Internet Appendix to
"An Explanation of Negative Swap Spreads Demand for Duration from Underfunded Pension Plans"

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A. Additional Results

This section contains three additional tables. Table IA.1 provides quotes from various sources that support our claim that pension funds use IRS for duration hedging.

[Table IA.1 here]

Table IA.2 repeats the analysis carried out in Section IV.C for 20-year swap spreads instead of 30-year swap spreads. The only difference between this analysis and the results in Section IV.C is that, because the 20-year swap spreads are computed relative to off-the run Treasuries, we remove the on-the-run off-the-run spread as control variable.

[Table IA.2 here]

Finally, because we have a total of 44 observations with underfunded pension plans for Japan, we run several additional robustness checks for this country. First, to investigate whether the effect of UFR_t^{Jap} is already captured by other variables, we add changes in Japanese stock market volatility, as captured by the VNikkei index, changes in broker-dealer EDFs, as well as changes in the level and the slope of the Japanese yield curve as control variables to the regression. As we can see from Column (2) of Table IA.3, adding these variables leaves our main results unchanged – ΔUFR_t^{Jap} remains a highly significant explanatory variable for Japanese swap spreads. Consistent with our analysis for the U.S. data, we also run a 2-stage least squares regression where we first regress ΔUFR_t^{Jap} on Japanese stock returns (as proxied by the returns of the Nikkei index) and then use the predicted ΔUFR_t^{Jap} from this regression as an explanatory variable. As before, we argue that the exclusion restriction is fulfilled because stock returns can directly affect the funding status of Japanese pension funds but are unlikely to influence the level of the swap spreads directly. A weak instruments test gives a p-value below 0.1%, implying that we can reject that our instrument is weak. Columns (3) and (4) of IA.3 show that the projected UFR, $\Delta U\hat{F}R_t^{Jap}$, is a statistically significant explanatory variable for Japanese swap spreads.

B. Illustrations for Vasicek Short Rate

In this section, we provide a numerical illustration of our model. We assume that the short rate follows a Vasicek process with the following dynamics:

$$dr = (\kappa(\bar{r} - r) - \lambda\nu)dt + \nu dw. \tag{IA.1}$$

With that, the cum-coupon dynamics of the perpetuity P are given as:

$$dP = \left(1 + P_r[(\kappa(\bar{r} - r) - \lambda\nu) + \frac{\nu^2}{2}P_{rr}\right) dt + \nu P_r dw$$

The bond also satisfies the following valuation equation:

$$rP - \lambda\nu P_r = 1 + \frac{\nu^2}{2}P_{rr} + P_r(\kappa(\bar{r} - r) - \lambda\nu)$$

and with that, we can compute μ_P and σ_P as:

$$\mu_P \equiv \mathbb{E}[dP/P] := r + \lambda\Theta, \text{ and } \sigma_P^2 \equiv var(dP/P) := \Theta^2,$$

where $\Theta := -\nu(\frac{\partial}{\partial r}P)/P$. Note that, since we are modeling coupon bonds, Θ is a function of the short rate r and needs to be approximated numerically. To obtain this numerical approximation, note that the price of a zero-coupon bond with time to maturity s in the Vasicek model is given as $p(s) = e^{A(s)-B(s)r}$ with

$$B(s) = \frac{1}{\kappa}(1 - e^{-\kappa s}) \text{ and } A(s) = \left(\bar{r} - \lambda\frac{\nu}{\kappa} - \frac{\nu^2}{2\kappa^2}\right)[B(s) - s] - \frac{\nu^2 B(s)^2}{4\kappa}.$$

Then, the price of the consol is $P = \int_t^\infty p(s)ds$ and Θ can be computed as:

$$\Theta = \frac{\nu}{P}\left[\int_t^\infty B(s)p(s)ds\right] \tag{IA.2}$$

To illustrate our results, we set the model parameters to $\kappa = 1$, $\bar{r} = 0.03$, $\nu = 0.1$, and $r = 0.02$. Furthermore, we assume that the pension fund does not associate a risk premium with holding bonds or swaps and set $\lambda = 0$. We further assume that the pension fund is risk averse with risk aversion $\gamma = 5$. In Figure IA.1, we illustrate the equilibrium swap spread for different values of a, ranging from $a = 0.5$ over $a = 1$ to $a = 2$. As we can see from the figure, the swap spread decreases as the level of pension fund underfunding increases. Moreover, the impact of UFR on the swap spread is more pronounced for higher values of a, that is, for regimes in which derivatives dealers are more constrained.

[Figure IA.1 here]

C. What keeps Arbitrageurs Away?

In this section we show that even if negative swap spreads are a textbook arbitrage opportunity, assuming no transaction costs and institutional frictions, the arbitrage strategy explained in Table IA.4 is still a risky strategy.[1] As pointed out by Shleifer and Vishny (1997), Liu and Longstaff (2004) and many others, even textbook arbitrage opportunities are subject to a risk, namely the possibility that the mispricing increases before it vanishes, thereby forcing the arbitrageur out of his position at a loss. With negative swap spreads arbitrage, we know that the mispricing vanishes after 30 years, but we do not know whether it vanishes within a much shorter and practical horizon.

[**Figure IA.4 here**]

To illustrate this point we provide some stylized sample calculations to approximate the excess returns of an arbitrageur engaging in the strategy, described in Table IA.4. We assume that the arbitrageur unwinds his position before maturity and consider two cases. In the first case, we assume that the arbitrageur unwinds the position after 3 months, in the second case we assume that he unwinds after 12 months. In both cases he receives a positive carry from the strategy but is exposed to the risk that the swap spread becomes even more negative. For simplicity, we ignore the ageing of the treasury and swap and simply assume that the arbitrageur unwinds the position by engaging in an opposite transaction where he sells a treasury bond with 30-years to maturity and receives fixed in an IRS with 30-years to maturity.[2] Every month, the arbitrageur observes the 30-year swap spread and engages in the transaction if the swap spread is negative. We illustrate the resulting excess returns of the two strategies in Figure IA.2. The Sharpe ratio for the 3-month and 12-month strategies are 0.86% and 5.03% respectively. Note that the Sharpe ratio for investing in the US stock market for the same time period is 29.39%

[**Figure IA.2 here**]

[1]We ignore potential issues with leverage constraints or frictions in the repo market and illustrate the returns to swap spreads arbitrage in a "best case".

[2]This simplification leads to a duration mistake of 3 months in case one and 1 year in case two. Since swap and treasury originally have 30 years to maturity this ageing effect is neglect-able for our approximation.

REFERENCES

Liu, Jun, and Francis A Longstaff, 2004, Losing money on arbitrage: Optimal dynamic portfolio choice in markets with arbitrage opportunities, *Review of Financial Studies* 17, 611–641.

Shleifer, Andrei, and Robert W Vishny, 1997, The limits of arbitrage, *Journal of Finance* 52, 35–55.

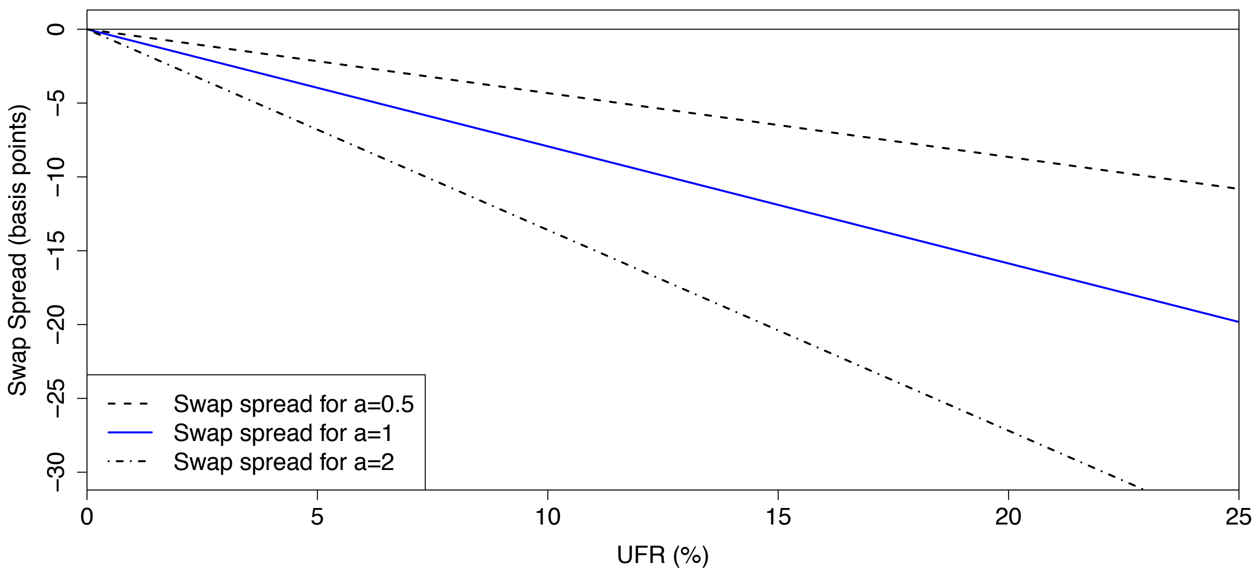
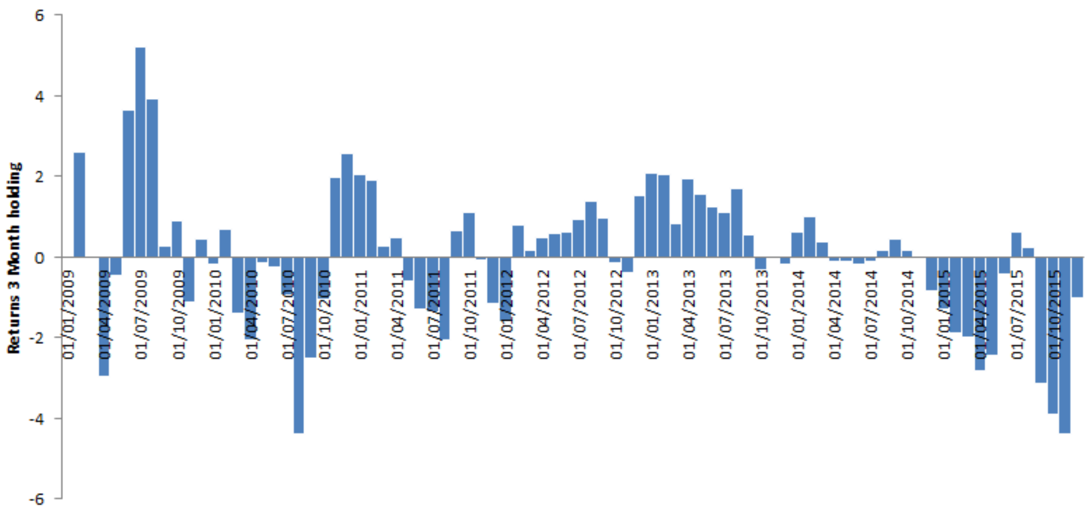
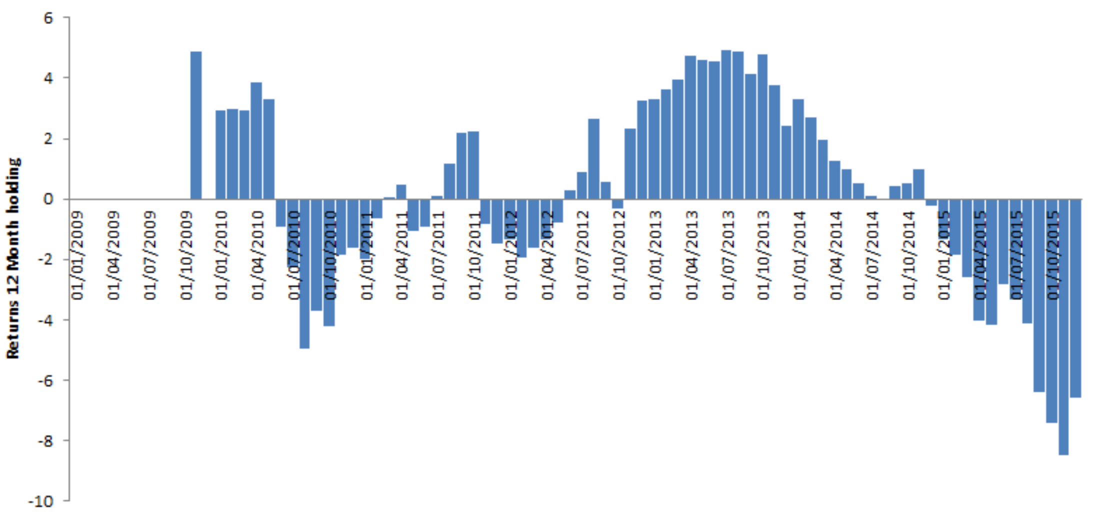


Figure IA.1. Numerical Illustration of the equilibrium swap spread. This graph shows the equilibrium swap spread as a function of pension fund's underfunding for three different levels of dealer balance sheet constraints, a. The short rate model is specified in Equation (IA.1) and the model parameters are $\kappa = 1$, $\bar{r} = 0.03$, $\nu = 0.1$, $r = 0.02$, and $\lambda = 0$. The pension fund's risk aversion is set to $\gamma = 5$.



(a) 3-months holding return



(b) 12-months holding return

Figure IA.2. Returns from 30-year swap spread arbitrage. The Figure shows the returns from engaging in swap spreads arbitrage. The Sharpe ratio of the two strategies are 0.86% and 5.03% respectively.

Table IA.1
Anecdotal Evidence of Pension Fund's Swap Usage

This table shows quotes from various sources, documenting that pension funds use interest rates swaps. *Highlights* by the authors.

Occasion	Source	Statement
Risk Magazine	"An imperfect solution: Derivatives create new challenges for buy side"	"Then you have *clients that are underfunded* and their balance sheet cannot afford a sterner contribution policy from employees, so they need to lean on the growth portfolio to fill the funding gap. *Their allocation to swaps tends to be greater.*"
New York Times	"A Strategy For Prudence On Pensions"	"To make a better match, Mr. Hunkeler [the Vice President for Investments at International Paper] needed bonds with longer durations. [...] Mr. Hunkeler *chose a second method, which used a financial instrument, interest rate swaps, to extend the duration of the bonds the pension fund already owned.*"
Ford	Annual Report, December 2012	"Worldwide, *our defined benefit pension plans were underfunded* by $18.7 billion at December 31, 2012 [...]" "Interest rate and foreign currency derivative instruments are used for the purpose of hedging changes in the fair value of assets that result from interest rate changes and currency fluctuations. *Interest rate derivatives also are used to adjust portfolio duration.*"
General Motors	10-K form, December 2013	"*Our defined benefit pension plans are currently underfunded* [...]" "*Interest rate derivatives may be used to adjust portfolio duration* to align with a plan's targeted investment policy."
Boeing	10-K form, December 2016	"At December 31, 2016 and 2015, *our pension plans were $20.1 billion and $17.9 billion underfunded* as measured under GAAP." "Derivatives are used to achieve the desired market exposure of a security or an index, transfer value-added performance between asset classes, achieve the desired currency exposure, *adjust portfolio duration* or rebalance the total portfolio to the target asset allocation."
Century Link	10-K Form, December 2014	"As of December 31, 2014, our pension plans and our other postretirement benefit plans *were substantially underfunded* from an accounting standpoint." "Derivative instruments are used to reduce risk as well as provide return. [...] *Interest rate swaps are used in the pension plans to reduce risk relative to measurement of the benefit obligation, which is sensitive to interest rate changes.*"

Table IA.2
20-Year Swap Spreads and Pension Fund Underfunding

This table reports results from regressions of quarterly changes in the 20-year swap spread on the indicated variables. ΔUFR_t is the change in the underfunding ratio of private and local government defined benefit pension funds, as defined in Equation (4), ΔUFR_t^+ (ΔUFR_t^-) is the change in UFR_t if $UFR_t > 0$ ($UFR_t \leq 0$) and zero otherwise. $\Delta LR\ Sprd_t$ is the change in the quarter-end difference between the 3-month Libor rate and 3-month General Collateral repo rate, $\Delta Debt/GDP_t$ is the change in the ratio of U.S. public debt to GDP, ΔEDF_t is the change in the Moody's expected default frequency of the 14 largest derivatives-dealing banks (G14 banks), $\Delta Move_t$ is the change in the 1-month implied volatility of U.S. Treasuries with 2,5, 10, and 30 years to maturity , $\Delta TERM_t$ measures changes in the slope of the yield curve, approximated as the difference between 30-year and 3-month treasury yields. A detailed description of the additional controls can be found in the caption of Table IV. All variables are quarter-end. The numbers in parenthesis are heteroskedasticity-robust t-statistics.***, **, and * indicate significance at a 1%, 5%, and 10% level respectively. The observation period is Q3 1994 – Q4 2015 with 5 missing observations between Q4 1997 and Q4 1998 due to missing repo rates.

	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	−0.31	−0.24	0.27	0.32	12.72	13.10
	(−0.24)	(−0.20)	(0.19)	(0.22)	(0.81)	(0.82)
ΔUFR_t	-1.21^{***}		-1.08^{***}		−0.69	
	(−2.95)		(−2.69)		(−1.46)	
ΔUFR_t^+		-1.39^{***}		-1.36^{***}		-1.02^{**}
		(−2.90)		(−3.61)		(−2.14)
ΔUFR_t^-		−0.81		−0.63		−0.41
		(−1.21)		(−0.92)		(−0.57)
$\Delta LR\ Spread_t$	0.07	0.07	0.04	0.04	0.02	0.02
	(1.00)	(1.06)	(0.61)	(0.66)	(0.22)	(0.23)
$\Delta Debt/GDP_t$			−1.17	−1.10	−0.69	−0.60
			(−0.97)	(−0.93)	(−0.74)	(−0.67)
ΔEDF_t			−0.03	−0.02	−0.03	−0.02
			(−0.68)	(−0.43)	(−0.50)	(−0.36)
$\Delta Move_t$			0.10	0.10	0.13	0.12
			(1.22)	(1.28)	(1.36)	(1.29)
$\Delta TERM_t$			−0.06	-0.06^{*}	-0.10^{**}	-0.10^{**}
			(−1.60)	(−1.81)	(−2.37)	(−2.38)
$\log(EPU_t^{DebtCeil})$					0.75	0.77
					(0.97)	(0.99)
Add. Controls?	*No*	*No*	*No*	*No*	*Yes*	*Yes*
Observations	81	81	81	81	81	81
Adjusted R^2	0.18	0.18	0.25	0.25	0.23	0.22

Table IA.3 Japanese Pension fund underfunding and Japanese swap spreads. This table reports results from regressions of quarterly changes in swap spread 30 years to maturity on the indicated variables. The swap spreads are computed as the difference between the fixed rate in an IRS based on Japanese LIBOR rates and Japanese government bond yields. ΔUFR_t^+ is the change in the underfunding ratio of Japanese pension funds, conditional on pension funds being underfunded at time t. There are no time periods where Japanese pension funds are fully funded. $\Delta LR\ Spread_t$ is the change in the quarter-end difference between the 6-month Japanese LIBOR rate and 6-month General Collateral repo rate. The numbers in parenthesis are heteroskedasticity-robust t-statistics. ***, **, and * indicate significance at a 1%, 5%, and 10% level respectively. The observation period is Q1 2005 – Q4 2015.

	OLS		*2 SLS*	
	(1)	(2)	(3)	(4)
Intercept	−1.06	−1.63	−1.07	−1.75
	(−0.84)	(−1.40)	(−0.85)	(−1.50)
ΔUFR_t^+	-2.02^{***}	-2.19^{***}	-2.05^{***}	-2.46^{***}
	(−4.45)	(−5.40)	(−3.97)	(−4.64)
$\Delta LRSpread_t$	0.06	−0.09	0.07	
	(0.38)	(−0.62)	(0.41)	
$\Delta VNikkei_t$		0.12		0.13
		(0.66)		(0.71)
ΔEDF_t		−0.01		−0.00
		(−0.35)		(−0.10)
$\Delta Level_t$		-0.25^{*}		-0.25^{**}
		(−2.00)		(−2.16)
$\Delta TERM_t$		0.03		0.04
		(0.21)		(0.25)
Observations	43	43	43	43
Adjusted R^2	0.34	0.45	–	–

Table IA.4
The Arbitrage Relationship Between Interest Rate Swaps and Treasuries

This table provides an arbitrage argument for positive swap spreads. s_0 denotes the fixed rate in an interest rate swap with maturity T, l_t denotes the variable Libor rate in month t, c_0 denotes the coupon of a treasury bond with maturity T, and r_t denotes repo rate in month t.

	$t = 0$	$t = 1$	...	$t = T$
Pay fixed rate s_0 in IRS	0	$-s_0$	...	$-s_0$
Receive Libor l_t from IRS	0	$+l_t$	...	$+l_T$
Buy bond with coupon c_0	-1	$+c_0$	...	$+1 + c_0$
Borrow at repo rate r_t	$+1$	$-r_t$	...	$-1 - r_T$
Payoff	0	$-(s_0 - c_0)$	...	$-(s_0 - c_0)$
		$+(l_t - r_t)$	...	$+(l_T - r_T)$