BANK FOR INTERNATIONAL SETTLEMENTS

BIS Working Papers
No 731

The likelihood of effective lower bound events

by Michal Franta

Monetary and Economic Department

June 2018

JEL classification: E37, E52, C11

Keywords: effective lower bound, ELB risk, mean adjustment, panel VAR, regime change

This publication is available on the BIS website (www.bis.org).

ISSN 1020-0959 (print)
ISSN 1682-7678 (online)

The likelihood of effective lower bound events

Michal Franta[1]

Abstract

This paper provides estimates of the probability of an economy hitting its effective lower bound (ELB) on the nominal interest rate and of the expected duration of such an event for eight advanced economies. To that end, a mean-adjusted panel vector autoregression with static interdependencies and the possibility of regime change is estimated. The simulation procedure produces ELB risk estimates for both the short term, where the current phase of the business cycle plays an important role, and the medium term, where the occurrence of an ELB situation is determined mainly by the equilibrium values of macroeconomic variables. The paper also discusses the ELB event probability estimates with respect to previous approaches used in the literature.

JEL codes: E37, E52, C11

Keywords: effective lower bound, ELB risk, mean adjustment, panel VAR, regime change

[1] Michal Franta, Czech National Bank, e-mail: michal.franta@cnb.cz.
I would like to thank Tomáš Adam, Leonardo Gambacorta, Elmar Mertens, Jouchi Nakajima, an anonymous referee and seminar participants at the Bank for International Settlements and the Czech National Bank for valuable comments. I completed this project while visiting the Bank for International Settlements under the Central Bank Research Fellowship program. The opinions expressed in this paper are those of the author and do not necessarily reflect those of the Czech National Bank or of the Bank for International Settlements.

1. Introduction

The recent experience of a prolonged period of extraordinarily low monetary policy rates in advanced countries has rekindled interest in macroeconomic issues relating to the effective lower bound (ELB) on the nominal interest rate. Before the Great Financial Crisis (GFC), the only relevant example of an ELB event was Japan, which, however, was viewed as a peculiar case. It was not generally believed that advanced countries could experience long and recurrent periods of being stuck at the ELB. This opinion has now changed, and novel research relating to the ELB has been published.

There are two important policy questions underlying this research work: what is the probability of an ELB event at a given point in time, and how often can an ELB event be expected to occur in general? The usual approach to answering the latter involves conducting model-based stochastic dynamic simulations and estimating the stationary distribution of the short-term interest rate, with the area below a given lower bound defining the ELB risk. ELB risk estimates of this type are usually based on calibrated linear models and are highly sensitive to the equilibrium values of macroeconomic variables assumed (see, for example, Reifschneider and Williams, 2000, Coenen, 2003, and Kiley and Roberts, 2017).[2]

When the examination of the likelihood of an ELB event is related to a specific point in time, more data-driven approaches are employed. Statistical models are used to estimate a distribution forecast for the interest rate. In contrast to the other approach, this allows uncertainty in model parameters, latent variables and measurement errors to be taken into account, and enables some simple form of non-linearity (Chung et al., 2012). Nakata (2017a) employs survey data on macroeconomic projections to bring the standard stochastic simulations to actual data. However, data-driven approaches are not suitable for estimating ELB risk in the medium and long term, because they often do not possess well-defined unconditional moments.[3]

The aim of this paper is to draw on the above-mentioned approaches and to provide estimates of the probability and expected duration of ELB events. To that end, the paper combines stochastic simulations around equilibrium values and distribution forecasting reflecting current observed data. The results are therefore relevant to both the short term, where the ELB likelihood is driven mainly by the current phase of the business cycle, and the medium term, where the ELB risk is determined mainly by equilibrium values. Dealing with both time scales in one modeling framework makes the estimates for the short and medium term consistent with each other. Furthermore, the approach addresses some of the contentious issues of the previous approaches and should therefore deliver more accurate ELB likelihood estimates.

---

[2] A recent example of an estimated dynamic stochastic general equilibrium (DSGE) model used to estimate the ELB risk is Gust et al. (2017).

[3] Probit-type models could, in principle, be employed to estimate the ELB risk. However, different models have to be estimated for different horizons of interest. Moreover, duration of the ELB spell could not be examined within these types of model.

I adopt an empirical strategy and base ELB likelihood estimates on recent data from advanced countries, exploiting the fact that many of them have recently experienced, or are still experiencing, nominal interest rates at their ELB. As a consequence, the ELB risk estimates have to rely less on calibrated parameters and can be based more on estimated quantities. The framework thus accounts for parameter uncertainty, which has been found to be important for realistically assessing ELB likelihood (Chung et al., 2012). On the other hand, the time span of the macro data capturing ELB events is still short, suggesting that it is appropriate to exploit the panel nature of that data.

Furthermore, the data-driven approach depends less heavily on assumptions about the equilibrium values of macroeconomic variables than do studies based on stochastic simulations. The assumptions about the equilibrium values enter the estimation procedure in the form of priors and are confronted with the observed data during the Bayesian estimation process. Finally, as an ELB situation implies a possibility of regime change, the modeling framework allows for change in the shock transmission mechanism and shock volatilities.

To quantitatively assess the occurrence of ELB events, I employ the mean-adjusted panel vector autoregression technique, which allows for static interdependencies and threshold behavior. The model is estimated on data for eight advanced economies over the period 1999Q1–2016Q4 and provides estimates of ELB risk in both the short and the medium term. In addition, the modelling framework allows to analyze the impact of various assumptions employed by the techniques used to estimate ELB risk in previous studies.

It turns out that, in the short term, the ELB risk is currently the highest for Japan and Sweden and the lowest for Canada, Norway and the US. In the medium term, the probability of an ELB situation ranges from 0.01 for Canada to 0.16 for Japan. A supplementary analysis suggests that the calibration of the steady-state values of macroeconomic variables can lead to ELB risk underestimation due to the fact that the calibrated value of the equilibrium interest rate is high and is assumed to be known with certainty. It is also shown how the empirical approach gains from the panel nature of data by partially pooling country-specific information and by improving the efficiency of estimates.

From the policy maker's perspective, the estimates should be taken into account in the conduct of monetary policy. Higher ELB risk during a recession calls for more aggressive easing of monetary policy, whereas higher ELB risk during a boom justifies slower normalization of rates (see, for example, Williams, 2014, and references therein). Similarly, if there is agreement on the optimal size of the central bank balance sheet, the speed of balance sheet normalization from current levels should reflect the probability of return to the ELB and to unconventional measures that would increase the size of the sheet again.

The structure of the paper is as follows. Section 2 discusses recent monetary policy rates in advanced countries. Section 3 presents the model. Section 4 explains the estimation approach and describes the data. Section 5 presents the results and considers the role of calibration and regime change in ELB risk estimation. Section 6 concludes.

2. The ELB in advanced countries

Figure 1 shows monetary policy rates in eight advanced economies since 1999. In general, two cycles can be observed, one peaking around the year 2000 and the second peaking in 2007 just before the GFC hit the world economy. The observed monetary policy rate profiles also suggest a downward trend in the equilibrium interest rate. The short time span of the observations, however, makes any conclusion about the trend uncertain.

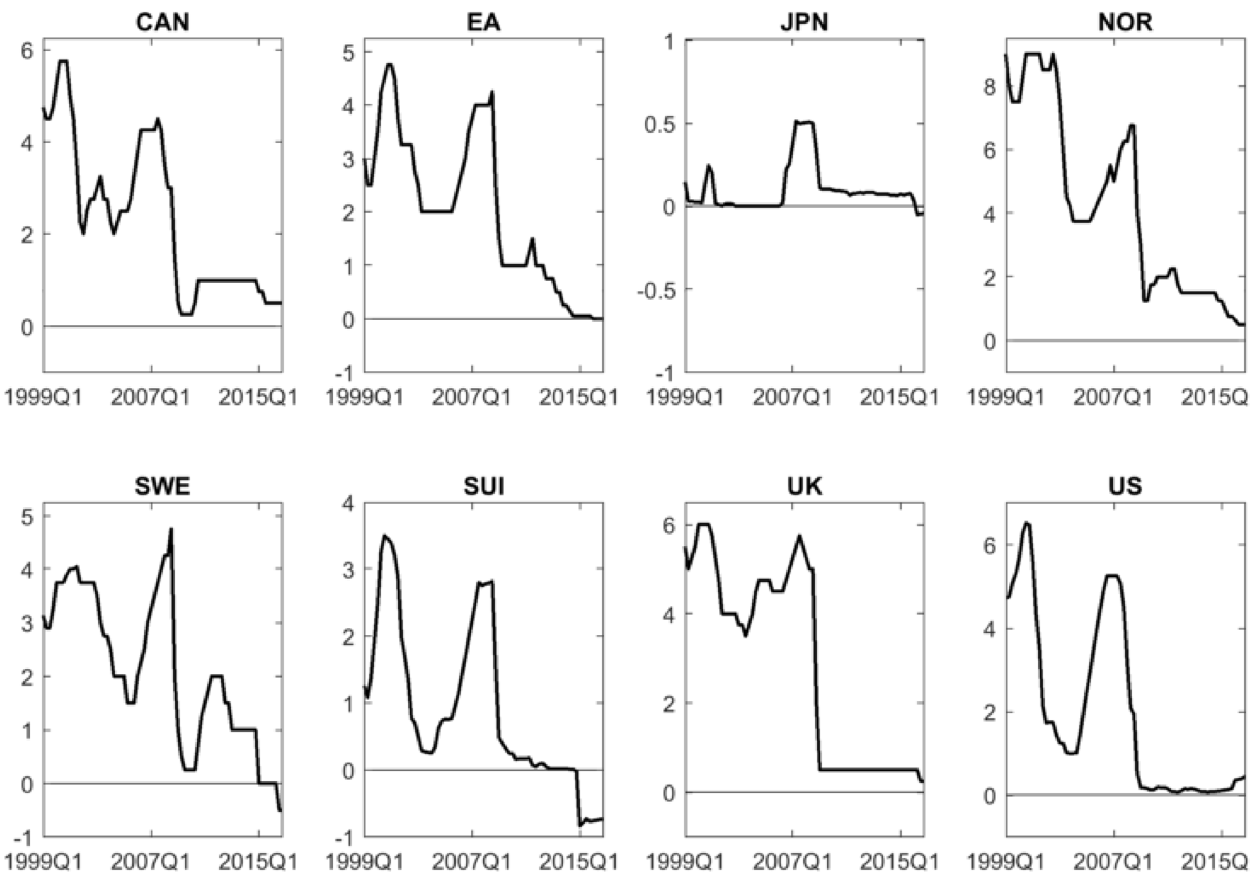

Figure 1: Monetary policy rates in advanced countries.

In the aftermath of the 2008 crisis, advanced economies approached or even crossed the zero bound with their interest rates. Once a country was not able or willing to ease monetary policy further using the policy rate, unconventional measures were employed. The United States, the United Kingdom, and the euro area used large-scale asset purchases aimed at lowering long-term interest rates. Japan initially adopted the same approach to quantitative easing, then followed up with additional quantitative and qualitative monetary easing (QQE) in 2013. It also introduced a negative monetary policy rate in mid-2016. Negative policy rates have been seen in Switzerland and Sweden recently as well. Furthermore, Switzerland imposed an exchange rate floor to prevent currency appreciation between 2011 and 2015.

Figure 1 suggests some degree of homogeneity and close interconnectedness across countries. On the other hand, significant heterogeneity is present in the data, especially after the start of the GFC. Monetary policy rules presumably changed when economies hit the ELB, and the changes differ across countries. The shock propagation mechanism and shock volatilities might also be different across countries after the GFC.

3. Model

The modeling framework is built on the panel vector autoregression (VAR) structure, which allows for cross-sectional heterogeneity and static interdependencies. Cross-sectional heterogeneity allows for differences in the dynamics of macroeconomic variables across countries. This option is especially important for the period of unconventional monetary policy, because countries employed different strategies for easing monetary policy further. Static interdependencies allow for correlation of reduced-form shocks across countries. The GFC spread across the advanced economies primarily through the financial markets within one quarter, so accounting for static interdependencies is crucial, probably more so than accounting for dynamic ones.[4] Ignoring static interdependencies, if present, would lead to less efficient estimates and consequently to less accurate ELB risk estimates. Note also that static interdependence is the only interconnection between the countries in the model. Without that, the model would be effectively a set of single-country models. The case of single-country VARs is discussed in Subsection 5.3.

The model is formulated in mean-adjusted form to deal directly with steady-state macroeconomic variables. Mean adjustment for VAR models is introduced in Villani (2009) and allows the steady state of a macroeconomic variable to be treated as a single parameter. As such, a prior on the steady state can be formulated and the parameter directly estimated. The posterior of the steady state then defines the long-run dynamics of the model. Explicit treatment of the steady state leads to well-defined ELB risk in the medium and long term.

For N countries and for $t = p + 1, \dots, T$, the model is formulated as follows:

[4] Another reason for not implementing dynamic interdependencies in our model is that allowing for them greatly increases the number of estimated parameters.

$$
\begin{pmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{N,t} \end{pmatrix} = \begin{pmatrix} A_1^1 & 0 & \cdots & 0 \\ 0 & A_2^1 & \cdots & 0 \\ \vdots & \vdots & 0 & \vdots \\ 0 & 0 & \cdots & A_N^1 \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{N,t-1} \end{pmatrix} + \cdots + \begin{pmatrix} A_1^p & 0 & \cdots & 0 \\ 0 & A_2^p & \cdots & 0 \\ \vdots & \vdots & 0 & \vdots \\ 0 & 0 & \cdots & A_N^p \end{pmatrix} \begin{pmatrix} y_{1,t-p} \\ y_{2,t-p} \\ \vdots \\ y_{N,t-p} \end{pmatrix} +
$$

$$
+ \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{pmatrix} x_t - \begin{pmatrix} A_1^1 F_1 \\ A_2^1 F_2 \\ \vdots \\ A_N^1 F_N \end{pmatrix} x_{t-1} - \cdots - \begin{pmatrix} A_1^p F_1 \\ A_2^p F_2 \\ \vdots \\ A_N^p F_N \end{pmatrix} x_{t-p} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{N,t} \end{pmatrix}. \tag{1}
$$

The $n \times 1$ vector $y_{c,t} = (y_{c1,t}, \dots, y_{cn,t})'$ denotes the vector of endogenous variables for country c and time t. The coefficient matrices A_c^l relating to the l-th lag of the vector of endogenous variables for country c are of dimension $n \times n$.

The vector of endogenous variables contains real GDP growth, CPI inflation, the short-term interest rate, and the spread between the ten-year government bond yield and the policy rate.[5] The set of endogenous variables is usually chosen according to the shocks that are to be identified. We do not perform structural shock identification in our exercise. The main aim of our choice of variables is to capture both conventional and unconventional monetary policy rules, i.e., the interest rate rule and the quantitative reaction function reflected by the spread.

The question is whether this minimal set can also represent the other unconventional monetary policy measures contained in our data set. Obviously, the vector of endogenous variables can deal with negative interest rates. There could be a problem with the Swiss exchange rate floor, as the exchange rate is not included in the vector of endogenous variables. The effect of this type of policy is captured to the extent to which the floor is reflected by long-term government bond yields.

The $m \times 1$ vector $x_{c,t} = (x_{c1,t}, \dots, x_{cm,t})'$ in (1) contains exogenous variables, which are the same across countries. In our application, the vector of exogenous variables comprises constant terms only. The coefficient matrices relating to exogenous variable F_c are of dimension $n \times m$. The country-specific matrices F_c include the steady states of the endogenous variables, because it follows from (1) that if the process for $y_{c,t}$ is stationary then

[5] The same vector of endogenous variables is employed in Baumeister and Benati (2013), who examine the effect of unconventional monetary policy based on large-scale asset purchases.

$$E\begin{pmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{N,t} \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{pmatrix}. \tag{2}$$

Finally, the vector of error terms $(\varepsilon_{1,t}, \dots, \varepsilon_{N,t})'$ in (1) is distributed independently and normally, with zero mean and covariance matrix Σ of dimension $Nn \times Nn$. The error covariance matrix is generally non-diagonal, allowing for residual correlation between countries. We assume two lags in the benchmark specification.

Let y denote a $NnT \times 1$ data vector for all countries:

$$\bar{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix},$$

where the $nT \times 1$ vector y_c is defined as $y_c = vec(Y_c)$. Y_c is a $T \times n$ matrix of observations for country c and the operator $vec(\cdot)$ denotes the columnwise transformation of the matrix into a vector. Let $\bar{X}$ denote an $(NnT) \times (Nn^2 p)$ data matrix

$$\bar{X} = \begin{pmatrix} \bar{X}_1 & 0 & \cdots & 0 \\ 0 & \bar{X}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \bar{X}_N \end{pmatrix},$$

where $\bar{X}_i = I_n \otimes X_i$, with X_i being a $T \times (np)$ country-specific data matrix comprising up to p lags of Y_i, and $\bar{Z} = I_{Nn} \otimes Z$, where

$$Z = \begin{pmatrix} x_1' & -x_0' & \cdots & -x_{1-p}' \\ x_2' & -x_1' & \cdots & -x_{2-p}' \\ \vdots & \vdots & \ddots & \vdots \\ x_T' & -x_{T-1}' & \cdots & -x_{T-p}' \end{pmatrix}.$$

Finally, define

$$\beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix}, \delta = \begin{pmatrix} \delta_1 \\ \vdots \\ \delta_N \end{pmatrix}, \text{and } \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{pmatrix},$$

where $\beta_c = vec\begin{pmatrix}(A_c^1)' \\ \vdots \\ (A_c^p)'\end{pmatrix}$, and $\delta_c = vec\begin{pmatrix}F_c' \\ F_c'(A_c^1)' \\ \vdots \\ F_c'(A_c^p)'\end{pmatrix}$ for $c = 1,...,N$.

The model (1) can then be written as follows:

$$\bar{y} = \bar{X}\beta + \bar{Z}\delta + \varepsilon, \qquad \varepsilon \sim N\left(0_{NnT\times 1}, \bar{\Sigma}\right). \tag{4}$$

As discussed in section 2, the data set presumably includes changes in policy rules when the ELB was hit and unconventional monetary policies were introduced. Moreover, as suggested by Clark (2011), allowing for changes in shock volatilities results in more accurate density forecasts, even though the time perspective of Clark (2011) is longer than in our case. Translating this into the context of our exercise, a more accurate density forecast means a more accurate estimate of ELB risk. Model (4) is therefore extended in such a way that it allows for two regimes in the form of possible threshold behavior driven by the endogenous variables. Due to the short time series available for the estimation, only one regime change is considered. Whether the change is driven by a change in dynamic coefficients (e.g., in policy rules), by a change in shock volatility, or by both is decided on the basis of model likelihood comparison within the estimation procedure.

Combining regime change with the mean-adjustment procedure forces us to take a stand on the issue of whether the mean-adjustment procedure should be applied to both regimes or to one regime only. Strong prior information exists for the "normal times" regime with standard monetary policy conduct, so mean adjustment is imposed in that regime only. For the other regime, a simple constant term is included in the data matrices.[6]

Allowing for two regimes with mean adjustment applied in one regime only, the model takes the form:

$$\begin{aligned} \bar{y}^{(1)} &= \bar{X}^{(1)}\beta^{(1)} + \varepsilon^{(1)} & \qquad y_t^{TR} &< r \\ \bar{y}^{(2)} &= \bar{X}^{(2)}\beta^{(2)} + \bar{Z}^{(2)}\delta + \varepsilon^{(2)} & \qquad y_t^{TR} &\geq r \end{aligned}, \tag{5}$$

where $\varepsilon^{(1)} \sim N\left(0, \bar{\Sigma}^{(1)}\right)$, $\varepsilon^{(2)} \sim N\left(0, \bar{\Sigma}^{(2)}\right)$, and the matrices $\bar{\Sigma}^{(1)}$ and $\bar{\Sigma}^{(2)}$ are in general non-diagonal. Data matrices and vectors with superscripts denoting the regime refer to the subsamples of $\bar{X}$ and $\bar{y}$ relating to the relevant regime. Note that the data matrix $\bar{X}^{(1)}$ includes the constant term.

[6] To simplify the notation, we assume that mean adjustment is applied to regime 2. The estimation procedure shows that this is indeed the case – the implied steady state of the threshold variable lies above the estimated threshold.

The switch between the two regimes is driven according to the threshold variable, y_t^{TR}, and the threshold r. The threshold variable is a function of the endogenous variables and is defined as the combination of the short-term interest rate and the spread (Figure 2). More precisely, the threshold variable is the average of the average short-term interest rate across countries and the average spread across countries, both lagged by one quarter. The definition of the threshold variable reflects the belief that a regime change is expected when the world economy is around the ELB and unconventional policies aimed at lowering the spread are employed. The threshold parameter r is estimated.

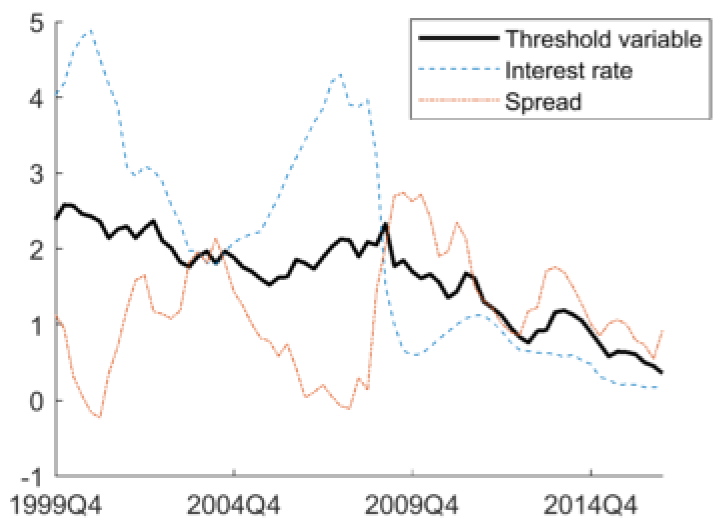

Figure 2: The threshold variable, the average short-term interest rate, and the average spread across countries.

4. Estimation

The estimation approach is Bayesian and combines estimation of a mean-adjusted VAR (Villani, 2009), a hierarchical linear model for VAR (Jarocinski, 2010), and Bayesian estimation of a threshold VAR (Chen and Lee, 1995, and Koop and Potter, 2003). To estimate the model, the country-specific estimates of some parameters exploit information from all countries by means of an exchangeable prior. We assume that the coefficients at the lagged values of the endogenous variables stacked in vector $\beta_c^{(r)}$ are distributed normally around a regime-specific common mean $b^{(r)}$:

$$\beta_c^{(r)} = b^{(r)} + b_c^{(r)}, \qquad c = 1, \ldots, N, r = 1,2 \tag{6}$$

where $b_c^{(r)} \sim N\left(0, \Sigma_c^{b,(r)}\right)$. The spread of the country-specific vectors of the dynamic parameters around the common mean is driven by the overall tightness parameter $\lambda^{(r)}$ defining the variance $\Sigma_c^{b,(r)}$ (see formula A2).

The exchangeable prior is not used for the parameters capturing the steady state, F_c. Strong prior information on the steady states of the endogenous variables is available, so there is no need to pool information across countries through the common mean for those parameters.

The estimation procedure simulates the posterior distributions based on likelihood, priors, and conditional priors, respectively. The vector of model parameters contains dynamic parameters $\beta^{(r)}$, hyperparameters $b^{(r)}$ and $\Sigma_c^{b,(r)}$, error variances $\overline{\Sigma}^{(r)}$, the steady states included in the vector δ, and the threshold r that determines the regimes. Using conditionally conjugate priors yields conditional posterior densities that are easy to draw from within the Gibbs sampler. The conditional posterior of the threshold cannot be expressed by a standard density function and a Metropolis step is used to take a draw of the parameter. The specification of the prior distributions and a description of the sampler can be found in Appendix A.

The likelihood of an ELB event is estimated during the estimation of the model. For a given draw of model parameters, iterated forecasts for up to 48 quarters are computed using random draws of shocks from a given distribution. Note that regime switching is allowed for during the simulation, because the threshold variable is a function of the endogenous variables. Then, for a given period, the proportion of forecasts that are below or at the ELB is computed to estimate the probability of an ELB event. The medium-term ELB risk is computed as the average ELB risk for last eight quarters, i.e., the 41st to 48th quarters. In addition to the estimate of the ELB risk, the expected duration of the ELB event is computed as the average number of quarters for which the simulated forecast remains at the ELB.

During the forecasting, the ELB constraint is imposed in a straightforward manner, because the model is backward looking. The value of the ELB is imposed whenever the one-period-ahead value of the interest rate falls below the ELB. This approach can be interpreted as passive conventional monetary policy. The monetary authority does not lower the interest rate when the estimated interest rate rule suggests doing so. Even with the interest rate rule switched off, unconventional monetary policy still affects the economy according to the estimated spread equation.

The numerical value of the ELB is set differently for each country. First of all, it is important to stress that we impose the ELB on the short-term nominal interest rate, because the policy rate is not directly included in the vector of endogenous variables. For those countries which have not experienced negative policy rates, the ELB is set to zero (Norway, the UK, and the US). In addition, the choice for the US is driven by comparability with other studies. For countries with negative policy rates, the ELB is set to the lowest value of the short-term interest rate in the sample. Finally, following Witmer and Yang (2016), the ELB for Canada is set to -0.5.[7] The ELB values are reported in Table 1.

Table 1: Imposed effective lower bounds.

	ELB
Canada	-0.5
Euro area	-0.3125
Japan	-0.0376
Norway	0
Sweden	-0.78
Switzerland	-0.84
United Kingdom	0
United States	0

Note: The ELB imposed on the short-term interest rate.

The ELB values do not enter the estimation procedure and thus do not affect the estimation of the model parameters. They enter the simulation of the ELB risk and the expected duration of the ELB spell only. As a consequence, a different ELB for a country only affects the country-specific results (the exception is regime change timing, which relates to all countries). Different reasoning for ELB values across countries therefore does not represent an obstacle.

4.1 Data

Real GDP and CPI are seasonally adjusted and enter the vector of endogenous variables as the first difference of their logs. The two series are downloaded from the BIS database. The spread variable is made up of the 10-year bond yield and the monetary policy rate. The bond yields are downloaded from the OECD MEI database. Euro area long-term bond yields are downloaded from the ECB FM database. They are constructed from AAA-rated bond yields.[8] Yields for the UK are downloaded from BoE Statistics and yields for the US from the FRED database. Monetary policy rates are obtained from the IMF IFS. The Swiss National Bank targets the three-month Swiss franc Libor, so the Libor is used as the policy rate. The policy rate for the euro area is the interest rate on main refinancing operations. Finally, short-term interest rates are 3-month interbank rates obtained from the OECD MEI database.

The data are quarterly and the data set covers the period 1999Q1–2016Q4. The choice of quarterly data, as opposed to monthly data, is driven by the fact that the panel VAR specification with static and without dynamic interdependencies is more reasonable in a setting with quarterly data.

---

[7] This value is derived from the cost of storing cash. The estimate is originally for the policy rate; we consider it as the ELB for the short-term interest rate.

[8] An alternative is to obtain yields from OIS rates – for details see ECB (2014).

5. Results

The results are based on 100,000 iterations, with 5,000 iterations as a burn-in period. Every tenth draw is used for inference to deal with autocorrelation of draws. Convergence diagnostics of the sampler and additional estimation results are presented in Appendix B.

The posterior mean of the threshold *r* is 1.27, implying that regime 1 covers the period 2012Q1–2016Q4 and regime 2 the period 1999Q2–2011Q4. Regime 1 thus covers the period when advanced economies hit or approached their ELBs, launched their unconventional monetary policy measures, and gradually reduced the spread between long- and short-term interest rates (Figure 3).

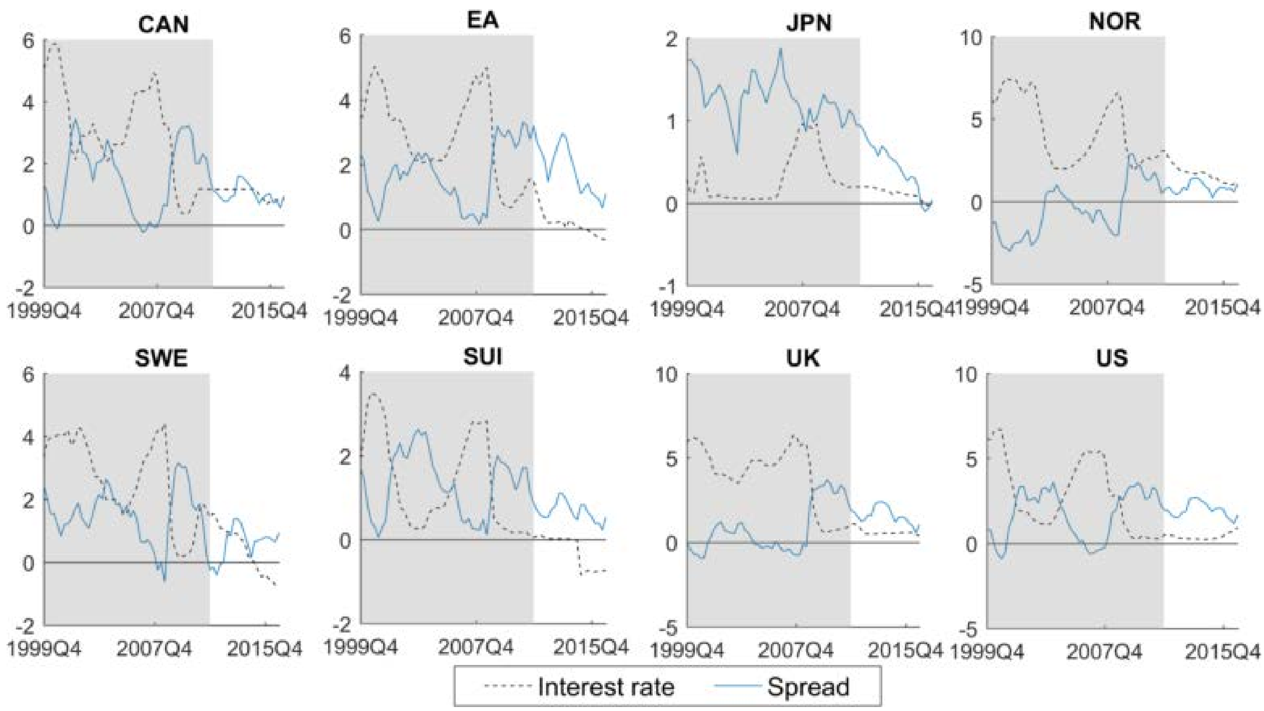

Figure 3: The interest rate and spreads in regime 1 (white) and regime 2 (grey).

The estimated ELB risk is presented in Table 2. The probability of the short-term interest rate being at the ELB in 2017Q1 is driven to a great extent by whether the relevant economy was stuck at the ELB in the last quarter used for the estimation, i.e., in 2016Q4. So, for the euro area, Japan, and Sweden, which are at their assumed ELBs, the ELB risk is in the range of 0.37–0.53. For the other countries, the ELB risk is lower. For Canada, Norway, and the US, it is virtually zero for 2017Q1.

Table 2: ELB risk.									
	2017Q1	2017Q2	2017Q3	2017Q4	2018Q1	2018Q2	2018Q3	2018Q4	MR
Canada (-0.50)	0.00	0.00	0.00	0.02	0.06	0.06	0.09	0.08	0.01
Euro area (-0.3125)	0.40	0.55	0.62	0.35	0.23	0.06	0.06	0.08	0.03
Japan (-0.0376)	0.37	0.49	0.48	0.49	0.30	0.50	0.65	0.53	0.16
Norway (0)	0.00	0.00	0.00	0.07	0.21	0.15	0.17	0.11	0.05
Sweden (-0.78)	0.53	0.37	0.43	0.52	0.47	0.13	0.17	0.20	0.03
Switzerland (-0.84)	0.23	0.52	0.46	0.29	0.31	0.32	0.30	0.21	0.04
United Kingdom (0)	0.02	0.06	0.08	0.16	0.14	0.14	0.09	0.08	0.02
United States (0)	0.00	0.01	0.10	0.24	0.26	0.22	0.19	0.16	0.06
Notes: The imposed ELB is indicated in parenthesis.									
MR denotes medium-term ELB risk.									

The ELB risk profile during the first eight forthcoming quarters (2017Q1–2018Q4) reflects the current phase of, and the outlook for, the business cycle. For the euro area, the ELB risk increases in the first three quarters and then declines due to a delay in its economic recovery with respect to other advanced economies.[9] For Canada, Norway, and the US, the ELB risk is very low for the first three quarters and rises after that, because these countries experience an expansionary phase of the business cycle in the first quarters of 2017. The comparison across countries is also influenced by the different ELBs. The lower the ELB, the lower the probability of the economy being at the ELB, all other things being equal.

The medium-term ELB risk is determined mainly by the posterior of the interest rate steady state, the assumed ELB, and the estimated average size of the shocks to macroeconomic variables. It ranges from 0.01 (Canada) to 0.16 (Japan) – see the MR column in Table 2. The numbers can be interpreted as meaning the percentage of the time (quarters) the ELB is binding. For Canada, the economy is at the ELB 1 percent of the time, i.e., for one quarter in 25 years. For Japan, on the other hand, the ELB is estimated to be binding 16 percent of the time, i.e., for four years every 25 years. By combining the short-run outlook based on the business cycle and medium-run simulations drawing on steady-state values, non-monotonicity of ELB risk measures can arise, as documented in Table 2.

The presented ELB risk estimates can be compared with the results provided by the recent literature. Assuming a steady-state nominal interest rate of 3 percent and imposing an ELB of zero, Kiley and Roberts (2017) estimate the frequency of ELB events for the US to be 17.4 percent based on the DSGE model of Lindé et al. (2016) and 31.7 percent based on the FRB/US model. The estimate based on the DSGE model is close to the estimates in Hills et al. (2016) and Nakata (2017b), which draw on calibrated DSGE models and provide an estimated

[9] Going back to the motivation mentioned in the Introduction, the ELB risk is estimated to decrease substantially by the beginning of 2018 in the euro area. Such timing justifies the reduction of the monthly pace of asset purchases planned for 2018.

probability of around 14 percent. If we assume the same ELB as in Kiley and Roberts (2017) and estimate the steady-state nominal interest rate at 2.79 percent, the ELB risk from our model is 5.71, which is much lower than the figures obtained by Kiley and Roberts (2017). Lower ELB risk estimates can also be found in the literature based on estimated nonlinear DSGE models. Gust et al. (2017) found for the US that the average probability of hitting the ZLB is 4 percent, while Richter and Throckmorton (2016) estimated the probability at around 5 percent.

Part of the difference can be explained by the fact that our ELB risk estimates include the effects of unconventional monetary policy, which lowers the probability of hitting/staying at the ELB. Whenever the US economy is simulated to be in regime 1, the estimated impact of unconventional monetary policy is present in the simulation. Unconventional measures are not included in Kiley and Roberts (2017). Another reason suggested by the plain comparison is that nonlinear models tend to deliver lower ELB risk – the effect of nonlinearity on ELB risk estimates is discussed in Subsection 5.2.

Nakata (2017a) combines survey-based macroeconomic projections and stochastic simulations of the FRB/US model and defines the ELB risk as the probability that the federal funds rate will be constrained by the ELB for at least one quarter in the next three years. For 2016Q4, the ELB risk is estimated to exceed 50 percent for all three survey-based projections considered (Survey of Primary Dealers, Survey of Professional Forecasters, and Summary of Economic Projections). Excluding the period of elevated macroeconomic volatility in the 1970s and 1980s, which reduces the estimated size of shocks, the ELB risk lies between 40 and 50 percent. Assuming the ELB consistent with Nakata (2017a), Nakata's ELB risk for the US estimated within our framework is 57 percent. Nakata's ELB risk measures for other countries are reported in Table 3.

Table 3: Probability of an ELB event in next three years.

CAN	EA	JPN	NOR	SWE	SUI	UK	US	Nakata (2017a) – US
0.20	0.96	0.97	0.40	0.93	0.77	0.43	0.57	0.40-0.50

The estimated ELB risk can be complemented with estimates of the expected duration of the ELB event, i.e., the average number of quarters the economy is stuck at the ELB given that it hits the ELB in a given quarter or remains at the ELB from the previous quarter (Table 4). The medium-term duration (column MR) is computed starting with the 11th quarter to filter out the initial conditions and the business cycle phase and ending with the 43rd quarter to filter out the effect of the maximum time period of 48 quarters used in the simulation procedure. Note that the expected duration cannot be computed if no simulated forecast hits the ELB (as is the case for Canada and Norway in 2017Q1).

Table 4: Expected duration.									
	2017Q1	2017Q2	2017Q3	2017Q4	2018Q1	2018Q2	2018Q3	2018Q4	MR
Canada	-	1.00	3.37	3.16	2.11	2.64	2.10	2.13	2.17
Euro area	2.16	2.01	1.81	1.65	1.20	3.78	4.45	4.25	3.04
Japan	2.39	2.78	2.73	1.86	2.65	2.56	2.50	2.55	2.78
Norway	-	2.13	2.92	2.74	2.10	2.13	1.93	2.59	2.94
Sweden	2.13	3.00	2.45	1.62	1.46	2.09	2.37	2.31	2.15
Switzerland	3.09	3.13	2.31	2.35	3.08	2.75	2.26	3.12	2.66
United Kingdom	1.77	1.85	2.23	1.79	1.85	1.80	2.29	2.28	2.51
United States	1.00	3.18	3.75	2.82	2.43	2.35	2.30	2.19	2.35

Note: MR denotes medium-term expected duration defined as the mean duration over the 11th to the 43rd quarter.

The highest medium-term expected duration (three quarters) is obtained for the euro area, while the lowest expected durations are observed for Canada, Sweden, and the US.

For the US, the medium-term expected duration is estimated to be 2.35 quarters. Hills et al. (2016) and Nakata (2017b) estimated the expected duration at 9 quarters. A much shorter duration of the ELB spell is estimated by Gust et al. (2017) and Richter and Throckmorton (2016), who give an average duration of around 3 quarters. Similarly to the estimates of the ELB risk, the estimate of 2.35 quarters for the US is closer to the literature dealing with estimated nonlinear DSGE models than calibrated linearized DSGE models.[10]

The ELB risk and duration estimates involve unconventional monetary policy, as it is reflected in the data used for the estimation. So, the estimate for the US assumes a relatively strong unconventional easing of monetary policy at the ELB, while the estimate for Canada draws on the estimated conventional monetary policy only, because the Bank of Canada has never employed unconventional measures in the post-2008 period. The two countries' ELB risk should be compared with this consideration in mind. If no unconventional monetary policy had been conducted in the US after 2008, the ELB risk would be higher and the difference with respect to the ELB risk in Canada would be more profound.

5.1 The role of mean adjustment

The extensive literature dealing with equilibrium values of macroeconomic variables in advanced countries allows us to formulate informative priors on the steady-state parameters. Regarding the

[10] When comparing predictions of ELB spell duration, estimates from calibrated linearized DSGE models seem to be closer to surveys than estimates based on estimated nonlinear models. For example, Survey of Professional Forecasters and Primary Dealers Surveys suggest for the US an expected duration starting at 5 quarters at the beginning of 2011, increasing close to 10 quarters in 2012 and 2013, and then steadily decreasing toward zero at the end of 2015. The expected duration based on panel VAR does not exceed 2 quarters throughout the ZLB period in the US. In addition to the simplistic dynamics in the panel VAR model, forward guidance not captured by endogenous variables could explain the difference between the estimated expected duration and survey data.

informativeness of the priors, two extreme cases can be distinguished. First, imposing a very tight prior is similar to the situation where the steady-state values are calibrated. Second, imposing a very loose prior resembles the case where no mean adjustment is conducted and allows us to discuss the role of mean adjustment in the estimation of ELB risk. Examining the two extreme cases can shed some light on, respectively, the effect of calibration and the effect of not imposing any equilibrium values on the ELB risk estimates in the previous literature.

Table 5: ELB risk in the medium term for different priors on the steady-state parameters.

	Benchmark	Tight	Loose
Canada	0.01	0.01	0.03
Euro area	0.03	0.02	0.06
Japan	0.16	0.06	0.17
Norway	0.05	0.03	0.05
Sweden	0.03	0.03	0.06
Switzerland	0.04	0.04	0.07
United Kingdom	0.02	0.02	0.05
United States	0.06	0.04	0.10

Note: The tight prior is given by a 95 percent confidence band of width 0.1 and the loose prior by a 95 percent confidence band of width 10.

Table 5 reports the medium-term probabilities of an ELB event for the benchmark case (a 95 percent confidence band for the prior mean on the steady-state parameters of width 2 for GDP growth, inflation, and the interest rate and of width 1.5 for the spread), the case with a very tight prior on the steady state (a 95 percent confidence interval of width 0.1 for all variables), and the case with a loose prior (width 10). The tight prior results in the same or a lower ELB risk. The difference is driven to a great extent by the posterior of the interest rate steady state. With the exception of Norway, the tight prior implies a higher steady-state interest rate (Table 6) and, ceteris paribus, a lower ELB risk. The comparison in Table 5 suggests that the calibration of steady states plays an important role in ELB risk estimation. For example, calibrating the interest rate steady state to 1 percent for Japan implies a reduction in the ELB risk of more than half.

Table 6: Prior mean and posterior means of the steady-state interest rate.

	Prior	Posterior		
		Benchmark	Tight	Loose
Canada	4.00	3.38	4.00	2.22
Euro area	3.00	2.81	3.00	2.47
Japan	1.00	0.34	1.00	0.24
Norway	3.00	3.83	3.00	4.03
Sweden	3.00	2.70	3.00	1.62
Switzerland	3.00	2.82	3.00	1.48
United Kingdom	4.00	3.82	3.99	3.16
United States	3.00	2.79	3.00	1.99

Comparing the estimates for the loose prior on the steady states with the benchmark setting demonstrates the role of mean adjustment. It turns out that without treating the steady state explicitly, the estimated ELB risk is higher for some countries. For example, the estimate for the US suggests that it would be stuck at the ELB 10 percent of the time. With mean adjustment, the ELB situation is estimated to be observed only 6 percent of the time. In the short term, the effect of mean adjustment is negligible. Within a period of one year, the distance between the ELB event probabilities in the benchmark and the case without mean adjustment does not exceed 0.01 in absolute terms.

Table 7: ELB risk in the medium term for the benchmark and the tight prior centered on the estimated steady state.

	Benchmark	Tight on SS
Canada	0.01	0.01
Euro area	0.03	0.03
Japan	0.16	0.16
Norway	0.05	0.04
Sweden	0.03	0.03
Switzerland	0.04	0.04
United Kingdom	0.02	0.02
United States	0.06	0.05

Note: “Tight on SS” means that the prior on the steady state is centered on the posterior mean of the benchmark case and the prior is tight (the width of the 95 percent confidence band is 0.01).

As noted above, the change in the ELB risk estimates for tight priors with respect to the benchmark is influenced by the change in the posterior of the interest rate. In addition, part of the difference can be explained by the fact that the benchmark case allows some uncertainty of the steady state, which is not the case when the steady state is calibrated. The strength of the effect is demonstrated in Table 7. The role of uncertainty in the benchmark estimation is filtered out by employing the tight prior centered on the posterior mean of the benchmark specification. In other words, the steady state is calibrated to the benchmark posterior values and the uncertainty of the steady state is negligible. The table shows that the influence on the ELB risk is of order 0.01 at most. Ignoring uncertainty of the steady state leads to underestimation of the ELB risk.

5.2 The role of regime change

In addition to the theoretical reasoning for allowing regime change due to a structural change in monetary policy conduct, the estimation results can justify the chosen modeling framework ex post. A natural question is whether there are changes in the estimates of the model parameters between regimes. Furthermore, one can ask whether the regime change is driven by a change in shock volatilities, a change in dynamic coefficients, or both.

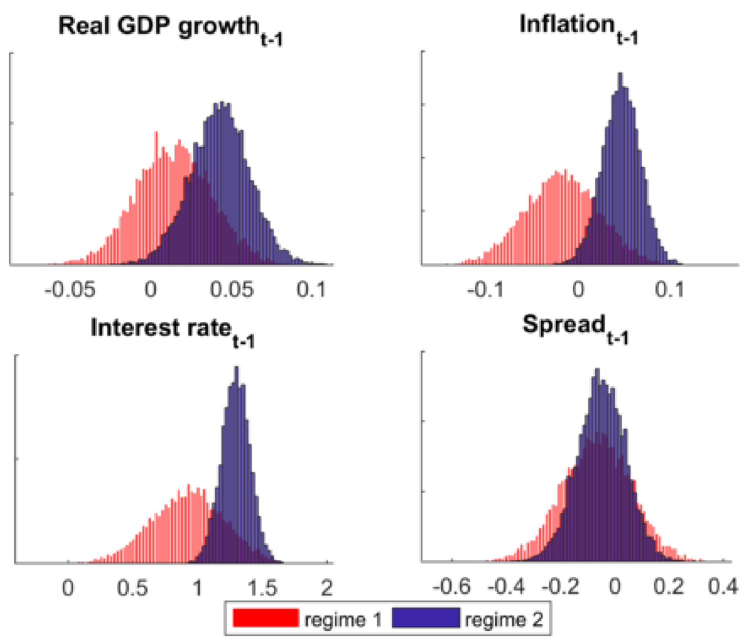

Figure 4: The posterior distribution of the parameters at the first lags of the endogenous variable in the interest rate equation for the US.

Figures 4, 5, and 6 present the posterior distributions of the parameters at the first lag of the endogenous variables in the equation for the interest rate and spread for the US and the posterior distributions of the diagonal elements of the error covariance matrix for the US. It turns out that the regime change is driven primarily by a change in shock volatilities, with the posterior distributions covering mostly different parameter values in the two regimes (Figure 6). The volatility of the shocks to the four equations for the US is much lower in regime 1, when the interest rate is stuck at, or close to, the ELB, and the main GFC-related drop in real economy variables happened before 2012, i.e., in regime 2.

The passive conventional monetary policy for regime 1 is manifested by posteriors centered on zero for all parameters at the first lag except for the interest rate (Figure 4). In regime 2, mainly positive numbers are covered by the posterior distributions of the parameters at the first lag of real growth and inflation, which is reminiscent of the standard backward-looking Taylor rule. In the spread equation, the lagged interest rate is found to have a zero effect on the spread in regime 1, while regime 2 exhibits the standard negative relationship between the two. Lagged real GDP growth and inflation do not affect the spread much (Figure 5).

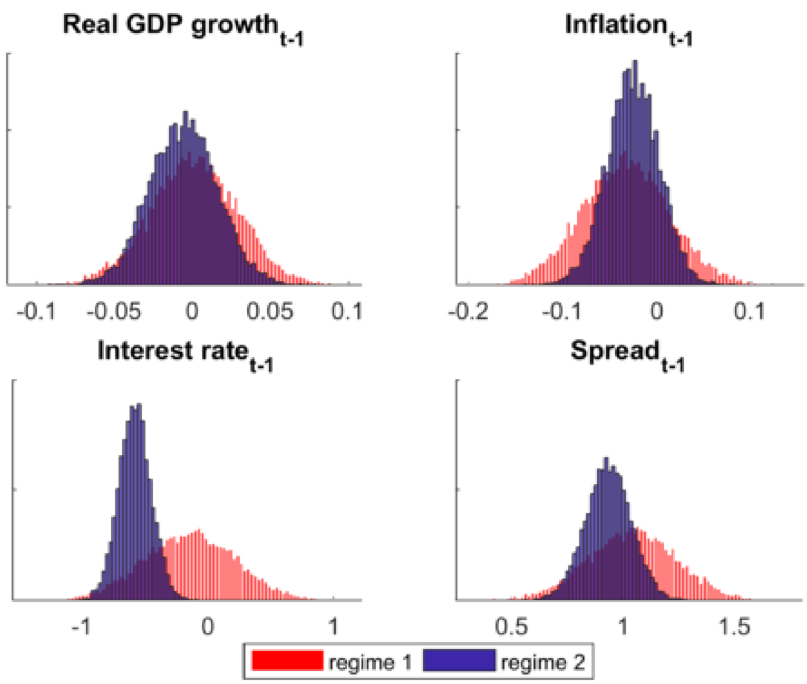

Figure 5: The posterior distribution of the parameters at the first lags of the endogenous variable in the spread equation for the US.

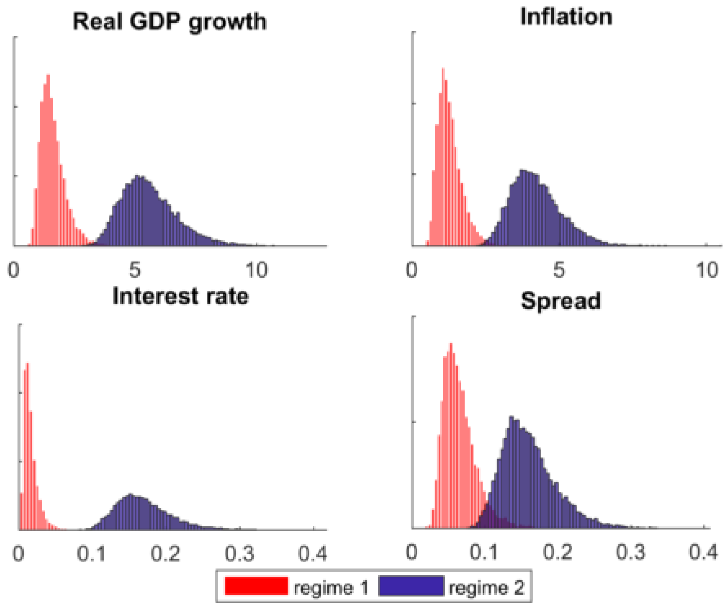

Figure 6: The posterior distribution of the diagonal elements of the shock volatilities for the US.

Allowing for regime change is one possible explanation for the different ELB risks estimated in the literature. The difference between estimates based on linear models (Kiley and Roberts, 2017, Hills et al., 2016, and Nakata, 2017b) and those based on nonlinear models (Gust et al., 2017, Richter and Throckmorton, 2016) exceeds 10 percentage points. Table 8 compares the medium-term ELB risk in the benchmark model and the linear model that does not allow for regime change.

The ELB event probabilities estimated with the linear model are different but not systematically moved in any specific direction. So, the nonlinearity seems not to be the primary reason for the different estimates provided by the literature. However, the differences may be relevant from the policy maker's point of view. For example, based on the linear model the euro area medium-term ELB risk is estimated at 0.05, while the model with regime change suggests that the risk is 0.03. The frequency of an ELB episode for the euro area decreases from more than one year in 25 to less than one year in 25.

Table 8: ELB risk in the medium term for the benchmark and the model without regime change.

	Regime change	No regime change
Canada	0.01	0.02
Euro area	0.03	0.05
Japan	0.16	0.22
Norway	0.05	0.03
Sweden	0.03	0.04
Switzerland	0.04	0.05
United Kingdom	0.02	0.05
United States	0.06	0.05

5.3 The role of panel structure

Exploiting the panel structure of data with observed ELB spells allows for estimation of the whole set of model parameters despite the short time series available for a single country. In addition, taking into account the correlation of shocks across countries leads to more efficient estimates, which, in turn, result in more accurate simulation of the ELB risk due to the fact that the parameters' uncertainty enters the simulation procedure. On the other hand, if the heterogeneity between countries is substantial and the interdependence is weak, the panel approach may not be preferable.

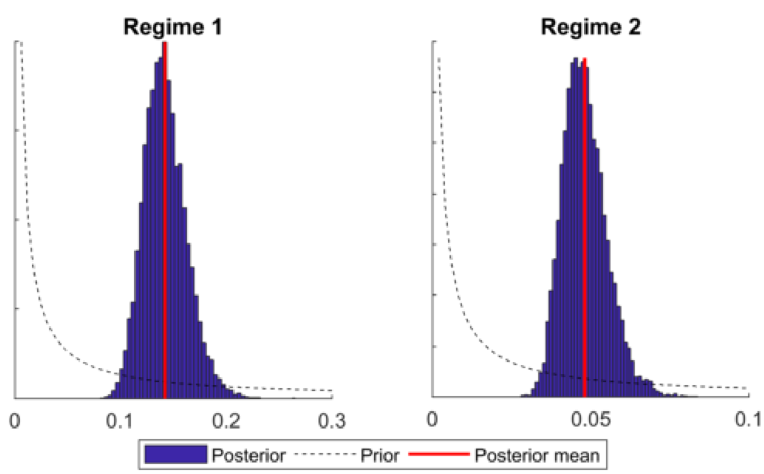

Figure 7: The posterior distribution of parameter λ in the two regimes.

The degree of commonality of the macroeconomic dynamics across countries in the two regimes is driven by the overall tightness parameter $\lambda^{(r)}$. As shown in Figure 7, the posterior mean of the parameter is higher in regime 1 than in regime 2. A higher $\lambda^{(r)}$ suggests more divergent coefficients across countries or more tightly estimated country coefficients. There are 20 observations in regime 1 and 49 in regime 2 and, as suggested by the posteriors presented in Figures 4 and 5, we do not expect the coefficients in regime 1 to be more tightly estimated. Therefore, the differences in dynamics across countries in regime 1 are probably substantial. This should not come as a surprise, because regime 1 contains different unconventional monetary policy measures employed during the Great Recession.

The posterior mean of $\lambda^{(1)}$ is 0.14 and that of $\lambda^{(2)}$ is 0.05. Following the discussion in Jarocinski (2010), the square roots of the posterior means are comparable to the usual overall tightness used when setting the Minnesota prior. In our case, the square roots are 0.38 and 0.22, close to the interval of 0.1–0.2 covering the usual values used for overall tightness. This gives us some confidence that the corresponding prior mean provides valuable information. The estimated $\lambda^{(r)}$ suggests that some pooling of information across countries is present and justifies the use of a common mean when the country-specific dynamic parameters are estimated.

The parameter $\lambda^{(r)}$, however, does not capture the possible heterogeneity in the timing of the regime change, which is assumed to be the same for all countries. If there are significant differences in the regime change timing across countries, the ELB measures could be inaccurate. To set up the model, we face a trade-off between the possibility of pooling information across countries in the model with regime change and inaccuracy due to country differences in the regime change date.

To shed some light on the issue of country heterogeneity with respect to regime change, single-country mean-adjusted VARs are estimated in this subsection. The single-country VARs are estimated similarly to their panel VAR counterpart. The Normal-inverse Wishart prior is assumed for the dynamic coefficients and the error covariance matrix. The prior on the dynamic coefficients is the same as that imposed on the common mean in the panel VAR case. The priors on the steady state and the threshold remain the same. The threshold variable is now defined as the single country average of the short-term interest rate and spread.

Table 9: Switch date.	
Panel VAR	2011Q4
Single-country VARs:	
Canada	2010Q4
Euro area	2010Q4
Japan	2010Q2
Norway	2009Q2
Sweden	2011Q2
Switzerland	2010Q4
United Kingdom	2011Q1
United States	2011Q4

Table 9 reports the switch dates – the last quarter of regime 2 – for each country estimated separately. It turns out that all countries switch their regimes within four quarters (2010Q4–2011Q4), except Japan (2010Q2) and Norway (2009Q2). For those two countries, the additional accuracy gained by allowing for regime change may not be high. Especially for Japan, the medium-run ELB risk found within the panel VAR (0.16) can be viewed as too low, especially when the fact that the ZLB event is observed for almost the whole data sample is taken into account.

Table 10: ELB risk in the medium term for the benchmark and the model without regime change.

	panel VAR	single-country VAR
Canada	0.01	0.03
Euro area	0.03	0.13
Japan	0.16	0.35
Norway	0.05	0.07
Sweden	0.03	0.14
Switzerland	0.04	0.15
United Kingdom	0.02	0.26
United States	0.06	0.08

Table 10 shows the change in the medium-term ELB risk estimates if single-country VAR is employed. The comparison with the benchmark panel VAR shows higher probabilities of an ELB event based on single-country VAR in the medium term. The difference is not driven by the steady-state estimates. For example, for the UK the medium-term ELB risk increases from 0.03 to 0.37 while the posterior mean of the interest rate steady state increases from 3.82 to 3.92 and thus, ceteris paribus, should imply a lower ELB risk. The effect of the improved efficiency of the estimates in the panel VAR seems to dominate and justifies the use of the panel VAR even though some evidence of heterogeneity of regime-switching dates is found.

6. Conclusions

This paper provides new estimates of the probability and expected duration of an ELB event for eight advanced economies. Such estimates are very important for policy makers, especially those in central banks, because the likelihood of the economy being stuck at the ELB is related to monetary policy conduct and central bank balance sheet size management.

On the one hand, the task is simple, because the employed methodology always results in a number, and the available data, which cover only a few ELB events, preclude any rigorous ex-post assessment of the accuracy of the ELB risk estimates. On the other hand, without such a forecasting accuracy exercise, the reasoning for the approach underlying the estimates has to be very clear and sound. In this paper, I motivate all the features of the model with the aim of obtaining accurate ELB risk estimates. In addition, the model features are related to the contentious aspects of previous approaches to estimating ELB risk.

So, mean adjustment is present to obtain well-behaved long-run dynamics and to incorporate out-of-data information on equilibrium values less strictly than during the calibration of the model. As a consequence, uncertainty relating to the equilibrium values is present in the ELB risk estimates. Next, allowing for regime change seems to be necessary, as I work with a statistical model, not a structural one. Finally, the panel nature of the data is a particular advantage in the situation, where only a short time span of data is available and accounting for static interdependencies is necessary (and sufficient) if financial markets play an important role in shock transmission between countries. The multi-country perspective may enrich the country-specific estimates available so far in the literature.

Some methodological issues are still open. The ELB situation can be viewed as a tail event. As such, the focus of the modeling procedure should be on accurate modeling of distribution tails, especially for the interest rate distribution. While the regime change can be viewed as an attempt to go in this direction, the assumption of normally distributed errors may not be too realistic. Extending the methodology to include shocks exhibiting fat tails represents a natural next step for future research.

References

Baumeister, C., and L. Benati (2013): "Unconventional Monetary Policy and the Great Recession: Estimating the Macroeconomic Effects of Spread Compression at the Zero Lower Bound," *International Journal of Central Banking*, 9(2), 165–212.

Canova, F., and M. Ciccarelli (2013): "Panel Vector Autoregressive Models: A Survey," ECB Working Paper No. 1507.

Chen, C. W. S., and J. C. Lee (1995): "Bayesian Inference of Threshold Autoregressive Models," *Journal of Time Series Analysis*, 16, 483–492.

Chung, H., Laforte, J. P., Reifschneider, D., and J. C. Williams (2012): "Have We Underestimated the Likelihood and Severity of Zero Lower Bound Events?" *Journal of Money, Credit and Banking*, 44(1), 47–82.

Clark, T. E. (2011): "Real-Time Density Forecasts from Bayesian Vector Autoregressions with Stochastic Volatility," *Journal of Business & Economic Statistics*, 29(3), 327–341.

Coenen, G. (2003): "Zero Lower Bound: Is It a Problem in the Euro Area?" ECB Working Paper No. 269.

Dieppe, A., Lagrand, R., and B. van Roye (2016): "The BEAR Toolbox," ECB Working Paper No. 1934.

ECB (2014): "Euro Area Risk-Free Interest Rates: Measurement Issues, Recent Developments and Relevance to Monetary Policy," ECB Monthly Bulletin, July 2014.

Fujiwara, S., Iwasaki, Y., Muto, I., Nishizaki, K., and N. Sudo (2016): "Developments in the Natural Rate of Interest in Japan," Bank of Japan Review, 2016-E-12.

Gust, C., Herbst, E., López-Salido, D., and M. E. Smith (2017): "The Empirical Implications of the Interest-Rate Lower Bound," *American Economic Review*, 107(7), 1971–2006.

Hills, T. S., Nakata, T., and S. Schmidt (2016): "The Risky Steady State and the Interest Rate Lower Bound," Finance and Economics Discussion Series 2016-009, Board of Governors of the Federal Reserve System (U.S.).

Jarocinski, M. (2010): "Responses to Monetary Policy Shocks in the East and the West of Europe: A Comparison," *Journal of Applied Econometrics*, 25(5), 833–868.

Kiley, M. T., and J. M. Roberts (2017): "Monetary Policy in a Low Interest Rate Environment," Brookings Papers on Economic Activity Conference, March 23–24, 2017.

Koop, G., and S. M. Potter (2003): "Bayesian Analysis of Endogenous Delay Threshold Models," *Journal of Business & Economic Statistics*, 21(1), 93–103.

Laubach, T., and J. C. Williams (2003): "Measuring the Natural Rate of Interest," *Review of Economics and Statistics*, 85, 1063–1070.

LeSage, J. (1999): "Applied Econometrics Using MATLAB," University of Toledo.

Lindé, J., Smets, F., and R. Wouters (2016): "Challenges for Central Bank Macro Models," prepared for the Handbook of Macroeconomics.

Nakata, T. (2017a): "Model-Based Measures of ELB Risk," FEDS Notes, Washington: Board of Governors of the Federal Reserve System, August 23, 2017.

Nakata, T. (2017b): "Uncertainty at the Zero Lower Bound," *American Economic Journal: Macroeconomics*, 9(3), 186–221.

Raftery, A. E., and S. Lewis (1992): "How Many Iterations in the Gibbs Sampler?" In: Bernardo, J., J. Berger, A. P. Dawid, and A. F. M. Smith (eds.), Bayesian Statistics, 763–773, Oxford University Press.

Reifschneider, D. L., and J. C. Williams (2000): "Three Lessons for Monetary Policy in a Low-Inflation Era," *Journal of Money, Credit and Banking*, 32, 936–966.

Villani, M. (2009): "Steady-State Priors for Vector Autoregressions," *Journal of Applied Econometrics*, 24, 630–650.

Williams, J. C. (2014): "Monetary Policy at the Zero Lower Bound: Putting Theory into Practice," Hutchins Center on Fiscal & Monetary Policy at Brookings, January 2014.

Witmer, J., and J. Yang (2016): "Estimating Canada's Effective Lower Bound," Bank of Canada Review, spring 2016.

Appendix A: Bayesian estimation

The posterior distributions of the model parameters are computed employing the Bayes formula. The prior distribution for $\beta^{(1)}$ and $\beta^{(2)}$ is formulated conditional on hyperparameters $b^{(1)}$, $b^{(2)}$, $\Sigma_c^{b,(1)}$, and $\Sigma_c^{b,(2)}$ for $c = 1,...,N$ and the priors on the hyperparameters enter the formula according to the definition of the conditional probabilities:

$$\pi\left(\beta^{(1)},\beta^{(2)},b^{(1)},b^{(2)},\Sigma_c^{b,(1)},\Sigma_c^{b,(2)},F,\overline{\Sigma}^{(1)},\overline{\Sigma}^{(2)},r \mid \overline{y}\right) \propto \pi\left(\overline{y} \mid \beta^{(1)},\beta^{(2)},F,\overline{\Sigma}^{(1)},\overline{\Sigma}^{(2)},r\right) *$$
$$* \pi\left(\beta^{(1)} \mid b^{(1)},\Sigma_c^{b,(1)}\right)\pi\left(\beta^{(2)} \mid b^{(2)},\Sigma_c^{b,(2)}\right)\pi\left(b^{(1)}\right)\pi\left(b^{(2)}\right)\prod_{c=1}^{N}\left[\pi\left(\Sigma_c^{b,(1)}\right)\pi\left(\Sigma_c^{b,(2)}\right)\right]\pi\left(\overline{\Sigma}^{(1)}\right)\pi\left(\overline{\Sigma}^{(2)}\right)\pi(F)\pi(r) \quad \text{(A1)}$$

In the following subsections, the particular ingredients of the Bayes rule are discussed in turn. Subsection A.4 presents the sampler.

A.1 Likelihood

The likelihood of the model (4) is as follows:

$$\pi\left(\overline{y} \mid \beta^{(1)},\beta^{(2)},F,\overline{\Sigma}^{(1)},\overline{\Sigma}^{(2)},r\right) = \left|\overline{\Sigma}^{(1)}\right|^{-\frac{1}{2}}\left|\overline{\Sigma}^{(2)}\right|^{-\frac{1}{2}}$$
$$\exp\left\{-\frac{1}{2}\sum_{r=1}^{2}\left(\overline{y}^{(r)} - \overline{X}^{(r)}\beta^{(r)} - I_t(r=2)\overline{Z}^{(2)}\delta\right)'\left(\overline{\Sigma}^{(r)}\right)^{-1}\left(\overline{y}^{(r)} - \overline{X}^{(r)}\beta^{(r)} - I_t(r=2)\overline{Z}^{(2)}\delta\right)\right\},$$

where the indicator of regime 2, $I(r=2)$, equals one if the system is in regime 2 at time *t* and zero otherwise.

A.2 Priors

From (6) it follows that for country *c* and regime *r*, the vector $\beta_c^{(r)}$, made up of parameters relating to the lagged vectors of the endogenous variables (and of a constant term in the case of regime 1), is distributed normally with mean $b^{(r)}$ and country-specific variance $\Sigma_c^{b,(r)}$. The common mean error covariance matrix $\Sigma_c^{b,(r)}$ is treated as a proportion of the $\left(n^2 p + n\right)\times\left(n^2 p + n\right)$ diagonal matrix $\Omega_c^{b,(1)}$ and the $\left(n^2 p\right)\times\left(n^2 p\right)$ diagonal matrix $\Omega_c^{b,(2)}$, which are defined in the manner of the Minnesota prior: the variance of the parameter in the *i*-th equation at the lagged values of the *j*-th variable is given by:

$$\frac{\sigma_{c,i}^2}{\sigma_{c,j}^2},$$

where $\sigma_{c,i}$ and $\sigma_{c,j}$ are estimated standard errors from univariate AR(p) models for the corresponding endogenous variables and serve as scaling parameters to account for the different sizes of the parameters. The standard errors are estimated on the whole sample and are the same for both $\Omega_c^{b,(r)}$. In addition, the matrix $\Omega_c^{b,(1)}$ includes the variance for the intercept, which is set to $10^2\sigma_{c,i}^2$ for the i-th equation.

Given $\Omega_c^{b,(r)}$, the common mean covariance matrix is then defined as:

$$\Sigma_c^{b,(1)} = \left(\lambda_1^{(1)} \otimes I_{n^2 p+n}\right)\Omega_c^{b,(1)} \text{ and } \Sigma_c^{b,(2)} = \left(\lambda_1^{(2)} \otimes I_{n^2 p}\right)\Omega_c^{b,(2)}. \tag{A2}$$

The regime-specific parameter $\lambda_1^{(r)}$ plays a role in the prior's overall tightness and its posterior distribution drives the extent to which the vector $\beta_c^{(r)}$ is allowed to change across countries. Moreover, the parameter $\lambda_1^{(r)}$ is the only parameter in $\Sigma_c^{b,(r)}$ that is not fixed and is estimated. The prior on $\lambda_1^{(r)}$ is assumed to be inverse-Gamma distributed:

$$\lambda_1^{(r)} \sim IG\left(\frac{s_0}{2}, \frac{v_0}{2}\right),$$

with $s_0 = 0.001$ and $v_0 = 0.001$.

The prior on the common mean $b^{(r)}$ is assumed to have the standard form of the Minnesota prior. The prior is normal centered around the AR(1) process:

$$b^{(r)} \sim N\left(B^{(r)}, \Xi^{b,(r)}\right),$$

with the prior mean $B^{(r)}$ made up of zeros, except for the coefficient at the first own lag of the variables in levels (interest rate, spread), which is 0.9.[11] The prior variance $\Xi^{b,(r)}$ is set using matrices $\Omega_c^{b,(r)}$, which define the variance of country-specific $\beta_c^{(r)}$ around the common mean. We define

$$\Xi^{b,(r)} = \left(\lambda_1^b \otimes I_{n^2 p}\right)\frac{1}{N}\sum_{c=1}^{N}\Omega_c^{b,(r)} .$$

The overall tightness parameter λ_1^b is set to the standard value of 0.01.

The prior on the error covariance matrix $\Sigma^{(r)}$ is assumed be inverse-Wishart:[12]

$$\Sigma^{(r)} \sim W^{-1}\left(0.01 I_{Nn}, Nn+1\right).$$

For regime 2, the prior on the coefficients capturing the steady state for country c, F_c, is distributed normally with mean $\psi_{c,0}$ and variance $\Lambda_{c,0}$. The parameters of the prior distribution are given by their 95 percent confidence bands for F_c and are reported in Table A1.

Table A1: Steady-state priors – 95 percent confidence bands.

	Real GDP growth		Inflation		Interest rate		Spread	
	left	right	left	right	left	right	left	right
Canada	1	3	1	3	3	5	0.6	2.1
Euro area	0.5	2.5	1	3	2	4	0.98	2.48
Japan	0	2	0	0	0	2	0.3	1.8
Norway	0.5	2.5	1.5	3.5	2	4	-0.95	0.55
Sweden	0.5	2.5	1	3	2	4	0.52	2.02
Switzerland	0.5	2.5	1	3	2	4	0.38	1.88
United Kingdom	1	3	1	3	3	5	0.2	1.7
United States	1	3	1	3	2	4	0.98	2.48

The confidence bands are set to imply prior means that coincide with the long-run equilibrium values available in the literature. Holston et al. (2016) provide estimates of real equilibrium GDP

[11] We cannot use a noninformative prior on the hyperparameter b as is usual in panel VARs, because such prior would not be consistent with mean adjustment, which assumes the existence of a steady state. This is also why we use a value of 0.9 for the coefficient at the first own lag for variables in levels instead of the standard value of 1.

[12] The usually employed diffuse prior is not suitable for our case because of the low number of observations that there can be in one regime.

growth and the real equilibrium interest rate for Canada, the euro area, the UK, and the US. Other European countries are assumed to have the same equilibrium growth and real interest rate as the euro area. Equilibrium growth and real interest rate estimates for Japan can be found in Fujiwara (2016). The inflation steady state is assumed to follow the country's definition of price stability, i.e., except for Norway and Japan a price growth target of 2 percent is assumed. Norway's inflation target is 2.5 percent. Japan introduced an explicit CPI inflation target in January 2013 with two-year time span to achieve it. We set the prior on Japanese steady-state price growth at 1 percent. For the steady state of the nominal interest rate we add the inflation steady state to the equilibrium real rate. The confidence bands for the steady-state interest rate spread are based on the average spread over the period 1999Q1–2016Q4 and the width of the band is assumed to be 1.5.

Finally, the prior distribution assumed for the threshold parameter r is uniform on the interval $[r_{\min}, r_{\max}]$, where the bounds of the interval are defined such that at least 20 observations remain in a regime if the threshold takes an extreme value.

A.3 Conditional posteriors

Drawing on formula (A1) and ignoring terms that do not involve the parameter we are deriving, conditional posterior distributions emerge for the following formulas. To a great extent, the specifications of the conditional posteriors follow the formulas in Dieppe et al. (2016), Villani (2009), Chen and Lee (1995), and Koop and Potter (2003) and derivations are not presented. The main difference from the above-mentioned papers with respect to the derivations is that static interdependencies are allowed for by dealing with data for all countries at once when the vectors of parameters relating to the lagged values of endogenous variables and the error covariance matrix are drawn. Note that mean adjustment is applied to regime 2 only.

The conditional posterior of the common mean $b^{(r)}$ is distributed normally with mean

$$\left(\left(\frac{1}{N^2}\sum_{c=1}^{N}\Sigma_c^{b,(r)}\right)^{-1} + \left(\Xi^{b,(r)}\right)^{-1}\right)^{-1}\left(\left(\frac{1}{N^2}\sum_{c=1}^{N}\Sigma_c^{b,(r)}\right)^{-1}\left(\frac{1}{N}\sum_{c=1}^{N}\beta_c^{(r)}\right) + \left(\Xi^{b,(r)}\right)^{-1}B^{(r)}\right) \tag{A3}$$

and variance

$$\left(\left(\frac{1}{N^2}\sum_{c=1}^{N}\Sigma_c^{b,(r)}\right)^{-1} + \left(\Xi^{b,(r)}\right)^{-1}\right)^{-1}. \tag{A4}$$

The mean of the conditional posterior for $b^{(r)}$ is thus a weighted average of the prior mean $B^{(r)}$ and the average vector of dynamic coefficients across countries, with weights given by the reciprocal of the variances of the two.

Next, the covariance matrix that drives the dispersion of the country-specific vectors of the dynamic coefficients around the common mean, $\Sigma_c^{b,(r)}$, is defined in (A2). The conditional posterior of $\lambda_1^{(r)}$ is inverse-Gamma distributed with the shape parameter

$$\frac{1}{2}\left(h^{(r)}+s_0\right), \text{ where } h^{(1)}=Nn(np+1) \text{ and } h^{(2)}=Nn^2p, \tag{A5}$$

and the scale parameter

$$\frac{1}{2}\left(v_0+\sum_{c=1}^{N}\left\{\left(\beta_c^{(r)}-b^{(r)}\right)'\left(\Omega_c^{b,(r)}\right)^{-1}\left(\beta_c^{(r)}-b^{(r)}\right)\right\}\right). \tag{A6}$$

The conditional posterior of the vector of dynamic coefficients $\beta_c^{(r)}$ is distributed normally with mean

$$\left[\overline{X}^{(r)'}\left(\overline{\Sigma}^{(r)}\right)^{-1}\overline{X}^{(r)}+\left(\overline{\Sigma}^{b,(r)}\right)^{-1}\right]^{-1}\left[\overline{X}^{(r)'}\left(\overline{\Sigma}^{(r)}\right)^{-1}\overline{X}^{(r)}\left(\overline{X}^{(r)'}\overline{X}^{(r)}\right)^{-1}\overline{X}^{(r)'}\overline{y}^{(r)}+\left(\overline{\Sigma}^{b,(r)}\right)^{-1}b^{(r)}\right] \tag{A7}$$

and variance

$$\left[\overline{X}^{(r)'}\left(\overline{\Sigma}^{(r)}\right)^{-1}\overline{X}^{(r)}+\left(\overline{\Sigma}^{b,(r)}\right)^{-1}\right]^{-1}, \tag{A8}$$

where $\overline{\Sigma}^{b,(1)}$ is defined as follows

$$\overline{\Sigma}^{b,(r)}=\begin{pmatrix}\overline{\Sigma}_1^{b,(r)} & 0 & \cdots & 0\\ 0 & \overline{\Sigma}_2^{b,(r)} & \cdots & \vdots\\ \vdots & \vdots & \ddots & 0\\ 0 & \cdots & 0 & \overline{\Sigma}_N^{b,(r)}\end{pmatrix}.$$

Data matrices $\overline{X}^{(r)}$ and vectors $\overline{y}^{(r)}$, $r=1,2$, are defined in (5). Importantly, $\overline{X}^{(2)}$ and $\overline{y}^{(2)}$ are demeaned, i.e., the vector of steady states F_c is subtracted from each $y_{c,t}$ when the data vectors and matrices are constructed.

The mean of the conditional posterior (A7) is a weighted average of the common mean $b^{(r)}$ and the maximum likelihood estimator $\left(\overline{X}^{(r)\prime}\overline{X}^{(r)}\right)^{-1}\overline{X}^{(r)\prime}\overline{y}^{(r)}$, with the weights being the reciprocal of the variances of the two.

The conditional posterior of the error covariance matrix $\Sigma^{(r)}$ is inverse-Wishart distributed with scale parameter

$$0.01 I_{Nn} + \left(y^{(r)} - X^{(r)} \underset{c=1,\ldots,N}{blkdiag}\left(\beta_c^{(r)}\right)\right)' * \left(y^{(r)} - X^{(r)} \underset{c=1,\ldots,N}{blkdiag}\left(\beta_c^{(r)}\right)\right), \tag{A9}$$

and degrees of freedom

$$Nn + 1 + t^{(r)}, \tag{A10}$$

where $t^{(r)}$ denotes the number of observations in regime r. The data matrix $X^{(r)}$ is defined as an $Nt^{(r)} \times Nnp$ block diagonal matrix with $X_c^{(r)}$, $c = 1,\ldots,N$, on its diagonal, where only observations from regime r are taken. Similarly, $y^{(r)}$ is an $Nt^{(r)} \times Nn$ block diagonal matrix with $y_c^{(r)}$ on its diagonal. The data vectors and matrices for regime 2 are demeaned by the steady-state vector F_c.

Next, the conditional posterior of the vector F_c of steady states in regime 2 is distributed normally with mean

$$\begin{aligned}&\left[\Lambda_{c,0}^{-1} + U_c'\left(Z^{(2)\prime} Z^{(2)} \otimes \left(\Sigma_c^{(r)}\right)^{-1}\right)U_c\right] * \\ &\left[\Lambda_{c,0}^{-1}\psi_{c,0} + U_c' vec\left(\left(\Sigma_c^{(2)}\right)^{-1}\left(y_c^{(2)} - X_c^{(2)}\left[A_c^{1,(2)},\ldots,A_c^{p,(2)}\right]'\right)' Z^{(2)}\right)\right]\end{aligned} \tag{A11}$$

and variance

$$\left[\Lambda_{c,0}^{-1} + U_c'\left(Z^{(2)\prime} Z^{(2)} \otimes \left(\Sigma_c^{(2)}\right)^{-1}\right)U_c\right]^{-1}, \tag{A12}$$

where

$$U_c = \begin{pmatrix} I_n \\ A_c^{1,(2)} \\ \vdots \\ A_c^{p,(2)} \end{pmatrix}.$$

Data matrices $X_c^{(2)}$ and $y_c^{(2)}$ are not demeaned. Note that draws of steady-state vectors are carried out country-by-country, i.e., the covariance structure relating countries is not exploited. In the case of steady states, this simplifying assumption is probably reasonable.

Finally, regarding the threshold parameter r, the conditional posterior is not available in analytical form. We therefore employ a Metropolis step based on the conditional posterior probability of the threshold:

$$p\left(r \mid \beta^{(1)}, \beta^{(2)}, \overline{\Sigma}^{(1)}, \overline{\Sigma}^{(2)}, F, \overline{y}, \overline{X}\right) \propto$$
$$\left|\overline{\Sigma}^{(1)}\right|^{-\frac{t^{(1)}}{2}} \left|\overline{\Sigma}^{(2)}\right|^{-\frac{t^{(2)}}{2}} \exp\left\{-\frac{1}{2} tr\left[\sum_{r=1}^{2} \left(\overline{y}^{(r)} - \overline{X}^{(r)}\beta^{(r)}\right)' \left(\overline{\Sigma}^{(r)}\right)^{-1} \left(\overline{y}^{(r)} - \overline{X}^{(r)}\beta^{(r)}\right)\right]\right\}. \tag{A13}$$

A.4 The Gibbs sampler with a Metropolis step

The sample from the joint posterior of the model parameters is obtained by sampling from the conditional posteriors in the following steps:

1) Initial values: the country-specific vectors of dynamic parameters $\beta_c^{(r)}$ are initialized at their respective OLS estimates, the common mean $b^{(r)}$ at the prior mean, the overall tightness for the common mean error covariance matrix $\lambda^{(r)}$ at the value of 0.01, and the error covariance matrix $\Sigma^{(r)}$ at the OLS estimate based on pooled data for all countries in a regime given by the initial draw of the threshold parameter. The threshold is initialized at a random draw from the uniform distribution on the interval $\left[r_{\min}, r_{\max}\right]$, the vector of steady states F at the respective prior means.
2) Given $\beta_c^{(r)}$, $c = 1, \ldots, N$, and $\Sigma_c^{b,(r)}$ from the previous iteration, the conditional posterior of the common mean $b^{(r)}$ is normally distributed with the mean given by (A3) and the variance given by (A4). For a given draw of the common mean $b^{(1)}$ and $b^{(2)}$ a check is done of whether the eigenvalues of VAR with $b^{(1)}$ and $b^{(2)}$ in the companion form are less than or equal to one. If they are not, another draw is taken. The maximum of tries to obtain a stable VAR with $b^{(1)}$ is set to 100 and with $b^{(2)}$ is set to 20.
3) Given $\beta_c^{(r)}$, $c = 1, \ldots, N$, and $b^{(r)}$ from the previous iteration, the covariance matrix $\Sigma_c^{b,(r)}$ equals $\left(\lambda_1^{(1)} \otimes I_{n^2 p+n}\right)\Omega_c^{b,(1)}$ or $\left(\lambda_1^{(2)} \otimes I_{n^2 pn}\right)\Omega_c^{b,(2)}$, where the overall tightness parameter $\lambda_1^{(r)}$

is inverse-gamma distributed with the shape parameter defined in (A5) and the scale parameter defined in (A6).

4) Given $\Sigma_c^{b,(r)}$, $b^{(r)}$, $\overline{\Sigma}^{(r)}$, and F from the previous iteration, the country-specific vectors of the dynamic parameters for regime r, $\beta_c^{(r)}$, are multivariate-normal distributed with mean (A7) and variance (A8). Similarly to $b^{(r)}$ in the first step, a stability check of the VAR structure for the drawn $\beta_c^{(1)}$ and $\beta_c^{(2)}$ is conducted. The draws are taken until a stable VAR is implied, the maximum number of tries being 50,000 and 200, respectively. The total number of tries is driven by the aim to get the ratio of unstable draws below 1 percent of the total draws.
5) Given $\Sigma_c^{b,(r)}$, $b^{(r)}$, $\beta_c^{(r)}$, and F_c from the previous iteration, the error covariance matrix is inverse-Wishart distributed with scale parameter (A9) and degrees of freedom (A10).
6) Given $\Sigma_c^{b,(r)}$, $b^{(r)}$, and $\Sigma^{(r)}$ from the previous iteration, the vector F_c is distributed normally with mean (A11) and variance (A12).
7) Metropolis step:
    a. Draw a proposed value r^* from the prior distribution for the parameter.
    b. Compare the log of the conditional probability (A13) of the proposed value with the log of the conditional probability (A13) for the original value r from the previous iteration.
    c. Accept the proposed value with a probability of $\min\{1, p(r^* \mid ...)/ p(r \mid ...)\}$, where $p(\cdot \mid ...)$ is defined in (A13) (i.e., if the difference between the two logs of the conditional probabilities is larger than the logarithm of a draw from a standard uniform).
8) Repeat steps 2–7 until convergence is achieved.

Appendix B: Post-estimation diagnostics

Two measures of convergence of the Gibbs part of the sampler are used: autocorrelation of the draws at a distance of 10, and the Raftery and Lewis (1992) estimate of the number of draws from the conditional posteriors needed to obtain a stationary distribution.[13] For the Metropolis step, the usual acceptance ratio – the ratio of the number of draws accepted to the total number of attempts – is reported.

[13] The Econometrics Toolbox for Matlab by LeSage (1999) is used.

For parameters $\lambda^{(1)}$, $\lambda^{(2)}$, and F_c, the autocorrelation of the draws is less than 0.01 in absolute terms and the number of suggested runs is less than 1,000. Similar results are obtained for the rest of the parameters, as reported in Figures B1–B3 for a selection of the parameter set.

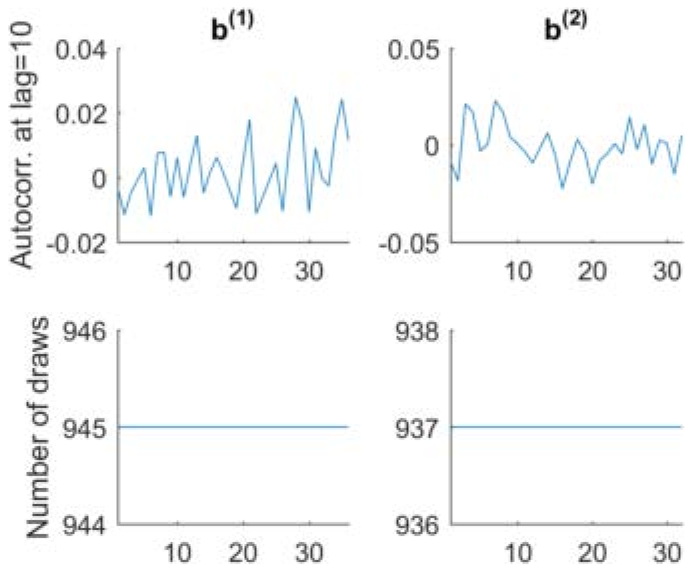

Figure B1: Convergence diagnostics for the common mean $b^{(r)}$.
Note: The parameters are stacked on the x-axis (36 parameters in regime 1 and 32 parameters in regime 2).

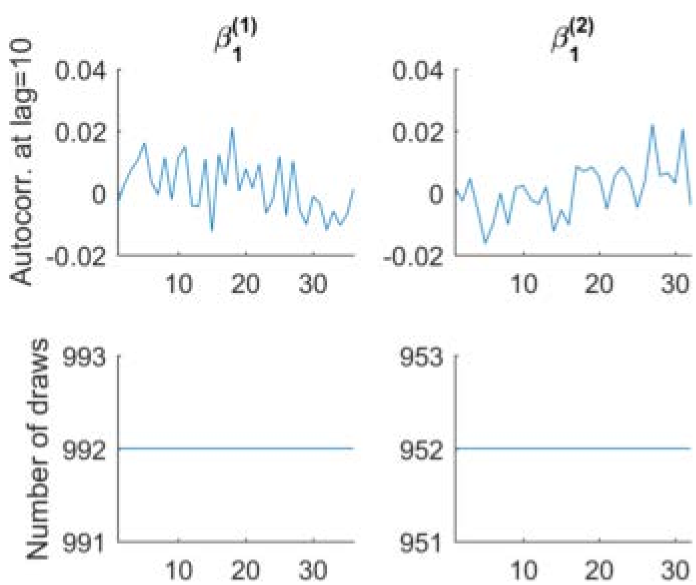

Figure B2: Convergence diagnostics for the vector of dynamic coefficients for Canada $\beta_1^{(r)}$.
Note: The parameters are stacked on the x-axis (36 parameters in regime 1 and 32 in regime 2).

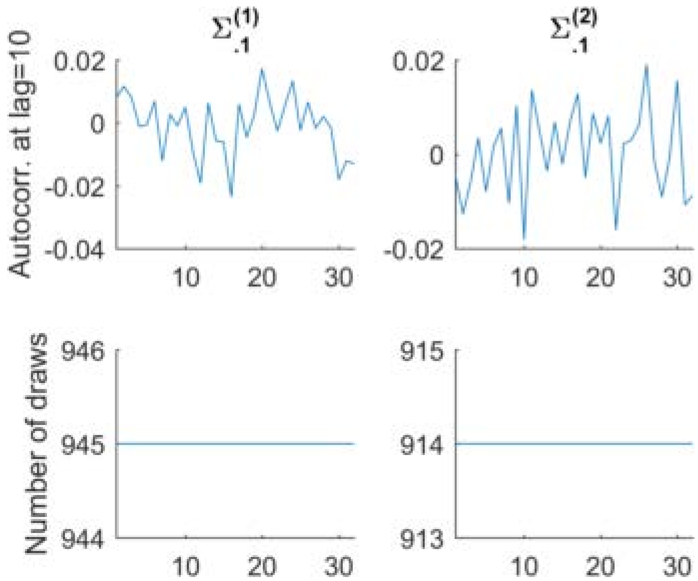

Figure B3: Convergence diagnostics for the first column of the error covariance matrix $\Sigma^{(r)}$.
Note: The parameters are stacked on the x-axis (32 parameters in each regime).

Finally, the acceptance ratio for the threshold parameter r is 0.25. The posterior distribution of the threshold parameter is presented in Figure B4. The posterior distributions of the steady-state parameters are shown in Figure B5.

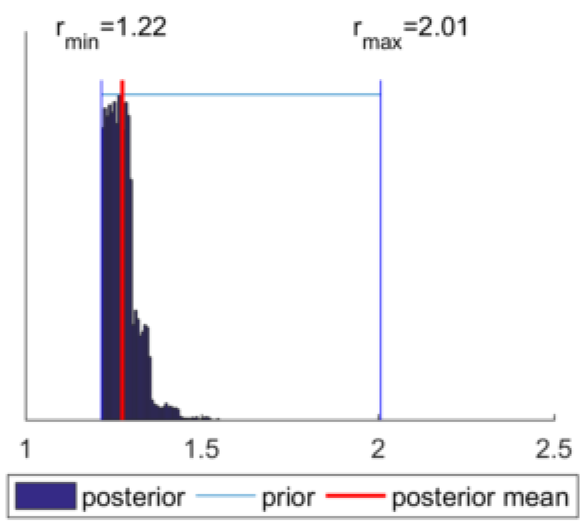

Figure B4: The posterior distribution of the threshold parameter.

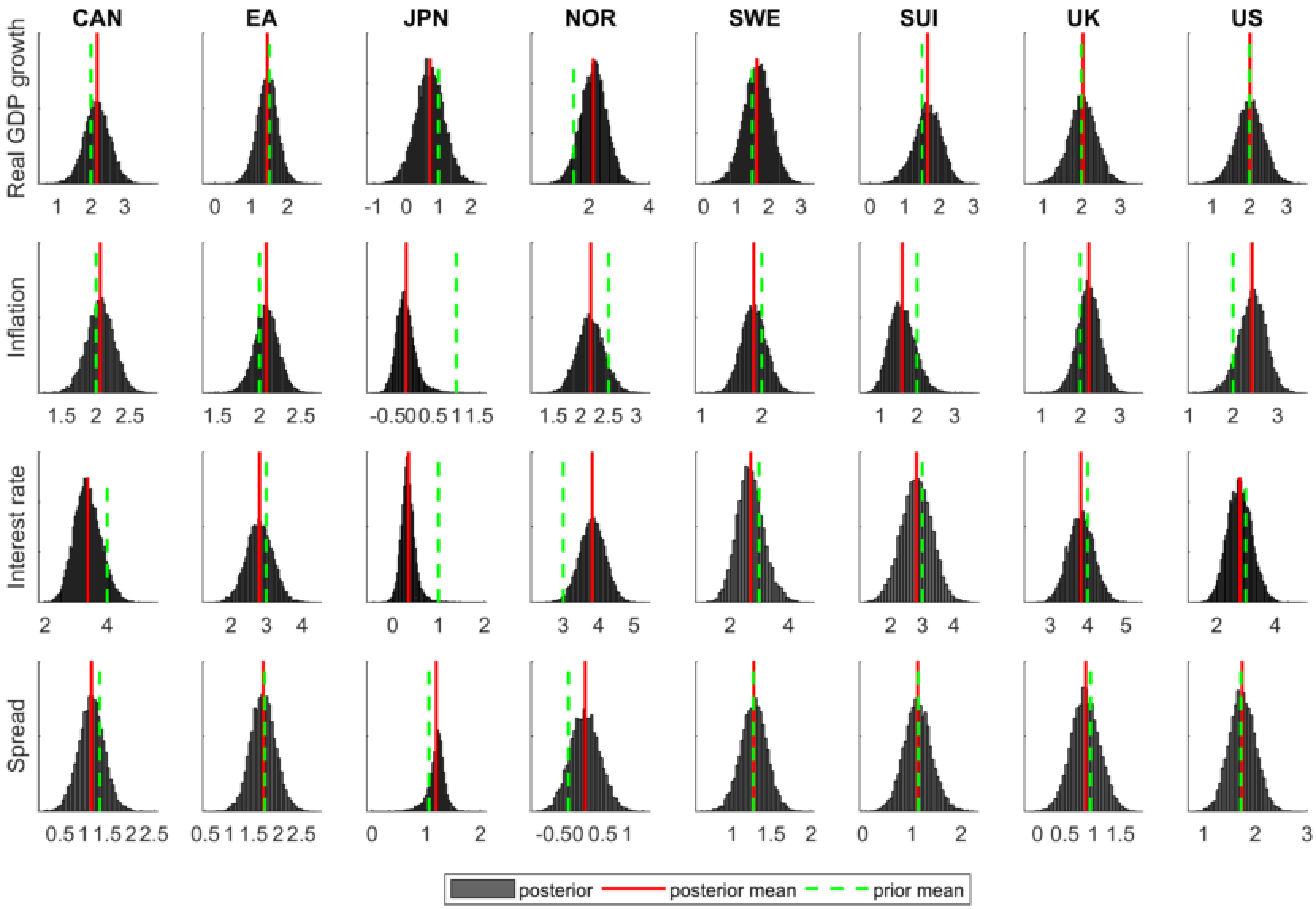

Figure B5: Posterior distributions of the steady-state parameters.

Previous volumes in this series

729 June 2018	Has inflation targeting become less credible?	Nathan Sussman and Osnat Zohar
728 June 2018	Accumulation of foreign currency reserves and risk-taking	Rasmus Fatum and James Yetman
727 June 2018	Recent RMB policy and currency co-movements	Robert N McCauley and Chang Shu
726 June 2018	Residential investment and economic activity: evidence from the past five decades	Emanuel Kohlscheen, Aaron Mehrotra and Dubravko Mihaljek
725 May 2018	Identifying oil price shocks and their consequences: the role of expectations in the crude oil market	Takuji Fueki , Hiroka Higashi , Naoto Higashio , Jouchi Nakajima , Shinsuke Ohyama and Yoichiro Tamanyu
724 May 2018	Do small bank deposits run more than large ones? Three event studies of contagion and financial inclusion	Dante B Canlas, Johnny Noe E Ravalo and Eli M Remolona
723 May 2018	The cross-border credit channel and lending standards surveys	Andrew Filardo and Pierre Siklos
722 May 2018	The enduring link between demography and inflation	Mikael Juselius and Előd Takáts
721 May 2018	Effects of asset purchases and financial stability measures on term premia in the euro area	Richhild Moessner
720 May 2018	Could a higher inflation target enhance macroeconomic stability?	José Dorich, Nicholas Labelle St-Pierre, Vadym Lepetyuk and Rhys R. Mendes
719 May 2018	Channels of US monetary policy spillovers to international bond markets	Elias Albagli, Luis Ceballos, Sebastian Claro, and Damian Romero
718 May 2018	Breaking the trilemma: the effects of financial regulations on foreign assets	David Perez-Reyna and Mauricio Villamizar-Villegas
717 May 2018	Financial and price stability in emerging markets: the role of the interest rate	Lorenzo Menna and Martin Tobal
716 April 2018	Macro-financial linkages: the role of liquidity dependence	Alexey Ponomarenko, Anna Rozhkova and Sergei Seleznev

All volumes are available on our website www.bis.org.