Online Appendices: Not for Publication

A. Appendix: Omitted Derivations for the Two Period Model

This appendix presents the derivations and proofs omitted from the main text for the two period model that we analyze in Section 2. We start by the case analyzed in the main text. We then analyze the case in which EIS is different than one, as well as the case with belief disagreements. Throughout, recall that the market portfolio is the claim to all output at date 1. Combining Eqs. (1) and (2), the return on the market portfolio is also log normally distributed, that is,

$$r^m(z_1) = \log\left(\frac{Q_1}{z_1}\right) \sim N\left(g - \log Q - \frac{\sigma^2}{2}, \sigma^2\right). \tag{A.1}$$

A.1. Baseline two period model

For this case, most of the analysis is provided in the main text. Here, we formally state the investor's problem and derive the optimality conditions. The investor takes the returns as given and solves the following problem,

$$\begin{aligned}
&\max_{c_0,a_0,\omega^m} \log c_0 + e^{-\rho}\log U_1 \\
&\text{where } U_1 = \left(E\left[c_1(z_1)^{1-\gamma}\right]\right)^{1/(1-\gamma)} \\
\text{s.t.}\quad & c_0 + a_0 = y_0 + Q \\
\text{and}\quad & c_1(z_1) = a_0\left(\omega^m \exp\left(r^m(z_1)\right) + (1-\omega^m)\exp\left(r^f\right)\right).
\end{aligned}$$

Here, $c_1(z_1)$ denotes total financial wealth, which equals consumption (since the economy ends at date 1). Note that the investor has Epstein-Zin preferences with EIS coefficient equal to one and the RRA coefficient equal to $\gamma > 0$. The case with $\gamma = 1$ is equivalent to time-separable log utility as in the dynamic model.

In view of the Epstein-Zin functional form, the investor's problem naturally splits into two steps. Conditional on savings, a_0, she solves a portfolio optimization problem, that is, $U_1 = R^{CE}a_0$, where

$$\begin{aligned}
R^{CE} &= \max_{\omega^m}\left(E\left[\left(R^p(z_1)\right)^{1-\gamma}\right]\right)^{1/(1-\gamma)} \qquad \text{(A.2)}\\
\text{and } R^p(z_1) &= \left(\omega^m \exp\left(r^m(z_1)\right) + (1-\omega^m)\exp\left(r^f\right)\right).
\end{aligned}$$

Here, we used the observation that the portfolio problem is linearly homogeneous. The variable, $R^p(z_1)$, denotes the realized portfolio return per dollar, and R^{CE} denotes the optimal certainty-equivalent portfolio return. In turn, the investor chooses asset holdings, a_0, that solve the intertemporal problem,

$$\max_{a_0} \log\left(y_0 + Q - a_0\right) + e^{-\rho}\log\left(R^{CE}a_0\right). \tag{A.3}$$

The first order condition for this problem implies Eq. (4) in the main text. That is, regardless of her certainty-equivalent portfolio return, the investor consumes and saves a constant fraction of her lifetime wealth.

It remains to characterize the optimal portfolio weight, ω^m, as well as the certainty-equivalent return, R^{CE}. Even though the return on the market portfolio is log-normally distributed (see Eq. $(A.1)$), the portfolio return, $R^p(z_1)$, is in general not log-normally distributed (since it is the sum of a log-normal variable and a constant). Following Campbell and Viceira (2002), we assume the investor solves an approximate version of the portfolio problem $(A.2)$ in which the log portfolio return is also normally distributed. To state the problem, let $\pi^p \equiv \log E[R^p] - r^f$ and $(\sigma^p)^2 \equiv var(\log R^p)$ to denote respectively the risk premium and the variance of the market portfolio (measured in log returns). Then, the approximate portfolio return satisfies,

$$\begin{aligned} \pi^p &= \omega^m \pi^k \\ \text{where } \pi^k &\equiv \log\left(E\left[\exp\left(r^m(z_1)\right)\right]\right) - r^f = E\left[r^m(z_1)\right] - r^f + \frac{\sigma^2}{2}. \end{aligned} \tag{A.4}$$

Hence, the risk premium on the portfolio return depends linearly on the risk premium on the market portfolio (measured in log returns). We also have,

$$\sigma^p = \omega^m \sigma. \tag{A.5}$$

Thus, the volatility of the portfolio also depends linearly on the volatility of the market portfolio (measured in log returns). These identities hold exactly in continuous time. In the two period model, they hold approximately when the period time-length is small. Moreover, they become exact for the level the risk premium that ensures equilibrium, $\omega^m = 1$, since in this case the portfolio return is actually log-normally distributed.

Taking the log of the objective function in problem $(A.2)$, and using the log-normality assumption, the problem can be equivalently rewritten as,

$$\log R^{CE} - r^f = \max_{\omega^m} \pi^p - \frac{1}{2}\gamma\left(\sigma^p\right)^2, \tag{A.6}$$

where π^p and σ^p are defined in Eqs. $(A.4)$ and $(A.5)$. It follows that, up to an approximation (that becomes exact in equilibrium), the investor's problem turns into standard mean-variance optimization. Taking the first order condition, we obtain Eq. (6) in the main text. Substituting $\omega^m = 1$ and $E[r^m(z_1)] = g - \log Q - \frac{\sigma^2}{2}$ [cf. Eq. $(A.1)$] into this expression, we further obtain Eq. (7) in the main text. Substituting these expressions into $(A.6)$, we also obtain the closed form solution for the certainty-equivalent return in (11).

A.2. More general EIS

In this case, the representative investor solves the following problem,

$$\begin{aligned} \max_{c_0, a_0, \omega^m, \{c_1(z_1)\}} \quad & U_0 = \frac{c_0^{1-1/\varepsilon} - 1}{1 - 1/\varepsilon} + e^{-\rho}\frac{U_1^{1-1/\varepsilon} - 1}{1 - 1/\varepsilon} \\ \text{where} \quad & U_1 = \left(E\left[c_1(z_1)^{1-\gamma}\right]\right)^{1/(1-\gamma)} \\ \text{s.t.} \quad & c_0 + a_0 = y_0 + Q \\ \text{and} \quad & c_1(z_1) = a_0\left(\omega^m \exp\left(r^m(z_1)\right) + (1 - \omega^m)\exp\left(r^f\right)\right). \end{aligned}$$

Here, ε denotes the elasticity of substitution. The case with $\varepsilon = 1$ is equivalent to the earlier problem.

Most of the analysis remains unchanged. As before, the investor's problem splits into two parts. The portfolio problem $(A.2)$ as well as its solution remains unchanged. In particular, Eqs. (6), (7), (11) from the main text continue to apply.

The main difference concerns the intertemporal problem $(A.3)$, which is now given by,

$$\max_{a_0} (y_0 + Q - a_0)^{1-1/\varepsilon} + e^{-\rho} \left(R^{CE} a_0\right)^{1-1/\varepsilon}.$$

Taking the first order condition and rearranging terms, we obtain the consumption function,

$$c_0 = \frac{1}{1 + e^{-\rho\varepsilon} \left(R^{CE}\right)^{(\varepsilon-1)}} (y_0 + Q).$$

Combining this expression with the aggregate resource constraint, $y_0 = c_0$, we obtain the output-asset price relation (10) in the main text. The main difference from the earlier analysis is that consumption (and savings) also depends on income and substitution effects, in addition to the wealth effect in the main text. When $\varepsilon > 1$, the substitution effect dominates and all else equal an increase in the certainty-equivalent return reduces consumption (increases savings). This in turn lowers aggregate demand and output. Conversely, when $\varepsilon < 1$, the income effect dominates and an increase in certainty-equivalent return increases consumption, aggregate demand, and output.

The equilibrium is found by jointly solving Eq . (10) together with Eqs. (7) and (11), as well as the constrained policy interest rate. Collecting the equations together, the equilibrium tuple, $\left(y_0, Q, R^{CE}, r^f\right)$, is the solution to the following system,

$$\begin{aligned} \log y_0 &= \rho\varepsilon + (1-\varepsilon)\log R^{CE} + \log Q \qquad \text{(A.7)} \\ \log R^{CE} &= g - \log Q - \frac{1}{2}\gamma\sigma^2 \\ \sigma &= \frac{1}{\gamma}\frac{g - \log Q - r^f}{\sigma} \\ r^f &= \max\left(r^{f*}, 0\right) \text{ where } r^{f*} \text{ ensures } y_0 = z_0. \end{aligned}$$

To characterize the solution further, consider the case in which the equilibrium is supply determined, $y_0 = z_0 = 1$. Substituting this into the first two equations, we solve for the first-best price level of the market portfolio as,

$$\log Q^* = -\rho + \frac{(\varepsilon-1)}{\varepsilon}\left(g - \frac{1}{2}\gamma\sigma^2\right). \qquad \text{(A.8)}$$

Substituting this into the last equation, we further obtain an expression for "rstar",

$$\begin{aligned} r^{f*} &= \rho + g - \gamma\sigma^2 - \frac{(\varepsilon-1)}{\varepsilon}\left(g - \frac{1}{2}\gamma\sigma^2\right) \qquad \text{(A.9)} \\ &= \rho + \frac{g}{\varepsilon} - \frac{1}{2}\gamma\left(1 + \frac{1}{\varepsilon}\right)\sigma^2. \end{aligned}$$

Note that setting $\varepsilon = 1$ recovers Eq. (8) in the main text. The main difference is that "rstar" is now also influenced by the attractiveness of investment opportunities, captured by the term $g - \frac{1}{2}\gamma\sigma^2$ (that shifts $\log R^{CE}$). When $\varepsilon > 1$, reducing the attractiveness of investment opportunities induces the representative household to consume more and save less due to a substitution effect. This requires an increase in the risk-free rate to equilibrate the goods market. In this case, a risk premium shock that increases γ or σ reduces aggregate wealth, which tends to reduce the interest rate as before, but it also reduces the attractiveness of investment opportunities, which tends to raise the interest rate. When $\varepsilon < 1$, the two channels work in the same direction. The second line of Eq. $(A.9)$ collects similar terms together and shows that the risk shocks lower "rstar" as in the baseline setting regardless of the level of ε. When $\varepsilon > 1$, the effect is quantitatively weaker due to the substitution channel but it is qualitatively the same.

Now consider the case in which the interest rate is at its lower bound, $r^f = 0$. Substituting this into the equation system $(A.7)$, we obtain,

$$\begin{aligned} \log Q &= g - \gamma\sigma^2 \\ \text{and } \log y_0 &= \varepsilon\left(\rho + \log Q - \frac{(\varepsilon-1)}{\varepsilon}\left(g - \frac{1}{2}\gamma\sigma^2\right)\right) \\ &= \varepsilon\left(\rho + \frac{g}{\varepsilon} - \frac{1}{2}\gamma\left(1+\frac{1}{\varepsilon}\right)\sigma^2\right). \end{aligned} \tag{A.10}$$

In this case, the additional effect of the changes in the attractiveness of investment opportunities is absorbed by output, because the interest rate does not respond. An increase in $\gamma\sigma^2$ tends to reduce the output by reducing the aggregate wealth, as in the baseline setting, but it also affects output through substitution or income effects. The last line in $(A.10)$ illustrates that the wealth effect dominates regardless of the level of ε. When $\varepsilon > 1$, the substitution effect mitigates the quantitative impact of the wealth effect relative to the baseline setting but it does not overturn it. When $\varepsilon < 1$, the income effect amplifies the quantitative impact of the wealth effect.

A.3. Belief disagreements and speculation

We denote optimists and pessimists respectively with superscript $i \in \{o, p\}$. With a slight abuse of notation, we also let $\alpha^o \equiv \alpha$ and $\alpha^p \equiv 1 - \alpha$ denote respectively optimists' and pessimists' wealth shares. Recall that investors are identical except possibly their beliefs about aggregate growth. Then, type i investors solve the following problem,

$$\begin{aligned} \max_{c_0, a_0, \omega^m, \{c_1(z_1)\}} \quad & \log c_0 + e^{-\rho}\log U_1 \\ \text{where} \quad & U_1 = \left(E^i\left[c_1(z_1)^{1-\gamma}\right]\right)^{1/(1-\gamma)} \\ \text{s.t.} \quad & c_0 + a_0 = \alpha^i(y_0 + Q) \\ \text{and} \quad & c_1(z_1) = a_0\left(\omega^m \exp\left(r^m(z_1)\right) + (1-\omega^m)\exp\left(r^f\right)\right). \end{aligned} \tag{A.11}$$

Note that we set the EIS equal to one as in the baseline setting. Note also that the asset market clearing condition requires,

$$\omega^{m,o} a_0^o + \omega^{m,p} a_0^p = Q, \tag{A.12}$$

that is, the total amount of wealth invested in the market portfolio equals the value of the market portfolio. The rest of the model is the same as in the baseline setting.

In this case, the investor's portfolio problem $(A.2)$ remains unchanged. Applying the log-normal approximation that we described previously, we obtain Eq. (6) as in the main text, that is,

$$\omega^{m,i}\sigma \simeq \frac{1}{\gamma}\frac{E^i\left[r^m(z_1)\right] + \frac{\sigma^2}{2} - r^f}{\sigma}.$$

Substituting $E^i[r^m(z_1)] = g^i - \log Q - \frac{\sigma^2}{2}$ [cf. Eq. $(A.1)$] into this expression, we further obtain,

$$\omega^{m,i}\sigma \simeq \frac{1}{\gamma}\frac{g^i - \log Q - r^f}{\sigma}. \tag{A.13}$$

As before, investors choose their share of the market portfolio so that their optimal portfolio risk is proportional to the Sharpe ratio. The difference is that the Sharpe ratio is calculated according to investors' own beliefs (and it is greater for optimists since $g^o > g^p$).

The intertemporal problem $(A.3)$ also remains unchanged. Taking the first order condition, we obtain,

$$c_0^i = \frac{1}{1+e^{-\rho}}\alpha^i(y_0 + Q) \tag{A.14}$$

Aggregating this equation across investors, and using the aggregate resource constraint (3), shows that the output-asset price relation (5) continues to apply in this setting. Belief heterogeneity does not affect this equation since investors share the same discount rate, ρ.

Next note that combining $(A.12), (A.14)$ and (5), the asset market clearing condition can be rewritten as,

$$\alpha\omega^{m,o} + (1-\alpha)\omega^{m,p} = 1. \tag{A.15}$$

Investors' wealth-weighted average portfolio weight on the market portfolio is equal to one. Combining this with Eq. $(A.13)$, we obtain the following analogue of Eq. (7),

$$\sigma \simeq \frac{1}{\gamma}\frac{\alpha^o g^o + \alpha^p g^p - \log Q - r^f}{\sigma}. \tag{A.16}$$

Hence, the risk balance condition continues to apply with the difference that the expected growth rate is determined according to a weighted average belief. Another difference is that the condition is typically not exact because investors' shares of the market portfolio typically deviate from one (and thus, their return is typically not log-normal). Specifically, the equilibrium portfolio shares satisfy, $\omega^o > 1 > \omega^p$: optimists' make a leveraged investment in the market portfolio by issuing some risk-free debt, whereas pessimists invest only a fraction of their wealth in the market portfolio (and invest the rest of their wealth in the risk-free asset issued by optimists).

Next consider the supply-determined equilibrium in which output is equal to its potential, $y_0 = z_0 = 1$. By Eq. (5), this requires the asset price to be at a particular level, $Q^* = e^{-\rho}$. Combining this with Eq. $(A.16)$ we obtain Eq. (12) in the main text that characterizes "rstar." The level of "rstar" is increasing in optimists' wealth share, α. This is because increasing optimists' wealth share tends to increase asset prices, aggregate demand, and output. In a supply-determined equilibrium, the monetary policy increases the interest rate to neutralize the impact of optimists on aggregate demand and output.

Finally, consider the case in which the interest rate is at its lower bound, $r^f = 0$. Substituting this into the risk balance condition $(A.16)$, and using the output-asset price relation (5), we obtain Eq. (9) in the main text that characterizes the equilibrium level of output in a demand recession. In this case, increasing optimists' wealth share translates into an actual increase in asset prices, aggregate demand, and output, because the monetary policy cannot neutralize these effects due to the constraint on the interest rate.

B. Appendix: Omitted Derivations for the Dynamic Model

This appendix presents the derivations and proofs omitted from the main text for the dynamic model that we present and analyze in Sections 3-5. The subsequent appendix C presents the details of the welfare analysis for the same model.

B.1. Omitted derivations in Section 3

B.1.1. Portfolio problem and its recursive formulation

The investor's portfolio problem (at some time t and state s) can be written as,

$$V_{t,s}^i\left(a_{t,s}^i\right) = \max_{\left[\tilde{c}_{\tilde{t},\tilde{s}},\tilde{\omega}_{\tilde{t},\tilde{s}}^m,\tilde{\omega}_{\tilde{t},\tilde{s}}^{\tilde{s}'}\right]_{\tilde{t}\geq t,\tilde{s}}} E_{t,s}^i\left[\int_t^\infty e^{-\rho\tilde{t}}\log\tilde{c}_{\tilde{t},\tilde{s}}^i d\tilde{t}\right]$$

$$\text{s.t.}\ \begin{cases} da_{t,s}^i = \left(a_{t,s}^i\left(r_{t,s}^f + \tilde{\omega}_{t,s}^m\left(r_{t,s}^m - r_{t,s}^f\right) - \tilde{\omega}_{t,\tilde{s}}^{\tilde{s}'}\right) - \tilde{c}_{t,s}\right)dt + \tilde{\omega}_{t,s}^m a_{t,s}^i\sigma_s dZ_t & \text{absent transition,} \\ a_{t,s'}^i = a_{t,s}^i\left(1 + \tilde{\omega}_{t,s}^m\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}} + \tilde{\omega}_{t,s}^{s'}\frac{1}{p_{t,s}^{s'}}\right) & \text{if there is a transition to state } s' \neq s. \end{cases} \tag{B.1}$$

Here, $E_{t,s}^i[\cdot]$ denotes the expectations operator that corresponds to the investor i's beliefs for state transition probabilities. The HJB equation corresponding to this problem is given by,

$$\begin{aligned} \rho V_{t,s}^i\left(a_{t,s}^i\right) = \max_{\tilde{\omega}^m,\tilde{\omega}^{s'},\tilde{c}} & \log\tilde{c} + \frac{\partial V_{t,s}^i}{\partial a}\left(a_{t,s}^i\left(r_{t,s}^f + \tilde{\omega}^m\left(r_{t,s}^m - r_{t,s}^f\right) - \tilde{\omega}^{s'}\right) - \tilde{c}\right) \\ & + \frac{1}{2}\frac{\partial^2 V_{t,s}^i}{\partial a^2}\left(\tilde{\omega}^m a_{t,s}^i\sigma_s\right)^2 + \frac{\partial V_{t,s}^i\left(a_{t,s}^i\right)}{\partial t} \\ & + \lambda_s^i\left(V_{t,s'}^i\left(a_{t,s}^i\left(1 + \tilde{\omega}^m\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}} + \frac{\tilde{\omega}^{s'}}{p_{t,s}^{s'}}\right)\right) - V_{t,s}^i\left(a_{t,s}^i\right)\right). \end{aligned} \tag{B.2}$$

In view of the log utility, the solution has the functional form in (44), which we reproduce here,

$$V_{t,s}^i\left(a_{t,s}^i\right) = \frac{\log\left(a_{t,s}^i/Q_{t,s}\right)}{\rho} + v_{t,s}^i.$$

The first term in the value function captures the effect of holding a greater capital stock (or greater wealth), which scales the investor's consumption proportionally at all times and states. The second term, $v_{t,s}^i$, is the normalized value function when the investor holds one unit of the capital stock (or wealth, $a_{t,s}^i = Q_{t,s}$). This functional form also implies,

$$\frac{\partial V_{t,s}^i}{\partial a} = \frac{1}{\rho a_{t,s}^i} \text{ and } \frac{\partial^2 V_{t,s}^i}{\partial a^2} = \frac{-1}{\rho\left(a_{t,s}^i\right)^2}.$$

The first order condition for $\tilde{c}$ then implies Eq. (22) in the main text. The first order condition for $\tilde{\omega}^m$ implies,

$$\frac{\partial V_{t,s}^i}{\partial a}a_{t,s}^i\left(r_{t,s}^m - r_{t,s}^f\right) + \lambda_s^i\frac{\partial V_{t,s'}^i\left(a_{t,s'}^i\right)}{\partial a}a_{t,s}^i\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}} = -\frac{\partial^2 V_{t,s}^i}{\partial a^2}\omega_{t,s}^m\left(a_{t,s}^i\sigma_s\right)^2.$$

After substituting for $\frac{\partial V_{t,s}^i}{\partial a}, \frac{\partial V_{t,s'}^i}{\partial a}, \frac{\partial^2 V_{t,s}^i}{\partial a^2}$ and rearranging terms, this also implies Eq. (23) in the main text.

Finally, the first order condition for $\tilde{\omega}^{s'}$ implies,

$$\frac{p_{t,s}^{s'}}{\lambda_s^i} = \frac{\frac{\partial V_{t,s'}^i\left(a_{t,s'}^i\right)}{\partial a}}{\frac{\partial V_{t,s}^i\left(a_{t,s}^i\right)}{\partial a}} = \frac{1/a_{t,s'}^i}{1/a_{t,s}^i},$$

which is Eq. (24) in the main text. This completes the characterization of the optimality conditions.

B.1.2. New Keynesian microfoundation for nominal rigidities

The supply side of our model features nominal rigidities similar to the standard New Keynesian setting. There is a continuum of measure one of monopolistically competitive production firms denoted by ν. These firms own the capital stock (in equal proportion) and produce differentiated goods, $y_{t,s}(\nu)$, subject to the technology,

$$y_{t,s}(\nu) = A\eta_{t,s}(\nu)\, k_{t,s}. \tag{B.3}$$

Here, $\eta_{t,s}(\nu) \in [0,1]$ denotes the firm's choice of capital utilization. We assume utilization is free up to $\eta_{t,s}(\nu) = 1$ and infinitely costly afterwards. The production firms sell their output to a competitive sector that produces the final output according to the CES technology,

$$y_{t,s} = \left(\int_0^1 y_{t,s}(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\varepsilon/(\varepsilon-1)}, \tag{B.4}$$

for some $\varepsilon > 1$. Thus, the demand for the firms' goods implies,

$$y_{t,s}(\nu) \leq p_{t,s}(\nu)^{-\varepsilon} y_{t,s}, \text{ where } p_{t,s}(\nu) = P_{t,s}(\nu)/P. \tag{B.5}$$

Here, $p_{t,s}(\nu)$ denotes the firm's relative price, which depends on its nominal price, $P_{t,s}(\nu)$, as well as the ideal nominal price index, $P_{t,s} = \left(\int P_{t,s}(\nu)^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)}$. We write the demand constraint as an inequality because an individual firm can in principle refuse to meet the demand for its goods.

Without price rigidities, the firm chooses $\eta_{t,s}(\nu) \in [0,1], y_{t,s}(\nu), p_{t,s}(\nu)$ to maximize its earnings, $p_{t,s}(\nu)\, y_{t,s}(\nu)$, subject to the supply constraint in $(B.3)$ and the demand constraint, $(B.5)$. In this case, the demand constraint holds as equality (because otherwise the firm can always raise its price to keep its production unchanged and raise its earnings). By combining the constraints, the firm's problem can be written as,

$$\max_{p_{t,s}(\nu),\eta_{t,s}(\nu)} p_{t,s}(\nu)^{1-\varepsilon} y_{t,s} \text{ s.t. } 0 \leq \eta_{t,s}(\nu) = \frac{p_{t,s}(\nu)^{-\varepsilon} y_{t,s}}{Ak_{t,s}} \leq 1.$$

Inspecting this problem reveals that the solution features full factor utilization, $\eta_{t,s}(\nu) = 1$. This is because, when $\eta_{t,s}(\nu) < 1$, the marginal cost of production is zero. Thus, the firm can always lower its price and increase its demand and production, which in turn increases its earnings. Hence, at the optimum, the firms set $\eta_{t,s}(\nu) = 1$ and $y_{t,s}(\nu) = Ak_{t,s}$. To produce at this level, they set the relative price level, $p_{t,s}(\nu) = \left(\frac{y_{t,s}}{Ak_{t,s}}\right)^{-1/\varepsilon}$. Since all firms are identical, we also have $p_{t,s}(\nu) = 1$ and $y_{t,s} = y_{t,s}(\nu) = Ak_{t,s}$. In particular, output is determined by aggregate supply at full factor utilization.

Now consider the alternative setting in which firms have a preset nominal price that is equal for all firms, $P_{t,s}(\nu) = P$. This also implies the relative price of a firm is fixed and equal to one, $p_{t,s}(\nu) = 1$. The firm chooses the remaining variables, $\eta_{t,s}(\nu) \in [0,1], y_{t,s}(\nu)$, to maximize its earnings, $y_{t,s}(\nu)$, subject to the supply constraint in $(B.3)$ and the demand constraint, $(B.5)$. Combining the constraints and using

$p_{t,s}(\nu) = 1$, the firm's problem can be written as,

$$\max_{\eta_{t,s}(\nu)} A\eta_{t,s}(\nu) k_{t,s} \text{ s.t. } 0 \le \eta_{t,s}(\nu) \le 1 \text{ and } A\eta_{t,s}(\nu) k_{t,s} \le y_{t,s}.$$

The solution is given by, $\eta_{t,s}(\nu) = \min\left(1, \frac{y_{t,s}}{Ak_{t,s}}\right)$. Intuitively, when $\eta_{t,s}(\nu) < 1$ and $A\eta_{t,s}(\nu) k_{t,s} < y_{t,s}$, the marginal cost of production is zero and there is some unmet demand for firms' goods. The firm optimally increases its production until the supply or the demand constraints bind. Combining this observation with the production technology for the final output, we also obtain, $y_{t,s} \le Ak_{t,s}$. This implies that the demand constraint holds as equality also in this case. In particular, we have $\eta_{t,s}(\nu) = \frac{y_{t,s}}{Ak_{t,s}} \le 1$.

In sum, when the firms' nominal prices are fixed, aggregate output is determined by aggregate demand subject to the capacity constraint, which verifies Eq. (19) in the dynamic model (and Eq. (3) in the two period model).

Note also that, in equilibrium, firms' equilibrium earnings are equal to aggregate output, $y_{t,s}$. Since firms own the capital (and there is no rental market for capital), the division of these earnings between return to capital and monopoly profits is indeterminate. This division does not play an important role in our baseline model but it matters when we introduce investment. In Appendix D with endogenous investment (that we present subsequently), we use slightly different microfoundations that ensure earnings accrue to firms in the form of return to capital, i.e., there are no monopoly profits, which helps to simplify the exposition.

B.2. Omitted derivations in Section 4

Proof of Proposition 1. Provided in the main text.

B.3. Omitted derivations in Section 5

We derive the equilibrium conditions that we state and use in Section 5. First note that, using Eq. (24), the optimality condition (23) can be written as,

$$\omega_{t,s}^{m,i}\sigma_s = \frac{1}{\sigma_s}\left(r_{t,s}^m - r_{t,s}^f + p_{t,s}^{s'}\frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s}}\right). \tag{B.6}$$

Note also that Eq. (25) implies,

$$\omega_{t,s}^{m,o} = \omega_{t,s}^{m,p} = 1. \tag{B.7}$$

Next note that by definition, we have

$$a_{t,s}^o = \alpha_{t,s} Q_{t,s} k_{t,s} \text{ and } a_{t,s}^p = (1 - \alpha_{t,s}) Q_{t,s} k_{t,s} \text{ for each } s \in \{1, 2\}.$$

After plugging these into Eq. (24), using $k_{t,s} = k_{t,s'}$ (since capital does not jump), and aggregating over optimists and pessimists, we obtain,

$$p_{t,s}^{s'} = \overline{\lambda}_{t,s}\frac{Q_{t,s}}{Q_{t,s'}}, \tag{B.8}$$

where $\overline{\lambda}_{t,s}$ denotes the wealth-weighted average belief defined in (37).

Next, we combine Eqs. $(B.6), (B.7)$, and $(B.8)$ to obtain

$$\sigma_s = \frac{1}{\sigma_s}\left(r_{t,s}^m - r_{t,s}^f + \overline{\lambda}_{t,s}\left(1 - \frac{Q_{t,s}}{Q_{t,s'}}\right)\right) \text{ for each } s \in \{1, 2\}. \tag{B.9}$$

Substituting for $r^m_{t,s}$ from Eq. (28), we obtain the risk balance condition (38) in the main text.

We next characterize investors' equilibrium positions. Combining Eq. $(B.1)$ with Eqs. $(B.7)$ and $(B.8)$, investors' wealth after transition satisfies,

$$\frac{a^i_{t,s'}}{a^i_{t,s}} = \frac{Q_{t,s'}}{Q_{t,s}}\left(1 + \frac{\omega^{s',i}_{t,s}}{\overline{\lambda}_{t,s}}\right). \tag{B.10}$$

From Eq. (24), we have $\frac{p^{s'}_{t,s}}{\lambda^i_s} = \frac{1/a^i_{t,s'}}{1/a^i_{t,s}}$. Substituting this into the previous expression and using Eq. $(B.8)$ once more, we obtain,

$$\omega^{s',i}_{t,s} = \lambda^i_s - \overline{\lambda}_{t,s} \text{ for each } i \in \{o,p\}. \tag{B.11}$$

Combining this with Eq. (37) implies Eq. (39) in the main text.

Finally, we characterize the dynamics of optimists' wealth share. Combining Eqs. $(B.10)$ and $(B.11)$ implies,

$$\frac{a^i_{t,s'}}{a^i_{t,s}} = \frac{\lambda^i_s}{\overline{\lambda}_{t,s}}\frac{Q_{t,s'}}{Q_{t,s}}. \tag{B.12}$$

Combining this with the definition of wealth shares as well as $k_{t,s} = k_{t,s'}$, we further obtain,

$$\frac{\alpha_{t,s'}}{\alpha_{t,s}} = \frac{\lambda^o_s}{\overline{\lambda}_{t,s}}. \tag{B.13}$$

Thus, it remains to characterize the dynamics of wealth conditional on no transition. To this end, we combine Eq. $(B.1)$ with Eqs. $(B.7)$, (28), (22) to obtain,

$$\frac{da^o_{t,s}}{a^o_{t,s}} = \left(g + \mu^Q_{t,s} - \omega^{s',i}_{t,s}\right)dt + \sigma_s dZ_t.$$

After substituting $a^o_{t,s} = \alpha_{t,s}Q_{t,s}k_{t,s}$, and using the observation that $\frac{dQ_{t,s}}{Q_{t,s}} = \mu^Q_{t,s}dt$ and $\frac{dk_{t,s}}{k_{t,s}} = gdt + \sigma_s dZ_t$, we further obtain,

$$\frac{d\alpha_{t,s}}{\alpha_{t,s}} = -\omega^{s',o}_{t,s}dt = -\left(\lambda^o_s - \overline{\lambda}_{t,s}\right)dt. \tag{B.14}$$

Combining Eqs. $(B.13)$ and $(B.14)$ implies Eq. (40) in the main text.

Proof of Proposition 2. First consider the high-risk-premium state, $s = 2$. Combining Eqs. (40) and (41), we obtain the differential equation system,

$$\begin{aligned} \dot{q}_{t,2} &= -\left(\rho + g + \overline{\lambda}_2(\alpha_{t,2})\left(1 - \frac{\exp(q_2)}{Q^*}\right) - \sigma_2^2\right), \\ \dot{\alpha}_{t,2} &= -\left(\lambda^o_2 - \lambda^p_2\right)\alpha_{t,2}\left(1 - \alpha_{t,2}\right). \end{aligned} \tag{B.15}$$

This system describes the joint dynamics of the price and optimists' wealth share, $(q_{t,2}, \alpha_{t,2})$, conditional on there not being a transition. We next analyze the solution to this system using the phase diagram over the range $\alpha \in [0,1]$ and $q_2 \in [q^p_2, q^o_2]$. Here, recall that q^i_2 corresponds to the equilibrium log price with common beliefs characterized in Section 4 corresponding to type i investors' belief.

First note that the system has two steady states given by, $(\alpha_{t,2} = 0, q_{t,2} = q^p_2)$, and $(\alpha_{t,2} = 1, q_{t,2} = q^o_2)$. Next note that the system satisfies the Lipschitz condition over the relevant range. Thus, the vector flows that describe the law of motion do not cross. Next consider the locus, $\dot{q}_2 = 0$. By comparing Eqs. (41)

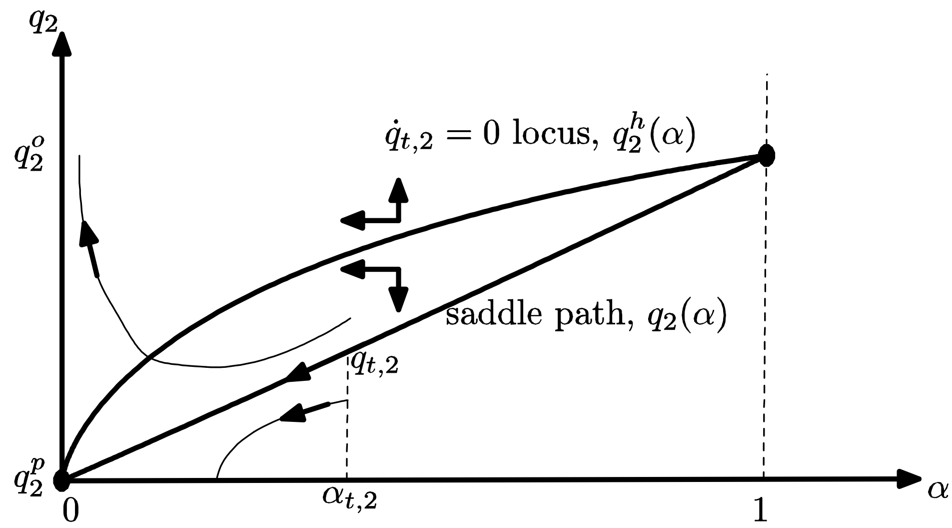

Figure 8: The phase diagram that describes the equilibrium with heterogeneous beliefs.

and (32), this locus is exactly the same as the price that would obtain if investors shared the same wealth-weighted average belief, denoted by $q_2 = q_2^h(\alpha)$. Using our analysis in Section 4, we also find that $q_2^h(\alpha)$ is strictly increasing in α. Moreover, $q_2 < q_2^h(\alpha)$ implies $\dot{q}_{t,2} < 0$ whereas $q_2 > q_2^h(\alpha)$ implies $\dot{q}_{t,2} > 0$. Finally, note that $\dot{\alpha}_{t,2} < 0$ for each $\alpha \in (0,1)$.

Combining these observations, the phase diagram has the shape in Figure 8. This in turn implies that the system is saddle path stable. Given any $\alpha_{t,2} \in [0,1)$, there exists a unique solution, $q_{t,2}$, which ensures that $\lim_{t\to\infty} q_{t,2} = q_2^p$. We define the price function (the saddle path) as $q_2(\alpha)$. Note that the price function satisfies $q_2(\alpha) < q_2^h(\alpha)$ for each $\alpha \in (0,1)$, since the saddle path cannot cross the locus, $\dot{q}_{t,2} = 0$. Note also that $q_2(1) = q_2^o$, since the saddle path crosses the other steady-state, $(\alpha_{t,2} = 1, q_{t,2} = q_2^o)$. Finally, recall that $q_2 < q_2^h(\alpha)$ implies $\dot{q}_{t,2} < 0$. Combining this with $\dot{\alpha}_{t,2} < 0$, we further obtain $\frac{dq_2(\alpha)}{d\alpha} > 0$ for each $\alpha \in (0,1)$.

Next note that, after substituting $\dot{q}_{t,2} = q_2'(\alpha)\dot{\alpha}_{t,2}$, Eq. $(B.15)$ implies the differential equation (42) in α-domain. Thus, the above analysis shows there exists a solution to the differential equation with $q_2(0) = q_2^p$ and $q_2(1) = q_2^o$. Moreover, the solution is strictly increasing in α, and it satisfies $q_2(\alpha) < q_2^h(\alpha)$ for each $\alpha \in (0,1)$. Note also that this solution is unique since the saddle path is unique. The last part of the proposition follows from Eqs. (26) and (27).

Next consider the low-risk-premium state, $s = 1$. In the conjectured equilibrium, we have $Q_{t,1} = Q^*$, which also implies $\mu_{t,1}^Q = 0$. Substituting these expressions into Eq. (38), we obtain the risk balance condition in this state,

$$\sigma_1 = \frac{1}{\sigma_1}\left(g + \rho - r_{t,1}^f + \overline{\lambda}_{t,1}\left(1 - \frac{Q^*}{Q_{t,2}}\right)\right).$$

Writing the equilibrium variables as a function of optimists' wealth share, we obtain $r_{t,1}^f = r_1^f(\alpha)$ and $\overline{\lambda}_{t,1} = \overline{\lambda}_1(\alpha)$ and $Q_{t,2} = \exp(q_2(\alpha'))$, where $\alpha' = \alpha\lambda_1^o/\overline{\lambda}_1(\alpha)$ denotes optimists' wealth share after a transition [cf. Eq. (40)]. Substituting these expressions into the risk balance condition and rearranging terms, we obtain Eq. (43) in the main text that, which we replicate here,

$$r_1^f(\alpha) = \rho + g - \overline{\lambda}_1(\alpha)\left(\frac{Q^*}{\exp(q_2(\alpha'))} - 1\right) - \sigma_1^2.$$

Note also that $\frac{dr_1^f(\alpha)}{d\alpha} > 0$ because $\overline{\lambda}_1(\alpha)$ is decreasing in α (in view of Assumption 4), and $q_2(\alpha')$ is strictly increasing in α. The latter observation follows since $\alpha' = \frac{\alpha\lambda_1^o}{\alpha\lambda_1^o+(1-\alpha)\lambda_1^p}$ is increasing in α (in view of Assumption 4) and $q_2(\cdot)$ is a strictly increasing function. Note also that $r_1^f(\alpha) > r_1^f(0) > 0$, where the latter inequality follows since Assumptions 1-3 holds for the pessimistic belief. Thus, the interest rate in state 1 is always positive, which verifies our conjecture and completes the proof. □

C. Appendix: Omitted Derivations for the Welfare Analysis

This appendix presents the omitted derivations and proofs for the welfare analysis of the dynamic model that we present in Section 6. Section C.1 establishes the properties of the equilibrium value functions that are used in the main text. Section C.2 describes the details of the equilibrium with macroprudential policy, presents the analyses omitted from the main text (e.g., macroprudential policy in the high-risk-premium state), and presents omitted proofs.

C.1. Value functions in equilibrium

We first derive the HJB equation that describes the normalized value function in equilibrium and derive Eqs. (45). We then derive the differential equations in α-domain that characterize the value function and its components, and derive Eq. (48). We then prove Lemmas 1 and 2 that are used in the analysis.

Characterizing the normalized value function in equilibrium. Consider the recursive version of the portfolio problem in $(B.2)$. Recall that the value function has the functional form in Eq. (44). Our goal is to characterize the value function per unit of capital, $v_{t,s}^i$ (corresponding to $a_{t,s}^i = Q_{t,s}$). To facilitate the analysis, we define,

$$\xi_{t,s}^i = v_{t,s}^i - \frac{\log Q_{t,s}}{\rho}. \tag{C.1}$$

Note that $\xi_{t,s}^i$ is the value function per unit wealth (corresponding to $a_{t,s}^i = 1$), and that the value function also satisfies $V_{t,s}^i\left(a_{t,s}^i\right) = \frac{\log\left(a_{t,s}^i\right)}{\rho} + \xi_{t,s}^i$. We first characterize $\xi_{t,s}^i$. We then combine this with Eq. $(C.1)$ to characterize our main object of interest, $v_{t,s}^i$.

Consider the HJB equation $(B.2)$. We substitute the optimal consumption rule from Eq, (22), the contingent allocation rule from Eq. (24), and $a_{t,s}^i = 1$ (to characterize the value per unit wealth) to obtain,

$$\begin{aligned}\rho\xi_{t,s}^i = \log\rho + \frac{1}{\rho}\left(r_{t,s}^f + \omega_{t,s}^{m,i}\left(r_{t,s}^m - r_{t,s}^f\right) - \frac{1}{2}\left(\omega_{t,s}^{m,i}\right)^2\sigma_s^2 - \rho - \omega_{t,s}^{s',i}\right) \\ + \frac{\partial\xi_{t,s}^i}{\partial t} + \lambda_s^i\left(\frac{1}{\rho}\log\left(\frac{\lambda_s^i}{p_{t,s}^{s'}}\right) + \xi_{t,s'}^i - \xi_{t,s}^i\right).\end{aligned} \tag{C.2}$$

As we describe in Section 5, the market clearing conditions imply the optimal investment in the market portfolio and contingent securities satisfies, $\omega^m = 1$ and $\tilde{\omega}_{t,s}^{s',i} = \lambda_s^i - \overline{\lambda}_{t,s}$, and the price of the contingent security is given by, $p_{t,s}^{s'} = \overline{\lambda}_{t,s}\frac{1/Q_{t,s'}}{1/Q_{t,s}}$. Here, $\overline{\lambda}_{t,s}$ denotes the weighted average belief defined in (37). Using these conditions, the HJB equation becomes,

$$\begin{aligned}\rho\xi_{t,s}^i = \log\rho + \frac{1}{\rho}\begin{pmatrix} r_{t,s}^m - \rho - \frac{1}{2}\sigma_s^2 \\ -\left(\lambda_s^i - \overline{\lambda}_{t,s}\right) + \lambda_s^i\log\left(\frac{\lambda_s^i}{\overline{\lambda}_{t,s}}\right)\end{pmatrix} \\ + \frac{\partial\xi_{t,s}^i}{\partial t} + \lambda_s^i\left(\frac{1}{\rho}\log\left(\frac{Q_{t,s'}}{Q_{t,s}}\right) + \xi_{t,s'}^i - \xi_{t,s}^i\right).\end{aligned} \tag{C.3}$$

After substituting the return to the market portfolio from (28), the HJB equation can be further simplified as,

$$\rho\xi^i_{t,s} = \begin{array}{c} \log\rho + \frac{1}{\rho}\left(\begin{array}{c} g+\mu^Q_{t,s}-\frac{1}{2}\sigma_s^2 \\ -\left(\lambda^i_s - \overline{\lambda}_{t,s}\right) + \lambda^i_s \log\left(\frac{\lambda^i_s}{\overline{\lambda}_{t,s}}\right)\end{array}\right) \\ +\frac{\partial \xi^i_{t,s}}{\partial t} + \lambda^i_s\left(\frac{1}{\rho}\log\left(\frac{Q_{t,s'}}{Q_{t,s}}\right) + \xi^i_{t,s'} - \xi^i_{t,s}\right)\end{array}.$$

Here, the term inside the summation on the second line, $-\left(\lambda^i_s - \overline{\lambda}_{t,s}\right) + \lambda^i_s \log\left(\frac{\lambda^i_s}{\overline{\lambda}_{t,s}}\right)$, is zero when there are no disagreements, and it is strictly positive when there are disagreements. This illustrates that speculation increases the expected value for optimists as well as pessimists.

We finally substitute $v^i_{t,s} = \xi^i_{t,s} + \frac{\log Q_{t,s}}{\rho}$ (cf. $(C.1)$) into the HJB equation to obtain the differential equation,

$$\rho v^i_{t,s} = \begin{array}{c} \log\rho + \log\left(Q_{t,s}\right) + \frac{1}{\rho}\left(\begin{array}{c} g-\frac{1}{2}\sigma_s^2 \\ -\left(\lambda^i_s - \overline{\lambda}_{t,s}\right) + \lambda^i_s \log\left(\frac{\lambda^i_s}{\overline{\lambda}_{t,s}}\right)\end{array}\right) \\ +\frac{\partial v^i_{t,s}}{\partial t} + \lambda^i_s\left(v_{t,s'} - v_{t,s}\right)\end{array}.$$

Here, we have canceled terms by using the observation that $\frac{\partial \xi^i_{t,s}}{\partial t} = \frac{\partial v^i_{t,s}}{\partial t} - \frac{1}{\rho}\frac{\partial \log Q_{t,s}}{\partial t} = \frac{\partial v^i_{t,s}}{\partial t} - \frac{1}{\rho}\mu^Q_{t,s}$. We have thus obtained Eq. (45) in the main text.

Differential equations for the value functions in α-domain. The value function and its components, $\left\{v^i_{t,s}, v^{i,*}_{t,s}, w_{t,s}\right\}_{s,i}$, can be written as functions of optimists' wealth share, $\left\{v^i_s(\alpha), v^{i,*}_s(\alpha), w_s(\alpha)\right\}_{s,i}$, that solve appropriate ordinary differential equations. We next represent the value functions as solutions to the differential equations in α-domain. Recall that the price level in each state can be written as a function of optimists' wealth shares, $q_{t,s} = q_s(\alpha)$ (where we also have, $q_1(\alpha) = q^*$). Plugging in these price functions, and using the dynamics of $\alpha_{t,s}$ from Eq. (40), the HJB equation (45) can be written as,

$$\rho v^i_s(\alpha) = \begin{array}{c} \log\rho + q_s(\alpha) + \frac{1}{\rho}\left(\begin{array}{c} g-\frac{1}{2}\sigma_s^2 \\ -\left(\lambda^i_s - \overline{\lambda}_s(\alpha)\right) + \lambda^i_s \log\left(\frac{\lambda^i_s}{\overline{\lambda}_s(\alpha)}\right)\end{array}\right) \\ -\frac{\partial v^i_s}{\partial\alpha}\left(\lambda^o_s - \lambda^p_s\right)\alpha(1-\alpha) + \lambda^i_s\left(v^i_{s'}\left(\alpha\frac{\lambda^o_s}{\overline{\lambda}_s(\alpha)}\right) - v^i_s(\alpha)\right)\end{array}.$$

For each $i \in \{o,p\}$, the value functions, $\left(v^i_s(\alpha)\right)_{s\in\{1,2\}}$, are found by solving this system of ODEs. For $i = o$, the boundary conditions are that the values, $\{v^o_s(1)\}_s$, are the same as the values in the common belief benchmark characterized in Section 4 when all investors have the optimistic beliefs. For $i = p$, the boundary conditions are that the values, $\{v^p_s(0)\}_s$, are the same as the values in the common belief benchmark when all investors have the pessimistic beliefs.

Likewise, the first-best value functions, $\left(v^{i,*}_s(\alpha)\right)_{s\in\{1,2\}}$, are found by solving the analogous system after replacing $q_s(\alpha)$ with q^* (and changing the boundary conditions appropriately). Finally, substituting the price functions into Eq. (47), the gap-value functions, $\left(w^i_s(\alpha)\right)_{s,i}$, are found by solving the system in (48).

For the proofs in this section (as well as in some subsequent sections), we find it useful to work with the transformed state variable,

$$b_{t,s} \equiv \log\left(\frac{\alpha_{t,s}}{1-\alpha_{t,s}}\right), \text{ which implies } \alpha_{t,s} = \frac{1}{1+\exp\left(-b_{t,s}\right)}. \tag{C.4}$$

The variable, $b_{t,s}$, varies between $(-\infty,\infty)$ and provides a different measure of optimism, which we refer to as "bullishness." Note that there is a one-to-one relation between optimists' wealth share, $\alpha_{t,s} \in (0,1)$, and the bullishness, $b_{t,s} \in \mathbb{R} = (-\infty, +\infty)$. Optimists' wealth dynamics in (40) become particularly simple when expressed in terms of bullishness,

$$\begin{cases} \dot{b}_{t,s} = -\left(\lambda_s^o - \lambda_s^p\right), & \text{if there is no state change,} \\ b_{t,s'} = b_{t,s} + \log \lambda_s^o - \log \lambda_s^p, & \text{if there is a state change.} \end{cases} \tag{C.5}$$

With a slight abuse of notation, we also let $q_2(b), w_s^i(b)$, and so on, denote the equilibrium functions in terms of bullishness. Note also that, since $\frac{db}{d\alpha} = \frac{1}{\alpha(1-\alpha)}$, we have the identities,

$$\frac{\partial q_2(b)}{\partial b} = \alpha(1-\alpha)\frac{\partial q_2(\alpha)}{\partial b} \text{ and } \frac{\partial w_s^i(b)}{\partial b} = \alpha(1-\alpha)\frac{\partial w_s^i(\alpha)}{\partial \alpha}. \tag{C.6}$$

Using this observation, the differential equation for the price function, Eq. (42), can be written in terms of bullishness as,

$$\frac{\partial q_2(b)}{\partial b}\left(\lambda_2^o - \lambda_2^p\right) = \rho + g + \overline{\lambda}_2(\alpha)\left(1 - \frac{Q_2}{Q^*}\right) - \sigma_2^2. \tag{C.7}$$

Likewise, the differential equation for the gap value function, Eq. (48), can be written in terms of bullishness as,

$$\rho w_s^i(b) = q_s(b) - q^* - \left(\lambda_s^o - \lambda_s^p\right)\frac{\partial w_s^i(b)}{\partial b} + \lambda_s^i\left(w_{s'}^i(b') - w_s^i(b)\right). \tag{C.8}$$

Proof of Lemma 1. To show that the gap value function is increasing, consider its representation in terms of bullishness, $w_s^i(b)$ [cf. $(C.4)$], which solves the system in $(C.8)$. We will first describe this function as a fixed point of a contraction mapping. We will then use this contraction mapping to establish the properties of the function.

Recall that, in the time domain, the gap value function solves the HJB equation (47). Integrating this equation forward, we obtain,

$$w_s^i(b_{0,s}) = \int_0^\infty e^{-\left(\rho+\lambda_s^i\right)t}\left(q_s(b_{t,s}) - q^* + \lambda_s^i w_{s'}^i(b_{t,s'})\right)dt, \tag{C.9}$$

for each $s \in \{1,2\}$ and $b_{0,s} \in \mathbb{R}$. Here, $b_{t,s}$ denotes bullishness conditional on there not being a transition before time t, whereas $b_{t,s'}$ denotes the bullishness if there is a transition at time t. Solving Eq. $(C.5)$ (given beliefs, λ^i) we further obtain,

$$\begin{aligned} b_{t,s} &= b_{0,s} - t\left(\lambda_s^o - \lambda_s^p\right), \\ b_{t,s'} &= b_{0,s} - t\left(\lambda_s^o - \lambda_s^p\right) + \log \lambda_s^o - \log \lambda_s^p. \end{aligned} \tag{C.10}$$

Hence, Eq. $(C.9)$ describes the value function as a solution to an integral equation given the closed form solution for bullishness in $(C.10)$.

Implicitly differentiating the integral equation $(C.9)$ with respect to $b_{0,s}$, and using Eq. $(C.10)$, we also obtain,

$$\frac{\partial w_s^i(b_{0,s})}{\partial b} = \int_0^\infty e^{-\left(\rho+\lambda_s^i\right)t}\left(\frac{\partial q_s(b_{t,s})}{\partial b} + \lambda_s^i\frac{\partial w_{s'}^i(b_{t,s'})}{\partial b}\right)dt. \tag{C.11}$$

We next let $B\left(\mathbb{R}^2\right)$ denote the set of bounded value functions over $\mathbb{R}^2$. Given some continuation value function, $\left(\frac{\partial \tilde{w}_s^i(b)}{\partial b}\right)_s \in B\left(\mathbb{R}^2\right)$, we define the function, $\left(T\frac{\partial \tilde{w}_s^i(b)}{\partial b}\right)_s \in B\left(\mathbb{R}^2\right)$, so that

$$T\frac{\partial \tilde{w}_s^i\left(b_{0,s}\right)}{\partial b} = \int_0^\infty e^{-\left(\rho+\lambda_s^i\right)t}\left(\frac{\partial q_s\left(b_{t,s}\right)}{\partial b} + \lambda_s^i \frac{\partial \tilde{w}_{s'}^i\left(b_{t,s'}\right)}{\partial b}\right)dt,$$

for each s and $b_{0,s} \in \mathbb{R}$. Note also that the resulting value functions are bounded since the derivative of the price functions, $\left(\frac{\partial q_s(b_{t,s})}{\partial b}\right)_s$, are bounded (see Eq. $(C.7)$). Thus, Eq. $(C.11)$ describes the derivative functions, $\left(\frac{\partial w_s^i(b_{0,s})}{\partial b}\right)_s$, as a fixed point of a corresponding operator T over bounded functions. It can be checked that this operator is a contraction mapping with respect to the sup norm. Thus, it has a unique fixed point that corresponds to the derivative functions. Moreover, since $\frac{\partial q_s(b_{t,s})}{\partial b} > 0$ for each b, and $\lambda_s^i > 0$ for each s, it can further be seen that the fixed point satisfies, $\frac{\partial w_s^i(b_{0,s})}{\partial b} > 0$ for each b and $s \in \{1, 2\}$. Using Eq. $(C.6)$, we also obtain $\frac{\partial w_s^i(\alpha)}{\partial \alpha} > 0$ for each $\alpha \in (0, 1)$ and $s \in \{1, 2\}$, completing the proof. □

Proof of Lemma 2. Consider the analysis in Lemma 1 for the special case, $\lambda_1^o = \lambda_1^p$. Applying Eq. $(C.11)$ for $s = 1$, we obtain [since $q_1\left(b_{t,s}\right) = q^*$ is constant],

$$\frac{\partial w_1^i\left(b_{0,1}\right)}{\partial b} = \int_0^\infty e^{-\left(\rho+\lambda_1^i\right)t}\lambda_1^i \frac{\partial w_2^i\left(b_{t,2}\right)}{\partial b}dt.$$

Note also that $\lambda_1^o = \lambda_1^p$ and Eq. $(C.10)$ imply $b_{t,2} = b_{0,1}$ (since there is no speculation). Substituting this into the displayed equation, we obtain $\frac{\partial w_1^i(b_{0,1})}{\partial b} = \frac{\lambda_1^i}{\rho+\lambda_1^i}\frac{\partial w_2^i(b_{0,1})}{\partial b} < \frac{\partial w_2^i(b_{0,1})}{\partial b}$. Combining this with Eq. $(C.6)$ completes the proof. □

C.2. Equilibrium with macroprudential policy

Recall that macroprudential policy induces optimists to choose allocations as if they have more pessimistic beliefs, $\lambda^{o,pl} \equiv \left(\lambda_1^{o,pl}, \lambda_2^{o,pl}\right)$, that satisfy, $\lambda_1^{o,pl} \geq \lambda_1^o$ and $\lambda_2^{o,pl} \leq \lambda_2^o$. We next show that this allocation can be implemented with portfolio restrictions on optimists. We then show that the planner's Pareto problem reduces to solving problem (50) in the main text. We also derive the equilibrium value functions that result from macroprudential policy. We then analyze macroprudential policy in the recession state, which complements the analysis in the main text (that focuses on the boom state), and present Proposition 4. Finally, we present the proofs of Propositions 3 and 4.

Implementing the policy with risk limits. Consider the equilibrium that would obtain if optimists had the planner-induced beliefs, $\lambda_s^{o,pl}$. Using our analysis in Section 5, optimists' equilibrium portfolios are given by,

$$\omega_{t,s}^{m,o,pl} = 1 \text{ and } \omega_{t,s}^{s',o,pl} = \lambda_s^{o,pl} - \overline{\lambda}_{t,s}^{pl} \text{ for each } t, s. \tag{C.12}$$

We first show that the planner can implement the policy by requiring optimists to hold exactly these portfolio weights. We will then relax these portfolio constraints into inequality restrictions (see Eq. $(C.14)$).

Formally, an optimist solves the HJB problem $(B.2)$ with the additional constraint $(C.12)$. In view of log utility, we conjecture that the value function has the same functional form (44) with potentially different normalized values, $\xi_{t,s}^o, v_{t,s}^o$, that reflect the constraints. Using this functional form, the optimality condition for consumption remains unchanged, $c_{t,s} = \rho a_{t,s}^o$ [cf. Eq. (22)]. Plugging this equation and the portfolio holdings in $(C.12)$ into the objective function in $(B.2)$ verifies that the value function has the conjectured functional form. For later reference, we also obtain that the optimists' unit-wealth value function satisfies [cf. Eq. $(C.1)$],

$$\xi^o_{t,s} = \log\rho + \frac{1}{\rho}\left(r^f_{t,s} + \omega^{m,o,pl}_{t,s}\left(r^m_{t,s} - r^f_{t,s}\right) - \rho - \omega^{s',o,pl}_{t,s}\right) \tag{C.13}$$
$$-\frac{1}{2\rho}\left(\omega^{m,o,pl}_{t,s}\sigma_s\right)^2 + \frac{\partial\xi^o_{t,s}}{\partial t} + \lambda^o_s\left(\frac{1}{\rho}\log\left(\frac{a^o_{t,s'}}{a^o_{t,s}}\right) + \xi^o_{t,s'} - \xi^o_{t,s}\right).$$

Here, $\frac{a^o_{t,s'}}{a^o_{t,s}} = 1 + \omega^{m,o,pl}_{t,s}\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}} + \frac{\omega^{s',o,pl}_{t,s}}{p^{s'}_{t,s}}$ in view of the budget constraints $(B.1)$. Hence, the value function has a similar characterization as before [cf. Eq. $(C.2)$] with the difference that optimists' portfolio holdings reflect the portfolio constraints.

Since pessimists are unconstrained, their optimality conditions are unchanged. It follows that the equilibrium takes the form in Section 5 with the difference that investors' beliefs are replaced by their as-if beliefs, $\lambda^{i,pl}_s$. This verifies that the planner can implement the policy using the portfolio restrictions in $(C.12)$. We next show that these restrictions can be relaxed to the following inequality constraints,

$$\omega^{m,o,pl}_{t,s} \leq 1 \text{ for each } s, \tag{C.14}$$
$$\omega^{2,o,pl}_{t,1} \geq \underline{\omega}^{2,o}_{t,1} \equiv \lambda^{o,pl}_1 - \overline{\lambda}^{pl}_{t,1} \text{ and } \omega^{1,o,pl}_{t,2} \leq \overline{\omega}^{1,o}_{t,2} \equiv \lambda^{o,pl}_2 - \overline{\lambda}^{pl}_{t,2}.$$

In particular, we will establish that all inequality constraints bind, which implies that optimists optimally choose the portfolio weights in Eq. $(C.12)$. Thus, our earlier analysis continues to apply when optimists are subject to the more relaxed restrictions in $(C.14)$.

The result follows from the assumption that the planner-induced beliefs are more pessimistic than optimists' actual beliefs, $\lambda^{o,pl}_1 \geq \lambda^o_1$ and $\lambda^{o,pl}_2 \leq \lambda^o_2$. To see this formally, note that the optimality condition for the market portfolio is given by the following generalization of Eq. (23),

$$\omega^{m,o,pl}_{t,s}\sigma_s \leq \frac{1}{\sigma_s}\left(r^m_{t,s} - r^f_{t,s} + \lambda^o_s\frac{a^o_{t,s}}{a^o_{t,s'}}\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}}\right) \text{ and } \omega^{m,o,pl}_{t,s} \leq 1, \tag{C.15}$$

with complementary slackness. Note also that,

$$\lambda^o_s\frac{a^o_{t,s}}{a^o_{t,s'}}\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}} = \lambda^o_s\frac{\overline{\lambda}^{pl}_{t,s}}{\lambda^{o,pl}_s}\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s'}} \geq \overline{\lambda}^{pl}_{t,s}\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s'}} \text{ for each } s.$$

Here, the equality follows because Eq. $(B.12)$ in Appendix B.3 applies with as-if beliefs. The inequality follows by considering separately the two cases, $s \in \{1,2\}$. For $s = 2$, the inequality holds since $Q_{t,s'} - Q_{t,s} > 0$ and the beliefs satisfy, $\lambda^o_s \geq \lambda^{o,pl}_s$. For $s = 1$, the inequality holds since $Q_{t,s'} - Q_{t,s} < 0$ and the beliefs satisfy, $\lambda^{o,pl}_s \geq \lambda^o_s$. Note also that in equilibrium the return to the market portfolio satisfies Eq. $(B.9)$, which we replicate here,

$$\sigma_s = \frac{1}{\sigma_s}\left(r^m_{t,s} - r^f_{t,s} + \overline{\lambda}^{pl}_{t,s}\left(1 - \frac{Q_{t,s}}{Q_{t,s'}}\right)\right).$$

Combining these expressions implies, $\sigma_s \leq \frac{1}{\sigma_s}\left(r^m_{t,s} - r^f_{t,s} + \lambda^o_s\frac{a^o_{t,s}}{a^o_{t,s'}}\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}}\right)$, which in turn implies the optimality condition $(C.15)$ is satisfied with $\omega^{m,o,pl}_{t,s} = 1$. A similar analysis shows that optimists also choose the corner allocations in contingent securities, $\omega^{2,o,pl}_{t,1} = \underline{\omega}^{2,o}_{t,1}$ and $\omega^{1,o,pl}_{t,2} = \overline{\omega}^{1,o}_{t,2}$, verifying that the portfolio constraints $(C.12)$ can be relaxed to the inequality constraints in $(C.14)$.

Simplifying the planner's problem. Recall that, to trace the Pareto frontier, we allow the planner to do a one-time wealth transfer among the investors at time 0. Let $V^i_{t,s}\left(a^i_{t,s}|\left\{\lambda^{o,pl}_t\right\}\right)$ denote type i investors' expected value in equilibrium when she starts with wealth $a^i_{t,s}$ and the planner commits to implement the policy, $\left\{\lambda^{o,pl}_t\right\}$. Then, the planner's Pareto problem can be written as,

$$\max_{\tilde{\lambda}^{o,pl},\tilde{\alpha}_{0,s}} \gamma^o V^o_{0,s}\left(\tilde{\alpha}_{0,s}Q_{0,s}k_{0,s}|\tilde{\lambda}^{o,pl}\right) + \gamma^p V^p_{0,s}\left(\left(1-\tilde{\alpha}_{0,s}\right)Q_{0,s}k_{0,s}|\tilde{\lambda}^{o,pl}\right). \tag{C.16}$$

Here, $\gamma^o, \gamma^p \geq 0$ (with at least one strict inequality) denote the Pareto weights, and $Q_{0,s}$ denotes the endogenous equilibrium price that obtains under the planner's policy.

Next recall that the investors' value function with macroprudential policy has the same functional form in (44) (with potentially different $\xi^o_{t,s}, v^o_{t,s}$ for optimists that reflect the constraints). After substituting $a^i_{t,s} = \alpha^i_{t,s}k_{t,s}Q_{t,s}$, the functional form implies,

$$V^i_{t,s} = v^i_{t,s} + \frac{\log\left(\alpha^i_{t,s}\right) + \log\left(k_{t,s}\right)}{\rho}.$$

Using this expression, the planner's problem (C.16) can be rewritten as,

$$\max_{\tilde{\lambda}^{o,pl},\tilde{\alpha}_{0,s}} \left(\gamma^o v^o_{0,s} + \gamma^p v^p_{0,s}\right) + \frac{\gamma^o \log\left(\tilde{\alpha}^o_{0,s}\right) + \gamma^p \log\left(1-\tilde{\alpha}^o_{0,s}\right)}{\rho} + \frac{\left(\gamma^o+\gamma^p\right)\log\left(k_{0,s}\right)}{\rho}.$$

Here, the last term (that features capital) is a constant that doesn't affect optimization. The second term links the planner's choice of wealth redistribution, $\alpha^o_{0,s}, \alpha^p_{0,s}$, to her Pareto weights, γ^o, γ^p. Specifically, the first order condition with respect to optimists' wealth share implies $\frac{\gamma^o}{\gamma^p} = \frac{\alpha_{0,s}}{1-\alpha_{0,s}}$. Thus, the planner effectively maximizes the first term after substituting γ^o and γ^p respectively with the optimal choice of $\alpha_{0,s}$ and $1-\alpha_{0,s}$. This leads to the simplified problem (50) in the main text.

Characterizing the value functions with macroprudential policy. We first show that the normalized value functions, $v^i_{t,s}$, are characterized as the solution to the following differential equation system,

$$\rho v^i_{t,s} - \frac{\partial v^i_{t,s}}{\partial t} = \log\rho + q_{t,s} + \frac{1}{\rho}\left(\begin{array}{c} g - \frac{1}{2}\sigma^2_s \\ -\left(\lambda^{i,pl}_s - \overline{\lambda}^{pl}_{t,s}\right) + \lambda^i_s \log\left(\frac{\lambda^{i,pl}_s}{\overline{\lambda}^{pl}_{t,s}}\right) \end{array}\right) + \lambda^i_s\left(v^i_{t,s'} - v^i_{t,s}\right). \tag{C.17}$$

This is a generalization of Eq. (45) in which investors' positions are calculated according to their as-if beliefs, $\lambda^{i,pl}_s$, but the transition probabilities are calculated according to their actual beliefs, λ^i_s.

First consider the pessimists. Since they are unconstrained, their value function is characterized by solving the earlier equation system (C.13). In this case, equation (C.17) also holds since it is the same as the earlier equation.

Next consider the optimists. In this case, the analysis in Appendix B.3 applies with as-if beliefs. In particular, we have [cf. Eqs. (B.12) and (B.13)],

$$\frac{a^o_{t,s'}}{a^o_{t,s}} = \frac{\lambda^{o,pl}_s}{\overline{\lambda}^{pl}_{t,s}}\frac{Q_{t,s'}}{Q_{t,s}}.$$

Plugging this expression as well as Eq. $(C.12)$ into Eq. $(C.13)$, optimists' unit-wealth value function satisfies,

$$\begin{aligned} \xi^o_{t,s} &= \log\rho + \frac{1}{\rho}\left(\begin{array}{c} r^m_{t,s} - \rho - \frac{1}{2}\sigma_s^2 \\ -\left(\lambda_s^{o,pl} - \overline{\lambda}^{pl}_{t,s}\right) + \lambda^o_s \log\left(\frac{\lambda^{o,pl}_{t,s}}{\overline{\lambda}^{pl}_{t,s}}\right) \end{array} \right) \\ &+ \frac{\partial \xi^o_{t,s}}{\partial t} + \lambda^o_s \left(\frac{1}{\rho}\log\left(\frac{Q_{t,s'}}{Q_{t,s}}\right) + \xi^o_{t,s'} - \xi^o_{t,s} \right), \end{aligned}$$

This is the same as Eq. $(C.13)$ with the difference that the as-if beliefs, $\lambda^{o,pl}_s$, are used to calculate their positions on (and the payoffs from) the contingent securities, whereas the actual beliefs, λ^o_s, are used to calculate the transition probabilities. Using the same steps after Eq. $(C.13)$, we also obtain $(C.17)$ with $i = o$.

We next characterize the first-best and the gap value functions, $v^{i,*}_{t,s}$ and $w^i_{t,s}$, that we use in the main text. By definition, the first-best value function solves the same differential equation $(C.17)$ after substituting $q_{t,s} = q^*$. It follows that the gap value function $w^i_{t,s} = v^i_{t,s} - v^{i,*}_{t,s}$, solves,

$$\rho w^i_{t,s} - \frac{\partial w^i_{t,s}}{\partial t} = q_{t,s} - q^* + \lambda^i_s \left(w^i_{t,s'} - w^i_{t,s}\right),$$

which is the same as the differential equation (47) without macroprudential policy. The latter affects the path of prices, $q_{t,s}$, but it does not affect how these prices translate into gap values.

Note also that, as before, the value functions can be written as functions of optimists' wealth share, $\left\{v^i_s(\alpha), v^{i,*}_s(\alpha), w_s(\alpha)\right\}_{s,i}$. For completeness, we also characterize the differential equations that these functions satisfy in equilibrium with macroprudential policy. Combining Eq. $(C.17)$ with the dynamics of optimists' wealth share conditional on no transition, $\dot{\alpha}_{t,s} = -\left(\lambda^{o,pl}_s - \lambda^p_s\right)\alpha_{t,s}(1-\alpha_{t,s})$, the value functions, $\left(v^i_s(\alpha)\right)_{s,i}$, are found by solving,

$$\rho v^i_s(\alpha) = \left[\begin{array}{c} \log\rho + q^{pl}_s(\alpha) + \frac{1}{\rho}\left(\begin{array}{c} g - \frac{1}{2}\sigma_s^2 \\ -\left(\lambda^{i,pl}_s - \overline{\lambda}^{pl}_{t,s}\right) + \lambda^i_s \log\left(\frac{\lambda^{i,pl}_s}{\overline{\lambda}^{pl}_{t,s}}\right) \end{array}\right) \\ -\frac{\partial v^i_s}{\partial \alpha}\left(\lambda^{o,pl}_s - \lambda^p_s\right)\alpha(1-\alpha) + \lambda^i_s\left(v^i_{s'}\left(\alpha\frac{\lambda^{o,pl}_s}{\overline{\lambda}^{pl}_{t,s}}\right) - v^i_s(\alpha)\right) \end{array} \right],$$

with appropriate boundary conditions. As in the main text, we denote the price functions with $q^{pl}_s(\alpha)$ to emphasize that they are determined by as-if beliefs. Likewise, the first-best value functions, $\left(v^{i,*}_s(\alpha)\right)_{s\in\{1,2\}}$, are found by solving the analogous system after replacing $q_s(\alpha)$ with q^*. Finally, combining Eq. (47) with the dynamics of optimists' wealth share, the gap-value functions, $\left(w^i_s(\alpha)\right)_{s,i}$, are found by solving Eq. (49) in the main text

Macroprudential policy in the recession state. The analysis in the main text concerns macroprudential policy in the boom state and maintains the assumption that $\lambda^{o,pl}_2 = \lambda^o_2$. We next consider the polar opposite case in which the economy is currently in the recession state $s = 2$, and the planner can apply macroprudential policy in this state, $\lambda^{o,pl}_2 \le \lambda^o_2$ (she can induce optimists to act as if the recovery is less likely), but not in the other state, $\lambda^{o,pl}_1 = \lambda^o_1$. We obtain a sharp result for the special case in which optimists' wealth share is sufficiently large.

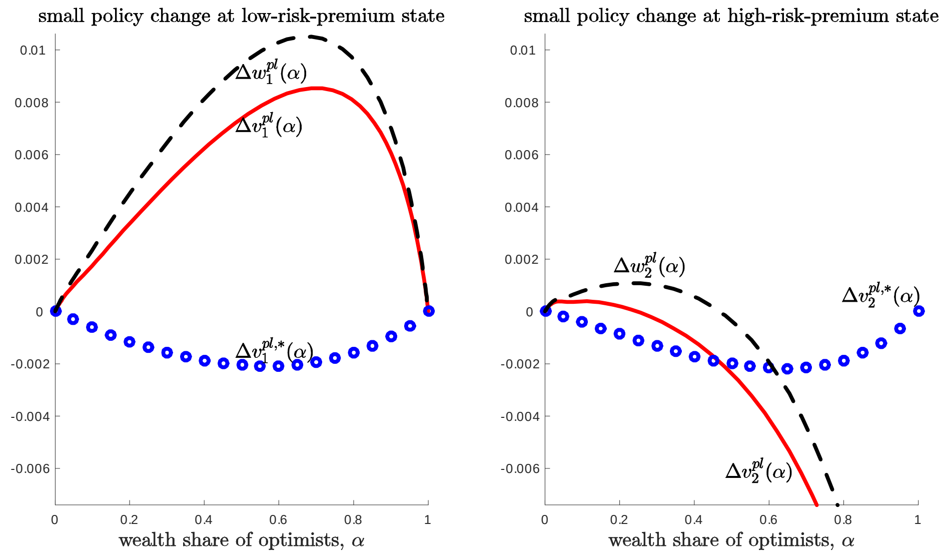

Figure 9: The left (resp. the right) panel illustrates the effect of a small change in macroprudential policy in the boom (resp. the recession) state.

Proposition 4. *Consider the model with two belief types. Consider the macroprudential policy in the recession state, $\lambda_2^{o,pl} \leq \lambda_2^o$ (and suppose $\lambda_1^{o,pl} = \lambda_1^o$). There exists a threshold, $\overline{\alpha} < 1$, such that if $\alpha \in (\overline{\alpha}, 1]$, then the policy reduces the gap value according to each belief, that is,*

$$\left.\frac{\partial w_2^i(\alpha)}{\partial\left(-\lambda_2^{o,pl}\right)}\right|_{\lambda_2^{o,pl}=\lambda_2^o} < 0 \text{ for each } i \in \{o,p\}.$$

Thus, for $\alpha \in (\overline{\alpha}, 1]$, the policy also reduces the planner's value, $\left.\frac{\partial v_s^{pl}(\alpha)}{\partial\left(-\lambda_2^{o,pl}\right)}\right|_{\lambda_2^o} = \left.\frac{\partial w_s^{pl}(\alpha)}{\partial\left(-\lambda_2^{o,pl}\right)}\right|_{\lambda_2^o} < 0.$

Thus, in contrast to Proposition 3, macroprudential policy in the recession state can actually reduce the social welfare. The intuition can be understood by considering two counteracting forces. First, as before, macroprudential policy in the recession state is potentially valuable by reallocating optimists' wealth from the boom state $s = 1$ to the recession state $s = 2$. Intuitively, optimists purchase too many call options that pay if there is a transition to the boom state but that impoverish them in case the recession persists. They do not internalize that, if they keep their wealth, they will improve asset prices if the recession lasts longer.

However, there is a second force that does not have a counterpart in the boom state: Macroprudential policy in the recession state also affects the current asset price level, with potential implications for social welfare. It can be seen that making optimists less optimistic in the recession state shifts the price function downward, $\frac{\partial q_2^{pl}(\alpha)}{\partial\left(-\lambda_2^{o,pl}\right)} < 0$ (as in the common-belief benchmark we analyzed in Section 4). Hence, the price impact of macroprudential policy is welfare reducing. Moreover, as optimists dominate the economy, $\alpha \to 1$, the price impact of the policy is still first order, whereas the beneficial effect from reshuffling optimists' wealth is second order. Thus, when optimists' wealth share is sufficiently large, the net effect of macroprudential policy is negative.

This analysis also suggests that, even when the policy in the recession state exerts a net positive effect, it would typically increase the welfare by a smaller amount than a comparable policy in the boom state. Figure

9 confirms this intuition. The left panel plots the change in the planner's value function in the boom state resulting from a small macroprudential policy change. Note that the policy slightly reduces the planner's first-best value function but increases the gap value function as well as the actual value function, illustrating Proposition 3 (see also Figure 5). The right panel illustrates the effect of the macroprudential policy in the recession state that would generate a similar distortion in the first-best equilibrium as the policy in the boom state.[26] Note that a small macroprudential policy in the recession state has a smaller positive impact when optimists' wealth share is small, and it has a negative impact when optimists' wealth share is sufficiently large, illustrating Proposition 4.

Proof of Proposition 3. We will prove the stronger result that

$$\left.\frac{\partial w_s^i(\alpha)}{\partial \lambda_1^{o,pl}}\right|_{\lambda^{o,pl}=\lambda^o} > 0 \text{ for each } i, s \text{ and } \alpha \in (0,1)\,. \tag{C.18}$$

That is, a marginal amount of macroprudential policy in the low-risk-premium state increases the gap value according to each investor (and in either state). Combining this with the definition of the planner's gap value function in (51) implies $\left.\frac{\partial w_s^{pl}(\alpha)}{\partial \lambda_1^{o,pl}}\right|_{\lambda^{o,pl}=\lambda^o} > 0$. Combining this with $\left.\frac{\partial v_{0,s}^{pl,*}}{\partial \lambda^{o,pl}}\right|_{\lambda^{o,pl}=\lambda^o} = 0$ (which follows from the First Welfare Theorem) and $v_{0,s}^{pl} = v_{0,s}^{pl,*} + w_{0,s}^{pl}$ implies $\left.\frac{\partial w_s^{pl}(\alpha)}{\partial \lambda_1^{o,pl}}\right|_{\lambda^{o,pl}=\lambda^o} = \left.\frac{\partial v_s^{pl}(\alpha)}{\partial \lambda_1^{o,pl}}\right|_{\lambda^{o,pl}=\lambda^o}$ for each s and $\alpha \in (0,1)$. Applying this result for state $s=1$ proves the proposition.

It remains to prove the claim in (C.18). To this end, fix a belief type i and consider the representation of the gap value function in terms of bullishness, $w_s^i(b)$ [cf. (C.4)]. Following similar steps as in Lemma 1, we describe this as solution to the integral function,

$$w_s^i(b_{0,s}) = \int_0^\infty e^{-\left(\rho+\lambda_s^i\right)t}\left(q_s^{pl}(b_{t,s}) - q^* + \lambda_s^i w_{s'}^i(b_{t,s'})\right)dt, \tag{C.19}$$

for each $s \in \{1,2\}$ and $b_{0,s} \in \mathbb{R}$, where the bullishness has the closed form solution,

$$\begin{aligned} b_{t,s} &= b_{0,s} - t\left(\lambda_s^{o,pl} - \lambda_s^p\right), \\ b_{t,s'} &= b_{0,s} - t\left(\lambda_s^{o,pl} - \lambda_s^p\right) + \log\lambda_s^{o,pl} - \log\lambda_s^p. \end{aligned} \tag{C.20}$$

The main difference from the analysis in Lemma 1 is that the dynamics of bullishness is influenced by policy, as illustrated by the as-if beliefs in (C.10). In addition, we denote the price functions with $q_s^{pl}(b)$ to emphasize they are in principle determined by as-if beliefs.

Next note that in this case the price functions $q_s^{pl}(b)$ are actually not affected by the as-if belief, $\lambda_1^{o,pl}$. The price function in the low-risk-premium state is not affected because $q_1^{pl}(b) = q^*$ (because the beliefs continue to satisfy Assumption 3 for small changes). The price function in the high-risk-premium state is also not affected because $\lambda_1^{o,pl}$ does not enter the differential equation that characterizes $q_2^{pl}(b)$ [see. Eq. (42) or Eq. (C.7)].

Using this observation, we implicitly differentiate the integral equation (C.19) with respect to $\lambda_1^{o,pl}$, and

[26] Specifically, we calibrate the policy-induced belief change in the recession state so that the maximum decline in the planner's first-best value function is the same in both cases plotted in Figure 9, $\max_\alpha \left|\Delta v_2^{pl,*}(\alpha)\right| = \max_\alpha \left|\Delta v_1^{pl,*}(\alpha)\right|$.

use Eq. $(C.20)$, to obtain,

$$\begin{aligned}
\frac{\partial w_1^i(b_{0,1})}{\partial \lambda_1^{o,pl}} &= \int_0^\infty e^{-(\rho+\lambda_1^i)t}\lambda_1^i\left(\frac{\partial w_2^i(b_{t,2})}{\partial \lambda_1^{o,pl}} + \frac{\partial w_2^i(b_{t,2})}{\partial b}\frac{db_{t,2}}{d\lambda_1^{o,pl}}\right)dt, \\
\frac{\partial w_2^i(b_{0,2})}{\partial \lambda_1^{o,pl}} &= \int_0^\infty e^{-(\rho+\lambda_1^i)t}\lambda_2^i\frac{\partial w_1^i(b_{t,1})}{\partial \lambda_1^{o,pl}}dt.
\end{aligned}$$

Note also that, using Eq. $(C.20)$ implies, $\frac{db_{t,2}}{d\lambda_1^{o,pl}} = -t + \frac{1}{\lambda_1^{o,pl}}$. Plugging this into the previous system, and evaluating the partial derivatives at $\lambda_1^{o,pl} = \lambda_1^o$, we obtain,

$$\begin{aligned}
\frac{\partial w_1^i(b_{0,1})}{\partial \lambda_1^{o,pl}} &= h(b_{0,1}) + \int_0^\infty e^{-(\rho+\lambda_1^i)t}\lambda_1^i\frac{\partial w_2^i(b_{t,2})}{\partial \lambda_1^{o,pl}}dt, \\
\frac{\partial w_2^i(b_{0,2})}{\partial \lambda_1^{o,pl}} &= \int_0^\infty e^{-(\rho+\lambda_1^i)t}\lambda_2^i\frac{\partial w_1^i(b_{t,1})}{\partial \lambda_1^{o,pl}}dt, \\
\text{where } h(b_{0,1}) &= \int_0^\infty e^{-(\rho+\lambda_1^i)t}\lambda_1^i\frac{\partial w_2^i(b_{t,2})}{\partial b}\left(-t+\frac{1}{\lambda_1^o}\right)dt.
\end{aligned} \tag{C.21}$$

Note that the function, $h(b)$, is bounded since the derivative function, $\frac{\partial w_2^i(b)}{\partial b}$, is bounded (see $(C.11)$). Hence, Eq. $(C.21)$ describes the partial derivative functions, $\left(\frac{\partial w_s^i(b)}{\partial \lambda_1^{o,pl}}|_{\lambda_1^{o,pl}=\lambda_1^o}\right)_s$, as a fixed point of a corresponding operator T over bounded functions. Since $h(b)$ is bounded, it can be checked that the operator T is also a contraction mapping with respect to the sup norm. In particular, it has a fixed point, which corresponds to the partial derivative functions.

The analysis so far applies generally. We next consider the special case, $\lambda_1^o = \lambda_1^p$, and show that it implies the partial derivatives are strictly positive. In this case, $\lambda_1^i \equiv \lambda_1$ for each $i \in \{o,p\}$. In addition, Eq. $(C.10)$ implies $b_{t,2} = b_{0,2}$. Using these observations, for each $b_{0,1}$, we have,

$$\begin{aligned}
h(b_{0,1}) &= \frac{\partial w_2^i(b_{0,2})}{\partial b}\int_0^\infty e^{-(\rho+\lambda_1)t}\lambda_1\left(-t+\frac{1}{\lambda_1}\right)dt \\
&= \frac{\partial w_2^i(b_{0,2})}{\partial b}\left(-\frac{\lambda_1}{\rho+\lambda_1}\frac{1}{\rho+\lambda_1}+\frac{1}{\rho+\lambda_1}\right) > 0.
\end{aligned}$$

Here, the inequality follows since $\frac{\partial w_2^i(b_{0,2})}{\partial b} > 0$ [cf. Lemma 1]. Since $h(b) > 0$ for each b, and $\lambda_s^i > 0$, it can further be seen that the fixed point that solves $(C.21)$ satisfies $\frac{\partial w_s^i(b)}{\partial \lambda_s^{o,pl}} > 0$ for each b and $s \in \{1,2\}$. Using Eq. $(C.6)$, we also obtain $\frac{\partial w_s^i(\alpha)}{\partial \lambda_1^{o,pl}} > 0$ for each $s \in \{1,2\}$ and $\alpha \in (0,1)$. Since the analysis applies for any fixed belief type i, this establishes the claim in $(C.18)$ and completes the proof. □

Proof of Proposition 4. A similar analysis as in the proof of Proposition 3 implies that the partial derivative function, $\frac{\partial w_s^i(b)}{\partial\left(-\lambda_2^{o,pl}\right)}$, is characterized as the fixed point of a contraction mapping over bounded functions (the analogue of Eq. $(C.21)$ for state 2). In particular, the partial derivative exists and it is bounded. Moreover, since the corresponding contraction mapping takes continuous functions into continuous functions, the partial derivative is also continuous over $b \in \mathbb{R}$. Using Eq. $(C.6)$, we further obtain that the partial derivative, $\frac{\partial w_s^i(\alpha)}{\partial\left(-\lambda_2^{o,pl}\right)}$, is continuous over $\alpha \in (0,1)$.

Next note that $w_s^i(1) \equiv \lim_{\alpha\to 1} w_s^i(\alpha)$ exists and is equal to the value function according to type i beliefs when all investors are optimistic. In particular, the asset prices are given by $q_1^{pl} = q^*$ and $q_2^{pl} = q^o$, and the transition probabilities are evaluated according to type i beliefs. Then, following the same steps as in our analysis of value functions in Appendix C.1, we obtain,

$$\begin{aligned} \rho w_s^i(1) &= \beta_s^i q_s^o + \left(1-\beta_s^i\right) q_{s'}^o - q^*, \\ \text{where } \beta_s^i &= \frac{\rho + \lambda_{s'}^i}{\rho + \lambda_{s'}^i + \lambda_s^i}. \end{aligned}$$

Here, β_s^i can be thought of as the expected discount time the investor spends in state s according to type i beliefs. We consider this equation for $s = 2$ and take the derivative with respect to $\left(-\lambda_2^{o,pl}\right)$ to obtain,

$$\frac{\partial w_2^i(1)}{\partial\left(-\lambda_2^{o,pl}\right)} = \beta_2^i \frac{dq_2^o}{d\left(-\lambda_2^{o,pl}\right)} < 0.$$

Here, the inequality follows since reducing optimists' optimism reduces the price level in the common belief benchmark (see Section 4).

Note that the inequality, $\frac{\partial w_2^i(1)}{\partial\left(-\lambda_2^{o,pl}\right)} < 0$, holds for each belief type i. Using the continuity of the partial derivative function, $\frac{\partial w_2^i(\alpha)}{\partial\left(-\lambda_2^{o,pl}\right)}$, we conclude that there exists $\overline{\alpha}$ such that $\left.\frac{\partial w_2^i(\alpha)}{\partial\left(-\lambda_2^{o,pl}\right)}\right|_{\lambda_2^{o,pl}=\lambda_2^o} < 0$ for each i, s and $\alpha \in (\overline{\alpha}, 1)$, completing the proof. □

D. Appendix: Extension with investment and endogenous growth

Our baseline setup in the main text assumes there is no investment and the expected growth rate of capital is exogenous. In this appendix, we analyze a more general environment that relaxes these assumptions. We first present the environment, define the equilibrium, and provide a partial characterization. We then characterize this equilibrium when investors have common beliefs and generalize Proposition 1 to this setting.

D.1. Environment and equilibrium with investment

We focus on the components that are different than the baseline setting described in Section 3.

Potential output and endogenous growth. We modify the equation that describes the dynamics of capital (14) as follows,

$$\frac{dk_{t,s}}{k_{t,s}} = g_{t,s}dt + \sigma_s dZ_t \qquad \text{where } g_{t,s} \equiv \varphi\left(\iota_{t,s}\right) - \delta. \tag{D.1}$$

Here, $\iota_{t,s} = \frac{i_{t,s}}{k_{t,s}}$ denotes the investment rate, $\varphi\left(\iota_{t,s}\right)$ denotes a neoclassical production function for capital (we will work with a special case that will be described below), and δ denotes the depreciation rate. Hence, the growth of capital is no longer exogenous: it depends on the endogenous level of investment as well as depreciation.

Investment firms. To endogenize investment, we introduce a new set of firms, which we refer to as investment firms, that own and manage the aggregate capital stock. These firms rent capital to production firms to earn the instantaneous rental rate, $R_{t,s}$. They also make investment decisions to maximize the value of capital. Letting $\tilde{Q}_{t,s}$ denote the price of capital, the firm's investment problem can generally be written as,

$$\max_{\iota_{t,s}} \tilde{Q}_{t,s}\varphi\left(\iota_{t,s}\right)k_{t,s} - \iota_{t,s}k_{t,s}. \tag{D.2}$$

As before, we denote the price of the market portfolio per unit of capital with $Q_{t,s}$. In this case, the market portfolio represents a claim on investment firms as well as production firms. Hence, we have the inequality $\tilde{Q}_{t,s} \leq Q_{t,s}$, where the residual price, $Q_{t,s} - \tilde{Q}_{t,s}$, corresponds to the value of production firms per unit of capital. We make assumptions (that we describe below) so that output accrues to the investment firms in the form of return to capital, $y_{t,s} = R_{t,s}k_{t,s}$, and there are no monopoly profits. This in turn implies that the value of the market portfolio is equal to the value of capital (and the value of production firms is zero), that is,

$$Q_{t,s} = \tilde{Q}_{t,s}. \tag{D.3}$$

This simplifies the analysis by ensuring that we have only one price to characterize. Considering a different division of output between return to capital and profits will have a quantitative effect on investment, as illustrated by problem (D.2), but we conjecture that it would leave our qualitative results on investment unchanged. We leave a systematic exploration of this issue for further research.

Return of the market portfolio. The price of the market portfolio per unit of capital follows the same equation (15) as in the main text. The volatility of the market portfolio (absent state transitions) is also unchanged and given by σ_s. However, the return on the market portfolio conditional on no transition is slightly modified and given by,

$$r_{t,s}^{m} = \frac{y_{t,s} - \iota_{t,s} k_{t,s}}{Q_{t,s} k_{t,s}} + \left(g_{t,s} + \mu_{t,s}^{Q}\right). \tag{D.4}$$

Hence, the dividend yield is now net of the investment expenditures the (investment) firms undertake. In addition, the expected growth of the price of the market portfolio is now endogenous and given by $g_{t,s}$.

Nominal rigidities and equilibrium in goods markets. As before, the supply side of our model features nominal rigidities similar to the standard New Keynesian model that ensure output is determined by aggregate demand. In this case, demand comes from investment as well as consumption so we modify Eq. (19) as,

$$y_{t,s} = \eta_{t,s} A k_{t,s} = \int_I c_{t,s}^{i} di + k_{t,s} \iota_{t,s}, \text{ where } \eta_{t,s} \in [0,1]. \tag{D.5}$$

We also modify the microfoundations that we provide in Section B.1.2 so that all output accrues to investment firms as return to capital and there are no monopoly profits, that is,

$$R_{t,s} = A\eta_{t,s} \text{ and thus } y_{t,s} = R_{t,s} k_{t,s}. \tag{D.6}$$

We relegate a detailed description of these microfoundations to the end of this appendix.

Combining Eqs. $(D.5)$, $(D.4)$, (22) and (17), we can also rewrite the instantaneous (expected) return to the market portfolio as,

$$r_{t,s}^{m} = \rho + g_{t,s} + \mu_{t,s}^{Q}.$$

Hence, as in the main text, the equilibrium dividend yield is equal to the consumption rate ρ.

The rest of the model is the same as in Section 3. We formally define the equilibrium as follows.

Definition 2. *The equilibrium with investment and endogenous growth is a collection of processes for allocations, prices, and returns such that capital evolves according to* (14)*, the price of market portfolio per capital evolves according to* (15)*, its instantaneous return (conditional on no transition) is given by* $(D.4)$*, investment firms maximize (cf. Eqs.* $(D.7)$*), investors maximize (cf. Appendix B.1.1), asset markets clear (cf. Eqs.* (17) *and* (18)*), production firms maximize (cf. Appendix D.3), goods markets clear (cf. Eq.* (19)*), all output accrues to agents in the form of return to capital* $(D.6)$*, the price of the market portfolio per unit of capital is the same as the price of capital (cf. Eq.* $(D.3)$*), and the interest rate policy follows the rule in* (21)*.*

We next provide a partial characterization of the equilibrium with investment.

Investors' optimality conditions. Eqs. $(22-25)$ in the main text remain unchanged.

Investment firms' optimality conditions. Under standard regularity conditions for the capital production function, $\varphi(\iota)$, the solution to problem $(D.2)$ is determined by the optimality condition,

$$\varphi'(\iota_{t,s}) = 1/Q_{t,s}.$$

We will work with the special and convenient case proposed by Brunnermeier and Sannikov (2016b): $\varphi(\iota) = \psi \log\left(\frac{\iota}{\psi} + 1\right)$. In this case, we obtain the closed form solution,

$$\iota(Q_{t,s}) = \psi(Q_{t,s} - 1). \tag{D.7}$$

The parameter, ψ, captures the sensitivity of investment to asset prices.

Growth-asset price relation. Note also that the amount of capital produced is given by,

$$\varphi(\iota(Q_{t,s})) = \psi q_{t,s}, \text{ where } q_{t,s} \equiv \log(Q_{t,s}). \tag{D.8}$$

The log price level, $q_{t,s}$, will simplify some of the expressions. Combining Eq. $(D.8)$ with Eq. (14), we obtain Eq. (36) in the main text, which we replicate here for ease of exposition,

$$g_{t,s} = \psi q_{t,s} - \delta.$$

Hence, the expected growth rate of capital (and potential output) is now endogenous and depends on asset prices. Lower asset prices reduce investment, which translates into lower growth and lower potential output in future periods. As we will describe, this mechanism provides a new source of amplification.

Output-asset price relation. As in the main text, there is a tight relationship between output and asset prices as in the two period model. Specifically, Eq. (26) in the main text continues to apply and implies that aggregate consumption is a constant fraction of aggregate wealth. Plugging this into Eq. (19) and using the investment equation $(D.7)$, we obtain Eq. (35) in the main text, which we replicate here for ease of exposition,

$$A\eta_{t,s} = \rho Q_{t,s} + \psi(Q_{t,s} - 1) = (\rho + \psi) Q_{t,s} - \psi.$$

In this case, factor utilization (and output) depends on capital not only because consumption depends on asset prices through a wealth effect but also because investment depends on asset prices through a standard marginal-Q channel. Full factor utilization, $\eta_{t,s} = 1$, obtains only if the price of capital is at a particular level

$$Q^* \equiv \frac{A + \psi}{\rho + \psi}.$$

This is the efficient price level that ensures that the implied consumption and investment clear the goods market. Likewise, the economy features a demand recession, $\eta_{t,s} < 1$, if and only if the price of capital is strictly below Q^*.

Combining the output-asset price relation (together with $y_{t,s} = A\eta_{t,s} k_{t,s}$) with Eq. $(D.7)$, we obtain $\frac{y_{t,s} - \iota_{t,s} k_{t,s}}{Q_{t,s} k_{t,s}} = \rho$. Using this expression along with Eq. (36), we can rewrite Eq. (16) as,

$$r^m_{t,s} = \rho + \psi q_{t,s} - \delta + \mu^Q_{t,s}. \tag{D.9}$$

Hence, a version of Eq. (28) in the main text continues to apply. In equilibrium, the dividend yield on the market portfolio is equal to the consumption rate ρ. Moreover, the growth rate of dividends is endogenous and is determined by the growth-asset price relation.

Combining the output-asset price relation with the interest rate policy in (21), we also summarize the goods market side of the economy with (29) as in the main text. In particular, the equilibrium at any time and state takes one of two forms. If the natural interest rate is nonnegative, then the interest rate policy ensures that the price per unit of capital is at the efficient level, $Q_{t,s} = Q^*$, capital is fully utilized, $\eta_{t,s} = 1$, and output is equal to its potential, $y_{t,s} = Ak_{t,s}$. Otherwise, the interest rate is constrained, $r^f_{t,s} = 0$, the price is at a lower level, $Q_{t,s} < Q^*$, and output is determined by aggregate demand according to Eq. (27).

As a benchmark, we characterize the first-best equilibrium without interest rate rigidities. In this case, there is no lower bound constraint on the interest rate, so the price of capital is at its efficient level at all times and states, $Q_{t,s} = Q^*$. Combining this with Eq. (D.9), we obtain $r^m_{t,s} = \rho + \psi q^* - \delta$, where $q^* = \log Q^*$. Substituting this into Eq. (23) and using Eq. (25), we solve for "rstar" as,

$$r^{f*}_s = \rho + \psi q^* - \delta - \sigma^2_s \text{ for each } s \in \{1, 2\}. \tag{D.10}$$

Hence, in the first-best equilibrium the risk premium shocks are fully absorbed by the interest rate. We next characterize the equilibrium with interest rate rigidities for the case in which investors have common beliefs.

D.2. Common beliefs Benchmark with Investment

Suppose investors have common beliefs (that is, $\lambda^i_s \equiv \lambda_s$ for each i). Substituting Eq. (D.9) into (23), we obtain the following analogue of the risk balance conditions (31),

$$\sigma_s = \frac{\rho + \psi q_s - \delta + \lambda_s \left(1 - \frac{Q_s}{Q_{s'}}\right) - r^f_s}{\sigma_s} \text{ for each } s \in \{1, 2\}. \tag{D.11}$$

The only difference is that the growth rate in each state is endogenous and described by the growth-asset price relation, $g_s \equiv \psi q_s - \delta$, where recall that $q_s = \log Q_s$ [cf. Eq. (36)]. We also make the following analogue of Assumption 1.

Assumption 1$^{\text{I}}$. $\sigma^2_2 > \rho + \psi q^* - \delta > \sigma^2_1$.

With this assumption, we conjecture that the low-risk-premium state 1 features positive interest rates, efficient asset prices, and full factor utilization, $r^f_1 > 0, q_1 = q^*$ and $\eta_1 = 1$, whereas the high-risk-premium state 2 features zero interest rates, lower asset prices, and imperfect factor utilization, $r^f_2 = 0, q_2 < q^*$ and $\eta_2 < 1$.

Equilibrium in the high-risk-premium state and amplification from the growth-asset price relation. Under our conjecture, the risk balance condition (D.11) for the high-risk state $s = 2$ can be written as,

$$\sigma_2 = \frac{\rho + \psi q_2 - \delta + \lambda_2 \left(1 - \frac{Q_2}{Q^*}\right)}{\sigma_2}. \tag{D.12}$$

As before, this equation illustrates an amplification mechanism: Since the recession reduces firms' earnings, a lower price level does not increase the dividend yield (captured by the constant dividend yield, $\rho = \frac{\rho Q_2}{Q_2}$). Unlike before, Eq. ($D$.12) illustrates a second amplification mechanism captured by the growth-asset price relation, $g_2 = \psi q_2 - \delta$. In particular, a lower price level lowers investment, which reduces the expected growth of potential output and profits, which in turn lowers the return to capital. The strength of this second mechanism depends on the sensitivity of investment to asset prices, captured by the term ψq_2. Figure 1 in the introduction presents a graphical illustration of the two amplification mechanisms.

The stabilizing force from price declines comes from the expected transition into the low-risk-premium state captured by the term, $\lambda_2 \left(1 - \frac{Q_2}{Q^*}\right)$. As before, to ensure that there exists an equilibrium with positive prices, we need a minimum degree of optimism, which we capture with the following analogue of Assumption 2.

Assumption 2^{I}. $\lambda_2 \geq \lambda_2^{\min}$, where $\lambda_2^{\min}$ is the unique solution to the following equation over the range $\lambda_2 \geq \psi$:

$$\rho + \psi q^* - \delta + \lambda_2^{\min} - \psi + \psi \log\left(\psi/\lambda_2^{\min}\right) = \sigma_2^2.$$

This assumption ensures that there exists a unique $Q_2 \in (0, Q^*)$ that solves Eq. $(D.12)$ (see the proof at the end of this section).

Equilibrium in the low-risk-premium state. Under our conjecture, the risk balance condition $(D.11)$ can be written as,

$$r_1^f = \rho + \psi q^* - \delta - \sigma_1^2 + \lambda_1 \left(1 - \frac{Q^*}{Q_2}\right) \tag{D.13}$$

As before, the interest rate adjusts to ensure that the risk balance condition is satisfied with the efficient price level, $Q_1 = Q^*$. For our conjectured equilibrium, we also assume an upper bound on λ_1 so that the implied interest rate is positive, $r_1^f > 0$, which we capture with the following analogue of Assumption 3.

Assumption 3^{I}. $\lambda_1 < \left(\rho + \psi q^* - \delta - \sigma_1^2\right) / \left(Q^*/Q_2 - 1\right)$, where $Q_2 \in (0, Q^*)$ solves Eq. (33).

As before, Eq. $(D.13)$ implies that r_1^f is decreasing in the transition probability, λ_1, as well as in the asset price drop conditional on transition, Q^*/Q_2.

The following result summarizes the characterization of equilibrium and generalizes Proposition 1. The testable predictions regarding the effect of risk premium shocks on consumption, investment, and output follow by combining the characterization with Eqs. $(26), (D.7), (35)$, and .

Proposition 5. *Consider the extended model with investment with two states, $s \in \{1, 2\}$, with common beliefs and Assumptions 1^I-3^I. The low-risk-premium state 1 features a positive interest rate, efficient asset prices and full factor utilization, $r_1^f > 0, Q_1 = Q^*$ and $\eta_1 = 1$. The high-risk state 2 features zero interest rate, lower asset prices, and a demand-driven recession, $r_2^f = 0, Q_2 < Q^*$, and $\eta_2 < 1$, as well as a lower level of consumption, $c_{t,2}/k_{t,2} = \rho Q_2$, investment, $i_{t,2}/k_{t,2} = \psi (Q_2 - 1)$, output, $y_{t,2}/k_{t,2} = (\rho + \psi) Q_2 - \psi$, and growth, $g_2 = \psi q_2 - \delta$. The price of capital in state 2 is characterized as the unique solution to Eq. $(D.12)$, and the risk-free rate in state 1 is given by Eq. $(D.13)$.*

Proof. Most of the proof is provided in the discussion leading to the proposition. The remaining step is to show that Assumptions 1^{I}-2^{I} ensure there exists a unique solution, $Q_2 \in (0, Q^*)$ (equivalently, $q_2 < q^*$) to Eq. $(D.12)$.

To this end, we define the function,

$$f(q_2, \lambda_2) = \rho + \psi q_2 - \delta + \lambda_2 \left(1 - \frac{\exp(q_2)}{Q^*}\right) - \sigma_2^2.$$

The equilibrium price is the solution to, $f(q_2, \lambda_2) = 0$ (given λ_2). Note that $f(q_2, \lambda_2)$ is a concave function of q_2 with $\lim_{q_2 \to -\infty} f(q_2, \lambda_2) = \lim_{q_2 \to \infty} f(q_2, \lambda_2) = -\infty$. Its derivative is,

$$\frac{\partial f(q_2, \lambda_2)}{\partial q_2} = \psi - \lambda_2 \exp(q_2 - q^*).$$

Thus, for fixed λ_2, it is maximized at,

$$q_2^{\max}(\lambda_2) = q^* + \log(\psi/\lambda_2).$$

Moreover, the maximum value is given by

$$\begin{aligned} f\left(q_2^{\max}\left(\lambda_2\right), \lambda_2\right) &= \rho - \delta + \psi\left(q^* + \log\left(\psi/\lambda_2\right)\right) + \lambda_2\left(1 - \exp\left(\log\left(\psi/\lambda_2\right)\right)\right) - \sigma_2^2 \\ &= \rho - \delta + \psi q^* + \psi \log\left(\psi/\lambda_2\right) + \lambda_2 - \psi - \sigma_2^2. \end{aligned}$$

Next note that, by Assumption 1^{I}, the maximum value is strictly negative when $\lambda_2 = \psi$, that is, $f\left(q_2^{\max}\left(\psi\right), \psi\right) < 0$. Note also that $\frac{df\left(q_2^{\max}\left(\lambda_2\right), \lambda_2\right)}{d\lambda_2} = 1 - \frac{\psi}{\lambda_2}$, which implies that the maximum value is strictly increasing in the range $\lambda_2 \geq \psi$. Since $\lim_{\lambda_2 \to \infty} f\left(q_2^{\max}\left(\lambda_2\right), \lambda_2\right) = \infty$, there exists $\lambda_2^{\min} > \psi$ that ensures $f\left(q_2^{\max}\left(\lambda_2^{\min}\right), \lambda_2^{\min}\right) = 0$. By Assumption 2^{I}, the transition probability satisfies $\lambda_2 \geq \lambda_2^{\min}$, which implies that $f\left(q_2^{\max}\left(\lambda_2\right), \lambda_2\right) \geq 0$. By Assumption 1^{I}, we also have that $f\left(q^*, \lambda_2\right) < 0$. It follows that, under Assumptions 1^{I}-2^{I}, there exists a unique price level, $q_2 \in [q_2^{\max}, q^*)$, that solves the equation, $f\left(q_2, \lambda_2\right) = 0$. □

D.3. New Keynesian microfoundations for nominal rigidities with investment

In the rest of this appendix, we present the microfoundations for nominal rigidities that lead to Eqs. $(D.5)$ and $(D.6)$. The production structure is the same as in Appendix B.1.2. Specifically, there is a continuum of monopolistically competitive production firms that produce intermediate goods according to $(B.3)$, and there is a competitive sector that produces the final good according to $(B.4)$. This also implies the demand for production firms is given by$(B.5)$. One difference is that production firms do not own the capital but they rent it from investment firms at rate $R_{t,s}$. Hence, they choose how their capital input $k_{t,s}\left(\nu\right)$, in addition to their factor utilization rate, $\eta_{t,s}\left(\nu\right)$, as well as production and pricing decisions, $y_{t,s}\left(\nu\right), p_{t,s}\left(\nu\right)$.

These features ensure that the production firm's output will be split between their capital expenditures (that they pay to investment firms) and monopoly profits. To simplify the analysis, we make assumptions so that there are no monopoly profits in equilibrium (and all output accrues to investment firms as return to capital). Specifically, we assume the government taxes the firm's profits lump sum, and redistributes these profits to the firms in the form of a linear subsidy to capital.

Formally, we let $\Pi_{t,s}\left(\nu\right)$ denote the equilibrium pre-tax profits of firm ν (that will be characterized below). We assume each firm is subject to the lump-sum tax determined by the average profits of all firms,

$$T_{t,s} = \int_{\nu} \Pi_{t,s}\left(\nu\right) d\nu. \tag{D.14}$$

We also let $R_{t,s} - \tau_{t,s}$ denote the after-subsidy cost of renting capital, where $R_{t,s}$ denotes the equilibrium rental rate paid to investment firms, and $\tau_{t,s}$ denotes a linear subsidy paid by the government. We assume the magnitude of the subsidy is determined by the government's break-even condition,

$$\tau_{t,s} \int_{\nu} k_{t,s}\left(\nu\right) d\nu = T_{t,s}. \tag{D.15}$$

Without price rigidities, the firm chooses $p_{t,s}\left(\nu\right), k_{t,s}\left(\nu\right), \eta_{t,s}\left(\nu\right) \in [0,1], y_{t,s}\left(\nu\right)$, to maximize its (pre-tax) profits,

$$\Pi_{t,s}\left(\nu\right) \equiv p_{t,s}\left(\nu\right) y_{t,s}\left(\nu\right) - \left(R_{t,s} - \tau_{t,s}\right) k_{t,s}\left(\nu\right), \tag{D.16}$$

subject to the supply constraint in $(B.3)$ and the demand constraint in $(B.5)$. As in Appendix B.1.2, the demand constraint holds as equality. Then, the optimality conditions imply,

$$\eta_{t,s}(\nu) = 1 \text{ and } p_{t,s}(\nu) = \frac{\varepsilon}{\varepsilon - 1} \frac{R_{t,s} - \tau_{t,s}}{A}.$$

That is, the firm utilizes its capital at full capacity (as before) and it increases its capital input and production up to the point at which its price is a constant markup over its after-subsidy marginal cost. In a symmetric-price equilibrium, we further have, $p_{t,s}(\nu) = 1$. Using Eqs. $(B.3)$ and $(D.15)$, this further implies,

$$y_{t,s}(\nu) = y_{t,s} = Ak_{t,s} \text{ and } R_{t,s} = \frac{\varepsilon - 1}{\varepsilon} A + \tau_{t,s} = A. \tag{D.17}$$

That is, output is equal to potential output, and capital earns its marginal contribution to potential output (in view of the linear subsidies).

Now consider the alternative setting in which the firms have a preset nominal price that is equal across firms, $P_{t,s}(\nu) = P$. In particular, the relative price of a firm is fixed and equal to one, $p_{t,s}(\nu) = 1$. The firm chooses the remaining variables, $k_{t,s}(\nu), \eta_{t,s}(\nu) \in [0,1], y_{t,s}(\nu)$, to maximize its (pre-tax) profits, $\Pi_{t,s}(\nu)$, subject to the supply constraint in $(B.3)$ and the demand constraint, $(B.5)$. Combining the constraints and using $p_{t,s}(\nu) = 1$, the firm's problem can be written as,

$$\max_{\eta_{t,s}(\nu), k_{t,s}(\nu)} A\eta_{t,s}(\nu) k_{t,s}(\nu) - (R_{t,s} - \tau_{t,s}) k_{t,s}(\nu) \text{ s.t. } 0 \leq \eta_{t,s}(\nu) \leq 1 \text{ and } A\eta_{t,s}(\nu) k_{t,s}(\nu) \leq y_{t,s}.$$

We conjecture an equilibrium in which $R_{t,s} = \tau_{t,s}$ and firms choose symmetric capital inputs, $k_{t,s}(\nu) = k_{t,s}$. Under this equilibrium, the marginal cost of renting capital is zero, $R_{t,s} - \tau_{t,s} = 0$. This verifies that it is optimal for firms to choose symmetric inputs, $k_{t,s}(\nu) = k_{t,s}$. After substituting these expressions, the firm's problem becomes equivalent to its counterpart in Appendix B.1.2. Following the same steps there, the optimal factor utilization is given by $\eta_{t,s}(\nu) = \frac{y_{t,s}}{Ak_{t,s}} \leq 1$. Hence, output is determined by aggregate demand, $y_{t,s}$, subject to the capacity constraint, $\eta_{t,s}(\nu) \leq 1$.

In the conjectured equilibrium, the production firms choose the same level of inputs and factor utilization rates and produce the same level of output as each other. Therefore, they also have the same level of pre-tax profits. Using Eqs. $(D.16)$ together with $R_{t,s} = \tau_{t,s} = 0$, we also calculate the pre-tax profit level as $\Pi_{t,s} = y_{t,s}$. Substituting this into Eqs. $(D.14)$ and $(D.15)$, we obtain $\tau_{t,s} = y_{t,s}/k_{t,s} = \eta_{t,s}A$. Substituting this into Eq. $(D.16)$, we further obtain $R_{t,s} = y_{t,s}/k_{t,s} = \eta_{t,s}A$. This verifies the conjecture, $R_{t,s} = \tau_{t,s}$.

In sum, when the firms' nominal prices are fixed, aggregate output is determined by aggregate demand subject to the capacity constraint, which verifies Eq. $(D.5)$. Moreover, thanks to lump-sum costs to profits and linear subsidies to capital, all output accrues to the investment firms as return to capital, which verifies Eq. $(D.6)$.

E. Appendix: Data Details and Omitted Empirical Results

This appendix presents the details of the data sources and variable construction used in Section 7, and presents the empirical results (tables and figures) omitted from the main text.

House price index. We rely on the cross-country quarterly panel dataset described in Mack et al. (2011). The dataset is regularly updated and publicly available at https://www.dallasfed.org/institute/houseprice. We use the inflation-adjusted (real) house price index measure to construct the shock variable in our regression analysis (see (53)). Our country coverage is to a large extent determined by the availability of this measure, e.g., we exclude a few developed countries such as Portugal and Austria for which we do not have consistent data on real house prices.

Euro or Exchange Rate Mechanism (Euro/ERM) status. We hand-collect this data from various online sources. A country-quarter is included in the Euro/ERM sample if the country is a member of the Euro or the European Exchange Rate mechanism in most of the corresponding calendar year. Table 1 describes the Euro/ERM status by year for all countries in our sample.

GDP, consumption, investment. We obtain this data from the OECD's quarterly national accounts dataset (available at https://stats.oecd.org). We use the variables calculated according to the expenditure approach. The corresponding OECD subject codes are as follows:

- GDP: "B1_GE" (Gross domestic product – expenditure approach).
- Consumption: "P31S14_S15" (Private final consumption expenditure)
- Investment: "P51" (Gross fixed capital formation)

For each of these variables, we use the measures that are adjusted for inflation as well as seasonality. The OECD measure code is: "LNBQRSA" (National currency, chained volume estimates, national reference year, quarterly levels, seasonally adjusted).

Relative GDP (with PPP-adjusted prices in a common base year). We obtain an alternative GDP measure from the OECD's annual national accounts dataset (available at https://stats.oecd.org). We use the variable calculated according to the expenditure approach (with subject code "B1_GE"), measured with PPP-adjusted prices in a common base year. The OECD measure code is: "VPVOB" (Current prices, constant PPPs, OECD base year). We use the value of this measure in 1990 to weight all of our regressions (see (53)).

CPI. We obtain this data from the OECD's prices and purchasing power parities dataset (available at https://stats.oecd.org). We use the core CPI measure that excludes food and energy. The OECD subject code is: "CPGRLE" (Consumer prices - all items non-food, non-energy). We use the annual measure, which is less subject to seasonality, and we linearly interpolate this to obtain a quarterly measure.

Unemployment rate. We obtain this data from the OECD's key short-term economic indicators database (available at https://stats.oecd.org). We use the harmonized unemployment rate measure with seasonal adjustment and at quarterly frequency. The OECD subject code is "LRHUTTTT" (Harmonised unemployment rate: all persons, s.a).

The policy interest rate. Obtaining the policy interest rate is not as trivial as it might sound since different central banks conduct monetary policy in terms of different target rates (and sometimes without specifying a target rate, or by monitoring multiple rates). On the other hand, the selection does not substantially affect the results since short-term risk-free rates within a developed country are often highly correlated. Following Romer and Romer (2018), we use announced policy target rates when available, and otherwise we use collateralized short-term market rates (such as Repo rates or Lombard rates). For Eurozone countries, we use the local collateralized rate until the country joins the Euro, and we switch to the European Central Bank's (ECB) main refinancing operations (MRO) rate after the country joins the Euro.

For most of the countries, we construct our own measure of the policy interest rate according to the above selection criteria by using data from the Global Financial Data's GFDATABASE (GFD). This is a proprietary database that contains a wealth of information on various asset prices (see https://www.globalfinancialdata.com for details).

For a few countries (specified below), we instead rely on the Bank for International Settlements's (BIS) database on central bank policy interest rates (publicly available at https://www.bis.org/statistics/cbpol.htm). We switch to the BIS measure when we cannot construct an appropriate measure using the GFD; or when the BIS measure has greater coverage than ours and the two measures are highly correlated. From either database, we obtain monthly data and convert to quarterly data by averaging over the months within the quarter.

- United States: GFD ticker "IDUSAFFD" (USA Fed Funds Official Target Rate).
- United Kingdom: GFD ticker "IDGBRD" (Bank of England Base Lending Rate).
- Australia: GFD ticker "IDAUSD" (Australia Reserve Bank Overnight Cash Rate).
- South Korea: GFD ticker "IDKORM" (Bank of Korea Discount Rate).
- Germany: GFD ticker "IDDEULD" (Germany Bundesbank Lombard Rate) until the country joins the Euro. Afterwards, we use the ECB MRO rate. The corresponding GFD ticker is: "IDEURMW" (Europe Marginal Rate on Refinancing Operations).
- New Zealand: GFD ticker "IDNZLD" (New Zealand Reserve Bank Official Cash Rate).
- France: GFD ticker "IDFRARD" (Bank of France Repo Rate) until the country joins the Euro.
- Denmark: We use the BIS measure (highly correlated with our measure and greater coverage).
- Finland: GFD ticker "IDFINRM" (Bank of Finland Repo Rate) until the country joins the Euro.
- Sweden: GFD ticker "IDSWERD" (Sweden Riksbank Repo Rate).
- Israel: GFD ticker "IDISRD" (Bank of Israel Discount Rate).
- Italy: GFD ticker "IDITARM" (Bank of Italy Repo Rate) until the country joins the Euro.
- Spain: GFD ticker "IDESPRM" (Bank of Spain Repo Rate) until the country joins the Euro.
- Ireland: GFD ticker "IDIRLRD" (Bank of Ireland Repo Rate) until the country joins the Euro.
- Belgium: GFD ticker "IDBELRM" (Belgium National Bank Repo Rate) until the country joins the Euro.
- Greece: GFD ticker "IDGRC"D (Bank of Greece Discount Rate) until the country joins the Euro.

- Netherlands: GFD ticker "IDNLDRD" (Netherlands Bank Repo Rate) until the country joins the Euro.
- Norway: GFD ticker "IDNORRD" (Bank of Norway Sight Deposit Rate).
- Japan: GFD ticker "IDJPNCM" (Japan Target Call Rate). GFD data is missing from March 2001 until July 2006. BIS data is also missing for most of this period. We use other sources to hand-fill the interest rate over this period as being equal to 0% (see for instance, the data from St. Louis Fed at https://fred.stlouisfed.org/series/IRSTCI01JPM156N).
- Switzerland: We use the BIS measure (cannot identify an appropriate rate from the GFD).
- Canada: We use the BIS measure (highly correlated with our measure and greater coverage).

Stock prices. We obtain this data from the GFD. For each country, we try to pick the most popular stock price index (based on Internet searches). We obtain daily data and convert to quarterly data by averaging over all (trading) days within the quarter. We then divide this with our core CPI measure (see above) to obtain a real stock price series.

- United States: GFD ticker "_SPXD" (S&P500 Index)
- United Kingdom: GFD ticker "_FTSED" (UK FTSE100 Index).
- Australia: GFD ticker "_AXJOD" (Australia S&P/ASX 200 Index).
- South Korea: GFD ticker "_KS11D" (Korea SE Stock Price Index (KOSPI)).
- Germany: GFD ticker "_GDAXIPD" (Germany DAX Price Index).
- New Zealand: GFD ticker "_NZ15D" (NZSX-15 Index).
- France: GFD ticker "_FCHID" (Paris CAC-40 Index).
- Denmark: GFD ticker "_OMXC20D" (OMX Copenhagen-20 Index).
- Finland: GFD ticker "_OMXH25D" (OMX Helsinki-25 Index).
- Sweden: GFD ticker "_OMXS30D" (OMX Stockholm-30 Index).
- Israel: GFD ticker "_TA125D" (Tel Aviv SE 125 Broad Index).
- Italy: GFD ticker "_BCIJD" (Milan SE MIB-30 Index).
- Spain: GFD ticker "_IBEXD" (Madrid SE IBEX-35 Index).
- Ireland: GFD ticker "_ISEQD" (Ireland ISEQ Overall Price Index).
- Belgium: GFD ticker "_BFXD" (Belgium CBB Bel-20 Index).
- Greece: GFD ticker "_ATGD" (Athens SE General Index).
- Netherlands: GFD ticker "_AEXD" (Amsterdam AEX Stock Index).
- Norway: GFD ticker "_OSEAXD" (Oslo SE All-Share Index).
- Japan: GFD ticker "_N225D" (Nikkei 225 Stock Index).
- Switzerland: GFD ticker "_SSMID" (Swiss Market Index).

- Canada: GFD ticker "_GSPTSED" (Canada S&P/TSX 300 Index).

Earnings. We obtain monthly data on the price-earnings ratio of publicly traded firms from the GFD (typically constructed for a broad sample of stocks chosen by the GFD). We then combine this information with our nominal price index (using the price at the last trading day of the month) to construct a monthly series for earnings. We convert this to a quarterly measure by averaging over the months within the quarter. We then divide this by our core CPI measure to obtain a quarterly real earnings series for publicly traded firms.

GFD ticker for the price earnings ratio typically has the form "SY-three digit country code-PM" (e.g., the ticker for the United States is "SYUSAPM"). One exception is the United Kingdom for which the corresponding GFD code is "_PFTASD" (UK FT-Actuaries PE Ratio).

Credit expansion. Our measure of bank credit is based on Baron and Xiong (2017), who construct a variable, *credit expansion*, defined as the annualized past three-year change in bank credit to GDP ratio. Mathematically, it is expressed as

$$\text{credit expansion} = \frac{\Delta\left(\frac{\text{bank credit}}{\text{GDP}}\right)_t - \Delta\left(\frac{\text{bank credit}}{\text{GDP}}\right)_{t-12}}{12} \times 4, \tag{E.1}$$

where t denotes a quarter. Baron and Xiong (2017) construct this measure by merging data from two sources. Their main source is the "bank credit" measure from the BIS, which covers a large set of countries but is generally available only for postwar years. For this reason, Baron and Xiong (2017) also supplement it with the "bank loans" measure from Schularick and Taylor (2012), which covers fewer countries but more years. Since our panel starts in 1990, we ignore the second source and rely entirely on the BIS measure.

Specifically, we use the quarterly BIS database on credit to the nonfinancial sector (publicly available at https://www.bis.org/statistics/totcredit.htm). We obtain the measure "bank credit to the private nonfinancial sector" expressed in units of percentage of GDP (the corresponding BIS code is "Q:5A:P:B:M:770:A"), which enables us to construct the variable in (E.1). We verify that our variable is highly correlated with the measure constructed by Baron and Xiong (2017) (who generously shared their data with us)—the correlation coefficient for the available country-quarters is 0.975.

Following Baron and Xiong (2017), we also construct a "credit expansion-std" variable by standardizing the measure in (E.1) by its mean and standard deviation within each country. Since Baron and Xiong (2017) focus on predicting stock prices, they calculate the mean and the standard deviation using only past data so as to avoid any look-ahead bias. Since our focus is different, we ignore this subtlety and calculate the sample statistics using the entire data for the corresponding country (in the BIS database).

Table 1: Euro/ERM status by country and year

Country	1990	1991	1992	1993	1994	1995	1996	1997-2017
Belgium	1	1	1	1	1	1	1	1
Denmark	1	1	1	1	1	1	1	1
Finland	0	0	0	0	0	0	0	1
France	1	1	1	1	1	1	1	1
Germany	1	1	1	1	1	1	1	1
Greece	0	0	0	0	0	0	1	1
Ireland	1	1	1	1	1	1	1	1
Italy	1	1	1	0	0	0	0	1
Netherlands	1	1	1	1	1	1	1	1
Spain	1	1	1	1	1	1	1	1
Australia	0	0	0	0	0	0	0	0
Canada	0	0	0	0	0	0	0	0
Israel	0	0	0	0	0	0	0	0
Japan	0	0	0	0	0	0	0	0
Korea	0	0	0	0	0	0	0	0
NZL	0	0	0	0	0	0	0	0
Norway	0	0	0	0	0	0	0	0
Sweden	0	1	1	0	0	0	0	0
Switzerland	0	0	0	0	0	0	0	0
UK	0	1	1	0	0	0	0	0
USA	0	0	0	0	0	0	0	0

Euro status. Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Spain adopted the Euro in 1999. Greece adopted in 2001. Denmark hasn't adopted the Euro but is a member of the ERM.

Table 2: Summary statistics by ERM for the baseline regression sample

	ERM sample		Non-ERM sample		Difference	
	Mean	Std.Deviation	Mean	Std.Deviation	Mean	Std.Error
Δ log house prices (real)	0.0040	0.0183	0.0053	0.0181	-0.0013	(0.0023)
Δ log GDP (real)	0.0043	0.0128	0.0065	0.0093	-0.0022	(0.0010)
policy interest rate (nominal)	0.0232	0.0194	0.0352	0.0288	-0.0119	(0.0038)
Δ log CPI (core)	0.0041	0.0029	0.0046	0.0039	-0.0004	(0.0005)
Δ unemployment rate	-0.0000	0.0042	-0.0002	0.0030	0.0002	(0.0004)
Δ log investment (real)	0.0030	0.0535	0.0070	0.0297	-0.0040	(0.0021)
Δ log consumption (real)	0.0035	0.0103	0.0069	0.0095	-0.0034	(0.0008)
earnings to price ratio	0.0616	0.0409	0.0585	0.0227	0.0031	(0.0039)
Δ log stock prices (real)	0.0011	0.0974	0.0108	0.0820	-0.0097	(0.0047)
credit expansion	0.0175	0.0557	0.0136	0.0298	0.0040	(0.0079)
credit expansion-std	0.1715	1.2657	-0.0346	1.1128	0.2062	(0.1957)
Observations	821		1120		1941	

Δ represents quarterly change. Standard errors are Newey-West standard errors with a bandwidth of 20 quarters.

Table 3: Private housing wealth in 2005 (% of GDP) by Euro/ERM status

Country (Euro/ERM)	Housing wealth	Country (Non-Euro/ERM)	Housing wealth
Spain	414.33	Australia	301.32
Italy	271.25	USA	199.77
France	253.74	Korea	179.55
Netherlands	222.03	Japan	169.74
Germany	186.77	Canada	146.51
Denmark	168.45	Norway	139.48
		Sweden	132.10
Average	252.76	Average	181.21
GDP-weighted average	255.29	GDP-weighted average	191.64

Table 4: Stock market capitalization in 2005 (% of GDP) by Euro/ERM status

Country (Euro/ERM)	Market cap	Country (Non-Euro/ERM)	Market cap
Finland	102.48	Switzerland	229.68
Netherlands	87.37	Canada	129.84
Spain	82.95	UK	126.75
France	80.07	Australia	121.32
Belgium	74.47	Sweden	116.08
Denmark	67.30	USA	103.83
Greece	58.57	Korea	96.16
Ireland	53.90	Israel	86.04
Italy	43.08	Norway	79.94
Germany	42.01	Japan	61.89
Average	69.22	Average	115.16
GDP-weighted average	61.84	GDP-weighted average	120.26

Data sources. We obtain housing wealth to GDP ratio from the World Inequality Database (WID) which is publicly available at https://wid.world/. We construct the ratio by combining yearly series on "private housing assets" (WID indicator, "mpwhou") and "gross domestic product (WID indicator, "mgdpro").

We obtain stock market capitalization to GDP ratio as yearly series from the GFD. The corresponding ticker has the form "CM.MKT.LCAP.GD.ZS three digit country code" (e.g., the ticker for the United States is "CM.MKT.LCAP.GD.ZS USA").

For both tables, we construct the GDP-weighted averages by using our relative GDP measure (in 2005) described earlier in this appendix.

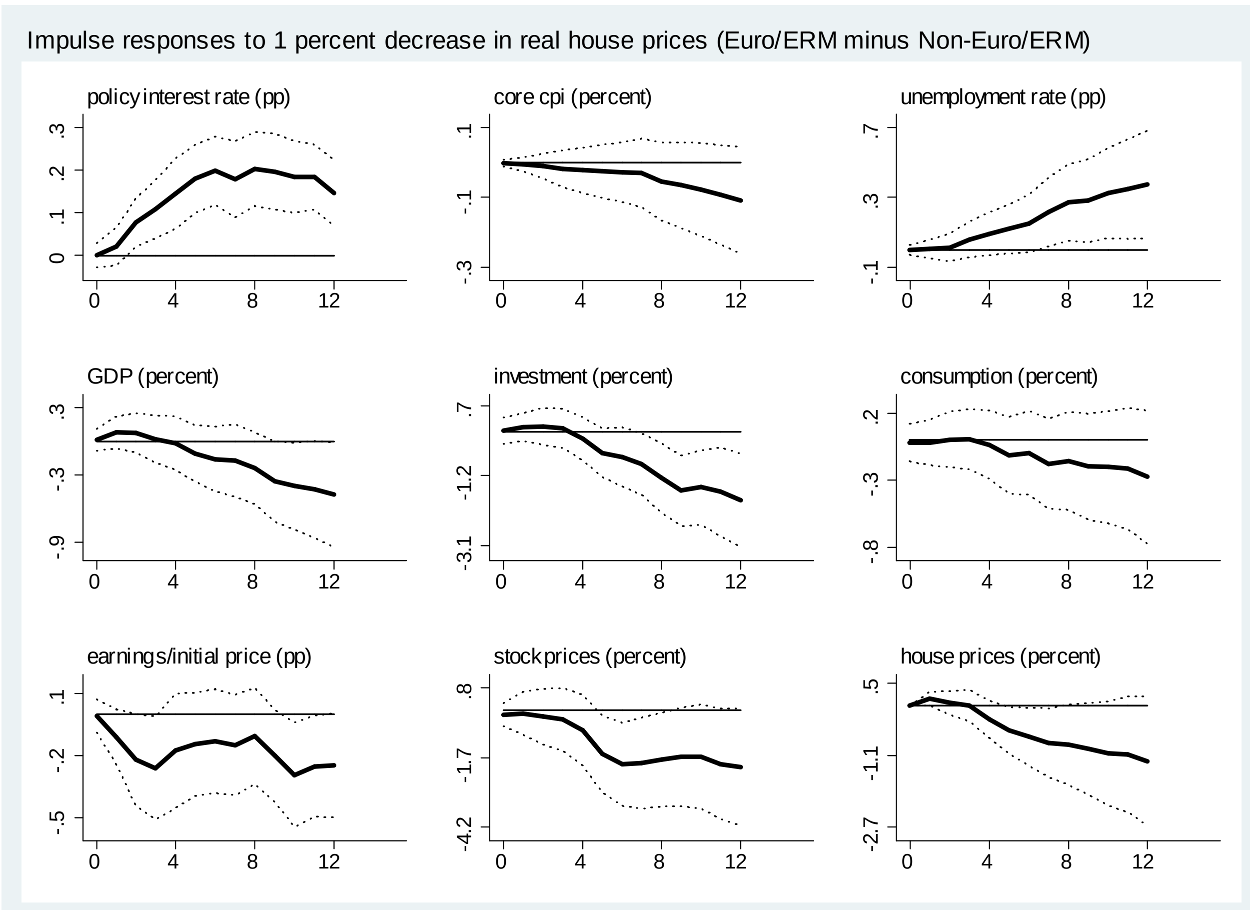

Figure 10: Differences in coefficients between the ERM and the non-ERM samples corresponding to the baseline regression results in Figure 6.

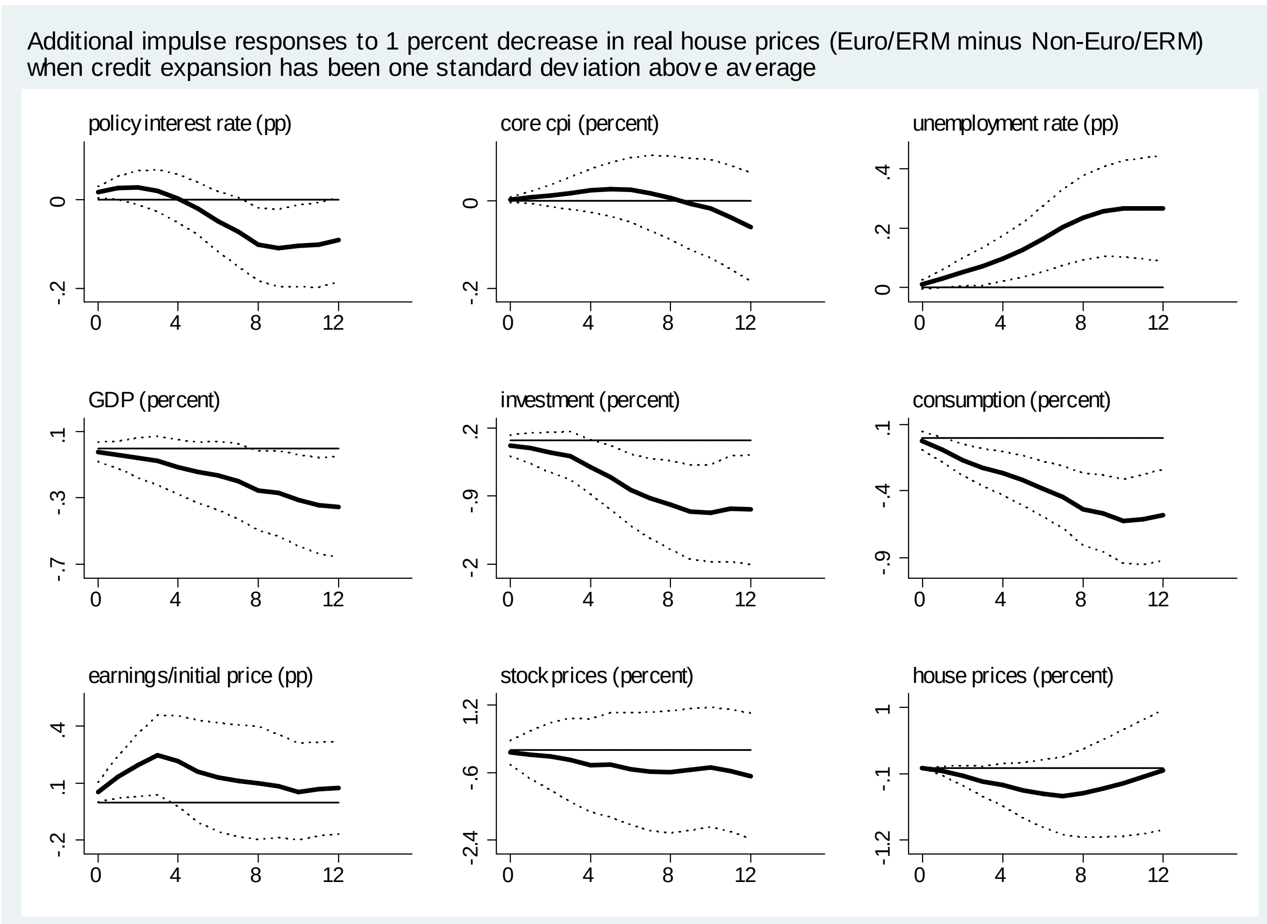

Figure 11: Differences in coefficients between the ERM and the non-ERM samples corresponding to the regression results with credit interaction in Figure 7.

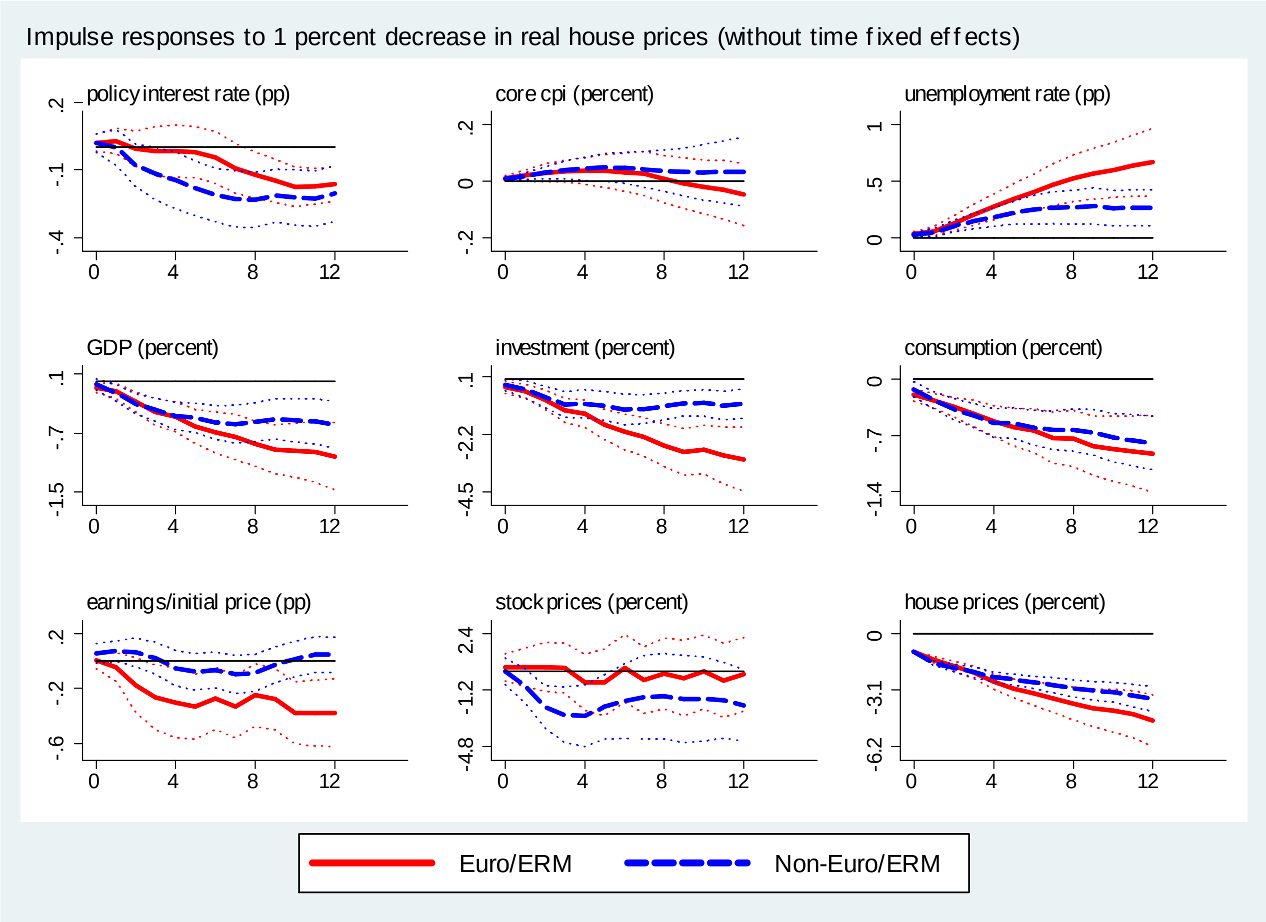

Figure 12: The analogues of the baseline regression results in Figure 6 with the difference that time fixed effects are excluded from the regressions.

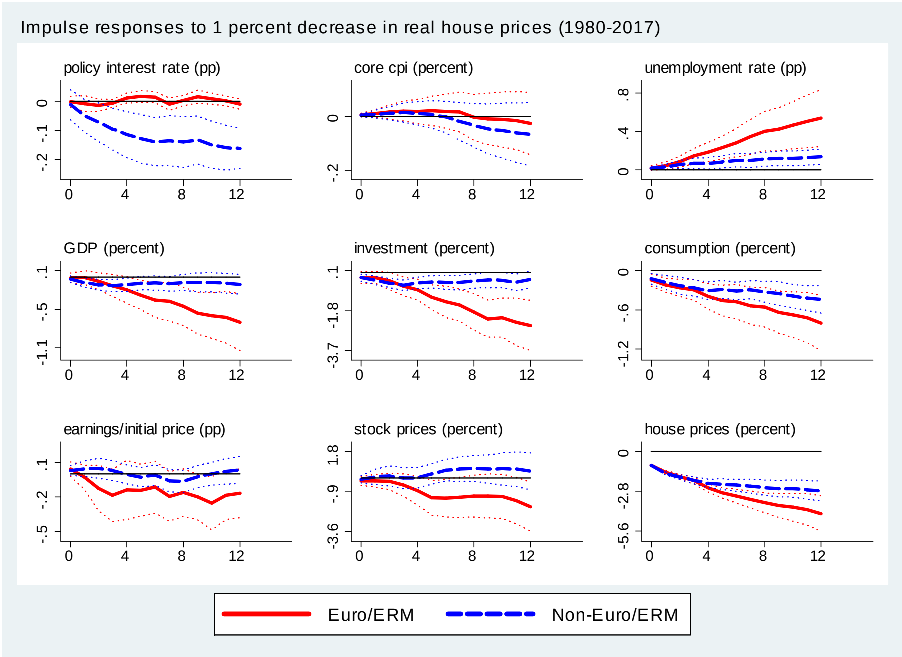

Figure 13: The analogues of the results in Figure 6 with a sample that starts in 1980Q1 (as opposed to 1990Q1).

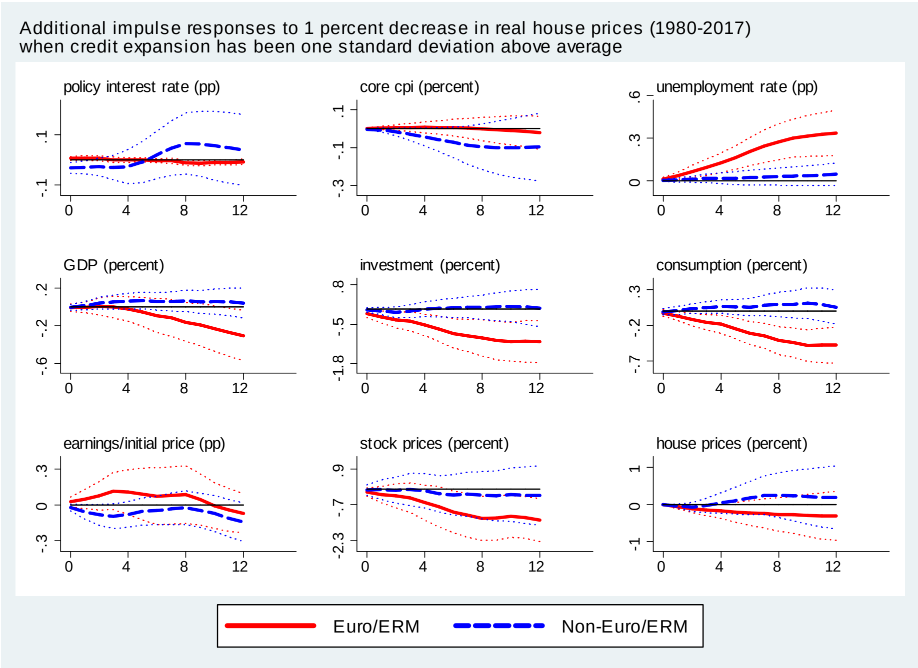

Figure 14: The analogues of the results in Figure 7 with a sample that starts in 1980Q1 (as opposed to 1990Q1).

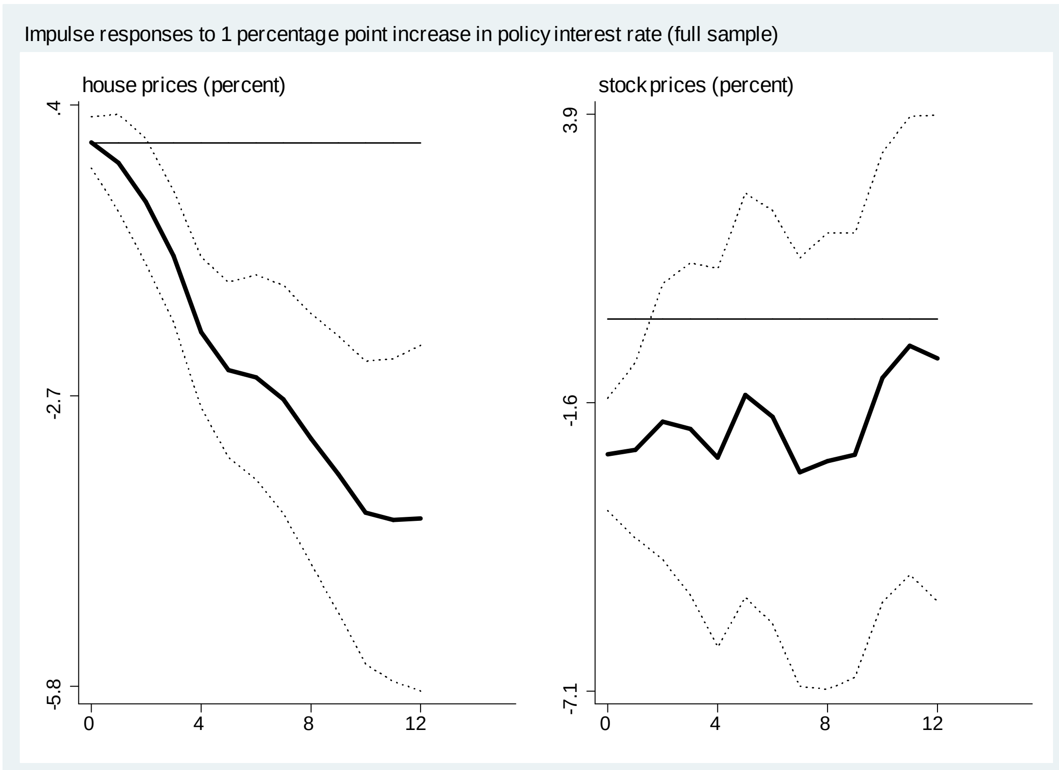

Figure 15: The analogues of the baseline regression results in Figure 6, where we consider shocks to the policy interest rate as opposed to house prices. Specifically, we run the analogue of the specification in (53) (on the full sample) where the shock variable is the level of the policy interest rate and the outcome variable is log house prices (left panel) or log stock prices (right panel). The solid lines plot the coefficients corresponding to the the policy interest rate variable. All regressions include time and country fixed effects; 12 lags of the level of the policy interest rate, 12 lags of the first difference of log GDP, 12 lags of the first difference of log house prices, and 12 lags of the first difference of log stock prices. The dotted lines show 95% confidence intervals calculated according to Newey-West standard errors with a bandwidth of 20 quarters. All regressions are weighted by countries' PPP-adjusted GDP in 1990. Data is unbalanced quarterly panel that spans 1990Q1-2017Q4. All variables except for the policy interest rate are adjusted for inflation. The sources and the definitions of variables are described earlier in this appendix.