BANK FOR INTERNATIONAL SETTLEMENTS

BIS Working Papers

No 929

Greening (runnable) brown assets with a liquidity backstop

by Eric Jondeau, Benoit Mojon and Cyril Monnet

Monetary and Economic Department

March 2021

JEL classification: G01, G18, G28.

Keywords: financial stability, runs, brown assets, liquidity provision.

This publication is available on the BIS website (www.bis.org).

ISSN 1020-0959 (print)
ISSN 1682-7678 (online)

Greening (Runnable) Brown Assets with a Liquidity Backstop*

Eric Jondeau Benoit Mojon

Cyril Monnet

March 2021

Abstract

The momentum toward greening the economy implies transition risks that are new threats to financial stability. In particular, the expectation that other investors may exclude high carbon corporate emitters from their portfolio creates a risk of runs on brown assets. We show that runs can be contained by a liquidity backstop with an access fee that depends on the firm's carbon intensity, while the interest rate on the liquidity lent through this facility is independent from its carbon intensity.

JEL: G01, G18, G28

Keywords: Financial stability, Runs, Brown assets, Liquidity provision

*E. Jondeau: Swiss Finance Institute and Faculty of Business and Economics (HEC Lausanne), University of Lausanne, Extranef 232, CH-1015 Lausanne, Switzerland (e-mail: eric.jondeau@unil.ch). B. Mojon: Bank for International Settlements, Centralbahnplatz 2, 4051 Basel, Switzerland (e-mail: benoit.mojon@bis.org). C. Monnet: University of Bern and Study Center Gerzensee, Schanzenheckstr 1, 3001 Bern, Switzerland (e-mail: cyril.monnet@vwi.unibe.ch). We thank Fernando Alvarez, Torsten Elhers, Kumar Jegarasasingam, Simone Manganelli, Luiz Pereira and participants to the BIS webinar series for useful comments.

1 Introduction

Reducing carbon emissions to limit the increase in global temperature to 2 degrees is a major challenge for the world. There is a consensus that we collectively fail to deliver sufficient mitigation because the negative externalities of carbon usage are underpriced. In particular, market prices fail to correctly reflect the social cost of carbon and capture the long-term benefits of the transition. Introducing a widely-accepted Pigouvian carbon tax mechanism and promoting and subsidizing green energies at a large scale could be effective tools (Lagarde and Gaspar, 2019) but so far governments have failed to coordinate their taxing of carbon emissions. Many observers and a majority of central banks communicate on the need to adapt the financial system to mitigate climate change. Solutions include, among others, promoting green investments, supporting the development of green financial securities, and integrating climate risk into the central bank collateral framework (see Krogstrup and Oman, 2019, Bolton et al., 2020, Villeroy de Galhau, 2020, and references therein).[1]

From the buy side, investors are increasingly reluctant to finance carbon-intensive projects or firms and a growing proportion of investors are demanding greener portfolios.[2] This evolution in investors' preferences can become a powerful driver to a greener economy through the rebalancing of their portfolio toward firms with low carbon emissions. An important argument in favor of this approach is that carbon emissions are extremely skewed across firms. The most polluting 1% of firms account for 40% of total Scopes 1 to 3 carbon emissions in 2018 (Ehlers et al., 2020). As a consequence, asset managers can effectively reduce the carbon footprint of their portfolio by excluding (or reducing its exposure to) a moderate number of firms with limited impact on the financial performance and tracking error of their portfolio (Andersson et al., 2016, Jondeau et al., 2021). Such a rebalancing of large investors' portfolios toward green assets may however severely impact potentially excluded firms.

In addition to a change in investors' preferences, climate policies may try to mitigate the consequences of climate change by forcing highly polluting firms to let a substan-

[1]Technical progress is also a major contributor to the low-carbon transition. The reduction in the cost of producing energy with renewable means collapsed in the last decade. From 2010 to 2019, the cost of photovoltaic solar panels dropped by 82% and onshore wind by 38% (IRENA, 2021).

[2]Blackrock (2020) reports that 88% of its investors are expressing a preference for Environmental, Social, and Governance (ESG) portfolios with the Environmental pillar being a much bigger concern than either Social or Governance pillars.

tial fraction of their assets (fossil fuel reserves, for instance) to be stranded. Both the rebalancing of investors' portfolios and the risk of stranded assets could result in a run by investors on potentially affected firms. Indeed, if investors realize that the secondary market for stocks of these firms may collapse, they will seek to reduce their exposures to these assets before all other investors do. In view of the market capitalization of brown firms, this risk is a major threat to financial stability, one that central banks and agencies that guard financial stability cannot ignore (Bolton et al., 2020).[3]

Against the background of increasing social demand to green the economy and the increasing odds of runs on brown assets, how can should one design a policy that preserve financial stability? We argue first that the Damocles sword hanging over brown firms is an opportunity to incentivize these firms to switch to greener production processes. Second, the increase in transition risk due to the stigma on stranded assets, should also be managed. We design a policy that addresses both issues. More precisely, we deal with the following questions: What are the implications of designing a framework targeting the greening of the economy? How should regulators and/or central banks respond to the threat of a run on brown assets? Which combination of instruments would help restore efficiency?

To address these questions, we consider the following model. Firms produce a consumption good with a technology that uses labor and emits pollution. The intensity of pollution is idiosyncratic and private information to each firm initially but it becomes public as soon as firms invest. Firms can use a costly abatement technology to reduce their emissions from their "natural" level. The project is implemented in two rounds of investment. In the initial stage, firms borrow funds from savers to pay wages. Following the initial investment stage, the intensity of pollution becomes public. In addition, some firms need a second round of financing to pay the wage bill and bring their project to fruition. Some of these firms may experience a refinancing shock if a fraction of lenders decide not to roll over their loan to firms whose carbon intensity is above a threshold that is considered unacceptable.

The logic behind the two rounds of investment is the following. When the project starts, potential lenders do not know the carbon intensity of the project. After the project is launched, this information becomes public and lenders may be reluctant to roll over

[3] Estimates of the potential loss in stranded assets vary from USD 1 trillion (Mercure et al., 2018) to 18 trillion (IRENA, 2017).

their loans if the project turns out to have high carbon intensity. In this setting, there is a risk of a run in the second financing stage, which would be harmful for investors or financial intermediaries that lent to these firms in the first place.

The two-round framework allows us to sketch different types of situation that are insightful in the context of climate change. In the first case, firms in the fossil fuel extraction industry start by exploring new fields and if these fields are promising go for the second round of investment with the exploitation of the fields. The roll-over risk that we introduce in the model corresponds to a situation in which the exploitation phase cannot be financed and the fossil fuel reserves are de facto stranded. A second case corresponds to start-ups in the renewable energy industry. Again, there is a first phase, here consisting of research and development, and a second phase that would correspond to the production and diffusion of the new technology (say, solar panels). In this context, we do not expect any roll-over risk due to pollution but clearly the risk that the project would not be pursued would have detrimental consequences for the economy. A third case is an asset manager's portfolio. The carbon footprint of the portfolio depends on the investment decisions of all the firms it is exposed to. This footprint may be revealed only over time as the various firms held in the portfolio reveal the true carbon impact of their production technology. Investors may be reluctant to keep assets that could bear the stigma of a high-carbon footprint, a situation that may trigger a bank run.

In this setting, we solve for the optimal allocation of resources and pollution emission. The first-best allocation has the following characteristics: (1) all firms must clean their act; (2) all firms have the same marginal benefit of production (net of the marginal cost of cleaning their act); (3) low-emission firms reduce their emissions to zero; (4) high-emission firms reduce their emissions and reduce their production scale.

We then show that the laissez-faire market allocation cannot decentralize the efficient allocation. First, firms have no incentive to reduce their emission. Second, there is a pooling equilibrium where low-emission and high-emission firms have no incentive to differentiate from each other when borrowing from the cash market. This is a self-fulfilling equilibrium where high-emission firms that need refinancing cannot borrow funds in the second cash market and default, although their projects have positive net present value. This equilibrium gives rise to the need for a liquidity backstop for "brown" corporates.

We design the optimal liquidity backstop facility. A liquidity backstop can sustain the optimal allocation provided it uses a fee structure similar to a two-part tariff. First, access to the liquidity backstop is charged a lump-sum fee that is contingent on effective emissions, but not on the borrowed amount. Once a firm pays this access fee, the interest rate on the borrowed amount is independent from its effective emissions. We show that the policy can be designed in such a way that in equilibrium all firms with access to a dedicated liquidity backstop will clean their act according to the optimal allocation.

The remainder of the paper is organized as follows. In Section 2, we describe the model. In Section 3, we solve the social planner problem for the efficient outcome. Section 4, we show that absent policies, the laissez-faire is inefficient. In Section 5, we analyze the design of the best policy for the operator of the liquidity backstop. Finally, we discuss several empirical implications of the model in Section 6.

Related literature. Our paper belongs to the literature that investigates the design of financial and monetary policies to mitigate the impact of climate change. As put forward by Carney (2016), being successful at mitigating climate change may result in the paradox that "success is failure," as a rapid low-carbon transition may have undesirable consequences on financial stability. Reducing the access of companies with high carbon intensity to financial resources may generate so-called stranded assets, which may compromise financial stability by reducing the value of such assets held by financial institutions (Bolton et al., 2020). Our model is designed to break this paradox, as it aims at designing a policy that incentivizes firms to reduce their carbon emissions, while preventing runs on brown/stranded assets.

Van der Ploeg and Rezai (2018) analyze the effect of various climate policies (such as immediate or delayed carbon tax and renewable energy subsidy). They find that an unanticipated policy would severely affect the value of assets in fossil energy firms, while a delayed policy would foster extraction and accelerate global warming in the short term. In contrast, Barnett (2019) demonstrates that a climate policy that would restrict fossil energy use at an unknown arrival time would imply a run on fossil energy. By their impact on the value of stranded assets, climate policies can severely affect the value of financial institutions' assets. An evaluation of this impact can be obtained using climate stress tests, see Battiston et al. (2017) and Vermeulen et al. (2019). Reinders et al. (2020) measure the impact of climate policy on the assets of the Dutch banking system and find, in the most severe scenario, a market value loss equal to 3.2% of total assets or 63% of common equity Tier 1 capital.

In addition to climate policies, changes in investors' preference may severely affect the value of stranded assets. The trend in favor of ESG investing is already massive. Theoretical asset pricing models (Pastor et al., 2021, and Pedersen et al., 2021) show that green assets should provide a lower expected return because investors enjoy holding these assets and because green assets provide a hedge to climate risk. Görgen et al. (2020a) and Bolton and Kacperczyk (2020) empirically demonstrate that investors are already demanding a compensation for being exposed to carbon emission risk. Ehlers et al. (2020) discuss the benefits of a firm-level rating based on carbon intensity to complement existing project-based green labels and find that such a rating system could provide a useful signal to investors and encourage firms to reduce their carbon footprint.

2 The Model

The model is an adapted version of Monnet and Temzelides (2016). Time is discrete, denoted by $t = 0, 1, 2, ...$, and continues forever. The economy is populated by a $[0, 1]$-continuum of bank/firm pairs, a $[0, 1]$-continuum of workers, and a $[0, 1]$-continuum of financiers that we call savers. For short, we will use "firms" to denote the bank/firm pairs. All agents discount future cash flows at a rate $\beta \in (0, 1)$. Each period, there are three goods: labor, a consumption good, and a numéraire good. Each firm has a technology that transforms labor into the consumption good. Using q units of labor, each firm can produce $f(q)$ units of the consumption good. The production function f is smooth, strictly increasing, and strictly concave, with $f(0) = 0$. Workers supply labor to firms by incurring a linear disutility of working. They derive utility $u(c)$ from consuming c units of the consumption good and linear disutility q from working q hours each period. Savers produce n units of the numéraire by operating a linear technology at the cost $-n$. They also derive linear utility X from consuming X units of the numéraire. For simplicity, we assume that there is no storage technology allowing to transfer the consumption of the numéraire good across periods.

Production is costly for society, as each operating firm creates harmful emissions of some pollutants (emissions for short). When the level of overall emissions is E, the utility of workers from consuming c units of the consumption good, and working q hours is

$U(c,q,E) = u(c) - q - \frac{1}{2}E$. The utility of savers when they consume X units of the numéraire and producing n units of the numéraire is $U_F(X,n,E) = X - n - \frac{1}{2}E$.

The emission intensity of a firm's operations is subject to a random shock.[4] More precisely, we assume that in each period, each firm receives a shock, denoted by θ, which determines the intensity of emissions generated by its production activity. In a given period, the amount of emissions generated by a firm that receives shock θ and uses q units of labor is θq. For simplicity, we assume that θ is *i.i.d.* across time and across firms. We denote the cumulative distribution of θ by $G(\theta)$ with support $\left[0, \bar{\theta}\right]$. In order to capture the fact that firms have more information than the regulating authority about their emission intensity, we assume that these shocks are private information. In particular, workers and savers do not know the emission intensity at the time firms invest in their projects.[5] However, there is a costly information production technology: by paying cost γ (a reduction in profit), a firm can reveal its true emission intensity. This can be interpreted as the cost to become transparent to the authorities by paying, for example, a rating agency certifying the emission intensity of the firm.

While all producing firms create pollution, they can also reduce their emissions at some cost. Emissions of a firm with emission intensity θ and labor input q would naturally be θq. However, for given θ and q, the firm can reduce its natural emissions to an amount e by incurring the cost $h(\theta q - e)$, where $h(\cdot)$ is the same convex function for all firms, with $h(0) = 0$ and $h'(0) = 0$. We refer to the difference $\theta q - e$ as the effort made by the firm to reduce its emissions. The function $h(\cdot)$ represents a resource cost, such as an iceberg cost, so it will reduce the amount of resources available for consumption.

Finally, firms pay the real wage w to workers in cash. To do so, firms have to borrow cash either from savers (at rate i_m) or directly access to the liquidity backstop facility (at rate i), by pledging their future output as collateral. While i_m is endogenous, i is a policy variable that is part of the design of the liquidity backstop facility. Firms do not retain earnings and distribute all their profits to their owners (savers).[6]

In the second stage, all firms face a refinancing risk similar to Holmström and Tirole

[4]For example, this shock may represent the need to use energy for transportation, or for cooling or heating due to weather conditions, which are inherently random.

[5]See Monnet and Temzelides (2016) for a more general specification, also for the cost to reduce emissions that we introduce below. The results would not change if emissions were assumed to be proportional to output. The case with private information and serially correlated shocks is not analytically tractable. This is an important avenue for future research.

[6]We could assume firms can retain some limited earnings. Here, we set this limit to zero.

(1998) but without the moral hazard problem: with probability ρ, they need to re-hire a share $s \in (0,1]$ of workers in order to bring their project to fruition (otherwise their project yields nothing), while with probability $1-\rho$ they will not. This re-hiring does not affect effective emissions. At this stage, the emission intensity of each productive firm and therefore aggregate emissions are now known. Later, we will consider a "run" risk, which is related to the emission intensity θ of each firm: with probability $r(\theta)$, a firm with emission intensity θ will not be able to access the cash market, where $r'(\theta) \geq 0$. For simplicity, we set $r(\theta) = 0$ for all $\theta < \theta^r$ and $r(\theta) = 1$ for all $\theta \geq \theta^r$. The threshold θ^r is exogenous but can be interpreted as the level of emission intensity above which assets may be stranded at some point.

The central bank manages the stock of money in the economy and ensures that the money stock grows at a constant rate π, which will be the rate of inflation.

The sequence of events and markets for the producing firms is summarized in Figure 1 and it is as follows. First, firms learn their shock θ. Then, they hire workers and decide whether to pay the revelation cost γ and make their emission intensity public, which allows them to be considered as borrowers from the liquidity backstop facility. To pay wages, firms can borrow from the liquidity backstop facility or in the cash market, or use their cash holdings from the previous period. Their emission intensity becomes known. Then, firms can incur a refinancing shock. If they cannot refinance – either by borrowing from the cash market or by self-refinancing – they have to liquidate their investment for nothing and they default on all their liabilities. Otherwise, firms proceed with production. They can reduce emissions to $e(\theta) \leq \theta q$ and produce and sell their production to workers for cash. They reimburse their loans and pay dividends to equity holders. Then, the next period begins.

We note that banks are not explicitly introduced in the model, while firms have direct access to the liquidity backstop facility or the cash (interbank) market. In fact, our set-up naturally maps into one with banks operating on different islands (or catering to specific industrial sectors). The model would be as follows: banks do not compete with each other in the market for loans and they can extract all the rents from firms. Each period, banks on each island issue deposits to savers and decide to fund q firms, which can be more of less polluting as captured by their emission intensity θ. In that version, each firm uses a technology that requires one unit of labor, with the same labor cost w. Labor

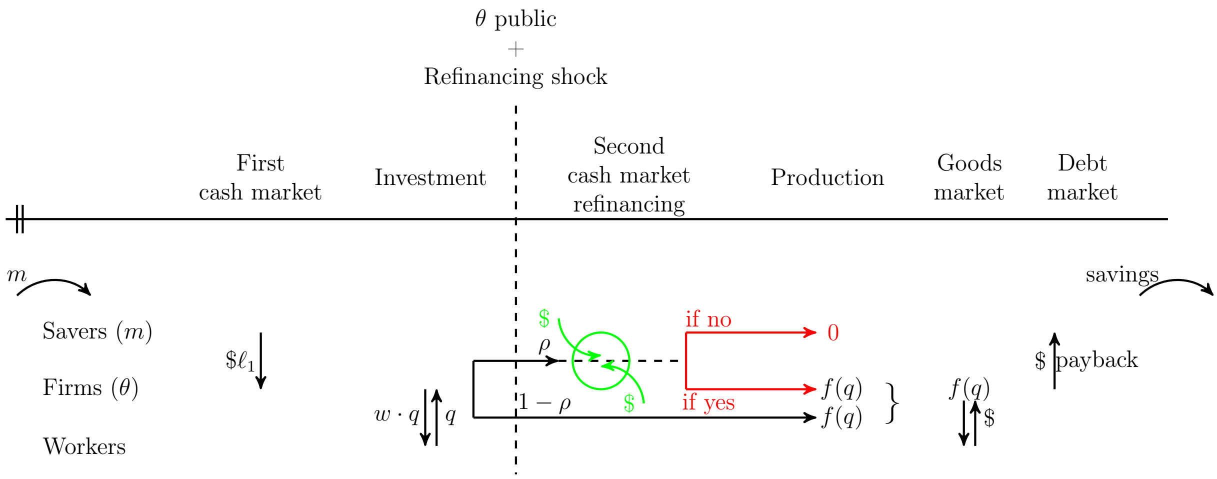


Figure 1. Timeline

is mobile across islands, so that the unit wage is the same across all islands. Each firm asks the bank for a loan w, which the bank funds using deposits, borrowing from the interbank market (which would operate across islands), or borrowing directly from the liquidity backstop facility. The return to the bank of financing q firms is $f(q)$, which is the aggregate production of all q firms. Finally, by being on the board of these firms, the bank forces its pool of borrowers to reduce their emissions to $e(\theta)$. Since the bank has monopoly power and extracts all rents from firms, it internalizes the cost of reducing emissions. All in all, our results extend directly to this more natural and maybe more realistic setup. We will however keep our original setup, which is slightly easier to go through.

3 Benchmark: Social Planner's Problem

A social planner takes emissions into account when solving for the efficient outcome. Since firms vary in their degree of emissions, the first-best solution should induce different production levels across firms. The social planner maximizes the expected surplus from production net of total emissions and subject to the resource constraint. Given the choice $q(\theta)$, the social planner will refinance a firm with emission factor θ only if $f(q(\theta))u'(c) \geq sq(\theta)$, i.e., if the consumption value of this firm's production compensates for the additional hours worked in that firm. We guess and verify that, if s is low enough, this inequality always holds. Then, the social planner's problem is

$$\max_{q(\theta),0\leq e(\theta)\leq\theta q(\theta)} u\left(c\right)-\int(1+\rho s)q\left(\theta\right)dG(\theta)-\int e\left(\theta\right)dG\left(\theta\right) \tag{1}$$
$$s.t. \quad c=\int\left[f\left(q\left(\theta\right)\right)-h\left(\theta q-e\left(\theta\right)\right)\right]dG\left(\theta\right).$$

Effective emissions $e\left(\theta\right)$ cannot be negative and are bounded above by the amount of natural emissions θq. Let λ_θ be the Lagrange multiplier on $e\leq\theta q\left(\theta\right)$ and λ_0 be the one on $e\geq 0$. We denote the efficient production scale by $q^*\left(\theta\right)$ and the efficient level of emissions by $e^*\left(\theta\right)$. The schedule (q^*,e^*) satisfies the following first-order conditions for all θ:

$$\left[f'\left(q^*\left(\theta\right)\right)-\theta h'\left(\theta q^*\left(\theta\right)-e^*\left(\theta\right)\right)\right]u'\left(c^*\right)+\theta\lambda_\theta = 1+\rho s \tag{2}$$

$$h'\left(\theta q^*\left(\theta\right)-e^*\left(\theta\right)\right)u'\left(c^*\right)-\lambda_\theta+\lambda_0 = 1. \tag{3}$$

In that case, consumption c^* is found by solving Program (1). We obtain the following characterizations of the efficient allocation.

1. **All firms must partially clean their act:** $e^*(\theta)<\theta q^*(\theta)$ for all $\theta>0$. Suppose $\lambda_\theta>0$ for some θ, so that firms with θ do not clean their act and their ex-post emission $e(\theta)$ is the same as their ex-ante level $\theta q(\theta)$. Then $1+\lambda_\theta=\lambda_0>0$ and hence, $e(\theta)=0$, which can only be the case iff $\theta=0$. Therefore, as $h'\left(0\right)=0$, it is efficient for all firms to reduce emissions by a small amount and all active firms must reduce their emissions at the optimum. Therefore $\lambda_\theta=0$.

2. **Once firms clean their act, all firms have the same marginal benefit of production net of the marginal cost of cleaning their act:** This result follows directly from Equation (2), setting $\lambda_\theta=0$. The intuition is that the planner equates the (expected) marginal benefit of production (as valued by workers) to the marginal cost of working (here equal to $1+\rho s$ because the cost of working is a linear function of hours worked).[7] However, the planner also takes into account that more resources will be devoted to reduce emissions if more hours are worked in firms with emission factor θ, which reduces the marginal production gains. So while in a standard neoclassical model the planner only takes the marginal gain $f'(q)$ into account,

[7]Assuming a convex effort cost $\kappa(\cdot)$ instead of a linear one would not change that result. Indeed, Equation (2) would then be: $\left[f'\left(q^*\left(\theta\right)\right)-\theta h'\left(\theta q^*\left(\theta\right)-e^*\left(\theta\right)\right)\right]u'\left(c^*\right)+\theta\lambda_\theta=\kappa'\left(\int q(\theta)\right)$.

here the planner also needs to adjust for the marginal cost of reducing emissions $\theta h'(\theta q - e)$.

3. **Some firms must reduce their emissions to zero:** For all $\theta \leq \tilde{\theta}$ and $\theta \geq \hat{\theta}$, $e(\theta) = 0$. Suppose $\lambda_0 = 0$ for all firms. Then from Equation (3), we have $h'(\theta q(\theta) - e(\theta)) u'(c^*) = 1$, and from Equation (2), we have $f'(q(\theta)) u'(c^*) = 1 + \rho s + \theta$. Therefore, $q(\theta)$ is finite for all θ and smaller than $q(0)$.[8] Combining the last two equations gives $f'(q(\theta)) = (1 + \rho s + \theta) h'(\theta q(\theta) - e(\theta))$. Since $h'(0) = 0$, this last equation cannot be satisfied for $\theta > 0$ small unless $q(\theta) \to \infty$, which cannot be as $q(\theta) \leq q(0)$. By continuity, it cannot be satisfied when θ is smaller than some threshold $\tilde{\theta}$ and for these firms we have $e(\theta) = 0$. Now consider the case where the maximum factor of emission $\bar{\theta}$ is large. There is $\hat{\theta}$ such that all firms with $\theta > \hat{\theta}$ reduce their emissions to zero. Suppose instead $\lambda_0 = 0$ for θ large. Then $q^*(\theta)$ solves $f'(q^*(\theta)) u'(c^*) = 1 + \theta + \rho s$, so by concavity of the production function, we obtain $q^*(\theta) \to 0$ when $\theta \to \infty$. If $\theta q^*(\theta)$ is decreasing then there is $\hat{\theta}$ such that $e^*(\theta) = 0$ for all $\theta \geq \hat{\theta}$. Since $q'(\theta) < 0$ and the derivative of $\theta q(\theta)$ is $q(\theta) + \theta q'(\theta)$, the expression $\theta q(\theta)$ will be decreasing with θ whenever $1 + \frac{\theta}{1+\theta+\rho s} \frac{f'(q)}{f''(q) q}$ is negative. For example, with $f(q) = q^\alpha$, $\theta q(\theta)$ is decreasing whenever $\alpha\theta > (1-\alpha)(1+\rho s)$.

4. **Other firms make the same effort to reduce their level of emissions:** For all $\theta \in [\tilde{\theta}, \hat{\theta}]$, $\theta q^*(\theta) - e^*(\theta)$ is constant. This result is obvious from Equation (3) when $\lambda_\theta = \lambda_0 = 0$. Above the threshold $\tilde{\theta}$, the optimal ex-post emissions are positive and proportional to the ex-ante emissions. The reason is that the marginal benefit from reducing emissions is the same regardless of whether the reduction comes from a firm with high or low emissions. On the other hand, the cost of reducing emissions (in terms of consumption loss) is small if firms are already relatively clean.

5. **High-emission firms must reduce their production scale, and more so as they emit more:** For all $\theta > \tilde{\theta}$, $q(\theta)$ is decreasing in θ. Combining Equations (2) and (3) with $\lambda_\theta = \lambda_0 = 0$, we obtain $f'(q(\theta)) u'(c^*) = 1 + \theta + \rho s$, from which the result follows. So the efficient allocation dictates that some firms reduce their emissions, by *both* reducing their production scale *and* by cleaning their act.

The optimal level of total emissions is given by $E^* = \int e^*(\theta) \, dG(\theta)$. Finally, we verify

[8] When there is no emission, labor $q(0)$ is given by $f'(q(0)) = (1 + \rho s)/u'(c^*)$.

that it is optimal to refinance all firms when they hire $q^*(\theta)$ units of labor. To show this, note that $f'(q(\theta))\,u'(c^*) = 1+\theta+\rho s$, as shown in point 5 above. Now a property of a concave and differentiable function f is that $f'(q)q \leq f(q)$ for all q. Therefore, $f'(q(\theta))u'(c)q(\theta) = (1+\theta+\rho s)q(\theta) \leq f(q(\theta))u'(c)$, which verifies that $f(q(\theta))u'(c) \geq sq(\theta)$ as $s \leq 1$.

Figure 2 represents the optimal investment and carbon emission resulting from the optimal social planner's allocation. The figure illustrates that, as the emission intensity θ increases (x-axis), the investment by low-emission firms (green segment of investment curve) decreases less than the investment by high-emission firms (brown segment). The magenta curve represents the ex-ante "natural' level of emissions by firms with emission intensity θ, in the absence of any carbon-reduction measure. The red curve represents the emission under the social planner's optimal policy. Emissions go to zero for low-emission firms and are substantially reduced for high-emission firms.

Figure 3 represents the optimal investment and carbon emission resulting from the optimal social planner's allocation for a different parametrization. This example illustrates the case where high-emission firms both reduce their production and take measures to bring their effective emissions down. In the limit, firms with very high levels of emission reduce their production to such a level that they do no longer emit pollution.

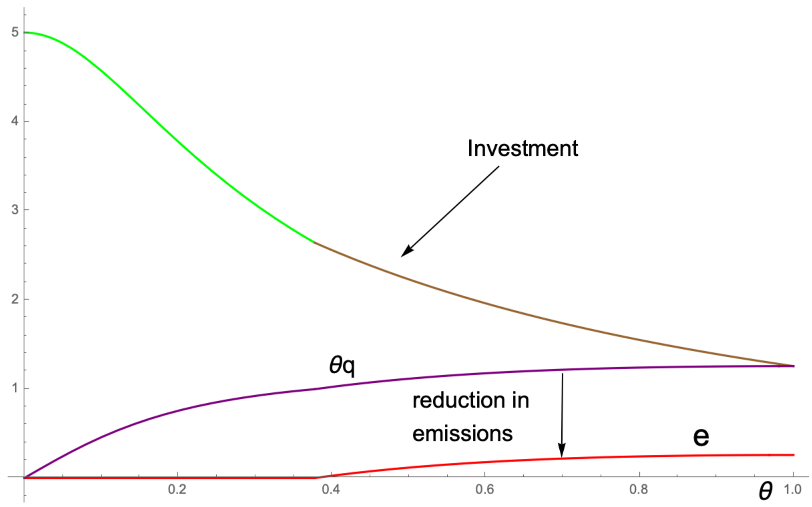


Figure 2. Social Planner's Allocation

Note: The figure illustrates the investment and the emissions of firms with intensity θ. θq represents emissions when no reduction is undertaken, while e represents effective emissions, so that the difference between the two curves corresponds to the reduction in emissions. In this paramerization, firms with θ below 0.4 reduce their emissions to 0, which we represent in the investment curve with the green segment.

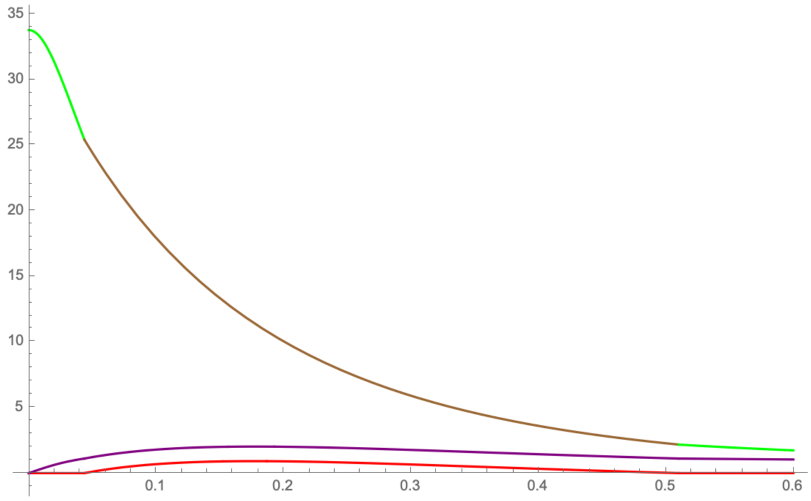


Figure 3. Social Planner's Allocation

Note: The figure illustrates the investment and the carbon emissions of firms with intensity θ with a more extreme parametrization. As in Figure 3, we have from top to bottom the investment of firms with carbon intensity θ, the emissions when no reduction is undertaken, and the effective emissions. In this paramerization, firms with θ below 0.05 reduce their emissions to 0, and firms with θ above 0.5 also reduce their emissions to 0, through a reduction in pollution and a reduction in production.

4 Laissez-faire Benchmarks

We first show that the laissez-faire solution is inefficient when the policy is solely focused on inflation (whether or not it would provide a liquidity backstop facility). We consider the laissez-faire equilibrium when firms that need refinancing can be excluded from borrowing in the cash market with a probability $r(\theta)$, where $r'(\theta) \geq 0$. For simplicity here, we set $r(\theta) = 0$ for all $\theta < \theta^r$ and $r(\theta) = 1$ for all $\theta \geq \theta^r$.[9] Therefore, a firm that draws emission intensity $\theta \geq \theta^r$ knows that it will not be able to refinance should it need to, but it can still lend to firms with $\theta < \theta^r$ on the second cash market. A high-emission firm that needs to refinance, but cannot, must terminate its project and gets no resources from it. In this case, the firm defaults on any loan it took in the first cash market.

Savers, who do not know the type of firms in the first cash market, may price in the risk of lending to a high-emission firm (with $\theta \geq \theta^r$). Now recall that firms can reveal their type θ by paying a cost γ before they make their initial investment. Low-emission firms (with $\theta < \theta^r$) will never terminate their project and may be willing to pay that cost in order to distinguish themselves from high-emission firms and economize on the risk

[9] To understand the meaning of the assumption $r'(\theta) \geq 0$, it is best to consider the banking interpretation of our model: At the investment stage, savers do not know how the bank will use the funds it borrows from depositors or bondholders. However, when the bank needs to refinance its portfolio, creditors observe the industrial components of this portfolio, but it is still difficult to obtain information on the direct emissions of firms constituting that portfolio.

premium.

We show that there are two types of equilibrium: In a pooling equilibrium, γ is high enough, so that low-emission firms do not want to pay this cost to reveal their type, and they get the same funding conditions as high-emission firms. In a separating equilibrium, low-emission firms are willing to pay the cost γ in order to benefit from the lower market rate. As the pooling equilibrium gives rise to a possible run on brown assets, we mainly focus in this section on the description of the pooling equilibrium and we relegate all the details to the Appendix.

We should note that irrespective of the equilibrium, θ becomes publicly known once a firm has made its initial investment decision. Therefore, there is no longer private information in the second cash market. We first solve for the problem of low-emission firms and then the problem of high-emission firms. Also, firms can only clean their act after contracting loans in cash market. Therefore, the loan conditions cannot be made contingent on effective emissions because firms naturally cannot commit to reduce emissions.

4.1 Problem of Workers

The problem of workers does not depend on the type of equilibrium in the economy. Taking the level of total emissions E and the wage w as given, workers choose to consume c and supply q units of labor to maximize their utility $U(c, q, E) = u(c) - q - \frac{1}{2}E$, subject to their budget constraint $c \leq wq$. The first-order conditions of workers give: $w = \frac{1}{u'(c)}$.

4.2 Problem of Low-emission Firms ($\theta < \theta^r$)

Firms with emission intensity $\theta < \theta^r$ can always borrow in the second cash market. We guess that these firms will want to refinance rather than terminate their project. Then, at the refinancing stage, these firms have an alternative: (1) they have to borrow a fraction or all of their refinancing need swq, paying the second cash market rate $i_{m,2}$, or (2) they have idle cash and they lend it in the second cash market to earn $i_{m,2}$. When considering their initial investment decision, low-emission firms will consider the marginal cost of funds in the first and second cash markets. The cost of borrowing to invest q in the first cash market is $wq(1 + i_{m,1})$, while the cost of refinancing q in the second cash market is $swq(1 + i_{m,2})$. Then, low-emission firms choose q to equate their marginal benefit of investment to their expected marginal borrowing cost,

$$f'(q) \;\; = \;\; w\,(1 + i_{m,1}) + w\rho s\,(1 + i_{m,2})\,. \tag{4}$$

So, the marginal cost of production equals the cost of the loan to fund the wage necessary to pay the last unit of production in the investment stage as well as the expected cost to refinance the project in the second cash market. It is noteworthy that, whenever $i_{m,1} > i_{m,2}$, low-emission firms only borrow what they need to invest in the initial investment stage (i.e., wq) and not more. Otherwise when $i_{m,1} = i_{m,2}$, they can hold money for precautionary reasons to self-refinance: if they do not need to refinance, they will lend their idle cash at $i_{m,2}$, so they make no losses from borrowing at $i_{m,1}$ as long as they can lend at $i_{m,2} = i_{m,1}$ in the second cash market. Notice that $i_{m,2} > i_{m,1}$ cannot hold in equilibrium because all firms with $\theta < \theta^r$ would borrow as much as possible at $i_{m,1}$ in the first cash market and then lend at $i_{m,2}$ in the second cash market.

4.3 Problem of High-emission Firms ($\theta \geq \theta^r$)

In the pooling equilibrium, high-emission firms (with $\theta \geq \theta^r$) pretend to be clean and behave as low-emission firms do. Therefore, they borrow the same amount as cleaner firms in the first cash market and they invest the same amount q in the initial investment stage. We already know that, if $i_{m,1} > i_{m,2}$, low-emission firms just borrow what they need for their initial investment in the first cash market. This means that, whenever $i_{m,1} > i_{m,2}$, a high-emission firm mimicking a low-emission firm will not be able to refinance and will default on the first cash-market loan. If a high-emission firm does not have to refinance, it will lend its idle cash, if any, in the second cash market.

4.4 Savings Decision

We now consider the problem of savers. In the equilibrium where $i_{m,1} > i_{m,2}$, savers know that firms only borrow what they need for their initial investment and so high-emission firms that need to refinance will default. Savers are diversifying the default risk by lending to the entire $[0, 1]$-continuum of firms in the first cash market. Therefore, they correctly anticipate that the loan they made in the first cash market will only be repaid by the $(1 - \rho)$ firms that do not suffer a refinancing shock and by the $\rho G(\theta^r)$ low-emission firms that suffered one. In other words, savers expect a return $\left(1-\rho+\rho G(\theta^r)\right)(1+i_{m,1})$ from lending one dollar in the first cash market. If lenders were lending that dollar in the second cash market, they would only lend to low-emission firms that never default with a return $1+i_{m,2}$. As savers have to be indifferent between both options as they are the ones supplying funds in the second cash market (when $i_{m,1} > i_{m,2}$, no firm has idle cash), in a pooling equilibrium we obtain:

$$\left(1-\rho+\rho G(\theta^r)\right)(1+i_{m,1}) = (1+i_{m,2})$$

with $i_{m,1} > i_{m,2}$ as we guessed. Finally, savers have to be compensated for the cost of holding nominal balances across periods, which depreciate at the rate of inflation π, and so we obtain,

$$1+i_{m,2} = \frac{1+\pi}{\beta} \equiv 1+i.$$

Then, Equation (4) gives

$$f'(q) = w\frac{(1+i)}{\left(1-\rho+\rho G(\theta^r)\right)} + w\rho s\,(1+i)\,, \tag{5}$$

and the expected payoff of low-emission firms in a pooling equilibrium is

$$V^P = f(q) - wq\left(\frac{1}{1-\rho+\rho G(\theta^r)} + \rho s\right)(1+i). \tag{6}$$

In the Appendix, we also describe how to find the equilibrium amount of real money balances.

4.5 Separating Equilibrium

In a separating equilibrium, low-emission firms pay the cost γ and benefit from the risk-free rate in the first and second cash markets. Once low-emission firms have paid the revelation cost, and given the interest rates, their decision is unchanged from the one in a pooling equilibrium. Therefore, in an equilibrium where $i_{m,1} > i_{m,2}$, low-emission firms just borrow in the first cash market what they need to invest. Otherwise, when $i_{m,1} = i_{m,2}$, they are indifferent as to when they borrow funds. Then, as above, the initial investment of low-emission firms when they choose to separate, q^s, equates the marginal benefit to the expected marginal cost of investment, so that

$$f'(q^s) = w(1 + i_{m,1}) + w\rho s(1 + i_{m,2}). \tag{7}$$

The expected payoff of low-emission firms then is

$$V^S = f(q^s) - wq^s(1 + i_{m,1}) + wq^s\rho s(1 + i_{m,2}) - \gamma. \tag{8}$$

The problem of high-emission firms is more involved in a separating equilibrium because those firms have to decide whether to borrow in the first period for precautionary reasons and self-refinance if need be, or to liquidate in case they are hit by the refinancing shock.

When a high-emission firm does not need to refinance, it will lend its idle cash in the second market at the market rate $i_{m,2}$. A high-emission firm that needs to refinance either has enough cash to self-refinance or has to terminate its project. In the Appendix, we show that high-emission firms will always borrow sufficiently in the first cash market to refinance when the need arises, and they never terminate their projects in equilibrium. Then the investment level of high-emission firms is given by:

$$f'(q^r) = (1 + i^r_{m,1})w(1 + s) - (1 - \rho)(1 + i_{m,2})sw,$$

where $i^r_{m,1}$ is the interest rate charged by lenders to high-emission firms in the first cash market. Therefore, a firm with $\theta \geq \theta^r$ will equate the marginal benefit of investment with its expected marginal cost, which is computed as follows: it consists of the marginal cost of funding the project including the cost of refinancing $(1 + s)w$ net of the saving sw that the firm makes when it does not need to refinance (with probability $1 - \rho$) and that it can lend in the second cash market at rate $i_{m,2}$. In this equilibrium, high-emission firms do not default because they always have cash to self-refinance.

Since there is no default risk, lenders do not charge a risk premium, hence $i_{m,1} = i^r_{m,1}$. Savers lend all their savings in the first cash market whenever $i_{m,1} > i_{m,2}$. It is possible that this inequality holds in equilibrium since high-emission firms with no refinancing need would now be able to lend to low-emission firms with a refinancing need (even if savers have no more cash to lend in the second cash market). From the intertemporal decision of savers, we obtain

$$1 + i_{m,1} = 1 + i.$$

In the Appendix, we show that the equilibrium where $i_{m,1} > i_{m,2}$ exists whenever the measure of high-emission firms is larger than the measure of firms that need refinancing,

$$1 - G(\theta^r) > \rho. \tag{9}$$

Intuitively, the supply of funds in the second cash market by high-emission firm will then be a force to decrease the lending rate. In this equilibrium we can show that low-emission firms invest more than high-emission firms, i.e., $q^s > q^r$. However, when the inequality (9) is reversed, the equilibrium rates satisfy $i_{m,1} = i_{m,2} = i$, and savers are indifferent lending in the first or second cash market. Then $q^s = q^r$.

4.6 Pooling or Separating?

When deciding whether to pay the revelation cost γ, low-emission firms understand that they will be charged the risk free rate when their emission intensity θ is revealed. When $i_m^1 = i_m^2$, low-emission firms gain nothing by paying the revelation cost γ. In this case, the equilibrium is necessarily a pooling one. Now consider the case where $i_m^1 > i_m^2$. Suppose that we are in a pooling equilibrium. It must be that a low-emission firm does not prefer to pay the revelation cost γ to benefit from a lower lending rate in the first cash market. Precisely, we obtain a pooling whenever $V^P > V^S$, where we use $i_{m,1} = i_{m,2} = i$ in Equation (8), or

$$f(q) - wq\left(\frac{1}{1-\rho+\rho G(\theta^r)} + \rho s\right)(1+i) > f(q^s) - wq^s(1+\rho s)(1+i) - \gamma,$$

with q solving Equation (5) and q^s solving Equation (7) with $i_{m,1} = i_{m,2} = i$. This inequality will hold whenever ρ is small enough or γ is large enough.[10] The other condition is that high-emission firm must prefer to act like low-emission firms rather than borrowing $(1+s)w$ in the first cash market at rate i_m^1 to be able to self refinance when needed. So,

[10] The inequality is satisfied when $\rho = 0$ and by continuity it will be for ρ small enough.

the following inequality must hold:

$$(1-\rho)\left[f(q)-\frac{wq}{1-\rho+\rho G(\theta^r)}(1+i)\right] > f(\tilde{q})-(1+s)\frac{w\tilde{q}}{1-\rho+\rho G(\theta^r)}+(1-\rho)sw\tilde{q}(1+i),$$

where $\tilde{q}$ maximizes the right hand side of the inequality above.[11] Hence $\tilde{q} \leq q$ and strictly so when $\rho > 0$. Notice that, as i increases, $\tilde{q}$ tends to zero faster than q. Therefore, holding everything else constant, there is some level of i such that above it, high-emission firms prefer to act like low-emission firms do. Similarly for low-emission firms: their profit tends to zero as i increases, but the fixed cost of information revelation would induce low-emission firms to prefer the pooling equilibrium because difference in profit is too small to justify paying to reveal information about their emission intensity θ. Hence, we obtain the following result.

Lemma 1. *There is $\bar{i}$ such that the pooling equilibrium with $i_{m,1} > i_{m,2}$ exists whenever $i > \bar{i} \geq 0$. Otherwise the equilibrium is separating.*

Figure 4 illustrates the regions where the equilibrium is pooling or separating, depending on the value of the parameters $(i_{m,2} = i, \theta^r, \rho)$. For this illustration, we used $f(q) = Aq^\alpha$ and $h(x) = x^2/2$ where $A = 2$, $\alpha = 0.85$, and $s = 0.25$ and the utility function of workers is selected so that $w = 1$. Finally $G(\theta)$ is a Normal distribution with mean 0.5 and standard deviation of 0.5. The figure shows that, as soon as the refinancing risk is not too high ($\rho = 0.02$), the equilibrium is more likely to be pooling. As expected, for high ρ, low-emission firms have an incentive to reveal their type to the savers and the equilibrium will be separating: high-emission firms will need to borrow a sufficient amount of cash in the first cash market and therefore they will not suffer from run risk in the second period.

We obtain the following additional characterization of the equilibrium of the laissez-faire case:

6. **The laissez-faire economy with run risk can be in one of two regimes.** In the pooling regime, high- and low-emission firms invest the same amount. When $i_{m,1} = i_{m,2}$ high-emission firms can self refinance and they do not default. When

[11] Precisely, $\tilde{q}$ is the solution to: $f'(\tilde{q}) = w\frac{(1+i)}{1-\rho+\rho G(\theta^r)} + \rho sw(1+i) + \left(\frac{1}{1-\rho+\rho G(\theta^r)} - 1\right) sw(1+i)$.

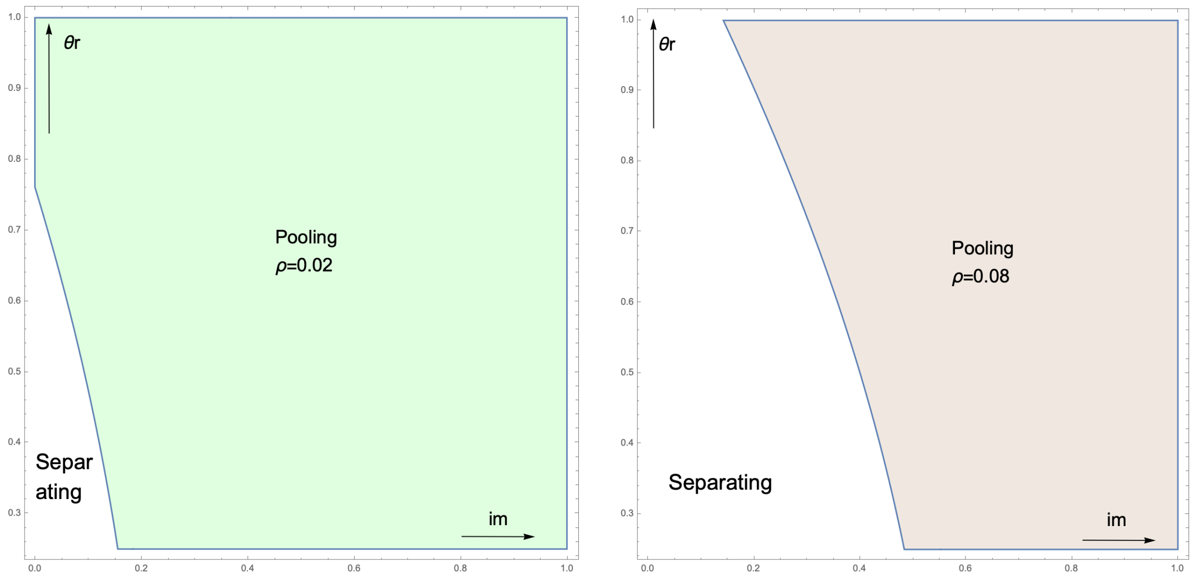


Figure 4. Pooling vs Separating

Note: The figure illustrates the regions where the equilibrium is pooling or separating, depending on the value of the parameters. The x-axis corresponds to the second-market interest rate i_m and the y-axis corresponds to the carbon threshold θ^r. The left-hand-side plot corresponds to low refinancing risk ($\rho = 0.02$) and the rig ht-hand-side plot corresponds to high refinancing risk ($\rho = 0.08$).

$i_{m,1} > i_{m,2}$, they cannot self refinance and they do default. This latter equilibrium is self-fulfilling. In the separating regime, low-emission firms pay the information cost γ. High-emission firms always borrow enough in the first cash market to self-refinance. When the refinancing shock is not likely ($1 - G(\theta^r) > \rho$), there is an equilibrium where $i_{m,1} > i_{m,2}$, and when the refinancing is more likely ($\rho > 1 - G(\theta^r)$), the equilibrium has $i_{m,1} = i_{m,2}$. Both regimes are inefficient.

Figure 5 compares the production in the laissez-faire economy (blue line) to the optimal production in the social planner's allocation. It clearly illustrates that production in this economy is inefficient: high-emission firms produce too much because they have no incentive to reduce their emission. Figure 6 represents the gap in terms of carbon emission due to the laissez-faire solution.

Figure 5 may be misleading, as it suggests that a large range of values of θ is associated with a higher production in the laissez-faire economy. However, in facts carbon intensity is not Normally distributed, but it rather looks like a Pareto distribution with a very fat right tail. We have used Trucost measures of firm-level Scope 1 carbon intensity to estimate the shape of the distribution. With this data, we have estimated the shape parameter of the Pareto distribution in the cross section for several regions (as of 2018). Estimates are equal to 0.27 for the world, with 0.25 for emerging countries, 0.29 for the

United States and Europe, and 0.30 for Pacific.[12] Using this estimation as well as an arbitrary parametrization for the rest of the model, Figure 7 shows the production level in the laissez-faire economy (straight blue line) and with a social planner (red dots) for the different regions. This example shows that the distribution of carbon intensity that we estimate worldwide, the reduction in production by high-emission firms is more than compensated by the increase in production by low-emission firms, resulting in a higher level of aggregate production under the efficient allocation than under laissez-faire.

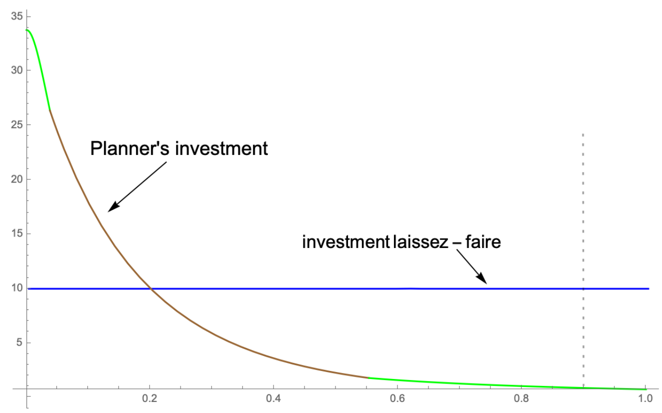


Figure 5. Production in Laissez-faire Economy

Note: The figure illustrates the firms' production in the social planner's economy and laissez-faire economy, depending on their emission intensity θ. In the laissez-faire economy, production is not affected by the emission intensity of the firm. In the planner's economy, production decreases with emission intensity.

5 Green Liquidity Backstop Policy

In this section, we analyze the design of the best liquidity backstop policy for an operator that lends money to firms directly. Savers, being atomistic agents, take market interest rates as given when they lend to firms. Therefore, each of them cannot make interest rate dependent on effective emissions and they have no incentive to do so either, as they understand that their (small at the aggregate level) lending does not impact pollution. However, they could condition their refinancing strategy on effective emissions but the result is qualitatively similar to the equilibrium that we have described above. The operator of the liquidity backstop, in contrast, can make its loan conditions conditional on

[12] Other parameters are as follows: the production function is $f(q) = Aq^{\alpha}$, with $A = 2$ and $\alpha = 0.95$. The revelation cost is $\gamma = 0.2$. Total labor is $w = 1$, the probability of refinancing is $\rho = 0.1$ and the fraction of labor to re-hire is $s = 0.05$. Last, the carbon intensity threshold that triggers a run is $\theta^r = 0.95$.

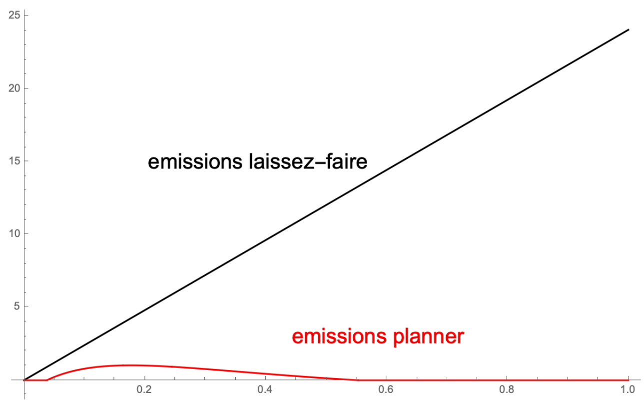


Figure 6. Emission in Laissez-faire Economy

Note: The figure illustrates the firms' emissions in the social planner's economy and laissez-faire economy, depending on their emission intensity θ. In the laissez-faire economy, emissions are just increasing with the emission intensity of the firm. In the planner's economy, emissions are reduced to 0 for low-emission firms, positive for intermediate-emission firms, and again equal to 0 for high-emission firms.

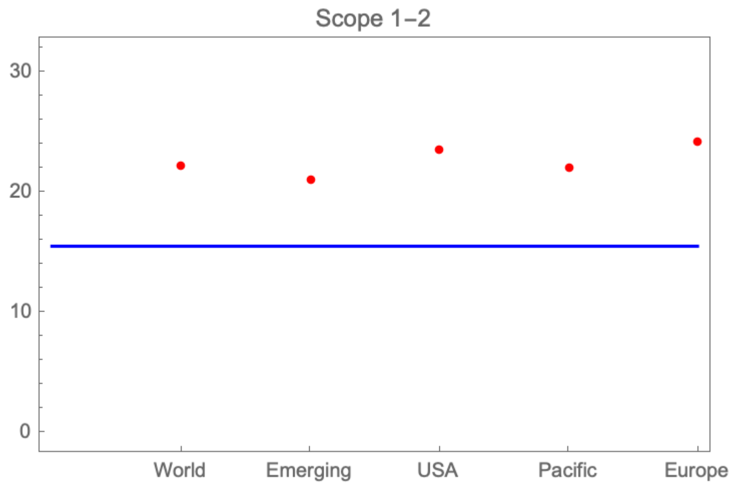


Figure 7. Production level for different estimations of the distribution of carbon intensity

Note: The figure illustrates the aggregate production in the social planner's economy and laissez-faire economy, depending on the distribution of the carbon intensity in some regions, for an arbitrary parametrization of the model. In the laissez-faire economy (blue line), the aggregate production is not affected by the carbon intensity of firms in the economy. In the planner's economy (red dots), aggregate production depends on the distribution of carbon intensity.

effective emissions.

We start from the premise that the operator charges an access fee as a function of effective emissions, $\tau(e)$. Once a firm pays this access fee (which we do not restrict to be positive), the operator lends at rate $i(e)$, again conditional on effective emissions. Alternatively, $i(e)$ could also be interpreted as a haircut on bonds. A policy is a list $(\pi, \tau(e), i(e))$. Our goal in this section is to find the optimal design of the policy, that is the characterization of the level of inflation π and of the functions $\tau(\cdot)$ and $i(\cdot)$. Finally, we assume that the operator is conditioning its green loans on the firms investing the funds in their project rather than, for instance, lending the borrowed funds on the cash market.

Taking the policy as given, we first investigate the problem of the firms that decide to pay the access fees $\gamma + \tau(e)$ and can borrow from the operator, given real wage, the inflation level, and the interest rate i_m in the cash market. Then, we consider the problem of the other firms and finally the general equilibrium prices and allocations.

5.1 Problem of Firms that Decide to Access the Liquidity Backstop

We denote by $V_1(\theta)$ the value of a firm with emission intensity θ, which paid the cost γ and accesses the liquidity backstop. We guess and verify later that it is optimal for the operator to always lend at or below the market rate. Therefore, a firm will not want to borrow from the cash market while paying the access cost.

We proceed backward. First, we consider the problem of a firm with access that has to refinance, given its initial investment q, its cash holding m, its borrowing at the liquidity backstop facility in the first investment stage ℓ^0, and its emissions e. If we assume again that the project is brought to fruition even with a refinancing need, the value of that firm is defined as:

$$W_1(m; q, \ell^1, e) = \max_{\ell^2} f(q) + \phi\ell^1 - swq - \phi(1 + i(e))\ell^1 - \phi(1 + i(e))\ell^2,$$

subject to $swq \leq \phi(m + \ell^2)$, where ℓ^1 denotes borrowing in the investment stage and ℓ^2 denotes borrowing in the refinancing stage. Either $swq \leq \phi m$ and the firm does not borrow from the liquidity backstop facility, or the firm borrows $\phi\ell^2 = swq - \phi m$. Denoting by λ_{ℓ^2} the Lagrange multiplier on the refinancing constraint, the first-order condition gives $i(e) = \lambda_{\ell^2}$. The envelope conditions are:

$$\frac{\partial W_1}{\partial m} = -\frac{\partial W_1}{\partial \ell^1} = \phi(1+i(e)), \quad \frac{\partial W_1}{\partial q} = f'(q) - (1+i(e))sw, \quad \text{and} \quad \frac{\partial W_1}{\partial e} = -\phi i'(e)(\ell^1 + \ell^2).$$

Then, the value of that firm when it does not incur the refinancing shock is simply:

$$W_0(m; q, \ell^1, e) = f(q) + \phi m - \phi(1 + i(e))\ell^1.$$

Note that we assume that loans are contingent on being invested in the real economy and we do not allow firms to borrow from the backstop facility to then lend on the cash market. The envelope conditions are

$$\frac{\partial W_0}{\partial m} = \phi, \quad \frac{\partial W_0}{\partial q} = f'(q), \quad \frac{\partial W_0}{\partial \ell^1} = -\phi\left(1+i(e)\right), \quad \text{and} \quad \frac{\partial W_0}{\partial e} = -\phi i'(e)\ell^1.$$

Turning to the investment decision, the firm solves:

$$\begin{aligned} V_1(\theta) \quad &= \quad \max_{q,e,\ell^1} \quad -h(\theta q - e) - \tau(e) + \rho W_1(\ell^1 - wq/\phi; q, \ell^1, e) \qquad (10) \\ &\qquad\qquad +(1-\rho)W_0(\ell^1 - wq/\phi; q, \ell^1, e) \\ s.t. \quad &\quad wq \leq \phi\ell^1 \quad \text{and} \quad 0 \leq e \leq \theta q. \qquad (11) \end{aligned}$$

We have used the fact that the wage is paid in cash and diminishes cash holdings. In addition, the cost of reducing emissions from their natural level and the access fee $\tau(e)$ are subtracted from the profit of the firm, and therefore act as a penalty relative to market funding. Also, when the firm borrows ℓ^1 from the liquidity backstop facility, its debt is $(1+i(e))\ell^1$. Therefore, the operator of the backstop facility offers a schedule of interest rate that depends on realized emissions as observed at the time the firm has to repay the operator. The two constraints are the cash in advance constraint on wages as well as the constraint on effective emissions. Let λ, λ_θ, and λ_0 be Lagrange multipliers on the cash constraint, the upper and the lower bounds on e, respectively. Combining the first-order conditions gives the following equations characterizing the choice of firms that have access to and borrow from the liquidity backstop facility:

$$\begin{aligned} f'(q) - \theta h'(\theta q - e) + \theta\lambda_\theta &= (1+i(e))w(1+\rho s), \\ h'(\theta q - e) + \lambda_0 - \lambda_\theta &= \tau'(e) + i'(e)\left(\phi\ell^1 + \rho\phi\ell^2\right). \end{aligned}$$

These conditions provide us with the additional following characterizations:

1. **If $\tau'(e) > 0$, all firms with access to the liquidity backstop facility will partially clean their act.**

2. **If $i'(e) > \tau'(e) = 0$, all firms with access to the liquidity backstop will clean their act *if and only if* they borrow from the facility.** Set $i'(e) > 0$ but suppose $\ell^1 = \ell^2 = 0$. Then, the second condition above gives $\theta q = e$. For the "only

if" part, suppose firms clean their act $\theta q > e$. Then, we have $h'(\theta q - e) > 0$ and it must be that $\ell^1, \ell^2 \geq 0$ with one strict inequality.

3. **To be effective in reducing emissions, the policy must depend on effective emissions.** In other words, suppose that $\tau'(e) = i'(e) = 0$. Then, firms with access to the liquidity backstop facility do *not* clean their act. We already showed that if $\tau'(e) > 0$ or $i'(e) > 0$, then firms will clean their act. Now suppose $\tau'(e)$ and $i'(e) \leq 0$, then from the second condition above, $\theta q = e$ because $h'(0) = 0$ and $h'(\cdot) \geq 0$.

From characterizations 1 and 2, it follows that $e < \theta q$, that is $\lambda_\theta = 0$, whenever $\tau' > 0$ or $i' > 0$, or both. In other words, all firms will reduce their emissions.

Since we showed that $\lambda_\theta = 0$, we are now left with two cases related to whether firms will choose zero or strictly positive effective emissions. First, suppose $\lambda_0 = 0$ so that $e > 0$. Then, the solution for (q, e) is characterized by:

$$\begin{aligned} f'(q) - \theta h'(\theta q - e) &= w(1+\rho s)(1+i(e)) \\ h'(\theta q - e) &= \tau'(e) + i'(e)\left(\phi\ell^1 + \rho\phi\ell^2\right). \end{aligned}$$

Otherwise, suppose $\lambda_0 > 0$ so that $e = 0$. Then, the solution for q is characterized by:

$$\begin{aligned} f'(q) - \theta h'(\theta q) &= (1+i(e))w(1+\rho s) \\ h'(\theta q - e) + \lambda_0 &= \tau'(0) + i'(0)\left(\phi\ell^1 + \rho\phi\ell^2\right). \end{aligned}$$

Comparing these first-order conditions to Equations (2)–(3) with $\lambda_\theta = 0$, we obtain the important following result:

Lemma 2. *A necessary condition for the operator of the liquidity backstop to achieve the efficient allocation is $i(e) = \bar{\imath}$ and $\tau'(e) > 0$, for all e.*

In words, the optimal policy is a two-part tariff policy consisting of a lump-sum effective emissions-contingent fee $\tau(e)$ and a non-contingent interest rate $\bar{\imath}$. Comparing the first-order conditions above to Equations (2)–(3), it is easy to see that the operator can achieve the efficient allocation only if $(1 + i(e))w(1 + \rho s)$ is constant and equal to $(1 + \rho s)u'(c^*)^{-1}$. So $i(e)$ should be a constant for all e. Making the facility lending rate a function of emission would be effective in reducing emissions, as we have shown above. However, it would do so by distorting the hiring decision of firms relative to the first best: If $i'(e) > 0$, high-emission firms would factor in that their hiring cost is higher. As a consequence, they would reduce hiring (as well as emissions) so that their marginal (net) benefit of hiring would equal their marginal cost (the cost of borrowing at $i(e)$ to pay wages). This is inefficient as we have seen that the social planner would prefer to have the same marginal (net) benefit across all firms. Therefore, the only lending rate that allows the operator of the liquidity backstop to achieve the first-best solution is one that does not depend on effective emissions.

Since the first-best policy consists in all firms reducing their level of emissions, and since the lending rate cannot depend on effective emissions, it must be that $\tau'(e) > 0$. In fact, $\tau(e)$ should be an affine function for some $\bar{\tau} \in \mathbb{R}$,

$$\tau(e) = \bar{\tau} + \frac{e}{u'(c)}, \tag{12}$$

so that the fee to access the liquidity backstop facility should also depend on aggregate economic conditions (here, summarized by the wage $w = 1/u'(c)$). Taking $c = c^*$, the policy achieves the efficient allocation as long as all firms pay the access fee to the operator.

The payoff of firms with access to the liquidity backstop is

$$V^A(\theta) = -h(\theta q - e) - \tau(e) + f(q) - (1 + i(e))(1 + \rho s)wq. \tag{13}$$

5.2 Decision to Access the Liquidity Backstop

Instead of paying the access fee to the liquidity backstop facility, firms can opt to borrow from the cash market at rate $i_{m,1}$ during the investment stage and at rate $i_{m,2}$ during the refinancing stage, as in Section 4. Those firms will not reduce their emissions, because none of the market prices depends on their effective emissions.

Here, we solve for the decision to pay the access fee γ for a specific firm with emission intensity θ. This firm will prefer to pay the access fee whenever its expected payoff is higher than borrowing from the cash markets,

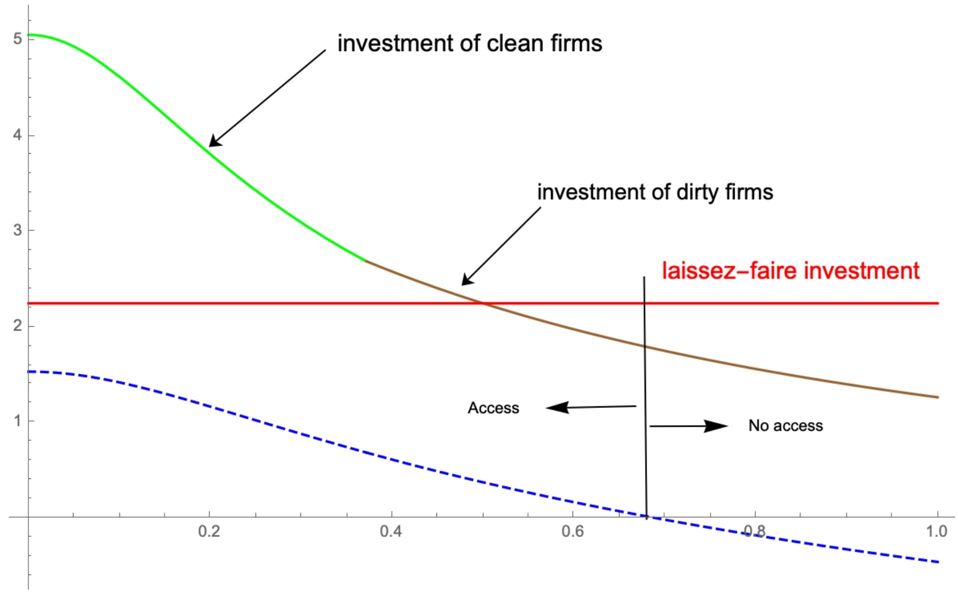
$$V^A(\theta) - \gamma > \max\{V^P, V^S(\theta)\}. \tag{14}$$

where V^P , V^S, and $V^A(\theta)$ are defined in Equations (6), (8), and (13), respectively. Since $V^A(\theta)$ is decreasing in θ, we obtain our second result, which we prove in the remainder of this section.

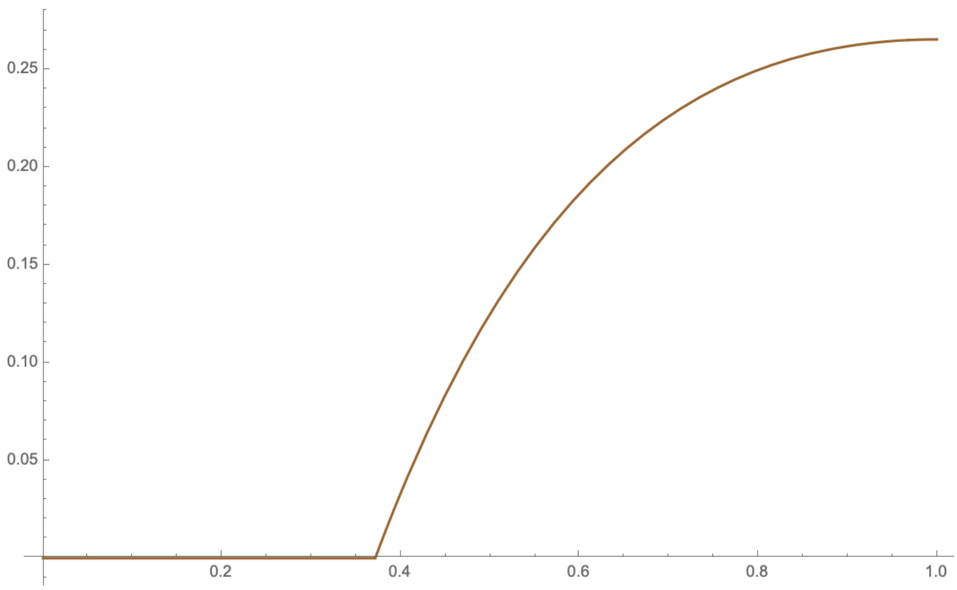
Lemma 3. *Firms with relatively low emissions (with $\theta \leq \hat{\theta}$, for some $\hat{\theta}$ defined in the proof) prefer to access the liquidity backstop facility, firms with relatively high emissions do not. The cutoff $\hat{\theta}$ depends on the cost of funding in the money market, i. The operator can entice firms to acquire access, either by increasing the cost of market liquidity (increasing i), by using inflation (and encouraging access, which reduces the amount of cash in the cash market), and/or by reducing the fixed part of the access fee $\bar{\tau}$ (which can become negative). The operator can achieve the efficient allocation when all firms prefer to have access. The operator can mop up excess liquidity in the money market by offering a deposit facility with a remuneration rate $1 + i$.*

Proof: See Appendix C.

Figure 8 illustrates the access and investment choices of firms (Panel a), as well as their effective level of emissions (Panel b). The green and brown segments depict the investment levels of respectively low-emission (i.e., $e(\theta) = 0$) and high-emission (i.e., $e(\theta) > 0$) firms, when they would borrow from the liquidity backstop facility (also the first-best levels). The horizontal red line corresponds to the investment level in the laissez-faire economy, which does not depend on the emission intensity θ. The monetary conditions here imply an interest rate in the cash market such that all firms with $\theta \leq 0.5$ would hire more when accessing the liquidity backstop than under laissez-faire and the other way around with $\theta > 0.5$. The blue dotted curve summarizes the access decision: When the curve shows a positive value given θ, firms with this emission intensity prefer to access the liquidity backstop facility. Otherwise, they prefer to finance their production in the cash market to hire the laissez-faire quantity of labor. Notice that some firms prefer to access the liquidity backstop facility, although they would invest less than the level under laissez-faire. The reason is that in this example the operator sets $\bar{\tau} < 0$ to entice more firms to access. When $\bar{\tau} \geq 0$, no firm will access the liquidity backstop facility to invest a lower amount than in the laissez-faire economy. Finally, a large fraction of investment will take place in low-emission firms.



(a) Investment levels



(b) Emission levels

Figure 8. Equilibrium levels of hires and emissions

Note: In the figure, Panel a illustrates the firms' investment in the case of an optimal liquidity backstop facility (with the green segment corresponding to low-emission firms and the brown segment to high-emission firms) and in the laissez-faire economy (horizontal red line). the blue dotted curve represents the decision of firms to access the liquidity backstop facility: firms access the facility when the value is positive (up to $\theta = 0.68$) and do not access the facility when the value is negative. Panel b represents the emission level of firms with emission intensity θ. Emission is equal to 0 for firms with $\theta \leq \theta^r$.

6 Discussion

In this section, we discuss some implications of the optimal two-part tariff highlighted in the previous section. There are several dimensions that need to be discussed.

First, we have assumed that threshold θ^r, which separates firms experiencing a run and firms with no run, is exogenously given. Firms with an emission intensity θ above θ^r are the most likely to suffer from stranded assets. In fact, the threshold θ^r could be determined by the central bank or the regulator to correspond to an acceptable level of carbon emissions, for instance the level of emissions that would limit the increase in temperature to 2 degrees. Andersson et al. (2016),Ehlers et al. (2020), and Jondeau et al. (2021) examine alternative strategies to determine such a desirable level of carbon intensity. It should be noted that, as the distribution of carbon intensity is extremely right skewed, it would be possible to substantially reduce the amount of carbon emitted in the production process by forcing a limited number of firms to reduce their carbon emissions.

Second, our policy can be viewed as the design of a collateral framework. It is worth mentioning that the European Central Bank has recently decided that bonds with coupon structures linked to the satisfaction of some sustainability performance targets will be eligible as collateral for Eurosystem credit operations and also for Eurosystem outright purchases for monetary policy purposes (provided they comply with all other eligibility criteria). For sustainability-linked bonds, the performance target refers to some environmental objectives, such as those defined by the United Nations Sustainable Development Goals. Oustry et al. (2020) also explore an approach to factoring climate-related transition risks into a central bank's collateral framework. The approach consists in aligning the collateral pools pledged by the counterparty with a given climate target.

Third, in our two-part tariff policy, firms have to pay a fixed cost, determined according to their emission intensity, and then they can obtain from the regulator the amount of cash that they need (with no ex-ante restriction on the amount of money that they borrow). This two-level mechanism is relatively close to the logic of regulatory capital, imposed by regulators, that financial institutions need to hold to preserve financial stability. Capital requirements are determined by the risk-weighted assets of individual banks but then all banks have the same access to central bank liquidity if needed. Deposits insurance, such as implemented by Federal Deposit Insurance Corporation (FDIC) in the United States, relies on a similar mechanism: banks have to satisfy some liquidity and reserve requirements to benefit from the insurance mechanism. Once the bank is insured, the

FDIC deposit insurance covers all deposit accounts (up to a certain limit).[13]

Fourth, in the model we assume that firms decide to disclose their emission intensity θ if such revelation allows them to benefit from better conditions in the first cash market. This optional disclosure raises two issues. On the one hand, we could imagine that a mandatory disclosure should raise welfare in equilibrium. In fact, Alvarez and Barlevy (2015) find that, in the context of bank balance sheet information, mandatory disclosure is not always desirable for financial stability. In our context, ex-ante mandatory disclosure would result in the separating equilibrium discussed in Section 4.5: as high-emission firms are initially forced to borrow more in the first period to avoid a default if refinancing risk materializes, this would in principle result in a lower investment by high-emission firms. Of course, imposing disclosure after the first stage would probably result in a dramatic run on all high-emission firms. This mechanism would be close to the "paradox is that success is failure" (Carney, 2016), as full information would result in a rapid adjustment and run, which would jeopardize financial stability. On the other hand, disclosure would require some certification by external parties, such as rating agencies. Several data providers are already involved in the process of assessing the carbon footprint of firms. Such data can be used by investors to reduce the exposure of their portfolio to carbon risk (Bolton and Kacperczyk, 2020, and Görgen et al., 2020b).

In our model, the access fee τ could in principle be negative to incentivize all firms to access the liquidity backstop facility. In a dynamic setting, the operator of the facility could impose a positive access fee that would be reimbursed to firms that have managed to reduce their carbon emissions. Again, a verification device would be necessary.

[13] Relatedly, capital requirements could be determined according to the exposure of banks to climate risk. For instance, Thomä and Hilk (2018) and Berenguer et al. (2020) propose a mechanism by which a higher exposure to green assets and a lower exposure to brown assets would reduce banks' capital reserve requirements. Kocherlakota (2010) drew the reverse parallel by proposing that risk taking be taxed in the same way that an externality, such as pollution, should be taxed.

References

Alvarez, F., Barlevy, G., 2015. Mandatory disclosure and financial contagion. NBER Working Paper No 21328.

Andersson, M., Bolton, P., Samama, F., 2016. Hedging climate risk. Financial Analysts Journal 72(3).

Barnett, M., 2019. A run on oil: Climate policy, stranded assets, and asset prices. Working Paper.

Battiston, S., Mandel, A., Monasterolo, I., Schütze, F., Visentin, G., 2017. A climate stress-test of the financial system. Nature Climate Change 7(4), 283–288.

Berenguer, M., Cardona, M., Evain, J., 2020. Integrating climate-related risks into banks' capital requirements. Institute for Climate Economics Report.

Blackrock, 2020. Blackrock survey shows acceleration of sustainable investing. Available at https://www.blackrock.com/corporate/newsroom/press-releases/article/corporate-one/press-releases/blackrock-survey-shows-acceleration-of-sustainable-investing.

Bolton, P., Despres, M., Pereira Da Silva, L.A., Samama, F., Svartzman, R., 2020. The green swan: Central banking and financial stability in the age of climate change. Bank for International Settlements Working Paper.

Bolton, P., Kacperczyk, M., 2020. Do investors care about carbon risk? NBER Working Paper No. 26968 .

Carney, M., 2016. Resolving the climate paradox. Text of the Arthur Burns Memorial Lecture, Berlin, 22 September 2016. Available at: https://www.bis.org/review/r160926h.pdf.

Ehlers, T., Mojon, B., Packer, F., 2020. Green bonds and carbon emissions: Exploring the case for a rating system at the firm level. BIS Quarterly Review, September 2020 .

Villeroy de Galhau, F., 2020. Paris 2020 climate finance day. Speech at the Paris 2020 Climate Finance Day.

Görgen, M., Jacob, A., Nerlinger, M., Riordan, R., Rohleder, M., Wilkens, M., 2020a. Carbon risk. Working Paper.

Görgen, M., Jacob, A., Nerlinger, M., Riordan, R., Rohleder, M., Wilkens, M., 2020b. Carbon risk. Working Paper, Available at SSRN: https://ssrn.com/abstract=2930897.

Holmström, B., Tirole, J., 1998. Private and public supply of liquidity. Journal of Political Economy 106(1).

IRENA, 2017. Stranded assets and renewables: How the energy transition affects the value of energy reserves, buildings and capital stock. International Renewable Energy Agency (IRENA) Working Paper.

IRENA, 2021. Global trends. Available at https://www.irena.org/Statistics/View-Data-by-Topic/Costs/Global-Trends.

Jondeau, E., Mojon, B., Pereira Da Silva, L.A., 2021. Excluding firms with large carbon emission for portfolio investment. Bank for International Settlements Working Paper.

Kocherlakota, N., 2010. Taxing risk and the optimal regulation of financial institutions. Federal Reserve Bank of Minneapolis Economic Policy Paper 10-3, Available at https://www.minneapolisfed.org/article/2010/taxing-risk-and-the-optimal-regulation-of-financial-institutions-epp.

Krogstrup, S., Oman, W., 2019. Macroeconomic and financial policies for climate change mitigation: A review of the literature. IMF Working Paper No. 19/185.

Lagarde, C., Gaspar, V., 2019. Getting real on meeting paris climate change commitments. IMF Blog, May 3.

Mercure, J.F., Pollitt, H., Viñuales, J.E., Edwards, N.R., Holden, P.B., Chewpreecha, U., Salas, P., Sognnaes, I., Lam, A., Knobloch, F., 2018. Macroeconomic impact of stranded fossil fuel assets. Nature Climate Change 8(7).

Monnet, C., Temzelides, T., 2016. Monetary emissions trading mechanisms. International Journal of Economic Theory 12, 85–100.

Oustry, A., Erkan, B., Svartzman, R., Weber, P.F., 2020. Climate-related risks and central banks' collateral policy: A methodological experiment. Banque de France Working Paper No 790.

Pastor, L., Stambaugh, R.F., Taylor, L.A., 2021. Sustainable investing in equilibrium. Journal of Financial Economics .

Pedersen, L.H., Fitzgibbons, S., Pomorski, L., 2021. Responsible investing: The ESG-efficient frontier. Journal of Financial Economics .

Reinders, H.J., Schoenmaker, D., van Dijk, M., 2020. A finance approach to climate stress testing. Working Paper.

Thomä, J., Hilk, A., 2018. The green supporting factor: Quantifying the impact on european banks and green finance. 2 Degrees Investing Initiative Report.

Van der Ploeg, F., Rezai, A., 2018. Macroeconomic and financial policies for climate change mitigation: A review of the literature. Working Paper.

Vermeulen, R., Schets, E., Lohuis, M., Kölbl, B., Jansen, D., 2019. The heat is on: A framework measuring financial stress under disruptive energy transition scenarios. DNB Working Paper No. 625.

Appendices

A Pooling Equilibrium

A.1 Problem of Low-emission Firms ($\theta < \theta^r$)

Firms with emission factor $\theta < \theta^r$ can always borrow on the second cash market. We guess that these firms will want to refinance rather than terminate their project. Then at the refinancing stage, the value of a firm that holds m units of cash, used q units of labor in the initial investment stage and borrowed ℓ_1 in the first cash market, and **incurs the refinancing shock**, is

$$V_1(m; q, \ell_1) = \max_{\ell_2} f(q) - swq + \phi(m + \ell_2) - \phi(1 + i_{m,1})\ell_1 - \phi(1 + i_{m,2})\ell_2 \tag{A.1}$$

subject to $swq \leq \phi(m + \ell_2)$. Wages from the first round of hiring have already been paid. So either $swq \leq \phi m$ and the firm does not borrow on the cash market (but lends), or the firm borrows its cash shortfall $\phi\ell_2 = swq - \phi m$.

Then the problem of a firm that **does not incur the refinancing shock** is

$$V_0(m; q, \ell_1) \quad = \quad \max_{\ell_2} f(q) + \phi(m + \ell_2) - \phi(1 + i_{m,1})\ell_1 - \phi(1 + i_{m,2})\ell_2 \tag{A.2}$$

subject to $\phi(m + \ell_2) \geq 0$. This firm can lend on the cash market.

Then, we can define the problem of low-emission firms in the first cash market and prior to incurring the refinancing shock (these firms are not holding cash but borrow ℓ_1 to invest q), as

$$V = \max_{q,\ell_1} (1 - \rho)V_0(\ell_1 - wq/\phi; q, \ell_1) + \rho V_1(\ell_1 - wq/\phi; q, \ell_1) - \phi(1 + i_{m,1})\ell_1 \tag{A.3}$$

subject to $wq \leq \phi\ell_1$. Using λ_q as the Lagrange multiplier on the cash constraint, and using the first-order and envelope conditions for problems (A.1) and (A.2), the first-order condition with respect to investment is

$$f'(q) - w(1 + \rho s)(1 + i_{m,2}) - w\lambda_q = 0,$$

and the one with respect to borrowing/lending gives

$$\lambda_q = i_{m,1} - i_{m,2}.$$

Therefore, $\lambda_q > 0$ whenever $i_{m,1} > i_{m,2}$ – and low-emission firms only borrow what they need to invest in the initial investment stage – and $\lambda_q = 0$ otherwise, and low-emission firms hold money for precautionary reasons. Note that $i_{m,2} > i_{m,1}$ cannot be in equilibrium (as all firms with $\theta < \theta^r$ would borrow as much as possible at $i_{m,1}$ in the first cash market and then lend at $i_{m,2}$ in the second cash market). Now, as $\lambda_q = i_{m,2} - i_{m,1}$, we obtain

$$f'(q) \quad = \quad w(1 + i_{m,1}) + w\rho s(1 + i_{m,2}). \tag{A.4}$$

So the marginal cost of production equals the cost of the loan to fund the wage necessary to pay the last unit of production in the investment stage as well as the expected cost to refinance the project in the second cash market.

A.2 Problem of High-emission Firms ($\theta \geq \theta^r$)

In the pooling equilibrium, high-emission firms with $\theta \geq \theta^r$ pretend to be clean. Therefore, they borrow the same amount ℓ_1 as low-emission firms in the first cash market and they invest the same amount q in the initial investment stage. We already know that if $i_{m,1} > i_{m,2}$ low-emission firms just borrow what they need for their initial investment in the first cash market. This means that a high-emission firm mimicking a low-emission firm will not be able to refinance and will default whenever $i_{m,1} > i_{m,2}$.

We first focus on the case of high-emission firms that do not have to refinance. Then, in the second cash market, the value of a firm, that holds balances m, invested q, borrowed ℓ_1 in the first cash market, and **does not incur the refinancing shock** is the same as the one of low-emission firms. A high-emission firm will lend its idle cash in the second cash market for $i_{m,2}$. Its payoff then is given by

$$V_0^r(m; q, \ell_1) = \max_{\ell_2} f(q) + \phi(m + \ell_2) - \phi(1 + i_{m,1})\ell_1 - \phi(1 + i_{m,2})\ell_2$$

subject to $\phi(m + \ell_2) \geq 0$. A high-emission firm that does not need to refinance will lend all its idle cash on the cash market.[14]

We now turn to the problem of high-emission firms with a refinancing need. They can be in two possible states: they can either hold enough cash to self-refinance or not in which case they default.

A.2.1 High-emission firms hold enough cash to self-refinance (equilibrium where $i_{m,1} = i_{m,2}$)

We first concentrate on the equilibrium where high-emission firms ($\theta \geq \theta^r$) that need refinancing **borrowed enough cash in the first period to be able to refinance if the need ever arose**. Then the value of a firm with $\theta \geq \theta^r$, which holds balances m, invested q, borrowed ℓ_1 in the first cash market, and that *needs refinancing* is

$$V_1^r(m; q, \ell_1) = \max_{\ell_2} f(q) - wsq + \phi m - \phi(1 + i_{m,1})\ell_1 - \phi(1 + i_{m,2})\ell_2^r$$

subject to $wsq^r \leq m + \ell_2^r$ and $\ell_2^r \leq 0$, where ℓ_2^r cannot be positive since these firms cannot borrow on the cash market, but they can certainly lend cash there. Since ℓ_2^r is large enough to cover the refinancing need, from the first-order condition we obtain that a firm that incurs the refinancing shock either lends all its idle cash on the cash market, or has not cash to lend and uses all its cash to refinance. Notice that, again, we do not consider the envelope conditions because in a pooling equilibrium, m, q, and ℓ_1 are not choice variables for the high-emission firms.

[14] We do not need consider the envelope conditions because in a pooling equilibrium, m, q, and ℓ_1 are not choice variables for high-emission firms.

A.2.2 High-emission firms do not have enough cash to self-refinance (equilibrium where $i_{m,1} > i_{m,2}$)

We now analyze the equilibrium where firms with $\theta \geq \theta^r$ and that need refinancing **do not have enough cash to self-refinance**. Then these firms have to liquidate their investment for which they get nothing and they default on the loan they contracted in the first cash market.[15]

A.3 Savings Decision

We now consider the problem of savers. In the equilibrium where $i_{m,1} > i_{m,2}$, savers now anticipate that the loan they made in the first cash market will only be repaid by the $(1-\rho)$ firms that do not suffer a refinancing shock and by the $\rho G(\theta^r)$ low-emission firms that suffered one. The other (high-emission) firms will default. Savers are diversifying the default risk by lending to the entire $[0,1]$-continuum of firms in the first cash market. Hence, they solve

$$\begin{aligned}
V(m) \quad &= \quad \max_{\ell_1,\ell_2,m_+} \phi(m-\ell_1-\ell_2) + (1-\rho+\rho G(\theta^r))\,\phi(1+i_{m,1})\ell_1 + \phi(1+i_{m,2})\ell_2 - \phi m_+ + \beta V(m_+) + P \\
s.t. \quad & \qquad \phi\ell_1 \leq \phi m \qquad\qquad (\lambda_{\ell,1}) \\
& \qquad \phi\ell_2 \leq \phi\,(m-\ell_1) \qquad (\lambda_{\ell,2})
\end{aligned}$$

where $\lambda_{\ell,1}$ and $\lambda_{\ell,2}$ are the Lagrange multipliers on the first and second constraint, respectively. Combining the first and second first-order conditions, we obtain

$$(1-\rho+\rho G(\theta^r))\,(1+i_{m,1}) - (1+i_{m,2}) = \lambda_{\ell_1}.$$

We note that, when savers do not lend all their cash in the first cash market, we have $(1-\rho+\rho G(\theta^r))\,(1+i_{m,1}) = (1+i_{m,2})$, which implies that $i_{m,1} > i_{m,2}$. Therefore, in any pooling equilibrium, $i_{m,1} > i_{m,2}$, which implies that firms only borrow what they need for their initial investment in the first cash market, and high-emission firms will default in case they have to refinance. Combining the envelope condition and the first-order condition with respect to savings gives

$$1+i_{m,1} = \frac{1+i}{1-\rho+\rho G(\theta^r)},$$

so the first cash market rate is the risk-free rate adjusted for the default risk.

The only equilibrium is the one where savers do not lend all their cash: since firms never carry idle cash into the second cash market, they would not be able to refinance if savers were not carrying cash. Savers do not lend all their cash whenever

$$(1-\rho+\rho G(\theta^r))\,(1+i_{m,1}) = (1+i_{m,2}) = \frac{1+\pi}{\beta} = 1+i$$

[15] We assume the liquidation value of the investment is zero. We could assume it is a small fraction of $f(q)$.

and Equation (A.4) gives

$$f'(q) = w\frac{(1+i)}{(1-\rho+\rho G(\theta^r))} + w\rho s\,(1+i)\,.$$

This is an equilibrium whenever savers have enough real balances to cover the funding needs of all firms, that is $\phi m - wq \geq s\rho G(\theta^r) wq$. We can find ϕm by setting this equation with equality.

Finally, in order to analyze the decision of low-emission firms to pay the revelation cost, we need to know they expected payoff in a pooling equilibrium. It is given by:

$$f(q) - wq\left(\frac{1}{1-\rho+\rho G(\theta^r)} + \rho s\right)(1+i).$$

B Separating Equilibrium

B.1 Problem of Low-emission Firms ($\theta < \theta^r$)

The problem of low-emission firms is the same as with no run risk. So if $i_{m,1} > i_{m,2}$, these firms just borrow in the first cash market what they need to invest. Using the expression for λ_q, we obtain the investment of firms as

$$f'(q) = w(1+i_{m,1}) + w\rho s(1+i_{m,2}). \tag{A.5}$$

If $i_{m,1} = i_{m,2} = i_m$, their investment is given by

$$f'(q) = w(1+\rho s)(1+i_m),$$

and the expected payoff is

$$f(q) - w(1+i_{m,1}) + w\rho s(1+i_{m,2}) - \gamma.$$

B.2 Problem of High-emission Firms ($\theta \geq \theta^r$)

The problem of high-emission firms is more involved because those firms have to decide whether to borrow in the first period for precautionary reasons and self-refinance if need be, or to liquidate in case they are hit by the refinancing shock.

In the second cash market, the value of a firm with $\theta \geq \theta^r$, that holds balances m, invested q^r, borrowed ℓ^r in the first cash market, and that *does **not** need refinancing* is

$$V_0^r(m; q^r, \ell^r) = \max_{\ell_2^r} f(q^r) + \phi(m+\ell^r) - \phi(1+i_{m,1}^r)\ell_1^r - \phi(1+i_{m,2})\ell_2^r$$

subject to $\phi(m+\ell_2^r) \geq 0$, where $i_{m,1}^r$ is the interest rate charged to high-emission firms in the first cash market. Notice that this firm can lend its idle cash to low-emission firms in the second cash market at the market rate $i_{m,2}$. Using λ_m as the Lagrange multiplier on the cash constraint, the first-order condition with respect to ℓ_2^r is

$$\lambda_m = i_{m,2},$$

so a high-emission firm that does not need to refinance will lend all its idle cash on the cash market. The envelope conditions then are

$$\frac{\partial V_0^r}{\partial m} = \phi(1 + i_{m,2}), \quad \frac{\partial V_0^r}{\partial q^r} = f'(q^r), \quad \text{and } \frac{\partial V_0^r}{\partial \ell_1^r} = -\phi(1 + i_{m,1}^r).$$

Now consider the problem of a high-emission firms who need to refinance.

B.2.1 High-emission firms hold enough cash to self-refinance

We first concentrate on the case where firms with $\theta \geq \theta^r$ and that need refinancing **have enough cash to self-refinance**. Then the value of a firm with $\theta \geq \theta^r$, which holds balances m, invested q^r, borrowed ℓ_1^r in the first cash market, and *does need refinancing* is

$$V_1^r(m; q^r, \ell_1^r) = \max_{m_+,\ell} f(q^r) - wsq^r + \phi m - \phi(1 + i_{m,1}^r)\ell_1^r - \phi(1 + i_{m,2})\ell_2^r$$

subject to

$$\begin{aligned} wsq^r &\leq m + \ell_2^r \\ \ell_2^r &\leq 0 \end{aligned}$$

where ℓ_2^r cannot be positive since these firms cannot borrow on the cash market, but they can certainly lend cash there. Using λ_m as the Lagrange multiplier on the cash constraint, and λ_0 the one on the lending constraint, the first-order condition is

$$\lambda_m - \lambda_0 = i_{m,2}.$$

So a firm that needs refinancing either lend all its idle cash on the cash market, or has not cash to lend ($\lambda_0 > 0$) and uses all its cash to refinance. The envelope conditions then are

$$\frac{\partial V_1^r}{\partial m} = \phi(1+i_{m,2}+\lambda_0), \quad \frac{\partial V_1^r}{\partial q^r} = f'(q^r) - ws(1+i_{m,2}+\lambda_0), \quad \text{and} \quad \frac{\partial V_1^r}{\partial \ell_1^r} = -\phi(1+i_{m,1}^r).$$

Turning to the decision of firms with $\theta \geq \theta^r$ at the time of making their investment, they maximize their expected payoff by choosing the appropriate level of investment q^r and borrowing on the first cash market ℓ_1^r, and solve the following problem

$$V_r(\theta) = \max_{q^r,\ell_1^r}(1 - \rho)V_0^r(\ell_1^r - wq^r/\phi; q^r, \ell_1^r) + \rho V_1^r(\ell_1^r - wq^r/\phi; q^r, \ell_1^r). \tag{A.6}$$

Since these firms save some cash in case they need to refinance, their cash constraint does not bind in the first cash market. The first-order conditions give

$$\rho\lambda_0 = i^r_{m,1} - i_{m,2},$$

so that, naturally firms with $\theta \geq \theta^r$ will not participate in the second cash market (even as lenders) whenever the rate in the first cash market rate, $i^r_{m,1}$, is greater than the one in the second, $i_{m,2}$ (these firms are better off carrying just enough cash and lending in the first cash market). And the investment level is given by

$$\begin{aligned} f'(q^r) &= (1+i_{m,2})w(1+\rho s) + \left(i^r_{m,1} - i_{m,2}\right) w(1+s) \\ &= \left(1+i^r_{m,1}\right)(1+s)\, w - (1-\rho)(1+i_{m,2})sw. \end{aligned}$$

Therefore a firm with $\theta \geq \theta^r$ will equate the marginal benefit of investment with its expected marginal cost, which consists of the opportunity cost of funding the project with refinancing $(1+s)w$ net of the saving sw the firm makes in case the refinancing shock does not happen that can be lent on the second cash market at rate $i_{m,2}$.

In this equilibrium, high-emission firms do not default since they always have cash to self-refinance. Therefore, the problem of savers when lending to a firm of type θ is (where $i^\theta_{m,1} = i^r_{m,1}$ if $\theta \geq \theta^r$ and $i_{m,1}$ otherwise).

$$\begin{aligned} V(m;\theta) &= \max_{\ell_1,\ell_2,m_+} \phi(m - \int \ell_1^\theta dG(\theta) - \ell_2) + \int \phi(1+i^\theta_{m,1})\ell_1^\theta dG(\theta) + \phi(1+i_{m,2})\ell_2 - \phi m_+ + \beta V(m_+) + P \\ s.t. &\quad \phi \int \ell_1^\theta dG(\theta) \leq \phi m \\ &\quad \phi\ell_2 \leq \phi\left(m - \int \ell_1^\theta dG(\theta)\right). \end{aligned}$$

From the first-order conditions, we obtain $i_{m,1} = i^r_{m,1}$ and there are no risk premium attached to lending to high-emission firms. Savers lend all their savings in the first cash market whenever $i_{m,1} > i_{m,2}$. It is now possible that $i_{m,1} > i_{m,2}$ since high-emission firms with no refinancing need would now be able to lend to low-emission firms with a refinancing need (even if savers have no more cash to lend in the second cash market). Combining the savings decision together with the envelope condition, we obtain the Fisher equation: $1 + i_{m,1} = 1 + i$.

When $i_{m,1} > i_{m,2}$, the market clearing condition in the first cash market gives

$$\phi m = G(\theta^r)wq + (1 - G(\theta^r))(1+s)wq^r$$

and the market clearing condition in the second cash market gives

$$(1 - G(\theta^r))(1-\rho)swq^r = G(\theta^r)\rho swq$$

and the equilibrium is given by

$$\begin{aligned} f'(q) &= w(1+i) + w\rho s(1+i_{m,2}) \\ f'(q^r) &= w(1+i) + w\rho s(1+i_{m,2}) + (i - i_{m,2})sw \end{aligned}$$

so that $q > q^r$. The last three equations can be jointly solved for $i_{m,2}$, q, and q^r. Hence this equilibrium exists whenever $(1 - G(\theta^r))(1 - \rho) > G(\theta^r)\rho$, or $1 - G(\theta^r) > \rho$. In this equilibrium, the expected payoff of low-emission firms is

$$f(q) - wq(1 + i) - \rho swq(1 + i_{m,2}) - \gamma.$$

When $i_{m,1} = i_{m,2} = i$, savers are indifferent lending in the first or second cash market, and $q = q^r$. The market clearing condition in the first cash market gives

$$\phi m \geq G(\theta^r)wq + (1 - G(\theta^r))(1 + s)wq$$

and in the second cash market,

$$\phi m - wq(1 + (1 - G(\theta^r))\rho s) = G(\theta^r)\rho swq$$

or

$$\phi m = wq\left(1 + \rho s\right).$$

This is an equilibrium whenever

$$\begin{aligned} wq\left(1 + \rho s\right) &\geq G(\theta^r)wq + (1 - G(\theta^r))(1 + s)wq \\ (1 + \rho s) &\geq 1 + (1 - G(\theta^r))s \\ \rho &\geq 1 - G(\theta^r). \end{aligned}$$

In this equilibrium, the expected payoff of cleaner firms is

$$f(q) - wq(1 + \rho s)(1 + i) - \gamma.$$

B.2.2 High-emission firms do not have enough cash to self-refinance (not an equilibrium)

We now analyze the equilibrium where firms with $\theta \geq \theta^r$ and that need refinancing **do not have enough cash to self-refinance**. We concentrate on the case where high-emission firms simply holds not cash in the second cash market – this is without loss of generality because the risk premium implies $i^r_{m,1} > i_{m,2}$ and high-emission firms have no incentive to borrow in the first cash market to lend in the second. Then the value of a firm with $\theta \geq \theta^r$, who holds no balances $m = 0$, invested q^r, borrowed ℓ^r_1 in the first cash market, and that *does need refinancing* is simply zero.

Turning to the decision of firms with $\theta \geq \theta^r$ at the time of making their investment, they maximize their expected payoff by choosing the appropriate level of investment q^r and borrowing on the first cash market ℓ^r_1, and solve the following problem

$$V_r(\theta) \;=\; \max_{q^r, \ell^r_1} (1 - \rho)V^r_0(\ell^r_1 - wq^r/\phi; q^r, \ell^r_1) \tag{A.7}$$

subject to $\phi \ell_1^r - wq^r \geq 0$. Since we assume this constraint binds, these firms solve

$$\max_{q^r} (1-\rho)\left[f(q^r) - wq^r(1+i_{m,1}^r)\right]$$

with first order condition

$$f'(q^r) = w(1+i_{m,1}^r)$$

Therefore a firm with $\theta \geq \theta^r$ will equate the expected marginal benefit of investment with its marginal cost, which consists of the opportunity cost of funding the project. These firms default in case they suffer a refinancing shock.

The problem of savers when lending to a firm of type θ is (where $i_{m,1}^\theta = i_{m,1}^r$ if $\theta \geq \theta^r$ and $i_{m,1}$ otherwise) now has to take into account the default probability of loans to high-emission firms, $\delta^\theta = \rho$ whenever $\theta \geq \theta^r$:

$$\begin{aligned}
V(m;\theta) = & \max_{\ell_1,\ell_2,m_+} \phi\left(m - \int \ell_1^\theta dG(\theta) - \ell_2\right) + \int (1-\delta^\theta)\phi(1+i_{m,1}^\theta)\ell_1^\theta dG(\theta) + \phi(1+i_{m,2})\ell_2 \\
& -\phi m_+ + \beta V(m_+) + P \\
s.t. \quad & \phi \int \ell_1^\theta dG(\theta) \leq \phi m \\
& \phi \ell_2 \leq \phi\left(m - \int \ell_1^\theta dG(\theta)\right)
\end{aligned}$$

The first order conditions are

$$\begin{aligned}
\ell_1^{\theta<\theta^r} : -\phi + \phi(1+i_{m,1}) - \phi\lambda_{\ell_1} - \phi\lambda_{\ell_2} &= 0 \\
\ell_1^{\theta\geq\theta^r} : -\phi + (1-\rho)\phi(1+i_{m,1}^r) - \phi\lambda_{\ell_1} - \phi\lambda_{\ell_2} &= 0 \\
-\phi + \phi(1+i_{m,2}) - \phi\lambda_{\ell_2} &= 0 \\
\beta V'(m_+) &= \phi.
\end{aligned}$$

Hence $i_{m,1}^r > i_{m,1}$ and there is a risk premium attached to lending to high-emission firms. Savers lend all their savings in the first cash market whenever $i_{m,1} > i_{m,2}$, as combining the first and second first-order conditions, we get

$$i_{m,1} - i_{m,2} = \lambda_{\ell_1}.$$

Note that it is not possible that $i_{m,1} > i_{m,2}$ since here there would not be any cash on offer in the second cash market (no firm is holding cash, and savers prefer to lend in the first cash market). Therefore the only equilibrium is when $i_{m,1} = i_{m,2} = i$ from combining the Euler and the envelope conditions. Then

$$1 + i_{m,1}^r = \frac{1+i}{1-\rho}.$$

Market clearing in the first cash market gives

$$\phi m \geq G(\theta^r)wq + (1-G(\theta^r))wq^r$$

and in the second cash market,

$$\phi m - [G(\theta^r)wq + (1 - G(\theta^r))wq^r] = G(\theta^r)\rho swq$$
$$\phi m = G(\theta^r)(1 + \rho s)wq + (1 - G(\theta^r))wq^r$$

This is an equilibrium whenever high-emission firms prefer to hold no cash, rather than holding enough cash to self-refinance, or

$$\begin{aligned} (1-\rho)\left[f(q^r) - (1+i^r_{m,1})wq^r\right] &= \\ (1-\rho)f(q^r) - (1+i)wq^r &> \\ f(\tilde{q}^r) - w\tilde{q}^r(1+i) - w\tilde{q}^r\rho s(1+i_{m,2}) - (i - i_{m,2})sw\tilde{q}^r &= \\ f(q) - wq(1+\rho s)(1+i) & \end{aligned}$$

where the last inequality follows from the fact that $i_{m,2} = i_{m,1}$ in this equilibrium and so the investment of high-emission firms who choose to hold enough cash to refinance is the same as the one of low-emission firms. Since low-emission firms can always choose to not refinance but do not, this inequality is never satisfied. Therefore, there is no separating equilibrium where high-emission firms would default.

C Proof of Lemma 3

We consider the pooling equilibrium where $i_{m,1} = i_{m,2} = i$, since this equilibrium gives the higher expected payoff for all firms. Let $q_0(i)$ denote the investment decision of firms without access. Then $q_0(i)$ satisfies

$$f'(q_0(i)) = (1 + i)w(1 + \rho s),$$

where $1 + i = (1 + \pi)/\beta$. And their payoff is given by (6). Then, with q denoting the investment decision of the firms with access, firms will decide to access the liquidity backstop facility whenever

$$f(q) - h(\theta q - e) - \tau(e) - (1 + i(e))(1 + \rho s)wq - \gamma \quad > \quad f(q_0(i)) - (1 + i)(1 + \rho s)wq_0(i).$$

We can show that $V^A(\theta)$ is decreasing in θ, as (using the first order condition of the firm with $\lambda_\theta = 0$), since

$$\frac{\partial V^A(\theta)}{\partial \theta} = -h'(\theta q - e)q(\theta) - \lambda_0 e'(\theta).$$

If $\lambda_0 = 0$ then it is direct that $\frac{\partial V^A(\theta)}{\partial \theta} < 0$ as $h'(\cdot) > 0$. If $\lambda_0 > 0$ we showed $e'(\theta) = 0$ almost surely as $e(\theta) = 0$, so that $\frac{\partial V^A(\theta)}{\partial \theta} \leq 0$. Now consider the firm with $\theta = 0$. The first-order condition for this firm gives

$$f'(q(0)) = w(1 + \rho s)(1 + i(0)).$$

Hence $q(0) > q_0(i)$ whenever $i(0) < i$. Suppose this is the case, the firm with $\theta = 0$ accesses the liquidity backstop facility whenever

$$f\left(q(0)\right) - (1 + i(0))(1 + \rho s)wq(0) - \tau(0) - \gamma > f\left(q_0(i)\right) - (1 + i)(1 + \rho s)wq_0(i)$$

If this is the case, there is $\hat{\theta}$ (possibly the upper-bound of the support of emission factors, $\bar{\theta}$) such that all firms with $\theta \leq \hat{\theta}$ accesses the liquidity backstop facility and all firm with $\theta > \hat{\theta}$ do not. The threshold $\hat{\theta}$ is an endogenous variable that depends on monetary conditions, as captured by the level of the interest rate on the cash market i. As i increases, $q_0(i)$ tends to zero and $\hat{\theta}$ then satisfies

$$f\left(q(\hat{\theta})\right) - h(\hat{\theta}q(\hat{\theta}) - e(\hat{\theta})) - \tau(e(\hat{\theta})) - (1 + \rho s)wq(\hat{\theta})(1 + i(e(\hat{\theta}))) = \gamma.$$

For a given i, all firms access the liquidity backstop facility, i.e. $\hat{\theta} = \bar{\theta}$ whenever

$$\begin{aligned} f\left(q(\bar{\theta})\right) - h(\bar{\theta}q(\bar{\theta}) - e(\bar{\theta})) - \tau(e(\bar{\theta})) - (1 + i(e(\bar{\theta})))(1 + \rho s)wq(\bar{\theta}) - \gamma = \\ f\left(q_0(i)\right) - (1 + i)(1 + \rho s)wq_0(i). \end{aligned}$$

Using the fact that

$$\tau(e) = \bar{\tau} + \frac{e}{u'(c)} = \tau + we,$$

we obtain the value for $\bar{\tau}$ as a function of market conditions i so that all firms access the liquidity backstop facility. If all firms get access, it is straightforward that all firms will produce the efficient amount and emissions will be at the efficient level, with $w = u'\left(c^*\right)^{-1}$. Finally, note that in our economy savers do not face any demand for cash in the money market. To sustain the money market rate to the right level of $1 + i$, the operator of the liquidity backstop facility offers a deposit facility where savers can deposit their idle cash to earn $1 + i$.

Previous volumes in this series

All volumes are available on our website www.bis.org.