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Monetary Policy Hysteresis and the Financial Cycle

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Abstract

A long tradition of macroeconomic analysis accords monetary policy only a transient role in driving real outcomes. At the same time, a large body of evidence highlights the persistent impact of financial cycles, particularly those that end in crises. We present a model where monetary policy, through its impact on the financial cycle, influences long-term economic trajectories. The model has two distinguishing features. First, financing underpins economic activity, with bank loans and the associated creation of inside money acting as the critical impetus for production and consumption. Under monetary exchange, the goods market always clears and there is no natural rate of interest. Instead, the central bank anchors the real interest rate, even in the long run. Second, externalities in the loan market generate an endogenous boom-bust cycle in bank lending. A forward-looking policymaker optimally leans against the build-up of financial imbalances during the boom, trading off short-term activity with longer-term stability. An inordinate focus on short-term outcomes can lead to ‘monetary policy hysteresis’, where low interest rates increase the vulnerability to financial busts over successive cycles. As a result, low rates can beget lower rates.

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On my view, there is no unique long-period position of equilibrium equally valid regardless of the character of the policy of the monetary authority. On the contrary there are a number of such positions corresponding to different policies.

J. M. Keynes (1979, p.35)

1 Introduction

What is the role of monetary policy in driving economic fluctuations? A long tradition of macroeconomic analysis accords monetary policy, and financial factors more broadly, only a transient effect on real outcomes. Under ‘monetary neutrality’, the equilibrium paths of all real variables are determined independently of the monetary and financial regime. This is reflected prominently in presumptions that real interest rates are tied to some ‘natural rate of interest’ whose variations are dictated purely by real saving-investment factors. At the same time, major economic dislocations, such as those that follow financial crises, are generally seen as being driven by exogenous shocks which are amplified by various propagation mechanisms. This is evident in much of the literature on macro-financial linkages spurred by the great financial crisis (GFC) as reviewed, for example, by Brunnermeier et al. (2013) and Claessens and Kose (2017).

We argue that this approach to modelling the role of monetary and financial factors may be overly restrictive. The influence of monetary policy, and particularly of the policy regime which governs the systematic component of monetary policy, may cast long shadows on real economic trajectories. A growing literature documents that monetary policy has a material influence on the financial cycle, not least through its impact on risk-taking, and that the financial cycle, in turn, has a long-lasting impact on output.1 There are a number of potential mechanisms that could be at work. Some may be the familiar hysteresis effects operating through the impact of slumps on labour and capital markets (e.g. Reifschneider et al. (2015)). But others may include the much less recognised impact of financial booms and subsequent busts on resource misallocations (Cecchetti and Kharroubi (2015), Borio et al. (2016)). The potential for monetary policy to have long-lasting effects on the path of real variables through its impact on the financial cycle represents a new form of ‘monetary hysteresis’ that deserves greater attention.

At the same time, the focus on shocks and their propagation neglects the crucial role of underlying vulnerabilities that accumulate over time and influence the economy’s proclivity, as well as response, to financial busts. Under this alternative view, major economic fluctuations arise from forces that are internal to the economy rather than exogenous shocks, resulting in recurrent periods of boom and bust (Beaudry et al. (2020)). This follows in the spirit of the long tradition of work on the financial cycle that highlight the critical interaction between financial fragility and the ebb and flow of investor sentiment (Kindleberger (2000), Minsky (1982, 1986)). To the extent that the build-up of vulnerabilities is endogenous to policy, omitting the channel understates the importance of policy frameworks for long-run financial and macroeconomic stability. Linking financial busts to the preceding booms is therefore crucial for a more complete

1A number of studies document the fact that financial busts tend to generate long-lasting, if not permanent, output costs (e.g. Basel Committee on Banking Supervision (BCBS) (2016), Cerra and Saxena (2008), Blanchard et al. (2015)).
assessment of the trade-offs policymakers face.

In this paper we study the role of monetary policy in a setting where, owing to financial frictions related to credit supply, lower interest rates boost risk-taking, thereby generating financial fragility. Financial fragility, in turn, generates sizeable and persistent output losses when stress materialises. As a result, monetary policy regimes have a pivotal role in determining the economy’s vulnerability to boom-bust cycles and its long-run evolution. This form of monetary policy non-neutrality represents a key departure from typical frameworks, in which the long-run path is exogenous to monetary policy and the problem is one of cyclical stabilisation around that path.

For the central bank, this set-up generates path dependence – in the sense that future policy depends on constraints that are determined by policy actions in the past – which gives rise to an intertemporal policy trade-off. Easier policy today boosts output in the short run but at the expense of the build-up of financial imbalances and large output losses in the future. By changing its policy rule, the central bank can influence the economy’s vulnerability to boom-bust cycles, and hence the long-run range of outcomes for output and real interest rates. While optimal policy involves both leaning and cleaning, it is the degree of leaning during the booms that generates substantive differences in long-run economic outcomes. We illustrate the possibility of a ‘low interest rate trap’, whereby monetary policy does not lean sufficiently against the build-up of financial imbalances resulting in a growing vulnerability to financial busts over successive cycles, which in turn leads to lower policy rates. In this sense, low rates beget lower rates.

The model features three key building blocks. The first two generate long-run non-neutrality while the third gives rise to financial cycles.

First, finance plays an integral role in the economy: it both underpins and coordinates economic activity. Financing – a cash flow concept – is essential because firms need to pay for factors of production before output can be produced. Firms can do so only by borrowing from banks, which generate purchasing power or inside money. Banks here do not just intermediate saving or endowments, they create money (deposits) in tandem with loans that enable the production of goods on which money has a claim. Goods production is perfectly synchronised with the flow of income necessary to purchase the goods – financing coordinates supply and demand, and hence, goods market clearing.

Second, we allow a given balanced growth path to be compatible with many interest rates levels by adopting an overlapping generations setup where agents have finite planning horizons. This assumption breaks the link between consumption growth at the individual level as governed by intertemporal substitution and aggregate consumption growth, enabling many equilibria to be sustained at different real interest rates. It also frees up the interest rate from having to equilibrate the goods market. By contrast, representative agent setups assume a uniform infinite planning horizon, which implies that aggregate demand growth is tied rigidly to individuals’ intertemporal substitution decisions. In this case, there can only be one real interest rate level consistent with the goods market equilibrium.²

²In a similar spirit, much recent work has sought to address the unrealistic implications of representative agent intertemporal substitution, either through the introduction of heterogeneous agents (e.g. Kaplan et al. (2018)), departures from rational expectations (e.g. Gabaix (2019)), or finite planning horizons (e.g. Woodford (2018)).
Third, the banking system is inherently instability-prone, exhibiting phases of excessive risk-taking followed by periods of risk aversion. The corresponding financial cycle reflects two key frictions in the banking system. First, heterogeneous firms have limited liability and cannot be sorted from one another, so that even those with negative-value projects obtain loans. Second, short-lived bank managers focus on short-term profits and, in the presence of externalities, may inadvertently take excessive risk in competing for market share when interest rates are low. Depending on the level of bank capital, the loan market may be subject to multiple equilibria – a boom and a bust.

These features raise the possibility that the economy, including real interest rates, do not converge to a fixed steady state in the long run. Rather, the economy alternates between boom and bust regimes. Because interest rates influence risk-taking, the nature of these fluctuations depends on the central bank’s reaction function given that monetary policy sets banks’ funding costs. The monetary regime is an integral part of the economy’s equilibrium. The interaction between monetary policy and the leverage cycle has a long-term impact on the economy, which is of first-order importance in the evaluation of policy trade-offs. Of course, this is not to say that other trade-offs, not least between inflation and output, are less important. We are simply abstracting from these to focus on a particular mechanism that has received less consideration.

Our paper is related to several literature strands. The first is the burgeoning macro-finance literature spurred by the GFC. Much of this work has stressed the potential for financial frictions to amplify the impact of financial shocks. The vast majority of studies overlay those frictions on an otherwise standard real business-cycle framework. Gertler and Kiyotaki (2010) and Cúrdia and Woodford (2016) show how frictions on the lender side can amplify the effects of the traditional financial accelerator mechanism, which focuses on borrower-side frictions (e.g. Bernanke et al. (1999)). Gertler et al. (2017) and Brunnermeier and Sannikov (2014) introduce further amplification through non-linearities and feedback mechanisms. In all of these cases, financial factors simply increase the persistence of the effects of the shocks, rather than generating endogenous boom-bust cycles.³

The second literature strand is the growing body of empirical research on the financial cycle. In contrast to the theoretical work, this work strongly suggests that financial busts are linked to the booms that precede them. Many studies have found that strong credit and/or asset prices increases, beyond historical norms, are useful leading indicators of subsequent busts and financial crises (e.g. Borio and Lowe (2002), Borio and Drehmann (2009), Aldasoro et al. (2018), Reinhart and Rogoff (2009), Schularick and Taylor (2012)). It has also become increasingly evident that strong credit growth and/or easy financial conditions carry information about subsequent economic slowdowns (Mian and Sufi (2015), Mian et al. (2017), Claessens et al. (2012), Jordà et al. (2016), Drehmann et al. (2017)), large negative output gaps or possibly deeper recessions (Borio and Lowe (2004), Krishnamurthy and Muir (2017), Jordà et al. (2016), Borio et al. (2018)), or downside risks to output (Adrian et al. (2018)).⁴

³Relatively small shocks may however be sufficient to cause a boom to implode, given the amplification effect – see Boissay et al. (2016)
⁴In particular, Adrian et al. (2018) find that the unconditional distribution of output is highly skewed to the left owing to the impact of financial conditions. Specifically, financial conditions boost growth in the near term but sap it in the longer term.
A third literature strand is the one that has begun to explore the role of monetary policy in boom-bust cycles by focusing on endogenous vulnerabilities (see Adrian and Liang (2016) for a recent review). Building on the ‘risk-taking channel’ of monetary policy (Borio and Zhu (2012)), Adrian and Duarte (2017) present a New Keynesian model with financial vulnerabilities in the form of endogenous second moments. Within such a setup, it is optimal for monetary policy to lean against the build-up of financial vulnerabilities during times of easy financial conditions as they cause greater future downside risks to growth. Coimbra and Rey (2017) develop a model with time-varying endogenous macroeconomic risk that arises from the risk-shifting behaviour of heterogeneous financial intermediaries. They show that when interest rates are low, further monetary stimulus can increase systemic risk, resulting in a trade-off between growth and financial stability. Neither of these papers, however, deals directly with booms and busts, as we do here. Closer in spirit to ours is Filardo and Rungcharoensittikul (2016), who examine optimal monetary policy in the context of an endogenous financial cycle and macroeconomic feedback.

A final literature strand includes studies that explore sources of long-run monetary policy non-neutrality. Benigno and Fornaro (2018) and Garga and Singh (2016) generate monetary policy hysteresis in New Keynesian models with endogenous growth. In both cases, deficient aggregate demand hampers aggregate supply by reducing innovation. Caballero and Simsek (2018) also focus on the possibility of a prolonged demand shump when interest rates are at the zero lower bound and the economy is caught in a negative feedback loop between pessimism and asset price declines. With respect to the long-run real interest rate, our work complements a recent set of papers that show how the natural rate of interest can be endogenous to policy in the presence of financial frictions (De Fiore and Tristani (2011), Benigno et al. (2014), Cúrdia and Woodford (2016), Sheedy (2018), and Vines and Wills (2018)). In contrast to these papers, the present one focuses on the non-neutrality that arises from the economy’s vulnerability to boom-bust cycles in a setting where monetary policy, rather than saving-investment fundamentals, pins down the real interest rate path.

By exploring the role of monetary policy in influencing boom-bust cycles that have persistent impact on real outcomes, our paper combines these strands of research. It builds on previous empirical work that documents a significant and persistent impact of monetary policy on output through its impact on leverage and debt service burdens (see Juselius et al. (2017) and Juselius and Drehmann (2015)). It also complements the empirical findings of Borio et al. (2017), who document a robust link between monetary policy regimes and real interest rates over the long run. This paper can be seen as providing a theoretical counterpart to these empirical results.

The rest of the paper is organised as follows. The next section sets out the building blocks of a financing-based economy, deriving the absence of a natural rate of interest and policy nonneutrality results. Section 3 introduces lending externalities, and shows how they lead to a financial cycle and an intertemporal policy tradeoff. Section 4 investigates properties of the calibrated model, and characterises the optimal monetary policy and the associated dynamic equilibrium. Section 5 analyses the effects of different policy regimes on the economy’s proclivity to boom-bust cycles, illustrating the possibility of ‘monetary policy hysteresis’. The
2 A Financing-based Economy

This section describes the basic ‘financing-based economy’, featuring an endogenous creation of bank credit in the context of overlapping generations. We show that any arbitrary level of interest rate chosen by the central bank can be sustained in equilibrium. As there is no well-defined natural rate of interest towards which the economy gravitates, it is up to the central bank to anchor the interest rate at the welfare-maximising level. This setup lays the groundwork for the next section, where we introduce defaults and externalities in the loan market, which cause boom-bust swings in bank loans and lending rate and leads to an intertemporal policy trade-off for the central bank.

2.1 The basic model

The basic setup consists of overlapping generations of households, firms, and bank managers who live for two periods, with a new cohort of equal size born every period. There is a single consumption good that firms can produce with a one-period lag using the labour input that young households provide. Households have to be paid upfront. As a result, production requires financing because it takes one period to complete and firms cannot credibly promise to pay for labour in advance. Banks, through their ability to create risk-free deposits, provide this financing by extending loans. Households allocate part of their wage receipts to current consumption, buying goods from old firms, and save the rest to finance consumption when they are old (and do not work). Old firms use the receipts from their sales to pay back bank loans.

Financing is a distinguishing feature of the model. It is a repetitive process of creating new credit flows to enable production, assigning claims on the resulting output, and specifying how to settle these claims. By extending new loans, banks create purchasing power in the form of deposits which function as a generally accepted means of payment (inside money). Firms use the deposits to acquire labour and start production; households use them to buy goods from firms; and banks accept them from firms as a means to repay loans. Loan repayment removes money from the economy, completing the financing process. This feature contrasts with the ‘real exchange’ approach that treats borrowing and lending as an exchange between an endowed stock of financial asset against goods. In the latter case, the natural interest rate arises because the goods market equilibrium determines the financial asset’s price by virtue of the Walras’ law. In our financing model by contrast, bank financing plays a coordinating role, simultaneously generating new supply of goods as well as purchasing power for them. With production, income, and consumption plans perfectly synchronised, supply and demand are matched at inception. The goods market clears independently of the interest rate level. By relieving the interest rate from having to clear the goods market, the natural interest rate concept becomes extraneous. To close the model, the central bank sets the risk-free rate.

Our emphasis on the primacy of financing builds on body of research which highlights the importance of the underlying frictions that give rise to money (see Wright (2018), Jakab and Kumhof (2018), Disyatat (2011), and Kiyotaki and Moore (2002)).
The timeline of each period is as follows (see Figure 1). First, the central bank sets the risk-free interest rate, which becomes the deposit rate that banks take as given. Banks then issue new loans and creating new deposits for young firms, who transfer them to young households in exchange for labour and start a new project. The project initiated last period is then complete, and the goods market opens – both old and young households purchase goods using their deposits. Old firms repay banks with these deposits, settling their loans. Appendix A walks through the balance sheet mechanics underlying a financing round in more details. With this financing mechanism in mind, we now proceed to characterise the objectives of each agent in the economy.

**Households**

Every period, a new generation of households is born. These agents live for two periods and can work only when young. They choose how much labour to supply in the first period and how to allocate wage income, which they receive immediately, to consumption over their lifetime. Risk-free bank deposit is the only vehicle for saving labour income (households have no storage technology). It pays a real gross interest rate $R^d$. Households own the firms, which transfer any profits to them when old. Thus, in the second period, households’ consumption is financed by running down their deposits and any profits received from firms. Households then die and are replaced by an identical new generation, who inherits the ownership of the firms.

The maximisation problem is as follows. Each household chooses labour supply in period $t$ and consumption in periods one and two so as to maximise utility

$$\max_{C_t, C_{2t+1}, l_t} \left( \frac{C_{1t}^{1-\sigma}}{1-\sigma} + \rho \frac{C_{2t+1}^{1-\sigma}}{1-\sigma} - \frac{\Phi}{1+\chi} L_t^{1+\chi} \right)$$

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*See also Jakab and Kumhof (2018) and McLeay et al. (2014) for detailed discussion on bank financing and its distinction from other modelling approaches.*
subject to the budget constraint
\[ C_{2t+1} \leq R^d_t(W_tL_t - C_{1t}) + \pi_{t+1}^F \] (2.2)
where \( C_{1t} \) and \( C_{2t+1} \) are the consumption of young and old households, respectively. \( L_t \) denotes labour services, \( W_t \) is the real wage, and \( \pi_{t+1} \) is firms’ profit (dividend). \( \sigma \) and \( \chi \) determine the curvature of the utility of consumption and disutility of labour, respectively, while \( \Phi \) represents the relative weight of the two. \( \rho \) is the household’s discount factor. Households face no uncertainty regarding their consumption, as deposits are risk-free and firms’ profits are known ex ante.

**Firms**

Firms live for two periods and are owned by households. They have a production technology of the form
\[ F(L_t) = AL_t^\alpha \] (2.3)

The total labour cost is \( L_tW_t \), and has to be paid upfront. Therefore, firms must obtain one-period bank loans in order to contract labour and initiate production. This represents the *financing constraint*. Given the real lending interest rate, \( R_t \), firms maximise
\[ \pi_{t+1}^F = F(L_t) - R_tW_tL_t \] (2.4)

**Competitive banks**

Banks issue loans and in so doing create risk-free deposits, or inside money, which function as the economy’s means of payment. Loans and deposits are financial claims and liabilities created by banks, and while denominated in real terms, neither are physical goods. This realistic feature breaks from many models that assume banks collect goods from households and lend them to firms, or treat financial assets as endowed stocks without any financial liability counterpart. The bank financing assumption will play a key role for the (non)existence of the natural interest rate as derived later.

Lending to firms in a given period earns banks a net interest income
\[ NII_t = (R_t - 1)W_tL_t - (R^d_t - 1)(W_tL_t - C_{1t}) \] (2.5)

The first term on the right hand side is the interest income on the loan, while the second term is the interest payment to (old) depositors. The latter is calculated based on an outstanding deposit balance \( W_tL_t - C_{1t} \), rather than \( W_tL_t \). When banks create deposits \( W_tL_t \) that firms use to pay young households as wages, part of this, \( C_{1t} \), is used immediately to finance their consumption (buying goods from old firms that in turn use the deposits to repay loans, extinguishing both). Only the remaining banks’ deposit base carried over to the next period, namely \( (W_tL_t - C_{1t}) \), is interest bearing.

Banks’ problem is to set the lending interest rate. Banks then *elastically* supply loans
at that rate and create deposits in the process. We assume that banks engage in Bertrand competition and bid down the lending rate $R_t$ to the effective funding cost, i.e. until $NII_t = 0$. The zero-profit condition pins down the equilibrium lending interest rate $R_t$ for a given $R^d_t$. Banks take the deposit rate $R^d_t$ as given, which is directly set by the central bank (we elaborate below how). If any bank sets a deposit rate lower than this market funding rate, all of its deposits will flow to other banks.

For completeness, we spell out other assumptions that will be important later on. Households own the banks (transferring ownership to the next generation as before), and receive fixed but negligible dividends from banks. This means households care about the solvency of banks, but otherwise have no preferences over how the banking business is run. We assume banks have access to a goods storage technology, which they use to hold claims against the rest of the economy. This stock of goods occupies the asset side of banks’ balance sheets, which reflect banks’ retained earnings on the liability side. At present, zero bank profit implies that bank equity is time-invariant and irrelevant to the analysis. Later, when defaults are allowed, bank capital will be a key state variable.

**Central bank**

To close the model, we let the central bank set the deposit rate $R^d_t$ as its policy rate, by acting as the intermediary between banks in the interbank market. We assume that banks settle payments among themselves by transferring deposits at the central bank. Any bank that needs to transfer funds to another bank must therefore borrow from the central bank to do so. To facilitate this settlement, the central bank stands ready to lend to banks at an interest rate $R^d_t + \epsilon$ and remunerates balances held with them at an interest rate $R^d$ (where $\epsilon > 0$ is small). Such a setup, which resembles ‘corridor systems’ used in practice by central banks to set overnight interest rates, ensures that no bank has an incentive to set deposit rates different from $R^d_t$.

To see this, suppose that all banks initially set deposit rates at $R^d_t$. If a bank sets a deposit rate lower than its peers, it will suffer a deposit outflow to other banks which it must fund by borrowing from the central bank at $R^d_t + \epsilon$. This higher funding cost means there is no incentive for any bank to deviate. Conversely, any bank that sets a higher deposit rate than $R^d_t$ (say $R^d_t + \Delta$) will attract deposits from other banks and see correspondingly higher balances at the central bank remunerated at $R^d_t$. The negative spread ($-\Delta$) rules out such a deviation. In the limit of $\epsilon \to 0$, the equilibrium deposit rate is exactly equal to the policy rate $R^d$, and the central bank neither requires nor earns real resources in running this operation.\[7\]

The model is cast in real terms, but there are two ways to think about nominal prices. One is to assume that the central bank sets the nominal interest rate and invoke price rigidity. Given the lack of any demand pressure as the goods market always clears at any interest rate, an equilibrium with a fixed price level can always be sustained. Alternatively one could allow prices to adjust flexibly and let inflation be determined by the Fisher equation. This case requires the

\[7\] There is no other equilibrium. If all banks set deposit rates below $R^d_t$, each bank will have an incentive to deviate and set a slightly higher rate to attract deposits from other banks which they will receive in the form of balances at the central bank earning $R^d_t$, earning a positive spread. Similarly, if all banks set rates above $R^d_t$, each bank will want to lower their deposit rate, incur outflows of funds which they more than make up by borrowing from the central bank at $R^d_t + \epsilon$. 

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central bank to move the nominal rate in lock step with the targeted real rate, which would ensure a stable inflation under the Taylor principle. Appendix C shows the derivation in this case.

2.2 Financing equilibrium

An absence of the natural interest rate

The first key result is the absence of a natural interest rate. Start with the households’ budget constraint in equation 2.2, and substituting for firms’ profit using equation 2.4 assuming a steady state (hence dropping time subscripts), we get

\[ C_2 = R^d(WL - C_1) + F(L) - RWL \]
\[ = F(L) - C_1 + (R^d - 1)(WL - C_1) - (R - 1)WL \]
\[ = F(L) - C_1 - NII \]

which is exactly the goods market equilibrium condition, \( F(L) = C_1 + C_2 + NII \). Note that this relationship holds as an identity, as we have not used any first-order conditions. This leads to the following result.

Result 1. The goods market clears at any given interest rate level \( R^d \) in the steady state. There is no natural rate of interest.

Intuitively, this result follows from the combination of financing and the overlapping-generations structure. With bank financing, purchasing power (deposits) is simultaneously generated as banks make loans to enable firms’ new production. These IOUs establish, at the outset, the amount of goods to be produced as well as the distribution of claims on them conditional on the level of the interest rate. Financing thus coordinates aggregate production with aggregate demand. Meanwhile, the overlapping-generations structure preserves this supply-demand balance even in the presence of intertemporal substitution. Under this assumption, each cohort can freely adjust its lifetime consumption plan in response to different interest rates, while aggregate consumption (the sum of old and young households’ consumption) remains matched with the steady-state output.\(^8\)

A natural interest rate concept regains relevance if one sufficiently changes either of the two assumptions. Replacing bank financing with an exogenous medium of exchange such as a stock of cash (similar to standard overlapping generations models with fiat money) would imply a natural rate of interest of zero, \( R^d = 1 \) (see Result 8 in Appendix 8). Outside cash is essentially a commodity that is traded alongside and against goods. In such a ‘real exchange’ model, Walras’ law applies and the goods market equilibrium condition pins down the price of the commodity, namely the equilibrium interest rate. With the financing assumption, Result 1 holds generally

\(^8\)In other words, while there are many steady states corresponding to different interest rate values, all of these entail stable aggregate consumption equal to aggregate output. With a higher steady-state \( R^d \), each generation of households consumes relatively more when old than young, but the sum of young and old households’ consumption will be equal to the (lower) aggregate output. That is, consumption growth of each cohort can adjust without necessarily changing aggregate consumption growth, thus loosening the link between the Euler equation and the interest rate rate.
under other symmetric overlapping generations structures, since the cross-generational sum of consumption in any period would mirror each household’s total income, hence total output. But in the case of an infinitely-lived representative household, the natural interest rate exists and is equal to $1/\rho$ (see Result 9 in Appendix B). This is the standard result that follows directly from the Euler equation. Intuitively, when a single agent’s intertemporal substitution dominates aggregate demand, only one level of interest rate can be consistent with consumption being in a steady state. Table 1 summarises the results under different combinations of assumptions. See Appendix B for detailed comparisons of these and other cases.

### Closed-form solutions

We now proceed to characterise the equilibrium solutions. The first-order conditions of the household problem imply the Euler equation and the optimal work-leisure choice

$$C_{2t+1}^\sigma = \rho R_t^d C_{1t}^\sigma$$  \hspace{1cm} (2.9)

$$\Phi L_t^\chi = \frac{W_t}{C_{1t}^\sigma}$$  \hspace{1cm} (2.10)

Solving these together with the budget constraint 2.2 and firms’ profit 2.14 (to be derived shortly), one obtains the labour supply function

$$\Phi L_t^{\chi+\sigma} = W_t^{1-\sigma} \psi_t^{\sigma}$$  \hspace{1cm} (2.11)

where

$$\psi_t = \frac{1 + \rho R_t^d}{1 + 1-\alpha \frac{R_t}{R_d}}$$  \hspace{1cm} (2.12)

and $R_t$ is the lending rate on bank loans.

Turning to the firms’ problem, the profit-maximising amount of labour hired is

$$L_t = \left(\frac{\alpha A}{R_t W_t}\right)^{\frac{1}{1-\alpha}}$$  \hspace{1cm} (2.13)

which implies firms’ profit, to be distributed to households, of

$$\pi_t^F = \frac{(1-\alpha)}{\alpha} R_t W_t L_t$$  \hspace{1cm} (2.14)
Lastly from the banking side, the net interest margin can be written as

$$NII_t = (R_t - \psi_{2t})W_tL_t$$  \hspace{1cm} (2.15)$$

where the unit cost of deposit is

$$\psi_{2t} \equiv R^d_t - \frac{R^d_t - 1}{\psi_{1t}}$$  \hspace{1cm} (2.16)$$

For positive net interest rate $R^d > 1$, the unit cost of deposit in 2.16 is less than $R^d$ because for each unit of deposits created in making loans, a part of it is extinguished and thus not liable for interest payments. Perfect price competition in the loan market then implies that the lending rate is set at the marginal financing cost

$$R_t = \psi_{2t}$$  \hspace{1cm} (2.17)$$

so that banks make zero profit every period.

The equilibrium can be solved by combining the labour supply equation 2.11, the labour demand 2.13 and the lending rate 2.17. The resulting equilibrium employment (hence output) is a function of the policy interest rate

$$\Phi L_t^{\chi+\sigma+(1-\alpha)(1-\sigma)} = (\alpha A)^{1-\sigma} \left( \frac{\psi_{1t}}{\psi_{2t}} \right)^{1-\sigma}$$  \hspace{1cm} (2.18)$$

where $\psi_{1t} = \alpha \left( 1 + \rho^1 R^d_t \frac{1}{\psi_{2t}} - 1 \right)$ and $\psi_{2t} = \alpha \left( 1 + \rho^1 R^d_t \frac{1}{\psi_{2t}} \right) / \psi_{1t}$. Using this equation together with the budget constraint 2.2, the Euler equation 2.9 and firms’ profit 2.14, household consumption is given by

$$C_{1t} = \frac{\alpha A L_t^\alpha}{\psi_{1t} \psi_{2t}}$$  \hspace{1cm} (2.19)$$

$$C_{2t+1} = \frac{\alpha A L_t^\alpha}{\psi_{2t}} \left( R^d_t \left( 1 - \frac{1}{\psi_{1t}} \right) + \frac{1 - \alpha}{\alpha} \psi_{2t} \right)$$  \hspace{1cm} (2.20)$$

These solutions lead to the second key result.

**Result 2.** *Monetary policy determines real allocations, and is non-neutral.*

In this financing-based economy, interest rates anchors the economy, coordinating production and consumption decisions. This monetary non-neutrality result does not hinge on the assumption of nominal rigidities. In fact, it is perfectly consistent with fully flexible prices. In Appendix C, we invoke inflation through the Fisher equation in close parallel with classical monetary theory. Unlike the classical theory where the central bank can only choose the inflation target (or equivalently the nominal interest rate), here the central bank has an extra degree of freedom and can target any two variables: the real interest rate, the nominal interest rate or inflation. The central bank can thus design a nominal interest rate rule that implements any path of the real policy rate $R^d$ it desires while at the same time delivering the chosen inflation rate.

As a result of this non-neutrality, monetary policy has long-run welfare implications. The
first-best outcome is to set $R^d$ to maximise household utility, subject to equilibrium conditions 2.18-2.20. Intuitively, the central bank should set the economy’s financing cost such that households’ marginal utility from additional lifetime consumption is equal to their marginal disutility from supplying labour.

Results 1 and 2 hark back to the older ‘monetary analysis’ literature of Wicksell, Hawtrey, Robertson, and eventually Keynes that emphasized the importance of money flows (here financing) in driving real transactions. The ebb and flow of financing impinges on real outcomes because they underlie production and consumption decisions. As Keynes put it, “The theory which I desiderate would deal...with an economy in which money plays a part of its own and affects motives and decisions and is, in short, one of the operative factors in the situation, so that the course of events cannot be predicted, either in the long period or in the short, without knowledge of the behaviour of money...” (Keynes (1973) pp. 408-411). In our framework, the central bank, through its reaction function, pins down the interest rate. For Keynes, interest rate determination was achieved through the introduction of his ‘liquidity preference’ theory. The outcome in terms of an elevated role of monetary factors in the long-run evolution of the economy is similar, as reflected in the quote at the beginning of the paper.

3 The Economy with a Financial Cycle

We now introduce frictions that lead to boom-bust cycles in bank credits. On the production side, we add an unproductive sector that causes loan defaults which weaken banks’ balance sheets. On the financial side, banks now operate in an imperfect competition environment and can price loans strategically to gain market share. The overall quality of the borrower pool deteriorates as lending interest rates decline, but which no single bank internalises. Lower lending rates, moreover, make unproductive firms less interest-sensitive when shopping for loans, a credit satiating effect. As a result, strategic complementarity arises among profit-maximising banks – both high and low lending rates (conservative and aggressive lending) can be supported in equilibrium.

We introduce an agency friction between bank owners (households) and managers, which selects an equilibrium and generates a cycle. Bank managers are delegated to run the lending business, but have innate preferences for gaining market share via a low-rate strategy. Left to their own devices, managers tend to over-compete which deplete bank capital, possibly leading to bankruptcy. Owners prevent this by penalising a low-rate strategy increasingly as capital runs low. Managers incur a cost when changing strategy, and only coordinate on a ‘regime switch’ when capital is sufficiently low (owners’ penalties dominate) or high (innate preferences dominate). For intermediate bank capital, history dictates equilibrium strategy due to the switching cost. Persistent spells of conservative and aggressive lending give rise to a financial

\[9\text{Kohn (1986) provides an exceptionally clear exposition of monetary analysis and particularly Keynes’ contribution and approach to it.}\]

\[10\text{See Leijonhufvud (1981) for an insightful discussion of the role of Keynes’ liquidity preference theory in pinning down the interest rate in place of the loanable funds framework.}\]

\[11\text{Our focus is on the frictions arising from the credit supply side. But a similar mechanism could well operate on the credit demand side. In reality, credit demand frictions are likely to be just as important, e.g. due to over-borrowing and balance sheet overhangs in the non-financial sector.}\]
cycle. \(^\text{12}\)

## 3.1 Adding lending frictions

We start by describing the unproductive sector and the new bank profit-maximisation problem, before laying out the agency frictions between owners and managers and how the outcome gives rise to a cycle equilibrium.

### Unproductive sector

The unproductive sector consists of a continuum of unproductive firms and asset owners who are endowed with an asset that unproductive firms require to produce. Banks have no means, directly or otherwise, of differentiating unproductive firms from productive ones. Unproductive firms can utilise one unit of an asset to produce consumption goods, but only succeed with an infinitesimal probability. Due to limited liability, these firms would seek bank loans and produce as long as the best-case return exceeds the total cost of production. Each unproductive firm incurs a unit production cost \(R_t q_t\), where \(q_t\) is the rental cost of the asset.

Unproductive firms differ in their productivities \(x > 0\), the density function of which is given by \(\omega d N_d/x^{1+\omega_d}\), where \(\tilde{N}_d\) captures the total number of unproductive firms and \(\omega_d\) regulates the shape of the distribution (a higher \(\omega_d\) implies that the density is more concentrated at lower values of \(x\)). Integrating this density function yields the aggregate asset demand

\[
N_d(R_t q_t) = \frac{\tilde{N}_d}{(R_t q_t)^{\omega_d}} \tag{3.1}
\]

As a result, the number of unproductive firms entering the loan market, and hence asset demand, rises as the production cost \(R_t q_t\) falls.

On the input side, asset owners are one-period-lived agents who expend effort to prepare the assets before renting them out. Once they receive the rent, they spend all bank deposits to consume immediately.\(^\text{13}\) They solve

\[
\max_{N_t} \mathbb{E}_t \left( C_t^{1-\sigma} - \frac{\Phi}{1 + \chi} N_t^{1+\chi} \right) \tag{3.2}
\]

subject to

\[
C_t \leq q_t N_t \tag{3.3}
\]

The first-order condition gives rise to the asset supply schedule

\[
N_s(q_t) = \left[ \frac{1}{\Phi} \right]^{\frac{1}{1+\sigma}} q_t^{\frac{1+\sigma}{1+\sigma}} \tag{3.4}
\]

Equating asset demand and supply, equations 3.1 and 3.4, one obtains the equilibrium

\(^{12}\)The mechanism echoes the financial instability hypothesis of Minsky (1982, 1986), though in place of fluctuations in investor sentiment, here it is shifts in bank lending behaviour that drive boom-bust cycles.

\(^{13}\)It is also possible to assume, less restrictively, that asset owners live for two period and face a well-defined optimisation problem. Doing so does not affect the key results.
asset price \(q_t\). This price is decreasing in \(R_t\). The equilibrium amount of loans unproductive firms obtain also declines in \(R_t\), so that

\[
q_t \tilde{N}_t = \frac{\tilde{N}}{R_t}
\]

where

\[
\tilde{N} \equiv \tilde{N} \frac{1+\chi}{1-\sigma} \Phi \frac{1-\omega_d}{1-\sigma}
\]

(3.6)

\[
\omega \equiv \frac{(1+\chi)\omega_d}{(1-\sigma + (\chi + \sigma)\omega_d)}
\]

(3.7)

Note that \(\omega\) is rising in \(\omega_d\) (with \(\omega = 1\) when \(\omega_d = 1\)), and the elasticity of asset demand translates into the elasticity of unproductive firms’ demand for loans (i.e. the extensive margin’s sensitivity to the production cost). We will calibrate \(\omega\) directly below, on the understanding that it reflects the deep parameter \(\omega_d\).

### Imperfectly competitive banks

Banks cannot differentiate firm types but know the aggregate quantity of loans demanded by unproductive firms at any given lending interest rate.\(^{14}\) Thus aggregate bank profits depend on underlying loan quality, and are equal to net interest income from productive loans \((R_t - \psi_{2t})L_tW_t\) (equation 2.15) minus the loss on the loans to unproductive firms of \(q_t \tilde{N}_t = \frac{\tilde{N}}{R_t^d}\) (equation 3.5). To allow loan losses to matter for bank profits, we depart from the baseline case and assume that the loan market is imperfectly competitive. Individual banks may thus set different lending rates from their peers in an attempt to capture market share and influence loan composition. Specifically, we assume each small bank \(i\) takes the aggregate variables \(L_t, W_t, R_{t}^d\), as well as the average lending rate set by its competitor banks \(R_t\) as given, and chooses its lending rate \(R_{it}\) to maximise its profit

\[
\Pi(R_{it}, R_t) = (R_{it} - \psi_{2t})L_tW_t \left( \frac{R_t}{R_{it}} \right)^{\theta_1} - \frac{\tilde{N}}{R_{it}^d} \left( \frac{R_t}{R_{it}} \right)^{\theta_2}
\]

(3.8)

which is the sum of net interest income on the loans granted to productive firms and the loss on the loans granted to unproductive firms, adjusted by the corresponding market shares.\(^{15}\) The market shares depend on how each bank prices its loans relative to competitors. \(\theta_1 > 1\) and \(\theta_2 \geq 0\) capture the interest sensitivities of loan demand for productive and unproductive firms, respectively, and represents banks’ market power. Appendix D discusses the microfoundations for the market shares in equation 3.8.

\(^{14}\)If banks were able to specify both the loan amount and the interest rate, they could in principle exploit the heterogeneity of loan demand across types and screen for productive firms. We rule out this possibility by assuming that banks meet any loan demand at the chosen lending rate. More generally, what is central for the result is that the size of the loans extended to productive and unproductive activities are correlated. For instance, this would be the case if banks can only lend to a consortium of firms or production units.

\(^{15}\)Each bank takes its funding cost \(\psi_{2t}\) as given when optimising, as \(\psi_{2t}\) depends only on \(R_{t}^d\) and the banking system’s average lending rate \(R_t\) which an individual bank cannot influence. Note that there is no deposit funding cost for loans to unproductive firms because the corresponding deposits are paid out to asset owners who, in turn, spend them immediately, thereby extinguishing the deposits within the period.
Note the first source of externality – no single bank can on its own influence the overall quality of the pool of borrowers $\bar{N}/R_t^\omega$, which is determined by the aggregate lending rate $R_t$. Banks take $R_t$ as given, failing to internalise the effect of their decision on it and on aggregate loan quality. This creates an incentive to over-lend, in line with the standard ‘tragedy-of-the-commons’ problem.

The second externality we introduce weakens banks’ incentives to poach firms from their competitors when interest rates are high, and strengthen them when rates are low. When overall lending interest rates are high and credit relatively scarce, we assume that unproductive firms are more willing to switch to banks that offer cheaper loans – their loan demand is more price elastic at higher interest rates.\(^{16}\) The assumption allows us to introduce strategic complementarity in a simple way. If a bank were to undercut competitors in this environment, it would disproportionately attract unproductive firms, leading to higher loan losses. If all banks lend conservatively and set a high lending rate $R$, then any individual bank would have little incentive to poach.

Specifically, we let $\theta_2(R_t)$ be an increasing step-function of $R_t$

$$
\theta_2 = \begin{cases} 
\theta_H & \text{under high-$R_t$ equilibrium} \\
\theta_L & \text{under low-$R_t$ equilibrium} 
\end{cases}
$$

(3.9)

where $\theta_H > \theta_L$. The first-order condition from equation 3.8 implies bank $i$’s best-response function

$$
R_{it} = \theta_1\psi_2t \frac{\theta_2\bar{N}}{\theta_1 - 1} + \left(\theta_1 - 1\right)\theta_2 L_t W_t R_t^\omega \left(\frac{R_t}{R_{it}}\right)^{\theta_2 - \theta_1}
$$

(3.10)

In the symmetric Nash equilibrium where $R_{it} = R_t$, the aggregate loan supply function dictates that the lending rate must solve

$$
R_t = \theta_1\psi_2t \frac{\theta_2\bar{N}}{\theta_1 - 1} + \left(\theta_1 - 1\right)\theta_2 L_t W_t R_t^\omega
$$

(3.11)

The first term shows that banks exercise market power by setting a mark-up over the funding cost $\psi_2t$ when lending to productive firms. As individual banks’ incentive to steal market share increases ($\theta_1$ is higher), the Nash equilibrium mark-up declines towards 1. The second term reflects incentives to contain losses from lending to unproductive firms, internalised up to the parameter $\theta_2$. Strategic complementarity arises because $\theta_2$ is increasing in $R_t$. At higher interest rates, the loan demand by unproductive firms is more interest rate sensitive, reducing individual banks’ incentive to undercut competitors and hence limiting loan losses. Thus when average interest rates are high, each bank wants to maintain a high rate. There are two Nash equilibria: a ‘bust’ with high $R_t$, where banks are conservative and careful about undercutting each other ($\theta_2 = \theta_H$), and a ‘boom’, with low $R_t$, where banks compete more

\(^{16}\)One interpretation is that firms face a trade-off between their preferences for bank diversity (the source of banks’ market power) and obtaining the cheapest possible loans (see Appendix D). We assume that the shape of preferences is such that the second consideration tends to dominate when overall financial conditions are tight. This is consistent, for example, with empirical evidence of a higher price elasticity at higher interest rates, leading to a ‘kinked’ loan demand function – see Karlan and Zinman (2008).
aggressively for customers ($\theta_2 = \theta_L$).

Despite profit maximisation, a bank may well make a loss in equilibrium, because each has no control over the aggregate share of unproductive firms. It must take the system-wide profit or loss as given. Raising own lending rate $R_{it}$ may deter unproductive firms, but only at a cost of turning away productive borrowers. Doing so may in fact lower profit even further if unproductive firms are relatively price-insensitive, as is the case in a boom. As a result, a bank cannot guarantee itself a non-negative profit. We assume $R_{it}$ is the only choice variable of a bank, and there is no option to shut down the business temporarily to avoid a bad equilibrium (as Charles Prince famously remarked, “As long as the music is playing, you’ve got to get up and dance”).

**Agency frictions and equilibrium selection**

We now describe agency frictions between ownership and management. Households own the banks and delegate the running of them to two-period-lived managers, who receive a (small) fraction $\gamma$ of profit $\Pi_{t+1}(R_{it}, R_t)$ as remuneration. This incentivises managers to maximise bank profit and follow the best-response rule derived in equation 3.10. The equilibrium lending rate $R_t$ is thus one of the two solutions to equation 3.11, with lower and higher values denoted by $R_L$ and $R_H$ respectively (dropping time subscript to ease notation, with an understanding that these depend on $R^d_t$). As it is common knowledge that all managers pursue to same objective, the strategic interaction between managers is reduced to a 2-by-2 coordination game, with action space $\{R_L, R_H\}$.

We assume that bank managers have an innate predisposition to expand market share, which can be achieved by setting low interest rates. This is captured by assuming that managers derive a positive utility $D$ from lending at rate $R_L$. Bank owners have an incentive to counter this bias to guard against bankruptcy that may arise from over-competition. They do so by imposing a penalty $H(K_t)$ on managers for each period that they choose $R_L$, more so at lower bank capital levels $K_t$—formally $H'(K) < 0$ and $\lim_{K \to 0} H(K) = \infty$ so that bankruptcy never occurs.\(^{17}\) \(^{18}\) Lastly managers incur a switching cost $C$ each time they shift between $R_L$ and $R_H$ (e.g. the cost of having to explain to the board and customers the reasons for such change). A bank manager’s payoff is thus given by

$$\Pi^M(R_{it}, R_{it-1}|R_t) = \gamma \Pi(R_{it}, R_t) + H(K) I(R_{it} = R_H)$$
$$+ DI(R_{it} = R_L) - CI(R_{it} \neq R_{it-1})$$

(3.12)

where $I(\cdot)$ is an indicator function. We assume that $\gamma, C, D, H(K)$ are all small and can be ignored when computing bank profits or social welfare. $\gamma$ is also small relative to $C, D, H(K)$ (see below).

For a bank manager, switching to a different lending strategy yields a payoff that depends

---

\(^{17}\) Consistent with this agency cost assumption is some recent empirical evidence that higher bank capital leads to greater risk-taking. See Dell’Ariccia et al. (2017).

\(^{18}\) The fact that $H(K)$ is state contingent could be justified by owners having to incur monitoring and due diligence costs each time they decide to assess banks’ insolvency risk and managers’ strategy. $H(K)$ could thus represent the expected penalty, decreasing in $K$ partly because owners are more likely to begin due diligence process when bank capital runs low.
not only on her private cost $H(K)$ and $C$, but also what she expects other managers to do. Determining the switching point in equilibrium is thus an equilibrium selection problem. We adopt a dominant strategy criterion, according to which the equilibrium switches, say from $R_L$ to $R_H$, when each manager finds it optimal to do so regardless of what others choose.\textsuperscript{19} That is

$$\Pi^M(R_H, R_L \mid \check{R}) \geq \Pi^M(R_L, R_L \mid \check{R})$$

(3.13)

for $\check{R} \in \{R_L, R_H\}$. Combining this with equation 3.12 and exploiting the assumption that $\gamma$ is small (relative to $C$, $D$, and $H(K)$), the switching condition from $R_L$ to $R_H$ boils down to

$$H(K) - D \geq C$$

(3.14)

Conversely, the switching from the $R_H$ equilibrium to the $R_L$ counterpart occurs when

$$H(K) - D \leq -C$$

(3.15)

These conditions define the threshold levels of bank capital at which equilibrium switches take place, namely

$$K_c \equiv H^{-1}(C + D)$$

(3.16)

$$K_r \equiv H^{-1}(D - C)$$

(3.17)

where $K_c < K_r$ (subscripts standing for ‘crisis’ and ‘recovery’) because $H'(K) < 0$.

Intuitively, when bank capital falls below $K_c$, the owner penalty dominates and pushes all bank managers to coordinate on $R_H$. While this conservative lending equilibrium prevails, all banks benefit from lower loan losses, earn positive profits and build up capital. Higher capital level lowers the threat of penalty relative to managers’ preferences for competition ($H(K) - D$ declines). But due to the switching cost, it takes time before managers switch back to aggressive lending at $H(K) - D \leq -C$. For intermediate bank capital $K_t \in (K_c, K_r)$, the prevailing equilibrium regime thus persists into the next period, a form of hysteresis. See Bebchuk and Goldstein (2011) and Rajan (1994) for a related approach in dealing with multiplicity.

We allow $C$ to be a random variable, and hence equilibrium switches stochastic. This is the only source of uncertainty in the model. We assume that the stochastic properties of $C$, through equations 3.16-3.17, induce a logistic probability distribution of regime switches over bank capital $K_t$. Parameters $K_c$ an $K_r$ are the centres of these distribution, and the transition probabilities between the two equilibria or regimes are

$$P(s_t = 2 \mid s_{t-1} = 1, K_t) = G(a_c(K_t - K_c))$$

$$P(s_t = 1 \mid s_{t-1} = 2, K_t) = G(a_r(K_t - K_r))$$

(3.18)

where $G(x) = e^x / (1 + e^x)$ is the logistic function, $a_c < 0, a_r > 0$. The regime $s_t = 1$ (a boom) corresponds to the equilibrium where banks compete actively with $R_t = R_L$. The regime

\textsuperscript{19}When $\gamma$ is small, as we assume, it turns out that the Harsanyi-Selten risk dominance equilibrium selection concept delivers exactly the same outcome.
$s_t = 2$ (a bust) is associated with the conservative lending equilibrium with $R_t = R_H$. This formulation nests the deterministic case when $\sigma_c$ and $\sigma_r$ are large, though we will focus on the more interesting case where both are finite.\footnote{The deterministic case implies predictable crises, and the central bank can always stabilise bank capital just above $K_c$ to avoid them. There is no financial cycle and policy tradeoff in such case.}

Bank capital is a key state variable in the economy, which in a symmetric Nash equilibrium evolves according to

$$K_{t+1} = K_t + \Pi(R_t, R_t)$$  \hspace{1cm} (3.19)

Bank capital reflects the various stages of the financial cycle. In the initial stages of a boom, bank capital is ample and banks can withstand an extended period of losses. In the early stages of a bust, bank capital is low and takes time to build up. Despite the persistence of bank risk-taking, regime switches occasionally reverse the bank capital dynamics, ensuring that in equilibrium banks never become bankrupt nor accumulate net worth indefinitely.

The absence of a natural interest rate applies also in this richer model with lending frictions. Note that banks hold goods on the asset side of their balance sheets, which mirror bank equity capital. When banks make profits, they are acquiring claims on goods in the economy. Firms transfer these goods to banks as part of their loan settlement, when their loan obligations exceed the deposits they acquire through sales of goods. Similarly, when banks make losses, they run down their goods inventory to meet the excess of deposit withdrawals over loan repayments. This serves to make deposits risk-free. The evolution of bank capital then reflects banks’ profit and loss – banks accumulate capital when making profit, and run down capital when making losses.\footnote{In Appendix A, we explain the balance sheet mechanics in more detail via explicit numerical examples.} Bank inventories of goods serve the same function as inventories in national income accounting, where output equals to consumption plus change in inventories. Financing coordinates aggregate production with aggregate demand, now defined to include changes in inventories. Goods market clearing again obtains without the need for the interest rate or other prices to adjust.

### 3.2 Equilibrium conditions and the financial cycle

The equilibrium conditions consist of the labour supply in equation 2.11, labour/credit demand in equation 2.13, and the regime-specific lending interest rate in equation 3.11:

$$L_t = \left( \frac{W_t^{1-\sigma} \psi_t^\sigma}{\Phi} \right)^{\frac{1}{\chi+\sigma}}$$  \hspace{1cm} (3.20)

$$L_t = \left( \frac{\alpha A}{R_t W_t} \right)^{\frac{1}{\alpha}}$$  \hspace{1cm} (3.21)

$$R_t = \begin{cases} \frac{\theta_1 \psi_2 t}{\theta_1 - 1} + \frac{\theta_t \bar{N}}{(\theta_1 - 1)L_t W_t R_t^2}, & \text{for } s_t = 1 \\ \frac{\theta_1 \psi_2 t}{\theta_1 - 1} + \frac{\theta_H \bar{N}}{(\theta_1 - 1)L_t W_t R_t^2}, & \text{for } s_t = 2 \end{cases}$$  \hspace{1cm} (3.22)
For a given regime, these three conditions pin down a unique equilibrium in $L_t$, $W_t$ and $R_t$ period by period, at any level of policy interest rate $R^d_t$. The solutions imply all other variables, including consumption, output as well as firms’ and banks’ profits. Finally, the laws of motion for bank capital in equation 3.19 and the regime-switching process in equation 3.18 govern the economy’s evolution.

The relationship between output and interest rates is subject to two opposing forces. In equation 3.20, labour supply depends positively on $R^d_t$ due to an income effect – an increase in interest income makes households more willing to work. This dampens the usual labour demand effect, which lowers employment as rates rise. For some parameter configurations, the income effect could dominate the labour demand effect, so that higher rates would go hand-in-hand with higher employment and output. This is more likely in a bust because the lending rate, and hence labour demand, is less responsive to policy. We rule out this possibility of an upper ‘reversal interest rate’, where a sufficiently high interest rate could be expansionary, by adding a technical assumption that the labour supply is perfectly elastic at $W_{min}$, calibrated to always be binding in a bust. As a result, changes in employment during a bust stem entirely from labour demand shifts.

A financial boom-bust cycle exists when bank profits are persistently negative during the boom $s_t=1$, and positive during the bust $s_t=2$. Under such condition, a boom would deplete bank capital and inevitably trigger a bust, while a bust would restore bank capital and eventually usher in a boom. The economy never reaches a resting steady state as a result, but goes through phases of declining and rising bank capital, aggressive and conservative lending as well as high and low economic activity. A financial cycle is more likely to emerge the stronger the lending frictions, which under a special case can be expressed as simple parametric restrictions.

Result 3. When $\theta_L = 0$ and $R^d_t = 1$, sufficient conditions for a financial cycle to exist are

$$\theta_H > \theta_1 - 1 \quad (3.23)$$
$$\kappa > 0 \quad (3.24)$$
$$\theta_1 > \theta_1^* \quad (3.25)$$

where $\kappa \equiv \omega - \frac{1+\chi}{\chi+\sigma+(1-\alpha)(1-\sigma)}$ and $\theta_1^*$ solves

$$\left(\frac{\theta_1^* - 1}{\theta_1}\right)^{\frac{1}{\theta_1^*}} = \frac{1}{N} \left[ (\alpha A)^{1+\chi} \left( \frac{\psi}{\Phi} \right) \right]^{\frac{1}{\chi+\sigma+(1-\alpha)(1-\sigma)}}$$

Proof. See Appendix E. \qed

In other words, a financial cycle tends to arise when $\theta_H$ is high relative to $\theta_1$, and $\theta_1$ is high relative to $\theta_L$ (assumed zero here). Intuitively, high $\theta_H$ and low $\theta_L$ implies that unproductive firms can have very different effects on the lending market equilibrium and bank profitability depending on which regime prevails. In the next section, we calibrate the parameters to satisfy these sufficient conditions as a basic requirement. However, in the case of optimal time-varying $R^d_t$, the conditions for financial cycle to exist become more complicated and cannot be pinned
Table 2: Calibration assumption

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Note: This table reports the baseline parameters used to calibrate the model and compute equilibrium as well as optimal policy.

down in closed-form.\textsuperscript{22}

4 Simulation and the Optimal Policy

4.1 Period equilibrium

We numerically solve for the equilibrium values of endogenous variables conditional on the level of $R^d$, using the set of parameters in Table 2. Those parameters pertaining to households’ intertemporal problem are standard. Others are calibrated such that a financial cycle does arise in equilibrium, to illustrate the full implications of the model with banking sector externalities.\textsuperscript{23} Figure 2 plots the equilibrium values of the endogenous variables in an arbitrary period (vertical axis), as a function of the contemporaneous policy interest rate $R^d$ (horizontal axis). Each panel displays the equilibrium functions for both boom and bust regimes (solid and dash lines, respectively). A number of features are worth highlighting.

First, monetary policy influences the real economy not only through aggregate demand, but also through supply. A lower $R^d$ pushes down the lending rate $R$ in equilibrium. This raises productive firms’ demand for labour, which in turn increases wages, employment and output. And as $R^d$ falls, households bring forward consumption – a standard Euler equation result. In the figure, this is reflected in the rise in consumption of the young ($C_{11}$) and decline in those of the old ($C_{22}$). Alongside the expansion of aggregate consumption demand, productive firms’ production increases resulting in a boost to output, as firms take advantage of lower funding costs to borrow more and scale up production. This is consistent with the ‘cost channel’ of monetary transmission, which emphasises the real effects of monetary policy working through the supply side, here in the form of variations in the marginal cost of production linked to changes in interest expenses (e.g. Ravenna and Walsh (2006) and Gaiotti and Secchi (2006)).

\textsuperscript{22}The challenge is the mutual dependence between optimal policy and the financial cycle. As the optimal policy problem considered here is highly nonlinear and not available in closed-form, it is also impossible to characterise the financial cycle analytically. We rely on numerical methods in the following.

\textsuperscript{23}The model clearly nests the case where a financial cycle does not arise, e.g. when $\bar{N} = 0$ so that there are no unproductive firms.
Note: This figure plots the equilibrium values of endogenous variables in each period (vertical axis), for any given level of policy interest rate $R^d$ (horizontal axis). Solid and dash lines correspond to boom and bust regimes, respectively.

**Figure 2:** Period equilibrium and policy interest rate
Second, while lower interest rates expand firm profits, they generally reduce bank profits (bottom right panel of Figure 2). This is due to the higher participation of unproductive firms, which depresses the overall quality of the borrower pool. In this calibration, there exists a cut-off level for $R^d$ below which bank profits become negative. This creates a key tension: a more accommodative monetary policy boosts economic activity today, but saps bank profitability and drains banks’ net worth. Banks take on too much risk amidst intense competition.24

Third, in the bust regime (dashed lines), banks are more conservative and set a higher $R$ for any given $R^d$. In other words, the interest rate spread $R - R^d$ jumps as the economy moves from a boom to a bust, reflecting banks’ increased conservatism.25 Consequently, output, employment, consumption and firms’ profits are lower for any given level of $R^d$. Because of the fall in labour demand, the minimum wage becomes binding at every level of $R^d$. The binding minimum wage ensures that a lower policy rate is still expansionary for the economy during the bust, even though the policy transmission to the lending rate $R$ is weaker. However, bank profits are higher in this regime, as banks seek to contain bad loans. A bust is painful while it lasts, but allows banks to re-build their capital and repair their balance sheets even at a low risk-free rate.

Finally, note that monetary policy is subject to an effective lower bound. Figure 2 shows that for sufficiently low $R^d$ (approximately less than 1), the lending rate $R$ increases as monetary policy is eased. This second instance of a reversal rate is directly related to bank intermediation (similar to Brunnermeier and Koby (2019)), and thus we let it define the effective lower bound for monetary policy. To see why it exists, note that the effective deposit cost for the bank $\psi_{2t}$ (equation 2.16) is increasing in $\psi_{1t}$ when $R^d_t > 1$, and decreasing when $R^d_t < 1$. Intuitively, below this inflection point, banks are earning rather than paying money to depositors (because of the negative net interest rates) so that a reduction in the policy rate leads to lost income on deposits rather than lower costs. Banks try to offset this by increasing lending rates, leading to perverse output effects.

4.2 Optimal monetary policy

We next endogenise the setting of $R^d_t$ in each period, and let the central bank set the policy rate $R^d_t$ to maximise the social welfare function. This is defined in each period to be an equally-weighted sum of the welfare of households and asset owners – the only agents that consume in the economy – discounted over the infinite future. The policy problem at each time $t$ is

$$\max_{R^d_t} E_t \sum_{\tau=t}^{\infty} \beta^\tau (W_{H,\tau} + W_{A,\tau})$$ (4.1)

24Negative bank profits during the boom may seem contrary to the observation that banks usually report strong earnings during the upswing. But it is precisely during the upswing that key fragilities build up, obscured because either asset prices are inflated or accounting valuations are backward-looking. Thus, one can interpret our results as simply reflecting a more accurate measurement of the true underlying health of banks, incorporating also the vulnerabilities usually hidden from view. Put differently, our model indicates that even if there is no bias in the measurement of risk and valuations, boom-bust dynamics can arise. For recent empirical evidence, see Haubrich (2015) who documents a countercyclical behaviour of bank capital.

25The spread is in general non-monotonic in the policy rate $R^d$. On the one hand, a higher interest rate improves the loan pool quality, lowering the required compensation for loan defaults and the spread. On the other hand, a higher interest rate induces households to save more, increases the proportion of deposits that are held to maturity, and raises banks’ funding cost $\psi_{2t}$, putting an upward pressure on the spread.
The left panel shows the period payoff, the contemporaneous aggregate utility of the agents in the economy as a function of the policy interest rate $R^d$ (horizontal axis), corresponding to boom and bust regimes (solid and dashed lines, respectively). The right panel shows the transition probabilities between boom and bust regimes, depending on the initial regime (blue for boom, orange for bust) and the level of bank capital $K$ (horizontal axis).

**Figure 3:** Period welfare and regime switching probabilities

where

$$W_{H,t} = E_t \left( \frac{C_{H,t}^{1-\sigma}}{1-\sigma} + \frac{C_{H,t}^{2t+1}}{1-\sigma} - \frac{\Phi}{1+\chi} L_t^{1+\chi} \right)$$  \hspace{1cm} (4.2)$$

$$W_{A,t} = E_t \left( \frac{C_{A,t}^{1-\sigma}}{1-\sigma} - \frac{\Phi}{1+\chi} N_t^{1+\chi} \right)$$  \hspace{1cm} (4.3)$$

are the present value lifetime utilities of households and asset owners in period $t$, respectively. The discount factor $\beta$ describes how the central bank compares the payoffs and hence consumption of different generations. It is a normative parameter, and need not equal households’ rate of time preference $\rho$.

The setup gives rise to an intertemporal trade-off for monetary policy. As described above, monetary policy does not just affect output and consumption, but also the degree of risk-taking during booms and hence bank profits and the evolution of bank capital. A higher policy rate today restrains the boom, at the cost of more subdued growth in the short run. At the same time, this ‘leaning’ policy promotes a more robust financial sector and reduces the risk of a bust with severe output consequences down the road. The optimal policy problem involves managing this trade-off.

The left panel of Figure 3 shows the short-term benefit in terms of the period payoff

$$W(R^d_t, s_t, K_t) \equiv W_H(R^d_t, s_t) + W_A(R^d_t, s_t)$$  \hspace{1cm} (4.4)$$

corresponding to each regime $s_t$. As can be seen, the period payoff in the boom dominates that in the bust for any policy interest rate $R^d$. And in both regimes, there is an incentive to set a low policy interest rate and stimulate the economy in the short-term.

Implementing a low $R^d$, however, feeds excessive competition and risk-taking, which
adversely affects bank earnings. At a low enough $R^d$, bank profits turn negative, eroding banks’ net worth and pushing the economy closer to a bust. The probability of this regime switch rises as $K$ declines (right panel of Figure 3, blue line). In addition, the probability of recovering from a bust is positive only after $K$ has reached a sufficiently high level (same panel, red line). Because the recovery threshold is higher than the bust threshold ($K_r > K_c$), both boom and bust regimes are persistent states. Once in a bust, it takes time to escape from it. And during the boom, policy has a window of opportunity to avoid a bust.

The resulting policy problem, which is captured in equation 4.1, can be formally written in recursive form as

$$V(K_t, s_t) = \max_{R^d_t} \left[ W(R^d_t, s_t, K_t) + \beta \left( \sum_{s_{t+1} \in \{1,2\}} P(s_{t+1}|s_t, K_t)V(K_{t+1}(R^d_t), s_{t+1}) \right) \right]$$

(4.5)

subject to the dynamic evolution of the regime and bank capital in equations 3.18 and 3.19, reproduced here.

$$P(2|1, K_t) = G(a_c(K_t - K_c))$$  \quad (4.6)$$

$$P(1|2, K_t) = G(a_r(K_t - K_r))$$  \quad (4.7)$$

$$K_{t+1} = K_t + \Pi(R_t(R^d_t), R_t(R^d_t))$$  \quad (4.8)$$

What does optimal monetary policy look like? The answer is that it ‘leans against the wind’. The solution to this dynamic programming problem is shown in Figure 4. The left-panel presents the optimal choice of $R^d$ as a function of bank capital $K$ and regime $s$. In a boom, the optimal policy generally involves giving up some short-term gains by setting $R^d$ at a higher level than that which maximises the period payoff (as we saw in the left hand side of Figure 3, optimal short-term policy would set $R^d$ close to 1). This ‘leaning-against-the-wind’ policy confers two key benefits. First, it promotes a stronger banking system and lowers the probability of entering a costly bust, effectively internalising the effects of excessive bank competition. Second, by moderating the boom and maintaining higher bank capital on average, it strengthens the resilience of the banking system to face a bust, should it occur. This, in turn, shortens the expected duration of a bust, which boosts welfare.

The optimal policy has a non-linear profile. The central bank leans moderately when bank capital is ample, and steps up the degree of leaning as bank capital declines. At its peak, leaning almost completely stabilises bank capital and fully offsets the bankers’ incentives to over-compete (we consider the case where bank capital is indeed fully stabilised below). For a sufficiently low $K$, when a bust becomes imminent, optimal policy calls for a sharp reduction in the leaning intensity. In this region, the cost of leaning no longer justifies the benefit, and the central bank switches to implementing a very low interest rate to stimulate short-term activity.

In the bust regime, the central bank has significantly less control over the financial system’s evolution, as banks enter a high risk-aversion and balance-sheet-repair mode – equivalent to pushing on a string. Here bank profits are much less sensitive to changes in $R^d$ (see bottom right panel of Figure 2). In addition, bank profits are now positive at any level of the policy rate, which implies that there is no financial stability benefit of setting a high $R^d_t$. In fact, a
lower interest rate, by stimulating economic activity at a time when banks are conservative, provides additional support to bank profits and helps the economy recover faster.

**Result 4.** *During booms, optimal policy leans against the wind. In busts, optimal policy eases to stimulate economic activity to the extent possible.*

Thus, the results support both ‘leaning’ and ‘cleaning’ policies. During a boom, it is optimal to lean against the build-up of financial imbalances (deterioration in bank capital), especially as these grow. And once the economy is in a bust, it is optimal to clean, quite aggressively, to put the economy back on its feet as quickly as possible. In practice, the challenge is to determine when the bust phase is over and to move from cleaning to leaning in a timely manner. Moving too early would prolong the slump. Moving too late would jeopardise the sustainability of the recovery and make it more likely that the economy ends up in a bust again.

The welfare consequences are substantial. The right panel of Figure 4 shows the value functions $V(K,s)$ – the discounted sum of period payoffs under the optimal policy, corresponding to each regime. Welfare in the boom phase strictly dominates that in the bust phase, reflecting the fact that regimes are persistent – once a bust ensues, not only is current consumption low, it is also expected to be low for a protracted period. As bank capital increases and a boom phase edges closer, the discounted sum of the payoffs rises, so that the value function is upward sloping. The virtue of leaning is that it shores up bank capital and thus the continuation value, even as it sacrifices some short-term payoff.

### 4.3 Cycle dynamics and the equilibrium distribution

A key feature of the model is that it accommodates the possibility of recurrent financial cycles. To illustrate, we conduct dynamic simulations for our baseline calibration over 100 periods and under different random draws that determine the exact timing of regime switches. The top
Note: Dynamic simulation under optimal monetary policy, when $\beta = 0.9$. Histograms based on 100 rounds of simulations, each lasting 100 periods. The economy is initialised with $K = 10$ and a boom regime.

Figure 5: Dynamic simulation and equilibrium distribution

Panels in Figure 5 show the evolution of bank capital $K_t$ (top left panel) and output $Y_t$ (top right panel) from two different simulations. Both $K_t$ and $Y_t$ cycle endogenously over time, as banks switch between spells of aggressive and conservative lending equilibria. The economy starts in a boom, with bank capital equal to 10. Aggressive lending leads to loan losses, initially reducing $K_t$. The fall in bank capital accelerates at lower values of $K_t$, as the central bank optimally stops leaning at this point, and the economy eventually enters a bust. In this phase, bank capital is rebuilt until $K_t$ rises sufficiently and there is a recovery into a boom phase. The cycle then continues. There is a similar periodic swing in the level of output (top right panel), with high output in booms and low output in busts.

Due to stochastic regime switching, individual cycles vary, hence the difference between the two simulations. In the long run, the distribution of equilibrium variables converges to a stationary one – the model’s analogue of a ‘steady state’. This distribution can be approximated numerically (thanks to the ergodic theorem) by simulating the model and obtaining the empirical distribution. The histograms for output and bank capital are shown in the two lower panels of Figure 5, computed from 100 rounds of simulations each for 100 periods.

Because of the recurrent financial cycle, the stationary distribution for output is bi-modal. In a bust, the distribution of output is concentrated in a very narrow range, as it is hardly affected by policy. In a boom, the distribution is skewed, with the mode being lower than the mean. This reflects the leaning policy, as the central bank gives up some upside for output to curtail the financial boom. The equilibrium distribution for $K_t$ is unimodal and shows a similar skew for the same reason – policy tends to favour higher capital and stabilise it during
the boom.

A distinctive feature of the economy’s evolution is path dependence (Figure 5). Past decisions influence the trade-offs policy faces today. Bygones are not bygones. This dependence reflects the path of bank capital and the sequence of boom-bust phases – the state variables in our model that describe the evolution of financial imbalances. The structural link between the booms and the subsequent busts generates a powerful and rather neglected monetary policy transmission channel. We next explore its implications.

5 The role of monetary policy frameworks

In our setup, the monetary policy framework acts as the anchor for the economy. Absent a unique ‘natural interest rate’ to which the economy gravitates, the long-run evolution of interest rates depends on the monetary regime. Moreover, such a regime determines the economy’s proclivity to boom-bust cycles – a key property that influences the output path in the long-run.

What are the implications of different policy frameworks? We next derive two main results. First, the extent to which the central bank discounts future welfare has a bearing on how much it is willing to lean against the boom, which in turn has different long-term consequences for the real economy. Second, the degree of policy inertia has non-trivial implications, but these depend critically on whether the central bank takes future inertia into account when setting policy today.

5.1 Central banks’ discount factors

Given the intertemporal trade-off, the key element of the monetary policy framework is the relative weight assigned to the welfare of agents of different generations. We therefore compute optimal policy for central banks with various discount factors $\beta$. The higher the $\beta$, the more forward-looking – and hence more ‘egalitarian’ – the central bank is in assessing welfare. Conversely, as $\beta$ falls, the greater is the relative weight placed on short-term outcomes.

In the baseline calibration above, $\beta = 0.9$. Here we explore the system’s behaviour for $\beta = 0.95$, $\beta = 0.85$, and $\beta = 0.8$. Figure 6 plots the optimal policy corresponding to the four policymakers during the boom regime. A lower discount factor leads to a weaker leaning policy for every $K$. The optimal timing is also different. Short-termist central banks start leaning late, preferring to wait until bank fragility becomes more evident ($K$ is lower). They also give up leaning more quickly as fragility worsens, preferring to stimulate the economy even when the probability of a bust remains relatively low.

The implications of monetary policy frameworks for the evolution of the economy can be significant. Consider two simulation draws over 100 periods corresponding to central banks with $\beta = 0.95$ and $\beta = 0.8$, shown in Figure 7. Clearly, outcomes under the more myopic central bank with $\beta = 0.8$ are more volatile, with output fluctuating much more as the economy alternates frequently between booms and busts. Policy leans very little, with interest rates set at low levels most of the time to maximise short-run payoffs. The more rapid decline in bank capital then triggers the bust, forcing the central bank to lower the policy rate again. Here,

\[\text{In a bust, all central banks set the interest rate at a similarly low level to maximise short-term payoff.}\]
the interest rate is the same on average over successive cycles. In this sense, low interest rates beget low interest rates. We show later a case where it actually declines.

In contrast, a central bank with $\beta = 0.95$ places a greater weight on future outcomes and implements a higher interest rate for any given level of bank capital. In this case, monetary policy is actually able to completely stabilise bank capital and the economy remains in a boom for an extended period. As a result, the interest rate is higher on average, since busts are few and far between. In this particular simulation, it takes a large negative shock near the middle of the sample to trigger a bust. The bottom right-hand panel of Figure 7 confirms that the relative time spent in a bust is much shorter.

Another way to illustrate the importance of monetary policy frameworks is to examine long-term equilibrium outcomes. As before, we conduct 100 simulations, each lasting 100 periods, for the economy under the three different policy regimes. Figure 8 shows histograms serving as numerical proxies for the ergodic distribution.

The more forward-looking the central bank, the higher is the interest rate, on average (top-left panel). This results in a more stable financial system with bank capital generally higher and less volatile (top-right panel). The economy therefore spends less time in a bust (bottom-left panel) and output is generally high and stable, though slightly below the highest level achieved in the other regimes given the short-run trade-off (bottom-right panel).

**Result 5.** More forward-looking (egalitarian) central banks with a higher discount factor $\beta$ lean more and earlier against the booms. As a consequence, they deliver on average more stable financial systems with more prolonged booms and higher real interest rates.

### 5.2 Policy inertia

The analysis so far assumes that optimal monetary policy adjusts freely over time in either direction. As a result, policy rate adjustments are generally very sharp. In practice, central
Note: Dynamic simulation under optimal monetary policy, when the central bank’s discount rate is $\beta = 0.95$ and $\beta = 0.8$. Both simulations last for 100 rounds, and are based on the same sequence of random numbers used to determine regime switches (i.e. identical shocks). The economy is initialised with $K = 10$ and a boom regime.

Figure 7: Simulated time-series under two monetary policy frameworks

banks typically adjust policy gradually and in small steps. Coibion and Gorodnichenko (2012), for example, provide evidence of strong policy inertia and discuss possible rationales. And such interest rate smoothing is a standard feature in Taylor rule estimates, (e.g. Carlstrom and Fuerst (2014)). A key question, then, is whether policy inertia contributes to policy being ‘behind the curve’ and hence to raising the economy’s proclivity to experiencing boom-bust cycles.

To examine this issue, we explore the implications of assuming that changing the policy rate involves adjustment costs. We conduct two exercises, which lead to very different outcomes. In one, the central bank attempts to track the first-best optimal policy but is forced to adjust policy only gradually in small steps. In this case, the boom-bust cycle becomes more virulent over time as policy is hindered in leaning against the upswing. As a result, interest rates drift downwards and the economy spends more and more time in the bust phase, despite no asymmetry in the gradualism constraint.

In the second exercise, the central bank is subject to quadratic adjustment costs but fully anticipates and internalises these costs, current and prospective, in setting policy today. In this case, the central bank leans even more strongly against the upswing than under the baseline. In doing so, it avoids busts and the associated policy adjustment costs down the road.
Note: Histograms based on 100 rounds of simulations, each lasting 100 periods. The economy is initialised with \( K = 10 \) and a boom regime.

**Figure 8:** Equilibrium distribution under different monetary policy frameworks

### 5.2.1 Constrained policy adjustments

Consider the first case. Here, the central bank cannot adjust the policy rate by more than a fixed amount \( \delta \) each period. Let \( R^d_t^* \) denote the optimal policy. Specifically, the central bank sets the interest rate according to

\[
R^d_t = \begin{cases} 
R^d_t^*, & \text{if } R^d_t^* \in [R^d_{t-1} - \delta, R^d_{t-1} + \delta] \\
R^d_{t-1} + \delta, & \text{if } R^d_t^* > R^d_{t-1} + \delta \\
R^d_{t-1} - \delta, & \text{if } R^d_t^* < R^d_{t-1} - \delta
\end{cases}
\]

In other words, the central bank attempts to mimic the baseline optimal policy as much as possible, subject to changing its policy rate by at most \( \delta \) per period.

Figure 9 depicts the simulated time series of key variables. The economy starts with \( R^d_t \) close to the baseline optimal policy. Initially, the constrained central bank is able to lean against the financial upswing not very differently from the unconstrained one (top left panel). Once a bust ensues, the central bank lowers the interest rate, albeit more gradually and by less than in the baseline. As the economy recovers and enters a boom again, the central bank finds itself with a lower starting interest rate and a binding adjustment constraint which curtails the degree of leaning against the boom. Indeed, before the central bank can raise the policy rate enough to curb the boom or normalise the policy rate level, another bust occurs, which forces another round of rate cuts.

This ratcheting process continues to push the interest rate down over successive cycles,
Dynamic simulation when the central bank attempts to implement an interest rate as close as possible to the baseline optimal policy, subject to a constraint that it cannot change the policy rate by more than $\delta = 0.01$ per period. The simulation lasts for 100 rounds, and are based on a sequence of random numbers used to determine regime switches. The economy is initialised with $K_0 = 10$, a boom regime and $R_{0} = 1.3$.

**Figure 9:** Simulation with constrained policy adjustments

with busts becoming not only more frequent but also more drawn out. The constrained central bank in effect falls into a ‘low interest rate trap’, with an increasingly potent financial cycle and a downward trend in interest rate reinforcing each other, rendering an escape impossible. Here low rates beget lower rates.

**Result 6.** Failure to address the booms over successive cycles can lead to more frequent busts and, as a result, a secular decline in the real interest rate.

### 5.2.2 Optimal policy under adjustment costs

A qualitatively different result emerges in the second scenario, in which the central bank fully internalises the effects of adjustment costs associated with its future decisions when formulating policy today. It is convenient to consider this problem in the context of quadratic adjustment costs, which simplifies the computation of interior optimal policy. Specifically, the central bank’s period payoff is assumed to take the form

$$W^*(R_t^d, R_{t-1}^d, s_t, K_t) = W(R_t^d, s_t, K_t) - c_{I(R_t^d > R_{t-1}^d)}(R_t^d - R_{t-1}^d)^2$$

where the adjustment cost $c_{I(R_t^d > R_{t-1}^d)}$ is equal to $c_1$ when the interest rate is increased and $c_0$ when it is cut. This formulation allows for an asymmetry in the adjustment cost if $c_0 \neq c_1$. For illustrative purposes, we assume that it is costlier to increase the interest rate than to cut it by
The left panel shows the optimal policy surfaces as a function of $K_t$ and $R_{t-1}^d$ during a boom, corresponding to costly policy adjustment case and the baseline. For costly adjustment, it is assumed that $c_1 = 10$ and $c_0 = 2$. The right panel shows two simulations under an identical draw of shock and initial conditions ($K_0 = 10$, a boom regime and $R_0^d = 1.3$).

Figure 10: Optimal policy under costly policy adjustments

The dynamic programming problem now involves an additional state variable, $R_{t-1}^d$, reflecting an additional form of history-dependence. The left panel of Figure 10 shows the optimal policy surface under adjustment costs as a function of bank capital $K_t$ and the previous period interest rate level $R_{t-1}^d$. For comparison, the panel shows also the optimal policy from the baseline case without adjustment costs, which by construction does not vary with $R_{t-1}^d$. Qualitatively, the result appears quite intuitive. The optimal policy varies positively with the initial level of the policy rate, implying that monetary policy now involves some inertia.

Strikingly, this case strengthens the case for leaning against the wind. The reason is that acting pre-emptively helps the central bank avoid large policy adjustments associated with booms and busts. Optimal policy actually involves more leaning at the outset. This is illustrated in the right panel of Figure 10, which shows two simulations of $R_t^d$ corresponding to the baseline and the adjustment cost case, both with identical initial conditions, parameters and shock sequence. Despite a penalty on interest rate increases, the central bank sets interest rate to a higher level right at the outset relative to the baseline. The intuition is that leaning earlier, and incurring adjustment costs immediately, reduces the risk of a future bust and the associated need for large and costly interest rate adjustments. Taken together, the two cases suggest that policy inertia per se is not sufficient for policymakers to fall behind the curve with respect to the financial cycle. Indeed, were policymakers to internalise adjustment costs, they would lean even more today to avoid situations where large interest adjustments are needed down the road.

Policy inertia is more problematic when it constrains large required policy actions in real time in ways not anticipated beforehand. This is arguably the more realistic case, as policymakers can hardly foresee how their hands may be tied in the future (e.g. how uncertainty

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27 The case with symmetric adjustment costs yields qualitatively similar results.
may force them to act cautiously in the future). The analysis points to systematic deviations from optimal policy that is not internalised as the root cause of policy falling behind the curve, with the effects cumulating over time. The predictions of the model in this case, with interest rate normalisation being repeatedly stunted by financial dislocations, looks eerily similar to developments over the last two decades.

5.3 Policy discussion

Our analysis highlights a number of key policy issues.

Most prominently, our framework underlines the path-dependence of policy. With financial fragility acting as a state variable that links developments today to outcomes in the distant future, policy options available today depend on the long sequence of previous policy decisions. This sequence of decisions does not just have a transient impact on the economy, but leaves a long-lasting imprint.

A case in point is the long-run evolution of real interest rates. Prevailing frameworks presume that their evolution is determined exclusively by real saving-investment factors, such as preferences, productivity growth and demographics. These factors are seen as determining some notional equilibrium natural rate of interest. By contrast, in our framework, the monetary policy regime plays a crucial role in steering real interest rates over time, through its influence of the economy’s proclivity to boom-bust cycles. A policy rule that places a smaller weight on future outcomes, and hence leans less against the financial cycle, not only prescribes a lower interest rate in normal times, but also leads to a higher incidence of busts and hence lower interest rates in the future. From this perspective, the trend decline in real interest rates in recent decades could be viewed, at least in part, as a consequence of financial boom-bust cycles. Examples include those generating the financial strains in the early 1990s and the GFC (e.g. Drehmann et al. (2012), Borio (2014b)). As higher debt and other distortions weigh on the economy over successive cycles, it becomes increasingly hard to raise rates without damaging the economy and policy runs out of ammunition.

Recognising the impact that monetary policy may have on longer-run outcomes through its impact on financial fragility significantly changes the policy trade-off associated with leaning against the wind. In prevailing approaches, risks are not expected to grow over time in the absence of leaning. This is because crises are effectively seen as the result of exogenous shocks, so that leaning only reduces the probability of a crisis at any point in time – always a remote tail event – but does not have a cumulative impact over time. There is thus little or no cost to waiting. This encourages the view that a financial stability-oriented monetary policy follows a traditional strategy most of the time and then deviates from it only once the signs of financial imbalances become evident. Our analysis indicates that when policy influences risk cumulatively, and hence how vulnerabilities build over time, the cost of waiting can be large and grows as time passes. This constitutes a hitherto neglected channel of transmission that makes the intertemporal policy trade-off much more acute.

Importantly, in our analysis financial stability matters for macroeconomic policy because it affects the feasible set of output and consumption in the future. The fundamental trade-off is

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28See Borio et al. (2018) and Adrian and Liang (2016) for a review.
an intertemporal one, between short-run and long-run economic activity. Financial stability is simply a means to an end. In this sense, ‘financial stability’ in our context is synonymous with ‘macroeconomic stability’ (indeed, in our model there are no bank failures). This contrasts with the popular view that sees financial stability as an additional, separate objective to be traded off with output.

In standard setups, the only variable that monetary policy affects in the long run is inflation. To the extent that high and variable inflation harms long-run output, this justifies a primary focus on inflation as a target of policy. But if monetary policy can also affect the economy’s proclivity to boom-bust cycles, and if these cycles, in turn, affect output persistently – and possibly inflation, too – this conclusion is less justifiable. Under such circumstances, it stands to reason that financial stability considerations cannot be ignored. That is, both inflation and financial cycles are, to varying degrees, a monetary phenomenon. Monetary policy cannot simply aim to be neutral, to ‘do no harm’ as it were, because it cannot. To put it starkly, the issue may not be so much whether monetary policy should lean against the wind, but rather that monetary policy is the wind.

Could macroprudential tools solve the problem set out in our model? While a precise answer depends on explicit modelling assumptions, which we leave for future research, we note a couple of general observations. First, given that there are no sectoral imbalances in our framework, a key benefit of macroprudential policy in dealing with specific segments of the economy would be absent. Second, conventional macroprudential tools could not surgically remove the underlying friction at play here – that banks do not internalise how their interest rate settings collectively affect the aggregate quality of borrowers. Standard measures such as minimum capital requirements would effectively entail forcing banks to set interest rates high enough, which result in lower output. As such, macroprudential measures would face the same trade-off as monetary policy. That said, macroprudential tools would add an extra degree of freedom, enabling the policymaker to regulate both risk-taking (the lending rate) and household intertemporal consumption allocation (the deposit rate). But this would be second-order relative to the welfare consequences of boom-bust cycles. So long as the intertemporal trade-off could not be eliminated by any policy tool, the first-order welfare gain from leaning is likely to justify a combination of (mutually reinforcing) policies.

6 Conclusion

The debate on the appropriate role of monetary policy with respect to financial stability has been an active one, spurred once more by the GFC. Views necessarily differ depending on one’s underlying model of the economy. We have argued that by assuming long-run money neutrality, prevailing frameworks neglect a key, and potentially important, transmission channel. By relaxing this assumption, the monetary policy regime becomes a significant factor in influencing longer-run economic outcomes, including the real interest rate. We are by no means arguing that it is the only factor – central banks set policy in response to economic developments and are hence influenced by a confluence of factors related to the structure of the economy. Rather, we simply offer a complementary explanation to that based on traditional saving-investment
factors, whose empirical performance in explaining real interest rates has been rather weak (e.g. Borio et al. (2017)).

In drawing this conclusion, it is also important to distinguish between the specific model developed in this paper and broader considerations. The model is purely illustrative. It focuses on a specific source of instability – incentive problems in the supply of credit. In practice, there are other sources of instability, such as limitations in the measurement of risk owing to imperfect information and problems on the demand side, which can result in over-indebtedness and debt overhangs (Borio et al. (2001)). Moreover, as empirical evidence indicates, financial cycles may have persistent first-order effects on the misallocation of resources and hence productivity. Such considerations, ignored in our analysis, would strengthen our conclusions concerning the long-term impact of monetary policy.

Our goal in this paper was simply to shed light on a set of theoretical conditions that could justify the implementation of a more financial stability-oriented monetary policy. Of course, reaching a proper conclusion requires consideration of a broader set of issues, many of which are empirical and extend to the analysis of alternative instruments, such as prudential policies (e.g. Borio (2014a) and Borio et al. (2018)). The paper provides just one more piece of the puzzle.
References


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Appendix A  Balance sheet mechanics

This appendix details the book-keeping of all agents in the simple financing model. Figure 11 shows entries in a given period, assuming a steady state. We consolidate old and young firms’ balance sheets for brevity.

**Figure 11: Balance sheet entries**

<table>
<thead>
<tr>
<th>Banks</th>
<th>Firms (consolidated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset</td>
<td>Liability</td>
</tr>
<tr>
<td>①</td>
<td>RWL</td>
</tr>
<tr>
<td></td>
<td>R^d(WL − C_1)</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>−C_1</td>
</tr>
<tr>
<td></td>
<td>−C_1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Young Households</th>
<th>Old Households</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset</td>
<td>Liability</td>
</tr>
<tr>
<td>①</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>②</td>
<td>WL</td>
</tr>
<tr>
<td></td>
<td>WL</td>
</tr>
<tr>
<td>③</td>
<td>−C_1</td>
</tr>
<tr>
<td></td>
<td>−C_1</td>
</tr>
</tbody>
</table>

Note: This figure traces steady-state balance sheet entries of all agents in a given period. The first-row entries, marked by ①, are initial balances at the beginning of each period. Entries ② correspond to new bank lending and creation of new bank deposits. Entries ③ document the book entries after the goods market opens, relating to the payments of goods by households to firms and settlement of bank loans by firms to banks. As the entries represent steady-state transactions, the aggregate positions net out to equal the initial conditions in row ① after taking interest rates into account.

The initial positions at the beginning of the period are marked by ①. Banks’ assets are loans extended last period plus interest, RWL, which also appear as firms’ liabilities. Banks owe deposit plus interest to old households worth R^d(WL − C_1) (appearing in old households’ books as equity), where the principal is WL − C_1 because households withdrew C_1 last period for immediate consumption. These outstanding claims and liabilities earn banks net interest income of NII, which is zero in equilibrium. Recall from equation 2.5 that NII is defined as (R − 1)WL − (R^d − 1)(WL − C_1) = RWL − R^d(WL − C_1) − C_1. Equating it to zero implies RWL = R^d(WL − C_1) + C_1, namely that banks have a residual liability term of C_1 as of stage ①. This represents bank equity which will always be carried forward. It arises because, in any period, part of newly created deposit is used to extinguish old loans, so that bank assets exceed liabilities. ② Finally, firms have output ALα coming due as their assets, and owe dividend π^F

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29 This can also be viewed as a consequence of the timing feature of overlapping generations setups. In these models, there is always a question of how the young households obtain the goods at the very first date when there is not yet any production. One solution is to assume that banks supply these initially out of their...
to old households.

In step 2, banks grant new loans to firms worth $WL$, creating new financial claims in the economy. Banks’ assets represent 1-period loans to firms, while liabilities represent obligations to depositors. Young firms transfer this deposit to young households for labour services. Accrual accounting convention means interest rates are not included until next period, while young firms’ assets are booked at cost $WL$ for the moment as production only finishes next period. Young households now have $WL$ deposits, representing their equity.

In the final step 3, the goods market opens and loan repayments take place. Old households transfer the remaining deposits of $R^d(WL - C_1)$ to firms in exchange for goods, while young households similarly transfer $C_1$. Households consume these goods (plus dividend $\pi^F$ in the case of old households). Firms receive the deposit transfers totalling $R^d(WL - C_1) + C_1$, just enough to repay maturing bank loans of $RWL$ (again using the fact that $R^d(WL - C_1) + C_1 = RWL$ when $NII = 0$). Firms distribute the output $AL^\alpha$ to households corresponding to deposit transfers, and rebate the remaining $\pi^F$ to old households as dividend. Entries at step 3 exactly cancel the initial positions at step 1. For banks, all outstanding loans in 1 are settled by firms’ repayments in 3 leaving only new loans of $WL$, while deposit liability nets to $WL - C_1$. All agents’ balance sheets contract at this final stage, as a round of financing runs its course.

The net balance sheet positions, when shifted forward one period, are exactly the initial positions at step 1, consistent with the steady-state situation. Banks’ assets, once including interests, become $RWL$. For liability, banks now owe $R^d(WL - C_1)$ of deposits to households, earn $NII$ and carry over equity of $C_1$. For firms, assets become output worth $AL^\alpha$ with profit $\pi^F$ booked as the difference over liability plus interest of $RWL$. Previously young households become old, and their net positions plus interest becomes $R^d(WL - C_1)$ plus the firm dividend.

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endowment, which is the source of capital $C_1$. 41
Appendix B  The illusive natural rate of interest

This appendix examines the mechanism behind the ‘no natural rate’ result through variations of the simple financing model. We contrast our financing model with the usual approach of embedding a financial asset in a real exchange model. We then examine the role of the overlapping generations structure versus the representative household with an infinite horizon, in the context of the financing model. Finally, we consider the financing model under overlapping generations, but where households do not engage in intertemporal substitution.

B.1 Financing versus real exchange models

Consider first the financing variant, where a financial asset is created to facilitate trade and destroyed after that trade is complete. To expose the mechanism and allow for a direct comparison with the real exchange model, we abstract from banks and make the strong assumption that firms perform the banking role including the issuance and settlement of inside money. We replace bank deposits by divisible IOU contracts issued by firms to young households in exchange for their labour, with a promise to repay in goods. There are no agency costs, and firms are committed to accept these IOUs in exchange for goods when they complete production next period. Each firm accepts IOUs of other firms including those of the next cohort. Thus these IOUs serve as both a store of value and a medium of exchange, allowing young households to purchase goods immediately or save for future consumption. As with banks, the IOUs have a term structure – if households hold on to them until the next period, they are entitled to an interest rate of \( R^d_t \) on the remaining balance.

This stripped-down financing model shares the same core properties as the original one with banks. In particular, the goods market is guaranteed to be in equilibrium at any steady state interest rate \( R^d_t \). To see this, note that the household problem is identical to the original model with banks, involving the same budget constraint:

\[
C_{2t+1} \leq R^d_t(W_tL_t - C_{1t}) + \pi^F_{t+1}
\]

What changes is the firms’ profit function, which now reflects firms’ commitment to honour IOUs on demand. For a firm born at date \( t \), its realised profit is its final output minus goods demanded by the young households in period \( t+1 \), minus goods owed to the old households inclusive of interests on their first-period balances. Thus,

\[
\pi^F_{t+1} = F(L_t) - C_{1t+1} - R^d_t(W_tL_t - C_{1t})
\]

Substituting firm profit into the household budget constraint and focusing on a steady state, one gets the goods market equilibrium condition

\[
F(L) = C_1 + C_2
\]

which is automatically satisfied regardless of the steady state \( R^d_t \). Analogous to the case with banks, the financing contract issued by firms spells out \textit{ex ante} how much output is to be
produced as well as how it should be distributed among different agents. As a result, the goods market is always in equilibrium, and does not require the interest rate $R^d$ to clear it.

**Result 7.** *In a financing model with inside money creation and overlapping generations of households, there is no natural interest rate and there is room for policy non-neutrality.*

Consider next the real exchange model where a financial asset is not constantly created and destroyed, but a commodity in fixed supply that can be traded against goods. We assume that firms are endowed with a fixed amount of cash, universally accepted as the means of payment. Young firms pay cash to young households in exchange for labour, to initiate production. When production is complete, old firms sell goods to households, re-acquire cash, and bequest it to young firms to start a new round of production. Cash and goods are thus two commodities that can be traded for one another. The return from holding cash is the interest rate $R^d_t$, which is the rate of deflation as in a standard overlapping generations model with money. Households’ problem is identical to before, except that any saving is in cash rather than IOUs. For firms, the cost of production is the opportunity cost of not holding money, so their period profit is given by

$$\pi^F_{t+1} = F(L_t) - R^d_t W_t L_t$$ \hspace{1cm} (B.4)

Substituting this into the households’ budget constraint, re-arranging and looking at the steady state, one gets

$$F(L) = R^d C_1 + C_2$$ \hspace{1cm} (B.5)

In this case, the goods market equilibrium condition is satisfied only if $R^d = 1$ in the steady state, namely the natural rate of interest is zero. This is the same result as in a standard overlapping generations model with fixed money supply, because inflation rate is zero.

**Result 8.** *In a real exchange model with overlapping generations of households, there is a natural rate of interest (zero under fixed money supply and no income growth).*

Comparing these two modelling approaches highlights the key assumption underlying the natural rate concept. In the real exchange modelling approach, the risk-free financial asset is a commodity that is traded alongside goods. The real-side equilibrium conditions thus have a direct bearing on the price of this financial asset by virtue of the Walras’ law. This is why the natural rate of interest is often defined as the level that equilibrates the goods market. In the financing model by contrast, the creation of the financial asset generates purchasing power that contractually arranges for the production, while also specifying relevant obligations that dictate how the goods were to be distributed. This guarantees *ex ante* the goods market will always clear *ex post*. The budget constraint of the money issuer indeed ensures that the goods market clears as a matter of an accounting identity. Because the goods market equilibrium condition does not impinge on the interest rate determination, the natural interest rate fails to exist.

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30 We assume firms are run by generations of entrepreneurs with altruistic motives, so that old entrepreneurs have an incentive to accept cash.
B.2 Bank financing with a representative agent

The overlapping generations structure plays a key part in ensuring that multiple steady state interest rates are possible, because it introduces household heterogeneity that weakens the aggregate intertemporal substitution effect. To see this, consider the case of an infinitely-lived representative household in a financing economy. This household maximises life-time utility by choosing consumption and labour supply plans

$$\max_{C_t, L_t} \sum_{t=0}^{\infty} \rho^t \left( \frac{C_{t+1}^{1-\sigma} - \Phi}{1+\chi} L_t^{1+\chi} \right)$$  \hspace{1cm} (B.6)

subject to the budget constraint

$$C_t + D_t \leq R_t D_{t-1} + W_t L_t + \pi_{t+1}^F$$  \hspace{1cm} (B.7)

where $D_t$ denotes bank deposit balance. This household problem appears isomorphic to the standard consumption-saving problem. But as discussed in the previous section, the economic interpretation and the rest of the model are fundamentally different from an endowment or a real exchange economy. In a financing economy, $D_t$ is in positive supply, created by banks as the counterpart of loans to finance new production every period. In the real exchange model, $D_t$ would have been a financial asset, typically a bond in net zero supply, issued by the household to borrow and lend with itself. This distinction has important implications for the general equilibrium, as explained above.

Nevertheless, the representative household assumption implies that the solution to the household problem alone is sufficient to pin down the natural interest rate. Note that the first-order condition of the household problem gives rise to the familiar Euler equation

$$\left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} = \frac{1}{\rho R_t^d}$$  \hspace{1cm} (B.8)

In the steady state with constant consumption level, this implies immediately the existence of the natural interest rate.

**Result 9.** With bank financing and a representative household, the steady-state real interest rate is exogenously given by $R_t^d = 1/\rho$. There are both the natural interest rate and policy neutrality.

With the representative agent, intertemporal substitution dominates aggregate demand determination. While a new round of financing initially introduces a balanced increase in output and purchasing power through the dual process of credit and deposit creation, changes in the interest rate still induce the representative household to exercise its purchasing power at different dates. By construction, only the interest rate can play the role of regulating aggregate demand and ensuring a goods market equilibrium. Hence the notion of the natural interest rate becomes relevant despite bank financing assumption.

The rest of the model mirrors that of the previous section and pins down the steady-state level of consumption, employment and output, but does not affect the existence of a natural
interest rate and monetary policy neutrality. Competitive banks, as in the previous case, charge
lending rate at funding cost, \( R_t = R^d_t \), because the entire deposit balance is held to maturity by
the representative agent. In each period, the household uses the maturing deposit plus interests
to purchase goods that are just finished in the current period, and deposit all income earned in
this period with the banks (the proceeds of which are used to pay for next-period goods).
Appendix C  Anchoring real interest rate under flexible prices

This appendix shows that the policy non-neutrality result does not require price stickiness assumption. We consider the case of fully flexible prices and show that the central bank can achieve any inflation target by systematically anchoring the nominal interest rate to the real policy rate it wishes to implement.

We follow closely the approach of the classical monetary theory described in Galí (2015) and Woodford (2011), by introducing inflation through the Fisher equation:

\[ i_t = R^d_t + E_t(π_{t+1}) \]  
\[ (C.1) \]

where \( i_t \) is the nominal policy rate, \( R^d_t \) is the real interest rate that the central bank wishes to implement and \( π_t \) is the inflation rate. The usual interpretation of the Fisher equation applies – for a given real interest rate determined by the central bank, the nominal interest rate must move one-to-one with the expected inflation. Let the central bank adopt the following policy rule when setting the nominal interest rate \( i_t \)

\[ i_t = \bar{π} + φ_π(π_t - \bar{π}) + R^d_t + v_t \]  
\[ (C.2) \]

where \( \bar{π} \) is the central bank’s inflation target, \( φ_π \) is the policy loading on the inflation gap, and \( v_t \) is a white-noise policy error. Thus, the central bank anchors the nominal rate \( i_t \) to the real interest rate it intends to set, as well as the deviation of inflation from its target.

The Fisher equation \( C.1 \) and the policy rule \( C.2 \) together imply

\[ π_t = \left( \frac{φ_π - 1}{φ_π} \right) \bar{π} + \frac{1}{φ_π} (E_t(π_{t+1}) - v_t) \]  
\[ (C.3) \]

Solving forward, one obtains an explicit form for inflation

\[ π_t = \bar{π} + \sum_{k=0}^{∞} \left( \frac{1}{φ_π} \right)^{k+1} E_t(v_{t+k}) \]  
\[ (C.4) \]

\[ = \bar{π} \]  
\[ (C.5) \]

The central bank can therefore achieve any inflation target \( \bar{π} \) it wishes, as in the classical theory, by following an appropriate nominal interest rate rule.

Price level determinacy is guaranteed when the Taylor principle is satisfied, namely when \( φ_π > 1 \). Under this condition, the influence of future expected inflation on the current inflation weakens with the horizon, so that forward iteration converges. Note that in the standard model, the summation on the right hand side of equation \( C.4 \) would also include the real interest rate gap term (the difference between the real interest rate and the exogenous natural rate), and \( φ_π > 1 \) also ensures that a non-explosive weighted sum exists, and hence inflation determined.
Appendix D  Banks’ market shares

This appendix provides a microfoundation for the bank-firm matching assumption under imperfect lending competition, deriving banks’ market shares in equations 3.8 and 3.9. We present two complementary approaches to motivate market power that allows each bank to charge a different interest rate from its competitors.

D.1 Preferences for diversity

The first approach follows closely the modelling of pricing power in the New Keynesian literature and is grounded on firms’ preferences for bank diversity. Let a representative firm have a CES preference over services from different banks, reflecting firms’ desire for a diverse set of relationships with banks. Consider first the case of productive firms. Let $M_{it}$ denote the amount of loans a firm obtains from bank $i$, and $R_{it}$ the corresponding lending interest rate. Total loan $L_t W_t$ and average interest rate $R_t$ are aggregate indices of the bank-level variables:

$$L_t W_t \equiv \left( \int_0^1 M_{it}^{\frac{\theta_1}{\theta_1-1}} di \right)^{\frac{\theta_1}{\theta_1-1}}$$  \hspace{1cm} (D.1)

$$R_t \equiv \left( \int_0^1 R_{it}^{1-\theta_1} di \right)^{\frac{1}{1-\theta_1}}$$  \hspace{1cm} (D.2)

where $\theta_1 > 1$ represents the elasticity of substitution, and the number of banks is normalised to one (hence the integral limits). The total cost of borrowing for a given firm is

$$\bar{R} = \int_0^1 R_{it} M_{it} di$$  \hspace{1cm} (D.3)

Firms decide how much to borrow from each bank by maximising equation D.1 subject to D.3. Solving this problem gives rise to the standard demand equation for bank $i$’s loan:

$$M_{it} = \left( \frac{R_t}{R_{it}} \right)^{\theta_1} L_t W_t$$  \hspace{1cm} (D.4)

which depends on the relative price $R_t / R_{it}$ and the degree of bank market power $\theta_1$. This establishes the first part of bank profit function in equation 3.8. Note that the restriction $\theta_1 > 1$ ensures that the interest rate markup, $\theta_1 / (\theta_1 - 1)$ in equation 3.11, is sensibly positive.

Identical derivation applies in the case of unproductive firms, except the aggregate loan is given by $N/R_t^\omega$, and $\theta = \theta_2$. In this case, we assume in addition that unproductive firms value bank diversity less when the average lending rate $R_t$ is higher – in other words, when credit is scarce, they care more about obtaining cheapest possible loans than having an access to a diverse group of banks. For simplicity, the diversity preference parameter $\theta_2$ is set at a constant $\theta_H$ for higher values of $R_t$, and $\theta_L$ otherwise, where $\theta_H > \theta_L > 1$.

D.2 Preferred habitat

The second approach motivates market power directly as arising from firms’ preference to be matched with their chosen banks. Assume that each firm is initially assigned to its first-choice
bank $i$, but has an option to switch to a second-choice bank $j$ – call this firm $(i,j)$. When firm $(i,j)$ decides to move to bank $j$, it incurs a switching cost $c \in [0, \infty)$ (in utility unit) and receives a utility $v(R_j/R_i) \geq 0$ that is strictly positive if bank $j$ offers a cheaper loan, with $v(1) = 0$ and $v' < 0$. Firm $(i,j)$ will thus leave bank $i$ for bank $j$ if and only if

$$v\left(\frac{R_j}{R_i}\right) > c$$

This assumption endows bank $i$ with market power, as firm $(i,j)$ will stay with bank $i$ even if it can obtain a lower rate from bank $j$, provided the rate differential is not too large. We assume that for each bank $i$ there is a continuum of firms $(i,j)$, where $j$ is uniformly distributed across all non-$i$ banks and the switching costs $c$ are heterogeneous with density function $h(c)$.

With these assumptions, one can derive the size of market share that bank $i$ will capture as a function of its lending rate $R_i$ and the market rate $R$. For a market lending rate $R$ and an aggregate loan amount $LW$ (or $N/R^\omega$ for unproductive firms), loan demand facing bank $i$ is $s_iLW$ (or $s_iN/R^\omega$), where market share $s_i$ is given by

$$s_i\left(\frac{R}{R_i}, h\right) = \begin{cases} 1 - \int_{v_i}^{v(R_i)} h(c) dc \; dj & = 1 - \int_0^{v(R_i)} h(c) dc \quad \text{for } R_i > R \\ 1 + \int_{v_i}^{v(R_i)} h(c) dc \; dj & = 1 + \int_0^{v(R_i)} h(c) dc \quad \text{for } R_i < R \end{cases}$$

The second equalities are obtained from integrating over evenly distributed $j$ using $R_j = R$. Note the symmetry of $s_i$ as a function of $R/R_i$ around $R/R_i = 1$. For a small deviation of $R_i$ around $R$, we can then approximate the market share function $s_i$ for any given $h$ by

$$s_i\left(\frac{R}{R_i}, h\right) \approx \left(\frac{R}{R_i}\right)^{\theta(h)}$$

by choosing an appropriate $\theta > 0$. To motivate the two regimes $\theta_2 \in [\theta_L, \theta_H]$, unproductive firms are assumed to have their utility $v()$ or switching cost density $h(c)$ being regime specific and dependent on $R$, such that $\theta_2$ is a step function of $R$.

Note that this modelling approach permits any $\theta \geq 0$, and is more general than the first approach which requires $\theta > 1$. This provides a foundation for the case of $\theta_2 = \theta_L = 0$ considered in the main text. The case corresponds to $h(c)$ being non-zero only at very high $c$, i.e. the switching cost is prohibitively high such that all firms stay with their first-choice banks.
Appendix E  Proofs

Result 3. When $\theta_L = 0$ and $R^d_i = 1$, sufficient conditions for a financial cycle to exist are

$$
\theta_H > \theta_1 - 1 \quad (3.23)
$$

$$
\kappa > 0 \quad (3.24)
$$

$$
\theta_1 > \theta_1^* \quad (3.25)
$$

where $\kappa \equiv \omega - \frac{1+\chi}{1+\sigma+(1-\alpha)(1-\sigma)}$ and $\theta_1^*$ solves

$$
\frac{(\theta_1^*-1)^{\kappa+1}}{\theta_1^{\kappa}} = \frac{1}{N} \left( (\alpha A)^{1+\chi} \left( \frac{\psi}{\Phi} \right)^{\alpha} \right) \theta_1^{1+\sigma+(1-\alpha)(1-\sigma)}
$$

Proof. With a constant policy rate, all endogenous variables are constant in any given regime, and hence we drop the time subscript, with an understanding that $L, W$ and $R$ depend on the prevailing regime. The existence of a financial cycle at $R^d = 1$ requires

$$
\Pi(R_H, R_H) > 0 > \Pi(R_L, R_L)
$$

At $R^d = 1$, we have

$$
\psi_1 = \alpha (1 + \rho^{1/\sigma})
$$

$$
\psi_2 = 1
$$

implying bank profit and lending rate of

$$
\Pi(R, R) = (R - 1)LW - \frac{N}{R^{\omega}}
$$

$$
R = \frac{\theta_1}{\theta_1 - 1} + \frac{\theta_2 N}{(\theta_1 - 1)LWR^{\omega}}
$$

Bank profit $\Pi(R, R)$ is positive when

$$
(R - 1)LWR^{\omega} > N
$$

Substituting for $R$, the left-hand side can be expanded to

$$
(R - 1)LWR^{\omega} = \left( \frac{\theta_1}{\theta_1 - 1} - 1 + \frac{\theta_2 N}{(\theta_1 - 1)LWR^{\omega}} \right) LWR^{\omega}
$$

$$
= \left( \frac{1}{\theta_1 - 1} \right) (LWR^{\omega} + \theta_2 N)
$$

so that bank profit is positive when

$$
LWR^{\omega} > (\theta_1 - 1 - \theta_2)\bar{N} \quad (E.1)
$$

and negative when inequality is reversed.
In the bust regime, \( \theta_2 = \theta_H > \theta_1 - 1 \) is sufficient to ensure E.1, thus proving the first inequality 3.23. In the boom regime where \( \theta_2 = \theta_L = 0 \) and \( R = \theta_1/(\theta_1 - 1) \), a sufficient condition for the reverse of E.1 is

\[
 LW < (\theta_1 - 1)\bar{N}/R^\omega
\]  

(E.2)

We solve for \( LW \) under the boom by equating the labour demand and supply functions (equations 3.20 and 3.21), yielding

\[
 LW = \left( \frac{\alpha A}{R} \right)^{1+\chi} \left( \frac{\psi_1^\sigma}{\Phi} \right)^{\alpha \left( \frac{1}{\chi+\sigma+(1-\alpha)(1-\sigma)} \right)}
\]

Substituting this into E.2 and using \( R = \theta_1/(\theta_1 - 1) \), we get

\[
 \frac{(\theta_1 - 1)^{\kappa+1}}{\theta_1^\kappa} > \frac{1}{N} \left( \frac{\alpha A}{R} \right)^{1+\chi} \left( \frac{\psi_1^\sigma}{\Phi} \right)^{\alpha \left( \frac{1}{\chi+\sigma+(1-\alpha)(1-\sigma)} \right)}
\]  

(E.3)

where

\[
 \kappa \equiv \omega - \frac{1+\chi}{\chi+\sigma+(1-\alpha)(1-\sigma)}
\]  

(E.4)

Differentiating the left-hand side of equation E.3 gives

\[
 \frac{\partial}{\partial \theta_1} \left[ \frac{(\theta_1 - 1)^{\kappa+1}}{\theta_1^\kappa} \right] = \left[ \frac{\theta_1 - 1}{\theta_1} \right]^\kappa \left( \frac{1}{\theta_1} \right)
\]

This is positive when \( \kappa > -1/\theta_1 \), or in the limit of large \( \theta_1 \), when \( \kappa > 0 \). When satisfied, the left hand side of E.3 is both strictly increasing and continuous, so that there exists \( \theta_1^* > 1 \) such that \( (\theta_1^* - 1)^{\kappa+1}/\theta_1^\kappa \) is exactly equal to the right hand side of E.3. Together, \( \kappa > 0 \) and \( \theta_1 > \theta_1^* \) imply that E.3 holds. Thus conditions 3.24 and 3.25 ensure that bank profit is indeed negative during the boom regime. This completes the proof. \( \Box \)
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