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Do Term Premiums Matter? Transmission via Exchange Rate Dynamics*

Mitsuru Katagiri† Koji Takahashi‡

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Abstract

The macroeconomic effect of term premiums is a controversial issue both theoretically and quantitatively. In this paper, we explore the possibility that term premiums affect inflation and the real economy via exchange rate dynamics. For this purpose, we construct a small open economy model with limited asset market participation, focusing on the empirical observation that uncovered interest parity holds better for longer-term interest rate differentials. A quantitative exercise using Japanese and U.S. data shows that changes in term premiums, particularly those made by the central bank's bond purchases, have sizable effects on Japanese inflation rates via exchange rate dynamics.

Keywords: Term premium; Uncovered interest rate parity; Quantitative easing

JEL Classification: E31, E52, E58

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1 Introduction

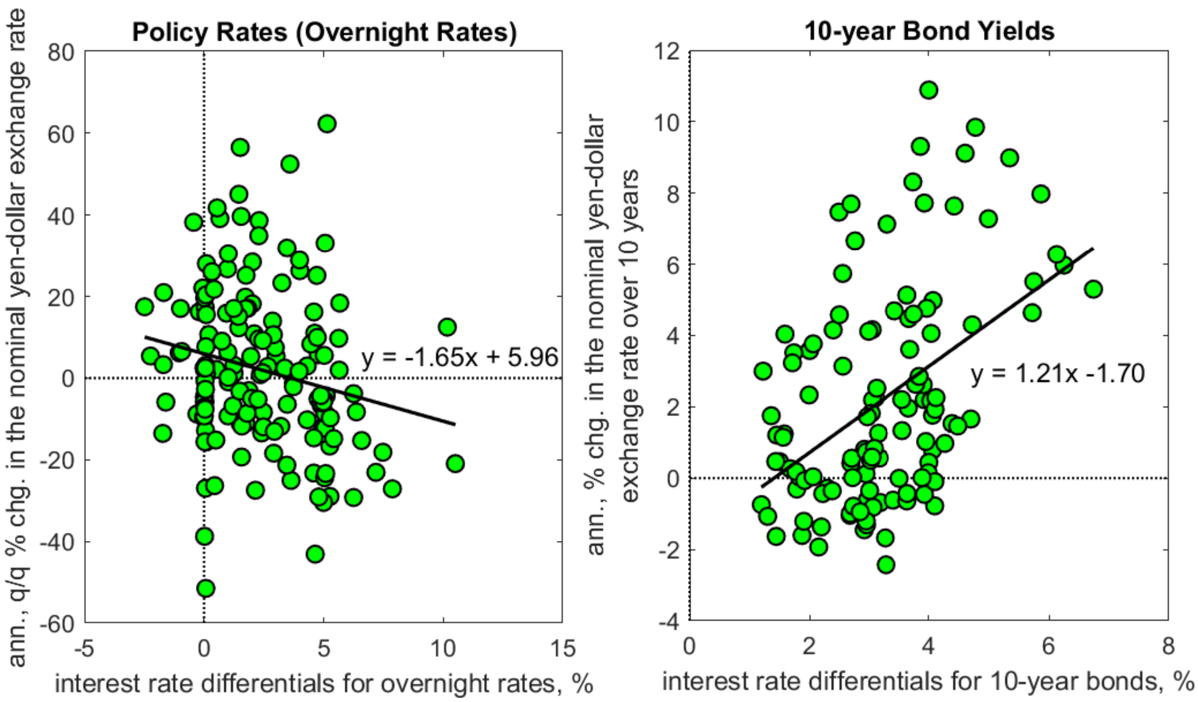
The macroeconomic effect of term premiums is a controversial issue both theoretically and quantitatively. While financial economists and practitioners have been keen to discuss the term premium and the factors behind its development, a modern macroeconomic model developed under the dynamic stochastic general equilibrium (DSGE) framework describes long-term yields just as a future course of short-term interest rates, and, thus, has to have neither long-term yields nor term premiums in addition to the policy interest rate. Against this backdrop, Stein (2012) and Faust (2015) claim there is no rigid theoretical background on the relationship between term premiums and economic activity.[1] However, many central banks in advanced economies have conducted long-term bond purchase programs to lower long-term interest rates mainly by reducing term premiums (see, e.g., D'Amico et al., 2012; Bank of Japan, 2015, 2016). Given this fact, there has emerged an interest in examining by macroeconomic models how significantly changes in term premiums affect the economy and what is the transmission mechanism of these measures.

This study contributes to the literature by constructing a small open economy DSGE model with long-term bonds and examining whether term premiums affect the inflation rate and the real economy via exchange rate dynamics. Our motivation for exploring the exchange rate channel comes from the observed relationship between long-term interest rate differentials and exchange rate dynamics. Figure 1 plots the interest rate differentials of short- and long-term interest rates between Japan and the U.S. against corresponding future changes in the yen–dollar exchange rate. While yen–dollar exchange rate dynamics are almost uncorrelated (or negatively correlated) with the interest rate differentials of overnight rates, they are positively correlated with those of ten-year government bond yields, and the slope is close to one. The data in the figure suggest that the theoretical prediction of uncovered interest parity (UIP) (i.e., a positive relationship between interest rate differentials and the future exchange rate changes) holds better for long-term interest rates than short-term interest rates.[2] As the term premiums are the gap between long-term interest rates and

[1]Stein (2012) takes a simple corporate finance example to examine the effect of term premiums on corporate investment and concludes that "[...] investment spending is decoupled from the term premium and is determined instead by the expected future path of short rates."

[2]The null hypothesis that the slope is equal to one is statistically rejected for the case of short-term interest rates (p-value < 0.01) but not rejected for the case of long-term interest rates (p-value $= 0.8$). The empirical result that the UIP holds better for longer-term interest rates is obtained not only for the yen–

Figure 1: Interest Rate Differentials and Exchange Rate Dynamics



Note: The figure shows scatter plots for interest rate differentials and exchange rate changes. In the case of short-term interest rates (the left panel), we calculated the quarterly averages of overnight lending rates and daily changes in exchange rates. In the case of long-term interest rates (the right panel), we use 10-year bond returns and changes in realized exchange rates over ten years. The sample periods for both panels span from 1980Q1 through 2019Q4.

the future path of short-term interest rates, the above empirical observation suggests that changes in term premiums may influence the exchange rate and, thus, affect inflation and the real economy.

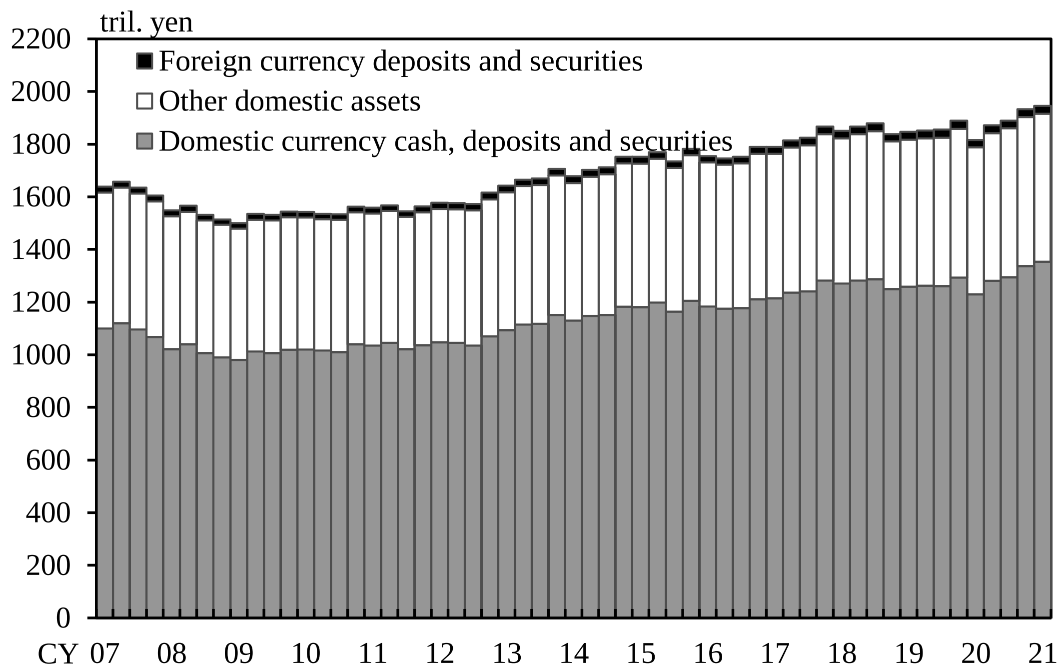
As a mechanism to potentially account for the observation that UIP holds better for long-

dollar exchange rate but also for the exchange rate of other currencies against the U.S. dollar (e.g., Chinn and Meredith (2004) and Chinn (2006)). However, Chinn and Quayyum (2012) and Kano and Wada (2017) show that the slope of the UIP regression for long-term interest rate differentials varies across currencies as well as across sample periods, implying that the UIP does not perfectly hold for long-term interest rates. These results, including the case of yen–dollar exchange rates in this study, may be influenced by the fact that long-term forecasting regressions using overlapping data tend to provide significant coefficients, as pointed out by Valkanov (2003).

term interest rate differentials, we incorporate the following two assumptions on limited asset market participation, both of which are observed in data, into a standard small open economy DSGE model. First, the model assumes that households do not invest in foreign bonds and save only by investing in domestic short- and long-term bonds. This assumption, which can be interpreted as a kind of "home bias," is in line with Japanese households' investment behavior. That is, the composition of Japanese household assets from the Flow of Funds statistics in Figure 2 indicates that foreign assets account for less than 2 percent of Japanese households' total asset holdings and that this extreme home bias in Japan has changed little over the years. Second, the model assumes that bond investors do not trade short-term domestic and foreign bonds but trade only long-term domestic and foreign bonds. We justify this second assumption based on the fact that most Japanese institutional investors, which are the major investors in foreign bond markets in Japan, mostly hold long-term bonds rather than short-term bonds. A breakdown of the bond portfolios of institutional investors (financial institutions other than deposit-taking corporations) in Figure 3 shows that they hold almost negligible amounts of short-term bonds and that their portfolio mostly consists of long-term bonds—thus empirically justifying our second assumption. While investigating the factors that explain the investment behaviors of households and institutional investors in Japan is an interesting economic topic, we do not intend to propose detailed micro-foundations thereof. Rather, as in the literature on macroeconomic analyses with limited asset market participation (e.g., Mankiw and Zeldes, 1991; Chen et al., 2012), we simply assume the above limited asset market participation of households and investors and focus on its macroeconomic implications under incomplete arbitrage in the foreign exchange market by taking those investment behaviors as given.

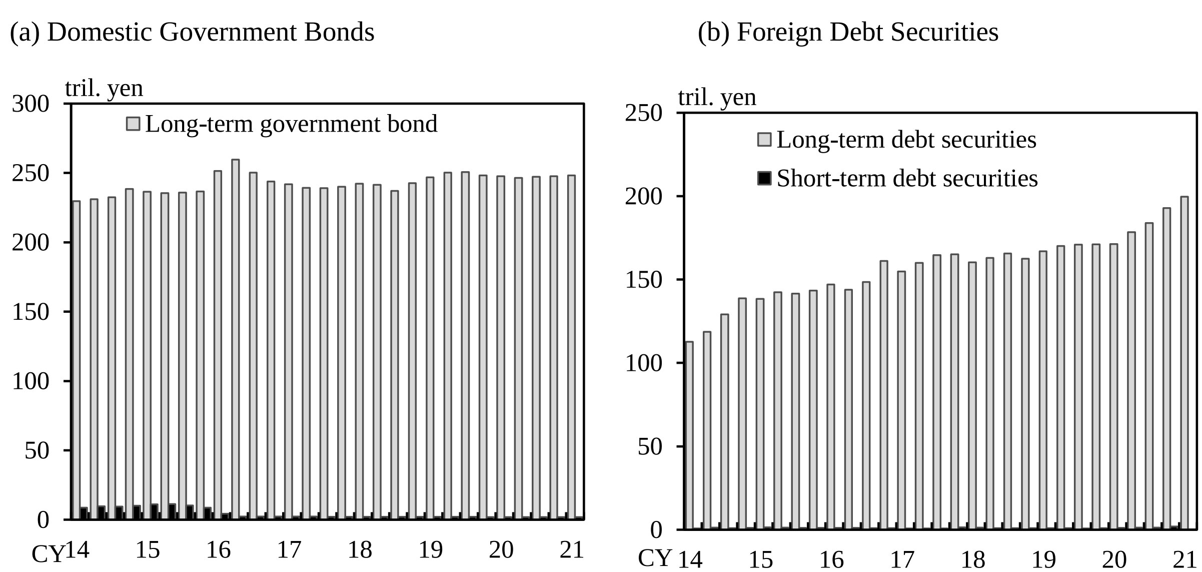
The above two assumptions on limited asset market participation lead to the following two macroeconomic consequences in the model: First, as expected, they make the model consistent with the observation that the UIP holds better for long-term interest rate differentials. Because we assume a wedge between the short-term interest rate and the exchange rate as in the literature including Adolfson et al. (2007, 2008), the UIP does not hold for short-term interest rate differentials in our model by construction. Differently, a novelty of our model is that it shows that the UIP holds for long-term interest rate differentials even with the wedge if the model assumes limited asset market participation for households and investors. It is worth noting that such contrasting results between the UIP regression for short- and long-term interest rates are obtained only if sufficiently large fluctuations exist in

Figure 2: Composition of Japanese Household Assets



Source: Bank of Japan, "Flow of Funds."

Figure 3: Bond Portfolio of Japanese Institutional Investors



Note: Japanese institutional investor consists of bond investments trusts, and insurance and pension funds. Long-term domestic government bonds are composed of central government securities, fiscal investments, and local program bonds.

Sources: Bank of Japan, "Flow of Funds"; Ministry of Finance, "International Investment Position."

term premiums. Second, and more importantly, the above limited asset market participation assumptions lead term premiums to influence inflation and the real economy via exchange rate dynamics in the model. As pointed out by Andrés et al. (2004) and Chen et al. (2012), changes in term premiums do not have any effects on the economy unless there exists an agent who has access only to long-term bonds and whose actions are relevant to the economy. In our model, bond investors in the foreign exchange market play this role. As discussed, bond investors can trade only domestic and foreign long-term bonds in our model, and their actions are relevant to the economy because it is the sole determinant of foreign exchange rates in the model because of the limited participation assumption for households. In other words, if households had access to foreign bonds (or the bond investor had access to short-term bonds), changes in term premiums would not have any effects on the inflation rate and the real economy, as in a standard DSGE model.

Finally, we estimate the model parameters using the yen–dollar exchange rate and data for the Japanese and U.S. economies, including the data for the central bank's bond purchases, and then quantitatively investigate the effects of term premiums on the exchange rate and inflation in Japan. The historical decomposition of inflation for the last decade indicates that the decline in U.S. term premiums put downward pressure on the Japanese inflation rate through the appreciation of the yen against the US dollar since 2008, after expanding large-scale asset purchase (LSAP) by the Federal Reserve (Fed). However, since 2013, the decrease in term premiums in Japan, induced by the Bank of Japan's (BOJ) quantitative and qualitative easing (QQE) policy, raised the inflation rate by causing a depreciation of the exchange rate. Hence, the quantitative results imply that the exchange rate has been affected by both U.S. and Japanese term premiums, leading to non-negligible effects on inflation in Japan.

Our study is related to Andrés et al. (2004) and subsequent studies that investigate the role of term premiums in a DSGE model. Chen et al. (2012) constructed a closed-economy DSGE model with long-term bonds, where a part of the households do not have access to short-term bonds. They estimated the model parameters using U.S. data and showed that the LSAP had non-negligible effects on the U.S. economy through changes in term premiums. Following their work, other scholars examined the role of term premiums in an open economy DSGE model. Alpanda and Kabaca (2020) examined the role of term premiums in a two-country DSGE model by introducing government bonds in the utility function; they argue that the U.S. LSAP had international spillover effects. Kolasa and Wesolowski

(2020) constructed a two-country DSGE model with limited asset market participation and accounted for the massive capital flows following the U.S. LSAP and the comovement of term premiums across countries. Wesolowski (2018) investigated the effects of term premiums in the context of a small open economy model and estimated the model using data for Poland. Similar to these studies, we too investigate the role of term premiums in an open economy DSGE model. However, we propose a different, but compatible, limited participation assumption and then examine the validity of our assumption by showing data in Figures 2 and 3 and estimating the UIP regressions over different horizons rather than providing only some anecdotal observations. In the empirical literature, Bauer and Neely (2014) investigated the transmission mechanism of unconventional monetary policy through international bond markets. Dedola et al. (2021), similarly, showed that QE policies have sizable effects on exchange rates, which is consistent with our findings. Another strand of literature related to our study is the relationship between interest rate differentials and exchange rates; Engel (2014, 2016) provides recent advances in this regard. Of particular relevance in this context are the studies by Chinn and Meredith (2004) and Chinn (2006), who investigated the UIP for long-term interest rate differentials. Kano and Wada (2017) also examined the long-term UIP for the yen–dollar exchange rate.

The remaining paper proceeds as follows. In section 2, we present a small open economy model with limited bond market participation. In section 3, we estimate the model parameters and examine the policy effects of term premiums based on impulse response analyses. In section 4, we model term premiums as a function of the central bank's bond purchases and explore what drives inflation and exchange rates in Japan through historical decomposition. Finally, we conclude the study in section 5.

2 Model

Our model follows a standard small open economy DSGE framework with long-term bonds. The home country economy consists of households, an arbitrager, and several types of firms, which produce consumption goods, intermediate goods, export goods, and import goods. The economy in the rest of the world (foreign economy) is described by a small-scale New Keynesian model. In the spirit of small open economy models, the foreign economy is assumed to be independent of the home economy, while the home economy is assumed to be influenced by the foreign economy through the exchange rate, demand for export goods, and prices of import goods. Each type of agent"s behavior in the home and foreign economies is described in turn.

2.1 Households

There is a continuum of households in the home economy that supplies a differentiated labor force indexed by $h \in (0,1)$ to obtain wage income $W_t(h)L_t(h)$, where $W_t(h)$ denotes the nominal wage and $L_t(h)$ denotes the hours worked of each household h. In addition, because all firms in the economy are owned by households and households also invest in the arbitrager, households obtain profits D_t from firms and the arbitrager as another source of their income. Households allocate their income to the consumption basket C_t and savings. The consumption basket consists of domestic and foreign consumption goods,

$$C_t = \left[(1-\delta)^{\frac{1}{\eta}} C_{d,t}^{\frac{\eta-1}{\eta}} + \delta^{\frac{1}{\eta}} C_{f,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

where $C_{d,t}$ and $C_{f,t}$ are domestic and imported consumption goods, respectively. δ and η are the parameters for the share of imported consumption goods in the consumption basket and for the elasticity between domestic and foreign goods, respectively. The price level of the consumption basket (i.e., the consumer price index, CPI) is given by

$$P_t C_t = P_{d,t} C_{d,t} + P_{f,t} C_{f,t},$$

where $P_{d,t}$ and $P_{f,t}$ are the prices of domestic and imported consumption goods. Savings take two forms: nominal one-period domestic bonds, B_t, and long-term domestic bonds, B_t^L. Following Woodford (2001), long-term bonds take the form of perpetuities which pay a decaying coupon κ^s at $t+1+s$. Further, following Chen et al. (2012), households are assumed to pay time-varying transaction cost ζ_t per unit of long-term bonds. This transaction cost, which is introduced to describe the preferred habitat behavior of investors, is a source of term premiums in this model, and follows the exogenous process

$$\zeta_t - \zeta = \rho_\zeta \left(\zeta_{t-1} - \zeta \right) + \varepsilon_{\zeta,t}. \tag{2}$$

In the policy analysis in Section 4, the transaction cost ζ_t is reformulated as a function of the central bank's bond holdings rather than the exogenous process to decompose the changes in term premiums into policy effects and others.

Households face the following budget constraint:

$$P_t C_t + B_t + (1+\zeta_t) P_t^L B_t^L = R_{t-1} B_{t-1} + P_t^L R_t^L B_{t-1}^L + W_t(h) L_t(h) + D_t, \tag{3}$$

where P_t^L is the price of long-term bonds and $R_t^L = \frac{1}{P_t^L} + \kappa$ is the long-term interest rate. Note that households can only invest in domestic bonds and not foreign bonds. This limited bond market participation assumption for households is based on the empirical fact that Japanese households hold almost negligible amounts of foreign assets, as shown in Figure 2. Households choose their consumption C_t and short-term and long-term bonds, B_t and B_t^L, to maximize their lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log \left(C_t - \varkappa C_{t-1} \right) - \psi \frac{L_t(h)^{1+\nu}}{1+\nu} \right],$$

subject to constraints (1) and (3). $\beta \in (0,1)$ is the constant discount factor and $\varkappa$ is the parameter for habit formation.

There are competitive labor agencies that aggregate the labor services provided by each household h into homogeneous labor L_t based on the following constant elasticity of substitution (CES) function:

$$L_t = \left(\int_0^1 L_t(h)^{\frac{1}{\lambda_w}} dh \right)^{\lambda_w},$$

where $\lambda_w > 1$ is the markup parameter. Let $W_{h,t}$ be the nominal wage rate for household h. The aggregate nominal wage W_t is defined as

$$W_t = \left(\int_0^1 W_t(h)^{\frac{1}{1-\lambda_w}} dh \right)^{1-\lambda_w},$$

and the demand function for each household's labor services is then derived as a result of profit maximization by the labor agencies,

$$L_t(h) = \left(\frac{W_t(h)}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_t. \tag{4}$$

Given this demand function for their labor services, households monopolistically supply differentiated labor $L_t(h)$ and set their wages $W_t(h)$ on a staggered basis à la Calvo (1983). In each period, a fraction of households, $0 < \xi_w < 1$, set their wages based on the partial indexation rule $W_t(h) = \left(\pi_{t-1} e^{\gamma_{t-1}}\right)^{\iota_w} \left(\pi e^{\gamma}\right)^{1-\iota_w} W_{t-1}(h)$, where π_{t-1} and $e^{\gamma_{t-1}}$ are the aggregate inflation rate and the aggregate productivity growth rate in period $t-1$, and π and e^{γ} are their steady state values. The remaining fraction $1 - \xi_w$ of households chooses $\tilde{W}_t(h)$ to maximize

$$\max_{\tilde{W}_t(h)} E_t \sum_{s=0}^{\infty} \left(\beta \xi_w \right)^s \left[\Xi_{t+s} \tilde{W}_{t+s}(h) L_{t+s}(h) - \psi \frac{L_{t+s}(h)^{1+\nu}}{1+\nu} \right],$$

where Ξ_t is the marginal utility of consumption in nominal terms,

$$P_t \Xi_t \equiv \frac{1}{C_t - \varkappa C_{t-1}} - \frac{\beta \varkappa}{C_{t+1} - \varkappa C_t}.$$

Nominal wage rates $\tilde{W}_{t+s}(h)$ are determined by the following law of motion:

$$\tilde{W}_{t+s}(h) = \left(\pi_{t+s-1} e^{\gamma_{t+s-1}}\right)^{\iota_w} \left(\pi e^{\gamma}\right)^{1-\iota_w} \tilde{W}_{t+s-1}(h),$$

for $s \geq 1$, and labor demand $L_{t+s}(h)$ is determined by (4) and $\tilde{W}_{t+s}(h)$. Given each household's optimization, wage inflation dynamics can be described by a recursive structure with the two auxiliary variables, $x^w_{1,t}$ and $x^w_{2,t}$, as follows:

$$\left[\frac{1 - \xi_w \left(\frac{w_{t-1}}{w_t} \Pi^*_{w,t}\right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w}\right]^{1-\lambda_w} = \left(\frac{\lambda_w}{w_t} \frac{x^w_{1,t}}{x^w_{2,t}}\right)^{\frac{1-\lambda_w}{1-(1+\nu)\lambda_w}},$$

where $w_t = W_t/P_t$ and $\Pi^*_{w,t} = \left(\pi_{t-1} e^{\gamma_{t-1}}\right)^{\iota_w} \left(\pi e^{\gamma}\right)^{1-\iota_w} / \left(\pi_t e^{\gamma_t}\right)$. These two auxiliary variables, $x^w_{1,t}$ and $x^w_{2,t}$, follow the following laws of motion:

$$x^w_{1,t} = \psi L_t^{1+\nu} + \beta \xi_w E_t \left[\left(\frac{w_t}{w_{t+1}} \Pi^*_{w,t+1}\right)^{\frac{\lambda_w (1+\nu)}{1-\lambda_w}} x^w_{1,t+1}\right],$$

$$x^w_{2,t} = \Lambda_t L_t + \beta \xi_w E_t \left[\left(\frac{w_t}{w_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\Pi^*_{w,t+1}\right)^{\frac{1}{1-\lambda_w}} x^w_{2,t+1}\right],$$

where $\Lambda_t = P_t A_t \Xi_t$.

2.2 Consumption Good Firms

The domestic consumption good firms produce the final goods, $Y_{d,t}$, by aggregating the intermediate goods, $Y_{d,t}(i)$, based on the following CES production function in a competitive market:

$$Y_{d,t} = \left(\int_0^1 Y_{d,t}(i)^{\frac{1}{\lambda_c}} di\right)^{\lambda_c},$$

where $\lambda_c > 1$ is the markup parameter. Let $P_{d,t}(i)$ be the price of each intermediate good. The price index for domestic intermediate goods, $P_{d,t}$, is then defined as

$$P_{d,t} = \left(\int_0^1 P_{d,t}(i)^{\frac{1}{1-\lambda_c}} di\right)^{1-\lambda_c},$$

and the demand for each intermediate good is derived based on profit maximization by consumption good firms,

$$Y_{d,t}(i) = \left(\frac{P_{d,t}(i)}{P_{d,t}}\right)^{\frac{\lambda_c}{1-\lambda_c}} Y_{d,t}. \tag{5}$$

2.3 Intermediate Goods Firms

A continuum of intermediate goods firms indexed by i produces differentiated intermediate goods using labor $L_t(i)$ and imported intermediate inputs $Z_t(i)$ based on the following technology:

$$Y_{d,t}(i) = [Z_t(i)]^{\alpha} [A_t L_t(i)]^{1-\alpha}, \tag{6}$$

where A_t is labor-augmenting technology in period t. Let $\gamma_t = A_t/A_{t-1}$ and assume that γ_t follows the process

$$\log\left(\frac{\gamma_t}{\gamma}\right) = \rho_\gamma \log\left(\frac{\gamma_{t-1}}{\gamma}\right) + \varepsilon_{\gamma,t}.$$

Intermediate goods firms' optimization problem means that nominal marginal cost $MC_{d,t}$ is given by

$$MC_{d,t} = \frac{(P_{f,t})^{\alpha} W_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha} A_t^{1-\alpha}},$$

where $P_{f,t}$ is the price of imported goods.

Under monopolistic competition, intermediate goods firm i faces consumption goods firms' demand $Y_{d,t}(i) = (P_{d,t}(i)/P_{d,t})^{\frac{\lambda_c}{1-\lambda_c}} Y_{d,t}$ and maximizes its discounted profits by setting the price of its differentiated product on a staggered basis à la Calvo (1983). In each period, a fraction of intermediate good firms, $1 - \xi_d \in (0,1)$, reoptimizes prices while the remaining fraction ξ_d indexes prices to a weighted average of past and steady-state inflation $(\pi_{d,t-1})^{\iota_d} \pi^{1-\iota_d}$, where $\iota_d \in [0,1]$ is the relative weight on past inflation and $\pi_{d,t} = P_{d,t}/P_{d,t-1}$. Hence, the intermediate good firm i that reoptimizes prices in the current period chooses its prices so as to maximize,

$$\max_{\tilde{P}_{d,t}(i)} E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s \, \Xi_{t+s} \left[\tilde{P}_{d,t+s}(i) - v_{d,t} MC_{d,t+s}\right] Y_{d,t+s}(i),$$

where $v_{d,t}$ is a marginal cost shock for domestic intermediate good firms, and it follows the process,

$$\log(v_{d,t}) = \rho_d \log(v_{d,t-1}) + \varepsilon_{d,t}.$$

Also, $\tilde{P}_{d,t+s}(i)$ is determined by the following law of motion,

$$\tilde{P}_{d,t+s}(i) = (\pi_{d,t+s-1})^{\iota_d} (\pi)^{1-\iota_d} \tilde{P}_{d,t+s-1}(i),$$

for $s \geq 1$, and the demand for $Y_{d,t+s}(i)$ is determined by (5) and $\tilde{P}_{d,t+s}(i)$. As a result of the intermediate goods firms' optimization, inflation dynamics $\pi_{d,t}$ can be described by a recursive structure with the two auxiliary variables, $x_{1,t}^d$ and $x_{2,t}^d$, as follows:

$$\left[\frac{1-\xi_d \left(\Pi_{d,t}^*\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right]^{1-\lambda_d} = \lambda_c \frac{x_{1,t}^d}{x_{2,t}^d},$$

where $\Pi_{d,t}^* = (\pi_{d,t-1})^{\iota_d} \pi^{1-\iota_d}/\pi_{d,t}$. Those two auxiliary variables, $x_{1,t}^d$ and $x_{2,t}^d$, follow the following laws of motion:

$$x_{1,t}^d = \Lambda_t p_{d,t} y_{d,t} v_{d,t} mc_{d,t} + \beta \xi_d E_t[\left(\Pi_{d,t+1}^*\right)^{\frac{\lambda_c}{1-\lambda_c}} x_{1,t+1}^d],$$
$$x_{2,t}^d = \Lambda_t p_{d,t} y_{d,t} + \beta \xi_d E_t[\left(\Pi_{d,t+1}^*\right)^{\frac{1}{1-\lambda_c}} x_{2,t+1}^d],$$

where $y_{d,t} = Y_{d,t}/A_t$, $mc_{d,t} = MC_{d,t}/P_t$, and $p_{d,t} = P_{d,t}/P_t$.

2.4 Imported Goods Firms

The imported goods firms are classified into two groups: intermediate goods firms and final goods firms. A continuum of intermediate goods firms indexed by f purchases foreign goods from abroad at the foreign price P_t^*, and sells them to final goods firms as differentiated foreign goods $Y_{f,t}(f)$ at the price of $P_{f,t}(f)$. Then, the final goods firms aggregate these differentiated imported goods into the final imported good $Y_{f,t}$ using the CES aggregator, and sell it at the price of $P_{f,t}$. The final imported good is used for consumption $C_{f,t}$ or intermediate inputs for domestic intermediate goods firms, Z_t, as is shown in (6).

The demand function for each intermediate good $Y_{f,t}(f)$ is derived as a result of profit maximization of the final goods firm as,

$$Y_{f,t}(f) = \left(\frac{P_{f,t}(f)}{P_{f,t}}\right)^{\frac{\lambda_f}{1-\lambda_f}} Y_{f,t}. \tag{7}$$

Given this demand function, the intermediate goods firms monopolistically supply differentiated intermediate goods. Since the intermediate goods firms in the imported sector purchase foreign goods from abroad, their nominal marginal cost, $MC_{f,t}$, is defined as,

$$MC_{f,t} = \frac{P_t^*}{Q_t},$$

where Q_t is the nominal exchange rate.

Under monopolistic competition, intermediate goods firm f for imported goods faces the final goods firms' demand (Equation (7)) and maximizes its discounted profits by setting the price of its differentiated products on a staggered basis à la Calvo (1983). In each period, a fraction of intermediate goods firms, $1-\xi_f \in (0,1)$, reoptimizes prices while the remaining fraction ξ_f indexes prices to a weighted average of past and steady-state inflation $(\pi_{f,t-1})^{\iota_f}\,\pi^{1-\iota_f}$, where $\iota_f \in [0,1]$ is the relative weight on past inflation and $\pi_{f,t} = P_{f,t}/P_{f,t-1}$. Hence, the intermediate goods firm f that reoptimizes prices in the current period chooses its prices so as to maximize,

$$\max_{\tilde{P}_{f,t}(f)} E_t \sum_{s=0}^{\infty} (\beta\xi_f)^s\, \Xi_{t+s} \left[\tilde{P}_{f,t+s}(f) - v_{f,t} MC_{f,t+s}\right] Y_{f,t+s}(f),$$

where $v_{f,t}$ is a marginal cost shock for intermediate goods firms for imported goods, and it follows the process,

$$\log(v_{f,t}) = \rho_f \log(v_{f,t-1}) + \varepsilon_{f,t}.$$

Also, $\tilde{P}_{f,t+s}(f)$ is determined by the following law of motion,

$$\tilde{P}_{f,t+s}(f) = (\pi_{f,t+s-1})^{\iota_f}\,(\pi)^{1-\iota_f}\,\tilde{P}_{f,t+s-1}(f),$$

for $s \geq 1$, and the demand for $Y_{f,t+s}(f)$ is determined by (7) and $\tilde{P}_{f,t+s}(f)$. Then, by assuming the optimal price setting under the staggered prices with partial indexation, the inflation dynamics of the imported price, $\pi_{f,t} = P_{f,t}/P_{f,t-1}$, are described by a recursive structure with two auxiliary variables similarly to those of domestic good prices in the previous subsection. Since the imported goods firms set the imported prices in the home currency on a staggered basis, they cannot reflect all of the fluctuations in their marginal cost caused by exchange rate changes, thus making the pass-through to the imported prices not perfect in the short run.

2.5 Exported Goods Firms

The exported goods firms purchase domestic consumption goods $Y_{d,t}$, and sell them to foreign customers at the price of $P^*_{x,t}(x)$ on a foreign currency basis. The demand function for exported goods $Y_{x,t}(x)$ in an international market is assumed to depend on the relative price of exported goods to foreign price level, $P^*_{x,t}(x)/P^*_t$, the foreign output gap, y^*_t, and a trade balance shock, v_x, and it is determined by the following reduced form demand function,

$$Y_{x,t}(x) = \left(\frac{P^*_{x,t}(x)}{P^*_t}\right)^{\frac{\lambda_x}{1-\lambda_x}} (y^*_t)^{\theta} \exp(v_{x,t}), \tag{8}$$

where λ_x and θ are parameters for elasticity of demand to relative price of exported goods and foreign output gap, respectively. Also, the trade balance shock follows

$$\log(v_{x,t}) = \rho_x \log(v_{x,t-1}) + \varepsilon_{x,t}.$$

Given this demand function, the exported goods firms monopolistically supply exported goods. Since the exported good firms purchase domestic consumption goods and sell them to foreign customers, their nominal marginal cost, $MC_{x,t}$, is defined as,

$$MC_{x,t} = P_{d,t}.$$

The exported goods firms are assumed to set the exported prices *in the foreign currency* on a staggered basis with partial indexation, and maximize their profits *in the home currency.* Therefore, the exchange rate has effects on the export prices measured by home currency and consequently, on the amount of exports as well as their profits in home currency. More concretely, under monopolistic competition, exported good firm x faces final goods firms' demand (Equation (8)) and maximizes its discounted profits by setting the price of its differentiated products on a staggered basis à la Calvo (1983). In each period, a fraction of exported goods firms, $1-\xi_x \in (0,1)$, reoptimizes prices while the remaining fraction ξ_x indexes prices to a weighted average of past and steady-state inflation $\left(\pi^*_{x,t-1}\right)^{\iota_x} \pi^{*1-\iota_x}$, where $\iota_x \in [0,1]$ is the relative weight on past inflation and $\pi^*_{x,t} = P^*_{x,t}/P^*_{x,t-1}$. Hence, exported goods firm x that reoptimizes prices in the current period chooses its prices so as to maximize,

$$\max_{\tilde{P}^*_{x,t}(x)} E_t \sum_{s=0}^{\infty} (\beta\xi_x)^s \,\Xi_{t+s} \left[\tilde{P}^*_{x,t+s}(x)/Q_{t+s} - MC_{x,t+s}\right] Y_{x,t+s}(x).$$

Also, $\tilde{P}^*_{x,t+s}(x)$ is determined by the following law of motion,

$$\tilde{P}^*_{x,t+s}(x) = \left(\pi^*_{x,t+s-1}\right)^{\iota_x} (\pi^*)^{1-\iota_x} \tilde{P}^*_{x,t+s-1}(x),$$

for $s \geq 1$, and the demand for $Y_{x,t+s}(x)$ is determined by (8) and $\tilde{P}^*_{x,t+s}(x)$.

As a result of profit maximization of the exported firms, the inflation dynamics of exported price, $\pi^*_{x,t} = P^*_{x,t}/P^*_{x,t-1}$, are described by a recursive structure with two auxiliary variables similar to those of domestic prices.

2.6 Central Bank

The central bank in the home country sets short-term nominal interest rates, depending on the year-on-year inflation rate and output growth rate. In particular, it follows the Taylor-type policy rule with interest rate smoothing,

$$R_t = (R_{t-1})^{\rho_R} \left[R \left(\frac{\left[\Pi_{j=1}^{4} \pi_{t-j+1}\right]^{1/4}}{\bar{\pi}_t} \right)^{1+\phi_\pi} \left(\frac{\left[\Pi_{j=1}^{4} g_{t-j+1}^{y}\right]^{1/4}}{e^{\gamma}} \right)^{\phi_y} \right]^{1-\rho_R} v_{m,t},$$

where $g_t^y = Y_{d,t}/Y_{d,t-1}$ and $\bar{\pi}_t$ is the target inflation rate for the central bank. The target inflation rate is assumed to be time varying and follow the process,

$$\log\left(\frac{\bar{\pi}_t}{\bar{\pi}}\right) = \rho_{tp} \log\left(\frac{\bar{\pi}_{t-1}}{\bar{\pi}}\right) + \varepsilon_{tp,t}.$$

The central bank can deviate from the rule by adjusting a short-term interest rate shock, $v_{m,t}$, which follows the AR(1) process,

$$\log\left(v_{m,t}\right) = \rho_m \log\left(v_{m,t-1}\right) + \varepsilon_{m,t}$$

where $\varepsilon_{m,t}$ is an iid shock.[3]

2.7 Arbitrager

To close the international bond market, we introduce a risk-neutral arbitrager who trades domestic and foreign bonds to maximize his or her instantaneous profits. Based on the empirical facts shown in Figure 3, Japanese institutional investors almost exclusively hold long-term bonds. We thus assume that the arbitrager trades only domestic and foreign

[3] In a working paper version of this paper, an interest rate news shock in $t-12$ is introduced to capture the effects stemming from the existence of the zero lower bound, as proposed by Bodenstein et al. (2013), in addition to the temporary interest rate shock in period t. We cease to assume the news shock in the current version because the zero lower bound is not binding except for the short periods of time around the global financial crisis in our quantitative analysis and thus has only negligible effects on our quantitative results. Ignoring the zero lower bound may cause a bias in a quantitative analysis (i.e., Richter and Throckmorton, 2016), but we think that the benefit of explicitly modeling it in a non-linear model is not large enough for our analysis, given that the computational burden of solving a non-linear DSGE model is really high unless we use a small-scale model (e.g., Fernández-Villaverde et al., 2015) or a model without rational expectations (e.g., Guerrieri and Iacoviello, 2015).

long-term bonds and does not trade domestic or foreign short-term bonds. That is, the arbitrager's optimization problem is formulated as

$$\max_{B_t^L, B_t^{L*}} E_t \left[\beta_{A,t} \left\{ (R_{t+1}^{L*} + \phi_{t+1}) \frac{P_{t+1}^{L*}}{Q_{t+1}} B_t^{L*} + R_{t+1}^L P_{t+1}^L B_t^L \right\} - \left(\frac{P_t^{L*}}{Q_t} B_t^{L*} + P_t^L B_t^L \right) \right],$$

where P_t^{L*}, R_{t+1}^{L*}, and B_t^{L*} are the price of, the return on, and the amount of foreign long-term bonds held by the arbitrager, respectively, and $\beta_{A,t}$ is the discount factor for the arbitrager. Foreign long-term bonds, similar to domestic long-term bonds, take the form of perpetuities that pay a decaying coupon κ^s at $t+1+s$; thus, the relationship $R_t^{L*} = 1/P_t^{L*} + \kappa$ is satisfied. The arbitrager's profits are assumed to be distributed to households in lump-sum payments.

To ensure the existence of a steady state in the small open economy and add a wedge to fit the exchange fluctuations in the data, we assume that there exists a tiny time-varying risk premium, ϕ_t, which is given by the following rule:

$$\phi_t - \phi = -\Phi \left(B_t^{L*} - B^{L*} \right) + v_{q,t} - \chi \varepsilon_{m,t}^*. \tag{9}$$

Here, $v_{q,t}$ is a real exchange rate shock and assumed to follow the process

$$v_{q,t} = \rho_q v_{q,t-1} + \varepsilon_{q,t},$$

where $\varepsilon_{q,t}$ is an iid shock and $\varepsilon_{m,t}^*$ is a monetary policy shock in the foreign economy, as described in the next section. One interpretation of the time-varying risk premium ϕ_t is the negative transaction cost involved in trading foreign bonds, given that the (domestic) arbitrager earns $R_{t+1}^{L*} + \phi_{t+1}$ on foreign bonds, while foreign investors earn R_{t+1}^{L*}. Hence, the above rule for ϕ_t in (9) means that the transaction cost for foreign bonds increases as net foreign assets held by the arbitrager increase, thus decreasing the return of foreign bonds and pushing back the amount of foreign assets to their steady-state value. Without this risk premium, there would not exist a steady state for foreign assets.[4] Furthermore, the risk premium on foreign bonds ϕ_t is time-varying and faces stochastic shocks, thus helping the model account for deviations from the UIP in a quantitative analysis. In particular, the risk premium is assumed to be negatively correlated with the monetary policy shock in the foreign economy, $\varepsilon_{m,t}^*$, as described in (9). As the negative correlation with the foreign monetary policy shock implies that the transaction cost for foreign bonds is high when their

[4] See Schmitt-Grohé and Uribe (2003) for more details on ways to close a small open economy model, including the assumption of the existence of a risk premium on the foreign bonds assumed here.

return is high, it helps the model, by construction, account for the systematic deviations from the UIP for short-term interest rate differentials in data.[5] Finally, in the steady state, ϕ_t offsets the effects stemming from the difference in the steady-state level of productivity growth across countries, which helps the real exchange rate remain stationary.

By combining the arbitrager's first order conditions with respect to B_t^L and B_t^{L*}, and deleting the discount factor $\beta_{A,t}$, the following UIP condition *with respect to the long-term interest rate differentials* is derived:

$$E_t \left[\frac{P_{t+1}^{L*}}{P_t^{L*}} \left(R_{t+1}^{L*} + \phi_{t+1} \right) \frac{Q_t}{Q_{t+1}} \right] = E_t \left[\frac{P_{t+1}^{L}}{P_t^{L}} R_{t+1}^{L} \right]. \tag{10}$$

This UIP condition, with respect to long-term interest rates, shows that the expected change in the exchange rate is determined by the difference in the one-period expected returns between domestic long-term bonds and foreign long-term bonds. Note that the same UIP condition is not necessarily satisfied for short-term interest rate differentials owing to the existence of term premiums. Hence, under limited asset market participation with term premiums, the UIP for short-term interest rates may behave differently from the UIP for long-term interest rates. This issue is discussed in more detail in the quantitative analysis in section 3.3.

2.8 Market Clearing

To close the model, the market clearing conditions for the domestic and imported goods markets need to be satisfied. The market clearing condition for domestic goods is

$$P_{d,t}Y_{d,t} = P_{d,t}C_{d,t} + I_d + P_{d,t}Y_{x,t},$$

where I_d is the nominal corporate investment. This market clearing condition states that domestic goods are allocated to consumption, investment, or export. As capital accumulation is not explicitly modeled here, nominal corporate investment is assumed to be constant. The market clearing condition for imported goods, in contrast, is formulated as

$$Y_{f,t} = C_{f,t} + Z_t,$$

[5] This type of wedge to account for the systematic deviations from the UIP is common in the literature. See, for instance, Adolfson et al. (2007, 2008) for more details on how to interpret the wedge.

which states that imported goods are used for consumption or as intermediate input.

In addition to these two market clearing conditions being satisfied, the capital market for foreign assets needs to be balanced. This condition is given by the following current account condition:

$$P_t^{L*} B_t^{L*} - \left(R_t^{L*} + \phi_t\right) P_t^{L*} B_{t-1}^{L*} = P_{x,t}^{*} Y_{x,t} - P_t^{*} Y_{f,t}.$$

This current account condition is derived by aggregating households' budget constraints and the arbitrager's profits, and by assuming a zero net supply of domestic short- and long-term bonds.[6] The left-hand side of this condition represents the income balance plus net increases in foreign assets, whereas the right-hand side represents the trade balance. Note that in this model, the income balance is determined only by long-term bonds.B_t^{L*}, reflecting the assumption that households cannot access foreign bonds and the arbitrager trades only long-term bonds.

2.9 Foreign Economy

The foreign economy is given by a small-scale New Keynesian model. In the spirit of small open economy models, the foreign economy is assumed not to be influenced by economic activity in the home economy, while it influences the home economy through imports/exports and the exchange rate.

The foreign economy's output gap y_t^* and inflation rate π_t^* are described by the following IS curve,

$$\Lambda_t^* = \beta E_t \left[\Lambda_{t+1}^* \frac{R_t^* e^{\gamma_{t+1}^*}}{\pi_{t+1}^*} \right],$$

and the New Keynesian Phillips curve,

$$\frac{\pi_t^*}{\pi^*} = \beta E_t \left(\frac{\pi_{t+1}^*}{\pi^*} \right) \left(y_t^* \right)^{\varpi^*} v_{p,t}^*,$$

where ϖ^* is the parameter for the elasticity of inflation to the output gap. The marginal utility of consumption, Λ_t^*, is defined by

$$\Lambda_t^* \equiv \frac{1}{y_t^* - \varkappa^* y_{t-1}^* e^{\gamma_t^*}} - \frac{\beta \varkappa^*}{y_{t+1}^* e^{-\gamma_{t+1}^*} - \varkappa^* y_t^*},$$

---

[6]Note that the zero net supply of domestic bonds is assumed here because the steady-state level of domestic bonds did not influence the dynamics of the model.

where $\varkappa^*$ is the parameter for habit formation. Here γ_t^* and $v_{p,t}^*$ are shocks to productivity growth and markups, respectively, and follow the process,

$$\log\left(\frac{\gamma_t^*}{\gamma^*}\right) = \rho_\gamma^* \log\left(\frac{\gamma_{t-1}^*}{\gamma^*}\right) + \varepsilon_{\gamma,t}^*,$$
$$\log\left(v_{p,t}^*\right) = \rho_p^* \log\left(v_{p,t-1}^*\right) + \varepsilon_{p,t}^*.$$

Finally, nominal interest rates are determined by the central bank following the Taylor-type policy rule as in the home economy,

$$R_t^* = \left(R_{t-1}^*\right)^{\rho_R^*} \left[R^* \left(\frac{\left[\Pi_{j=1}^4 \pi_{t-j+1}^*\right]^{1/4}}{\bar{\pi}_t^*}\right)^{1+\phi_\pi^*} \left(\frac{\left[\Pi_{j=1}^4 g_{t-j+1}^{y*}\right]^{1/4}}{e^{\gamma*}}\right)^{\phi_y^*}\right]^{1-\rho_R^*} v_{m,t}^*,$$

where $g_t^{y*} = y_t^*/y_{t-1}^*$ and $v_{m,t}^*$ is a short-term interest rate shock following the process,

$$\log\left(v_{m,t}^*\right) = \rho_m^* \log\left(v_{m,t-1}^*\right) + \varepsilon_{m,t}^*,$$

where $\varepsilon_{m,t}^*$ is an iid shock. Note that the monetary policy shock $\varepsilon_{m,t}^*$ also influences the risk premium of foreign bonds for the arbitrager in the home country, ϕ_t, as described in equation (9). Finally, as in the home economy, the target inflation rate is time varying and follows the process

$$\log\left(\frac{\bar{\pi}_t^*}{\bar{\pi}^*}\right) = \rho_{tp}^* \log\left(\frac{\bar{\pi}_{t-1}^*}{\bar{\pi}^*}\right) + \varepsilon_{tp,t}^*.$$

where $\varepsilon_{tp,t}^*$ is an iid shock.

The households in the foreign economy also hold foreign long-term bonds (B_t^{L*}), which are formulated as perpetuities that pay a decaying coupon κ^s at $t+1+s$. As the long-term bonds entail a transaction cost ζ_t^* per-unit, term premiums exist as in the home economy.[7] The prices and returns for the long-term bonds are denoted by P_t^{L*} and $R_t^{L*} = 1/P_t^{L*} + \kappa$, respectively. As foreign households trade only foreign short- and long-term bonds without paying a risk premium, ϕ, for foreign long-term bonds unlike the arbitrager, the hypothetical

[7] In contrast to the home country economy, term premiums do not have any effects on output or inflation in the foreign economy because we do not assume limited participation for the households in the foreign economy. We implicitly assume that foreign households do not hold short- and long-term bonds issued in the home country economy. While this assumption is a necessary condition to have term premiums influence the home country economy via the exchange rate, it is consistent with the fact that most Japanese bonds are held by domestic investors.

price of foreign long-term bonds *for the arbitrager*, $P_{A,t}^{L*}$, is different from that for the foreign households, and their relationship is described as

$$\frac{1}{P_{A,t}^{L*}} = \frac{1}{P_t^{L*}} + \phi_t.$$

That is, to ensure the existence of steady state, the hypothetical price of foreign bonds for the arbitrager is assumed to increase (decrease) relative to that for the foreign households if the net foreign assets held by the arbitrager increase (decrease).

3 Quantitative Analysis

In this section, we assume that the home and foreign countries in the model correspond to Japan and the U.S., respectively, and conduct a quantitative analysis to investigate the inflation rate in Japan and the yen–dollar exchange rate dynamics.[8] First, the model parameters are set by calibration or Bayesian estimation using Japanese and U.S. data. Then, the role of term premiums is quantitatively examined by conducting an impulse response analysis and examining whether our model replicates the observation that the UIP holds better for long-term interest rates relative to short-term interest rates.

3.1 Estimation

Before estimating the model parameters, some parameter values are calibrated such that the moment conditions are consistent with the Japanese data or simply set to conventional values. The quarterly discount factor β is set to $1.005^{-\frac{1}{4}}$ for both countries, and the decay rate for coupons of long-term bonds, κ, is set to satisfy $1/(1-\kappa) = 40$, which means that the maturity of long-term bonds is ten years. The steady-state growth rates of productivity, $\gamma = 1.01^{\frac{1}{4}}$ and $\gamma^* = 1.02^{\frac{1}{4}}$, are set to replicate the past performance of the Japanese and U.S. economies. The target inflation rates at the steady state, $\bar{\pi}$ and $\bar{\pi}^*$, are set to 1 percent for Japan and 2 percent for the U.S. based on the past policy conduct of the central banks of the two countries.[9] The sensitivity of the risk premium to the amount of foreign bonds

[8] As the Japanese economy is modeled as a small open economy, an implicit assumption here is that Japanese economy is influenced by the U.S. economy via the exchange rate and other factors, whereas the U.S. economy is not affected by the Japanese economy.

[9] In fact, the BOJ raised its target inflation rate to 2 percent in January 2013, when it formally adopted inflation target policy framework. In the quantitative analysis below, this effect should be captured by the

Table 1: Calibration

Parameter	Value or target
Discount factor, β and β^*	0.998
Decaying coupon, κ	$1-\frac{1}{40}$
Productivity growth (Japan γ and the U.S. γ^*)	$1.01^{\frac{1}{4}}$ and $1.02^{\frac{1}{4}}$
Target inflation rate (Japan $\bar{\pi}$ and the U.S. $\bar{\pi}^*$)	$1.01^{\frac{1}{4}}$ and $1.02^{\frac{1}{4}}$
Sensitivity of risk premium, Φ	0.001
Elasticity between domestic and foreign goods, η	1.076
Calvo parameter, ξ_w, ξ_f, and ξ_x,	0.8
Wage markup, λ_w	1.2
Wage indexation, ι_w	0.5
Share of imported consumption goods, δ	$P_{f,t}C_{f,t}/\left(P_tC_t\right)=0.063$
Share of imported intermediate goods, α	$P_{f,t}Z_t/\left(P_{d,t}Y_{d,t}\right)=0.1$
Nominal investment, I_d	$I_d/\left(P_{d,t}Y_{d,t}\right)=0.15$

held by the arbitrager is set to an arbitrary small value, $\Phi = 0.001$, to minimize its effects on economic dynamics. For the elasticity between domestic and imported consumption goods, η, we use the value from Bodenstein et al. (2013), as no estimates for Japanese households are available. The Calvo parameters, ξ, are fixed at 0.8, except for domestic consumption goods, and the markup and indexation rate for wages, λ_w and ι_w, are set to 1.2 and 0.5, respectively, both of which are conventional values. Finally, (i) the fraction of imported goods in the consumption basket, δ, (ii) the share of imported intermediate goods in the production function, α, and (iii) the nominal investment, I_d, are chosen using the following moments as calibration targets: (i)$P_{f,t}C_{f,t}/\left(P_tC_t\right)=0.063$, (ii) $P_{f,t}Z_t/\left(P_{d,t}Y_{d,t}\right)=0.1$, and (iii) $I_d/\left(P_{d,t}Y_{d,t}\right)=0.15$, which are taken from the data of Japan. Table 1 summarizes the calibration values and the targets.

The remaining parameters are estimated using the Bayesian approach. The model has 23 parameters for the domestic economy,

$$\left(\varkappa,\nu,\theta,\zeta,\chi,\lambda_d,\lambda_f,\lambda_x,\xi_d,\iota_d,\iota_f,\iota_x,\phi_\pi,\phi_y,\rho_{tp},\rho_m,\rho_R,\rho_\gamma,\rho_d,\rho_f,\rho_x,\rho_q,\rho_\zeta\right),$$

target inflation rate shock $\varepsilon_{tp,t}$.

and 11 parameters for the foreign economy,

$$\left(\varkappa^*, \varpi^*, \phi_\pi^*, \phi_y^*, \zeta^*, \rho_{tp}^*, \rho_m^*, \rho_R^*, \rho_\gamma^*, \rho_p^*, \rho_\zeta^*\right).$$

Those 34 parameters as well as the variance of the following 13 structural shocks,

$$\left(\varepsilon_\gamma, \varepsilon_d, \varepsilon_f, \varepsilon_x, \varepsilon_q, \varepsilon_\zeta, \varepsilon_{tp}, \varepsilon_m, \varepsilon_\gamma^*, \varepsilon_p^*, \varepsilon_\zeta^*, \varepsilon_{tp}^*, \varepsilon_m^*\right),$$

are estimated using the following 11 data sequences in Japan and the U.S.: Japanese GDP growth (*dGDP*), Japanese core CPI inflation (*dCPIXFV*), Japanese call rate (*CALL*), ten-year Japanese government bond yield (*Y40*), Japanese import price index inflation (*dIPI*), percent changes in the yen–dollar exchange rate (*dFXN*), Japanese net export-GDP ratio (*rTB*), U.S. GDP growth (*dUSGDP*), U.S. core CPI inflation (*dUSCPIXFV*), federal funds rate (*FF*), and ten-year US government bond yield (*FR*). Note that we use only 11 data sequences to identify 13 structural shocks. Among the 13 structural shocks, the trend inflation shock in Japan and the U.S., that is, ε_{tp} and ε_{tp}^*, are identified by decomposing the inflation rate into a persistent and transitory component using a tight prior distribution for their persistence and variance, as explained below. The measurement equations for each data sequence are formulated as:

$$\begin{aligned}
dGDP &= \frac{gdp_t}{gdp_{t-1}} e^{\gamma_t} \times 100 - 100 \\
dCPIXFV &= \pi_t \times 100 - 100 \\
CALL &= (R_t - 1) \times 400 \\
Y40 &= \left(R_t^L - 1\right) \times 400 \\
dIPI &= \pi_{f,t} \times 100 - 100 \\
dFXN &= \frac{Q_t}{Q_{t-1}} \times 100 - 100 \\
rTB &= \frac{\frac{P_{x,t}^*}{P_t Q_t} Y_{x,t} - \frac{P_{f,t}}{P_t} Y_{f,t}}{gdp_t} \times 100 \\
dUSGDP &= \frac{y_t^*}{y_{t-1}^*} e^{\gamma_t^*} \times 100 - 100 \\
dUSCPIXFV &= \pi_t^* \times 100 - 100 \\
FF &= (R_t^* - 1) \times 400 \\
FR &= \left(R_t^{L*} - 1\right) \times 400
\end{aligned}$$

where $gdp_t \equiv C_t + I_d/P_t + P_{x,t}^* Y_{x,t}/(P_t Q_t) - P_t^* Y_{f,t}/(P_t Q_t)$. By simultaneously using the call rate (i.e., the policy rate in Japan) and ten-year bond rates in the measurement equations, the term structure of interest rates is expected to identify the shock to the short-term interest rate (ε_m) and the term premium ($\varepsilon_{\zeta,t}$). To exclude the observations that are influenced by the level shift in the yen–dollar exchange rate in the mid-1980s, we use the data from 1987Q1 to 2016Q3.

Tables 2, 3, and 4 show the prior distribution for estimated parameters and their posterior mean. There are some features to be emphasized. First, a very persistent process with small variance for the target inflation rate is assumed for its prior distribution. These tight prior distributions make it possible to identify the target inflation rate shocks and thus identify 13 structural shocks using only 11 data sequences. Second, it is assumed that the prior distribution for the wedge parameter to disturb the UIP for short-term interest rate differentials, χ, has a positive mean. As shown in Figure 1, the UIP for short-term interest rate differentials are obviously not satisfied, and the positive value of the wedge helps the model replicate this stylized fact. Third, the estimated variance for the imported price shock σ_f is large. As energy imports have a large share in Japanese imports, this large variance of import price shocks seems to reflect large fluctuations in oil prices.[10] Fourth, the Calvo parameter for domestic goods ξ_d is larger than the conventional values. This estimation value probably reflects the fact that Japanese inflation barely changed, even with the large fluctuation of imported prices driven by oil prices.

3.2 Impulse Response Analysis of Term Premiums

In this section, we quantitatively examine the effect of exogenous changes in term premiums on inflation and the exchange rate by conducting an impulse response analysis. Before conducting the quantitative exercise, the term premium in the model is defined as follows: The first-order condition for domestic households with respect to long-term bonds B_t^L yields

$$1 + \zeta_t = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t \pi_{t+1}} e^{-\gamma_{t+1}} \frac{P_{t+1}^L R_{t+1}^L}{P_t^L} \right], \tag{11}$$

where ζ_t is the transaction cost incurred when trading long-term bonds. Given that the transaction cost is the only source of the term premium in this model, fictitious long-term

[10]Nominal energy import has a 23 percent share over all nominal imported goods on average from 1987Q1 through 2016Q3. The standard deviation of inflation rates of imported price index for energy is 25 percent during the sample period.

Table 2: Parameter Values for Japan

parameter	posterior mean	prior dist.	prior mean	prior stdev
$\varkappa$	0.28	Beta	0.4	0.1
ν	0.98	Gamma	1.0	0.5
θ	1.1	Gamma	1.0	0.5
ζ	0.003	Normal	0.01	0.01
χ	2.15	Gamma	3.0	0.2
λ_d	2.21	Gamma	1.2	0.5
λ_f	3.37	Gamma	1.2	0.5
λ_x	2.01	Gamma	1.2	0.5
ξ_d	0.91	Beta	0.66	0.1
ι_d	0.14	Beta	0.5	0.2
ι_f	0.19	Beta	0.5	0.2
ι_x	0.50	Beta	0.5	0.2
ϕ_π	0.24	Gamma	0.5	0.25
ϕ_y	0.32	Gamma	0.5	0.15
ρ_{tp}	0.99	Beta	0.97	0.02
ρ_m	0.76	Beta	0.5	0.2
ρ_R	0.81	Beta	0.8	0.05
ρ_γ	0.81	Beta	0.5	0.2
ρ_d	0.33	Beta	0.5	0.2
ρ_f	0.92	Beta	0.5	0.2
ρ_x	0.95	Beta	0.5	0.2
ρ_q	0.89	Beta	0.5	0.2
ρ_ζ	0.94	Beta	0.5	0.2

Table 3: Parameter Values for the U.S.

parameter	posterior mean	prior dist.	prior mean	prior stdev
$\varkappa^*$	0.72	Beta	0.4	0.1
ϖ^*	0.01	Beta	0.1	0.05
ϕ_π^*	0.88	Gamma	0.5	0.2
ϕ_y^*	0.53	Gamma	0.5	0.2
ζ^*	0.002	Normal	0.01	0.01
ρ_{tp}^*	0.99	Beta	0.97	0.02
ρ_m^*	0.66	Beta	0.5	0.2
ρ_R^*	0.80	Beta	0.8	0.1
ρ_γ^*	0.77	Beta	0.5	0.2
ρ_p^*	0.19	Beta	0.5	0.2
ρ_ζ^*	0.96	Beta	0.5	0.2

Table 4: Parameter Values for Standard Deviation

parameter	posterior mean	prior dist.	prior mean	prior stdev
σ_γ	0.43	Inv. Gamma	0.5	inf.
σ_d	18.03	Inv. Gamma	0.5	inf.
σ_f	4.41	Inv. Gamma	0.5	inf.
σ_x	0.13	Inv. Gamma	0.5	inf.
σ_q	0.20	Inv. Gamma	0.5	inf.
σ_ζ	0.18	Inv. Gamma	0.25	inf.
σ_{tp}	0.03	Inv. Gamma	0.05	inf.
σ_m	0.06	Inv. Gamma	0.5	inf.
σ_γ^*	2.00	Inv. Gamma	0.5	inf.
σ_p^*	0.11	Inv. Gamma	0.5	inf.
σ_ζ^*	0.15	Inv. Gamma	0.25	inf.
σ_{tp}^*	0.02	Inv. Gamma	0.05	inf.
σ_m^*	0.10	Inv. Gamma	0.75	inf.

interest rates *without the term premium*, $\tilde{R}_t^L$, can be computed as follows:

$$1 = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t \pi_{t+1}} e^{-\gamma_{t+1}} \frac{\tilde{P}_{t+1}^L \tilde{R}_{t+1}^L}{\tilde{P}_t^L} \right],$$

where $\tilde{P}_t^L$ denotes the fictitious price of long-term bonds without the term premium and $\tilde{R}_t^L = 1/\tilde{P}_t^L + \kappa$. Then, the term premium is defined as $R_t^L - \tilde{R}_t^L$.[11] Given this definition of the term premium, we can compute the size of the term premium shock $\varepsilon_{\zeta,t}$, which results in a one percentage point change in the term premium upon impact and use it in the following impulse response analysis.

Figure 4 shows the response of the term premium, the exchange rate, and the inflation rate in Japan to an exogenous decline (increase) in the term premium by one percentage point in Japan (in the U.S.). The responses of term premiums (the top two panels in Figure 4) indicate that term premiums are fairly persistent both in Japan and the U.S. As shown later, the persistence of the responses of term premiums is partially accounted for by the long-lasting unconventional monetary policies after the global financial crisis in both countries. The relationship between the estimated term premiums and the BOJ's and the Fed's bond purchasing programs is investigated in more detail in section 4.

The middle two panels in Figure 4 show the responses of the yen–dollar exchange rate. We see that a one percentage point decrease (increase) in the term premium for ten-year Japanese (U.S.) government bonds leads to a depreciation of the yen–dollar exchange rate by around 10 percent upon impact. Reflecting the persistent process of term premiums, the responses of exchange rates are also fairly persistent. The economic intuition behind these responses is simple: A decline (an increase) in the term premium in Japan (in the U.S.) leads to a decline (an increase) in the long-term interest rate differentials, R_t^L/R_t^{L*}, and, thus, entails a decline in Q_t (i.e., a depreciation of the home country's currency) through the UIP condition in (10). While the size of the effects on the exchange rate seems large at first glance, it is consistent with the UIP condition with respect to the long-term interest rate. That is, an interest rate differential in ten-year rates by one percentage point implies that the return on domestic long-term bonds should be complemented by an appreciation of the exchange rate *by one percentage point every year for ten years*, implying that the exchange rate depreciates upon impact by approximately ten percent in response to a one percentage

[11]Note that the term premiums associated with the covariance term are ignored here by solving the model using log-linearization around the steady state.

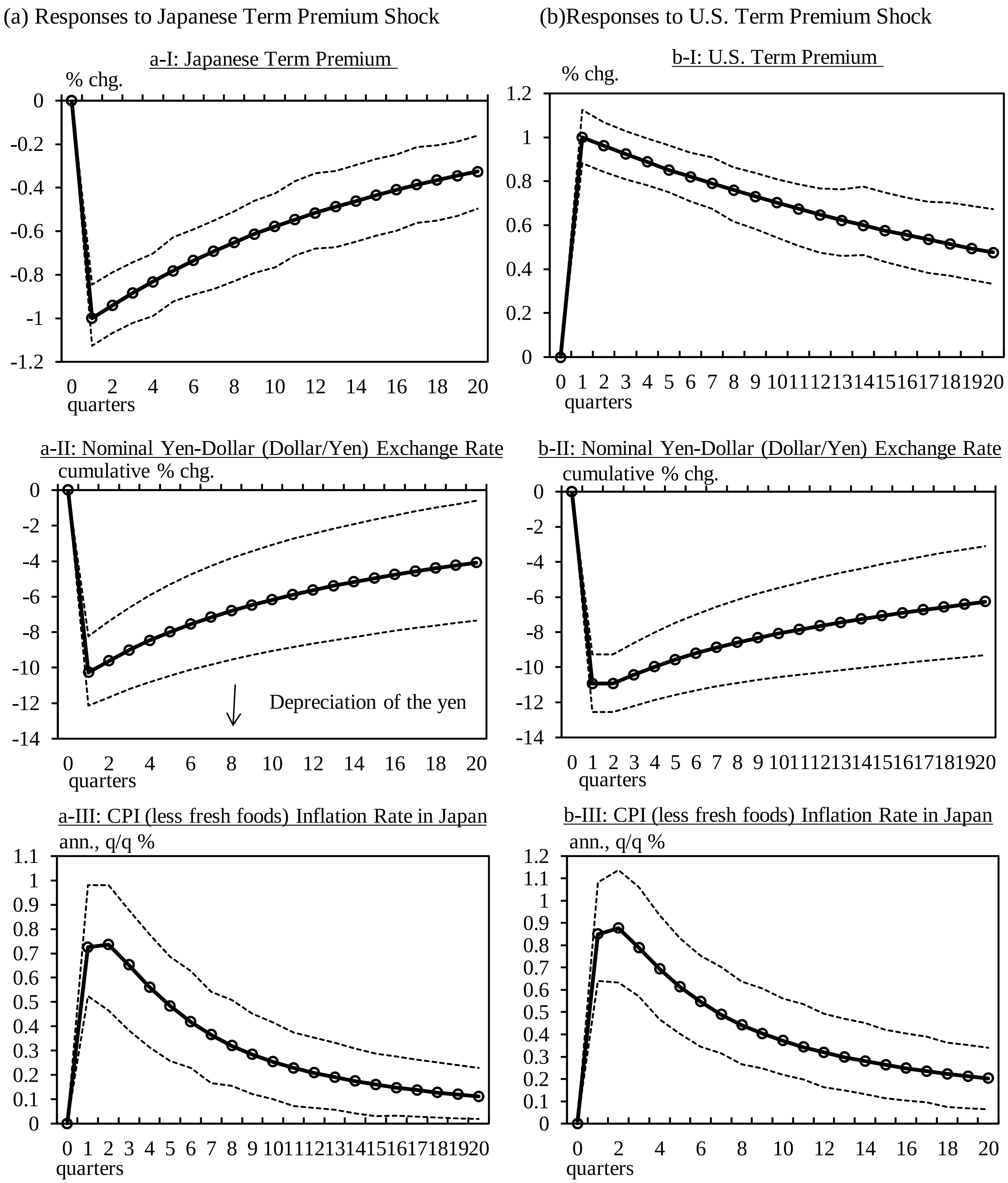
point decline (increase) in Japanese (the U.S.) term premiums.

The bottom two panels in Figure 4 show the response of inflation in Japan to a one percentage point decrease (increase) in the Japanese (U.S.) term premium. The panels indicate that (i) a one percentage point decline in term premiums in Japan, and (ii) a one-percentage-point increase in term premiums in the U.S. would push up inflation by around 0.7 and 0.8 percentage points in Japan, respectively. The upward pressure on inflation in response to changes in the term premium derives from the depreciation of the nominal exchange rate, Q_t, through the following two channels. First, the depreciation of the nominal exchange rate increases the marginal costs of imported goods firms, $MC_{f,t}$, and consequently leads to inflationary pressure on the price of imported consumption goods, $P_{f,t}$. The rise in imported goods prices directly raises the price of the consumption basket (i.e., the CPI) because part of the imported goods is consumed by households. As the rest of the imported goods are used as intermediate inputs (Z_t) by domestic firms, the increase in imported goods prices also indirectly leads to inflationary pressure on domestic goods prices, $P_{d,t}$, through the rise in marginal costs for domestic firms, $MC_{d,t}$. Second, as the depreciation in Q_t leads to a decline in marginal costs for exported goods firms, these firms will increase their exports, $Y_{x,t}$, by reducing export prices. Given that the increase in exports leads to tightening in the domestic consumption goods market, it pushes up domestic consumption goods prices, $P_{d,t}$. As the term premium in this model affects the inflation rate only through changes in the exchange rate, the small response of the inflation rate in the bottom two panels relative to the large response in the exchange rate in the middle two panels indicates that the pass-through rate to CPI in Japan is relatively small (around 6 percent upon impact).[12]

While the mechanism underlying the effect of term premiums on inflation and the exchange rate seems intuitive, the term premium does not have any effects on the exchange rate and, as a result, inflation, without the limited bond market participation assumptions for households and the arbitrager in our model. To see this, let us imagine an economy in which households can access foreign bonds. In such a hypothetical economy, the households' first-order condition with respect to foreign bonds provides the UIP condition with respect to short-term interest rate differentials,

[12]This small pass-through is consistent with empirical evidence for Japan. See Burstein and Gopinath (2014) for a survey of pass-through rates in major economies, including Japan.

Figure 4: Impulse Response Functions to Term Premium Shocks



Note: The figures on the left and right panel show the impulse response functions to a one percentage point decline in the domestic term premium and a one percent increase in the U.S. term premium, respectively. The dashed lines indicate 90% confidence intervals. A positive change in the yen–dollar exchange rate indicates an appreciation of the yen against the U.S. dollar.

$$1 = \beta R_t^* E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t \pi_{t+1}} e^{-\gamma_{t+1}} \frac{Q_t}{Q_{t+1}} \right], \tag{12}$$

instead of the UIP condition with respect to the long-term interest rate differentials in (10).[13] The first-order conditions with respect to domestic short- and long-term bonds provide the following two Euler equations for domestic households:

$$1 = \beta R_t E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t \pi_{t+1}} e^{-\gamma_{t+1}} \right], \tag{13}$$

$$1 + \zeta_t = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t \pi_{t+1}} e^{-\gamma_{t+1}} \frac{P_{t+1}^L R_{t+1}^L}{P_t^L} \right], \tag{14}$$

Suppose that the term premium drops because of a negative shock to the transaction cost, ζ_t. Equation (14) implies that the prices of long-term bonds P_t^L will rise (i.e., long-term interest rates R_t^L will decline) in response to the decline in ζ_t. However, equation (13) indicates that the stochastic discount factor $\beta\left[\Lambda_{t+1}/(\Lambda_t \pi_{t+1}) e^{-\gamma_{t+1}}\right]$ does not respond to the change in ζ_t, and equation (12) implies that without any changes in the stochastic discount factor, the nominal exchange rate Q_t does not respond at all to the decline in ζ_t.[14]

In other words, without the limited bond market participation assumptions for households or the arbitrager, the change in the term premium caused by a shock to ζ_t would just be a "sideshow" for the exchange rate dynamics. This can be understood as being in line with the finding of Chen et al. (2012). As they point out, changes in term premiums do not have any effect on the economy unless there is an agent who only has access to long-term bonds and whose actions are relevant to the economy. In our model, the domestic arbitrager, who trades only domestic and foreign long-term bonds, plays this role. The arbitrager's actions are relevant to the economy because the limited bond market participation assumption for households makes only the arbitrager's actions relevant to the exchange rate.[15]

[13] Note that the arbitrager's first-order condition cannot coexist with the UIP condition with respect to short-term interest rate differentials and has no role to determine the exchange rate in the economy without the limited bond market participation assumptions.

[14] Similarly, if the arbitrager had access to short-term bonds in addition to long-term bonds, changes in term premiums have no effect on the exchange rate.

[15] In Chen et al. (2012), a part of the households that does not have access to short-term bonds and thus borrows and lends only through long-term bonds takes this role. As argued above, without this limited participation assumption for a fraction of households, term premiums would not have any effect on inflation and the real economy in their model.

While the limited participation assumption is a key mechanism to make the term premium shock relevant to the economy, this assumption has little effect on the transmission mechanism of other shocks. Hence, the impulse responses to shocks other than the term premium shock, including the traditional monetary policy shock, are almost identical between the economy with and without the limited participation assumption. This is because the UIP condition for *short-term* interest rates can be derived by combining the UIP condition for *long-term* interest rates with the term structure of interest rates. That is, while the UIP condition for short-term interest rates is not satisfied in the face of the term premium shock, this fact does not change the transmission mechanism of other shocks. This result is also important when considering the difference between the empirical UIP relationship for short- and long-term interest rates in the next subsection.

3.3 Uncovered Interest Parity over Different Horizons

We now examine whether our model with limited participation assumptions is consistent with the observation that the UIP holds better for interest rate differentials over a longer horizon. This is an important step for assessing the validity of the effects of term premiums on exchange rates in the previous subsection, given that these effects rely entirely on the limited participation assumptions. For this purpose, we randomly generate a series of structural shocks and calculate the paths of endogenous variables based on the model dynamics for 300 quarters. Using the simulated data, we calculate the short- and long-term interest rate differentials as well as changes in nominal exchange rates, and regress future changes in yen–dollar exchange rates on short- and long-term interest rate differentials (i.e., the UIP regression).

Table 5 shows the estimated slope of the UIP regression using the actual data and the simulated data under various settings. In the table, "Short-term UIP" and "Long-term UIP" represent the slope of the UIP regression for short- and long-term interest rate differentials, respectively, and the 95 percent confidence interval in the bracket is computed by 1,000 simulations. To highlight the role of the limited participation assumption and the term premium shock, we run the UIP regression for the model with and without the limited participation assumption and the term premium shock, that is, we examine the four cases in total as shown in Table 5.

Table 5 indicates that our baseline model (i.e., the one with the limited participation

Table 5: UIP Regressions for Short- and Long-term Interest Rate

		With term premiums		No term premiums	
	Data	with LP	w/o LP	with LP	w/o LP
Short-term UIP	-1.65	-1.25	0.91	-1.32	0.91
	[-3.04,-0.45]	[-2.81,-0.18]	[0.52,1.38]	[-2.83,-0.20]	[0.52,1.36]
Long-term UIP	1.21	0.72	0.39	0.47	0.87
	[0.62,1.54]	[-0.02,1.47]	[-0.09,1.00]	[-0.57,1.26]	[0.21,1.51]

Note: The table shows the slope of the UIP regression with its 95% confidence interval in brackets. The confidence interval was computed by 1,000 simulations. "Short-term UIP" and "Long-term UIP" represent the relationship of short- and long-term interest rate differentials with future changes in foreign exchange rates for corresponding periods. The columns of "with LP" are the cases where the household and the arbitrager face the limited asset market participation assumption, while the columns of "w/o LP" 'are the cases where they can access all asset markets as in a standard model.

assumption and term premiums) can account for the observation that the UIP holds better for long-term interest rate differentials as in data. Specifically, the second column in the table shows that, with the limited participation assumption, the slope for the short-term UIP regression is -1.25 with a 95% confidence interval of $[-2.81, -0.18]$, while the slope for the long-term UIP regression is 0.72 with a 95% confidence interval of $[-0.02, 1.47]$. Hence, in the baseline model, the theoretical prediction of the UIP (namely, the slope of the UIP regression is equal to one) is clearly rejected for short-term interest differentials, but not for long-term interest rate differentials, which is in line with the empirical observation in the first column and Figure 1.

Table 5 suggests that the limited participation assumption is key to understanding the contrasting results between the short- and long-term UIP regressions in our baseline model. The result for the model *without* the limited participation assumption (the third column in the table) indicates that the slope of the short-term UIP regression is very close to one ($= 0.91$), while that for the long-term UIP regression is flatter and significantly lower than one ($= 0.39$), which is not consistent with the data. The economic intuition behind these results can be summarized as follows: With the limited participation assumption, investors and households cannot make arbitrage transactions to make profits by using the deviations from the UIP with respect to short-term interest rate differentials. Therefore, the slope of the short-term UIP regression becomes negative because of the wedge $\chi > 0$ in (9). In contrast, the slope for the long-term UIP regression becomes positive and close to one because the arbitrager can take the arbitrage opportunity with respect to long-term interest rate differentials. On the other hand, without the limited participation assumption, households can make arbitrage transactions with respect to the short-term interest rate differentials, thus making the slope of the short-term UIP regression close to one even with the wedge $\chi > 0$ (third and fifth columns). In addition, the comparison between the third and fifth columns indicates that fluctuations in term premiums lead to a flatter slope of the long-term UIP regression ($0.87 \rightarrow 0.39$) in the model without the limited participation assumption. This result contrasts with the case *with* the limited participation assumption, in which term premiums reinforce the long-term UIP ($0.47 \rightarrow 0.72$) through the arbitrager's optimization, as shown by the comparison between the second and fourth columns.

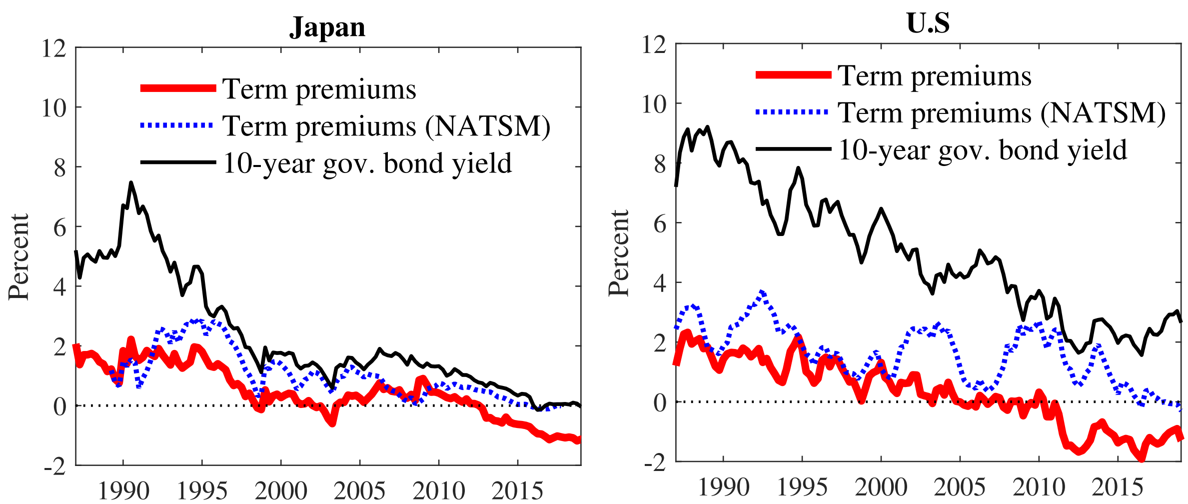
In summary, Table 5 implies that the relationship between the exchange rate and interest rate differentials over different horizons is consistent with the data only if the limited participation assumption is modeled together with the term premium shock as well as the wedge to the risk premium on foreign bonds. In other words, if either the limited participation assumption or the term premium shock is missing, the model cannot account for the relationship between the exchange rate and interest rate differentials over different horizons. Therefore, the quantitative result in this subsection can be interpreted as giving a certain validity to our assumption of limited participation in the bond market.

4 Policy Effects and Historical Decomposition

In the previous section, we showed that changes in term premiums have sizable effects on exchange rates and inflation under the limited participation assumption. Furthermore, we show that the limited participation assumption together with the term premium shock makes the model's behavior consistent with the relationship between the exchange rate and the interest rate differentials over different horizons, which gives a certain validity to our limited participation assumption.

Given the sizable effects of term premiums, a relevant policy question is the extent to which the BOJ's QQE and the Fed's LSAP account for changes in term premiums and, as a result, influence the exchange rate and inflation rate in Japan over the last decade. To answer this question, we first examine the term premiums estimated in the previous section,

Figure 5: Estimated Term Premiums



Note: The left and right panel show the estimation results for Japan and the U.S. The red bold line, dashed blue line, and black thin line show term premiums estimated by the model in this study, term premiums estimated by the no-arbitrage term structure model in Adrian et al. (2013) for the U.S. and Ichiue and Ueno (2013) for Japan, and the ten-year Japanese government bond yield, respectively.

particularly focusing on their relationship with the BOJ's and the Fed's bond purchasing programs. Then, we investigate the determinants of recent economic developments in Japan by decomposing changes in long-term bond yields, the yen–dollar exchange rate, and the inflation rate into the contribution of different structural shocks. The contribution of term premiums is further decomposed into the policy effects of bond purchasing programs and others by formulating the term premium as a function of the central bank's bond holdings, as in Chen et al. (2012), and then re-estimating the model parameters. Finally, we conduct a robustness check in which the correlation of term premiums between Japan and the U.S. is taken into account.

4.1 Quantitative Easing and Term Premium

The bold red lines in Figure 5 show the term premium for ten-year government bonds estimated by the model in the previous section. The left and right panels show the estimation results for Japan and the U.S., respectively. In Japan, term premiums had been more or less stable between 1% and 2% until the late-1990s, and then declined substantially in the late-

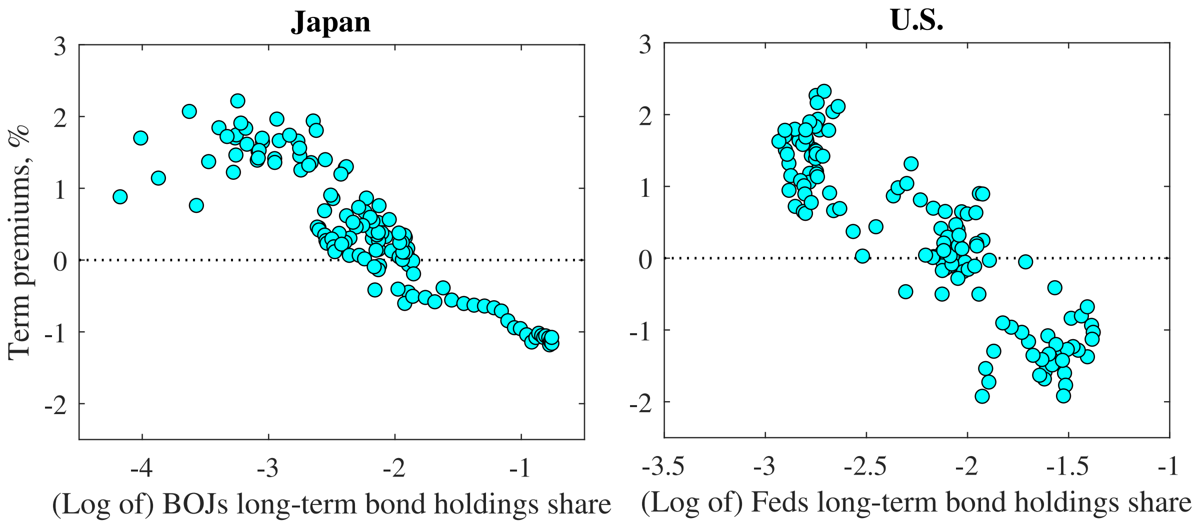
1990s and remained at around 0%. Then, at the onset of the BOJ's massive bond purchasing program under QQE in 2013, term premiums substantially declined again, and now they are around -1%. In the U.S., on the other hand, term premiums had been gradually declining until around 2011 from 2% to 0%, and then substantially declined in 2011 to the negative territory. While the ten-year government bond yields have moved quite differently in Japan and the U.S., the developments in term premiums have been surprisingly similar between the two countries, particularly after the central banks introduced bond purchasing programs.

To examine the relationship between term premiums and the central bank's bond purchasing program in more detail, we draw a scatter plot in Figure 6 between the BOJ's or Fed's long-term government bond holdings relative to the total outstanding stock (log scale, x-axis) and term premiums estimated in the model (y-axis). The left and right panels show the results for Japan and the U.S., respectively. These two variables are clearly and negatively correlated in both Japan and the U.S. not only during the periods under the bond purchasing programs, but also during the periods before the BOJ or the Fed started them, *even though we do not use any data on the central bank's bond holdings for estimation at this point*. This suggests that the estimated term premiums in Japan and the U.S. are strongly influenced by changes in the central bank's long-term bond holdings through demand and supply balance in the bond market, and that the decline in term premiums after the global financial crisis can be mostly accounted for by the policy effects of bond purchasing programs.

Finally, for comparison, Figure 5 shows the term premium estimated using a no-arbitrage term structure model (NATSM, dashed blue lines).[16] While these estimates and ours are basically in line with each other in both countries, the estimates by NATSM are more volatile than our estimates before the bond purchasing programs and show less substantial declines at the start of the bond purchasing programs. In fact, the estimates by NATSM have a much weaker correlation with the central bank's bond holdings than ours. More specifically, while the adjusted R^2 for the linear regression of the scatter plot in Figure 6 is 0.80 and 0.76 in Japan and the U.S., the adjusted R^2 for the same linear regression using the estimates by NATSM is 0.30 and 0.33 in Japan and the U.S., respectively. Hence, while term premiums estimated by different methods reflect different risk factors, our estimates can be interpreted as reflecting the demand and supply balance driven by the central bank's bond purchases.

[16]The term premiums estimated by NATSM are taken from Adrian et al. (2013) at the NY Fed website for the U.S. and from Ichiue and Ueno (2013) for Japan. We thank Yoichi Ueno for kindly sharing the estimated series of term premiums.

Figure 6: Term Premiums and the Central Bank's Bond Holdings



Note: The left and right panel show the estimation results for Japan and the U.S. The x-axis shows (log of) the share of BOJ's or Fed's long-term government bond holdings to the total outstanding stock, while the y-axis shows term premiums estimated by the model. The sample periods are from 1987Q1 to 2019Q4.

4.2 Historical Decomposition since the Global Financial Crisis

Given that term premiums in Japan and the U.S. are strongly influenced by the BOJ's or the Fed's bond purchasing programs, as shown in the previous subsection, this subsection formulates term premiums in Japan and the U.S. as a function of bond holdings by the BOJ and the Fed, respectively, and re-estimates the model parameters. Then, through historical decomposition, we investigate the factors that drive the long-term bond yield and the yen–dollar exchange rate for the last decade and examine how those factors influence inflation in Japan through exchange rate dynamics, particularly focusing on the contribution of the BOJ's and the Fed's bond purchasing programs.

4.2.1 New Formulation for Term Premiums

To explicitly model the effects of the central bank's bond purchases on term premiums, the transaction cost for holding long-term bonds, ζ_t, is reformulated as a function of the BOJ's bond holdings relative to the total outstanding stock, $B_{cb,t}$, rather than being assumed to follow the exogenous process (2). Specifically,

$$\zeta_t - \zeta = \Psi \log\left(\frac{B_{cb,t}}{\bar{B}_{cb}}\right) + \upsilon_{\zeta,t}, \tag{15}$$

where $B_{cb,t}$ and $\upsilon_{\zeta,t}$ are assumed to follow the AR(1) process,

$$\log\left(\frac{B_{cb,t}}{\bar{B}_{cb}}\right) = \rho_B \log\left(\frac{B_{cb,t-1}}{\bar{B}_{cb}}\right) + \varepsilon_{B,t}$$

and

$$\upsilon_{\zeta,t} = \rho_\zeta \upsilon_{\zeta,t-1} + \varepsilon_{\zeta,t}.$$

Here, $\bar{B}_{cb}$ is the steady-state value of the BOJ's bond holdings. Analogously, the transaction cost for holding long-term bonds in the U.S., ζ_t^*, is also reformulated as a function of the Fed's bond holdings relative to the total outstanding stock, $B_{cb,t}^*$.

Under this new formulation of the transaction cost in (15), all the parameters are re-estimated using a Bayesian method by employing the time series data for the BOJ's and the Fed's bond holdings relative to the total outstanding stock as well as the 11 data sequences used in the previous section. We also set a prior means for Ψ (and Ψ^*) in (15) based on the coefficient for the linear regression of the scatter plot in Figure 6. The estimation results including prior distributions are provided in the appendix. Because the model structure is unchanged except for the transaction costs in Japan and the U.S., the estimation results are almost identical to the baseline estimation in Tables 2, 3, and 4.

4.2.2 Historical Decomposition

Given the estimation results based on the new formulation of the transaction cost in (15), we investigate the factors that drive the long-term bond yield and the yen–dollar exchange rate for the last decade, and then examine how those factors influence inflation in Japan through exchange rate dynamics. Specifically, we categorize the structural shocks into six types of factors, namely, the policy effects of bond purchase program ($\varepsilon_{B,t}$), the other domestic term premium shock (ϵ_ζ), the domestic trend inflation (ϵ_{tp} and initial values), the domestic short-term interest rate shocks (ϵ_m), the foreign factors ($\epsilon_x, \epsilon_f, \epsilon_q, \epsilon_m^*, \epsilon_\gamma^*, \epsilon_p^*, \epsilon_{tp}^*$, $\varepsilon_{B,t}^*$ and ϵ_ζ^*), and the other domestic factors (ϵ_γ and ϵ_d). Among the six types of shocks listed above, the focus is on the policy effects of bond purchasing programs, $\varepsilon_{B,t}$, which includes the policy effects of the BOJ's QQE starting in 2013.

Long-term Bond Yield

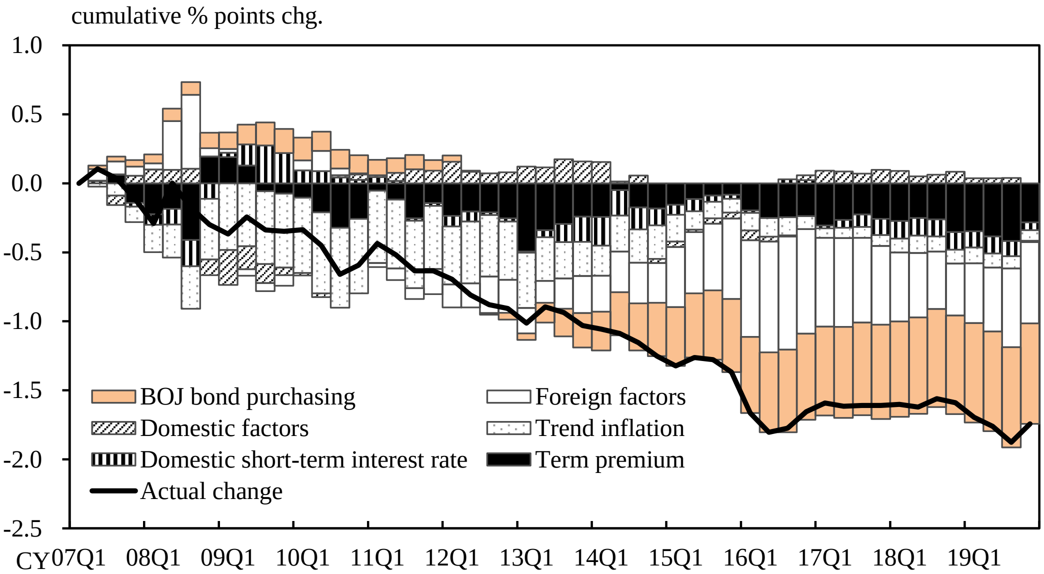
Figure 7(a) presents the decomposition of the cumulative change in Japanese ten-year bond yields from 2007 to 2019. Evidently, the decline in long-term bond yields since 2013 is mostly accounted for by the decline in term premiums entailed by the BOJ's bond purchasing program. Also, the foreign factors made a substantial contribution to the decline in Japanese long-term yields. In particular, as shown by the detailed decomposition of foreign factors in Figure 7(b), the Japanese long-term bond yields are faced with the negative pressure stemming from (i) the decline in the U.S. term premiums under the multiple LSAPs pursued by the Fed, and (ii) the decline in imported goods marginal costs mainly induced by oil prices. Those foreign factors placed deflationary pressure in Japan through the exchange rate dynamics, thus pulling down expected short-term rates and Japanese long-term interest rates from around 2011 onward.

Nominal Yen-Dollar Exchange Rate

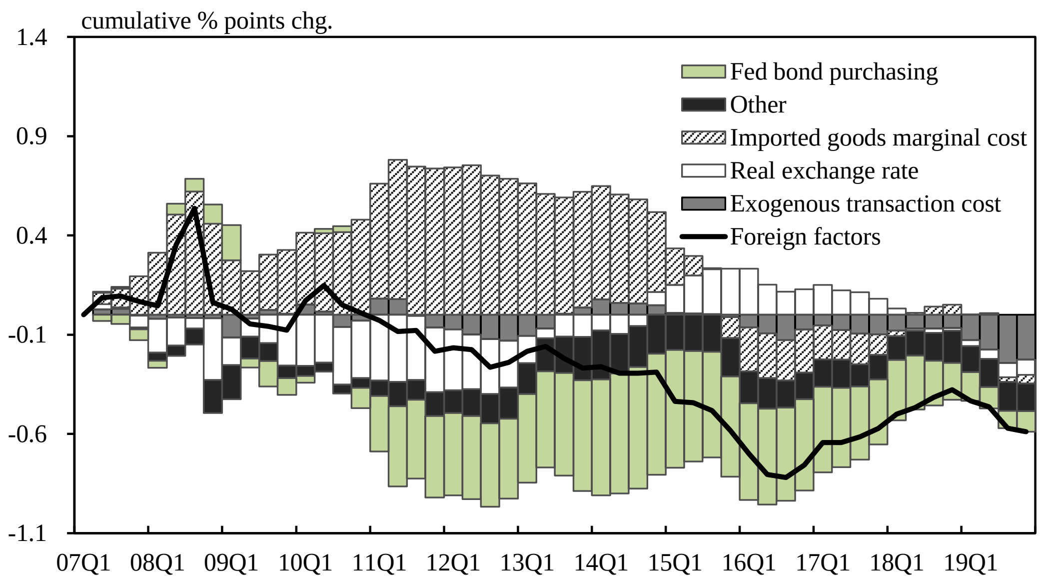
Figure 8(a) shows which factors explain cumulative changes in the yen–dollar exchange rate since 2007. The first notable feature is that the difference in trend inflation between the U.S. and Japan has exerted strong appreciation pressure on the yen–dollar exchange rate. Since the trend inflation during the sample periods in the U.S. and Japan is estimated around 2.0% and -0.5%, respectively, the nominal exchange rate has been appreciated by nearly 40% for the last 13 years in total solely because of the difference in trend inflation. Second, the decline in term premiums owing to the BOJ's bond purchases has made a sizable contribution to the depreciation of the yen, particularly since the introduction of BOJ's QQE in 2013. In addition, the monetary policy contributed to the depreciation in the yen–dollar exchange rate through the domestic short-term interest rate channel. This result seems a bit strange at first glance because the short-term interest rates have hardly moved since 2013 in Japan. However, the depreciation pressure caused by the short-term interest rate shocks can be interpreted as the effects of commitment policy. That is, since the BOJ has committed to keep interest rates low even in the face of moderate inflation, the model identifies such deviations from the monetary policy rule through the commitment policy as monetary policy shocks, which have had the depreciation pressure. The BOJ's monetary policy easing through the massive bond purchases under QQE and the low interest rate policy has contributed to the depreciation in the nominal exchange rate since 2013 by 27 percentage points in total,

Figure 7: Decomposition of Changes in the Japanese Ten-Year Government Bond Yields

(a) Japanese 10-year Government Bond Yields



(b) Details on the Contribution of Foreign Factors to Japanese 10-year Government Bond Yields



Note: "Foreign factors" include shocks to U.S. monetary policy, U.S. term premium, U.S. trend inflation, U.S. demand and supply, trade balance, imported good marginal cost, and real exchange rate. "Domestic factors" include domestic productivity shocks and price markup shocks. "Trend inflation" includes initial values, too. "Other foreign factors" in Figure 7(b) include shocks to U.S. monetary policy, U.S. trend inflation, U.S. demand and supply, and trade balance. Cumulative changes in the ten-year government bond yield were calculated from the first quarter of 2007.

thus mostly offsetting the appreciation pressure owing to the difference in trend inflation.

While the BOJ's monetary policy exerted a sizable depreciation pressure on the yen–dollar exchange rate, Figure 8 suggests that it has been substantially influenced by foreign factors as well.[17] To examine which foreign factors drove the yen–dollar exchange rate dynamics in more detail, Figure 8(b) provides a breakdown of the contribution of various foreign components. We see that the decline in the U.S. term premium has exerted substantial appreciation pressure on the yen–dollar exchange rate. The upward pressure increased particularly after the Fed started QE2 and the Maturity Extension Program starting in late late-2010 and 2011, and has cumulatively contributed to the appreciation of the yen by around 20 percentage points. Second, the deterioration of trade balance exerted downward pressure on the yen since 2012. This likely reflects the huge trade deficit caused by the increase in energy imports after the Great East Japan Earthquake. Finally, Figure 8(b) shows that a large share of the foreign factors is still unexplainable by the model and identified as a contribution of residuals, namely, the real exchange rate shock. In particular, the large depreciation from 2013 to 2015 is mostly unexplainable by the model and is attributed to the real exchange rate shock.

Inflation Rate

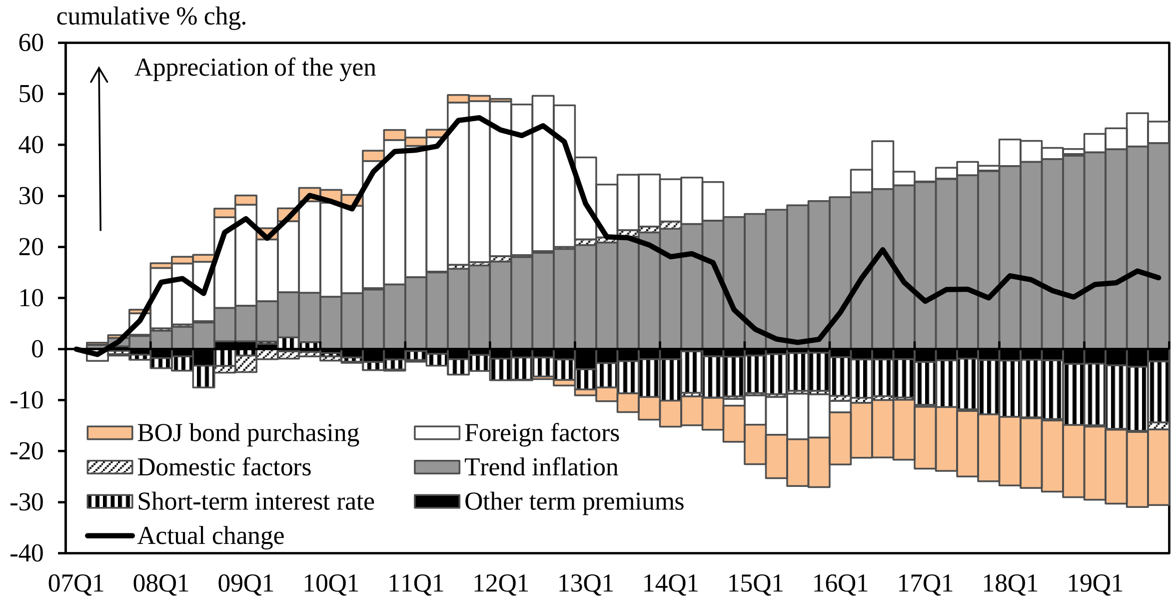
Consistent with the sizable effects of monetary policy on the yen–dollar exchange rate, the decomposition of CPI inflation in Figure 9(a) confirms that the monetary policy, as a form of both bond purchases and the low short-term interest rate, has helped increase inflation in Japan. Specifically, the positive contribution of the BOJ's bond purchasing program has increased since the onset of the BOJ's QQE in 2013 and raised CPI inflation by around 0.6% cumulatively. Moreover, the short-term interest rate shocks contributed to raising inflation around 2013 and 2018, by about 0.5 percentage points. As explained above, while short-term interest rates have remained almost unchanged during those periods, the positive contribution of short-term interest rate shocks can be interpreted as the effects of the BOJ's commitment policy.

While the accommodative monetary policy pursued by the BOJ has exerted upward pressure on inflation in Japan, foreign factors have placed considerable downward pressure

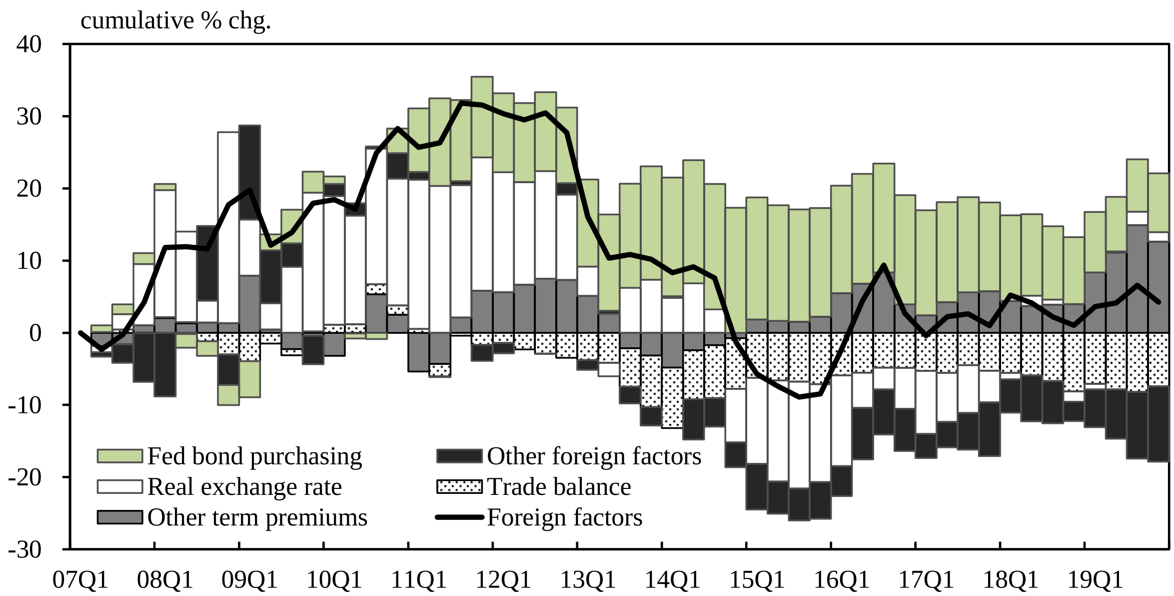
[17]The large effects of foreign factors on recent exchange dynamics are consistent with the findings of other empirical studies such as Kano and Wada (2017).

Figure 8: Decomposition of Changes in the Nominal Yen–Dollar Exchange Rate

(a) Nominal Yen-Dollar Exchange Rate



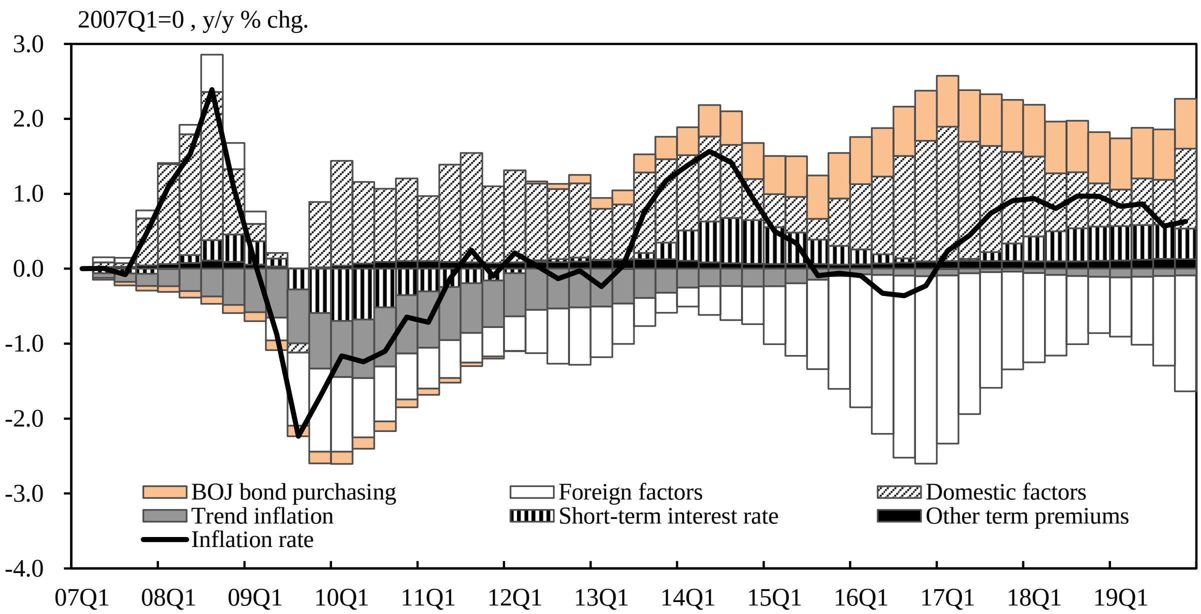
(b) Details on the Contribution of Foreign Factors to Nominal Yen-Dollar Exchange Rate



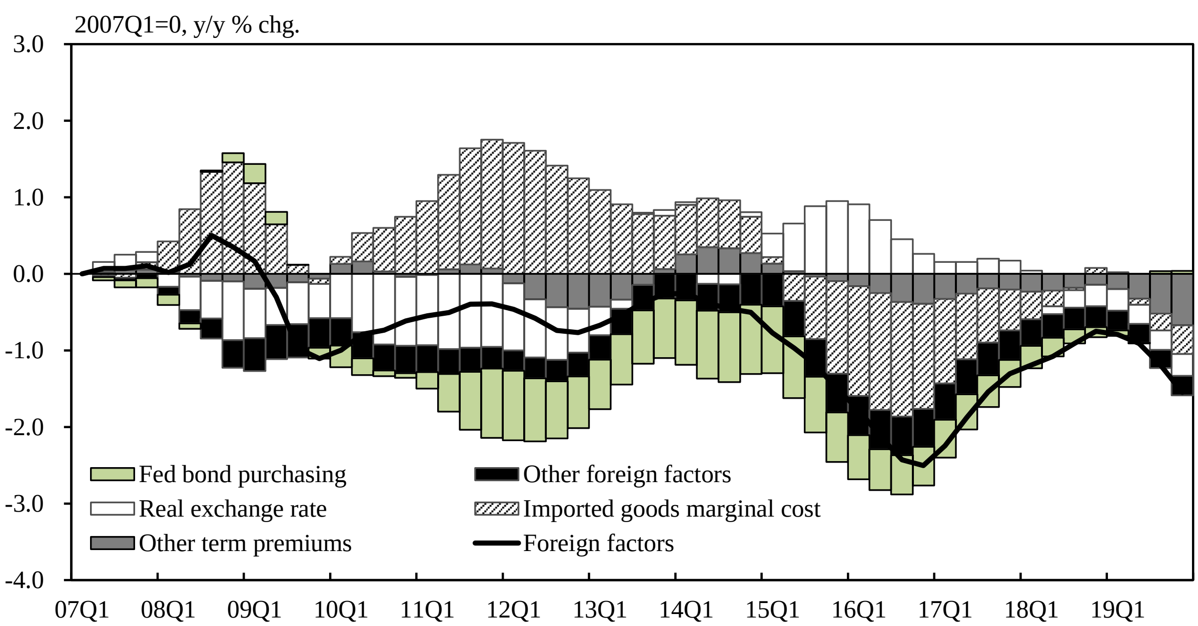
Note: See the note on Figure 7 for the definition of the factors in Figure 8(a). Note, however, that unlike in Figure 7, "Trend inflation" includes the contribution of U.S. trend inflation because the gap between trend inflation is relevant to the exchange rate. Accordingly, the "other foreign factors" in Figure 7(b) include shocks to U.S. monetary policy, U.S. demand and supply, and imported goods marginal cost. Cumulative changes in the ten-year government bond yield were calculated from the first quarter of 2007.

Figure 9: Decomposition of Changes in the Inflation Rate

(a) CPI (less fresh foods) Inflation Rate



(b) Details on the Contribution of Foreign Factors to the Inflation Rate



Note: See the note on Figure 7 for the definition of the factors in Figure 9(a). "Other foreign factors" in Figure 9(b) include shocks to U.S. monetary policy, U.S. trend inflation, U.S. demand and supply, and trade balance. The figures are adjusted to exclude the effects of the changes in the consumption tax rate.
Source: Ministry of Internal Affairs and Communications.

on inflation in Japan. Figure 9(b), which provides details on the contribution of the foreign factors, confirms that the decline in the U.S. term premium has exerted substantial downward pressure on inflation in Japan. That is, the decline in the U.S. term premiums owing to the Fed's multiple LSAPs has induced the appreciation of the yen–dollar exchange rate as explained above, thus exerting downward pressure on inflation in Japan cumulatively by around one percentage points at the peak 2013. Further, the cost of imported goods has generated large fluctuations of the inflation rate in Japan. Particularly, it exerted substantial downward pressure around 2016, in which the CPI inflation rate fell into a negative territory, reflecting a large decline in energy prices.[18]

In sum, we find that the combination of the BOJ's bond purchases under QQE and the low interest rate policy has significantly generated inflationary pressure from 2013 onward, cumulatively adding about one percentage point to the inflation rate. However, we also find that foreign factors play a crucial role in explaining developments in the Japanese inflation rate. In particular, the decrease in the U.S. term premium owing to the Fed's multiple LSAPs has exerted substantial downward pressure since 2011, and the changes in energy prices have generated large fluctuations.

4.3 Decomposition in the Long Run

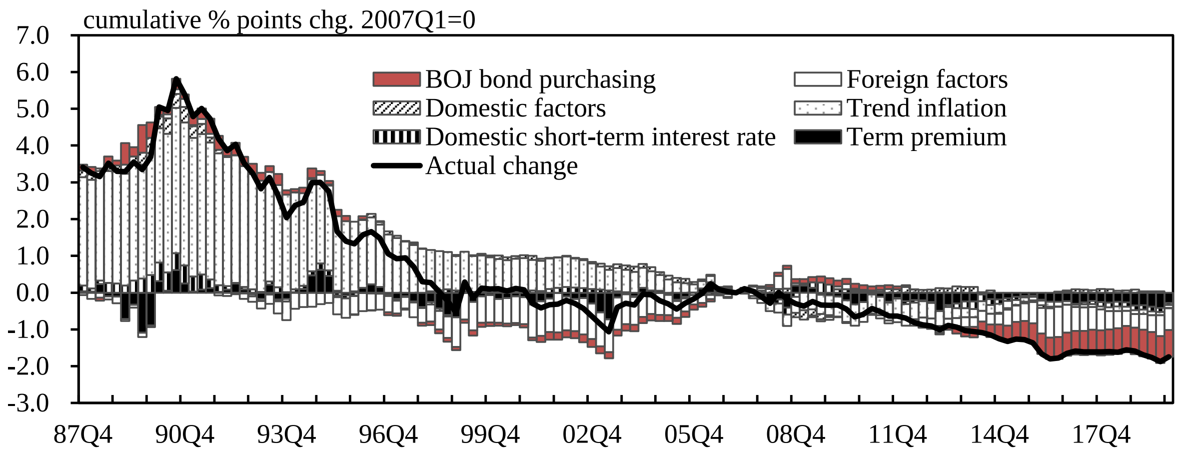
Figure 10 decomposes the cumulative change in Japanese ten-year bond yields, the yen–dollar exchange rate, and the inflation rate from 1987 to 2019 into the six factors listed above. Some comments are in order. First, Figure 10(a) indicates that, in the long run, the decline in trend inflation accounted for almost all of the substantial decline in Japanese long-term interest rates until the mid-2000s. This is in contrast with the result that the main driver of long-term interest rates for the last decade was the decline in term premiums as explained in the previous subsection.

Second, Figure 10(b) highlights that the yen–dollar exchange rate in Japan is highly susceptible to foreign shocks, which are completely exogenous to policymakers in Japan. Specifically, while the monetary easing in Japan has been putting depreciation pressure on the yen–dollar exchange rate recently, its main driver in the long run has been foreign factors, including the trade balance and U.S. term premiums. The difference in trend inflation

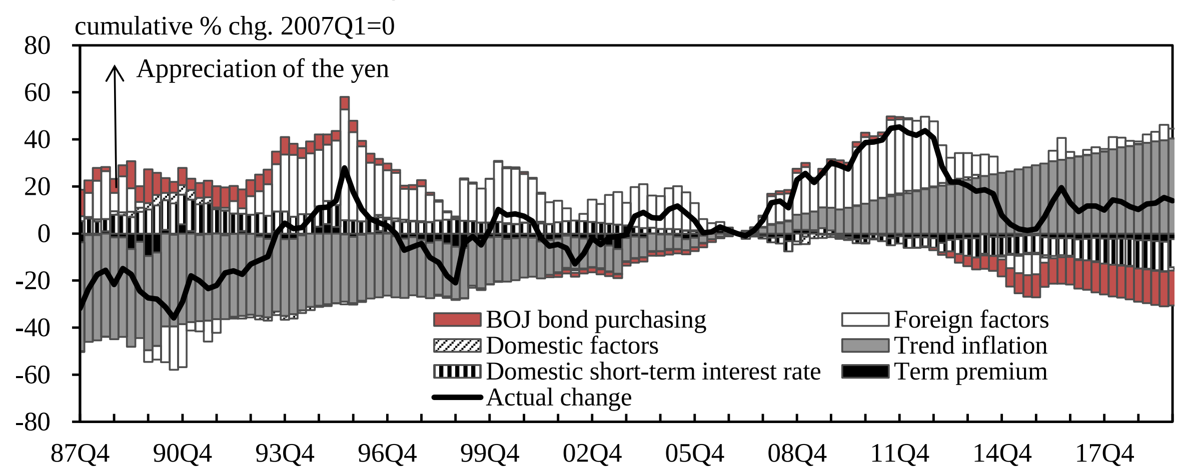
[18]The import price index for petroleum, coal, and natural gas, for instance, fell by 44 percent from December 2014 through September 2016.

Figure 10: Decomposition for Full Sample Period

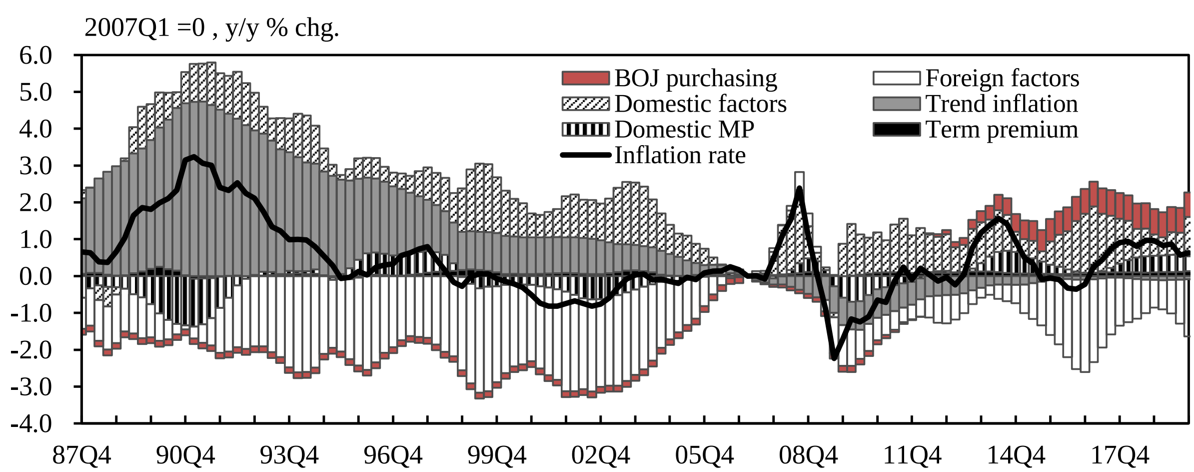
(a) Japanese 10-year Government Bond Yields



(b) Nominal Yen-Dollar Exchange Rate



(c) CPI (less fresh foods) Inflation Rate



between Japan and the U.S. has also created a long-run trend of appreciation in the Japanese yen against the U.S. dollar, making the yen–dollar rate appreciate by more than 70 percentage points since the early 1990s.

Finally, Figure 10(c) confirms that a significant decline in trend inflation pushed the Japanese economy into long-lasting deflationary periods in the early 1990s. While the model in this study assumes that trend inflation follows an exogenous process, exploring what drives the decline in trend inflation is an interesting economic issue. In addition, during the deflationary period from the late 1990s to the mid-2000s, the foreign factors continuously put downward pressure on inflation, while the BOJ's monetary policy put upward pressure on inflation. The main driver of the downward pressure from foreign factors during the deflationary period was the decline in import goods prices caused by weak oil prices. This implies that Japanese inflation rates have been significantly influenced by oil prices, which are exogenous to policymakers, throughout the sample period from the 1980s.

4.4 Robustness Check: Correlation of Term Premiums

In our specification of term premiums in both (2) and (15), term premiums in Japan and the U.S. are assumed to move independently. Nevertheless, some empirical studies, including Rogers et al. (2018), point to the significant correlation between term premiums across countries. In fact, our estimates of term premiums in Japan and the U.S. in Figure 5 show a significant correlation between them, even though they are estimated under the assumption that they move independently in the model.

Given that estimation under a misspecified shock structure may suffer from a bias, this subsection conducts a robustness check to quantitatively assess the size of bias owing to the misspecification of the shock structure. Specifically, while the quantitative analysis assumes that the bond purchasing shocks, $\varepsilon_{B,t}$ and $\varepsilon^*_{B,t}$, as well as the exogenous term premium shock, $\varepsilon_{\zeta,t}$ and $\varepsilon^*_{\zeta,t}$, in (15) are uncorrelated across the two countries, the robustness check in this subsection assumes that they follow a joint distribution, allowing the shocks to be correlated across countries, that is,

$$\begin{bmatrix} \varepsilon_{B,t} \\ \varepsilon^*_{B,t} \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_B & \sigma_{BB^*} \\ \sigma_{BB^*} & \sigma^*_B \end{bmatrix}\right) \quad \text{and} \quad \begin{bmatrix} \varepsilon_{\zeta,t} \\ \varepsilon^*_{\zeta,t} \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_\zeta & \sigma_{\zeta\zeta^*} \\ \sigma_{\zeta\zeta^*} & \sigma^*_\zeta \end{bmatrix}\right) \tag{16}$$

where σ_{BB^*} and $\sigma_{\zeta\zeta^*}$ are the covariances of $(\varepsilon_{B,t}, \varepsilon^*_{B,t})$ and $(\varepsilon_{\zeta,t}, \varepsilon^*_{\zeta,t})$, respectively. That is, $(B_{cb,t}, B^*_{cb,t})$ and $(v_{\zeta,t}, v^*_{\zeta,t})$ no longer follow independent AR(1) processes, but follow seemingly unrelated regression (SUR) processes.

Tables A4, A5, and A6 in the appendix show the estimation results under the new shock structure in (16). The tables show that the posterior mean of correlation between $\varepsilon_{B,t}$ and $\varepsilon^*_{B,t}$ and that between $\varepsilon_{\zeta,t}$ and $\varepsilon^*_{\zeta,t}$ are 0.13 and 0.26, respectively, while the prior means are set to zero. This estimation result suggests that those shocks, which drive term premiums in the model, should not be modeled by assuming independence across countries, but should be modeled by allowing correlation across countries.

Having said that, the tables indicate that the size of correlation between $\varepsilon_{B,t}$ and $\varepsilon^*_{B,t}$ is only around half of that between $\varepsilon_{\zeta,t}$ and $\varepsilon^*_{\zeta,t}$, and that it is not significantly different from zero. This implies that the correlation between term premiums across two countries is mainly attributed to the correlation of exogenous changes in term premiums rather than the correlation of bond purchasing policies.[19] Therefore, while the contribution of historical decomposition cannot be separately computed for correlated shocks in general, the quantitative contributions of bond purchasing programs by the BOJ and the Fed in Figures 7, 8, and 9 are not significantly influenced by the fact that term premiums are correlated across the two countries. Furthermore, the tables show that the estimated values of structural parameters under the new shock structure in (16) are not significantly different from the baseline in Tables 2, 3, and 4, which suggests that the misspecification of the shock structure probably does not lead to a serious bias in estimation.

5 Conclusion

This study explored the possibility that term premiums influence the economy via exchange rate dynamics, focusing in particular on the observation that the UIP holds better for longer-term interest rate differentials. In our small open economy DSGE model with the limited asset market participation assumption, changes in term premiums have sizable effects on the inflation rate via exchange rate dynamics, and a quantitative analysis using data for Japan and the U.S. shows that (i) the BOJ's monetary policy in the form of bond purchases under QQE and the commitment to low interest rates has contributed to the increase in inflation since 2013, and (ii) Japanese inflation rates have been substantially affected by foreign fac-

[19] While our model assumes an exogenous process with correlation, Kolasa and Wesolowski (2020) focus on the correlation between term premiums across countries and propose an endogenous mechanism to possibly explain it using a two-country DSGE model with limited asset market participation.

tors, especially by the energy prices as well as the decline in the U.S. term premiums owing to the Fed's bond purchases, through exchange rate dynamics.

For future research to deepen the study of term premiums, our analysis can be extended as follows: First, modeling the micro-foundations for the bond market limited participation for households and institutional investors can complement our study. Second, simultaneously incorporating other mechanisms to have nontrivial term premiums, such as the one in Chen et al. (2012), would help us quantitatively examine the relative importance of multiple mechanisms.

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Appendix

This appendix provides estimation results for different specifications. First, Tables A1, A2, and A3 show the estimation results under the specification in (15), in which the transaction cost is formulated as a function of bond purchasing. Second, Tables A4, A5, and A6 show the estimation results under the specification in (16), in which the shocks driving term premiums are allowed to be correlated across countries.

Table A1: Parameter Values for Japan for the Model with the BOJ and Fed's Bond Purchasing

parameter	posterior mean	prior dist.	prior mean	prior stdev
$\varkappa$	0.27	Beta	0.4	0.1
ν	1.1	Gamma	1.0	0.5
θ	1.2	Gamma	1.0	0.5
ζ	0.002	Normal	0.01	0.01
χ	2.16	Gamma	3.0	0.2
λ_d	2.30	Gamma	1.2	0.5
λ_f	3.39	Gamma	1.2	0.5
λ_x	1.74	Gamma	1.2	0.5
ξ_d	0.87	Beta	0.66	0.1
ι_d	0.15	Beta	0.5	0.2
ι_f	0.19	Beta	0.5	0.2
ι_x	0.50	Beta	0.5	0.2
ϕ_π	0.35	Gamma	0.5	0.25
ϕ_y	0.31	Gamma	0.5	0.15
ρ_{tp}	0.99	Beta	0.97	0.02
ρ_m	0.76	Beta	0.5	0.2
ρ_R	0.81	Beta	0.8	0.05
ρ_γ	0.87	Beta	0.5	0.2
ρ_d	0.40	Beta	0.5	0.2
ρ_f	0.91	Beta	0.5	0.2
ρ_x	0.96	Beta	0.5	0.2
ρ_q	0.87	Beta	0.5	0.2
ρ_ζ	0.70	Beta	0.5	0.2
Ψ	-0.0042	Normal	-0.0063	0.001
ρ_B	0.96	Beta	0.5	0.2

Table A2: Parameter Values for the U.S. for the Model with the BOJ and Fed's Bond Purchasing

parameter	posterior mean	prior dist.	prior mean	prior stdev
$\varkappa^*$	0.72	Beta	0.4	0.1
ϖ^*	0.01	Beta	0.1	0.05
ϕ_π^*	0.99	Gamma	0.5	0.2
ϕ_y^*	0.58	Gamma	0.5	0.2
ζ^*	0.001	Normal	0.01	0.01
ρ_{tp}^*	0.99	Beta	0.97	0.02
ρ_m^*	0.67	Beta	0.5	0.2
ρ_R^*	0.80	Beta	0.8	0.1
ρ_γ^*	0.78	Beta	0.5	0.2
ρ_p^*	0.18	Beta	0.5	0.2
ρ_ζ^*	0.90	Beta	0.5	0.2
Ψ^*	-0.008	Normal	-0.01	0.001
ρ_B^*	0.97	Beta	0.5	0.2

Table A3: Parameter Values for Standard Deviation for the Model with the BOJ and Fed's Bond Purchasing

parameter	posterior mean	prior dist.	prior mean	prior stdev
σ_γ	0.32	Inv. Gamma	0.5	inf.
σ_d	7.5	Inv. Gamma	0.5	inf.
σ_f	4.4	Inv. Gamma	0.5	inf.
σ_x	0.13	Inv. Gamma	0.5	inf.
σ_q	0.21	Inv. Gamma	0.5	inf.
σ_ζ	0.56	Inv. Gamma	0.25	inf.
σ_{tp}	0.02	Inv. Gamma	0.05	inf.
σ_{BOJ}	19.8	Inv. Gamma	1	inf.
$\tilde{\sigma}_m$	0.06	Inv. Gamma	0.5	inf.
σ^*_γ	1.74	Inv. Gamma	0.5	inf.
σ^*_p	0.11	Inv. Gamma	0.5	inf.
σ^*_{tp}	0.02	Inv. Gamma	0.05	inf.
σ^*_m	0.10	Inv. Gamma	0.75	inf.
σ^*_ζ	0.30	Inv. Gamma	0.25	inf.
σ^*_B	5.6	Inv. Gamma	1	inf.

Table A4: Parameter Values for Japan for the Model with Correlations

parameter	posterior mean	prior dist.	prior mean	prior stdev
$\varkappa$	0.26	Beta	0.4	0.1
ν	1.11	Gamma	1.0	0.5
θ	1.20	Gamma	1.0	0.5
ζ	0.002	Normal	0.01	0.01
χ	2.14	Gamma	3.0	0.2
λ_d	2.28	Gamma	1.2	0.5
λ_f	3.46	Gamma	1.2	0.5
λ_x	1.78	Gamma	1.2	0.5
ξ_d	0.86	Beta	0.66	0.1
ι_d	0.15	Beta	0.5	0.2
ι_f	0.18	Beta	0.5	0.2
ι_x	0.49	Beta	0.5	0.2
ϕ_π	0.34	Gamma	0.5	0.25
ϕ_y	0.30	Gamma	0.5	0.15
ρ_{tp}	0.99	Beta	0.97	0.02
ρ_m	0.75	Beta	0.5	0.2
ρ_R	0.81	Beta	0.8	0.05
ρ_γ	0.86	Beta	0.5	0.2
ρ_d	0.39	Beta	0.5	0.2
ρ_f	0.91	Beta	0.5	0.2
ρ_x	0.95	Beta	0.5	0.2
ρ_q	0.87	Beta	0.5	0.2
ρ_ζ	0.72	Beta	0.5	0.2
Ψ	-0.004	Normal	-0.006	0.001
ρ_B	0.95	Beta	0.5	0.2

Table A5: Parameter Values for the U.S. for Model with Correlations

parameter	posterior mean	prior dist.	prior mean	prior stdev
$\varkappa^*$	0.72	Beta	0.4	0.1
ϖ^*	0.01	Beta	0.1	0.05
ϕ_π^*	0.95	Gamma	0.5	0.2
ϕ_y^*	0.58	Gamma	0.5	0.2
ζ^*	0.002	Normal	0.01	0.01
ρ_{tp}^*	0.99	Beta	0.97	0.02
ρ_m^*	0.66	Beta	0.5	0.2
ρ_R^*	0.80	Beta	0.8	0.1
ρ_γ^*	0.78	Beta	0.5	0.2
ρ_p^*	0.20	Beta	0.5	0.2
ρ_ζ^*	0.86	Beta	0.5	0.2
Ψ^*	-0.008	Normal	-0.01	0.001
ρ_B^*	0.98	Beta	0.5	0.2

Table A6: Parameter Values for Standard Deviation for Model with Correlations

parameter	posterior mean	prior dist.	prior mean	prior stdev
σ_γ	0.31	Inv. Gamma	0.5	inf.
σ_d	7.45	Inv. Gamma	0.5	inf.
σ_f	4.40	Inv. Gamma	0.5	inf.
σ_x	0.14	Inv. Gamma	0.5	inf.
σ_q	0.21	Inv. Gamma	0.5	inf.
σ_ζ	0.55	Inv. Gamma	0.25	inf.
σ_{tp}	0.02	Inv. Gamma	0.05	inf.
σ_{BOJ}	19.84	Inv. Gamma	1	inf.
$\tilde{\sigma}_m$	0.06	Inv. Gamma	0.5	inf.
σ^*_γ	1.71	Inv. Gamma	0.5	inf.
σ^*_p	0.11	Inv. Gamma	0.5	inf.
σ^*_{tp}	0.03	Inv. Gamma	0.05	inf.
σ^*_m	0.10	Inv. Gamma	0.75	inf.
σ^*_ζ	0.37	Inv. Gamma	0.25	inf.
σ^*_B	5.57	Inv. Gamma	1	inf.
σ_{B,B^*}	0.13	Normal	0	0.2
σ_{ζ,ζ^*}	0.26	Normal	0	0.2

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