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Credit Supply Driven Boom-Bust Cycles*

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Abstract

Can shifts in the credit supply generate a boom-bust cycle similar to the one observed in the US around 2008? To answer this question, we develop a general equilibrium model that combines a rich heterogeneous agent overlapping-generations structure of households who make housing tenure decisions and borrow through long-term mortgages, firms that finance their working capital through short-term loans from banks, and banks whose ability to intermediate funds depends on their capital. Using a calibrated version of this framework, we find that shocks to banks' leverage can generate sizable boom-bust cycles in the housing market, the banking sector, and the rest of the macroeconomy, which provides strong support for the credit supply channel. The deterioration of bank balance sheets during the bust, the existence of highly leveraged households, and the general equilibrium feedback from the credit supply to household labor income significantly amplify the bust. Moreover, mortgage credit growth across the income distribution is consistent with recent findings that were otherwise argued to be against the credit supply channel. A comparison of the model outcomes across credit supply, house price expectation, and productivity shocks suggests that housing busts accompanied by severe banking crises are more likely to be generated by credit supply shocks.

JEL Codes: E21, E32, E44, E60, G20, G51.
Keywords: Credit Supply, House Prices, Financial Crises, Household and Bank Balance Sheets, Leverage, Foreclosures, Mortgage Valuations, Consumption, and Output.

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1 Introduction

The housing market in the US (and in many other countries) experienced a dramatic boom-bust cycle during the last two decades. Real house prices increased by more than 30 percent between 1995 and 2006, and then dropped by a similar amount between 2006 and 2011. Such a large decline in house prices pushed many homeowners with mortgages into negative equity, which then increased quarterly foreclosure rates from 1 to 5 percent. Not only the housing market but also the financial sector and the rest of the macroeconomy struggled: the losses in mortgage related assets weakened bank balance sheets and concerns about the value of these assets made creditors withdraw from the wholesale funding market, disrupting the credit flow to non-financial firms and households.[1] GDP contracted by about 6 percent, employment and consumption declined around 5 percent.

Several papers have studied the forces behind the boom and the subsequent collapse of the housing market. One line of research has emphasized the role of the credit supply during the boom period.[2] These papers argue that an increase in the loan supply lowers interest rates and increases both credit and house prices. Similarly, during the bust period, a decline in bank lending to firms has been effective on the worsening of consumption and employment dynamics (Chodorow-Reich (2013) and Jensen and Johannesen (2017)). However, another strand of literature argues that shifts in demand driven by changes in expectations of house prices have been the main force behind the boom-bust cycle (Adelino et al. (2016) and Kaplan et al. (2020)).

In this paper, we study how far shifts in the credit supply can generate boom-bust cycles in the housing market, the banking sector, and the macroeconomy, as observed in the US around 2008. For this purpose, we develop and study a quantitative general equilibrium model that combines three sectors of the economy that played critical roles during the boom-bust episode: (i) a rich heterogeneous agent overlapping-generations structure of households who face idiosyncratic income risk under incomplete markets and make housing tenure decisions, (ii) banks that issue short-term loans to firms and long-term mortgages to households and whose ability to intermediate funds depends on their capital, and (iii) firms that finance part of their wage bill (working capital) through short-term loans from banks.

We explicitly model the housing tenure choices of households by allowing them to choose between owning and renting a house of their desired size. Households can use long-term mortgages for their purchases and have the option to prepay and refinance. Households can default on the mortgage in any period throughout the life of the mortgage. As mortgage contracts internalize the default probabilities of households, each mortgage is individual specific, and borrowing limits endogenously arise via limited commitment by households.

The key theoretical contribution of our paper is to incorporate this rich mortgage structure into

[1]See Gertler and Gilchrist (2018) for an excellent review of the crisis and the literature, as well as for evidence on how the disruption in the banking sector affected overall employment.

[2]Prominent examples are Mian and Sufi (2009), Shin (2012), Favara and Imbs (2015), Justiniano et al. (2017), Landvoigt et al. (2015), Garriga et al. (2019), and Garriga and Hedlund (2020).

bank balance sheets. For this purpose, we assume a competitive banking industry with a continuum of identical banks. Banks fund themselves through international investors and household deposits, and can lend to firms, issue new mortgages, and invest in existing ones. We assume that bankers can steal a fraction of assets and default. As a result, to avoid such behavior in equilibrium, lenders limit their funding to banks, creating an endogenous constraint on bank leverage.

To study the role of shifts in the credit supply during the boom-bust episode, we assume that the economy is initially in the steady state and calibrate the model to match several US data moments—most importantly, regarding household and bank balance sheets—in 1995. We then give two subsequent unexpected leverage shocks to bank balance sheets. First, in 1996, banks start increasing their leverage gradually over time.Second, in 2008, however, the leverage constraint reverts back to its initial steady-state level. We calibrate the size of the boom shock such that the changes in the banks' book leverage matches the data during the boom and study the transition of our model economy in response to these shocks.

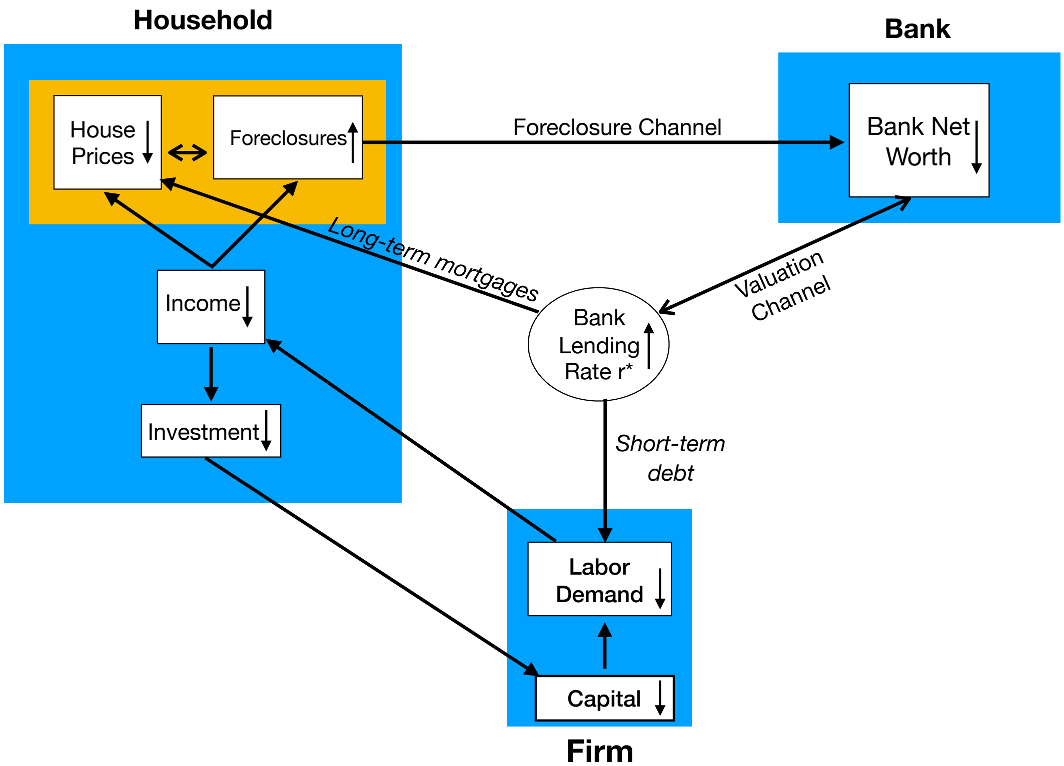
The main driver of the boom-bust cycle is the changes in the equilibrium bank lending rate in response to the credit supply shocks. With two unexpected and offsetting permanent shifts in bank leverage, the bank lending rate first decreases gradually by 0.6 percentage points until 2008 (and is expected to stay at that level permanently) and then unexpectedly reverts back to its initial steady-state level after a sharp jump (by 4.3 percent) in 2008 due to a sharp deterioration of bank balance sheets.

The changes in the bank lending rate generate a large boom-bust cycle in the housing market and the macroeconomy, and a slow recovery from the bust. During the boom, house prices increase around 12 percent and price-rent ratio increases by 7 percent. As house prices increase and borrowing rates decline, households borrow more by both lowering their down payments and tapping the refinancing option. As a result, household debt increases around 35 percent. However, household leverage increases less because of higher house prices. During the bust, house prices decline by 18.5 percent on impact, price-rent ratio declines by 15 percent, and the foreclosure rate jumps by 2.5 percentage points. On the real side of the economy, output and consumption expand by 3 and 4 percent in the boom and decline by about 5 and 7 percent in the bust, respectively.

The changes in the bank lending rate affect households both directly via borrowing costs and indirectly through general equilibrium effects. Most importantly, household labor income increases 4 percent during the boom and declines more than 9 percent during the bust as firms adjust their labor demand in response to the changes in the cost of funding and in the aggregate capital stock. Overall, we find that this general equilibrium effect accounts for about 50 percent of the house price and consumption dynamics, and the direct effect of the bank lending rate accounts for the rest. These findings underline the importance of modeling the feedback from the credit supply to labor income.

In the bust period, the credit supply declines not only because of the exogenous tightening of the

Figure 1: Linkages across sectors and amplification channels during the bust



bank leverage constraint but also because of the endogenous deterioration of bank balance sheets, which further tightens the leverage constraint and significantly amplifies the bust. Two, sometimes reinforcing, mechanisms drive the bank balance sheet amplification, as illustrated in Figure 1: (i) changes in mortgage valuations and (ii) foreclosures. First, when banks cut credit in response to the tightening of the leverage constraint, the equilibrium bank lending rate increases. But then, mortgage valuations decline and banks' net worth deteriorates. Hence, banks cut back credit more, which further increases the bank lending rate. Second, as house prices decline, a significant share of mortgage borrowers find themselves with negative equity and default. As a result, bank balance sheets worsen because of the rise in foreclosures. We find that the valuation losses account for more than two-thirds of the decline in bank net worth at the time of the bust, while the increase in foreclosures accounts for the rest, which is consistent with the evidence presented in IMF (2009). Overall, these two endogenous mechanisms cause a large but temporary spike in the bank lending rate, which amplifies the drop in house prices, consumption, and output by 25, 44, and 64 percent, respectively.

The temporary spike in the bank lending rate particularly amplifies the drop in variables that depend on short-term debt, such as output and labor income.[3] It does not affect mortgage costs

[3] Gertler and Gilchrist (2018) provide evidence that the disruption in banking, as in our model, was central to the overall employment contraction in the data.

significantly since mortgages are long-term. However, it reduces housing demand indirectly by lowering firms' labor demand and hence household labor income. Households reduce their savings, hence investment, in response to the decline in income at the time of the bust. The capital stock recovers slowly and the decline in income persists despite the quick recovery of the banking sector. The persistent decline in income amplifies the decline in house prices. This analysis suggests that firms' short-term liability structure is the key mechanism that translates the temporary spike in the bank lending rate to a significant and persistent decline in house prices.

The dynamics of interest rates and bank loans implied by our credit supply shock benchmark are supported by the empirical findings in the literature. Interest rates on firm loans and mortgages have declined during the boom (Glaeser et al. (2012a) and Justiniano et al. (2017)). On the effects of deregulation on interest rates, Jayaratne and Strahan (1997) and Favara and Imbs (2015) find significant declines in lending interest rates after the branching deregulation in the US. For the crisis period, Ivashina and Scharfstein (2010) document a more than 50 percent decline in bank real investment loans to corporations.[4] In parallel, Adrian et al. (2013) find that real investment loans to firms have declined substantially, while interest rates on loans more than quadrupled during the crisis.[5] Gilchrist and Zakrajšek (2012) show that credit spreads spike during downturns, predicting significant declines in subsequent economic activity. Together, these papers provide evidence for the distruption in the bank credit supply during the 2008 crisis.

The model's cross-sectional implications are also consistent with the recent evidence from detailed micro-level data analysis, some of which is argued to be inconsistent with the credit supply mechanism. In particular, we find that credit grows similarly across different income quantiles in our model over the boom episode, as shown to be the case in the data (Adelino et al. (2016) and Foote et al. (2016)). Consistent with the findings of Albanesi et al. (2017), our model implies that credit growth has been stronger for consumers with faster income growth. We also find that the higher leverage during the boom and the decline in income during the bust are the major factors that increased foreclosures. Taken altogether, these results provide support for our framework and the credit supply channel.

The rise of highly leveraged households during the boom causes a deeper contraction during the bust. To quantify its importance during the bust, we keep the aggregate debt constant but redistribute some part of the debt of households who fall into negative equity to the rest of the households. In this counterfactual economy, foreclosures do not increase during the bust, and as a result, house prices decline less: 15 percent with redistribution instead of 18.5 percent in the benchmark. Consumption and output also decline less, by about 1 percentage point.

We compare the model's dynamics across credit supply, productivity, and house price expectation

[4] Real investment loans include capital expenditure and working capital loans.

[5] Adrian et al. (2013) also report that non-financial US corporations counteracted the decline in the loan supply by increasing bond issuances. However, total credit (both loans and bonds) has declined. Thus, financial conditions must have tightened for non-corporate businesses, which do not have access to the bond market.

shocks. While we find many similarities, there are also several important differences. For example, with house price expectation shocks, households reduce capital accumulation, and thus output and labor income decline during the boom, and consumption barely rises in the short run and declines in the long run. In addition, the equilibrium bank lending rate does not increase significantly during busts with productivity and house price expectation shocks. This is because, in contrast to credit supply shocks, these shocks primarily reduce the credit demand. While increases in foreclosures cause losses in bank balance sheets and reduce the credit supply, the bank lending rate does not increase significantly at the time of the bust under these shocks unless they generate unrealistically high foreclosures. As a result, relative to the credit supply shock, mortgage valuations and, hence, bank net worth decline by significantly less. This result suggests that housing busts accompanied by severe banking crises are more likely to be generated by credit supply shocks rather than by house price expectation or productivity shocks.

Finally, our model allows us to study effects of both the ex ante and ex post policies on both household and bank balance sheets. For example, tighter LTV restrictions mitigate the increase in house prices by constraining household leverage, which subsequently reduces the fallout in the bust. Banks also become less vulnerable to declines in mortgage valuations and increases in foreclosures since the fraction of mortgages in bank portfolios are lower to start with and do not increase as much during the boom. Thus, overall, we find that stricter LTV requirements significantly reduce fluctuations in house prices, consumption, and output. Comparing capital injections to banks and household bailouts in a revenue-neutral fashion shows that capital injections to banks are more effective in eliminating the drop in bank net worth at the time of the bust and hence more effective in the short run, especially on variables that depend on short-term financing. The household bailout, on the other hand, is more effective in mitigating the drop in all variables in the longer run, the relative effectiveness appearing earlier on variables, such as house prices, that depend on long-term debt.

Related Literature

Our paper contributes to the literature that studies the dynamics of the housing market and the macroeconomy around the 2008 financial crisis.[6] Justiniano et al. (2017) and Greenwald (2016), using representative borrower and savers, and Huo and Rios-Rull (2013), Sommer et al. (2013), and Favilukis et al. (2017), using heterogeneous agent frameworks, show that credit conditions such as changes in maximum LTV or payment-to-income (PTI) ratios, and/or in credit supply can generate significant changes in house prices and consumption.[7] However, Kaplan et al. (2020)

[6]For excellent surveys, see Davis and Van Nieuwerburgh (2015), Piazzesi and Schneider (2016), and Guerrieri and Uhlig (2016).

[7]In Huo and Rios-Rull (2013), there is a feedback from household balance sheets to aggregate output because of good market frictions. In our model, feedback from household balance sheets to aggregate output goes through bank balance sheets that deteriorate because of higher foreclosures, which reduces the bank credit supply.

argue that the absence of the rental market and/or long-term defaultable mortgages are critical for obtaining large effects of credit conditions on house prices since, with rental markets, households can rent a house of their desired size if they are constrained in purchasing one. So, LTV and PTI constraints—even if they bind for some households—do not significantly affect the aggregate housing demand. Furthermore, defaultable mortgages generate endogenous borrowing limits that make the LTV constraint less relevant. With these extensions, Kaplan et al. (2020) argue that shifts in household demand due to shocks to house price expectations, rather than changes in credit conditions, were the driving force behind the boom-bust cycle in the housing market.

In this paper, similar to Kaplan et al. (2020), we model the rental market and long-term defaultable mortgages. However, in contrast to Kaplan et al. (2020), we find large effects of credit supply shocks because of two differences in our analysis. First, we consider permanent changes in bank leverage that essentially translate into permanent changes in the bank lending rate rather than the LTV, PTI, or temporary interest rate shocks considered in Kaplan et al. (2020). Second, the credit supply shock in our framework is not an isolated shock to households since we model the interaction between the bank credit supply and firms' production. Consequently, the permanent changes in the bank lending rate create large income and wealth effects on households, which then create boom-bust cycles in the housing market and the rest of the macroeconomy.

The degree of segmentation between owner-occupied and rental units matters for how far the changes in credit conditions (such as LTV limits) move house prices. For example, while Favilukis et al. (2017) assume a perfectly segmented housing market by assuming a fixed homeownership rate, Kaplan et al. (2020) assume a frictionless housing market where rental and owner-occupied units can be converted to each other without any cost. Partly because of the stark difference in this modeling choice, two papers reach opposing results. In a recent paper, Greenwald and Guren (2020) document empirical evidence that housing market is close to being fully segmented. In our model, the housing market is partially segmented as converting owner-occupied units to rental units is costly. We calibrate this cost parameter so that the degree segmentation in our benchmark is lower than the estimates of Greenwald and Guren (2020).[8] By doing so, we make sure that our results are not driven by a high degree of market segmentation.

Garriga and Hedlund (2018) also find that lower interest rates can account for the boom in house prices and consumption. There, the bust is generated through tighter down payment constraints and higher left tail income risk (see also Garriga and Hedlund (2020)). In our framework, as well, the credit supply expansion lowers the bank lending rate and creates a boom. The reversal of the credit supply shock by itself generates a deep bust in our model. The endogenous change in credit due to changes in bank balance sheets and firms' dependence on bank credit are the two key features of our framework that amplify the bust. Finally, all the aforementioned papers abstract from the

[8]We experimented with higher degree of housing market segmentation. The key difference in that case is the decline in the price-rent ratio becomes larger during the bust. The dynamics of other variables remain very similar.

bank balance sheet effects. By connecting the banking sector with the real sector, we can study housing and banking crises jointly. We can also compare the effectiveness of household versus bank bailout policies in a revenue-neutral fashion.

Our paper is also related to the literature that combines a banking sector that faces balance sheet constraints with household and/or production sectors. Landvoigt (2016) and Ferrante (2019) argue that credit supply shocks, along with shocks to house price uncertainty, play important roles for house prices changes. These papers assume within-sector perfect risk sharing so that each sector is represented by a single agent.[9] Compared to these papers, our paper's richer heterogeneity in the household sector allows us to compare our model's implications with cross-sectional facts that were argued to be against the credit supply channel. We also model the rental market for housing, which is important for analyzing house prices, as shown in Kaplan et al. (2020).

Our framework combines key elements from two strands of literature. On the one hand, an active literature has studied the pricing of default risk in the context of unsecured or mortgage debt. Prominent examples for unsecured credit are Chatterjee et al. (2007) and Livshits et al. (2010, 2007), and for mortgage debt are, Jeske et al. (2013), Corbae and Quintin (2015), Chatterjee and Eyigungor (2015), Arslan et al. (2015), Guler (2015), Hatchondo et al. (2015), Kaplan et al. (2020), and Garriga and Hedlund (2018, 2020). In this literature, banks are modeled as risk-neutral and zero-profit making competitive financial intermediaries. On the other hand, the literature on bank balance sheets has studied how depletion of a bank's capital reduces its ability to intermediate funds (Mendoza and Quadrini (2010), Gertler and Kiyotaki (2010, 2015), Gertler and Karadi (2011), He and Krishnamurthy (2012, 2013), Brunnermeier and Sannikov (2014), Bianchi and Bigio (2014), Boissay et al. (2016), and Navarro (2016)). However, in this literature, banks' asset structure typically takes a simple form such as one-period bonds or lacks the rich heterogeneity observed in banks' portfolios. By combining these two strands of the literature, our model allows us to study the rich interactions among households, firms, and banks.

2 Quantitative Model

The model economy is composed of five different sectors: (i) a unit measure of finitely lived households, (ii) a continuum of all-identical financial intermediaries, called banks, (iii) rental companies, (iv) final good producing firms, and (v) the government. We consider bankers as separate households in the economy.

We assume that total housing stock in the economy is fixed at $\bar{H}$, but the homeownership rate is not. This becomes possible as part of the housing stock is owned by homeowners and the rest

[9] Elenev (2017), Elenev et al. (2016), and Elenev et al. (2018) also use an approach similar to these papers to address different questions from ours. Elenev (2017) studies the effectiveness of large-scale asset purchases during busts. Elenev et al. (2016) and Elenev et al. (2018) study the incentive effects of government guarantees on financial sector risk taking and fragility.

is owned by rental companies who rent it to the households. There is perfect competition in all markets.

There is no aggregate uncertainty in the model. Boom-bust transitions are generated by two unexpected shocks, both of which are perceived as permanent shocks. Other than the periods that the shocks hit, there is perfect foresight. Since households are ex post heterogeneous in several dimensions, all the endogenous prices, value functions, and policy functions depend on the aggregate state of the economy and the distribution of households. For notational convenience, we suppress these dependencies.

2.1 Households

At the heart of the model economy is a rich household sector with realistic housing tenure and mortgage decisions.[10] We assume that households work until the mandatory retirement age J_r and live up to age J after the retirement. Working-age households are subject to idiosyncratic income uncertainty: before retirement, log labor income consists of a deterministic component $f(j)$, which only depends on age, and a stochastic component z_j, which is an AR(1) process. Thus, a household's income process $y(j, z_j)$ can be summarized by

$$y(j, z_j) = \begin{cases} w\,(1-\tau)\exp(f(j) + z_j), & \text{if } j \leq J_r \\ wy_R(z_{J_r}), & \text{if } j > J_r \end{cases} \tag{1}$$

$$z_j = \rho z_{j-1} + \varepsilon_j, \quad \varepsilon_j \sim i.i.d. \quad N(0, \sigma_\varepsilon^2),$$

where w is the wage per efficiency units of labor, τ is the tax rate, and $y_R(z_{J_r})$ is a function that approximates the US retirement system, as in Guvenen and Smith (2014). Households supply labor inelastically. However, the wage w depends on aggregate labor utilization rate as discussed in section 2.3.

We assume that there are two types of households: capitalists (K) and depositors (D). The key distinction between capitalists and depositors arises from the difference in savings options. Depositors can only save at the risk-free deposit rate r, while capitalists own the final good producing firm and the rental company, as we elaborate later, which give the same rate of return $\tilde{r}$.

Households receive utility from consumption and housing services and can choose between renting or owning a house of their desired size. Capitalists and depositors also have different discount factors. Thus, the preferences of a household of type $i \in \{K, D\}$ takes the following form:

$$E_0[\sum_{j=1}^{J} \beta_i^{j-1} u(c_j^i, s_j^i)],$$

[10] The household sector builds on the ones in Arslan et al. (2015) and Guler (2015) but is extended in some important ways, such as flexible housing and rental sizes, and refinancing options.

where β_i is the discount factor, c_j^i is consumption, and s_j^i is the housing services at age j for a type-i household.

Housing Choices: Households enter the economy as *active* renters and can stay as renters by renting a house at the desired size at the price p_r per unit of housing service. However, they can also purchase a house and become homeowners at any time. Purchasing a house is costly, especially for young households who do not have sufficient wealth to afford it. Although we do not allow unsecured borrowing in the model, we do allow households to have access to the mortgage market to finance their housing purchases. An important element of our model is that the terms of mortgage contracts, down payment and mortgage pricing, are endogenous and depend on household characteristics. Homeowners can choose to stay as homeowners or become renters again, by either selling their houses or defaulting on mortgage loans. Homeowners can refinance their houses at any point in time. Refinancing is the same as obtaining a mortgage at the time of purchase. Households also have the option of upgrading or downgrading the house size by selling the current house and buying a new one.

Several transaction costs are associated with owning a house. The purchase price of a house is p_h per unit of housing. To finance the purchase, the household can obtain a mortgage from banks. However, mortgages involve three types of costs. First, there is a fixed cost by the bank, φ_f, for originating a mortgage.[11] Second, banks charge a variable cost of origination for mortgages. This cost is φ_m fraction of the mortgage debt at the origination. Selling a house is also costly. A seller has to pay φ_s fraction of the selling price.[12] Lastly, since mortgages are risky, lenders charge a premium for the risk of defaulting. This premium shows up in the origination price of the mortgage.

Defaulting on a mortgage is possible, but it is costly. The cost is that after default, households become *inactive* renters; that is they temporarily lose access to the housing market. Inactive renters become active renters with probability π. Therefore, agents have three statuses regarding their housing decision: homeowner, active renter, or inactive renter.

Mortgage Payments: To keep the tractability in the model, we assume that mortgages are due by the end of life, which is deterministic, so that the household's age captures the maturity of the mortgage contract. We also allow for only fixed rate mortgages. The mortgage contract can be characterized by its maturity, the periodic mortgage payment m. We assume that the mortgage payments follow the standard amortization formula computed at the bank lending rate r^*. Thus, the relation between mortgage debt d and mortgage payment m in a period is given as

[11] Some examples of these costs are attorney fees, appraisal fees, and title company fees. These costs are fixed and do not depend on the size of the mortgage.

[12] Fees paid to real estate agents are the main part of these costs.

$$d = m \left(1 + \frac{1}{1+r^*} + \frac{1}{\left(1+r^*\right)^2} + ... + \frac{1}{\left(1+r^*\right)^{J-j}} \right) \;\; \Leftrightarrow \;\; m = d \frac{r^* \left(1+r^*\right)^{J-j}}{\left(1+r^*\right)^{J-j+1} - 1} \tag{2}$$

The remaining mortgage debt in the following period will be $(d-m)(1+r^*)$.

The mortgage interest rate differs across households since ex post households are heterogeneous. In principle, this should imply that the amortization schedule should be computed at the individual mortgage interest rate instead of r^*. However, to save from an additional state variable, we assume that mortgage amortization is computed at the risk-free mortgage rate, as in Hatchondo et al. (2015) and Kaplan et al. (2020). As will be clear later, individual default risk will show up in the pricing of the mortgages at the origination rather than in the mortgage interest rate. Thus, essentially all households pay points at the origination to reduce the mortgage interest rate to r^*.

2.2 Household's problem

2.2.1 Active Renters

An active renter has two choices: to continue to rent or purchase a house, that is, $V^r = \max\left\{V^{rr}, V^{rh}\right\}$ where V^{rr} is the value function if she decides to continue renting and V^{rh} is the value function if she decides to purchase a house. If she decides to continue to rent, she chooses rental unit size s at price p_r per unit, makes her consumption and saving choices, and remains as an active renter in the next period. After purchasing a house, she begins the next period as a homeowner. The value function of an active renter who decides to remain as a renter is given by

$$V_{ij}^{rr}(a,z) = \max_{c,s,a' \geq 0} \left\{u(c,s) + \beta_i E V_{j+1}^{r}(a',z')\right\} \tag{3}$$

subject to

$$c + \frac{a'}{1+r_i} + p_r s = w\left(1-\tau\right) y(j,z) + a,$$

where a is the beginning-of-period financial wealth, $p_r s$ is the rental payment, r_i is the return to savings, and w is the wage rate per efficiency unit of labor. Remember that capitalists have rate of return $r_K = \tilde{r}$ and depositors have rate of return $r_D = r$. The expectation operator is over the income shock z'.

If an active renter chooses to purchase a house, she can access the mortgage market to finance her purchase. She chooses a mortgage debt level d that determines $q^m(d; a, h, z, j)$, the price of the mortgage at the origination, which will be a function of the current state of the household (current wealth a, income realization z, and age j), house size h, and the amount of debt d. Then the value function of an active renter who chooses to buy a house is given by

$$V_{ij}^{rh}(a,z)=\max_{c,d,h,a'\geq 0}\left\{u(c,h)+\beta_i EV_{ij+1}^{h}(a',h,d,z')\right\} \tag{4}$$

subject to

$$\begin{aligned} c+p_h h+\delta_h p_h h+\varphi_f+\frac{a'}{1+r_i} &= w\,(1-\tau)\,y(j,z)+a+d\left(q^m(d;a',h,z,j)-\varphi_m\right) \\ d' &\leq p_h h\,(1-\varrho)\,, \end{aligned}$$

where p_h is the housing price, δ_h is the proportional maintenance cost of housing, φ_m is the variable cost of mortgage origination, φ_f is the fixed cost paid at the origination if the individual gets a mortgage, and ϕ is the minimum down payment required to get a mortgage.

2.2.2 Inactive Renters

Inactive renters are not allowed to purchase a house because of their default in previous periods. However, they can become active renters with probability π. Since they cannot buy a house, they only make rental size, consumption, and saving decisions. The value function of an inactive renter is given by

$$V_{ij}^{e}(a,z)=\max_{c,s,a'\geq 0}\left\{u(c,s)+\beta_i\left[\pi EV_{j+1}^{r}(a',z')+(1-\pi)EV_{ij+1}^{i}(a',z')\right]\right\} \tag{5}$$

subject to

$$c+\frac{a'}{1+r_i}+p_r s \;=\; w\,(1-\tau)\,y(j,z)+a.$$

2.2.3 Homeowners

The options of a homeowner are: 1) stay as a homeowner, 2) refinance, 3) sell the current house (become a renter or buy a new house), or 4) default. The value function of an owner is given as the maximum of these four options, that is, $V^h=\max\left\{V^{hh},V^{hf},V^{hr},V^{he}\right\}$, where V^{hh} is the value of staying as a homeowner, V^{hf} is the value of refinancing, V^{hr} is the value of selling, and V^{he} is the value of defaulting (being excluded from the ownership option).

A stayer makes a consumption and saving decision given his income shock, housing, mortgage debt, and assets. Therefore, the problem of the stayer can be formulated as follows:

$$V_{ij}^{hh}\left(a,h,d,z\right)=\max_{c,a'\geq 0}\left\{u\left(c,h\right)+\beta_i EV_{ij+1}^{h}\left(a',h,d',z'\right)\right\} \tag{6}$$

subject to

$$
\begin{aligned}
c+\delta_h p_h h+\frac{a'}{1+r_i}+m &= w\,(1-\tau)\,y\,(j,z)+a \\
d' &= (d-m)\,(1+r^*)\,,
\end{aligned}
$$

where m is the mortgage payment following the amortization schedule determined in equation 2.

The second choice for the homeowner is to refinance, which also includes prepayment. Refinancing requires paying the full balance of any existing debt and getting a new mortgage. We assume that refinancing is subject to the same transaction costs as new mortgage originations. So, we can formulate the problem of a refinancer as

$$
V_{ij}^{hf}(a,h,d,z)=\max_{c,d',a'\geq 0}\left\{u(c,h)+\beta_i E V_{ij+1}^{h}(a',h,d',z')\right\} \tag{7}
$$

subject to

$$
\begin{aligned}
c+d+\delta_h p_h h+\varphi_f+\frac{a'}{1+r_i} &= w\,(1-\tau)\,y(j,z)+a+d'\left(q^m(d';a,h,z,j)-\varphi_m\right) \\
d' &\leq p_h h\,(1-\varrho)\,.
\end{aligned}
$$

The third choice for the homeowner is to sell the current house and either stay as a renter or buy a new house. Selling a house is subject to a transaction cost that equals fraction φ_s of the selling price. Moreover, a seller has to pay the outstanding mortgage debt, d, in full to the lender. A seller, upon selling the house, can either rent a house or a buy a new one. Her problem is identical to a renter's problem. So, we have

$$
V_{ij}^{hr}\,(a,h,d,z)=V_{ij}^{r}\,(a+p_h h(1-\varphi_s)-d,z)\,.
$$

The fourth possible choice for a homeowner is to default on the mortgage, if she has one. A defaulter has no obligation to the bank. The bank seizes the house, sells it on the market, and returns any positive amount from the sale of the house, net of the outstanding mortgage debt and transaction costs, back to the defaulter. For the lender, the sale price of the house is assumed to be $(1-\varphi_e)\,p_h h$. Therefore, the defaulter receives $\max\{(1-\varphi_e)\,p_h h-d,0\}$ from the lender. The defaulter starts the next period as an active renter with probability π. With probability $(1-\pi)$, she stays as an inactive renter. The problem of a defaulter becomes the following:

$$
V_{ij}^{hi}\,(a,d,z)=\max_{c,s,a'\geq 0}\left\{u\,(c,s)+\beta_i E\left[\pi V_{ij+1}^{r}\left(a',z'\right)+(1-\pi)\,V_{ij+1}^{i}\left(a',z'\right)\right]\right\} \tag{8}
$$

subject to

$$c+\frac{a'}{1+r_i}+p_r s \quad = \quad a+w\left(1-\tau\right)y\left(j,z\right)+\max\left\{\left(1-\varphi_e\right)p_h h-d,0\right\}.$$

The problem of a defaulter is different from the problem of a seller in two ways. First, the defaulter receives $\max\left\{\left(1-\varphi_e\right)p_h h-d,0\right\}$ from the housing transaction, whereas a seller receives $\left(1-\varphi_s\right)p_h h-d$. We assume that the default cost is higher than the sale transaction cost, that is, $\varphi_e>\varphi_s$, the defaulter receives less than the seller as long as $\left(1-\varphi_s\right)p_h h-d\geq 0$ (i.e., the home equity net of the transaction costs for the homeowner is positive). Second, a defaulter does not have access to the mortgage in the next period with some probability. Such an exclusion lowers the continuation utility for a defaulter. In sum, since defaulting is costly, a homeowner will choose to sell the house instead of defaulting as long as $\left(1-\varphi_s\right)p_h h-d\geq 0$ (i.e., net home equity is positive). Hence, negative equity is a necessary, but not sufficient, condition for default in the model. Therefore, in equilibrium, a defaulter gets nothing from the lender.

2.3 Firms

A perfectly competitive firm produces final output by combining capital K_t and number of workers N_t. The firm can also choose the utilization rate per worker u_t. The wage per efficiency units of a worker is assumed to depend on the utilization rate, that is, $w\left(\bar{w}_t,u_t\right)=\bar{w}_t+\vartheta\frac{u_t^{1+\psi}}{1+\psi}$, where $w\left(\bar{w}_t,u_t\right)$ is the efficiency units of labor, same as w in previous sections, ϑ and ψ are constants, and $\bar{w}_t$ and u_t are determined in equilibrium. A household's labor income is given by $y\left(z,j\right)w\left(\bar{w}_t,u_t\right)$.

The firm has to finance a fraction μ of the wage payment in advance from banks and pay interest on that portion. Then, the firm's problem is given by

$$\max_{K_t,N_t,u_t}\mathbb{Z}_t K_t^{\alpha}\left(N_t u_t\right)^{1-\alpha}-(\tilde{r}_t+\delta)K_t-\left(1+\mu r_{t+1}^{*}\right)w\left(\bar{w}_t,u_t\right)N_t,$$

where $\tilde{r_t}$ is the rate of return to capital and δ is the depreciation rate. Since labor supply is exogenous, a worker's labor income depends on the firm's labor utilization rate. The basic idea behind this formulation is that the firm reduces labor utilization in response to an increase in bank lending rate r^*, which in turn reduces output.[13]

[13] We could have achieved the same effect without labor utilization but endogenous labor supply. In that case, the firm would reduce labor demand, which would reduce wages. Since households would reduce labor supply, aggregate output would decline. However, our formulation is easier to handle computationally.

The firm's first-order conditions are given as

$$\begin{aligned}
\alpha \mathbb{Z}_t \left(\frac{K_t}{N_t u_t}\right)^{\alpha-1} &= \tilde{r}_t + \delta \\
(1-\alpha)\,\mathbb{Z}_t u_t \left(\frac{K_t}{N_t u_t}\right)^{\alpha} &= \left(1+\mu r^*_{t+1}\right)\left(\bar{w}_t + \vartheta \frac{u_t^{1+\psi}}{1+\psi}\right) \\
(1-\alpha)\,\mathbb{Z}_t \left(\frac{K_t}{N_t u_t}\right)^{\alpha} &= \left(1+\mu r^*_{t+1}\right)\vartheta u_t^{\psi}.
\end{aligned}$$

2.4 Rental Companies

The rental company enters period t with $(1-\delta_h)\,H^r_{t-1}$ units of rental housing stock where δ_h is the depreciation rate of rental housing. Then it chooses H^r_t. In that period, the company receives net rent $(p^r_t - \kappa)\,H^r_t$ where p^r_t is the rental price per unit of housing and κ is the per-period maintenance cost and pays dividend $x^r_t = p^h_t(1-\delta)\,H^r_{t-1} - p^h_t H^r_t - \frac{\eta}{2}p^h_t\left(H^r_t - H^r_{t-1}\right)^2 + (p^r_t-\kappa)\,H^r_t$ to shareholders. $\frac{\eta}{2}p^h_t\left(H^r_t - H^r_{t-1}\right)^2$ is the quadratic adjustment cost of changing the rental supply (i.e. converting rental and owner-occupied units to each other). A higher value of η implies a more segmented housing market. Since both capital and rental company shares are riskless in a deterministic equilibrium, (i.e., in the steady state and along the transition path except for the unanticipated shock periods), both assets have to pay the same rate of return in equilibrium, which implies

$$1+\tilde{r}_t = \left(x^r_t + V^{rc}_{t+1}\left(H^r_t\right)\right)/V^{rc}_t\left(H^r_{t-1}\right),$$

where $V_{t+1}\left(H^r_t\right)$ is the post-dividend market value of the company at the end of period t.[14]

The objective of the company is to maximize its total market value $V_t\left(H^r_{t-1}\right)$:

$$\begin{aligned}
V^{rc}_t\left(H^r_{t-1}\right) &= \max_{H^r_t} \frac{1}{1+\tilde{r}_t}\left(x^r_t + V^{rc}_{t+1}\left(H^r_t\right)\right) \\
&\quad \text{s.t.} \\
x^r_t &= p^h_t(1-\delta)\,H^r_{t-1} - p^h_t H^r_t - \frac{\eta}{2}p^h_t\left(H^r_t - H^r_{t-1}\right)^2 + (p^r_t-\kappa)\,H^r_t.
\end{aligned}$$

The first-order condition to the above problem gives the rental price as functions of the house price

[14] At the time of an unexpected shock, capital and the rental housing return could be different. Then, the realized return of the capitalists, which will be different from the contracted return, would be given by

$$1+\tilde{r}'_t = \frac{K_t}{A_t}(1+\tilde{r}_t) + \left(1-\frac{K_t}{A_t}\right)\frac{x^r_t + V^{rc}_{t+1}\left(H^r_t\right)}{V_t\left(H^r_{t-1}\right)},$$

where A_t is the total assets of the capitalists, which is equal to the $K_t + V^{rc}_t\left(H^r_{t-1}\right)$, and $\tilde{r}_t = \alpha\mathbb{Z}_t K_t^{\alpha-1}N_t^{1-\alpha} - \delta$. The aggregate income of the capitalists is $\tilde{r}_t K_t + x^r_t + V^{rc}_{t+1}\left(H^r_t\right) - V^{rc}_t\left(H^r_{t-1}\right)$, which in steady state is $\tilde{r}K + x^r$: the return to capital plus the dividend from the rental company.

and rental housing stocks in periods $t-1$, t, and $t+1$.

$$p_t^r = \kappa + p_t^h + \eta p_t^h \left(H_t^r - H_{t-1}^r\right) - \frac{1}{1+\tilde{r}_{t+1}} \Big(\left(1-\delta_h\right) p_{t+1}^h + \eta p_{t+1}^h \left(H_{t+1}^r - H_t^r\right) \Big). \tag{9}$$

This is the supply equation for the rental housing. The demand for rental housing comes from households' housing choices.

In order to see how p_t^r is affected by p_t^h and homeownership rate, first consider the case where $\eta = 0$, which corresponds to the frictionless housing market explored in Kaplan et al. (2020). Equation 9 in this case becomes

$$p_t^r = \kappa + p_t^h - \frac{\left(1-\delta_h\right) p_{t+1}^h}{1+\tilde{r}_{t+1}}.$$

This equation implies that, for a given p_t^h, a higher future house price p_{t+1}^h reduces p_t^r. This is the main mechanism in Kaplan et al. (2020) that generates an increase in the price-rent ratio. However, the homeownership rate does not have any effect on rental price in this case. So, policies, such as relaxation of LTV limits that affect homeownership rate, does not move the price-rent ratio.

Next, consider a one-time permanent increase in homeownership rate in period t. Since homeownership rate increases in the current period, rental supply should decline for housing market to clear. As a result, we have $H_t^r < H_{t-1}^r$ and $H_{t+1}^r = H_t^r$.[15] Then, we can write equation 9 as

$$p_t^r = \kappa + \frac{\tilde{r} + \delta_h}{1+\tilde{r}} p_t^h + \eta p_t^h \left(H_t^r - H_{t-1}^r\right).$$

This equation shows that holding p_t^h fixed, an increase in the homeownership reduces p_t^r (since $H_t^r - H_{t-1}^r < 0$) if $\eta > 0$, and thus, the price-rent ratio increases. The higher the value of η, the higher is the increase in the price-rent ratio in response to a change in the homeownership rate. As it turns out, the leverage-shock-driven boom in our benchmark analysis does not affect homeownership rate significantly. Consequently, this mechanism is not significant during the boom. However, the decline in homeowneship during the bust is significant due to the increase in foreclosures and then, the price-rent ratio declines more with higher degree of market segmentation (see section 4.3).

[15] We would like to make a disclaimer here. Owner-occupied housing demand is not equal to homeownership rate because of differences in housing size per household. However, since homeownership rate and owner-occupied housing demand typically move together, we have chosen to explain this equation in terms of homeownership rate.

2.5 Banker

We assume a competitive banking industry with a continuum of identical banks that are risk-averse and maximize the discounted lifetime utility

$$\sum_{t=0}^{\infty} \beta_L^{t-1} \log\left(c_t^B\right),$$

where c_t^B is the banker's consumption. There is no entry to the banking sector. Banks fund their operations from their net worth ω_t and by borrowing B_{t+1} in the international market at a risk-free interest rate r_{t+1}, lend L_{t+1}^k to the firm at r_{t+1}^*, and issue mortgages and purchase existing mortgages.

Similar to Gertler and Kiyotaki (2015) and Gertler and Karadi (2011), we assume that banks can walk away at the beginning of a period without paying back their creditors. In that case, the bank can steal a fraction ξ of its assets but is excluded from banking operations in the future and can invest those assets at rate r_t. Knowing this, creditors lend to the bank to the extent that the bank does not walk away. Since the bank's outside option depends on its assets in this case, we need to keep track of assets and debt separately.

Letting $\theta = (d; a, h, z, j)$ define the type of a mortgage, ω_t be the bank's net worth, and $\ell_{t+1}(\theta)$ be the amount of investment in mortgage type θ (which includes any newly issued as well as existing mortgages), the budget constraint of the bank is given by

$$c_t^B + L_{t+1}^k + \int_\theta p_t(\theta)\, \ell_{t+1}(\theta) \quad = \quad \omega_t + B_{t+1}.$$

The bank's net worth evolves according to the following law of motion:

$$\omega_{t+1} \quad = \quad \int_\theta \int_{\theta'} v_{t+1}^l(\theta')\, \Pi(\theta'|\theta)\, \ell_{t+1}(\theta) + L_{t+1}^k\left(1 + r_{t+1}^*\right) - B_{t+1}(1 + r_{t+1}),$$

where $v_{t+1}^l(\theta') = m_{t+1}(\theta') + p_{t+1}(\theta')$ and $\Pi(\theta'|\theta)$ is the endogenous transition probability governed by exogenous household characteristics as well as endogenous choices.

If the bank defaults, it can steal a fraction ξ of its assets next period and save at interest rate r. We denote its value of default by $\tilde{\Psi}_{t+1}^D\left(\xi L'_{t+1}\right)$, where

$$L'_{t+1} = \left(\int_\theta \int_{\theta'} v_{t+1}^l(\theta')\, \Pi(\theta'|\theta)\, \ell_{t+1}(\theta) + L_{t+1}^k\left(1 + r_{t+1}^*\right)\right).$$

$L_{t+1} = L_{t+1}^k + \int_\theta p_t(\theta)\, \ell_{t+1}(\theta)$ is the investment in t and L'_{t+1} is the value of that investment in period $t+1$ after returns are realized. Investors lend to the bank up to a point where the bank does not steal in equilibrium. Denoting the value to the bank of honoring its obligations by

$\Psi_{t+1}(L_{t+1}, B_{t+1})$ where L_{t+1} is the bank's asset portfolio, the enforcement constraint is given as

$$\Psi_{t+1}(L_{t+1}, B_{t+1}) \geq \tilde{\Psi}^D_{t+1}\left(\xi L'_{t+1}\right).$$

The bank does not face any uncertainty in its net worth even though each mortgage is a risky investment. This is because we assume a continuum within each household type, which will translate into a continuum within each mortgage type θ. Thus, even if a bank invests in a particular type of mortgage θ by a tiny amount, its return is deterministic since a known fraction of θ-type households default and the remainder continue to pay their mortgages with certainty. The continuum assumption grants us tractability while keeping the rich heterogeneity in the household sector.

Since the bank does not face any uncertainty, an important property of the bank's problem is that all assets have to generate the same rate of return, which is equal to r^*_{t+1}. That is, the gross return on a mortgage of type θ is $\frac{\int_{\theta'} v^l_{t+1}(\theta')\Pi(\theta'|\theta)}{p_t(\theta)}$ and has to be equal to the gross return on loans to the firm $1 + r^*_{t+1}$. The price of the mortgage after that period's mortgage payment has been made is then given as

$$p_t(\theta) = \frac{1}{1 + r^*_{t+1}} \int_{\theta'} v^l_{t+1}(\theta')\, \Pi(\theta'|\theta) \text{ for all } \theta.$$

Since $v^l_{t+1}(\theta') = m_{t+1}(\theta') + p_{t+1}(\theta')$, the price of the mortgage is essentially the expected present discounted value of mortgage payments. As we will illustrate, the no-arbitrage condition greatly simplifies the problem of the bank. Since the bank is indifferent between investing in any asset, we do not have to keep track of its asset distribution in the bank's problem. Then, using $p_t(\theta) = \frac{1}{1+r^*_{t+1}} \int_{\theta'} v^l_{t+1}(\theta')\, \Pi(\theta'|\theta)$, we can simply show that $L'_{t+1} = \left(1 + r^*_{t+1}\right) L_{t+1}$. Then, the bank's problem can be written as

$$\Psi_t(L_t, B_t) = \max_{B_{t+1}, L_{t+1}, c^B_t} \left\{\log\left(c^B_t\right) + \beta_L \Psi_{t+1}(L_{t+1}, B_{t+1})\right\}$$

s.t.

$$\begin{aligned} c^B_t + L_{t+1} &= (1 + r^*_t) L_t - (1 + r_t) B_t + B_{t+1} \\ \Psi_{t+1}(L_{t+1}, B_{t+1}) &\geq \tilde{\Psi}^D_{t+1}\left(\xi\left(1 + r^*_{t+1}\right) L_{t+1}\right), \end{aligned}$$

where $\tilde{\Psi}^D_t(W) = \max_{W'} \log(W - W') + \beta_L \tilde{\Psi}^D_{t+1}((1 + r_{t+1})W')$. We can show that the enforcement constraint of the bank can be written as

$$(1 - \phi_{t+1})\left(1 + r^*_{t+1}\right) L_{t+1} \geq (1 + r_{t+1}) B_{t+1}$$

and implies an endogenous upper bound on bank leverage.[16] This leverage constraint is essentially

[16]Appendix C provides characterization of the bank's problem in detail.

a collateral constraint: it states that the bank can borrow up to a fraction of its assets and ϕ_{t+1} reflects the *haircut* on its collateral, where ϕ_t is defined recursively as follows:

$$\phi_t = \xi^{1-\beta_L} \left((1 + r_{t+1}) / \left(1 + r^*_{t+1}\right) - (1 - \phi_{t+1}) \right)^{\beta_L}. \tag{10}$$

If the bank was not able to steal, (i.e., $\xi = 0$), then $\phi_t = 0$ and $r^*_{t+1} = r_{t+1}$. Thus, the *collateral premium* $r^*_{t+1} - r_{t+1}$ would be zero.

Finally, perfect competition among banks implies that at the time of the mortgage initiation, the present value of mortgage payments should be equal to the loan amount

$$dq^m(d; a, h, z, j) = m + \frac{1}{1 + r^*_{t+1}} \int_{\theta'} v^l_{t+1}\left(\theta'\right) \Pi\left(\theta'|\theta\right). \tag{11}$$

Given d and m, this equation solves for $q^m(d; a, h, z, j)$.

2.5.1 Bank's solution

Given the collateral constraint the bank is facing, we can explicitly solve for the bank's problem, which is summarized in the following proposition.

Proposition 1. *The decision rules when the no-default constraint is binding (if $r^*_{t+1} > r_{t+1}$) are:*

$$\begin{aligned} L_{t+1} &= \frac{(1 + r_{t+1})}{1 + r_{t+1} - (1 - \phi_{t+1})(1 + r^*_{t+1})} \beta_L \omega_t \\ B_{t+1} &= \frac{(1 - \phi_{t+1})(1 + r^*_{t+1})}{1 + r_{t+1} - (1 - \phi_{t+1})(1 + r^*_{t+1})} \beta_L \omega_t, \end{aligned}$$

where $\omega_t = (1 + r^*_t)L_t - (1 + r_t)B_t$.

The decision rules when the no-default constraint is not binding (if $r^*_{t+1} \leq r_{t+1}$) are:

$$B_{t+1} = \begin{cases} \in \left[0, \frac{\beta_L(1-\phi_{t+1})(1+r^*_{t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r^*_{t+1})}\omega_t\right] & \text{if } r^*_{t+1} = r_{t+1} \\ 0 & \text{if } r^*_{t+1} < r_{t+1} \end{cases}$$

and

$$L_{t+1} = B_{t+1} + \beta_L \left((1 + r^*_t)L_t - (1 + r_t)B_t \right).$$

2.5.2 Characterization of the Bank's Problem in Stationary Equilibrium

We can further characterize the bank's problem under stationarity. Throughout the paper, we will focus on stationary equilibria where the capital requirement constraint is binding. If it were not, then bank balance sheets would not have any impact on the economy. However, we do not rule out the case that there might be some periods in the transition where this constraint becomes slack. Using the general formula capturing both the exogenous and endogenous capital requirement constraint, we have the following decision rules when the constraint is binding:

$$L_{t+1} = \beta_L \widehat{\lambda}_t \omega_t \quad \text{and} \quad B_{t+1} = \beta_L \left(\widehat{\lambda}_t - 1 \right) \omega_t,$$

where

$$\widehat{\lambda}_t = \frac{(1 + r_{t+1})}{1 + r_{t+1} - (1 - \phi_{t+1})(1 + r^*_{t+1})}. \tag{12}$$

Then the law of motion for net worth is given as

$$\omega_{t+1} \quad = \quad L_{t+1} \left(1 + r^*_{t+1}\right) - B_{t+1} \left(1 + r_{t+1}\right).$$

Then, we can obtain the next period's net worth as

$$\omega_{t+1} \quad = \quad \beta_L \left(\widehat{\lambda}_t \left(1 + r^*_{t+1}\right) - \left(\widehat{\lambda}_t - 1 \right) \left(1 + r_{t+1}\right) \right) \omega_t.$$

Imposing steady state $\omega_{t+1} = \omega_t$ and $\widehat{\lambda}_t = \widehat{\lambda}$ gives

$$r^* - r = \frac{1 - \beta_L (1 + r)}{\widehat{\lambda} \beta_L},$$

where $r^* - r$ is the premium due to the bank capital constraint. If $\beta_L (1 + r) < 1$ and $\widehat{\lambda} < \infty$, then $r^* - r > 0$. Thus, the capital constraint will be binding in the stationary equilibrium. To understand this point, assume that $\beta_L (1 + r) < 1$ but the bank starts with a high net worth so that the capital requirement constraint is not binding. In that case, $r^*_{t+1} = r$ and the bank's decision rule is $L_{t+1} - B_{t+1} = \beta_L \omega_t$. Using that, we can show that $\omega_{t+1} = (1 + r) \beta_L \omega_t < \omega_t$. Thus, the bank eats up its net worth until the capital constraint starts to bind. Thus, the economy will converge to a stationary equilibrium where it actually binds.

2.6 Symmetric Equilibrium

We focus on a symmetric equilibrium where each bank holds the market portfolio of mortgages. Thus, we have a representative bank. The definition of equilibrium is straightforward: all economic agents maximize their objectives given the exogenous price sequence $\{r_t\}_{t=1}^{\infty}$ and endogenous price sequences $\left\{r^*_t, \tilde{r}_t, \bar{w}_t, p^h_t, p^r_t\right\}_{t=1}^{\infty}$. The labor market clears in all periods, i.e. $N_t = 1$. We discuss the credit and housing market equilibrium conditions and the government budget next.

Credit market: Letting $\Gamma_t(\theta)$ be the distribution of available mortgages after HH's make their decisions at time t, the credit market clearing conditions can be summarized by the following conditions:

1. The representative bank holds the mortgage portfolio

$$\ell_{t+1}(\theta) \quad = \quad \Gamma_t(\theta).$$

2. Two credit market equilibrium conditions are

$$L_{t+1} = \mu w\left(\bar{w}_t, u_t\right) + \int_\theta p_t\left(\theta\right)\Gamma_t(\theta),$$

and

$$A_{t+1} = K_{t+1} + V_{t+1}^{rc}\left(H_t^r\right).$$

The first one determines the equilibrium r_{t+1}^*, and the second one determines the equilibrium $\tilde{r}_{t+1}$.

Housing market: Remember that total housing supply is fixed at H. Thus, the total demand of owners and renters should be equal to the supply, which determines house price $p_h(t)$. Given house prices $p_h\left(t\right)$ and $p_r\left(t\right)$, households solve their optimal housing choices, which gives the demand for owner-occupied units $H_t^{o,\mathrm{D}}$ and rental units $H_t^{r,\mathrm{D}}$. The supply of rental housing units is given by the first-order condition of the rental company, which is given as

$$p_t^r = \kappa + p_t^h + \eta\left(H_t^{r,\mathrm{S}} - H_{t-1}^{r,\mathrm{S}}\right) - \frac{1}{1+\tilde{r}_{t+1}}\left(\left(1-\delta_h\right)p_{t+1}^h + \eta\left(H_{t+1}^{r,\mathrm{S}} - H_t^{r,\mathrm{S}}\right)\right).$$

Then, the following two equilibrium conditions give the house price p_t^h and rental prices p_t^r:

$$\begin{aligned} H_t^{r,\mathrm{S}} &= H_t^{r,\mathrm{D}} \\ \bar{H} &= H_t^{r,\mathrm{D}} + H_t^{o,\mathrm{D}}. \end{aligned}$$

Government: The government runs a pay-as-you-go pension system. It collects social security taxes from working-age households and distributes to retirees. We assume the pension system runs a balanced budget:

$$\sum_{j=1}^{J_R}\sum_z \tau y\left(j,z\right)\pi_j\left(z\right) = \sum_{j=J_R+1}^{J}\sum_z y_R\left(j,z\right)\pi_j\left(z\right)$$

where $\pi_j\left(z\right)$ is the measure of individuals with income shock z at age j.

3 Calibration

Timing: The model period is two years. We assume that households start the economy at age 26 and work until age 65. After that, households retire and live until age 85.

Preferences: Households receive utility from consumption and housing services captured by the following CRRA specification:

$$u(c,s) = \frac{c^{1-\sigma}}{1-\sigma} + \gamma \frac{s^{1-\theta}}{1-\theta}.$$

We set $\theta = \sigma = 2$. We calibrate γ to match the share of housing services in aggregate income (including imputed income from housing services) as 15 percent. We assume 20 percent of the population is capitalist and the rest is depositor. These household types are drawn randomly at the beginning of life and are permanent. We calibrate the discount factor for the capitalists, β_k, to match a capital-output ratio of 1 in our biannual model. Lastly, we calibrate the discount factor for the depositors, β_D, so that the share of aggregate wealth that belongs to capitalists is 80 percent.[17]

Income Process: For the income process before retirement, we set the persistence parameter $\rho = 0.92$ and $\sigma_\varepsilon = 0.236$, which correspond to an annual persistence of 0.96 and a standard deviation of 0.17 following Storesletten et al. (2004). We approximate this income process with a 15-state first-order Markov process using the discretization method, as in Tauchen (1986). Retirement income approximates the US retirement system, as in Guvenen and Smith (2014). We adjust the retirement income level such that working age-households pay 12 percent tax.

Production Sector:

We assume the capital share in the final good production is $\alpha = 0.3$. Denoting Y as the final good or output, we target a capital-output ratio of $\frac{K}{Y} = 1$, which corresponds to a capital-output ratio of 2 in an annual model.[18] We normalize $N = 1$, $\mathbb{Z} = 1$, and target $u = 1$ at the steady state. Then, since $Y = \mathbb{Z}K^\alpha (Nu)^{1-\alpha}$, we get $Y = K = 1$.

We also target the share of housing services in aggregate income as 0.15. Since in our model aggregate income (including the imputed income from housing) corresponds to $\bar{Y} = Y + p_r\bar{H}$, this results in $\bar{Y} = \frac{1}{0.85}$ and $p_r\bar{H} = \frac{0.15}{0.85}$. In the data, the ratio of non-residential investment to aggregate income is 0.16. Since, at the SS, this ratio is $\frac{\delta_k K}{\bar{Y}}$, this gives us a capital depreciation rate of $\delta_k = \frac{0.16}{0.85}$. Given these targets, the model-implied biannual return to capital becomes $\tilde{r} = \alpha\frac{Y}{K} - \delta_k = 0.3 - \frac{0.16}{0.85} = 11$ percent. We set $\psi = 0.5$. Since at the steady state we target $u = 1$, from the firm's problem, we have $\vartheta = \left(\frac{1-\alpha}{1+\mu r^*}\right)\left(\frac{\alpha}{\tilde{r}+\delta}\right)^{\frac{\alpha}{(1-\alpha)}}$, which gives the calibrated value for ϑ.

Housing Market:

The probability of becoming an active renter, while the household is an inactive renter, is set to 0.265 to capture the fact that the bad credit flag remains, on average, for seven years in the credit

[17] The top 20 percent holds 80 percent of aggregate wealth in the US.

[18] This implies a capital-to-aggregate-income ratio (including the imputed income from housing) of 1.7. See the discussion in the next paragraph.

Table 1: Externally Set Parameters

Parameter	Explanation	Value
σ	risk aversion	2
α	capital share	0.3
ρ_ε	persistence of income	0.936
σ_ε	std of innovation to AR(1)	0.236
φ_h	selling cost for a household	7%
φ_e	selling cost for foreclosures	25%
ζ	fixed cost of mortgage origination	1
δ_h	housing depreciation rate	3%
τ	variable cost of mortgage origination	0.75
η	rental adjustment cost	3
π	prob. of being an active renter	0.265
ϱ	down payment requirement	0

history of the household. Consistent with the estimates of Gruber and Martin (2003), we set the selling cost (φ_s) to 7 percent, and for foreclosed properties, we set it to 25 percent, consistent with the estimates of Campbell et al. (2011). We set the fixed mortgage origination cost $\zeta = 1$ percent of the aggregate output, and the variable cost of mortgage origination $\tau = 0.75$ percent of the mortgage loan. We assume that there is no down payment requirement, that is $\varrho = 0$. We also assume that maximum payment to income ratio is 45%, which does not bind.

In the US data, the ratio of the house price to annual rental payments is around 11. So, in our biannual model, we target $\frac{p_h}{p_r} = 5.5$. This moment, together with the fact that the ratio of housing services to output is 0.15, implies $\frac{p_h \bar{H}}{Y} = 0.15 \times 5.5 = 8.2$ percent. So, we set $\bar{H}$ to match this ratio. We set the biennial depreciation rate for housing units as $\delta_h = 3$ percent. The steady-state relation between the rental price and house price is given by $p_r = \kappa + \frac{\tilde{r}+\delta_h}{1+\tilde{r}} p_h$. This gives us an estimate of κ given our target $\frac{p_h}{p_r} = 5.5$. We also restrict the minimum house size for owner-occupied units to be $\underline{h}$ to match a homeownership rate of 66 percent. Lastly, a higher level of the parameter governing the adjustment cost of rental supply (η) implies a higher degree of housing market segmentation. Using regional variation in credit supply, Greenwald and Guren (2020) find that rental markets are close to fully segmented. In our benchmark, we choose $\eta = 3$, which implies an intermediate level of housing market segmentation. In Appendix B.1, we report sensitivity analysis with respect to η. The key difference in that case is the decline in the price-rent ratio becomes larger during the bust. The dynamics of other variables remain very similar. Table 1 lists our parameter choices.

Financial Sector:

Since not only banks but other institutions as well, such as GSEs, hold large amounts of mortgage-related products, it is necessary to consider banks in our model as a collection of financial institutions that hold mortgages. With these considerations in mind, we follow Shin (2009) and include deposit-taking institutions (US chartered depository institutions and credit unions), issuers of asset backed securities, GSEs, and GSE-backed pools from FED Z1 data in our bank definition. Then we match bank balance sheets to the 1985-1994 average in the data. We use Tables L.218 and L.219 to obtain the total amount of home and multifamily residential mortgages held by banks. Banks on average hold \$2.117 trillion of these mortgages, which correspond to 86 percent of all mortgages. This 86 percent ratio is fairly constant from 1985 to 1994. To compute the amount of lending to non-financial firms, we use the balance sheets of non-financial firms (Table L.102). We use total loans (loans from depository institutions, mortgages, and other loans), which average to \$2.245 trillion and miscellaneous liabilities, which average to \$1.23 trillion. Residential mortgages would constitute 49 percent of banks' balance sheets if we include the loans only and 39 percent if we also include miscellaneous liabilities as firms financing from banks. Thus, we chose $\frac{\int_\theta p_t(\theta)\Gamma_t(\theta)}{\phi w(\bar{w}_t,u_t)N_t+\int_\theta p_t(\theta)\Gamma_t(\theta)}$ (the ratio of mortgages to banks' total financial assets) as 45 percent.[19]

In the steady state, we have $r^* - r = \frac{1-\beta_L(1+r)}{\widehat{\lambda}\beta_L}$, where $\widehat{\lambda} = \frac{(1+r)}{1+r-(1-\phi)(1+r^*)}$ is the endogenous leverage ratio and $\phi = \xi^{1-\beta_L}\left(\frac{1+r}{1+r^*} - (1-\phi)\right)^{\beta_L}$ is the haircut. We calibrate r to match a debt-output ratio of 40 percent (corresponding to an 80 percent ratio in an annual model), and we target $r^* - r = 3$ percent, representing the average biannual gap between the 30-year mortgage interest rate and the Treasury rate in the data. We also target $\widehat{\lambda}$ as 13.4. These two targets give us the bank's discount factor β_L and the bank's seizure rate ξ, which imply a steady state leverage ratio $(\beta_L\widehat{\lambda})$ of 9.1.

To summarize, overall we have 11 parameters that we calibrate internally: discount factor for capitalists (β_K), discount factor for depositors (β_D), minimum house size ($\underline{h}$), deposit rate (r), weight of housing services in utility (γ), housing supply $\bar{H}$, share of wage bill financed by banks (μ), bank's discount factor (β_L), bank's asset seizure rate (ξ), maintenance cost for rental units (κ), and capital depreciation rate (δ_k). The last four of these parameters are identified directly through analytical moments obtained through the model as discussed above. This leaves us with seven parameters that we calibrate using the model simulated data to jointly match the following seven data moments (Tables 2 and 3): 66 percent average homeownership rate, 40 percent mortgage-debt-to-output ratio, capital-goods production ratio of 1, house price-to-output ratio of 0.825, share of aggregate wealth that belongs to capitalists as 80 percent, share of mortgages in bank balance sheet as 45 percent, and share of housing services in GDP as 15 percent.

[19]There are obviously other items in banks' balance sheets that we do not model and do not take into account in these calculations. We will provide robustness of our results by including these residuals into banks' balance sheets. We did not take this approach as our benchmark since we do not model the demand for these residual assets and its dependence on the bank lending rate. We will provide robustness results based on two cases: 1) the dependence of the residual demand to bank lending rate is zero, so the residual demand is constant, and 2) the residual demand changes proportionally to mortgage and firms' demand for loans.

Table 2: Moments

Statistic	Data	Model
Capital-output ratio	1	1
Homeownership rate–aggregate	66 percent	66 percent
Share of wealth that belongs to capitalists	80 percent	80 percent
Debt-output ratio	40 percent	40 percent
House price-output ratio	0.825	0.825
Share of housing services in aggregate output	15 percent	15 percent
Ratio of mortgage loans to total loans in bank assets	0.45	0.45
Mortgage premium	0.03	0.03
Bank leverage ratio	10	10
House price-rental price ratio	5.5	5.5
Non-residential investment-output ratio	16 percent	16 percent

Note: Flow variables (output and rental price) are measured biannually.

Table 3: Internally Calibrated Parameters

Parameter		Value
β_K	discount factor–capitalist	1.05
β_D	discount factor–depositor	0.76
$\underline{h}$	minimum house size	0.51
r	deposit rate	0.07
γ	weight of housing services in utility	0.25
$\bar{H}$	housing supply	0.86
μ	share of wage bill financed from banks	0.67
β_L	bank discount factor	0.68
ξ	bank seizure rate	0.23
κ	rental maintenance cost	0.02
δ_k	capital depreciation rate	0.19

Leverage Shock:

Several shocks have been proposed for the boom and bust phases of the last housing cycle. For example, optimistic expectations, improved labor income prospects, lower regulation, and relaxed lending conditions. Even if all these shocks may have been important to some extent, we specifically aim to explore the credit supply channel and therefore pay less attention to other possible shocks.[20]

To study the role of bank credit supply, we study the following scenario. We assume that the economy is at steady state before 1996, but in 1996, unexpectedly, bank lending capacity gradually starts increasing. Each agent in the economy expects that it will take 25 years to reach the next steady state where the banks will have a higher leverage ratio. Unexpectedly, in 2008 however, the leverage reverts back. The parameter that controls the bank leverage is ξ: the fraction of assets that a bank can steal. A lower value reflects higher trust for banks and allows banks to have a higher leverage. To calibrate the changes in this parameter, we refer to two sources. First, Federal Reserve Bank of New York (2020) documents that the leverage ratio of the consolidated US banking organizations has increased by 25 percent from the first quarter of 1996 to the last quarter of 2007. We use the leverage ratio of all institutions (see page 34 of the report). Second, the Financial Stability Report by Federal Reserve Board (2019) documents that the leverage ratio of security brokers-dealers has increased by 50 percent from the first quarter of 1995 to the first quarter of 2008 (see figure 3-5 in the report). Both studies report marked-to-book leverage. However, in our model bank assets, L_{t+1}, and net worth, N_t, are in market values and the ratio L_{t+1}/N_t gives the marked-to-market leverage, which is the same as the book leverage when the economy is in steady state. However, after unexpected shocks, market and book values will no longer be equal. In order to compare to these two data sources, we compute the book values of bank loans and net worth, and calculate the corresponding book leverage in our model. We calibrate the changes in parameter ξ to have an increase in the financial system book leverage for 35 percent (from 1996 to 2006), which falls in the mid-range of 25 percent and 50 percent. We compare the book leverage from our model and from these sources in Figure 3.

While we consider changes in leverage constraints as the main driving force, an alternative scenario would be a decline in haircuts during the boom period and an increase during the bust period. We do not choose this path because of the limited availability of haircut data prior to the crisis. That said, available data (CGFS (2010)) suggest that haircuts more than doubled for most mortgage-related securities after the crisis. And for some nonprime products, the market ceased to exist. These changes in haircuts correspond to the leverage dynamics that we outlined above since in our framework the leverage constraint and haircuts on collateralized loans are equivalent.

Several changes in US legislation have deregulated the financial markets that provide explicit support to the leverage shock (see Sherman (2009)). Most relevant for our case is that, starting in 1986, the Federal Reserve gradually loosened the Glass-Steagall Act (the bill that strictly separates

[20] We compare our benchmark results with several alternative shocks in Section 7.

lending business from retail investment banking clients) several times, eventually, in 1996, allowing bank holding companies to earn up to 25 percent of their revenues in investment banking. The Gramm-Leach-Bliley Act repealed the Glass-Steagall Act completely in 1999, meaning that all restrictions against the combination of banking, securities, and insurance operations for financial institutions were removed. Therefore, it is reasonable to expect that banks could have projected to increase their leverage to the levels of investment banks, which was around 40 before the crisis. On the securitization side, from 1995 to 2005, the volume of private-label mortgage backed securities increased dramatically from negligible levels to $1.2 trillion, but disappeared with the crisis.[21]

One could also consider that after the bust period, policy makers became wary of banks and had the will and power to regulate the banks. Indeed, in the US, the Dodd-Frank Act (a federal law that passed in 2007) and the Federal Reserve's stress tests imply tighter regulation than the ones seen before 1995. At the global level, as well, the increase in the use of macroprudential policies and Basel III standards imply tighter regulation. All these developments suggest that the bank leverage ratio may have become even lower than it was in the pre-boom period. Therefore, the bust episode dynamics implied by our model can be thought of as a lower limit.

4 Results

Before turning to the analysis of transition dynamics, it is useful to check the model's performance in matching some key life-cycle statistics that may be important for the soundness of the quantitative exercise. The life-cycle implications of the model closely match the data (see Figure 4). The homeownership rate increases over the life cycle, similar to the data. Mortgage debt relative to housing value declines with age in both the data and the model. But it declines more in the model compared to the data. Average consumption and housing consumption in the model more than double over the life cycle and are very close to the values reported in the literature, such as Aguiar and Hurst (2013).

4.1 Transmission of the Shock and Banking Sector Dynamics

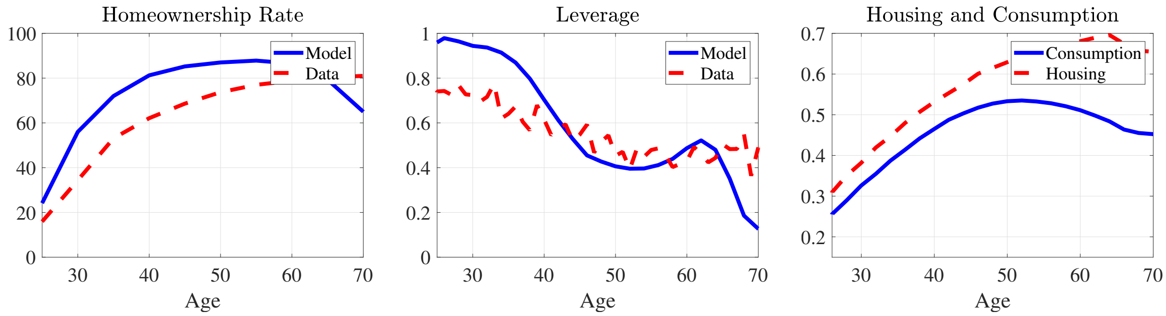
It will be instructive to illustrate how the leverage shock translates into changes and amplification in the bank lending rate. Focusing first on the steady state, an increase in bank leverage decreases the collateral premium

$$r^* - r = \frac{1 - \beta_L(1+r)}{\widehat{\lambda}\beta_L}.$$

Thus, a permanent increase in $\widehat{\lambda}$ will eventually lead the economy to a steady state with a lower interest rate. Moreover, when the bank net worth effects are absent, changes in $\widehat{\lambda}$ will translate

[21]The ease of securitization increased the liquidity in the housing market and led lenders to extend credit to marginal borrowers, increasing the credit supply (Keys et al. (2012)).

Figure 2: Life-Cycle Properties: Model vs. Data



Notes: The graph shows the life-cycle properties of housing and mortgage debt. The left panel plots the homeownership rate. The middle panel plots mortgage debt relative to housing value. The data come from 1995 Survey of Consumer Finances. The right panel plots the log difference of housing expenditure at a given age from its level at age 25.

into changes in r^* during the transition, as given by this equation. As a result, the equilibrium interest rate gradually falls during the boom and reverts back to the steady state level after the bust. However, changes in bank net worth amplify the changes in the collateral premium, which turns out to be significant during the bust. We will explain this amplification mechanism next.

Although all variables of interest affect each other simultaneously, we will proceed with an iterative approach in demonstrating the amplification mechanism. For this purpose, remember that the bank net worth in period t is given as

$$\omega_t = \int_{\theta} \int_{\theta'} \left(m_t\left(\theta'\right) + p_t\left(\theta'\right) \right) \Pi\left(\theta'|\theta\right) \Gamma_{t-1}\left(\theta\right) + L_t^k\left(1 + r_t^*\right) - B_t\left(1 + r_t\right).$$

The shock that generates the bust is essentially a decrease in $\widehat{\lambda}_t$ back to its steady-state level, which reduces the loan supply through $L_{t+1} = \beta_L \hat{\lambda}_t N_t$.[22] As a result, the equilibrium bank lending rate r_{t+1}^* increases. However, a higher r_{t+1}^* reduces the bank's net worth today by lowering mortgage valuations since

$$p_t\left(\theta\right) = \frac{1}{1 + r_{t+1}^*} \int_{\theta'} v_{t+1}^l\left(\theta'\right) \Pi\left(\theta'|\theta\right) \text{ for all } \theta,$$

where $v_{t+1}^l\left(\theta'\right) = m_{t+1}\left(\theta'\right) + p_{t+1}\left(\theta'\right)$. In response, loan supply L_{t+1} declines further and r_{t+1}^* increases more. With higher r_{t+1}^*, mortgage valuations and bank net worth declines further, which generates further increases in r_{t+1}^* and future bank lending rates. This is the key mechanism through which the deterioration of bank balance sheets amplifies the transmission of a shock to bank leverage.

Figure 3 shows the dynamics of the banking sector variables. In the top left panel of Figure 3, we report the evolution of the bank lending rate r_t^*; and in the top middle panel, we report evolution of the bank's net worth, which is intimately linked to r_t^*. Remember that r_t (the bank

[22] $\widehat{\lambda}_t$ is an endogenous object determined by equations 10 and 12. The parameter that goes back to its steady-state level is ξ, which decreases $\widehat{\lambda}_t$ to its steady-state level.

funding rate) is constant. The amplification arising from the bank balance sheet deterioration is the difference $\Delta r_t^* - \Delta r_t > 0$. Due to the decline in mortgage valuations (since r_t^* is higher) as well as the increase in foreclosures, bank net worth declines sharply at the time of the bust. However, the spike in r_t^* and the sharp drop in bank net worth turn out to be short-lived. This is because the amplification mechanism that creates the sharp drop works exactly the opposite way in the recovery. When r_t^* starts coming back, the market value of the bank's mortgage portfolio starts recovering, which increases the bank's net worth, which in turn allows the bank to further extend credit, reducing r_t^*'s even more. As a result, bank net worth recovers very quickly.

The model generates a 20 percent rise in bank assets in the boom. Since the loan supply increases with the extended leverage possibilities, the equilibrium bank lending rate declines. The value of the mortgage pool that the financial system holds increases. However, with a lower equilibrium bank lending rate, bank net worth declines during the boom. Overall, the banking sector supports more credit with lower bank net worth but with higher debt. Crisis occurs as the leverage constraint reverses to the initial steady-state level: the bank lending rate jumps to around 9 percent (Figure 3), and mortgage valuations and bank net worth sink substantially. However, as we discussed earlier, banks recover quickly. As mortgages are long-term assets, banks cannot flexibly adjust their balance sheets by issuing fewer mortgages. Therefore, they reduce their lending to firms (about 6.5 percent lower than the peak of the boom).

The dynamics of interest rates and bank loans are consistent with the findings in the literature. During the boom period, interest rates on firm loans and mortgages declined (Glaeser et al. (2012a) and Justiniano et al. (2017)). On the effects of deregulation on the interest rates, both Jayaratne and Strahan (1997) and Favara and Imbs (2015) find significant declines in lending interest rates after the branching deregulation in the US. For the crisis period, Ivashina and Scharfstein (2010) document a more than 50 percent decline in bank real investment loans to corporations.[23] In parallel, the lending interest rate on loans more than quadrupled (Adrian et al. (2013)) and credit spreads spiked during the financial crisis (Gilchrist and Zakrajšek (2012)).

The model implied bank leverage dynamics (based on both book and market values) are consistent with the data. The lower left panel shows that the marked-to-market bank leverage (L_{t+1}/N_t) increases during the boom, spikes at the time of the bust as asset valuations decline and bank net worth sinks, and declines afterward. Consistent with our model's implications, Begenau et al. (2018) find that the market leverage of listed banks increased during the boom and spiked during the bust.[24] We also construct book values of bank assets and equity, and compute marked-to-book leverage, which we have data for. The lower-middle panel compares the percentage change in book leverage in the model with those of commercial banks and security brokers-dealers in the data.[25]

[23]Real investment loans include capital expenditure and working capital loans.

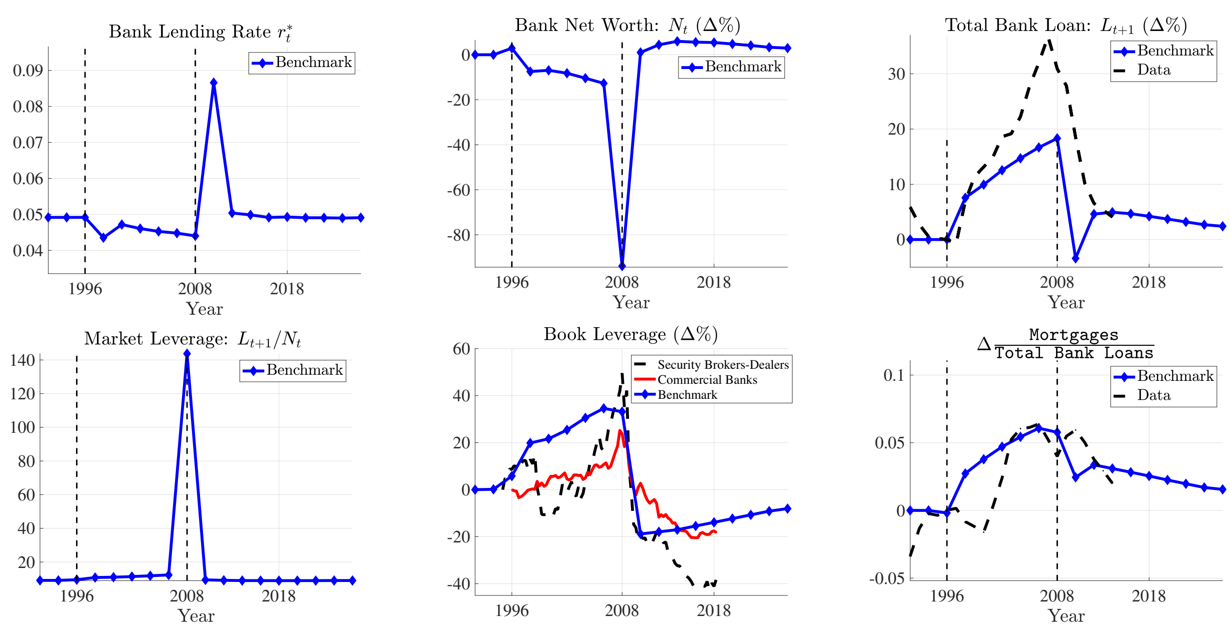
[24]See also He et al. (2010) for the role of asset valuations on bank leverage during the bust.

[25]Consistent with what we report here, there is broad agreement that marked-to-book leverage is procyclical (Adrian and Shin (2010), Nuno and Thomas (2017), and Coimbra and Rey (2017)).

During the boom, the leverage increases by 35 percent in the model as it is calibrated. The corresponding changes are 25 percent and 50 percent for commercial banks and securities brokers-dealers. The leverage does not change significantly at the time of the bust shock (2008), declines by 52 percent in the period after (2010), and recovers slowly. The leverage of commercial banks and security brokers-dealers decline by 27 and 71 percent from 2008 till 2010.

The share of mortgages in bank assets (the lower right panel) increases by 6 percent–matching the data counterpart–due to the rises in house prices, refinancing, and the decline in equilibrium down payment amounts. Finally, another implication of the leverage shock is an increase in capital inflow during the boom and a sharp decline in the bust. In the data too, the net capital inflow to GDP ratio increased from about 1 percent of GDP in 1996 to 5 percent of GDP in 2007 and declined to -0.3 percent by the first quarter of 2008. Since then, it has been hovering around 1 percent of GDP.

Figure 3: Bank Balance Sheet Dynamics



Notes: The graph plots the dynamics of key banking variables during the boom-bust episode. The shock is an unexpected ease and then an unexpected reversal of borrowing-lending constraints of the banks where bank leverage increases from 9 to 12.5 from 1995 to 2007. Total data for bank loans include home and multi-family residential mortgages, and firm loans and miscellaneous liabilities. The data for book leverage of banks is from Federal Reserve Bank of New York (2020) and of security brokers-dealers is from Federal Reserve Board (2019). For the mortgage share in bank loans, we report changes relative to 1990-1995 average.

4.2 Output Dynamics

The strong macroeconomic environment in the US during the boom period was partly seen as the driving force behind the boom in the housing market. For instance, per capita output was 6 percent and per capita labor income was 8 percent higher than their linear trends. Our model also features a strong macroeconomic outlook during its boom phase as a response to the relaxation of the bank leverage constraint (Figure 4). Output increases around 3 percent during the boom period. Capital, labor utilization, and wages increase around 2-4 percent.

With the crisis, macroeconomic conditions reverse sharply: output, and labor income decline steeply (5-10 percent). Even though the shock's impact on the banking sector is short-lived, surprisingly the real sector recovers very slowly. Overall, the credit supply shock can account for 40-60 percent (depending on the variable) of the increase during the boom period and more than 50 percent of the decline during the bust period.

In the data, too, there has been very slow recovery. Motivated by this slow recovery, the "secular stagnation" view suggests that some structural changes may have happened, and it may not be possible to reach the earlier trend. The findings in our paper present the possibility of an alternative view that builds on the sharp and temporary decline in capitalists' income. With a large loss in income, the capitalists reduce their investment by about 30 percent. As a result, the capital stock declines by 8 percent and recovers slowly. Even after 10 years, the capital stock is still more than 1 percent below its steady-state level. This persistent decline of capital is key for generating the persistent decline in output and wages.

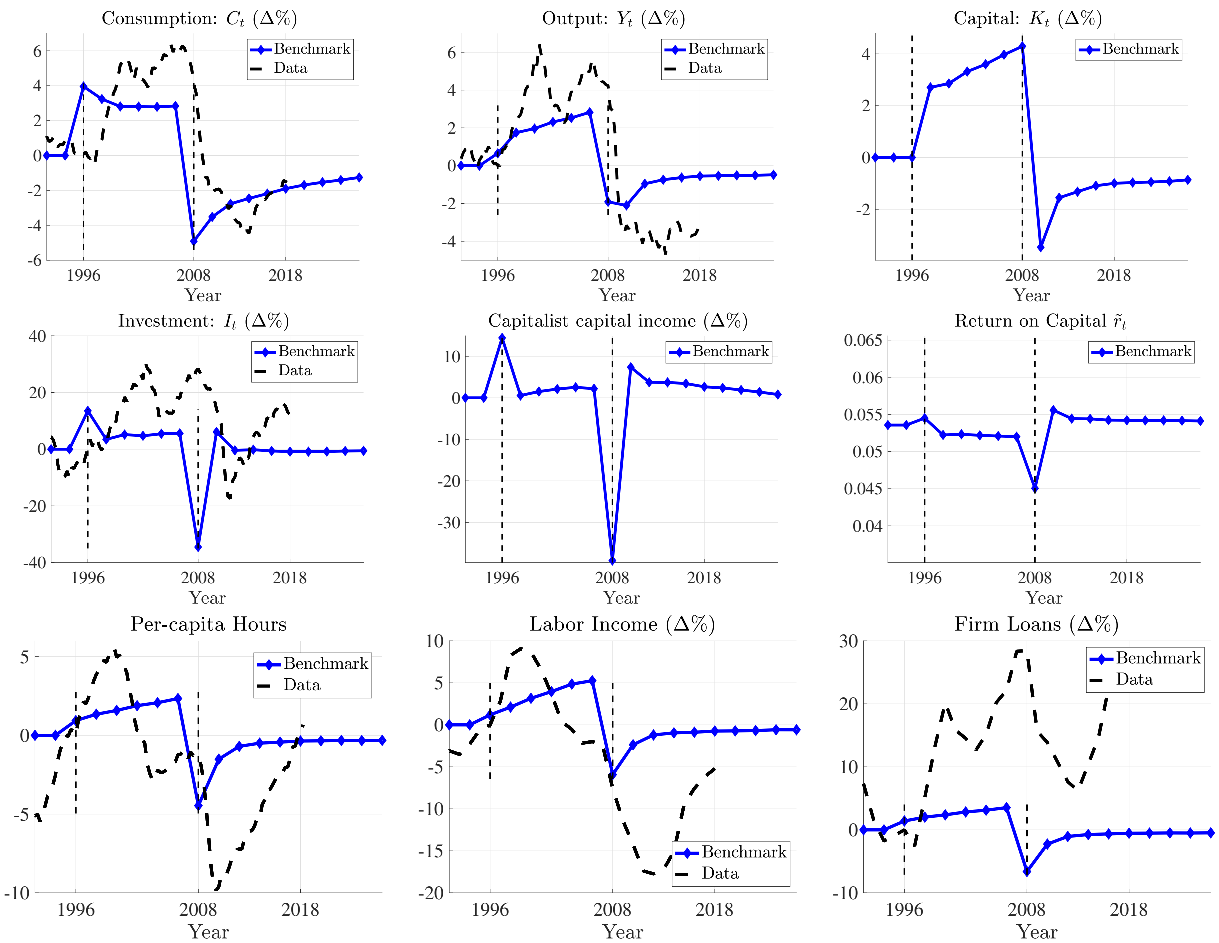
The response of the firms' labor demand to the changes in the bank lending rate is the key driver of the boom-bust in the production sector. Our model generates changes in total per capita hours worked, labor income, and firm loans qualitatively similar to the data.[26] There is extensive evidence that financial conditions indeed affect firm labor decisions, providing evidence for the mechanisms in our model. For example, Chodorow-Reich (2013) finds that firms that worked with weaker banks prior to the crisis, reduced employment more. Benmelech et al. (2019) find similar evidence from the depression era, and Popov and Rocholl (2015) bring evidence from Germany during the 2008 crisis. Finally, Ivashina and Scharfstein (2010) document a more than 50 percent decline in bank real investment loans to corporations, and Adrian et al. (2013) find that real investment loans to firms have declined substantially, while interest rates on loans more than quadrupled during the crisis.

4.3 Housing Market Dynamics

Figure 5 illustrates the dynamics of housing market variables. In response to the increase in bank lending, house prices increase around 12.5 percent in the model. With the reversal of the shock, it

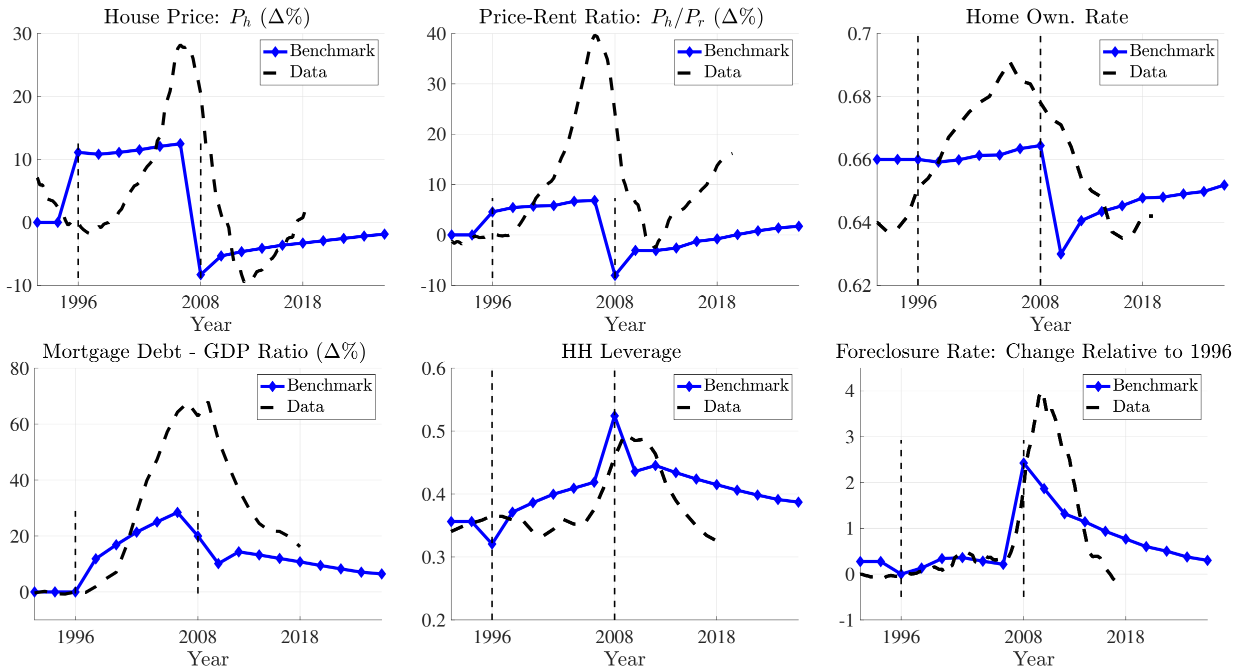
[26] We compare the labor utilization from the model to the hours per worked in the data.

Figure 4: Macroeconomic Dynamics



Notes: The graph plots the dynamics of production sector and consumption during the boom-bust episode. Consumption, output, investment, labor income, and firm loans data are percentage deviations from their linear trends obtained from 1985-2006 period.

Figure 5: Housing Market Dynamics



Notes: The graph plots the dynamics of housing market variables during the boom-bust episode. House price data is the percentage deviation from the linear trend obtained from 1985-2006 period.

undershoots and declines by 18.5 percent. Afterward, it slowly converges to its initial steady-state value. Overall, the model's implications regarding house prices are in line with the US housing price dynamics. Quantitatively, a credit supply shock by itself can generate more than one third of the boom and almost all of the bust in house prices.[27]

The model generates a modest rise in the homeownership rate compared to the data during the boom but generates a significant decline during the bust. The main reason is that while the decline in mortgage rates makes owning more affordable, the rise in the price-rent ratio increases the cost of owning a house relative to renting. We could have generated a further increase in the homeownership rate by imposing and then relaxing borrowing constraints on households (e.g., LTV or PTI or both). However, this extension would blur the effects of the leverage shock. Therefore, we choose not to have them in our benchmark.

The model generates a qualitatively similar but quantitatively smaller changes in the price-rent ratio: the rise in the price-to-rent ratio is 7 percent during the boom accounting for about 20 of the rise in the data. Several mechanisms are important for the dynamics of the price-to-rent ratio.

[27] As we have already mentioned, several factors might have contributed to the boom-bust in house prices: cheap borrowing conditions (Favara and Imbs (2015), Glaeser et al. (2012b), Garriga and Hedlund (2018), and Garriga et al. (2019)), securitization and subprime lending (Mian and Sufi (2009)), and optimistic expectations (Kaplan et al. (2020)). Therefore, it would be unrealistic to expect and/or force the changes in bank lending rate to account for all the movements in the house prices.

First, as can be seen from Equation 9, a rise in house prices in the following periods, a lower financing cost ($\tilde{r}_{t+1}$), and higher current homeownership rate decrease equilibrium rental prices in the current period. During the boom period, since house prices are not steep after the initial jump and the homeownership rate barely moves, the price-rent ratio does not increase as much as in the data. A lower financing cost helps to keep rental costs lower and contributes to the increase in the price-to-rent ratio. During the bust, the increase in foreclosures significantly lowers homeownership rate and thus, mitigates the decline in rental prices. As a result, the price-rent ratio declines by 15 percent, which accounts for almost half of the decline in the data.

Household debt increases 35 percent in the model during the boom period. Consistent with the data, household leverage increases less since house prices also increase. During the bust, both debt and leverage gradually converge to their steady-state levels. The rise of home equity extraction and refinancing activity during the boom period in the US were partly responsible for the rise of household debt and leverage (Mian and Sufi (2011)). In the model, we do not have home equity extraction. However, households can refinance and withdraw some cash from their home equity. During the initial boom period, refinancing activity jumps to 20 percent from 2.5 percent and returns to low levels in the following periods. Both higher house prices and lower interest rates cause an increase in refinancing volume. Unlike in the data, however, refinancing does not stay high for the whole boom episode, since after the initial shock there is perfect foresight.

The foreclosure rate has been very low in the data (on average, 1 percent annually) before the crisis. With the crisis, it increased by 4 percentage points (annual). The foreclosure rate in the model stays low during the boom and jumps by 2.5 percentage points in the bust period as the more than 18.5 percent decline in house prices, combined with 9 percent decline in income, during the bust pushes many households to negative equity, which makes default an attractive option.

4.4 Consumption Dynamics

The model generates a significant boom-bust in consumption: it increases by more than 4% during the boom and contracts by about 10% during the bust (Figure 4). Like many other macro variables in the model, the recovery takes a long time.

The declines in house prices and labor income are two important channels that derive consumption drop in the bust. To disentangle the role of each one, we ask how much the aggregate consumption would drop if we fix all prices at their boom levels and feed only equilibrium house prices or only equilibrium wages. The first analysis implies that the decline in house prices by itself generates 25% of the drop in consumption while the second one implies that the decline in wages by itself generates 74% of the total decline in consumption.[28] However, there is an indirect effect of labor income on consumption that this exercise does not capture because house prices are also

[28] The implied elasticity of consumption with respect to house prices is 0.1, which is at the lower end of the estimates reported in Berger et al. (2018).

endogenously affected by labor income. We analyze its total effect on consumption in the next section.

5 The Drivers of the Results

In this section, we explore three mechanisms that are relevant for the boom-bust in house prices and consumption: general equilibrium feedback from credit supply to household labor income, the amplifications arising from the deterioration of bank balance sheets during the bust, and the existence of highly leveraged households.

5.1 The Roles of Labor Income and the Bank Lending Rate

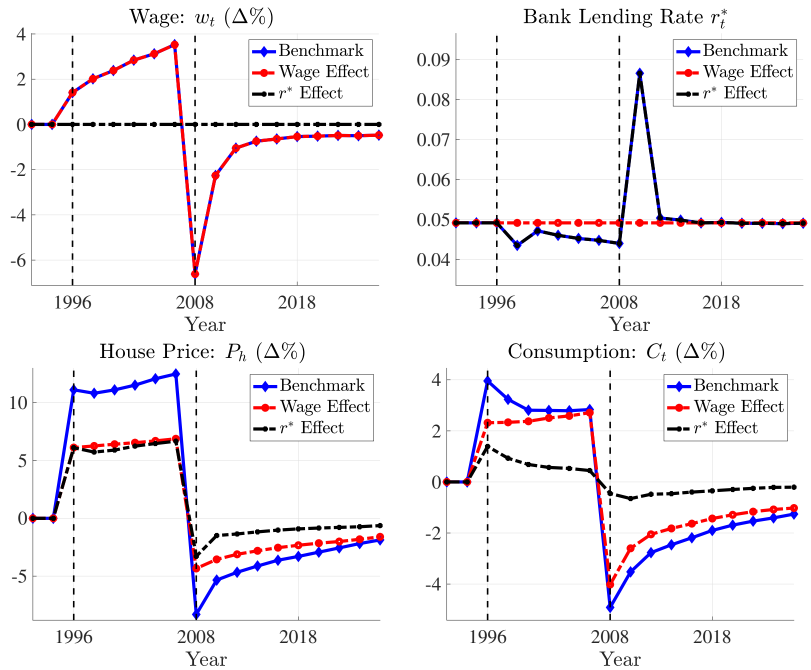
The changes in bank leverage influence model dynamics through their effects on the bank lending rate. The changes in the bank lending rate affect households both "directly" via borrowing costs and "indirectly" through affecting labor income. The 4 percent increase in labor income during the boom and the 9 percent decline during the bust, as firms adjust their labor demand in response to the changes in the cost of funding, affect households' consumption and housing demand. To isolate these direct and indirect effects, we solve two versions of our model where we keep wages and the bank lending rate constant separately at their initial steady-state levels and analyze how the boom-bust cycles differ from our benchmark economy. We present the results of this analysis in Figure 6.

Our results suggest that the changes in labor income have large effects on the dynamics of house prices and consumption. With labor income, house prices increase 6 percent during the boom and decline 8 percent at the bust, which is about half of the size of the boom-bust in house prices in the benchmark economy. The boom-bust in consumption is significantly reduced when we hold wages constant. The direct effect of the bank lending rate is equally important for house prices; however, its direct effect on consumption is limited: a 1 percent increase in consumption during the boom followed by around a 1 percent decline during the bust.

Kaplan et al. (2020) study a similar framework with aggregate uncertainty and reach the conclusion that credit conditions (LTV , PTI, mortgage origination, and temporary interest rate shocks) cannot generate a significant boom-bust cycle in house prices. Our analysis differs from theirs in three important aspects. First, the credit supply shock in our framework is not an isolated shock to the household borrowing rate; it also affects labor income. Our findings in this section suggest that about half of our results are driven by this channel. The second major difference is the persistence of the shocks. The shocks in our framework correspond to the changes in banking regulation that are more likely to be permanent. Not surprisingly, the boom-bust cycle gets amplified when the shock is more persistent.[29] Finally, there is a critical difference between LTV and PTI shocks and

[29] We have experimented with temporary shocks to bank leverage. The effects on housing market as well as on the rest of the economy are much smaller with temporary leverage shocks.

Figure 6: The Role of Labor Income and Bank Lending Rate r^*



Notes: The graph plots the dynamics of wages, house prices, and consumption. For the "Fixed Wages" exercise, we impose the same bank lending rate dynamics that arise in the benchmark economy. For the "r^* Effect" exercise, we keep the wages at the steady-state level, as shown in top left panel, and shock the economy with r^* boom and bust sequences of the benchmark economy (top right panel).

permanent changes in the bank lending rate. Relaxation of LTV and PTI constraints shift housing demand from renting to owning. Since households can rent a house of the desired size, these shocks do not significantly affect aggregate housing demand. On the other hand, a permanently lower bank lending rate creates a significant income effect—since mortgage payments decline for a given debt amount–and a wealth effect—since labor income permanently increases—, and thus, increases the total housing demand.

5.2 The Role of Bank Balance Sheet Deterioration in Amplifying the Bust

In the bust period, the credit supply declines not only because of the exogenous tightening of the bank leverage constraint but also because of the endogenous deterioration of bank balance sheets, which further tightens the banks' leverage constraint and significantly amplifies the bust. In this section, to quantify the role of the deterioration of bank balance sheets on the aggregates during the bust, we eliminate the decline in bank net worth in the bust and analyze the equilibrium transition.[30] The red line in Figure 7 shows the model dynamics for this exercise, and the blue line shows the

[30]Essentially, we solve the transition of the economy starting with the bust distribution but with bank net worth fixed at the boom level in the first period of the bust. We focus on the bust period only, since the bank balance sheet channel has a small effect during the bust period.

dynamics for the benchmark. The difference between the blue and red lines indicates the role of bank balance sheet deterioration in amplifying the bust, which we report under the "BBS" column in Table 4.

Overall, we find that the deterioration of the bank balance sheet significantly amplifies the bust. For example, we find that the bank balance sheet channel amplifies the drop in output by 66 percent. Since the deterioration of the bank balance sheet generates a spike in the bank lending rate at the time of the bust, it significantly lowers labor demand, compared to the counterfactual economy where there is no deterioration. This is the main driver of the decline in output at the time of the shock.

While the spike in r^* disappears rapidly, its effect on output lasts for a long time, which is illustrated by the persistent difference in the blue and red lines. This persistent effect on output comes from banks' role on the decline in the capital stock. During the crisis period, investment declines by 35 percent, and thus the capital carried to the next period declines by 7.5 percent. The bank balance sheet channel contributes 51 percent to this decline. Slow recovery of capital generates the slow recovery of output and the persistent amplification from the balance sheet deterioration.

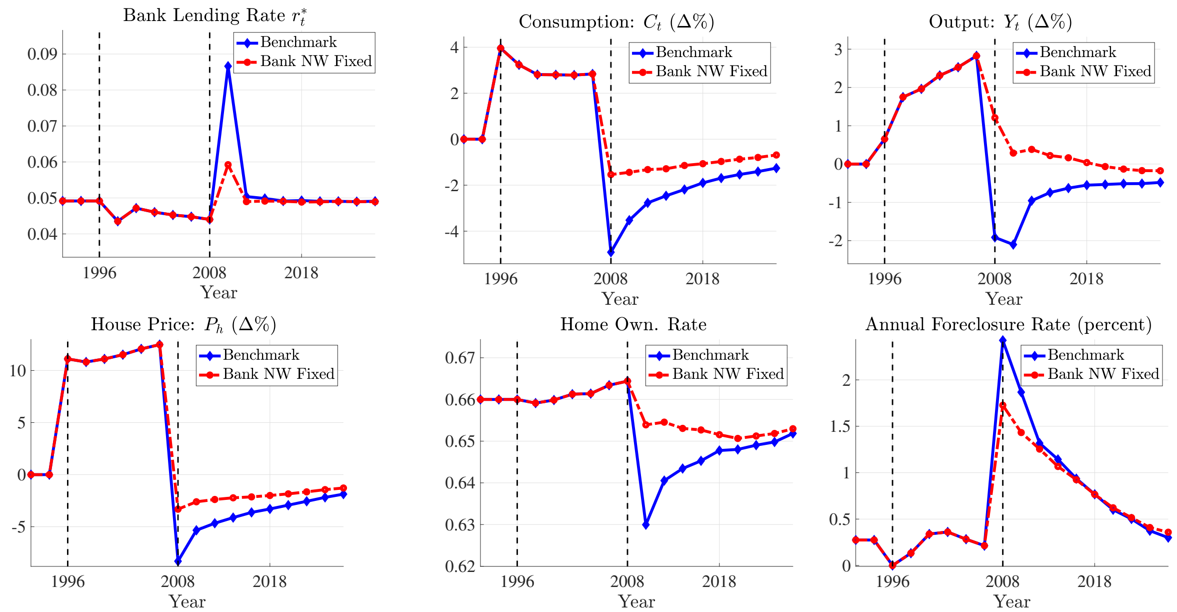
The bank balance sheet deterioration accounts for 25 percent of the 18.5 percent decline in house prices and 70 percent of the decline on the homeownership, 32 percent of the increase in foreclosures, and 44 percent to the decline in consumption. The effect on homeownership rate is higher because households benefit from delaying house purchases for one period.

The bank balance sheet amplification is larger on variables that depend on short-term debt, such as output, relative to those variables that depend on long-term debt, such as house prices. Still, we observe significant effects on house prices. To isolate how much of the bank balance sheet amplification is direct, that is, the spike in r^* making mortgages more expensive, versus indirect, that is, the spike reducing labor demand and labor income, we re-solve the full equilibrium transition of the model by keeping labor income at the steady-state value but with two different r^* sequences: the benchmark r^* and the r^* sequence obtained by fixing the bank net worth at the time of the bust to the boom value. The difference between these two experiments gives the direct effect of the the spike in r^*, which turns out to be negligible (see Figure 12 in Appendix B.2). Therefore, we conclude that the bank balance sheet amplification works through its indirect effects on short-term firm liabilities and household labor income.

Losses in Bank Balance Sheets (Mortgage Valuations versus Foreclosures): Bank balance sheets deteriorate at the time of the bust for two reasons: the increase in foreclosures and the decline in mortgage prices (valuations).[31] In this section, we decompose the relative importance of

[31]Since the equilibrium bank lending rate increases at the time of the bust, prices of mortgages—which are equal to present value of mortgage payments—decline. The "foreclosure effect" is computed as the difference in outcomes from our benchmark economy and the counterfactual economy, in which foreclosure losses in bank balance sheets are from "the bank NW fixed" experiments in Figure 7 while the mortgage values are computed using the benchmark r^* sequence. The "valuation effect" is computed as the difference in outcomes from our benchmark economy and the

Figure 7: The Effect of Bank Balance Sheet Amplification on the Bust



Notes: These graphs plot the contribution of bank balance sheets to some of the key variables of the model. For the benchmark economy, the shock is an ease and then the reversal of borrowing-lending constraints of the banks. Financial intermediaries leverage increases from 9 to 12.5. The blue diamond lines correspond to the benchmark economy. The red lines are the model-implied dynamics where we fix the bank net worth in 2008 (bust) to the 2006 (boom) level. The difference between the two lines measures the amplification arising from bank balance sheet deterioration.

these two channels for the bank balance sheet amplification.

Table 4 reports the results of these experiments. The "Benchmark Bust Δ%" column reports the percentage decline in a variable from its boom value to the bust. Under the "percent Amplification due to" column, the subcolumn "BBS" reports the contribution of the bank balance sheet deterioration to the "Benchmark Bust Δ%". The subcolumns "Valuation" and "Foreclosure" report the contribution of only valuation losses or only foreclosure losses in the bank balance sheets to the "Benchmark Bust Δ%".

We derive two main conclusions from Table 4. First, effects are highly nonlinear. If they were linear, the "BBS" column should have been equal to the summation of the "Valuation" and "Foreclosure" columns. For example, while the losses in mortgage valuations and foreclosures each contribute by 33 percent and 13 percent, respectively, to the total drop in output, their joint effect (the contribution of the bank balance sheet channel) is 66 percent. Second, losses in mortgage valuations are more than twice as important as foreclosure losses, which is consistent with the evidence presented in IMF (2009). As we will illustrate later in section 7, the losses in mortgage valuations are much lower under shocks to the credit demand such as house price expectation or productivity shocks.

counterfactual economy, in which foreclosure losses are computed from our benchmark economy while the mortgage values are computed using the r^* sequence from "the bank NW fixed" experiments in Figure 7.

Table 4: The Amplification through the Bank Balance Sheet: Leverage Shock

	Benchmark	% Amplification due to		
Variables	Bust Δ%	BBS	Valuation	Foreclosure
Output	–4.5	69	33	13
Consumption	–7.5	44	22	9
House Prices	–18.5	25	12	5

5.3 The role of debt distribution on the deepness of the bust

In our model, both age and idiosyncratic income risk generate rich heterogeneity in debt and asset holdings among households. In this section, we quantify the role of debt heterogeneity on the bust. For this purpose, we redistribute debt while keeping the aggregate debt constant during the bust period. In particular, we forgive the debt of households that have negative equity to the point where they have zero home equity. We finance this redistribution with a proportional tax on the assets of other households. As a result, aggregate household net worth remains constant, but the left tail of the debt distribution becomes truncated.

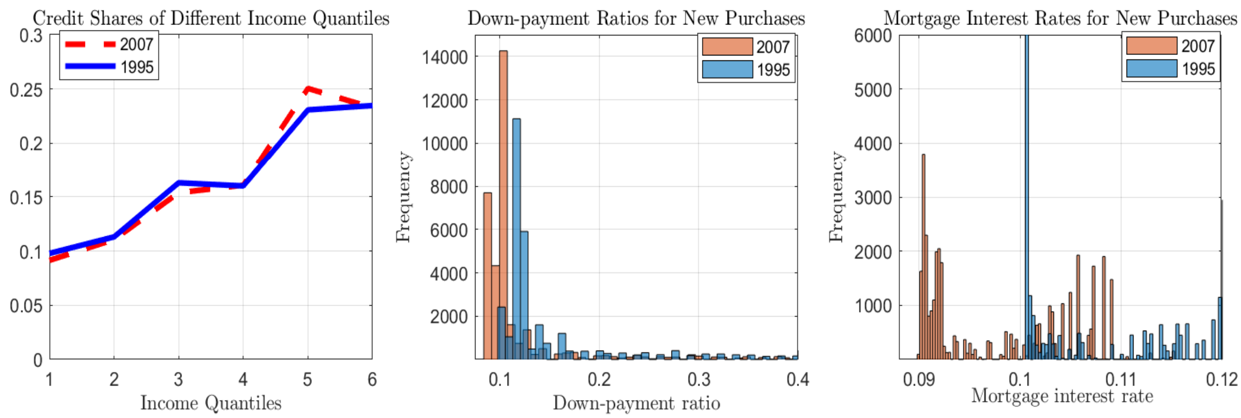
The biggest difference between the benchmark and the one with debt redistribution is the dynamics of foreclosures. With redistribution, foreclosures do not rise at all as there is no one with negative home equity, which is a necessary condition for default. With no rise in foreclosures, house prices decline less by 2 percentage points. Since foreclosures are lower, bank net worth does not deteriorate as much as in the benchmark economy. As a result, the bank lending rate increases to 7 percent during the bust, almost 2 percentage points lower than the benchmark economy. Both wages and labor utilization decline less with debt redistribution as firms' borrowing cost increases less. Thus, output and consumption (driven by output and house prices) decline less, by about 1.5 percentage points.[32]

6 Distributional Implications

Recent empirical findings that build on detailed micro-level data have generally been considered to be against the credit supply mechanism. In this section, given the rich heterogeneous agent structure of our framework, we compare the model's cross-sectional implications with the findings in the literature.

[32]Since the debt redistribution is financed by proportional taxes on assets, the aggregate capital stock is mechanically reduced in this experiment. Thus, the mitigating effect of redistribution on the decline in output can be viewed as a conservative estimate.

Figure 8: Cross-sectional Developments in Credit and Housing during the Boom



Notes: The graph plots several model-implied cross-sectional implications. The left panel plots the mortgage credit distribution across income quantiles in 1995 and 2007. The middle and right panels plot the histograms of thedown payment ratio and interest on mortgages in 1995 and at the peak of the boom in 2007.

6.1 Credit Growth across Income Groups during the Boom Period

The initial findings in Mian and Sufi (2009) were mostly interpreted as a strong indication that the financial crisis may be a consequence of an unprecedented increase in lending to low-income and subprime borrowers.[33] However, Adelino et al. (2016), Albanesi et al. (2017), and Foote et al. (2016) find that credit grew uniformly across income groups during the boom period. These findings have been considered to be more consistent with the house price boom driven by expectations of capital gains (i.e., a credit demand channel rather than a credit supply channel).

In this section, to check whether our model is consistent with these more recent empirical findings, we first analyze how credit shares of each income quantile evolved during the boom period. The left panel of Figure 8 plots the model's implications. As can be seen from the graph, the credit shares of each quantile remained mostly stable. Indeed, shares of only the fifth quantile, not the lower ones, increased during the boom period. Our model generates credit dynamics that are consistent with data because the lower bank lending rate and higher labor income affect all household segments, not just marginal ones. Consequently, the changes in these prices generate similar credit dynamics across the population.

During the boom episode, household debt in our framework increases both from lower down payment ratios and from higher house prices. In the steady state, the average down payment ratio is 13 percent. At the peak of the boom, it declines to 11 percent. However, not only the average but also the distribution is significantly different. In the model, before the crisis no one has a down payment ratio lower than 10 percent as shown in the middle panel of Figure 8. During the

[33] Mian and Sufi (2016) point out that the credit supply view of the mortgage boom does not imply that individuals with the lowest-income or lowest credit score were responsible for the aggregate rise in household debt.

Table 5: Change in Credit and Income Growth

Credit (2007)	Coefficient Estimates
Age	-0.02***
House	0.45***
Financial assets	0.06***
Income (2007)/income(1995)	0.08***

Notes: This table presents the results from a regression of credit growth on income growth using model-generated panel data. We restrict our sample to individuals who switch from renting to owning. Hence, credit in 1995 is zero. "House" variable corresponds to the level of housing services that households consume. All the coefficients are significant at 1 percent.

peak of the boom, about one-third of new mortgages have a down payment ratio below 10 percent. While the maximum LTV limit is fixed at 100 percent throughout the transition, our model implies increases in equilibrium LTV levels similar to the data since banks increase the credit supply during the boom and mortgage interest rates decline, as shown in the top right panel of Figure 8.[34] Thus, the increase in LTV's observed in the data might have been (at least partly) a consequence of the increase in the bank credit supply rather than regulatory changes.

Finally, Adelino et al. (2016) further show that income growth and credit growth were positively related during the boom period. To find out whether the model is consistent with the data in this dimension as well, we regress an individual's credit growth on his/her age, asset, housing, and income growth using model-generated panel data. We restrict our sample to individuals who switch from renting to owning to focus on new mortgage originations, as in Adelino et al. (2016). Table 5 shows that income growth has a positive coefficient in the regression. Thus, overall our model's implications are consistent with the evidence in Adelino et al. (2016). These results show the importance of using a structural model to interpret the data.

6.2 The Roles of Household Leverage and Income on Default

An additional support for our framework comes from the determinants of default. The literature, so far, has identified two major factors that derive foreclosures: leverage and unemployment (Gerardi et al. (2008), Foote et al. (2010), and Palmer (2015)). We use model-generated panel data and estimate a linear regression model to analyze the determinants of default in the model. In particular, we are interested in the roles of household leverage prior to the bust and the decline in income from boom to bust. We investigate the role of decline in income as we do not have unemployment in our framework. We find that households with higher leverage during the boom period were more likely to default (Table 6). Similarly, a sharper decline in household income is associated with higher default.

[34] See the evidence reported in Favilukis et al. (2017).

Table 6: Determinants of Default in the model

Default	Coefficient Estimates
Age	0.0026***
Financial assets	-0.0462***
Leverage	0.0946***
Income (bust)/income(boom)	-0.0126***

Notes: This table presents the results from a regression of default decision on several variables using model-generated panel data. Default takes a value of 1 if there is a default, 0 otherwise. Leverage is measured by mortgage debt divided by the house value. All the coefficients are significant at 1 percent..

7 Leverage versus House Price Expectation and Productivity Shocks

Alternative and sometimes competing macro shocks have been argued to be behind the boom-bust cycle around 2008. In this section, we compare our benchmark results with bank leverage shocks to the results with shocks to productivity and house price expectations. To make the model dynamics comparable across shocks, we choose the size of the shocks to generate a similar-sized boom in house prices in all economies.[35] Then, we revert the shocks to their initial steady-state values in the bust. Because of difficulties in judging the path of the macro shocks, we do not aim to have similar-sized busts. We report our results in Figure 9. Overall, our results suggest that, while there are many similarities, there are also important differences in the model dynamics across different shocks.

7.1 Banking Sector Dynamics

One of the key differences between the leverage shock and expectation and productivity shocks is that the leverage shock primarily affects the credit supply while productivity and expectation shocks primarily affect the credit demand. For example, a reversal in house price expectation in the bust lowers credit (mortgage) demand. The credit supply also declines since foreclosures worsen bank balance sheets. However, even though foreclosures increase more under the expectation and productivity shocks than under the leverage shock, the equilibrium bank lending rates increase by much less.[36] As a result, mortgage prices and hence bank net worth decline by much less under expectation and productivity shocks, by 20 and 35 percent respectively. Remember that, under the bank leverage shock, the decline in bank net worth is mostly driven by the decline in mortgage valuations. On the other hand, the valuation effect turns out to be much less important than

[35] In the house price expectation shock experiment, all economic agents incorrectly expect that house prices will be higher in 2008 and after. Given this off-the equilibrium expectation, we solve the full equilibrium until 2008. Once the economy is in 2008, agents realize that this expectation is not correct. From that point on, we again solve the full equilibrium with correct beliefs.

[36] Overall, the bank lending rate dclines during the boom and increases during the bust with the leverage shock, consistent with the evidence reported in Adrian et al. (2013) and Gilchrist and Zakrajšek (2012).

foreclosures under expectation and productivity shocks.[37] Thus, with relatively smaller valuation effects, these shocks can generate a much smaller deterioration in bank balance sheets.

We have also experimented with larger expectation and productivity shocks (see Appendix B.3). With larger shocks, it is possible to have larger losses in bank net worth and larger increases in the bank lending rate at the time of the bust. However, for that to happen, these shocks should generate more than twice the increase in foreclosures observed in 2008. Overall, these experiments suggest that housing busts accompanied by severe banking crises are more likely to be generated by credit supply shocks rather than expectation or productivity shocks.

7.2 Output, Consumption, and Housing Market Dynamics

Figure 9 shows that output increases with productivity and leverage shocks but declines with expectation shocks during the boom. This is mainly because, under the expectation shock, households dissave, and hence the capital stock declines during the boom. A similar but less stark difference arises in the consumption dynamics. While consumption increases strongly with productivity and leverage shocks during the boom, it barely increases with the expectation shock. This finding also suggests that the relationship between house prices and consumption may critically depend on the shock that generates the cycle. It will be low if the boom is generated by an expectation shock and will be high with productivity and leverage shocks.

Household mortgage debt dynamics, as well, differ across shocks, increasing by 35 percent with the leverage chock, 25 percent with the productivity shock, and 11 percent with the expectation shock during the boom. In the case of the leverage shock, a lower bank lending rate and around a 4 percent wage growth drive household debt. Strong income growth (7 percent) supports housing and credit demand in the productivity shock case. With the expectation shock, neither interest rates nor income growth increases demand. Household debt increases only because of the house price increase. As a result, credit growth remains low during the housing boom.

House prices rise 12 percent in all cases but decline less with the productivity shock. While calibrated to match the boom in house prices, it may first seem surprising to observe this much rise in house prices in response to a 2.1 percent increase in productivity. With a bigger rise in productivity (around 7 percent), Kaplan et al. (2020) do not find any significant rise in house prices. The key difference is that Kaplan et al. (2020) allow for housing production. More importantly, when a productivity shock hits their economy, it affects both the final good and the housing sectors. As a result, income increases along with housing production, which keeps prices stable. In our model, on the other hand, while we allow transformation between rental and owner-occupied housing units, aggregate housing supply is fixed, which is another extreme since we do not model housing production. Since the productivity growth in the construction sector has been mostly negative

[37] Losses due to foreclosures are three times more important than losses due valuations under the expectation shock and twice more important under the productivity shock.

during the housing boom (Harper et al. (2010)), we believe the reality falls between their choice and our choice of modeling.

The price-rent ratio increases with all shocks but much more with the expectation shock. This result is because the relatively steeper increase in house prices with the expectation shock implies a rise in prices relative to rents as show in equation 9. We also find that foreclosures rise by almost 5 percentage points with the expectation shock, 4 percentage points with the productivity shock, and 2.5 percentage points with the leverage shock.

All of these shocks might have contributed to the housing boom and the bust cycle. However, our results suggest that the leverage shock is the strongest candidate among the three. It generates reasonable fluctuations in almost all variables together with severe banking crises during the bust period.

8 Ex post versus Ex ante Government Policies

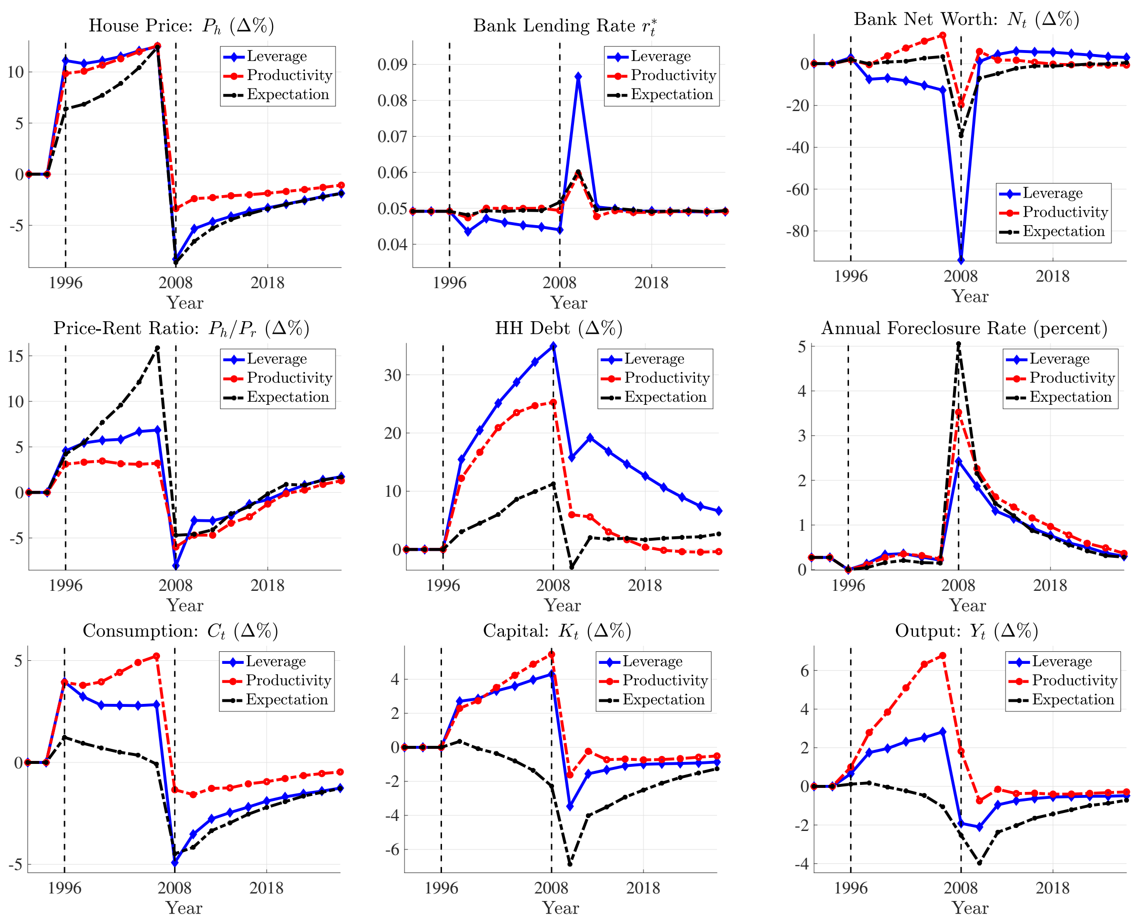
To avoid an economic collapse, governments intervened in many markets during the 2008 crisis. Capital injections to the banking sector and household debt bailouts have been major policy choices. Most governments used a combination of these policies since they lacked a clear understanding of their effectiveness. The severity of the crisis and the large cost of interventions afterward prompted policy makers to take prudential steps to limit the build up of risks. As a result, many countries started to use an extensive set of macroprudential policy tools (Claessens (2014), Galati and Moessner (2018), and BIS (2016, 2017)). Many specific policies can be classified as macroprudential policies, but LTV restrictions have become the most popular one. In this section, we use our framework to study both kinds of policies. We first compare two crises intervention policies, that is, capital injections and household debt bailouts at the time of the bust. Second, we evaluate the effect of a limit on LTV, a commonly implemented macro prudential policy tool especially since the 2008 crisis.

8.1 Capital Injections to Banks and Household Bailout

In this section, we make a cost-neutral comparison of two government interventions. In the first experiment, we make capital injections to banks at the time of the bust. In the second one, the government bails out household debt above 90 percent LTV at the time of the bust so that maximum LTV in the population is 90 percent, which benefits about 40 percent of households. The total size of the program is the same in both experiments: 5.75 percent of output that corresponds to \$620 billion.[38] We do not consider how these policies are financed. Therefore, these experiments do

[38] The output in our economy corresponds to GDP excluding total government expenditure and value added by finance, insurance, real estate, rental, and leasing industries. GDP is \$14.71 trillion, total government expenditure is \$1.98 trillion, and the value added by the finance and sectors is \$1.89 trillion. So, the output measure we use is \$10.84 trillion.

Figure 9: Credit Supply versus Credit Demand Shocks



Notes: The graph plots the dynamics of the model with three different shocks: leverage, productivity, and house price expectation. The sizes of productivity and expectation shocks are given to match the boom in the housing prices in our benchmark economy. The leverage shock is an ease and then the reversal of the borrowing-lending constraints of the banks where bank leverage increases from 9 to 12.5.

not provide information about the absolute benefits of these policies but allow us to compare their effectiveness given a certain set amount of government funds.

Figure 10 shows the results. The capital injection to banks reduces the decline in bank net worth at the time of the bust. As a result, the credit supply increases, and the sharp increase in the equilibrium bank lending rate at the time of the bust is significantly reduced. The household bailout policy also works like a capital injection to banks to some extent since it covers bank losses from otherwise defaulting households. However, this amount is small relative to the size of the total bailout since not all households with above 90 percent LTV default. Thus, the remainder of the bailout corresponds to the prepayment of some mortgage debt. As a result, the bailout policy is less effective than the capital injection in mitigating the decline in bank net worth. Total mortgage demand also goes down because of the prepayment by the government, but overall the decline in the bank credit supply dominates. Thus, the equilibrium bank lending rate remains high relative to the capital injection case.

Our findings suggest that, in the short run, the capital injection—relative to the household bailout–is more effective on variables that depend on short-term funding, such as output, wages, and the homeownership rate, since it almost eliminates the spike in r^*. For variables that are less dependent on short-term funding, the effectiveness of both policies becomes closer. For example, capital injection is still more effective for consumption, but not much so for house prices.

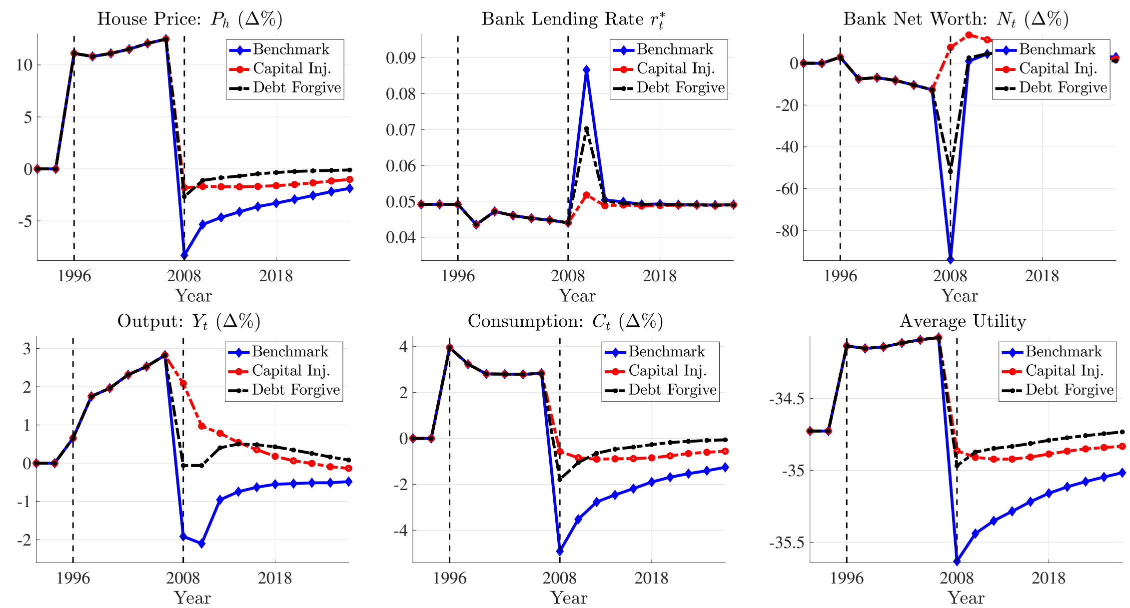
In the longer run, however, the household bailout is more effective in mitigating the drop in all variables, the relative effectiveness appearing earlier on variables that depend on long-term debt such as house prices. Overall, the average welfare of the present generation at the time of the bust is slightly higher with capital injection; however, all subsequent cohorts are better off with the debt forgiveness program.

8.2 The Role of Loan-to-Value Restrictions in Mitigating the Boom-Bust Cycle

In our benchmark economy, households could buy their houses with loans up to 100 percent of their house value. In this section, we analyze the effects of LTV limits on equilibrium outcomes by imposing a maximum LTV limit of 80 percent, that is, households should make at least a20 percent down payment to buy a house. To understand the role of this limit in mitigating the bust, we give the same leverage shock as in our benchmark.

We summarize our findings in Table 7. The first column reports the percentage changes in the steady-state values of variables from their benchmark values to the ones when the LTV limit is imposed. Since the steady-state value of the bank lending rate r^* is determined solely by the bank's preferences and its leverage ratio, the LTV limit has no effect on the steady-state value of r^*. The LTV limit eliminates all the foreclosures and reduces the steady state output by 1.2 percent. The reason for the decline in steady state-output is driven by the decline in aggregate capital. With the LTV limit, the demand for rental housing increases. Since capitalists allocate

Figure 10: Household Debt Forgiveness versus Capital Injection



Notes: The graph plots the dynamics of the model under the benchmark, capital injections to banks, and household bailout programs. The sizes of the government interventions are the same under both cases and equal 5.75 percent of output.

their saving between capital and rental housing stock, the equilibrium return to capital is higher and the aggregate capital stock is crowded out with the LTV limit. The higher return to capital, on the other hand, benefits capitalists and their overall saving increases. Overall, the aggregate consumption of capitalists increases, whereas it decreases for depositors. The LTV limit reduces the steady-state house prices by only 1.2 percent even though it reduces the homeownership rate by about 18 percent. This is because the aggregate housing demand in our model is not significantly affected by the homeownership rate since households can rent houses of similar sizes.

The second and third columns of Table 7 report how much the LTV limit mitigates the boom and bust compared to the benchmark economy. We find large mitigation effects. The boom (bust) in house prices is mitigated by 26 percent (36 percent). Both consumption and output grow less and contract less, the latter because of a smaller spike in r^*. Overall, we find that the LTV limit mitigates about half of the fluctuations in the benchmark economy.

The LTV limit smooths the cycle via two mechanisms. First, in the economy with the LTV limit, the fraction of households who refinance up does not increase as much (10 percentage points lower than the benchmark). Moreover, the fraction of those who pay all their debt does not decline as much. Together, these mechanisms mitigate the increase in mortgage debt. The second important effect of the LTV restriction is to significantly reduce the decline in bank net worth in the bust. This happens for two reasons. First, the fraction of mortgages in the bank's assets goes down from 50

Table 7: The Effects of 80 percent LTV Limit

Variable	Steady State percent Change (1995)	Boom mitigation (1995- 2007, percent)	Bust Mitigation (2008, percent)
r^* (Lending rate)*	0.0	0	45
Wage	–1.1	43	43
Capital	–3.8	73	53
Output	–1.2	55	44
Consumption	0.7	29	42
House Prices	–1.2	26	36
Home-own Rate*	–18.0	20	61
HH Debt	–34.0	41	24
Foreclosure Rate*	–0.4	0	100

*Since these variables are rates, the reported numbers are the percentage point changes.

percent to 39 percent since the homeownership rate declines from 66 percent to 48 percent. Thus, the bank's net worth is not as exposed to the changes in mortgage valuations as in the benchmark. As a result, the decline in bank net worth is smaller for a given increase in r^*. But then, the increase in equilibrium r^* is also smaller because of the feedback between bank net worth and r^*. Second, household mortgage debt is 34 percent lower with the LTV limit, foreclosures are eliminated, and there is no increase in foreclosures during the bust period.

Our results suggest that stricter LTV limits can be effective in mitigating both the boom and bust, especially by suppressing the increases in house prices and household mortgage debt during the boom and by reducing their subsequent busts. Our findings are, in general, in line with the recent findings in the empirical literature. For instance, Cerutti et al. (2017) find evidence that macroprudential policies can help manage financial cycles. Similarly, Richter et al. (2018) and Akinci and Olmstead-Rumsey (2018) find that tightening LTV limits reduces housing credit and house prices. However, these studies are naturally limited to studying short-term effects. With the benefit of the general equilibrium model, we have the opportunity to study a full boom-bust cycle.

9 Conclusions

In this paper, we show that shocks to banks' leverage can generate large fluctuations in the housing market and the macroeconomy. In addition, the model-generated changes in credit across different income and age groups are consistent with the recent empirical evidence. Overall, our findings provide a strong support for the credit supply channel.

During the boom period, an increase in credit supply from higher leverage opportunities lowers the equilibrium bank lending rate and initiates a chain of general equilibrium effects. First, the lower bank lending rate increases housing demand and house prices. Second, household labor income increases as firms demand more labor because of lower funding costs. As a result, household income permanently increases. Aggregate consumption rises from these two changes. During the bust, even if all exogenous variable return to their initial steady-state levels, house prices, output, and consumption all fall significantly below their steady-state levels. Our findings suggest that the deterioration of bank balance sheets during the bust, the existence of highly leveraged households, and the general equilibrium feedback from credit supply to household labor income significantly amplify the bust.

Many countries have started using macroprudential policies to have a more resilient financial system. However, as opposed to monetary policy analysis, these policies lack a widely accepted analytical framework in which to analyze their effectiveness. The framework developed in this paper is well-suited to macroprudential policy analysis as it has both households with limited commitment and banks with balance sheet constraints. In this paper, we scratch the surface and analyze only the role of LTV limits. However, it is possible to extend the analysis further to many of the other policy tools.

Even if we have developed a framework with more realistic bank balance sheets relative to those in the existing literature by incorporating a rich mortgage structure as well as loans to firms, we have abstracted from other features that may be relevant for quantitative results. Incorporating heterogeneity among banks and bank default as in Corbae and D'Erasmo (2013, 2019), modeling the maturity composition of firm debt and firm default, and introducing a feedback from consumption to output are important extensions that we leave for future work.

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Online Appendix

A Data

GDP Real gross domestic product per capita, Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate from FRED. We use quarterly data from 1985 to 2006 and linearly detrend it, and then take the percentage deviation of the data from 1985 to 2019 from this trend.

Consumption Real personal consumption expenditures percapita, Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate from FRED. We use quarterly data from 1985 to 2006 and linearly detrend it, and then take the percentage deviation of the data from 1985 to 2019 from this trend.

Labor income Total wages and salaries (Not Seasonally Adjusted Annual Rate from FRED) divided by working-age population and then divided by the price index for nondurable consumption (line 8 of NIPA Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product). We use annual data from 1985 to 2006 and linearly detrend it, and then take the percentage deviation of the data from 1985 to 2018 from this trend.

Hours per person Hours of Wage and Salary Workers on Nonfarm Payrolls (From FRED, Total, Billions of Hours, Quarterly, Seasonally Adjusted Annual Rate) divided by Working Age Population (From FRED, Aged 15-64: All Persons for the United States, Persons, Quarterly, Seasonally Adjusted).

Investment Private Nonresidential Fixed Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate from FRED. We use quarterly data from 1985 to 2006 and linearly detrend it, and then take the percentage deviation of the data from 1985 to 2019 from this trend.

Homeownership Rate Census Bureau Homeownership rate for the U.S. (Table 14) and by age of the householder (Table 19). Housing Vacancies and Homeownership (CPS/HVS) - Historical Tables.

House Prices House Price Index for the entire US (Source: Federal Housing Finance Agency) divided by the price index for nondurable consumption (line 6 of NIPA Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product). We use quarterly data from 1985 to 2006 and linearly detrend it, and then take the percent deviation of the data from 1985 to 2018 from this trend. To obtain the changes relative to GDP, we divide the real house price index by the real GDP series.

House Rent-Price Ratio Rents (Bureau of labor Statistics Consumer Price Index for All Urban Consumers: Rent of primary residence) divided by nominal house prices.

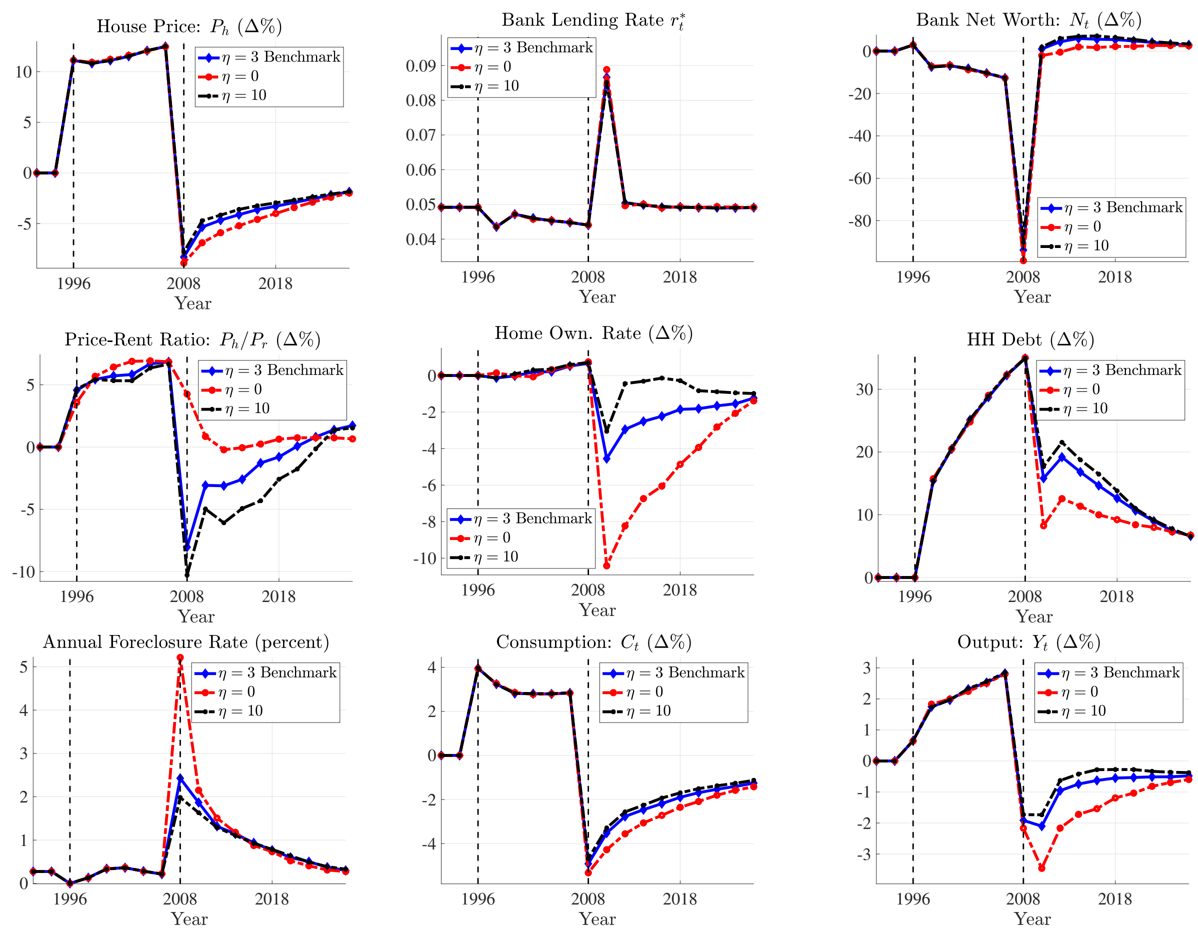
Household Leverage Home Mortgage Liabilities divided by Owner Occupied Housing Real Estate at Market Value. Source: Flow of Funds B.101 Balance Sheet of Households and Nonprofit Organizations.

B Extra Figures

B.1 Degree of Housing Market Segmentation

We report the boom-bust dynamics for various levels of housing market segmentation, captured by different values of η. $\eta = 3$ corresponds to our benchmark, $\eta = 0$ corresponds to frictionless housing market, and $\eta = 10$ corresponds to more frictional housing market than our benchmark. Surprisingly, the degree of housing market segmentation does not have a significant effect on the dynamics of the economy during the boom. In the bust also, the changes in house prices as well as in key macroeconomic and financial variables are not significantly affected by variations in η. However, key differences arise in the price-rent ratio, the homeownership rate, and the foreclosure rate in the bust. The price-rent ratio declines by 2, 15, and 17 percent, the homeownership rate declines by 12, 6, and 4.5 percent, and the foreclosure rate increases by 5, 2.2, and 1.7 percentage points for $\eta = 0$, $\eta = 3$, and $\eta = 10$, respectively.

Figure 11: Different Degree of Housing Market Segmentation

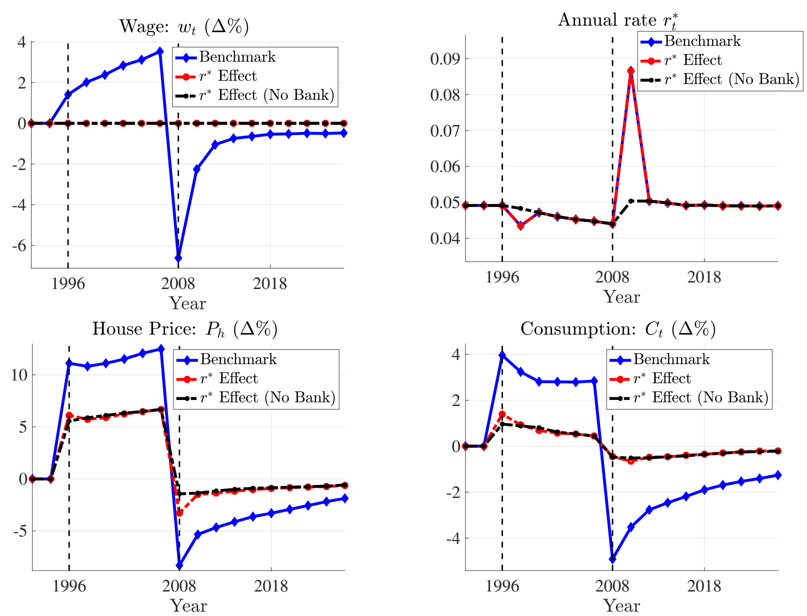


Notes: The graph plots the dynamics of the model with three different levels of housing market segmentation.

B.2 The Direct Effect of Bank Balance Sheet Amplification (Role of the Spike in r^*)

Figure 12 plots the dynamics (percentage changes from the initial steady state) house prices and consumption for given wage (w_t) and bank lending rate r_t^* sequences. For the "r^* Effect" exercise, we keep the wages at the steady-state level, as shown in top left panel, and shock the economy with r^* boom and bust sequences of the benchmark economy (top right panel). The "r^* Effect (No Bank)" is essentially the same exercise as the "r^* Effect" except that the spikes in r^* in the first period of the boom and the bust are eliminated.

Figure 12: The Role of the Spike in r^*

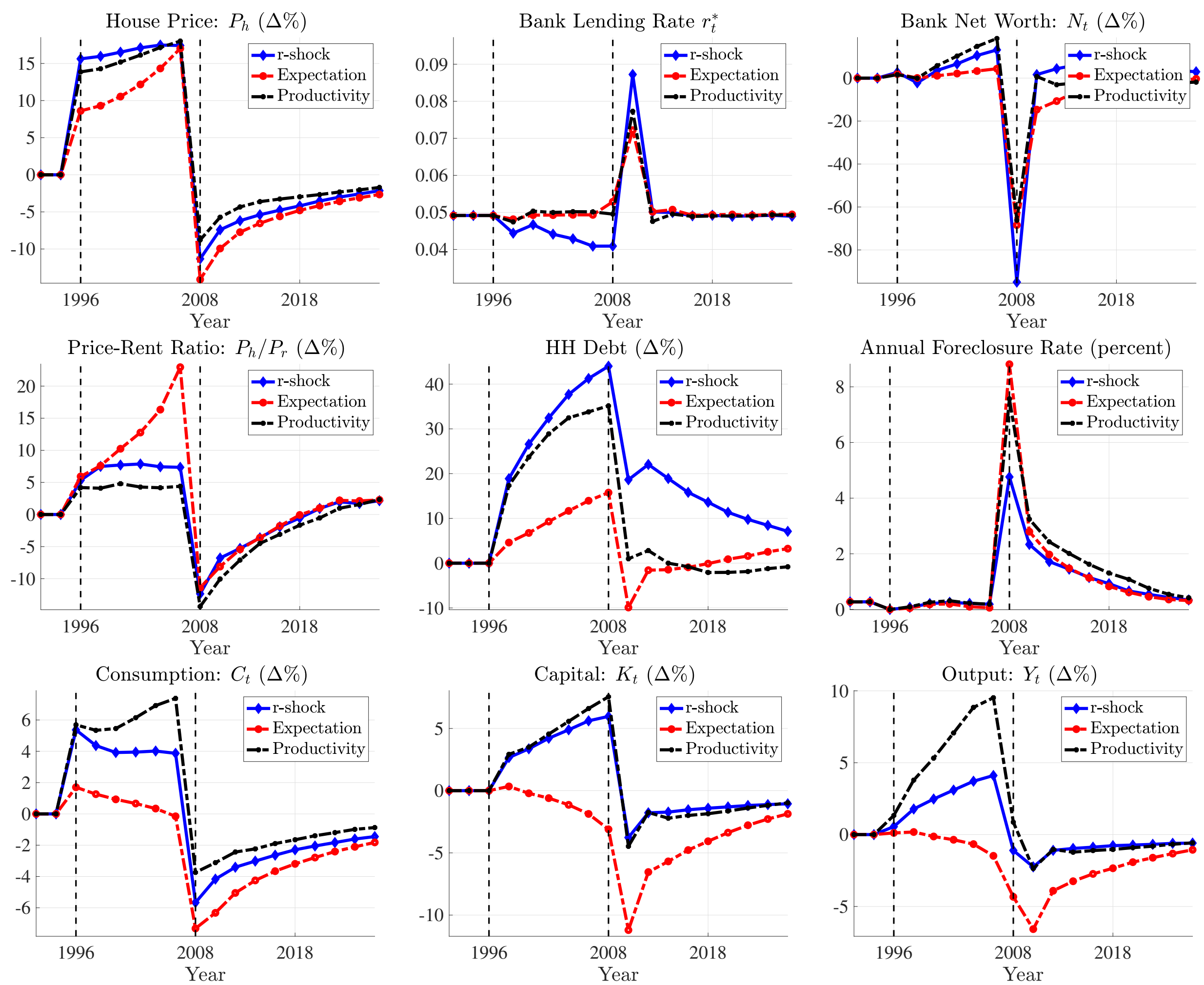


Notes: The graph plots the dynamics (percentage changes from the initial steady state) house prices and consumption for given wage (w_t) and bank lending rate r^* sequences.

B.3 Alternative and Larger Shocks

This figure presents results from three shocks: bank funding rate (r-shock), productivity, and house price expectation. We do not report results from a larger bank leverage shock since the bank net worth becomes negative in the bust period. Instead, we choose another credit supply shock—a shock to the bank funding rate that generates very similar results—for comparison. The sizes of shocks are given to match the same increase in the housing prices. The r-shock is primarily a shock to the credit supply, whereas the others are shocks to the credit demand. Compared to the analysis in the text, the boom-bust cycle is 60 percent larger.

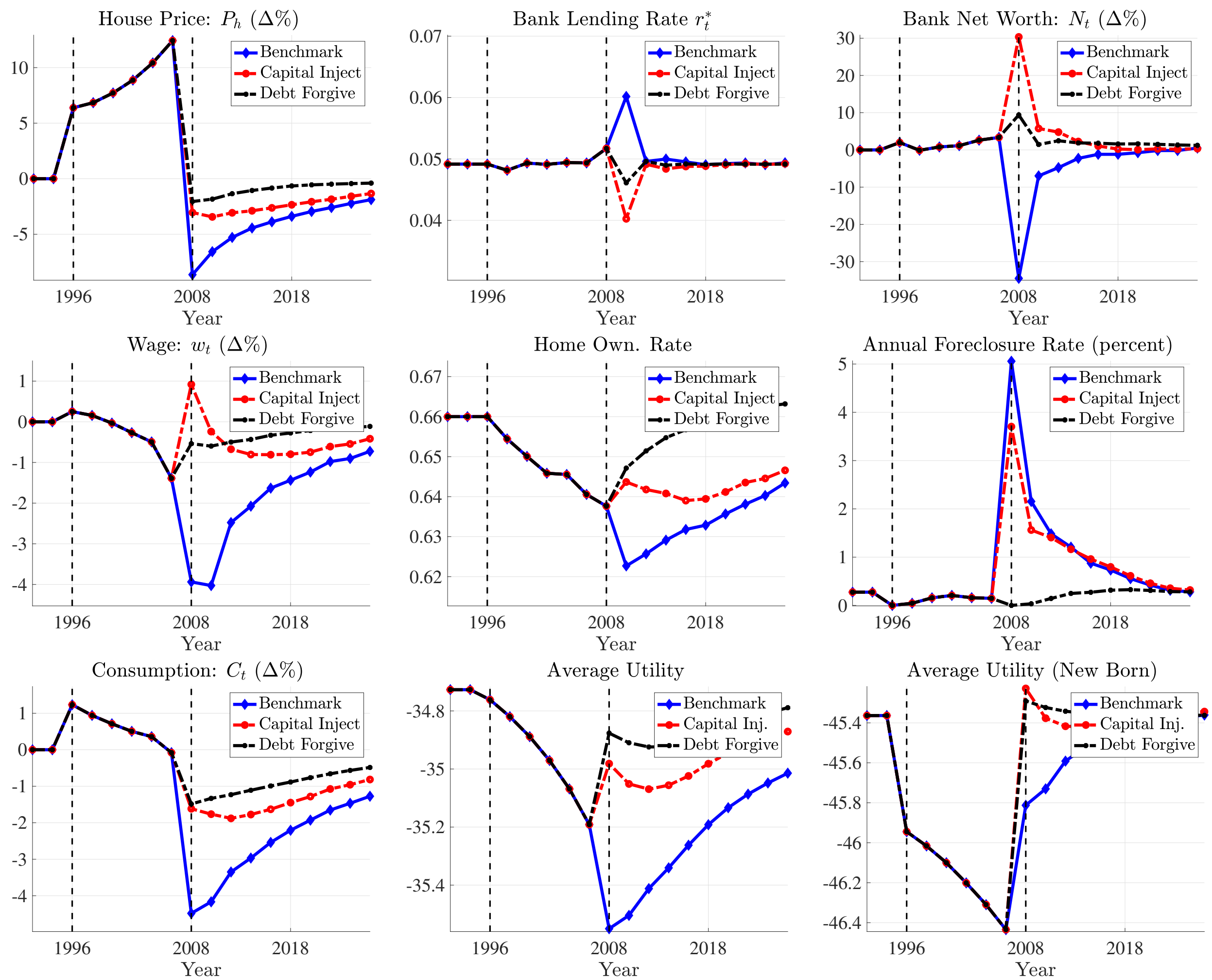
Figure 13: Credit Supply versus Credit Demand Shocks



Notes: The graph plots the dynamics of the model with three different shocks: bank funding rate (r-shock), productivity, and house price expectation. The sizes of shocks are given to match the same increase in the housing prices.

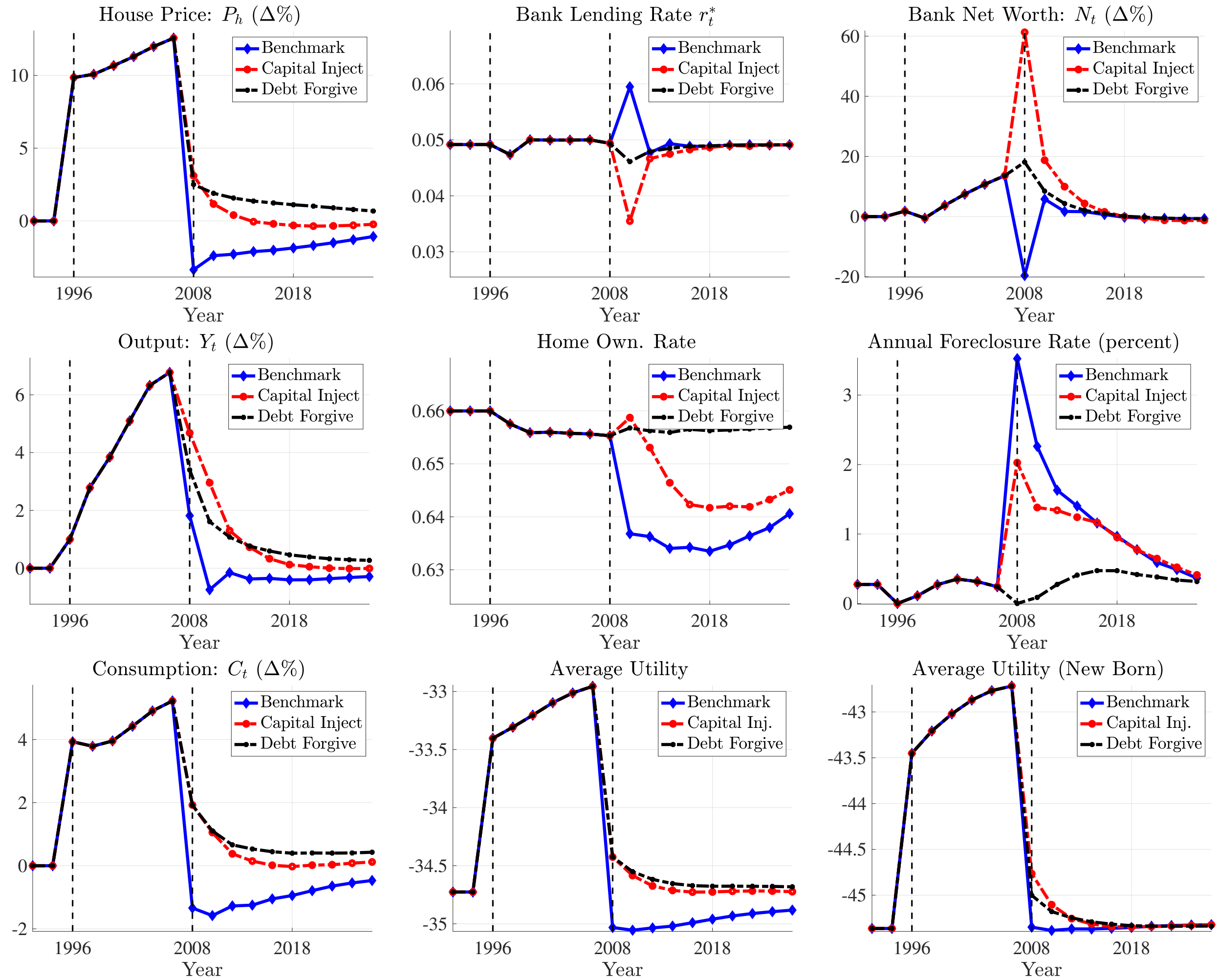
B.4 Capital Injections versus Household Bailout Comparison under Alternative Shocks

Figure 14: Government Interventions in the Crisis: Expectation Shock



Notes: The graph plots the dynamics of the model under the benchmark, capital injections to banks, and household bailout programs. The sizes of the government interventions are the same under both cases.

Figure 15: Government Interventions in the Crisis: Productivity Shock



Notes: The graph plots the dynamics of the model under the benchmark, capital injections to banks, and household bailout programs. The sizes of the government interventions are the same under both cases.

C Characterization of the Bank's Problem

In this section, we will provide proofs for the characterization of the bank's problem. We will start with the steady state value functions and decision rules and continue obtaining value functions in the transition by iterating backwards from the steady state.

C.1 Steady State with $r^* > r$

We will characterize the case $r^* > r$ and leave the cases for $r^* \leq r$ for brevity. We will start with the value function of the bank when it defaults.

Since the bank can steal a fraction ξ of assets after return has been realized and can continue saving at interest rate r, the bank's problem in the period of default is given as

$$\tilde{\Psi}^D\left(\xi L'\right) = \max_{s'} \log\left(\xi L' - W'\right) + \beta_L \Psi^D\left((1+r)W'\right)$$

and after default, it becomes

$$\tilde{\Psi}^D\left(W\right) = \max_{s'} \log\left(W - W'\right) + \beta_L \Psi^D\left((1+r)W'\right).$$

Lemma 1. *$\tilde{\Psi}^D\left(W\right)$ is given as*

$$\tilde{\Psi}^D\left(W\right) = \frac{1}{1-\beta_L}\log(W) + \frac{\beta_L}{(1-\beta_L)^2}\log(\beta_L(1+r)) + \frac{\log(1-\beta_L)}{1-\beta_L}.$$

The bank's problem in the no-default state solves

$$\Psi\left(L,B\right) = \max_{L',B'} \log\left((1+r^*)L - (1+r)B + B' - L'\right) + \beta_L \Psi\left(L',B'\right)$$

subject to

$$\Psi\left(L',B'\right) \geq \tilde{\Psi}^D\left(\xi(1+r^*)L'\right).$$

Proposition 2. *The solution to the bank's problem is given as follows:*

1. Value function:

$$\begin{aligned}\Psi\left(L,B\right) &= \frac{1}{1-\beta_L}\log\left((1+r^*)L - (1+r)B\right) \\ &+ \frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{(1+r)\left(1+r^*\right)\beta_L\phi}{1+r-\left(1+r^*\right)\left(1-\phi\right)}\right) + \frac{\log(1-\beta_L)}{1-\beta_L}.\end{aligned}$$

2. The no-default constraint can be written as

$$\left(1+r^*\right)\left(1-\phi\right)L' \geq \left(1+r\right)B'$$

where ϕ is given as

$$\phi = \xi^{1-\beta_L} \left(\frac{1+r}{1+r^*} - (1-\phi) \right)^{\beta_L}.$$

3. The bank's solution satisfies the following expression regardless of no-default constraint binding or not:
$$L' - B' = \beta_L \left((1+r^*)L - (1+r)B \right).$$

4. The decision rules when the no-default constraint is binding (if $r^* > r$):
$$\begin{aligned} L' &= \frac{(1+r)}{1+r-(1-\phi)(1+r^*)} \beta_L \left((1+r^*)L - (1+r)B \right) \\ B' &= \frac{(1-\phi)(1+r^*)}{1+r-(1-\phi)(1+r^*)} \beta_L \left((1+r^*)L - (1+r)B \right). \end{aligned}$$

Proof. (Proposition 2) We will use the expressions for value functions and verify the claims above. First, drive the capital requirement constraint:

$$\Psi \left(L', B' \right) \geq \tilde{\Psi}^D \left(\xi (1+r^*) L' \right).$$

$$\frac{1}{1-\beta_L} \log \left((1+r^*)L' - (1+r)B' \right) + \frac{\beta_L}{(1-\beta_L)^2} \log \left(\frac{(1+r^*)(1+r)\beta_L \phi'}{(1+r)-(1+r^*)(1-\phi')} \right) \geq \frac{1}{1-\beta_L} \log(\xi(1+r^*)L') + \frac{\beta_L}{(1-\beta_L)^2} \log(\beta_L(1+r))$$

where ϕ' is the capital requirement constraint in the next period. The expression above gives

$$\log \left(\frac{(1+r^*)L' - (1+r)B'}{\xi(1+r^*)L'} \right) \geq \frac{\beta_L}{1-\beta_L} \log \left(\frac{\beta \left((1+r) - (1+r^*)(1-\phi') \right)}{(1+r^*)\beta_L \phi'} \right)$$

$$\frac{(1+r^*)L' - (1+r)B'}{(1+r^*)L'} \geq \xi \left(\frac{((1+r) - (1+r^*)(1-\phi'))}{(1+r)\phi'} \right)^{\frac{\beta_L}{1-\beta_L}}$$

We will show below that the solution of ϕ' is the fixed point of

$$\phi = \xi \left(\frac{((1+r) - (1+r^*)(1-\phi'))}{(1+r)\phi'} \right)^{\frac{\beta_L}{1-\beta_L}}.$$

Then this constraint can be written as

$$(1+r^*)(1-\phi) L' \geq (1+r) B'.$$

Now, we can solve the bank's problem

$$\begin{aligned}
\Psi(L,B) &= \max_{L',B'} \log\left((1+r^*)L-(1+r)B+B'-L'\right)+\beta_L\Psi\left(L',B'\right) \\
&= \max_{L',B'} \log\left((1+r^*)L-(1+r)B+B'-L'\right) \\
&+ \frac{\beta_L}{1-\beta_L}\log\left((1+r^*)L'-(1+r)B'\right) \\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r^*)(1+r)\beta_L\phi'}{(1+r)-(1+r^*)(1-\phi')}\right)+\frac{\beta_L\log(1-\beta_L)}{1-\beta_L}
\end{aligned}$$

subject to

$$(1+r^*)(1-\phi)L' \geq (1+r)B'.$$

Imposing the balance sheet constraint, we obtain

$$\begin{aligned}
\Psi(L,B) &= \max_{L',B'} \log\left((1+r^*)L-(1+r)B+\frac{(1+r^*)(1-\phi)L'}{1+r}-L'\right) \\
&+ \frac{\beta_L}{1-\beta_L}\log\left((1+r^*)L'-(1+r)\frac{(1+r^*)(1-\phi)L'}{1+r}\right) \\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r^*)(1+r)\beta_L\phi'}{(1+r)-(1+r^*)(1-\phi')}\right)+\frac{\beta_L\log(1-\beta_L)}{1-\beta_L}
\end{aligned}$$

$$\begin{aligned}
\Psi(L,B) &= \max_{L'} \log\left((1+r^*)L-(1+r)B-\frac{(1+r)-(1+r^*)(1-\phi)}{1+r}L'\right) \\
&+ \frac{\beta_L}{1-\beta_L}\log\left((1+r^*)\phi L'\right) \\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r^*)(1+r)\beta_L\phi'}{(1+r)-(1+r^*)(1-\phi')}\right)+\frac{\beta_L\log(1-\beta_L)}{1-\beta_L}.
\end{aligned}$$

The first order condition is

$$\frac{\frac{(1+r)-(1+r^*)(1-\phi)}{1+r}}{(1+r^*)L-(1+r)B-\frac{(1+r)-(1+r^*)(1-\phi)}{1+r}L'}=\frac{\beta_L}{1-\beta_L}\frac{1}{L'},$$

which gives

$$\begin{aligned}
L' &= \frac{\beta_L(1+r)}{(1+r)-(1-\phi)(1+r^*)}\left((1+r^*)L-(1+r)B\right) \\
B' &= \frac{\beta_L(1-\phi')(1+r^*)}{(1+r)-(1-\phi')(1+r^*)}\left((1+r^*)L-(1+r)B\right).
\end{aligned}$$

Given these decision rules, the value function is given by

$$
\begin{aligned}
\Psi(L,B) &= \frac{1}{1-\beta_L}\log\left((1+r^*)L-(1+r)B\right) \\
&+ \frac{\beta_L}{1-\beta_L}\log\left(\frac{(1+r^*)(1+r)\beta_L\phi}{(1+r)-(1+r^*)(1-\phi')}\right) \\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r^*)(1+r)\beta_L\phi'}{(1+r)-(1+r^*)(1-\phi')}\right)+\frac{\log(1-\beta_L)}{1-\beta_L}.
\end{aligned}
$$

Equating this expression to our initial guess

$$
\frac{1}{1-\beta_L}\log\left((1+r^*)L-(1+r)B\right)+\frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{(1+r^*)(1+r)\beta_L\phi}{(1+r)-(1+r^*)(1-\phi)}\right)+\frac{\log(1-\beta_L)}{1-\beta_L},
$$

we obtain

$$
\begin{aligned}
\frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{(1+r^*)(1+r)\beta_L\phi}{(1+r)-(1+r^*)(1-\phi)}\right) &= \frac{\beta_L}{1-\beta_L}\log\left(\frac{(1+r^*)(1+r)\beta_L\phi}{(1+r)-(1+r^*)(1-\phi)}\right) \\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{(1+r^*)(1+r)\beta_L\phi'}{(1+r)-(1+r^*)(1-\phi')}\right),
\end{aligned}
$$

which gives

$$
\frac{\phi}{(1+r)-(1+r^*)(1-\phi)}=\frac{\phi'}{(1+r)-(1+r^*)(1-\phi')}.
$$

Since these expressions are monotone (and declining) in ϕ, they imply that $\phi=\phi'$. By imposing this into

$$
\phi=\xi\left(\frac{1+r-(1+r^*)(1-\phi')}{(1+r)\phi'}\right)^{\frac{\beta_L}{1-\beta_L}}.
$$

we obtain

$$
\phi=\xi^{1-\beta_L}\left(\frac{1+r-(1+r^*)(1-\phi)}{(1+r)}\right)^{\beta_L}.
$$

□

C.2 Transition

Assume that the last period of the transition is period T and the economy is in steady state with r^* and r from period $T+1$ and onward. The following proposition characterizes the bank's solution in the transition, where all prices r_t^* and r_t are potentially changing.

Proposition 3. *The solution to the bank's problem is given as follows:*

1. The value function:

$$\Psi_t\left(L_t, B_t\right) = \frac{1}{1-\beta_L}\log\left((1+r_t^*)L_t - (1+r_t)B_t\right) + \Omega_t + \frac{\log(1-\beta_L)}{1-\beta_L}$$

where

$$\Omega_t = \frac{\beta_L}{1-\beta_L}\log\left(\frac{\beta_L \phi_{t+1}(1+r_{t+1})1+r_{t+1}^*}{1+r_{t+1}-(1-\phi_{t+1})1+r_{t+1}^*}\right) + \beta_L \Omega_{t+1};$$

$$\Omega_T = \Omega = \frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{\beta_L \phi(1+r)\left(1+r^*\right)}{1+r-(1-\phi)\left(1+r^*\right)}\right);$$

$$\phi_t = \xi^{1-\beta_L}\left(\frac{1+r_{t+1}}{1+r_{t+1}^*} - (1-\phi_{t+1})\right)^{\beta_L};$$

and

$$\phi_T = \phi.$$

2. The no-default constraint in period t can be written as

$$\left(1+r_{t+1}^*\right)\left(1-\phi_{t+1}\right)L_{t+1} \geq \left(1+r_{t+1}\right)B_{t+1}.$$

3. The bank's solution satisfies the following expression regardless of the no-default constraint binding or not:

$$L_{t+1} - B_{t+1} = \beta_L\left((1+r_t^*)L_t - (1+r_t)B_t\right).$$

4. The decision rules when the no-default constraint is binding (if $r_{t+1}^* > r_{t+1}$):

$$\begin{aligned} L_{t+1} &= \frac{\beta_L(1+r_{t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{t+1}^*)}\left((1+r_t^*)L_t - (1+r_t)B_t\right) \\ B_{t+1} &= \frac{\beta_L(1-\phi_{t+1})(1+r_{t+1}^*)}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{t+1}^*)}\left((1+r_t^*)L_t - (1+r_t)B_t\right). \end{aligned}$$

5. The decision rules when the no-default constraint is not binding (if $r_{t+1}^* \leq r_{t+1}$):

$$B_{t+1} = \begin{cases} \in \left[0, \frac{\beta_L(1-\phi_{t+1})(1+r_{t+1}^*)}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{t+1}^*)}\left((1+r_t^*)L_t - (1+r_t)B_t\right)\right] & \text{if } r_{t+1}^* = r_{t+1} \\ 0 & \text{if } r_{t+1}^* < r_{t+1} \end{cases}$$

and

$$L_{t+1} = B_{t+1} + \beta_L\left((1+r_t^*)L_t - (1+r_t)B_t\right).$$

Proof. We are going to solve the problem backwards starting from period T.

Period T:

$$\begin{aligned}\Psi_T(L_T, B_T) &= \max_{L_{T+1}, B_{T+1}} \log\left((1+r_T^*)L_T - (1+r_T)B_T - (L_{T+1} - B_{T+1})\right) \\ &+ \frac{\beta_L}{1-\beta_L}\log\left((1+r^*)L_{T+1} - (1+r)B_{T+1}\right) \\ &+ \left(\frac{\beta_L}{1-\beta_L}\right)^2 \log\left(\frac{\beta_L\phi(1+r^*)(1+r)}{1+r-(1-\phi)(1+r^*)}\right) + \frac{\beta_L}{1-\beta_L}\log(1-\beta_L) \\ \text{s.t.} & \\ (1-\phi)(1+r^*)L_{T+1} &\geq (1+r)B_{T+1}\end{aligned}$$

The decision rules of this problem are given as

$$\begin{aligned}L_{T+1} &= \frac{\beta_L(1+r)}{1+r-(1-\phi)(1+r^*)}\left((1+r_T^*)L_T - (1+r_T)B_T\right) \\ B_{T+1} &= \frac{\beta_L(1-\phi)(1+r^*)}{1+r-(1-\phi)(1+r^*)}\left((1+r_T^*)L_T - (1+r_T)B_T\right) \\ L_{T+1} - B_{T+1} &= \beta_L\left((1+r_T^*)L_T - (1+r_T)B_T\right) \\ (1+r^*)L_{T+1} - (1+r)B_{T+1} &= \frac{\beta_L\phi(1+r^*)(1+r)}{1+r-(1-\phi)(1+r^*)}\left((1+r_T^*)L_T - (1+r_T)B_T\right)\end{aligned}$$

which give

$$\begin{aligned}\Psi_T(L_T, B_T) &= \frac{1}{1-\beta_L}\log\left((1+r_T^*)L_T - (1+r_T)B_T\right) \\ &+ \frac{\beta_L}{(1-\beta_L)^2}\log\left(\frac{\beta_L\phi(1+r^*)(1+r)}{1+r-(1-\phi)(1+r^*)}\right) + \frac{1}{1-\beta_L}\log(1-\beta_L).\end{aligned}$$

The value function when the banks defaults is

$$\tilde{\Psi}_T^D\left(\xi(1+r_T^*)L_T\right) = \frac{1}{1-\beta_L}\log\left(\xi(1+r_T^*)L_T\right) + \frac{\beta_L}{(1-\beta_L)^2}\log(\beta_L(1+r)) + \frac{\log(1-\beta_L)}{1-\beta_L}.$$

No default condition in period T can be written as

$$(1-\phi_T)(1+r_T^*)L_T \geq (1+r_T)B_T$$

where

$$\phi_T = \xi^{1-\beta_L}\left(\frac{1+r}{1+r^*} - (1-\phi)\right)^{\beta_L}.$$

Period $T-1$:

$$\begin{aligned}
\Psi_{T-1}\left(L_{T-1},B_{T-1}\right) &= \max_{L_T,B_T} \log\left((1+r^*_{T-1})L_{T-1}-(1+r_{T-1})B_{T-1}-(L_T-B_T)\right)\\
&+ \frac{\beta_L}{1-\beta_L}\log\left((1+r^*_T)L_T-(1+r_T)B_T\right)\\
&+ \left(\frac{\beta_L}{1-\beta_L}\right)^2\log\left(\frac{\beta_L\phi(1+r^*)(1+r)}{1+r-(1-\phi)(1+r^*)}\right)+\frac{\beta_L}{1-\beta_L}\log(1-\beta_L)\\
&\text{s.t.}\\
(1-\phi_T)(1+r^*_T)L_T &\geq (1+r_T)B_T.
\end{aligned}$$

The decision rules for this problem are given as

$$\begin{aligned}
L_T &= \frac{\beta_L(1+r_T)}{1+r_T-(1-\phi_T)(1+r^*_T)}\left((1+r^*_{T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right)\\
B_T &= \frac{\beta_L(1-\phi_T)(1+r^*_T)}{1+r_T-(1-\phi_T)(1+r^*_T)}\left((1+r^*_{T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right)\\
L_T-B_T &= \beta_L\left((1+r^*_{T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right)\\
(1+r^*_T)L_T-(1+r_T)B_T &= \frac{\beta_L\phi_T(1+r^*_T)(1+r_T)}{1+r_T-(1-\phi_T)(1+r^*_T)}\left((1+r^*_{T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right),
\end{aligned}$$

which give

$$\begin{aligned}
\Psi_{T-1}\left(L_{T-1},B_{T-1}\right) &= \frac{1}{1-\beta_L}\log\left((1+r^*_{T-1})L_{T-1}-(1+r_{T-1})B_{T-1}\right)\\
&+ \frac{\beta_L}{1-\beta_L}\log\left(\frac{\beta_L\phi_T(1+r^*_T)(1+r_T)}{1+r_T-(1-\phi_T)(1+r^*_T)}\right)\\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log\left(\frac{\beta_L\phi(1+r^*)(1+r)}{1+r-(1-\phi)(1+r^*)}\right)+\frac{1}{1-\beta_L}\log(1-\beta_L).
\end{aligned}$$

The value function when the bank defaults is

$$\begin{aligned}
\tilde{\Psi}^D_{T-1}\left(\xi(1+r^*_{T-1})L_{T-1}\right) &= \frac{1}{1-\beta_L}\log\left(\xi(1+r^*_{T-1})L_{T-1}\right)+\frac{\beta_L}{1-\beta_L}\log(\beta_L(1+r^D_T))\\
&+ \frac{\beta_L^2}{(1-\beta_L)^2}\log(\beta_L(1+r^D))+\frac{\log(1-\beta_L)}{1-\beta_L}.
\end{aligned}$$

No default condition in period $T-1$ can be written as

$$(1-\phi_{T-1})(1+r^*_{T-1})L_{T-1}\geq(1+r_{T-1})B_{T-1}$$

where

$$\phi_{T-1}=\xi^{1-\beta_L}\left(\frac{1+r_T}{1+r^*_T}-(1-\phi_T)\right)^{\beta_L}.$$

Period $T-2$:

$$\begin{aligned}
\Psi_{T-2}\left(L_{T-2}, B_{T-2}\right) &= \max_{L_{T-1}, B_{T-1}} \log\big((1+r^*_{T-2})L_{T-2} - (1+r_{T-2})B_{T-2} - (L_{T-1} - B_{T-1})\big) \\
&+ \frac{\beta_L}{1-\beta_L} \log\big((1+r^*_{T-1})L_{T-1} - (1+r_{T-1})B_{T-1}\big) \\
&+ \frac{\beta_L^2}{(1-\beta_L)^2} \log\left(\frac{\beta_L \phi_T (1+r^*_T)(1+r_T)}{1+r_T - (1-\phi_T)(1+r^*_T)}\right) \\
&+ \frac{\beta_L^3}{(1-\beta_L)^2} \log\left(\frac{\beta_L \phi (1+r^*)(1+r)}{1+r - (1-\phi)(1+r^*)}\right) + \frac{\beta_L}{1-\beta_L}\log(1-\beta_L) \\
\text{s.t.}& \\
(1-\phi_{T-1})(1+r^*_{T-1})L_{T-1} &\geq (1+r_{T-1})B_{T-1}.
\end{aligned}$$

The decision rules of this problem are given as

$$\begin{aligned}
L_{T-1} &= \frac{\beta_L(1+r_{T-1})}{1+r_{T-1} - (1-\phi)(1+r^*_{T-1})}\omega_{t-2} \\
B_{T-1} &= \frac{\beta_L(1-\phi_{T-1})(1+r^*_{T-1})}{1+r_{T-1} - (1-\phi_{T-1})(1+r^*_{T-1})}\omega_{t-2} \\
L_{T-1} - B_{T-1} &= \beta_L \omega_{t-2} \\
(1+r^*_{T-1})L_{T-1} - (1+r_{T-1})B_{T-1} &= \frac{\beta_L \phi_{T-1}(1+r^*_{T-1})(1+r_{T-1})}{1+r_{T-1} - (1-\phi_{T-1})(1+r^*_{T-1})}\omega_{t-2}, \\
\omega_{t-2} &= \big((1+r^*_{T-2})L_{T-2} - (1+r_{T-2})B_{T-2}\big)
\end{aligned}$$

which give

$$\begin{aligned}
\Psi_{T-2}\left(L_{T-2}, B_{T-2}\right) &= \frac{1}{1-\beta_L} \log\big((1+r^*_{T-2})L_{T-2} - (1+r_{T-2})B_{T-2}\big) \\
&+ \frac{\beta_L}{1-\beta_L} \log\left(\frac{\beta_L \phi_{T-1}(1+r^*_{T-1})(1+r_{T-1})}{1+r_{T-1} - (1-\phi_{T-1})(1+r^*_{T-1})}\right) \\
&+ \frac{\beta_L^2}{1-\beta_L} \log\left(\frac{\beta_L \phi_T (1+r^*_T)(1+r_T)}{1+r_T - (1-\phi_T)(1+r^*_T)}\right) \\
&+ \frac{\beta_L^3}{(1-\beta_L)^2} \log\left(\frac{\beta_L \phi (1+r^*)(1+r)}{1+r - (1-\phi)(1+r^*)}\right) + \frac{1}{1-\beta_L}\log(1-\beta_L).
\end{aligned}$$

The value function when bank defaults is

$$\begin{aligned}
\tilde{\Psi}^D_{T-2}\left(\xi(1+r^*_{T-2})L_{T-2}\right) &= \frac{1}{1-\beta_L}\log\left(\xi(1+r^*_{T-2})L_{T-2}\right) + \frac{\log(1-\beta_L)}{1-\beta_L} \\
&+ \frac{\beta_L}{1-\beta_L}\log(\beta_L(1+r_{T-1})) + \frac{\beta_L^2}{1-\beta_L}\log(\beta_L(1+r_T)) \\
&+ \frac{\beta_L^3}{(1-\beta_L)^2}\log(\beta_L(1+r)).
\end{aligned}$$

No default condition in period $T-2$ can be written as

$$(1-\phi_{T-2})(1+r^*_{T-2})L_{T-2} \geq (1+r_{T-2})B_{T-2}$$

where

$$\phi_{T-2} = \xi^{1-\beta_L}\left(\frac{1+r_{T-1}}{1+r^*_{T-1}} - (1-\phi_{T-1})\right)^{\beta_L}.$$

The derivations suggest that the value functions and decision rules have the same pattern. Thus, they will take the same form of the previous period. $\square$

D Computational Algorithm

Denote the state variable of the household as $\theta = (a, h, d, z, j, i, s)$ where s is the housing tenure, i is the indicator whether the individual is a depositor or a capitalist, j is the age of the household, z is the income efficiency shock, d is the ratio of mortgage debt to initial house price level, h is the size of owner-occupied unit, and a is the financial wealth after the return is realized. For active/inactive renters ($s \in \{r, i\}$) $h = d = 0$. We discretize a into 120 and d into 60 exponentially spaced points. The age j runs from 1 to 30 and h is linearly discretized into 5 points. Income shock z is discretized into 15 points, and grid points and transition probabilities are computed using Tauchen's method. Since this is a life-cycle model, the grid points for income shocks are age dependent to better approximate the AR(1) process with a Markov process.

D.1 Steady-State Computation

The steady state of the model is computed as follows:

1. From the bank's problem, the lending rate at the steady state is $r^* = r + \frac{1-\beta_L(1+r)}{\lambda\beta_L}$.

2. Make a guess on K and p_h.

3. Given these guesses, using the firm's problem, compute w and u:

$$\begin{aligned}
u &= \left(\frac{(1-\alpha)\,K}{(1+\phi r^*)\,\vartheta}\right)^{\frac{1}{\alpha+\psi}} \\
w &= \vartheta^{\frac{\alpha-1}{\alpha+\psi}}\left(\frac{(1-\alpha)\,K^{\alpha}}{(1+\phi r^*)}\right)^{\frac{1+\psi}{\alpha+\psi}} \\
\tilde{r} &= \alpha\left(\frac{K}{u}\right)^{\alpha-1} - \delta
\end{aligned}$$

4. Using the rental companies' problem, compute the rent price:

$$p_r = \kappa + \frac{1-\delta_h}{1+\tilde{r}} p_h$$

5. Given all these prices, solve the household's problem recursively:

   (a) Solve the terminal period problem where all dynamic choices are set to 0: $a' = d' = 0$. This gives the value for the household, $V_J(\theta)$, and the continuation value of the mortgage contract, $v_J^l(\theta)$.

   (b) Given $V_j(\theta)$ and $v_j^l(\theta)$, solve $V_{j-1}(\theta)$ and $v_{j-1}^l(\theta)$:

      i. Given $V_j(\theta)$ and $v_j^l(\theta)$, first solve the expected continuation values $EV_j(\theta)$ and $Ev_j^l(\theta)$.

      ii. Solve for mortgage prices at the origination, $q^m(\theta)$, using equations 2 and 11.

      iii. The solutions of the problems for the inactive renter and the active renter who decides to become a renter are straightforward. Their choices are housing services, consumption, and saving. We interpolate the expected value of the continuation value using linear interpolation, and to choose the optimal saving level, we first search globally over a finer discrete space for a' to bracket the maximum.[39] Once the maximum is bracketed, we solve for the optimum using Brent's method. Given the saving choice, we compute the optimal housing services using the analytical expression for it.[40] Then, we use the budget constraint to compute the consumption.

      iv. The most complex and time-consuming problem is the problem of the renter who decides to purchase a house. This household chooses consumption, saving, house size, and mortgage debt. We restrict the choice of down payment and house size to finite sets. For down payment, the grid points for d are the choices, and for house size the grid points for h are the choices.[41] For each down payment and house size choices, we solve household's objective function, $V_{j-1}^{d,h}$, by finding the optimal saving level as we discussed in point 5(b)iii. Given all household choices, we can obtain $q^m(\theta)$. We use linear interpolation for the points off the grid. Also given the choice of d and h, the mortgage debt becomes $dp_h^* h$ where p_h^* is the equilibrium price level at the initial steady state. Once the objective function is solved for a given down payment and house size choice, we set $V_{j-1}(\theta) = max_{d,h}\left\{V_{j-1}^{d,h}\right\}$.

      v. The solution of the homeowner's problem:

[39] For saving choice, we use 240 grid points.

[40] Since utility is Cobb-Douglas in non-durable consumption and housing services, we can obtain an analytical expression for optimal housing services.

[41] Increasing the number of grid points for d and h beyond the levels we set does not noticeablyc hange the results.

A. Stayer: The stayer's problem is simple since the household only chooses consumption and saving. It is solved similar to the inactive renter's problem. The only exception is that in the continuation value, the variable keeping track of the principal amount d will be adjusted. Given current d, $d' = (d - m)(1 + r^*)$ where $m = \frac{r^*(1+r^*)^{J-j}}{(1+r^*)^{J-j+1}-1}$. We use linear interpolation over d' to compute the expected continuation value for the household.

B. Seller: The seller's problem is the same as the problem of an active renter except for the fact that in the budget constraint, the household will have the term due to the proceedings from the sale of the house: $p_h h (1 - \varphi_s) - dhp_h^*$

C. Refinancer: The refinancer's problem is the same as the problem of a renter who purchases a house except for the fact that she is restricted to purchasing the same house.

D. Defaulter: The defaulter's problem is the same as the active renter's problem.

vi. Solving the homeowner's problem also gives us the mortgage payments for each type of mortgage contract and allows us to compute the continuation of the mortgage contract, $v_j^l(\theta)$:

$$v_{j-1}^l(\theta) = m(\theta) + \frac{1}{1+r^*} \int_{\theta'} v_j^l(\theta') \Pi(\theta'|\theta),$$

where

$$m(\theta) = \begin{cases} dhp_h^* & \text{if } s \in \{hr, hf\} \\ p_h h (1 - \varphi_e) & \text{if } s = he \\ \frac{r^*(1+r^*)^{J-j}}{(1+r^*)^{J-j+1}-1} dhp_h^* & \text{if } s = hh \end{cases}$$

(c) Repeat step (b) for each $j = \{J-1, ..., 1\}$.

6. Given the policy functions for the household, simulate the economy $N = 20,000$ individuals for $J = 30$ periods. This gives us aggregate saving, A, aggregate housing demand, H^d, and aggregate rental demand, H^r. Given aggregate saving, we update the aggregate capital guess as $K = (1 - \lambda_k) K + \lambda_k (A - V^{rc}(H^r))$ where $V^{rc} = \frac{p_r - \kappa - \delta_h p_h^0}{\tilde{r}} H^r$ is the value of rental companies. Given aggregate housing demand, we update the house price guess as $p_h = p_h \left(1 + \lambda_h \frac{H - \bar{H}}{\bar{H}}\right)$. We set $\lambda_k = \lambda_h = 0.1$. We continue this process until $max\left(|A - W(H^r) - K|, |H - \bar{H}|\right) < \epsilon$ where $\epsilon = 10^{-4}$.

7. Once equilibrium prices and allocations are solved, we solve for bank-related variables: bank net worth, bank assets, and bank liabilities using the steady-state analytical equations for these variables.

D.2 Transition Algorithm

The transitional problem has two main differences. First, we need to solve for a path of equilibrium prices and allocations along the transition. Second, we need to adjust the algorithm to capture the fact that the risk-free mortgage interest rate can change along the transition. This second point is important because in order to save from state variables, we assume individuals pay points at the origination time to reduce the risk-adjusted mortgage interest rate to the risk-free mortgage interest rate. This allows us to get rid of the mortgage interest rate as an additional state variable. However, since shocks are permanent, this assumption can artificially distort the equilibrium. Consider a decline in the risk-free mortgage interest rate from 5 percent to 4 percent. If we still assume all new mortgages are priced at 5 percent, this would imply that banks would be paid more than the principal amount if they still use the same amortization schedule we use in the steady-state algorithm. That will result inq^m being significantly larger than 1, implying a substantial subsidy from banks to individuals. More importantly, if we also apply this new risk-free mortgage interest rate to existing mortgages, that would imply a reduction of all the existing mortgage payments: a positive wealth shock to all existing mortgage owners and a negative shock to banks.[42]

To tackle this issue without further complicating the solution algorithm, we assume that after the shock is realized, all new mortgages will be priced at the new risk-free mortgage rate, whereas all existing mortgages will be still paid using the old risk-free mortgage rate. We also include an additional state variable to the household's problem to keep track of whether the household purchased a house before or after the shock is realized. This allows us to compute the mortgage payments more accurately without substantially distorting the solution algorithm.

Given these modifications, the rest of the algorithm is as follows:

1. Fix the time it takes for the transition to happen: T periods. We set $T = 60$ corresponding to 120 years.

2. Solve the initial steady state of the problem as outlined above. Store the initial steady-state distribution denoted as $\Gamma_0(\theta)$.

3. Given the boom shock, solve the final steady state of the problem as outlined above. Store $V_T(\theta)$ and $v_T^l(\theta)$.

4. Guess the path of aggregate capital stock, rental demand, house price and lending rate: $\left\{K_{t+1}, H_t^{r,0}, p_t^h, r_{t+1}^{*,0}\right\}_{t=1}^{T-1}$

5. Given these guesses, compute $\{w_t, \tilde{r}_{t+1}, p_t^r\}$ using the good-producing firm's and rental companies' problem. Compute V_t^{rc} using the rental companies' problem.

[42]Since we keep track of the principal balance as a state variable, we need to know the risk-free mortgage rate to compute the implied mortgage payments. Another formulation could be to keep track of the mortgage payments. However, in this case, we still need to know the risk-free mortgage rate in order to compute the implied principal amount since it affects the resources of homeowners in the event of selling/refinancing/defaulting.

6. Solve each cohort's problem for each period they are alive, starting from the cohort born in period $-J+2$ until the cohort born in period $T-1$[43]:

   (a) For each generation, given prices, solve the household's problem and the continuation value of the contract as in the steady-state problem above. The only difference is that for new mortgage buyers, the risk-free mortgage interest rate is the final steady-state risk-free mortgage interest rate, whereas for existing mortgage owners, it is the initial steady-state risk-free mortgage interest rate. This also affects the continuation value for households and mortgage contracts since we need to keep track of whether a mortgage originated before or after the shock.

   (b) Given the policy functions for each generation, simulate the economy starting from the initial steady-state distribution $\Gamma_0(\theta)$ for T periods. We fix the same random numbers for the idiosyncratic shocks to household.

   (c) Using the simulated path, compute the aggregates: $A_{t+1}, H_t^{r,1}, H_t^d$, $M_t = \int v_t^l(\theta)$.

   (d) Update guesses:

$$\begin{aligned}
K_{t+1} &= (1-\lambda_k) K_{t+1} + \lambda_k \left(A_{t+1} - V_{t+1}^{rc}\left(H_{t+1}^r\right)\right) \\
H_t^r &= (1-\lambda_{rc}) H_t^{r,0} + \lambda_{rc} H_t^{r,1} \\
p_t^h &= p_t^h \left(1 + \lambda_h \frac{H_t^d - \bar{H}}{\bar{H}}\right) \\
r_{t+1}^{*,0} &= (1-\lambda_r) r_{t+1}^{*,0} + \lambda_{rc} r_{t+1}^{*,1}
\end{aligned}$$

   where r_{t+1}^* solves

$$L_{t+1} = \frac{(1+r_{t+1})}{1+r_{t+1}-(1-\phi_{t+1})(1+r_{t+1}^*)} \beta_L N_t$$

   where $L_{t+1} = M_{t+1} + \phi w_{t+1}(\bar{w}, u_{t+1})$ and

$$N_t = \begin{cases} L_t\left(1+r_t^*\right) - B_t\left(1+r_t\right) & \text{if } t=1 \\ \left(1+r_t^{*,0}\right)\phi_t L_t & \text{if } t>1 \end{cases}$$

   (e) Iterate this process until convergence occurs on guesses. The convergence criteria are defined as $\max\left|K_{t+1} + V_{t+1}^{rc}\left(H_{t+1}^r\right) - A_{t+1}\right| < \epsilon_k$, $\max\left|H_t^{r,1} - H_t^{r,0}\right| < \epsilon_h$, $\max\left|H_t^d - \bar{H}\right| < \epsilon_h$, and $\max\left|r_t^{*,1} - r_t^{*,0}\right| < \epsilon_r$ where $\epsilon_k = \epsilon_h = 10^{-3}$ and $\epsilon_r = 10^{-4}$.

[43] A household of age j belonging to a cohort born in period $g \in \{-J+2, ..., T-1\}$ will be subject to prices p_{g+j-1}.

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