign goods in turn vary with the trade elasticity, the effect of openness on the gains from risk sharing need not be monotonic, but can be expected to depend on its interplay with elasticities and possibly other economic parameters.

Mirroring our discussion of size, we derive a proposition under symmetry in economic structure, to provide insight on the complex effect of openness on risk sharing.

Assumption 4.3 (Simplified model with home bias)**.** *Countries have identical TFP probability distributions, i.e.,* $\gamma_A = \gamma_B = \frac{\Gamma}{2}$, $\phi_A = \phi_B = \frac{\Phi}{2}$, $\eta_A = \eta_B = \frac{H}{2}$ *and equal size, i.e.,* $n_A = n_B$. $\lambda_A = \lambda_B = \lambda$ *and* λ *is a free parameter measuring the degree of home bias.*

Proposition 4.6 (Openness and Relative Gains from Risk Sharing)**.** *Under Assumption 4.3 and using a fourth-order approximation,*

$$\left.\frac{\partial\left(\mathcal{W}^{cm}_{A,0} - \mathcal{W}^{au}_{A,0}\right)}{\partial\lambda}\right|_{\lambda=1} \begin{cases} = 0 \quad if & \theta = 1 \\ > 0 \quad iff & \theta \neq 1 \,\&\, \Gamma > \frac{1}{2}(\rho-1)\Phi + \frac{1}{24\theta^2}Q(\rho,\theta)H, \end{cases} \tag{4.11}$$

[28] Appendix L studies the limit case of a small-open economy (SOE) with infinitesimal size, assuming that productivity in the rest of the world is constant. In this case the country wealth is predominantly driven by anticipated income flows (equilibrium output prices multiplied by equilibrium quantities) capitalized at constant discount rate, with convexities magnifying the present value of these flows.

where $Q(\rho, \theta) = 4 - \theta(5 + 2\rho + \rho^3) + \theta^2(-2 + 8\rho - 3\rho^2 + \rho^3)$.

Proof. The proof follows from direct calculation (see Appendix K). $\square$

Holding the conditions stated in Proposition 4.6, the benefits from risk sharing are increasing in openness in countries with a relatively small exposure to domestic risk and a relatively low degree of substitutability between domestic and foreign goods (relatively low θ) or when the price of risk is small (relatively low ρ). Section 5 will reconsider quantitatively these results, providing a more general characterization.

Together, Propositions 4.5 and 4.6 show that, while (at least locally) openness and size affect welfare gains from risk sharing in similar ways, there are significant differences in their impact. With $\Phi < 0$, provided $\rho > 1$, relative welfare gains are decreasing in size if $P(\rho, \theta) > 0$, while they are increasing in openness if $Q(\rho, \theta) < 0$. As we will see in the next section, these conditions are both satisfied in our model calibration. Yet, more generally, we may note that the condition $P(\rho, \theta) > 0$ on size is always satisfied for mildly high risk aversion (e.g., $\rho > 3$). In contrast, the condition $Q(\rho, \theta) < 0$ on openness depends on a complex interplay of ρ and θ (see Online Appendix K).

4.3 Welfare, asset prices, and the terms of trade

We conclude this section discussing the equilibrium implications of risk sharing for the international prices of assets and goods. Both prices contribute to the adjustment in relative wealth, hence are key drivers of the LE.

Asset prices. The following proposition establishes that welfare gains from risk sharing are positively related to domestic asset prices, although the link is not necessarily tight.

Proposition 4.7 (Relative welfare and asset prices). *Under 4th order approximation and holding Assumption 4.1, the difference in welfare in country A and country B under complete* *markets and financial autarky can be written as:*

$$\begin{gathered}\left(\mathcal{W}^{cm}_{A,0}-\mathcal{W}^{cm}_{B,0}\right)-E_0\left(P^{cm}_{A,K,t}-P^{cm}_{B,K,t}\right)= \\ \frac{\delta^2(\theta-1)^2(n_B-n_A)\rho}{8(1-\delta)\theta^4}\left(\theta(\rho-1)(\gamma_A-\gamma_B)-(n_B-n_A)\Gamma(\theta\rho-1)\right)^2+\mathcal{O}(\omega^5),\end{gathered} \tag{4.12}$$

and

$$\left(\mathcal{W}^{au}_{A,0}-\mathcal{W}^{au}_{B,0}\right)-\frac{E_0(P^{au}_{A,K,t}-P^{au}_{B,K,t})}{(1-\rho)}=\mathcal{O}(\omega^5). \tag{4.13}$$

Proof. The proof follows from direct calculation. □

Focusing on the case of equally sized countries, i.e., $n_A=\frac{1}{2}$, for analytical transparency, note that the right hand side of (4.12) is zero up to fourth order of approximation. Comparing countries with identical size, under perfect risk sharing, the one with higher asset valuation enjoys higher welfare. This result suggests that it may pay to be on the relatively low side of the risk distribution when integrating markets. The levels of welfare and wealth are simultaneously higher. This is in sharp contrast with the case of autarky. Under financial autarky, for standard model parameterization with $\rho>1$, welfare is negatively correlated with financial wealth. To appreciate why, recall that, without trade in financial assets, SDFs and risk-free rates are not equalized across borders (as they are under PPP if risk is efficiently shared). In countries with a higher variability of income (depressing welfare), the SDF will also be relatively more variable—implying a lower safe autarky interest rate and larger risk premium. Asset prices are thus driven by two opposing effects. On the one hand, a lower safe autarky interest rate raises the PDV of current and future income flows, boosting asset valuation. On the other hand, higher risk premia depress asset valuation. When $\rho>1$, the former effect dominates.[29]

However, the link between welfare and asset prices is not necessarily tight. As shown by Proposition 4.7, for $n_A\neq\frac{1}{2}$, cross country differences in welfare relative to asset

[29]We thank an anonymous referee for pointing out that the effect of risk on asset prices works through the intertemporal elasticity of substitution (EIS) which under CRRA is $EIS=1/\rho$.

prices become a function of both relative and total variance—other moments affect relative welfare and prices symmetrically.[30] For the larger country ($n_B > n_A$), the right-hand side of (4.12) is higher, the higher own output or total variance (γ_B or Γ). That said, observe that Proposition 4.7 and equation (4.2) together imply a positive relationship between relative welfare (across countries) and κ. Remarkably, in our quantitative assessment of Section 5 we find that, under complete markets, relative welfare, relative asset prices and κ's are all close to proportional.

Terms of trade. Differences in cross-border average consumption under perfect risk sharing may be expected to impact the international price of national outputs—i.e., the terms of trade. Although, as explained below, terms of trade cannot respond to risk insurance in the simplified specification used in this section, we find it useful to bring forward a discussion highlighting a key result of our quantitative exercises below, relying on the full model specification detailed in Section 2.

Under PPP (as assumed in this section), changes in relative consumption would not directly affect the composition of global demand. Even if a country gets a higher share of the global consumption basket, its relative demand for home and foreign goods remains unchanged. This is not the case when consumption is home biased: a rise in the average relative consumption of a country will also coincide with a rise in the global demand for its domestically produced goods—contributing to appreciating their relative price. This argument resonates with the classical controversy on the "transfer problem" between Keynes and Ohlin concerning the effects of the reparations imposed on Germany after World War I (Keynes, 1929; Ohlin, 1929). Bringing this controversy to bear on results in the next section: allowing for deviations from PPP, the asset repricing upon entering a risk-sharing agreement translates into a (market-based) "transfer" of purchasing power between countries (e.g., from a riskier to a safer economy)—which in turn drives up the price of the safer country output.

It is however worth noting that, with an endogenous labor supply, terms of trade

[30] Observe that the additional term in total variance is zero when either $\theta = 1$ or $\rho = 1$.

may respond to wealth transfers (in our case driven by risk sharing) also under PPP. To wit: under separable CRRA preferences, wealth effects impinge on labor supply. Because of this, labor falls in countries experiencing an increase in relative wealth. Even under PPP, while the relative demand for national goods is unaffected at the global level, international relative prices will need to adjust to cross-border changes in labor affecting the relative supply of national output.

5 Quantitative Analysis

In this section, we use global methods to pursue two goals. First, we revisit quantitatively the analytical results from applying perturbation methods. We do so by solving a two-country version of our model in Section 2 for a sample of artificial economies generated to best explore the range of mechanisms through which risk sharing affects welfare and the global allocation discussed so far. Second, we offer a numerical assessment of gains from risk sharing solving our three-country model in its general form and calibrating size (n_j), import share ($1 - \nu_j$) and higher order moments (γ_j, ϕ_j and η_j), to the countries in our sample.

Throughout our quantitative analysis, we set the frequency of the model to annual (the frequency of the Penn World Table database). The share of labor in production is $1 - \alpha = 0.7$ (in the ballpark of US labor share), the risk aversion parameter is $\rho = 4$ (e.g., Martin, 2008), the inverse Frisch elasticity of labor supply is $\varphi = 1.75$ (see Attanasio et al., 2018), the weight of disutility from working is normalized to $\chi = 1$, the trade elasticity of substitution is $\theta = 1.5$ (in the ballpark of the values used in the international macro literature), and the discount factor is $\delta = 0.98$. Concerning the import share $(1 - \nu_j)$, for transparency in the interpretation of the results from the simulated-data analysis, in the next subsection 5.1 we calibrate $\nu_j = 1 - (1 - n_j)\lambda_j$ using population data but setting $\lambda_j = 0.3\ \forall j$ (unless differently specified).[31] In the empirical application, instead, we calibrate

[31] Allowing for an independent degree of openness increases the number of determinants and their interactions, introducing complexities that make the interpretation considerably more involved.

λ_j using the share of imported consumption, measured as the foreign value-added content of final domestic demand from the OECD's Inter-Country Input–Output tables, averaged over 2016–2022.[32]

In line with the literature, we posit that output fluctuations are driven by standard autoregressive productivity shocks. In the first exercise, we set the autocorrelation coefficient of the TFP process to 0.5. This value, low relative to empirical estimates, improves the accuracy of discretization, without affecting inference. In the empirical application, we proxy the TFP process by an AR(1) process estimated on detrended real per-head GDP—hence the parameters of the TFP process are specific to each country in the model. The Online Appendix G provides details on the estimation of the AR(1) processes based on mixed-Gaussian distributions. Finally, we report absolute and relative welfare gains in permanent-consumption units (PCU), i.e., the change in the steady-state consumption necessary to bring about the level of welfare experienced in the stochastic regime. We measure welfare gains in permanent consumption units, PCU $= (\xi^{cm} - \xi^{au}) \times 100$, where ξ^X is the change in autarky steady-state consumption that achieves the welfare of regime X.

Two comments are in order. First, we are mindful that the distribution of GDP across countries is affected by shocks of different nature, as documented, e.g., by the SVAR and DSGE literature. We motivate our focus on productivity on the ground that our contribution is primarily theoretical. It is therefore natural for us to conduct our exercises in line with leading contributions that typically assume an exogenous productivity process as the source of macroeconomic risk. [33] Second, we have estimated our productivity process using GDP data. In principle, we could have calibrated our TFP processes to match the GDP's moments in the model with those observed in the data. However, given the large number of

[32]See Appendix P for details.

[33]The literature typically models output risk as arising either from endowment shocks (e.g., van Wincoop, 1999; Gourinchas and Rey, 2022) or, in production economies, from total factor productivity (TFP) shocks (e.g., Coeurdacier et al., 2020). For tractability and transparency, we focus on a single source of uncertainty, which allows us to map the model directly to the observed stochastic process for GDP. While calibrating the model to TFP shocks would provide a more direct comparison with the production-economy literature, sufficiently long and consistent TFP series are generally unavailable. Moreover, the quality of TFP estimates varies substantially across countries and is known to be poor in some cases.

countries in our sample and our global solution technique, computationally, this approach would have been extremely costly relative to the potential benefits.[34]

5.1 Numerical analysis under more general assumptions

We start by showing that our results derived using perturbation methods under educated restrictions on parameters carry over to a more general specification solved using global solution methods. In particular, allowing for endogenous labor supply and home bias in consumption, our exercise confirms that gains from risk sharing are higher for countries with smaller size, higher openness, larger variance, more negative skewness and kurtosis of their TFP process, and that the gains are larger with a larger trade elasticity. We also find that gains from risk sharing are tightly related to the level effect and to relative asset prices. Finally, we show that the level effect drives the average equilibrium terms of trade.[35]

To conduct the exercises in this subsection, we construct an ad hoc sample of artificial economies, specifically generated to produce a random sample of country size and cumulants (see the Online Appendix H for the construction of these data). As stressed by Bekaert and Popov (2019) among others, higher moments are strongly correlated in the data. Given a high correlation, the individual contribution of size and moments tends to be blurred in samples including only a relatively small number of countries. Our algorithm generates an effective sample of 477 economies (retaining those that display independent higher moments and that generate accurate solutions). Each of these differs in size and moments of TFP, allowing us to better distinguish their individual contribution to welfare gains and other variables of interest. In all the analysis in the rest of the section, each of these 477 countries

[34] In our exercises, model parameters other than TFP and country size are symmetric across countries. Differences in TFP estimates across methods may be expected to have minor effects on the welfare ranking of countries produced by our calibration. Similar considerations explain why we estimate the process of TFP separately for each country, rather than relying on VARs, given the high computational toll of discretizing the resulting multivariate processes. The Online Appendices G and F discuss the discretization approach and the case of correlated processes, respectively.

[35] Online Appendix I provides an indication that the perturbation-based solution for κ is similar to that obtained with the global methods approach.

Table 2: Quantiles of size and moments of generated data

Quantiles	**Size**	**Standard deviation**	**Skewness**	**Kurtosis**
0%	0.0005	0.0211	-2.9484	3.0157
25%	0.0034	0.0939	-0.7158	9.0786
50%	0.0259	0.1199	0.0571	13.3176
75%	0.1884	0.1357	0.8448	16.8538
100%	0.6643	0.1497	2.8778	19.9787
Base Country B	$1 - n$	0.1417	0.5992	3.7890

Note: The sample consists of 477 countries, obtained by drawing mixed-Gaussian distributions with mean zero and independent 2nd through 4th moments. Size is drawn randomly from our empirical dataset.

is matched with the same base country, labeled country B. Table 2 summarizes the DGP of the generated sample and of the base country B.

Qualitatively, the takeaway from our analysis is the same under our specification of CRRA and GHH preferences. To save space, the main text only reports results for the case of CRRA preferences—results for the GHH preferences are in the Online Appendix H.1. As expected, when the labor adjustment margin is shut down, fat tails—most markedly the role of kurtosis—are more consequential for the distribution of gains under GHH.

5.1.1 Relative welfare, κ, and the terms of trade as a function of cumulants and country size

To examine the relationship between endogenous variables and exogenous determinants, we estimate regression models using simulated data from our 477 surrogate two-country economies. The results are reported in Table 3. The three columns present, respectively, relative welfare gains, LE (relative to the autarky counterpart, as discussed in Section 4.1), and the terms of trade. Consistent with our analytical findings, each of these outcomes depends linearly on the relevant moments, country size, and their first-order interactions.[36] Both the dependent variables and the regressors are demeaned, with regressors expressed as differences between country A's and country B's values.[37]

Looking at the second row of the table, the regression results suggest that, holding cumulants constant and symmetric, the size of country A contributes negatively to its relative welfare gains (second column of the table). Remarkably, when moving from autarky to complete markets, the larger country A is, (a) the larger the wealth transfer to country B, i.e., the larger is $\kappa^{cm}_{A,B}$ relative to autarky (third column); and (b) the larger the terms of trade deterioration (last column). All these results are consistent with our analysis and propositions in Section 4. Rows 3 through 5 in the table show the effect of cross-border differentials in each moment in isolation, for countries with equal size ($n_A = 0.5$). Also in line with the results in Section 4, countries with larger variance and/or kurtosis and more negative skewness in the output distribution, gain more; have higher $\kappa_{A,B}$ and weaker terms of trade. The implicit wealth transfer through $\kappa_{A,B}$ is consistent with its interpretation as compensation for risk. The adjustment in the terms of trade aligns with the change in relative wealth/consumption.

Relative to the analytical section, our quantitative analysis sheds light on the interactions between size and cumulants—shown in rows 6 through 8 of the table. Using the results in these rows, we can indeed compute the critical value n^*_A, at which the marginal effect of the moments changes sign. To do so, we calculate, for each moment, $n^*_A = \bar{n}_A - \frac{d}{2f}$, where—recalling that the data is demeaned—$\bar{n}_A$ is the mean size, d is the coefficient on the moment differential (e.g., for variance, $d \approx 33.6$ in the second column of Table 3), f the coefficient on the interaction term of the same moment differential (for variance, $f \approx -46.6$) and we

[36]Proposition 4.3 implies that total moments (i.e., Γ, Φ, and H) vanish in the perturbation solution when computing the difference in welfare gains across countries. We therefore focus on *relative* rather than absolute welfare gains: changing the cumulants of country A's TFP, while holding those of country B fixed, also changes total riskiness.

[37]The fit is near-perfect ($R^2 \approx 1$): by construction only the listed regressors vary, though their exact non-linear mapping to the dependent variable is unknown. Openness is fixed at a common home bias $\lambda = 0.3$, so ν_A is deterministic in size (Section 2) and not a separate regressor. We use Bayesian regression for convenience; OLS with bootstrapped errors gives similar results.

Table 3: Relative welfare (PCU), κ, and the terms of trade as a function of cumulants and country size

	$PCU_A - PCU_B$	$\ln \frac{\kappa^{cm}_{A,B}}{\kappa^{au}_{A,B}} \times 100$	$\ln \frac{ToT_{cm}}{ToT_{au}} \times 100$
Constant	0.0 [0.0, 0.0]	0.0 [0.0, 0.0]	0.0 [0.0, 0.0]
$n_A - n_B$	-2.1 [-2.1, -2.1]	1.3 [1.3, 1.3]	2.1 [2.1, 2.2]
$\gamma_A - \gamma_B$	33.6 [33.2, 33.9]	5.6 [5.4, 5.7]	26.5 [20.7, 32.1]
$\phi_A - \phi_B$	-24.2 [-25.0, -23.4]	-3.8 [-4.2, -3.4]	-20.0 [-32.3, -6.8]
$\eta_A - \eta_B$	8.1 [6.1, 10.3]	0.3 [-0.6, 1.2]	12.0 [-22.0, 46.0]
$(n_A - n_B) \times (\gamma_A - \gamma_B)$	-46.6 [-47.6, -45.6]	28.8 [28.3, 29.3]	46.7 [30.8, 63.9]
$(n_A - n_B) \times (\phi_A - \phi_B)$	22.9 [20.8, 25.0]	-15.8 [-16.7, -14.9]	-14.4 [-48.0, 21.0]
$(n_A - n_B) \times (\eta_A - \eta_B)$	-5.9 [-11.9, 0.0]	4.4 [1.8, 7.2]	5.7 [-91.3, 100.2]
R^2	1.0	1.0	0.9

Note: Bayesian estimation of a linear model with interaction terms (CRRA preferences; welfare in permanent-consumption units, PCU). [95% Highest Posterior Density]. Number of observations: 477.

use the fact that $n_A-(1-n_A)-(\bar{n}_A-(1-\bar{n}_A)) = 2(n_A-\bar{n}_A)$, with $\bar{n}_A \approx 0.13$. For relative welfare (second column), the threshold for each moment is, respectively, $n^*_A \approx \{0.49, 0.66, 0.83\}$. Observe that, when countries have equal size, the second moment differential has no effect on the relative welfare gains, consistently with Proposition 4.3. Furthermore, in line with our analysis in Section 4.2.3, the table results confirm that the composition of the relative welfare gains, i.e., the SE/LE mix, can vary significantly with the size of the country.

5.1.2 Relative welfare, wealth and asset prices

The analytical results derived in Section 4 establish that, under PPP, there is a positive relationship between relative welfare and relative asset prices, tight under financial autarky, and possibly less so under complete markets unless countries are symmetric in size (Proposition 4.7). By the same token, if PPP holds and countries have equal size, there is a tight relation between κ and relative asset prices (equation 4.2). Together, these two results suggest that κ and relative welfare should also be related.

Consistent with our analytical propositions, the numerical results show that asset prices, relative welfare, and κ are almost perfectly linked. The correlations of $\ln\left(P_{A,K,0}/P_{B,K,0}\right)$ with $\left(\mathcal{W}_A^{cm}-\mathcal{W}_B^{cm}\right)$ and with $\ln\left(\kappa_{A,B}^{cm}/\kappa_{A,B}^{au}\right)$ are both approximately 1, while the latter two variables are correlated at -0.99.[38] Simulated data also support the transfer-effect mechanism discussed in Section 4.3: $\ln\left(\kappa_{A,B}^{cm}/\kappa_{A,B}^{au}\right)$ and $\ln\left(ToT_{cm}/ToT_{au}\right)$ are correlated at approximately 0.99, implying that under perfect risk sharing richer (safer) countries experience an appreciation of their terms of trade.

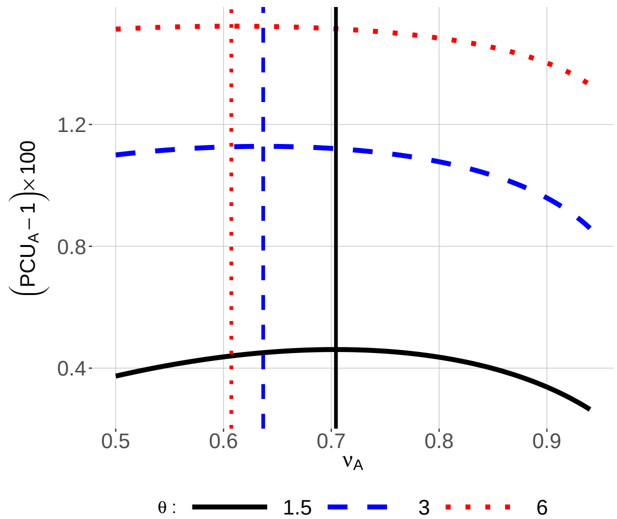
5.1.3 Openness

In Section 4, Proposition 4.6 illustrates how openness affects welfare gains around a symmetric benchmark. Building on this proposition we now construct a counterfactual in which all 477 economies face identical TFP shock distributions (same as our base country B), have equal size, but have different home bias—i.e., we only vary the parameter ν_A (keeping $\nu_B = n_B = 0.5$ and varying effectively λ_A). For these economies, Figure 1 plots the (smoothed) gains from risk sharing against their ν_A, for three alternative trade elasticities, $\theta = \{1.5, 3, 6\}$. For all θs, the pattern in the figure is a clear non-monotonic relationship. As discussed above, non-monotonicity can arise because a stronger home bias raises the exposure of a country to own income risk (thus raising the potential benefits from risk sharing), but at the same time reduces the scope for smoothing consumption by substituting domestic with foreign consumption goods (with opposite effects on the potential gains from insurance).

The takeaway from the exercise underscores the main conclusion from our analytical section: the effects of a single parameter on the gains from financial integration cannot be generally assessed independently of its interactions with other parameters and the distribution of the fundamental shocks. Observe that the value of ν_A at which welfare gains are at their maximum, marked by vertical lines (for respective values of θ) in Figure 1, is

[38] In line with Proposition 4.7, we also find that relative welfare and asset prices remain tightly related, with the predicted sign, under financial autarky.

Figure 1: Openness and trade elasticity: effects on welfare gains



Note: $\lambda_B = 1$, $n_A = n_B = 0.5$. All Markov processes are identical across cases and identical to the base-country's process. The vertical lines denote the value of ν_A that maximizes the gains.

model-dependent—that is, it is sensitive to alternative parameterizations. Similar to home bias, also the trade elasticity regulates the relative "exposure" of a country income and consumption to productivity risk (see Section 4.2.1). In the case illustrated in the figure, a higher trade elasticity—causing a higher exposure to own production risk—implies that the curve reaches a maximum at a lower level of home bias: it takes a lower home bias (which also impinges on the exposure to own production risk) to start seeing a deterioration in the gains from risk sharing. More in general, the effects of home bias on these gains may display different patterns depending on asymmetries in country size and shock distribution.

5.2 An evidence-based assessment of welfare gains

So far, we have explored in analytical and quantitative detail the mechanisms by which cross-country asymmetries in fundamental uncertainty in output shape income risk and welfare gains from risk sharing across countries differing in size and openness. This section brings this material together, with an attempt to gauge potential gains from efficient sharing of income risk across countries through the lens of our (full-fledged) model.

We construct our exercise by solving, with global methods, the three-country version of the model presented in Section 2, with moments and size for 39 countries calibrated using a combination of the Barro–Ursúa disaster-risk data (Barro and Ursúa, 2008) and Penn World Table (version 11) data, and consumption home bias calibrated using the OECD Inter-Country Input–Output (ICIO) tables. For each of the 39 countries,[39] we assess gains from sharing risk with the rest of the world, distinguishing two large foreign-economy blocks, AE and EME.[40] The population size corresponds to the aggregate population of each block, and the data-generating-process (DGP) of the TFP is calibrated using the aggregate real per-capita GDP of these regions.[41]

Following the rare-disaster literature (Barro, 2006; Barro and Ursúa, 2008), we estimate country-level moments based on long historical time series. As discussed by this literature, long time series allow analysts to avoid the sample-selection and survival biases inherent in shorter timelines, which may mistake periods of temporary macroeconomic stability for a permanent reduction in underlying risk. A consequence of this approach, however, is that the historical moments of some countries, e.g. Germany (DEU), may appear extreme when assessed conditional on their recent post-war experiences. Crucially, by no means does our study suggest that, today, modern Germany is inherently riskier than its peers. Rather, our analysis is meant to capture the case of a hypothetical country subject to the kind of large shocks that Germany experienced in the last 120 years.[42]

[39] We excluded countries with a poor convergence of the solution.

[40] The definition of advanced and emerging market economies is taken from the IMF's Fiscal Monitor Database. Weights are Penn World Table populations and, consistently with the construction of the regional endowment processes, each region excludes the domestic economy. For example, EME trading with China does not include China; AE trading with the USA does not include the latter, etc.

[41] Aggregating the rest of the world into two large regions allows us to demonstrate the extent to which gains from risk sharing, and their composition by LE and SE, are heterogeneous across both individual countries and the world they face, reflecting differences in cumulants, openness and size. For a more granular analysis, unveiling additional nuances in the distribution of gains, κ_s and consumption shares, see Appendix J.

[42] Barro (2006) and Nakamura et al. (2013) stress that disasters combine a strong global component with sizable idiosyncratic ones; jointly modeling both is problematic—treating them as permanent, as in Nakamura et al. (2013), would place countries off a common balanced-growth path. Our long-sample approach without a common component is a practical compromise (see the analytical section and Online Appendix F).

To enhance readability of our results, we color-code individual countries with meaningfully similar characteristics based on the result of a statistical clustering technique (Gaussian Mixture Model—GMM), accounting for country size, openness and moments (e.g., see Scrucca et al., 2023).[43] Table 4 provides a concise description of the clusters' determinants and the color coding. The procedure yields five groups. The group *Large* gathers the five biggest, relatively closed and thin-tailed economies (CHN, IND, BRA, IDN, USA); the group *Core*, by far the largest, includes mid-sized economies with average openness and low volatility; the group *Small open* includes small, very open and highly volatile economies; the group *Moderate tails* includes mid-sized, relatively closed economies with pronounced left skewness and moderately fat tails; and the *Fat-tailed* group includes small, open economies with very volatile, strongly left-skewed and fat-tailed output.

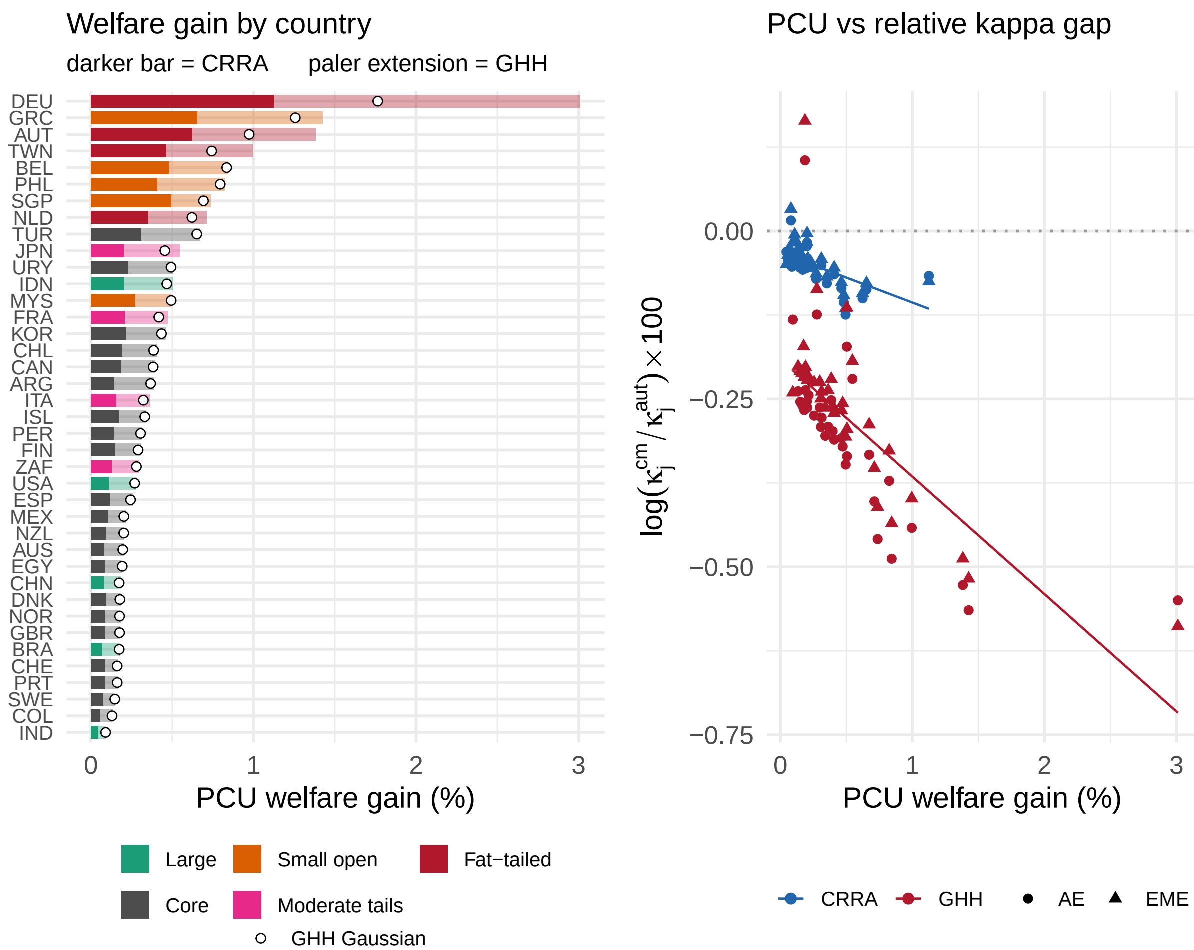
5.2.1 Gains from risk sharing and level effects across countries

The main results from our exercise are shown in the two panels of Figure 2. The left panel in Figure 2 plots, by country, the welfare gains from perfect risk sharing relative to autarky in permanent-consumption units (PCU, percent of steady-state consumption): the darker bar is computed under CRRA preferences and the paler extension is the additional gain under GHH preferences. As a reference point, a circle denotes the gains under GHH preferences using the Gaussian as opposed to the mixed-Gaussian distribution. The right panel plots, for both preference specifications, the country gains in PCU against the level effects, measured by the change in κ's from autarky to complete markets, i.e., $(\kappa_j^{cm} - \kappa_j^{aut}) \times 100$, separately for the CRRA (in blue) and the GHH (in red) preferences, and distinguishing between the AE and EME blocks, marked by, respectively, circles and triangles.

Consistently with the cross-border heterogeneity in fundamentals and economic structure (moments, size and openness) in the data, there is considerable variation in the

[43] We model the data as a weighted mix of Gaussian distributions. Each country is assigned to the Gaussian component that makes its features most likely. See Online Appendix M for a technical description.

Figure 2: Distribution of welfare gains and the level effect.



Note: Colors denote the groups described in Table 4. ISO country codes can be found here: https://www.iso.org/obp/ui/#search/code/.

potential welfare gains across countries. Under GHH preferences total gains vary from a minimum of 0.09% to a maximum of 3.0% (median 0.34%) in terms of permanent consumption units. The corresponding range for CRRA preferences is 0.04% − 1.13% (median 0.15%).

The GMM clustering accounts remarkably well for the welfare ranking and the composition of the gains, in line with our analytical results. In particular, the largest gains accrue to the fat-tailed economies (AUT, DEU, NLD, TWN). For three of these countries, DEU, AUT and TWN, gains under the mixed-Gaussian distribution (marked by the circle in the histogram bars) far exceed their Gaussian counterparts. At the top of the ranking

Table 4: Interpretation of country groups based on scaled distributional features

Group†	**Size**	**Openness**	**Standard deviation**	**Skewness**	**Kurtosis**	**Summary**
■ Large	$\uparrow\uparrow$	$\downarrow\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	Large, relatively closed, low volatility, thin tails.
■ Core	$\downarrow$	≈ 0	$\downarrow$	$\uparrow$	$\downarrow$	Mid-sized, average openness, low volatility, modest upside skewness, thin tails.
■ Small open	$\downarrow$	$\uparrow\uparrow$	$\uparrow\uparrow$	$\uparrow$	$\downarrow$	Small, very open, high volatility, near-Gaussian tails.
■ Moderate tails	≈ 0	$\downarrow$	≈ 0	$\downarrow$	$\uparrow$	Mid-sized, relatively closed, pronounced left skewness, moderately fat tails.
■ Fat-tailed	$\downarrow$	$\uparrow$	$\uparrow\uparrow$	$\downarrow\downarrow$	$\uparrow\uparrow$	Small, open, high volatility, strong left skewness, very fat tails.

Note: † Groups obtained via a two-stage GMM (model-based clustering, Scrucca et al., 2023) described in Online Appendix M. The standardized input features are: population size, openness (consumption import share) and standard deviation, skewness and kurtosis of the ergodic distribution of the discretized TFP process. Model selection is based on the Bayesian Information Criterion (BIC) together with a sufficiently large number of groups at each stage. We selected the EEI model (diagonal covariance with equal volume and shape); the two large-economy components of the first stage are merged into the single "Large" group, yielding five groups. Arrows denote group means of the standardized features: ≈ 0 for $|z| \leq 0.2$, single arrows for $0.2 < |z| \leq 1$, double arrows for $|z| > 1$.

these economies are interspersed with small, open and volatile ones (GRC, BEL, PHL, SGP). Unlike in the top group, the gains difference between Gaussian and Mixed Gaussian remains contained. Within the remaining groups the gains are more homogeneous—especially for the large and "core" economies—as size, openness and higher moments exert offsetting forces on the gains from risk sharing.[44] Importantly, the Gaussian calibration performs quite well for most of the countries (the median gain is about 0.32 vs 0.34 for the mixed-Gaussian).

Close inspection reveals that, perhaps unsurprisingly, risk sharing allows most countries to efficiently diversify the business cycle risk they face under financial autarky, i.e., $PCU = \xi^{cm} - \xi^{au} \approx -\xi^{au}$. By way of example, for the four largest-gain economies,

[44] A group-by-group description is detailed in the Online Appendix M.

under GHH preferences, the pairs ($\xi^{cm}\%, \xi^{au}\%$) are: $DEU = (-0.29, -3.30)$; $GRC = (-0.09, -1.52)$; $AUT = (-0.02, -1.40)$; $TWN = (-0.01, -1.00)$. These results highlight that, in our model, welfare costs of macroeconomic risk—which countries bear in full under financial autarky—are substantially larger than implied by traditional Lucas-type calculations. The latter, by relying on Gaussian shocks and a one-good economy, miss the implications of both higher-order cumulants and the distinction between output and income risk arising from relative price fluctuations.[45] It is worth emphasizing that the insurance provided by complete markets works best for most small and medium-sized countries: the residual (ξ^{cm}) is essentially zero for the median economy. In contrast, the largest economies (e.g., CHN and USA) cannot fully diversify their shocks internationally, as these constitute a sizable share of aggregate world risk. For these countries, a nontrivial fraction of the welfare cost of business-cycle risk remains even under complete markets—more than one-half in the case of CHN.

The cross-country variation in gains reflects on their level-effect component. The right panel of the figure shows that, in equilibrium, the relationship between PCU and $\ln\left(\kappa_j^{cm}/\kappa_j^{au}\right)$ is unconditionally negative (with a correlation coefficient about -0.73 for GHH and -0.62 for CRRA). Recall that the constant κ under complete markets is the implicit premium a country pays to its trading partners—so that a lower κ means that a country pays a lower (or, if negative, receives a higher) premium. In our sample, countries gaining a relatively larger premium under complete markets tend to benefit the most from risk sharing.

5.2.2 Quantitative importance of country size and home bias

We conclude with two model-based exercises that highlight the key role played by cross-country asymmetries in structural parameters. In both exercises, for brevity, we report these

[45] Yet, our costs are substantially smaller than the disaster-based, endowment estimates of Martin (2008) (about 14% of consumption). The difference reflects both our production and open-economy structure (with endogenous labor) and milder tail risk implied by estimating the shock process from GDP rather than calibrating consumption disasters directly.

results under GHH preferences; the corresponding tables under separable CRRA preferences are collected in Appendix N and deliver the same message.

In a first exercise we compare the gains from risk sharing and κ's in our baseline, and a counterfactual in which we impose PPP and equal size, while relying on the same mixed-Gaussian shocks as in our baseline. The comparison is shown in Table 5, which reports, for each variable, the minimum, median and maximum across the 39 countries for our baseline (first three columns in the table) and the counterfactual (last three columns). Comparing results on the first row of the table across columns clearly shows that removing asymmetries all but eliminates welfare gains. In permanent-consumption units, the median gain declines from approximately 0.34% to 0.03%, while the gain for the economy with the greatest exposure to tail risk falls from about 3.0% to 0.30%.

The relevance of asymmetries for the values of κ is apparent from the second and third row of the table. In the baseline calibration the (log) values of κ are sizable and display substantial cross-country dispersion. Once we impose purchasing power parity (PPP) and equal country size, the values of κ fall below 1% across all economies. Note further that, in our calibration, the values of κ are more negative vis-à-vis the EME region than the AE region, even though, in our long sample, it is the latter region that has the fatter tails. Specifically, $\kappa^{cm}_{j,EME}$ ranges between about -40% and -10%, and is negative for every country in the sample (the median is about -34%)—whereas $\kappa^{cm}_{j,AE}$ ranges between about -8% and 18%, with a median value close to zero. Heterogeneity in size is the main driver of these differences—as discussed in the comments to Table 3, size amplifies the risk inherent in cumulants. The EME block is about four times larger than the AE block. As a result, in spite of its smaller cumulants, the EME region tends to be riskier in equilibrium than the AE region, i.e., $\kappa^{cm}_{j,AE}$ is larger than $\kappa^{cm}_{j,EME}$. Indeed, eliminating asymmetries in size and openness, $\kappa^{cm}_{j,AE}$ falls systematically below $\kappa^{cm}_{j,EME}$. The takeaway is straightforward: a careful calibration of heterogeneity in expenditure baskets and country size is essential for assessing the welfare effects of financial integration and risk sharing.

Coming to the smoothing effect, Table 6 reports the changes in the second and higher moments of (log) consumption and labor when moving from autarky to complete markets under the same calibrations as in the previous table. The effects of risk sharing on consumption volatility are substantially larger in our baseline calibration: the median reduction in the standard deviation of consumption is 2.7 percentage points, compared with only 0.7 percentage points under PPP and equal country size. But note that, while complete markets reduce consumption volatility for almost every country, their effects on higher-order moments are far from uniform. For the median country, for instance, skewness rises even as variance falls. In general, changes in skewness and kurtosis take different signs across countries. Since welfare depends on the entire distribution of consumption—not just its variance—the standard deviation alone is an incomplete measure of the benefits of international risk sharing.

A final comment is in order concerning smoothing via adjustment at the labor margin. In our baseline, the pattern for labor is the same as for consumption: the standard deviation of labor supply falls, the change in the other moments may go either way. However, in the PPP and equal-size calibration, all changes in labor moments are identically zero. This is an implication of GHH preferences: under these preferences, labor depends exclusively on the real wage, and, under symmetric economic structures, real wages remain constant under both financial autarky and complete markets. It follows that there is no smoothing at the labor margin. This margin is active only in the presence of asymmetries in country size and home bias.

6 Conclusions

In this paper, we have reconsidered the welfare and macroeconomic effects of insuring fundamental output risk among countries differing in size and openness, offering a decomposition of welfare gains into a smoothing and a level effect, by moments (cumulants) of the distribution

Table 5: Percentiles of PCU and κ_s — GHH preferences

	mixed-Gaussian baseline			**mixed-Gaussian PPP and equal size**		
Variable[†]	min	median	max	min	median	max
PCU	0.09	0.34	3.01	0.01	0.03	0.30
$\kappa^{cm}_{j,AE}$	−8.41	0.10	17.65	−0.03	0.07	0.94
$\kappa^{cm}_{j,EME}$	−39.45	−33.55	−9.68	−0.02	0.10	0.95

[†] PCU = $(\xi^{cm} - \xi^{au}) \times 100$, percent of steady-state consumption; κ's in percent, in deviation from unity, i.e., $(x - 1) \times 100$.

of output, as a function of trade elasticities and risk aversion. Our analysis brings asymmetries in volatility, skewness and kurtosis—and especially their interactions with economic size and openness of a country—center stage in the assessment of risk sharing.

Our study contributes to the literature an in-depth and transparent analysis of the determinants of both the magnitude of, and a country's exposure to, income risk stemming from production uncertainty. Our results underscore the need for a careful calibration of cross-border heterogeneity in these dimensions to assess income risk and gains from risk sharing. In our quantitative exercise based on a long historical sample that encompasses catastrophic events, including major wars, these gains range from close to 0.1% to about 3% of permanent consumption. We find that fat tails account for a substantial share of the potential welfare gains from risk sharing. By clustering countries based on openness, size, and higher-order moments, we uncover a pattern closely aligned with the theoretical predictions of our model.

In economic and policy evaluations, risk sharing and the benefits of capital market integration are routinely assessed using indicators based on estimates of the volatility and cross-border correlation of consumption. Our analysis emphasizes that these indicators offer only a partial, potentially misleading, assessment. A key challenge for future research is to develop complementary indicators that both account for exposure to tail risks and capture level effects and relative wealth adjustments, thereby bridging the gap between macroeco-

Table 6: Percentiles of (log) consumption and labor moments — GHH preferences

	mixed-Gaussian baseline			mixed-Gaussian PPP and equal size		
Moment†	min	median	max	min	median	max
			Consumption			
$Stdev_C^{cm-au}$‡	−8.67	−2.73	−0.67	−2.39	−0.66	0.01
$Skew_C^{cm-au}$	−0.68	0.18	1.53	−0.31	0.08	0.49
$Kurt_C^{cm-au}$	−10.25	−1.04	0.20	−3.35	−0.36	0.27
			Labor			
$Stdev_L^{cm-au}$‡	−1.13	−0.38	−0.13	0.00	0.00	0.00
$Skew_L^{cm-au}$	−0.03	0.08	0.37	0.00	0.00	0.00
$Kurt_L^{cm-au}$	−2.00	−0.19	0.00	0.00	0.00	0.00

‡Percentage points. †$moment_X^{cm-au}$, with $X = \{C, L\}$, represents the difference between complete markets and autarky of the particular moment of (log) X computed using the Markov process ergodic distribution.

nomics and the finance and asset pricing literature. This line of inquiry could clarify the extent to which even relatively "safe" countries benefit from cross-border risk sharing, particularly during episodes of heightened tail risk—possibly based on richer models incorporating investment, financial frictions, and government activity, all of which play a central role in the risk-sharing debate. A related challenge is to bring new theoretical and applied work to bear on the scope for risk sharing in the context of rising trade and financial fragmentation, climate-related shocks, as well as heightened geopolitical risks, currently facing the global economy.

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