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by

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Abstract

There is a growing recognition in the literature on business cycles that production technologies may give rise to complicated interactions between seasonal and cyclical movements in economic time series, which can distort business cycle inference based on seasonally adjusted data. For the most part, however, the empirical research in this area has relied on standard univariate seasonal adjustment techniques that provide only a partial description of such interactions. In this paper, we develop an unobserved components model that explicitly accounts for the effects of business cycles on industry-level seasonality and for the potential feedback from seasonality to the aggregate business cycle. In particular, the model extracts an aggregate "common cycle" from industry-level data, allows formal statistical testing of seasonal differences in the comovement of an industry with the common cycle, and identifies economy-wide and industry-specific contributions to the seasonal and non-seasonal variation in the data. Applying the model to quarterly US payroll employment data, we frequently find evidence of statistically significant differences across seasons in the comovement between sectoral employment and the common cycle. On the other hand, we also find that seasonal fluctuations in employment at the industry level are largely idiosyncratic and that the proportion of the total variance of the common cycle accounted for by seasonality is much less than for aggregate employment. This suggests that seasonal shocks may have less of a business cycle element to them than one might infer from the seasonal movements in aggregate variables.

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Contents

1. Introduction

1.1 Overview

A growing segment of the literature on business cycle dynamics is turning to the large and relatively predictable seasonal movements in data as a means of obtaining insights into factors governing the more general dynamic properties of economic time series. One branch of this research has argued that complicated interactions may exist between seasonal and cyclical movements in data (as a result of, for example, non-linear marginal costs or seasonal shifts in production technologies), and that such interactions can cloud the distinction between seasonality and cyclicality and can distort business cycle inference based on seasonally adjusted data.[1]

For the most part, the empirical work in this area has relied on standard univariate seasonal adjustment techniques or simple single-equation regression models. These methods, however, can provide only a partial description of the potential interactions between seasonality and cyclicality. In this paper, we more fully account for seasonal/cyclical interactions in a model of quarterly US payroll employment data by parametrically specifying seasonal and non-seasonal components for an aggregate cycle, the comovement of disaggregated industry-level employment with the cycle, and the idiosyncratic variation in industry employment. These specifications are embedded in a simultaneous-equation unobserved components model along the lines of that proposed in Stock and Watson (1989, 1991). The model extracts an aggregate common cycle in employment, allows formal statistical tests of seasonal differences in the comovement of an industry with the cycle, and identifies economy-wide and industry-specific contributions to the seasonal and non-seasonal variation in industry employment.

In a number of the industries we consider, this model detects statistically significant differences across seasons in the comovement between sectoral employment and the common cycle. Furthermore, in more than half of the sectors, cyclical/seasonal interactions account for at least 20% of the variance remaining after the data have been adjusted for idiosyncratic seasonality. These results indicate that the presence of seasonal/cyclical interactions suggested by simpler models is borne out in a more fully specified parametric framework.

Nonetheless, the model also estimates that seasonal fluctuations in employment at the industry level are largely idiosyncratic. As a consequence, the proportion of the total variance of the common cycle accounted for by seasonality is much less than that for aggregate payroll employment; the quarterly

[1] Non-linear marginal costs imply that the response of production to a demand shock will depend on whether the shock occurs during a period of high or low seasonal demand. Seasonal technologies (regular shifts in production costs) can cause output to be concentrated during a particular season, so that the effects of a non-seasonal productivity shock may not become evident until that time of year.

patterns in the two series differ somewhat as well. This suggests that seasonal shocks may contain less of a business cycle element – in the sense of cohesion across sectors – than one might infer from the seasonal movements in aggregate variables.

We also find results that are relevant to economic forecasters and business cycle analysts. For example, the in-sample industry-specific seasonal factors generated by our model are very close to those produced by a univariate X-11 procedure similar to that used by the US Bureau of Labor Statistics (BLS) to seasonally adjust the payroll employment data. However, real-time out-of-sample experiments produce some noticeable differences between our seasonal factors and those produced by X-11. Furthermore, the non-seasonal component of the common cycle leads total seasonally adjusted employment growth around several business cycle peaks. These results indicate that the treatment of the cycle may be important to practitioners involved in the real-time analysis of seasonal time series, particularly around important periods such as turning-points in the business cycle.

1.2 Previous literature on the linkages between seasonality and the business cycle

Interest in linkages between seasonality and the business cycle was revived by Barsky and Miron (1989), who pointed out that correlations between the seasonal movements in time series often display characteristics that are qualitatively similar to their business cycle relationships: for example, at both seasonal and business cycle frequencies, output in numerous sectors moves together and labour productivity is positively correlated with output. Such similarities lead Barsky and Miron to conclude that the relationships between seasonal movements in data can provide useful insights into understanding business cycle behaviour.

In extending this analysis, other researchers have focused on testing for interactions between cyclicality and seasonality. Using the NBER business cycle chronology, Ghysels (1990) finds that the seasonal patterns in a number of US macroeconomic time series differ between recessions and expansions. The more general tests in Canova and Ghysels (1994) also provide evidence of structural instability in models that treat seasonality as constant or slow-changing, with the instability often associated with business cycle fluctuations. Canova and Ghysels (1994) further show how this instability can adversely affect the forecasts of econometric models that incorrectly assume invariant seasonal patterns.

Interactions between cycles and seasonals are one implication of earlier work by Plosser (1978, 1979), Wallis (1978), Ghysels (1988) and others. These papers show how seasonality in exogenous variables or in structural parameters imposes testable restrictions on seasonality in the transfer function and final equation time series specifications for endogenous variables. Such restrictions can be used to estimate underlying structural parameters or induced seasonality in endogenous variables (see Plosser (1978) and Krane (1993)).

Building on these more structural frameworks, Beaulieu, MacKie-Mason and Miron (1992) show how firm-level production functions can transform independent seasonal and non-seasonal variations into interactions between the business cycle and seasonality in macroeconomic aggregates. They further point out that the choice of technology implies that seasonal variations in demand can be a determinant of non-seasonal variations in output – and vice versa. Beaulieu et al. (1992) use these arguments to explain the strong positive correlations between the seasonal and non-seasonal variations in retail sales, employment and numerous other time series that they find across a variety of countries and industries.

In a related paper, Cecchetti, Kashyap and Wilcox (1997) look at interactions between cyclical and seasonal movements in production and inventories to infer the shapes of marginal cost curves. Using total manufacturing capacity utilisation as a proxy for the business cycle, they find statistically significant interactions between production seasonality and the cycle in nearly every major manufacturing industry. In some instances, cyclical strength is associated with muted production seasonality or inventory-building prior to high-output seasons, factors consistent with upward-sloping convex marginal costs. In one industry, the production and inventory patterns suggest a marginal cost curve that flattens at high levels of output. For most industries, however, the evidence regarding the shape of the marginal cost curve is inconclusive.

This literature indicates that business cycle developments can have important feedback on the seasonal movements in economic time series. And since business cycle inference is generally based on seasonally adjusted aggregates, feedback may also flow from sectoral seasonality to standard measures of business cycle activity. These issues lead Beaulieu et al. (1992) to conclude that "technological flexibility links seasonal and business cycles so that it is not possible to correctly study the two types of fluctuations separately" and Canova and Ghysels (1994) to argue that "cataloging business cycle facts with seasonally adjusted data is improper unless the seasonal adjustment takes into account the particular form of interaction existing among the components of the series (and this is seldom the case)".

Still, this literature generally estimates seasonal patterns using standard univariate methods such as X-11 or seasonal ARIMA models, or simple single-equation regressions that have seasonal dummies or dummies interacted with a business cycle proxy as explanatory variables. Univariate techniques may be adequate for calibrating average seasonal movements in data, but they make no attempt to structurally identify interactions between seasonals and cycles.[2] Regression models that condition

[2] As such, univariate methods can have difficulty disentangling seasonals from cycles. For example, with respect to X-11, Pierce and Cleveland (1981) point out that "it is difficult for X-11 to distinguish a run of consecutive unusual observations from the trend-cycle or seasonal elements of the series". Pierce and Cleveland (1981) recommend using intervention analysis to deal with unusual non-seasonal movements in data. Intervention analysis, which is an option in the new X-12 seasonal adjustment procedure, often amounts to calculating seasonal factors for an entire time series using observations from only non-recessionary periods. Such intervention would prevent distortion of seasonal factors during

seasonality on a business cycle proxy may capture some of the impact of cycles on seasonals, but even here the specification of the cyclical/seasonal interaction is incomplete, because potential feedback from seasonality to the cycle is omitted. In contrast, the model developed in this paper structurally estimates seasonal differences in the cyclical comovements of industry-level data and allows estimates of the aggregate cycle to be influenced by seasonality in the underlying component data.

2. Interactions between seasonality and the business cycle: some examples from US payroll employment data

The payroll employment data measure the number of employees on non-agricultural payrolls in the United States. These data are collected monthly by the BLS from a large sample of establishments (more than 350,000 in 1992). Aggregate and industry-level figures are generally published on the first Friday of the month following the survey reference period.

The employment data are particularly well suited for investigating interactions between seasonals and cycles for several reasons. First, the data span the spectrum of non-agricultural sectors in the US economy, whereas actual production measures are not generally available on a seasonally unadjusted basis outside the manufacturing sector.[3] Second, total employment can readily be disaggregated into sectors that exhibit a variety of combinations of cyclical and seasonal characteristics. Third, the employment data are some of those most closely scrutinised by financial markets, business economists and policy-makers tracking the course of the economy in real time. Finally, it is not unusual for market commentary on the high-frequency movements in employment to note that the seasonally adjusted data may be providing a misleading message because the seasonal factors are not properly accounting for the state of the business cycle.

Before turning to the specifics of our model, we present two real-world examples of interactions between seasonals and the business cycle found in the employment data. The first example illustrates how interactions between seasonality in demand and the business cycle can distort univariate seasonal factors, while the second considers interactions between production technology and the seasonal comovement of employment with the cycle.

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expansions, but would not capture potential changes in seasonal factors during recessions. Furthermore, some current indicator of recession needs to be available to make real-time intervention analysis operational. Since completion of the empirical work in this paper, the BLS has begun using X-12 to seasonally adjust payroll employment. However, it uses X-12 only to adjust for variation in the timing of the monthly employment survey; intervention analysis is not used to adjust for cyclicality.

[3] Sector-specific output measures are provided by the gross product originating data in the National Income and Product Accounts, but these data are only available on an annual basis.

2.1 Demand-driven interaction

Employment changes in retail trade are quite large around the turn of the year, with big increases in November and December in anticipation of the Christmas season and a sharp decline in January as these seasonal workers are let go. Naturally, the amount of Christmas hiring varies with the strength of the economy. If the seasonal factors do not adequately reflect the business cycle, weak Christmas hiring may be accompanied by inordinately sharp declines in preliminary seasonally adjusted retail employment in November and December, because the seasonal factors expect a gain in jobs based on the experience of an average year. In contrast, the subsequent decline in employment in January will be understated because retailers have fewer holiday workers to shed.

A particularly striking example of this problem occurred in early 1991. According to the initial seasonally adjusted estimates, employment in retail trade *increased* by 85,000 in January 1991, despite the fact that the economy was in the middle of a recession. At the time, analysts called the increase "artificial" and noted that "...stores added relatively few temporary workers over Christmas and thus the January cutbacks were smaller than assumed by the seasonal adjustment factor" (*New York Times*, 2 February 1991). Indeed, in her monthly testimony to Congress, BLS commissioner Janet Norwood cautioned that the retail data "were exaggerated by seasonal adjustment...[the] underlying trend in retail trade employment continues to be quite weak" (*Wall Street Journal*, 4 February 1991).

Such comments reflect, in part, the fact that analysts were observing large employment declines in other industries at the same time as the increase in seasonally adjusted retail employment, and thus had little doubt that the recession was continuing. A multivariate seasonal adjustment technique is one way to bring statistical content to such observations.

2.2 Technology-based interaction

An example of a technologically induced interaction between seasonals and cycles occurs in the motor vehicle sector. The timing of the introduction of new model year cars has been relatively constant over time, with September as the date for the sales launch of next year's models. As a result, motor vehicle assembly plants are typically shut down at some point during the third quarter for retooling, and employment in the sector drops sharply on a non-seasonally adjusted basis. Because plants are closed anyway, business cycle developments could be expected to have a relatively smaller effect on motor vehicle activity during the third quarter than during the fourth and first quarters, when car makers have a great deal of leeway in setting production schedules for new model year cars. This example highlights the importance of allowing for seasonal differences in the comovement between the common cycle and employment in industries for which the production function has an underlying seasonal component.

3. Model specification and estimation strategy

3.1 Preliminary data analysis

As will become evident, the computational burden associated with estimating our model grows quickly with the number of sectors and seasonal periods used in the analysis. To make estimation manageable, we restrict our empirical analysis to quarterly comovements in employment in nine sectors of the US economy: construction, motor vehicle manufacturing, durable goods manufacturing excluding motor vehicles, non-durable goods manufacturing, retail trade, other service-producing industries, federal government, state and local government, and mining. The combined sum of total employment in these sectors equals total non-farm payroll employment. The data have not been seasonally adjusted, but they have been adjusted to exclude the effects on employment of major strikes and temporary government workers hired as enumerators for the decennial census.

We decided to concentrate on comovements in the data at frequencies other than zero – both to simplify the analysis and because zero – frequency comovements in employment are likely to be dominated by demographic factors unrelated to the business cycle. With seasonal data, however, the correct filter is not obvious. Some practitioners, such as Box and Jenkins (1976) and Bell and Hillmer (1984), advocate taking both the first and seasonal differences of seasonal time series; that is, filtering quarterly data by $(1–L)(1–L^4)$, where L is the lag operator. It turns out, however, that this filter is not appropriate for our data: $(1–L)(1–L^4)$ has two roots at the zero frequency, but augmented Dickey-Fuller tests indicated that while, essentially, all of our (logged) series were $I(1)$, none were $I(2)$. Tests proposed by Hasza and Fuller (1982) also reject the joint differencing. Filtering by $(1–L^4)$ alone, which presumes the existence of roots at zero and at each of the seasonal frequencies π, $\pi/2$ and $3\pi/2$, also appears inconsistent with the time series properties of the employment data: although tests developed by Hylleberg, Engle, Granger and Yoo (1990) almost always fail to reject a zero-frequency root, they do reject the presence of at least one of the seasonal roots in most sectors. Accordingly, we chose to model the series as log differences and to use seasonal dummies to capture the average seasonality in each series.[4]

Table 1 shows the sample mean and variance of employment growth in each sector, the variance of the seasonal means and the variance explained by a univariate regression of job growth on a business cycle proxy (the percentage change in the Conference Board's coincident index) once the seasonal

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[4] An alternative strategy would have been to filter out those particular seasonal roots that were not rejected by the Hylleberg et al. (1990) tests on a series-by-series basis. We did not do so for two reasons. First, uniform prefiltering facilitates specification and interpretation of the common cycle. Second, tests of seasonal roots are problematic: Ghysels, Lee, and Noh (1994) show that the seasonal root tests of Hylleberg et al. (1990) and others can have low power against reasonable alternatives and that the existence of certain moving average terms can seriously distort the size of the tests. That said, our specifications of stochastic seasonality do not preclude the estimated model from containing seasonal roots (although it is not clear how our statistical inference would be affected by their presence).

means have been removed from the data. Our choice of sectoral disaggregation is designed to exploit four possible combinations of the relative intensities in seasonal and cyclical fluctuations. First, employment growth can exhibit both large seasonal and large cyclical variations; as is evident from the third and fourth columns, the motor vehicle sector is an example of an industry displaying such variations. Second, employment growth can be cyclically sensitive but not contain much seasonality; an example is manufacturing of durable goods other than motor vehicles. The third combination is found in state and local government, where employment has a large seasonal component caused by elementary and secondary school calendars but has essentially no cyclical component. Finally, in relative terms, as in the federal government sector, employment may not exhibit much seasonality or cyclicality.

Table 1

Growth rates in sectoral employment: summary statistics

Industry	**Mean (1)**	**Total variance (2)**	**Variance accounted for by: Seasonal dummies (3)**	**Variance accounted for by: Coincident index (4)**
Construction	0.45	109.26	102.70	0.96
Motor vehicles	–0.01	44.06	13.70	7.45
Durables except motor vehicles	0.10	3.48	0.42	1.55
Non-durables	0.04	3.35	2.34	0.33
Retail trade	0.64	11.35	10.02	0.13
Other services	0.76	1.29	1.07	0.07
Federal government	0.11	3.87	1.11	0.00
State and local government	0.85	17.50	16.45	0.00
Mining	–0.16	8.89	2.74	0.25
Total employment	0.52	2.83	2.24	0.24

Note: Summary statistics are computed from 1953:Q1 to 1989:Q4 and are based on log differences (multiplied by 100 to be comparable to percent changes). Variance accounted for by coincident index is for series with seasonal means removed.

3.2 Model specification

Let $\{Y_{it}\}$ be an $n \times 1$ vector of non-seasonally adjusted log differences in industry-specific employment. Following Stock and Watson, we assume the business cycle, C_t, can be defined by the comovement over time among the various sectors in the economy. The basic model is:

(1) $Y_{it} = \beta_i S_t + \gamma_i S_t C_t + u_{it}$ $\quad i = 1,2,...,n$

(2) $[1 - D_i(L)]u_{it} = \varepsilon_{it}$ $\quad i = 1,2,...,n$

(3) $C_t = \delta S_t + v_t$

(4) $[1 - \varphi(L)]v_t = \omega_t$

S_t is an s x 1 vector of seasonal dummies, and β_i, γ_i and δ are 1 x s vectors of coefficients, where s is the number of seasons in the year. $\beta_i S_t$ and δS_t capture the deterministic seasonal variation in $\{Y_{it}\}$ and C_t, respectively. Sectors' cyclical movements may be concentrated in different seasons, reflecting sectoral idiosyncrasies in production or demand. The model incorporates such interactions through $\gamma_i S_t C_t$, which measures the comovement between $\{Y_{it}\}$ and the common cycle and varies with the seasons according to γ_i.

The $\{u_{it}\}$ are idiosyncratic stochastic movements in the $\{Y_{it}\}$, and v_t is the stochastic variation in C_t. The $\{u_{it}\}$ and v_t are assumed to follow linear univariate autoregressive processes described by the lag polynomials $\{1\text{-}D_i(L)\}$ and 1 - $\varphi(L)$. Both the $\{u_{it}\}$ and v_t may contain stochastic seasonality. The innovations in $\{u_{it}\}$ and v_t, $\{\varepsilon_{it}\}$ and ω_t are assumed to be serially uncorrelated Gaussian processes with variances of $\{\sigma_{it}^2\}$ and σ_ω^2, respectively. As is usual in such models, the $\{\gamma_i\}$ and σ_ω^2 are not jointly identified; we follow the common practice of setting the variance of the innovation in the unobserved component, σ_ω^2, equal to unity.

The other standard assumption needed to identify the single cyclical index is that the $(v_t, u_{1t}, \ldots, u_{nt})$ are mutually uncorrelated at all leads and lags. This forces C_t to capture all of the stochastic comovement among the $\{Y_{it}\}$. Operationally, this is achieved by modelling the $\{u_{it}\}$ as univariate processes; that is, we assume $E[\varepsilon_{it}\varepsilon_{j\tau}] = 0$ for all t, τ and $i \neq j$, and $E[\varepsilon_{it}\omega_\tau] = 0$ for all t, τ and i (see Watson and Engle (1983) and Stock and Watson (1991)).

3.3 State-space form

Equations (1) - (4) form an unobserved components model. Equations (1) are observation equations relating the data $\{Y_{it}\}$ to the unobserved state variables, $(C_t, u_{1t}, \ldots, u_{nt})$. Equations (2), (3) and (4) are transition equations providing the laws of motion for the state variables. When written in state-space form, the order of the state vector for this model is rather large, n+1 plus the sum of the number of lags in the $\{D_i(L)\}$ and $\varphi(L)$. To ease the computational burden, we reduced the dimensionality of the state vector by prefiltering the observation equations by the $\{1\text{-}D_i(L)\}$, transforming (1) to:

$$(1') \qquad Y_{it} = [1\text{-}D_i(L)]\beta_i S_t + D_i(L) Y_{it} + [1 - D_i(L)]\gamma_i S_t C_t + \varepsilon_{it} \qquad i = 1,2,\ldots,n$$

This reduces the dimension of the state vector to 1 plus the maximum number of lags in the $\{D_i(L)\}$ and $\varphi(L)$. We call this dimension p. We also removed deterministic seasonal means, b_{iq}, from the Y_{it} (from (1), these means are $b_{iq} = \beta_{iq} + \gamma_{iq}\delta_q$, where β_{iq}, γ_{iq} and δ_q are the qth elements of β_i, γ_i and

δ, respectively) and from C_t (this mean is δ_q) before estimation; the mean-adjusted Y_{it} and C_t are denoted by $\tilde{Y}_{it}$ and $\tilde{C}_t$.[5]

Let $\tilde{Y}_t$ be the nx1 vector $(\tilde{Y}_{1t},\tilde{Y}_{2t},\ldots,\tilde{Y}_{nt})'$, $\tilde{C}_t^*$ be the (p+1)x1 vector $(\tilde{C}_t,\tilde{C}_{t-1},\ldots,\tilde{C}_{t-p})'$ and ε_t be the n x 1 vector $(\varepsilon_{1t},\varepsilon_{2t},\ldots,\varepsilon_{nt})'$. Equations (1′), (3) and (4) may then be written in the state-space form:

(5) $$\tilde{Y}_t = Z_t \tilde{C}_t^* + D(L)\tilde{Y}_{t-1} + \varepsilon_t$$

(6) $$\tilde{C}_t^* = \Phi \tilde{C}_{t-1}^* + \Omega_t$$

where: (5) is the (vector) observation equation; (6) is the (vector) state transition equation; Z_t is an n x p matrix of time-varying coefficients

$$Z_t = \begin{bmatrix} \gamma_1 S_t & -\gamma_1 d_{i1} S_{t-1} & \cdots & -\gamma_1 d_{ip} S_{t-p} \\ \vdots & \vdots & & \vdots \\ \gamma_n S_t & -\gamma_n d_{n1} S_{t-1} & \cdots & -\gamma_n d_{np} S_{t-p} \end{bmatrix}$$

where d_{ik} is the coefficient on L^k in $D_i(L)$; $D(L)$ is an n x n matrix with the polynomials $D_i(L)$ on the diagonals and zeros on the off diagonals; Φ is the p x p coefficient matrix

$$\Phi = \begin{bmatrix} \varphi_1 & \varphi_2 & \cdots & \varphi_{p-1} & \varphi_p \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

and Ω_t is the p x 1 vector $(\omega_t,0,0,\ldots,0)'$. Note that the time variation in Z_t is due only to the seasonality in the $\{\gamma_i\}$. In addition, $\phi_j = 0$ and $d_{ij} = 0$ for any j less than p but greater than the order of $\varphi(L)$ or $D_i(L)$, respectively.

Let θ be the parameters $[\{\gamma_{it}\}, \{d_{ij}\}, \{\sigma_{it}^2\}]'$. Given θ, the conditional moments of $\tilde{C}_t^*$ can be estimated by applying the Kalman filter to equations (5) and (6); we denote these moments as $\tilde{C}_{t|\tau}^* = E_\tau[\tilde{C}_t^*]$ and $\Sigma_{t|\tau} = E\left[\left(\tilde{C}_t^* - \tilde{C}_{t|\tau}^*\right)\left(\tilde{C}_t^* - \tilde{C}_{t|\tau}^*\right)'\right]$. See the Appendix for details.

[5] Results from a model estimated without removing deterministic means were similar to those reported below.

3.4 Estimation

Because of the large number of parameters in θ, the usual derivative-based methods for maximising the likelihood function associated with (5) and (6) would be computationally quite burdensome. Instead, we maximise the likelihood using the expectations maximisation, or EM, algorithm. The EM algorithm consists of (1) an expectations step to compute the expected value of the likelihood function conditional on the observed data and a set of parameter values, followed by (2) maximisation of this conditional expected likelihood with respect to θ. In state-space models, for a given set of parameters the expectations step first uses the Kalman filter to construct expectations of the state vector and prediction error variances conditional on the data up to time t (the $\tilde{C}^*_{t|t}$ and $\Sigma_{t|t}$ defined above with $\tau=t$) and then uses a Kalman smoother to construct expectations of these items based on the entire data set (denoted as $\tilde{C}^*_{t|T}$ and $\Sigma_{t|T}$).

In our model, the conditional expected likelihood constructed from $\tilde{C}^*_{t|T}$ and $\Sigma_{t|T}$ is a simple multi-equation system, which, because the $(\omega_t, \varepsilon_{1t}, ..., \varepsilon_{nt})$ are mutually uncorrelated, can be estimated on an equation-by-equation basis. In particular, θ and associated standard errors can be computed from single-equation regression models augmented by functions of $\Sigma_{t|T}$ to account for the uncertainty in $\tilde{C}^*_{t|T}$. These estimated parameter values become inputs into the next iteration of the algorithm, which proceeds until the likelihood function converges. The appendix provides further details on the application of the EM algorithm to our problem. Shumway and Stoffer (1982), Watson and Engle (1983), and Ruud (1991) provide more general treatments of the EM algorithm.

In specifying $\{ D_i(L) \}$ and $\varphi(L)$, we assume that the basic form of these lag polynomials is:

$$(7) \qquad 1\text{-}D_i(L) = \left(1\text{-}d_1L\text{-} \dots \text{-}d_rL^r\right)\left(1\text{-}d_sL^s\text{-} \dots \text{-}d_{sm}L^{sm}\right)$$

where $r < s$. The polynomial in L^s is meant to capture any stochastic seasonality in the data. Applying standard model selection criteria to estimates of (1′) based on an initial guess for $\tilde{C}_t$ (we used the deviation in total employment growth from seasonal means and a time trend) provides us with a basis for deciding upon initial guesses for the orders r and m. The resulting specifications were used to estimate the complete model. We then respecified (7) if correlograms or Q-statistics revealed residual serial correlation in the $\{ u_{it|t-1} \}$ or in $\tilde{C}_{t|t-1}$, if Wald tests indicated we could exclude terms, or if Lagrange multiplier tests suggested the presence of additional autoregressive terms or rejected the restrictions $d_{ks+j} = d_{ks} d_j$.[6] The models we eventually chose are listed in the Appendix. Note that, in many cases, the model failed some specification test unless an unrestricted autoregressive term $\tilde{d}_j L^j$ was added to (7).

Our modelling of $(\tilde{C}_t, u_{1t}, \ldots, u_{nt})$ as autoregressive processes differs from the canonical univariate specification for seasonal time series described in Hillmer and Tiao (1982) and Bell and Hillmer (1984). The canonical model assumes that the observed series is the sum of unobserved seasonal and non-seasonal components and imposes restrictions on these which imply that, at a minimum, the time series specification for the observed data contains a moving average (MA) term of order *s*. In our model, allowing the $\{u_{it}\}$ to follow MA processes greatly limits the extent to which we can reduce the dimensionality of the state-space model by prefiltering equations (1). Furthermore, as discussed in Ruud (1991), the usual state-space representations of moving average models cannot be estimated by the EM algorithm because the support of the expected likelihood is a function of the unknown parameters in the model.[7] For these reasons, we chose the AR representations for the $\{u_{it}\}$.

Although AR representation eases estimation, it does not lend itself to a parametric decomposition of u_{it} into a seasonal factor u_{it}^S and a seasonally adjusted piece u_{it}^N, as is possible in the canonical set-up. Instead, we run the $\{u_{it|T}\}$ through the X-11 program to separate them into seasonal and non-seasonal components.[8] X-11 should be adequate for this task: because the $\{u_{it}\}$ contain only idiosyncratic variations, there is no information in other series that would affect the decomposition performed by X-11. The stochastic seasonality in C_t is also identified by applying X-11 to $\tilde{C}_{t|T}$. Here again, because all of the common cycle in the employment data is contained in C_t, X-11 should do an adequate job of identifying its seasonal and non-seasonal components.

6 Note that standard Q-tests or correlograms using the Kalman smoother equation residuals $\hat{\varepsilon}_{it|T}$ are invalid, since even in a model with serially uncorrelated ε_{it}, $\hat{\varepsilon}_{it|T}$ and $\hat{\varepsilon}_{i\tau|T}$ will be correlated owing to conditioning on *T*.

7 Ruud (1988) offers an alternative state-space representation of MA processes that can be theoretically estimated using the EM algorithm. We were unable to successfully estimate such a formulation, however, possibly reflecting invertibility problems in the MA portions of the models.

8 Because it is a two-sided moving average filter, X-11 needs forecasts beyond the end of the sample to construct seasonal factors for data near the end of the sample. We generate forecasts using the AR models (7) for the $\{u_{it}\}$ and v_t. Ordinary X-11 forecasts by using previous averages, while X-11 ARIMA forecasts by building an ARIMA model.

4. Empirical results

4.1 Characteristics of the unobserved common cycle

Our estimate of the common cycle is $C_{t|T}$, which is constructed by adding the unconditional seasonal means δS_t to the $\tilde{C}_{t|T}$ generated by the Kalman filter and smoother.[9] The estimation period runs from the first quarter of 1953 to the fourth quarter of 1989 inclusive.

The top panel of Figure 1 shows the non-seasonal component of the common cycle, $C^N_{t|T}$ (the solid line), and compares it with quarterly log differences in total non-farm payroll employment as seasonally adjusted by X-11 (the broken line).[10] Although $C^N_{t|T}$ exhibits wider fluctuations, it picks up the same cyclical movements evident in the seasonally adjusted employment data. In particular, $C^N_{t|T}$ drops sharply during recessions. In addition, it turned down slightly earlier than payroll employment in the 1957, 1974 and 1982 recessions and began to decline in 1989 before the onset of the 1990 recession.

The bottom panel plots the seasonal factors in the common cycle (the dark line) and in payroll employment growth (the light line). The variability in $C^S_{t|T}$ is less pronounced than that in total employment. Moreover, the timing of seasonality in the common cycle differs from that in employment. For total employment, growth is lowest during the first and third quarters and highest during the second and fourth quarters. $C^S_{t|T}$ also declines during the first quarter and rises strongly in the second quarter. However, in contrast to total employment, growth in $C^S_{t|T}$ is well above average in the third quarter and below average in the fourth quarter.

Summary statistics for the common cycle and total employment shown in Table 2 confirm the impressions gained from Figure 1. The overall variance in $C_{t|T}$ is somewhat larger than that of total employment growth. However, the amount of the total variance accounted for by seasonality is much

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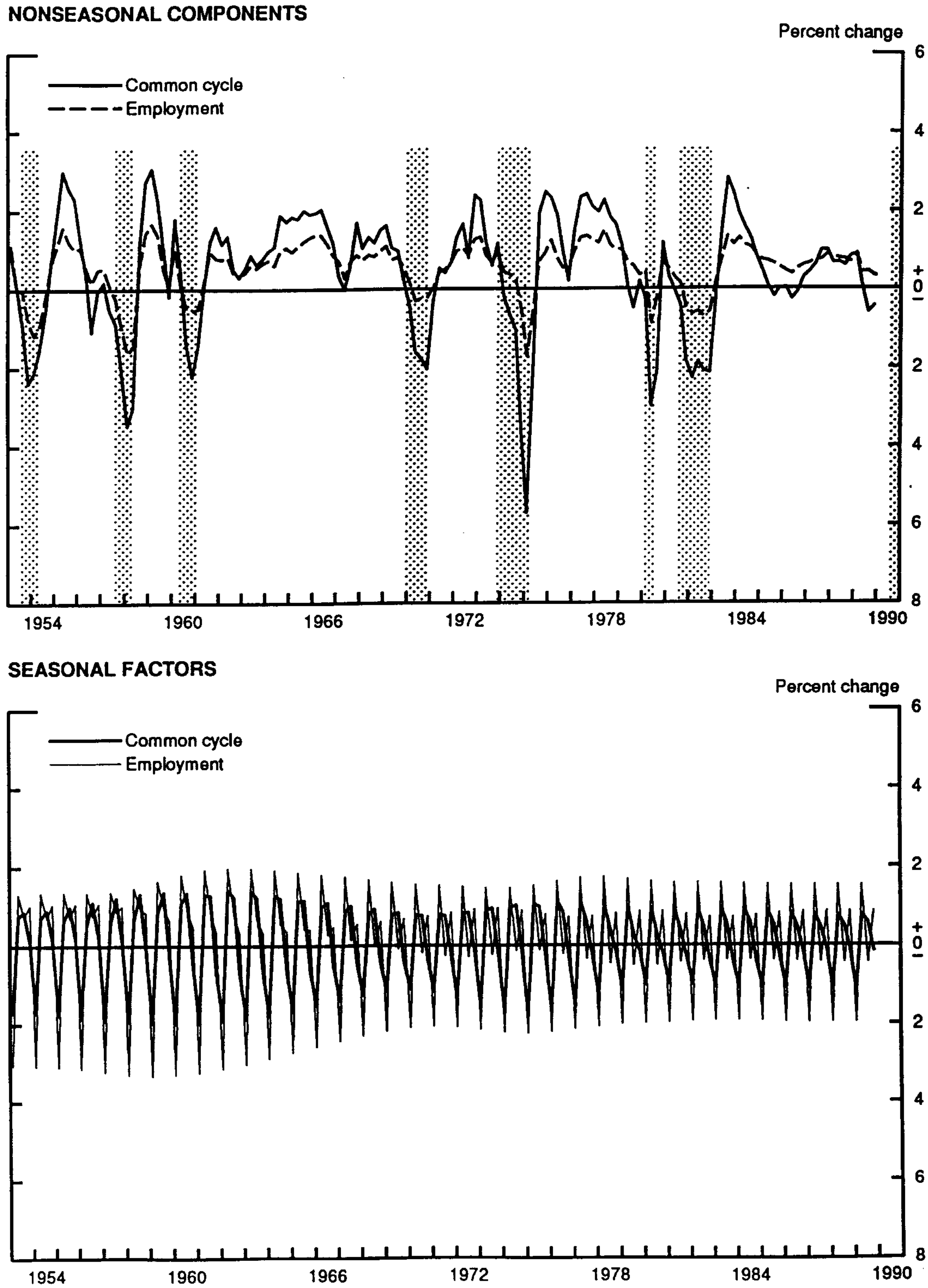
9 The δS_t were calculated from the unconditional means of the Y_{it} and the Kalman filter restrictions (see the Appendix, equation (A.3)). The seasonal component of $C_{t|T}$ is $C^S_{t|T} = \tilde{C}^S_{t|T} + (\delta - \bar{\delta})S_t$, where $\tilde{C}^S_{t|T}$ is the seasonal factor calculated by applying X-11 to $\tilde{C}_{t|T}$ and $\bar{\delta}$ is the average of the $\{\delta_q\}$. The seasonally adjusted common cycle is then $C^N_{t|T} = (\tilde{C}_{t|T} - \tilde{C}^S_{t|T}) + \bar{\delta}$.

10 The seasonally adjusted total employment series is not identical to the published BLS data, which are the sum of seasonally adjusted industry (two-digit SIC) series.

Figure 1

The common cycle and total employment



smaller for the common cycle – about 30% for $C_{t|T}$ as compared to nearly 85% for employment. This difference implies that a large portion of the seasonality in total payroll employment growth derives from sector-specific sources that do not have significant spillovers to the rest of the economy. More generally, it suggests caution when gauging the extent to which the simple seasonal variation in aggregate variables represents business cycle-like coherence in seasonal comovements across component sectors.

Table 2

The common cycle and total employment growth: summary statistics

	Common cycle	**Total employment**
Means:		
Total	0.64	0.52
Q1	–0.77	–1.90
Q2	1.53	2.09
Q3	1.49	0.63
Q4	0.36	1.28
Variances:		
Total	3.36	2.83
Seasonal dummies	0.90	2.24
Stochastic seasonal	0.08	0.16
Prediction error variances:		
$\Sigma_{t\|t\text{-}1}$	1.18	
$\Sigma_{t\|t}$	0.21	
$\overline{\Sigma}_{t\|T}$	0.17	

Note: $\overline{\Sigma}_{t|\tau}$ denotes the average value of $\Sigma_{t|\tau}$ over the 1953:Q1 to 1989:Q4 estimation period.

4.2 Comovements between the component series and the common cycle

The first four columns of Table 3 show the comovement between $\tilde{C}_t$ and the log difference in industry employment during each quarter of the year, γ_{iq}, q = 1, 2, 3, 4. Column 5 presents the test that this comovement is always zero ($\gamma_{i1} = \gamma_{i2} = \gamma_{i3} = \gamma_{i4} = 0$), and Column 6 the test that the comovement does not differ across seasons ($\gamma_{i1} = \gamma_{i2} = \gamma_{i3} = \gamma_{i4}$). Both hypotheses are tested using Wald tests based on the covariance matrix estimates described in the Appendix; the statistics are distributed χ^2 with four and three degrees of freedom, respectively.

Not surprisingly, the γ_{iq} coefficients are largest in construction and in the manufacturing industries, reflecting the significant degree of cyclicality in these sectors. In addition, the coefficients are larger in durables manufacturing than in non-durables manufacturing. Within the service-producing sector, retail trade exhibits the most cyclicality, while the γ_{iq} coefficients for the government sectors are close to zero. The $\{ \gamma_i \}$ are estimated fairly precisely, and in every industry except federal government and state and local government, we easily reject the null hypothesis of no cyclicality.

Table 3

Industry cyclicality

Industry	**Cyclical response by season**[1]				**Comovement with cycle**[2]	**Seasonality in comovement**[3]
	Q1 (1)	**Q2 (2)**	**Q3 (3)**	**Q4 (4)**	$\chi^2(4)$ **(5)**	$\chi^2(3)$ **(6)**
Construction	0.84	1.01	0.70	0.50	108.18	8.67
	(0.12)	(0.14)	(0.15)	(0.12)		
Motor vehicles	2.31	1.29	1.00	1.47	105.56	6.91
	(0.32)	(0.39)	(0.42)	(0.35)		
Durables except motor vehicles	0.93	0.93	0.71	0.87	498.03	15.86
	(0.05)	(0.06)	(0.06)	(0.05)		
Non-durables	0.58	0.49	0.34	0.47	388.33	22.13
	(0.03)	(0.04)	(0.04)	(0.04)		
Retail trade	0.35	0.32	0.30	0.26	240.37	2.92
	(0.04)	(0.04)	(0.05)	(0.04)		
Other services	0.22	0.23	0.21	0.15	154.10	8.88
	(0.02)	(0.03)	(0.03)	(0.02)		
Federal government	0.06	0.00	0.11	0.10	3.96	1.66
	(0.06)	(0.08)	(0.08)	(0.07)		
State and local government	–0.06	0.00	–0.04	–0.09	6.26	2.58
	(0.04)	(0.05)	(0.05)	(0.04)		
Mining	0.60	0.79	0.26	–0.16	24.34	16.85
	(0.19)	(0.21)	(0.23)	(0.20)		

[1]Standard errors are in parentheses. [2]The test is $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$. Critical values are 9.49 at the 5% significance level and 7.78 at the 10% level. [3]The test is $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4$. Critical values are 7.81 at the 5% significance level and 6.25 at the 10% level.

In some industries, the γ_{iq} coefficients are quite different across quarters, with the patterns potentially explainable by seasonal variations in the production technologies. For example, the correlation between motor vehicle employment and the cycle is smallest during the third quarter, when model year changeovers occur regardless of economic conditions. Carmakers have more leeway over how quickly they bring production of the new model year up to speed, and they often vary the Christmas and New Year holiday shutdowns according to economic conditions; this flexibility is reflected in the larger values for γ_{i4} and γ_{i1}. Note, however, that γ_{i3} is still large in comparison with the γ_{iq} coefficients in other industries, indicating that there remains substantial cyclicality in third-quarter motor vehicle employment relative to other sectors. Other manufacturing industries also exhibit a smaller correlation with the cycle in the third quarter, consistent with the traditional timing of vacations. The differences in γ_{iq} coefficients in construction and mining appear to be associated with seasonal weather patterns. In both industries, the correlations with the cycle are largest in the second quarter, when improvements in the weather bring down the entire production cost schedule. In each of these goods-producing sectors, the hypothesis that the comovement between employment growth and the cycle does not vary across seasons is rejected at the 10% level, and in all but motor vehicle manufacturing, the hypothesis is rejected at the 5% level.

In contrast, there is little evidence of variation across seasons in the comovement between employment growth and the cycle in service-producing industries. (Although we easily reject the null hypothesis of equal γ_{iq} coefficients in other services, the small numerical differences between the γ_{iq} coefficients suggest that this rejection has little economic content.) Given anecdotal evidence concerning Christmas hiring, this lack of seasonal variation seems surprising in retail trade. We suspect that seasonal comovements might be present in monthly data, but that this variation is not important at the quarterly frequency.[11]

4.3 Decomposition of the component series

Our specification allows Y_{it} to be partitioned into seasonal factors and seasonally adjusted pieces that, in turn, are determined by either the idiosyncratic or the common movement in the data. Let $\overline{\gamma_i\delta} = (1/4)\sum_{q=1}^{4}\gamma_{iq}\delta_q$; $\bar{\beta}_i = (1/4)\sum_{q=1}^{4}\beta_{iq}$; and $\bar{\gamma}_i = (1/4)\sum_{q=1}^{4}\gamma_{iq}$. Y_{it} can then be written as the sum of the following eight components:

	Seasonal factor	Seasonally adjusted component
Deterministic:		
Cyclical	$\gamma_{iq}\delta_q - \overline{\gamma_i\delta}$	$\overline{\gamma_i\delta}$
Idiosyncratic	$\beta_{iq} - \bar{\beta}_i$	$\bar{\beta}_i$
Stochastic:		
Cyclical	$\gamma_{iq}S_t\tilde{C}_t^S + (\gamma_{iq} - \bar{\gamma}_i)S_t\tilde{C}_t^N$	$\bar{\gamma}_i\tilde{C}_t^N$
Idiosyncratic	u_{it}^S	u_{it}^N

[11] As an informal test of this hypothesis, we regressed the monthly percentage change in retail employment on a set of monthly dummies and the dummies interacted with the percentage change in the Conference Board's index of coincident indicators. In both economic and statistical terms, the coefficient on the interaction term in December was significantly larger than the average of the 12 interaction coefficients. The interaction coefficient in January was much smaller than average, but we could not statistically reject equality with the mean. In contrast, when we ran the analogous regression using quarterly data, the coefficients on the interaction terms were quite similar in all four quarters.

Table 4

Decompositions of the variation in industry employment

4.1 Variation in seasonal means

Industry	**Cycle (1)**	**Idiosyncratic (2)**	**2*Covariance (3)**	**Total (4)**
Construction	0.84	84.63	16.53	102.00
Motor vehicles	3.24	15.51	–5.14	13.61
Durables except motor vehicles	0.87	0.13	–0.58	0.41
Non–durables	0.27	1.20	0.85	2.32
Retail trade	0.13	8.47	1.36	9.95
Other services	0.06	0.66	0.32	1.04
Federal government	0.01	1.01	0.09	1.10
State and local government	0.00	16.26	0.08	16.34
Mining	0.41	1.07	1.24	2.72

4.2 One-step-ahead forecast error variance decompositions

Industry	**Common cycle model**				**Time series models**	
	Cycle		**Idiosyn.**	**Total**	**AR**	**ARIMA**
	Seasonal (1)	**Nonseasonal (2)**	**(3)**	**(4)**	**(5)**	**(6)**
Construction	0.03	0.69	2.04	2.76	3.32	4.10
Motor vehicles	0.30	2.73	16.03	19.06	26.09	35.96
Durables except motor vehicles	0.01	0.87	0.29	1.17	1.32	2.24
Non–durables	0.01	0.26	0.15	0.42	0.50	0.69
Retail trade	0.00	0.11	0.20	0.31	0.53	0.50
Other services	0.00	0.05	0.06	0.11	0.12	0.14
Federal government	0.00	0.01	0.64	0.65	0.66	0.87
State and local government	0.00	0.00	0.26	0.27	0.27	0.29
Mining	0.15	0.17	3.88	4.20	4.56	5.44

Note: Forecast error variances for the seasonal and non–seasonal cycles are estimated by the sample averages of $\gamma_{is}^2 S_t \Sigma_{t|t-1} - \bar{\gamma}^2 \Sigma_{t|t-1}$ and $\bar{\gamma}^2 \Sigma_{t|t-1}$, respectively; the idiosyncratic error variance is set to $\hat{\sigma}_i^2$. *AR* models are $D(L)\tilde{Y}_i(t) = e_i(t)$, with $D(L)$ from (A.10). The ARIMA models are those chosen automatically by the X–11 ARIMA program.

Panel 4.1 in Table 4 presents the variability in the deterministic seasonal components of the $\{Y_{it}\}$. As shown in Columns 1 and 2, in most industries the cyclical variability due to the $\gamma_{iq}\delta_q$ components is relatively small, and the movement in deterministic seasonal means largely reflects idiosyncratic variation in β_{iq} coefficients. Because the seasonal means in $C_{t|T}$ are constructed from the seasonal means in the $\{Y_{it}\}$, the covariance between the idiosyncratic and cyclical means in Y_{it} – shown in the third column – provides a rough idea of the contribution of a sector to the seasonal mean in $C_{t|T}$. The most interesting result here pertains to our earlier finding that $C_{t|T}$ is above average in the third quarter, while growth in total payroll employment is about average at that time of year. Aggregate payroll employment growth in the third quarter is held down by job cuts in the state and local government sector. But because the γ_{iq} coefficients in this sector are all near zero, the covariance between the deterministic seasonal in $C_{t|T}$ and that in state and local government employment is quite low, so that this sector contributes little to seasonal movements in $C_{t|T}$. Total employment growth during the third quarter is also held down by declines in motor vehicle and other durable goods manufacturing. However, the patterns of the γ_{iq} coefficients in these industries produce negative covariances between the cyclical and idiosyncratic deterministic seasonal means, so that declines in the sectors' employment do not lower the third-quarter mean in the common cycle.

With regard to the stochastic variation in the $\{Y_{it}\}$, our identifying restrictions imply that the one-period-ahead forecast error variance in $\tilde{Y}_{it}$ is the simple sum of the forecast error variances in the stochastic seasonal and seasonally adjusted components related to the common cycle and in the (total) idiosyncratic error, $u_{t|t-1}$.[12] As seen in Columns 1 - 4 of Panel 4.2, despite the comovements with the common cycle evident in Table 3, in most cases the forecast error variance can largely be attributed to the idiosyncratic component of the series. Moreover, even in sectors where the common cycle makes a noticeable contribution, this reflects variations in the non-seasonal cyclical component rather than seasonal variations in the comovement with the common cycle. Nonetheless, identifying the common cycle is important. In sectors with a sizable non-seasonal cyclical component, the total one-step-ahead forecast variances of the common cycle model are smaller than those from two obvious univariate alternatives: an AR model for $\tilde{Y}_{it}$ with the same lag structure as specified in equations (A.10), Column 5, and the ARIMA model chosen by the X-11 ARIMA program, Column 6.

Finally, Panel 4.3 considers the reduction in the variance of the industry-level data obtained by seasonal adjustment. As can be seen by comparing Columns 1 and 2, with the exception of durables excluding motor vehicles, most of the reduction in variability is accounted for by the idiosyncratic seasonals. Nonetheless, a comparison of Columns 2 and 3 indicates that accounting for the cyclical sensitivity of the seasonal factors reduces the variability by an additional 20% (relative to the idiosyncratically seasonally adjusted data) in construction, durables manufacturing (excluding motor vehicles), non-durables manufacturing, retailing and other services.

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[12] Because the stochastic seasonal factors are calculated using the smoothed unobserved variables $\{u_{it|T}\}$ and $\tilde{C}_{t|T}$ – which are correlated over time – we can not construct a simple decomposition of the stochastic variation in $\tilde{Y}_{it}$ into its cyclical and idiosyncratic components.

4.3 Variance reduction due to seasonal adjustment

Industry	Variance in:			Additional reduction in variance from cycle during:		
		Seasonally adjusted by:				
	NSA data (1)	Idiosyn. only (2)	Idiosyn. and cycle (3)	Full sample (4)	Expan–sions (5)	Reces–sions (6)
Construction	109.26	3.75	2.96	0.79	0.80	0.67
Motor vehicles	44.06	25.98	22.20	3.78	0.03	15.32
Durables except motor vehicles	3.48	3.68	2.85	0.82	0.52	1.61
Non–durables	3.35	0.92	0.64	0.28	0.16	0.61
Retail trade	11.35	0.47	0.36	0.10	0.05	0.26
Other services	1.29	0.26	0.20	0.06	0.04	0.09
Federal government	3.87	0.65	0.64	0.01	0.01	0.00
State and local government	17.50	0.33	0.33	0.00	0.00	0.01
Mining	8.89	5.98	5.37	0.60	0.45	1.19

Note: Recessions refer to the period from the quarter of the cyclical peak to the quarter of the cyclical trough, inclusive.

The last three columns compare the reduction in variance associated with cyclical/seasonal interactions obtained over the entire sample period with those obtained during expansions and recessions separately. In a number of cases, most notably the three manufacturing industries, retailing and mining, the reductions are much more pronounced during recessions. As recessions are characterised by relatively sharp movements in $\tilde{C}^N_{t|T}$, the large effects during downturns probably reflect the tendency of comovements of these industries with the common cycle to be concentrated in certain seasons, a property captured by the term $(\gamma_{iq} - \bar{\gamma}_i)S_t\tilde{C}^N_{t|T}$ in our seasonal decomposition.

4.4 Comparison of multivariate and X-11 seasonally adjusted series

Table 5 shows R^2 values from the regressions

$$(8) \qquad Y^N_{it} = a + bY^{X11}_{it} + e_{it}$$

where Y^N_{it} is the seasonally adjusted series from our model, $Y^N_{it} = \bar{\beta}_i + \overline{\gamma_i\delta} + \bar{\gamma}_i\tilde{C}^N_{t|T} + u^N_{it}$, and Y^{X11}_{it} is the average of the deterministic means plus the seasonally adjusted series constructed by applying X-11 to $\tilde{Y}_{it}$. The table also compares $C^N_{t|T}$ with seasonally adjusted total employment growth. The first column presents results of (8) estimated over the 1953:Q1 to 1989:Q4 sample period used to identify and estimate our model. Most of these R^2 values are quite close to one, indicating that there is little within-sample difference between seasonal factors from our model and those from X-11. There is, however, a non-trivial difference between the seasonally adjusted common cycle and seasonally adjusted total payroll employment; this difference is also evident in the cyclical behaviour of the series depicted in Figure 1.

We also estimated equation (8) using data only from recessions. The R^2 values from these regressions (not shown) were quite similar to those obtained using the entire sample period, with most just 0.01 or 0.02 lower than those reported in Table 5. The one exception – also evident from Figure 1 – was the correlation between the common cycle and total employment, which fell from 0.90 over the entire sample to 0.76 during recessions.

It is perhaps not surprising that the R^2 values for the industry-level series are close to one, as most of the seasonal variation in the $\{Y_{it}\}$ is generally captured by the deterministic seasonal means. However, given the comparisons in Table 4.3 (Columns 5 and 6), the similarity of the R^2 values during recessions and expansions is somewhat surprising. One possible explanation might be related to the fact that both our multivariate model and X-11 are two-sided filters, so that seasonals for period τ use information not available until time $t > \tau$. Thus, the behaviour of employment *after* the beginning of a recession may help both filters identify total seasonality during the cyclical downturn.

Table 5

R^2 values from regressions of model-based seasonally adjusted series on X–11 seasonally adjusted series

	Log difference		Log level	
	In sample	**Out of sample**	**In sample**	**Out of sample**
Construction	0.96	0.93	0.96	0.90
Motor vehicles	0.96	0.76	0.96	0.79
Durables except motor vehicles	0.99	0.93	0.99	0.94
Non–durables	0.97	0.59	0.97	0.61
Retail trade	0.98	0.93	0.98	0.93
Other services	0.99	0.91	0.99	0.94
Federal government	0.99	0.79	0.99	0.87
State and local government	0.99	0.79	0.99	0.85
Mining	0.94	0.93	0.94	0.85
Common cycle and:				
Total employment	0.90	0.86	0.90	0.90
Constructed total	–	–	0.90	0.89

Note: The constructed total is seasonally adjusted total payroll employment growth calculated as the weighted sum of seasonally adjusted industry employment growth.

Because data are not available beyond time *T*, out-of-sample forecasts are needed to seasonally adjust variables near the end of the sample period. Differences in these forecasts could have a significant influence on seasonal factors towards the end of the sample. Indeed, given the differences in one-step-ahead forecast error variances shown in Table 4.2, we might expect some distinction between the seasonally adjusted series from our model and those from X-11 to emerge out of sample.

To examine this possibility, we conducted an out-of-sample experiment in which we constructed real-time seasonal factors from our model and compared them with the factors generated by an X-11 procedure similar to that used by the BLS. The BLS constructs seasonal factors for the subsequent six months each June and December. To mimic this procedure in our model, we first estimated $\{b_i\}$, δ and θ using data up to and including period *T*. Seasonal factors $\tilde{C}^S_{T+j|T}$ and $u^S_{iT+j|T}$ were then estimated for quarters j = 1, 2 using the model's forecasts of $\tilde{C}_{t+j|T}$ and $u_{t+j|T}$. Once the $\{Y_{it}\}$ are known, $C^N_{T+j|T+j}$ and the $\{Y^N_{iT+j}\}$ were calculated using:

$$C^N_{T+j|T+j} = \bar{\delta} + \tilde{C}^N_{T+j|T+j}$$

$$Y^N_{iT+j|T+j} = \bar{b}_i + \tilde{Y}^N_{iT+j|T+j}$$

where

$$\tilde{C}^N_{T+j|T+j} = \tilde{C}_{T+j|T+j} - \tilde{C}^S_{T+j|T}$$

$$\tilde{Y}^N_{iT+j|T+j} = \tilde{Y}_{iT+j} - \tilde{Y}^S_{iT+j|T} \quad \text{and}$$

$$\tilde{Y}^S_{iT+j|T} = \gamma_{iq} S_t \tilde{C}^S_{T+j|T} + (\gamma_{iq} - \bar{\gamma}_i)\, S_t \tilde{C}^N_{T+j|T} + (b_{iq} - \bar{b}_i) S_t + u^S_{iT+j|T}$$

for j = 1, 2. At the same time, we estimated $\tilde{Y}^{X11}_{iT+j|T}$ using X-11 seasonal factors calculated from data on $\tilde{Y}_{it}$ for $t = 1,2,...,T$. We repeated this experiment, re-estimating $\{b_i\}$, δ, θ and seasonal factors every two quarters between 1990:Q1 and 1994:Q4.

As can be seen from the second column of Table 5, the R^2 values generally fall when equation (8) is estimated using data from this real-time experiment.[13] There are quite noticeable differences in the seasonally adjusted series in motor vehicle and nondurables manufacturing, two sectors where the common cycle accounts for a significant portion of the one-step-ahead forecast variance. Curiously, the R^2 values between the seasonally adjusted government sectors also exhibit a marked decline, even though these series had little correlation with the common cycle. The correlation between the common cycle and total employment growth is about the same in this forecasting experiment as in the in-sample regression. But, as shown in Figure 2, $C^N_{T+j|T+j}$ leads total employment just prior to the

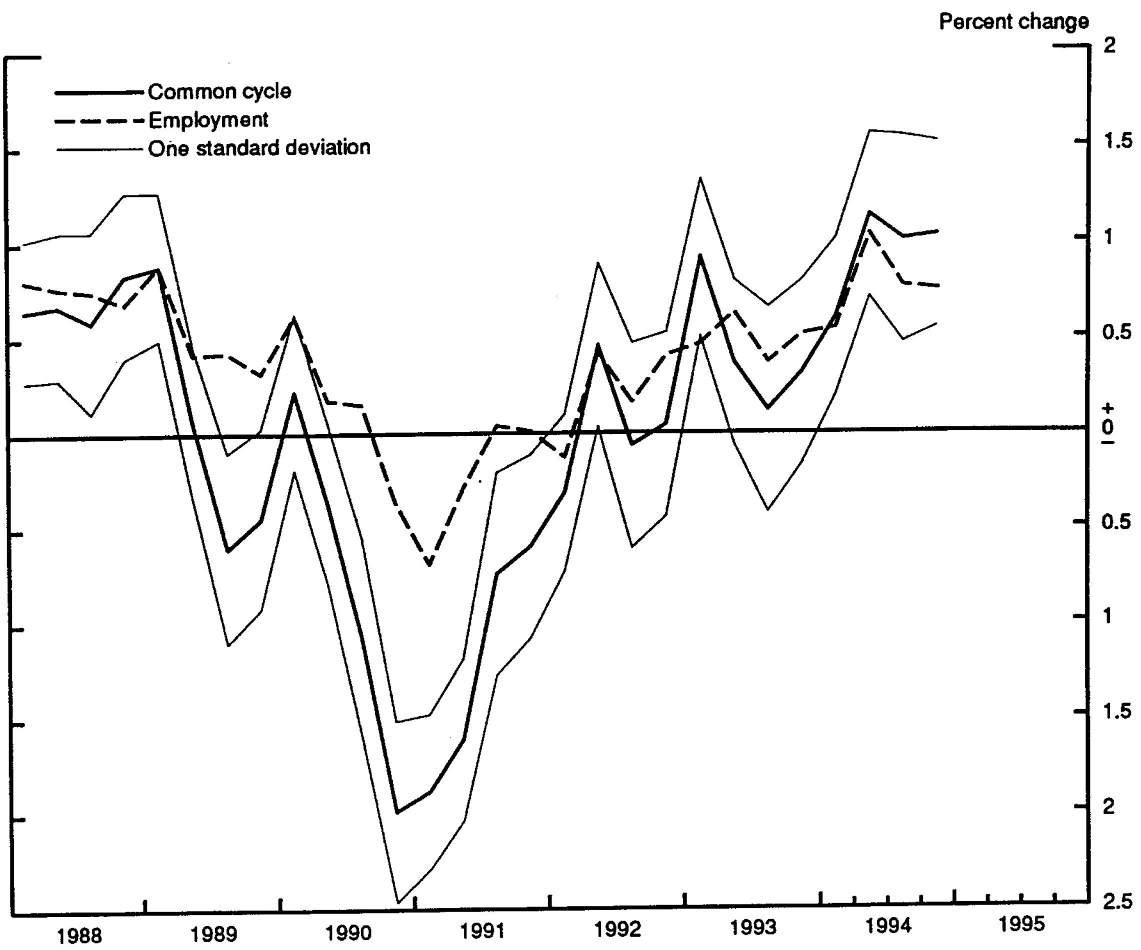
[13] In all but two cases, mining and the common cycle, the out-of-sample experiment produces higher regression standard errors.

1990 recession: $C^N_{T+j|T+j}$ turns negative in the second quarter of 1990, two quarters before the decline in total payroll employment.

Despite the lower R^2 values, there does not appear to be a significant difference in the stability of the two sets of industry-level seasonal factors: the average absolute revisions between the seasonal factors as first computed in our real-time experiment and those based on data up to end-1994 were not systematically different for the two methods of seasonal adjustment. The revisions in the seasonal factors for the common cycle tended to be somewhat larger than the revisions in the seasonals for total employment.

Figure 2

The common cycle and total employment: real time experiment



In practice, the BLS applies X-11 ARIMA to the log levels of employment rather than to the log differences. As shown in the two right-hand columns of Table 5, constructing Y_{it}^{X11} from the difference in seasonally adjusted log levels produces qualitatively the same results both in sample and in our real-time experiment. In addition, the BLS constructs seasonally adjusted total employment as the sum of the seasonally adjusted components. As can be seen from the last line in Table 5, constructing seasonally adjusted total employment growth in a similar manner does not change the relationship between the common cycle and total employment.

5. Comparisons with alternative cyclical indicators

An alternative strategy to the unobserved components model we employ would be to measure cyclical movements using an existing alternative time series. There are a host of possibilities to choose from, and Table 6 presents test statistics for the null hypotheses $\gamma_{i1} = \gamma_{i2} = \gamma_{i3} = \gamma_{i4} = 0$ and $\gamma_{i1} = \gamma_{i2} = \gamma_{i3} = \gamma_{i4}$ for four alternatives: (1) the Conference Board's index of coincident indicators, which is a weighted average of seasonally adjusted percentage changes in total non-farm payroll employment, constant-dollar personal income less transfers, industrial production, and constant-dollar manufacturing and trade sales; (2) a dummy variable set to one during NBER recessions; (3) the Conference Board's index of leading indicators, which is a weighted average of 11 series (nine of which are seasonally adjusted);[14] and (4) the coincident index developed by Stock and Watson for the NBER, using the same variables as in the Conference Board's coincident index, but derived from an unobserved components model similar to the one in equations (1) - (4) above.

In general, the hypothesis of no comovement between these alternative indicators and the sectoral employment series is still rejected for those sectors for which our model indicates comovement with the cycle. With respect to the tests for seasonality in the comovement, however, the alternative indicators produce a wide range of results. For example, using the NBER indicator, there appears to be statistically significant seasonality in the comovement between the cycle and employment growth in motor vehicles, other durables and retail trade, but little evidence of this interaction elsewhere. In contrast, using the Stock-Watson coincident index, significant effects show up in construction, other services and mining, but not in any other series. In general, the models using these alternative

[14] We used the leading indicator index as it was constructed prior to the revisions incorporated in December 1996. That index was comprised of seasonally adjusted percentage changes in manufacturing hours, initial claims for unemployment insurance, constant-dollar new orders for consumer goods and materials, vendor performance (diffusion index), constant-dollar orders for new plant and equipment, smoothed change in constant-dollar unfilled orders for durable goods, smoothed change in sensitive materials prices, and constant-dollar M2 (level), and non-seasonally adjusted stock prices and the index of consumer expectations.

indicators reject the hypothesis that there is no seasonality in the comovement less often than our model based on C_t, but the diversity of results makes them difficult to interpret.

Table 6

Industry cyclicality: alternative indicators

Chi–squared values

Industry	**Coincident index**		**NBER indicator**	
	Comovement with cycle	**Seasonality in comovement**	**Comovement with cycle**	**Seasonality in comovement**
Construction	64.6	9.5	22.5	0.9
Motor vehicles	141.4	8.9	59.1	15.4
Durables except motor vehicles	134.8	6.8	55.5	11.8
Non–durables	233.7	4.9	37.7	6.2
Retail trade	184.7	4.8	69.2	8.6
Other services	73.1	8.3	16.6	3.9
Federal government	8.0	6.1	6.5	5.9
State and local government	3.8	3.6	2.1	1.9
Mining	26.9	15.5	8.5	5.5
	Leading index		**S–W coincident index**	
	Comovement with cycle	**Seasonality in comovement**	**Comovement with cycle**	**Seasonality in comovement**
Construction	14.4	0.4	72.3	14.3
Motor vehicles	30.7	5.8	108.7	4.5
Durables except motor vehicles	13.4	3.7	106.4	0.9
Non–durables	24.6	3.1	151.6	0.8
Retail trade	18.1	7.6	97.9	5.8
Other services	0.9	0.5	45.1	12.4
Federal government	7.6	3.9	2.9	1.1
State and local government	3.6	2.2	5.1	4.2
Mining	10.4	7.3	15.9	15.1

Note: Tests of the comovement with the cycle are distributed $\chi^2(4)$, with critical values of 9.49 at the 5% significance level and 7.78 at the 10% level. Tests of seasonality in the comovement are distributed $\chi^2(3)$, with critical values of 7.81 and 6.25, respectively.

In any event, the use of such alternative time series as exogenous proxies for the business cycle strikes us as less desirable than our approach. As discussed in the introduction, most of these alternative indicators are based on seasonally adjusted data. Even if an alternative non-seasonally adjusted time series were an appropriate measure of the cycle, it would still be necessary to extract the comovement of that series with all of the data being seasonally adjusted in order to identify idiosyncratic seasonals. This would necessitate estimating an expanded version of the model presented in Section 3.

6. Conclusions

One strand of the recent literature on business cycle dynamics emphasises the potential interactions between seasonal and cyclical fluctuations in economic data. In this paper, we model such interactions using a multivariate unobserved components framework. Our model can be used to extract seasonal and cyclical components from a set of related time series, generate an index measuring a common cycle in these data and test for seasonal variations in the comovement between industry-level activity and the aggregate cycle. We apply this technique to a nine-sector disaggregation of quarterly total US payroll employment.

Overall, our results suggest that a sizable amount of the seasonal movements in industry-level employment is idiosyncratic. This finding helps explain why seasonality accounts for a smaller proportion of the variation in our model's common cycle than it does in total employment and why, within sample, the seasonal factors generated by our model are very close to those produced by a univariate X-11 filter.

However, our results also indicate that seasonal movements in the data can be considerably influenced by business cycle developments. In a number of sectors, the comovement of employment with the common cycle shows statistically significant variations over the year, and in some cases the estimates appear consistent with seasonality in the production technology affecting the responsiveness of employment to cyclical changes in demand. Furthermore, seasonal/cyclical interactions may influence the tracking of business cycle developments; our estimated common cycle leads movements in seasonally adjusted total employment around some business cycle peaks – a pattern that is also exhibited in a real-time, out-of-sample experiment run from 1990 to end-1994. This real-time simulation also produces some noticeable differences between the seasonal factors from our model and those from X-11, although our model's seasonals do not appear to be more stable than those produced by X-11.

A number of extensions of our model would make interesting topics for future research. Additional structural specifications of (1) might provide insights into the economic factors generating correlations between seasonality and cyclicality at the industry level, or one could specify a multi-index model where cyclical dynamics are also generated by sector-specific shocks that are directly correlated with shocks to the common cycle. However, these extensions come at the cost of additional (and potentially significant) computational complexity as long as one maintains a model with endogenous determination of the common cycle.

Appendix

A.1 The Kalman filter and the likelihood function

Our model is:

(5) $\tilde{Y}_t = Z_t \tilde{C}_t^* + D(L)\tilde{Y}_{t-1} + \varepsilon_t$

(6) $\tilde{C}_t^* = \Phi \tilde{C}_{t\text{-}1}^* + \Omega_t$

where $E[\varepsilon_t \varepsilon_t']$ = H and $E[\Omega_t \Omega_t']$ = M. Given the independence of each ε_{it} and $\varepsilon_{j\tau}$, H is a diagonal matrix with the terms σ_i^2 on the diagonals. M is a matrix with σ_ω^2 (1 in our model) in the (1,1) position and zeros elsewhere. Recall that θ is the vector of parameters $[\{\gamma_{it}\}, \{d_{ij}\}, \{\sigma_{it}^2\}]'$, $\tilde{C}_{t|\tau}^* = E_\tau[\tilde{C}_t^*]$ and $\Sigma_{t|\tau} = E[(\tilde{C}_t^* - \tilde{C}_{t|\tau}^*)(\tilde{C}_t^* - \tilde{C}_{t|\tau}^*)']$. The Kalman filter consists of the recursions

$$\tilde{C}_{t|t\text{-}1}^* = \Phi \tilde{C}_{t\text{-}1|t\text{-}1}^*$$

(A.1) $$\Sigma_{t|t\text{-}1} = \Phi \Sigma_{t\text{-}1|t\text{-}1} \Phi' + M$$

$$\tilde{C}_{t|t}^* = \tilde{C}_{t|t\text{-}1}^* + \Sigma_{t|t\text{-}1} Z_t' F_t^{\text{-}1} V_t$$

$$\Sigma_{t|t} = \Sigma_{t|t\text{-}1} - \Sigma_{t|t\text{-}1} Z_t' F_t^{\text{-}1} Z_t \Sigma_{t|t\text{-}1}$$

where $V_t = [\tilde{Y}_t - (Z_t \tilde{C}_{t|t\text{-}1}^* + D(L)\tilde{Y}_{t\text{-}1})]$, the one-period-ahead prediction error in $\tilde{Y}_t$, and $F_t = Z_t \Sigma_{t|t\text{-}1} Z_t' + H$. The joint log likelihood function can be written as

(A.2) $$L[\tilde{Y}_{it};\theta] = K - (1/2)\sum_{t=1}^{T}\left[\log(\det(F_t)) + V_t' F_t^{\text{-}1} V_t\right]$$

where K is the usual constant.

As noted in footnote 9, the unconditional seasonal means of $C_{t|T}$, δS_t, are calculated by solving the Kalman filter restrictions given by the third equation in (A.1),

(A.3) $$(C_{t|t}^* \text{-} \delta S_t) = (C_{t|t\text{-}1}^* \text{-} \delta S_{t\text{-}1}) + \Sigma_{t|t\text{-}1} Z_t' F_t^{\text{-}1}[(Y_t \text{-} b_i S_t) - (Z_t (C_{t|t\text{-}1}^* \text{-} \delta S_{t\text{-}1}) + D(L)(Y_{t\text{-}1} \text{-} b S_{t\text{-}1}))]$$

where the $\{b_i\}$ are the unconditional means of the $\{Y_{it}\}$.

A.2 *EM* algorithm estimation of the model

Assuming that $\tilde{C}_0^*$ is fixed, the unconditional log likelihood may be rewritten (with constant K') as

$$\text{(A.4)} \qquad L\left[\tilde{Y}_t;\theta\right] = K' - (T/2)\log\left|H\right| - (1/2)\sum_{t=1}^{T}\varepsilon_t' H^{-1}\varepsilon_t - (T/2)\log\left|M\right| - (1/2)\sum_{t=1}^{T}\Omega_t' M^{-1}\Omega_t$$

Given that H is a diagonal and the elements of M are all zero except for a 1 in the (1,1) slot, (A.4) simplifies to

$$\text{(A.5)} \qquad L\left[\tilde{Y}_t;\theta\right] = K' - (T/2)\log\prod_{i=1}^{n}\sigma_i^2 - (1/2)\sum_{i=1}^{n}\sum_{t=1}^{T}\varepsilon_{it}^2/\sigma_i^2 - (1/2)\sum_{t=1}^{T}\omega_t^2$$

The *EM* algorithm first takes the expectation of (A.5) conditioned on all available information. Replacing ε_{it} by $\varepsilon_{it|T} + (\varepsilon_{it} - \varepsilon_{it|T})$ and ω_{it} by $\omega_{it|T} + (\omega_{it} - \omega_{it|T})$ in (A.5) and taking expectations conditioned on the data in periods 1,2,...,T yields

$$\text{(A.6)} \qquad \begin{aligned} L_T\left[\tilde{Y}_t;\theta\right] &= K' - (T/2)\log\prod_{i=1}^{n}\sigma_i^2 - (1/2)\sum_{i=1}^{n}\left(1/\sigma_i^2\right)\sum_{t=1}^{T}\left\{\varepsilon_{it|T}^2 + Z_{it}\Sigma_{t|T}Z_{it}'\right\} - \\ &(1/2)\sum_{t=1}^{T}\left\{\omega_{t|T}^2 + \Sigma_{t|T} - \Phi\,\Sigma_{t\text{-}1,t|T} - \Sigma_{t-1,t|T}'\,\Phi' + \Phi\,\Sigma_{t\text{-}1|T}\,\Phi'\right\}\end{aligned}$$

where $L_T\left[\tilde{Y}_t;\theta\right] = E_T\left\{L\left[\tilde{Y}_t;\theta\right]\right\}$, Z_{it} is the ith row of Z_t, $\varepsilon_{it|T} = \tilde{Y}_{it} - Z_{it}\tilde{C}_{t|T}^* - D_i(L)\tilde{Y}_{it\text{-}1}$, $\omega_{t|T} = \tilde{C}_{t|T} - \Phi\tilde{C}_{t\text{-}1|T}$ and $\tilde{C}_{t|T}^*$, $\tilde{C}_{t|T}$, $\Sigma_{t|T}$, and $\Sigma_{t\text{-}1,t|T} = E\left[\left(\tilde{C}_{t\text{-}1}^* - \tilde{C}_{t\text{-}1|T}^*\right)\left(\tilde{C}_t^* - \tilde{C}_{t|T}^*\right)'\right]$ are outcomes of the Kalman smoother.[15] We use a fixed-interval Kalman smoother. For notational convenience, let $L_{t|T}\left[\tilde{Y}_t;\theta\right]$ be the time t observation of $L_T[\tilde{Y}_t;\theta]$ and $L_{it|T}\left[\tilde{Y}_{it};\theta_i\right] = \left[\varepsilon_{it|T}^2 + Z_{it}\,\Sigma_{t|T}\,Z_{it}'\right]/\sigma_i^2$.

The second step of the *EM* algorithm maximises (A.6) with respect to θ. Note that because there are no restrictions between the Z_{it} and Z_{jt}, $\{d_{ik}\}$ and $\{d_{jk}\}$, Z_{it} and Φ, or $D_i(L)$ and Φ, maximisation may be done on an equation-by-equation basis. That is, γ_i and d_{ik} are estimated by minimising each $\sum_{t=1}^{T} L_{it|T}\left[\tilde{Y}_{it};\theta_i\right]$ with respect to those parameters and σ_i^2 is estimated by $\left[\sum_{t=1}^{T}\left\{\hat{\varepsilon}_{it|T}^2 + \hat{Z}_{it}\,\Sigma_{t|T}\,\hat{Z}_{it}'\right\}\right]/T$. (Note that γ_i and d_{ik} enter this minimisation problem non-linearly.)

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[15] As discussed in Watson and Engle, $\Sigma_{t\text{-}1,t|T}$ can be estimated by running the Kalman filter and smoother with $\tilde{C}_t^*$ augmented by an additional lag of $\tilde{C}_t$.

Minimising $\sum_{t=1}^{T}\left\{\omega_{t|T}^{2}+\Sigma_{t|T}-\Phi\Sigma_{t\text{-}1,t|T}-\Sigma_{t\text{-}1,t|T}\Phi'+\Phi\Sigma_{t|T}\Phi'\right\}$ produces estimates of Φ. The resulting updated estimates of Z_t, $D(L)$ and M are then used in the next iteration of the *EM* algorithm.

A.3 Starting values

Starting values must be chosen for $\tilde{C}_{0|0}^{*}$ and $\Sigma_{0|0}$, both for the initial iteration in the *EM* algorithm and for the first pass through the Kalman filter at each iteration. After experimenting with some of the alternatives proposed in the literature, we decided to treat $\tilde{C}_{0|0}^{*}$ as fixed at its unconditional mean of 0 at each iteration. Treating $\tilde{C}_{0|0}^{*}$ as fixed implies $\Sigma_{0|0}$ = 0 at each iteration.

The estimates of θ and $\tilde{C}_{t|T}$ are not very sensitive to the starting values for $\tilde{C}_{0|0}^{*}$ and $\Sigma_{0|0}$. Nonetheless, the results obtained from fixed starting values had a few advantages over those from other possibilities, such as those proposed by Shumway, Olsen and Levy (1981) or Hamilton (1994), in terms of convergence speed, stability of model estimates (e.g. whether or not we removed unconditional seasonal means from the data prior to estimation) and time variation in $\Sigma_{t|T}$.

A.4 Testing

Because the *EM* algorithm is a maximum likelihood technique, the covariance matrix of $\hat{\theta}$ is given by the inverse information matrix. As discussed in Ruud (1991), the information matrix can be estimated by the expectation of $(1/T)\sum_{t=1}^{T}(\partial L_{t|T}[\tilde{Y}_t;\hat{\theta}]/\partial\theta)\bullet(\partial L_{t|T}[\tilde{Y}_t;\hat{\theta}]/\partial\theta)'$. Furthermore, because our problem reduces to equation-by-equation optimisation, the information matrix is block diagonal. Thus, the covariance matrix of the $\tilde{\theta}_i=\{\gamma_i,d_{ik}\}$ can be estimated by

$$V_{iT}=\hat{\sigma}_i^2\left[\sum_{t=1}^{T}\lambda_{it}\lambda_{it}'\right]^{-1} \tag{A.7}$$

where λ_{it} is the vector $\partial\varepsilon_{it|T}/\partial\tilde{\theta}_i$ + $(\partial Z_{it}/\partial\tilde{\theta}_i)\Sigma_{t|T}Z_{it}'$. We use the appropriate submatrices of the V_{iT} as the weighting matrices in the χ^2 statistics reported in Table 3.

In addition, we use Lagrange Multiplier (*LM*) tests to determine if the restrictions in $D_i(L)$ were satisfied and to test if terms d_k in $D_i(L)$ were insignificantly different from zero. In general, if d_j was insignificantly different from zero but d_k, $k > j$, was not, both terms were left in $D_i(L)$. The exceptions were cases where eliminating the insignificant d_j helped other specification problems, such as rejection of restrictions between seasonal and non-seasonal AR terms in (7).

The general form of the *LM* statistic for restrictions on equation (1′) is:

$$\text{(A.8)} \qquad (\sum_{t=1}^{T} \partial L_{it|T}[\tilde{Y}_t;\hat{\theta}_i]/\partial\bar{\theta}_j)'\,\Sigma_{iT}\,(\sum_{t=1}^{T} \partial L_{it|T}[\tilde{Y}_t;\hat{\theta}_i]/\partial\bar{\theta}_j)$$

where $\bar{\theta}_j$ is the parameter(s) being tested, Σ_{iT} is a consistent estimate of $[E(1/T)\sum_{t=1}^{T}(\partial L_{it|T}[\tilde{Y}_t;\hat{\theta}_i]/\partial\bar{\theta}_j)(\partial\ L_{it|T}[\tilde{Y}_t;\hat{\theta}_i]/\partial\bar{\theta}_j)']^{-1}$ and θ is set equal to the $\hat{\theta}$ estimated under the hypothesised restrictions. For example, suppose the maximum power of *L* in $D_i(L)$ is *k* and we wish to test for the inclusion of additional AR terms in $D_i(L)$. The *LM* test for the restriction that the coefficient on the *k*+1 lag, d_{k+1}, is zero is:

$$\text{(A.9)} \qquad \sum_{t=1}^{T}\left(\hat{\varepsilon}_{it}\lambda_{it}^{k+1}/\hat{\sigma}_i^2\right)\hat{\sigma}_i^2\left[\sum_{t=1}^{T}\left(\lambda_{it}^{k+1}\right)^2\right]^{-1}\sum_{t=1}^{T}\left(\hat{\varepsilon}_{it}\lambda_{it}^{k+1}/\hat{\sigma}_i^2\right)$$

where $\lambda_{it}^{k+1} = \tilde{Y}_{it\text{-}(k+1)} - \hat{\gamma}_i S_{t\text{-}(k+1)}\tilde{C}_{t\text{-}(k+1)|T} + (\partial\hat{Z}_{it}/\partial d_{k+1})\Sigma_{t|T}\hat{Z}'_{it}$. λ_{it}^{k+1} is replaced by λ_{it}^{sk+j} to test the restrictions $d_{sk+j} = d_{sk}d_j$ implied by (7). (A.9) is distributed χ^2 with one degree of freedom. (See Breusch and Pagan (1980).)

A.5 Autoregressive models

The basic general form of the autoregressive specifications used to model the { u_{it} } is:

$$\text{(A.10)} \qquad 1\text{-}D_i(L) = (1-d_1L-\ldots d_rL^r)(1-d_sL^s-\ldots d_{sm}L^{sm}) + \tilde{d}_qL^q + \tilde{d}_{sr}L^{sr}$$

The specific models chosen were:

Construction:	$(1\text{-}d_1L\text{-}d_3L^3)(1\text{-}d_4L^4\text{-}d_{12}L^{12}) + \tilde{d}_2L^2$
Motor vehicles:	$(1\text{-}d_1L\text{-}d_2L^2)(1\text{-}d_4L^4\text{-}d_8L^8)$
Durables ex. motor vehicles:	$(1\text{-}d_1L)(1\text{-}d_4L^4\text{-}d_8L^8) + \tilde{d}_4L^4$
Non-durables:	$(1\text{-}d_1L\text{-}d_2L^2)(1\text{-}d_4L^4\text{-}d_8L^8\text{-}d_{12}L^{12}) + \tilde{d}_1L$
Retail:	$(1\text{-}d_3L^3)(1\text{-}d_4L^4\text{-}d_8L^8\text{-}d_{12}L^{12})$
Other services[16]:	$(1\text{-}d_1L)(1\text{-}d_4L^4\text{-}d_8L^8\text{-}d_{12}L^{12}) + \tilde{d}_1L$
Federal:	$(1\text{-}d_1L\text{-}d_2L^2\text{-}d_3L^3)(1\text{-}d_4L^4\text{-}d_8L^8) + \tilde{d}_4L^4$

16 A deterministic linear time trend was also removed from this industry.

State and local: $(1\text{-}d_1 L)(1\text{-}d_4 L^4\text{-}d_8 L^8)+\tilde{d}_4 L^4$

Mining: $(1\text{-}d_1 L)(1\text{-}d_4 L^4\text{-}d_{12} L^{12})$

Common cycle: $(1\text{-}d_1 L\text{-}d_2 L^2)(1\text{-}d_4 L^4\text{-}d_8 L^8)+\tilde{d}_4 L^4$

Admittedly, some of these specification are somewhat ad hoc, and some may be overparameterised. However, we felt these models were preferable to simpler specifications that violated parameter restrictions or contained residual serial correlation that might confuse identification of cyclical and seasonal components.

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