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Exchange rate puzzles: evidence from rigidly fixed nominal exchange rate systems[1]

Charles Engel[2] and Feng Zhu[3]

Abstract

We examine several major exchange rate puzzles: the excess volatility of real exchange rates; their excess reaction to the real interest rate differentials; the uncovered interest rate parity (UIP) puzzle; the excess persistence of real exchange rates; the exchange rate disconnect puzzle; and the consumption correlation puzzle. We examine the behaviour of real exchange rates among pairs of economies that have rigidly fixed nominal exchange rates, e.g. countries within the euro area, regions in China and Canada, and Hong Kong SAR vis-à-vis the United States, compared to that among non-euro-area OECD economies.

Our results suggest that some of these puzzles are less puzzling under a rigidly fixed exchange rate regime. In particular, real exchange rates appear to have no or little excess volatility; excess reaction of the real exchange rate to real interest rates is less common; there is less disconnect between the real exchange rate and the economic fundamentals; and, uncovered interest rate parity appears to hold more frequently in these economies. However, real exchange rates are as persistent in these economies as in the floating-rate economies and there appears to be little difference in risk-sharing across countries with fixed versus floating nominal exchange rates. These results may have implications for exchange rate modelling.

Keywords: consumption correlation puzzle; excess volatility, exchange rate disconnect, exchange rate regime, real exchange rate, purchasing power parity, uncovered interest rate parity.

JEL classification: E43, F31.

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I. Introduction

The literature has named a number of exchange rate puzzles – involving the persistence and volatility of real exchange rates, the relationship between exchange rates and interest rates, aggregate consumption and productivity – and has offered many potential explanations for these puzzles.[4] We examine several major exchange rate puzzles: the excess volatility of real exchange rates; their excess reaction to the real interest rate differentials; the uncovered interest rate parity (UIP) puzzle; the excess persistence of real exchange rates; the exchange rate disconnect puzzle; and the consumption correlation puzzle.

We ask whether the nature of these puzzles is different under fixed versus free-floating exchange rate regimes. This paper focuses on six major exchange rate puzzles identified by the literature: the excess volatility of real exchange rates; their excess reaction to the real interest rate differentials; the uncovered interest rate parity (UIP) puzzle; the excess persistence of real exchange rates; the exchange rate disconnect puzzle; and the consumption correlation puzzle. We examine the behaviour of real exchange rates among pairs of economies that have rigidly fixed nominal exchange rates, *e.g.* countries within the euro area, regions in China and Canada, and Hong Kong SAR vis-à-vis the United States, compared to that among non-euro-area OECD economies.

For every puzzle that we investigate, the literature has offered theories to explain the puzzles based on real exchange rate determination in models in which nominal rigidities are absent. In such models, the nominal exchange rate *per se* does not play a role – the classical dichotomy holds, so real exchange rates are determined by real factors, and nominal exchange rate behaviour is not relevant for the proposed puzzle solution. It would not matter in those models whether the nominal exchange rate is fixed or floating, hence the "neutrality" of the nominal exchange rate regime.

4 Obstfeld and Rogoff (2001) list six challenging puzzles in international macroeconomics, namely the home-bias-in-trade puzzle, the Feldstein-Horioka (1980) puzzle, the home-bias portfolio puzzle, the consumption correlations puzzle, the purchasing-power-parity puzzle, the exchange-rate disconnect puzzle (including the Meese-Rogoff (1983) forecasting puzzle and the Baxter-Stockman (1989) neutrality-of-exchange-rate-regime puzzle). They suggest that trade costs could help resolve the core quantity puzzles. Eaton, Kortum and Neiman (2016) follow this line of argument and show that removing trade frictions helps quantitatively mitigate some puzzles, especially the Feldstein-Horioka (1980) puzzle and the exchange rate disconnect puzzle.

If the nominal exchange rate is fixed, then nominal prices are presumed to adjust freely to facilitate real exchange rate adjustment.

There is, of course, an abundance of open-economy macroeconomic models in which there is stickiness in nominal prices or wages of varying degrees. In that type of models, the behaviour of the nominal exchange rate does matter for the real exchange rates. The real exchange rate behaves very differently under fixed nominal exchange rates than under floating rates.[5] As nominal prices and wages adjust more slowly, real exchange rates and relative wages across economies may be driven by the same factors which affect nominal exchange rates. Therefore in the economies with fixed exchange rates, real exchange rate movements may actually be much more in line with the predictions of neoclassical models.

Our purpose is to see which of the puzzles, if any, are significantly different under rigidly fixed exchange rates versus floating exchange rates. We hope that this evidence will provide clues to the types of models that are useful for resolving the puzzles – and therefore, the types of models that are most useful for open-economy macroeconomic analysis.

We compare the degree to which the puzzles hold among pairs of economies with floating exchange rates (*e.g.*, among the pairs of OECD member countries that are not in the euro area) with pairs of economies which have rigidly fixed exchange rates (such as Hong Kong versus the United States and country pairs within the euro area). We also extend the analysis to intra-national data, such as for US states and Canadian and Chinese provinces, and examine at least some of these propositions, depending on data availability. Within the national borders, nominal exchange rates are irrevocably fixed, providing the best example of fixed exchange rates.

Our results suggest that, for those economies which have adopted a rigidly fixed nominal exchange rate arrangement, the excess volatility puzzle of real exchange rates practically disappears or becomes minor for the vast majority of the fixed-rate economies; there is less evidence for excess reaction of the real exchange

[5] Mussa (1986) observes that "real exchange rates typically show much greater short term variability than under a fixed exchange rate regime". He rejects the hypothesis of "nominal exchange rate regime neutrality" under which the behaviour of real exchange rates is not substantially and systematically affected by the choice of nominal exchange rate regime, and the models which do not embody the neutrality property include those which assume sluggish nominal price adjustment. Such models imply relatively slow real exchange rate movements under a fixed nominal exchange rate regime.

rate to the real interest rate differential; there is less disconnect between the real exchange rate and the economic fundamentals; and, the uncovered interest rate parity appears to hold more frequently in these economies. However, real exchange rates are as persistent in these economies as in the floating-rate economies, and the evidence on risk sharing shows little difference among countries with fixed versus floating nominal exchange rates.

The rest of the paper is organised as follows. Section II provides an overview of the related literature. In Section III, we describe each of the six major exchange rate puzzles in turn and provide a theoretical perspective on the puzzles wherever needed, and we present our empirical tests and results, highlighting the differences for pairs of currencies under the rigidly fixed and floating nominal exchange rate regimes. Section IV concludes. Section V contains appendices on our empirical methodology and data descriptions.

II. Selected literature

A vast literature exists on each of the exchange rate puzzles we examine in the paper. Here, we mention only a selected few that are especially helpful for understanding our analysis. A key focus of our work is the behaviour of such puzzles under a rigidly fixed nominal exchange rate regime. Helpman and Razin (1982) and Lucas (1982) show that in equilibrium models with flexible nominal goods prices, under certain conditions, the nominal exchange rate regime is neutral for the behaviour of real exchange rates. Frenkel (1981) and Mussa (1982) suggest that sluggish nominal goods price adjustment and rapid (floating) nominal exchange rate adjustment imply greater real exchange-rate variability under flexible than fixed nominal exchange rates, given the underlying shocks. Stockman (1983) finds a strong empirical relation between the nominal exchange rate system and real exchange rate variability. Mussa (1986) finds "substantial and systematic differences in the behaviour of real exchange rates" under fixed and floating nominal exchange rate regimes for pairs of economies with similar and moderate inflation rates.

On the other hand, Flood and Rose (1995, 1999), Jeanne and Rose (2002) and Itskhoki and Mukhin (2016) argue that one reason for real exchange rates being more volatile under the floating than fixed nominal exchange rate regime is that there is a volatile foreign exchange risk premium under floating rates, while that risk premium is zero under fixed exchange rates. This point does not get around the importance of sticky nominal prices in resolving the puzzles. If goods prices were flexible, then the volatility of the real exchange rate should not depend on the nominal exchange rate system. In a flexible-price world, the volatility of the real exchange rate would be the same under fixed or floating nominal exchange rates, and so the behaviour of the risk premium would be no different. A volatile risk premium may certainly play a role in accounting for real exchange rate volatility, but, as we find that the excess volatility of real exchange rates is generally smaller under fixed nominal exchange rates, there must be some interaction between nominal stickiness and the effects of changes in the risk premium on the nominal exchange rate that leads to more volatility under floating nominal exchange rates.

Berka, Devereux and Engel (2012, 2016) and Berka and Devereux (2013) have explored how productivity differences or income differences across countries have affected real exchange rates for countries within the euro area. They find that traditional theories of the long-run determinants of real exchange rates are more strongly supported among this set of countries that share a common currency than among countries with separate currencies and floating exchange rates.

Grilli and Kaminsky (1991) find evidence of the real exchange rate behaving like a random walk only in the post-World War II period. They suggest that what matters for the real exchange rate behaviour is the particular historical period rather than the nominal exchange rate arrangement. Yet Frankel and Rose (1996) find that there is little difference in the speed of adjustment of real exchange rates in post-World War II data in the Bretton Woods period versus the post-Bretton Woods era of floating exchange rates. Taylor (2002) finds that while there has been more real exchange rate volatility under floating nominal exchange rates, there is little difference in the persistence of real exchange rates between fixed and floating periods in his sample of 20 countries with over 100 years of data.

Lothian and Taylor (1996) use 200 years of data on the dollar-sterling and franc-sterling rates to compute the rate of real exchange rate convergence, but do not explicitly distinguish between periods of fixed and floating exchange rates. Similarly, Diebold, Husted and Rush (1990) find convergence of the real exchange rate under the gold standard, and note that their finding contrasts with findings of no convergence under floating exchange rates. Sarno and Valente (2006) examine the dynamics of convergence of real exchange rates under alternative exchange-rate regimes. They use 100 years of data for four major countries (France, the United Kingdom, Germany and Japan) relative to the United States. In contrast to Taylor (2002), they find that the speed of convergence is slower under fixed nominal exchange rates.

Many studies have examined the convergence of real exchange rates within the borders of a single country, so that there are no nominal exchange rate changes between locations. Parsley and Wei (1996) examine the rate of convergence of price differentials among US city pairs, and find the half-life of price deviations is lower than the typically measured half-life of real exchange rates. On the other hand, Engel, Hendrickson and Rogers (1997) find no difference in the persistence of real exchange rates among city pairs within countries and real exchange rates across countries. Ceglowski (2003) finds fast convergence of prices among Canadian cities, while Fan and Wei (2006) find similar results for China.

Flood and Rose (1996) consider the uncovered interest parity puzzle in fixed exchange rate regimes. The countries in their study do not have rigidly fixed exchange rates, but rather exchange rates that are allowed to float within narrow bands (in the European Monetary System, the forerunner of the euro area). They find that the forward premium anomaly is much less severe within the set of countries that have narrowly targeted exchange rates, compared to countries with freely floating currencies. The findings of Lothian and Wu (2011) are similar: using two hundred years of data for advanced countries, the study finds that the uncovered interest parity puzzle only emerges in the post-Bretton Woods era of floating exchange rates.

Backus, Kehoe and Kydland (1992) find lower correlation in consumption growth than in output growth in 11 OECD countries, contrary to theory predictions under complete international capital markets. Indeed, such a finding suggests that there is effectively no risk sharing at all. Backus and Smith (1993) introduce non-traded goods which are shown to reduce consumption growth correlation across countries. Yet contrary to their model predictions, growth rates of relative consumption tend to be negatively correlated with real exchange rates, while the complete-markets theory predicts positive correlation. There are a few papers that have studied the consumption correlation puzzle under fixed nominal exchange rates. Hess and Shin (2010) decompose the real exchange rate into a component attributable to the nominal exchange rate, and a part attributable to nominal prices, and then demonstrate that the puzzle arises from the movement of the nominal exchange rate. They find that within the United States, relative consumption and the real exchange rates are positively correlated – that is, the Backus-Smith (1993) puzzle disappears when the nominal exchange rate is fixed. Hadzi-Vaskov (2008) finds a very similar result by looking at euro area countries. Using within-country data for the United States, Canada, Germany and Spain, Devereux and Hnatkovska (2014) confirm this finding and build a sticky-price macroeconomic model to explain their findings.

Finally, the classic references in this area are Stockman (1983) and Mussa (1986). These two papers document the very different behaviour of real exchange rates under fixed and flexible nominal exchange rate regimes, while finding little systematic difference in other economic fundamentals. Baxter and Stockman (1989) find that the second moments of a wide range of macroeconomic variables are no different under fixed and floating regimes. Some of the exchange rate puzzles addressed here were examined in Obstfeld and Rogoff (2001).

III. Exchange rate puzzles under the fixed-rate regime

We study six major exchange rate puzzles in this section through the perspective of different nominal exchange rate regimes. We focus on the puzzle of excess volatility of real exchange rates; the puzzle of their excess reaction to the real interest rate differentials; the uncovered interest rate parity (UIP) puzzle; the puzzle of the excess persistence of real exchange rates; the exchange rate disconnect puzzle; and the consumption correlation puzzle. Some of these puzzles are closely related to each other or may be manifestations of one core puzzle. The purchasing power parity (PPP) puzzle and the UIP puzzle are two such core puzzles under consideration.

III.1. Excess volatility of real exchange rates

One of the main puzzles of real exchange rate behaviour (see, for example, Rogoff (1996) and Evans (2011)) is the "excess volatility" of real exchange rates. Let us define the real exchange rate Q_t as

$$Q_t \equiv \frac{S_t P_t^*}{P_t}$$

where S_t is the nominal exchange rate (the price of the foreign currency in home currency or the amount of the home currency that can be bought with one unit of foreign currency), P_t is the consumer price level in the home country, and P_t^* is the consumer price level in the foreign country. The real exchange rate is therefore a relative price, *i.e.* the price of the consumer basket in the foreign country relative to the price in the home country. Using lower case letters to denote the logs of variables written in upper case letters, we have

(1) $$q_t = s_t + p_t^* - p_t$$

A rise in q_t then indicates a real depreciation of the home currency. Note that under a rigidly fixed nominal exchange rate regime such as the system implemented in the euro area, the real exchange rate simply becomes the relative foreign-to-home price, *i.e.* $q_t = p_t^* - p_t$. Real exchange rate volatility can be defined in a number of ways, we focus on the following two: $\mathrm{var}\left(q_t\right)$ and $\mathrm{var}\left(q_t - q_{t-1}\right)$, *i.e.* the variance of the log of the real exchange rate and the variance of the change in the log of the real exchange rate, respectively. Whether a volatility measure is considered "excessive" is gauged relative to the predictions of a model, and we focus on three general models in this paper.

Many models of the real exchange rate put the spotlight on the relative price of nontraded goods. Specifically, we can write

$$p_t = \alpha_N p_{N,t} + \left(1 - \alpha_N\right) p_{T,t}$$

where $p_{N,t}$ is the log of the price of nontraded goods in the home country, and $p_{T,t}$ is the log of the price of traded goods. We define the traded price index by

$$p_{T,t} = \frac{\nu}{2} p_{H,t} + \frac{2-\nu}{2} p_{F,t}$$

where $p_{H,t}$ is the log of the price of the traded good produced in the home country and consumed in the home country, $p_{F,t}$ is the price of the traded good produced in the foreign country and consumed in the home country, and $\frac{\nu}{2}$ is the proportion of traded good consumed in the home country that is produced in the home country. There is "home bias" in consumption if $\nu > 1$. Foreign prices are defined analogously.

From these definitions, we can write

(2) $$q_t = s_{T,t} + p_{T,t}^{*} - p_{T,t} + \alpha_N \left(p_{N,t}^{*} - p_{T,t}^{*} - \left(p_{N,t} - p_{T,t} \right) \right)$$

Under the assumption of no home bias in consumption ($\nu = 1$), and if there is no pricing to market for traded goods, so $s_t + p_{H,t}^{*} - p_{H,t} = 0$ and $s_t + p_{F,t}^{*} - p_{F,t} = 0$, then $s_{T,t} + p_{T,t}^{*} - p_{T,t} = 0$. Since $\alpha_N < 1$, we must have

(3)

$$\operatorname{var}\left(q_t\right) < \operatorname{var}\left(p_{N,t}^{*} - p_{T,t}^{*} - \left(p_{N,t} - p_{T,t} \right) \right), \text{ and}$$

$$\operatorname{var}\left(\Delta q_t\right) < \operatorname{var}\Delta\left[\left(p_{N,t}^{*} - p_{T,t}^{*} - \left(p_{N,t} - p_{T,t} \right) \right) \right]$$

These are our first tests of excess volatility. Following much of the empirical literature and broadly speaking, we use the consumer price of goods as the price of traded goods, and the consumer price of services including housing as the measure of the price of nontraded goods.[6]

A popular approach to modelling real exchange rates is to observe that $s_{T,t} + p_{T,t}^{*} - p_{T,t} \neq 0$, and to attribute the inequality to the fact that consumer prices incorporate distribution services. Assuming those distribution services are nontraded, and their price is equal to the price of other nontraded services, we can write $p_{H,t} = \kappa \tilde{p}_{H,t} + \left(1-\kappa\right) p_{N,t}$ and $p_{F,t} = \kappa \tilde{p}_{F,t} + \left(1-\kappa\right) p_{N,t}$. Here, $\tilde{p}_{H,t}$ and $\tilde{p}_{F,t}$ are the prices of the home and foreign goods "at the dock", *i.e.* the price charged by the producer to the distributor. Assuming that there is no pricing to market for the actual good, then $s_{T,t} + \tilde{p}_{H,t}^{*} - \tilde{p}_{H,t} = 0$ and $s_{T,t} + \tilde{p}_{F,t}^{*} - \tilde{p}_{F,t} = 0$. Under these assumptions:

$$\begin{aligned} s_t + p_{T,t}^{*} - p_{T,t} &= s_t + \frac{\nu\kappa}{2}\tilde{p}_{F,t}^{*} + \frac{\left(2-\nu\right)\kappa}{2}\tilde{p}_{H,t}^{*} + \left(1-\kappa\right)p_{N,t}^{*} - \left(\frac{\nu\kappa}{2}\tilde{p}_{H,t} + \frac{\left(2-\nu\right)\kappa}{2}\tilde{p}_{F,t} + \left(1-\kappa\right)p_{N,t} \right) \\ &= \left(1-\kappa\right)\left[\left(p_{N,t}^{*} - \left(\frac{\nu}{2}\tilde{p}_{F,t}^{*} + \frac{\left(2-\nu\right)}{2}\tilde{p}_{H,t}^{*} \right) \right) - \left(p_{N,t} - \left(\frac{\nu\kappa}{2}\tilde{p}_{H,t} + \frac{\left(2-\nu\right)\kappa}{2}\tilde{p}_{F,t} \right) \right) \right] + \left(\nu-1\right)\left(\tilde{p}_{F,t} - \tilde{p}_{H,t}\right) \end{aligned}$$

We can express these relationships in terms of consumer prices, recognising for example that $p_{N,t} - \tilde{p}_{H,t} = \frac{1}{\kappa}\left(p_{N,t} - p_{H,t} \right)$:

6 A detailed account of classification of different items in a CPI consumption basket into tradable and non-tradable goods can be found in Appendix V.2.

$$s_t + p^*_{T,t} - p_{T,t} = (1-\kappa)\times$$
$$\left[\frac{\nu}{2\kappa}\left(p^*_{N,t} - p^*_{F,t}\right) + \frac{(2-\nu)}{2\kappa}\left(p^*_{N,t} - p^*_{H,t}\right) - \left(\frac{\nu}{2k}\left(p_{N,t} - p_{H,t}\right) + \frac{(2-\nu)}{2\kappa}\left(p_{N,t} - p_{F,t}\right)\right)\right] + (\nu-1)\left(\tilde{p}_{F,t} - \tilde{p}_{H,t}\right)$$
$$= \frac{1-\kappa}{\kappa}\left[p^*_{N,t} - p^*_{T,t} - \left(p_{N,t} - p_{T,t}\right)\right] + (\nu-1)\left(\tilde{p}_{F,t} - \tilde{p}_{H,t}\right)$$

Several studies have found that $\kappa \approx 0.5$. Then, under the assumption of no home bias in consumption ($\nu = 1$), we have

$$s_t + p^*_{T,t} - p_{T,t} = p^*_{N,t} - p^*_{T,t} - \left(p_{N,t} - p_{F,t}\right)$$

Substitute this into (1) and we obtain

$$q_t = \left(1+\alpha_N\right)\left(p^*_{N,t} - p^*_{T,t} - \left(p_{N,t} - p_{T,t}\right)\right)$$

This gives rise to a second set of variance bounds which we can test. Since $\alpha_N < 1$, we must have

(4)

$$\operatorname{var}\left(q_t\right) < 4\operatorname{var}\left(p^*_{N,t} - p^*_{T,t} - \left(p_{N,t} - p_{T,t}\right)\right)$$

and

$$\operatorname{var}\left(\Delta q_t\right) < 4\operatorname{var}\left[\Delta\left(p^*_{N,t} - p^*_{T,t} - \left(p_{N,t} - p_{T,t}\right)\right)\right]$$

Next, we consider a simple version of the Harrod-Balassa-Samuelson model. First, we assume that the basket of traded goods produced in each country is identical. Further assuming that goods are produced using only labour, then we have $p_{N,t} - \tilde{p}_{T,t} = a_{\mathrm{T},t} - a_{N,t}$ and $p^*_{N,t} - \tilde{p}^*_{T,t} = a^*_{T,t} - a^*_{N,t}$, where $a_{j,t}$ is the log of productivity in sector *j* in the home country and $a^*_{j,t}$ is the log of productivity in sector *j* in the foreign country, $j \in \{N, T\}$ and $\tilde{p}_{T,t}$ and $\tilde{p}^*_{T,t}$ refer to the price of the traded good "at the dock" in home and foreign currency, respectively.

From above, we have

$$q_t = \left[1 - \left(1-\alpha_N\right)\kappa\right]\left[a_{N,t} - a_{T,t} - \left(a^*_{N,t} - a^*_{T,t}\right)\right]$$

We use sectoral data from the Organisation for Economic Co-operation and Development (OECD) and other sources to measure the changes in the logs of productivity in traded and nontraded sectors, as in several studies in the literature.[7] We can then test for the following volatility bound

(5) $$\operatorname{var}\left(\Delta q_t\right) < \operatorname{var}\left[\Delta\left(a_{N,t}^{*} - a_{T,t}^{*} - \left(a_{N,t} - a_{T,t}\right)\right)\right]$$

A different approach starts by assuming uncovered interest parity holds, *i.e.*

$$i_t^{*} + E_t s_{t+1} - s_t = i_t$$

where i_t and i_t^{*} are one-period interest rates in the home and foreign country, respectively. Define real interest rates by $r_t \equiv i_t - E_t\pi_{t+1}$ and $r_t^{*} = i_t^{*} - E_t\pi_{t+1}^{*}$, where $\pi_{t+1} \equiv p_{t+1} - p_t$ and $\pi_{t+1}^{*} = p_{t+1}^{*} - p_t^{*}$. Then we can rewrite the uncovered interest parity relationship as

$$r_t^{*} + E_t q_{t+1} - q_t = r_t$$

Iterating this equation forward and assuming the real exchange rate is stationary with no drift, we find

$$q_t - \overline{q} = E_t \sum_{j=0}^{\infty} r_{t+j}^{*} - r_{t+j} - (\overline{r^{*} - r})$$

where the bar over the variable represents its unconditional mean. Define $q_t^{IP} \equiv E_t \sum_{j=0}^{\infty} r_{t+j}^{*} - r_{t+j} - (\overline{r^{*} - r})$ as the value that the real exchange rate takes on under uncovered interest parity, consequently, it must be the case that

(6)

$$\operatorname{var}\left(q_t\right) \le \operatorname{var}\left(q_t^{IP}\right)$$

and

$$\operatorname{var}\left(q_t - q_{t-1}\right) \le \operatorname{var}\left(q_t^{IP} - q_{t-1}^{IP}\right)$$

An estimate of the term q_t^{IP} can be obtained by constructing estimates of $r_t \equiv i_t - E_t\pi_{t+1}$ and $r_t^{*} = i_t^{*} - E_t\pi_{t+1}^{*}$ from an estimated core vector autoregressive

[7] We provide details on data sources for different countries and regions in Appendix V.1., which includes a list of sample periods and data frequencies for the analysis of the six puzzles we look at in the paper.

(VAR) model, which contains the stationary variables q_t, i_t, i_t^*, π_t, and π_t^*.[8] The core VAR model can be expanded to include other variables which are useful in determining the dynamics of these five core endogenous variables. We also examine the version where some variables enter the VAR model in home relative to foreign form, such as $i_t - i_t^*$ and $\pi_t - \pi_t^*$.

We first compute the variances of real exchange rates and relative foreign-to-home and non-tradable-to-tradable prices, both in levels and changes, for the euro area economies which have come under a rigidly fixed exchange rate regime and shared a common currency since 1999. Since Hong Kong Monetary Authority adheres to a Linked Exchange Rate System (essentially a currency board system) since 1983, pegging HK dollar to US dollar at the rate of HKD7.80/USD1, we include the HKD-USD pair in this group. We then do the same exercise for the OECD economies which do not belong to this group and of which the nominal exchange rates freely float. We also compute the variances for the real exchange rates between the core euro area economies under a rigidly fixed exchange rate regime and the other OECD economies under a floating regime. In the last step, we examine whether the two set of variance bounds (3) and (4) hold. The results of the variance comparisons are summarised in Table 1.

In terms of real exchange rate volatility, the computed variances suggest that the pairs of economies with rigidly fixed nominal exchange rates, including 19 euro area countries, stand in stark contrast to the non-euro area OECD economies with floating rates. For the economies under a fixed nominal exchange rate arrangement, the variance bound (3) in terms of the level of real exchange rate are satisfied in 154 out of 172 cases, but only 42 out of 423 cases for the economies under a floating regime.[9] The difference for the variance bound (3) in terms of the changes in real exchange rate is even more striking: the bound holds for all 172 pairs of fixed rate economy pairs, but for only 31 of the 423 pairs of floating-rate economies. Clearly, excess real exchange rate volatility is much less an issue for the economies under a rigidly fixed nominal exchange rate regime, but it remains a puzzle in those economies under a floating exchange rate regime.

[8] See Appendix V.4. for further details.

[9] In the latter calculation, we group together the "both floating" and the "fixed vs. floating" countries, since in fact the exchange rate floats between all country pairs in both groups.

Excess volatility of real exchange rates

Table 1

	Variance bounds (3)[1]					
	$\text{var}(q_t) < \text{var}\left(p^*_{N,t} - p^*_{T,t} - (p_{N,t} - p_{T,t})\right)$			$\text{var}(\Delta q_t) < \text{var}\left[\Delta\left(p^*_{N,t} - p^*_{T,t} - (p_{N,t} - p_{T,t})\right)\right]$		
	Both fixed[2]	Both floating[3]	Fixed vs floating[4]	Both fixed[2]	Both floating[3]	Fixed vs floating[4]
Within the bound	154	3	39	172	0	31
Above the bound	18	116	265	0	119	273
Total of pairs	172	119	304	172	119	304
	Variance bounds (4)[5]					
	$\text{var}(q_t) < 4\,\text{var}\left(p^*_{N,t} - p^*_{T,t} - (p_{N,t} - p_{T,t})\right)$			$\text{var}(\Delta q_t) < 4\,\text{var}\left[\Delta\left(p^*_{N,t} - p^*_{T,t} - (p_{N,t} - p_{T,t})\right)\right]$		
	Both fixed[2]	Both floating[3]	Fixed vs floating[4]	Both fixed[2]	Both floating[3]	Fixed vs floating[4]
Within the bound	172	31	144	172	6	137
Above the bound	0	88	160	0	113	167
Total of pairs	172	119	304	172	119	304

[1] Variance of real exchange rates relative to the variance of relative prices. [2] For the 19 Euro area countries, there are a total of (19 * 19 – 19)/2 = 171 pairs. In addition, we have the US-HK pair. [3] Four of the 19 non-Euro area OECD countries (Australia, Israel, Korea and New Zealand) have incomplete data. Hence, we have (15 * 15 – 15)/2 = 105 pairs. Plus 14 pairs with HK. [4] With data for 19 Euro area countries and 14 non-Euro area OECD countries, there are a total of 19 * 15 = 285 pairs. Plus 19 paris with HK. [5] Variance of real exchange rates relative to four times the variance of relative prices.

Sources: Eurostat; OECD; authors' calculations.

The results for the set of variance bounds in (4) are similarly striking. In fact, the bounds hold for all 172 pairs of real exchange rates in the 19 euro area economies and between Hong Kong SAR and the United States, in both levels and changes; but they only hold for 175 out of 423 pairs of real exchange rates among the 15 non-euro area OECD economies plus Hong Kong SAR in levels, and for 143 pairs in changes.

We apply the same analysis to intra-national data. Nominal exchange rates among the regions within the national borders are effectively and permanently fixed. We use data available for 10 provinces in Canada, 31 provinces in China and 27 metropolitan areas in the United States. The results, presented in Table 2, further strengthen the outcome we obtained from the international comparisons as shown above. In particular, the variance bounds in (3) hold in levels for the real exchange rates for 41 pairs out of the 45 pairs of provinces in Canada, 411 out of 465 pairs of provinces in China and 293 out of 351 pairs of metropolitan areas in the United States, and for 45, 454 and 351 pairs in changes, respectively. In terms of (4), the variance bounds are satisfied for 45 (45) out of 45 pairs of Canadian provinces in levels (changes), 458 (465) out of 465 pairs of Chinese provinces and 350 (351) out of 351 pairs of US metropolitan areas.

Excess volatility of real exchange rates: Canada, China and the United States[1]

Table 2

	Variance bounds (3)[2]					
	$\mathrm{var}(q_t) < \mathrm{var}\left(p^*_{N,t} - p^*_{T,t} - (p_{N,t} - p_{T,t})\right)$			$\mathrm{var}(\Delta q_t) < \mathrm{var}\left[\Delta\left(p^*_{N,t} - p^*_{T,t} - (p_{N,t} - p_{T,t})\right)\right]$		
	Canada[3]	China[4]	US[5]	Canada[3]	China[4]	US[5]
Within the bound	41	411	293	45	454	351
Above the bound	4	54	58	0	11	0
Total of pairs	45	465	351	45	465	351
	Variance bounds (4)[6]					
	$\mathrm{var}(q_t) < 4\,\mathrm{var}\left(p^*_{N,t} - p^*_{T,t} - (p_{N,t} - p_{T,t})\right)$			$\mathrm{var}(\Delta q_t) < 4\,\mathrm{var}\left[\Delta\left(p^*_{N,t} - p^*_{T,t} - (p_{N,t} - p_{T,t})\right)\right]$		
	Canada[3]	China[4]	US[5]	Canada[3]	China[4]	US[5]
Within the bound	45	458	350	45	465	351
Above the bound	0	7	1	0	0	0
Total of pairs	45	465	351	45	465	351

[1] Based on regional data for Canada, China and the United States. [2] Variance of real exchange rates relative to the variance of relative prices. [3] For the 10 Canadian provinces, there are a total of (10 * 10 – 10)/2 = 45 pairs. [4] For the 31 Chinese provinces, there are a total of (31 * 31 – 31)/2 = 465 pairs. [5] For the 27 Metropolitan area pairs, there are a total of (27 * 27 – 27)/2 = 351 pairs. [6] Variance of real exchange rates relative to four times the variance of relative prices.

Sources: Eurostat; OECD; authors' calculations.

Next, we examine real exchange rate volatility by comparing their variances with the computed variances of the relative foreign-to-home and non-tradable-to-tradable productivity as in (5).[10] Results presented in Table 3 suggest a similar picture. In particular, for the 136 euro area country pairs with fixed nominal exchange rates for which we are able to obtain productivity data, the bound is satisfied for about half of the country pairs. For OECD country pairs with floating rates, the variance bound holds for only 22 of 189 cases.

Table 4 presents the results of the comparison of the variance of real exchange rates with the variance of the real exchange rates constructed under the uncovered interest parity condition, as detailed in (6). The computed variances suggest a broadly similar picture to the patterns we see in previous variance bound analysis. Specifically, both in terms of levels and changes, the variance bounds hold for most of the pairs of economies with fixed nominal exchange rates (52 and 55 out

[10] We compute labour productivity based on available data as in Appendix V.3.

of 67 cases, respectively), but for very few pairs of economies with floating rates (24 and 17 out of 339 pairs, respectively.

Excess volatility of real exchange rates: productivity

Table 3

	Variance bounds (5)[1]		
	$\text{var}(\Delta q_t) < \text{var}\left[\Delta\left(a^*_{N,t} - a^*_{T,t} - \left(a_{N,t} - a_{T,t}\right)\right)\right]$		
	Both fixed[2]	Both floating[3]	Fixed vs floating[4]
Within the bound	64	12	10
Above the bound	72	24	143
Total of pairs	136	36	153

[1] Variance of change in real exchange rates relative to the variance of change in relative productivity. [2] For the 17 Euro area countries (except Cyprus and Malta), there are a total of (17 * 17 – 17)/2 = 136 pairs. [3] Data for only 9 non-Euro area OECD countries are available. We have a total of (9 * 9 – 9)/2 = 36 pairs.

Sources: Eurostat; OECD; authors' calculations.

To sum up, a clear picture emerges from our empirical analysis of various different variance bounds derived from the existing theories. The excess volatility puzzle of real exchange rates practically disappears or becomes minor for the vast majority of the economies which have adopted a rigidly fixed nominal exchange rate arrangement, including the 19 euro area countries which share a single currency, and Hong Kong which pegged its currency to the US dollar. The puzzle remains for most of the countries with floating nominal exchange rates such as the non-euro area OECD economies.

Excess volatility of real exchange rates: UIP

Table 4

	Variance bounds (6)[1]					
	$\text{var}(q_t) \le \text{var}(q_t^{IP})$			$\text{var}(\Delta q_t) \le \text{var}(\Delta q_t^{IP})$		
	Both fixed[2]	Both floating[3]	Fixed vs floating[4]	Both fixed[2]	Both floating[3]	Fixed vs floating[4]
Within the bound	52	4	20	55	2	15
Above the bound	15	131	184	12	133	189
Total of pairs	67	135	204	67	135	204

[1] Variance of real exchange rates relative to the variance of real exchange rate takes on under uncovered interest parity. [2] For the 12 Euro area countries, there are a total of (12 * 12 – 12)/2 = 66 pairs. Plus US-HK pair. [3] Three of the 19 non-Euro area OECD countries (Australia, Korea and New Zealand) have incomplete data. Hence, we have (16 * 16 – 16)/2 = 120 pairs. Plus 15 pairs with HK. [4] With data for 12 Euro area countries and 16 non-Euro area OECD countries, there are a total of 12 * 16 = 285 pairs. Plus 12 pairs with HK.

Sources: Eurostat; OECD; authors' calculations.

III.2. Excess reaction to the real interest rate differential

Engel (2016) notes that under uncovered interest parity (UIP), we should find that the following covariance equality holds

$$\text{cov}\left(q_t, r_t^* - r_t\right) = \text{cov}\left(q_t^{IP}, r_t^* - r_t\right) \tag{7}$$

Yet for many floating-rate economies, there tends to be excess comovement between the real exchange rate and the real interest rate differential, *i.e.*

$$\text{cov}\left(q_t, r_t^* - r_t\right) > \text{cov}\left(q_t^{IP}, r_t^* - r_t\right)$$

We compare these two covariances using the measures of $r_t^* - r_t$ and q_t^{IP} estimated from the VAR models discussed above. The same comparison is made in terms of the first differences of the real exchange rate and of the real interest rate differential as well, *i.e.* between $\text{cov}\left(\Delta q_t, \Delta\left(r_t^* - r_t\right)\right)$ and $\text{cov}\left(\Delta q_t^{IP}, \Delta\left(r_t^* - r_t\right)\right)$.

We summarise our analysis in Table 5. Here the results are similar to those we found in the previous section, though perhaps not as dramatic. The covariance bound in levels is satisfied for 54 of the 67 country pairs within the euro area for which we have data. In contrast, it is satisfied for only 92 of the 339 floating exchange rate pairs, meaning that there is excess reaction of the real exchange rate for most floating-rate pairs. The results are similar when we look at first differences. The covariance bound is satisfied for 55 of the 67 euro area pairs, but only 103 of the 339 floating-rate pairs.

Excess reaction to real interest rate differential — Table 5

	$\text{cov}(q_t, r_t^* - r_t) \le \text{cov}(q_t^{IP}, r_t^* - r_t)$			$\text{cov}(\Delta q_t, \Delta(r_t^* - r_t)) \le \text{cov}(\Delta q_t^{IP}, \Delta(r_t^* - r_t))$		
	Both fixed[1]	Both floating[2]	Fixed vs floating[3]	Both fixed[1]	Both floating[2]	Fixed vs floating[3]
Within the bound	54	26	66	55	30	73
Total of pairs	67	135	204	67	135	204

[1] For the Euro area-12, there are a total of (12 * 12 – 12)/2 = 66 pairs. Plus US-HK pair. [2] Three of the 19 non-Euro area OECD countries (Iceland, Israel and Korea) have incomplete data, therefore we have (16 * 16 – 16)/2 = 120 pairs. Plus 15 pairs with HK. [3] With data for Euro area-12 and 16 non-Euro area OECD countries, there are a total of 12 * 16 = 192 pairs. Plus 12 pairs with HK.

Sources: Eurostat; OECD; authors' calculations.

III.3. Uncovered interest rate parity puzzle

The uncovered interest rate parity (UIP) relationship postulates that the expected return on a home riskless short-term interest-bearing security (such as a Euro deposit) and a corresponding foreign security have the same expected return when returns are expressed in the same currency. Specifically,

(8) $$i_t^* + E_t s_{t+1} - s_t = i_t$$

Under the UIP, the null hypothesis in the Fama (1984) regression

(9) $$s_{t+1} - s_t = \alpha_0 + \beta_0 \left(i_t - i_t^*\right) + u_{0,t+1}$$

is that $\alpha_0 = 0$ and $\beta_0 = 1$. Yet in practice, it is well-known that for many pairs of economies under a floating exchange rate regime, the empirics actually suggest that $\beta_0 < 1$ and frequently $\beta_0 < 0$, hence the well-known UIP puzzle.

As Engel (2014, 2016) discusses, most models offered as explanations for the UIP puzzle, particularly those based on foreign exchange risk premiums, actually account for the comovement of the excess return with the real interest rate differential. That is, they imply that $\beta_1 < 0$ in the regression

(10) $$q_{t+1} - q_t = \alpha_1 + \beta_1 \left(r_t - r_t^*\right) + u_{1,t+1}$$

In practice, the existing models present theories constructed on real exchange rates in order to explain the UIP puzzle based on returns expressed in nominal terms. Effectively, the models treat one-period ahead inflation as known with certainty in advance, so that the only uncertainty about the real exchange rate arises from uncertainty about the nominal exchange rate. However, when we consider real exchange rates among pairs of economies which have a fixed nominal exchange rate, clearly the only source of variation in the real exchange rate resides in inflation movements. To study the UIP puzzle for economies under rigidly fixed nominal exchange rates, we therefore need to modify the interpretation of the UIP regression.

The key point to recognize, however, is that the countries that have fixed nominal exchange rates do not fit the paradigm of the literature on the UIP puzzle. That literature (see, for example, Verdelhan, 2010; Lustig, Roussanov and Verdelhan. 2011; Bansal and Shaliastovich, 2013) that for any given investor, the real return on a home short-term bond is different from the real return on a foreign short-term bond. In other words, that literature assumes that each bond pays off a riskless return in units of the consumption basket of the country where the bond is issued. A German bond, in those models, pays off in units of the German consumption basket, and a French bond pays off in units of the French consumption basket. Because the real exchange rate between France and Germany is a random variable, that model would imply that the real return to a French investor for a German bond is uncertain, while the real return on a French bond for a French investor is riskless.

In fact, French and German bonds pay off in euros. There is no risk to the nominal return, but there is real risk for either bond. But the real risk for a French and German bond is the same for a French investor, if we set aside any default or liquidity risk. Both bonds pay off in euros, and the inflation risk for the French investor arises from uncertainty in the French nominal consumer price level. Hence, the risk characteristics of the two bonds should be identical. Even for risk averse investors, the two bonds should have equal expected real rates of return, which implies that uncovered interest parity should hold *ex ante*. That is, we should find $\alpha_1 = 0$ and $\beta_1 = 1$ from equation (10) among country pairs with rigidly fixed nominal exchange rates.

Note that in the case of countries with fixed nominal exchange rates, the change in the real exchange rate on the left-hand-side of equation (10) is simply the foreign inflation rate minus the home inflation rate. Expanding out the expressions for the real exchange rate in this case, (10) would be written as:

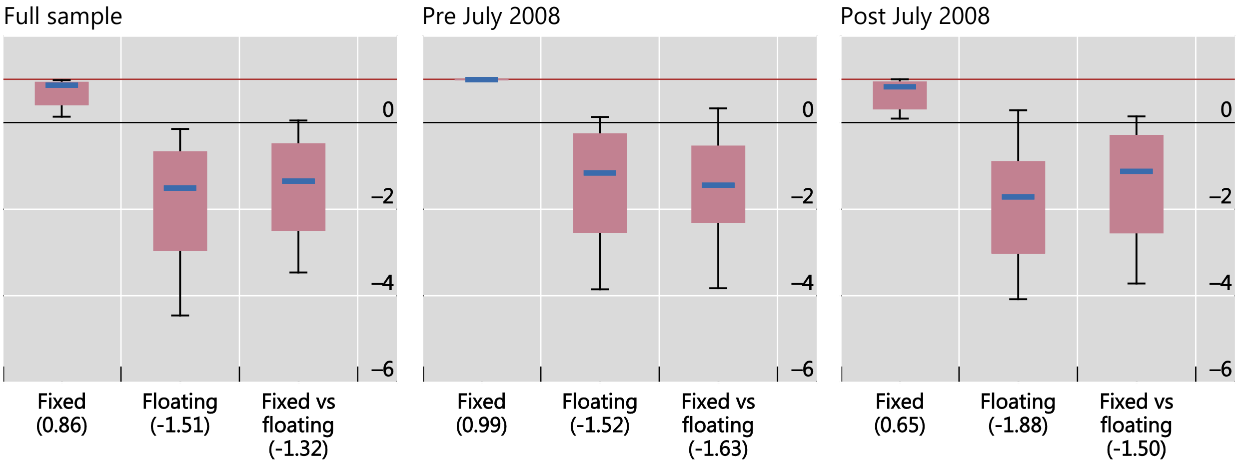
(11) $$\pi^*_{t+1} - \pi_{t+1} = \alpha_1 + \beta_1 \left(i_t - E_t \pi_{t+1} - \left(i^*_t - E_t \pi^*_{t+1} \right) \right) + u_{1,t+1}$$

If we were looking at euro area deposits for which $i^*_t = i_t$, then we would have $\beta_1 = 1$ by necessity. However, our interest-rate data is 3-month government bond yields, for which there is some difference in nominal interest rates. That difference in nominal rates may reflect differences in default or liquidity risk, which in turn could lead to deviations from uncovered interest parity.

We estimate the coefficients α_1 and β_1 for the full sample; the pre-crisis sample ending in July 2008, which precedes the market turmoil leading to the filing for Chapter 11 bankruptcy protection by Lehman Brothers on September 15, 2008; and the sample period starting after that. Presumably, the onset of the Global Financial Crisis and the ensuing implementation of near-zero interest rate policy and of unconventional monetary policies by the major advanced economies could have exerted a significant impact on the relationships between ex ante real returns that reflect factors not usually considered in the UIP literature, such as default risk and liquidity risk. It is also possible that "peso problems" characterise this period – that is, the sample of *ex post* realisations of exchange rates are a biased (in sample) measure of the *ex ante* expectations. Indeed the crisis period was marked by large swings in the nominal exchange rates of the major currencies such as the US dollar, euro, Japanese yen and sterling pound, while the corresponding nominal interest rates stayed close to zero for much of the post-July 2008 period. Splitting the sample may help us understand whether the dynamics of regressions have changed over time.

Uncovered interest rate parity puzzle[1]

Estimates of the coefficient β_1 Graph 1



The number in the parenthesis indicates the median estimate in each group.

[1] For 12 Euro area countries, there are a total of (12 * 12 – 12)/2 = 66 pairs; plus one pair US-HK, making it 67 pairs. Three of the 19 non-Euro area OECD countries (Iceland, Israel and Korea) have incomplete data, therefore we have (16 * 16 – 16)/2 = 120 pairs; plus 15 pairs between HK and OECD countries, making it 135 pairs. With data for Euro area-12 and 16 non-Euro area OECD countries, there are a total of 12 * 16 = 192 pairs; plus 12 pairs between HK and Euro area countries, making it 204 pairs. In the Tukey boxplots the bottom and top of the boxes indicate the first and third quartiles of the estimates within each set; the blue line indicates the median; and the bottom and top whiskers represent the 10th and 90th percentile of the estimates.

Source: authors' calculations.

We summarise the results in Tukey boxplots in Graph 1.[11] The results turn out to show a striking difference between the estimated slope coefficients for equation (10) when the country pairs have rigidly fixed nominal exchange rates versus floating exchange rates. While the estimates of the coefficient β_1 turn out to be negative for the pairs of economies under floating exchange rate regimes, the estimates are positive with the median estimates close to one for the pairs of economies with a rigidly fixed exchange rate arrangement.

The median slope coefficients for the countries with fixed exchange rates is around 0.99 for the pre-crisis sample, while the median estimate is 0.65 for the post-crisis sample and 0.86 for the complete sample. That difference arises because of the behaviour of interest rates on government bonds in several of the countries that ran into trouble during the European sovereign debt crisis. For example, if we restrict our focus to counties which were fiscally healthy and less affected by the debt crisis, such

[11] In Tukey boxplots, the centre boxes include the estimates ranging from the first to the third quartile of the estimates within each set of regressions, the blue line indicates the median estimate, and the bottom and top whiskers represent the 10th and 90th percentiles of the estimates.

as Austria, Belgium, Germany, Finland, France and the Netherlands, the median slope coefficient in the post-crisis years was 0.99 even in the post-crisis years.

Uncovered interest rate parity puzzle, null H_0: $\beta = 1$

Table 6

	Both fixed[1]			Both floating[1]			Fixed vs floating[3]		
	10%	5%	1%	10%	5%	1%	10%	5%	1%
Do not reject H_0: $\beta = 1$	35	37	39	18	31	55	27	39	71
Reject H_0: $\beta = 1$	31	29	27	102	89	65	165	153	121
Total	66			120			192		

Note: the numbers indicate the counts of observations for which the *p*-values are greater than 0.10, 0.05 and 0.01, respectively.

[1] For the Euro area-12, there are a total of (12 * 12 – 12)/2 = 66 pairs. [2] Three of the 19 non-Eurozone OECD countries (Iceland, Israel and Korea) have incomplete data, therefore we have (16 * 16 – 16)/2 = 120 pairs. [3] With data for Euro area-12 and 16 non-Eurozone OECD countries, there are a total of 12 * 16 = 192 pairs.

Sources: Eurostat; OECD; authors' calculations.

We summarise the results of the full-sample test for the null hypothesis of $\beta_1 = 1$ in Table 6, with the counts of pairs of economies which have the reported *p*-values being greater than 0.10 (10%), 0.05 (5%), and 0.01 (1%). For 12 euro area economies with fixed exchange rates, the null of $\beta_1 = 1$ can be rejected in 27 out of 66 cases at 1% significance level and in 31 cases at 10% significance level. Even though the estimated coefficients are close to one, the standard errors of the coefficient estimates tend to be very small for the countries with fixed exchange rates, leading to rejection of the null at the 10% level in nearly half the country pairs.

The important point to emphasise here is that the deviations of the slope coefficient from one in the case of the countries with rigidly fixed exchange rates all arise because of the interest rate behaviour in the set of countries that were affected by the European sovereign debt crisis: Spain, Greece, Ireland, Italy and Portugal. The failure of UIP in the fixed-exchange rate countries does not arise because of differences in real returns generated by real exchange rate uncertainty. The distinct behaviour arises because the nominal interest rates on government debt are different in these countries. This must represent some sort of financial market imperfection – differences in perceived default probabilities, or liquidity, or perhaps financial constraints on banks and households.

The null of $\beta_1 = 1$ can be rejected in 186 out 312 country pairs at 1% significance level and in 267 cases at 10% significance level among the floating rate pairs. These rejections are in line with other findings in the literature on the UIP puzzle, and are not driven by the inclusion of the European countries that came under siege during the debt crisis. Because the estimated slope coefficients are much lower than for the fixed nominal exchange rate country pairs, and the rejection of the null is much more frequent, we can conclude that there must be something else driving the rejections of UIP among country pairs that have floating nominal exchange rates.

III.4. Excess persistence of real exchange rates

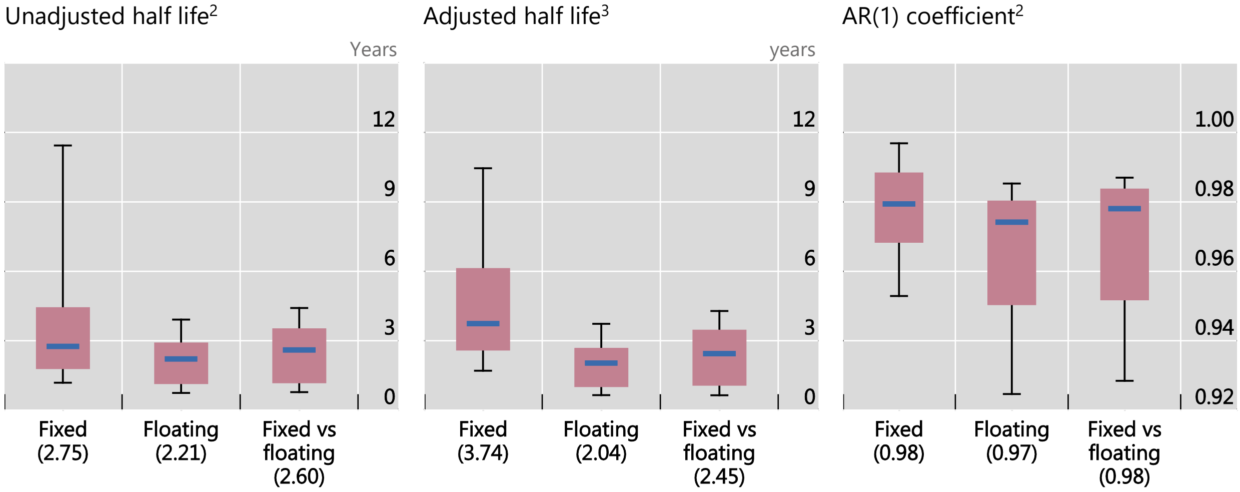
There is a large literature on the purchasing power parity (PPP) puzzle. Rogoff (1996) defines the puzzle as "how can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out?" That is, can we reconcile the high volatility of real exchange rates with their high persistence? Rogoff (1996) argues that the high volatility of real exchange rates might be explained in a monetary model with sticky prices, yet those models might imply that the real exchange rate's persistence is determined by the speed of adjustment of nominal prices. Rogoff (1996) notes that consensus estimates suggest very long half-lives for shocks to real exchange rates to be of approximately three to five years for floating-rate countries, "seemingly far too long to be explained by nominal rigidities." Indeed measures of price stickiness based on surveys of the frequency of price resetting suggest the half-life of nominal price levels is closer to nine months.

Nevertheless, much of the empirical research focused on the post-Bretton Woods period characterised by the floating exchange rate regime under which many economies operate. Does the PPP puzzle behave differently under a rigidly fixed nominal exchange rate regime? One direct test of excess persistence is to examine whether the half-life of real exchange rates is closer to nine months when nominal exchange rates are fixed. Alternatively, we compare the half-life of real exchange rates under fixed and floating nominal exchange rates to the half-life of $p^*_{N,t} - p^*_{T,t} - \left(p_{N,t} - p_{T,t}\right)$, based on equation (2).

Following Rogoff (1996), we first estimate an AR(1) process for the real exchange rates and then compute the half-life of real exchange rates based the estimate of the AR(1) coefficients. We then generalise the specification to an AR(*p*), *i.e.* a *p*-th order autoregressive process, and calculate the half-life accordingly.

The estimates of AR(1) coefficients in the right-hand-side panel of Graph 2 corroborate the finding that the real exchange rates appear to be generally more persistent in the economies where the nominal exchange rate is rigidly fixed. In all three groups of economies, independent of their nominal exchange rate regime, the median AR(1) coefficients are very close but smaller than one, the real exchange rates are apparently near-integrated processes with very high persistence.

Persistence of real exchange rates[1] Graph 2



The number in the parenthesis is the median in each group.

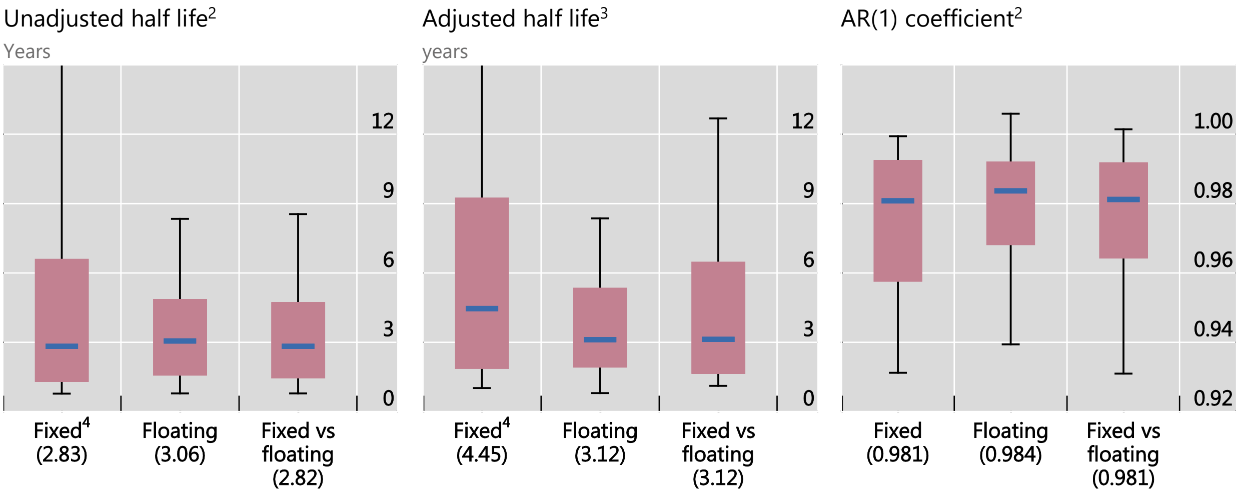
[1] For the Euro area, there are a total of (13 * 13 – 13)/2 = 78 pairs; plus one pair US-HK, making it 79 pairs. Three of the 19 non-Eurozone OECD countries (Iceland, Israel and Korea) have incomplete data, therefore we have (16 * 16 – 16)/2 = 120 pairs; plus 15 pairs between HK and OECD countries, making it 135 pairs. With data for Euro area and 16 non-Eurozone OECD countries, there are a total of 13 * 16 = 208 pairs; plus 13 pairs between HK and Eurozone countries, making it 221 pairs. In the Tukey boxplots the bottom and top of the boxes are the first and third quartiles of the estimates within each set; the blue line indicates the median; and the bottom and top whiskers represent the 10th and 90th percentile of the estimates. [2] From estimating the ar(1) of the real exchange rate regression. [3] From the regression on residuals.

Source: authors' calculations.

We present the results of the estimates of half-lives and AR(1) coefficients for the relative foreign-to-home and non-tradable-to-tradable prices, *i.e.* $p^*_{N,t} - p^*_{T,t} - \left(p_{N,t} - p_{T,t}\right)$, with boxplots in Graph 3. Comparing the same estimates of half-lives and AR(1) coefficients summarised in Graph 2 to those in Graph 3, it becomes clear that real exchange rates are actually less persistent than $p^*_{N,t} - p^*_{T,t} - \left(p_{N,t} - p_{T,t}\right)$, as all median estimates for half-lives and AR(1) coefficients turn out to be slightly smaller, and the ranges of the estimates in real exchange rates are actually much smaller.

We can conclude that the real exchange rate is quite persistent under both fixed and floating nominal exchange rates. It is not any more persistent, however, than $p^*_{N,t} - p^*_{T,t} - \left(p_{N,t} - p_{T,t}\right)$. These findings might suggest that there is some real factor driving the real exchange rate, and the persistence is not primarily determined by the behaviour of the nominal exchange rate. On the other hand, it may be the case that different factors determine persistence under fixed and floating exchange rates, but the persistence happens to be approximately the same.

Persistence of relative foreign-to-home and non-tradable-to-tradable prices[1] Graph 3



The number in the parenthesis is the median in each group.

[1] For the Euro area, there are a total of (13 * 13 – 13)/2 = 78 pairs; plus one pair US-HK, making it 79 pairs. Three of the 19 non-Eurozone OECD countries (Iceland, Israel and Korea) have incomplete data, therefore we have (15 * 15 – 15)/2 = 105 pairs; plus 14 pairs between HK and OECD countries, making it 119 pairs. With data for Euro area and 16 non-Eurozone OECD countries, there are a total of 13 * 15 = 195 pairs; plus 13 pairs between HK and Eurozone countries, making it 208 pairs. In the Tukey boxplots the bottom and top of the boxes are the first and third quartiles of the estimates within each set; the blue line indicates the median; and the bottom and top whiskers represent the 10th and 90th percentile of the estimates. [2] From estimating the ar(1) of the real exchange rate regression. [3] From the regression on residuals. [4] The maximum value for fixed regime in unadjusted half life and adjusted half life are 21 years and 44 years respectively.

Source: authors' calculations.

III.5. Exchange rate disconnect puzzle

One of the most puzzling aspects of exchange-rate behaviour is the seemingly rather weak relationship between the exchange rate and any economic fundamentals. We consider in this section two different expressions for fundamentals, related to the models presented above.

The first that we investigate is the relationship suggested by the Harrod-Balassa-Samuelson model as in Sub-section III.1, which leads to the equation

$$q_t = \left[1-\left(1-\alpha_N\right)\kappa\right]\left[a_{N,t} - a_{T,t} - \left(a^*_{N,t} - a^*_{T,t}\right)\right]$$

Based on this equation and using data on productivity in traded and nontraded sectors, we investigate whether changes in the real exchange rate are related to changes in relative non-traded-to-traded productivity. That is, for both pairs of economies with floating nominal exchange rates and with rigidly fixed nominal exchange rates, we study the short-run and long-run relationship between q_t and $a^*_{N,t} - a^*_{T,t} - \left(a_{N,t} - a_{T,t}\right)$ by estimating an error correction model (ECM).

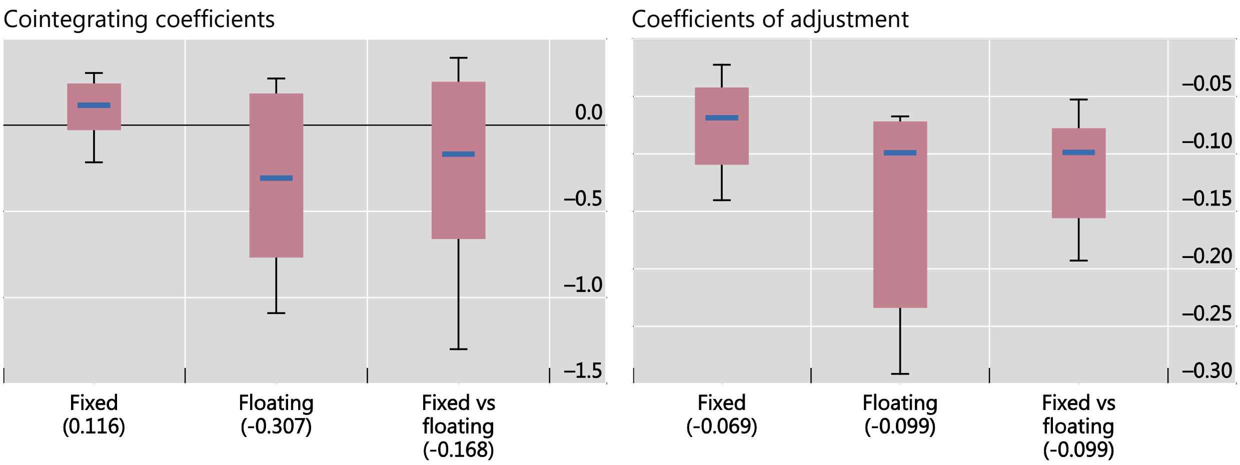
In this exercise, we limit ourselves to bivariate pairs of countries for which we have at least 15 years of data. Because the ECM examines long-run relationships, we require long enough time series. We have 55 country pairs within the euro area that satisfy this criterion, 23 pairs that are not in the euro area, and 88 that are a euro area country relative to one that is not in the euro area.

We look at the cointegrating relationship because the short-run relationship between the real exchange rate and the relative productivity variables is very weak. We are looking to see whether there is a difference in the tendency for these variables to converge in the long run. The first step is to estimate the cointegrating vector. A minimum criterion is that the variables be positively related. However, we find a very different pattern for the euro area countries with a rigidly fixed nominal exchange rate and those country pairs with floating exchange rates. Of the 55 euro area pairs, we find the estimated cointegrating vector has the correct sign in 38 cases. In contrast, of the 111 country pairs that have floating exchange rates, only 39 show the anticipated sign. In addition, we find the correct sign for 9 of the 23 pairs where neither country is in the euro area, and 30 of the 88 pairs where one of the countries is in the euro area nor the other is not.

Conditional on the cointegrating vector having the correct sign, the estimated error correction models for the countries with fixed nominal exchange rates and those with floating rates turns out to be slightly different. The estimated error-correction parameters are illustrated in Graph 4. The graphs show that the speed of adjustment is actually lower in the euro area countries on average. Perhaps one way to interpret the findings is this: on the one hand, when exchange rates are floating, there are many factors that affect the movements of the nominal exchange rate, unrelated to productivity. These could be monetary and financial shocks that have persistent effects on the real exchange rate, perhaps due to nominal price stickiness.

As a result, it is difficult to detect a cointegrating vector of the correct sign for floating countries. These problems are absent when nominal exchange rates are fixed. On the other hand, when we restrict our analysis to country pairs where we can estimate a cointegrating vector of the correct sign, adjustment is speedier under floating rates. So, tentatively, we might conclude that under floating rates, monetary and financial shocks lead to a disconnect between real exchange rates and productivity, but when those shocks are less prevalent, floating rates might lead to quicker adjustment compared to a regime of fixed nominal exchange rates. These tentative conclusions deserve further, more detailed, investigation.

Cointegration of real exchange rate and relative productivity Graph 4



The number in the parenthesis is the median in each group.

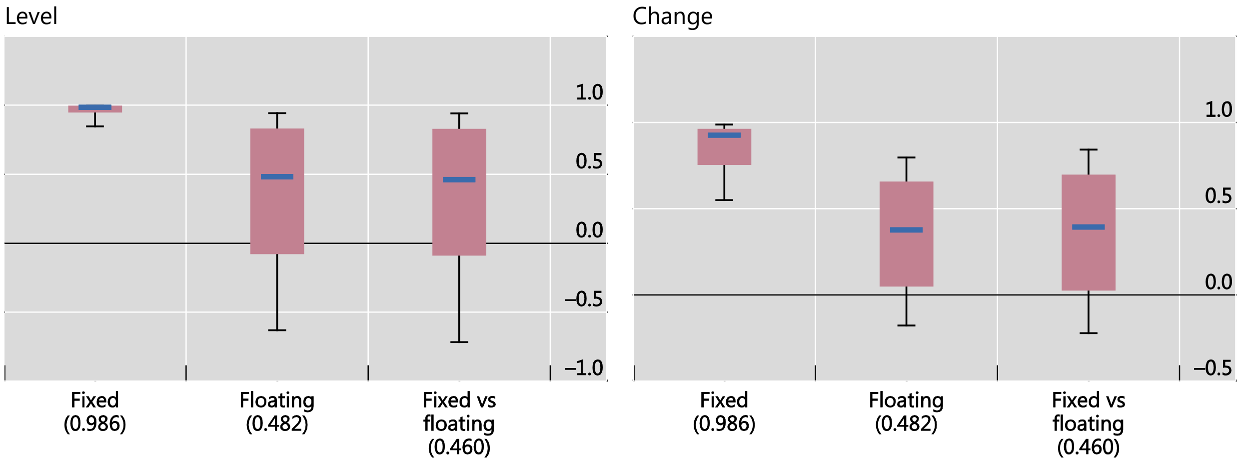
[1] For the Euro area, there are 55 and 38 pairs in the plot of cointegrating coefficients and coefficients of adjustments respectively. For the non-Eurozone OECD countries, there are 23 and 9 pairs in the plot of cointegrating coefficients and coefficients of adjustments respectively. For the fixed vs floating pairs, there are 88 and 30 pairs in the plot of cointegrating coefficients and coefficients of adjustments respectively. In the Tukey boxplots the bottom and top of the boxes are the first and third quartiles of the estimates within each set; the blue line indicates the median; and the bottom and top whiskers represent the 10th and 90th percentile of the estimates.

The second way in which we define economic fundamentals for the real exchange rate is to use the interest parity real exchange rate defined above:

$$q_t^{IP} \equiv E_t \sum_{j=0}^{\infty} r_{t+j}^* - r_{t+j} - \overline{(r^* - r)}$$

We can think of q_t^{IP} as being an index that captures the effect of measurable economic fundamentals on the real exchange rate. That is, factors such as monetary policy, fiscal policy, productivity changes, etc. – anything that affects the real exchange rate through the real interest rate channel rather than through the deviations from UIP – is captured by q_t^{IP}.

Correlation between q_t and q_t^{IP} Graph 5



The number in the parenthesis is the median in each group.

[1] For the Euro area, there are a total of (12 * 12 – 12)/2 = 66 pairs; plus one pair US-HK, making it 67 pairs. There are 16 non-Eurozone OECD countries, we have (16 * 16 – 16)/2 = 120 pairs; plus 15 pairs with HK. With data for Euro area and 16 non-Euro-area OECD countries, there are a total of 12 * 16 = 192 pairs; plus 12 pairs with HK. In the Tukey boxplots the bottom and top of the boxes are the first and third quartiles of the estimates within each set; the blue line indicates the median; and the bottom and top whiskers represent the 10th and 90th percentile of the estimates.

Source: authors' calculations.

In contrast to the relative productivity variable we discussed above, there is clearly a short-run relationship between q_t and q_t^{IP}. A simple way to compare the difference in the behaviour of real exchange rates in the fixed versus the floating regimes is to look at the correlation of these two variables. Graph 5 plots the distribution of correlation coefficients both for the levels of q_t and q_t^{IP} and their first differences, $\Delta q_t \equiv q_t - q_{t-1}$ and $\Delta q_t^{IP} \equiv q_t^{IP} - q_{t-1}^{IP}$. The difference between the fixed and floating regimes is striking. q_t and q_t^{IP} are very highly correlated in the country pairs with rigidly fixed nominal exchange rates, both in levels and first-differences. In levels, the correlations coefficients are very close to unity, and for changes, the median correlation coefficient is above 0.90.

For the floating rate countries, the correlation coefficients still tend to be quite high, with a median correlation coefficient around 0.5 for the levels of q_t and q_t^{IP}, and 0.40 for Δq_t and Δq_t^{IP}.

With both measures of fundamentals, there appears to be less disconnect between the real exchange rate and the economic variables under rigidly fixed nominal exchange rates than under floating rates.

III.6. Consumption correlation puzzle

A classic question in open economy macroeconomics is whether financial markets deliver risk sharing across countries. The original literature tests for risk sharing by examining whether consumption levels across pairs of countries are more highly correlated than income across those pairs. In the presence of financial integration and some capital mobility, one would expect some degree of consumption smoothing across countries, implying that the growth in real consumption of the home country representative agent should have a higher correlation with the real consumption growth in a foreign country than output growth in the two countries. Yet, as in Backus, Kehoe and Kydland (1992), the literature finds lower consumption growth correlation relative to output growth correlation.

A better approach to do the traditional test is to look at the correlation of the log of income available for consumption $\ln(Y-I-G)$, i.e. total income menus investment and government spending in the home country, with $\ln(Y^*-I^*-G^*)$ in the foreign country. The reason is that $Y-I-G$ is made available for private consumption in the home country were it closed, and likewise $Y^*-I^*-G^*$ is available for private consumption in the foreign country if it were closed. The same correlations can be analysed both in aggregate and *per capita* terms.

We present the evidence on this comparison in real terms in Table 7. We make the correlation comparisons for the pairs of 19 euro area economies with fixed nominal exchange rates; the pairs of 10 Canadian provinces; and the pairs of 19 non-euro-area OECD economies with floating rates. Although we present results for aggregate consumption and aggregate output, the more relevant results are the ones expressed in *per capita* terms, so we focus on those.

Consumption correlation puzzle: in real terms

Table 7

	In aggregate terms					
	Versus income correlation			Versus available consumption correlation[1]		
	Euro area [2]	Canada[3]	Floating[3]	Euro area [2]	Canada[3]	Floating[4]
Higher consumption correlation	49	45	59	113	45	133
Total of pairs	171	45	171	171	45	171
	In *per capita* terms					
	Versus income correlation			Versus available consumption correlation[1]		
	Euro area [2]	Canada[3]	Floating[3]	Euro area [2]	Canada[3]	Floating[4]
Higher consumption correlation	43	45	77	112	45	82
Total of pairs	171	45	171	171	45	171

[1] Available consumption is defined as output minus investment minus government expenditure. [2] For the 19 Euro area countries, there are a total of (19 * 19 – 19)/2 = 171 pairs. [3] For the 10 Canadian provinces, there are a total of (10 * 10 – 10)/2 = 45 pairs. [4] For the 19 non-Euro area OECD countries, there are a total of (19 * 19 -19)/2 = 171 pairs

Sources: Eurostat; OECD; authors' calculations.

Table 7 shows that the primary difference does not involve the nominal exchange rate system, but instead involves country borders. When we look at cross-country results, consumption correlation is higher than income correlation for 43 of 171 euro area pairs, and for 77 of the 171 OECD country pairs with floating exchange rates. However consumption correlation is higher than correlation of income available for consumption for 112 of 171 Eurozone pairs, and 82 of 171 floating exchange rate pairs. Roughly speaking, there appears to be evidence of some risk sharing for about half of the countries. In contrast, the consumption correlation is higher than income correlation for all 45 Canadian provinces, whether we look at total income or income available for consumption.

A strand of the literature has taken off from the fact that even if there were complete financial markets, so that a complete set of state-contingent bonds were traded, we might not see high consumption correlation across countries. The point arises because financial assets are denominated in currencies, not in units of aggregate consumption. If consumption baskets differ across countries, or there is pricing to market, so that purchasing power parity (PPP) does not hold, then complete markets actually imply a relationship between marginal utilities of consumption and real exchange rates.

To see this point, suppose that state-contingent bonds are denominated in the home currency – call it "dollar". Then in equilibrium, the marginal value of a dollar should be equal, up to a constant of proportionality, for consumers in the home country and the foreign country. In the home country, one dollar buys $\frac{1}{P_t}$ units of the aggregate consumption basket, where P_t is the dollar price of that consumer basket. The marginal utility of the last dollar in the home country is given by $\frac{U'(C_t)}{P_t}$, where $U(C_t)$ is utility of aggregate consumption, and $U'(C_t)$ is marginal utility.

In the foreign country, one dollar buys $\frac{1}{S_t P_t^*}$ units of the foreign consumption basket, where P_t^* is the foreign-currency price of the foreign consumption basket, and S_t is the amount of dollar per unit of foreign currency nominal exchange rate. The marginal utility of a dollar for foreign consumers is given by $\frac{U'(C_t^*)}{S_t P_t^*}$, where C_t^* is foreign consumption, and we have assumed for convenience that the form of the utility function is the same for home and foreign consumers.

Under complete markets, as Backus and Smith (1993) discuss, we should find

$$\frac{U'(C_t)}{P_t} = \kappa \frac{U'(C_t^*)}{S_t P_t^*}$$

This expression says that the marginal utility of a dollar should be the same for home and foreign residents under complete markets, up to a constant of proportionality, κ. The constant of proportionality arises because of differences in wealth for home and foreign consumers. If they were equally wealthy, then at the margin, complete markets insure that their marginal utilities are the same at all dates, and in all states of nature. Importantly, complete markets do not necessarily lead to complete risk sharing of consumption. If PPP held, the condition would reduce to

$$U'(C_t) = kU'(C_t^*)$$

This equality implies that the growth rates of marginal utilities of consumption are equal across countries. For example, when utility takes the standard form of constant relative risk aversion: $U(C_t)=\frac{1}{1-\gamma}C_t^{1-\gamma}$, the conditions implies $\Delta\left(c_t-c_t^*\right)=0$, where the lower-case c_t and c_t^* refer to the logs of consumption. In other words, when PPP holds, complete markets imply growth rates of consumption should be perfectly correlated across countries.

But if PPP does not hold, markets tend to allocate more resources to a country during periods in which its prices are relatively low. Accordingly, when utility is of the constant relative risk aversion form, the above condition implies

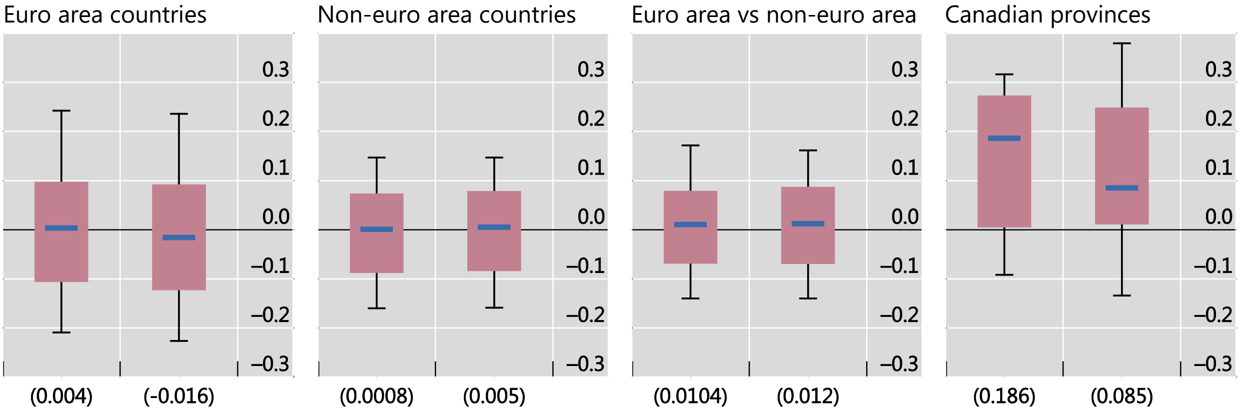
(12) $$\gamma\Delta\left(c_t-c_t^*\right)=\Delta q_t$$

That is, the condition implies that relative consumption growth rates should be perfectly positively correlated with the growth rate of the real exchange rate. The intuition is that, with efficient international risk sharing, home consumption grows at a faster pace than foreign consumption when home prices drop more rapidly than foreign prices, *i.e.* the home currency depreciates relative to the foreign currency and Δq_t rises. However, a fairly large empirical literature, including the earlier contribution from Backus and Smith (1993), has found that, among pairs of countries with floating nominal exchange rates, the correlation is actually low and negative, hence the consumption correlation puzzle or consumption-real-exchange-rate anomaly.

We re-examine this relationship for fixed-exchange rate pairs of countries, and compare it to that of floating exchange rate pairs, for relative consumption growth in both aggregate (left boxplot) and *per capita* (right boxplot) terms. The results are presented in Graph 6. Again, the key distinction turns out not to be whether the exchange rate is fixed or floating, but whether we are looking between countries or between regions within a country. When we look across countries, whether within the Euro area, or for floating exchange rate pairs, the average and median correlation is close to zero. The finding that the correlation is near zero for the Eurozone contrasts with the findings of Hadzi-Vaskov (2008) using an earlier data range.

Consumption correlation puzzle[1]

$corr(\Delta q_t, \Delta(c_{i,t} - c_{j,t}))$ Graph 6



[1] The first bar is for growth rate of aggregate consumption while the second one is for the *per capita* consumption. For the Euro area, there are a total of (19 * 19 – 19)/2 = 171 pairs; plus one pair US-HK, making it 79 pairs. For the 19 non-Eurozone OECD countries, we have (19 * 19 – 19)/2 = 171 pairs. With data for Euro area and 16 non-Eurozone OECD countries, there are a total of 19 * 19 = 361 pairs. In the Tukey boxplots the bottom and top of the boxes are the first and third quartiles of the estimates within each set; the blue line indicates the median; and the bottom and top whiskers represent the 10th and 90th percentile of the estimates.

Source: authors' calculations

Intra-national comparisons are different, in that the real exchange rates among Canadian provinces are mostly positively correlated with the relative consumption growth, with about 75% of the correlation coefficients being greater than zero and the median estimate being 0.19 for aggregate consumption. The results are broadly similar in terms of *per capita* consumption growth, although the median correlation estimate is now 0.09 for the Canadian provinces, about half the size of the median estimate in terms of aggregate consumption growth. This is consistent with the finding of Hess and Shin (2012) for US states.

Yet the traditional Backus-Smith (1993) breed of tests on consumption-real-exchange-rate correlation are hard to interpret. The test examines a condition that holds under complete markets, but markets are known to be incomplete. However, we can consider a test that is analogous to the tests of risk sharing in Table 7 if we make the special assumption of a logarithmic utility function. Log utility is the special case of the constant relative risk aversion utility function as $\gamma \to 1$. In that case, equation (12) becomes simply $\Delta\left(c_t - c_t^*\right) = \Delta q_t$.

Another way of writing that relationship is

$$\Delta\left(p_t + c_t\right) = \Delta\left(s_t + p_t^* + c_t^*\right)$$

This indicates that growth rates of nominal consumption that are expressed in a common currency should be perfectly correlated. Specifically, the change in nominal domestic consumption, *i.e.* $\ln\left(P \times C\right)$, should be perfectly correlated with the change in nominal foreign consumption expressed in the domestic currency, i.e. $\ln\left(S \times P^{*} \times C^{*}\right)$.

The traditional test of consumption correlation puzzle looks for evidence on whether any risk sharing is going on at all, so that the consumption levels become more correlated than the output levels. But once we take into account that the PPP does not hold, the analogous question becomes one of comparing the correlation of nominal domestic consumption $\ln\left(P \times C\right)$ with foreign consumption $\ln\left(S \times P^{*} \times C^{*}\right)$ to the correlation of nominal domestic available consumption $\ln\left(NY - NI - NG\right)$ with $\ln\left(S \times \left(NY^{*} - NI^{*} - NG^{*}\right)\right)$, where NY, NI and NG stand for domestic nominal income, nominal investment and nominal government spending, respectively, and * denotes the analogous expressions for the foreign country.

The results of the new correlation comparisons, in nominal terms, are presented in Table 8 and 9. They suggest that there is strong evidence of risk sharing by consumers among different countries and regions whether or not exchange rates are floating. Table 8 looks at the 171 pairs of 19 euro area economies with fixed nominal exchange rates and the 171 pairs of 19 non-euro-area economies with floating rates. When we focus on the comparison of nominal consumption correlations to the correlation in available nominal income for consumption, then 170 pairs out of 171 pairs of 19 euro area economies now have higher correlation in nominal consumption, in both aggregate and *per capita* terms. For the 171 floating-rate country pairs, higher correlation in nominal consumption is found in 142 cases in aggregate terms and 156 cases in *per capita* terms.

Table 9 adds further evidence from intra-national data on the tests based on nominal consumption, depending on data availability. First, there is strong evidence of risk sharing among Canadian provinces, and the consumption correlation puzzle goes away for most pairs of provinces when cross-region correlation in nominal consumption growth is compared to nominal income growth, both in aggregate and *per capita* terms. This is also true for the analysis on US states in aggregate terms for which we have data. But we find only limited evidence of risk sharing among the

Chinese provinces when it comes to the comparison to correlation in nominal income growth: the correlation in nominal consumption growth is only larger in 67 out of 465 pairs in aggregate terms, and in 71 cases in *per capita* terms.

Consumption correlation puzzle: in nominal terms

Table 8

	In aggregate terms			
	Versus income correlation		Versus available consumption correlation[1]	
	Euro area[2]	Floating[3]	Euro area[2]	Floating[3]
Higher consumption correlation	85	82	170	142
Total of pairs	171	171	171	171
	In *per capita* terms			
	Versus income correlation		Versus available consumption correlation[1]	
	Euro area[2]	Floating[3]	Euro area[2]	Floating[3]
Higher consumption correlation	86	82	170	156
Total of pairs	171	171	171	171

[1] Available consumption is defined as output minus investment minus government expenditure. [2] For the 19 Euro area countries, there are a total of (19 * 19 – 19)/2 = 171 pairs. [3] For the 19 non-Euro area OECD countries, there are a total of (19 * 19 -19)/2 = 171 pairs

Sources: Eurostat; OECD; authors' calculations.

But, consistent with the findings we have on correlation comparisons across countries, the comparison of the correlation in nominal consumption to the correlation in available nominal income for consumption suggests inter-regional risk sharing, as 42 (45) pairs out of 45 pairs of Canadian provinces have higher consumption correlation, in aggregate (*per capita*) terms. For the 465 pairs of Chinese provinces, higher nominal consumption correlation is found in 374 cases in aggregate terms and 370 cases in *per capita* terms. The evidence of risk sharing turns out to be rather strong for regions which have irrevocably fixed nominal exchange rates.

Consumption correlation puzzle (nominal): Canada, China and the United States — Table 9

	In aggregate terms					
	Versus income correlation			Versus available consumption correlation[1]		
	Canada[2]	China[3]	US[4]	Canada[2]	China[3]	US
Higher consumption correlation	40	67	1,197	42	374	NA
Total of pairs	45	465	1,225	45	465	NA
	In *per capita* terms					
	Versus income correlation			Versus available consumption correlation[1]		
	Canada[2]	China[3]	US	Canada[2]	China[3]	US
Higher consumption correlation	42	71	NA	45	370	NA
Total of pairs	45	465	NA	45	465	NA

[1] Available consumption is defined as output minus investment minus government expenditure. [2] For the 10 Canadian provinces, there are a total of (10 * 10 – 10)/2 = 45 pairs. 3 For the 31 Chinese provinces, there are a total of (31 * 31 – 31)/2 = 465 pairs. 4 For the 50 US states, there are a total of (50 * 50 – 50)/2 = 1,225 pairs,

Sources: Canada Statistics; Eurostat; OECD; authors' calculations.

IV. Conclusion

Do real exchange rates behave differently under fixed versus floating nominal exchange rates? Would the exchange rate puzzles disappear or become less severe in a rigidly fixed nominal exchange rate regime? We perform a number of empirical tests to examine six exchange rate puzzles, focusing on countries within the euro area, regions in China and Canada, and Hong Kong SAR vis-à-vis the United States. Our results suggest that indeed some of these puzzles are less "puzzling", *i.e.* less severe, under a rigidly fixed exchange rate regime, while other puzzles remain.

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V. Appendices

Appendix V.1. Data sources for economies under analysis

Data sources include the Bureau of Labour Statistics (BLS), Bureau of Economic Analysis (BEA), Eurostat, Federal Reserve Economic data (FRED), Hong Kong Census and Statistics Department (HK Census), Statistics Canada (CANSIM), International Monetary Fund's World Economic Outlook, Organisation for Economic Co-operation and Development (OECD), Bloomberg, CEIC, WIND and some national sources.

Table V.1		National data		Regional data		
Variable	Description	Euro area	Non-euro area	China	US	Canada
CPI, all items	Consumer price index, all items	Eurostat	Bloomberg, Eurostat, BLS, HK Census	WIND	BLS	CANSIM
CPI, goods	Consumer price index, goods	Eurostat	Bloomberg, Eurostat, BLS, HK Census	WIND, national source[4]	BLS	CANSIM
CPI, services	Consumer price index, services	Eurostat	Bloomberg, Eurostat, BLS, HK Census	WIND, national source[4]	BLS	CANSIM
Exchange rates	Bilateral exchange rates	Bloomberg	Bloomberg, FRED			
Interest rates	Three-month rates	Bloomberg	Bloomberg			
Consumption	Private consumption expenditure, national account	OECD/Eurostat	OECD	CEIC	BEA	CANSIM
Income	GDP output, national account	OECD/Eurostat	OECD	CEIC	BEA	CANSIM
Population		OECD/IMF	OECD/IMF	CEIC		CANSIM
Labour		OECD	OECD	CEIC		

We include in our analysis the following countries and regions:

Table V.2

	List of countries/regions
Euro area countries (19)	Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia, Spain
Non-euro-area OECD countries (19)	Australia, Canada, Chile, Czech Republic, Denmark, Hungary, Iceland, Israel, Japan, Korea, Mexico, New Zealand, Norway, Poland, Sweden, Switzerland, Turkey, United Kingdom, United States
Region with own currency (1)	Hong Kong SAR
Canada, provinces (10)	Alberta, British Columbia, Manitoba, New Brunswick, Newfoundland, Nova Scotia, Ontario, Prince Edward Island, Quebec, Saskatchewan
China, provinces (31)	Anhui, Beijing, Chongqing, Fujian, Gansu, Guangdong, Guangxi, Guizhou, Hainan, Hebei, Heilongjiang, Henan, Hubei, Hunan, Inner Mongolia, Jiangsu, Jiangxi, Jilin, Liaoning, Ningxia, Qinghai, Shaanxi, Shandong, Shanghai, Shanxi, Sichuan, Tianjin, Tibet, Xinjiang, Yunnan, Zhejiang
United States	
Metropolitan areas (27)	New York-Northern New Jersey-Long Island, NY-NJ-CT-PA; Philadelphia-Wilmington-Atlantic City, PA-NJ-DE-MD; Boston-Brockton-Nashua, MA-NH-ME-CT; Pittsburgh, PA; Chicago-Gary-Kenosha, IL-IN-WI; Detroit-Ann Arbor-Flint, MI; St. Louis, MO-IL; Cleveland-Akron, OH; Minneapolis-St. Paul, MN-WI; Milwaukee-Racine, WI; Cincinnati-Hamilton, OH-KY-IN; Kansas City, MO-KS; Washington-Baltimore, DC-MD-VA-WV; Dallas-Fort Worth, TX; Houston-Galveston-Brazoria, TX; Atlanta, GA; Miami-Fort Lauderdale, FL; Tampa-St. Petersburg-Clearwater, FL; Los Angeles-Riverside-Orange County, CA; San Francisco-Oakland-San Jose, CA; Seattle-Tacoma-Bremerton, WA; San Diego, CA; Portland-Salem, OR-WA; Honolulu, HI; Anchorage, AK; Phoenix-Mesa, AZ; Denver-Boulder-Greeley, CO
States (50)	Alabama, Alaska, Arizona, Arkansas, California, Colorado, Connecticut, Delaware, Florida, Georgia, Hawaii, Idaho, Illinois, Indiana, Iowa, Kansas, Kentucky, Louisiana, Maine, Maryland, Massachusetts, Michigan, Minnesota, Mississippi, Missouri, Montana, Nebraska, Nevada, New Hampshire, New Jersey, New Mexico, New York, North Carolina, North Dakota, Ohio, Oklahoma, Oregon, Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, Texas, Utah, Vermont, Virginia, Washington, West Virginia, Wisconsin, Wyoming

The data span and frequencies for our analysis of each exchange rate puzzle are listed as follows:

Table V.3	National	Regions		
		China provinces	US states	Canada provinces
Puzzle 1	Jan 1999 – Oct 2016	Jan 2002 – Dec 2016	H1 1984 – H2 2016	Jan 1999 – Dec 2016
Puzzle 2	Q1 1999 – Q3 2016			
Puzzle 3	Q1 1999 – Q3 2016			
Puzzle 4	Jan 1999 – Oct 2016			
Puzzle 5	1999 – 2015	2003 – 2016		
Puzzle 6	Q1 1999 – Q4 2016	1994 - 2015	1997 – 2015	1981 – 2015

Appendix V.2. Classification of tradables versus non-tradables

The tradable and non-tradable goods and services are classified according to the following list.

Table V.4	Description	Type
A	Agriculture, forestry and fishing	Tradable
B	Mining and quarrying	Tradable
C	Manufacturing	Tradable
D	Electricity, gas, steam and air conditioning supply	Tradable
E	Water supply, sewerage, waste management and remediation activities	Tradable
F	Construction	Tradable
G	Wholesale and retail trade, repair of motor vehicles and motorcycles	Non-tradable
H	Transportation and storage	Non-tradable
I	Accommodation and food service activities	Non-tradable
J	Information and communication	Non-tradable
K	Financial and insurance activities	Non-tradable
L	Real estate activities	Non-tradable
M	Professional, scientific and technical activities	Non-tradable
N	Administrative and support service activities	Non-tradable
O	Public administration and defence, compulsory social security	Non-tradable
P	Education	Non-tradable
Q	Human health and social work activities	Non-tradable
R	Arts, entertainment and recreation	Non-tradable
S	Other service activities	Non-tradable
T	Act. of HH as employers, undif. G&S-producing activities of HH for own use	Non-tradable

Appendix V.3. Labour productivity

Labour productivity measures the amount of goods and services produced by one unit of labour, often expressed as the ratio of gross domestic product or gross value added to the total number of hours worked or total employment. To obtain a measure of labour productivity in the tradable versus non-tradable sectors, we first divide the gross value added in a country or region into tradable and non-tradable components according to the structure of the International Standard Industrial Classification of All Economic Activities (ISIC) Rev. 4. Then the tradable and non-tradable components of the gross value added υ_i in each group *i* are added up and then divided by the sum of employment industry l_i in the tradable and non-tradable sector, respectively:

$$Prod_k = \frac{\sum_{i \in k} \upsilon_i}{\sum_{i \in k} l_i}$$

where the industry *i* belongs to sector $k = T, NT$, i.e. tradable or non-tradable.

Appendix V.4. Computing $q^{IP}{}_t$

For pairs of economies in and out of the euro area, we use VECM model estimates to obtain estimates for the term $r^*{}_t - r_t$, and then use these estimates to compute estimates of

$$q^{IP}{}_t \equiv E_t \sum_{j=0}^{\infty} r^*{}_{t+j} - r_{t+j} - \overline{(r^*{}_t - r_t)}$$

Let $x_t = \begin{bmatrix} s_t \\ p^R{}_t \\ i^R{}_t \end{bmatrix}$, the VECM model can be written as

$$\mathbf{x_t - x_{t-1} = C_0 + Gx_{t-1} + C_1(x_{t-1} - x_{t-2}) + C_2(x_{t-2} - x_{t-3}) + C_3(x_{t-3} - x_{t-4}) + u_t}$$

where **G** is restricted to be $G = \begin{bmatrix} g_{11} & -g_{11} & g_{13} \\ g_{21} & -g_{21} & g_{23} \\ g_{31} & -g_{31} & g_{33} \end{bmatrix}$ by assuming $i^R{}_t$ and $s_t - p^R{}_t$ are stationary.

For pairs of economies with rigidly fixed nominal exchange rates, *e.g.* those within the euro area, the VECM model remains the same but with the x_t term now reduced to $x_t = \begin{bmatrix} p^R{}_t \\ i^R{}_t \end{bmatrix}$.

Appendix V.5. Computing the half-life

In the half-life estimation for the puzzle of "excess persistence of real exchange rates", the raw price series we use as input are not seasonally adjusted. To deal with the data's seasonality, we first run the following regressions for the real exchange rate and relative price series adding monthly dummies as regressors:

$$q_t = \gamma_1 + \rho_1 q_{t-1} + \sum_{j=2}^{12} \kappa_{1,j} D_{\mathrm{j},t} + \zeta_{1,t}$$

and

$$x_t = \gamma_2 + \rho_2 x_{t-1} + \sum_{j=2}^{12} \kappa_{2,j} D_{\mathrm{j},t} + \zeta_{2,t}$$

where $x_t = p^*{}_{N,t} - p^*{}_{T,t} - (p_{\ N,T} - p_{N,t})$, and $\{D_{\mathrm{j},t}\}_{j=2}^{12}$ are the dummy variables for the months of February to December, $D_{\mathrm{j},t} = 1$ for the *j*-th month, and is zero otherwise. ρ_1 and ρ_2 are the AR(1) coefficients.

The half-life δ is then computed as

$$\delta = 12 \times \frac{-\log 2}{\log(\rho)}$$

Following Rossi (2005), we adjust the estimated AR(1) process, by regressing the residuals of the equations against their lags as follows:

$$\zeta_t = \gamma_1 \zeta_{t-1} + \gamma_2 \zeta_{t-2} + \gamma_3 \zeta_{t-3} + v_t$$

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