On the instability of banking and other financial intermediation

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Keywords: banking, financial intermediation, instability, volatility
ON THE INSTABILITY OF BANKING
AND OTHER FINANCIAL INTERMEDIATION*

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Abstract
Are financial intermediaries inherently unstable, and if so, why? To address this we analyze whether model economies with financial intermediation are particularly prone to multiple, cyclic, or stochastic equilibria. Several formalizations are considered: a dynamic version of Diamond-Dybvig banking incorporating reputational considerations; a model with fixed costs and delegated investment as in Diamond; one with bank liabilities serving as payment instruments similar to currency in Lagos-Wright; and one with intermediaries as dealers in decentralized asset markets, similar to Duffie et al. Although the economics and mathematics differ across specifications, in each case financial intermediation engenders instability in a precise sense.

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Banks, as several banking crisis throughout history have demonstrated, are fragile institutions. This is to a large extent unavoidable and is the direct result of the core functions they perform in the economy. Finance Market Watch Program @ Re-Define, Banks: How they Work and Why they are Fragile.

Introduction

The above quotation reflects an oft-heard view: banks, or financial intermediaries more generally, are inherently unstable and prone to volatility. This seems to be based on the notion that financial institutions are special compared to, say, producers or retailers, a position commonly associated with Keynes (1936), Kindleberger (1978) and Minsky (1992) (more recently, see Shleifer and Vishny 2010, Williams 2015, Akerlof and Shiller 2009 or Reinhart and Rogoff 2009). Rolnick and Weber (1986) provide evidence of the widespread acceptance of this view when they say: “Historically, even some of the staunchest proponents of laissez-faire have viewed banking as inherently unstable and so requiring government intervention.” As a leading case, Friedman (1960) defended unfettered markets in virtually all contexts, but advocated bank regulation in his program for monetary stability. As additional evidence, consider the large literature dedicated to the study of bank runs.\footnote{For now we discuss bank runs, panics, financial crises, etc. without defining these formally. As Rolnick and Weber (1986) put it, “There is no agreement on a precise definition of inherent instability in banking. However, the conventional view is that it means that general bank panics can occur without economy-wide real shocks.” They add “The usual explanation... involves a local real economic shock that becomes exaggerated by the actions of incompletely informed depositors,” and suggest this is consistent with Friedman and Schwartz’s (1963) view. In terms of models, Chari and Jagannathan (1988) have withdrawals by informed depositors lead to withdrawals by others, while Gu (2011) formalizes this as rational herding. Our approach is different, and avoids fixating only on runs, but does focus squarely on volatility “without economy-wide real shocks.”}

We are agnostic and do not know \textit{a priori} if financial intermediation engenders instability, or even if it does, we do not know if this is inherent to their activieis or induced by policy makers’ interventions (on that see, e.g., Lacker 2015 or Weinberg 2015). Nevertheless, it is a logical possibility that financial intermediation might be inherently unstable. This project is an attempt to investigate that possibility by analyzing several formal models of financial intermediation and asking if the equilibrium sets in these models exhibit instability, defined as a multiplicity
of Pareto-ranked equilibria, or volatile dynamics arising as self-fulfilling prophecies, including cyclic, chaotic or stochastic outcomes that entail fluctuations even when fundamentals are constant.

Thus, saying that financial intermediation engenders instability means that economic environments with this activity exhibit these types of equilibria for a larger set of parameters than otherwise-identical environments without it. Now it is well known that there are various ways to get instability without intermediaries – e.g., increasing returns in production or matching technologies – but we want to see what intermediation can generate without such devices. Also, it is important to consider several formal models of financial intermediaries since these agents in reality perform several distinct functions, and we want to know which if any lead to instability. Moreover, we make an effort to have intermediaries arise endogenously. As Gorton and Whinton (2002) say at the start of their well-known survey, crucial questions are: “Why do financial intermediaries exist? What are their roles? Are they inherently unstable?” In this spirit we endeavor to model why they exist and what they do as explicitly as possible before studying stability.²

There is no generally-accepted, all-purpose model of financial intermediaries because, as mentioned, these institutions perform a myriad of functions that are difficult to capture in a single setup: they serve as middlemen between savers and borrowers or asset sellers and buyers; they find, screen and monitor investment opportunities on behalf of depositors; they issue liabilities like demand deposits that facilitate third-party transactions; they provide liquidity insurance or maturity transformation; they are safe keepers of cash and other valuables; and they main-

²Without making too much of this, one might say that we want models of intermediation, not just with intermediation. It does not suffice to simply assert that households lend to banks and banks lend to firms but households do not lend to firms – that is a model with banking but not of banking. By analogy, Clower (1965) said money buys goods and goods buy money but goods do not buy goods, and while a popular shortcut, it is hard to argue that Clower (cash-in-advance) constraints constitute the last word in monetary theory. We feel similarly about banking theory (see Wright 2017 for more on this). Now it is not necessary for every study to have everything endogenous – e.g., Debreu (1959) makes progress in a model with firms and households but not of firms and households, which is fine for his purposes, if not for industrial organization or family economics. But surely it is desirable for financial institutions to emerge endogenously when asking if they are unstable as “the direct result of the core functions they perform” (from the epigraph).
tain privacy (secrecy) about their assets or customers. As different approaches are naturally used to formalize these diverse activities, we consider several distinct specifications, and while they all are constructed using building blocks from off-the-shelf models, the ways in which we combine and apply them are somewhat novel.

The first formulation extends Diamond and Dybvig’s (1983) insurance-based model to an infinite horizon, to highlight bankers’ reputation as in Gu et al. (2013a), based on Kehoe and Levine (1993). The second features fixed costs of investment as in Diamond (1984) or Huang (2017). The third, an adaptation of Nosal et al. (2017), puts middlemen like those in Rubinstein and Wolinsky (1987) into an OTC asset market similar to Duffie et al. (2005), where such agents are called market-makers, dealers or brokers (so while they are not banks, it is standard to interpret them as asset-market intermediaries). The fourth has bank liabilities serving as payment instruments, similar to cash in Lagos and Wright (2005) or Berentsen et al. (2007), in an environment where these liabilities are less susceptible to loss or theft, as in He et al. (2007) and Sanches and Williamson (2010), or less sensitive to information, as in Andolfatto and Martin (2013) and Dang et al. (2017).

We find in each case that financial intermediaries can indeed engender instability: an economy with these institutions is more likely to have multiple Pareto-ranked equilibria or volatile dynamics than the same economy without them. In some cases, without intermediation there is a unique equilibrium and it is stable, but with it there are multiple or volatile equilibria; in other cases both can have multiple or volatile equilibria, but intermediation expands the set of parameters for which this is true. Further, while the logic differs across models, in each case the results are directly related to the *raison d’être* for intermediation. Yet while financial intermediation may be fragile in this sense, we emphasize that it still tends to increase welfare.

As the intermediation literature is vast, for that we refer to standard sources (e.g., Freixas and Rochet 2008; Calomiris and Haber 2014; Vives 2016). Different from some work in the area, we always use infinite-horizon models, since we are interested in economic dynamics, and since finite-horizon models are ill suited for
capturing many phenomena – e.g., unsecured credit, which we use extensively below, and fiat (outside) currency, which we do not use but in principle could (we do use inside money in some cases). We also try to minimize exogenous restrictions on prices, contracts or behavior. This is to see if instability, if it does arise, arises from intermediation *per se*, not from convenient-but-extraneous features like noise traders, sticky prices, arbitrary expectations or *ad hoc* restrictions on contracts.

To be clear, we have frictions like limited commitment, imperfect information, and spatial separation, but they are imposed on the environment, not on prices, contracts or behavior. By way of example, while there seems to be confusion about this, one can show the baseline Diamond-Dybvig model *cannot* generate bank runs unless one rules out contracts with suspension clauses (the proof is actually in their original paper). The goal here is to rely less on those kind of restrictions and proceed more along the lines of mechanism design, where any resource- and incentive-compatible arrangement is allowed. Relatedly, as mentioned in fn. 1, we downplay bank runs to focus on other types of instability. The rest of the paper involves laying out four specifications, and in some cases multiple subcases, where financial intermediation arises endogenously, and in each case asking about instability.

While some readers may find four-plus models too much for one paper, the presentations are self contained, and each case could be skipped without loss of continuity. More importantly, a main point is to see if the instability results transcend different ways of formalizing the role of financial intermediaries. Yet there is a common thread across setups: First, it should be clear that one must model frictions explicitly to get an essential role for financial intermediaries, as such a role is absent in frictionless GE theory. Now consider what Shell (1992) calls the Philadelphia Pholk Theorem: in all models where the First Welfare Theorem does not hold there can be multiplicity/volatility.\(^3\) It is hard to prove this in general, as the statement concerns *all* models; hence, corroboration comes from showing that it works in a

\(^3\)Note that Shell was thinking about sunspot equilibria, but the idea also applies to deterministic cycles. Also note that the converse of the Pholk Theorem is easy to prove: if the First Welfare Theorem holds there cannot be sunspot or cyclic equilibria.
series of models. We consider a series of models capturing many of the usual ways of thinking about financial intermediaries. In each case, we find the frictions making intermediation useful make multiplicity/volatility more likely.

Model 1: Insurance

The first specification extends Diamond and Dybvig (1983) to a discrete-time, infinite-horizon setting, which as in Gu et al. (2013a) lets us incorporate reputational considerations à la Kehoe and Levine (1993). The key feature of this framework is that different agents are more or less trustworthy. To capture this in a stark way, suppose some live forever, while at each date a $[0, 1]$ continuum of other agents are around only for that period. In fact the latter can be around for any $N < \infty$ periods, but $N = 1$ is obviously easiest way to make them care less about reputation. Indeed, in Gu et al. (2013a), everyone lives forever, but some are less trustworthy because they are less patient, cannot be as easily monitored, have greater gains from trade, face better investment opportunities, or get fewer opportunities to misbehave.

Given this is understood, for present purposes it suffices to consider agents around for only $N = 1$ period. However, each period has two subperiods, to capture the usual Diamond-Dybvig idea that the short-lived agents, while homogeneous \textit{ex ante}, face idiosyncratic shocks: they are impatient with probability $\pi$ and patient with probability $1 - \pi$, where impatient (patient) agents derive utility only from consumption in the first (second) subperiod. The shock is private information, and conditional on it an agents has utility $u_j(c_j)$, $j = 1, 2$, where $c_j$ is consumption in subperiod $j$, with $u'_j > 0$ and $u''_j < 0$.\footnote{Many applications of Diamond-Dybvig assume $u_1(\cdot) = u_2(\cdot)$, but not all (e.g., Peck and Shell 2003). The flexibility of the general version is useful for constructing examples.} Infinitely-lived agents have period utility $v(c)$ for $c$ in either subperiod, with $v' > 0$, $v'' \leq 0 = v(0)$.

Short-lived agents have an endowment of 1; infinitely-lived agents have 0. The standard technology is this: a unit of the good invested at the start of the first subperiod yields $R > 1$ units in the second subperiod; or, the investment can be terminated at the end of the first subperiod to get back the input. The good can
also be stored one-for-one across subperiods. As in any Diamond-Dybvig model, to
insure against the shocks, the short-lived agents can form a coalition that resembles
a banking arrangement. Thus, they design a contract \((c_1, c_2)\) to solve

\[
\begin{aligned}
\max_{c_1, c_2} & \left\{ \pi u_1(c_1) + (1 - \pi) u_2(c_2) \right\} \\
\text{st} & \quad (1 - \pi) c_2 = (1 - \pi c_1) R \\
\end{aligned}
\]

\((1)\)

\[
\begin{aligned}
& \quad c_2 \geq c_1,
\end{aligned}
\]

\((2)\)

\((3)\)

where \((2)\) is feasibility and \((3)\) is a truth-telling constraint (if \(c_2 < c_1\) patient agents
would claim to be impatient, get \(c_1\) and store it to the next subperiod). There are
also nonnegativity constraints omitted to save space.

This problem is well understood. One result is: assuming \(u'_1(1) > u'_2(R) R\) and
\(u'_1(c) \leq u'_2(c) R\) at \(c = R / (1 - \pi + \pi R)\), we get \(1 < c_1^* < c_2^* < R\), so \((3)\) is not
binding, and full insurance/efficiency obtains. However, this requires commitment;
otherwise, when they learn they are patient and are supposed to make transfers
to the impatient, agents will renege. Given our short-lived agents cannot commit,
naturally, there emerges a role for long-lived agents as bankers who accept deposits,
invest them, and pay off depositors on demand at terms to be determined. Importantly,
bankers do not have exogenous commitment ability – it is endogenous and
based on reputation. Thus, bankers honor obligations lest they get identified as
renegers, whence they are punished to autarky, which is a credible threat because
there are many perfect substitutes for any given banker.

However, a banker may be tempted to misbehave as in the “cash diversion”
models in Biais et al. (2007) or DeMarzo and Fishman (2007): if he misappropriates
d deposits, he gets payoff \(\lambda d\), where \(\lambda\) is not too big, so this is socially inefficient, but
he might do it opportunistically. As in Gu et al. (2013a,b), the risk is that he gets
caught, and punished, with probability \(\mu \leq 1\), where one interpretation is that \(\mu\) is
the probability one generation of depositors can communicate his misbehavior to the
next generation.\(^5\) Now depositors may set \(d < 1\), and invest \(1 - d\) on their own, to

\(^5\)While \(\mu = 1\) is fine, it does not simplify things much, and it is known from other applications
that \(\mu < 1\) can be interesting (e.g., the extension of Kocherlakota 1998 in Gu et al. 2016).
reduce bank incentives to misbehave, different from most papers that simply assume $d = 1$, but similar to Peck and Setayesh (2019). In addition to $d$, the contract now specifies payouts per deposit contingent on withdrawal time $r_j$, $j = 1, 2$, and the banker’s income $b \in [0, d]$, which he invests for utility $v(bR)$.

Since there is more than one long-lived agent, the short-lived agents can choose any of them to act as banker, and in the spirit of Diamond-Dybvig they make this choice as a coalition. However, we assume they can choose only one, to avoid determining the optimal number of bankers, something we do in Model 2, but would be a distraction here; it can be rationalized by assuming that it is too costly to monitor more than one banker. Still, since they can choose any one, for reasons often summarized as Bertrand competition the contract maximizes the expected utility of the depositors. Still, a banker may get a positive surplus – a rent on his option to act opportunistically – because the contract must give him incentive to not misuse deposits for his own gain.

The banker’s incentive constraint is

$$v(b_t R) + \beta V_{t+1} \geq \lambda d_t + \beta (1- \mu) V_{t+1},$$

(4)

where $\beta$ is his discount factor, $V_t$ is his equilibrium payoff, and the RHS is the deviation payoff, including $\lambda d$ for sure and $V_{t+1}$ iff he is not caught.\(^6\) Note that $V_{t+1}$ is his valuation next period, facing a new generation of depositors, and hence is taken as given when designing a contract at $t$. Also note that bankers do not misuse $d$ on the equilibrium path, but if one were to, he would get $\lambda d$ but not $v(Rd) + \lambda (d - b)$.

This leads to the contracting problemm

$$\max_{d_t, r_{1t}, r_{2t}, b_t} \left\{ \pi u_1 (d_t r_{1t} + 1 - d_t) + (1- \pi) u_2 [d_t r_{2t} + (1 - d_t) R] \right\}$$

(5)

subject to

$$r_{2t} \geq r_{1t}$$

(6)

$$\lambda d_t - v(b_t R) \leq \phi_t,$$

(8)

\(^6\)Here the actions of the bank are observable and the investment return is deterministic, excluding the strategic concern of the reputation as in Ordonez (2013, 2018).
where (8) rewrites (4) using $\phi_t \equiv \beta \mu V_{t+1}$. Note that $\phi_t$ is a bank’s *franchise value*, capturing the banker’s reputation for trustworthiness. Substituting (6) into (5) to eliminate $r_2$, ignoring $t$ subscripts for now, and taking FOC’s wrt $(r_1, d, b)$ we get

$$
    r_1 : d \{ \pi [u'_1(c_1) - Ru'_2(c_2)] - \eta_1 (1 - \pi + R\pi) \} r_1 = 0
$$

$$
    d : \{(r_1 - 1) \pi [u'_1(c_1) - Ru'_2(c_2)] + \eta_1 [R - (1 - \pi + R\pi) r_1] - \eta_2 \lambda \} d = 0
$$

$$
    b : [-u'_2(c_2) - \eta_1 + \eta_2 \nu'(bR)] b = 0,
$$

where $c_1 = dr_1 + 1 - d$ and $c_2 = dr_2 + (1 - d) R$, while $\eta_1$ and $\eta_2$ are multipliers for constraints (7) and (8).

These FOC’s yield two critical values, $\phi^* > 0$ and $\hat{\phi} < \phi^*$, delineating three regimes: (i) If $\phi \geq \phi^*$ then (8) is slack, and $b = 0$, since the franchise value keeps the banker honest without $b > 0$. In this case there is a continuum of contracts achieving the full-insurance outcome, because depositors can have the bank invest a lot or a little, and in the latter case invest the rest on their own (exactly as in Peck and Setayesh 2019). (ii) If $\phi \in [\hat{\phi}, \phi^*)$ we must either lower $d < d^*$ or raise $b > 0$ to satisfy (8). While lowering $d$ from $d^*$ means less-than-full insurance, this is a second-order cost by the envelope theorem, so the contract sets $d = \phi/\lambda$ and keeps $b = 0$. (iii) If $\phi < \hat{\phi}$, lowering $d$ further entails too much risk, so it sets $b > 0$. In case (i), one of the payoff-equivalent contracts has $r_1 = r_2$, and (ii)-(iii) the unique contract has $r_1 = r_2$; hence wlog we set $r_1 = r_2 = r$ from now on.

In regime (iii), $(d, r, b)$ satisfies

$$
    b = d/R [R - (1 - \pi + R\pi) r]
$$

$$
    \phi = \lambda d - v(bR)
$$

$$
    \frac{u'_2(c_2)}{u'_1(c_1) - Ru'_2(c_2)} = \frac{\pi}{1 - \pi + R\pi} \left[ \frac{(R - 1) (1 - \pi)}{\lambda} \nu'(bR) - 1 \right]
$$

with $c_1 = 1 - d + (d - b) R/ (1 - \pi + R\pi)$, $c_2 = (1 - d) R + (d - b) R/ (1 - \pi + R\pi)$.

These and the analogs from regimes (i)-(ii) characterize the contract given $\phi$, and in particular, one can easily check $b'(\phi) < 0 \forall \phi < \hat{\phi}$, which is important below. This is shown in Fig. 1 for the following parameterization:

\[\text{Notice } \hat{\phi} > 0 \text{ here (in fact, for the example } \hat{\phi} = 0.3257 \text{ and } \phi^* = 0.600) \text{; the case } \hat{\phi} < 0 \text{ is less}\]
Example 1: Let $v(b) = BB$, 

$$u_1(c_1) = A_1\left(\frac{c_1 + \varepsilon}{1 - \sigma_1} - \varepsilon^{1 - \sigma_1}\right) \quad \text{and} \quad u_2(c_2) = A_2\left(\frac{c_2 + \varepsilon}{1 - \sigma_2} - \varepsilon^{1 - \sigma_2}\right),$$

where $B = 0.95$, $\sigma_1 = \sigma_2 = 2$, $\varepsilon = 0.01$, $A_1 = 1$, $A_2 = 0.1$, $R = 2.1$, $\mu = 0.7$, $\pi = 0.25$, $\lambda = 0.6$ and $\beta = 0.99$.

As mentioned, the contract takes $\phi$ as given. To embed this in general equilibrium, use $\phi_t \equiv \beta \mu V_{t+1}$ to write 

$$V_t = v(b_t R) + \beta V_{t+1} \quad \text{as a dynamical system,}$$

$$\phi_{t-1} = f(\phi_t) \equiv \beta \mu v[b(\phi_t) R] + \beta \phi_t,$$

(12)

where $b(\phi_t)$ comes from the contracting problem. Equilibrium is defined as a nonnegative, bounded path for $\phi_t$ solving (12), from which the other endogenous variables follow using the FOC’s. A stationary equilibrium, which is the same as a steady state here, solves $\bar{\phi} = f(\bar{\phi})$. The nature of steady state depends on whether $\hat{\phi} \leq 0$ or $\hat{\phi} > 0$ (conditions for which are given in fn. 7). Appendix A proves:

Proposition 1 If $\hat{\phi} \leq 0$ the unique steady state has no banking, $d = 0$. If $\hat{\phi} > 0$ the unique steady state has $\bar{\phi} \in (0, \hat{\phi})$ and banking, $d > 0$.

For dynamic equilibria, first note from (12) that $f(\phi_t)$ has a linear term that is increasing and a nonlinear term that is decreasing because $b'(\phi) < 0$. If the net effect interesting because it never has banking in steady state. In terms of primitives, one can show that $\hat{\phi} > 0$ iff $\pi [u'(1) - R u'(R)] [(R - 1)(1 - \pi) v'(0) - \lambda] > u'_2(R) (1 - \pi + R\pi) \lambda$. 

Figure 1: Model 1, bank contract vs $\phi$
implies $f'(\phi_t) < 0$ over some range the system can exhibit nonmonotone dynamics. For Example 1, Fig. 2a shows $f$ and $f^{-1}$, which cross on the 45° line at $\bar{\phi} = 0.3215$. In this case the system is monotone and there is a unique equilibrium – the steady state, because it is the only bounded path solving (12). But now consider:

**Example 2:** Same as above except $\sigma_1 = 10$ and $\mu = 1$.

As Fig. 2b shows, now $f'(\phi_t) < -1$, and so $f$ and $f^{-1}$ intersect not only on the 45°, but also off it, at $(\phi_L, \phi_H)$ and $(\phi_H, \phi_L)$ with $\phi_H = 0.0696$ and $\phi_L = 0.0689$. As is standard (see Azariadis 1993), this means there is a two-cycle equilibrium where $\phi_t$ oscillates deterministically between $\phi_L$ and $\phi_H$. It also means there are sunspot equilibria where $\phi_t$ fluctuates randomly between values close to $\phi_L$ and $\phi_H$ (see Appendix B). Thus we can get deterministic or stochastic volatility with banking and not without it. That does not mean banking is a bad idea, as it provides insurance to agents who cannot insure each other due to commitment issues. To be clear, it is obvious that if we were to eliminate banking, say through taxation or regulation, the unique outcome is autarky and all agents are worse off.

![Figure 2a: Model 1, monotone $f$](image)

![Figure 2b: Model 1, nonmonotone $f$](image)

The intuition is straightforward: if next period $V_{t+1}$ is high then this period $\phi_t$ is high and we can discipline bankers with low $b_t$; but that makes the current $V_t$ and hence $\phi_{t-1}$ low. This induces a tendency towards oscillations, but for a cycle the
effect has to dominate the linear term in $f(\phi_t)$, which is why parameters matter. Fig. 3 plots time series of $(\phi, d, b, r)$ over the cycle in Example 2. Notice $r$ moves with $\phi$ and $b$ against $\phi$. Whether $d$ moves with or against $\phi$ depends on parameters, but here it is the latter. While the point is not to take this example seriously in a quantitative sense, it is worth noting that the theory does make qualitative predictions, and does not say “anything goes.”

Figure 3: Model 1, time series for a two-cycle

Fig. 4 displays the existence of two-cycles in a different way, as fixed points of the second iterate $f^2 = f \circ f$, for another parameterization:

**Example 3:** $B = 1$, $\sigma_1 = 14$, $\sigma_2 = 1.5$, $\varepsilon = 0.01$, $A_1 = 1$, $A_2 = 0.075$, $R = 2.2$, \( \mu = 1, \pi = 0.28, \lambda = 0.75 \) and $\beta = 0.76$.

Notice $f^2$ has three fixed points, $\bar{\phi}$, plus $\phi_L$ and $\phi_H$ from the two-cycle. Also shown is $f^3$, which has seven fixed points, $\bar{\phi}$ plus a pair of three-cycles. Standard results (again see Azariadis 1993) say the existence of three-cycles implies the existence of $n$–cycles for any integer $n$, plus chaos, which is basically a cycle with $n = \infty$.$^8$

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$^8$Benhabib et al. (2016) consider a banking model with potential risk-shifting problem. Multiple steady states and chaotic dynamics can arise in such an economy. Our model provides a different mechanism for exotic dynamics.
To summarize, banking can introduce many equilibria, including deterministic, stochastic and chaotic dynamics, directly attributable to the idea that banks depend on trust, and to some extent that is a self-fulfilling prophecy. However, in any equilibrium with banking all agents are better off than they are without it.

Model 2: Delegated Investment with Fixed Costs

The next formulation has intermediation originating from economies of scale, based on Diamond (1984) and Huang (2017) (see also Leland and Pyle 1976 or Boyd and Prescott 1986 on the bigger picture). Time is discrete and continues forever as in Model 1, but here all agents are infinitely lived. Also, they are now spatially separated – say, across a large number of islands – and randomly relocated at the end of each period, following a literature on banking including Champ et al. (1996), Bencivenga and Smith (1991), Smith (2002) and Bhattacharya et al. (2005). Economies of scale are captured as follows: agents must pay a fixed cost $\kappa$. 

\footnote{The main function of random relocation here is to let us avoid long-term contracting considerations, which are interesting but complicated (e.g., in Gu et al. 2013a, bankers’ rewards can be backloaded over multi-period contracts). Elsewhere in the paper we avoid those issues using short-lived agents, but here we want all agents to be long-lived, so that \textit{ex ante} anyone can potentially be a banker. In any case, it is important to emphasize that these are not restrictions on contracting \textit{per se}, but assumptions on the environment that impinge on the contract. Does it matter? Yes, because without making them explicit one cannot know, in general, how these}
in terms of goods, to locate/evaluate/monitor investment projects, after which any project returns $R$ per unit invested.

Period utility is $u(x) - c(d)$, where $x$ is consumption and $d$ investment, with $u', c' > 0$ and $c'' > 0 > u''$ and $u(0) = c(0) = 0$. Also, $u'(0) R > c'(0)$, so that agents invest if $κ = 0$. If $κ > 0$ the payoff is

$$W_1 = \max_{x,d} \{u(x) - c(d)\} \text{ st } x = Rd - κ,$$  

(13)

from investing on one’s own (omitting nonnegativity constraints as above). Suppose $κ$ is too high to support this, so $W_1 < 0$, while the autarky payoff is $W_0 = 0$. Now consider agents forming a coalition where some, that we call depositors, delegate their investment to others, that we call bankers, to share the fixed cost.

As is standard in models with nonconvexities, the coalition uses a lottery to chose a subset of members to act as bankers.$^{10}$ Thus, $ω_t$ is the probability of being a banker, equal to the measure of bankers if the island population is normalized to 1. As in Model 1, bankers have the option to misbehave, with $λ$ and $μ$ playing similar roles. The relevant incentive condition is therefore

$$βV_{t+1} \geq \frac{λ(1 - ω_t)}{ω_t} \frac{x_t}{ω_t} + (1 - μ)βV_{t+1},$$  

(14)

where the RHS is the deviation payoff, given each depositor is promised $x_t$ and each banker controls $(1 - ω_t) x_t/ω_t$ of the resources. The trade-off, emphasized in Huang (2017), is that having fewer banks saves on fixed costs but raises their temptation to misbehave, because they must be larger, given total deposits. The contract maximizes the payoff to the representative agent on an island

$$W(φ) = \max_{ω,X,D,x,d} \{ω[u(X) - c(D)] + (1 - ω)[u(x) - c(d)]\}$$

st $ωX + (1 - ω)x = R[ωD + (1 - ω)d] - κω$  

(15)  

$$u(x) - c(d) \geq 0$$

(16)  

$$\frac{1 - ω}{ω}x \leq φ,$$  

(17)  

assumptions impinge on all endogenous variables.

$^{10}$This is similar to, e.g., Rogerson’s (1988) indivisible-labor model, except unlike his, our agents cannot commit, so our contracts must be incentive compatible before and after the lottery.
where $\phi_t \equiv \mu \beta V_{t+1}/\lambda$, while $(X, D)$ and $(x, d)$ are the consumption/investment allocations of bankers and depositors.\textsuperscript{11}

Substituting (16) into (15) to eliminate $X$, and letting $\eta$ and $\gamma$ be multipliers, we get the FOC's:

$$D : u'(X) R - c'(D) = 0$$
$$d : (1 - \omega) [u'(X) R - c'(d)] - \eta c'(d) = 0$$
$$x : (1 - \omega) [u'(x) - u'(X)] + \eta u'(x) - \gamma \frac{1 - \omega}{\omega} = 0$$
$$\omega : \omega \left\{ u(X) - c(D) - [u(x) - c(d)] + u'(X) \frac{x - Rd}{\omega} + \gamma \frac{x}{\omega^2} \right\} = 0$$

One can check $W'(\phi) \geq 0$. Moreover, $W(0) = 0$, so we get no banking at $\phi = 0$. In the limit as $\phi \to \infty$ we get $\omega \to 0$, which means very few banks but they are huge. Also as $\phi \to \infty$ we get $W(\phi) \to W^* \equiv \max_{x,d} [u(x) - c(d)]$ s.t $x = Rd$, which totally dissipates the fixed cost (i.e., delivers the same payoff as $\kappa = 0$).

Fig. 5 shows the contract given $\phi$ for the following parameterization:

**Example 4:** Let

$$u(x) = A \frac{(x + \varepsilon)^{1-\sigma} - \varepsilon^{1-\sigma}}{1 - \sigma} \text{ and } c(d) = Bd,$$

with $A = \varepsilon = 0.001$, $\sigma = 2$, $B = 0.1$, $\kappa = 230$, $R = 1.2$, $\beta = 0.76$, $\mu = 0.95$ and $\lambda = 9$.

Notice that there is a cutoff $\bar{\phi}$, which is $\bar{\phi} = 0.0182$ in this example, and banking is viable iff $\phi \geq \bar{\phi}$.

\textsuperscript{11}Here (16) is the resource constraint, (17) is the incentive constraint for depositors, and (18), which rewrites (14), is the incentive constraint for bankers.
Using $V_t = W(\phi_t) + \beta V_{t+1}$ and emulating the methods from Model 1 we get

$$\phi_t = f(\phi_{t+1}) = \frac{\beta t}{\lambda} W(\phi_{t+1}) + \beta \phi_{t+1}. \quad (19)$$

Equilibrium is a bounded, nonnegative solution to (19). Notice $f(\phi) = \beta \phi$ for $\phi \leq \overline{\phi}$, and $f(\phi) < \phi$ for big $\phi$ due to the fact that $W \leq W^*$. Then we have:

**Proposition 2** There is a steady state at $\overline{\phi} = 0$, without banking. There can be steady states with banking, generically an even number that alternate between stable and unstable.

Fig. 6 shows Example 4 has three steady states, $\phi = 0$, plus two with banking, $\phi_2 > \phi_1 > 0$. This is different from Model 1, which has a unique steady state $\overline{\phi}$, and has nonstationary equilibria iff $f'(\overline{\phi}) < -1$. Now $f'(\phi) > 0$, so deterministic cycles are impossible, but if there are multiple steady states we can use a different approach to construct sunspot equilibria around the stable ones.\(^{12}\) Appendix B shows there are equilibria where $\phi$ fluctuates between $\phi_A$ and $\phi_B$ for any $\phi_A \in (0, \phi_1)$ and $\phi_B \in (\phi_1, \overline{\phi})$. In particular, $\phi_A < \overline{\phi}$ means we switch stochastically between $d_t > 0$

\(^{12}\)To give credit where credit is due, in Model 2 we use the method in Azariadis (1981), while in Model 1 we use the method in Azariadis and Guesnerie (1986).
and $d_t = 0$—i.e., random episodes of crises, where deposits dry up and banking shuts down, due to sunspots, which are fundamentally irrelevant events. This is again different from Model 1, where $d_t$ can fluctuate, but only with $d_t > 0 \forall t$.

![Figure 6: Model 2, monotone $f$ with multiple steady states](image)

While Models 1 and 2 are different, in terms of economics and mathematics, Appendix C presents an environment that integrates elements of both. It has two agents on each island, one that is infinitely lived and one that is only around for one period, who negotiate the contract using generalized Nash bargaining (having just two is simpler, but we also considered many depositors and one banker, with multilateral bargaining, and got similar results). There are gains to delegating investment due to $\kappa > 0$, as in Model 2, but only long-lived agents can act as bankers, as in Model 1. Letting $\theta$ denote bankers’ bargaining power, we get a dynamical system $\phi_t = f(\phi_{t+1})$ that can be nonmonotone for $\theta < 1$. Appendix C shows we can have multiple steady states, with $f'(\phi) > 0$ around the stable ones and hence sunspot equilibria as in the benchmark Model 2, as well as $f'(\phi) < -1$ around the unstable steady states and hence cycles and sunspots as in Model 1.

The reason $f(\phi)$ can be decreasing in Appendix C is a well-known (see Kalai
feature of Nash bargaining: agents with bargaining power $\theta < 1$ can get a smaller surplus when the bargaining set expands. Here bankers’ surplus can fall with $\phi$, similar to $b'(\phi) < 0$ in Model 1. That does not happen in the baseline Model 2, where agents are ex ante identical and all get a bigger surplus when $\phi$ increases. Details aside, the point is that there are distinct ways for banking to engender instability based on trust issues, but it is still the case that agents are better off in any equilibrium with banking than they are without it.\(^{13}\)

### Model 3: Asset Market Intermediation

Banks are not the only interesting financial intermediaries. Work following Duffie et al. (2005) studies OTC asset markets using search theory, where agents may trade with each other, or with middlemen/dealers. However, most of these papers give middlemen access to a frictionless interdealer market, so they never hold assets in inventory (with exceptions, e.g., Weill 2007). Our middlemen are more like those in Rubinstein and Wolinsky (1987), who buy goods from producers and hold inventories until they sell to consumers, except here they trade assets and not goods, which Nosal et al. (2017) argue matters a lot. The presentation below builds on that work, but amends the setup in many ways – e.g., it adds heterogeneity, modifies the market composition conditions (see fn. 15), and switches from continuous to discrete time, all of which affect the results significantly.\(^{14}\)

There are large numbers of three risk-neutral types, $B$, $S$ and $M$, for asset buyers, sellers and middlemen. Type $M$ agents stay in the market forever, while type $S$ and $B$ stay for one period (we also studied alternatives, like letting everyone stay forever, with similar results). Upon exit $S$ and $B$ are replaced by “clones” to maintain stationarity (a device borrowed from Burdett and Coles 1997). Type $B$ agents, sometimes called end users, want to acquire an asset – let’s call it capital –

\(^{13}\)Model 1 can be interpreted as bargaining where banks have $\theta = 0$. Hence, one may conjecture that dynamics like those in Appendix C emerge in Model 1 if we allow $\theta \in (0, 1)$, but then Model 1 becomes intractable; the setup in Appendix C is relatively tractable.

\(^{14}\)A key margin in this framework concerns entry by some types, as in many search models going back to Diamond (1982). Somewhat relatedly, Admati and Pfeiderer (1988), Pagano (1989) and Allen and Gale (1994) also show that entry results in endogenous market composition, and that can induce volatility in prices.
to implement a project for profit $\pi > 0$, where $\pi$ is observable when agents meet, but random across end users with CDF given by $F(\pi)$. The originators of capital, type $S$, if they enter the market each bring 1 indivisible unit; those that stay out put their capital to alternative uses, defining their opportunity cost of participation, denoted $\kappa_s > 0$, which for simplicity is the same for all $S$.

Type $M$ agents, who are always in the market, can acquire capital from $S$, but as usual in these models their inventories are restricted to $k \in \{0, 1\}$ (with exceptions, e.g., Lagos and Rocheteau 2009, but they do not study the issues analyzed here). Let $n_t$ be the measure of type $M$ at $t$ with $k = 1$. Capital held by $M$ depreciates by disappearing each period with probability $\delta \geq 0$, but while he holds it $M$ gets a return $\rho > 0$. His crucial choice is then, if he has $k = 1$ and meets $B$, should he trade or keep $k$ for himself? This depends on fundamentals, of course, including the end user’s $\pi$, but as we show below, it can also depend on beliefs.

Market composition is determined as follows: the measures $n_m$ and $n_b$ of types $M$ and $B$ are fixed, but entry by $S$ makes $n_s$ endogenous. Given this, the meeting technology is standard: each period everyone in the market contacts someone with probability $\alpha$, and each contact is a random draw from the participants. In particular, if $N_t$ is the total measure of participants then types $M$ and $S$ both meet type $B$ with probability $\alpha n_b / N_t$, so $M$ has no advantage over $S$ in that regard. When $B$ and $S$ meet they trade for sure since this is $S$’s only chance and cost $\kappa_s$ is sunk. Similarly, when $S$ meets $M$ with $k = 0$ they trade for sure. When $M$ with $k = 1$ meets $B$, however, they may or may not trade.

As regards prices, if type $j$ gives $i$ capital the latter pays $p_{ij}$ (in terms of transferable utility) determined by bargaining. Thus, if $\Sigma_{ij}$ is the total surplus available when $i$ and $j$ meet, as long as $\Sigma_{ij} > 0$ they trade, and type $i$’s surplus is $\theta_{ij} \Sigma_{ij}$, where $\theta_{ij} \in [0, 1]$ is his bargaining power. To flesh this out, let $V_{s,t}$ and $V_{b,t}$ be

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15Entry by $S$ is nice because it lets us compare economies with the same entry conditions with and without middlemen. Still, results for entry by $M$ are given in Appendix D; entry by $B$ is less interesting and hence omitted. These alternatives are all better than Nosal et al. (2017), where agents choose to be either type $M$ or $S$. That is awkward because in cyclic equilibria they switch back and forth over time between being $M$ and $S$. Here no one switches, but participation by a type can vary, as in conventional search theory (e.g., Pissarides 2000).
value functions for types $S$ and $B$; let $V_{k,t}$ be the value function for $M$ when he has $k \in \{0, 1\}$; and let $\Delta_t = V_{1,t} - V_{0,t}$ be $M$’s gain from having inventory. Then

$$\Sigma_{bs,t} = \pi, \Sigma_{ms,t} = (1 - \delta) \beta \Delta_{t+1}, \Sigma_{bm,t} = \pi - (1 - \delta) \beta \Delta_{t+1},$$

where $\beta \in (0, 1)$ is $M$’s discount factor. Note there are no continuation values or threat points for $S$ and $B$, as they are in the market for just one period, but while that simplifies the algebra it is not otherwise important. In any case, bargaining yields

$$p_{bs,t} = \theta_{sb}\pi, p_{ms,t} = \theta_{sm} (1 - \delta) \beta \Delta_{t+1}, p_{bm,t} = \theta_{mb}\pi + \theta_{mb} (1 - \delta) \beta \Delta_{t+1}. \quad (20)$$

When $M$ with $k = 1$ and $B$ with project $\pi$ meet, they trade with probability $\tau_t = \tau (\pi, R_t)$, where

$$\tau (\pi, R) = \begin{cases} 
0 & \text{if } \pi < R \\
[0, 1] & \text{if } \pi = R \\
1 & \text{if } \pi > R
\end{cases} \quad (21)$$

and $R_t \equiv (1 - \delta) \beta \Delta_{t+1}$ is the reservation value of a project making $\Sigma_{mb} = 0$. Hence, the market payoff for $B$ with project $\pi$ is

$$V_{b,t} (\pi) = \frac{\alpha n_{s,t}}{N_t} \theta_{bs}\pi + \frac{\alpha n_t}{N_t} \tau (\pi, R_t) \theta_{bm} [\pi - (1 - \delta) \beta \Delta_{t+1}] . \quad (22)$$

The first term on the RHS is the probability $B$ meets $S$, times his share of the surplus; the second is the probability he meets $M$ with $k = 1$, times the probability they trade, times his share of the surplus; and note prices do not appear since they were eliminated using (20). Similarly, the market payoff for $S$ is

$$V_{s,t} = \frac{\alpha n_{b}}{N_t} \theta_{sb} \int_0^{\infty} \pi dF (\pi) + \frac{\alpha (n_m - n_t)}{N_t} \theta_{sm} (1 - \delta) \beta \Delta_{t+1} . \quad (23)$$

The payoff for $M$ depends on inventory. Using $R_t = (1 - \delta) \beta \Delta_{t+1}$, we have

$$V_{0,t} = \frac{\alpha n_{s,t}}{N_t} \theta_{ms}R_t + \beta V_{0,t+1} \quad (24)$$

$$V_{1,t} = \rho + \frac{\alpha n_{b}}{N_t} \theta_{mb} \int_{R_t}^{\infty} (\pi - R_t) dF (\pi) + (1 - \delta) \beta V_{1,t+1} + \delta \beta V_{0,t+1} . \quad (25)$$

Subtracting and simplifying with integration by parts, we get

$$R_{t-1} = (1 - \delta) \beta \left\{ \rho + R_t + \frac{\alpha n_{b} \theta_{mb}}{N_t} \int_{R_t}^{\infty} [1 - F (\pi)] d\pi - \frac{\alpha n_{s,t} \theta_{ms}}{N_t} R_t \right\} , \quad (26)$$
giving the evolution of $R$ over time. The evolution of inventories held by $M$ is

$$n_{t+1} = n_t (1 - \delta) \left[ 1 - \frac{\alpha n_b}{N_t} \mathbb{E}_{\tau} (\pi, R) \right] + \frac{(n_m - n_t) \alpha n_{s,t}}{N_t} (1 - \delta), \quad (27)$$

where $\mathbb{E}_{\tau} (\pi, R) = \text{prob}(\pi > R)$ is the unconditional probability that $M$ and $B$ trade. The first term on the RHS is current $n$ times the probably a unit of $k$ does not depreciate or get traded; the second is current $n_m - n$ times the probability $M$ acquires $k$ and it does not depreciate.

We can eliminate $N_t$ in (26)-(27) using $S$’s entry condition, $V_{s,t} = \kappa_s$, which reduces to

$$n_{s,t} = \frac{\alpha n_b \theta_{sb} \mathbb{E}_{\pi} + \alpha (n_m - n_t) \theta_{sm} R_t}{\kappa_s} - n_b - n_m. \quad (28)$$

What’s left is a two-dimensional dynamical system that is compactly written as

$$\begin{bmatrix} n_{t+1} \\ R_{t-1} \end{bmatrix} = \begin{bmatrix} f(n_t, R_t) \\ g(n_t, R_t) \end{bmatrix}, \quad (29)$$

Given an initial $n_0$, equilibrium is a nonnegative, bounded path for $(n_t, R_t)$ solving (29).\(^{16}\)

With no intermediaries, $n_m = 0$, the equilibrium is basically static and it is easy to check that it is unique. With $n_m > 0$, first note that the locus of points satisfying $n = f(n, R)$, called the $n$-curve, and the locus satisfying $R = g(n, R)$, called the $R$-curve, both slope up in $(n, R)$ space. Then, to develop some intuition, consider the special case where $\pi = \bar{\pi}$ is constant. As shown in Fig. 7, for this case there are three possible regimes: (i) $R < \bar{\pi}$, so $M$ with $k = 1$ and $B$ trade with probability $\tau = 1$; (ii) $R > \bar{\pi}$, so they trade with probability $\tau = 0$; and (iii) $R = \bar{\pi}$, so they trade with probability $\tau \in (0, 1)$. Appendix A proves:

**Proposition 3** With $\pi = \bar{\pi}$ there exist $\bar{\rho} > 0$ and $\bar{\hat{\rho}} > \bar{\rho}$ such that: (i) if $\rho \in [0, \bar{\rho})$ there is a unique steady state and it has $R < \bar{\pi}$; (ii) if $\rho \in (\bar{\rho}, \infty)$ there is a unique

\(^{16}\)A distinction between this model and others in the paper is that this system is two dimensional: $R$ is a jump variable, like $\phi$ in the previous sections, while $n$ is a (predetermined) state variable, so transitions are nontrivial. The version of Model 3 in Appendix D, with entry by $M$ instead of $S$, is different: there one can solve a univariate system $R_{t-1} = G(R_t)$ to get the path for $R_t$, after which $n_t, N_t$ etc. follow from simple conditions. Intuitively, with entry by $M$ (entry by $S$) the model is (is not) block recursive, as discussed in Shi (2009). Hence, Appendix D delivers more results, including chaotic dynamics, but we still prefer entry by $S$ as a benchmark model.
steady state and it has $R > \bar{\pi}$; (iii) if $\rho \in (\hat{\rho}, \breve{\rho})$ there are three steady states, $R < \bar{\pi}$, $R > \bar{\pi}$, and $R = \bar{\pi}$.

Figure 7: Model 3, phase plane

For several reasons we prefer a nondegenerate $F(\pi)$.\textsuperscript{17} So, consider a smooth mean-preserving spread of the degenerate case, shown in Fig. 8.

**Example 6:** Let

$$F(\pi) = \begin{cases} 
\pi_1 \pi / \pi_0 & \text{if } 0 \leq \pi \leq \pi_0 \\
\pi_1 + (\pi_3 - \pi_1)(\pi - \pi_0) / (\pi_2 - \pi_0) & \text{if } \pi_0 < \pi \leq \pi_2 \\
\pi_2 + (1 - \pi_3)(\pi - \pi_2) / (\pi_4 - \pi_2) & \text{if } \pi_2 < \pi \leq \pi_4 
\end{cases}$$

with $\pi_0 = 0.99$, $\pi_1 = 0.05$, $\pi_2 = 1.01$, $\pi_3 = 0.95$ and $\pi_4 = 2$. Also, let $\alpha = 1$, $\kappa_s = 0.1$, $n_b = 0.05$, $n_m = 0.5$, $\theta_{sm} = 0.5$, $\theta_{sb} = 1$, $\theta_{mb} = 0.7$, $\beta = 1/1.04$, $\delta = 0.008$ and $\rho = 0.2$.

\textsuperscript{17}For the nondegenerate $F(\pi)$ studied below, the flat portion of the $n$-curve in Fig. 7 is eliminated. Then in any steady state $M$ and $B$ are indifferent to trade only in the rare event $\pi = R$, in contrast to the mixed-strategy equilibrium in the degenerate case, where they are always indifferent. Moreover, with nondegenerate $F(\pi)$, if $R$ varies across pure-strategy steady states intermediation activity does too, but not necessarily to the extreme extent of the degenerate case, where it is either $\tau = 1$ or $\tau = 0$. Similarly, for real-time dynamics, cycles with nondegenerate $F(\pi)$ have fluctuations in intermediation activity but not necessarily between $\tau = 0$ and $\tau = 1$. 

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Fig. 8 is similar to Fig. 7, except the slope of the $n$-curve is always positive. Hence the results are similar to Proposition 3, including multiplicity.

![Phase Plane Diagram]

Figure 8: Model 3, phase plane with two-cycle

Here is the intuition. First suppose $R$ is low, so $M$ trades $k$ to $B$ with a high probability. Then the probability $M$ has $k = 0$ is high, which is good for sellers, so $n_s$ is high. That makes it easy for $M$ to get $k$ and rationalizes a low $R$. Now suppose $R$ is high, so $M$ trades $k$ to $B$ with a low probability. Then the probability $M$ has $k = 0$ is low, which is bad for sellers, so $n_s$ is low. That makes it hard for $M$ to get $k$ and rationalizes a high $R$. Moreover, market liquidity – i.e., the ease with which agents can buy and sell $k$ – is high (low) if $R$ is low (high). Multiplicity means liquidity is not pinned down by fundamentals, a recurring theme in monetary theory (e.g., Kiyotaki and Wright 1989), but the intuition here is different. In monetary economies, whether an agent accepts something as medium of exchange depends on what others accept. Here, whether type $M$ agents trade away $k$ depends on $n_s$, and $n_s$ depends on whether type $M$ trade away $k$, which is a different idea. In particular, our result requires endogenous market composition, something that is not true in most monetary models.

Now consider a two-cycle oscillating between a liquid regime with low $R$ and an
illiquid regime with high $R$, or $(R^L, n^L)$ and $(n', R')$ solving

$$\begin{bmatrix} n' \\ R' \end{bmatrix} = \begin{bmatrix} f(n^L, R^L) \\ g(n^L, R^L) \end{bmatrix}$$ \quad and \quad $$\begin{bmatrix} n^L \\ R^L \end{bmatrix} = \begin{bmatrix} f(n', R') \\ g(n', R') \end{bmatrix}.$$ \quad (31)

One solution is $(R^L, n^L) = (0.9862, 0.4504)$ and $(R^H, n^H) = (1.0103, 0.4312)$. Hence, we have real-time dynamics (not just multiple steady states) with excess volatility in liquidity, trade volume, prices and quantities. Fig. 9 shows the times series. In the liquid regime: $R$ is low, making $M$ more inclined to trade with $B$; $n$ is high, because $M$ and $B$ traded less last period; and $n_s$ is low, because low $R$ and high $n$ discourage entry by $S$. The illiquid regime has the opposite properties.

![Graphs showing time series for Model 3](image)

Figure 9: Model 3, time series for a two-cycle

We do not claim that actual data are best explained by a two-cycle, but suggest if such a simple model can deliver equilibria where endogenous variables vary over time, as self-fulfilling prophecies, it lends credence to the notion that intermediated asset markets in the real world might be prone to similar instability.\footnote{Prices are also shown in Fig. 9 (averaged over $\pi$ when $B$ trades). The price $B$ pays $S$ is constant over time, as it depends only on fundamentals, but the price $M$ pays $S$ or $B$ pays $M$ moves with $R$. The spread can go either way, but here it moves against $R$. This is all broadly consistent with the data discussed in Comerton-Forde et al. (2010), and other stylized facts (e.g., inventories are volatile than output). While this is obviously not a calibration, the finding that it is qualitatively more consistent with observations may lend further credence to the story.} A final point
about this setup is that, as in some other models of middlemen, welfare can be higher or lower with intermediation than without it. The reason is that while $M$ perform a real service by getting assets from $S$ to $B$, their activity depends on bargaining power – they like to buy low and sell high – and hence they may operate even if they are not socially efficient, say because $\rho$ is very low.

Model 4: Safety and Secrecy

An important role of banks is that their liabilities facilitate third-party transactions. Indeed, some say that is their defining characteristic: “banks are distinguished from other kinds of financial intermediaries by the readily transferable or ‘spendable’ nature of their IOUs, which allows those IOUs to serve as a means of exchange, that is, money... Commercial bank money today consists mainly of deposit balances that can be transferred either by means of paper orders known as checks or electronically using plastic ‘debit’ cards” (Selgin 2018). We pursue this in a model with an explicit need for payment instruments, building on the framework surveyed by Lagos et al. (2017) and Rocheteau and Nosal (2017), with banks added in two ways.

In Model 4a, bank liabilities are safe relative to other assets in the sense that they are less susceptible to theft or loss, as in He et al. (2007) or Sanches and Williamson (2010). Traveler’s checks are an example, but more generally, it is obviously worse to have your cash lost or stolen than your checkbook or debit card. Also, if merchandise turns out to be fraudulent or defective (a form of theft) it is easier to stop payment if you use a check or credit card than if you use cash.\footnote{Safety was a critical feature of banks historically. Consider the British goldsmiths: “At first [they] accepted deposits merely for safe keeping; but early in the 17th century their deposit receipts were circulating in place of money” (Encyclopedia Britannica, quoted in He et al. 2005; emphasis added). Also, “In the 17th century, notes, orders, and bills (collectively called demandable debt) acted as media of exchange that spared the costs of moving, protecting and assaying specie” (Quinn 1997; emphasis added). Safety was also crucial for earlier bankers, including the Templars (Sanello 2003), who specialized in moving purchasing power over dangerous territory.}

Model 4b builds on the alternative idea that payment instruments originating with banks can be, as Dang et al. (2017) put it, \textit{informationally insensitive} when these institutions act as secret keepers. See also Gorton and Pennachi (1990), Andolfatto and Martin (2013), Andolfatto et al. (2014) and Monnet and Quintin (2017).
While there are different approaches to modeling media of exchange, one based on Lagos and Wright (2005) is convenient for both Models 4a and 4b. In that setup, in each period of discrete time two markets convene sequentially: a decentralized market, or DM, with frictions detailed below; and a frictionless centralized market, or CM. There are two types of infinitely-lived agents, a measure 1 of buyers and a measure n of sellers. Their roles differ in the DM, but they are similar in the CM, where they all trade a numeraire consumption good x and labor ℓ for utility $U(x) - \ell$, with $U'>0>U''$. They also trade assets in the CM, like the trees in the standard Lucas (1978) model, giving off a dividend $\rho > 0$ in the CM in numeraire. All agents discount by $\beta \in (0, 1)$ between one CM and the next DM, but wlog they do not discount between the DM and CM.

In the DM agents meet bilaterally, where sellers can provide a good $q$ (different from $x$) that buyers want. Let $\alpha$ be the probability a buyer meets a seller, so that $\alpha/n$ is the probability a seller meets a buyer. In any meeting, if a seller produces for a buyer the former incurs cost $c(q)$ and the latter gets utility $u(q)$, where $c(0) = u(0) = 0, c', u' > 0$ and $c'' \geq 0 > u''$. Also, let $q^*$ satisfy $u'(q^*) = c'(q^*)$. Goods $q$ and $x$ are nonstorable, so they cannot serve as commodity money. Credit is not viable because there is limited commitment and DM trading is anonymous. Hence, as is standard in these models, sellers only produce if they get assets in exchange.

Let the terms of trade be given by a generic mechanism, as in Gu and Wright (2016), meaning this: for a buyer to get $q$, he must give the seller assets worth $v(q)$ in CM numeraire, for some function with $v(0) = 0$ and $v'(q) > 0$. A simple example is Kalai’s (1977) proportional bargaining solution, $v(q) = \theta c(q) + (1 - \theta) u(q)$. Generalized Nash is similar but the formula for $v(q)$ is more complicated when liquidity constraints are operative. For a fairly general class of mechanisms, Gu and Wright (2016) show this: if a buyer has enough assets to make his liquidity constraint slack, he gets the efficient $q = q^*$ and pays $p^* = v(q^*)$; but if he has assets worth $p < p^*$, he gives them all to the seller and gets $q = v^{-1}(p) < q^*$.

In Model 4a, assets can be held in forms that differ in safety and liquidity,
where safety is captured by the probability of being stolen (or lost), and liquidity is captured by whether it can be used as means of payment in the DM. To maintain stationarity, any assets that are stolen (or lost) return to the system next period, say because thieves (or finders) bring them to the CM. Let \( a = (a_1, a_2) \) be a buyer’s portfolio: \( a_1 \) denotes assets held in a safe but illiquid form, say hidden in one’s basement, meaning that it cannot be stolen (or lost) but also cannot be used in the DM; and \( a_2 \) denotes assets held in a liquid form, which means they are brought to the DM, where can be used as payment instruments, but there is a probability \( \delta > 0 \) of being stolen (or lost).

The *ex dividend* price of the asset in terms of numeraire is \( \psi \) independent of whether it is held as \( a_1 \) or \( a_2 \). A buyer’s CM value function is \( W(A) \) where \( A = (\psi + \rho)^j a_j \) is wealth. His DM value function is \( V(a) \), which depends on his portfolio and not just its value. A buyer’s CM problem is then

\[
W_t(A_t) = \max_{x_t, \ell_t, \hat{a}_t} \{ U(x_t) - \ell_t + \beta V_t(\hat{a}_t) \} \text{ st } x_t = A_t + \ell_t - \psi_t \Sigma_j \hat{a}_{j,t}
\]

where \( \hat{a} = (\hat{a}_1, \hat{a}_2) \) is his updated portfolio, and the CM real wage is 1 assuming that \( x \) is produced one-for-one with \( \ell \). Given an interior solution, several standard results are immediate: (i) \( x_t = x^* \) solves the FOC \( U'(x^*) = 1 \); (ii) \( \hat{a}_t \) solves the FOC’s \( \beta \partial V_{t+1}/\partial \hat{a}_{j,t} \leq \psi_{t+1} \) if \( \hat{a}_{j,t} > 0 \), which is independent of \( a_t \), so all buyers exit the CM with the same portfolio; and (iii) \( W'_t(A_t) = 1 \), so \( W_t(A_t) \) is linear in wealth. A seller’s CM problem (not shown) is similar, with a CM payoff again linear in wealth.

A buyer’s value function is

\[
V_{t+1}(\hat{a}_t) = (1 - \delta) \left\{ \alpha [u(q_{t+1}) - v(q_{t+1})] + W_{t+1}(\hat{A}_{t+1}) \right\} + \delta W_{t+1}[\psi_{t+1} + \rho] \hat{a}_{1,t}
\]

where \( \hat{A}_{t+1} \) is the wealth implied by \( \hat{a}_t \), with \( q_{t+1} \) solving \( v(q_{t+1}) = (\psi_{t+1} + \rho) \hat{a}_2 \) if \( (\psi_{t+1} + \rho) \hat{a}_2 < v(q^*) \), and \( v(q_{t+1}) = v(q^*) \) otherwise. The buyer’s surplus in a DM transaction is \( u(q) - v(q) \), because of the result that \( W(\cdot) \) is linear. Equilibrium is described by the Euler equations, which come from inserting the derivatives of \( V \)
into the FOC’s from the CM:

\[ 0 = \hat{a}_{1,t} [\beta (\psi_{t+1} + \rho) - \psi_t] \quad (32) \]
\[ 0 = \hat{a}_{2,t} \{ \beta (\psi_{t+1} + \rho) (1 - \delta) [1 + \alpha \lambda (q_{t+1})] - \psi_t \}, \quad (33) \]

where \( \lambda (q) = u'(q)/v'(q) - 1 > 0 \) is the liquidity premium on assets in the DM.

If we normalize the aggregate asset supply to 1, the dynamical system implied by the model is described as follows. At any \( t \), there are three possible regimes: (i) \( \hat{a}_{2,t} = 0 \); (ii) \( 0 < \hat{a}_{2,t} < 1 \); and (iii) \( \hat{a}_{2,t} = 1 \). In regime (i), inserting \( \hat{a}_{1,t} = 1 \) and \( \hat{a}_{2,t} = 0 \) into (32) and (33), we get \( \psi_t = \beta (\psi_{t+1} + \rho) \) and \( (1 - \delta) [1 + \alpha \lambda (0)] \leq 1 \), with the latter equivalent to

\[ \delta \geq \hat{\delta} \equiv \frac{\alpha \lambda (0)}{1 + \alpha \lambda (0)}. \quad (34) \]

Thus, agents bring no assets to the DM if the probability of theft is high. If (34) holds, the DM shuts down and the only possible equilibrium has \( \psi_t = \psi^F \forall t \), where \( \psi^F \equiv \beta \rho / (1 - \beta) \) might be called the fundamental price of the asset.\(^{20}\)

Now assume \( \delta < \hat{\delta} \), and consider regime (ii), where agents hold some but not all their assets in liquid form. Inserting \( \hat{a}_{1,t} \hat{a}_{2,t} > 0 \) into (32) and (33), we get \( \psi_t = \beta (\psi_{t+1} + \rho) \) and \( (1 - \delta) [1 + \alpha \lambda (q_{t+1})] = 1 \), which means \( q_{t+1} = \bar{q} \) where

\[ \alpha \lambda (\bar{q}) = \frac{\delta}{1 - \delta}. \quad (35) \]

One can show regime (ii) obtains iff \( \psi_{t+1} + \rho > \hat{a}_{2,t} (\psi_{t+1} + \rho) = v (\bar{q}) \) and \( \delta < \hat{\delta} \).

Finally, consider regime (iii). Inserting \( \hat{a}_{1,t} = 0 \) and \( \hat{a}_{2,t} = 1 \) into (32) and (33), we get \( \psi_t \geq \beta (\psi_{t+1} + \rho) \) and

\[ \psi_t = \beta (\psi_{t+1} + \rho) (1 - \delta) [1 + \alpha \lambda (q_{t+1})], \quad (36) \]

where \( q_{t+1} = v^{-1} (\psi_{t+1} + \rho) < \bar{q} \). This last condition is equivalent to \( \psi_{t+1} \leq \tilde{\psi} \equiv v (\bar{q}) - \rho \). Hence if \( \delta < \hat{\delta} \) the dynamic system is \( \psi_t = f (\psi_{t+1}) \) where:

\[ f (\psi) \equiv \begin{cases} 
\beta (\psi + \rho) (1 - \delta) [1 + \alpha \lambda \circ v^{-1} (\psi + \rho)] & \text{if } \psi < \tilde{\psi} \\
\beta (\psi + \rho) & \text{if } \psi \geq \tilde{\psi}
\end{cases} \quad (37) \]

\(^{20}\)One might argue that \( (1 - \delta) \beta (\psi + \rho) \), and not \( \beta (\psi + \rho) \), is the fundamental price, since an asset holder only gets the return when it is not stolen. A rebuttal is that someone always gets the payoff, even if it is the thief. To avoid this issue simply interpret \( \psi^F \) as notation for \( \beta \rho / (1 - \beta) \).
Equilibrium is a nonnegative and bounded path for $\psi_t = f(\psi_{t+1})$.

**Proposition 4** Steady state exists, is unique, and is described as follows. Define $\delta \in [0, \tilde{\delta})$ by

$$
\tilde{\delta} = \frac{\alpha \lambda \circ v^{-1}(\psi^F + \rho)}{1 + \alpha \lambda \circ v^{-1}(\psi^F + \rho)}.
$$

Then (i) $\delta \geq \tilde{\delta}$ implies $\hat{a}_1 = 1$, $\hat{a}_2 = 0$ and $\tilde{\psi} = \psi^F$; (ii) $\delta \in (\tilde{\delta}, \hat{\delta})$ implies $\hat{a}_1 > 0$, $\hat{a}_2 > 0$ and $\tilde{\psi} = \psi^F$; and (iii) $\delta \leq \hat{\delta}$ implies $\hat{a}_1 = 0$, $\hat{a}_2 = 1$ and $\tilde{\psi} > \psi^F$.

Fig. 10 shows how steady state depends on $\delta$. In regime (i) the DM is inactive and $\tilde{\psi} = \psi^F$, because assets are not safe enough to use as payment instruments. In regime (ii) the DM is active at $q = \tilde{q} > 0$, but since some wealth is parked in illiquid $\hat{a}_1$, we again have $\tilde{\psi} = \psi^F$, with $\partial q / \partial \delta < 0$. Thus DM output goes down with $\delta$ because it reduces output per trade $\tilde{q}$, as well as the number of trades $(1 - \delta)\alpha$. In regime (iii), which it maximizes “cash in the market” with $\hat{a}_2 = 1$, we get $\tilde{\psi} > \psi^F$. Here $\partial q / \partial \delta < 0$ not because $\hat{a}_2$ falls, but because $\psi$ falls, with $\delta$.

A steady state can either be on the linear or the nonlinear branch of $f(\cdot)$. In the former case, $\tilde{\psi} = \psi^F > \psi$ and steady state is the only equilibrium (any other path for $\psi_t$ is unbounded). In the latter case, $\psi^F < \tilde{\psi} < \psi$, and we potentially have cyclic, chaotic or stochastic equilibria where $\psi_t$ oscillates around $\tilde{\psi}$. As usual, this occurs
when $f' (\psi) < -1$. So the economy may have asset prices above their fundamental value, and these prices may exhibit excess volatility, even without banks, as is true in many monetary models.

Now consider banks that take assets as deposits and issue receipts, claims on these deposits. Let $a_3$ denote assets on deposit and assume they are safer than $a_2$ but still liquid – i.e., they are spendable. However, deposits may entail a lower yield than the original assets, since banks may have operating costs and depositors may be willing to sacrifice return for safety. If $\iota$ is the interest rate on deposits then bank profit is

$$\Pi (a_3) = a_3 (\rho - \iota) - k (a_3),$$

where $k (a_3)$ is the cost of managing deposits, with $k', k'' \geq 0 = k(0)$. Maximization of $\Pi$ equates the spread $\rho - \iota$ to marginal cost $k' (a_3)$.

Now $\hat{a}_j > 0 \ \forall j$ is possible because there are three asset characteristics – namely, liquidity, safety and rate of return. However, to begin, suppose deposits are perfectly liquid and safe, and set $k (a_3) = 0$ so that $\iota = \rho$ (we return to the general case

---

21One can make deposits less than perfectly liquid – i.e., some sellers do not accept them or accept them only up to a limit – using private information (Lester et al. 2012; Li et al. 2012).
below). Then $a_3$ strictly dominates $a_2$, and the economy looks like one without banking where $\delta = 0$. This is shown in Fig. 11, where $f_1$ and $f_0$ are the dynamical systems with and without banking. Adding banks shifts up in the nonlinear branch of $f(\cdot)$, which increases $\bar{\psi}$ and $\tilde{\psi}$, since agents can now keep assets in a safe place and still use them for DM transactions.

Banking increases DM output, and hence welfare, because it increases the size and number of trades. What does it do for volatility? Starting without banks, suppose steady state is on the linear branch of $f(\cdot)$, so there is a unique equilibrium $\psi_t = \psi^F \forall t$. Then adding banks can shift $f(\cdot)$ up by enough that the new steady state is on the nonlinear branch. Thus, banking can make possible cyclic, chaotic and stochastic equilibria that were impossible without it. For some parameters such outcomes are also possible without banking, but if the economy has a unique equilibrium with banking the same is true without banking.

Fig. 11 is drawn for the following specification:

**Example 7:** Let $k(a_3) = 0$, $c(q) = q$,

$$u(q) = \frac{A}{1-\sigma}[(q+\varepsilon)^{1-\sigma} - \varepsilon^{1-\sigma}],$$

and use bargaining with $\theta = 1$. Also set $A = 0.15$, $\sigma = 3.1$, $\varepsilon = 0.16$, $\rho = 0.033$, $\beta = 0.8333$, $\delta = 0.85$ and $\alpha = 1$.

Without banks there is a unique equilibrium, the steady state $\bar{\psi} = \psi^F = 0.1650$. With banks there is a steady state $\bar{\psi} = 0.3183 > \psi^F$ plus a two-cycle where $\psi_L = 0.3193$ and $\psi_H = 0.3502$.

While this example makes our main point, it is worth asking what else the model can do. We now show it generates something realistic but uncommon in economic theory: the concurrent circulation of assets and bank liabilities as payment instruments. So that $\hat{a}_3$ does not strictly dominate $\hat{a}_2$, consider a more general cost function $k(a_3)$. Then bank’s FOC defines a supply curve that is increasing in $\iota$, which is endogenous in equilibrium but taken as given by individuals. Equilibrium is characterized by (32)-(33) with

$$v(q_{t+1}) = (\psi_{t+1} + \rho) \hat{a}_2 + [\psi_{t+1} + \iota (\hat{a}_3)] \hat{a}_3,$$
since DM purchases now use $a_2$ and $a_3$. The demand for $a_3$ satisfies

$$0 = \hat{a}_{3,t} \left\{ \beta \left( \psi_{t+1} + \iota_t \right) \left[ 1 + \delta \alpha \lambda \left( q'_{t+1} \right) + (1 - \delta) \alpha \lambda \left( q_{t+1} \right) \right] - \psi_t \right\}$$  \hspace{1cm} (40)

where $v \left( q'_{t+1} \right) = \left[ \psi_{t+1} + \iota \left( \hat{a}_3 \right) \right] \hat{a}_3$ and $q'_{t+1}$ is the DM purchase when $a_2$ is stolen.

Consider the following example:

**Example 8:** Let $A = 2.5$, $\sigma = 2.5$, $\varepsilon = 0.001$, $\rho = 0.04$, $\beta = 0.8$, $\delta = 0.01$, $\alpha = 1$, and $k \left( a_3 \right) = 0.03a_3$.

There is a unique steady state in which $\bar{\psi} = 1.3125$, $\bar{\hat{a}} = (0, 0, 1)$ and $\iota = 0.01$. There is also a two-cycle with $\psi_L = 1.2128$, $\hat{a}_L = (0, 0, 1)$, $\psi_H = 1.4760$ and $\hat{a}_H = (0.0384, 0.2293, 0.7323)$. In the $L$ state, $\psi$ is low, all assets are deposited, and only bank liabilities are used in the DM; in the $H$ state, $\psi$ is high, assets are held in all three forms, with both $a_2$ and $a_3$ used in the DM. Fig. 12 shows the price $\psi$, deposits $a_3$, their value $(\psi + \iota) a_3$, and the surplus $u \left( q \right) - c \left( q \right)$ over the cycle.

![Figure 12: Model 4a, time series for a two-cycle](image)

Now consider Model 4b, based on secrecy rather than safety. First, following Hu and Rocheteau (2015) or Lagos and Zhang (2019), assume Lucas trees die (disappear) with probability $\delta$ at the beginning of each CM.\(^{22}\) To maintain stationarity,
dead trees are replaced by new ones, distributed across agents as lump sum transfers from nature. Also, this is an aggregate shock (all or no assets survive each period). Moreover, information about the shock in the next CM is revealed in the current DM, before agents trade, which is a hindrance to having assets serve as media of exchange. This specification is extreme, in that the asset value drops to 0 after a shock; all we really need, however, is that it goes down.

The CM problem is

$$W_t(a_t) = \max_{x_t, \ell_t, \hat{a}_t} \{U(x_t) - \ell_t + \beta V_{t+1}(\hat{a}_t)\} \text{ st } x_t = (\psi_t + \rho)a_t + \ell_t - \psi_t\hat{a}_t + T$$

where $T$ denotes transfers. Here the asset is the only DM means of payment, and it is only usable when it is revealed that it will survive to the next CM. Hence,

$$V_{t+1}(\hat{a}_t) = (1 - \delta) \{\alpha [u(q_{t+1}) - v(q_{t+1})] + W_{t+1}(\hat{a}_t)\} + \delta W_{t+1}(0)$$

where, as in Model 4a, $v(q_{t+1}) = (\psi_{t+1} + \rho) \hat{a}_t$ if $(\psi_{t+1} + \rho) \hat{a}_t < v(q^*)$ and $v(q_{t+1}) = v(q^*)$ otherwise. Again, we get $\psi_t = f_0(\psi_{t+1})$, where the subscript 0 indicates there are no banks for now, and

$$f_0(\psi) = \beta (1 - \delta) (\psi + \rho) \left[1 + \alpha \lambda \circ v^{-1}(\psi + \rho)\right]. \quad (41)$$

Now introduce banks that take assets on deposit and issue receipts. These deposits are not insured – they are claims to the asset, and if the asset dies the claim is worthless. By design, the role of banks in this formulation is not to provide insurance, but to capture secrecy as follows: while an agent holding an asset can see if it will die in the next CM, once he deposits it in a bank he cannot, and although the banker holding the asset can see it, he may or may not inform people. This is the idea in the literature, discussed above, where some assets are more informationally insensitive than others and banks’ role as secret keepers. Agents like to use bank liabilities as DM payment instruments, rather than the original assets, since the former trade at their expected value rather than their realized value. This bank money provides a steadier stream of liquidity.
With banks, the DM value function is

\[
V_{t+1}(\hat{a}_t) = \alpha [u(q_{t+1}) - v(q_{t+1})] + (1 - \delta)W_{t+1}(\hat{a}_t) + \delta W_{t+1}(0)
\]

This leads to \( \psi_t = f_1(\psi_{t+1}) \), where

\[
f_1(\psi) = \beta (1 - \delta) (\psi + \rho) \left\{ 1 + \alpha \lambda \circ v^{-1}[(1 - \delta) (\psi + \rho)] \right\}.
\]

(42)

As \( \lambda(\cdot) \) is decreasing, \( f_1 \) lies above \( f_0 \) on the nonlinear branch, and hence \( f_1 \) reaches a higher steady state. It can be shown that the liquidity provided by deposits in steady state is lower than that provided by the asset when the asset does not die, but of course is higher when it dies. On net, banking can improve welfare, but it can also engender instability.

This is shown in Fig.13 for the following parameterization:

**Example 9** Same as Example 7 except \( A = 0.5, \sigma = 3.5, \varepsilon = 0.15, \rho = 0.5, \beta = 0.9 \) and \( \delta = 0.5 \).

Without banking, the unique equilibrium is a steady state where \( \psi = 0.4091 \), and \( q = q^* = 0.6703 \) if the asset survives while \( q = 0 \) otherwise. With banking, there is a steady state where \( \psi = 0.7187 \) and \( q = 0.6093 \), and welfare is higher, but there is also a two-cycle where \( \psi_L = 0.6081 \) and \( \psi_H = 0.8514 \). The time series (not shown) in this case is simple since all variables move with \( \psi \). Thus, banking eliminates fundamental cycles induced by information about realized asset values, but introduces volatility as a self-fulfilling prophecy.
Figure 13: Model 4b, dynamic equilibrium

The first part of Proposition 5 below says banks may engender volatility. The second part says they cannot eliminate volatility due to self-fulfilling prophecies, since if there is a unique equilibrium $\psi_t = \psi^F \forall t$ with banking, there is also a unique equilibrium with $\psi_t = \psi^F \forall t$ without it.

**Proposition 5** When the steady state $\bar{\psi} = \psi^F$ is the unique equilibrium without banking, introducing banks can introduce nonstationary equilibria. When the steady state $\bar{\psi} = \psi^F$ is the unique equilibrium with banking, steady state is the unique equilibrium without banking.

Models 4a and 4b have similar results and intuition. In both, due to liquidity considerations, $\psi_t = f(\psi_{t+1})$ has two terms: one reflects a store-of-value component making price today increasing in the price tomorrow; the other reflects a medium-of-exchange component making price today generally nonmonotone in the price tomorrow. If the second term is decreasing and dominates the first, $f' (\bar{\psi}) < -1$ and hence endogenous dynamics are possible. In Model 4a, without banks we have $f_0 (\psi)$, and with banks we have $f_1 (\psi) = f_0 (\psi) / (1 - \delta)$ on the nonlinear branch. This is why we can get $f'_0 (\psi) > -1$ without banks and $f'_1 (\psi) < -1$ with banks.
at a given $\psi$. In addition, steady state moves from $\tilde{\psi}_0$ without banking to $\tilde{\psi}_1$ with banking, which can also make $f_1'(\tilde{\psi}) < -1$ more likely.

In Model 4a banks make the asset better as a store of value and as a medium of exchange by reducing the risk of theft. In Model 4b banks do not make the asset a better store of value, because there is no way to avoid the loss if the tree dies, but they make it a better medium of exchange by keeping information secret. Hence, in Model 4b agents unambiguously put more weight on the nonmonotone medium-of-exchange component, making it more likely that $f_1'(\psi) < -1$. Details aside, these results show again how banking can engender instability, although again agents are better off with banking than without, it at least near steady states.\(^{23}\)

**Conclusion**

This paper demonstrates that the varied activities of financial intermediaries make multiplicity/volatility more likely in a precise sense. The result is true in Models 1 and 2, both involving trust but differing in the reason for banking (insurance vs fixed costs); in Model 3, capturing not banks but dealers in OTC markets; and in Model 4, featuring the use of bank liabilities as inside money.\(^{24}\) The different specifications therefore all lend support to the notion that financial intermediation engenders instability, although again we emphasize that this does not make it bad: intermediaries improve welfare in Models 1 and 2 for sure; in Model 3 at least for some parameters; and in Model 4 at least near steady states.\(^{25}\) The presentation involved many examples, naturally, since the claim is that financial intermediation

\(^{23}\)This is not necessarily true in general – e.g., it is possible that a two-cycle equilibrium with banking is worse than the outcome with no banking if we start the cycle in the low $\psi$ state.

\(^{24}\)The frictions considered in our paper are not the only ones that can generate multiple equilibria and interesting dynamics in the financial market. Other frictions and mechanisms, including adverse selection (Benhabib et al 2018, Myerson 2012), information frictions (Benhabib et al. 2015), a fixed loan collection cost (Benhabib and Wang 2013), etc. can give rise to instability in the economy.

\(^{25}\)Rajan (2005) argues that volatility, which he takes to be self-evidently bad, has emerged from recent financial innovation. This is consistent with our theoretical findings, but we tend to agree with Summers’ comment: Financial innovations are like improvements in transportation technology, which have an overwhelmingly positive impact on welfare even if they increase the possibility of, say, plane crashes. Clearly, financial markets, like the airline industry, may need some regulation, but too much can be counterproductive (e.g., Lacker 2015; Weinberg 2015).
may generate instability, not that it must.\textsuperscript{26} Also, the claim is not that the data are best explained by cycles or sunspots, but that when rudimentary models can have equilibria where liquidity, prices, quantities and welfare vary as self-fulfilling prophecies, it seems more likely that actual economies can, too.

We close by reiterating how the different setups are related. First, we believe theories of financial intermediaries should use environments with explicit frictions that give rise to an endogenous role for these institutions, as they can give rise to monetary exchange and other arrangements meant to ameliorate the frictions. At the same time, frictions mean there can be multiple Pareto-ranked equilibria and belief-based dynamics. Recall the Pholk Theorem, that all models where equilibria may be inefficient can display multiplicity/volatility. Again, corroboration of this idea consists of showing that it works in a series of environments. Our formulations used different frictions capturing different aspects of financial intermediaries, and we tried to cover all of the standard ways of studying them. While there may be still other ways to model these institutions, we hope the above results are enough to make a case for this: endogenizing financial intermediaries using explicit frictions, in various ways, leads to the conclusion that they can be unstable.

\textsuperscript{26}Also, the examples are not calibrated because we are interested here mainly in logical possibilities, but future work could see if instability arises for more realistic specifications and parameters. Other future work could put the various models, all of which are somewhat novel, to work in other applications – e.g., designing regulations to improve bank performance – but the goal here was to focus exclusively on stability.
Appendix A: Proofs of Nonobvious Results

**Proposition 1:** If $\hat{\phi} \leq 0$ then (12) reduces to $\phi_{t-1} = \beta \phi_t$ and the only equilibrium is the steady state with $\tilde{\phi} = 0$. If $\hat{\phi} > 0$ then $f(0) > 0$ and $f(\hat{\phi}) = \beta \hat{\phi} < \hat{\phi}$ implies $\tilde{\phi} \in (0, \hat{\phi})$ exists. To see it is unique, first solve (12) for $\phi = \beta \mu v (bR) / (1 - \beta)$ and substitute it into (10) to get $\lambda d = (1 - \beta + \beta \mu) v (bR) / (1 - \beta)$. This implies $d$ is increasing in $b$. But (11) implies $d$ is decreasing in $b$, so if they have a solution $(\bar{b}, \bar{d})$ it is unique.

**Proposition 3:** First, for uniqueness, note that when $\tau = \pi$ the equations for the $R$-curve and $n$-curve are defined by

$$
\left( \frac{r + \delta}{1 - \delta} + \alpha \theta_{ms} \right) R - \rho - \frac{\alpha n_b \theta_{mb} \tau (\tilde{\pi} - R) + \alpha (n_b + n_m) \theta_{ms} R}{N} = 0 \tag{43}
$$

$$
\delta n + n (1 - \delta) \frac{\alpha n_m \tau}{N} - (n_m - n) (1 - \delta) \alpha \left( 1 - \frac{n_b + n_m}{N} \right) = 0 \tag{44}
$$

where

$$
N = \frac{\alpha n_b \theta_{sb} \bar{\pi} + \alpha (n_m - n) \theta_{sm} R}{\kappa_s}.
$$

In the region where $R > \tilde{\pi}$, where $\tau = 0$ combine (43) and (44) to eliminate $N$,

$$
\left( \frac{r + \delta}{1 - \delta} + \frac{\theta_{ms} \delta n}{n_m - n} \right) R = \rho (1 - \delta) \tag{45}
$$

This implies

$$
\frac{\partial R}{\partial n} = -\frac{\theta_{ms} \delta n}{(n_m - n)^2 (r + \delta) + (n_m - n) \theta_{ms} \delta n} < 0.
$$

Thus we transform the system (43)-(44) to (45)-(44). As (45) is downward sloping and (44) is upward sloping, there exists at most one steady state with $R > \tilde{\pi}$.

In the region where $R < \tilde{\pi}$, where $\tau = 1$, combine (43) and (44) to get

$$
\left( \frac{r + \delta}{1 - \delta} + \alpha \theta_{ms} \right) R = \rho + \frac{n_b \theta_{mb} (\tilde{\pi} - R) + (n_b + n_m) \theta_{ms} R}{(1 - \delta) n_m (n_b + n_m - n)} \left[ (n_m - n) (1 - \delta) \alpha - n \delta \right].
$$

This implies

$$
\frac{\partial R}{\partial n} = -\frac{n_b \theta_{mb} (\tilde{\pi} - R) + (n_b + n_m) \theta_{ms} R}{r + \delta + (\theta_{ms} + n_b \theta_{mb}) (1 - \delta) \alpha / N} \frac{\delta (n_b + n_m) + (1 - \delta) \alpha n_b}{n_m (n_b + n_m - n)^2} < 0 \tag{46}
$$

Again, since (46) is downward and (44) upward sloping, there is at most one steady state with $R < \pi$. Similarly, when $R = \tilde{\pi}$ and $\tau \in (0, 1)$, the $n$-curve is flat and $R$-curve is upward sloping. Hence, there again is at most one steady state.
For existence, first, it is easily verified that the $R$- and $n$-curve are upward sloping. At $n = 0$ the $R$-curve implies $R > 0$ and the $n$-curve implies $n > 0$. At $R = \infty$ the $R$-curve implies $n = n_m$ and the $n$-curve implies $n = \bar{n} \equiv \alpha n_m (1 - \delta) / [\delta + (1 - \delta) \alpha] < n_m$. Hence the curves cross at least once, and generically an odd number of times. Since we already established that there cannot be multiple steady states in the same regime, if there is a steady state at $R = \bar{\pi}$, there must exist two other steady states, one with $R < \bar{\pi}$ and one with $R > \bar{\pi}$. Routine calculation implies $\partial R / \partial \rho > 0$, and so there exist $\bar{\rho}, \bar{\rho} \geq 0$ with the properties specified in Proposition 3. ■

Appendix B: Sunspot Equilibria

A dynamical system allows for a two-state sunspot equilibrium solves

$$\phi_{s,t-1} = \zeta_s f(\phi_{s,t}) + (1 - \zeta_s) f(\phi_{s,t})$$  \hspace{1cm} (47)

where $s = A, B$ denotes two states in the sunspot equilibrium, $\zeta_s \in (0,1)$ is the probability of staying in the same state, and $f$ is the dynamical system in the deterministic case. We seek a pair of probabilities $(\zeta_A, \zeta_B) \in (0,1)^2$ satisfying (47) in stationary equilibrium.

To proceed, rewrite (47) as

$$\zeta_A = \frac{f(\phi_B) - \phi_A}{f(\phi_B) - f(\phi_A)} \quad \text{and} \quad \zeta_B = \frac{\phi_B - f(\phi_A)}{f(\phi_B) - f(\phi_A)}$$

Consider wlog $\phi_B > \phi_A$. If $f$ is decreasing on $(\phi_A, \phi_B)$, the denominator is negative. Then $\zeta_A, \zeta_B \in (0,1)$ iff $f(\phi_A) > \phi_B > \phi_A > f(\phi_B)$, which implies that $f$ crosses the 45 degree line from above and $[f(\phi_A) - f(\phi_B)] / (\phi_A - \phi_B) < -1$. Therefore, in Model 1 where $f$ is decreasing around the steady state, there exist sunspot equilibria if $f(\bar{\phi}) < -1$.

Similarly, if $f$ is increasing on $(\phi_A, \phi_B)$, the denominator is positive. Then $\zeta_A, \zeta_B \in (0,1)$ iff $f(\phi_B) > \phi_B > \phi_A > f(\phi_A)$, which implies $f$ crosses the 45° line from below on $[\phi_A, \phi_B]$. Therefore, in Model 2 where $f$ is increasing, there exist sunspot equilibria around a stable steady state $\phi_1$ for any $\phi_A \in (0, \phi_1)$ and $\phi_B \in (\phi_1, \phi_2)$.

Appendix C: Bargaining in Model 2

There are two agents on each island, one who lives for one period and one who lives forever, so the former should be the depositor and the latter the banker. Assume

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the cost $\kappa$ is too high for them to invest individually. If the banker’s bargaining power is $\theta$, the generalized Nash problem is

$$W(\phi) = \max_{X,x,D,d} [U(X) - C(D)]^\theta [u(x) - c(d)]^{1-\theta}$$

subject to $X + x = R(D + d) - \kappa$ \hspace{1cm} (49)

$$u(x) - c(d) \geq 0$$

$$x_t \leq \phi_t.$$ \hspace{1cm} (51)

The last constraint is from the banker’s incentive condition $\beta V_{t+1} \geq \lambda x_t + (1 - \mu) \beta V_{t+1}$ rewritten using $\phi_t \equiv \beta \mu V_{t+1}/\lambda$. Notice $W'(\phi) > 0$ if (51) binds, and there is a cutoff $\tilde{\phi}$ above which banking is viable and below which it is not.

Denote the solution ignoring (50) and (51) by $(X^*, x^*, D^*, d^*)$. Further, consider the case $u(x^*) > c(d^*)$ and let $\phi^* = x^*$. Substituting (49) into the objective function and taking FOC’s, we get

$$D : U'(X) R - C'(D) = 0$$

$$d : \theta U'(X) R[u(x) - c(d)] - (1 - \theta) c'(d) [U(X) - C(D)] - \eta_1 c'(d) = 0$$

$$x : -\theta U'(X) [u(x) - c(d)] + (1 - \theta) u'(x) [U(X) - C(D)] + \eta_1 u'(x) - \eta_2 = 0$$

where $\eta_1$ and $\eta_2$ are multipliers. From this one can see the banker’s surplus may decrease with $\phi$ at least close to $\phi^*$:

$$\frac{\partial(U(X) - C(D))}{\partial \phi} \bigg|_{\phi \to \phi^*} = \frac{(1-\theta)U''(U-C)(R\theta U''-C''\theta)}{(C''-R\theta U'')(C\theta''+(1-\theta)\theta U'-\theta R\theta U''C''(u-c))} < 0.$$

The banker’s value function is $V_t = U(X_t) - C(D_t) + \beta V_{t+1}$, and using $\phi_t = \beta \mu V_{t+1}/\lambda$ we have

$$\phi_{t-1} = \frac{\beta \mu}{\lambda} [U(X_t) - C(D_t)] + \beta \phi_t.$$ \hspace{1cm} (52)

Now (52) can be written as

$$\phi_{t-1} = \begin{cases} 
\beta \phi_t & \text{if } \phi_t < \tilde{\phi} \\
\frac{\beta \mu}{\lambda} [U \circ X (\phi_t) - C \circ D (\phi_t)] + \beta \phi_t & \text{if } \tilde{\phi} \leq \phi_t < \phi^* \\
\frac{\beta \mu}{\lambda} [U(X^*) - C(D^*)] + \beta \phi_t & \text{if } \phi_t \geq \phi^*
\end{cases}$$

Fig. AC shows the dynamical system for the following parameterization:

**Example AC:** Let $U(x) = u(x) = Ax$ and $C(d) = c(d) = Bd^\gamma/\gamma$, where $A = 1$, $B = 0.5$, $\gamma = 5$, $R = 2$, $k = 1.5$, $\theta = 0.01$, $\lambda = 0.01$, $\mu = 1$ and $\beta = 0.35$. 

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There are three steady states, $\phi = 0$ and $\phi_2 > \phi_1 > 0$, with $f$ crossing the $45^\circ$ line from below at $\phi_1$ and from above at $\phi_2$. Hence there are sunspot equilibria around $\phi_1$ fluctuating between any $\phi_A \in (0, \phi_1)$ and $\phi_B \in (\phi_1, \phi_2)$, similar to the baseline version of Model 2, and since $f'(\phi_2) < -1$ there is a two-cycle with periodic points $\phi_L$ and $\phi_H$, plus sunspot equilibria for any $\phi_A \in (\phi_L, \phi_2)$ and $\phi_B \in (\phi_2, \phi_H)$, similar to Model 1.

Appendix D: Entry by Type M in Model 3

Consider entry by $M$ instead of $S$. The equations (23)-(27) are the same but now $n_s$ is fixed while $n_{m,t}$ is endogenous. Also $V_s,t = \kappa_s$ is replaced by $V_{0,t} = \kappa_m$. Then (24) yields $N_t$ in terms of $R_t$,

$$N_t = \frac{\alpha n_s \theta_{ma} R_t}{(1 - \beta) \kappa_m}. \quad (53)$$

From (53), $N_t$ depends only on $R_t$, while with entry by $S$, it depends on $R_t$ and $n_t$. Substituting (53) into (26), after some algebra we get $R_{t-1} = G(R_t)$, where

$$G(R) \equiv \beta (1 - \delta) \left\{ \rho + R + \frac{(1 - \beta) \kappa_m n_{b} \theta_{mb}}{n_s \theta_{ma} R} \int_{R}^{\infty} [1 - F(\pi)] d\pi - (1 - \beta) \kappa_m \right\}. $$

From this, $R_{t-1}$ depends only on $R_t$, while in the version with entry by $S$, it depends on $R_t$ and $n_t$.

Hence we now get a univariate system $R_{t-1} = G(R_t)$, which determines the path for $R_t$, after which $N_t$ follows from (53), $n_t$ from (27), etc. Given a fixed point
\[ R = G(R), \]

\[ N = \frac{\alpha n_s \theta_{ms} R_t}{(1 - \beta) \kappa_m} \]

\[ n_m = \frac{\alpha n_s \theta_{ms} R_t}{(1 - \beta) \kappa_m} - n_s - n_b \]

\[ n = \frac{n_s (1 - \delta) [\alpha n_s \theta_{ms} R - (n_b + n_s) (1 - \beta) \kappa_m]}{\delta n_s \theta_{ms} R + (1 - \delta) [n_b \tau(R) + n_s] (1 - \beta) \kappa_m} \]

To guarantee the fixed point is a steady state we must check \( n_m, n \geq 0 \), both of which hold iff \( R \geq R \equiv (n_s + n_b) (1 - \beta) \kappa_m / \alpha n_s \theta_{ms} \) (we also need \( n \leq n_m \) but that never binds). Hence, a solution to \( R = G(R) \geq R \) is a steady state with type \( M \) active; otherwise, there is no intermediation.

![Figure AD: Model 3, cycles with entry by \( M \)](image)

One can check \( G(0) = \infty, G'(R) < 1 \) and \( G''(R) \geq 0 \). Also, \( \forall R > \max(\pi) \), \( G \) is linear with slope \( \beta (1 - \delta) \). This is shown in the left panel of Fig. AD, from which it is clear that there exists a unique fixed point, say \( \hat{R} \). In any case, we can have \( \hat{R} > \max(\pi) \) on the linear part of \( G(R) \) or \( \hat{R} < \max(\pi) \), on the nonlinear part of \( G(R) \). If \( G'(\hat{R}) < -1 \) then \( \hat{R} \) is locally stable, and there exist cycles and sunspots.

There is a threshold \( \rho_1 \) such that \( G'(\hat{R}) < -1 \) iff \( \rho < \rho_1 \). We do not know if \( \rho_1 > 0 \) or \( \rho_1 < 0 \), in general, but all our examples gave \( \rho_1 < 0 \). Still we have to verify \( R \geq R \), as discussed above. Is \( G'(\hat{R}) < -1 \) and \( \hat{R} \leq R \) possible? Yes, as we now show by way of example.

**Example AD:** Consider \( \alpha = 1, \delta = 0.01, \beta = 0.99, n_b = n_s = 1, \theta_{mb} = \theta_{ms} = 0.5, \kappa_m = 0.1, \rho = -0.1, \) and use the \( F(\pi) \) in (30), with \( \pi_0 = 4.95, \pi_1 = 0.05, \pi_2 = 5.05, \)

\( \pi_3 = 0.95 \) and \( \pi_4 = 10. \)
This is the example used in Fig. AD, where it can be easily checked that $G'(\hat{R}) < -1$ and $\hat{R} < \bar{R}$. Hence this admits a two-cycle. Note that $\rho < 0$ in this example. If we lower $\rho$ a little more, we can get higher-order cycles and chaotic dynamics. This is shown in the right panel of Fig. AD, where we plot $G^3(\hat{R})$ and see that there exist fixed points other than $\hat{R}$, namely a pair of three cycles. Hence, we can explicitly construct higher order cycles. Finally, one more result is that $\rho < 0$ implies $M$ and $B$ must trade for some $\pi$, $\Pr(R < \pi) > 0$, if $M$ is in the market – just like in the other version, a buy-and-hold-forever strategy is never a good idea at $\rho < 0$. 
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