Online Appendix to "An Intermediation-Based Model of Exchange Rates" *

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A Online Appendix

The Appendix contains additional material:

- Section A.1 reviews the frictionless model.
- Section A.2 discusses exchange rate disconnect.
- Section A.3 discusses crash risk.
- Section A.4 provides an alternative foundation for intermediation frictions based on portfolio constraints.
- Section A.5 contains proofs of all results.

A.1 Frictionless Economy

In this section, we solve for the equilibrium in the special case when there are no intermediation frictions and customers can freely trade with each other. This analysis serves as an important benchmark for the analysis in the main text. In this case, market completeness implies that all local nominal pricing kernels are linked through the state-by-state relationship with the US dollar pricing kernel:

$$M^H_{i,0,t} \;=\; M^H_{\$,0,t}\mathcal{E}_{i,t}/\mathcal{E}_{i,0}\,.$$

Furthermore, local nominal pricing kernels are determined by the cash-in-advance constraint,

$$\sum_i C^H_{i,0}\Psi_{i,t}\,(M^H_{i,0,t})^{-1}\theta_{i,k}\mathcal{E}_{i,t} \;=\; \mathcal{M}_{k,t}\mathcal{E}_{k,t}\,.$$

so that

$$M^H_{k,0,t} \;=\; (\mathcal{M}_{k,t})^{-1}\,\Theta_{k,t}\,, \tag{A.1}$$

while the exchange rates are then given by

$$\mathcal{E}_{k,t} \;=\; \frac{M^H_{k,0,t}}{M^H_{\$,0,t}} \;=\; \frac{\mathcal{M}_{\$,t}}{\mathcal{M}_{k,t}}\,\frac{\Theta_{k,t}}{\Theta_{\$,t}}\,, \tag{A.2}$$

where we have defined

$$\Theta_{k,t} \;\equiv\; \sum_i \mathcal{E}_{i,0} C_{i,0} \Psi_{i,t}\, \theta_{i,k},\;\; k = 1, \cdots, N$$

to be the international wealth-weighted discount factor for goods of country k.

Money is super-neutral[1] in the frictionless economy, and both goods prices and nominal stock prices are proportional to the money supply. That money super-neutrality holds in frictionless cash-in-advance economies is well known: Money simply serves as a numraire and has no impact on real asset prices. Similar arguments concern the other phenomena: Exchange rates exhibit trivial behavior and simply reflect preferences for local goods, with the parameters $\theta_{k,t}$ being the primitive drivers of exchange rate dynamics. Furthermore, exchange rates perfectly perform their role of shock absorbers: Flexible exchange rates and capital flows guarantee monetary policy independence, as in Obstfeld and Taylor (2004) and in complete agreement with the Mundellian trilemma.

These simplistic features of the benchmark frictionless model are useful for analysis of the model with intermediation frictions: Indeed, they immediately imply that any interesting dynamic properties of prices and exchange rates are due solely to the intermediation frictions. We summarize these observations in the following proposition.

Proposition 1 (Frictionless economy) *The following is true in a frictionless economy in which customers can freely trade all securities with each other:*

(1) Money is super-neutral: The nominal pricing kernels (A.1) *are inversely proportional*

[1] Money is said to be super-neutral when neither the current money supply nor the expectations about the future monetary policy have any impact on real (inflation-adjusted) asset prices.

to the money supply, while the nominal prices of real goods as well as stock prices are proportional to the money supply:

$$
\begin{aligned}
P_{i,k,t} &= \frac{\mathcal{M}_{i,t}}{X_{k,t}} \frac{\Theta_{k,t}}{\Theta_{i,t}^H} \\
S_{i,t} &= \mathcal{M}_{i,t}\, E_t \left[\sum_{\tau=t}^{T} \frac{\Theta_{i,\tau}^H}{\Theta_{i,t}^H} \right]
\end{aligned}
$$

In particular, domestic inflation, stock prices, and the domestic pricing kernel are independent of foreign monetary policy shocks.

(3) Exchange rates are given by (A.2).

The following corollary summarizes the basic properties of exchange rates in the frictionless economy.

Corollary 2 *In a frictionless economy,*

- *The exchange rate $\mathcal{E}_{i,t}$ always scales inversely with the relative money supply. In particular, if country i expands the monetary base more than the US, then its currency always depreciates relative to the US dollar.*
- *Expectations about future monetary policy (forward guidance) have no impact on exchange rates: They depend only on the current money supply.*
- *Monetary shocks outside the US and country i have no impact on $\mathcal{E}_{i,t}$.*

A.2 The disconnect of exchange rates and consumption

As in Gabaix and Maggiori (2015), in our model, intermediaries are marginal investors in the international financial markets and, hence, exchange rates are determined by their marginal utilities, which can be quite different from those of households. Specifically, we have

$$M^I_{i,t,t+1} \;=\; \Psi_{i,t,t+1}(C^I_{i,t+1}/C^I_{i,t})^{-1} \;\neq\; \Psi_{i,t,t+1}(C^H_{i,t+1}/C^H_{i,t})^{-1}\,,$$

Hence,

$$\mathcal{E}_{i,t+1}/\mathcal{E}_{i,t} \;=\; \frac{M^I_{i,t,t+1}}{M^I_{\$,t,t+1}} \;=\; \frac{\Psi_{i,t,t+1}(C^I_{i,t+1}/C^I_{i,t})^{-1}}{\Psi_{\$,t,t+1}(C^I_{\$,t+1}/C^I_{\$,t})^{-1}} \;\neq\; \frac{\Psi_{i,t,t+1}(C^H_{i,t+1}/C^H_{i,t})^{-1}}{\Psi_{\$,t,t+1}(C^H_{\$,t+1}/C^H_{\$,t})^{-1}}\,.$$

Thus, our model is naturally able to generate deviations from the one-to-one relationship between exchange rates and consumption, known as the Backus and Smith (1993) puzzle.

Consider a simplified setup in which two countries, i and j, have identical discount factors $\Psi^H_{i,t} = \Psi^H_{j,t}$ and, hence, their only differences stem from monetary policies. By the cash-in-advance constraint, aggregate nominal consumption $C_{i,t} = C^I_{i,t} + C^H_{i,t}$ coincides with the money supply and, hence, $C_{i,t+1}/C_{i,t} = \mathcal{N}_{i,t+1}$. As a result, in the frictionless model, the correlation of exchange rates with relative consumption growth equals one, in stark contrast to the empirical evidence where this correlation is almost always negative (see, e.g., Backus and Kehoe (1992)). Here, we note that our model is also able to generate a zero or negative correlation. For example, if the countries have identical monetary policies, so that $\mathcal{N}_{i,t+1} = \mathcal{N}_{\$,t+1}$, then $(C_{i,t+1}/C_{i,t})/(C_{\$,t+1}/C_{\$,t}) = 1$ and, hence, its correlation with exchange rates is zero. At the same time, if intermediaries in the two countries are different, then exchange rates will exhibit non-trivial dynamics, unrelated to relative consumption.

A.3 Crash risk

As explained above, the state-contingent intermediation markups represent the cost of insurance in the D2C market segment: When this cost is high, customers reduce their consumption in those states, driving down the value of the local currency. This fact has an important link with the empirical regularity known as the negative currency skew: That is, for many currencies, implied volatilities for out-of-the-money put options tend to be higher than those for out-of-the-money calls (see, e.g., Farhi et al. (2015) and Chernov et al. (2017)), implying that the costs of insurance against currency depreciation are high relative to those for currency appreciation. Indeed, in our model, states with low shadow costs $\Lambda_{i,t}$ are costly to insure against and correspond to states with depressed exchange rates.

Thus, customers that want to buy insurance against currency depreciation states using out-of-the-money put options in the D2C markets will observe highly skewed quotes. We have

$$M^I_{i,t,t+1} \;=\; (\Psi_{i,t,t+1}\, D_{i,t,t+1})^{-1}\, (M^H_{i,t,t+1})^2\, (\lambda_{i,t}(S_{i,t+1}/S_{i,t}) + \mu_{i,t})\,,$$

and therefore

$$\mathcal{E}_{i,t,t+1} \;=\; \frac{(\Psi_{i,t,t+1}\, D_{i,t,t+1})^{-1}\, (M^H_{i,t,t+1})^2\, (\lambda_{i,t}(S_{i,t+1}/S_{i,t}) + \mu_{i,t})}{(\Psi_{\$,t,t+1}\, D_{\$,t,t+1})^{-1}\, (M^H_{\$,t,t+1})^2\, (\lambda_{\$,t}(S_{\$,t+1}/S_{i,t}) + \mu_{\$,t})}\,.$$

Thus, we arrive at the following result:

Corollary 3 *Suppose that $M^H_{i,t,t+1}$ stays bounded. If time t expectations lead customers into a risk-on regime so that $\lambda_{i,t} > 0 > \mu_{i,t}$, then a large enough drop in the country i stock market price $S_{i,t+1}$ at time $t+1$ always leads to a currency crash.*

Corollary 3 highlights an important boom and bust feature of currency crashes in the model. A "boom" that leads to a buildup of optimistic expectations and drives customers into a "risk-on" regime leads to an endogenous buildup of risk in intermediaries' balance sheets. In such episodes, strong drops in asset prices go hand in hand with currency crashes. This finding suggests that it may make sense to differentiate between "good" and "bad" crashes: A good crash (e.g., like the one following a dot com bubble) hits only customers but has no systemic implications; a bad crash hits intermediaries and, therefore, comes with "systemic" implications.

A.4 Market Power Versus Collateral Constrains

Suppose that each trading round t is split into two sub-periods. At time $t-$, customers contact intermediaries and trade state-contingent claims with them in a centralized competitive market. However, this market is subject to collateral constraints for intermediaries: They need to hold enough of liquid assets (stocks and bonds in this example) to cover their trades and incur a regulatory cost at time $t+1$ (through, e.g., capital requirements or leverage ratio constraints), that are given by $-K_{t+1}\log(\alpha_t^I+\beta_t^I S_{t+1}-Y_{t+1})$. Here, the cost factor K_{t+1} accounts for the fact that regulatory requirements and/or the impact of these requirements in the intermediary balance sheets can be time varying. We also assume that these firms are short-lived. Then, the maximization problem is given by

$$\begin{aligned}
&\max_{X,\alpha_{t-}^I,\beta_{t-}^I,Y_{t+,t+1}}\Bigg(E_t[(M_{t,t+1}^H-M_{t,t+1}^I)(Y_{t+1}-\alpha_t^I-\beta_t^I S_{t+1})]\\
&+\ E_t[M_{t,t+1}^I Y_{t+1}]\ +\ E_t[M_{t,t+1}^I K_{t+1}\log(\alpha_t^I+\beta_t^I S_{t+1}-Y_{t+1})]\\
&-\ E_t[M_{t,t+1}^I Y_{t+,t+1}]+E_t[M_{t,t+1}^I Y_{t+,t+1}]\Bigg)
\end{aligned}$$

where $\alpha_t^I,\ \beta_t^I$ are arbitrary and satisfy that the market price of the claim, $E_t[M_{t,t+1}^H(\alpha_t^I+\beta_t^I S_{t+1})]\le W_t^I$, where W_t^I is intermediary wealth. Note that we assume that the time $t+$ market is free from any collateral constraints and, hence, the choice of collateral $\alpha_t^I,\ \beta_t^I$ has no impact on the choice of the claim $Y_{t+,t+1}$ traded in the D2D market. Also, we assume that this claim imposes no regulatory costs on the firm. Thus, its choice is irrelevant. In addition, we assume that the I agents can also trade stocks and bonds at both time $t-$ and t, but they incur no regulatory cost and, thus, can perfectly arbitrage away any price discrepancies. As a result, stocks and bonds are priced fairly across the two markets and the maximization problem takes the form

$$\max_{X,\alpha^I_{t-},\beta^I_{t-},Y_{t+,t+1}} \Bigg(E_t[M^H_{t,t+1} Y_{t+1}]$$
$$+ \ E_t[M^I_{t,t+1} K_{t+1} \log(\alpha^I_t + \beta^I_t S_{t+1} - Y_{t+1})] \ - \ E_t[M^I_{t,t+1} Y_{t+,t+1}] + E_t[M^I_{t,t+1} Y_{t+,t+1}] \Bigg) \, .$$

Clearly, the optimal choice always satisfies $E_t[M^H_{t,t+1}(\alpha^I_t + \beta^I_t S_{t+1})] = W^I_t$. The first-order condition gives

$$M^H_{t,t+1} \ = \ M^I_{t,t+1} K_{t+1} (\alpha^I_t + \beta^I_t S_{t+1} - Y_{t+1})^{-1} \, ,$$

while we know that $Y_{t+1} = W^H_t \Psi_{t,t+1} D_{t,t+1} (M^H_{t,t+1})^{-1}$. Substituting, we get

$$M^H_{t,t+1} \ = \ M^I_{t,t+1} K_{t+1} (\alpha^I_t + \beta^I_t S_{t+1} - W^H_t \Psi_{t,t+1} D_{t,t+1} (M^H_{t,t+1})^{-1})^{-1} \, ,$$

which gives

$$M^H_{t,t+1} \ = \ \frac{W^H_t \Psi_{t,t+1} D_{t,t+1} + M^I_{t,t+1} K_{t+1}}{\alpha^I_t + \beta^I_t S_{t+1}}$$

Importantly, as in the markups case, the D2C pricing kernel explodes when $\alpha^I_t + \beta^I_t S_{t+1}$ goes to zero because the intermediary is not willing to provide insurance against states in which the value of collateral deteriorates.

A.5 Proofs

Proof of Lemma 1. The customer rationally anticipates that he will be consuming as follows: Given the time $t+1$ wealth $W_{i,t+1}$, the agent will consume according to

$$C_{i,t+\tau} \ = \ \frac{W_{i,t+1}}{D_{i,t+1}} \Psi_{i,t+1,t+\tau} (M^H_{t+1,t+\tau})^{-1}, \ \tau \in [1, \cdots, T-t] \, .$$

Therefore, the agent's future value function is given by

$$U_{t+1}(W_{i,t+1}) \;=\; E_{t+1}\left[\sum_{\tau=1}^{T-t} \Psi_{i,t+1,t+\tau} \log C_{i,t+\tau}\right] \;=\; D_{i,t+1} \log W_{i,t+1} \;+\; Const_{i,t+1}\,.$$

Thus, the optimization problem of the customer as a function of the quoted pricing kernel $M_{H,t,t+1}$ takes the form

$$U_{i,t}(W_{i,t}, M_{H,t,t+1}) \;=\; \max_{W_{i,t+1}} \left(\log(W_{i,t} \;-\; E_t[M_{H,t,t+1} W_{i,t+1}]) \;+\; E_t[\Psi_{i,t,t+1} U_{t+1}(W_{i,t+1})]\right)$$

and the first-order condition implies

$$C_{i,t}^{-1} M_{H,t,t+1} \;=\; \Psi_{i,t,t+1} D_{i,t+1} W_{i,t+1}^{-1}$$

Hence,

$$W_{i,t+1} \;=\; \Psi_{i,t,t+1} D_{i,t+1} C_{i,t} M_{H,t,t+1}^{-1} \;=\; \Psi_{i,t,t+1} D_{i,t+1} W_{i,t} D_{i,t}^{-1} M_{H,t,t+1}^{-1}\,.$$

Q.E.D.

Proof of Proposition 2. Substituting the identity

$$M_{i,t,t+1}^{I} \;=\; M_{i,t,t+1}^{H} (W_{i,t,t+1}^{H})^{-1} (\lambda_{i,t}(S_{i,t+1}/S_{i,t}) + \mu_{i,t})\,.$$

into the system, we get a linear system for the Lagrange multipliers which we solve explicitly. Q.E.D.

Proof of Proposition 3. The proof follows directly because we can rewrite the markup as

$$\frac{E_t\left[M_{i,t,t+1}^{H} \frac{M_{i,t,t+1}^{I}}{M_{i,t,t+1}^{H}}\right]}{E_t\left[M_{i,t,t+1}^{H}\right]} \cdot E_t\left[M_{i,t,t+1}^{H} Y_{t+1}\right] - E_t[\frac{M_{i,t,t+1}^{I}}{M_{i,t,t+1}^{H}} M_{i,t,t+1}^{H} Y_{t+1}]\,.$$

Q.E.D.

Proof of Lemma 4, Proposition 5 and Theorem 6. Denote

$$\bar{C}_{i,t} \;\equiv\; \Psi_{i,t}\left(C^H_{i,0}\,(M^H_{i,0,t})^{-1} \;+\; C^I_{i,0}\;(M^I_{i,0,t})^{-1}\right)\mathcal{E}_{i,t}.$$

Then, we get the linear system

$$(1-\beta_k)\bar{C}_{k,t} \;+\; \theta_k\sum_i \beta_i\bar{C}_{i,t} \;=\; \mathcal{M}_{k,t}\mathcal{E}_{k,t}\,.$$

Multiplying by $(1-\beta_k)^{-1}\beta_k$ and summing, we get

$$\sum_k \beta_k\bar{C}_{k,t} \;+\; \bar{B}\sum_i \beta_i\bar{C}_{i,t} \;=\; \sum_k(1-\beta_k)^{-1}\beta_k\mathcal{M}_{k,t}\mathcal{E}_{k,t} \;=\; -(1+\bar{B})Dollar_t\,,$$

where

$$\bar{B} \;\equiv\; \sum_k(1-\beta_k)^{-1}\beta_k\theta_k\,.$$

Thus, we get

$$\sum_k \beta_k\bar{C}_{k,t} \;=\; -Dollar_t\,.$$

Hence,

$$\bar{C}_{k,t} \;=\; (1-\beta_k)^{-1}(\mathcal{M}_{k,t}\mathcal{E}_{k,t} \;+\; \theta_k Dollar_t)\,.$$

Substituting the expressions for pricing kernels, we get

$$(\mathcal{M}_{k,t+1}\mathcal{E}_{k,t+1} \;+\; \theta_k Dollar_{t+1}) \;=\; (1-\beta_k)\Psi_{k,t,t+1}\,(C^H_{k,t}(M^H_{k,t,t+1})^{-1}+C^I_{k,t}(M^I_{k,t,t+1})^{-1})\mathcal{E}_{k,t+1}\,.$$

This completes the proof.

Substituting the expression for $M^H_{k,t,t+1}$, we get

$$\begin{aligned}
&\Psi_{k,t,t+1}\,(C^H_{k,t}(M^I_{k,t,t+1})^{-1/2}(\lambda_{k,t}S_{k,t,t+1}+\mu_{k,t})^{1/2}(\Psi_{k,t,t+1}D_{k,t,t+1})^{-1/2}+C^I_{k,t}(M^I_{k,t,t+1})^{-1})\\
&=\ (1-\beta_k)^{-1}(\mathcal{M}_{k,t+1}\ +\ \theta_k Dollar_{t+1}\mathcal{E}^{-1}_{k,t+1})\,.
\end{aligned}$$

This is a quadratic equation for $(M^I_{k,t,t+1})^{-1/2}$, which gives

$$\begin{aligned}
&(M^I_{k,t,t+1})^{-1/2}\\
&=\ \Psi^{1/2}_{k,t,t+1}\Bigg(-C^H_{k,t}(\lambda_{k,t}S_{k,t,t+1}+\mu_{k,t})^{1/2}(D_{k,t,t+1})^{-1/2}\\
&+\sqrt{\big(C^H_{k,t}(\lambda_{k,t}S_{k,t,t+1}+\mu_{k,t})^{1/2}(D_{k,t,t+1})^{-1/2}\big)^2+4C^I_{k,t}(1-\beta_k)^{-1}(\mathcal{M}_{k,t+1}\ +\ \theta_k Dollar_{t+1}\mathcal{E}^{-1}_{k,t+1})}\Bigg)\\
&\times(2C^I_{k,t}\Psi_{k,t,t+1})^{-1}\,.
\end{aligned}$$

Using the identity

$$-a+b^{1/2}\ =\ \frac{b-a^2}{b^{1/2}+a}$$

we get

$$\begin{aligned}
&(M^I_{k,t,t+1})^{-1/2}\\
&=\ \Psi^{-1/2}_{k,t,t+1}\frac{(\mathcal{M}_{k,t+1}\ +\ \theta_k Dollar_{t+1}\mathcal{E}^{-1}_{k,t+1})}{Y_{k,t+1}+\sqrt{(Y_{k,t+1})^2+C^I_{k,t}(1-\beta_k)(\mathcal{M}_{k,t+1}\ +\ \theta_k Dollar_{t+1}\mathcal{E}^{-1}_{k,t+1})}}
\end{aligned}$$

and the claim follows.

The first claim of the Theorem follows then directly from the identity

$$\mathcal{E}_{k,t,t+1} \;=\; \frac{M^I_{k,t,t+1}}{M^I_{\$,t,t+1}}\,.$$

The fact that the solution g_k to the fixed point equation is unique follows because the right-hand side is clearly monotone decreasing in z for $z<0$. The last claim follows directly from the definition of the dollar index.

Q.E.D.

Proof of Proposition 7. We have

$$(1-\beta_k)(C^H_{k,t+\tau} \,+\, C^I_{k,t+\tau})\mathcal{E}_{k,t+\tau} + \,\theta_k(1+\bar{B})^{-1}\sum_j(1-\beta_j)^{-1}\beta_j\mathcal{M}_{j,t+\tau}\mathcal{E}_{j,t+\tau} \;=\; \mathcal{M}_{k,t+\tau}\mathcal{E}_{k,t+\tau}$$

Multiplying by $M^I_{\$,t,t+\tau}$ and taking expectations and summing over τ, we get

$$(1-\beta_k)(\widetilde{W}^H_{k,t} \;+\; W^I_{k,t})\mathcal{E}_{k,t} \;+\; \theta_k\,(1+\bar{B})^{-1}\sum_i(1-\beta_i)^{-1}\beta_i S^{\$}_{i,t} \;=\; S^{\$}_{k,t}\,,$$

where we have defined $\widetilde{W}^H_{k,t}$ to be the present value of household consumption under the intermediary pricing kernel. The claim now stems from the following lemma.

Lemma 4

$$\widetilde{W}^H_{k,t} \;=\; W^H_{k,t} \;-\; PV_t(Markups_k)\,.$$

Proof. We prove the result by backward induction. For simplicity, we omit the index k. For $t=T$, we have $C^H_T = W^H_T$, and, hence, the result holds for $t=T-1$. Suppose now we have proven the result for $t+1$, so that

$$\widetilde{W}^H_{t+1} \;=\; W^H_{t+1} \;-\; PV_{t+1}(Markups_k)\,.$$

and let us prove it for t. We have

$$\begin{aligned}
\tilde{W}^H_{k,t} &= C^H_t + E_t[M^I_{t,t+1}\widetilde{W}^H_{t+1})] \\
&= W^H_t - E_t[M^H_{t,t+1}W^H_{t+1}] + E_t[M^I_{t,t+1}(W^H_{t+1} \;-\; PV_{t+1}(Markups))] \\
&= W^H_t \;+\; E_t[(M^I_{t,t+1} - M^H_{t,t+1})W^H_{t+1}] \;-\; E_t[M^I_{t,t+1}PV_{t+1}(Markups)] \\
&= W^H_t \;-\; PV_t(Markups_k)\,,
\end{aligned}$$

and the claim follows. Q.E.D.

Q.E.D.

A.6 Proofs for Substantial Consumption Home Bias

Theorem 5 *Equilibrium domestic stock prices are given by*

$$S_{i,t} \approx S^*_{i,t}\left(1 \;+\; \bar{\theta}_i\left(\frac{\bar{S}^\$_t}{S^\$_{i,t}} + \frac{Dollar_t}{\mathcal{M}_{i,t}\,\mathcal{E}^*_{i,t}}\right)\right)\,,$$

while the country i D2D pricing kernel is given by

$$\begin{aligned}
M^I_{i,t,t+1} &\approx \mathcal{N}^{-1}_{i,t+1}\Psi_{i,t,t+1} \\
\times&\left(\frac{1}{2w^*_i+1}\underbrace{\left(\lambda_{i,t} \;+\; \mu_{i,t}(S^*_{i,t,t+1})^{-1}\right)}_{Shadow\ Cost\ of\ Intermediation} - \bar{\theta}_i\frac{Dollar_t}{\mathcal{M}_{i,t}\mathcal{E}^*_{i,t}}\underbrace{\left(\frac{Dollar_{t,t+1}}{\mathcal{N}_{i,t+1}\mathcal{E}^*_{i,t,t+1}} - 1\right)}_{Dollar\ Factor}\right. \\
&\left.+\quad \underbrace{\frac{\bar{\theta}_i}{2w^*_i+1}\frac{\bar{S}^\$_t}{S^\$_{i,t}}\left(\frac{\bar{S}^\$_{t,t+1}}{S^\$_{i,t,t+1}} - 1\right)}_{Global\ Trade-Weighted\ Stock\ Market\ Portfolio}\right)
\end{aligned}$$

while exchange rates changes are given by

$$\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} \;=\; \frac{M^I_{i,t,t+1}}{M^I_{\$,t,t+1}}\,.$$

Proof of Theorem 5. We can rewrite market clearing as

$$
\begin{aligned}
&((1-\beta_k)\, C^H_{k,0}\, \Psi_{k,0,t}(M^H_{k,0,t})^{-1} \;+\; (1-\beta_k)\, C^I_{k,0}\, \Psi_{k,0,t}(M^I_{k,0,t})^{-1})\mathcal{E}_{k,t} \\
&+\; \bar\theta_k \sum_j \beta_j \left(C^H_{j,0}\Psi_{j,0,t}\,(M^H_{j,0,t})^{-1} \;+\; C^I_{j,0}\Psi_{j,0,t}\,(M^I_{k,0,t})^{-1}\right)\mathcal{E}_{j,t} \;=\; \mathcal{E}_{k,t}\mathcal{M}_{k,t}\,.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
M^H_{k,0,t} \;=\; &(\mathcal{E}_{k,t}\mathcal{M}_{k,t})^{-1} M^H_{k,0,t}\Bigg(((1-\beta_k)\, C^H_{k,0}\, \Psi_{k,0,t}(M^H_{k,0,t})^{-1} \;+\; (1-\beta_k)\, C^I_{k,0}\, \Psi_{k,0,t}(M^I_{k,0,t})^{-1})\mathcal{E}_{k,t} \\
&+\; \bar\theta_k \sum_j \beta_j \left(C^H_{j,0}\Psi_{j,0,t}\,(M^H_{j,0,t})^{-1} \;+\; C^I_{j,0}\Psi_{j,0,t}\,(M^I_{j,0,t})^{-1}\right)\mathcal{E}_{j,t}\Bigg) \\
=\; &\mathcal{M}^{-1}_{k,t}\Big((1-\beta_k)\, C^H_{k,0}\, \Psi_{k,0,t} \;+\; (1-\beta_k)\, C^I_{k,0}\, \Psi_{k,0,t}(M^H_{k,0,t}/M^I_{k,0,t})\Big) \\
&+ (\mathcal{E}_{k,t}\mathcal{M}_{k,t})^{-1} M^H_{k,0,t}\bar\theta_k \sum_j \beta_j \left(C^H_{j,0}\Psi_{j,0,t}\,(M^H_{j,0,t})^{-1} \;+\; C^I_{j,0}\Psi_{j,0,t}\,(M^I_{j,0,t})^{-1}\right)\mathcal{E}_{j,t}\,.
\end{aligned}
$$

Let us make an ansatz

$$
M^J_{i,0,t} \;\approx\; M^{J,*}_{i,0,t}(1+M^{J,(1)}_{i,0,t})
$$

and recall that

$$
\mathcal{E}_{j,t} \;=\; \frac{M^I_{i,0,t}}{M^I_{\$,0,t}} \;\approx\; \mathcal{E}^*_{j,t}(1+\mathcal{E}^{(1)}_{j,t})
$$

with

$$
\mathcal{E}^{(1)}_{j,t} \;=\; M^{I,(1)}_{i,0,t} - M^{I,(1)}_{\$,0,t}\,.
$$

Recall that

$$M^{I,*}_{k,0,t} = M^*_{k,0,t} = C_{k,0}\Psi_{k,0,t}\mathcal{M}^{-1}_{k,t}.$$

Thus,

$$\begin{aligned}
&M^*_{k,0,t}(1 + M^{H,(1)}_{k,0,t} + M^{H,(2)}_{k,0,t})\\
&\approx\ \mathcal{M}^{-1}_{k,t}\Big((1-\beta_k)\,C^H_{k,0}\,\Psi_{k,0,t}\\
&+\ (1-\beta_k)\,C^I_{k,0}\,\Psi_{k,0,t}(1 + M^{H,(1)}_{k,0,t} - M^{I,(1)}_{k,0,t} + M^{H,(2)}_{k,0,t} - M^{I,(2)}_{k,0,t} + (M^{I,(1)}_{k,0,t})^2)\Big)\\
&+ (\mathcal{E}^*_{k,t}\mathcal{M}_{k,t})^{-1}M^{H,*}_{k,0,t}(1 + M^{H,(1)}_{k,0,t} - \mathcal{E}^{(1)}_{k,t})\bar{\theta}_k\\
&\times \sum_j \beta_j\left(C^H_{j,0}\Psi_{j,0,t}\,(M^{H,*}_{j,0,t})^{-1}(1 - M^{H,(1)}_{j,0,t})\ +\ C^I_{j,0}\Psi_{j,0,t}\,(M^{I,*}_{j,0,t})^{-1}(1 - M^{I,(1)}_{j,0,t})\right)\mathcal{E}^*_{j,t}(1 + \mathcal{E}^{(1)}_{j,t})\\
&\approx\ \mathcal{M}^{-1}_{k,t}\Psi_{k,0,t}\Big((1-\beta_k)\,C^H_{k,0}\ +\ (1-\beta_k)\,C^I_{k,0}\,(1 + M^{H,(1)}_{k,0,t} - M^{I,(1)}_{k,0,t} + M^{H,(2)}_{k,0,t} - M^{I,(2)}_{k,0,t} + (M^{I,(1)}_{k,0,t})^2)\Big)\\
&+ M^*_{k,0,t}(1 + M^{H,(1)}_{k,0,t} - \mathcal{E}^{(1)}_{k,t})\bar{\theta}_k\\
&\times \sum_j \beta_j C^{-1}_{j,0}\left(C^H_{j,0}(1 - M^{H,(1)}_{j,0,t})\ +\ C^I_{j,0}(1 - M^{I,(1)}_{j,0,t})\right)\mathcal{E}_{j,0}\frac{C_{j,0}\Psi_{j,0,t}}{C_{k,0}\Psi_{k,0,t}}(1 + \mathcal{E}^{(1)}_{j,t})
\end{aligned}$$

Dividing by $M^*_{k,0,t}$, we get

$$\begin{aligned}
&M^{H,(1)}_{k,0,t} + M^{H,(2)}_{k,0,t}\\
&\approx\ C^{-1}_{k,0}\Big(-\beta_k\,C^H_{k,0}\ - C^I_{k,0} + (1-\beta_k)\,C^I_{k,0}\,(1 + M^{H,(1)}_{k,0,t} - M^{I,(1)}_{k,0,t} + M^{H,(2)}_{k,0,t} - M^{I,(2)}_{k,0,t} + (M^{I,(1)}_{k,0,t})^2)\Big)\\
&+ (1 + M^{H,(1)}_{k,0,t} - \mathcal{E}^{(1)}_{k,t})\bar{\theta}_k\\
&\times \sum_j \beta_j C^{-1}_{j,0}\left(C^H_{j,0}(1 - M^{H,(1)}_{j,0,t})\ +\ C^I_{j,0}(1 - M^{I,(1)}_{j,0,t})\right)\mathcal{E}_{j,0}\frac{C_{j,0}\Psi_{j,0,t}}{C_{k,0}\Psi_{k,0,t}}(1 + \mathcal{E}^{(1)}_{j,t})
\end{aligned}$$

First, we write down the system for the first-order corrections:

$$M^{H,(1)}_{k,0,t} \;=\; C^{-1}_{k,0}\Big(-\beta_k\, C_{k,0} + C^{I}_{k,0}(M^{H,(1)}_{k,0,t} - M^{I,(1)}_{k,0,t})\Big) \;+\; \bar{\theta}_k \sum_j \beta_j \mathcal{E}_{j,0} \frac{C_{j,0}\Psi_{j,0,t}}{C_{k,0}\Psi_{k,0,t}} \tag{A.3}$$

Denote

$$\Xi_t \;\equiv\; \sum_j \beta_j C_{j,0} \mathcal{E}_{j,0} \Psi_{j,0,t}\,.$$

Then,

$$M^{H}_{k,t,\tau} \;\approx\; \mathcal{N}^{-1}_{t,\tau}\Psi_{k,t,\tau}\left((M^{H,(1)}_{k,t,\tau} - M^{I,(1)}_{k,t,\tau}) \;+\; \theta_k(\Xi_\tau \Psi^{-1}_{k,0,\tau} - \Xi_t \Psi^{-1}_{k,0,t})\right)$$

Note that

$$\Delta C^{H/k,*}_{t,\tau} \;=\; \Xi_\tau \Psi^{-1}_{k,0,\tau} - \Xi_t \Psi^{-1}_{k,0,t}\,.$$

At the same time,

$$\begin{aligned}
&M^{I}_{k,t,t+1} \;=\; (M^{H}_{k,t,t+1})^2(\Psi_{k,t,t+1}\, D_{k,t,t+1})^{-1}(\lambda_{k,t}(S_{k,t+1}/S_{k,t}) + \mu_{k,t})\\
&\approx\; \mathcal{N}^{-2}_{k,t+1}(1 + 2(M^{H,(1)}_{k,t,t+1} + M^{H,(2)}_{k,t,t+1}) + (M^{H,(1)}_{k,t,t+1})^2)\Psi_{k,t,t+1}(D_{k,t,t+1})^{-1}\\
&\times((1 + \lambda^{(1)}_{k,t} + \lambda^{(2)}_{k,t})\mathcal{N}_{k,t+1}D_{k,t,t+1}(1 + S^{(1)}_{k,t,t+1} + S^{(2)}_{k,t,t+1}) + \mu^{(1)}_{k,t} + \mu^{(2)}_{k,t})\\
&\approx\; M^{I,*}_{k,t,t+1}(1 + 2(M^{H,(1)}_{k,t,t+1} + M^{H,(2)}_{k,t,t+1}) + (M^{H,(1)}_{k,t,t+1})^2)\\
&\times\left(1 + (\lambda^{(1)}_{k,t} + S^{(1)}_{k,t,t+1} + \mu^{(1)}_{k,t}(\mathcal{N}_{k,t+1}D_{k,t,t+1})^{-1}) + (\lambda^{(2)}_{k,t} + \lambda^{(1)}_{k,t}S^{(1)}_{k,t,t+1} + S^{(2)}_{k,t,t+1} + \mu^{(2)}_{k,t}(\mathcal{N}_{k,t+1}D_{k,t,t+1})^{-1})\right)\\
&\approx\; M^{I,*}_{k,t,t+1}\Bigg(1 + 2(M^{H,(1)}_{k,t,t+1} + (\lambda^{(1)}_{k,t} + S^{(1)}_{k,t,t+1} + \mu^{(1)}_{k,t}(\mathcal{N}_{k,t+1}D_{k,t,t+1})^{-1})\\
&+ \lambda^{(2)}_{k,t} + \lambda^{(1)}_{k,t}S^{(1)}_{k,t,t+1} + S^{(2)}_{k,t,t+1} + \mu^{(2)}_{k,t}(\mathcal{N}_{k,t+1}D_{k,t,t+1})^{-1}\\
&+ 2M^{H,(1)}_{k,t,t+1}(\lambda^{(1)}_{k,t} + S^{(1)}_{k,t,t+1} + \mu^{(1)}_{k,t}(\mathcal{N}_{k,t+1}D_{k,t,t+1})^{-1}) + 2M^{H,(2)}_{k,t,t+1} + (M^{H,(1)}_{k,t,t+1})^2\Bigg)\,.
\end{aligned}$$

(A.4)

Now, using that $S_{k,t}$ is priced correctly under both the D2C and D2D kernels and iterating the identity

$$S_{k,t} \; = \; \mathcal{M}_{k,t} \; + \; E_t[M^H_{t,t+1} S_{k,t+1}]\,,$$

we get

$$S_{k,t} \approx \mathcal{M}_{k,t} D_{k,t}\,(1+\bar{\theta}_k(\bar{W}_t^{H/k,*} - (\bar{\Psi}_t/\Psi_{k,t})))\,,$$

where we have defined

$$\bar{W}_t \; \equiv \; \sum_j \beta_j C_{j,0} \mathcal{E}_{j,0}\, \Psi_{j,0,t} D_{j,t} \; = \; \mathcal{M}^{-1}_{\$,t} C_{\$,0} \Psi_{\$,0,t} \sum_j \beta_j \mathcal{E}_{j,t} S^*_{j,t}$$

and

$$\bar{W}_t^{H/k,*} \; \equiv \; \frac{\bar{W}_t}{\Psi_{k,0,t} D_{k,t}} \; = \; \frac{\bar{S}_t^{\$}}{S_{k,t}^{\$}}$$

Hence,

$$\begin{aligned} M^I_{k,t,t+1} \; &\approx \; \mathcal{N}^{-1}_{k,t+1} \Psi_{k,t,t+1} \\ &\times (1 + 2M^{H,(1)}_{k,t,t+1} - \bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} + \bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1}) \end{aligned}$$

Therefore,

$$\begin{aligned} M^{H,(1)}_{k,t,t+1} \; &= \; C^{-1}_{k,0} C^I_{k,0} (M^{H,(1)}_{k,t,t+1} - M^{I,(1)}_{k,t,t+1}) \; + \; \bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} \\ &= \; C^{-1}_{k,0} C^I_{k,0} (-M^{H,(1)}_{k,t,t+1} + \bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} - \bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} - \lambda^{(1)}_{k,t} - \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1}) \; + \; \bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} \end{aligned} \tag{A.5}$$

Hence,

$$M^{H,(1)}_{k,t,t+1} \;=\; \bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} - \frac{C^I_{k,0}}{2C^I_{k,0} + C^H_{k,0}} (\bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1})\,.$$

Therefore,

$$\begin{aligned}
M^{I,(1)}_{k,t,t+1} \;&=\; 2M^{H,(1)}_{k,t,t+1} - \bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} + \bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1} \\
&=\; 2\bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} - 2\frac{C^I_{k,0}}{2C^I_{k,0} + C^H_{k,0}} (\bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1}) \\
&- \bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} + \bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1} \\
&=\; \bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} + \frac{C^H_{k,0}}{2C^I_{k,0} + C^H_{k,0}} (\bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1})
\end{aligned}$$

and

$$\begin{aligned}
&M^{H,(1)}_{k,t,t+1} \;-\; M^{I,(1)}_{k,t,t+1} \\
&=\; -M^{H,(1)}_{k,t,t+1} + \bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} - \bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} - \lambda^{(1)}_{k,t} - \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1} \\
&=\; -(\bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} - \frac{C^I_{k,0}}{2C^I_{k,0} + C^H_{k,0}} (\bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1})) \\
&+ \bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} - \bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} - \lambda^{(1)}_{k,t} - \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1} \\
&=\; -\frac{C^I_{k,0} + C^H_{k,0}}{2C^I_{k,0} + C^H_{k,0}} (\bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1})
\end{aligned} \tag{A.6}$$

Therefore, the equations for the Lagrange multipliers are

$$\begin{aligned}
&E_t[M^*_{k,t,t+1}(1 + M^{H,(1)}_{t,t+1} + M^{H,(2)}_{t,t+1})] \;=\; E_t[M^*_{k,t,t+1}(1 + M^{I,(1)}_{t,t+1} + M^{I,(2)}_{t,t+1})] \\
&E_t[M^*_{k,t,t+1}(1 + M^{H,(1)}_{t,t+1} + M^{H,(2)}_{t,t+1}) \mathcal{M}_{k,t+1} D_{k,t+1} (1 + S^{(1)}_{k,t+1} + S^{(2)}_{k,t+1})] \\
&=\; E_t[M^*_{k,t,t+1}(1 + M^{I,(1)}_{t,t+1} + M^{I,(2)}_{t,t+1}) \mathcal{M}_{k,t+1} D_{k,t+1} (1 + S^{(1)}_{k,t+1} + S^{(2)}_{k,t+1})]\,.
\end{aligned} \tag{A.7}$$

To the first order, this gives

$$
\begin{aligned}
E_t[M^*_{k,t,t+1} M^{H,(1)}_{t,t+1}] \;&=\; E_t[M^*_{k,t,t+1} M^{I,(1)}_{t,t+1}] \\
E_t[M^*_{k,t,t+1} \mathcal{M}_{k,t+1} D_{k,t+1} (M^{H,(1)}_{t,t+1} + S^{(1)}_{k,t+1})] \;&=\; E_t[M^*_{k,t,t+1} \mathcal{M}_{k,t+1} D_{k,t+1} (M^{I,(1)}_{t,t+1} + S^{(1)}_{k,t+1})]\,,
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
E_t[M^*_{k,t,t+1} (M^{H,(1)}_{t,t+1} - M^{I,(1)}_{t,t+1})] \;&=\; 0 \\
E_t[M^*_{k,t,t+1} \mathcal{M}_{k,t+1} D_{k,t+1} (M^{H,(1)}_{t,t+1} - M^{I,(1)}_{t,t+1})] \;&= 0\,.
\end{aligned}
$$

Substituting the expression for the difference in pricing kernel corrections, we get

$$
\begin{aligned}
E_t[\mathcal{N}^{-1}_{k,t+1} \Psi_{k,t,t+1} (\bar\theta_k \Delta \bar W^{H/k,*}_{t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1})] \;&=\; 0 \\
E_t[\mathcal{N}^{-1}_{k,t+1} \Psi_{k,t,t+1} \mathcal{N}_{k,t+1} D_{k,t,t+1} (\bar\theta_k \Delta \bar W^{H/k,*}_{t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1})] \;&= 0\,.
\end{aligned}
$$

and the claim follows in complete analogy with formula (A.8). Q.E.D.

The equations for the Lagrange multipliers are

$$
\begin{aligned}
&E_t[M^*_{k,t,t+1} (1 + M^{H,(1)}_{t,t+1})] \;=\; E_t[M^*_{k,t,t+1} (1 + M^{I,(1)}_{t,t+1})] \\
&E_t[M^*_{k,t,t+1} (1 + M^{H,(1)}_{t,t+1}) \mathcal{M}_{k,t+1} D_{k,t+1} (1 + S^{(1)}_{k,t+1})] \\
&\;=\; E_t[M^*_{k,t,t+1} (1 + M^{I,(1)}_{t,t+1}) \mathcal{M}_{k,t+1} D_{k,t+1} (1 + S^{(1)}_{k,t+1})]\,.
\end{aligned}
$$

To the first order, this gives

$$
\begin{aligned}
E_t[M^*_{k,t,t+1} M^{H,(1)}_{t,t+1}] \;&=\; E_t[M^*_{k,t,t+1} M^{I,(1)}_{t,t+1}] \\
E_t[M^*_{k,t,t+1} \mathcal{M}_{k,t+1} D_{k,t+1} (M^{H,(1)}_{t,t+1} + S^{(1)}_{k,t+1})] \;&=\; E_t[M^*_{k,t,t+1} \mathcal{M}_{k,t+1} D_{k,t+1} (M^{I,(1)}_{t,t+1} + S^{(1)}_{k,t+1})]\,,
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
E_t[M^*_{k,t,t+1}(M^{H,(1)}_{t,t+1} - M^{I,(1)}_{t,t+1})] &= 0 \\
E_t[M^*_{k,t,t+1}\mathcal{M}_{k,t+1}D_{k,t+1}(M^{H,(1)}_{t,t+1} - M^{I,(1)}_{t,t+1})] &= 0\,.
\end{aligned}
$$

Substituting the expression for the difference in pricing kernel corrections, we get

$$
\begin{aligned}
E_t[\mathcal{N}^{-1}_{k,t+1}\Psi^H_{k,t,t+1}(\Delta W^{*,I/H}_{k,t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t}(\mathcal{N}_{k,t+1}D_{k,t,t+1})^{-1})] &= 0 \\
E_t[\mathcal{N}^{-1}_{k,t+1}\Psi^H_{k,t,t+1}\mathcal{N}_{k,t+1}D_{k,t,t+1}(\Delta W^{*,I/H}_{k,t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t}(\mathcal{N}_{k,t+1}D_{k,t,t+1})^{-1})] &= 0\,,
\end{aligned}
\tag{A.8}
$$

and solving this system we arrive at the required result.

Proposition 6 *Suppose that the variance of all shocks is small. The following is true if and only if either (a) the stabilization policy in country i is mild and country i has low sensitivity to global shocks or (b) the stabilization policy in country i is strong and country i has high sensitivity to global shocks.*

(1) The exchange rate $\mathcal{E}_{i,t}$ "overshoots" in response to country i monetary shocks.

(2) The total country i US dollar wealth, $(W^H_{i,t+1} + W^I_{i,t+1})\mathcal{E}_{i,t+1}$, decreases in country i monetary shocks.

The strength of these effects is decreasing in country i intermediation capacity, w^*_i.

Proof of Proposition 6. The total dollar wealth of country k (normalized by the time zero level of exchange rates) equals (using the assumed normalization $C^H_{k,0} + C^I_{k,0} = 1$ as well as formula (A.6))

$$
\begin{aligned}
&(W^H_{k,t} \;+\; W^I_{k,t})\mathcal{E}_{k,0,t}\\
&= \; (C^H_{k,0}\,\Psi_{k,0,t}(M^H_{k,0,t})^{-1} \;+\; C^I_{k,0}\,\Psi_{k,0,t}(M^I_{k,0,t})^{-1})D_{k,t}\mathcal{E}_{k,t}\\
&\approx \; D_{k,t}(C^H_{k,0}\,\Psi_{k,0,t}(M^*_{k,0,t})^{-1}(1-M^{H,(1)}_{k,0,t}) \;+\; C^I_{k,0}\,\Psi_{k,0,t}(M^*_{k,0,t})^{-1}(1-M^{I,(1)}_{k,0,t}))\mathcal{E}^*_{k,0,t}(1+M^{I,(1)}_{k,0,t}-M^{I,(1)}_{\$,0,t})\\
&= \; D_{k,t}(C^H_{k,0}\,(1-M^{H,(1)}_{k,0,t}) \;+\; C^I_{k,0}\,(1-M^{I,(1)}_{k,0,t}))\frac{\Psi_{k,0,t}}{\Psi_{\$,0,t}\mathcal{N}^{-1}_{\$,0,t}}(1+M^{I,(1)}_{k,0,t}-M^{I,(1)}_{\$,0,t})\\
&= \; D_{k,t}\frac{\Psi_{k,0,t}}{\Psi_{\$,0,t}\mathcal{N}^{-1}_{\$,0,t}}\left(1+C^H_{k,0}(M^{I,(1)}_{k,0,t}-M^{H,(1)}_{k,0,t})-M^{I,(1)}_{\$,0,t}\right)
\end{aligned}
$$

Therefore, total return on wealth is given by

$$
\begin{aligned}
&\frac{(W^H_{k,t+1} \;+\; W^I_{k,t+1})\mathcal{E}_{k,0,t+1}}{(W^H_{k,t} \;+\; W^I_{k,t})\mathcal{E}_{k,0,t}}\\
&\approx \; D_{k,t,t+1}\frac{\Psi_{k,t,t+1}}{\Psi_{\$,t,t+1}\mathcal{N}^{-1}_{\$,t,t+1}}\left(1+C^H_{k,0}(M^{I,(1)}_{k,t,t+1}-M^{H,(1)}_{k,t,t+1})-M^{I,(1)}_{\$,t,t+1}\right)\\
&= \; D_{k,t,t+1}\frac{\Psi_{k,t,t+1}}{\Psi_{\$,t,t+1}\mathcal{N}^{-1}_{\$,t,t+1}}\\
&\times\Bigg(1+\frac{C^H_{k,0}}{2C^I_{k,0}+C^H_{k,0}}(\bar{\theta}_k\Delta\bar{W}^{H/k,*}_{t,t+1}+\lambda^{(1)}_{k,t}+\mu_{k,t}(\mathcal{N}_{k,t+1}D_{k,t,t+1})^{-1})\\
&-\left(\bar{\theta}_\$\Delta\bar{C}^{H/\$,*}_{t,t+1}+\frac{C^H_{\$,0}}{2C^I_{\$,0}+C^H_{\$,0}}(\bar{\theta}_\$\Delta\bar{W}^{H/\$,*}_{t,t+1}+\lambda^{(1)}_{\$,t}+\mu_{\$,t}(\mathcal{N}_{\$,t+1}D_{\$,t,t+1})^{-1})\right)\Bigg)\,,
\end{aligned}
$$

Hence, the sign of the response to domestic monetary shocks $\mathcal{N}_{k,t+1}$ coincides with that of $-\mu_{k,t}$. The proof is complete.

Q.E.D.

Proof of Proposition 11. Theorem 5 implies that the appreciation rate of the foreign currency is equal to

$$
\begin{aligned}
\mathcal{E}_{i,t,t+1} &= \frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} \approx \frac{\mathcal{N}_{i,t+1}^{-1}}{\mathcal{N}_{\$,t+1}^{-1}}\Bigg(1+\frac{1}{2w_i^*+1}\lambda_{i,t}-\frac{1}{2w_\$^*+1}\lambda_{\$,t} \\
&+ (D_{t,t+1}^*)^{-1}\,\big(\frac{1}{2w_i^*+1}\mu_{i,t}\mathcal{N}_{i,t+1}^{-1}-\frac{1}{2w_\$^*+1}\mu_{\$,t}\mathcal{N}_{\$,t+1}^{-1}\big)\Big) \\
&+ \bar{\theta}\left(\frac{1}{2w_i^*+1}-\frac{1}{2w_\$^*+1}\right)\frac{\bar{S}_t^\$}{S_{i,t}^\$}\left(\frac{\bar{S}_{t,t+1}^\$}{S_{i,t,t+1}^\$}-1\right)\Bigg).
\end{aligned}
\tag{A.9}
$$

Suppose first that $w_i^* = w_\$^*$. Then, absent monetary shocks, we have $\mathcal{N}_{i,t+1}^{-1} = e^{\alpha^{\mathcal{N}}\delta\omega_{t+1}}$, whereas $D_{t+1}^* \approx e^{\delta\omega_{t+1}}$. Thus,

$$
\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} \approx \left(1+\frac{1}{2w^*+1}\Big(\lambda_{i,t}-\lambda_{\$,t}+e^{(\alpha^{\mathcal{N}}-1)\,\delta\,\omega_{t+1}}\,(\mu_{i,t}-\mu_{\$,t})\Big)\right). \tag{A.10}
$$

By assumption, both countries have low sensitivity to global shocks and, hence, the sign of $\mu_{i,t}$ coincides with that of $1-\alpha^{\mathcal{N}}$. If stabilization policies are mild ($\alpha^{\mathcal{N}} < 1$), we get that the US dollar is a safe haven relative to currency i if and only if $0 < \mu_{i,t} < \mu_{\$,t}$: Indeed, in this case, formula (A.10) implies that $\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}}$ is monotone increasing in ω_{t+1}, implying that the US dollar value is decreasing in ω. In contrast, if $\alpha^{\mathcal{N}} > 1$, then the dollar is a safe haven if and only if $0 > \mu_{i,t} > \mu_{\$,t}$.

Recall that

$$
\mathcal{N}_{i,t+1}^{-1} = e^{\alpha^{\mathcal{N}}\delta\omega_{t+1}-\varepsilon_{i,t+1}^{\mathcal{N}}}
$$

Since all expressions are homogeneous of degree zero in $E_t[e^{\varepsilon_{i,t+1}^{\mathcal{N}}}]$, we can impose the normalization $E_t[e^{\varepsilon_{i,t+1}^{\mathcal{N}}}] = 1$. Under the independence assumption and the identical discount factors assumption, the solution to (A.8) is given by

$$\begin{aligned}
\mu_{i,t} &= \frac{E_t[\mathcal{N}_{i,t+1}^{-1}\Psi_{i,t+1}]E_t[\frac{\bar{S}^{\$}_{t+1}}{S^{\$}_{i,t+1}}D_{i,t+1}\Psi_{i,t+1}] - E_t[\mathcal{N}_{i,t+1}^{-1}\Psi_{i,t+1}\frac{\bar{S}^{\$}_{t+1}}{S^{\$}_{i,t+1}}]E_t[D_{i,t+1}\Psi_{i,t+1}]}{E_t[\Psi_{i,t+1}D_{i,t+1}]E_t[\Psi_{i,t+1}(D_{i,t+1})^{-1}\mathcal{N}_{i,t+1}^{-2}] - (E_t[\Psi_{i,t+1}\mathcal{N}_{t+1}^{-1}])^2} \\
&= \frac{E_t[e^{\alpha^{\mathcal{N}}\delta\omega_{t+1}}\Psi_{i,t+1}]E_t[\frac{\bar{S}^{\$}_{t+1}}{S^{\$}_{i,t+1}}D_{i,t+1}\Psi_{i,t+1}] - E_t[e^{\alpha^{\mathcal{N}}\delta\omega_{t+1}}\Psi_{i,t+1}\frac{\bar{S}^{\$}_{t+1}}{S^{\$}_{i,t+1}}]E_t[D_{i,t+1}\Psi_{i,t+1}]}{E_t[e^{2\varepsilon^{\mathcal{N}}_{i,t+1}}]\,E_t[\Psi_{i,t+1}D_{i,t+1}]E_t[\Psi_{i,t+1}(D_{i,t+1})^{-1}e^{2\alpha^{\mathcal{N}}\delta\omega_{t+1}}] - (E_t[\Psi_{i,t+1}e^{\alpha^{\mathcal{N}}\delta\omega_{t+1}}])^2} \\
&= \frac{\alpha}{\mathrm{Var}_t[e^{\varepsilon^{\mathcal{N}}_{i,t+1}}]\beta+\gamma}
\end{aligned}$$

for some constants $\beta, \gamma > 0$ that are independent of the country identity, while the sign of α depends on whether the policy is mild. At the same time,

$$\begin{aligned}
\lambda_{i,t} &\approx 1 + \frac{\bar{S}^{\$}_t}{S^{\$}_{i,t}} - \frac{\mathrm{Cov}^H_t(S^*_{i,t+1}\frac{\bar{S}^{\$}_{t+1}}{S^{\$}_{i,t+1}}, 1/S^*_{i,t+1})}{\mathrm{Cov}^H_t(S^*_{i,t+1}, 1/S^*_{i,t+1})} \\
&= 1 + \frac{\bar{S}^{\$}_t}{S^{\$}_{i,t}} \\
&+ \frac{E_t[\mathcal{N}_{i,t+1}^{-1}\Psi_{i,t+1}]E_t[\frac{\bar{S}^{\$}_{t+1}}{S^{\$}_{i,t+1}}\mathcal{N}_{i,t+1}^{-1}\Psi_{i,t+1}] - E_t[D_{i,t+1}\Psi_{i,t+1}\frac{\bar{S}^{\$}_{t+1}}{S^{\$}_{i,t+1}}]E_t[\Psi_{i,t+1}(D_{i,t+1})^{-1}\mathcal{N}_{i,t+1}^{-2}]}{E_t[\Psi_{i,t+1}D_{i,t+1}]E_t[\Psi_{i,t+1}(D_{i,t+1})^{-1}\mathcal{N}_{i,t+1}^{-2}] - (E[\Psi_{i,t+1}\mathcal{N}_{t+1}^{-1}])^2}
\end{aligned}$$

Suppose now that the two countries only differ in intermediation capacity. Then, using formula (A.11) below, modified for the effect of monetary policy uncertainty, we get that, with strictly positive monetary policy uncertainty, the effect of the second term in (A.9) is always stronger than that of the first term.

Q.E.D.

Define an auxiliary object[2]

$$Q_i \equiv \frac{0.5\bar{\theta}_i(w_i^*+2)}{w_i^*+0.5},$$

Proof of Proposition 12. We prove the following result:

[2]Clearly, Q_i is monotone decreasing in w_i^*, the intermediation capacity of country i.

Proposition 7 *Suppose that the variances of all shocks are small. Then, we have*[3]

$$Basis^{\$}_{i,t} \approx 0.5\frac{\bar{S}^{\$}_{t}}{S^{\$}_{\$,t}}\left(\delta^S_i Q_i \frac{S^{\$}_{\$,t}}{S^{\$}_{i,t}}(\delta^S_i - \delta^{\mathcal{N}}_i) - \delta^S_{\$} Q_{\$}(\delta^S_{\$} - \delta^{\mathcal{N}}_{\$})\right) E_t[(\Delta\omega_{t+1})^2]\,.$$

If $\delta^S_k(\delta^S_k - \delta^{\mathcal{N}}_k) > 0,\ k = i, \$$, *then a positive basis emerges if countries differ in only one of the following:*

(1) Country i *has smaller intermediation capacity than the US.*

(2) The US has a higher market capitalization than country i.

Furthermore, the basis is monotone increasing in the aggressiveness $\alpha^{\mathcal{N}}_{\$}$ *of the US monetary policy if and only if the US has low sensitivity to global shocks.*[4]

[3] Note that, by definition, $S^{\$}_{\$,t} = S_{\$,t}$ because stock prices $S_{i,t}$ are in the domestic currency.

[4] That is, when $\delta^S_{\$} < 0$.

Indeed,

$$
\begin{aligned}
&- e^{-r^{I,H}_{\$,t}} + e^{-r_{\$,t}} \;=\; -E_t[M^H_{i,t,t+1}\,(\mathcal{E}_{i,t}/\mathcal{E}_{i,t+1})] \;+\; E_t[M^H_{\$,t,t+1}] \\
&\approx\; -E_t[M^{H,*}_{i,t,t+1}(1+M^{H,(1)}_{i,t,t+1})\,\frac{M^*_{\$,t,t+1}}{M^*_{i,t,t+1}}(1+M^{I,(1)}_{\$,t,t+1}-M^{I,(1)}_{i,t,t+1})] \;+\; E_t[M^{H,*}_{\$,t,t+1}(1+M^{H,(1)}_{\$,t,t+1})] \\
&\approx\; E_t[M^{H,*}_{\$,t,t+1}(M^{H,(1)}_{\$,t,t+1}-M^{H,(1)}_{i,t,t+1}-(M^{I,(1)}_{\$,t,t+1}-M^{I,(1)}_{i,t,t+1}))] \\
&=\; E_t\left[M^{H,*}_{\$,t,t+1}\left(M^{H,(1)}_{\$,t,t+1}-M^{H,(1)}_{i,t,t+1}-2(M^{I,(1)}_{\$,t,t+1}-M^{I,(1)}_{i,t,t+1})+\theta_\$\Delta C^{H/\$}_{t,t+1}-\theta_i\Delta C^{H/i}_{t,t+1}\right)\right] \\
&=\; E_t\Bigg[M^{H,*}_{\$,t,t+1}\Bigg(\bar\theta_\$\Delta\bar C^{H/\$,*}_{t,t+1}-\frac{C^I_{\$,0}}{2C^I_{\$,0}+C^H_{\$,0}}(\bar\theta_\$\Delta\bar W^{H/\$,*}_{t,t+1}+\lambda^{(1)}_{\$,t}+\mu_{\$,t}(\mathcal{N}_{\$,t+1}D^H_{\$,t,t+1})^{-1}) \\
&- \bar\theta_i\Delta\bar C^{H/i,*}_{t,t+1}+\frac{C^I_{i,0}}{2C^I_{i,0}+C^H_{i,0}}(\bar\theta_i\Delta\bar W^{H/i,*}_{t,t+1}+\lambda^{(1)}_{i,t}+\mu_{i,t}(\mathcal{N}_{i,t+1}D^H_{i,t,t+1})^{-1}) \\
&- 2\Bigg(\bar\theta_\$\Delta\bar C^{H/\$,*}_{t,t+1}+\frac{C^H_{\$,0}}{2C^I_{\$,0}+C^H_{\$,0}}(\bar\theta_\$\Delta\bar W^{H/\$,*}_{t,t+1}+\lambda^{(1)}_{\$,t}+\mu_{\$,t}(\mathcal{N}_{\$,t+1}D^H_{\$,t,t+1})^{-1}) \\
&- \bar\theta_i\Delta\bar C^{H/i,*}_{t,t+1}-\frac{C^H_{i,0}}{2C^I_{i,0}+C^H_{i,0}}(\bar\theta_i\Delta\bar W^{H/i,*}_{t,t+1}+\lambda^{(1)}_{i,t}+\mu_{i,t}(\mathcal{N}_{i,t+1}D^H_{i,t,t+1})^{-1})\Bigg) \\
&+ \theta_\$\Delta C^{H/\$}_{t,t+1}-\theta_i\Delta C^{H/i}_{t,t+1}\Bigg)\Bigg] \\
&=\; E_t\Bigg[\frac{C^I_{i,0}+2C^H_{i,0}}{2C^I_{i,0}+C^H_{i,0}}(\bar\theta_i\Delta\bar W^{H/i,*}_{t,t+1}+\lambda^{(1)}_{i,t}+\mu_{i,t}(\mathcal{N}_{i,t+1}D^H_{i,t,t+1})^{-1}) \\
&- \frac{C^I_{\$,0}+2C^H_{\$,0}}{2C^I_{\$,0}+C^H_{\$,0}}(\bar\theta_\$\Delta\bar W^{H/\$,*}_{t,t+1}+\lambda^{(1)}_{\$,t}+\mu_{\$,t}(\mathcal{N}_{\$,t+1}D^H_{\$,t,t+1})^{-1})\Bigg]
\end{aligned}
$$

Suppose first that there is no noise in monetary policy. Using the approximation

$$E[X] \;=\; E[e^{\log X}] \;\approx\; e^{E[\log X]+0.5\mathrm{Var}[\log X]} \;\approx e^{E[\log X]}(1+0.5\mathrm{Var}[\log X])$$

that holds in the limit of small variance, we get

$$\frac{\bar S^\$_{t+1}}{S^\$_{i,t+1}} \;\approx\; e^{-\delta_i^S\omega_{t+1}} \;=\; e^{-\delta_i^S/\delta_i\log D_{i,t+1}}$$

Hence, defining $\alpha_i^I = -\delta_i^S/\delta_i$, we get

$$\begin{aligned}
\theta_i^{-1}\mu_{i,t} &= \frac{E_t[\mathcal{N}_{i,t+1}^{-1}\Psi_{i,t+1}]E_t[\frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}}D_{i,t,t+1}\Psi_{i,t+1}] - E_t[\mathcal{N}_{i,t+1}^{-1}\Psi_{i,t+1}\frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}}]E_t[D_{i,t,t+1}\Psi_{i,t+1}]}{E_t[\Psi_{i,t+1}D_{i,t,t+1}]E_t[\Psi_{i,t+1}(D_{i,t,t+1})^{-1}\mathcal{N}_{i,t+1}^{-2}] - (E[\Psi_{i,t+1}\mathcal{N}_{t+1}^{-1}])^2} \\
&= \frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}}\frac{E_t[e^{\psi+\alpha_i^{\mathcal{N}}d}]E_t[e^{d+\alpha_i^I d+\psi}] - E_t[e^{\alpha_i^{\mathcal{N}}d+\psi+\alpha_i^I d}]E_t[e^{d+\psi}]}{E_t[e^{\psi+d}]E_t[e^{\psi-d+2\alpha_i^{\mathcal{N}}d}] - E_t[e^{\psi+\alpha_i^{\mathcal{N}}d}]^2} \\
&= \frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}}\frac{\mathrm{Var}_t[\psi+\alpha_i^{\mathcal{N}}d] + \mathrm{Var}_t[(1+\alpha_i^I)d+\psi] - \mathrm{Var}_t[(\alpha_i^{\mathcal{N}}+\alpha_i^I)d+\psi] - \mathrm{Var}_t[d+\psi]}{\mathrm{Var}_t[\psi+d] + \mathrm{Var}_t[\psi+(2\alpha_i^{\mathcal{N}}-1)d] - 2\mathrm{Var}_t[\psi+\alpha_i^{\mathcal{N}}d]} \\
&= \frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}}\frac{(\alpha_i^{\mathcal{N}})^2 + (1+\alpha_i^I)^2 - (\alpha_i^{\mathcal{N}}+\alpha_i^I)^2 - 1}{1 + (2\alpha_i^{\mathcal{N}}-1)^2 - 2(\alpha_i^{\mathcal{N}})^2} \;=\; \frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}}\frac{\alpha_i^I}{1-\alpha_i^{\mathcal{N}}}
\end{aligned} \tag{A.11}$$

Similarly,

$$\begin{aligned}
&-\frac{\mathrm{Cov}_t^H(S_{i,t+1}^*\frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}}, 1/S_{i,t+1}^*)}{\mathrm{Cov}_t^H(S_{i,t+1}^*, 1/S_{i,t+1}^*)} \\
&= \frac{E_t[\mathcal{N}_{i,t+1}^{-1}\Psi_{i,t+1}]E_t[\frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}}\mathcal{N}_{i,t+1}^{-1}\Psi_{i,t+1}] - E_t[D_{i,t+1}\Psi_{i,t+1}\frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}}]E_t[\Psi_{i,t+1}(D_{i,t+1})^{-1}\mathcal{N}_{i,t+1}^{-2}]}{E_t[\Psi_{i,t+1}D_{i,t+1}]E_t[\Psi_{i,t+1}(D_{i,t+1})^{-1}\mathcal{N}_{i,t+1}^{-2}] - (E[\Psi_{i,t+1}\mathcal{N}_{t+1}^{-1}])^2} \\
&\approx \frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}}\frac{\mathrm{Var}_t[\psi+\alpha_i^{\mathcal{N}}d] + \mathrm{Var}_t[(\alpha_i^{\mathcal{N}}+\alpha_i^I)d+\psi] - \mathrm{Var}_t[(1+\alpha_i^I)d+\psi] - \mathrm{Var}_t[(2\alpha_i^{\mathcal{N}}-1)d+\psi]}{\mathrm{Var}_t[\psi+d] + \mathrm{Var}_t[\psi+(2\alpha_i^{\mathcal{N}}-1)d] - 2\mathrm{Var}_t[\psi+\alpha_i^{\mathcal{N}}d]} \\
&= \frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}}\frac{(\alpha_i^{\mathcal{N}})^2 + (\alpha_i^{\mathcal{N}}+\alpha_i^I)^2 - (1+\alpha_i^I)^2 - (2\alpha_i^{\mathcal{N}}-1)^2}{2(\alpha_i^{\mathcal{N}}-1)^2} \;=\; \frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}}\frac{\alpha_i^I - \alpha_i^{\mathcal{N}} + 1}{\alpha_i^{\mathcal{N}} - 1},
\end{aligned}$$

Hence,

$$\begin{aligned}
\lambda_{i,t} &\approx 1 + \theta_i\left(\frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}} - \frac{\mathrm{Cov}_t^H\left(S_{i,t+1}^*\frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}}, 1/S_{i,t+1}^*\right)}{\mathrm{Cov}_t^H(S_{i,t+1}^*, 1/S_{i,t+1}^*)}\right) \\
&= 1 + \theta_i\frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}}\left(1 + \frac{\alpha_i^I - \alpha_i^{\mathcal{N}} + 1}{\alpha_i^{\mathcal{N}} - 1}\right) \;=\; 1 + \theta_i\frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}}\frac{\alpha_i^I}{\alpha_i^{\mathcal{N}} - 1}.
\end{aligned}$$

If C_0^I/C_0^H is the same across the two countries, then

$$
\begin{aligned}
& -e^{-r_{\$,t}^{I,H}}+e^{-r_{\$,t}} \\
\approx\ & \frac{C_0^I+2C_0^H}{2C_0^I+C_0^H}\bar{\theta}E_t\Bigg[\mathcal{N}_{\$,t+1}^{-1}\Psi_{\$,t,t+1}\Bigg((\Delta W_{t,t+1}^{*,H/i}-\Delta W_{t,t+1}^{*,H/\$}) \\
+\ & \Big(\frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}}\frac{\alpha_i^I}{\alpha_i^{\mathcal{N}}-1}(1-(D_{i,t,t+1})^{\alpha_i^{\mathcal{N}}-1})-\frac{\bar{S}_t^{\$}}{S_{\$,t}^{\$}}\frac{\alpha_{\$}^I}{\alpha_{\$}^{\mathcal{N}}-1}(1-(D_{\$,t,t+1})^{\alpha_{\$}^{\mathcal{N}}-1})\Big)\Bigg)\Bigg].
\end{aligned}
$$

If $D_{i,t,t+1}=D_{\$,t,t+1}$, then we get

$$
\begin{aligned}
& -e^{-r_{\$,t}^{I,H}}+e^{-r_{\$,t}} \\
\approx\ & \frac{C_0^I+2C_0^H}{2C_0^I+C_0^H}\bar{\theta}E_t\Bigg[\mathcal{N}_{\$,t+1}^{-1}\Psi_{\$,t,t+1}\Big(W_t^{*,H/i}\alpha^I(F(\alpha_i^{\mathcal{N}},D_{t,t+1})-F(\alpha_{\$}^{\mathcal{N}},D_{t,t+1}))\Big)\Bigg].
\end{aligned}
$$

with $F(\alpha,x)=(1-x^{\alpha-1})/(\alpha-1)$ and the claim follows because F is monotone decreasing in α for x close to one.[5] More generally, substituting

$$
D_{i,t,t+1}=e^{\delta_i\Delta\omega_{t+1}},\ (\frac{S_{i,t+1}^{\$}}{\bar{S}_{t+1}^{\$}})\frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}}\ =\ e^{\delta_i^S\Delta\omega_{t+1}},
$$

and denoting

$$
Q_i=\bar{\theta}_i\frac{C_{i,0}^I+2C_{i,0}^H}{2C_{i,0}^I+C_{i,0}^H},
$$

[5]Since $D_{i,t,t+1}$ has a small variance and takes a finite number of values, it is close to one with a probability that is close to one.

we get

$$
\begin{aligned}
& - e^{-r^{I,H}_{\$,t}} + e^{-r_{\$,t}} \\
\approx\ & E_t\Bigg[\mathcal{N}^{-1}_{\$,t+1}\Psi_{\$,t,t+1}\Bigg((Q_i \Delta W^{*,H/i}_{t,t+1} - Q_\$ \Delta W^{*,H/\$}_{t,t+1}) \\
+\ & \Big(\frac{\bar{S}^\$_t}{S^\$_{i,t}} Q_i \frac{\alpha^I_i}{\alpha^{\mathcal{N}}_i - 1}(1 - (D_{i,t,t+1})^{\alpha^{\mathcal{N}}_i - 1}) - W^{*,H/\$}_t Q_\$ \frac{\alpha^I_\$}{\alpha^{\mathcal{N}}_\$ - 1}(1 - (D_{\$,t,t+1})^{\alpha^{\mathcal{N}}_\$ - 1})\Big)\Bigg)\Bigg] \\
=\ & \frac{\bar{S}^\$_t}{S^\$_{\$,t}} E_t\Bigg[\mathcal{N}^{-1}_{\$,t+1}\Psi_{\$,t,t+1}\Bigg(Q_i \frac{S^\$_{\$,t}}{S^\$_{i,t}}(e^{-\delta^S_i \Delta\omega_{t+1}} - 1) - Q_\$(e^{-\delta^S_\$ \Delta\omega_{t+1}} - 1)) \\
+\ & \Big(\frac{S^\$_{\$,t}}{S^\$_{i,t}} Q_i \frac{\alpha^I_i}{\alpha^{\mathcal{N}}_i - 1}(1 - e^{\delta_i \Delta\omega_{t+1}(\alpha^{\mathcal{N}}_i - 1)}) - Q_\$ \frac{\alpha^I_\$}{\alpha^{\mathcal{N}}_\$ - 1}(1 - e^{\delta_\$ \Delta\omega_{t+1}(\alpha^{\mathcal{N}}_\$ - 1)})\Big)\Bigg]\ .
\end{aligned}
$$

We can rewrite this as

$$
\begin{aligned}
& - e^{-r^{I,H}_{\$,t}} + e^{-r_{\$,t}} \\
\approx\ & E_t\Bigg[\mathcal{N}^{-1}_{\$,t+1}\Psi_{\$,t,t+1}\Bigg((Q_i\Delta W^{*,H/i}_{t,t+1} - Q_\$\Delta W^{*,H/\$}_{t,t+1}) \\
+\ & \Big(\frac{\bar S^\$_t}{S^\$_{i,t}}Q_i\frac{\alpha^I_i}{\alpha^{\mathcal{N}}_i-1}(1-(D_{i,t,t+1})^{\alpha^{\mathcal{N}}_i-1}) - W^{*,H/\$}_t Q_\$\frac{\alpha^I_\$}{\alpha^{\mathcal{N}}_\$-1}(1-(D_{\$,t,t+1})^{\alpha^{\mathcal{N}}_\$-1})\Big)\Bigg)\Bigg] \\
=\ & \frac{\bar S^\$_t}{S^\$_{\$,t}}E_t\Bigg[\mathcal{N}^{-1}_{\$,t+1}\Psi_{\$,t,t+1}\Bigg(Q_i\frac{S^\$_{\$,t}}{S^\$_{i,t}}(e^{-\delta^S_i\Delta\omega_{t+1}}-1) - Q_\$(e^{-\delta^S_\$\Delta\omega_{t+1}}-1)) \\
+\ & \Big(\frac{S^\$_{\$,t}}{S^\$_{i,t}}Q_i\frac{\alpha^I_i}{\alpha^{\mathcal{N}}_i-1}(1-e^{\delta_i\Delta\omega_{t+1}(\alpha^{\mathcal{N}}_i-1)}) - Q_\$\frac{\alpha^I_\$}{\alpha^{\mathcal{N}}_\$-1}(1-e^{\delta_\$\Delta\omega_{t+1}(\alpha^{\mathcal{N}}_\$-1)})\Big)\Bigg)\Bigg] \\
=\ & \frac{\bar S^\$_t}{S^\$_{\$,t}}E_t\Bigg[\mathcal{N}^{-1}_{\$,t+1}\Psi_{\$,t,t+1}\Bigg(Q_i\frac{S^\$_{\$,t}}{S^\$_{i,t}}\Bigg((e^{-\delta^S_i\Delta\omega_{t+1}}-1)\ +\ \frac{-\delta^S_i/\delta_i}{\alpha^{\mathcal{N}}_i-1}(1-e^{\delta_i\Delta\omega_{t+1}(\alpha^{\mathcal{N}}_i-1)})\Bigg) \\
& - Q_\$\Bigg((e^{-\delta^S_\$\Delta\omega_{t+1}}-1) + \frac{-\delta^S_\$/\delta_\$}{\alpha^{\mathcal{N}}_\$-1}(1-e^{\delta_\$\Delta\omega_{t+1}(\alpha^{\mathcal{N}}_\$-1)})\Bigg)\Bigg)\Bigg] \\
=\ & \frac{\bar S^\$_t}{S^\$_{\$,t}}E_t\Bigg[\mathcal{N}^{-1}_{\$,t+1}\Psi_{\$,t,t+1}\Bigg(Q_i\frac{S^\$_{\$,t}}{S^\$_{i,t}}\Bigg((e^{-\delta^S_i\Delta\omega_{t+1}}-1)\ +\ \frac{-\delta^S_i/\delta_i}{\alpha^{\mathcal{N}}_i-1}(1-e^{\delta_i\Delta\omega_{t+1}(\alpha^{\mathcal{N}}_i-1)})\Bigg) \\
& - Q_\$\Bigg((e^{-\delta^S_\$\Delta\omega_{t+1}}-1) + \frac{-\delta^S_\$/\delta_\$}{\alpha^{\mathcal{N}}_\$-1}(1-e^{\delta_\$\Delta\omega_{t+1}(\alpha^{\mathcal{N}}_\$-1)})\Bigg)\Bigg)\Bigg] \\
=\ & \frac{\bar S^\$_t}{S^\$_{\$,t}}E_t\Bigg[\mathcal{N}^{-1}_{\$,t+1}\Psi_{\$,t,t+1}\Bigg(\delta^S_i Q_i\frac{S^\$_{\$,t}}{S^\$_{i,t}}\Bigg(-F(-\delta^S_i)+\ F(\delta_i(\alpha^{\mathcal{N}}_i-1))\Bigg) \\
& - \delta^S_\$ Q_\$\Bigg(-F(-\delta^S_\$)+\ F(\delta_\$(\alpha^{\mathcal{N}}_\$-1))\Bigg)\Bigg)\Bigg] \\
\approx\ & 0.5\frac{\bar S^\$_t}{S^\$_{\$,t}}\Bigg(\delta^S_i Q_i\frac{S^\$_{\$,t}}{S^\$_{i,t}}(\delta^S_i+\delta_i(\alpha^{\mathcal{N}}_i-1)) - \delta^S_\$ Q_\$(\delta_\$(\alpha^{\mathcal{N}}_\$-1)+\delta^S_\$)\Bigg)\Bigg)E_t[(\Delta\omega_{t+1})^2]\,,
\end{aligned}
$$

where we have used the Taylor approximation

$$
F(\alpha) = (e^{\Delta\omega_{t+1}\alpha}-1)/\alpha\ \approx\ \frac{\Delta\omega_{t+1}\alpha + 0.5(\Delta\omega_{t+1}\alpha)^2}{\alpha}\ =\ \Delta\omega_{t+1} + 0.5\alpha(\Delta\omega_{t+1})^2\,.
$$

Q.E.D.

Proposition 8 *Ceteris paribus, the sensitivity of a recipient country (a) nominal bond prices and (b) customer net worth to a US monetary shock is monotone increasing in*

(1) Country's intermediation capacity, w_i^*.

(2) Country's stock market capitalization, $S_{i,t}^{\$}$.

Proof of Proposition 8. We now go to the second order. In this case, we get from (A.5) that

$$
\begin{aligned}
& M_{k,0,t}^{H,(2)} \\
& \approx\; C_{k,0}^{-1}\Big(C_{k,0}^{I}(M_{k,0,t}^{H,(2)} - M_{k,0,t}^{I,(2)} + (M_{k,0,t}^{I,(1)})^2) - \beta_k C_{k,0}^{I}\,(1 + M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)})\Big) \\
& + \bar{\theta}_k \sum_j \beta_j C_{j,0}^{-1} \mathcal{E}_{j,0} \frac{C_{j,0}\Psi_{j,0,t}}{C_{k,0}\Psi_{k,0,t}} \\
& \times \Big(C_{j,0}^{H}(\mathcal{E}_{j,t}^{(1)} - M_{j,0,t}^{H,(1)} + M_{k,0,t}^{H,(1)} - \mathcal{E}_{k,t}^{(1)})\; + C_{j,0}^{I}(M_{k,0,t}^{H,(1)} - \mathcal{E}_{k,t}^{(1)} + \mathcal{E}_{j,t}^{(1)} - M_{j,0,t}^{I,(1)})\Big) \\
& =\; C_{k,0}^{-1}\Big(C_{k,0}^{I}(M_{k,0,t}^{H,(2)} - M_{k,0,t}^{I,(2)} + (M_{k,0,t}^{I,(1)})^2) - \beta_k C_{k,0}^{I}\,(M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)})\Big) \\
& + \bar{\theta}_k \sum_j \beta_j C_{j,0}^{-1} \mathcal{E}_{j,0} \frac{C_{j,0}\Psi_{j,0,t}}{C_{k,0}\Psi_{k,0,t}} \\
& \times \Big(C_{j,0}^{H}(M_{j,0,t}^{I,(1)} - M_{k,0,t}^{I,(1)} - M_{j,0,t}^{H,(1)} + M_{k,0,t}^{H,(1)})\; + C_{j,0}^{I}(M_{k,0,t}^{H,(1)} + M_{j,0,t}^{I,(1)} - M_{k,0,t}^{I,(1)} - M_{j,0,t}^{I,(1)})\Big) \\
& =\; C_{k,0}^{-1}\Big(C_{k,0}^{I}(M_{k,0,t}^{H,(2)} - M_{k,0,t}^{I,(2)} + (M_{k,0,t}^{I,(1)})^2) - \beta_k C_{k,0}^{I}\,(M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)})\Big) \\
& + \bar{\theta}_k \sum_j \beta_j C_{j,0}^{-1} \mathcal{E}_{j,0} \frac{C_{j,0}\Psi_{j,0,t}}{C_{k,0}\Psi_{k,0,t}} \\
& \times \Big(C_{j,0}^{H}(-(M_{j,0,t}^{H,(1)} - M_{j,0,t}^{I,(1)}) + (M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)}))\; + C_{j,0}^{I}(M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)})\Big)\,.
\end{aligned}
$$

Rewriting, we get

$$
\begin{aligned}
& M^{H,(2)}_{k,0,t} \\
&= C^{-1}_{k,0}C^{I}_{k,0}(M^{H,(2)}_{k,0,t} - M^{I,(2)}_{k,0,t}) + C^{-1}_{k,0}C^{I}_{k,0}(M^{I,(1)}_{k,0,t})^2 \\
&+ (M^{H,(1)}_{k,0,t} - M^{I,(1)}_{k,0,t})\Bigg(- \beta_k C^{-1}_{k,0}C^{I}_{k,0} + \bar{\theta}_k \sum_j \beta_j \mathcal{E}_{j,0}\frac{C_{j,0}\Psi_{j,0,t}}{C_{k,0}\Psi_{k,0,t}}\Bigg) \\
&- \bar{\theta}_k \sum_j \beta_j C^{-1}_{j,0}\mathcal{E}_{j,0}\frac{C_{j,0}\Psi_{j,0,t}}{C_{k,0}\Psi_{k,0,t}} C^{H}_{j,0}(M^{H,(1)}_{j,0,t} - M^{I,(1)}_{j,0,t}) \\
&= C^{-1}_{k,0}C^{I}_{k,0}(M^{H,(2)}_{k,0,t} - M^{I,(2)}_{k,0,t}) + C^{-1}_{k,0}C^{I}_{k,0}(M^{I,(1)}_{k,0,t})^2 \\
&+ (M^{H,(1)}_{k,0,t} - M^{I,(1)}_{k,0,t})\Bigg(- \beta_k C^{-1}_{k,0}C^{I}_{k,0} + \bar{\theta}_k \bar{W}^{H/k,*}_t\Bigg) \\
&+ \bar{\theta}_k \Xi^{/k}_t
\end{aligned}
$$

where we have defined

$$
\begin{aligned}
\Xi^{/k}_t &= \sum_j \beta_j C^{-1}_{j,0}\mathcal{E}_{j,0}\frac{C_{j,0}\Psi_{j,0,t}}{C_{k,0}\Psi_{k,0,t}} C^{H}_{j,0}(M^{H,(1)}_{j,0,t} - M^{I,(1)}_{j,0,t}) \\
&= -\sum_j \beta_j C^{-1}_{j,0}\mathcal{E}_{j,0}\frac{C_{j,0}\Psi_{j,0,t}}{C_{k,0}\Psi_{k,0,t}} C^{H}_{j,0}\frac{C^{I}_{j,0}+C^{H}_{j,0}}{2C^{I}_{j,0}+C^{H}_{j,0}}\bar{\theta}_j\Bigg(\Delta \bar{W}^{H/j,*}_t + \sum_{\tau=0}^{t-1}\Big(\lambda^{(1)}_{j,\tau} + \mu_{j,\tau}(\mathcal{N}_{j,\tau+1}D^{H}_{j,\tau,\tau+1})^{-1}\Big)\Bigg)
\end{aligned}
$$

Therefore,

$$
M^{H}_{k,t,t+1} \approx M^{*}_{k,t,t+1}(1 + M^{(1)}_{k,t,t+1} + (M^{H,(2)}_{k,0,t+1} - M^{H,(2)}_{k,0,t} + (M^{H,(1)}_{k,0,t})^2))
$$

Hence,

$$
M^{(2)}_{k,t,t+1} = M^{H,(2)}_{k,0,t+1} - M^{H,(2)}_{k,0,t} + (M^{H,(1)}_{k,0,t})^2 \tag{A.12}
$$

Now, the second-order correction in equations (A.13) can be rewritten as

$$
\begin{aligned}
E_t[M^*_{k,t,t+1}(M^{H,(2)}_{t,t+1} - M^{I,(2)}_{t,t+1})] \;&=\; 0 \\
E_t[M^*_{k,t,t+1}\mathcal{N}_{k,t+1}D_{k,t,t+1}\Big((M^{H,(1)}_{t,t+1} - M^{I,(1)}_{t,t+1})S^{(1)}_{k,t+1} + (M^{H,(2)}_{t,t+1} - M^{I,(2)}_{t,t+1})\Big)] \;&=\; 0\,.
\end{aligned}
\tag{A.13}
$$

Thus, we have, using the second of the identities (A.13), that

$$
\begin{aligned}
S_{k,t} \;&=\; S^*_{k,t}(1+S^{(1)}_{k,t}) \;+\; E_t[M^{H,*}_{k,t,t+1}S^*_{k,t+1}S^{(2)}_{k,t+1}] + E_t[M^{H,(1)}_{k,t,t+1}S^{(1)}_{k,t+1}] \;+\; E_t[M^{H,*}_{k,t,t+1}M^{H,(2)}_{k,t,t+1}S^*_{k,t+1}] \\
&=\; S^*_{k,t}(1+S^{(1)}_{k,t}) \;+\; E_t[M^{H,*}_{k,t,t+1}S^*_{k,t+1}S^{(2)}_{k,t+1}] + E_t[M^{H,(1)}_{k,t,t+1}S^{(1)}_{k,t+1}] \\
&+\; E_t\Bigg[\Bigg((M^{H,(1)}_{k,0,t})^2 - C^{-1}_{k,0}C^I_{k,0}(M^{H,(1)}_{t,t+1} - M^{I,(1)}_{t,t+1})S^{(1)}_{k,t+1} \\
&+ C^{-1}_{k,0}C^I_{k,0}((M^{I,(1)}_{k,0,t+1})^2 - (M^{I,(1)}_{k,0,t})^2) \\
&+ (M^{H,(1)}_{k,0,t+1} - M^{I,(1)}_{k,0,t+1})\Bigg(-\beta_k C^{-1}_{k,0}C^I_{k,0} + \bar\theta_k \bar W^{H/k,*}_{t+1}\Bigg) - (M^{H,(1)}_{k,0,t} - M^{I,(1)}_{k,0,t})\Bigg(-\beta_k C^{-1}_{k,0}C^I_{k,0} + \bar\theta_k \bar W^{H/k,*}_{t}\Bigg) \\
&+ \bar\theta_k \Delta\Xi^{/k}_{t,t+1}\Bigg)M^{H,*}_{k,t,t+1}S^*_{k,t+1}\Bigg]
\end{aligned}
$$

Define

$$
\begin{aligned}
A_{k,t} \;&\equiv\; E_t[M^{H,(1)}_{k,t,t+1}S^{(1)}_{k,t+1}] \\
&+\; E_t\Bigg[\Bigg((M^{H,(1)}_{k,0,t})^2 - C^{-1}_{k,0}C^I_{k,0}(M^{H,(1)}_{t,t+1} - M^{I,(1)}_{t,t+1})S^{(1)}_{k,t+1} \\
&+ C^{-1}_{k,0}C^I_{k,0}((M^{I,(1)}_{k,0,t+1})^2 - (M^{I,(1)}_{k,0,t})^2) \\
&+ (M^{H,(1)}_{k,0,t+1} - M^{I,(1)}_{k,0,t+1})\Bigg(-\beta_k C^{-1}_{k,0}C^I_{k,0} + \bar\theta_k \bar W^{H/k,*}_{t+1}\Bigg) \\
&- (M^{H,(1)}_{k,0,t} - M^{I,(1)}_{k,0,t})\Bigg(-\beta_k C^{-1}_{k,0}C^I_{k,0} + \bar\theta_k \bar W^{H/k,*}_{t}\Bigg)\Bigg)M^{H,*}_{k,t,t+1}S^*_{k,t+1}\Bigg]
\end{aligned}
$$

and note that $A_{k,t}$ only depends on the domestic monetary policy in country k (though in a quite complicated fashion). Then, we can rewrite the equation for $S_{k,t}$ as

$$S^*_{k,t} S^{(2)}_{k,t} \;=\; E_t[M^{H,*}_{k,t,t+1} S^*_{k,t+1} S^{(2)}_{k,t+1}] \;+\; A_{k,t} \;+\; E_t\left[\bar{\theta}_k \Delta \Xi^{/k}_{t,t+1}\, M^{H,*}_{k,t,t+1} S^*_{k,t+1}\right],$$

which defines $S^{(2)}_{k,t}$. Thus,

$$S^{(2)}_{k,t} \;=\; -\bar{\theta}_k \Xi^{/k}_t E_t[M^{H,*}_{k,t,t+1} S^*_{k,t+1}]/S^*_{k,t} \;=\; -\bar{\theta}_k \Xi^{/k}_t (1 - \mathcal{M}_{k,t}/S^*_{k,t}) + Z_{k,t}$$

where $Z_{k,t}$ only depends on the domestic monetary policy as well as expectations about future policy. Now, from (A.4), we get that

$$M^{I,(2)}_{k,t,t+1} \;=\; 2M^{H,(2)}_{k,t,t+1} + S^{(2)}_{k,t,t+1} + Q_{k,t,t+1}$$

where $Q_{k,t,t+1}$ is a (complicated) expression that depends only on the domestic monetary policy.

Substituting into (A.15), we get

$$\begin{aligned} M^{H,(2)}_{k,t,t+1} \;&=\; C^{-1}_{k,0} C^I_{k,0} (M^{H,(2)}_{k,t,t+1} - M^{I,(2)}_{k,t,t+1}) + \tilde{Z} + \bar{\theta}_k \Delta \Xi^{/k}_{t,t+1} \\ =\; C^{-1}_{k,0} C^I_{k,0} (M^{H,(2)}_{k,t,t+1} &- 2M^{H,(2)}_{k,t,t+1} - S^{(2)}_{k,t,t+1} - Q_{k,t,t+1}) + \tilde{Z} + \bar{\theta}_k \Delta \Xi^{/k}_{t,t+1}\,, \end{aligned} \tag{A.14}$$

where $\tilde{Z}$ does not depend on foreign monetary shocks. Hence,

$$M^{H,(2)}_{k,t,t+1} \;=\; (1 + C^{-1}_{k,0} C^I_{k,0})^{-1} (-C^{-1}_{k,0} C^I_{k,0} S^{(2)}_{k,t,t+1} + \bar{\theta}_k \Delta \Xi^{/k}_{t,t+1}) + \hat{Z}$$

where

$$S^{(2)}_{k,t,t+1} \;=\; S^{(2)}_{k,t+1} - S^{(2)}_{k,t} + (S^{(1)}_{k,t})^2\,.$$

Substituting, we get

$$\begin{aligned}
M^{H,(2)}_{k,t,t+1} &= (1+C^{-1}_{k,0}C^{I}_{k,0})^{-1}(-C^{-1}_{k,0}C^{I}_{k,0}(S^{(2)}_{k,t+1}-S^{(2)}_{k,t}+(S^{(1)}_{k,t})^2)+\bar{\theta}_k\Delta\Xi^{/k}_{t,t+1})+\hat{Z} \\
&= (1+C^{-1}_{k,0}C^{I}_{k,0})^{-1}(-C^{-1}_{k,0}C^{I}_{k,0}(-\bar{\theta}_k\Xi^{/k}_{t+1}(1-\mathcal{M}_{k,t+1}/S^{*}_{k,t+1})+\bar{\theta}_k\Xi^{/k}_{t}(1-\mathcal{M}_{k,t}/S^{*}_{k,t}))+\bar{\theta}_k\Delta\Xi^{/k}_{t,t+1})+\hat{Q} \\
&= (1+C^{-1}_{k,0}C^{I}_{k,0})^{-1}\bar{\theta}_k\Xi^{/k}_{t+1}\Big(1+C^{-1}_{k,0}C^{I}_{k,0}(1-\mathcal{M}_{k,t+1}/S^{*}_{k,t+1})\Big) \\
&- (1+C^{-1}_{k,0}C^{I}_{k,0})^{-1}\bar{\theta}_k\Xi^{/k}_{t}\Big(1+C^{-1}_{k,0}C^{I}_{k,0}(1-\mathcal{M}_{k,t}/S^{*}_{k,t})\Big) \;+\; Q^{H},
\end{aligned} \tag{A.15}$$

where none of the Q and Z terms depends on the foreign shocks, but rather they only depend on their expectations. Thus,

$$\begin{aligned}
M^{I,(2)}_{k,t,t+1} &= 2M^{H,(2)}_{k,t,t+1}+S^{(2)}_{k,t,t+1}+Q_{k,t,t+1} \\
&= 2(1+C^{-1}_{k,0}C^{I}_{k,0})^{-1}\bar{\theta}_k\Xi^{/k}_{t+1}\Big(1+C^{-1}_{k,0}C^{I}_{k,0}(1-\mathcal{M}_{k,t+1}/S^{*}_{k,t+1})\Big) \\
&- 2(1+C^{-1}_{k,0}C^{I}_{k,0})^{-1}\bar{\theta}_k\Xi^{/k}_{t}\Big(1+C^{-1}_{k,0}C^{I}_{k,0}(1-\mathcal{M}_{k,t}/S^{*}_{k,t})\Big) \\
&- \bar{\theta}_k\Xi^{/k}_{t+1}(1-\mathcal{M}_{k,t+1}/S^{*}_{k,t+1})+\bar{\theta}_k\Xi^{/k}_{t}(1-\mathcal{M}_{k,t}/S^{*}_{k,t})+Q^{**} \\
&= (1+C^{-1}_{k,0}C^{I}_{k,0})^{-1}\bar{\theta}_k\Xi^{/k}_{t+1}\Big(2-(1-C^{-1}_{k,0}C^{I}_{k,0})(1-\mathcal{M}_{k,t+1}/S^{*}_{k,t+1})\Big) \\
&- 2(1+C^{-1}_{k,0}C^{I}_{k,0})^{-1}\bar{\theta}_k\Xi^{/k}_{t}\Big(2-(1-C^{-1}_{k,0}C^{I}_{k,0})(1-\mathcal{M}_{k,t}/S^{*}_{k,t})\Big) \;+Q^{***}.
\end{aligned}$$

Thus, the shock to the exchange rate $\mathcal{E}_{i,t+1}/\mathcal{E}_{i,t}$ is given by

$$\begin{aligned}
&-\Bigg((1+C^{-1}_{k,0}C^{I}_{k,0})^{-1}\bar{\theta}_k\Big(2-(1-C^{-1}_{k,0}C^{I}_{k,0})(1-\mathcal{M}_{k,t+1}/S^{*}_{k,t+1})\Big)\frac{1}{C_{k,0}\Psi_{k,0,t+1}} \\
&- (1+C^{-1}_{\$,0}C^{I}_{\$,0})^{-1}\bar{\theta}_\$\Big(2-(1-C^{-1}_{\$,0}C^{I}_{\$,0})(1-\mathcal{M}_{\$,t+1}/S^{*}_{\$,t+1})\Big)\frac{1}{C_{\$,0}\Psi_{\$,0,t+1}}\Bigg) \\
&\times\sum_j\beta_j C^{-1}_{j,0}\mathcal{E}_{j,0}C_{j,0}\Psi_{j,0,t}C^{H}_{j,0}\frac{C^{I}_{j,0}+C^{H}_{j,0}}{2C^{I}_{j,0}+C^{H}_{j,0}}\bar{\theta}_j\mu_{j,\tau}(\mathcal{N}_{j,\tau+1}D^{H}_{j,\tau,\tau+1})^{-1}
\end{aligned}$$

Similarly, the sensitivity of the relative net worth of customers in countries i and j, $W^{H}_{i,t+1}/W^{H}_{j,t+1}$, to US monetary policy shocks $\mathcal{N}_{i,t+1}^{-1}$ is given by

$$\frac{C_{i,0}^H}{C_{j,0}^H}\Bigg((1+C_{i,0}^{-1}C_{i,0}^I)^{-1}\bar{\theta}_i\Big(1+C_{i,0}^{-1}C_{i,0}^I(1-D_{i,t+1}^{-1})\Big)\frac{1}{C_{i,0}\Psi_{i,0,t}}$$
$$-(1+C_{j,0}^{-1}C_{j,0}^I)^{-1}\bar{\theta}_j\Big(1+C_{j,0}^{-1}C_{j,0}^I(1-D_{j,t+1}^{-1})\Big)\frac{1}{C_{j,0}\Psi_{j,0,t}}\Bigg)$$
$$\times\beta_\$ C_{\$,0}^H\frac{C_{\$,0}^I+C_{\$,0}^H}{2C_{\$,0}^I+C_{\$,0}^H}\bar{\theta}_\$\mu_{\$,t}(\mathcal{N}_{\$,\tau+1}D_{j,t,t+1}^H)^{-1}\mathcal{E}_{\$,0}\Psi_{\$,0,t}$$

Q.E.D.

The following auxiliary lemma shows that stock prices inherit the one-factor structure of discount rates.

Lemma 9 *Suppose that the transition density of ω_t has the monotone likelihood property: $\frac{\partial}{\partial\omega_t}\log p(\omega_t,\omega_{t+1})$ is strictly monotone increasing in ω_{t+1} for almost every (ω_t,δ_{t+1}). Then,*

- *There exist strictly monotone increasing functions $d_i(\omega,t)$ such that $\log D_{i,t} = d_i(\omega_t,t)$.*[6]
- *$S_{i,t}^\$/S_{j,t}^\$$ is monotone increasing in ω_t if and only if $\delta_i^\Psi > \delta_j^\Psi$.*

The proof is straightforward and follows by standard arguments.

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[6]The dependence on t arises due to the finite horizon T and vanishes as $T \to \infty$.

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