

Online Appendix for Understanding Bank and Nonbank Credit Cycles: A Structural Exploration*

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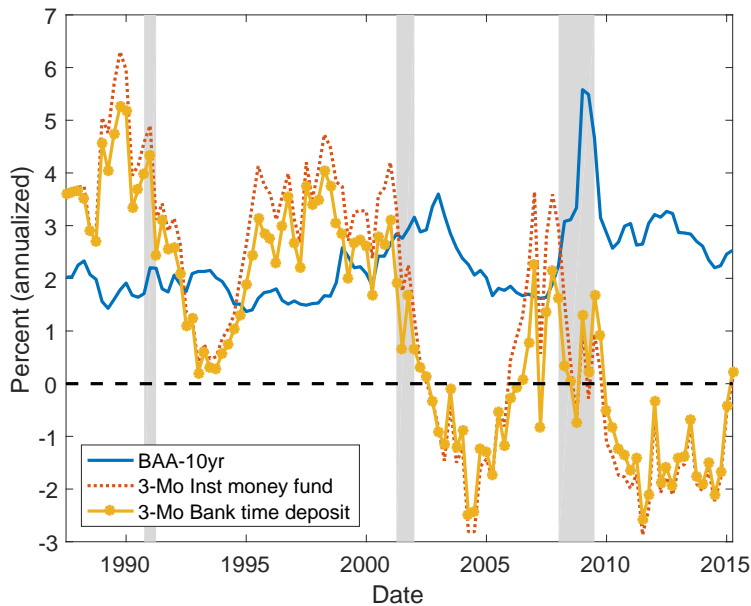
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Appendix

A Data

For our deposit rate data, we use three-month deposit rates for commercial bank time deposits collected from call reports and three-month rates of institutional only money market funds from iMoneyNet. We turn these nominal rates to real rates by subtracting the GDP price deflator inflation from FRED. We use the Moody's Seasoned BAA Corporate bond yield over the 10-year Treasury constant maturity data from FRED to inform our unregulated intermediary lending spreads (see Figure 1).¹

Figure 1: Spreads on Entrepreneurial Loans and Real Rates on Deposits



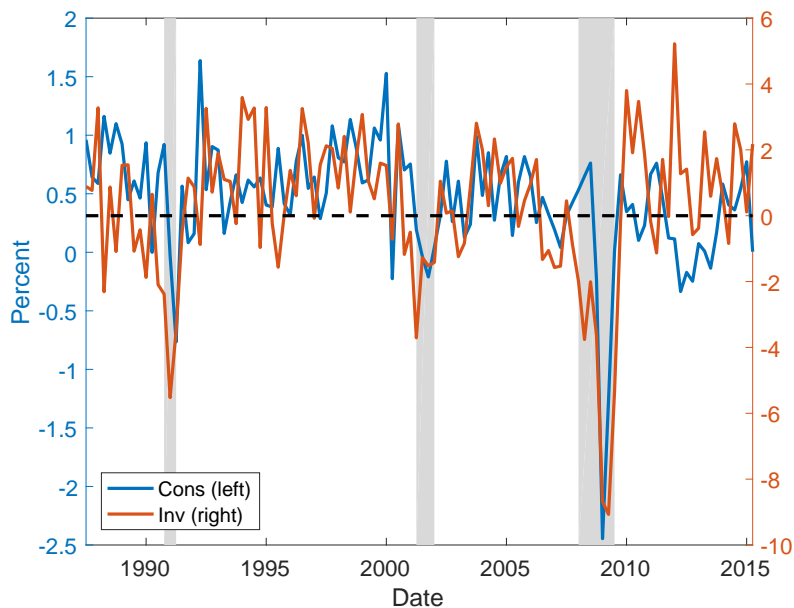
SOURCE: MMF: iMoneyNet, Inc., iMoneyNet Bulk Data - Offshore Analyzer and Gold Analyzer, BAA: Moody's, Moody's Seasoned Baa Corporate Bond Yield [BAA], Commercial bank time deposits: Board of Governors of the Federal Reserve System (US), Call report data.

NOTE: Our deposit rate data is a real rate, as we subtract the GDP price deflator inflation. Gray shaded areas denote NBER recessions.

Our consumption and investment growth data both come from FRED. Consumption is defined as the sum of personal consumption expenditures (PCE) services and nondurables whereas investment is the sum of PCE durables and domestic private investment. We deflate the series using the GDP price deflator and turn them into per capita values by dividing by the civilian noninstitutionalized population aged 16 or over from FRED (see Figure 2).

¹Note that as we are constructing real deposit rates, the three-month Inst money fund and three-month bank time deposit series can go negative.

Figure 2: Growth Rate of Consumption and Investment



NOTE: This data is the real per capita consumption and investment growth series from FRED. Consumption is defined as the sum of PCE services and nondurables, whereas investment is the sum of PCE durables and domestic private investment. We deflate the series using the GDP price deflator and turn them into per capita values by dividing by the civilian noninstitutionalized population aged 16 or over from FRED. Gray shaded areas denote NBER recessions.

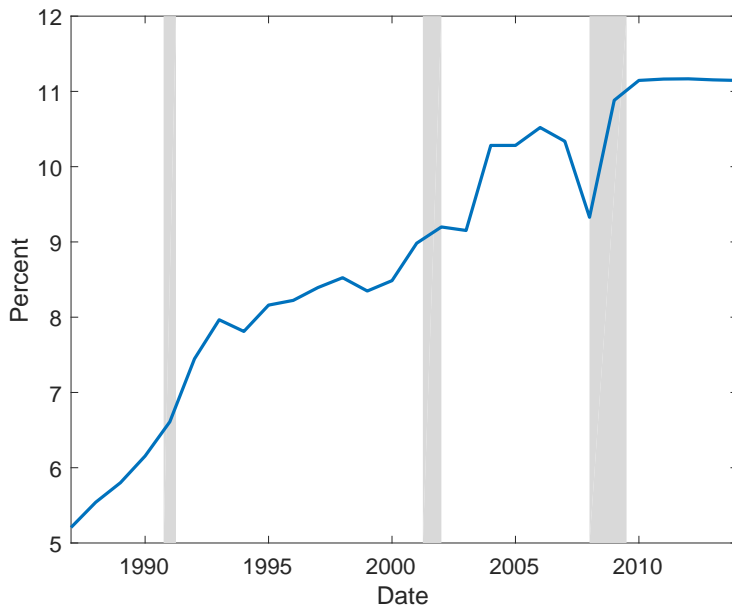
The bank equity-to-lending ratio data is constructed as the ratio between the total equity capital of commercial banks and savings institutions (defined as the sum of perpetual preferred stock, common stock, surplus, undivided profits, and other capital) and the total equity capital and liabilities (liabilities are the sum of total deposits, borrowed funds, subordinated notes, and other liabilities, see Figure 3). This data are from the FDIC.

A.1 Details on the Calculation of Bank and Nonbank Debt Growth

The construction of our bank and nonbank nonfinancial business debt growth data closely follows the methodology of Gallin (2013), which uses the Z.1. Financial Accounts of the United States. Gallin (2013) decomposes the credit from nonfinancial sector lenders to nonfinancial sector borrowers as flowing through five categories of financial intermediaries: traditional banks (commercial banks, savings institutions, and credit unions), government (federal government and the monetary authority), foreign entities, long-term funders (mutual funds, pension funds, insurance companies), and short-term funders (money market mutual funds).² He calls these financial intermediaries as "terminal funders." Broadly speaking, these terminal funders borrow from the nonfinancial sector and fund both other financial intermediaries and nonfinancial sector borrowers. The objective of Gallin (2013) is to trace each unit of debt provided to nonfinancial sector borrowers through the intermediation chains in the financial system back to one of these terminal funders. For the purposes

²A full list of the definitions for each category can be found in Table 4.1 of Gallin (2013).

Figure 3: Equity-to-Lending Ratio Commercial Bank Lending



NOTE: This figure is constructed using annual data of total equity capital of commercial banks and savings institutions (sum of perpetual preferred stock, common stock, surplus, undivided profits, and other capital) and the total equity capital and liabilities (liabilities are the sum of total deposits, borrowed funds, subordinated notes, and other liabilities). The data are from the FDIC. Gray shaded areas denote NBER recessions.

of our paper, this measure is especially appropriate as it attempts to resolve any double counting in the amount of credit provided by the financial system to the nonfinancial sector from grossing up the aggregate debt holdings of different financial intermediary entities.

Relative to Gallin (2013), which constructs this decomposition for the nonfinancial sector as a whole, we do so for only the nonfinancial business sector. We define banks as the traditional banks in Gallin (2013). The nonbanks are the sum of long-term funders and short-term funders. As we are primarily concerned with the domestic private provision of credit, we exclude from our calculations the government and foreign entities.

In our paper, we provide a short description of our implementation of the empirical strategy of Gallin (2013).

The Z.1. Tables give a breakdown of total nonfinancial sector liabilities into several instruments. They also provide information on the holders of each instrument. Gallin (2013) allocates the holders of each instrument into terminal funders and intermediate funders. Intermediate funders include financial institutions that are generally thought of as borrowing from other financial institutions (e.g. government-sponsored enterprises, or private-label issuers of asset-backed securities). For the nonfinancial sector liabilities held by the intermediate funders, Gallin (2013) uses information on the funding structure of the intermediate funders to allocate these liabilities further along the intermediation chain. Specifically, the nonfinancial sector liabilities held by the intermediate funders are allocated proportionally to the holders of the liabilities issued by the intermediate funders. The

process abstracts away from the equity claims issued by the intermediate funders. It finishes when all nonfinancial sector liabilities are allocated to only terminal funders.

We follow the same strategy but focus on the nonfinancial business sector. A complication, however, is that we only have terminal and intermediate funders' holdings by instrument of the overall nonfinancial sector liabilities but not the nonfinancial business sector components of these instruments from the Z.1. tables. We do, however, have data on the total liabilities of the nonfinancial business sector broken down by instrument. Therefore, an assumption we make is that each type of funder (terminal and intermediate) holds the same fraction of each instrument for the nonfinancial business sector as it does for the overall nonfinancial sector. This allows us to back out the amount of nonfinancial business sector liabilities by instrument held by each funder from only the total nonfinancial business sector liabilities by instrument and terminal and intermediate funders' holdings of total nonfinancial sector liabilities by instrument.

Our bank and nonbank lending data therefore capture differences in the importance of terminal funders for the nonfinancial business sector relative to the nonfinancial sector due to the differing mix of the liability instruments issued. For example, the nonfinancial business sector is funded by commercial paper and corporate bonds, whereas the household sector is not. What our assumption misses, however, is any differences in the importance of terminal funders due to differing terminal funder holdings of nonfinancial business versus household debt instruments. For instance, we would not capture any relative differences in traditional bank holdings of household mortgages versus business mortgages.

B Details on Market Clearing

The following market clearing conditions must be satisfied in equilibrium. The consumption good market clearing implies

$$y_t = c_t + \left[1 + g \left(\frac{I_t}{I_{t-1}} \right) \right] I_t + \mu^{e,B} G^{e,B} (\bar{\omega}_t^{e,B}) R_t^{K,B} q_{t-1}^K k_{t-1}^B + \mu^{e,N} G^{e,N} (\bar{\omega}_t^{e,N}) R_t^{K,N} q_{t-1}^K k_{t-1}^N + \mu^B G^B (\bar{\omega}_t^B) \tilde{R}_t^B b_{t-1}^B + \mu^N G^N (\bar{\omega}_t^N) \tilde{R}_t^N b_{t-1}^N. \quad (1)$$

The consumption good market clearing implies that total output must equal total consumption, total investment taking into account adjustment costs, and monitoring costs from the defaulting entrepreneurs and intermediaries in both the B and N sectors.

Labor market clearing implies

$$\begin{aligned}
(1 - \alpha^B) \frac{y_t^B}{w_t^B} &= l_t^B \\
(1 - \alpha^N) \frac{y_t^N}{w_t^N} &= l_t^N \\
l_t^B + l_t^N &= l_t.
\end{aligned} \tag{2}$$

Capital market clearing implies

$$\begin{aligned}
\alpha^B \frac{y_t^B}{r_t^B} &= k_t^B \\
\alpha^N \frac{y_t^N}{r_t^N} &= k_t^N \\
k_t^B + k_t^N &= k_t \\
k_t &= (1 - \delta)k_{t-1} + I_t.
\end{aligned} \tag{3}$$

Market clearing in the bank and nonbank deposit markets implies the following conditions

$$\begin{aligned}
d_t^B &= (1 - \phi_t^B)b_t^N \\
d_t^N &= b_t^N - e_t^N.
\end{aligned} \tag{4}$$

Market clearing in the entrepreneurial loan markets implies

$$\begin{aligned}
q_t^K k_t^B - b_t^B &= n_t^{e,B} \\
q_t^K k_t^N - b_t^N &= n_t^{e,N}.
\end{aligned} \tag{5}$$

The intermediary equity market clearing condition implies

$$(1 - \chi_t^b) W_t^b = \phi_t^B b_t^B + b_t^N - d_t^N. \tag{6}$$

Finally, the deposit insurance agency needs to have a balanced budget, hence the taxes collected from households need to be equal to the insurance provided to the regulated intermediary deposits, e.g.,

$$T_t = [\bar{\omega}_t^B - \Gamma^B (\bar{\omega}_t^B) + \mu_t^B G^B (\bar{\omega}_t^B)] \tilde{R}_t^B b_{t-1}^B. \tag{7}$$

C Model Equations

We list here the full set of detrended equilibrium conditions implied by the model. The variables that are trending are detrended by A_t .

Households

$$\lambda_t = \frac{\tilde{\beta}_t}{C_t - h C_{t-1} \exp(-\Delta \log A_t)} - h \beta E_t \left[\frac{\tilde{\beta}_{t+1} \exp(-\Delta \log A_{t+1})}{C_{t+1} - h \exp(-\Delta \log A_{t+1}) C_t} \right]$$

$$UL_t = \chi L_t^\eta$$

$$UL_t = w_t \lambda_t$$

$$\lambda_t = \chi h \frac{(d_t^B)^{\alpha_h - 1}}{\Lambda_{N,t} (d_t^N)^{\alpha_h} + (d_t^B)^{\alpha_h}} + \beta E_t \left[\exp(-\Delta \log A_{t+1}) \lambda_{t+1} R_t^D \right]$$

$$\lambda_t = \chi h \frac{\Lambda_{N,t} (d_t^N)^{\alpha_h - 1}}{\Lambda_{N,t} (d_t^N)^{\alpha_h} + (d_t^B)^{\alpha_h}} + \beta E_t \left[\exp(-\Delta \log A_{t+1}) \lambda_{t+1} \tilde{R}_{t+1}^{D,N} \right]$$

$$\lambda_t = \beta E_t \left[\exp(-\Delta \log A_{t+1}) \lambda_{t+1} R_t^f \right]$$

$$R_t^{D,N} = \frac{(\Gamma_t^N - \mu^N G_t^N) \tilde{R}_t^N}{1 - \phi_{t-1}^N}$$

$$c_t + d_t^B + d_t^N = w_t l_t + R_{t-1}^D d_{t-1}^B + \tilde{R}_t^{D,N} d_{t-1}^N - T_t + \Pi_t$$

Entrepreneurs

$$W_t^{e,B} = (1 - \Gamma_t^{e,B}) \exp(-\Delta \log A_t) R_t^{K,B} q_{t-1}^K K_{t-1}^B$$

$$n_t^{e,B} = (1 - \chi_t^{e,B}) W_t^{e,B}$$

$$E_t \left[\left((1 - \Gamma_{t+1}^{e,B}) R_{t+1}^{K,B} + \lambda_t^{e,B} \left(\frac{1 - \Gamma_{t+1}^B}{\phi_t^B} (\Gamma_{t+1}^{e,B} - m_t^{e,B} G_{t+1}^{e,B}) R_{t+1}^{K,B} - \rho_{t+1}^B \right) \right) \right] = 0$$

$$E_t \left[-\Gamma_{t+1}^{e,B,1} \right] + \lambda_t^{e,B} E_t \left[\frac{1 - \Gamma_{t+1}^B}{\phi_t^B} (\Gamma_{t+1}^{e,B,1} - m_t^{e,B} G_{t+1}^{e,B,1}) \right] = 0$$

$$\rho_t^{e,B} = \frac{(1 - \Gamma_t^B) \tilde{R}_t^B}{\phi_{t-1}^B}$$

$$\bar{\omega}_t^{e,B} = \frac{x_{t-1}^{e,B}}{R_t^{K,B}}$$

$$x_t^{e,B} = \frac{R_t^B \left(q_t^K K_t^B - n_t^{e,B} \right)}{q_t^K K_t^B}$$

$$R_t^{K,B} = \frac{r_t^{K,B} + q_t^K (1 - \delta)}{q_{t-1}^K}$$

$$W_t^{e,N} = \exp(-\Delta \log A_t) \left(1 - \Gamma_t^{e,N} \right) R_t^{K,N} q_{t-1}^K K_{t-1}^N$$

$$n_t^{e,N} = \left(1 - \chi_t^{e,N} \right) W_t^{e,N}$$

$$E_t \left[\left(1 - \Gamma_{t+1}^{e,N} \right) R_{t+1}^{K,N} + \lambda_t^{e,N} \left(\frac{1 - \Gamma_{t+1}^N}{\phi_t^N} \left(\Gamma_{t+1}^{e,N} - m_t^{e,N} G_{t+1}^{e,N} \right) R_{t+1}^{K,N} - \rho_{t+1}^N \right) \right] = 0$$

$$E_t \left[\left(-\Gamma_{t+1}^{e,N,1} \right) + \lambda_t^{e,N} \frac{1 - \Gamma_{t+1}^N}{\phi_t^N} \left(\Gamma_{t+1}^{e,N,1} - m_t^{e,N} G_{t+1}^{e,N,1} \right) \right] = 0$$

$$\rho_t^{e,N} = \frac{\tilde{R}_t^N (1 - \Gamma_t^N)}{\phi_{t-1}^N}$$

$$\bar{\omega}_t^{e,N} = \frac{x_{t-1}^{e,N}}{R_t^{K,N}}$$

$$x_t^{e,N} = \frac{R_t^N \left(q_t^K K_t^N - n_t^{e,N} \right)}{q_t^K K_t^N}$$

$$R_t^{K,N} = \frac{r_t^{K,N} + q_t^K (1 - \delta)}{q_{t-1}^K}$$

$$\tilde{R}_t^B = \frac{K_{t-1}^B q_{t-1}^K R_t^{K,B} \left(\Gamma_t^{e,B} - m_t^{e,B} G_t^{e,B} \right)}{q_{t-1}^K K_{t-1}^B - n_{t-1}^{e,B}}$$

$$\tilde{R}_t^N = \frac{K_{t-1}^N q_{t-1}^K R_t^{K,N} \left(\Gamma_t^{e,N} - m_t^{e,N} G_t^{e,N} \right)}{q_{t-1}^K K_{t-1}^N - n_{t-1}^{e,N}}$$

$$\Gamma_t^{e,i} = \Phi \left(\frac{\log \left(\bar{\omega}_t^{e,i} \right) - \frac{(\sigma_t^{e,i})^2}{2}}{\sigma_t^{e,i}} \right) + \bar{\omega}_t^{e,i} \left(1 - \Phi \left(\frac{\log \left(\bar{\omega}_t^{e,i} \right) + \frac{(\sigma_t^{e,i})^2}{2}}{\sigma_t^{e,i}} \right) \right)$$

$$\Gamma_t^{e,i,1} = 1 - \Phi \left(\frac{\log(\bar{\omega}_t^{e,i}) + \frac{(\sigma_t^{e,i})^2}{2}}{\sigma_t^{e,i}} \right) + \frac{\phi \left(\frac{\log(\bar{\omega}_t^{e,i}) - \frac{(\sigma_t^{e,i})^2}{2}}{\sigma_t^{e,i}} \right) - \bar{\omega}_t^{e,i} \phi \left(\frac{\log(\bar{\omega}_t^{e,i}) + \frac{(\sigma_t^{e,i})^2}{2}}{\sigma_t^{e,i}} \right)}{\bar{\omega}_t^{e,i} \sigma_t^{e,i}}$$

$$G_t^{e,i} = \Phi \left(\frac{\log(\bar{\omega}_t^{e,i}) - \frac{(\sigma_t^{e,i})^2}{2}}{\sigma_t^{e,i}} \right)$$

$$G_t^{e,i,1} = \frac{\phi \left(\frac{\log(\bar{\omega}_t^{e,i}) - \frac{(\sigma_t^{e,i})^2}{2}}{\sigma_t^{e,i}} \right)}{\bar{\omega}_t^{e,B} \sigma_t^{e,B}}$$

Investors

$$W_t^b = \exp(-\Delta \log A_t) (\rho_t^B \phi_{t-1}^B b_{t-1}^B + \rho_t^N e_{t-1}^N)$$

$$n_t^b = b_t^N + \phi_t^B b_t^B - d_t^N$$

$$n_t^b = (1 - \chi_t^b) W_t^b$$

$$E_t [\rho_{t+1}^N] = E_t [\rho_{t+1}^N]$$

Intermediaries

$$\bar{\omega}_t^B = \frac{R_{t-1}^D (1 - \phi_{t-1}^B)}{\tilde{R}_t^B}$$

$$\bar{\omega}_t^N = \frac{R_{t-1}^{D,N} (1 - \phi_{t-1}^N)}{\tilde{R}_t^N}$$

$$\phi_t^N = \frac{e_t^N}{b_t^N}$$

$$E_t \left[\Gamma_{t+1}^{N,1} \right] = \beta \frac{\lambda_t^N}{d_t^N} E_t \left[\exp(-\Delta \log A_{t+1}) \lambda_{t+1} \left(\Gamma_{t+1}^{N,1} - \mu^N G_{t+1}^{N,1} \right) \right]$$

$$E_t \left[(1 - \Gamma_{t+1}^N) \tilde{R}_{t+1}^N + \lambda_t^N \left(\chi_h \Lambda_{N,t} \frac{(-\Lambda_{N,t}) - (1 - \alpha^h) \left(\frac{d_t^B}{d_t^N} \right)^{\alpha^h}}{\left(d_t^N \Lambda_{N,t} + d_t^N \left(\frac{d_t^B}{d_t^N} \right)^{\alpha^h} \right)^2} + \beta E_t \left[\frac{\tilde{R}_{t+1}^N \exp(-\Delta \log A_{t+1}) \lambda_{t+1} \left(\Gamma_{t+1}^N - \mu^N G_{t+1}^N \right) (d_t^N - b_t^N)}{(d_t^N)^2} \right] \right) \right] = 0$$

$$\Gamma_t^i = \Phi \left(\frac{\log(\bar{\omega}_t^i) - \frac{(\sigma_t^i)^2}{2}}{\sigma_t^i} \right) + \bar{\omega}_t^i \left(1 - \Phi \left(\frac{\log(\bar{\omega}_t^i) + \frac{(\sigma_t^i)^2}{2}}{\sigma_t^i} \right) \right)$$

$$\Gamma_t^{i,1} = 1 - \Phi \left(\frac{\log(\bar{\omega}_t^i) + \frac{(\sigma_t^i)^2}{2}}{\sigma_t^i} \right) + \frac{\phi \left(\frac{\log(\bar{\omega}_t^i) - \frac{(\sigma_t^i)^2}{2}}{\sigma_t^i} \right) - \bar{\omega}_t^i \phi \left(\frac{\log(\bar{\omega}_t^i) + \frac{(\sigma_t^i)^2}{2}}{\sigma_t^i} \right)}{\bar{\omega}_t^i \sigma_t^i}$$

$$G_t^i = \Phi \left(\frac{\log(\bar{\omega}_t^i) - \frac{(\sigma_t^i)^2}{2}}{\sigma_t^i} \right)$$

$$G_t^{i,1} = \frac{\phi \left(\frac{\log(\bar{\omega}_t^i) - \frac{(\sigma_t^i)^2}{2}}{\sigma_t^i} \right)}{\bar{\omega}_t^i \sigma_t^i}$$

Final Good Production

$$Y_t^B = \exp(-\alpha^{yB} \Delta \log A_t) (K_{t-1}^B)^{\alpha^{yB}} (L_t^B)^{1-\alpha^{yB}}$$

$$r_t^{K,B} = \alpha^{yB} \exp(\Delta \log A_t) \frac{Y_t^B}{K_{t-1}^B}$$

$$w_t = (1 - \alpha^{yB}) \frac{Y_t^B}{L_t^B}$$

$$Y_t^N = \exp(-\alpha^{yN} \Delta \log A_t) (K_{t-1}^N)^{\alpha^{yN}} (L_t^N)^{1-\alpha^{yN}}$$

$$r_t^{K,N} = \alpha^{yN} \exp(\Delta \log A_t) \frac{Y_t^N}{K_{t-1}^N}$$

$$w_t = (1 - \alpha^{yN}) \frac{Y_t^N}{L_t^N}$$

Capital Good Production

$$g_t^I = \frac{\psi^i}{2} \left(\exp(\Delta \log A_t) \frac{I_t}{I_{t-1}} - \exp(\Delta \log A_t) \right)^2$$

$$g_t^{I,1} = \psi^i \left(\exp(\Delta \log A_t) \frac{I_t}{I_{t-1}} - \exp(\Delta \log A_t) \right)$$

$$K_t = I_t + (1 - \delta) \exp(-\Delta \log A_t) K_{t-1}$$

$$q_t^K = EK_t \left(1 + g_t^I + \exp(\Delta \log A_t) \frac{I_t}{I_{t-1}} g_t^{I,1} \right) - \beta E_t \left[\exp(\Delta \log A_{t+1}) EK_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{I_{t+1}}{I_t} \right)^2 g_{t+1}^{I,1} \right]$$

Market Clearing

$$d_t^B = (1 - \phi_t^B) b_t^B$$

$$d_t^N = b_t^N - e_t^N$$

$$b_t^B + b_t^N = b_t$$

$$Y_t = Y_t^B + Y_t^N$$

$$K_t = K_t^B + K_t^N$$

$$L_t = L_t^B + L_t^N$$

$$q_t^K K_t^B - b_t^B = n_t^{e,B}$$

$$q_t^K K_t^N - b_t^N = n_t^{e,N}$$

Deposit Insurance

$$-Tr_t + \exp(-\Delta \log A_t) b_{t-1}^B \tilde{R}_t^B (\bar{\omega}_t^B - \Gamma_t^B + \mu^B G_t^B) = 0$$

Observation Equations

$$Cgr_t = 100 (\Delta \log A_t + \log(C_t) - \log(C_{t-1}))$$

$$Igr_t = 100 (\Delta \log A_t + \log(I_t) - \log(I_{t-1}))$$

$$bCgr_t = 100 (\Delta \log A_t + \log(bC_t) - \log(bC_{t-1}))$$

$$bSgr_t = 100 (\Delta \log A_t + \log (bS_t) - \log (bS_{t-1}))$$

$$R_{obs,t}^{B,N,spr} = 400 \left(R_t^{B,N} - R_t^f \right)$$

$$R_{obs,t}^D = 400 \left(R_t^D - 1 \right)$$

$$R_{obs,t}^{D,N} = 400 \left(R_t^{D,N} - 1 \right)$$

$$\phi_{obs,t}^C = 100 \phi_t^C$$

D Outside Equity Issuance by Banks and Nonbanks

We extend our model to allow for outside equity issuance by banks and nonbanks. Outside equity issuance allows the financial sector in our model to obtain funding through an additional channel beyond retained earnings and deposits. The effects of costly equity issuance on macro-financial dynamics and macroprudential policy have been previously examined in the context of general equilibrium models by Gertler et al. (2012); Kiley and Sim (2014). In addition, this channel potentially becomes especially relevant when exploring the effects of changes in bank capital requirements, as highlighted by discussions in the policy and academic circles (Admati et al., 2010; BIS, 2010).

We first discuss the key changes to our benchmark model. Then, we discuss the calibration and estimation strategy. Finally, we delve into the results. The results in large part cohere with those from the benchmark model. We also briefly discuss the model's implications for equity issuance.

D.1 Model Changes

Our modeling strategy closely follows that of Covas and Den Haan (2012). We extend the choice of the bank and nonbank intermediaries to include raising outside equity from the household at a cost. This decision occurs after the investors have allocated inside equity between the banks and nonbanks.

Households. Households maximize the following problem:

$$\begin{aligned}
& \max_{c_t, l_t, d_t^B, d_t^N, s_t^B, s_t^N} E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[\tilde{\beta}_{t+i} \log(c_{t+i} - hc_{t+i-1}) - \frac{\chi}{1+\psi} l_{t+i}^{1+\psi} + \chi_h \log(h_{t+i}) \right] \\
& \text{s.t. } c_t + d_t^B + d_t^N + s_t^B + s_t^N \leq w_t l_t + R_{t-1}^D d_{t-1}^B + \tilde{R}_t^{D,N} d_{t-1}^N + \\
& \quad \tilde{R}_t^{B,s} s_{t-1}^B + \tilde{R}_t^{N,s} s_{t-1}^N - T_t + \Pi_t \\
& \quad h_t = \left[\Lambda_{N,t} (d_t^N)^{\alpha_h} + (d_t^B)^{\alpha_h} \right]^{1/\alpha_h}
\end{aligned} \tag{8}$$

Relative to before, households have an additional choice of determining the amount of outside equity shares in banks, s_t^B , and nonbanks, s_t^N , in which to invest. Outside equity shares in banks and nonbanks give a time t return of $\tilde{R}_t^{B,s}$ and $\tilde{R}_t^{N,s}$, respectively. These returns will be defined later in the banks' and nonbanks' problems.

Banks. In addition to choosing deposits and lending, banks now also decide how much outside equity s_t^B to raise and the share of their profits given to the outside equity holders \tilde{s}_t^B . Banks have to pay a cost $\lambda_0^B (s_t^B)^2$ to raise outside equity. This outside equity can be used to meet the capital requirements that are imposed on them. Therefore, the term $\tilde{\phi}_t^B$, which includes both inside and outside equity, is determined by capital requirement policy.

$$\begin{aligned}
& \max_{d_t^B, b_t^B, s_t^B, \tilde{s}_t^B} E_t \left[(1 - \tilde{s}_t^B) \left(1 - \Gamma^B \left(\frac{x_t^B}{\tilde{R}_{t+1}^B} \right) \right) \tilde{R}_{t+1}^B b_t^B \right] \\
& \text{s.t.} \\
& b_t^B = e_t^B + d_t^B + s_t^B - \lambda_0^B (s_t^B)^2 \\
& \tilde{\phi}_t^B = \frac{e_t^B + s_t^B - \lambda_0^B (s_t^B)^2}{b_t^B} \\
& \lambda_t = \beta E_t \left[\lambda_{t+1} \left(1 - \Gamma^B \left(\frac{x_t^B}{\tilde{R}_{t+1}^B} \right) \right) \tilde{R}_{t+1}^B \tilde{s}_t^B \frac{b_t^B}{s_t^B} \right]
\end{aligned} \tag{9}$$

where $x_t^B = \frac{R_t^D d_t^B}{b_t^B}$

The last equation in the constraint is given by the first order condition of the households with respect to banks outside equity. When making the financing and lending decisions, banks must satisfy this constraint.

The return on banks' outside equity is given by:

$$\tilde{R}_{t+1}^{B,s} = \left(1 - \Gamma^B \left(\frac{x_t^B}{\tilde{R}_{t+1}^B} \right) \right) \tilde{R}_{t+1}^B \tilde{s}_t^B \frac{b_t^B}{s_t^B}. \tag{10}$$

Nonbanks. We extend the nonbanks' problem in an analogous fashion. Nonbanks now also have the choice to raise outside equity s_t^N from the household at a cost $\lambda_0^N (s_t^N)^2$. They choose the share

of profits given to the outside equity holders \tilde{s}_t^N . Similar to the banks' problem, the households' first order condition with respect to nonbanks deposits shows up as a constraint.

$$\begin{aligned}
& \max_{x_t^N, d_t^N, b_t^N, s_t^N, \tilde{s}_t^N} E_t \left[(1 - \tilde{s}_t^N) \left(1 - \Gamma^N \left(\frac{x_t^N}{\tilde{R}_{t+1}^N} \right) \right) \tilde{R}_{t+1}^N b_t^N \right] \\
& \text{s.t.} \\
& b_t^N = e_t^N + d_t^N + s_t^N - \lambda_0^N (s_t^N)^2 \\
& \lambda_t - \chi h \frac{\Lambda_{N,t} (d_t^N)^{\alpha_h - 1}}{\Lambda_{N,t} (d_t^N)^{\alpha_h} + (d_t^B)^{\alpha_h}} = \dots \\
& \beta E_t \left[\lambda_{t+1} \left(\left(\Gamma^N \left(\frac{x_t^N}{\tilde{R}_{t+1}^N} \right) - \mu^N G^N \left(\frac{x_t^N}{\tilde{R}_{t+1}^N} \right) \right) \tilde{R}_{t+1}^N \frac{b_t^N}{d_t^N} \right) \right] \\
& \lambda_t = \beta E_t \left[\lambda_{t+1} \left(1 - \Gamma^N \left(\frac{x_t^N}{\tilde{R}_{t+1}^N} \right) \right) \tilde{R}_{t+1}^N \tilde{s}_t^N \frac{b_t^N}{s_t^N} \right]
\end{aligned} \tag{11}$$

where $x_t^N = \frac{R_t^{D,N} d_t^N}{b_t^N}$.

We define the return on nonbanks outside equity as:

$$\tilde{R}_{t+1}^{N,s} = \left(1 - \Gamma^N \left(\frac{x_t^N}{\tilde{R}_{t+1}^N} \right) \right) \tilde{R}_{t+1}^N \tilde{s}_t^N \frac{b_t^N}{s_t^N}. \tag{12}$$

Entrepreneurs The problem of the entrepreneurs in each sector has to be modified as well. This is because the entrepreneurs are only concerned with satisfying the incentive compatibility constraint of the inside equity holders. Therefore, only those payoffs are included in the incentive compatibility constraint. Note that $\phi_t^i = \frac{e_t^i}{b_t^i}$ as before.

$$\begin{aligned}
& \max_{k_t^i, b_t^i, R_t^i} E_t \left[\left(1 - \Gamma^{e,i} \left(\frac{x_t^{e,i}(\cdot)}{R_{t+1}^{K,i}} \right) \right) R_{t+1}^{K,i} q_t^K k_t^i \right] \\
& \text{s.t. } q_t^K k_t^i - b_t^i = n_t^{e,i} \\
& E_t \left[(1 - \tilde{s}_t^B) (1 - \Gamma^i(\bar{\omega}_{t+1}^i)) \left(\Gamma^{e,i} \left(\frac{x_t^{e,i}(\cdot)}{R_{t+1}^{K,i}} \right) - \mu^{e,i} G^{e,i} \left(\frac{x_t^{e,i}(\cdot)}{R_{t+1}^{K,i}} \right) \right) R_{t+1}^{K,i} q_t^K k_t^i \right] \geq \rho_t \phi_t^i b_t^i,
\end{aligned} \tag{13}$$

Discussion We think of inside equity as funds provided by management of the banks and nonbanks whereas outside equity as funds from investors. Management works to maximize the value of inside equity. The ultimate owners of both the inside and outside equity are the households. Inside equity is accumulated as retained earnings and paid out to the households as dividends. Outside equity is a one period contract that pays out a slice of firm profits in that quarter. We pursue this modeling strategy as it is a straightforward way to extend our benchmark model.

Furthermore, note that the equity issuance cost is modeled so our original model is nested as a special case with infinite outside equity cost.

D.2 Calibration and Estimation

With the new elements of the model, we have two extra parameters that we need calibrate: λ_0^B and λ_0^N . We follow Gertler et al. (2012) and set the ratio of outside to inside equity for the banks and nonbanks to be 2/3. Then, we follow the same strategy as in our benchmark model, additionally calibrating the two equity issuance cost parameters. We have the same calibration targets as in Table 2 of the main text.³

For the parameters that we do not calibrate, we use Bayesian methods to estimate the extended model on the same data series as for our benchmark model. Table 1 shows the posterior mode results.

D.3 Results

What drives bank and nonbank lending growth? The extension to allow for bank and nonbank outside equity issuance does not impact the conclusions on the drivers of bank and nonbank credit growth. Table 2 shows the unconditional variance decomposition of bank and nonbank lending growth. Credit growth in both sectors are driven largely by financial shocks. The most important source of bank lending growth fluctuations is from nonbank entrepreneur dividend policy shocks and vice versa. Capital requirements have negligible effects on credit growth in either sector. These results are largely similar to those implied by the benchmark model.

We also do a decomposition by frequency, with the results from the benchmark model holding up qualitatively and quantitatively. At business cycle frequencies, sectoral entrepreneur dividend policy shocks are the dominant drivers of credit fluctuations. At longer horizons, aggregate entrepreneur risk shocks gain importance.

Finally, we also look at what the model implies for investment growth (Table 4). TFP growth continues to play a major role in investment growth fluctuations. One difference from the benchmark model is that in the extended model, MEI shocks gain importance, driving around 15% of investment growth fluctuations. The other three important shocks are financial in nature.

What drives outside equity issuance? In the extended model, outside equity issuance by both the bank and nonbank sectors are almost entirely driven by investor dividend policy shocks. Around 90% of equity issuance fluctuations in both sectors are caused by these dividend policy shocks. Positive dividend policy shocks decrease the amount of inside investor equity available to back lending to the entrepreneurs. In response, banks and nonbanks raise outside equity to partially make up for the shortfall.

³One difference is that instead of matching the quarterly returns of investor equity to U.S. commercial bank equity returns, we match the returns on outside equity issued by the banks. We believe that with the introduction of inside and outside equity into the model, outside equity more closely matches the U.S. commercial bank equity returns series.

β	Discount factor**	0.9965
h	Habits	0.67
η	Frisch elasticity of labor**	1
α_h	Substitutability between bank and nonbank deposits**	0.999
χ	Importance of labor disutility*	7.67
χ_h	Importance of liquidity service*	0.014
Λ_N	Importance of nonbank deposits in liquidity*	0.92
$\chi_{e,B}(N)$	Entrepreneur dividend policy*	0.022(0.028)
χ_b	Banker dividend policy*	0.075
$\alpha^B(N)$	Capital share in production**	0.33
δ_K	Depreciation rate**	0.025
Φ_I	Investment adj. cost	1.92
$\mu_{e,B}(N)$	Monitoring cost entrepreneur B(N)**	0.30(0.30)
$\mu_B(N)$	Monitoring cost B(N) intermediary**	0.30(0.25)
$\sigma_{e,B}(N)$	Std. of idio. shock B(N) entrepreneurs*	0.52(0.28)
$\sigma_B(N)$	Std. of idio. shock B(N) bank*	0.033(0.041)
$\bar{\phi}_0^B$	B Bank capital requirement*	0.088
λ_0^B	B Bank outside equity issuance cost*	0.83
λ_0^N	N Bank outside equity issuance cost*	0.28
Λ_A	Steady state TFP growth**	0.004
ρ_A	Persistence TFP growth	0.30
ρ_{EK}	Persistence MEI	0.86
ρ_β	Persistence pref.	0.51
$\rho_{\Lambda,N}$	Persistence nonbank liquidity demand shock	0.91
$\rho_{\sigma,e,Agg}$	Persistence economy-wide entrepreneur risk shock	0.98
$\rho_{\sigma,e,B}(N)$	Persistence bank (nonbank) sector entrepreneur risk shock	0.50(0.47)
$\rho_{\chi,e,Agg}$	Persistence aggregate entrepreneur dividend policy shock	0.5
$\rho_{\chi,e,B}(N)$	Persistence bank (nonbank) sector entrepreneur dividend policy shock	0.50(0.65)
$\rho_{\chi,b}$	Persistence investor dividend policy shock	0.30
ρ_η	Persistence capital requirements shock**	0.999
σ_A	Std. TFP	0.010
σ_{EK}	Std. MEI	0.0114
σ_β	Std. preference	0.018
$\sigma_{\Lambda,N}$	Std. nonbank liquidity demand shock	0.10
$\sigma_{\sigma,e,Agg}$	Std. aggregate entrepreneur risk shock	0.019
$\sigma_{\sigma,e,B}(N)$	Std. bank (nonbank) sector entrepreneur risk shock	0.0(0.01)
$\sigma_{\chi,e,Agg}$	Std. aggregate entrepreneur dividend policy shock	0.0
$\sigma_{\chi,e,B}(N)$	Std. bank (nonbank) sector entrepreneur dividend policy shock	0.006(0.006)
$\sigma_{\chi,b}$	Std. investor dividend policy shock	0.11
σ_η	Std. capital requirements shock	0.002

Table 1: Alternative Parameters: * denotes that the parameter is calibrated from the data moments. ** denotes that the parameter is calibrated from the previous literature. The rest of the parameters are estimated.

Table 2: Unconditional Variance Decomposition of Bank and Nonbank Lending Growth: Alternative Model

Variable	TFP gr.	Liq. N	Ent. risk EW	Ent. risk N	Ent. div B	Ent. div N	Cap. req.
Bank lending gr.	3	8	12	10	17	43	3
Nonbank lending gr.	6	6	16	11	45	5	1

NOTE: This table shows the unconditional variance decomposition of bank and nonbank lending growth at the posterior mode parameters for a selected set of structural shocks. EW denotes economy-wide. The shocks which are not included here were estimated to be unimportant.

Table 3: Variance Decompositions at Business Cycle (6-32 qtrs.) and Medium-Frequency Cycle (32-200 qtrs.) Frequencies: Alternative Model

	Variable	Ent. risk EW	Ent div B	Ent div N
6-32 qtrs	Bank lending gr.	9	20	59
	Nonbank lending gr.	5	69	2
32-200 qtrs	Bank lending gr.	46	13	26
	Nonbank lending gr.	34	26	18

NOTE: This table shows the variance decompositions at the posterior mode parameters at business cycle (6-32 quarters) and medium-frequency cycle (32-200 quarters) frequencies. The isolation of the frequencies is done by applying a bandpass filter. EW means economy-wide.

Capital requirements changes play a secondary role in outside equity issuance fluctuations. A tighter capital requirement leads to inside equity flowing from nonbanks to banks. In addition, banks and nonbanks seek outside equity to help cushion the equity funding shortfall from the tighter requirements. Because the size of capital requirements changes are much smaller than dividend policy shocks, they play a much less important role.

At business cycle frequencies, outside equity issuance is negatively correlated with real activity. Bank and nonbank outside equity issuance has a correlation with investment growth of around -0.22 . Investor dividend policy shocks increase outside equity issuance in both sectors while depressing investment at business cycle frequencies, leading to the observed negative correlation. The countercyclical nature of financial sector equity issuance is consistent with what other have documented in the data (Adrian et al., 2015; Baron, 2020).

Table 4: Unconditional Variance Decomposition of Investment Growth: Alternative Model

Variable	TFP gr.	MEI	Ent. risk EW	Ent. div. B	Ent. div. N
Inv. gr.	33	15	21	10	9

NOTE: This table shows the unconditional variance decomposition of investment growth at the posterior mode parameters for interesting structural shocks. EW means economy-wide. The shocks not on this list were estimated to be unimportant.

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