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What share for gold? On the interaction of gold and foreign exchange reserve returns

Omar Zulaica[1]

Abstract

Almost five decades after the collapse of the Bretton Woods system, gold continues to form an important share of global foreign exchange reserves. This may be because gold has traditionally offered reserve managers many benefits, such as the absence of default risk. This paper explores whether these large investment shares in gold are also justified from a risk-return standpoint, or whether any other explanations have to be brought to bear. To do this, we go beyond the simple application of portfolio optimisation techniques, comprehensively analysing all possible long-only combinations of gold and representative fixed income reserve portfolios. We conclude that the market risk associated with gold is substantial when evaluated against a broad range of criteria, such as mitigating portfolio volatility, tail-risk, the probability of loss and measures of diversification. This will tend to limit overall allocations. Nonetheless, for portfolios with higher sensitivity to interest rates (duration) and for reserve managers who measure their returns in a non-reserve currency, we find evidence that gold may function as a hedge, making it easier to justify sizeable gold holdings from a purely quantitative perspective.

JEL classification: E58, F31, G11, G17.

[1] The author is a member of Reserve Management Advisory Services (RMAS), a Unit within the BIS Banking Department. The views expressed are those of the author and do not necessarily reflect those of the BIS. I would like to thank Mikael Charozé, Stijn Claessens, Benoit Gilson, Emanuel Kohlscheen, Mike McMorrow, Prakash, Michela Scatigna, Dora Xia, Peter Zöllner and especially, Ingo Fender. We also thank participants of the various BIS Banking Department seminars for comments.

1. Introduction

Gold allocations in foreign exchange (FX) reserve porfolios are an issue of great relevance for both reserve managers – the main audience for this paper – and the broader investor community. According to the World Gold Council (WGC), the official sector holds about 17% of all the gold that has ever been mined[2], a percentage that can be assumed to be managed mostly vis-à-vis bond benchmarks of limited interest rate risk (or "duration", as labelled in the fixed income space). This raises a host of questions, such as why do central banks and other reserve managers hold such large amounts of gold and why do they hold gold in the first place? What has portfolio theory to say about these choices?

The answer starts with the definition of FX reserve assets. According to the International Monetary Fund (IMF)'s Balance of Payments Manual (BoPM), (foreign exchange) reserve assets are those external assets that are readily available to, and controlled by, monetary authorities for meeting balance of payments financing needs, for intervention in exchange markets to affect the currency exchange rate, and for other related purposes.[3] Nowadays, the term *FX reserve asset* is routinely associated with cash deposits in foreign currencies and highly liquid tradeable instruments, such as bills, notes and bonds issued by the US Treasury or other highly-rated government entities. Somewhat peculiarily, however, at least by today's standards, the listing of FX reserve assets in the BoPM indeed names monetary gold as the first component of reserves. Money market and fixed income instruments fall into the last category: *other reserve assets*, listed after Special Drawing Right (SDR)[4] holdings and IMF reserve positions.

This ordering highlights the historical importance that gold has had as a part of global FX reserves. In fact, in defining what qualifies as a reserve asset, the BoPM explicitly exempts gold from some of the associated criteria.[5] This is best summarised in the following statement: "Gold bullion is not a claim and does not have a corresponding liability. It is treated as a financial asset, however, because of its special role as a means of financial exchange in international payments by monetary authorities and as a reserve asset held by monetary authorities."[6]

But what is this special role? According to a recent survey by the World Gold Council, the top five reasons why central banks hold gold are: (1) its historical position (ie legacy holdings), (2) its status as a long-term store of value, (3) its performance during times of crisis, (4) its lack of default risk and (5) its effectivity as a portolio diversifier.[7] Of course, some of these notions are a matter of perception, but at least one is more contested and can be empirically explored: the diversifying properties of gold. Our paper aims to do just that, in the context of FX reserves.

Plenty has been said in the literature about the properties of gold as a diversifying asset. The question was popular in the eighties in the context of developed economy equity markets (for example, Herbst

2 Based on total above-ground stocks as of end-2019. For the latest World Gold Council data, click here.

3 IMF (2013), Chapter 6, Paragraph 6.64 "a. General definition" and Box 6.5 "Components of Reserve Assets and Reserve-Related Liabilities".

4 SDRs are supplementary FX reserve assets created and maintained by the International Monetary Fund. Under its Articles of Agreement, the IMF may allocate SDRs to members according to their quotas, who can then buy or sell these assets among each other. Values are determined on the basis of a basket of currencies with predetermined weights; see Appendix 1.

5 For example, it states that reserve assets should typically be claims on non-residents except in the case of gold, whose account may be located anywhere; of course, metals are not a claim on any legal person and are therefore free of credit risk. Furthermore, it delineates that reserve assets must be denominated and settled in convertible foreign currencies – indeed, gold is typically quoted and traded in US dollars. Yet, assets denominated *in gold*, which is not a currency, may also qualify as reserve assets. See for example, IMF (2013), Chapter 6, Paragraph 6.73 "f. Further clarification on reserve assets".

6 IMF (2013), Chapter 3, Paragraph 3.24.

7 World Gold Council (2020), Central Bank Gold Reserve Survey. https://www.gold.org/goldhub/data/2020-central-bank-gold-reserve-survey. Reasons listed are the five top considered "highly relevant" to central banks.

(1983) and Chua, et al (1990)) and, more recently, gained traction in the context of emerging markets, such as China (Arouri, et al (2015)) and the BRICS (Chkili (2016)). Generally, this research showed that gold, while being rather volatile, has a low correlation with stock prices, and that holding at least a small share can function as a hedge against stock market crashes.

More relevant in the FX reserve context are analyses of gold's potential value in fixed income and multi-asset portfolios. Jaffe (1989), for example, explores the risk-return properties of gold in portfolios with different degrees of risk appetite, each represented by a different asset mix of stocks and bonds. He finds that gold allocations of five to ten percent can improve the portfolio's Sharpe ratio. An extended and updated version of this study by Emmrich and McGroarty (2013) – whose work also includes a comprehensive literature review – points to 10% as the appropriate portfolio weight, and suggests that gold remains an attractive investment counterbalancing equity exposures. The more widely quoted Bauer and Lucey (2010), in turn, explore whether gold may be considered a safe haven[8] in the stock and bond markets of the US, the UK and Germany. They conclude that gold is nowhere a safe haven for bonds and that it may not serve as a safe haven for stocks at all times; only if the associated negative market shock is sufficiently extreme and for a relatively short amount of time (~15 trading days).[9]

A key shortcoming of these studies, from a reserve management perspective, is their focus on equity markets and long-duration fixed income indices (usually 10 years or above). Short rates, if included, typically feature as a separate, risk-free asset. In addition, the optimal or adequate share of gold is normally understood as the weight that either minimises the variance or maximises the Sharpe ratio of the portfolio being considered. While these measures are drawn naturally from the traditional Markowitz optimisation problem, other metrics – such as value-at risk (VaR) or conditional value–at-risk (cVaR) are likely to be more relevant for reserve managers.

Against this background, our study's contribution is threefold: (1) it focuses exclusively on the fixed income space where most, if not all, reserve managers operate; (2) it addresses the question of choosing the appropriate gold allocation for an array of portfolio durations spanning the *shorter* term (6 months to 10 years) typically associated with FX reserve portfolios; and (3) it defines optimality across a range of criteria, including – but not limited to – portfolio volatility and the Sharpe ratio.

Our approach is simple. Taking the historical behaviour of the prices of gold and fixed income instruments as given, we calculate their codependency structure and then explore how alternative long-only combinations of gold and fixed income behave across different scenarios. This allows us to draw inference on their interaction, highlighting some key observations for reserve managers facing questions about the appropriate weight of gold in their respective portfolios.

Our main findings are the following:

- From a market risk perspective, a low-duration, reserve currency fixed income portfolio may benefit from very small gold allocations (between 0% and 5%) on average. This is mainly due to the volatile behaviour of the precious metal.

- For higher durations, gold can provide an effective hedge against the portfolio's sensitivity to changes in yields. This is evidenced by increases in portfolio duration beyond 2 years, which can give support to gold holdings above 10% of total FX reserves, on average.

8 Defined as a negatively correlated asset in terms of market stress.

9 See also Baur and McDermott (2010) and Reboredo (2013) on gold and safe havens, and Sjaastad and Scacciavillani (1996) and Kohlscheen (2020a) on gold and FX rates.

- Gold's diversification benefits are enhanced in the presence of foreign exchange risk, especially for currencies that are highly volatile vis-à-vis the portfolio's unit of account (or numeraire).[10] This is apparent from an exercise varying the numeraire currency, which shows that holding more than 20% on average may be optimal in cases where the numeraire is a commodity currency or one from an Emerging Market economy.

- Disaggregating the results to analyse the benefits of gold as a hedge against an extremely rainy day (ie, a tail event) suggests that high allocations of between 20% and 50% may be adequate in the presence of elevated yield sensitivity or exchange rate risks. This is, for durations above 5 years, or when measuring the performance of a portfolio in a non-reserve currency numeraire.

- Our results, altogether, reveal that choosing an adequate share for gold in reserve portfolios is non-trivial and depends crucially on both the purpose (policy objectives) and implementation (numeraire, risk tolerance, etc) of reserve management. In addition, that both quantitative and qualitative considerations may matter.

The remainder of the paper is structured as follows: section 2 provides a brief recapitulation of the official sector's holdings of gold, highlighting key stylised facts relevant for our study. Section 3 explains the methodology used for the empirical analysis, which is based on a range of risk-return statistics, calculated for combinations of representative reserve portfolios and gold. We start by exploring the risk-return properties of gold for a single, representative reserve portfolio; then, we extend the analysis to portfolios with different maturities and numeraires; finally, we take a look at the full distribution of appropriate gold holdings across our simulated scenarios. Section 4 summarises the results and adds some reflections that go beyond statistical analysis.

[10] See, for example, McCauley (2008). The term numeraire derives from the French *numéraire*, or the Latin *numerarius*.

2. Gold as a reserve asset: a brief historical context

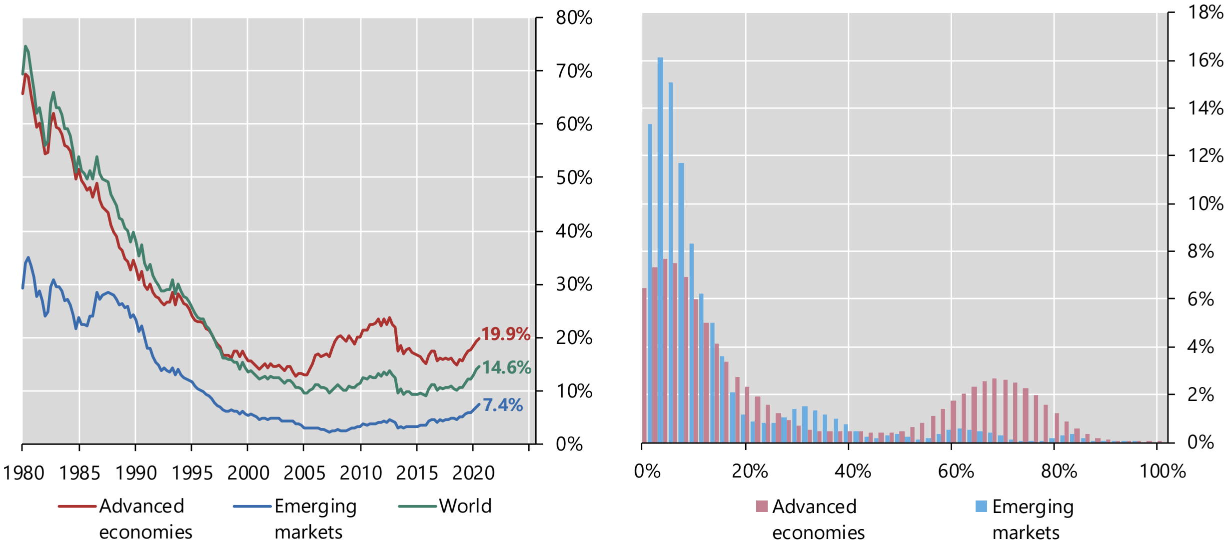
Almost five decades after the collapse of the Bretton Woods system, gold continues to form an important part of many countries' stock of foreign reserves. According to data from the International Monetary Fund, the metal constitutes 14.6% of total world FX reserves. And, even though this percentage has remained broadly stable at least since the late 1990s, it masks important differences in the behaviour of central banks in emerging countries versus those in advanced economies.

For example, by the end of the third quarter of 2020, advanced economies altogether held around 20% of gold as a percentage of their official assets, while the share in emerging and developing countries' portfolios was only 7.4% (Graph 1, left-hand panel).[11] Looking more closely, it is also evident that, individually, advanced economies hold greater amounts of gold than emerging markets as a fraction of reserves. Estimating the empirical distribution of the share of gold most recently reported by reserve managers (Graph 1, right-hand panel), reveals two modes in the distribution of advanced economies (red bars): one approximately between 0% to 10% (including countries as Singapore, Ireland and Australia), and another between 60% and 80% (eg, Italy, Germany and the US). Noticeably different results are observed for emerging markets, where we find much more density in the lower end of the spectrum (blue bars).

Holdings of gold in market value terms — Graph 1

Gold as a fraction of total reserve assets by country grouping and the world (as of Q3-2020)
In per cent of total reserves

Distribution of the fraction of reserves invested in gold (as of Q2-2020)
Probability density



Sources: International Monetary Fund (IMF); World Gold Council; BIS calculations. Emerging markets includes countries classified as emerging and developing economies by the IMF.

History suggests that this divergence is explained by at least two factors: first, large legacy holdings, in tonnes, on the balance sheets of central banks in many advanced economies (an inheritance of what once was the gold standard); second, the protracted growth of the FX reserve portfolios of emerging market countries after the 2007/08 global financial crisis (which rose by 2 trillion USD over the last 10 years in market value terms). The latter trend reverses the result of aggregate net selling by central banks and other official sector accounts in the decade prior to the crisis, taking the total gold stock held by the

[11] Since the 2000s, the stock of world reserve assets including gold has generally seen an upward trend. One important exception is in 2013, where a drop of 40% in the price of the metal affected the market value of holdings. The event coincided with news that Cyprus might sell its gold reserves as part of its bailout plan (Reuters (2013)).

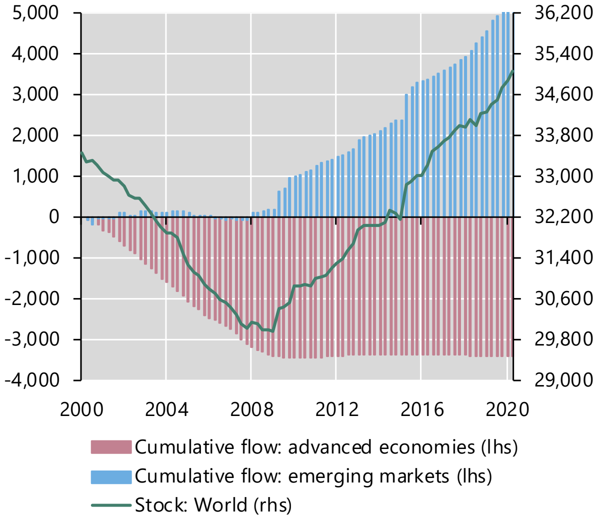
official sector back to levels around 35 thousand tonnes (Graph 2, left-hand panel). Indeed, country grouping-level flows confirm that the recent reserve accumulation has been driven mostly by reserve managers from emerging markets (blue bars), whereas those from advanced economies appear to have been the driving force behind the earlier de-accumulation (red bars).[12]

At the same time, gold holdings tend to be generally stable at the individual country level, whether it's in the advanced country grouping or not. The standard deviation of each country's holdings of gold reported quarterly between Q1-2000 and Q2-2020 is in a range between 0 and 615 tonnes (and at 28 on average). Indeed, there are strong signs of concentration effects. When removing the four biggest outliers in terms of standard deviations (Russia, China, Switzerland and France[13]), the cross-country average decreases to 14 tonnes (Graph 2, right-hand table).

Holdings of gold in volume terms

In tonnes | Graph 2

Official sector stock of gold and cumulative flows by country grouping since the year 2000



Summary statistics of reported quarterly gold holdings across countries since the year 2000

Full sample

	Average	Std. deviation
Min	0	0
Mean	227	28
Max	8,135	615

Excluding outliers*

	Average	Std. deviation
Min	0	0
Mean	187	14
Max	8,135	115

Sources: International Monetary Fund (IMF); World Gold Council; BIS calculations. Emerging markets includes countries classified as emerging and developing economies by the IMF. *Central banks with a standard deviation of holdings above 250 tonnes were excluded from the sample (Russia, China, Switzerland and France).

Changes in gold holdings, therefore, tend to accumulate gradually, suggesting that reserve managers generally maintain a buy-and-hold strategy for their gold. From a portfolio perspective, this could be an indication that reserve managers do not dynamically determine these exposures, but rather take them as given. Perhaps, it does not form part of their asset allocation analyses in practice. Nevertheless, any such decision has implications for a reserve portfolio's overall risk and return properties (and, therefore, official financial statements), especially as the price of gold is known to be highly volatile.

[12] For some of the reasons behind the earlier deaccumulation period, see Henderson et al (1997).

[13] The same countries have historically been some of the biggest holders of gold.

3. Gold as a reserve asset: empirical analysis

The ability of gold to act as a diversifying asset will depend primarily on two factors: its codependency with other instruments in the portfolio, and the asset manager's definition of risk. In practice, central banks and governments have similar objectives when managing FX reserves: liquidity, safety and return (see Fender et al (2019)). Yet, how the concept of *safety* (or *risk*) is understood may differ, depending on the central bank's investment preferences. For example, for those institutions whose tolerance for mark-to-market price variation is low, safety may effectively mean minimising portfolio volatility. Yet, there are other ways for risk managers to express their risk tolerance. Here, we keep the definition as broad as possible, and look at return variation, measures of tail-risk, the probability of loss and beyond.

We now proceed to explore the adequate weight for gold in FX reserve portfolios. To do so, we engage in an exercise of comparative statics, which we call "experiments". We sample gold and a fixed income portfolio, synthesising thousands of different combinations of the two, analysing the statistical properties of each combination (ie, of different asset class weights). Our objective is to elucidate how a fixed income allocation reacts to the presence of the precious metal.

3.1 Methodology

Our methodology consists of combining the returns of two assets: gold and a generic fixed income instrument. With return samples from each, we can create a sample of portfolio returns by setting weights respectively – in other words, by choosing the asset allocation. To enable this, we must first decide on the data and the investment horizon that will be used for the analysis.

We choose gold futures prices to model the commodity's returns and construct an SDR government bond portfolio with a constant maturity of 2 years to model the fixed income portion.[14] We consider that this duration approximates the one used by reserve managers in practice relatively well (see RAMP et al (2019)). In addition, as reserve managers tend to hold gold over long periods of time (see section 2), we consider ten years a reasonable investment horizon. To enrich the distribution of possible outcomes, we do this across 5,000 different scenarios.

In more technical terms, let the returns of gold and the fixed income part at time t be denoted by r_t^G and r_t^{FI}, respectively. For a given weight of gold $w \in [0,1]$, the return of a model FX reserve portfolio r_t^P is given by:

$$r_t^P = wr_t^G + (1-w)r_t^{FI}$$

where $(1-w)$ is the complementary weight in fixed income. Using monthly returns, this calculation is done for a sample of size $\tau = 120$ months.[15]

Obtaining these scenarios, of course, requires a model that allows us to simulate from the return distributions, ideally in a multivariate fashion. We therefore fit the joint distribution from the historical monthly returns using a copula.[16] This method is fit for purpose as it captures the high non-normality of financial market data, while allowing us to preserve the individual empirical distributions of the assets and to capture the non-linear dependency in the tails.[17]

14 Gold price data comes from Bloomberg. For details on the construction of the SDR portfolio, see Appendix 1.

15 We repeat this for $J = 5{,}000$ different scenarios, which translates to a total of $\tau \times J$ simulated returns. For simplicity and realism, our analysis is solely based on unlevered positions, so $0 \leq w \leq 1$.

16 Historical monthly returns are calculated from January 1981 (when SDR data becomes available from the IMF) to July 2020.

17 The technical details of this process are described in Appendix 2.

With our simulations of FX reserve portfolio returns, we can estimate a summary statistic across all the different scenarios for each possible combination of gold and fixed income. We work with a total of eight alternative metrics: volatility, Value-at-Risk (VaR), expected shortfall, the probability of loss or negative return, expected return, the Sharpe ratio, the ratio of expected return to expected shortfall and a measure of risk factor diversification (which is explained in more detail below).

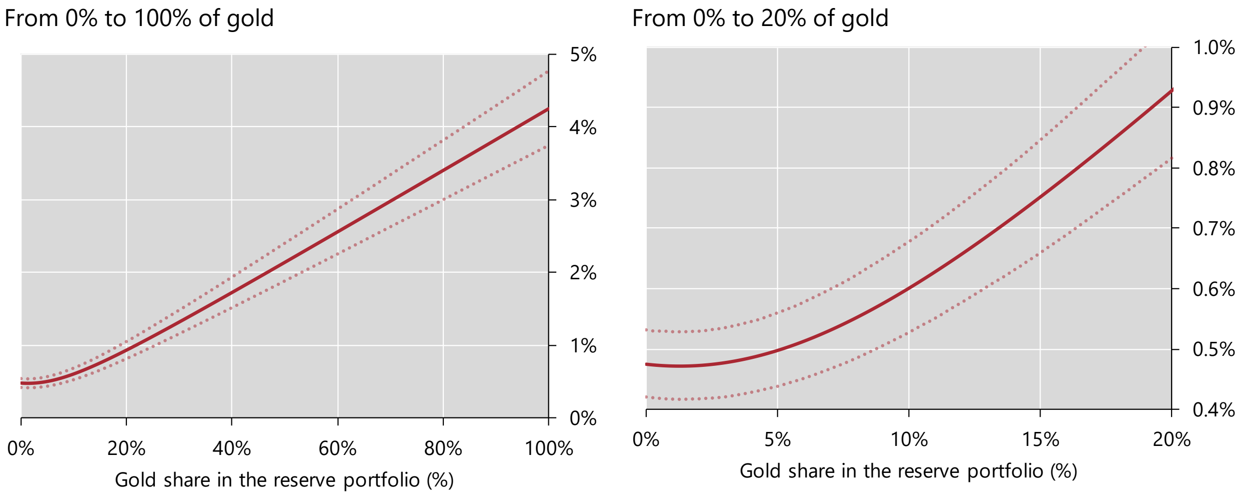
3.2 A first experiment: gold and a 2-year SDR portfolio

To set the scene, we first explore what the addition of gold does to reserve portfolio volatility, a commonly used risk tolerance measure. Graph 3 summarises what expected volatility would be for our model FX reserve portfolio under each possible combination of gold and a 2-year constant maturity SDR government bond. The representative composition of the reserve portfolio was set to the SDR because, according to IMF Currency Composition of Official Foreign Exchange Reserves (COFER) data, the top five currencies forming part of world FX reserves are the SDR-five.[18]

Gold's market risk profile is palpable in this exercise. An addition of the metal (ie, a positive per cent share in gold) quickly increases the variability of overall portfolio returns, making the relationship between increased gold allocations and volatility nearly a straight line (Graph 3, left-hand panel). If a reserve manager's objective is to decrease return variation, these results would hardly support investments in gold. However, upon closer visual examination (Graph 3, right-hand panel), it turns out that there are some (small) allocations where, on average, diversification benefits can be reaped. For our model portfolio, a share of gold above 0%, but below 5%, would help mitigate some of the return variability of the 2-year SDR fixed income instruments. Indeed, the global minimum in terms of portfolio volatility is reached for a gold weight at 1.35%.

FX reserve portfolio volatility combining gold and 2-year SDR government bonds[1]

In per cent — Graph 3



[1] These calculations are based on 5,000 simulations with 10 years of monthly data each. The dotted lines denote a 95% confidence band around the mean estimate.

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

The above result may be better understood when looking at the share of total volatility contributed by gold. (This is the concept of total risk contribution used in Maillard et al (2010)). Graph 4 (top left panel,

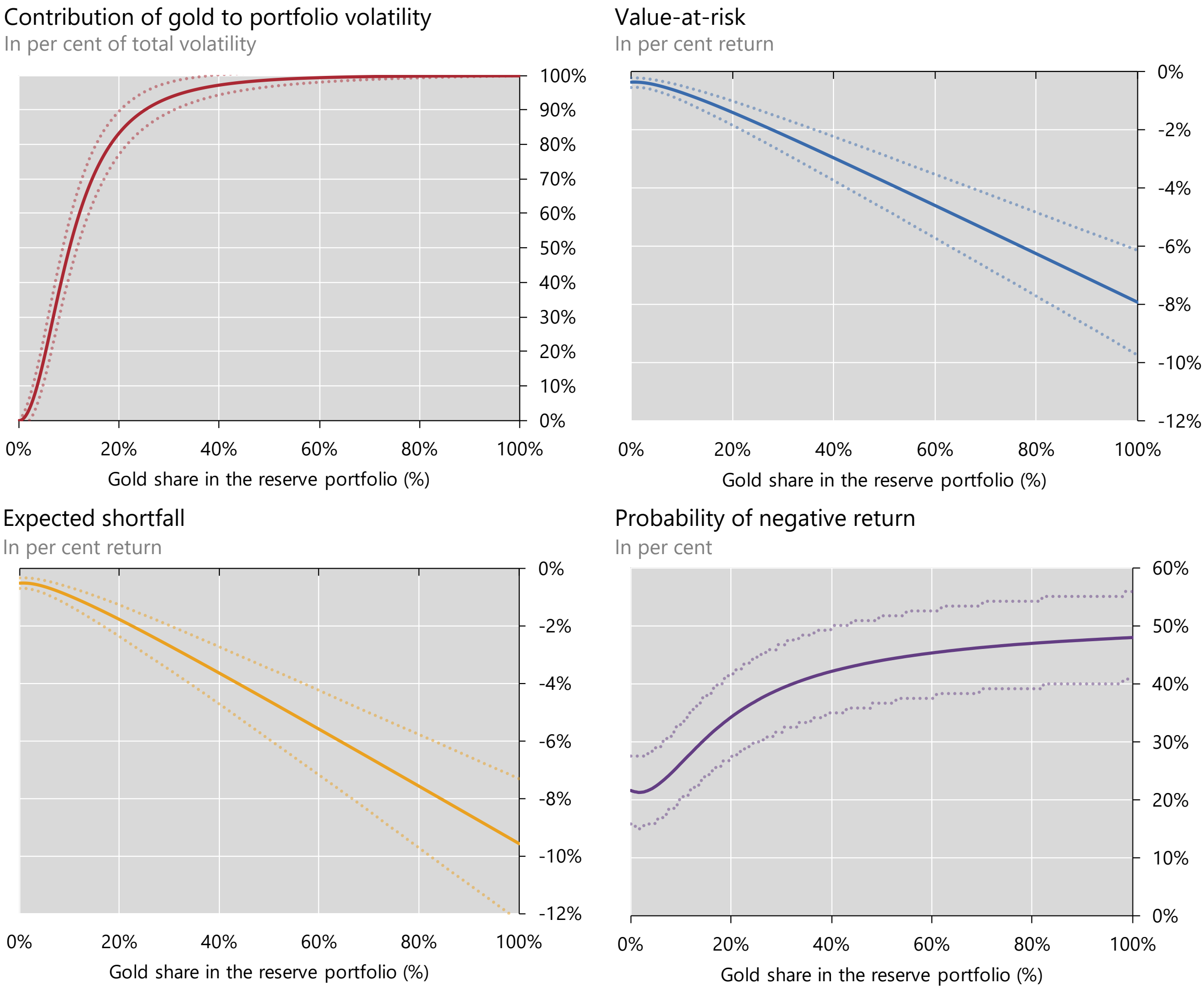
[18] For this exercise, all returns are measured in SDR percentage terms. The idea behind this assumption is to start off the analysis without the effect of foreign exchange rate risk. The implications of varying this unit of account are discussed in section 3.3.

red lines) shows that, with only a share of 10% invested in the metal, a full half of total portfolio volatility is already explained by it. And should a reserve manager decide to hold a proportion of gold above 20%, nearly all of the changes in value of the model portfolio are likely to be explained by fluctuations in gold prices. A key reference point used by reserve managers is *risk parity*, which is defined as the point where the two asset classes each contribute 50% to total portfolio volatility. In our example, risk parity is reached at 10% of gold. This indicates where risk sources may be balanced (yet not minimised).

In practice, volatility is not the only (or even the key) risk tolerance measure employed by reserve managers. For those concerned with the realisation of a low probability event, for example, VaR and expected shortfall may offer superior insight. In turn, for those who interpret their objective of capital preservation as the avoidance of negative returns at any cost, calculating the probability of loss may be more useful.

Against this background, Graph 4 illustrates the impact of rising gold allocations on four commonly employed risk metrics. In addition to volatility, as discussed above, the top right and bottom left-hand panels (blue and yellow lines) focus on measures of portfolio tail risk. The results are quite striking.

FX reserve portfolio statistics combining gold and 2-year SDR government bonds[1] Graph 4



[1] Calculations are based on 5,000 simulated scenarios with 10 years of monthly data each. Dotted lines denote a 95% confidence band. Value-at-risk and expected shortfall are estimated at 97.5% confidence, the level used by the Basel Committee on Banking Supervision (2019).

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

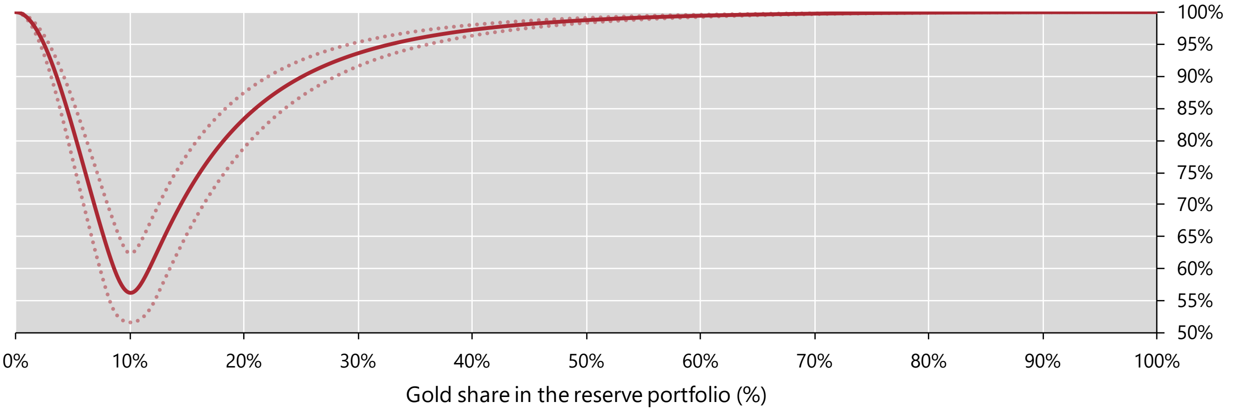
The VaR metric turns out to be roughly a monotonic function of the amount invested in gold. Even though the fixed income portfolio has a 97.5 percentile return close to zero, according to our estimates, an addition of gold quickly brings this number to negative territory. Gold appears to have a VaR (97.5% confidence) of about 8% in monthly return terms on a standalone basis. Results are similar for expected shortfall. When the the VaR threshold is breached, the average loss a reserve manager can expect is about 10% when only holding gold (ie for a portfolio weight of 100%). Still, there is support for a positive, yet small, gold allocation: the global maximum for both the VaR and expected shortfall metrics – where a weight in gold minimises tail risk – is reached at about 1.2%. The fourth metric, the probability of facing a negative return (Graph 4, bottom right-hand panel), yields slightly less dramatic results. The adequate proportion would be about 2%, if reserve managers wanted to avoid experiencing losses in any given month.

Graph 5 explores a fifth risk metric. We call it the single factor concentration measure (*SFC* measure), which is based on the application of principal component analysis (PCA).[19] To calculate it, we first weight the time series simulations of gold and the SDR portfolio; then, we run PCA on the weighted data – this results in a set of unobservable and independent factors; finally, we calculate how much of the total variance is explained by the first factor.[20] The higher it is, the more portfolio risk can be assumed to be concentrated in a single factor, regardless of what this latent variable is. After all, the total return of the reserve portfolio will be a linear combination of this and other quantitites.

Calculating the SFC measure for every combination of gold and the 2-year SDR government bond in our model portfolio yields a pronounced "V" shape. This implies that the addition of gold does help decrease concentration in a single risk factor, but only to a certain extent. In our example, allocating about 10% to gold minimises the SFC's value, but going any further will gradually fade the associated diversification effect. Indeed, the first PCA factor explains more than three quarters of the variance when a share of 20% of the model portfolio is invested in gold.

SFC measure for combinations of 2-year SDR and government bonds and gold[1]

In per cent of total variance — Graph 5



[1] These calculations are based on 5,000 simulations with 10 years of monthly data each. The dotted lines denote a 95% confidence band around the mean estimate. A higher value of the SFC measure is associated with an increased concentration of risk in a single factor.

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

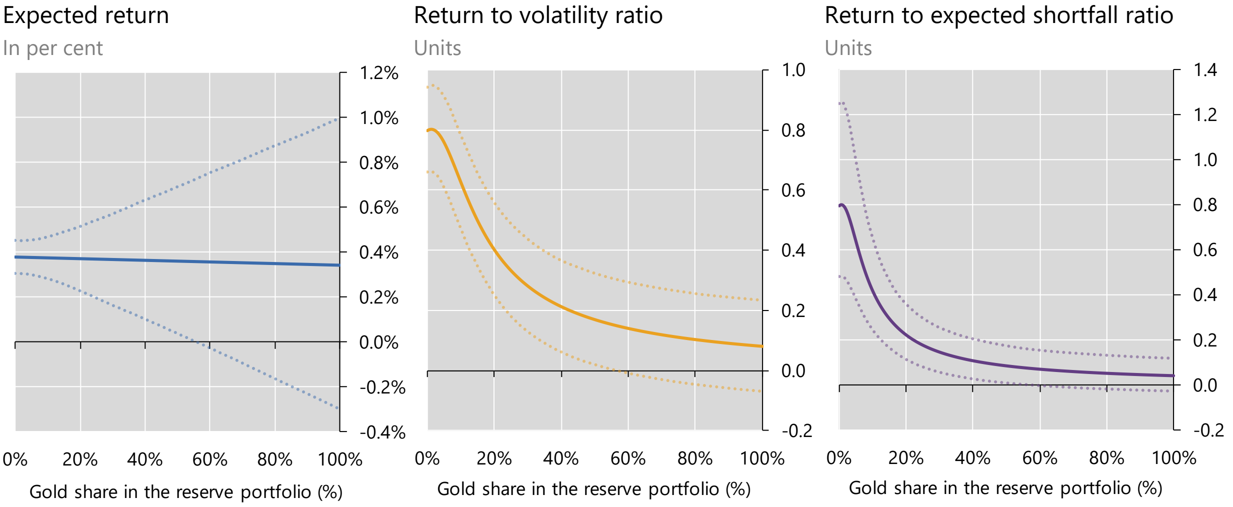
[19] In financial economics, one of the most commonly applied techniques to estimate unobservable risk factors that underlie asset returns is principal component analysis. In brief, it helps calculate the latent variables that can best explain the variance of returns. However, interpretation may be challenging in practice. For a comprehensive review, see Bai and Ng (2008).

[20] In PCA, the first factor is always the one to explain the greatest fraction of total covariance. In more technical terms, the proportion of total variance that each eigenvector represents (in this case, for the first factor) can be calculated by dividing the eigenvalue corresponding to that eigenvector by the sum of all eigenvalues.

Let us now take a return rather than a risk perspective. Under our simulations, the average (or *expected*) return of gold seems to be close to that of the 2-yr SDR fixed income benchmark. This translates into very small changes in the expected return of the FX reserve portfolio as the exposure to gold is being raised (Graph 6, left-hand panel) – a pattern driven by the historical behaviour of both assets.[21] Arguably however, the most important result relates to the confidence bands around these estimates. First, they appear the widest of any exercise thus far, highlighting the difficulty of predicting financial asset returns. Second, as the exposure to gold is increased, uncertainty around the portfolio's expected return rises precipitously. This is in line with the previous results relating to the market risk of gold, and suggests that one needs to be careful not to draw conclusions based solely on estimates of the mean return.

We can put the risk and return angles together by looking at ratios of expected return over a given risk metric (a measure of risk-adjusted returns). Graph 6 illustrates the results based on ratios over volatility and expected shortfall (middle and right-hand panel, yellow and purple lines), both of which are commonly used by reserve managers in practice. In both cases, risk-adjusted returns are increasing only for very small portfolios weights of gold and are declining steeply for higher weights, especially those beyond 10%. Indeed, taking a closer look, it turns out that the return gained per unit of risk taken is maximised at a weight somewhout below 2% in both cases.

FX reserve portfolio statistics combining gold and 2-year SDR government bonds[1] Graph 6



[1] Monthly figures. These calculations are based on 5,000 simulations with 10 years of monthly data each. The dotted lines denote a 95% confidence band around the mean estimate.

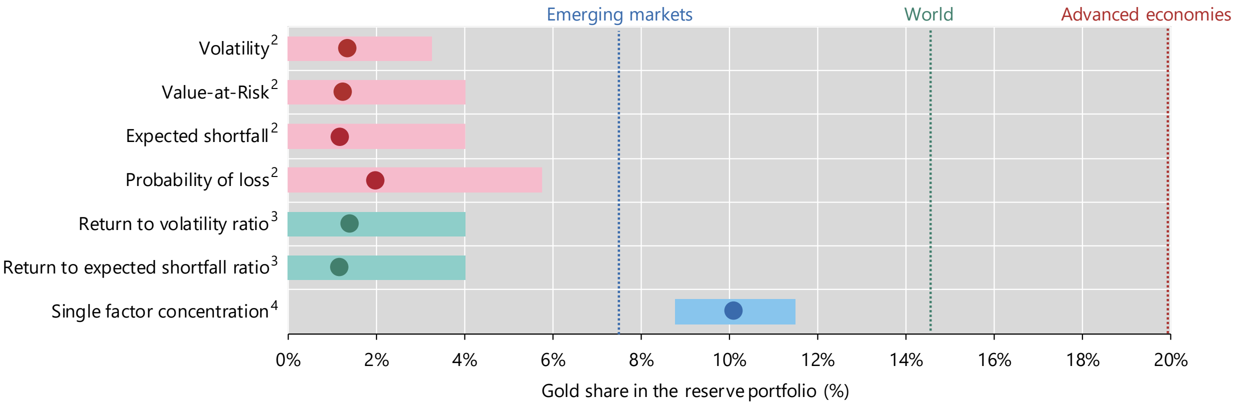
Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

As a result, a reserve manager with a 2-year government bond portfolio of SDR bonds would find it hard to argue in favour of sizeable gold holdings, at least based on the criteria reviewed above. Graph 7 summarises these findings. Minimising portfolio risk, reserve managers would find shares below 2% of gold an adequate choice on average, and up to 6% considering a 95% confidence interval (red dots and bars). Similar results are obtained when risk-adjusted returns are being maximised (green dots and bars). Only if reserve managers were to target a minimisation of the number of statistical factors explaining the variance of portfolio returns (ie the SFC metric), allocations of around 10% might be appropriate. It is unlikely, however, that a criterion like this would be the primary guide in choosing an optimal reserve portfolio. It seems difficult, therefore, to explain real world choices of gold reserve shares, as indicated by the dotted vertical lines, without further consideration of other choices, such as portfolio duration and the portfolio's numeraire.

[21] In practice, it is ill-advised to use historical average returns as a predictor of future performance.

Optimal allocation to gold according to multiple quantitative criteria[1]

In per cent of the market value of the reserve portfolio — Graph 7



[1] The bars denote a 95% confidence band around the mean estimate. [2] Allocation to gold that minimises the measure. [3] Allocation to gold that maximises the measure. [4] Allocation to gold that minimises the variance explained by the first principal component of a PCA analysis.

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

3.3 Further experiments: SDR portfolios with different durations

Reserve managers – like any asset manager – occassionally ask themselves what the appropriate interest rate sensitivity (or duration) of their portfolio should be and, thus, tend to vary it. A simple analysis of the holdings of US Treasury bills, notes and bonds helps us illustrate this point.[22] Through the Global Financial Crisis (GFC), the official sector's holdings of US Treasury notes and bonds (instruments with maturities above 1 year) vis-à-vis the rest (bills, whose maturity is 12 months or below) fell from a high of 90% in 2007 to a low of 74% by mid-2008. Notwithstanding, by end-2019, 90% of the US debt held by the official sector was again in maturities beyond one year.

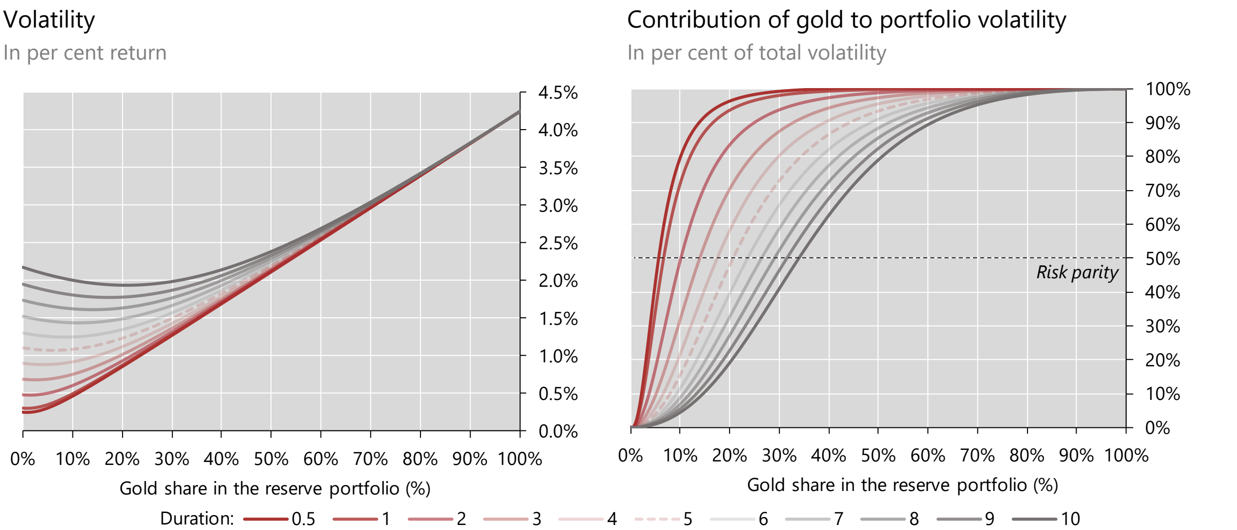
Basic fixed income wisdom suggests that a higher weighted maturity on any bond portfolio exposes investors to increased price sensitivity due to changes in interest rates. Whether exposures to longer duration are rewarded remains an empirical question (see Cohen, Hördahl and Xia (2018) for a discussion on term premia). However, one can safely assume that, regardless of the rationale, reserve managers will actively think about this important variable. We thus extend our analysis to a series of SDR government bond portfolios with different maturities: 6 months, 1 year, 3 years, and maturities up to 10 years using one-year increments. We contrast this with our original 2-year exercise.[23] Our prior is that the higher the interest rate risk exposure, the less risky gold will look in relative terms.

Graph 8 (left-hand panel) tracks changes in total portfolio volatility across model portfolios with different durations and for rising shares invested in gold. A darker shade of red denotes lower durations, while a darker shade of grey denotes higher durations; the dashed line in the middle denotes the 5-year case. As previously stipulated, when the duration of the portfolio increases, the fixed income investor is exposed to higher risk. Indeed, for a portfolio fully invested in fixed income (0% in gold), total volatility is increasing in maturity. But what about gold allocations? Effects differ across portfolios. Indeed, it appears that, for low durations (below 5 years) higher gold allocations will raise overall portfolio volatility, while the reverse may be true as duration increases.

[22] See U.S. Department of Treasury data, Securities (B): Portfolio Holdings of U.S. and Foreign Securities.

[23] We work with generic zero-coupon bonds, which allows us to use the duration and maturity concepts interchangeably.

FX reserve portfolio statistics combining gold and SDR government bonds of different maturities[1] Graph 8



[1] Monthly figures. These calculations are based on 5,000 simulations with 10 years of monthly data each.

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

Specifically, as the duration of the fixed income portfolio is raised, adding gold more easily *decreases* the variability of the model portfolio. For the 10-year duration case (the darkest grey line), for example, there is a dip in volatility for gold investments in a 10-30% range, whereas portfolio volatility increases almost in a linear fashion in the 6-month case (the darkest red line) as allocations to gold are being raised. The effect is even clearer when expressed in terms of gold's contribution to total portfolio volatility (Graph 8, right-hand panel). For fixed income porfolios with a duration of 6 months, risk parity (the point where a given gold weight explains half of the portfolio's variance) is reached for a gold allocation of 6.5%. For a portfolio with a duration of 5 years, in contrast, it takes an allocation of 20.75%; and 33.75% for a duration of 10 years. This validates our prior that, with higher exposure to interest rate risk, gold appears *relatively* less risky, even as total portfolio risk is still increasing in duration.[24]

Graph 9 takes the analysis one step further by looking at the tail risk-mitigating properties of gold holdings (left-hand panel). Again, the benefits of gold investments are larger for higher-duration portfolios. However, regardless of maturity, replacing 20% or more of the model portfolio's fixed income instruments by gold quickly takes the average expected loss to very negative territory. Similar caution is advised for reserve managers whose key risk measure is the probability of loss (right-hand panel). In this case, even without any gold allocation (weight of 0%), higher durations can markedly elevate the possibility of facing a negative return during any given month. Adding gold can help mitigating the possibility of facing a loss for these higher durations, but the effect is not strong and fades as allocations approach a gold weight of 10%.[25]

[24] In other words, increasing the duration of the portfolio to cushion the effect of gold could leave the portfolio exposed to a much higher level of market risk due to the high volatility of both assets.

[25] Results for risk-return ratios based on portfolio volatility and expected shortfall are similar, but omittted for brevity.

FX reserve portfolio statistics combining gold and SDR government bonds of different maturities[1]

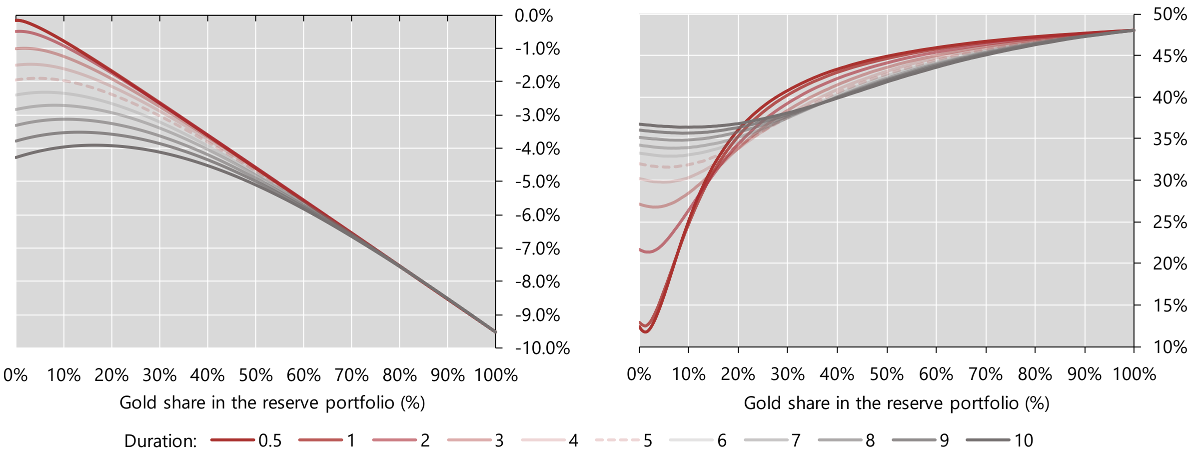
Graph 9

Expected shortfall

In per cent return

Probability of negative return

In per cent



[1] Monthly figures. These calculations are based on 5,000 simulations with 10 years of monthly data each.

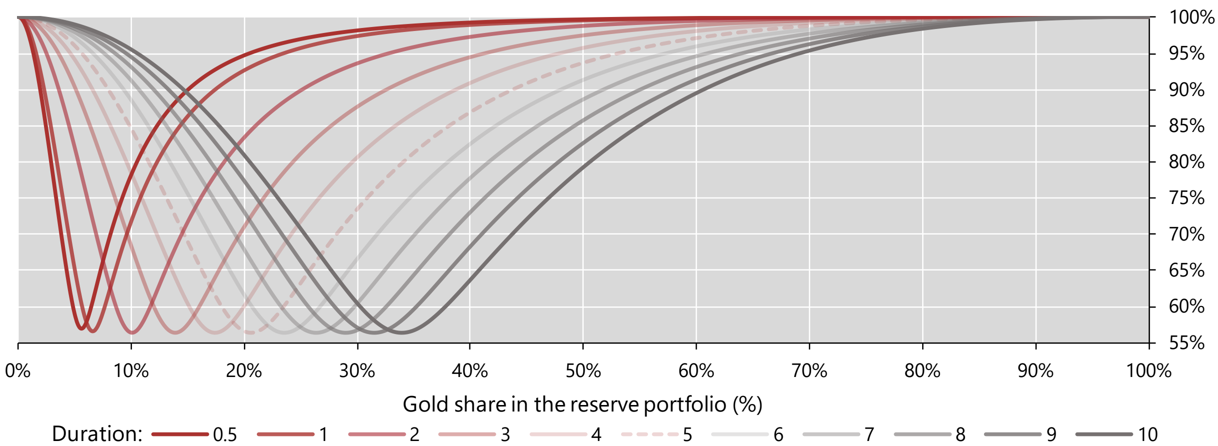
Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

An interesting case is the SFC measure, which might support somewhat higher gold allocations in some cases. Graph 10 shows the variance of the portfolio that is explained by a single risk factor, and how it varies with changing gold allocations. Just like in the 2-year SDR case discussed above, the estimated relationships display a pronounced "V" shape. Crucially, observed minima (marking the spot where one should "jump the rope") span a rather wide range of gold allocations between about 5% (low duration) and 35% (high duration), depending on the duration of the fixed income tranche. In addition, how quickly these diversification benefits are given up beyond that point is curbed by the effect of higher maturities, since the portfolio is more sensitive to changes in interest rates.

SFC measure combining gold and SDR gov. bond portfolios with different maturities[1]

In per cent of total variance

Graph 10



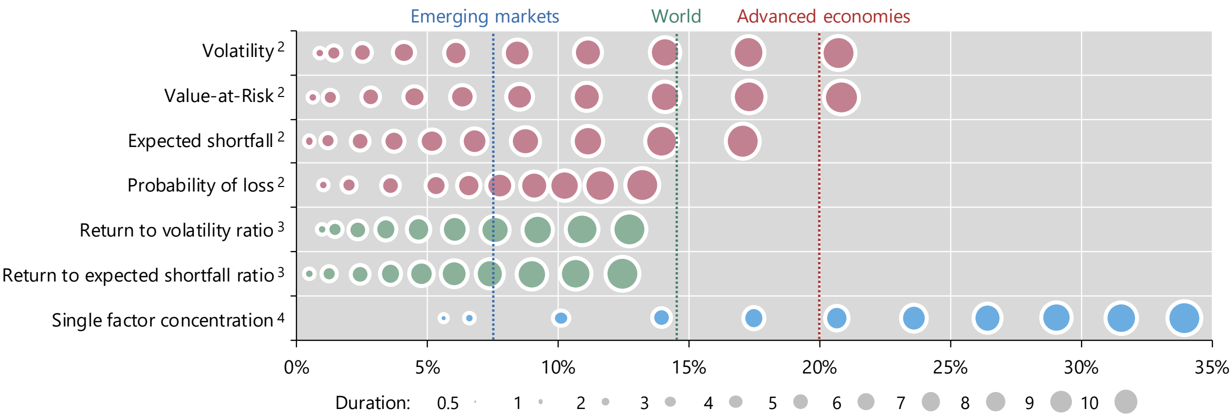
[1] These calculations are based on 5,000 simulations with 10 years of monthly data each. The dotted lines denote a 95% confidence band around the mean estimate. A higher value of the SFC measure is associated with an increased concentration of risk in a single factor.

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

Graph 11 provides a graphical summary of all these results, benchmarked against the observed gold holdings of reserve managers in emerging markets, advanced economies and the entire world. It highlights the key role of duration in making decisions about appropriate gold allocations. In cases where appetite for interest rate risk is high (bigger bubbles/higher durations), gold can help cushion the sensitivity of reserve portfolios to changes interest rates. Yet, for the lower durations characteristic of FX reserve portfolios (smaller bubbles), gold's diversification benefits are limited, and quickly dissipate for holdings beyond 5%, far below the observed world average. For these lower duration portfolios, observed gold holdings are thus hard to justify from this purely quantitative standpoint.

Optimal allocation to gold according to multiple quantitative criteria[1]

In per cent of the market value of the reserve portfolio — Graph 11



[1] Bubbles denote the average optimal weight across 5,000 simulations. [2] Allocation to gold that minimises the measure. [3] Allocation to gold that maximises the measure. [4] Allocation to gold that minimises the variance explained by the first principal component of a PCA analysis.

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

3.3 Yet more experiments: different *numéraires*

According to the BoPM, the values of positions and transactions in FX reserves may be expressed in a variety of currencies or in other standards of value, such as the SDR. However, the conversion of all these values into a single reference unit of account (the numeraire) is a requisite for the construction of consistent and analytically meaningful accounts.[26]

In asset allocation, the chosen numeraire currency is used to measure all returns during the portfolio optimisation exercise and it may also be the currency in which all FX reserve returns are reported internally, for investment and performance management purposes.[27] The numeraire also implicitly defines the risk-free rate. For example, central banks with a US dollar numeraire will find that investments in any instrument denominated in other currencies will expose them to mark-to-market profit and loss due to variation in nominal exchange rates (which is commonly said to "expose them to currency risk").

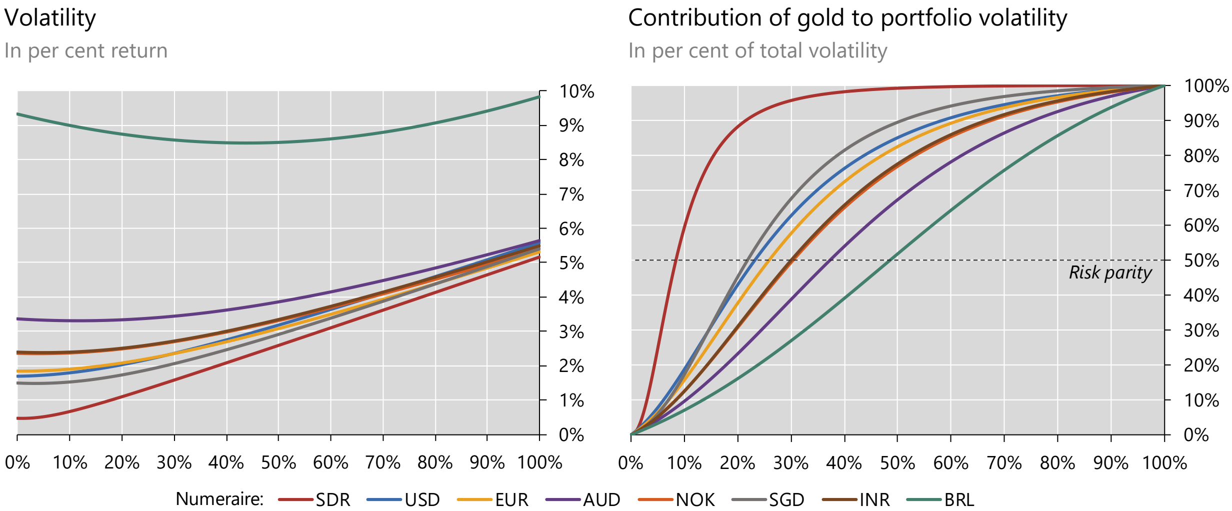
On this basis, reserve managers' numeraire choice will affect the observed correlation between the return on their fixed income assets and the price of gold, which is commonly quoted (and perhaps, also funded) in US dollars. In addition, central banks are known to have different numeraires, depending on their

[26] See paragraph 3.92 of section "4. Unit of account and currency conversion", applicable to all concepts of the balance of payments.

[27] Whereas national accounts (including the value of FX reserves) are commonly reported in domestic currency.

objectives and practices.[28] With this in mind, comparative statics for the representative 2-year duration SDR portfolio defined above are now performed for a range of alternative numeraires: the US dollar (USD), euro (EUR), Australian dollar (AUD), Norwegian krone (NOK), Singaporean dollar (SGD), Indian rupiah (INR) and Brazilian real (BRL).[29] The results are contrasted with those from the initial SDR numeraire exercise, which is a currency-basket unit of account.

FX reserve portfolio statistics combining gold and 2-yr SDR government bonds measured in different numeraires[1] Graph 12



[1] Monthly figures. These calculations are based on 5,000 simulations with 10 years of monthly data each.

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

Again, FX reserve portfolio risk is analysed first. Graph 12 (left-hand panel) shows the volatilities of the model reserve portfolio under different numeraire assumptions. In contrast to many of the previous charts, the starting point, where gold is held at 0%, differs across cases. Indeed, the same SDR portfolio's risk and return statistics will change when measured in a different numeraire currency, which are represented by a different colour. Sitting at the bottom of the chart, as might be expected for the numeraire that reflects the same currency composition of the portfolio's fixed income instruments, SDR returns display the lowest volatility. It is followed, in ascending order, by SGD, USD, EUR, NOK, INR, AUD, and distantly, the BRL. Two factors drive this result: first, commodity and emerging market currency exchange rates tend to be more volatile than those of reserve currencies; second, the fixed income portfolio is formed of instruments denominated in reserve currencies.

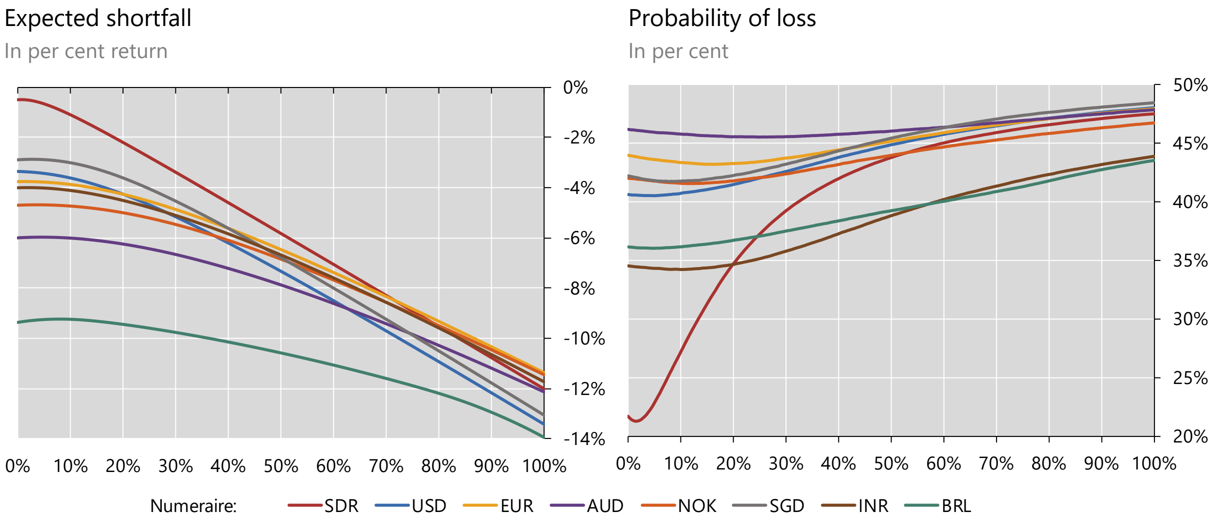
What happens when gold is added? Portfolios with more volatile numeraires and/or higher exposure to exchange rate risk are less sensitive to increased gold allocations. In other words: there is a base effect (whether the original portfolio was *already* highly volatile) as well as a relative effect (whether the portfolio is more or less volatile *than* the gold allocation). For example, moving from left to right, BRL-numeraire portfolio volatility reaches its minimum at around 50% of gold; yet, the volatility of an all-gold BRL portfolio has a similar level of volatility as the original one (at 0% gold). For all other numeraires, shares above 10 to 20% are seemingly less supported and tend to increase portfolio variance.

[28] For reserves used mainly to intervene in FX markets, reserve currencies such as US dollars and euros may be used; for a liability management objective, the currency composition of liabilities may be ideal; when the goal is to preserve national wealth, domestic currencies. In practice, the USD, the EUR and domestic currencies are most common (see RAMP et al. (2019)).

[29] Our selection is a mix of reserve-, commodity-, free floating-, managed float-, advanced- and emerging economy- currencies.

This can be further appreciated when measuring the contribution of gold allocations to total portfolio variance (Graph 12, right-hand panel). The two extremes are again the SDR numeraire (where volatility is more *easily* explained by gold) and the BRL (where total risk is *less* easily explained by gold), with risk parity at a gold share of around 50%. Based on this, it is natural to expect reserve managers with riskier numeraires to have a stronger tolerance for higher gold allocations. Indeed, Graph 13 shows similar results for a measure of tail risk (expected shortfall, left-hand panel). For the probability of loss (right-hand panel), the most sensitive case is again the SDR numeraire. However, the probability of observing a negative return appears to be minimised with shares of between 10% and 30% in all other cases – providing relatively strong support for gold allocations.

FX reserve portfolio risk measures combining gold and 2-yr SDR government bonds measured in different numeraires[1] Graph 13



[1] Monthly figures. These calculations are based on 5,000 simulations with 10 years of monthly data each.

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

Graph 14, finally, offers the full spectrum of results across numeraires and portfolio metrics. The colored dots (or "confetti") represent optimal allocations to gold across different numeraires (colours) and metrics (rows). There are at least five main conclusions:[30]

- The majority of confetti remains behind (ie to the left) the advanced economies' share of gold (dotted red line). Thus, across numeraires, holding such a high amount of precious metal is not well supported, even as the weight is often well above 0% (though not in the SDR case).

- Results for portfolio metrics focused on risk (results for the first four rows, which are the *safety* measures analysed here) suggest that holding shares above 15% of gold may compromise risk minimisation objectives. Nonetheless, gold fits better in portfolios exposed to FX risk.

[30] We make a note that all these experiments would look different under alternative reserve portfolio compositions. For example, an investor with an EUR numeraire might find less diversification benefits should their fixed income allocation be fully constituted of bonds issued in euro, as opposed to the SDR mix applied here. This is because less exchange rate risk (and therefore, less base market risk) would be present at inception.

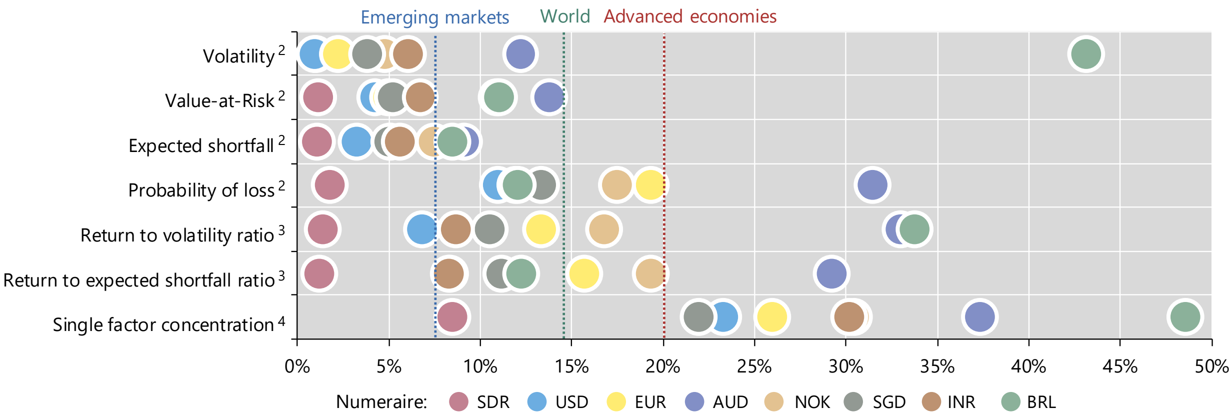
- When adding the return angle (fifth and sixth rows), the optimal range of gold allocations becomes wider. From a historical return perspective, reserve portfolios with European numeraires and those based on commodity currencies show broader diversification benefits. [31]

- In terms of risk factor diversification (row seven), it seems the metal has excellent benefits in the presence of exchange rate risk. Indeed, optimal shares under this criterion appear more spaced out and mostly beyond the weight advanced economies altogether place on gold.[32]

- Finally, across results, there seems to be broader support for gold when FX reserve portfolio returns are measured in more volatile numeraires. Commodity currencies, such as the Australian dollar (purple bubbles), or Emerging Market currencies, such as the Brazilian real (green bubbles) and the Indian rupiah (brown bubbles), tend to sit further on the right. This is similar to the interest rate risk results presented above (section 3.2), where higher duration provided a bigger cushion for holdings of gold.

Optimal allocation to gold according to multiple quantitative criteria[1]

In per cent of the market value of the reserve portfolio — Graph 14



[1] Bubbles denote the average optimal weight across 5,000 simulations. [2] Allocation to gold that minimises the measure. [3] Allocation to gold that maximises the measure. [4] Allocation to gold that minimises the variance explained by the first principal component of a PCA analysis.

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

3.4 Beyond means: the value of gold in tail risk scenarios

So far in the analysis, we've focused on what the optimal share of gold would be in expectation. Indeed, after creating each of the 5,000 scenarios, we estimated the relevant risk and return statistic for the model portfolio under each of these, and then aggregated the results by taking the mean. In proceeding this way, we've placed the focus of the analysis on the optimal share of gold for each criterion *on average*. In this section, the focus is placed on unaggregated results, highlighting other, higher moments of the relevant distribution of adequate gold holdings. We find that, under highly improbable scenarios, investors may want to hold gold in proportions different from those reviewed in the previous sections.

The analysis starts by returning to the 2-year SDR fixed income model portfolio reviewed earlier. For this purpose, Graph 15 takes a close look at the distribution of gold holdings that minimise portfolio volatility

31 Nonetheless, as previously noted, caution is advised as expected returns are hard to forecast, even more so when nominal exchange rate components are an additional part of them.

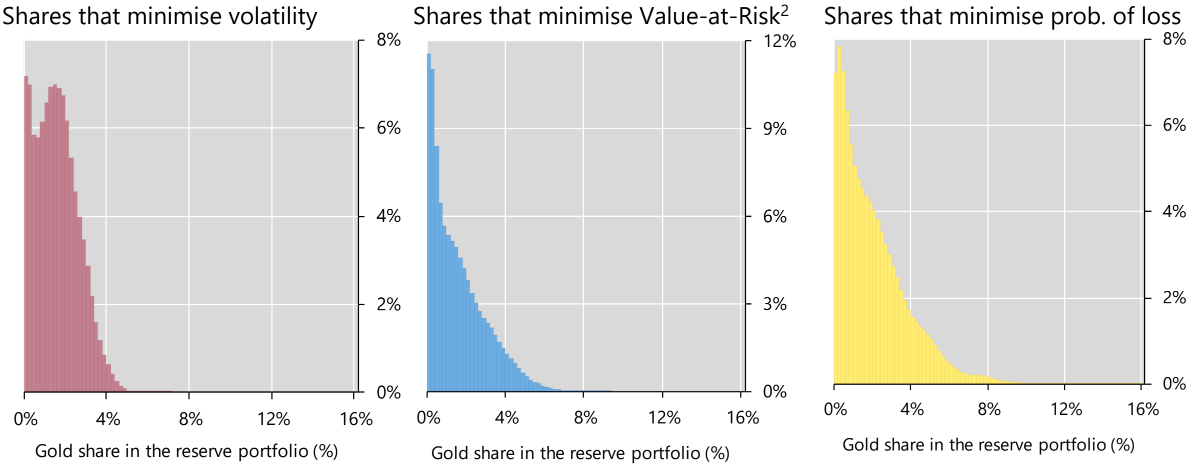
32 This is also a function of the rank correlation between gold and exchange rate returns. Pairwise rank correlations between gold and each currency (measured in US dollars) are: SDR 33%, EUR 30%, AUD 35%, NOK 34%, SGD 37%, INR 18%, BRL 17%.

(left-hand panel). It suggests that, although the density across the 5,000 scenarios is quite concentrated in the range between 0% and 2%, gold holdings of up to 4.75% may be helpful to reduce portfolio variance during an extremely unlikely event (ie the right tail of the density function).

Optimal shares of gold across scenarios for the 2-year SDR portfolio[1]

Probability density

Graph 15



[1] These distributions are based on 5,000 simulations with 10 years of monthly data each. One optimal weight of gold results from each simulated scenario. The density functions have been normalised to facilitate reading and comparison. [2] Estimated at 97.5% confidence.

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

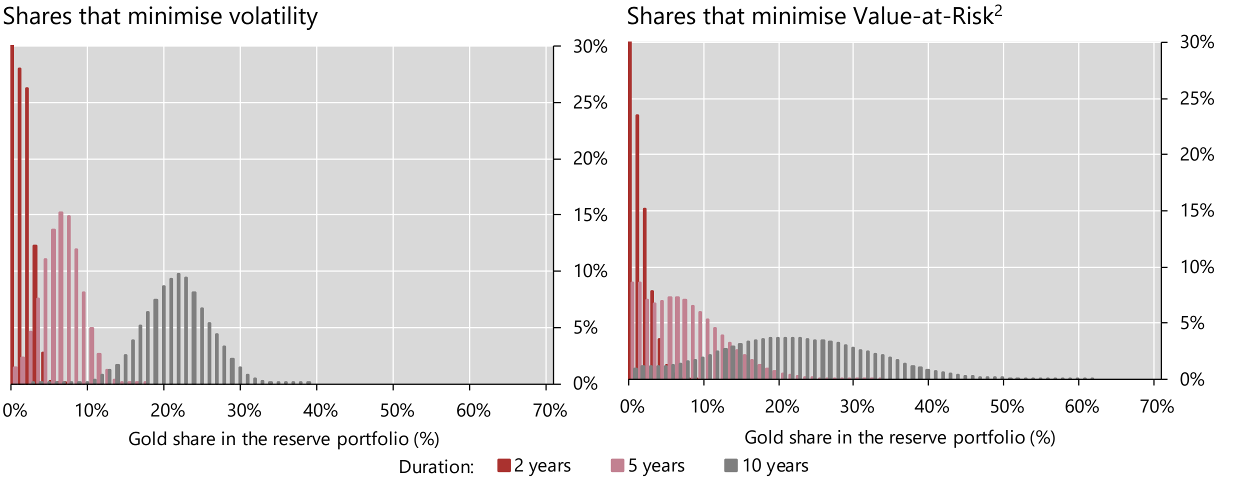
The centre panel repeats the same analysis for reserve managers seeking to minimise portfolio VaR. In this case, the adequate share of gold is more concentrated at 0%, suggesting that gold holdings will be less beneficial than in the earlier example, at least most of the time, in line with the results derived earlier. However, it also appears that extreme tail events could support gold allocations of up to a 9.25%. Similarly, in cases where the reserve manager seeks to mitigate the probability of observing negative returns (right-hand panel), results are tilted even further to the right, supporting shares of gold of up to 14.5% of the model portfolio.

As a result, a focus on tail scenarios can help support higher shares of gold than an analysis based entirely on observed means. The exact allocation, in turn, will depend on the institution's chosen definition of safety and the associated risk metric (that is, say, whether portfolio volatility, VaR, or something else is being minimised). Nonetheless, even though the resulting distributions for the representative 2-year SDR model portfolio are highly skewed to the right, gold shares above 5% appear adequate only under very few highly adverse scenarios.

But what about the comparative statics of changing assumptions about the portfolio's exposure to interest rate risk (duration)? And those of the chosen currency numeraire? Graph 16 (left-hand panel) answers the first of these questions by looking at the distribution of gold holdings that minimise volatility for portfolios with 2, 5 and 10 year durations. It is apparent that, starting from the original result for the 2 year portfolio, increases in duration lead to distributions that are less skewed and would tend to support higher gold weights. Indeed, moving to a 10 year duration portfolio (grey bars) leads to a distribution that is rather symmetric, and centered around gold allocations well in positive territory. Results based on the VaR metric are qualitatively similar (right-hand panel), with gold holdings making tail-risk events progressively easier to face when the model portfolio's sensitivity to yields is increased. Overall, therefore, investors with higher tolerance for interest rate risk might find bigger diversification benefits in gold than their lower duration peers; both on average and during tail events.

Optimal shares of gold across scenarios for the 2- 5- and 10-year SDR portfolios[1]

Probability density — Graph 16

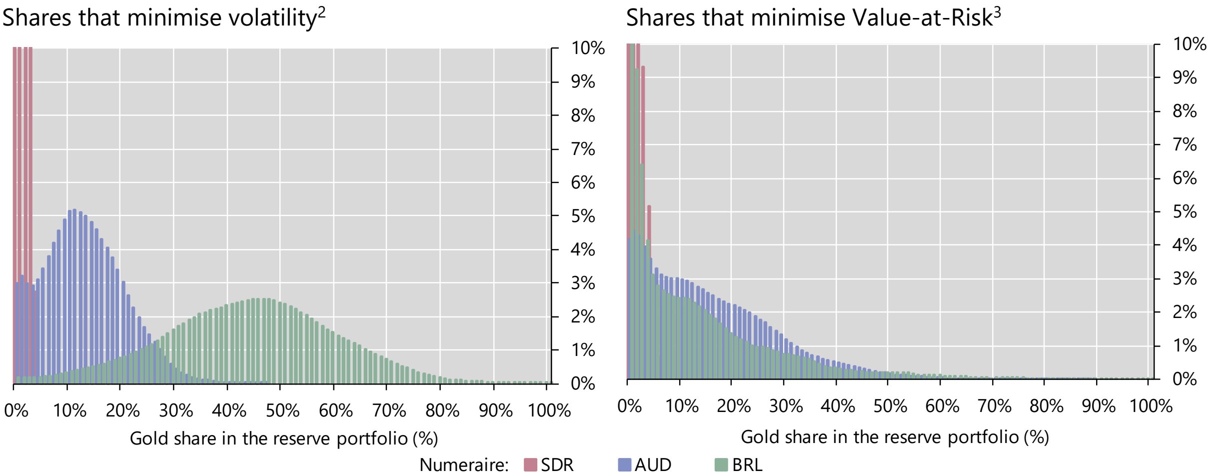


[1] These distributions are based on 5,000 simulations with 10 years of monthly data each. One optimal weight of gold results from each simulated scenario. The density functions have been normalised to facilitate reading and comparison. [2] Estimated at 97.5% confidence.

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

Optimal shares of gold across scenarios for the 2-year SDR portfolio measured in different numeraires[1]

Probability density — Graph 17



[1] These distributions are based on 5,000 simulations with 10 years of monthly data each. One optimal weight of gold results from each simulated scenario. The density functions have been normalised to facilitate reading and comparison. [2] Vertical axis was cropped at 10% for ease of comparison; maximum value is 35% for the SDR case. [3] Estimated at 97.5% confidence. Vertical axis was cropped at 10% for ease of comparison; maximum value is 31% for the SDR case.

Sources: Board of Governors of the Federal Reserve System; Deutsche Bundesbank; Bank of Japan; Bloomberg; BIS calculations.

Graph 17 answers the second question by performing comparative statics on the chosen portfolio numeraire. For this purpose, the benchmark 2-year SDR model portfolio is assessed on the basis of returns measured in Australian dollar (AUD) and Brazilian real (BRL) terms; two of the more interesting cases from section 3.3 and, in both cases, currencies that are not part of the SDR basket. Results are reported both for the volatility (left-hand panel) and VaR metric (right-hand panel).

In both of these cases, non-SDR numeraires would tend to support higher allocations to gold as a hedge against adverse scenarios than the SDR numeraire. Indeed, when the reserve manager seeks to minimise portfolio volatility, the distribution of adequate gold shares appears to become more symmetric and evenly spread out across the 0% to 100% range. In contrast, when portfolio VaR is being minimised, the distribution of gold allocations retains its asymmetric shape as the chosen numeraire is varied. However, right-hand tails are extended relative to the SDR case, supporting relatively high "extreme tail risk hedging" allocations in the case of the BRL numeraire (green bars). This bears special importance when we consider that FX reserves, as a measure of sovereign risk premia, matter the most – if not only – during stressful times (Kohlscheen (2020b)).

4. Conclusions

The analysis revealed to us that choosing an adequate share to invest in gold is not trivial and depends on purpose (policy objectives) and practice (target duration, numeraire choice, risk tolerance metric, etc). Given the volatility of gold returns, only a very small share of gold appears quantitatively adequate under most circumstances. However, there is evidence of a potential insurance value of gold in adverse scenarios, which can support higher allocations of gold in cases where the protection against tail risks is a key reserve management consideration. This applies, in particular, in those cases where the target duration of the fixed income part of the reserve portfolio is very high or when certain currencies are chosen as the portfolio numeraire.[33] Under the current interest rate environment (at the zero or effective lower bound for many reserve currencies), expectations of gold price appreciation also have the potential to drive gold demand upwards and, thereby, total central bank holdings.[34]

Yet, a portfolio's risk and return properties are only one set of considerations when evaluating the size of gold holdings. One issue is accounting rules. Indeed, gold's market risk may not be apparent in the reserve portfolio's performance if accounted for at historical cost. Another issue is intangible benefits that go beyond FX reserve diversification and that aren't easily captured by standard metrics.

In practice, therefore, careful risk-return analysis may have to be supplemented by more qualitative considerations before a decision about (the size of) gold reserves is being made. Various such additional arguments favouring gold holdings have been advanced in the literature:

- First, gold is an asset viewed by many as durable and largely imperishable (Erb and Harvey (2013)), which renders it free of default risk. The BoPM clearly establishes that gold bullion is the only case of a financial asset with no counterpart liability.[35]
- Second, unlike currencies and debt – which are claims on foreign governments or institutions – gold kept at home is not subject to political manipulation (for a World War II example, see Bernholz (2002)).
- Third, gold has been empirically proven to serve as an inflation hedge, at least over longer periods of time (see for example: Worthington and Pahlavani (2007), Shafiee and Topal (2010), Erb and Harvey (2013) and Aye et al (2016)).
- Last, its most widely recognised feature is its potential value in highly adverse scenarios. This is the so-called "war chest" argument (Nugee (2000)).

[33] Our analysis also highlights the broader limitations of portfolio optimisation algorithms, which generally focus on adequate asset allocations based on average returns. In practice, investors may wish to extend their considerations beyond these averages, tilting their models to better suit their specific views and needs. In practice this is done, for example, by introducing constraints to the optimisation exercise.

[34] At the zero or effective lower bound, correlations between gold and interest rate instruments may also differ vis-à-vis estimates using the full history – an important consideration to make when conducting prospective asset allocation analysis.

[35] See IMF (2013), Chapter 7, Paragraph 7.10.

In fact, factors beyond purely financial ones (eg, experience) may better explain central banks' holdings of gold. Recently, it has been shown that habits of the gold standard transmitted over time and affected monetary policy decisions decades later. In other words, both the memory of a central bank and its individuals have an impact on the share of gold held (Monnet and Puy (2020)). These behavioural aspects are not captured by our analysis.

Nonetheless, the possibility of large-scale, adverse non-financial events may lead to further reflection. For example, a major war, a period of very high inflation, or – speaking to more recent events – a systemic cyberattack or a global pandemic may give central banks enough reason to hold on – if not add – to their bars of gold. Indeed, in highly adverse scenarios, a country's stock of the precious metal could be one of the ultimate means to ensure confidence in the stability of its finances.[36] The relatively high gold allocations observed in practice, therefore, may be a measure of the importance attached to these broader considerations within the reserve mangement community. There is more to gold than risk and return.

[36] A recent survey by the World Gold Council (2020) suggests that gold's performance during times of crisis, historical factors (ie legacy stocks) and long-term store of value considerations are the top-three reasons behind central banks' gold holdings.

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Appendix 1. Building an SDR fixed income portfolio

The SDR is an international reserve asset, created by the International Monetary Fund in 1969 to supplement its member countries' official reserves.[37] The value of the SDR is based on a basket of five currencies – the US dollar, the euro, the Chinese renminbi, the Japanese yen, and the British pound sterling. The SDR also offers a good picture of the currencies held by central banks in practice. According to IMF Currency Composition of Official Foreign Exchange Reserves (COFER) data, by the end of Q1-2020, the top five currencies forming part of world FX reserves were the aforementioned five.[38] This is why we choose the composition of the SDR to form our representative FX reserve portfolio.

To build the historical returns of a generic, d-year constant maturity SDR fixed income portfolio:

1. We use historical, monthly US Treasury yields to build a history of US Treasury zero-coupon curves. To do so, we apply the version of the Nelson-Siegel model of Diebold and Li (2006).[39]
2. From each zero-coupon yield curve at month t, we price the d-year zero-coupon bond, as well as the $\left(d-\frac{1}{12}\right)$ −year bond.
3. For every month t, we calculate the return of the d-year zero-coupon bond as its price return:

$$r_t^{d,USD} = \frac{P_t^{\left(d-\frac{1}{12}\right),USD} - P_{t-1}^{d,USD}}{P_{t-1}^{d,USD}}$$

   where $P_t^{(d)}$ is the original price of a d-year zero-coupon bond. Since our portfolio is constant maturity, we assume that at the end of every month, when it has rolled into a $\left(d-\frac{1}{12}\right)$-year bond, it is automatically rebalanced back to d years.
4. We repeat steps (1) to (3) for German Bunds ($r_t^{d,EUR}$), Japanese government bonds ($r_t^{d,JPY}$), British pound Gilts ($r_t^{d,GBP}$), and Chinese government bonds ($r_t^{d,CNY}$). The data are sourced from the Board of Governors of the Federal Reserve System, the Deutsche Bundesbank, the Bank of Japan and Bloomberg.
5. For every month t, we calculate the return of the d-year zero-coupon SDR portfolio as:

$$r_t^{d,SDR} = w_t^{USD} r_t^{d,USD} + w_t^{EUR} r_t^{d,EUR} + w_t^{JPY} r_t^{d,JPY} + w_t^{GBP} r_t^{d,GBP} + w_t^{CNY} r_t^{d,CNY}$$

   where w_t^{USD} is the SDR weight of the US dollar at time t. The source of the data is the International Monetary Fund (see link for SDR valuation).

37 For a detailed definition, click here.

38 To view the latest data, click here.

39 Historical data of the SDR valuation is available since January 1981 from the IMF.

Appendix 2. Copula estimation and simulation

To perform our Monte-Carlo simulations, we must make two choices: how to model the stochastic behavior of an individual asset's returns (ie, their marginal distribution); and how to model the dependencies between the returns of all the assets in our system (ie, their joint distribution).

- For the first, we opt for empirical distribution functions, Epanechnikov kernels (Silverman (1992)), which preserve the observed non-normality of financial market data, such as asymmetry and heavy tails.

- For the second, we follow the line of reasoning of Embrechts et al (2008), who discuss the problems that may arise when applying the concept of linear correlation to the modeling of non-elliptical multivariate functions. To address this issue, we estimate a multivariate *t-copula* using maximum likelihood, and simulate from it as outlined below.

Theory on t-copulas

Let $\mathrm{R} = (R_1, \dots, R_n)$ be the random vector of returns of assets $1, \dots, n$ in our system, and $F(R_1, \dots, R_n)$ their joint distribution function, with margins $F_1, \dots, F_n$. An n-dimensional copula C is an n-dimensional distribution function $[0,1]^n$ with standard uniform marginal distributions. Sklar's Theorem (Sklar (1959)) states that every distribution function F can be written as:

$$F(r_1, \dots, r_n) = C(F_1(r_1), \dots, F_n(r_n)) \qquad (1)$$

for distributions F_i with continuous margins.

From (1) above, the copula C of their joint distribution function can be defined by using:

$$C(u) \coloneqq C(u_1, \dots, u_2) = F(F_1^{-1}(u_1), \dots, F_n^{-1}(u_n)) \qquad (2)$$

where F_i^{-1} is the quantile function of margin i.

The n-dimensional vector R is said to have a multivariate t-distribution with ν degrees of freedom, mean vector μ and positive-definitive scatter matrix Σ, when its density is given by:

$$f(\mathrm{r}) = \frac{\Gamma\left(\frac{\nu + n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{(\pi\nu)^n|\Sigma|}}\left(1 + \frac{(\mathrm{r}-\mu)^T\Sigma^{-1}(\mathrm{r}-\mu)}{\nu}\right)^{-\frac{\nu+n}{2}} \qquad (3)$$

This is denoted $R \sim t_n(\nu, \mu, \Sigma)$. Since copulas remain invariant under a standardisation of the marginal distributions (Demarta and McNeil (2004)), this means that the copula of a $t_n(\nu, \mu, \Sigma)$ is identical to that of a $t_d(\nu, 0, S)$ distribution where, in practitioner's terms, S is the rank correlation matrix of asset returns R, one of our quantities of interest.

Thus, the unique t-copula is given by:

$$C_{\nu,S}^t(\mathrm{u}) = \int_{-\infty}^{t_\nu^{-1}(u_1)} \dots \int_{-\infty}^{t_\nu^{-1}(u_n)} \frac{\Gamma\left(\frac{\nu + n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{(\pi\nu)^n|S|}}\left(1 + \frac{r^T S^{-1} r}{\nu}\right)^{-\frac{\nu+n}{2}} dr \qquad (4)$$

where t_ν^{-1} represents the quantile function of a standard univariate t_ν distribution. Parameters ν and Σ can be estimated via maximum likelihood.

Simulating from t-copulas

Based on the above, simulations are performed as follows:

1. We estimate the empirical marginal distribution of each random variable in $\mathrm{R} = (R_1, \dots, R_n)$ using historical data.
2. We build a matrix $\widetilde{U}$ of pseudo-observations of the unit hypercube by evaluating the marginal cumulative distribution functions in the historical realisations of R.

$$\widetilde{U} = (\tilde{u}_1, \dots, \tilde{u}_2) = F_1(r_1), \dots, F_n(r_n))$$

3. We use matrix $\widetilde{U}$ to estimate values for parameters (ν, S) of the t-copula $C^t_{\nu,S}(\cdot)$ via maximum likelihood, obtaining $(\hat{\nu}, \hat{S})$.
4. From our estimated copula $C^t_{\hat{\nu},\hat{S}}(\cdot)$, we draw 600,000 n-variate simulations to build matrix $U = (u_1, \dots, u_2)$. This is, five thousand scenarios of ten years of monthly data (5000 x 120).
5. Given marginal quantile functions $F_i^{-1}(\cdot)$, we construct our return sample for the analysis as:

$$\hat{R} = (\hat{r}_1, \dots, \hat{r}_n) = (F_1^{-1}(u_1), \dots, F_n^{-1}(u_n)).$$

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