Model Notes: What Comes Next?

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Abstract

These notes describes the model underlying the paper "What Comes Next?".

1 The Model

The economy is made up of $\mathcal{F}$ industries. I use the letter j to describe individual industries and the letter ι to describe individual firms within each industry. With some abuse of notation, I also use ι to denote household-level variables.

1.1 Nonlinear model

1.1.1 Households

The economy features two types of households: Ricardian households, who have access to financial markets and non-Ricardian households, who don't. The share of the two household types is ω_r and $1 - \omega_r$.

Ricardian Households

There is a continuum of identical households indexed by ι (which I suppress when not important). The household's problem is to choose aggregate and industry-level consumption, investment and capital, household-by-industry level wages and aggregate bond holdings to maximise utility:

$$\sum_{t=0}^{\infty} \beta^t \left[e^{\xi_{c,t}} \log(C_t^r - hC_{t-1}^r) - \frac{A_N}{1+\nu} N_t^r(\iota)^{1+\nu} \right] \tag{1}$$

subject to the budget constraint:

$$\begin{aligned} P_{C,t}C_t^r + P_{I,t}I_t^r + \frac{B_{t+1}}{R_t} \leq & B_t + \sum_{j=1}^{\mathcal{F}} \left(P_{C,t} \frac{r_{j,t}^K k_{j,t} u_{j,t}}{\mathcal{M}} + w_{j,t}(\iota) n_{j,t}^r(\iota) - a(u_{j,t}) k_{j,t} \right) \\ & + T_t^r \end{aligned} \tag{2}$$

and capital accumulation constraints for each industry:

$$k_{j,t+1} = (1-\delta)k_{j,t} + \left(1 - \mathcal{S}\left(\frac{z_{j,t}}{z_{j,t-1}}\right)\right) z_{j,t} \tag{3}$$

where C_t is aggregate consumption, $\xi_{c,t}$ is a consumption preference shifter, I_t is aggregate investment, $k_{j,t}$ is the capital stock of industry j, $u_{j,t}$ is the utilisation of capital in industry j and $z_{j,t}$ is gross investment in industry j. $w_{j,t}$ is the wage in industry j, which is distinct from the wage paid to household ι in industry j, $w_{j,t}(\iota)$. Similarly $n_{j,t}(\iota)$ is hours worked by household ι in industry j, while $n_{j,t}$ is total hours worked in industry j. The household takes industry-level wages and hours worked as given in making its decisions. T_t are lump sum transfers to the government. $\mathcal{M}$ is a wedge between the return on capital paid by firms and the amount received by households. It can

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be viewed as a reduced form for firm defaults or other factors that cause investors to demand a risk premium on lending to corporates.

Aggregate consumption and investment consist of bundles of consumption and investment goods sourced from each industry:

$$C_t^r = \left[\sum_{j=1}^{\mathcal{F}} \omega_{cj}^{\frac{1}{\eta}} c_{j,t}^{r\,\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{4}$$

$$I_t^r = \left[\sum_{j=1}^{\mathcal{F}} \omega_{ij}^{\frac{1}{\eta}} i_{j,t}^{r\,\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{5}$$

The price indices accompanying the consumption and investment aggregates are:

$$P_{C,t} = \left[\sum_{j=1}^{\mathcal{F}} \omega_{c,j} p_{j,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{6}$$

$$P_{I,t} = \left[\sum_{j=1}^{\mathcal{F}} \omega_{i,j} p_{j,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{7}$$

where $p_{j,t}$ is the price of the good produced by industry j.

It follows that the demand functions for the output of individual industries are:

$$c_{j,t} = \omega_{cj} \left(\frac{p_{j,t}}{P_{C,t}} \right)^{-\eta} C_t \tag{8}$$

$$i_{j,t} = \omega_{ij} \left(\frac{p_{j,t}}{P_{I,t}} \right)^{-\eta} I_t \tag{9}$$

Similarly, total labour supply, $N_t(\iota)$ is a bundle of labour supplied to each sector:

$$N_t^r(\iota) = \left[\sum_{j=1}^{\mathcal{F}} \omega_{nj}^{-\frac{1}{\xi}} n_{j,t}^r(\iota)^{\frac{\xi+1}{\xi}} \right]^{\frac{\xi}{\xi+1}} \tag{10}$$

A labour packer aggregates the labour supply of individual households in each industry according to:

$$n_{j,t} = \left(\int_0^1 n_{j,t}(\iota)^{\frac{\epsilon_w-1}{\epsilon_w}} du \right)^{\frac{\epsilon_w}{\epsilon_w-1}} \tag{11}$$

Consequently, demand for different types of labour is given by:

$$n_{j,t}(\iota) = \left(\frac{w_{j,t}(\iota)}{w_{j,t}} \right)^{-\epsilon_w} n_{j,t} \tag{12}$$

where $w_{j,t}$ is the aggregate wage index in industry j. The household takes this labour demand function into account when making its wage decisions.

Price and wage inflation is given by:

$$\Pi_{C,t} = \frac{P_{C,t}}{P_{C,t-1}} \tag{13}$$

$$\Pi_{I,t} = \frac{P_{I,t}}{P_{I,t-1}} \tag{14}$$

Market clearing for investment goods requires the aggregate volume of investment goods demanded by households to equal the sum of investment in all of the industries, that is:

$$I_t^r = \sum_{j=1}^{\mathcal{F}} z_{j,t}^r \tag{15}$$

Letting the Lagrange multipliers for the constraints on the budget constraint and the capital accumulation condition be $\Lambda_t/P_{C,t}$ and $\Lambda_t q_{j,t}$, the first order conditions for the household's problem are:

$$\frac{e^{\xi_{c,t}}}{C_t^r - hC_{t-1}^r} = \Lambda_t^r + \beta E_t \left\{ \frac{he^{\xi_{c,t+1}}}{C_{t+1}^r - hC_t^r} \right\} \tag{16}$$

$$\Lambda_t^r = \beta R_t E_t \left\{ \frac{\Lambda_{t+1}^r}{\Pi_{C,t+1}} \right\} \tag{17}$$

$$\Lambda_t^r q_{j,t} = \beta E_t \left\{ (1-\delta_j)\Lambda_{t+1}^r q_{j,t+1} + \Lambda_{t+1}^r \frac{r_{j,t+1}^K u_{j,t+1}}{\mathcal{M}} \right\} \tag{18}$$

$$\begin{aligned} \Lambda_t^r =& \Lambda_t^r q_{j,t} \left[1 - \mathcal{S}\left(\frac{z_{j,t}}{z_{j,t-1}}\right) - \mathcal{S}'\left(\frac{z_{j,t}}{z_{j,t-1}}\right) \frac{z_{j,t}}{z_{j,t-1}} \right] \\ &+ \beta E_t \left\{ \Lambda_{t+1}^r q_{j,t+1} \mathcal{S}'\left(\frac{z_{j,t+1}}{z_{j,t}}\right) \frac{z_{j,t+1}^2}{z_{j,t}^2} \right\} \end{aligned} \tag{19}$$

$$r_{j,t}^k = a(u_{j,t}) \tag{20}$$

Non-Ricardian households
Non-Ricardian households maximise the utility function:

$$\sum_{t=0}^{\infty} \beta^t \left[e^{\xi_{c,t}} \log(C_t^{nr} - hC_{t-1}^{nr}) - \frac{A_N}{1+\nu} N_t^{nr}(\iota)^{1+\nu} \right] \tag{21}$$

subject to the budget constraint:

$$P_{C,t} C_t^{nr} \leq \sum_{j=1}^{\mathcal{F}} w_{j,t} n_{j,t}^{nr} + T_t^{nr} \tag{22}$$

The first order conditions for their problem are:

$$\frac{e^{\xi_{c,t}}}{C_t^{nr} - hC_t^{nr}} = P_{C,t}\Lambda_t^{nr} + \beta E_t \left\{ \frac{e^{\xi_{c,t+1}} h}{C_{t+1}^{nr} - hC_t^{nr}} \right\} \tag{23}$$

which defines the marginal utility of consumption for non-Ricardian households

1.1.2 Aggregate consumption and marginal utility of consumption

The 'aggregate' marginal utility of consumption, Λ_t is a weighted average of the marginal utilities of the Ricardian and non-Ricardian households:

$$\Lambda_t = \omega_r \Lambda_t^r + (1-\omega_r)\Lambda_t^{nr} \tag{24}$$

Similarly, aggregate consumption is a weighted average of Ricardian and non-Ricardian consumption:

$$C_t = \omega_r C_t^r + (1-\omega_r) C_t^{nr} \tag{25}$$

1.1.3 Labour market:

In each industry, a continuum of perfectly competitive labour hiring firms combine the specialised labour types according to:

$$n_{j,t} = \left(\int_0^1 n_{j,t}(s)^{\frac{\epsilon_w - 1}{\epsilon_w}} ds \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{26}$$

The hiring firm's demand for each labour type j is given by:

$$n_{j,t}(s) = \left(\frac{w_{j,t}(s)}{w_{j,t}} \right)^{-\epsilon_w} n_{j,t} \tag{27}$$

where $w_{j,t}$ is the industry wage index given by:

$$w_{j,t} = \left(\int_0^1 w_{j,t}(s)^{1-\epsilon_w} ds \right)^{\frac{1}{1-\epsilon_w}} \tag{28}$$

Workers of type s unionise in order to take advantage of their monopoly power. These unions set nominal wages subject to the labour demand constraint and a Calvo friction that means that a random proportion, $\theta_{w,j}$ of households cannot re-optimise their wage each period.

Unions that do not re-optimise their wages re-scale them according to the indexation rule that depends on industry-specific lagged wage inflation $(\pi_{t-1}^{w,j})$:

$$w_{j,t}(s) = (\pi_{j,t-1}^{w})^{\chi_{w,j}} w_{j,t-1}(s)$$

Define:

$$\Omega_{j,t,t+s} = \prod_{m=t}^{t+s-1} (\pi_{j,m}^{w})^{\chi_{w,j}}$$

to be the total indexation in period s of a union that last updated its wage in period t.

Unions choose $w_{j,t}(s)$ to maximise:

$$\begin{aligned} \mathcal{L} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_{w,j})^s \Bigg[& \Upsilon_j^w \frac{\Lambda_{t+s}}{P_{C,t+s}} w_{j,t}(s) \Omega_{j,t,t+s} \left(\frac{w_{j,t}(s)\Omega_{j,t,t+s}}{w_{j,t+s}} \right)^{-\epsilon_w} n_{j,t+s} \\ & - \frac{A_N}{1+\nu} \left[\sum_{j=1}^{\mathcal{F}} \left(\int_0^1 \left(\frac{w_{j,t}(k)\Omega_{j,t,t+s}}{w_{j,t+s}} \right)^{-\epsilon_w} n_{j,t+s} dk \right)^{\frac{1+\xi}{\xi}} \right]^{\frac{\xi(1+\nu)}{1+\xi}} \Bigg] \end{aligned} \tag{29}$$

where Υ^w is a wage subsidy calibrated to offset the effect of imperfect labour market competition on employment.

The first order condition for this problem is:

$$\begin{aligned} 0 = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_{w,j})^s \Bigg[& -(1-\epsilon_w) \Upsilon_j^w \frac{\Lambda_{t+s}}{P_{C,t+s}} \Omega_{j,t,t+s} \left(\frac{w_{j,t}(s)\Omega_{j,t,t+s}}{w_{j,t+s}} \right)^{-\epsilon_w} n_{j,t+s} \\ & + \epsilon_w A_N N_{t+s}^{\nu - \frac{1}{\xi}} n_{j,t+s}^{\frac{1+\xi}{\xi}} \left(\frac{\Omega_{j,t+s}}{w_{j,t+s}} \right)^{-\epsilon_w \frac{1+\xi}{\xi}} w_{j,t}(k)^{-\epsilon_w \frac{1+\xi}{\xi} - 1} \Bigg] \end{aligned} \tag{30}$$

which we can re-arrange to:

$$\left[\frac{w_{j,t}(k)}{w_{j,t}}\right]^{\frac{\xi+\epsilon_w}{\xi}} = \frac{H_{w1,t}}{H_{w2,t}} \tag{31}$$

where:

$$H_{w1,t} = \sum_{s=0}^{\infty}(\beta\theta_{w,j})^s A_N N_{t+s}^{\nu-\frac{1}{\xi}} n_{j,t+s}^{\frac{1+\xi}{\xi}} \left(\frac{\Omega_{j,t+s}}{\pi_{w,j,t,t+s}}\right)^{-\epsilon_w \frac{1+\xi}{\xi}} \tag{32}$$

$$H_{w2,t} = \sum_{s=0}^{\infty}(\beta\theta_{w,j})^s \Lambda_{t+s} \frac{w_{j,t+s}}{P_{C,t+s}} n_{j,t+s} \left(\frac{\Omega_{j,t+s}}{\pi_{w,j,t,t+s}}\right)^{1-\epsilon_w} \tag{33}$$

We can-re-write $H_{w1,t}$ and $H_{w2,t}$ as:

$$H_{w1,t} = A_N N_t^{\nu-\frac{1}{\xi}} n_{j,t}^{\frac{1}{\xi}+1} + \beta\theta_{w,j}\mathbb{E}_t \left\{ \left(\frac{\pi_{j,t}^{w\chi_w}}{\pi_{j,t+1}^{w}}\right)^{-\epsilon_w \frac{1+\xi}{\xi}} H_{w1,t+1} \right\} \tag{34}$$

$$H_{w2,t} = \Lambda_t \frac{w_{j,t}}{P_{C,t}} n_{j,t} + \beta\theta_{w,j}\mathbb{E}_t \left\{ \left(\frac{\pi_{j,t}^{w\chi_w}}{\pi_{j,t+1}^{w}}\right)^{1-\epsilon_w} H_{w2,t+1} \right\} \tag{35}$$

From the definition of the wage index, we also know that:

$$1 = (1-\theta_{w,j})\left(\frac{w_{j,t}(k)}{w_{j,t}}\right)^{1-\epsilon_w} + \theta_{w,j}\left(\frac{\pi_{j,t-1}^{w\chi_w}}{\pi_{j,t}^{w}}\right)^{1-\epsilon_w} \tag{36}$$

1.1.4 Firms:

Firms in industry j produce output using capital, labour and intermediate goods according to the multi-layered production function:

$$y_{j,t}^{va}(\iota) = \left[\omega_{n,j}^{\frac{1}{\zeta}} n_{j,t}(\iota)^{\frac{\zeta-1}{\zeta}} + (1-\omega_{n,j})^{\frac{1}{\zeta}} k_{j,t}^{s}(\iota)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}} \tag{37}$$

$$x_{j,t}(\iota) = \left[\sum_{k=1}^{\mathcal{F}} \omega_{k,j}^{\frac{1}{\psi}} x_{k,j,t}(\iota)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}} \tag{38}$$

$$y_{j,t}(\iota) = a_j\left[\omega_{y,j}^{\frac{1}{\varphi}} y_{j,t}^{va}(\iota)^{\frac{\varphi-1}{\varphi}} + (1-\omega_{y,j})^{\frac{1}{\varphi}} x_{j,t}(\iota)^{\frac{\varphi-1}{\varphi}}\right]^{\frac{\varphi}{\varphi-1}} \tag{39}$$

where $y_{j,t}^{va}(\iota)$ is the value added of firm ι in industry j, $k_{j,t}^{s}(\iota)$ is the amount of capital hired by the firm, $x_{j,t}(\iota)$ is the amount of intermediate goods used by the firm and $y_{j,t}(\iota)$ is gross output of the firm. The total capital hired by industry j and total capital available to be hired is related by:

$$k_{j,t}^{s} = u_{j,t}k_{j,t} \tag{40}$$

Marginal costs (deflated by industry-specific final prices) and the resulting demand functions are:

$$mc_{j,t}(\iota) = \frac{1}{a_j}\left[\omega_{y,j}(p_{j,t}^{yva}(\iota)/p_{j,t})^{1-\varphi} + (1-\omega_{y,j})(p_{j,t}^{x}(\iota)/p_{j,t})^{1-\varphi}\right]^{\frac{1}{1-\varphi}} \tag{41}$$

$$(p_{j,t}^{yva}(\iota))^{\varphi} y_{j,t}^{va}(\iota) = \frac{\omega_{y,j}}{1-\omega_{y,j}}(p_{j,t}^{x}(\iota))^{\varphi} x_{j,t}(\iota) \tag{42}$$

where the price indices for value-added and intermediate goods in each industry are given by:

$$p_t^{yva} = \left[\omega_n w_{j,t}^{1-\zeta} + (1-\omega_{n,j}) r_{j,t}^{k1-\zeta}\right]^{\frac{1}{1-\zeta}} \tag{43}$$

$$p_{j,t}^{x} = \left[\sum_{k=1}^{\mathcal{F}} \omega_{k,j} p_{k,t}^{1-\psi}\right]^{\frac{1}{1-\psi}} \tag{44}$$

These imply the demand functions:

$$n_{j,t} = \omega_{n,j} \left(\frac{w_{j,t}}{p_{j,t}^{yva}}\right)^{-\zeta} y_{j,t}^{va} \tag{45}$$

$$k_{j,t} = (1-\omega_{n,j}) \left(\frac{r_{j,t}^{k}}{p_{j,t}^{yva}}\right)^{-\zeta} y_{j,t}^{va} \tag{46}$$

$$x_{kj,t} = \omega_{k,j} \left(\frac{p_{k,t}}{p_{j,t}^{x}}\right)^{-\psi} x_{j,t} \tag{47}$$

In each industry, individual firms face price stickiness a la Calvo. Each period a fraction of firms, $1-\theta_{pj}$ are able to change their prices. The remainder follow an indexing rule:

$$p_{j,t}(\iota) = (\pi_{j,t-1})^{\varphi_{j,p}} \, p_{j,t-1}(\iota)$$

Define:

$$\Omega_{j,t,t+s} = \prod_{m=t}^{t+s-1} (\pi_{j,m})^{\varphi_{j,p}}$$

as the cumulative change in prices between t and m, conditional on not re-optimising.
The problem for a firm that is able to reset its prices at time t is:

$$\max_{p_{j,t}^{*}(\iota)} E_t \sum_{s=0}^{\infty} (\beta\theta_{pj})^s \left\{\Lambda_{t+s}\left[\frac{p_{j,t}^{*}(\iota)\gamma_{j,t+s}\Omega_{j,t,t+s}}{p_{j,t+s}} y_{j,t+s}(\iota) - \frac{1}{1+\phi_{pj}} mc_{j,t+s}\gamma_{j,t+s}y_{j,t+s}(\iota)\right]\right\}$$

where $\gamma_{j,t+s} = p_{j,t+s}/P_{C,t+s}$ is the relative price of goods in industry j, subject to the demand condition given above. The parameter ϕ_j is a production subsidy to offset the steady-state distortion from imperfect competition. The first order condition for this problem is:

$$E_t \sum_{s=0}^{\infty} (\beta\theta_{pj})^s \left\{\Lambda_{t+s}\left[\frac{1-\epsilon_{jp}\gamma_{j,t+s}}{p_{j,t}(\iota)} \left(\frac{p_{jt}(\iota)\Omega_{j,t,t+s}}{p_{j,t+s}}\right)^{1-\epsilon_{jp}} y_{j,t+s} \right.\right.$$
$$\left.\left. + \frac{\epsilon_{jp}}{1+\phi_{jp}} \frac{mc_{j,t+s}}{p_{j,t}(\iota)} \left(\frac{p_{j,t}(\iota)\Omega_{j,t,t+s}}{p_{j,t+s}}\right)^{-\epsilon_{jp}} \gamma_{j,t+s}y_{j,t+s}\right]\right\} = 0 \tag{48}$$

Re-arranging gives:

$$\frac{p_{j,t}(\iota)}{p_{j,t}} = \frac{\epsilon_{j,p}}{(1+\phi_{jp})(\epsilon_{jp}-1)} \frac{h_{j,p1,t}}{h_{j,p2,t}}$$

where

$$h_{j,p1,t} = E_t \sum_{s=0}^{\infty} (\beta\theta_{pj})^s \Lambda_{t+s} \left(\frac{\Omega_{j,t,t+s}}{\pi_{j,t,t+s}}\right)^{-\epsilon_{jp}} mc_{j,t+s}\gamma_{j,t+s}y_{j,t+s} \tag{49}$$

$$h_{j,p2,t} = E_t \sum_{s=0}^{\infty} (\beta\theta_{pj})^s \Lambda_{t+s} \left(\frac{\Omega_{j,t,t+s}}{\pi_{j,t,t+s}}\right)^{1-\epsilon_{jp}} \gamma_{j,t+s}y_{j,t+s} \tag{50}$$

Note that:

$$h_{j,p1,t} = \Lambda_t mc_{j,t}\gamma_{j,t}y_{j,t} + \beta\theta_{pj}E_t \left(\frac{\Omega_{j,t,t+1}}{\pi_{j,t,t+1}}\right)^{-\epsilon_{jp}} h_{j,p1,t+1} \tag{51}$$

$$h_{j,p2,t} = \Lambda_t \gamma_{j,t}y_{j,t} + \beta\theta_{pj} \left(\frac{\Omega_{j,t,t+1}}{\pi_{j,t+1}}\right)^{1-\epsilon_{jp}} h_{j,p2,t+1} \tag{52}$$

Note also that the domestic price index can be expressed as:

$$1 = (1-\theta_{jp}) \left(\frac{p_{j,t}(\iota)}{p_{j,t}}\right)^{1-\epsilon_p} + \theta_{jp} \left(\frac{(\pi_{j,t-1})^{\varphi_{jp}}}{\pi_{j,t}}\right)^{1-\epsilon_{jp}} \tag{53}$$

1.1.5 Market clearing and aggregate price indices:

Goods market clearing requires that:

$$y_{j,t} = c_{j,t} + i_{j,t} + \sum_{k=1}^{\mathcal{F}} x_{j,k} \tag{54}$$

1.1.6 Monetary policy:

The monetary policy authority follows the policy rule:

$$\frac{R_t}{\bar{R}} = \left[\frac{R_{t-1}}{\bar{R}}\right]^{\rho^R} \left[(\Pi_t^{ye})^{\phi^\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi^y}\right]^{1-\rho^R} e^{\varepsilon^R_{j,t}} \tag{55}$$

1.1.7 Fiscal policy:

The government budget constraint is:

$$\frac{B_{t+1}}{R_t} = B_t + P_{G,t}G_t - T_t \tag{56}$$

I assume that in steady state, government bonds are in zero net supply, so that $B_t = 0 \forall t$. Fiscal policy purchases goods and services, G_t, according to the aggregate:

$$G_t = \left[\sum_{j=1}^{\mathcal{F}} \omega_{g,j}^{\frac{1}{\eta}} g_{j,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{57}$$

Implying the price index:

$$P_t^g = \left[\sum_{j=1}^{\mathcal{F}} \omega_{g,j} p_{j,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{58}$$

and final output demands:

$$g_{j,t} = \omega_{g,j} \left(\frac{p_{j,t}}{P_t^g} \right)^{-\eta} G_t \tag{59}$$

Aggregate government spending evolves according to:

$$\frac{G_t}{G} = \left[\frac{G_{t-1}}{G} \right]^{\rho_g} \exp(\varepsilon_t^g) \tag{60}$$

Transfers consist of transfers to Ricardian and non-Ricardian households:

$$T_t = T_t^r + T_t^{nr} \tag{61}$$

1.2 Steady state

The steady state of the system is given by:
From the first order condition for bond holdings:

$$R = 1/\beta \tag{62}$$

From the first order condition for investment

$$q_j = 1 \tag{63}$$

From the first order condition for capital:

$$r_j^k = \mathcal{M} \left(\frac{1}{\beta} - 1 + \delta \right) \tag{64}$$

From the consumption choice for Ricardian households:

$$\frac{1 - \beta h}{C^r (1 - h)} = \frac{\Lambda^r}{e^{\xi_c}} \tag{65}$$

From the budget constraint for for non-Ricardian households:

$$C_t^{nr} = W_t N_t^{nr} + T_t^{nr} \tag{66}$$

From the consumption choice of non-Ricardian households

$$\frac{1 - \beta h}{C^{nr} (1 - h)} = \frac{\Lambda^{nr}}{e^{\xi_c}} \tag{67}$$

I set the level of transfers so that the marginal utility of consumption for Ricardian and non-Ricardian households are equal, i.e. $\Lambda^r = \Lambda^{nr} = \Lambda$.
From the definition of aggregate marginal utility:

$$\Lambda = \omega_r \Lambda^r + (1 - \omega_r) \Lambda^{nr} \tag{68}$$

From the definition of aggregate consumption:

$$C = \omega_r C^r + (1 - \omega_r) C^{nr} \tag{69}$$

From the wage choice:

$$\omega_{wj}^{\frac{1}{\xi}} A_N N^{\nu+\frac{1}{\xi}} n_j^{-\frac{1}{\xi}} = \Lambda w_j \tag{70}$$

From the definition of aggregate labour supply:

$$N = \left[\sum_{j=1}^{\mathcal{F}} \omega_{nj}^{-\frac{1}{\xi}} n_j^{\frac{\xi+1}{\xi}} \right]^{\frac{\xi}{\xi+1}} \tag{71}$$

From the capital accumulation condition:

$$z_j = \delta k_j \tag{72}$$

From the market clearing condition for investment:

$$I = \sum_{j=1}^{\mathcal{F}} z_j \tag{73}$$

From the demand function for consumption:

$$c_j = \omega_{cj} \gamma_j^{-\eta} C \tag{74}$$

From the demand function for investment:

$$i_j = \omega_{ij} \left(\frac{\gamma_j}{\gamma_I} \right)^{-\eta} I \tag{75}$$

From the demand function for government expenditure:

$$g_j = \omega_{g,j} \left(\frac{\gamma_j}{\gamma_G} \right)^{-\eta} G \tag{76}$$

From the price index for consumption:

$$1 = \left[\sum^{\mathcal{N}} \omega_{c,j} \gamma_j^{1-\eta} \right] \tag{77}$$

From the price index for investment:

$$\gamma_I = \left[\sum^{\mathcal{N}} \omega_{i,j} \gamma_j^{1-\eta} \right] \tag{78}$$

From the price index for government expenditure:

$$\gamma_G = \left[\sum_{j=1}^{\mathcal{F}} \omega_{g,j} \gamma_j^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{79}$$

From the definition of aggregate wages:

$$W = \left[\sum^{\mathcal{N}} \omega_{l,j} w_j^{1+\xi} \right]^{\frac{1}{1+\xi}} \tag{80}$$

From the production function:

$$y_j = a_j \left[\omega_{y,j}^{\frac{1}{\varphi}} y_j^{va\,\frac{\varphi-1}{\varphi}} + (1-\omega_{y,j})^{\frac{1}{\varphi}} x_j^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}} \tag{81}$$

From the demand functions for intermediate goods:

$$x_{kj} = \omega_{kj} \left(\frac{\gamma_k}{\gamma_j} \right)^{-\psi} x_j \tag{82}$$

From the demand function for capital:

$$k_j = (1-\omega_n) \left(\frac{r_j^k}{\gamma_j^{yva}} \right)^{-\zeta} y_j^{va} \tag{83}$$

From the demand function for labour:

$$n_j = \omega_{nj} \left(\frac{w_j}{\gamma_j^{yva}} \right)^{-\zeta} y_j^{va} \tag{84}$$

From the definition of the price index of intermediate goods:

$$\gamma_j^x = \left[\sum_{j=1}^{\mathcal{F}} \omega_{kj} \gamma_k^{1-\psi} \right]^{\frac{1}{1-\psi}} \tag{85}$$

From the definition of the price of value added:

$$\gamma_j^{yva} = \left[\omega_{nj} w_j^{1-\zeta} + (1-\omega_n j) r_j^{k1-\zeta} \right]^{\frac{1}{1-\zeta}} \tag{86}$$

From the relative demand for inputs:

$$(\gamma_j^{yva})^{\varphi} y_j^{va} = \frac{\omega_{y,j}}{1-\omega_{y,j}} (\gamma_j^x)^{\varphi} x_j \tag{87}$$

From goods market clearing:

$$y_j = c_j + i_j + g_j + \sum_{k=1}^{\mathcal{F}} x_{j,k} \tag{88}$$

From the definition of $h_{j,p2}$:

$$h_{j,p2} = \frac{\Lambda \gamma_j y_j}{1-\beta\theta_{pj}} \tag{89}$$

From the price index for good j:

$$h_{j,p2} = h_{j,p1} \tag{90}$$

From the definition of $h_{j,p1}$:

$$mc_j = 1 \tag{91}$$

From the definition of marginal costs:

$$a_j = \left[\omega_{y,j} (\gamma_j^{yva})^{1-\varphi} + (1-\omega_{y,j} (\gamma_j^x)^{1-\varphi} \right]^{\frac{1}{1-\varphi}} \tag{92}$$

1.3 Linearised equations

Capital accumulation:

$$\hat{k}_{j,t+1} - \delta \hat{z}_{j,t} = (1-\delta)\hat{k}_{j,t} \tag{93}$$

Consumption price index:

$$0 = \sum_{j=1}^{\mathcal{F}} \omega_{c,j} \gamma_j^{1-\eta} \hat{\gamma}_{j,t} \tag{94}$$

Investment price index:

$$\gamma_I^{1-\eta} - \sum_{j=1}^{\mathcal{F}} \omega_{i,j} \gamma_j^{1-\eta} \hat{\gamma}_{j,t} = 0 \tag{95}$$

Government price index:

$$\gamma_G^{1-\eta} - \sum_{j=1}^{\mathcal{F}} \omega_{g,j} \gamma_j^{1-\eta} \hat{\gamma}_{j,t} = 0 \tag{96}$$

Consumption variety choice:

$$\hat{c}_{j,t} = \hat{C}_t - \eta \hat{\gamma}_{j,t} \tag{97}$$

Investment variety choice:

$$\hat{i}_{j,t} = \hat{I}_t - \eta(\hat{\gamma}_{j,t} - \hat{\gamma}_{I,t}) \tag{98}$$

Government expenditure variety choice:

$$\hat{g}_{j,t} = \hat{G}_t - \eta(\hat{\gamma}_{j,t} - \hat{\gamma}_{G,t}) \tag{99}$$

Aggregate labour supply:

$$N^{\frac{\xi+1}{\xi}} \hat{n}_t - \sum_{j=1}^{\mathcal{F}} \omega_{nj}^{-\frac{1}{\xi}} n_j^{\frac{\xi+1}{\xi}} \hat{n}_{j,t} = 0 \tag{100}$$

Investment price inflation:

$$\hat{\pi}_{I,t} - \hat{\pi}_t - \hat{\gamma}_{I,t} = -\hat{\gamma}_{I,t-1} \tag{101}$$

Wage inflation:

$$\hat{\pi}_{W,t} - \hat{\pi}_t - \hat{w}_t = -\hat{w}_{t-1} \tag{102}$$

Aggregate wage index:

$$W^{1+\xi} \hat{w}_t = \sum_{j=1}^{\mathcal{F}} \omega_{nj} w_j^{1+\xi} \hat{w}_{j,t} \tag{103}$$

Investment market clearing:

$$I \hat{i}_t - \sum_{j=1}^{\mathcal{F}} z_j \hat{z}_{j,t} = 0 \tag{104}$$

Consumption choice for Ricardian consumers:

$$h\hat{c}_{t-1}^r + \beta h E_t\{\hat{c}_{t+1}^r\} = (1+\beta h^2)\hat{c}_t^r + (1-h)(1-\beta h)\hat{\lambda}_t^r - (1-h)(\hat{\xi}_{c,t} - \beta \hat{\xi}_{c,t+1}) \tag{105}$$

Euler equation for Ricardian consumers:

$$\hat{\lambda}_t^r = \hat{r}_t + E_t\{\hat{\lambda}_{t+1}^r\} - E_t\{\hat{\pi}_{t+1}\} \tag{106}$$

Capital stock choice for Ricardian consumers:

$$\hat{\lambda}_t^r + \hat{q}_{j,t} = E_t\{\hat{\lambda}_{t+1}^r\} + \beta(1-\delta)E_t\{\hat{q}_{j,t+1}\} + \frac{\beta r_j^K}{\mathcal{M}} E_t\{\hat{r}_{j,t+1}^k\} \tag{107}$$

Relationship between capital supplied to firms and total capital stock:

$$\hat{k}_{j,t}^s = u_{j,t} + \hat{k}_{j,t} \tag{108}$$

Capital utilisation in industry j:

$$\mathcal{A}\hat{r}_{j,t}^k = \hat{u}_{j,t} \tag{109}$$

where $\mathcal{A}$ controls the degree of capital utilisation costs.
Investment choice:

$$(1+\beta)\hat{z}_{j,t} = \frac{\hat{q}_{j,t}}{S''} + \beta E_t\{\hat{z}_{j,t+1}\} + \hat{z}_{j,t-1} \tag{110}$$

Consumption choice for non-Ricardian consumers

$$C^{nr}\hat{c}_t^{nr} - WN(\hat{w}_t + \hat{n}_t) - TRANS\hat{trans}_t = 0 \tag{111}$$

Marginal utility of consumption for non-Ricardian consumers

$$h\hat{c}_{t-1}^{nr} + \beta h E_t\{\hat{c}_{t+1}^{nr}\} = (1+\beta h^2)\hat{c}_t^{nr} + (1-h)(1-\beta h)\hat{\lambda}_t^{nr} - (1-h)(\hat{\xi}_{c,t} - \beta\hat{\xi}_{c,t+1}) \tag{112}$$

Aggregate consumption:

$$\hat{c}_t - \omega_r \frac{C^r}{C}\hat{c}_t^r - (1-\omega_r)\frac{C^{nr}}{C}\hat{c}_t^{nr} = 0 \tag{113}$$

Aggregate marginal utility:

$$\hat{\lambda}_t - \omega_r \frac{\lambda^r}{\Lambda}\hat{\Lambda}_t^r - (1-\omega_r)\frac{\Lambda^{nr}}{\Lambda}\hat{\lambda}_t^{nr} = 0 \tag{114}$$

Wages choice:

$$\hat{\pi}_{j,t}^w - \frac{\beta}{1+\beta\chi_w}E_t\{\hat{\pi}_{j,t+1}^w\} - \frac{\kappa_{w,j}}{(1+\beta\chi_w)}\left[-\hat{\lambda}_t + (\nu - \frac{1}{\xi})\hat{n}_t + \frac{1}{\xi}\hat{n}_{j,t} - \hat{w}_{j,t}\right] = \frac{\chi_w}{1+\beta\chi_w}\hat{\pi}_{j,t-1}^w \tag{115}$$

where $\kappa_{w,j} = \frac{\xi}{\xi+\epsilon_w}(1-\beta\theta_{w,j})(1-\theta_{w,j})/\theta_{w,j}$
Gross output in sector j:

$$y_j^{\frac{\varphi-1}{\varphi}}\hat{y}_{j,t} - a_j^{\frac{\varphi-1}{\varphi}}\hat{a}_{j,t} - \omega_{y,j}^{\frac{1}{\varphi}}(y_j^{va})^{\frac{\varphi-1}{\varphi}}\hat{y}_{j,t}^{va} - (1-\omega_{y,j})^{\frac{1}{\varphi}}x_j^{\frac{\varphi-1}{\varphi}}\hat{x}_{j,t} = 0 \tag{116}$$

Marginal costs in sector j:

$$\begin{aligned}\hat{mc}_{j,t} + \hat{a}_{j,t} - \omega_{y,j}(\gamma_j^{va}/a_j\gamma_j)^{1-\varphi}\hat{\gamma}_{j,t}^{va} \\ - (1-\omega_{y,j})(\gamma_j^x/a_j\gamma_j)^{1-\varphi}\hat{\gamma}_{j,t}^x + \hat{\gamma}_{j,t} = 0\end{aligned} \tag{117}$$

Factor demand in sector j:

$$\varphi\hat{\gamma}_{j,t}^{va} + \hat{y}_{j,t}^{va} - \varphi\hat{\gamma}_{j,t}^x - \hat{x}_{j,t} = 0 \tag{118}$$

Value added price index in sector j:

$$\left(\gamma_j^{va}\right)^{1-\zeta}\hat{\gamma}_{j,t}^{va} - \omega_{nj} w_j^{1-\zeta}\hat{w}_{j,t} - (1-\omega_{nj})(r_j^k)^{1-\zeta}\hat{r}_{j,t}^k = 0 \tag{119}$$

Intermediate good price index in sector j:

$$(\gamma_j^x)^{1-\psi}\hat{\gamma}_{j,t}^x - \sum_{k=1}^{\mathcal{F}} \omega_{k,j}(\gamma_k)^{1-\psi}\hat{\gamma}_{k,t} = 0 \tag{120}$$

Labour demand in sector j:

$$\hat{y}_{j,t}^{va} - \zeta\hat{w}_{j,t} + \zeta\hat{\gamma}_{j,t}^{va} - \hat{n}_{j,t} = 0 \tag{121}$$

Capital demand in sector j:

$$\hat{y}_{j,t}^{va} - \zeta\hat{r}_{j,t}^k + \zeta\hat{\gamma}_{j,t}^{va} = \hat{k}_{j,t} \tag{122}$$

Intermediate good k demand in sector j:

$$\hat{x}_{j,t} - \hat{x}_{kj,t} - \psi\hat{\gamma}_{k,t} + \psi\hat{\gamma}_{j,t}^x = 0 \tag{123}$$

Defintion of relative price in sector j:

$$\hat{\gamma}_{j,t} - \hat{\pi}_{j,t} + \hat{\pi}_t = \hat{\gamma}_{j,t-1} \tag{124}$$

Phillips curve in sector j:

$$\hat{\pi}_{j,t} - \frac{\beta}{1+\beta\chi_p}E_t\{\hat{\pi}_{j,t+1}\} - \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\beta\chi_p)}\hat{mc}_{j,t} = \frac{\chi_p}{1+\beta\chi_p}\hat{\pi}_{j,t-1} \tag{125}$$

Link between wage inflation and real wages in sector j:

$$\pi_{j,t}^w - w_{j,t} - \pi_t = -w_{j,t-1} \tag{126}$$

Market clearing in sector j:

$$y_j\hat{y}_{j,t} - c_j\hat{c}_{j,t} - i_j - \hat{i}_{j,t} - g_j\hat{g}_{j,t} - \sum_{k=1}^{\mathcal{F}} x_{jk}\hat{x}_{jk,t} = 0 \tag{127}$$

Taylor rule:

$$\hat{r}_t - (1-\rho_r)(\phi_\pi\hat{\pi}_t^{ye} + \phi_y\hat{y}_t^{va}) + \varepsilon_{r,t} = \rho_r\hat{r}_{t-1} \tag{128}$$

Additional aggregate variables:

Year-ended inflation:

$$\hat{\pi}_t^{ye} = \hat{\pi}_t + \hat{\pi}_{t-1} + \hat{pi}_{t-2} + \hat{\pi}_{t-3} \tag{129}$$

Aggregate value added:

$$y_t^{va} - \sum_{j=1}^{\mathcal{F}} nva_j\hat{y}^v a_{j,t} = 0 \tag{130}$$

where nva_j is the steady-state share of sector j in nominal GDP.

Shock processes:

Productivity in sector j:

$$\hat{a}_{j,t} = \rho_{aj}\hat{a}_{j,t-1} + \varepsilon_{aj,t} \tag{131}$$

Aggregate government expenditure:

$$\hat{g}_t = \rho_g \hat{g}_{t_1} + \varepsilon_{gt} \tag{132}$$

Transfers:

$$trans_t = \rho_{trans} trans_{t-1} + \varepsilon_{trans,t} \tag{133}$$