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Steady-State Growth*

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Abstract

We compute steady-state economic growth - defined as the rate of growth that the economy would converge to in the absence of new shocks. This rate can be computed in real-time by means of a parsimonious time-varying parameter (TVP) VAR model. Our procedure offers a relatively agnostic estimation of benchmark equilibrium growth rates. Estimates show that the steady-state GDP growth rate in the case of the United States declined from just above 3% per year in the 1990s to 2.4% at present. Results for other six advanced economies and the euro area indicate that the steady-state growth rate, which is consistent with stable inflation and financial conditions, has been relatively stable since 2010 in most cases in spite of a recent slowdown in actual GDP growth rates.

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Keywords: economic growth; financial conditions; inflation; monetary policy; potential output; time-varying parameter VAR; trend growth.

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1 Introduction

What is the medium to long-term GDP growth rate of an economy? And against which benchmark should current growth rates be assessed at any point in time? Policy makers are continuously confronted with these questions. The answer to them is crucial for the appropriate calibration of monetary and fiscal policy. This paper presents a relatively agnostic estimation of *steady-state growth rates*, based on a time-varying parameter (TVP) structural VAR model.

Our definition of steady-state growth is simply the growth rate to which an economy would converge to in the absence of any new shocks. An economy is modelled by means of a parsimonious and agnostic TVP-VAR model. The steady-state growth rates can be easily recomputed in real time after each new observation, making it a useful reference in practice.

In comparison with standard HP-filter techniques, an important advantage of the TVP-VAR methodology is that it is not haunted by end-point distortions (see Hamilton (2018)). At the same time, our benchmark growth rate relies on less *a priori* theoretical restrictions than the well-known model of Laubach and Williams (2003) for the natural rate of interest. Furthermore, the allowance for variation of the parameters over time enables the model to continuously adapt to changes in policy or private sector behavior (Primiceri (2005)). [1] The TVP-VAR model features stochastic volatility of

[1] This implies that the model is more robust to potential mutations in the Phillips

innovations, so that changes in the properties of the shocks are well captured.

We apply our method to assess the steady-state growth rates for seven OECD countries and the euro area. The estimates show that the steady-state growth in the case of the United States declined from just above 3% per year in the 1990s to 2.4% at present. Results for other countries indicate that the steady-state growth rate - which is consistent with stable inflation and normal financial conditions - has been relatively stable in most cases in recent years despite a recent slowdown of the actual GDP growth rates. The methodology can be easily replicated for other countries.

Relation to the literature. Several studies have tried to estimate equilibrium growth rates in the past using a variety of approaches. Methods differ on how much they rely on theoretical models. Our approach, which is strictly empirical, uses the TVP-VAR model developed by Primiceri (2005) and del Negro and Primiceri (2015). Lubik and Matthes (2015) have already employed this methodology to compute natural interest rates in the United States. Earlier, Laubach and Williams (2003) constructed a small-scale macroeconomic model and used a Kalman filtering method to compute the natural rate of interest and trend growth rates. Their crucial assumption is that the output gap is determined by its own lags and the *ex ante* real funds rate gap. Holston, Laubach and Williams (2017) discuss how estimates of that model may vary according to model specification. Our work is inspired

curve.

by this evolving literature.

The main advantage of our analysis is that the model parameters are not assumed to be fixed, but are allowed to vary over time as new data points come in. In this sense, the approach is far less prescriptive, and the model is simpler and more flexible. It is also more appropriate if behaviors, policies and responses change over time. On the whole, the contribution of this paper is to provide a benchmark steady-state growth rate that can be easily estimated in real-time.

Outline. The paper proceeds as follows. Section 2 explains the econometric specification. Section 3 presents estimated steady-state growth rates for several major OECD countries and the robustness of these estimates. We show the differences between real time and full sample estimates and the sensitivity to alternative prior specifications. The last section offers some concluding remarks.

2 Computing Steady-State Growth Rates

We estimate the standard TVP-VAR model

$$\begin{bmatrix} \Delta y_t \\ \Delta p_t \\ m_t \end{bmatrix} = c_t + B_{1,t} \begin{bmatrix} \Delta y_{t-1} \\ \Delta p_{t-1} \\ m_{t-1} \end{bmatrix} + B_{2,t} \begin{bmatrix} \Delta y_{t-2} \\ \Delta p_{t-2} \\ m_{t-2} \end{bmatrix} + \upsilon_t, \qquad (1)$$

where Δy_t stands for the log change in output, Δp_t for the log change in the core CPI, and m_t represents an indicator of monetary policy or financial conditions; [2] c_t is the time-varying intercept; and $B_{1,t}$ and $B_{2,t}$ are matrices of coefficients, which are time varying. υ_t represents the vector of heteroskedastic disturbances, which follows the multivariate normal distribution as $\upsilon_t \sim N(0, G_t)$, where G_t is the time-varying covariance matrix. G_t is decomposed to the variance and covariance components as $G_t = A_t^{-1} S_t S_t' A_t^{-1\prime}$, where S_t is the diagonal matrix with the diagonal elements corresponding to the variance of structural shocks, and A_t is a lower-triangular matrix with diagonal elements equal to one. We denote as $\gamma_{i,t}$ the vector of elements in $B_{i,t}$ for $i = 1, 2$; σ_t as the vector of diagonal elements of S_t; and a_t as the vector of free parameters in A_t.

As in Primiceri (2005), the time varying parameters follow the first-order autoregressive process

$$
\begin{aligned}
c_t &= c_{t-1} + \varphi_t, \\
\gamma_{1,t} &= \gamma_{1,t-1} + \chi_{1,t}, \\
\gamma_{2,t} &= \gamma_{2,t-1} + \chi_{2,t}, \\
a_t &= a_{t-1} + \psi_t, \\
\log \sigma_t &= \log \sigma_{t-1} + \omega_t,
\end{aligned}
$$

where φ_t, $\chi_{i,t}$, ψ_t and ω_t are innovations. In other words, the coefficient vectors are modelled as random walks and the standard deviations are geometric random walks - making it a stochastic volatility model. [3]

[2] The selection of variables is such that it allows the model to have one implicit IS curve, a Phillips curve and a Taylor rule.

[3] A more detailed description of the model specification and estimation methodology

In what follows, we use the *Goldman Sachs* financial conditions index or the 5-year government bond yield as m_t. The latter is preferred to the short-term policy rate because it better reflects the overall stance of monetary policy during the recent period of policy rates at or close to the zero lower bound.

The steady-state or equilibrium values can be derived from (1). More specifically, after rearranging (1), the steady-state values have to satisfy

$$\begin{bmatrix} \overline{\Delta y_t} \\ \overline{\Delta p_t} \\ \overline{m_t} \end{bmatrix} = (I - \widehat{B}_{1,t} - \widehat{B}_{2,t})^{-1}\widehat{c}_t.$$

In what follows, we focus on the benchmark growth rate $\overline{\Delta y_t}$.

3 Estimation

3.1 Benchmark Results

To obtain the steady-state growth rates, the TVP-VAR models were estimated for seven OECD countries and the euro area. Estimates were based on quarterly data. Our benchmark steady state growth rate estimation uses the *Goldman Sachs* financial conditions indices (FCIs). Later, as a robustness check, we show estimates when the 5-year government bond yield is used as the variable m. Due to the availability of the FCI, the sample period starts

for the TVP-VAR model can be found in Primiceri (2005) and Nakajima (2011).

in 1990Q3 and ends in 2018Q4. For Canada, Sweden and Switzerland, the starting point of sample period varies according to data availability. For the euro area, time series were taken from the AWM database with the sample period spanning from 1990Q3 to 2017Q4.

Following Primiceri (2005) and Nakajima (2011), the prior distribution for the variances of innovations φ_t, $\chi_{i,t}$, ψ_t and ω_t is set as an inverse gamma distribution with the hyperparameters calibrated by a pre-sample period. The number of lags of the VAR is two based on the Bayesian Information Criterion (BIC).

[**TABLE 1 about here**]

Table 1 reports steady-state growth rates for the 7 largest advanced economies and the euro area just before the GFC (2006Q4) and at the end of our sample (i.e. 2018Q4). Equilibrium growth in the United States declined from an estimated 2.8% per year in 2006 to 2.4% in 2018. Similarly, in the United Kingdom it declined from 2.3% per year to 2.0%. Estimates were less affected by the crisis in the cases of the euro area and Japan, where they stand at, respectively, 2.0% and 1.2%. Canada and Switzerland also exhibit a slight decline in the steady-state growth rate between 2006 and 2018.

[**GRAPH 1 about here**]

Graph 1 plots the evolution of the estimated steady-state growth rates over time. The solid lines show the mean estimate and the shadow areas

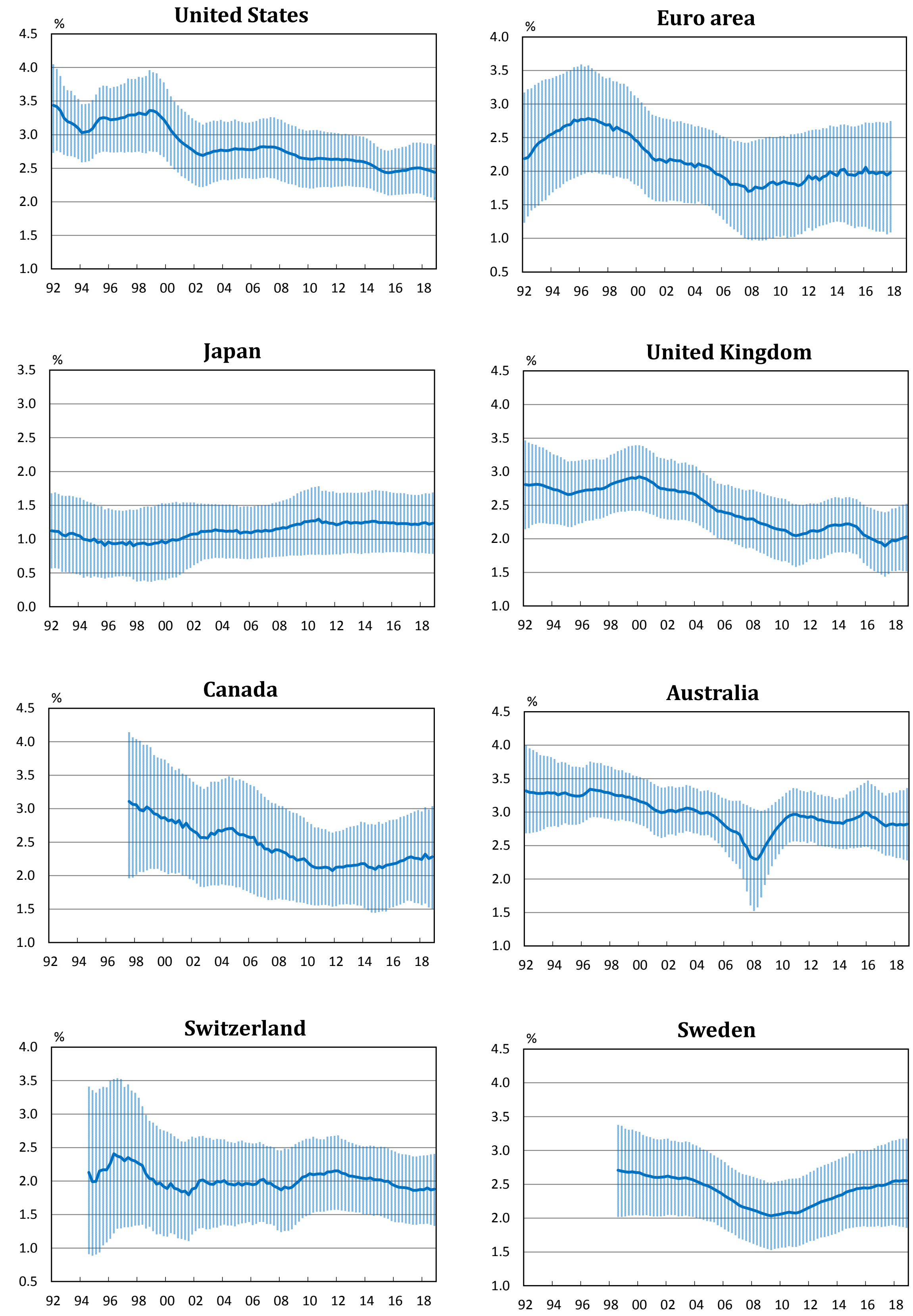
Table 1. Steady-state growth rates.

%

	2006Q4	2018Q4
United States	2.8	2.4
Euro area	1.8	2.0
Japan	1.1	1.2
United Kingdom	2.3	2.0
Canada	2.5	2.3
Australia	2.7	2.8
Switzerland	2.0	1.9
Sweden	2.2	2.6

Note: 1. For the euro area and Switzerland, 10-year yield is used. 2. For the euro area, the latest estimate refers to 2017Q4.

Figure 1. Steady-state growth rate (VAR with FCI).



Note: For the euro area (up to Q4 2017) and Switzerland, 10-year yield is used. Shadow area indicates 68% confidence intervals.

indicate the 68% confidence intervals. Estimates show that steady-state GDP growth rate in the case of the United States declined from just above 3% per year in the 1990s to the current 2.4%. For the euro area, one noteworthy aspect is that the confidence band is substantially wider than elsewhere. Large uncertainty in this case hardly comes as a surprise given the shorter time series and the regional heterogeneity (e.g. in labor markets).

Graph A1 in the Appendix plots the evolution when the 5-year government bond yield is used as m instead. By and large, the patterns are found to be similar. [4] Further, Graph A2 shows the evolution of the stochastic volatility of growth shocks. As can be seen in the plots, the financial crisis led to a spike in the volatility of growth innovations in most cases. [5]

3.2 Comparison with other Approaches

It may also be useful to compare the estimates obtained from our methodology to existing potential growth estimates. The OECD's production function based estimates of potential growth for the median country in 2006 are exactly equal to the median delivered by the TVP-VAR approach for that year (i.e. 2.3% per year). [6] However, while the median potential output

[4] We also find similar patterns and levels when we use the shadow interest rates of Wu and Xia (2016) for the United States, United Kingdom and the euro area. The confidence bands are however wider under this choice during the period that interest rates are close to zero.

[5] The exception is Australia, where core inflation increased most after the crisis, to above 5%. The model considered this unsustainable. The trend growth rate recovered when core inflation dropped.

[6] Based on data from the OECD Economic Outlook no.105, May 2019.

growth has declined 69 basis points according to the OECD, our econometric methodology leads to a reduction of only 10 basis points by 2018 for the median country. The methodology seems to reflect that higher growth rates are compatible with stable inflation and financial conditions. We find that the bulk of the reduction in trend growth rates in most countries predates the GFC (as can clearly be seen in Graph 1). In the case of the United States, however, we find that the trend growth rate diminished by a further 40 basis points since 2006. [7]

The current rate of 2.4% for the United States is somewhat higher than the 2.0% estimated by Holston, Laubach and Williams (2017). Also our estimate for the euro area trend growth is above theirs. In contrast, the estimate for the United Kingdom is virtually identical as our estimate is 2.0% while theirs is 1.9%.

3.3 Sensitivity to Priors

The estimated steady-state growth rate could be affected by the prior specification for the innovation variance of the time-varying intercept c_t and the lagged coefficients $\gamma_{1,t}$ and $\gamma_{2,t}$. In the baseline model, we use the inverse gamma distribution, IG($\alpha = 25$,$\beta = 0.0025$). To check for robustness of the estimates, we estimate the model with alternative prior hyperparameters: (1) IG(10, 0.0025), (2) IG(50, 0.0025), (3) IG(25, 0.001), and (4) IG(25, 0.005).

[7] Put differently, the model does not interpret the financial crisis as a very substantial change in the transmission mechanism, but it sees it as a large negative output shock.

Table 2 reports the posterior means and 68% confidence intervals of the steady-state growth rate for the Unitest States in 2006Q4 and 2018Q4. The estimates turn out to be only marginally different, which implies that the estimation methodology is quite robust. We also examined the sensitivity for the other countries and found that the posterior estimates of steady-state growth rate do not change much with alternative prior specifications.

[**TABLE 2 about here**]

3.4 Real-Time

Table 2 reports the difference between real-time and full-sample estimates in each case. In the case of the G-3 economies (the United States, the euro area and Japan) corrections for steady-state growth rates are generally found to be modest. For 2016Q4 and 2017Q4, for instance, they are never above 15 basis points. Larger corrections for 2016Q4 are found in the case of the United Kingdom.

[**TABLE 3 about here**]

On average, two-years ago real-time steady state growth rates are adjusted by 19 basis points, while one-year ago estimates are adjusted by 12 basis points. All in all, this assessment suggests that a reasonably reliable estimator of equilibrium growth rates is obtained in real-time in most cases.

Table 2. Prior sensitivity: US steady-state growth rate estimates

%

	2006Q4			2018Q4		
	Lower	Mean	Upper	Lower	Mean	Upper
Baseline	2.36	2.82	3.24	2.02	2.44	2.85
Prior 1	2.24	2.84	3.37	1.88	2.32	2.77
Prior 2	2.39	2.78	3.16	2.12	2.50	2.87
Prior 3	2.41	2.79	3.16	2.14	2.53	2.91
Prior 4	2.26	2.84	3.37	1.92	2.36	2.79

Note: "Lower" and "Upper" correspond to 68% confidence intervals.

Table 3. Difference in steady-state growth rate estimates between real time and full sample

%

	2016Q4	2017Q4
United States	-0.13	0.10
Euro area	-0.11	-0.04
Japan	0.05	0.10
United Kingdom	0.58	0.06
Canada	-0.06	0.34
Australia	0.44	-0.01
Switzerland	0.03	0.29
Sweden	-0.11	0.01

Note: Real-time estimate less the full-sample (up to 2018Q4). For the euro area, 2015Q4 and 2016Q4.

4 Concluding Remarks

We presented a new benchmark growth rate. The approach to obtain the steady-state growth rate is purely data driven, and does not rely on fixed relations between economic variables. The model is adaptative, as changing coefficients are updated after each new data point. The TVP-VAR based methodology is not subject to end-of-sample distortions and offers flexibility as it captures for instance changing Phillips curves. An additional advantage is that the steady-state growth rate can easily be computed in real time.

The estimates show that steady-state GDP growth rate for the United States declined from just above 3% per year in the 1990s to 2.4% at present. In six other advanced economies and the euro area the steady-state growth rate, which is consistent with stable inflation and normal financial conditions, has been relatively stable after the crisis in most cases, despite a recent slowdown in actual GDP growth rates.

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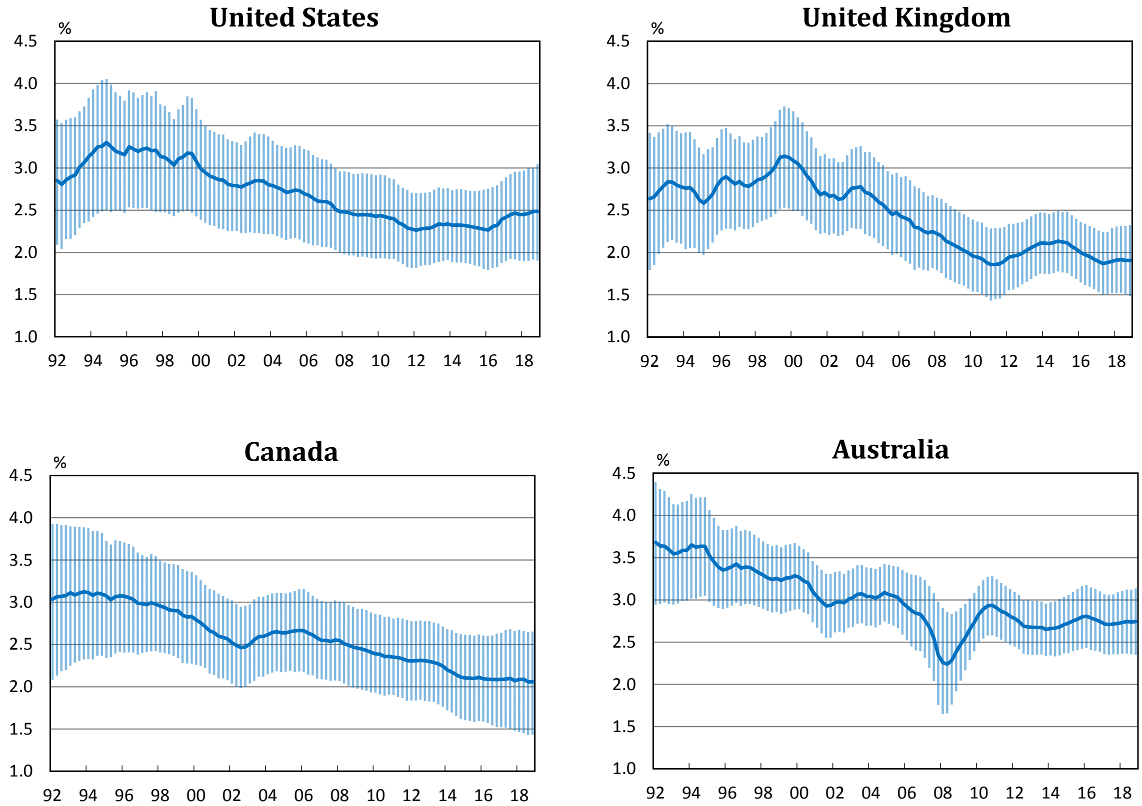
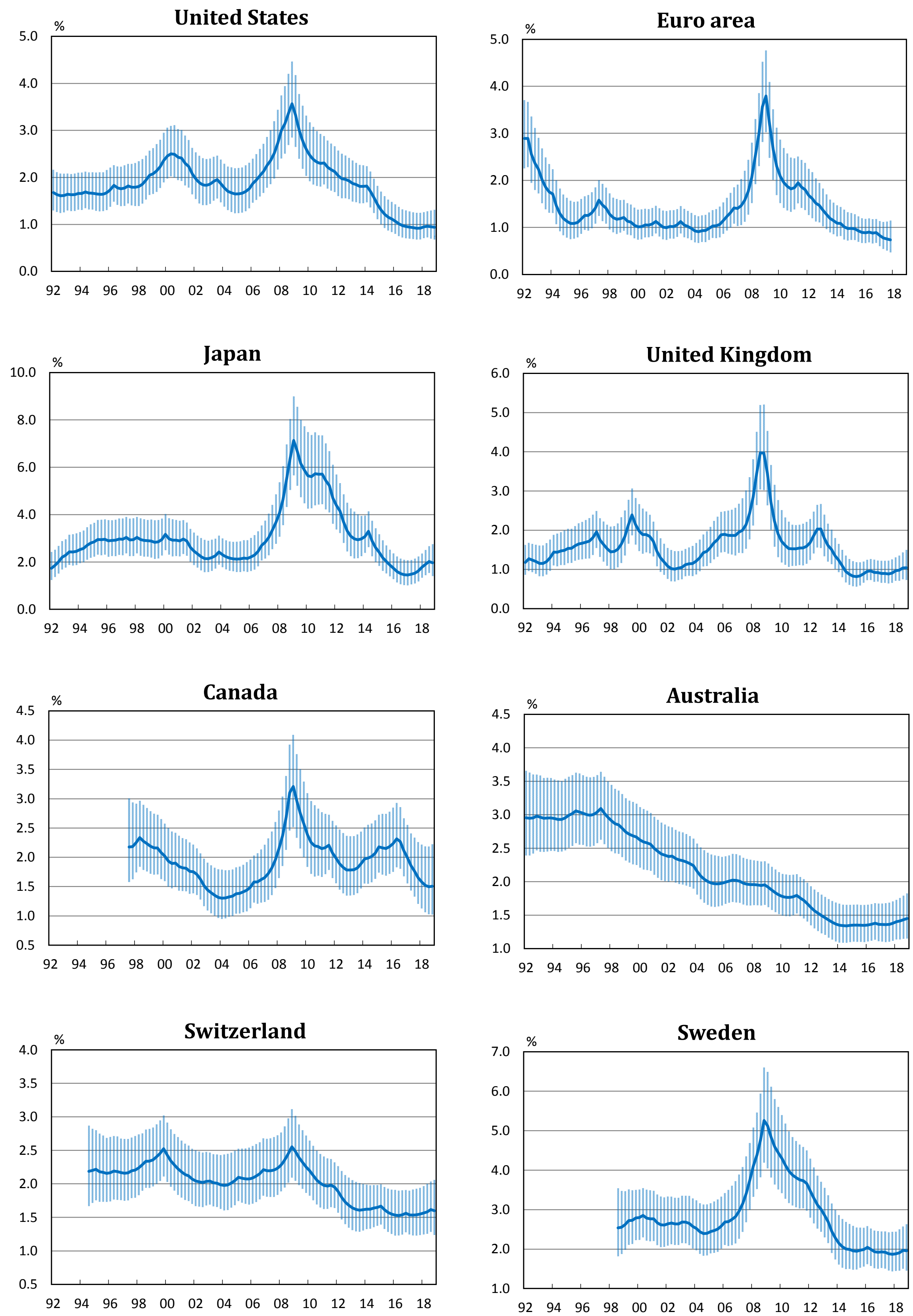
Figure A1. Steady-state growth rate (VAR with 5-year yields).



Note: Shadow area indicates 68% confidence intervals.

Figure A2. Stochastic volatility for the growth rate (VAR with FCI).



Note: For the euro area (up to Q4 2017) and Switzerland, 10-year yield is used. Shadow area indicates 68% confidence intervals.

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