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Central Counterparty Capitalization and Misaligned Incentives

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Central Counterparty Capitalization and Misaligned Incentives

Abstract

Financial stability depends on the effective regulation of central counterparties (CCPs), which must take account of the incentives that drive CCP behavior. This paper studies the incentives of a for-profit CCP with limited liability. It faces a trade-off between fee income and counterparty credit risk. A better-capitalized CCP sets a higher collateral requirement to reduce potential default losses, even though it forgoes fee income by deterring potential traders. I show empirically that a 1% increase in CCP capital is associated with a 0.6% increase in required collateral. Limited liability, however, creates a wedge between its capital and collateral policy and the socially optimal solution to this trade-off. The optimal capital requirements should account for clearing fees.

Keywords: central counterparties (CCPs), capital requirement, financial stability.
JEL classifications: G01, G12, G21, G22.

1 Introduction

Central counterparties (CCPs) are systemically important. First, the outstanding positions cleared by CCPs are enormous. For over-the-counter (OTC) interest rate derivatives, the notional amount of centrally cleared contracts was USD 366 trillion in June 2018, accounting for 76% of global outstanding positions (BIS, 2018). Second, CCPs and other financial institutions are highly interconnected via clearing membership, custodianship and credit relationships (BCBS-CPMI-IOSCO-FSB, 2017, 2018).

Unfortunately, CCPs are not invulnerable and their failure could threaten financial stability. There were several clearinghouse failures in the 1970s and 1980s, which led to market shutdowns.[1] More recently, stress in the European repo market in 2016 suggests that market participants did indeed price in the probability of a CCP failure (Boissel et al., 2017). In September 2018, a single trader's default wiped out two-thirds of the default fund of Nasdaq Clearing, a CCP that the Financial Stability Board (FSB) identifies as systemically important.

For-profit CCPs have incentives that are misaligned with financial stability. Since the demutualization trend in 1990s, many CCPs have operated as for-profit publicly listed financial firms. These include the Chicago Mercantile Exchange (CME) in the U.S. and Eurex Clearing in Europe. There are public debates as to whether these CCPs have enough capital to align incentives appropriately (see, e.g., Bignon and Vuillemey, 2018; Giancarlo and Tuckman, 2018). Furthermore, clearing members with exposures to CCPs have called for them to hold more capital, arguing that CCPs are not adequately incentivized to manage risk (see, e.g., Financial Times, 2014).

This paper investigates CCP capital and incentives. In particular, it explains how profit maximization and limited liability give rise to incentives for CCPs that are misaligned from the viewpoint of a benevolent social planner. The paper further shows how optimal capital requirements might be designed to correct the misaligned incentives. Finally, it provides new empirical evidence that more CCP capital leads to more prudent risk management.

[1] There have been at least four cases of clearinghouse failure: the French Caisse de Liquidation (1973), the Kuala Lumpur Commodities Clearing House (1983), the Hong Kong Futures Exchange (1987), and the New Zealand Futures and Options Exchange (1989). Interested readers can refer to Hills et al. (1999), Buding, Cox, and Murphy (2016) and Bignon and Vuillemey (2018).

The model has three types of agent: risk-averse protection buyers, risk-neutral protection sellers, and a risk-neutral for-profit CCP. Each risk-averse buyer is endowed with a unit of a risky asset, such as a bond. To hedge the risk, buyers purchase insurance from sellers via a derivatives contract, such as a credit default swap (CDS), that is cleared by the CCP (Biais, Heider, and Hoerova, 2012, 2016). Buyers and sellers are clearing members of the CCP. They trade only when there are utility gains from trading net of clearing costs. The total welfare surplus is equal to the CCP's expected utility plus the traders' utility gains from trading.

Sellers have heterogeneous capacities to reduce their losses in the bad state when the risky asset suffers a negative shock (Perez Saiz, Fontaine, and Slive, 2013). While the distribution of the loss-reduction capacity is common knowledge, the CCP does not observe individual sellers' loss-reduction capacities. As a result, the CCP sets a *uniform* collateral requirement, i.e., an initial margin requirement, for all sellers. In practice, CCPs generally seek to overcome the asymmetric information problem by (i) setting relatively strict membership requirements and (ii) charging credit add-ons to reflect the creditworthiness of clearing members. Although such measures may reduce the asymmetry of information between CCPs and members, they are not perfect. As the recent Nasdaq Clearing episode shows, CCPs can substantially misjudge the creditworthiness of their clearing members.

A seller defaults strategically when his out-of-the-money position, net of his loss reduction, is larger than the collateral he has posted. In other words, a seller's loss-reduction capacity determines his creditworthiness as a counterparty.

The for-profit CCP in the model has three important characteristics. First, it always has a matched book and is not directly exposed to market risk (Cox and Steigerwald, 2017). Instead, it is exposed to counterparty credit risk because it needs to cover the losses when some traders default. Second, the CCP relies on a so-called default waterfall to mutualize counterparty credit risk (Duffie, 2015). In case of defaults, the waterfall specifies different layers of prefunded resources contributed by the defaulting traders, the CCP and the non-defaulting traders. Without capital regulation, as is currently the case, the CCP chooses how much capital to contribute to the default waterfall, which is often called skin-in-the-game.[2] Third, the CCP's profit comes solely from fees that are

[2]For this model, I use skin-in-the-game and capital interchangeably. In practice, this may not be the case as skin-

set exogenously per unit of cleared contracts. The CCP's expected utility is the volume-based fee income, minus the capital cost and the potential loss of its capital as a result of defaults by clearing members.

Main findings. The model shows that a CCP with more capital requires more collateral from its clearing members. A higher collateral requirement lowers the default rate as well as the loss-given-default. This does, however, cause profitable trades to be forgone, hence reducing fee income. When a CCP has a higher level of capital, it is more concerned about the losses from counterparty risk that eats into this capital. Hence, it will set a higher collateral requirement to disincentivize defaults.

High skin-in-the-game has an ambiguous impact on traders' utility gains. On the one hand, since collateral is costly, a higher level of CCP capital means that traders must bear higher collateral costs. In fact, CCPs use this point to argue against high skin-in-the-game (see, e.g., LCH, 2014). On the other hand, a better-capitalized CCP is less likely to impose losses on surviving traders (CPMI-IOSCO, 2012). Depending on which channel dominates, increasing CCP skin-in-the-game can increase or decrease the utility gains from trading.

A key insight of the model is that, if a for-profit CCP faces no capital requirements, the amount of capital it chooses to hold is lower than the socially optimal level. As the CCP is protected by limited liability, its expected utility decreases with the capital cost and the potential loss on this capital. As a result, without capital requirements, the CCP chooses zero capital. In this case, when the negative shock is realized, the CCP finds itself with insufficient prefunded resources and becomes insolvent. This means that buyers are not fully insured, leading to a loss of economic efficiency.

The socially optimal capital requirement for a for-profit CCP needs to take into account the per-unit clearing fee.[3] The higher the fee, the higher the temptation for a for-profit CCP to increase

in-the-game can be a fraction of CCP operational capital (e.g., the European Market Infrastructure Regulation). Since the model does not feature operational risk, however, there are no additional insights to be gained from distinguishing a CCP's skin-in-the-game from its capital.

[3] Clearing fees can vary significantly. For the standard plan in LCH SwapClear, clearing fees for interest rate derivatives are \$0.9-\$18 per million for a new trade with maturity ranging from overnight to 50 years (see http://www.wip.swapclear.com/fees/llc/swapclear.asp). For the standard plan in LCH EquityClear, however, fees for

trading volume. When the fee is below a certain threshold, a high capital requirement is effective *ex ante* in incentivizing the CCP to eliminate the counterparty credit risk it is exposed to. The CCP does so by increasing its collateral requirement which reduces trading volume. When the fee is above the threshold, the CCP maximizes its trading volume by charging its clearing members a low collateral requirement. In this case, the capital requirement cannot rule out defaults, but it serves as loss-absorbing capacity *ex post* in the event of defaults. Although the optimal capital requirement can avoid the dead-weight loss of an insolvent CCP, it is worthwhile pointing out that it does not achieve the first-best outcome because of the costliness of the collateral and capital.

The model also allows me to study the optimization problem of a CCP that is owned by all clearing members, and hence maximizes the total welfare surplus (instead of its own expected utility as in the case of a for-profit CCP). In line with Cox and Steigerwald (2016) who study CCPs with different ownership structures, I call this type of CCP "user-owned".[4] Compared with a for-profit CCP, a user-owned one holds more capital and sets a lower collateral requirement. In real operations, user-owned CCPs are owned mainly by a small number of large clearing members. Hence, there could be misaligned incentives between the larger members (who are CCP owners) and the smaller members (who have no say in deciding CCP capital and collateral policy). As my aim is to contrast a for-profit CCP against a user-owned CCP, I assume away this layer of misaligned incentives.

Finally, the paper provides empirical evidence from CCP quantitative disclosure data (CPMI-IOSCO, 2015) and CCP ownership information from public sources.[5] In total, there are 16 CCPs at the group level and 44 CCPs at the entity level, which captures the majority of the clearing industry. The data are at a quarterly frequency and range from 2015 Q3 to 2017 Q4.

The empirical results support the theoretical model. First, panel regressions show that there is a significantly positive relationship between CCP skin-in-the-game and required initial margin for the for-profit CCPs in the sample. A 1% increase of CCP skin-in-the-game is associated with an increase of more than a 0.6% in the required initial margin. Second, ownership structures matter

cash equities are less than \$0.003 per million (see https://www.lch.com/services/equityclear/equityclear-ltd/fees).

[4] Appendix C shows the different ownership structure for different CCPs.

[5] The CCP quantitative disclosure data are from the CCPView of Clarus Financial Technology: https://www.clarusft.com/products/data/ccpview/.

for CCP skin-in-the-game. The for-profit CCPs in the sample have significantly lower skin-in-the-game than the user-owned ones.

Relevant literature. Central clearing has three main features: multilateral netting across counterparties, a central data warehouse of outstanding position information, and mutualization of counterparty credit risk. While the first two features are reminiscent of other payment and settlement systems, the mutualization feature is unique to CCPs.[6]

This paper contributes to the fast-growing literature on incentives and risks resulting from the mutualization of counterparty credit risk. The basic setup is similar to Biais, Heider, and Hoerova (2012) and Biais, Heider, and Hoerova (2016). But the key economic frictions are different. This paper focuses on a for-profit CCP's incentives while theirs study traders' risk-shifting incentives. Biais, Heider, and Hoerova (2012) explain the risk allocation implications of central clearing. Their model suggests that, although central clearing brings diversification benefits by mutualizing counterparty credit risk, a CCP should not offer full insurance against counterparty credit risk due to moral hazard problems. Central clearing reduces traders' incentives to acquire information and monitor counterparty credit risk, leading to a higher aggregate risk. Biais, Heider, and Hoerova (2016) show that margin requirements, together with central clearing, can preserve the risk-prevention incentives by inducing the optimal level of risk monitoring and can exploit the mutualization benefits of risk-sharing.

In this paper, the key frictions are that (i) the heterogeneous loss-reduction capacities (i.e., the counterparty credit risk) of individual sellers are not observable to the CCP; and (ii) the for-profit CCP does not internalize its impact on the traders' utilities. The modeling of heterogeneity is built on Perez Saiz, Fontaine, and Slive (2013). However, their focus is on the impact of central clearing on dealers' competition and profits. Koeppl (2013) also studies the unobservable counterparty credit risk in the context of central clearing. In his setup, the CCP does not chase profit but

[6]The netting benefits and data warehouse functions of CCPs are important factors when comparing different clearing systems. This paper, however, focuses on a for-profit CCP's incentives, which are not affected by these two features. Interested readers can refer to Duffie and Zhu (2011); Cont and Kokholm (2014); Duffie, Scheicher, and Vuillemey (2015) for netting benefits across bilateral and central clearing; and refer to Acharya and Bisin (2009, 2014); Koeppl and Monnet (2010, 2013); Koeppl, Monnet, and Temzelides (2012) for how central clearing can alleviate the externalities that are associated with the opacity of OTC derivatives positions.

minimizes counterparty risk. Hence, the incentives of the CCP are very different from those of the for-profit CCP in this paper.

This paper explains how using the prefunded financial resources sequentially (i.e., following the default waterfall) can create intertwined incentives between the CCP and the traders, adding to the literature on CCP prefunded financial resources, which emphasizes the overall loss allocation rules (Elliott, 2013; Cumming and Noss, 2013), the adequacy of the default fund (Murphy and Nahai-Williamson, 2014; Capponi, Wang, and Zhang, 2018), and the roles of CCP skin-in-the-game (Carter, Hancock, and Manning, 2016; Carter and Garner, 2016; Murphy, 2016).

The existing literature considers many critical aspects of central clearing but, fundamentally, previous researchers have assumed that CCPs are benevolent organizations, which could be true in some cases. However, given that many for-profit CCPs are publicly listed financial firms, one should not overlook their incentives. This paper takes a different approach from the literature, as it explicitly models CCPs' for-profit incentives. It also provides empirical evidence that a higher level of CCP capital is associated with a higher required collateral.

The remainder of this paper is as follows. Section 2 introduces the model. Section 3 focuses on the misaligned incentives for a for-profit CCP. Section 4 analyses the optimal capital requirement of the for-profit CCP. Section 5 studies the case of a user-owned CCP. Section 6 provides empirical evidence from CCP quantitative disclosure data. Section 7 concludes.

2 Model

2.1 Model setup

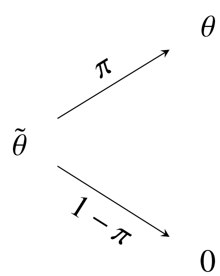
The two-period model has three types of agent: protection buyers, protection sellers and a for-profit CCP.[7] At $t = 0$, the CCP chooses its capital and sets the collateral requirement. Observing the CCP's capital and the collateral requirement, the buyers and sellers decide whether to trade a standard protection contract that is cleared by the CCP. At $t = 1$, uncertainty is resolved and

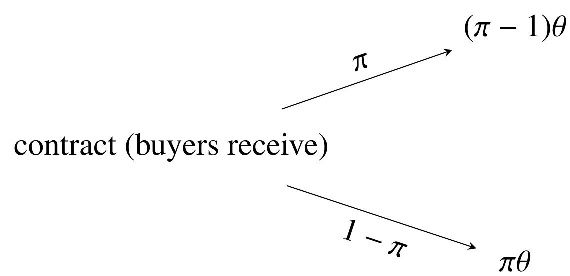
[7]Since the focus of the paper is the misaligned incentives of for-profit CCPs, "CCP" refers to a for-profit CCP unless specified otherwise.

payoffs are realized. All the variables are summarized in Appendix A.

Protection buyers. There is a unit mass of *homogeneous* protection buyers who are risk-averse. They are endowed with one unit of a risky asset at $t = 0$. The asset has a random return $\tilde{\theta}$ at $t = 1$. $\tilde{\theta}$ can take on two values: θ (> 0) in the good state with probability π and 0 in the bad state with probability $1 - \pi$.

The risk-averse buyers purchase insurance from sellers via a protection contract. The contract has zero-mean and provides full insurance to the buyers. In other words, the contract specifies that the buyers pay the sellers $(1 - \pi)\theta$ in the good state; and the sellers pay the buyers $\pi\theta$ in the bad state.





Protection sellers. There is a unit mass of *heterogeneous* protection sellers who are risk-neutral and have limited liability. They are endowed with loss-reduction capacity in the bad state. The capacity to reduce loss in the bad state varies across sellers. Let r_j denote the loss-reduction capacity of seller j. This means that, for each protection contract, seller j can reduce his loss by $r_j\pi\theta$ in the bad state. Instead of paying $\pi\theta$ to his buyer in the bad state, seller j pays $(1 - r_j)\pi\theta$. For simplicity, I assume that r_j is uniformly distributed on an interval of $(0, 1)$. The distribution of r_j is common knowledge; but the individual seller's r_j is observable to the buyers but *not* observable to the CCP (elaborated below). The assumption of heterogeneous loss-reduction capacity is not far from reality. Dealers in derivatives markets normally have their own specialty in managing their position risk (Perez Saiz, Fontaine, and Slive, 2013).

Each protection buyer is randomly matched with one protection seller who has one unit of contract to sell.[8] However, matching does not guarantee trading. A buyer and a seller decide to trade or not depending on their utility improvement from trading (and clearing). If their utility improvement from trading is positive, they will trade and Nash-bargain to split the positive utility improvement. As long as both buyers and sellers have positive bargaining power, their individual utility improvement is positive when the joint utility improvement is positive. Hence, the trading volume between a buyer and a seller can be either zero or one, depending on the joint utility improvement for this pair of traders.

CCP. The contract is required to be centrally cleared. A representative competitive CCP clears all the trades. Through novation, the protection contract is split into two contracts: one is between the protection buyer and the CCP, and the other is between the protection seller and the CCP. If traders default, they default on the CCP.

The CCP demands collateral to disincentivize defaults. As specified by the protection contract, the buyers are out-of-the-money in the good state and the sellers in the bad state. Since the buyers receive a high payoff from the risky asset in the good state, they can settle the out-of-the-money positions smoothly. Although the sellers can reduce the downside risk, their loss-reduction capacities are not large enough to settle the out-of-the-money positions ($r_j < 1$). Hence, the sellers have incentives to default in the bad state. To protect itself from the sellers' defaults, the CCP requires that the sellers post collateral.[9] Since individual sellers' loss-reduction capacities are not observable to the CCP, it sets a *uniform* collateral requirement based on the distribution of sellers' loss-reduction capacities. Hence, a seller with loss-reduction capacity r_j will not default when his collateral with the CCP is larger or equal to his loss from the out-of-the-money position.[10] Let c denote the collateral requirement for each unit of outstanding position. The per-unit collateral cost

[8] I adopt the most simplified search model here: random matching. Introducing more advanced search models will definitely have a better approximation of derivatives markets. But that complicates the model unnecessarily, since the focus of my paper is the CCP's incentives, not those of the trading parties. Interested readers could refer to Koeppl, Monnet, and Temzelides (2012), for example.

[9] In the current setup, the buyers do not need to pledge collateral with the CCP. The implicit assumption is that the CCP could seize the buyers' risky asset if they default. This is similar to the setup in Koeppl, Monnet, and Temzelides (2012). The benefit of such a setup is that it separates losses borne by two groups of surviving members: the non-defaulting sellers and the buyers. Requiring the buyers to pledge collateral will not change the results qualitatively.

[10] As discussed later, sellers also need to deposit default fund with the CCP. Hence, the "collateral" includes not only the collateral requirement but also the default fund requirement.

borne by sellers is δ.

Apart from the collateral requirement, the CCP has other prefunded financial resources: the default fund and the CCP's capital. Each seller's default fund contribution is proportional to his collateral, i.e., αc, where α is an exogenous parameter.[11] Without capital regulation, the CCP chooses its own capital K. The per-unit capital cost borne by the CCP is φ. In short, the CCP has the following default waterfall to allocate losses (Duffie, 2015):

1. the collateral contributed by defaulting sellers;

2. the default fund contribution by defaulting sellers;

3. the CCP's capital K;

4. the default fund contributed by non-defaulting sellers.

When the default fund contributed by the non-defaulting sellers is used to cover default losses, the non-defaulting sellers share the losses evenly. Let d denote the default fund losses of each non-defaulting seller. At the end of the default waterfall, the remaining loss will be borne by the buyers evenly, meaning the buyers are only partially insured. Let w denote the wedge between the required payment (specified by the contract) and the actual payment.[12]

The CCP is a risk-neutral and for-profit financial firm.[13] The CCP's income comes from a volume-based fee. Both the buyers and sellers need to pay $\frac{f}{2}$ for each unit cleared. The fee level f is exogenous as the CCP is a price-taker. Instead of increasing the fee level, the CCP can increase the trading volume by changing the collateral requirement c, since the high collateral cost could deter some sellers from trading. The CCP is a limited liability entity, which means the maximum loss that it needs to cover will not exceed its own capital. Let L denote the total default loss that needs to be borne by the CCP, and v denote the trading volume. The risk-neutral CCP maximizes the following expected utility:

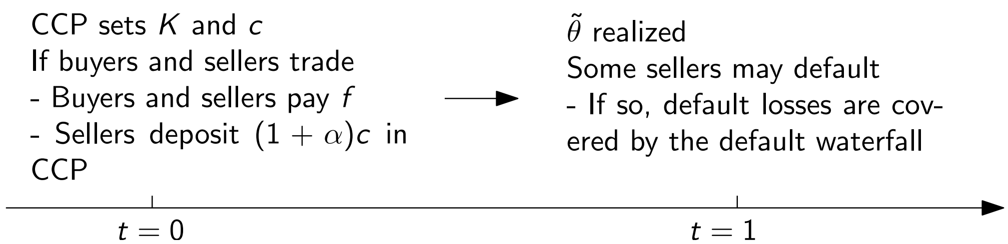
[11] The overall size of the default fund could be determined by the "Cover 2" standard, for instance.

[12] It is not far from reality. In the recovery plan outlined by CPMI-IOSCO (2014), one way to recover an insolvent CCP is variation margin gains haircutting (VMGH), which essentially asks the winning side (protection buyers) to bear the losses caused by the losing side's (protection sellers') defaults.

[13] In Section 5, I analyze the case of a CCP that is owned by all clearing members. In that case, the user-owned CCP maximizes the total social welfare.

$$U_{CCP} = \underbrace{fv}_{\text{fee income}} + (1-\pi)\underbrace{\max(-L,-K)}_{\text{limited liability}} - \underbrace{\varphi K}_{\text{capital cost}}. \quad (1)$$

Timeline. At $t = 0$, the CCP chooses its own capital and the collateral requirement to maximize its expected utility. The sellers and buyers are randomly matched and they observe the CCP's capital and the collateral requirement. If they decide to trade, they pay the CCP the clearing fee and the sellers deposit $(1 + \alpha)c$ with the CCP. At $t = 1$, the payoff of the risky asset is realized. If the bad state is realized, some of the sellers may default (depending on how high the collateral is). If so, the CCP will allocate the default losses in accordance with the default waterfall.



Traders' state-contingent payoffs and expected utilities. The default waterfall changes the state-contingent payoffs of the buyers and sellers.[14] For the buyers, they are fully insured only if the prefunded resources can cover all default losses. In those cases, they will receive $\pi\theta$ in both states. Otherwise, they will receive $\pi\theta$ in the good state and $\pi\theta - w$ in the bad state.

The sellers all receive $(1 - \pi)\theta$ in the good state. In the bad state, if nobody defaults, seller j has negative payoff of $-(1 - r_j)\pi\theta$. If some sellers default, the sellers' payoffs in the bad state vary across the defaulting sellers and the non-defaulting sellers.The defaulting sellers in the bad state have a negative payoff of $-(1 + \alpha)c$. The payoffs of the non-defaulting sellers in the bad state depend on whether their default fund contributions will be used to cover the losses:

- If the default fund is not used, the non-defaulting sellers have a negative payoff: $-(1 - r_j)\pi\theta$.
- If the default fund is partly used, they have a negative payoff: $-(1 - r_j)\pi\theta - d$.
- If the default fund is depleted, they have a negative payoff: $-(1 - r_j)\pi\theta - \alpha c$.

[14] Appendix B shows traders' payoffs in different states when different layers of the waterfall are affected.

Let $\tilde{b}$ denote the state-contingent payoffs for the homogeneous buyers and $\tilde{s}_j$ denote the state-contingent payoffs for seller j. The buyers are risk-averse and have mean-variance utility.[15] Let γ denote the risk aversion of the buyers. The expected utility of a buyer is

$$U^b = \mathrm{E}(\tilde{b}) - \frac{\gamma}{2}\mathrm{var}(\tilde{b}) - \underbrace{\frac{f}{2}}_{\text{clearing cost}}. \tag{2}$$

The sellers are risk-neutral. Their expected utility is the expected value of their payoffs minus the cost associated with clearing, i.e., the collateral cost and the clearing fee.

$$U^{s_j} = \mathrm{E}(\tilde{s}_j) - \underbrace{\left[(1+\alpha)\delta c + \frac{f}{2}\right]}_{\text{clearing cost}}. \tag{3}$$

The utilities that a buyer and a seller can derive from their outside options (i.e., no trade) are

$$D^b = \pi\theta - \frac{\gamma}{2}(1-\pi)\pi\theta^2, \qquad D^{s_j} = 0. \tag{4}$$

Thus, the total welfare surplus from trading is

$$W = U_{CCP} + \int_0^1 \left(U^b + U^{s_j} - D^b - D^{s_j}\right)\mathrm{d}r_j. \tag{5}$$

2.2 The parameter assumptions

In what follows, I focus on the relevant cases where collateral and capital matter. Hence, the following assumptions are imposed. Assumption 1 specifies that the collateral cost is not negligible. If collateral cost is so low that every seller can provide full collateral, nobody will default when the bad state is realized. To exclude that scenario, it is necessary to establish some lower bound for the collateral cost.

The loss of the seller with zero loss-reduction capacity ($r_j = 0$) in the bad state is largest among the sellers: $\pi\theta$. If this seller would provide full collateral, the associated collateral cost is $\delta\pi\theta$. I

[15] All the results are preserved with concave utility functions. However, for tractability purposes, I use mean-variance utility in the model.

assume that such cost is larger than the utility gain from the buyer's risk aversion, which ensures that the utility improvement for this pair of traders is negative[16]:

$$\frac{\gamma}{2}\pi(1-\pi)\theta^2 < \delta\pi\theta.$$

This establishes the lower bound for the collateral cost in assumption 1.

Assumption 1. *The collateral cost is large enough so that at least some sellers cannot provide full collateral to cover their loss in the bad state.*

$$\delta > \frac{(1-\pi)\gamma\theta}{2} \equiv \underline{\delta}. \tag{6}$$

It is only meaningful to talk about capital when the capital cost is not so large that it could be destructive for the total welfare surplus. If the capital cost is so large that holding capital itself is costly enough to cancel out the utility gain from trading, it is optimal for the CCP not to hold capital. To exclude such a scenario, assumption 2 establishes an upper bound for the capital cost.

When all sellers default in the bad state (i.e., collateral is zero), the amount of capital that would be needed to cover the losses reaches the maximum: $\int_0^1 (1-r_j)\pi\theta\,\mathrm{d}r_j$. The cost of holding capital is $\varphi\int_0^1 (1-r_j)\pi\theta\,\mathrm{d}r_j$. Such a level of capital can ensure that the buyers are fully insured. The utility gain from the risk-averse buyers is $\frac{\gamma}{2}\pi(1-\pi)\theta^2$. For the capital cost not to be welfare-destructive, the utility gain should outweigh the associated capital cost:

$$\frac{\gamma}{2}\pi(1-\pi)\theta^2 > \varphi\int_0^1 (1-r_j)\pi\theta\,\mathrm{d}r_j.$$

This establishes the upper bound for the capital cost.

[16]Since the clearing fee f would be a deduction from the utility improvement, when the inequality holds, the utility improvement is negative at any fee level.

Assumption 2. *The capital cost is small enough that it will not destroy welfare.*

$$\varphi < (1 - \pi)\gamma\theta \equiv \bar{\varphi}. \tag{7}$$

3 A for-profit CCP

In this section, I study the case of a for-profit CCP protected by limited liability. The CCP chooses capital K and collateral requirement c to maximize its expected utility U_{CCP} as specified in equation 1. The key trade-off is between fee income and counterparty risk.

I solve the for-profit CCP's optimal problem by backward deduction. I first study whether the buyers and sellers will trade or not when c and K are given. To achieve this, I show when the sellers will default, and how the default losses will affect the traders' utility improvement as they eat up different layers of the default waterfall. This determines trading volume v as functions of c and K.

With the trading volume, I can derive the optimal collateral and capital for the CCP. To elaborate the underlying intuitions, I study the optimal collateral policy conditional on a given amount of capital. Then I solve the optimal capital and the associated optimal collateral.

3.1 Traders' utility at different layers of the default waterfall

Collateralized financial resources. When a seller defaults, both his collateral c and default fund contribution αc will be used to cover his default loss. Hence, both the collateral and the default fund contributed by a defaulting seller are *collateralized financial resources*. Correspondingly, the default fund contributed by the non-defaulting sellers is *mutualized financial resources*.

Sellers default strategically. When the payment a seller needs to make exceeds his collateralized financial resources, he defaults. For this reason, seller j with loss-reduction capacity r_j will not default if and only if

$$(1 + \alpha)c \geq (1 - r_j)\pi\theta.$$

Reorganizing the inequality above, with a given c, seller j with loss-reduction capacity higher than $\frac{\pi\theta-(1+\alpha)c}{\pi\theta}$ ($\equiv \hat{r}$) will not default in the bad state. Let's call seller j with loss-reduction capacity $\hat{r}$ the "marginal seller". The loss-reduction capacity r_j can be interpreted as seller j's creditworthiness as a counterparty.

When seller j does not default, i.e., $r_j \geq \hat{r}$, the buyer of seller j receives $\pi\theta$ in both states. Seller j receives $(1-\pi)\theta$ in the good state and pays $(1-r_j)\pi\theta$ in the bad state. To clear the trade, both parties need to pay the clearing fee $\frac{f}{2}$ and seller j needs to bear the collateral cost. The utilities of trading for the buyer and seller are

$$U^b_{ND} = \pi\theta - \frac{f}{2}, \qquad U^{s_j}_{ND} = (1-\pi)r_j\pi\theta - \frac{f}{2} - (1+\alpha)\delta c$$

Equation 8 shows the utility improvement from trading for a pair of traders. Even though all buyers face the same counterparty risk from their centrally cleared trades, a specific trade takes place if and only if the seller extracts positive utility from it. This is why the seller's idiosyncratic loss reduction capacity enters in (8), and hence determines whether a trade takes place or not.

$$\begin{aligned} \Delta U_{ND} &= U^b_{ND} + U^{s_j}_{ND} - D^b - D^{s_j} \\ &= \underbrace{\frac{\gamma}{2}(1-\pi)\pi\theta^2}_{\text{utility gain}} + \underbrace{(1-\pi)r_j\pi\theta}_{\text{Expected return from loss-reduction capacity}} - \underbrace{(1+\alpha)\delta c}_{\text{collateral cost}} - \underbrace{f}_{fee}. \end{aligned} \tag{8}$$

When seller j has a loss-reduction capacity lower than $\hat{r}$, he defaults if the bad state is realized. In that case, both the payoff of the loss-reduction capacity $r_j\pi\theta$ and the collateralized financial resources $(1+\alpha)c$ are seized by the CCP. The remaining loss is $(1-r_j)\pi\theta - (1+\alpha)c$. Hence the default loss that needs to be borne by the CCP L is a function of c:

$$\begin{aligned} L(c) &= \int_0^{\hat{r}} \left[(1-r_j)\pi\theta - (1+\alpha)c\right] \mathrm{d}r_j \\ &= \frac{[\pi\theta - (1+\alpha)c]^2}{2\pi\theta}. \end{aligned} \tag{9}$$

According to the default waterfall, the default losses will be covered first by the collateralized financial resources and the CCP's capital. From equation 9, the mutualized financial resources are untouched when the following relationship holds:

$$K \geq \frac{[\pi\theta - (1+\alpha)c]^2}{2\pi\theta} \equiv \bar{K}(c). \tag{10}$$

In this case, as the remaining loss is covered by the CCP capital, the buyer's payoffs remain the same. The costs due to clearing are the same. However, seller j only needs to pay $(1+\alpha)c$ in the bad state due to the strategic default. Equation 11 shows the utility improvement for this pair of traders.

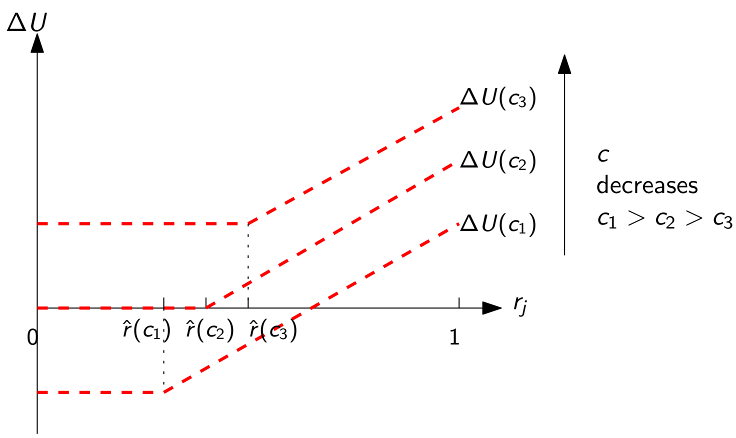
$$\Delta U_D = \underbrace{\frac{\gamma}{2}\pi(1-\pi)\theta^2}_{\text{utility gain}} + \underbrace{(1-\pi)(\pi\theta - (1+\alpha)c)}_{\text{expected gain from default}} - \underbrace{(1+\alpha)\delta c}_{\text{collateral cost}} - \underbrace{f}_{fee}. \tag{11}$$

The traders' utility improvement from trading decreases in collateral c, as shown in equation 8 and 11. If the CCP sets a high collateral requirement, traders need to bear a high collateral cost. Moreover, for a seller who has a low loss-reduction capacity, the high collateral cost will drive the utility improvement of trading to zero (or negative). Hence, the trading volume is a decreasing function of collateral.

The trading volume, however, does not *strictly* decrease in collateral, given that defaulting sellers have a "floor" for their downside risk: the maximum they can lose is the collateralized resources. Figure 1 shows the relationship between the utility improvement and the loss-reduction capacity. There is a kink at $\hat{r}$. The kink means that the trading volume will jump to 1 when the collateral is below some threshold. Let $\bar{r}$ denote the loss-reduction capacity threshold above which a seller will trade and not default, i.e., $\Delta U_{ND}(\bar{r}, c) = 0$. This means $\bar{r}$ is a function of c. When $\bar{r}(c) = \hat{r}(c)$, the trading volume will jump to 1 because of the kink. Thus, this determines a collateral threshold $\bar{c}$ below which the trading volume reaches the maximum. Lemma 1 formalizes the idea.

Figure 1: Utility improvement from trading with different collateral

This figure shows the utility improvement when only the collateralized resources and the CCP's capital are used to cover the total default loss. $\hat{r}$ is the loss-reduction capacity of the "marginal seller" that is indifferent between defaulting and non-defaulting. There are three levels of collateral: $c_1 > c_2 > c_3$, where c_2 is the threshold of collateral level above which only non-defaulting sellers and their buyers will have a positive utility improvement from trading.



Lemma 1. ***The maximum trading volume (collateralized financial resources and CCP's capital used)***

When $K \geq \bar{K}(c)$, only the collateralized financial resources and the CCP's capital are used to cover the total default loss. The maximum trading volume is achieved when $0 \leq c < \bar{c}$.

Proof. See Appendix D.[17]

Mutualized financial resources. When $K < \bar{K}(c)$, the mutualized financial resources are used to cover the remaining loss. As long as the mutualized resources are large enough, the buyers are fully insured. Hence, K should satisfy the following condition:

$$\tilde{K}(c) \leq K < \bar{K}(c), \tag{12}$$

where $\tilde{K}$ is

[17] The functional form of $\bar{c}$ is in Appendix D.

$$\tilde{K}(c) = \underbrace{\frac{[\pi\theta - (1+\alpha)c]^2}{2\pi\theta}}_{L(c)} - \underbrace{\alpha c(1-\tilde{r})}_{\text{Default fund contributed by the non-defaulting sellers}}, \tag{13}$$

and $\tilde{r}$ stands for the loss-reduction capacity threshold at which a seller will trade and not default (elaborated later in equation 15). Note that $\tilde{r}$ is different from $\bar{r}$: The utility improvement of a non-defaulting seller and his buyer is smaller in this case because of the expected loss from default fund contribution.

As specified in the default waterfall, the non-defaulting sellers share the remaining loss evenly. d is the default fund loss for each non-defaulting seller:

$$d = \frac{L(c) - K}{1 - \tilde{r}}. \tag{14}$$

Thus, the utility improvement for a non-defaulting seller and his buyer is

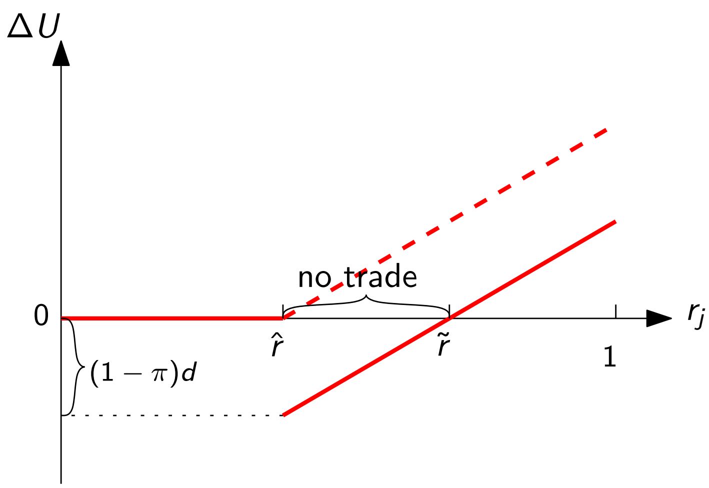
$$\Delta U_{ND,M} = \frac{\gamma}{2}\pi(1-\pi)\theta^2 + (1-\pi)r_j\pi\theta - (1+\alpha)\delta c - f - \underbrace{\boldsymbol{(1-\pi)d}}_{\text{Expected loss from default fund}}. \tag{15}$$

Given the definition of $\tilde{r}$, the following condition holds: $\Delta U_{ND,M}(\tilde{r}) = 0$, which makes it an implicit equation that pins down $\tilde{r}$. Moreover, since $\Delta U_{ND,M}$ is a function of both c and K, $\tilde{r}$ is not only a function of c (as is $\bar{r}$) but also a function of K.

Figure 2 shows the utility improvement when the mutualized financial resources are used. As in Figure 1, seller j with loss-reduction capacity lower than $\hat{r}$ are the defaulting sellers. Unlike in Figure 1, not all sellers with loss-reduction capacity higher than $\hat{r}$ will trade because of the expected loss from their default fund contribution. The sellers with loss-reduction capacity between $\hat{r}$ and $\tilde{r}$ will not trade.

Figure 2: Utility improvement when the mutualized resources are used

This figure shows the utility improvement from trading when mutualized resources are used to cover the total default loss. $\hat{r}$ is the loss-reduction capacity of the "marginal seller" that is indifferent between defaulting and non-defaulting. The dashed line shows the utility improvement of the non-defaulting sellers and their buyers when the mutualized resources are not used while the solid line shows when the mutualized resources are used. The difference between the two lines is the expected losses from the default fund usage. $\tilde{r}$ is the loss-reduction capacity of the non-defaulting seller with zero utility improvement.



The utility improvement from trading indicates how the trading volume v varies given c and K. When $c \geq \bar{c}$, only non-defaulting sellers will trade with their buyers. The trading volume is $1 - \bar{r}$. When $0 \leq c < \bar{c}$, the trading volume is not always one because the non-defaulting sellers anticipate losses from their default fund contributions. In this case, the collateral affects the trading volume via two channels. First, the collateral cost reduces the utility improvement. The trading volume decreases as the collateral increases. Second, the higher the collateral requirement, the lower the remaining loss that would need to be covered by the mutualized resources. The trading volume increases along with the collateral requirement. When $\tilde{r}(c, K) = \hat{r}(c)$, the trading volume is one and that pins down the collateral that achieves the maximum trading volume: $\tilde{c}(K)$. Lemma 2 summarizes the trading volume when mutualized resources are used to cover the remaining loss.

Lemma 2. ***Trading volume (mutualized financial resources used)***

When $\tilde{K}(c) \leq K < \bar{K}(c)$, the CCP does not have enough capital to cover the total default loss. The mutualized financial resources are used to cover the remaining loss. The trading volume is:

$$v(c, K) = \begin{cases} 1 - \bar{r}, & \text{if } c \geq \bar{c}; \\ 1, & \text{if } c = \tilde{c}(K). \end{cases} \tag{16}$$

Proof. See Appendix D.

End of the default waterfall. When $0 \leq K < \tilde{K}(c)$, all the prefunded resources are not enough to cover the default losses. At the end of the default waterfall, the buyers will bear the rest of the losses evenly. In other words, they are not fully insured: they receive less than $\pi\theta$ in the bad state. For each buyer, w is the wedge between the contracted payment and the actual payment in the bad state.

$$w = \frac{L(c) - K - \alpha c(1 - \underline{r})}{v(c, K)}, \tag{17}$$

where $\underline{r}$ is the loss-reduction capacity threshold at which a seller will trade and not default. $\underline{r}$ will be determined by the utility improvement of a non-defaulting seller and his buyer which is defined later in equation 19.

As the buyers are not fully insured now, the utility improvement of a defaulting seller and his buyer is

$$\begin{aligned} \Delta U_{D,E} &= \frac{\gamma}{2}\pi(1 - \pi)(\theta^2 - \boldsymbol{w^2}) + (1 - \pi)(\pi\theta - (1 + \alpha)c - \boldsymbol{w}) - (1 + \alpha)\delta c - f \\ &= \Delta U_D - E(w). \end{aligned} \tag{18}$$

where $E(w)$ stands for the utility loss from partial insurance:

$$E(w) = \frac{\gamma}{2}\pi(1-\pi)w^2 + (1-\pi)w.$$

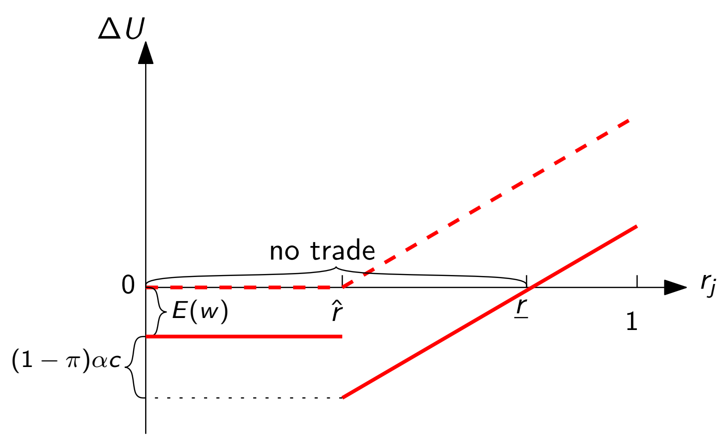
As for the non-defaulting sellers, they lose the whole amount that they have contributed to the default fund. Hence, the utility improvement of a non-defaulting seller and his counterparty is

$$\begin{aligned}\Delta U_{ND,E} &= \frac{\gamma}{2}\pi(1-\pi)(\theta^2 - w^2) - (1-\pi)w + (1-\pi)r_j\pi\theta - (1+\alpha)\delta c - f - \boldsymbol{(1-\pi)\alpha c} \\ &= \Delta U_{ND} - E(w) - (1-\pi)\alpha c.\end{aligned} \tag{19}$$

$\Delta U_{ND,E}(r_j, c, K) = 0$ determines the loss-reduction capacity threshold $\underline{r}(c, K)$ at which a seller will trade and not default. Figure 3 shows the utility improvement from trading when all prefunded resources are exhausted.

Figure 3: Utility improvement when all prefunded resources are exhausted

This figure shows utility improvement when all prefunded resources are exhausted. $\hat{r}$ is the loss-reduction capacity of the "marginal seller" that is indifferent between defaulting and non-defaulting. The dashed line shows the utility improvement when only collateralized resources and CCP capital are used while the solid line shows that when all the resources are used. $\underline{r}$ is the loss-reduction capacity of the non-defaulting seller with zero utility improvement.



When $c \geq \bar{c}$, the trading volume is $1 - \bar{r}$ because only non-defaulting sellers will trade. When $0 \leq c < \bar{c}$, the trading volume could be affected by the collateral in the following ways. First, the collateral cost reduces utility improvement. Thus, trading volume decreases as the collateral requirement increases. Second, the higher the collateral requirement, the larger the default fund losses that the non-defaulting sellers need to bear. So the trading volume of the non-defaulting sellers decreases in the collateral requirement as well. Third, the higher the collateral requirement, the lower the utility loss from partial insurance. The trading volume will be one when $\hat{r}(c) = \underline{r}(c, K)$, which determines the collateral that achieves the maximum trading volume: $\underline{c}(K)$. Lemma 3 summarizes the results.

Lemma 3. ***Trading volume (insolvent CCP)***

When $0 \leq K < \tilde{K}(c)$, the buyers are partially insured. The trading volume is

$$v(c, K) = \begin{cases} 1 - \bar{r}, & \textit{if } c \geq \bar{c}; \\ 1, & \textit{if } c = \underline{c}(K). \end{cases} \tag{20}$$

Proof. See Appendix D.

Four cases with different combinations of c and K. The default waterfall specifies the sequence in which resources contributed by the CCP and the traders are used, which gives rise to the intertwined incentives between the CCP and the traders. Panel A of Table 1 presents the traders' utility improvement from trading when the default losses eat up different layers of the default waterfall. It shows how the utility improvement depends on the sellers' heterogeneous loss-reduction capacity (r_j) and the choice variables of the CCP (c, K). Based on the utility improvement, Panel B shows the thresholds that separate traders who will trade and those who will not. Because only the traders with positive utility improvement will trade.

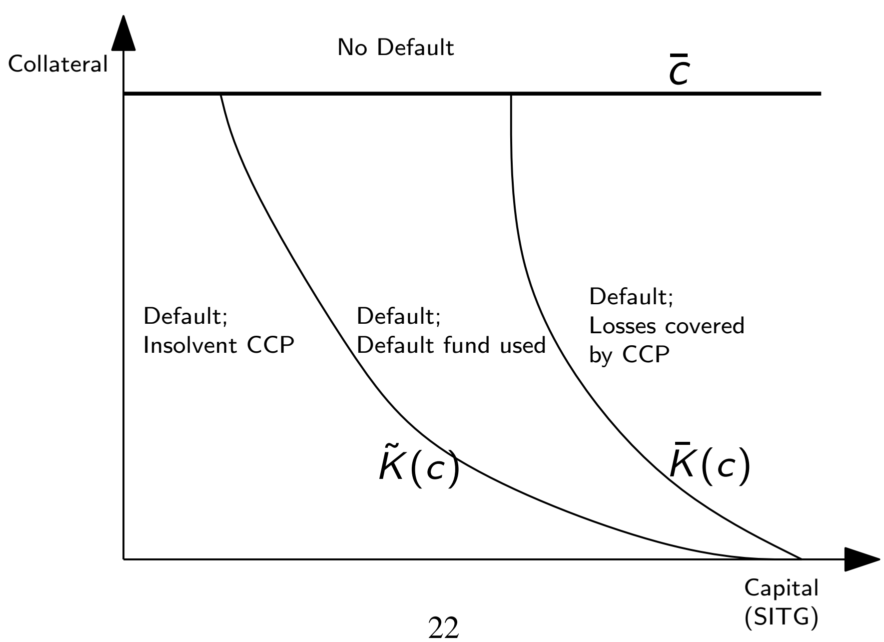
Table 1: Different layers of the default waterfall

This table summarizes the analysis of the default waterfall. Panel A shows the traders' utility improvement as functions of loss-reduction capacity, collateral and capital. Based on the utility improvement, Panel B presents the thresholds of loss-reduction capacity above which the non-defaulting sellers and their buyers will have positive utility and would like to trade. When the thresholds of trading coincide with the loss-reduction capacity of the margin seller that is indifferent between defaulting and non-defaulting, one could pin down the collateral thresholds that lead to the maximum trading volume.

	Collateral/SITG $\left(K \geq \bar{K}(c)\right)$	Default fund $\left(\tilde{K}(c) \leq K < \bar{K}(c)\right)$	End-of-waterfall $\left(0 \leq K < \tilde{K}(c)\right)$
	Panel A: Utility improvement for a pair of traders		
Non-defaulting ones	$\Delta U_{ND}(r_j, c)$	$\Delta U_{ND,M}(r_j, c, K)$	$\Delta U_{ND,E}(r_j, c, K)$
Defaulting ones	$\Delta U_D(c)$	$\Delta U_D(c)$	$\Delta U_{D,E}(c, K)$
	Panel B: Key thresholds		
Threshold of trading	$\Delta U_{ND}(r_j, c) = 0 \rightarrow \bar{r}(c)$	$\Delta U_{ND,M}(r_j, c, K) = 0 \rightarrow \tilde{r}(c, K)$	$\Delta U_{ND,E}(r_j, c, K) = 0 \rightarrow \underline{r}(c, K)$
Collateral threshold	$\hat{r}(c) = \bar{r}(c) \rightarrow \bar{c}$	$\hat{r}(c) = \tilde{r}(c, K) \rightarrow \tilde{c}(K)$	$\hat{r}(c) = \underline{r}(c, K) \rightarrow \underline{c}(K)$

Figure 4: Four combinations of collateral and capital

This figure shows the four different combinations of collateral and capital. Case 1 is when no sellers default. Case 2 is when some sellers default and the CCP capital is large enough to cover the default losses. Case 3 is when some sellers default and default losses are covered by both the CCP capital and the default fund. Case 4 is when some sellers default and all the prefunded resources are not enough to cover the losses.



With all these elements in place, one could have four cases with different combinations between c and K, as shown in Figure 4. Given a pair of (c, K) at $t = 0$, both the CCP and the traders can "foresee" what would happen at $t = 1$ if the bad state is realized. Depending on whether the traders have a positive utility improvement, they will decide to trade or not to trade, which in turn determines the volume-based fee income of the CCP.

3.2 Optimal collateral and capital for a for-profit CCP

The expected utility of the CCP depends on c and K. When $c \geq \bar{c}$, there is no default loss for the CCP at $t = 1$. Hence, the expected value of the CCP only consists of the volume-based fee income and the cost of capital. When $0 \leq c < \bar{c}$, U_{CCP} takes two different expressions, depending on how large the CCP capital is. When $K \geq \bar{K}(c)$, defaulting sellers and their counterparties would like to trade. The CCP will cover the total default loss, i.e., $\frac{(\pi\theta-(1+\alpha)c)^2}{2\pi\theta}$, at $t = 1$ if the bad state is realized. When $0 < K < \bar{K}(c)$, the CCP contributes only its capital but does not cover all default losses when the bad state is realized.

$$U_{CCP} = \begin{cases} fv(c, K) - \varphi K, & \text{if } c \geq \bar{c}; \\ fv(c, K) - (1-\pi)\frac{(\pi\theta-(1+\alpha)c)^2}{2\pi\theta} - \varphi K, & \text{if } 0 \leq c < \bar{c}, K \geq \bar{K}(c); \\ fv(c, K) - (1-\pi)K - \varphi K, & \text{if } 0 \leq c < \bar{c}, 0 \leq K < \bar{K}(c). \end{cases} \tag{21}$$

Optimal collateral policy when the CCP's own capital is given. Although the CCP chooses the optimal collateral and capital simultaneously, I divide the decision procedure into two steps in order to facilitate the comparison between the CCP's choice and the optimal collateral and capital in terms of maximizing social welfare, which will be discussed in Section 4.

There are several important observations from equation 21. First, when $K \geq \bar{K}(c)$, the CCP trades off between high fee income and high counterparty risk. On the one hand, the CCP could set collateral higher than $\bar{c}$ to minimize the counterparty risk. However, the volume-based fee income will be low. On the other hand, the CCP could set collateral lower than $\bar{c}$ to maximize the trading volume, hence maximizing the fee income. But the default losses will be high.

The optimal collateral depends on which leads to a higher expected value of CCP. As a result, the fee level is a crucial element in determining the optimal collateral. There exists some threshold $\underline{f}$ where $\underline{f}v(c,K) = \underline{f}v(c,K) - (1-\pi)\frac{(\pi\theta-(1+\alpha)c)^2}{2\pi\theta}$. Intuitively, when the fee level is low, i.e., $f \leq \underline{f}$, the temptation for the CCP to increase the trading volume is small because the sensitivity of the CCP's expected utility to the trading volume is low. The CCP cares more about the expected default losses and will set a high collateral. However, when the fee level is high, i.e. $f > \underline{f}$ the CCP has a strong incentive to maximize the trading volume and will go for a low collateral.

Second, when $0 \leq K < \bar{K}(c)$, there is no trade-off (in setting collateral) between large trading volume and large default losses, as the CCP is protected by the limited liability and does not cover all the default losses. As K decreases, the CCP tends to chase high trading volume since it has very little to lose. Thus, when K is smaller than some threshold $\hat{K}$ where $fv(c,\hat{K}) = f - (1-\pi)\hat{K}$, the CCP will set collateral c to reach the maximum trading volume.

Proposition 1. ***Optimal collateral policy given specific capital***

The optimal collateral policy when the clearing fee is lower and higher than $\underline{f}$*:*

$$c^*(K) = \begin{cases} \bar{c}, & \text{if } K \geq \hat{K}(\bar{c}); \\ \tilde{c}(K), & \text{if } \tilde{K}(\hat{c}) \leq K < \hat{K}(\bar{c}); \\ \underline{c}(K), & \text{if } 0 \leq K < \tilde{K}(\hat{c}); \end{cases} \qquad c^*(K) = \begin{cases} [\bar{c}]^-, & \text{if } K \geq \bar{K}(\bar{c}); \\ \tilde{c}(K), & \text{if } \tilde{K}(\hat{c}) \leq K < \bar{K}(\bar{c}); \\ \underline{c}(K), & \text{if } 0 \leq K < \tilde{K}(\hat{c}); \end{cases} \tag{22}$$

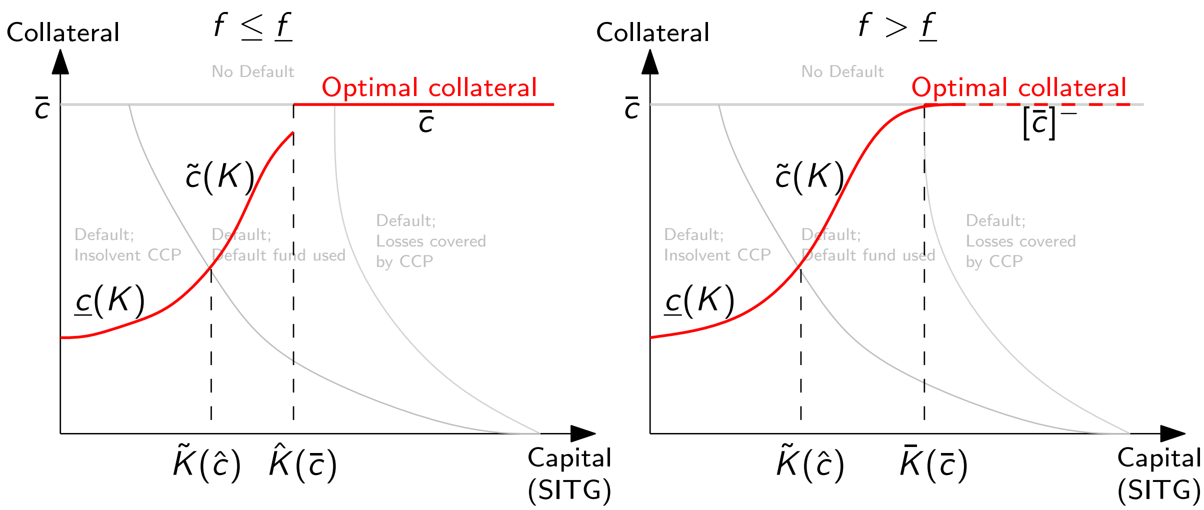
where $\hat{c} = \tilde{c}(\tilde{K}(\hat{c}))$ *and the thresholds are*

$$\begin{aligned} \underline{f} &= \frac{(1-\pi)\pi\theta[2\delta - (1-\pi)\gamma\theta]}{2 - 2\pi + 4\delta}; \\ \hat{K} &= f(1 - \frac{(1+\alpha)c}{\pi\theta}). \end{aligned} \tag{23}$$

Proof. See Appendix D.

Figure 5: Optimal collateral policy

This figure shows the optimal collateral as a piece-wise function of capital. The left subplot shows it when the fee level is low and the right one shows it when the fee level is high. The dashed line shows the slightly smaller amount. In the case of the right subplot, it stands for $[\bar{c}]^-$.



Proposition 1 summarizes the optimal collateral policy when K is given.[18] Figure 5 visualizes the optimal collateral policy.

Optimal capital for a for-profit CCP. With the optimal collateral policy when the capital is given, one could have the CCP's expected utility as functions of the capital. When $f \leq \underline{f}$, the CCP's expected utility is

$$U_{CCP}(c^*(K)) = \begin{cases} f(1-\bar{r}(\bar{c})) - \varphi K, & \text{if } K \geq \hat{K}(\bar{c}); \\ f - (1-\pi)K - \varphi K, & \text{if } 0 \leq K < \hat{K}(\bar{c}). \end{cases} \tag{24}$$

When $f > \underline{f}$, the CCP's expected utility is

$$U_{CCP}(c^*(K)) = \begin{cases} f - (1-\pi)\frac{(\pi\theta-(1+\alpha)\bar{c})^2}{2\pi\theta} - \varphi K, & \text{if } K \geq \bar{K}(\bar{c}); \\ f - (1-\pi)K - \varphi K, & \text{if } 0 \leq K < \bar{K}(\bar{c}). \end{cases} \tag{25}$$

[18] I use the notation $[X]^-$ to denote the amount that is slightly smaller than X and $[X]^+$ to denote the amount that is slightly larger than X.

Proposition 2 presents the optimal capital and the associated optimal collateral for a for-profit CCP.

Proposition 2. ***A for-profit CCP's optimal capital and collateral***

The optimal capital and collateral for a for-profit CCP are

$$K^* = 0, \qquad c^* = \underline{c}(0). \tag{26}$$

Proof. As the CCP's expected utility is a decreasing function of K, the minimum capital leads to the highest expected utility for the CCP.

In this section, I solve the optimal capital and collateral for a for-profit CCP with limited liability. The current design of the default waterfall, coupled with the central clearing mandate, creates intertwined incentives between the CCP and the traders. Since it is the traders who decide whether they would like to trade (and clear) the contract, they are effectively the risk takers. However, their risk-taking behavior is constrained by the CCP's collateral policy. Hence, the CCP is a gate-keeper that sets the risk management standards. Nonetheless, the default waterfall of the for-profit CCP puts the traders in the position of residual risk-bearers, since the remaining loss that exceeds the CCP's capital will be covered by the surviving traders. This separates the (risk management) decision-making and the residual risk-bearing. As a result, the optimal capital and collateral chosen by a for-profit CCP are not the socially optimal ones.

4 Optimal capital regulation for a for-profit CCP

In the previous section, there is no capital regulation for a for-profit CCP. The CCP chooses the amount of capital and collateral that will maximize its own expected utility. In this section, I introduce a regulator who maximizes the total welfare surplus by setting the capital for the CCP:

$$\max_K \quad \Delta U(c^*(K), K) + U_{CCP}(c^*(K), K).$$

Note that the optimal capital regulation will take the CCP's optimal collateral policy as given, which is written in Proposition 1. Proposition 3 summarizes the optimal capital requirements for a for-profit CCP. The optimal capital requirement depends on the fee level f. When $f > \underline{\underline{f}}$, the clearing business is so profitable that the fee income of an additional trading volume is larger than the potential losses of that additional volume. The for-profit CCP will always chase a high trading volume to maximize its profits. In this case, a high level of capital does not help to reduce default losses in the bad state. Instead, a high capital requirement leads to a high collateral and makes clearing expensive for traders. The optimal capital in this case is to maintain a safe CCP with the least collateral possible. Hence, $K^* = \tilde{K}(\hat{c})$. When $f \leq \underline{\underline{f}}$, a high level of capital weights in the for-profit CCP's optimal collateral policy. When the total welfare surplus with no default is larger than that with defaults, $K^* = \hat{K}(\bar{c})$ leads to a high level of collateral $\bar{c}$ which disincentivizes sellers' defaults.

Proposition 3. ***Optimal capital requirement for a for-profit CCP***

The optimal capital requirement for a for-profit CCP depends on the fee level f.

(i) When $f > \underline{\underline{f}}$, the optimal capital requirement is $K^* = \tilde{K}(\hat{c})$.

(ii) When $f \leq \underline{\underline{f}}$, the optimal capital requirement is $K^* = \hat{K}(\bar{c})$.

Proof. See Appendix D.

Although clearing fees in general are not policy instruments, they could be indicative policy variables for regulators. Clearing fees vary a lot. For the standard plan in LCH SwapClear,the clearing fees for interest rate derivatives are \$0.9-\$18 per million for a new trade with maturity ranging from overnight to 50 years.[19] For the standard plan in LCH EquityClear, however, the fees for cash equities are less than \$0.003 per million.[20] Thus, the temptation for a for-profit CCP to lower its collateral requirement and increase trading volumes varies with different levels of clearing fees. Hence, clearing fees could be informative policy variables for regulators when setting optimal capital requirements for for-profit CCPs.

[19] Information from the website of LCH SwapClear: http://www.wip.swapclear.com/fees/llc/swapclear.asp.

[20] Information from the website of LCH EquityClear: https://www.lch.com/services/equityclear/equityclear-ltd/fees.

5 Comparison with a user-owned CCP

In this section, I analyze the optimal collateral and capital for a user-owned CCP. Some CCPs are indeed owned by their clearing members. These include, for example, Japanese Security Clearing Corporations (JSCC) and Swiss SIX X-clear Ltd. When a CCP is owned by the users, i.e., the buyers and sellers, it will maximize the total welfare surplus W, including the utility improvement of the buyers and sellers and the CCP's expected utility, by setting the optimal collateral and capital:

$$\max_{K,c} \quad \Delta U(c, K) + U_{CCP}(c, K).$$

Similar to the analysis of a for-profit CCP, the traders' decisions depend on the relationship between collateral requirement (c) and the CCP's capital (K), as shown in Figure 4. However, the different objective function in the case of a user-owned CCP leads to different optimal collateral and capital policies.

When no seller defaults ($c \geq \bar{c}$), the total welfare surplus decreases in both collateral and capital because of the collateral and capital costs. The optimal collateral and capital would be the lower bound. Lemma 4 summarizes the optimal capital and collateral in this case. Compared with the case with the for-profit CCP, this total welfare surplus is larger because there is no capital cost.

Lemma 4. ***No default case (user-owned CCP)***

(i) For the user-owned CCP, the optimal capital and collateral in the no default case are

$$K^*_{ND} = 0, \qquad c^*_{ND} = \bar{c}. \tag{27}$$

(ii) The total welfare surplus $W^{ND}(c^*_{ND}, K^*_{ND})$ is

$$W^{ND}(c^*_{ND}, K^*_{ND}) = \underbrace{\frac{(1+\alpha)\bar{c}}{2}(1-\pi)\left[\gamma\theta + \frac{(1+\alpha)\bar{c}}{\pi\theta}\right]}_{\textit{Realized gains from trade}} - \underbrace{\frac{(1+\alpha)\bar{c}}{\pi\theta}(1+\alpha)\delta\bar{c}}_{\textit{collateral cost}}. \tag{28}$$

Proof. See Appendix D.

When $0 \le c < \bar{c}, K \ge \tilde{K}(c)$, the sellers' default losses would be covered by prefunded resources. The expected gains from defaults for the defaulting sellers are subsidized by the user-owned CCP's capital and the default fund contributed by the non-defaulting sellers. Since capital is costly, the constraint that $K \ge \tilde{K}(c)$ should be binding. One can substitute K with $\tilde{K}(c)$ and maximize the total welfare surplus with respect to c. Lemma 5 shows the optimal capital and collateral in this case. It turns out that the optimal collateral is zero when the capital cost is smaller than the collateral cost. In order to cover the default losses, the user-owned CCP needs to hold a capital of $\frac{\pi\theta}{2}$. When the capital cost is larger than the collateral cost, it is better to charge some collateral so that less capital is needed to cover default losses.

Lemma 5. ***Default case (user-owned CCP)***

(i) For the user-owned CCP, the optimal capital and collateral in the default case are

$$K_D^* = \begin{cases} \frac{\pi\theta}{2}\frac{\delta^2}{\varphi^2}, & \text{if } \varphi > \delta; \\ \frac{\pi\theta}{2}, & \text{if } \varphi \le \delta; \end{cases} \qquad c_D^* = \begin{cases} \frac{\pi\theta}{1+\alpha}\frac{\varphi-\delta}{\varphi}, & \text{if } \varphi > \delta; \\ 0, & \text{if } \varphi \le \delta. \end{cases} \tag{29}$$

(ii) The total welfare surplus $W^D(c_D^*, K_D^*)$ is

$$W^D(c_D^*, K_D^*) = \begin{cases} \underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{\frac{\pi\theta}{2}\frac{\delta^2}{\varphi}}_{\text{cost of capital}} - \underbrace{\frac{\delta\pi\theta(\varphi-\delta)}{\varphi}}_{\text{cost of collateral}}, & \text{if } \varphi > \delta; \\ \underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{\frac{\pi\theta}{2}\varphi}_{\text{cost of capital}}, & \text{if } \varphi \le \delta. \end{cases} \tag{30}$$

Proof. See Appendix D.

When $0 \le c < \bar{c}, 0 \le K < \tilde{K}(c)$, the total welfare surplus is always lower than in the previous case because of the utility loss from the partial insurance. Hence comparing the total welfare surplus in the first two cases leads to the optimal collateral and capital policy for a user-owned CCP. Proposition 4 summarizes the optimal capital and collateral for a user-owned CCP.

Proposition 4. ***Optimal capital and collateral for a user-owned CCP***

The optimal capital and collateral of a user-owned CCP are

$$K^* = \begin{cases} \frac{\pi\theta}{2}\frac{\delta^2}{\varphi^2}, & \text{if } \varphi > \delta; \\ \frac{\pi\theta}{2}, & \text{if } \varphi \leq \delta; \end{cases} \qquad c^* = \begin{cases} \frac{\pi\theta}{1+\alpha}\frac{\varphi-\delta}{\varphi}, & \text{if } \varphi > \delta; \\ 0, & \text{if } \varphi \leq \delta. \end{cases} \tag{31}$$

Proof. See Appendix D.

This section studies the optimal capital and collateral for a user-owned CCP with costly collateral and capital. The user-owned CCP, unlike the for-profit CCP, always maintains sufficient prefunded resources. Hence, there is no dead-weight loss associated with the user-owned CCP.

6 Empirical results

According to the model, there are following testable hypotheses:

1. *A better capitalized for-profit CCP sets a higher collateral requirement. (Proposition 1)*
2. *A user-owned CCP has a higher level of capital than a for-profit CCP does. (Proposition 2 and 4)*

The quantitative disclosure data from CCPs enable some empirical tests of these hypotheses. CCPs have been required to disclose quantitative information related to the PFMI since the second half of 2015 (CPMI-IOSCO, 2015). Following the CPMI-IOSCO disclosure framework, information is available on a CCP's skin-in-the-game and its clearing members' total default fund contribution (item 4.1), and the total required initial margin at each quarter-end (item 6.1). The CCP quantitative disclosure data are from the CCPView of Clarus Financial Technology.[21] On top of that, I collect CCP ownership information from public sources. Appendix C shows the list of CCPs and their ownership structure. In total, there are 16 CCPs at the group level and 44 CCPs at

[21] https://www.clarusft.com/products/data/ccpview/

the entity level, covering the majority of the clearing industry. The data are at quarterly frequency and range from 2015 Q3 to 2017 Q4.

It is noteworthy that, for CCP skin-in-the-game, there are different concepts in the quantitative disclosure framework:

- Item 4.1.1 Prefunded - Own Capital Before (Default Fund)
- Item 4.1.2 Prefunded - Own Capital Alongside (Default Fund)
- Item 4.1.3 Prefunded - Own Capital After (Default Fund)
- Item 4.1.7 Committed - Own/parent funds that are committed to address a participant default

For the theoretical model, item 4.1.1 is what I defined as CCP skin-in-the-game, which is the CCP's resources that would be used before the surviving members' default fund contributions. To serve as robustness checks, I also run regressions based on the other items. Let $SITG^{pre}_{before}$ denote item 4.1.1; $SITG^{pre}_{before+alongside}$ denote the sum of item 4.1.1 and 4.1.2; $SITG^{pre}_{before+alongside+after}$ denote the sum of item 4.1.1, 4.1.2 and 4.1.3; $SITG^{committed}$ denote the sum of all the four items.

Figure 6 shows the time series of CCP skin-in-the-game ($SITG^{pre}_{before}$) and total initial margin. The red line stands for for-profit CCPs and the blue line for user-owned CCPs. This suggests that (i) user-owned CCPs have higher capital than for-profit CCPs have; and (ii) for-profit CCPs impose a much larger initial margin than user-owned CCPs do.

Figure 6: Time series of CCP skin-in-the-game and total initial margin

This figure plots the time series of CCP skin-in-the-game ($SITG^{pre}_{before}$) and total initial margin. The red stars stand for for-profit CCPs and the blue circles are user-owned CCPs.

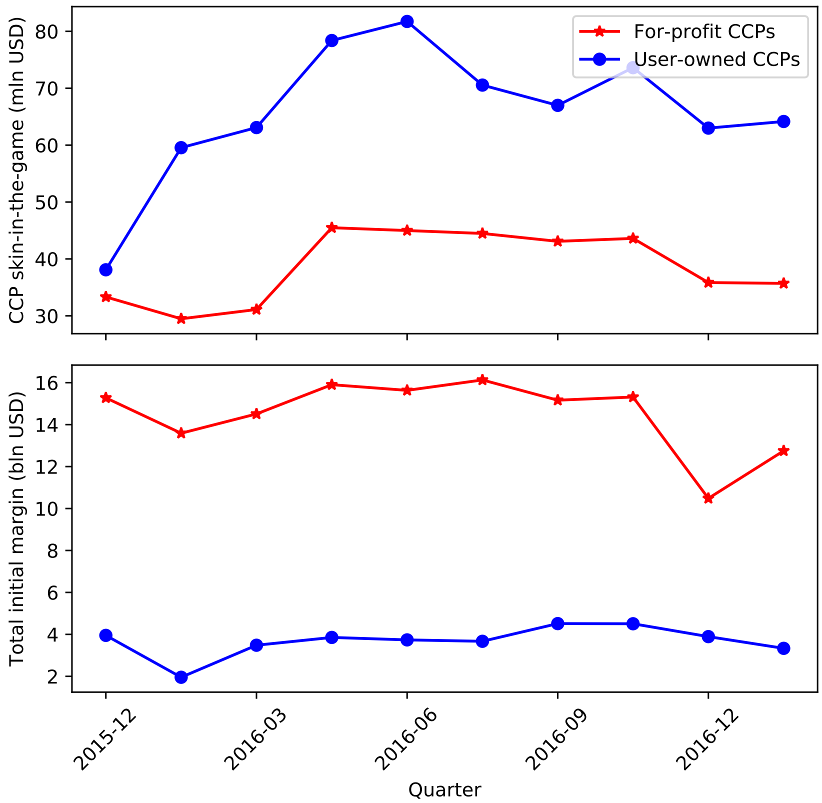
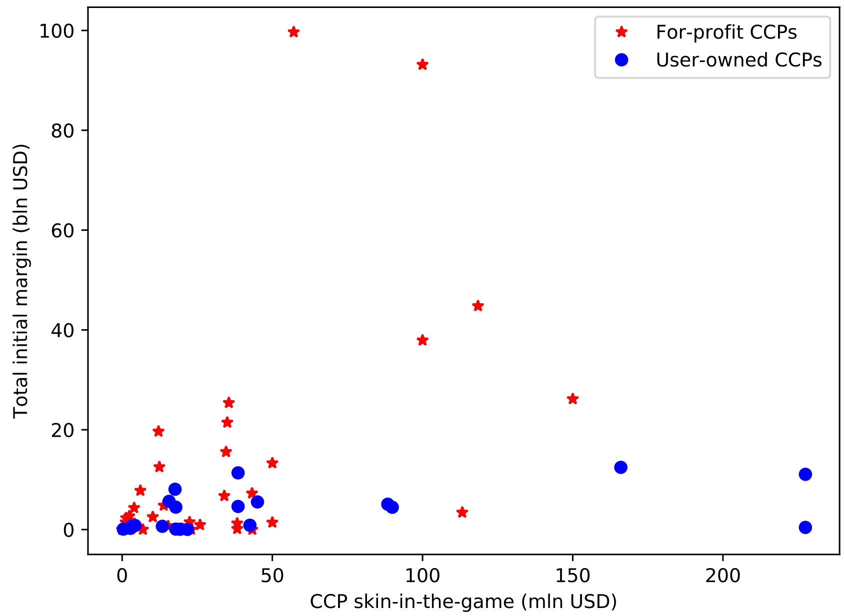


Figure 7 shows cross-sectional variations of CCP skin-in-the-game and total initial margin. The red stars stand for for-profit CCPs and the blue circles are user-owned CCPs. The scatter plots confirm the messages in the time series. In addition, the scatter plot suggests that there is a positive relationship between CCP skin-in-the-game and total initial margin for for-profit CCPs, but not for user-owned CCPs. Figures 6 and 7, are generally in line with the two hypotheses. In the rest of this section, I formally test these hypotheses with regressions.

Figure 7: Scatter plot of total initial margin against CCP skin-in-the-game

This figure plots total initial margin against CCP skin-in-the-game ($SITG^{pre}_{before}$) for each CCP. The red stars stand for for-profit CCPs and the blue circles are user-owned CCPs.



6.1 Impact of CCP skin-in-the-game on required collateral

To test the first hypothesis, I utilize the panel data for the for-profit CCPs. Let i denote CCPs, j denote jurisdictions and t denote quarters. For CCP i in jurisdiction j at quarter t, $IM_{i,j,t}$ is the total initial margin required by the CCP and $SITG_{i,j,t}$ is the CCP's skin-in-the-game.

One potential factor that drives the heterogeneity of CCP capital and margin is that CCPs in different jurisdictions may be subject to different levels of regulatory requirement and supervision. Hence, "better" CCPs have higher capital and higher collateral requirements because of "better" supervision, which has nothing to do with their incentives. To control for such time-varying jurisdictional differences, I include Jurisdiction $\times$ Time fixed effect $\delta_{j,t}$. Moreover, as different CCPs (at the entity level) clear different products, there could be product-specific or entity-specific features. To capture these features, I include a CCP fixed effect α_i. To account for the scale differences, I take the natural logarithm of $IM_{i,t}$ and $SITG_{i,t}$. The panel regression model is

$$log(IM)_{i,j,t} = \beta_0 + \beta_1 log(SITG)_{i,j,t} + \delta_{j,t} + \alpha_i + \varepsilon_{i,j,t}. \tag{32}$$

Table 2: Impact of for-profit CCPs' skin-in-the-game on initial margin requirement

This table shows the panel regression results of skin-in-the-game on required initial margin, controlling for fixed effects of time and CCPs. The sample contains all the for-profit CCPs.

	(1) $log(IM)$	(2) $log(IM)$	(3) $log(IM)$	(4) $log(IM)$
$log(SITG^{pre}_{before})$	0.6704***			
	(2.81)			
$log(SITG^{pre}_{before+alongside})$		0.7034***		
		(2.79)		
$log(SITG^{pre}_{before+alongside+after})$			0.7034***	
			(2.79)	
$log(SITG^{pre}_{before+alongside+after} + SITG^{committed})$				0.8736***
				(4.28)
constant	6.0873***	5.9496***	5.9241***	5.1868***
	(10.61)	(9.43)	(9.25)	(8.75)
CCP FE	Yes	Yes	Yes	Yes
Jurisdiction × Time FE	Yes	Yes	Yes	Yes
R-sq	0.167	0.170	0.170	0.195
N	253	253	253	253
t statistics in parentheses	$^*p < 0.10$	$^{**}p < 0.05$	$^{***}\ p < 0.01$	

Table 2 shows the regression results. Across different concepts of CCP skin-in-the-game, the impact of skin-in-the-game on the CCPs initial margin requirement is significantly positive, which supports the empirical implication from the theoretical model. The estimated coefficient is the elasticity of the required initial margin with respect to CCP skin-in-the-game. It shows that, for a 1% increase of CCP skin-in-the-game, the initial margin requirement increases more than 0.6%, controlling for the fixed effects of CCP and time. For the for-profit CCPs in the sample, the average skin-in-the-game (based solely on item 4.1.1) is about USD 38 million and the average required initial margin is about USD 14 billion. The elasticity suggests that, for a USD 0.38 million increase of skin-in-the-game for for-profit CCPs, the required initial margin will increase by USD 9.1 million.

6.2 Impact of ownership structure on CCP's skin-in-the-game

To test the second hypothesis, I utilize the cross-sectional data for both the for-profit CCPs and the user-owned CCPs at the end of 2017. Let $D_i^{\textit{for-profit}}$ denote the dummy variable for CCP i's ownership structure: 1 stands for for-profit and 0 stands for user-owned. CV_i is the control variable. The cross-sectional regression model is

$$log(SITG)_i = \gamma_0 + \gamma_1 D_i^{\textit{for-profit}} + \gamma_2 CV_i + \varphi_i. \tag{33}$$

I use the size of initial margin and the default fund contributions as proxies for the size of a CCP. Moreover, I also calculate the ratio between a CCP's skin-in-the-game to the financial resources contributed by clearing members, and see if this ratio varies across ownership structure.

Table 3: Impact of ownership structure on CCP skin-in-the-game

This table shows the cross-sectional regression results of the dummy variable of ownership structure on CCP skin-in-the-game. $D_i^{\textit{for-profit}}$ is 1 for for-profit CCPs and 0 for user-owned CCPs.

	(1)	(2)	(3)	(4)	(5)	(6)
	$log(SITG^{pre}_{before})$	$log(SITG^{pre}_{before})$	$log(SITG^{pre}_{before})$	$\frac{SITG^{pre}_{before}}{IM}$	$\frac{SITG^{pre}_{before}}{DF}$	$\frac{SITG^{pre}_{before}}{IM+DF}$
$D^{\textit{for-profit}}$	-0.9619**	-1.2261***	-1.1020***	-0.1138	-1.1555**	-0.0715*
	(-2.45)	(-2.70)	(-2.82)	(-1.57)	(-2.33)	(-1.91)
$log(IM)$	0.5008***					
	(6.26)					
$log(DF)$		0.5226***				
		(5.01)				
$log(IM+DF)$			0.5741***			
			(6.49)			
constant	-0.4109	0.5467	-1.0461	0.1346**	1.2167***	0.0805***
	(-0.67)	(0.94)	(-1.53)	(2.41)	(3.19)	(2.80)
R-sq	0.494	0.387	0.512	0.055	0.114	0.080
N	44	44	44	44	44	44
t statistics in parentheses		$^{*}p < 0.10$	$^{**}p < 0.05$	$^{***}p < 0.01$		

Table 3 reports the results for the cross-sectional regressions. Controlling for the size of financial resources contributed by clearing members, a for-profit CCP holds less skin-in-the-game than a user-owned CCP. Such a relationship is also statistically significant. The regressions of the ownership structure dummy on the ratio between CCP skin-in-the-game and clearing members' resources also lead to a similar relationship.

7 Conclusion

Following the post-crisis regulatory reforms, standardized OTC derivatives are now centrally cleared. This puts CCPs in the spotlight, as potential pressure points in the financial system. At the same time, many CCPs are for-profit public companies with limited liability and face a trade-off between fee income and counterparty risk. The theoretical model presented in this paper shows that, when there is no capital requirement, a for-profit CCP chooses to hold no capital and to set a low collateral requirements for its clearing members. By contrast, a user-owned CCP chooses to hold a high level of capital. This is supported by the empirical evidence.

The current static model does not take into account the disciplinary effect of the franchise value. Franchise value, which is defined as the present value of the future profits that a firm is expected to earn as a going concern, can indeed play the role of skin-in-the-game. One could argue that, in a dynamic setup, a for-profit CCP will charge a prudent level of collateral to protect itself from insolvency even with the minimum capital it chooses to hold. However, franchise value does not always provide self-disciplinary incentives when excessive risk goes hand-in-hand with franchise-enhancing expansions (see, e.g., Hughes et al., 1996). One example for a for-profit CCP is whether or not it should provide clearing services for a new product that will increase its franchise value and the overall riskiness of the clearing system, such as Bitcoin futures. The static model could serve as the first step. It would be an interesting avenue for future research to extend the model, with a view to investigating under what conditions the franchise value of CCPs could safeguard them from failure.

Appendix

A Notation summary

Below is a table that summarizes all variables used in the model.

Variable	Definition
	Panel A: exogenous variables
α	Ratio between default fund contribution and initial margin
π	Probability of the good state
θ	Payoff of the risky asset in the good state
δ	per-unit collateral cost of the sellers
φ	per-unit capital cost of the CCP
γ	Risk-aversion of the buyers
f	per-unit clearing fee charged by the CCP
	Panel B: other variables
c	Collateral requirement for the sellers
$\bar{c}, \tilde{c}, \underline{c}$	Thresholds of collateral that maximizes volume when default fund is not used, when default fund is used, and when default fund is depleted
d	Loss of each non-defaulting seller's default fund contributions
K	CCP capital/skin-in-the-game(SITG)
$\bar{K}, \tilde{K}$	Thresholds of CCP capital that determines whether default fund is used and whether default fund is depleted
L	Total default loss
$\hat{r}$	Threshold of loss-reduction capacity that determines whether a seller will default or not
$\bar{r}, \tilde{r}, \underline{r}$	Thresholds of loss-reduction capacity that determines whether a non-defaulting seller will trade or not when default fund is not used, when default fund is used, and when default fund is depleted
ΔU	Utility improvement from trading for a pair of matched-traders
U_{ccp}	Expected utility of the CCP
v	Trading volume
w	Wedge between the required payment and the available financial resources
W	Total welfare surplus

B Traders' payoffs when different layers of default waterfall are affected

Table 4 shows the traders' payoffs when (i) there is no default and (ii) seller j defaults. In the case of default, traders' payoffs vary as the default losses are covered by (a) collateralized resources and skin-in-the-game, (b) mutualized resources contributed by non-defaulting sellers, and (c) the counterparty of the defaulting seller.

Table 4: Traders' payoffs

This table shows the traders' state-contingent payoffs depending on whether there is default and if so which layer(s) of the default waterfall would be used to cover the default losses. For the default cases, seller j stands for a defaulting seller and seller j' stands for a non-defaulting one.

	Prob.	No default	Default Collateralized/SITG	 Default fund	 End-of-waterfall
Seller j	π	$(1-\pi)\theta$	$(1-\pi)\theta$	$(1-\pi)\theta$	$(1-\pi)\theta$
(defaulting)	$1-\pi$	$-(1-r_j)\pi\theta$	$-(1+\alpha)c$	$-(1+\alpha)c$	$-(1+\alpha)c$
Buyer of	π	$\pi\theta$	$\pi\theta$	$\pi\theta$	$\pi\theta$
seller j	$1-\pi$	$\pi\theta$	$\pi\theta$	$\pi\theta$	$\pi\theta - w$
Seller j'	π	$(1-\pi)\theta$	$(1-\pi)\theta$	$(1-\pi)\theta$	$(1-\pi)\theta$
(non-defaulting)	$1-\pi$	$-(1-r_{j'})\pi\theta$	$-(1-r_{j'})\pi\theta$	$-(1-r_{j'})\pi\theta - d$	$-(1-r_{j'})\pi\theta - \alpha c$
Buyer of	π	$\pi\theta$	$\pi\theta$	$\pi\theta$	$\pi\theta$
seller j'	$1-\pi$	$\pi\theta$	$\pi\theta$	$\pi\theta$	$\pi\theta$

C CCP ownership

In the CCP quantitative disclosure dataset provided by Clarus, there are in total 16 CCPs at the group level and 44 at the entity level. I collect the ownership information from the public sources for these CCPs. The table below shows the ownership structure for the 16 CCPs at the group level, which is then mapped to the 44 entities. $D_i^{for\text{-}profit}$ is a dummy variable of ownership structure: 1 for for-profit CCPs and 0 for user-owned CCPs.

CCP	Ownership structure	$D^{for\text{-}profit}$
ASXCLF	ASX is a market operator, clearing house and payments system facilitator. It is a publicly listed company	1
BME Clearing	BME is the operator of all stock markets and financial systems in Spain. BME Group was constituted in 2002 and it is publicly listed since 2006.	1
CDCC	CDCC is a wholly owned subsidiary of the Montreal Exchange (MX), which has itself been owned by the TMX Group since May 2008. TMX is a publicly listed company.	1
CME	Owned by the CME Group, a publicly listed company	1
DTCC	User-owned and directed	0
Eurex Clearing	Owned and operated by Deutsche Borse, a publicly listed company	1
EuroCCP	Since 2016, ABN AMRO Clearing Bank, Nasdaq, DTCC, Euronext own an equal share of 20% in EuroCCP.	0
HKSCC/HKCC/ OTC Clearing/ SEOCH	All four clearing houses are owned and operated by HKEx, whose main equity holder is the Hong Kong government.	0
ICE	Operated by Intercontinental Exchange, a publicly listed company.	1
JSCC	Owned by the Japan Stock Exchange, other exchanges in Japan and users	0
LCH	LCH is majority-owned by the London Stock Exchange, with the remainder being owned by its users and other exchanges. The London Stock Exchange is a publicly listed company.	1
LME	Since 2012, it is owned and operated by HKEx, whose main equity holder is the Hong Kong government.	0
NCC	NCC is owned and operated by Moscow Exchange Group, which has been publicly listed since 2015.	1
Nodal Clear	Nodal Clear is owned and operated by EEX, whose major equity holder is Deutsche Brse AG, a publicly listed company.	1
SGX	Its major equity holder is Temasek, a sovereign wealth fund.	0
SIX	SIX is owned by around 130 national and international banks in Switzerland that are also the main users of its services. SIX is not listed on the stock exchange.	0

Data sources: company websites, annual reports and Bloomberg Business

D Proof

Lemma 1. Trading volume (collateralized financial resources and CCP's capital used)

Proof. As Figure 1 shows, as collateral decreases, traders' utility improvement increases. I first get the loss-reduction capacity $\bar{r}$ of the non-defaulting sellers with zero utility by setting $\Delta U_{ND}(r_j, c) = 0$. From equation 8, I have

$$\bar{r} = \frac{2(1+\alpha)\delta c + 2f - \gamma\pi(1-\pi)\theta^2}{2(1-\pi)\pi\theta}. \tag{A1}$$

Also, one needs to take into account the fact that $\bar{r}(\bar{c}) = \hat{r}(\bar{c})$. Thus, equation A1 and $\hat{r} = \frac{\pi\theta-(1+\alpha)c}{\pi\theta}$ pin down a threshold $\bar{c}$:

$$\bar{c} = \frac{\pi\theta(1-\pi)(\gamma\theta+2) - 2f}{2(1+\alpha)(1-\pi+\delta)}. \tag{A2}$$

Hence, when $c \geq \bar{c}$, only non-defaulting sellers have a positive utility improvement from trading. Hence, the trading volume is $1-\bar{r}$. When $0 \leq c < \bar{c}$, both default and non-defaulting sellers will trade since they both have a positive utility improvement. Trading volume is 1.

■

Lemma 2. Trading volume (mutualized financial resources used)

Proof.

The non-defaulting seller with loss-reduction capacity $\tilde{r}$ should satisfy $\Delta U_{ND,M}(\tilde{r}, c, K) = 0$ where $\Delta U_{ND,M}$ is specified in equation 15.

$$\begin{aligned}
\frac{\gamma}{2}\pi(1-\pi)\theta^2 + (1-\pi)\tilde{r}\pi\theta - (1+\alpha)\delta c - f - (1-\pi)\frac{L(c)-K}{1-\tilde{r}} &= 0; \\
[(1-\pi)\pi\theta]\,\tilde{r}^2 + \left[\frac{\gamma}{2}\pi(1-\pi)\theta^2 - (1+\alpha)\delta c - f - (1-\pi)\pi\theta\right]\tilde{r} & \\
+(1-\pi)(L(c)-K) - \frac{\gamma}{2}\pi(1-\pi)\theta^2 + (1+\alpha)\delta c + f &= 0.
\end{aligned} \tag{A3}$$

Equation A3 leads to two important observations: First, one could have $\frac{\partial\tilde{r}}{\partial K} < 0$. Because when K increases, given other parameters unchanged, the expected default fund losses of the non-defaulting sellers decrease, which means $\tilde{r}$ should decrease as well. Second, equation A3 is a quadratic equation with respect to $\tilde{r}$. Writing it in the form of $A\tilde{r}^2 + B\tilde{r} + C = 0$, one could have the following:

$$
\begin{aligned}
A &= (1-\pi)\pi\theta; \\
B &= \frac{\gamma}{2}\pi(1-\pi)\theta^2 - (1+\alpha)\delta c - f - (1-\pi)\pi\theta; \\
C &= (1-\pi)(L(c) - K) - \frac{\gamma}{2}\pi(1-\pi)\theta^2 + (1+\alpha)\delta c + f.
\end{aligned}
$$

The solution to the quadratic equation is $\frac{-B \pm \sqrt{B^2-4AC}}{2A}$. Hence, $\frac{\partial \tilde{r}}{\partial K} = \pm \frac{-4A\frac{\partial C}{\partial K}}{2A\sqrt{4AC-B^2}}$. Since $A > 0$, $\frac{\partial C}{\partial K} < 0$, to have $\frac{\partial \tilde{r}}{\partial K} < 0$, the solution to the quadratic equation should be $\frac{-B-\sqrt{B^2-4AC}}{2A}$.

Collateral affects trading volume through two channels: collateral cost $(1+\alpha)\delta c$ and remaining loss $L(c) - K$. As long as assumption 1 holds, $A > 0, B < 0, C > 0$, $\frac{\partial B}{\partial c} < 0$, and $\frac{\partial C}{\partial c} > 0$. Hence, one could have

$$
\begin{aligned}
\frac{\partial \frac{-B}{2A}}{\partial c} &= -\frac{1}{2A}\frac{\partial B}{\partial c} > 0; \\
\frac{\partial (B^2 - 4AC)}{\partial c} &= 2B\frac{\partial B}{\partial c} - 4A\frac{\partial C}{\partial c} < 0.
\end{aligned}
$$

Thus, $\frac{\partial \tilde{r}}{\partial c} = \frac{\partial \frac{-B-\sqrt{B^2-4AC}}{2A}}{\partial c} > 0$. In other words, the higher is the collateral, the higher is $\tilde{r}$ and the lower is the trading volume. Plug in A, B, and C, one could have the explicit form of $\tilde{r}$ as follows:

$$
\begin{aligned}
\tilde{r} =& \frac{(1+\alpha)\delta c + f + (1-\pi)\pi\theta - \frac{\gamma}{2}\pi(1-\pi)\theta^2}{2(1-\pi)\pi\theta} \\
&- \frac{\sqrt{\left[(1+\alpha)\delta c + f(1-\pi)\pi\theta - \frac{\gamma}{2}\pi(1-\pi)\theta^2\right]^2 - 4(1-\pi)\pi\theta\left[(1-\pi)(L(c)-K) - \frac{\gamma}{2}\pi(1-\pi)\theta^2 + (1+\alpha)\delta c + f\right]}}{2(1-\pi)\pi\theta}.
\end{aligned} \tag{A4}
$$

Substitute $\tilde{r}$ in equation 13, one could have the implicit function that defines $\tilde{K}$:

$$
L(c) - \alpha c(1 - \tilde{r}(c, \tilde{K})) - \tilde{K} = 0.
$$

Unfortunately, I cannot find the explicit form of $\tilde{K}$. Still, given the implicit function, I could derive the first-order derivatives of $\tilde{K}$ with respect to c:

$$\frac{dL(c)}{dc}dc - \frac{d\alpha c}{dc}dc + \frac{\partial \alpha c\tilde{r}(c,\tilde{K}))}{\partial c}dc + \frac{\partial \alpha c\tilde{r}(c,\tilde{K}))}{\partial \tilde{K}}d\tilde{K} - d\tilde{K} = 0;$$

$$\left[\underbrace{\frac{dL(c)}{dc}}_{<0} - \underbrace{\alpha(1-\tilde{r}(c,\tilde{K}))}_{>0} - \underbrace{\alpha c\frac{\partial \tilde{r}(c,\tilde{K}))}{\partial c}}_{>0}\right]dc + \left[\underbrace{\alpha c\frac{\partial \tilde{r}(c,\tilde{K}))}{\partial \tilde{K}}}_{<0} - 1\right]d\tilde{K} = 0.$$

Thus, $\frac{d\tilde{K}}{dc} < 0$. $\tilde{K}$ decreases in c. In addition, solving $\tilde{r}(\tilde{c}, K) = \hat{r}(\tilde{c})$, I have

$$\tilde{c} = \frac{[\pi\theta(1-\pi)(\gamma\theta+2) + 2\pi\theta - 2f] + \sqrt{[\pi\theta(1-\pi)(\gamma\theta+2) + 2\pi\theta - 2f]^2 + 4\pi\theta(3-2\pi+2\delta)(2K-\pi\theta)}}{2(1+\alpha)(3-2\pi+2\delta)}. \tag{A5}$$

Hence, $\tilde{c}$ is increasing in K. Moreover, $\tilde{c}(\bar{K}(\bar{c})) = \bar{c}$.

■

Lemma 3. Trading volume (insolvent CCP)

Proof. When $c \geq \bar{c}$, only non-defaulting sellers will trade with their buyers. The trading volume is $1 - \bar{r}$. When$c < \bar{c}$ and $\Delta U_{D,E} \geq 0$, some non-defaulting sellers won't trade but all the defaulting sellers will trade with their buyers. Hence, the trading volume is $1 - \underline{r} + \hat{r}$. In this case, the wedge between the required payment and the actual payment is

$$w = \frac{L(c) - K - \alpha c(1-\underline{r})}{1 - \underline{r} + \hat{r}}.$$

$\underline{r}(c, K)$ is pinned down by $\Delta U_{ND,E}(\underline{r}, c, K) = 0$ where $\Delta U_{ND,E}$ is given in equation 19. There is no explicit form of $\underline{r}(c, K)$. Still, one can still get the implicit form of $\underline{c}(K)$ by replacing $\underline{r}$ with $\hat{r}$ in $\Delta U_{ND,E}(\underline{r}, c, K) = 0$.

$$\begin{aligned}\pi\theta - (1+\alpha)\underline{c} + \frac{(1-\alpha^2)\underline{c}^2}{2\pi\theta} + \frac{1}{\pi\gamma} \\ - \frac{1}{\pi\gamma}\sqrt{1 + \gamma\theta\pi^2(\gamma\theta+2) - \frac{2\pi\gamma}{1-\pi}(f + (1+\alpha)\delta\underline{c}) - 2\pi\gamma\underline{c}(1+2\alpha)} - K &= 0.\end{aligned}$$

Since the utility of a defaulting seller and the associated buyer is always larger than $\Delta U_{ND,E}(\hat{r}, c, K)$, when the above relationship is satisfied, all the defaulting sellers could benefit from trading. Hence, both the defaulting sellers and the non-defaulting sellers will trade. The trading volume is one.

■

Proposition 1. Optimal collateral given specific capital

Proof. To solve the optimal collateral policy, I first replace $v(c, K)$ in equation 21 with the explicit form. When $c \geq \bar{c}$, trading volume is $1 - \bar{r}(c)$. When $0 \leq c < \bar{c}$ and $K \geq \bar{K}$, trading volume is always 1. When $0 \leq c < \bar{c}$ and $K \geq \bar{K}$, trading volume could be either 1 or smaller than 1, depending on the collateral. But since the other parts of the expected value in this case do not depend on c. To maximize the expected value, trading volume must be 1. Hence, equation 21 could be simplified as

$$U_{CCP} = \begin{cases} f(1 - \bar{r}(c)) - \varphi K, & \text{if } c \geq \bar{c}; \\ f - (1 - \pi)\frac{(\pi\theta - (1+\alpha)c)^2}{2\pi\theta} - \varphi K, & 0 \leq c < \bar{c}, \text{if } K \geq \bar{K}; \\ f - (1 - \pi)K - \varphi K, & 0 \leq c < \bar{c}, \text{if } 0 \leq K < \bar{K}. \end{cases} \tag{A6}$$

From equation A6, it is obvious that for the first case U_{CCP} decreases in c and for the second case it increases in c. Hence, the optimal collateral in the first case is $\bar{c}$ while that for the second case is $[\bar{c}]^-$ which is a slightly smaller amount than $\bar{c}$. For the third case, U_{CCP} does not change in c. Hence, the optimal collateral is either $\tilde{c}$ or $\bar{c}$ depending whether K is larger or smaller than $\tilde{K}$. But to get the optimal collateral policy for any given K, we have to compare the expected value with the optimal collateral when K is given.

When $K \geq \bar{K}$, the optimal collateral depends on whether the first case or the second case lead to a higher expected value. It means that the CCP faces the trade-off between large trading volume and large default losses.Let $h_1(f)$ denote the difference between $f(1 - \bar{r}(\bar{c})) - \varphi K$ and $f - (1 - \pi)\frac{(\pi\theta - (1+\alpha)\bar{c})^2}{2\pi\theta} - \varphi K$.

$$\begin{aligned} h_1(f) &= (f(1 - \bar{r}(\bar{c})) - \varphi K) - \left(f - (1 - \pi)\frac{(\pi\theta - (1 + \alpha)\bar{c})^2}{2\pi\theta} - \varphi K\right) \\ &= (1 - \pi)\frac{(\pi\theta - (1 + \alpha)\bar{c})^2}{2\pi\theta} - f\bar{r}(\bar{c}). \end{aligned} \tag{A7}$$

As f increases, $\bar{c}$ decreases and $\bar{r}(\bar{c})$ increases. Thus, $h_1(f)$ is decreasing in f. Solving $h_1(f) \geq 0$, I have $f \leq \underline{f}$, where $\underline{f}$ is the threshold of fee level above which the expected CCP value in the second case will be larger.

$$\underline{f} = \frac{(1-\pi)\pi\theta[2\delta - (1-\pi)\gamma\theta]}{2 - 2\pi + 4\delta}. \tag{A8}$$

When $0 \leq K < \bar{K}$, the optimal collateral depends on whether the first case or the third case lead to a higher expected CCP value. There is no trade-off in setting collateral because the expected value in the third case is invariant to collateral. Instead, there is a threshold of K that determines which case leads to a higher expected value for the CCP. Let $h_2(f, K)$ denote the difference between $f(1 - \bar{r}(\bar{c})) - \varphi K$ and $f - (1-\pi)K - \varphi K$.

$$\begin{aligned} h_2(f, K) &= f(1 - \bar{r}(\bar{c})) - \varphi K - (f - (1-\pi)K - \varphi K) \\ &= (1-\pi)K - f\bar{r}(\bar{c}). \end{aligned} \tag{A9}$$

$h_2(f, K)$ decreases in f and increases in K. Solving $h_2(f, K) \geq 0$, I have $K \geq \hat{K}$, where $\hat{K}$ is the threshold of capital level below which the expected CCP value in the third case will be larger.

$$\hat{K} = f\left(1 - \frac{(1+\alpha)c}{\pi\theta}\right). \tag{A10}$$

Hence, when $f \leq \underline{f}$, the optimal collateral is

$$c^*(K) = \begin{cases} \bar{c}, & \text{if } K \geq \hat{K}(\bar{c}); \\ \tilde{c}, & \text{if } \tilde{K}(\underline{c}) \leq K < \hat{K}(\bar{c}); \\ \underline{c}, & \text{if } 0 \leq K < \tilde{K}(\underline{c}). \end{cases} \tag{A11}$$

When $f > \underline{f}$, the optimal collateral is

$$c^*(K) = \begin{cases} [\bar{c}]^-, & \text{if } K \geq \bar{K}(\bar{c}); \\ \tilde{c}, & \text{if } \tilde{K}(\underline{c}) \leq K < \bar{K}(\bar{c}); \\ \underline{c}, & \text{if } 0 \leq K < \tilde{K}(\underline{c}). \end{cases} \tag{A12}$$

■

Proposition 3. Optimal capital requirement for a for-profit CCP

Proof.

From proposition 1, I have optimal collateral with given capital K. I plug in the optimal collateral into equation A17.

When $f \leq \underline{f}$ and $K \geq \hat{K}(\bar{c})$, the optimal collateral is $\bar{c}$. No sellers default at $t = 1$ when the negative shock is realized. The welfare surplus consists of two parts: the utility improvement from the non-defaulting sellers and the CCP value. Since fee is a pure transfer from the traders to the CCP, it has no impact on the total welfare surplus directly. Hence, f disappears in the final expression. However, note that fee does matter for the thresholds on collateral and capital. Thus, the level of fee has indirect impact on the welfare surplus.

$$\begin{aligned} W &= \int_0^{\frac{(1+\alpha)\bar{c}}{\pi\theta}} \Delta U_{ND} dr_j + f v(\bar{c}) - \varphi K \\ &= \underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta\bar{c}\frac{(1+\alpha)\bar{c}}{\pi\theta}}_{\text{collateral cost}} - \underbrace{(1-\pi)\frac{(\pi\theta-(1+\alpha)\bar{c})^2}{2\pi\theta}}_{\text{not all traders trade}} - \underbrace{\varphi K}_{\text{capital cost}} . \end{aligned} \tag{A13}$$

When $f > \underline{f}$ and $K \geq \bar{K}(\bar{c})$, the optimal collateral is $[\bar{c}]^-$. The trading volume is one and some sellers default at $t = 1$. The welfare surplus is the sum of the utility improvement from the default and non-defaulting sellers and the CCP value.

$$\begin{aligned} W &= \int_0^{\frac{(1+\alpha)\bar{c}}{\pi\theta}} \Delta U_{ND} dr_j + \int_0^{\frac{(1+\alpha)\bar{c}}{\pi\theta}} \Delta U_D dr_j + f - (1-\pi)\frac{(\pi\theta-(1+\alpha)c)^2}{2\pi\theta} - \varphi K \\ &= \underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta\bar{c}}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}} . \end{aligned} \tag{A14}$$

When $f > \underline{f}$ and $\tilde{K}(\underline{c}) \leq K < \bar{K}(\bar{c})$, the optimal collateral is $\tilde{c}$. The same optimal collateral also applies to the case when $f \leq \underline{f}$ and $\tilde{K}(\underline{c}) \leq K < \hat{K}(\bar{c})$. In this case, the trading volume is one and some sellers default at $t = 1$. The utility improvement for the defaulting sellers and their counterparties are the same as before; but that for the non-defaulting sellers and their counterparties is different from the previous case because of the losses from default fund contribution.

$$\begin{aligned} W &= \int_0^{\frac{(1+\alpha)\tilde{c}}{\pi\theta}} \Delta U_{ND,M} dr_j + \int_0^{\frac{(1+\alpha)\tilde{c}}{\pi\theta}} \Delta U_D dr_j + f - (1-\pi)K - \varphi K \\ &= \underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta\tilde{c}}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}} . \end{aligned} \tag{A15}$$

When $0 \le K < \tilde{K}(\underline{c})$, the optimal collateral is $\underline{c}$. In this case, the CCP becomes insolvent. The utility improvement for the defaulting sellers and their counterparties is different because of the losses from partial insurance. For the non-defaulting sellers and their counterparties, they lose all the default fund contribution and have a lower utility improvement as well. But the CCP in this case has a higher expected value. That is also why they will choose minimum capital if there is no capital requirement.

$$\begin{aligned} W &= \int_0^{\frac{(1+\alpha)\underline{c}}{\pi\theta}} \Delta U_{ND,E} dr_j + \int_0^{\frac{(1+\alpha)\underline{c}}{\pi\theta}} \Delta U_{D,E} dr_j + f - (1-\pi)K - \varphi K \\ &= \underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta\underline{c}}_{\text{collateral cost}} - \underbrace{\frac{\gamma}{2}\pi(1-\pi)w^2}_{\text{loss from partial insurance}} - \underbrace{\varphi K}_{\text{capital cost}} . \end{aligned} \tag{A16}$$

Putting the optimal collateral policy into the total welfare surplus W, it could be written as follows.

$$W = \int_0^{\frac{(1+\alpha)c^*(K)}{\pi\theta}} \Delta U^{ND}(r_j, c^*(K), K)\, \mathrm{d}r_j + \int_{\frac{(1+\alpha)c^*(K)}{\pi\theta}}^{v(c^*(K),K)} \Delta U^D(c^*(K), K)\, \mathrm{d}r_j + U_{CCP}(c^*(K), K). \tag{A17}$$

To summarize, when $f \leq \underline{f}$, the total welfare surplus is

$$W = \begin{cases} \underbrace{\frac{(1+\alpha)\bar{c}}{2}(1-\pi)\left[\gamma\theta + \frac{(1+\alpha)\bar{c}}{\pi\theta}\right]}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta\bar{c}\frac{(1+\alpha)\bar{c}}{\pi\theta}}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}}, & \text{if } K \geq \hat{K}(\bar{c}); \\ \underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta\tilde{c}}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}}, & \text{if } \tilde{K}(\hat{c}) \leq K < \hat{K}(\bar{c}); \\ \underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta\underline{c}}_{\text{collateral cost}} - \underbrace{\frac{\gamma}{2}\pi(1-\pi)w^2}_{\text{loss from partial insurance}} - \underbrace{\varphi K}_{\text{capital cost}}, & \text{if } 0 \leq K < \tilde{K}(\hat{c}). \end{cases} \tag{A18}$$

When $f > \underline{f}$, the total welfare surplus is

$$W = \begin{cases} \underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta\bar{c}}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}}, & \text{if } K \geq \bar{K}(\bar{c}); \\ \underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta\tilde{c}}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}}, & \text{if } \tilde{K}(\hat{c}) \leq K < \bar{K}(\bar{c}); \\ \underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta\underline{c}}_{\text{collateral cost}} - \underbrace{\frac{\gamma}{2}\pi(1-\pi)w^2}_{\text{loss from partial insurance}} - \underbrace{\varphi K}_{\text{capital cost}}, & \text{if } 0 \leq K < \tilde{K}(\hat{c}). \end{cases} \tag{A19}$$

I compare the total welfare surplus with different capitals. Assumption 2 specifies that $\varphi < \bar{\varphi} \equiv (1-\pi)\gamma\theta$. Thus the case when $0 \leq K < \tilde{K}(\underline{c})$ is not optimal because the utility loss from partial insurance is larger than the reduction of capital cost.

When $f > \underline{f}$, the total welfare surplus decreases in collateral and capital based on equation A19. Given the positive correlation between capital and collateral, the optimal capital is $\tilde{K}(\bar{c})$ where both capital and collateral reach the lower bounds.

For the case when $f \leq \underline{f}$, I need to compare the welfare surplus when $K = \tilde{K}(\hat{c})$ and that when $K = \hat{K}(\bar{c})$. Let $l(f)$ denote the difference between these two welfare surplus. From equation A18, I have the following:

$$
\begin{aligned}
l(f) &= -(1+\alpha)\delta\bar{c}\frac{(1+\alpha)\bar{c}}{\pi\theta} - (1-\pi)\frac{(\pi\theta-(1+\alpha)\bar{c})^2}{2\pi\theta} - \varphi\bar{K}(\bar{c}) + (1+\alpha)\delta\hat{c} + \varphi\tilde{K}(\hat{c}) \\
&= (1+\alpha)\delta\hat{c} - (1+\alpha)\delta\bar{c}\frac{(1+\alpha)\bar{c}}{\pi\theta} - (1-\pi)\frac{(\pi\theta-(1+\alpha)\bar{c})^2}{2\pi\theta} + \varphi(\tilde{K}(\hat{c}) - \bar{K}(\bar{c})).
\end{aligned}
\tag{A20}
$$

Solve $l(f) = 0$, I have

$$
\begin{aligned}
\underline{\underline{f}} =& \frac{\pi\gamma\theta^2(1-\pi)(\gamma\theta+2)(\gamma\theta+2+\alpha(\gamma\theta+6))}{4(\gamma\theta+1)(\gamma\theta+2+\alpha(\gamma\theta+4))} \\
&- \frac{\sqrt{(\pi\gamma\theta^2(1-\pi)(\gamma\theta+2)(\gamma\theta+2+\alpha(\gamma\theta+6)))^2 - 8\alpha\gamma\pi^2\theta^3(\gamma\theta+1)(\gamma\theta+2)^2(\gamma\theta+2+\alpha(\gamma\theta+4))}}{4(\gamma\theta+1)(\gamma\theta+2+\alpha(\gamma\theta+4))}.
\end{aligned}
\tag{A21}
$$

Hence, when $f \leq \underline{\underline{f}}$, $l(f) \geq 0$, which means the total welfare surplus without default is higher. The optimal capital requirement is $\hat{K}(\bar{c})$. When $f > \underline{\underline{f}}$, $l(f) < 0$. The optimal capital requirement is $\tilde{K}(\hat{c})$. ■

Lemma 4. No default case (user-owned CCP)

Proof.

The total welfare surplus for a user-owned CCP is

$$
W = \begin{cases}
\underbrace{\frac{(1+\alpha)c}{2}(1-\pi)\left[\gamma\theta + \frac{(1+\alpha)c}{\pi\theta}\right]}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta c\frac{(1+\alpha)c}{\pi\theta}}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}}, & \text{if } c \geq \bar{c}; \\
\underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta c}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}}, & \text{if } 0 \leq c < \bar{c}, K \geq \tilde{K}(c); \\
\underbrace{\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)}_{\text{Realized gains from trade}} - \underbrace{(1+\alpha)\delta c}_{\text{collateral cost}} - \underbrace{\frac{\gamma}{2}\pi(1-\pi)w^2}_{\text{loss from partial insurance}} - \underbrace{\varphi K}_{\text{capital cost}}, & \text{if } 0 \leq c < \bar{c}, 0 \leq K < \tilde{K}(c).
\end{cases}
\tag{A22}
$$

In the case of no default at $t = 1$, holding capital is only adding cost for the user-owned CCP. Hence, the optimal capital in this case is 0. As to collateral, to have no default at $t = 1$, collateral needs to satisfy $c \geq \bar{c}$. Take the first-order derivative of W^{ND} with respect to c leads to

$$\frac{\partial W^{ND}}{\partial c} < 0, \quad \text{if } c \geq \bar{c}. \tag{A23}$$

Thus, the optimal c is $\bar{c}$. In addition, one can have $\bar{r} = \hat{r}$ when $c = \bar{c}$, which means that

$$\bar{r}(\bar{c}) = 1 - \frac{(1+\alpha)\bar{c}}{\pi\theta}.$$

Plug in c^*_{ND} and K^*_{ND}, I directly have

$$\begin{aligned} W^{ND}(c^*_{ND}, K^*_{ND}) \quad &= (1-\bar{r})(\frac{\gamma}{2}\pi(1-\pi)\theta^2 - (1+\alpha)\delta\bar{c}) + (1-\pi)\pi\theta \int_{\bar{r}}^{1} r\,\mathrm{d}r \\ &= \frac{1}{2}(1-\pi)(1+\gamma\theta)\pi\theta - (1-\bar{r})(1+\alpha)\delta\bar{c} - \bar{r}\left(\frac{1}{2}(1-\pi)(1+\gamma\theta)\pi\theta - \frac{1}{2}(1-\pi)(1+\alpha)\bar{c}\right) \\ &= \frac{(1+\alpha)\bar{c}}{2}(1-\pi)\left[\gamma\theta + \frac{(1+\alpha)\bar{c}}{\pi\theta}\right] - \frac{(1+\alpha)\bar{c}}{\pi\theta}(1+\alpha)\delta\bar{c}. \end{aligned} \tag{A24}$$

■

Lemma 5. Default case (user-owned CCP)

Proof. Simplifying the total welfare surplus, one could have the following optimization problem for the user-owned CCP.

$$\begin{aligned} \max_{K,c} \quad & \frac{\gamma}{2}\pi(1-\pi)\theta^2 + \frac{1}{2}(1-\pi)\pi\theta - (1+\alpha)\delta c - \varphi K \\ s.t. \quad & K \geq \frac{(\pi\theta - (1+\alpha)c)^2}{2\pi\theta}. \\ & 0 \leq c < \bar{c} \end{aligned} \tag{A25}$$

Since the objective function is decreasing in K, the optimal K is achieved when $K \geq \frac{(\pi\theta-(1+\alpha)c)^2}{2\pi\theta}$ is binding. Hence I could plug in $K = \frac{(\pi\theta-(1+\alpha)c)^2}{2\pi\theta}$ into the objective function. Take the first- and second-order derivative of the objective function with respect to c and I could see that the optimal c depends on how large is φ:

$$\begin{aligned}\frac{\partial W^D}{\partial c} &= (1+\alpha)\left(\frac{\pi\theta-(1+\alpha)c}{\pi\theta}\varphi-\delta\right);\\ \frac{\partial (W^D)^2}{\partial^2 c} &= -\frac{(1+\alpha)^2}{\pi\theta}<0.\end{aligned} \tag{A26}$$

Hence, the optimal collateral is when $\frac{\mathrm{d}W^D}{\mathrm{d}c}=0$, i.e.,

$$c_D^*=\frac{\pi\theta}{1+\alpha}\frac{\varphi-\delta}{\varphi}.$$

Since c_D^* should always be non-negative. When $\varphi \le \delta$, the objective function is decreasing in c. Thus, the optimal collateral is zero.

With the optimal c_D^*, I could have the optimal K_D^* by plugging c_D^* in $K=\frac{(\pi\theta-(1+\alpha)c)^2}{2\pi\theta}$. Hence, K_D^* is $\frac{\pi\theta}{2}$ when $\varphi \le \delta$ and is $\frac{\pi\theta}{2}\frac{\delta^2}{\varphi^2}$ when $\varphi > \delta$.

With the c_D^* and K_D^*, I have the total welfare surplus as

$$W^D(c_D^*,K_D^*)=\begin{cases}\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)-\frac{\pi\theta}{2}\frac{\delta^2}{\varphi}-\frac{\delta\pi\theta(\varphi-\delta)}{\varphi}, & \text{if } \varphi>\delta,\\ \frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)-\frac{\pi\theta}{2}\varphi, & \text{if } \varphi\le\delta.\end{cases}$$

■

Proposition 4. Optimal capital and collateral for a user-owned CCP

Proof. From lemma 4 and 5, I have the optimal capital and collateral for a user-owned CCP in the no default case and default case, respectively. Which case leads to a higher total welfare surplus depends on how large is the capital cost φ. Because the total welfare surplus in the default case $W^D(c_D^*,K_D^*)$ is decreasing in φ, while that in the no default case $W^{ND}(c_{ND}^*,K_{ND}^*)$ is invariant in φ. I first discuss the situation that $\varphi \le \delta$.

When $\varphi \le \delta$, the total welfare surplus in the default case $W^D(c_D^*,K_D^*)$ is

$$\begin{aligned}W^D(c_D^*,K_D^*) &= \frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)-\frac{\pi\theta}{2}\varphi\\ &\ge \frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)-\frac{\pi\theta}{2}\delta.\end{aligned} \tag{A27}$$

Let $f(\delta)$ denote the function of the difference between $W^{ND}(c^*_{ND}, K^*_{ND})$ and $\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1)-\frac{\pi\theta}{2}\delta$.

$$\begin{aligned} f(\delta) \quad =& W^{ND}(c^*_{ND}, K^*_{ND}) - (\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1) - \frac{\pi\theta}{2}\delta) \\ =& \frac{\pi\theta}{2}\delta - \frac{(1+\alpha)\bar{c}}{\pi\theta}(1+\alpha)\delta\bar{c} - \frac{\pi\theta-(1+\alpha)\bar{c}}{\pi\theta}\left(\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1) - \frac{1}{2}(1-\pi)(1+\alpha)\bar{c}\right). \end{aligned} \tag{A28}$$

Since $\bar{c} = \frac{\pi\theta(1-\pi)(\gamma\theta+2)-2f}{2(1+\alpha)(1-\pi+\delta)}$, I have the first-order derivative of $f(\delta)$ w.r.t. δ as

$$\begin{aligned} \frac{\partial f(\delta)}{\partial \delta} \quad =& \frac{1}{2}\frac{(\pi\theta)^2 - 2((1+\alpha)\bar{c})^2}{\pi\theta} \\ <& 0. \end{aligned} \tag{A29}$$

As I assume the collateral cost is large enough that $\delta > \underline{\delta}$; and $f(\underline{\delta}) < 0$, I have $f(\delta) < 0$ for $\delta > \underline{\delta}$. In other words, $W^{ND}(c^*_{ND}, K^*_{ND}) \leq \frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1) - \frac{\pi\theta}{2}\delta \leq W^{D}(c^*_{D}, K^*_{D})$, when $\varphi \leq \delta$. The default case leads to a higher total welfare surplus for the user-owned CCP.

When $\varphi > \delta$, the total welfare surplus in the default case $W^{D}(c^*_{D}, K^*_{D})$ is

$$W^{D}(c^*_{D}, K^*_{D}) = \frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1) - \frac{\pi\theta}{2}\frac{\delta^2}{\varphi} - \frac{\delta\pi\theta(\varphi-\delta)}{\varphi}. \tag{A30}$$

Let $g(\varphi)$ denote the function of the difference between $W^{ND}(c^*_{ND}, K^*_{ND})$ and $W^{D}(c^*_{D}, K^*_{D})$.

$$\begin{aligned} g(\varphi) \quad =& W^{ND}(c^*_{ND}, K^*_{ND}) - W^{D}(c^*_{D}, K^*_{D}) \\ =& \frac{\pi\theta}{2}\frac{\delta^2}{\varphi} + \frac{\delta\pi\theta(\varphi-\delta)}{\varphi} - \frac{(1+\alpha)\bar{c}}{\pi\theta}(1+\alpha)\delta\bar{c} - \frac{\pi\theta-(1+\alpha)\bar{c}}{\pi\theta}\left(\frac{\pi\theta}{2}(1-\pi)(\gamma\theta+1) - \frac{1}{2}(1-\pi)(1+\alpha)\bar{c}\right). \end{aligned} \tag{A31}$$

Take the first-order derivative of $g(\varphi)$ w.r.t. φ, I have

$$\frac{\partial g(\varphi)}{\partial \varphi} > 0. \tag{A32}$$

Given the assumption that $\varphi < \bar{\varphi} \equiv (1-\pi)\gamma\theta$, plugging in $\bar{\varphi}$ into $g(\varphi)$ lead to $g(\bar{\varphi}) < 0$. Hence, for $\varphi < \bar{\varphi}$, $g(\varphi)$ is always smaller than 0, which means the default case leads to a higher total welfare surplus for the user-owned CCP. Hence, the optimal capital and collateral of a user-owned

CCP are

$$K^* = \begin{cases} \frac{\pi\theta}{2}\frac{\delta^2}{\varphi^2}, & \text{if } \varphi > \delta; \\ \frac{\pi\theta}{2}, & \text{if } \varphi \le \delta; \end{cases} \qquad c^* = \begin{cases} \frac{\pi\theta}{1+\alpha}\frac{\varphi-\delta}{\varphi}, & \text{if } \varphi > \delta; \\ 0, & \text{if } \varphi \le \delta. \end{cases} \tag{A33}$$

■

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