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Bank Capital Allocation under Multiple Constraints

Tirupam Goel, Ulf Lewrick, and Nikola Tarashev*

Abstract

Banks allocate capital across business units while facing multiple constraints that may bind contemporaneously or only in future states. When risks rise or risk management strengthens, a bank reallocates capital to the more efficient unit. This unit would have generated higher constraint- and risk-adjusted returns while satisfying a tightened constraint at the old capital allocation. Calibrated to US data, our model reveals that, when credit or market risk increases, market-making attracts capital and lending shrinks. Leverage constraints affect banks only when measured risks are low. At low credit risk, tighter leverage constraints may reduce market-making but support lending.

JEL Codes: G21, G28, G3
Keywords: Internal capital market, Value-at-Risk, Leverage ratio, Risk-adjusted return on capital

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1 Introduction

Banks – as complex firms – optimise the size and composition of their balance sheets while facing internal and regulatory constraints. Some constraints restrict the riskiness of each business unit for a given amount of equity capital. Other constraints limit banks' size, irrespective of measured risks. While existing research has typically focused on one type of constraint at a time, we study the two types within a single model. This allows us to study how different constraints affect banks' value-optimisation problem. We derive a general rule for reallocating capital from one business unit to another when risk increases or internal risk management tightens. We also calibrate the model to study the relative relevance of different constraints, and illustrate cross-unit spillovers stemming from optimal capital re-allocation.

The banking industry develops risk-based constraints that regulators elevate to national and international capital standards. Seeking to ensure that a bank can withstand adverse shocks with a high probability, the risk-based constraints rely on statistical concepts such as value-at-risk and expected shortfall. They underpin well-established and broadly monitored metrics, such as risk-adjusted return on capital and economic value added, which have been guiding banks' risk management and capital planning over the past three to four decades.[1] To ensure that such constraints contribute to levelling the playing field globally and are calibrated in light of the externalities of bank distress, the Basel Committee on Banking Supervision (BCBS) has embedded risk metrics in the minimum standards for internationally active banks (BCBS [2005, 2009, 2011]).

Size-based constraints limit leverage. Such constraints are also rooted in the private sector's risk management practices. In reports to shareholders in the 1970s, banks referred to low leverage as indicating financial robustness.[2] More recently, haircuts on collateral values – which are insensitive to small changes in market risk (Gorton and Metrick [2012]) – have restrained the leverage of counterparties in collateralised lending transactions. National and international regulatory standards have emulated this market practice by adopting leverage ratio requirements as a backstop to risk-based capital requirements (BCBS [2014]).[3] The need for such a backstop stems from inherent deficiencies in risk measurement (Tarashev [2010]) as well as from banks' incentives to under-report their risks (Behn et al. [2016], Begley et al. [2017]).

To study the interaction of risk- and sized-based constraints, we incorporate them in a model of a bank that runs two business units. One of them is a "lending" (or loan) unit. The other unit holds a securities inventory to make markets: the "market-making" unit. The uncertainty of the two units' cash flows stems from credit and market risk, respectively. A separate value-at-risk (VaR) constraint applies to each unit, in line with current regulation. And even though regulation imposes a leverage ratio (LR) constraint at the bank level (BCBS [2014]), we apply it to each business unit separately, in line with reported industry practice (Bank of England [2016]). These constraints could be (i) *binding*; or (ii) non-binding contemporaneously but nonetheless *influencing* the bank's decisions because of the likelihood to bind in the future.

In choosing its optimal balance sheet, the bank ensures that the last increment of its capital

[1] As discussed in Guill [2016], the origin of many risk management practices can be traced back to approaches pioneered by Bankers Trust starting from the mid-1970s. Practical applications of these concepts are discussed in, for example, James [1996], Zaik et al. [1996], Nishiguchi et al. [1998] or Ita [2016].

[2] According to company annual reports, at least some banks have been actively managing their leverage ratios as far back as the early 1970s (e.g. Wells Fargo [1974]), i.e. around the time major banks started to implement risk-sensitive measures (Guill [2016]).

[3] US authorities introduced a leverage limit in the early 1980s (Wall and Peterson [1987]). The leverage ratio is expected to be fully integrated in international standards by 2018 (BCBS [2014]). For a discussion of the Basel III leverage ratio see, for example, Fender and Lewrick [2015].

generates the same profits, irrespective of which business unit it is deployed in.[4] The bank's choices are steered by two sets of drivers. First, there are the "production technologies," which map the size of each unit into profits. They feature the standard property whereby successive expansions add less and less to a unit's expected profits, i.e. there are diminishing marginal returns. The second set of drivers comprises the constraints. A binding constraint limits the extent to which a unit can expand, thus affecting the profits that the last increment of capital generates. An influencing constraint, by comparison, does not bind contemporaneously. Yet there is a risk that it binds in the future if adverse shocks materialise. The bank's optimal choices anticipate the straitjacket that such shocks would place on it. Putting it all together, we derive that the bank equalises *constraint- and risk-adjusted marginal returns* (CRAR) across business units.

Our main interest is in the bank's response to an increase in risk, stricter supervision or more conservative internal risk management, any of which would tighten a constraint. A *passive* response would be to downsize the business units so that the new constraint is satisfied at the old capital allocation. Given diminishing marginal returns, this downsizing would raise marginal profitability. But the increase would typically differ across units, thus driving a wedge between the respective CRARs and calling for a re-allocation of capital.

Capital re-allocation would depend on the relative efficiency with which the two units raise their profitability on the back of the passive response. *If both business units were downsized to levels that satisfy the new constraint at the old capital allocation, the more efficient unit would generate higher profits with the last increment of its capital.* An actively optimising bank would then re-allocate capital to the more efficient unit. We refer to this as the *capital re-allocation rule*.

Two business-unit characteristics are key determinants of efficiency. The first one is the sensitivity of the unit's marginal return to changes in the unit's size. All else equal, a higher sensitivity implies that less downsizing is needed to generate a given return with the last increment of allocated capital. The second key determinant is the unit's size itself. For a given tightening of either a size-based or a risk-based constraint, a larger unit needs to downsize more in order to continue satisfying the constraint. And given diminishing marginal returns, more downsizing results in higher profits for the last increment of capital. Thus, the more efficient unit would tend to be larger or have a marginal return that is more sensitive to downsizing, or both.

The capital re-allocation rule applies generally. When the bank has a short decision horizon, we derive analytically that the rule applies to the tightening of either a risk-based or a size-based constraint. And we verify numerically that this continues to be the case for longer, multi-period decision horizons. Namely, the bank reallocates capital towards the more efficient unit when the tightened constraint does not bind contemporaneously but influences the bank's decisions as it may bind in the future. What matters here is the units' relative efficiency in those future states in which the influencing constraint does bind.

We calibrate the model to data on large US banks. The benchmark calibration is consistent with common perceptions about a key feature of the two business units: in comparison to the lending unit, the market-making unit generates lower expected return on the back of higher leverage. Assuming that each business unit faces a contemporaneously binding VaR constraint and risks decline from their benchmark values, we obtain that the LR binds with a material probability in the future and is thus an influencing constraint. The impact of the LR constraint diminishes if the bank applies it to its overall balance sheet – as allowed by international regulatory standards – instead of separately to each business unit.

[4] In this sense, the bank resembles a discriminating monopolist who equalises marginal revenues across segmented markets. Armstrong and Vickers [1991] and Schmalensee [1981], for example, study a discriminating monopolist.

The benchmark calibration also implies that market-making attracts capital when *either* of the two VaR constraints tightens. This reflects the higher sensitivity of the unit's marginal return to downsizing and occurs despite the unit's smaller size. The re-allocation of capital generates spillovers across the business units. For example, an increase in market risk tightens the VaR constraint on the market-making unit and results in less market-making. But, because the bank reallocates capital from the loan to the market-making unit, lending declines as well. And if credit risk increases, a similar re-allocation of capital surfaces as less lending and more market-making.

That said, the units' relative efficiency changes away from the benchmark calibration. At sufficiently low credit risk, the lending unit is sufficiently large to be more efficient than the market-making unit. Thus, if a constraint tightens in such an environment, the bank's optimal response is to reallocate capital to lending. Symmetrically, capital is re-allocated away from market-making when a constraint tightens at a high level of market risk.

In line with its role as a backstop, the LR constraint influences the bank's decisions only when risks are below their calibration benchmarks. For low credit risk, the LR is an influencing constraint and the lending unit is more efficient. Tightening the LR constraint in this environment results in more lending and less market-making.

To streamline the capital allocation problem, we make a number of modelling choices. For instance, we allow the bank to satisfy a tightened constraint only through capital re-allocation, without having the option to raise capital externally. In addition, we do not consider market frictions that generate losses for the bank if it needs to downsize. Such frictions would strengthen the constraints' bite – as they would make it costlier to shed assets if a constraint binds down the road – but would not affect our qualitative results. That said, as done also in Erel et al. [2015] for instance, we neglect a potential relationship between risk-taking and the cost of funding. If this relationship is positive, it would ease the effect of constraints on the bank's balance sheet. Importantly, we do not derive optimal capital requirements, as this would require a model that accounts for the social implications of the bank's decisions. Instead, we take capital constraints as given and study the bank's responses to their tightening.

The rest of the paper is organised as follows. We provide an overview of the related literature in Section 2 and present the setup of the model in Section 3. In Section 4, we study the bank's capital allocation problem, and derive analytical conditions that characterise the implications of changes in risk or risk management. Numerical examples, which we present in Section 5, serve to complement our analytical results. Section 6 concludes. Analytical proofs and additional results are in the Appendix.

2 Related literature

Our paper draws on two strands of the literature. It relates to studies on the optimal allocation of capital within complex financial institutions. In addition, it contributes to a growing literature on the impact of recent global regulatory reforms on the functioning of financial markets, most notably on market liquidity.

An early theoretical contribution on internal capital allocation is Stein [1997]. This paper provides a rationale for the establishment of internal capital markets, which enable a firm's headquarters to shift resources across business units according to their profitability. In a related paper, Froot and Stein [1998] assess the optimal capital allocation when a bank is exposed to non-hedgeable risks and faces increasing costs of raising new capital. The benefits of centralised decision-making arise from interdependencies across investment decisions on the back of correlated investment returns and risk aversion that depends on new exposures. In turn, Perold [2005] studies a US investment bank and shows that accounting for diversification benefits can significantly reduce the bank's capital needs. His analysis suggests that banks should evaluate business activities based on their marginal contribution to expected operating profits and to the bank's required risk capital.

Against this analytical backdrop, we point to a complementary source of interdependencies across business units: capital constraints that also need to be factored into capital allocation decisions. This leads us to derive a new optimality metric: constraint- and risk-adjusted net marginal return – or CRAR. Our metric is similar to that proposed by Perold [2005] as both do not simply compare profitability to the *level* of risk capital but consider the *marginal* contribution of a business unit's expansion to required capital. A novel element of CRAR is its flexibility as regards the binding constraint. The optimality condition that equates CRARs across business units accommodates cases in which the binding constraints are risk-based, or size-based, or risk-based for some business units and size-based for others. This allows us to study a whole spectrum of bank reactions to a tightening constraint.

Market frictions represent another key consideration for capital allocation. With such frictions in mind, we assume that the bank cannot raise additional capital after the realisation of shocks. With this assumption, we follow closely Stoughton and Zechner [2007], who stress banks' (almost) continuous access to debt markets and argue that the cost of debt – rather than the cost of equity – drives capital allocation.

Our analysis also relates to the capital allocation problem in Baud et al. [2000], which studies constraints arising from regulation or investor objectives. The common element stems from the dynamic nature of capital allocation, whereby today's decisions influence tomorrow's optimal allocation. In our paper, this surfaces as the bank responding to the tightening of a constraint because the constraint may bind tomorrow, not necessarily because it binds contemporaneously.

The need to better understand the interaction of new regulatory standards and their market implications has spurred research on the effects of the post-crisis regulatory reforms. To this strand of the literature belong Chami et al. [2017], who also study several regulatory standards and consider a trading and a lending unit within a bank holding company. In contrast to our focus on capital re-allocations in response to a tightened constraint, they focus on the principal-agent conflict that may arise because the bank management has incomplete control over the risk-taking activities of the trading desk. Their analysis illustrates how bank governance measures, such as choosing appropriate traders or imposing risk limits on the trading desks, can help sustain the benefits of having multiple business lines within the bank holding company.

Cecchetti and Kashyap [2016] study the interaction of regulatory metrics from a bank-wide perspective. They argue that the regulatory framework encourages banks to choose similar business models because the tightness of individual regulatory requirements depends on the bank's balance sheet choices. Expanding on this, we highlight that the interaction of constraints with the underlying risks of different business units is a key driver of the bank's balance sheet composition.

Several studies consider the effect of recent regulatory reforms on market liquidity. A common thread in the analysis is the link between banks' willingness to maintain securities inventory in order to make markets in less liquid markets, such as those for corporate bonds. Empirical analysis by, for example, Bao et al. [2016] suggests that restrictions on banks' proprietary trading under the Volcker rule reduced dealers' willingness to warehouse US corporate bonds and can be associated with a decline in market liquidity at times of stress. By comparison, Adrian et al. [2017] document a reversal in the relationship between US dealer capitalisation and corporate bond liquidity. While bonds traded by weakly capitalised dealers enjoyed better liquidity before the Great Financial Crisis, bonds traded by better capitalised banks are found to be relatively more liquid post-crisis as new regulatory requirements are phased in. Another example is Baranova et al. [2017], who model the response of dealer-banks to an increase in market volatility. Their model suggests that leverage regulation induces banks to operate with less leverage, which reduces their market-making capacity. As a result, liquidity premia in secondary markets rise. From a financial stability perspective, it would be important to establish whether the reduction in leverage translates into greater balance sheet capacity that can serve to accommodate liquidity demand at times of stress. Against this background, our paper sheds light on the link between risk management and market liquidity by analysing optimal capital re-allocation when financial conditions change.

3 Model

We model a bank with two business units: market-making and lending. After outlining the building blocks of the model in Section 3.1, we zoom in sequentially on each of the two units in Section 3.2. We describe these units' risk- and size-based constraints in Section 3.3.

3.1 Setup

The bank operates on three dates: 0, 1 and 2. Subject to size- and risk-based constraints, it optimises its balance sheet with the goal of maximising the expected final value of its capital.

The timeline is portrayed in Figure 1. On date 0, the bank chooses the sizes of its loan and market-making units – L_0 and M_0, respectively – while facing unit-specific LR and VaR constraints. The L_0 loans are valued at 1 each, whereas the M_0 bonds in the market-making inventory are valued at p_0. The bank finances the two units with its capital endowment, K_0, and a one-period debt, D_0. Then, the bank enters date 1 and a first set of credit and market shocks materialises. These shocks determine the end-period value of the two business units, a part of which pays off the debt. The remaining value equals the new level of capital, K_1. At the end of date 1, the bank chooses new sizes for its units, L_1 and M_1, funding them with capital, K_1, and another issuance of one-period debt, D_1. Again, it needs to satisfy LR and VaR constraints on each unit. Then, the bank enters date 2 and a second set of credit and market shocks materialises. These shocks determine the end-period value of the two units, which is used to pay off debt. The remainder is the final value of capital, K_2.

The balance sheet identity on each date $t \in \{0, 1\}$ implies

$$\begin{aligned} L_t + p_t M_t &= D_t + K_t \\ K_t &= K_{tL} + K_{tM} \end{aligned}$$

where K_{tL} and K_{tM} are the amounts of capital allocated to the loan and market-making units, respectively. While our model does not exclude an absorbing default state, our discussion focuses exclusively on $K_t > 0$ for all t.

Throughout the paper, we will rule out external capital raising (as well as disbursements to shareholders). This is motivated by the observation of capital stickiness, reflecting banks' infrequent access to capital markets (Stoughton and Zechner [2007]). Effectively, we follow the spirit of Adrian and Shin [2011] in viewing capital as predetermined over sufficiently short periods.

We conclude this subsection by stressing a key difference between the date-0 and date-1 problems.

On date 1, provided that the units' marginal profitability is sufficiently high, the bank would expand its balance sheet by as much as the constraints allow it. Thus, there would be binding

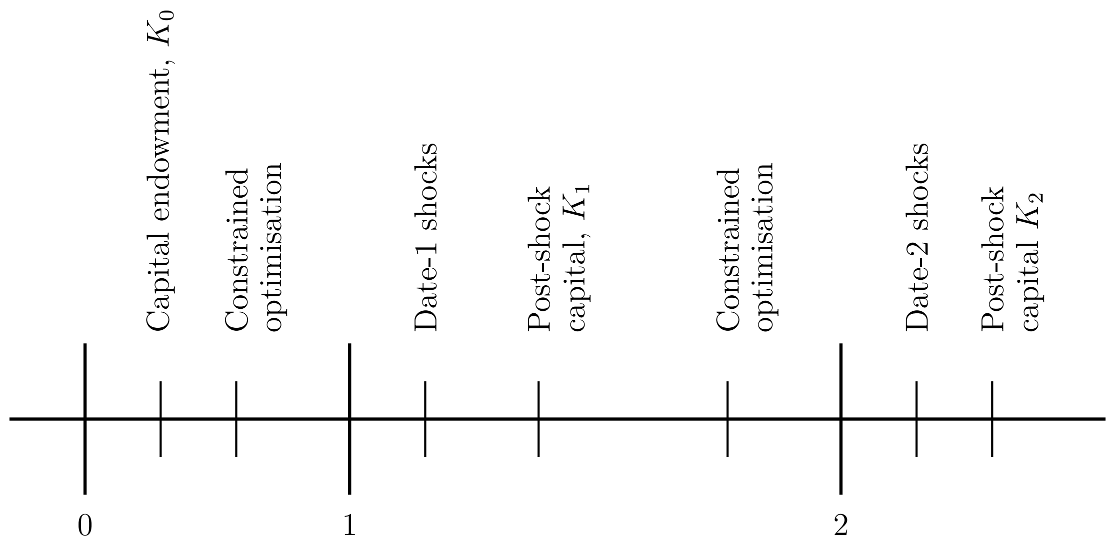

Figure 1: Timeline

constraints, and tightening or loosening them would affect the bank's decisions. But there would also be slack constraints on date 1. These would be either the weaker of the constraints in each unit, or, for sufficiently low profitability of loans and market-making, all the constraints. Marginally changing a slack constraint on date 1 would be inconsequential.

On date 0, the bank acts in anticipation of date-1 shocks. Even though some of the constraints would be slack on date 0, adverse date-1 shocks might turn them into binding constraints on date 1. And, as we show below, the bank's optimal date-0 exposure to date-1 shocks depends on the tightness of all the constraints that could bind on date 1. Thus, in contrast to date 1, changing a contemporaneously non-binding constraint could influence the bank's decision on date 0.

3.2 Risk-return profile: market-making and lending

The two business units share two features in common. First, the bank is a monopolist in both lending and market-making.[5] Second, cash flows increase in each unit's size but by less and less as a result of successive expansions: i.e. there are decreasing marginal returns. We now provide further detail, dropping time subscripts from this point on where this does not create confusion.

Market-making unit We model the bank's market-making in the spirit of Garman [1976]. Concretely, the bank needs to hold a securities inventory of M at date-$(t-1)$ in order make markets for a total transaction volume of λM at date-t, where $\lambda > 1$. An interpretation of λ is that it is inversely related to the expected security holding period. See Figure 2. Suppressing time subscripts, the transaction volume λM pins down the bid and ask prices p_b and p_a of the traded inventory, which come from the associated supply and demand:

$$p_b = \gamma + \delta\lambda M + \epsilon \quad \text{and} \quad p_a = \alpha - \beta\lambda M, \tag{1}$$

[5]For examples of a monopolistic market maker, see Kyle [1989], O'Hara and Oldfield [1986], Glosten and Milgrom [1985].

where the supply shock, ϵ, is normally distributed

$$\epsilon \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$$

and i.i.d. over time. It is ϵ that generates market risk.

The bid-ask spread s is given as:

$$s = p_a - p_b = (\alpha - \gamma) - \lambda M(\beta + \delta) - \epsilon. \tag{2}$$

In turn, the post-shock value of the inventory is given by the "fair" price p, at the intersection of the demand and supply schedules:

$$p : p_a = p_b \implies p = \alpha - \beta\left(\frac{a - \gamma - \epsilon}{\beta + \delta}\right). \tag{3}$$

The market-making unit's net cash flow is given by:[6]

$$V_m(M, K_m; \epsilon) = s\lambda M + pM - R(p_{-1}M - K_m). \tag{4}$$

and comprises three terms. The first one, $s\lambda M$, represents the revenue from charging a spread on the volume of intermediated transactions. The second term, pM, is equal to the fair value at which the bank sells the inventory. The third term, $R(p_{-1}M - K_m)$, captures the cost of funding the inventory M at the interest rate R. The funding needs are given by the inventory purchase value at the previous date, $p_{-1}M$, less the amount that is funded by capital K_m.

The supply shock has two opposing effects on the net cash flow. As revealed by equation (2), $\epsilon < 0$ increases the bid-ask spread, s. At the same time, it lowers the inventory value, as seen in equation (3) and in the right-hand panel of Figure 2. Ultimately, substituting for the bid-ask spread and the inventory price, we obtain

$$\begin{aligned} V_m(M, K_m, \epsilon) &= \underbrace{-\lambda^2(\beta + \delta)}_{f_2} M^2 + \underbrace{\left((\alpha - \gamma)\lambda + \alpha - \beta\frac{\alpha - \gamma}{\beta + \delta}\right)}_{f_1} M - RM - \epsilon\Big(\lambda - \frac{\beta}{\beta + \delta}\Big)M + RK_m \\ &= \underbrace{F(M) - \epsilon\Big(\lambda - \frac{\beta}{\beta + \delta}\Big)M}_{\text{Market-making revenue}} - \underbrace{R\,(M - K_m)}_{\text{Cost of debt}}, \end{aligned} \tag{5}$$

defining the gross cash flow as $F(M) \equiv f_1 M + f_2 M^2$, where $f_1 > 0$ and $f_2 < 0$. The latter inequality implies diminishing marginal returns.

Lending unit The gross contractual payment on each loan – the loan interest rate, $H(L)$ – is a decreasing function of the loan volume:

$$H(L) = g_1 + g_2 L,$$

where $g_1 > 0$ and $g_2 < 0$. The gross contractual payment on *all* the loans is given by $G(L) \equiv H(L)L$. The downward sloping loan demand ($g_2 < 0$) implies diminishing marginal returns for the loan unit.

[6]Throughout the paper, we use lower case (m, l) to denote a business unit, and we use upper case (M, L) to denote business unit size. Likewise, when we do not specify the exact unit, we use x or y (respectively, X or Y).

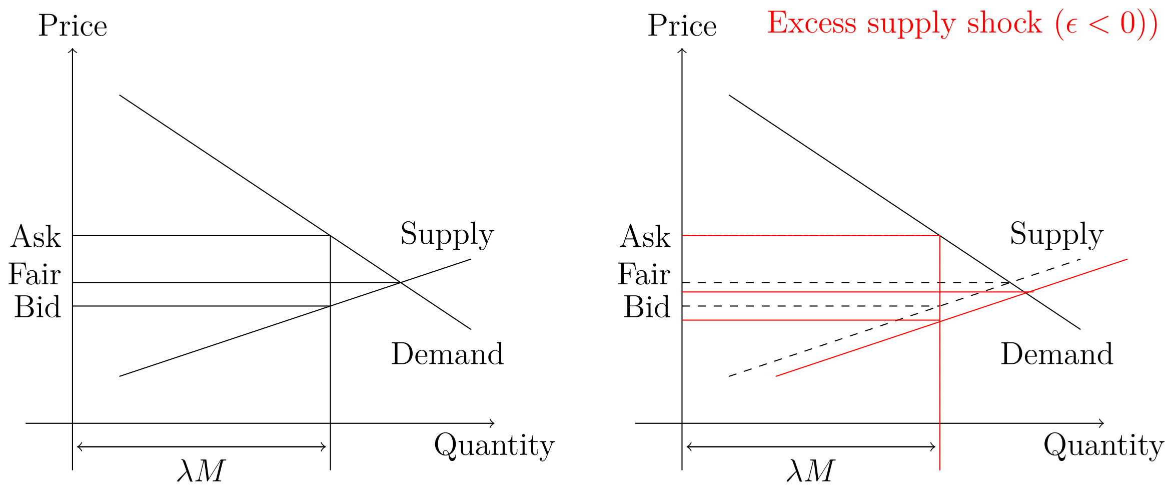

Figure 2: Market-making, and the determination of the bid-ask spread

Credit risk implies that the actual payments on the loans would deviate from the contractual ones. The deviations could be due to loans that default or do not perform, or pre-pay. To capture this parsimoniously, we assume that the actual payment is $G(L) - ZL$, where

$$Z \sim \mathcal{N}\left(0, \sigma_Z^2\right)$$

and Z is i.i.d. over time and independent of ϵ.[7] We note that a higher Z means a lower loan revenue.

The net cash flow from lending is:

$$V_l(L, K_l; Z) = \underbrace{G(L) - ZL}_{\text{Loan revenue}} - \underbrace{R(L - K_l)}_{\text{Cost of debt}} \tag{6}$$

3.3 VaR and leverage ratio constraints

We study the interaction of risk-based and size-based constraints.

VaR constraints The VaR constraint for the market-making unit (VaR_m) and the lending unit (VaR_l) are defined in terms of the respective net cash flows in equations (5) and (6):

$$VaR_m : \ Pr(V_m(M, K_m; \epsilon) \leq 0) \leq a_m \quad \text{and} \quad VaR_l : \ Pr(V_l(L, K_l; Z) \leq 0) \leq a_l.$$

When a business unit $x \in \{m, l\}$ satisfies VaR_x, this unit's capital is sufficiently high to ensure that a negative net cash flow occurs with a probability not greater than a_x.

We rewrite each unit's VaR constraint in terms of the implied *minimum required capital* (MRC). For the market-making unit:

$$\begin{aligned} MRC_m^{VaR} &= \underbrace{\frac{RM + \Omega M - F(M)}{R}}_{\text{Minimum required capital}} \leq \underbrace{K_m}_{\substack{\text{Capital} \\ \text{allocated}}} \quad (7) \\ \text{for } \Omega &\equiv \Big(\lambda - \frac{\beta}{\beta + \delta}\Big)\mathcal{N}^{-1}(1 - a_m, 0, \sigma_\epsilon^2), \end{aligned}$$

[7]Despite this admittedly strong assumption, we obtain spillover effects across the two business units.

where $\mathcal{N}^{-1}(1-a_m, 0, \sigma_\epsilon^2)$ is the inverse CDF of the market shock, ϵ, evaluated at the $1-a_m$ percentile (confidence level). Given that the size of the market-making unit is M, the capital allocated to it is K_m and the debt-funding cost is R, Ω denotes the maximum value of the random loss that the market-making unit can incur per unit of inventory and still generate a non-negative net cash flow.

In turn, the MRC of the lending unit is given by:

$$\begin{aligned} MRC_l^{VaR} &= \underbrace{\frac{RL + \Theta L - G(L)}{R}}_{\text{Minimum required capital}} \leq \underbrace{K_l}_{\substack{\text{Capital}\\\text{allocated}}} \\ \text{for } \Theta &\equiv \mathcal{N}^{-1}(1-a_l; 0, \sigma_Z^2), \end{aligned} \tag{8}$$

where $\mathcal{N}^{-1}(1-a_l; 0, \sigma_Z^2)$ is the inverse CDF of the credit shock, Z, evaluated at the $1-a_l$ percentile. Given that the size of the lending unit is L, the capital allocated to it is K_l and the debt-funding cost is R, Θ denotes the maximum value of the random loss per loan that is consistent with a non-negative net cash flow.[8]

Leverage ratio (LR) constraint If a minimum LR (χ) is imposed on each business unit, the two LR constraints and the corresponding levels of minimum required capital (MRC) are:

$$\begin{aligned} LR_m: \ \chi M \leq K_m \quad &\text{and} \quad LR_l: \ \chi L \leq K_l, \\ \text{where } MRC_m^{LR} = \chi M \quad &\text{and} \quad MRC_l^{LR} = \chi L. \end{aligned} \tag{9}$$

For future reference, we also record an LR constraint applied at the *bank-wide* level:

$$\chi(L+M) \leq K. \tag{10}$$

4 Bank's Problem

We solve the bank's optimization problem by backward induction. That is, we begin by solving the date-1 problem. Then, we embed the solution in the date-0 problem. In the main text, we present the case where the bank applies the LR constraint at the business-unit level.

Date 1. Starting with the current capital stock, K_1 – which is the result of date-0 decisions and date-1 shocks – the bank chooses the size of each unit and the associated capital allocation – that is, L_1, M_1, K_{m1} and K_{l1} – to maximize the expected value of the end capital stock, $E(K_2)$. Denoting the maximised value of $E(K_2)$ by $U_1(K_1)$, we write

$$U_1(K_1) = \max_{L_1, M_1, K_{m1}, K_{l1}} \Delta \left\{ E\Big[V_{m2}(M_1, K_{m1}; \epsilon_2) + V_{l2}(L_1, K_{l1}; Z_2) \Big] \right\}, \tag{11}$$

where Δ is the discount factor and the expectation is taken over the distributions of the date-2 credit and market shocks, Z_2 and ϵ_2. Equations (5) and (6) deliver the business units' cash flows, V_{m2} and V_{l2}, for specific values of M_1, K_{m1}, ϵ_2, L_1, K_{l1} and Z_2. The constraints are $K_{m1}+K_{l1} \leq K_1$ and as given by expressions (7), (8) and (9).

[8]The application of the VaR constraint at a business unit level is in line with regulatory standards. These standards incorporate the conservative assumption of no diversification benefits in credit and market risks.

Date 0. The date-0 objective is to maximise the discounted expected value of the date-1 maximisation of $E(K_2)$. The bank chooses L_0, M_0, K_{m0} and K_{l0} while taking the capital endowment K_0 as given:

$$\max_{L_0,M_0,K_{m0},K_{l0}} \Delta E\Big[U_1\Big[V_{m1}(M_0,K_{m0};\epsilon_1)+V_{l1}(L_0,K_{l0};Z_1)\Big]\Big] \tag{12}$$

where $U_1(\cdot)$ is as defined by the date-1 problem in expression (11) and the expectation is taken over the distributions of the date-1 credit and market shocks, Z_1 and ϵ_1. Equations (5) and (6) deliver the business units' cash flows, V_{m1} and V_{l1}, for specific values of M_0, K_{m0}, ϵ_1, L_0, K_{l0} and Z_1. The constraints are $K_{m0}+K_{l0} \leq K_0$ and as given by expressions (7), (8) and (9). We will see in Section 4.1 that, in contrast to the date-1 problem, the date-0 problem incorporates contemporaneously non-binding but influencing constraints.

4.1 Optimality conditions

In Appendix A.1, we derive the following optimality conditions. When no constraint binds contemporaneously, the first-order conditions constitute the optimality conditions, and consist of setting the expected net marginal cash flow for each business unit equal to zero on each date:

$$\begin{aligned}
\frac{dE\Big[V_{m2}(M_1,K_{m1};\epsilon_2)\Big]}{dM_1} &= \frac{dE\Big[V_{l2}(L_1,K_{l1};Z_2)\Big]}{dL_1}=0 \quad \text{and} \\
\frac{dE\Big[U_1\Big[V_{m1}(M_0,K_{m0};\epsilon_1)+V_{l1}(L_0,K_{l0};Z_1)\Big]\Big]}{dM_0} &= \frac{dE\Big[U_1\Big[V_{m1}(M_0,K_{m0};\epsilon_1)+V_{l1}(L_0,K_{l0};Z_1)\Big]\Big]}{dL_0}=0.
\end{aligned}$$

If constraints i and j bind on date 1 for the market-making and loan units, respectively, whereas constraints i' and j' bind on date 0, then

$$\begin{aligned}
MRC^{i}_{m1} &= K_{m1},\ MRC^{j}_{l1}=K_{l1}, \\
MRC^{i'}_{m0} &= K_{m0} \text{ and } MRC^{j'}_{l0}=K_{l0}, \text{ where } i,j,i',j' \in \{VaR, LR\}
\end{aligned} \tag{13}$$

The first-order conditions under binding constraints are

$$\frac{dE\Big[V_{m2}(M_1,K_{m1};\epsilon_2)\Big]}{dM_1}\Bigg/\frac{dMRC^{i}_{m1}}{dM_1}=\frac{dE\Big[V_{l2}(L_1,K_{l1};Z_2)\Big]}{dL_1}\Bigg/\frac{dMRC^{j}_{l1}}{dL_1} \quad \text{and,} \tag{14}$$

$$\frac{dE\Big[U_1\Big[V_{m1}(M_0,K_{m0};\epsilon_1)+V_{l1}(L_0,K_{l0};Z_1)\Big]\Big]}{dM_0}\Bigg/\frac{dMRC^{i'}_{m0}}{dM_0}=$$
$$\frac{dE\Big[U_1\Big[V_{m1}(M_0,K_{m0};\epsilon_1)+V_{l1}(L_0,K_{l0};Z_1)\Big]\Big]}{dL_0}\Bigg/\frac{dMRC^{j'}_{l0}}{dL_0}.$$

These mean that on any given date, *the last increment of required capital delivers the same value, irrespective of which unit the bank deploys it in.*

Several remarks are in order. First, capital fungibility makes it impossible to have only one unit with a binding constraint. If such a situation were to arise, capital would be transferred to this unit so that either zero or two units end up with binding constraints in equilibrium. Second, the conceptual difference between the optimality condition on date 1 and that on date 0 stems from how they incorporate cash flows. On date 1, the bank considers the *expected* value of *unit-specific* marginal net cash flows on date 2.

On date 0, by contrast, it considers how changing the size of a business unit affects a *non-linear transformation* of aggregate net cash flows, i.e. $U_1(\cdot)$. To see where the non-linearity comes from, note that adverse date-1 shocks would effectively tighten any binding constraints and necessitate a contraction of the balance sheet. Because of diminishing marginal returns, the resulting drop in cash flows would be larger than the corresponding rise triggered by favourable shocks. This lack of symmetry is reinforced if the adverse shocks transform non-binding constraints into binding ones. From the standpoint of date 0, these would be non-binding but influencing constraints. In sum, the combination of diminishing marginal returns and capital constraints implies that adverse date-1 shocks have an asymmetrically larger impact on the maximised expected value of end capital, K_2. At date 0, this leads the bank to consider not only the expected value but also the *riskiness* of the shock-driven date-1 cash flows.

From this point on, we alleviate the notation with the following shortcuts. When no constraint binds contemporaneously, the bank equalises business units' expected *r*isk-*a*djusted net marginal *r*eturns (RAR), where:

$$RAR_{x,t} \equiv \frac{dE\Big[U_{t+1}\Big[V_{m,t+1}(M_t, K_{m,t}; \epsilon_{t+1}) + V_{l,t+1}(L_t, K_{l,t}; Z_{t+1})\Big]\Big]}{dX_t}, \tag{15}$$

where $x \in \{m, l\}$, $X \in \{M, L\}$, $t \in \{0, 1\}$, U_1 is given by (11) and U_2 is the identity function. When there are binding constraints, we say that the bank equalises the expectation of business units' *c*onstraint- and *r*isk-*a*djusted net marginal *r*eturns (CRAR). In this case:

$$CRAR^i_{x,t} \equiv \frac{dE\Big[U_{t+1}\Big[V_{m,t+1}(M_t, K_{m,t}; \epsilon_{t+1}) + V_{l,t+1}(L_t, K_{l,t}; Z_{t+1})\Big]\Big]}{dX_t} \Bigg/ \frac{dMRC^i_{x,t}}{dX_t}, \tag{16}$$

where $x \in \{m, l\}$, $X \in \{M, L\}$, $t \in \{0, 1\}$, U_1 is given by (11) and U_2 is the identity function.

4.2 Defining efficiency

When a constraint tightens, capital becomes scarcer and the overall balance sheet has to shrink. A passive response would be to maintain the old capital allocations and simply shrink each business unit to the extent required to satisfy the new constraint. However, the resulting outcome would typically be suboptimal as the CRARs would differ across units. This implies that the optimising bank needs to re-allocate capital across business units. And one would expect the re-allocation to result in a smaller reduction (or even an expansion) of the unit that makes a better use of the last increment of the scarcer capital.

To derive a concrete expression for this intuition, we define two concepts that help describe the passive response. First, we denote by $\frac{\partial CRAR^i_x}{\partial i}$ the change in the CRAR of unit $x \in \{m, l\}$ when constraint $i \in \{VaR, LR\}$ tightens and all else stays the same. In the light of expressions (7), (8) and (9), this tightening corresponds to an increase in Ω, Θ or χ. Second, we denote by $\dot{X}|_{\overline{MRC^i_x}}$ the downsizing that restores the initial MRC, where $x \in \{m, l\}$, $X \in \{M, L\}$, $i \in \{VaR, LR\}$. Here and below, the dot notation denotes the first derivative of a unit's size with respect to a parameter of the tightening constraint and an upper bar indicates that the corresponding object remains constant.

Definition. *Let constraint i and j bind for business units x and y, respectively, where $x, y \in \{m, l\}$ and $i, j \in \{VaR, LR\}$. For a tightening of constraint i, business unit x is more efficient if*

$$\frac{\partial CRAR_x^i}{\partial i} + \frac{\partial CRAR_x^i}{\partial X}\dot{X}|_{\overline{MRC_x^i}} > \frac{\partial CRAR_y^j}{\partial i} + \frac{\partial CRAR_y^j}{\partial Y}\dot{Y}|_{\overline{MRC_y^j}} \tag{17}$$

In other words, if both business units were downsized to levels that satisfy the tightened constraint at the old capital allocation, the more efficient unit would increase by more the profits it generates with the last increment of capital.

In the next subsections, we show that relative efficiency guides capital re-allocation.

4.3 Response to a tighter binding constraint on date 1

On date 1, the bank's response reflects expected net cash flows:

$$\begin{aligned} E\Big[V_{m2}(M_1, K_{m1}; \epsilon_2)\Big] &= F(M) - (M - K_m)\, R \text{ and} \\ E\Big[V_{l2}(L_1, K_{l1}; Z_2)\Big] &= G(L) - (L - K_l)R. \end{aligned} \tag{18}$$

Combined with (13) and (14), these cash flows are at the centre of the bank's response to the tightening of a binding constraint.

4.3.1 Different constraints bind for the two business units

We consider cases in which a binding constraint tightens. For example, χ increases (for the LR constraint), or σ_ϵ increases (for the market-making VaR constraint), or σ_Z increases (for the lending unit's VaR constraint). For these cases, we prove the following proposition.

Proposition 1.1. *When a constraint tightens, the bank's optimal response is to reduce the size of the business unit for which this constraint binds.*

Proof. (by contradiction): Assume that the bank were to expand unit x for which a binding constraint has tightened. Since MRC is an increasing function of the unit's size – see Appendix A.1 – the bank would now need to allocate more capital to unit x. At the same time, the increased size would lower the unit's CRAR because of diminishing marginal returns – recall (7), (8), (9), (16) and (18). This adds to the CRAR decline that is due to the tightening of the constraint. For the optimality condition (14) to be restored, the CRAR of the other unit, y, must decline as well, necessitating an increase in the size of y because of diminishing marginal returns. Since the binding constraint of y has not weakened, the bank would need to allocate more capital to y as well. Yet, for a given amount of capital and binding constraints, the bank cannot allocate more capital to both units at the same time – a contradiction. ■

The unit with a tightened constraint shrinks, but what happens to the size of the other unit? The answer to this question depends on the bank's optimal re-allocation of capital. If this involves a shift of capital towards (respectively, away from) the unit with a tightened constraint, the other unit shrinks (respectively, expands). The following proposition – proved in Appendix A.2 – states formally the capital re-allocation rule.

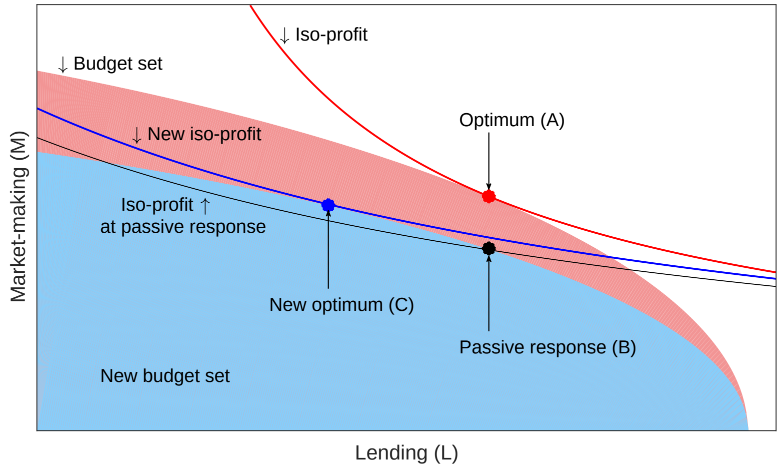

Figure 3: The bank's response to an increase in market risk when market-making is the more efficient unit.

Proposition 1.2. *Assume that the binding constraints, i and j, differ across the two business units, x and y. Suppose that only the binding constraint i for business unit x is tightened, which implies $\frac{\partial CRAR_y^i}{\partial i} + \frac{\partial CRAR_y^i}{\partial Y}\dot{Y}|_{\overline{MRC_y^i}} = 0$. In line with expression (17), capital is allocated to business unit x – i.e. $\dot{K}_x > 0$, $\dot{K}_y < 0$ and $\dot{Y} < 0$ – if this unit is more efficient, i.e. if $\frac{\partial CRAR_x^i}{\partial i} + \frac{\partial CRAR_x^i}{\partial X}\dot{X}|_{\overline{MRC_x^i}} > 0$. Otherwise, $\dot{K}_x < 0$, $\dot{K}_y > 0$ and $\dot{Y} > 0$.*

To illustrate the mechanisms at work, Figure 3 focuses on a case in which the two VaR constraints bind. The bank equalises CRARs across units if and only if the relevant iso-profit curve is tangent to the relevant budget set in this figure. Initially, the relevant budget set combines the red and blue areas and the optimum choice of M and L is at point A. Suppose then that the market-making constraint tightens, which contracts the budget set to the blue area, so that point A is no longer feasible. The bank's passive response, point B, would be to only reduce M in order to satisfy the new constraint at the old capital allocation. This is suboptimal, as the iso-profit curve is not tangent at B to the blue budget set. The optimal response is at point C, where the bank attains the highest feasible iso-profit curve. In moving from B to C, M increases and L declines, indicating a re-allocation of capital to the market-making unit. Per Proposition 1.2, the capital re-allocation reveals that the market-making unit is the more efficient one. That said, the move from the old optimum (A) to the new optimum (C) results in a net decline in M, in line with Proposition 1.1.

To establish which business-unit features drive relative efficiency, we write explicitly changes of specific CRARs, triggered by a passive response to the tightening of specific constraints:

$$\begin{aligned}
\frac{\partial CRAR_m^{VaR}}{\partial \Omega} + \frac{\partial CRAR_m^{VaR}}{\partial M}\dot{M}|_{\overline{MRC_m^{VaR}}} &= -\frac{F'(M) - R}{\Omega^2} - \frac{F''(M)}{\Omega}\frac{M}{R + \Omega - F'(M)} \\
\frac{\partial CRAR_m^{LR}}{\partial \Omega} + \frac{\partial CRAR_m^{LR}}{\partial M}\dot{M}|_{\overline{MRC_m^{LR}}} &= -\frac{F'(M) - R}{\chi^2} - \frac{F''(M)M}{\chi^2} \\
\frac{\partial CRAR_l^{VaR}}{\partial \Theta} + \frac{\partial CRAR_l^{VaR}}{\partial L}\dot{L}|_{\overline{MRC_l^{VaR}}} &= -\frac{G'(L) - R}{\Theta^2} - \frac{G''(L)}{\Theta}\frac{L}{R + \Theta - G'(L)} \\
\frac{\partial CRAR_l^{LR}}{\partial \Theta} + \frac{\partial CRAR_l^{LR}}{\partial L}\dot{L}|_{\overline{MRC_l^{LR}}} &= -\frac{G'(L) - R}{\chi^2} - \frac{G''(L)L}{\chi^2}
\end{aligned} \tag{19}$$

where the prime and double-prime notation denotes first and second derivatives, respectively. For the interpretation of these expressions, we note that:

- The gross marginal return is higher than the marginal cost of debt when the unit is constrained: $F'(M) > R$ and $G'(L) > R$.
- There are diminishing marginal returns: $F''(M) < 0$ and $G''(L) < 0$.
- MRCs in (7) and (8) increase with the unit's size (see Appendix A.1): $R + \Omega > F'(M)$ and $R + \Theta > G'(L)$.
- The VaR losses and the LR constraint's parameter are positive: $\Omega > 0$, $\Theta > 0$ and $\chi > 0$.
- Units' sizes are positive: $M > 0$ and $L > 0$.

We can now discuss the drivers of efficiency. For one, efficiency increases with the sensitivity of a unit's gross marginal returns to the unit's size, i.e. with $|F''(M)|$ or $|G''(L)|$. This is because the higher this sensitivity, the greater the increase in CRAR for a given downsizing. In addition, efficiency increases with the unit's size, i.e. with M or L. This stems from the fact that a tightening of a constraint raises the MRC of a larger unit by more. All else the same, this implies that a larger unit needs to downsize more in order to satisfy a tightened constraint, which, given diminishing marginal returns, raises the unit's CRAR by more.

Expression (19) indicates that other business unit characteristics play a role as well. That said, the VaR losses entering the tightened constraint, i.e Ω or Θ, as well as the gross marginal return, $F'(M)$ or $G'(L)$, have an ambiguous net effect on efficiency. In turn, parameters of the unchanged constraint play an indirect role in (19), only through their impact on units' sizes.

4.3.2 The same constraint binds in both business units

If the same constraint is to bind in both business units, this must be the LR constraint, as the VaR constraints differ across units. In this case, the CRAR and MRC conditions – (9) and (16) – take on particularly simple forms:

$$\frac{G'(L) - R}{\chi} = \frac{F'(M) - R}{\chi} \tag{20}$$

$$\chi M + \chi L = K. \tag{21}$$

We now have a variant of Proposition 1.1:

Proposition 2.1. *When the LR constraint binds for both business units, an increase in χ implies that both business units contract.*

Proof. The proof of Proposition 1.1 still goes through, given that, by (20), a change in χ does not affect the CRAR-equality condition. ■

Capital re-allocation follows the same rule as before, thus leading to the following proposition, which we prove in Appendix A.3:

Proposition 2.2. *Assume that the LR constraint binds for two business units, x and y, and is tightened. Capital is allocated to business unit x – i.e. $\dot{K}_x > 0$, $\dot{K}_y < 0$ and $\dot{Y} < 0$– if inequality (17) holds, thus implying that unit x is more efficient. Otherwise, $\dot{K}_x < 0$, $\dot{K}_y > 0$ and $\dot{Y} > 0$.*

To establish what business unit characteristics drive relative efficiency in this case, we refer to the definitions of $F(\cdot)$ and $G(\cdot)$ in Section 3.2, the second and fourth lines in (19), and the equilibrium conditions in (20) and (21). We then obtain that inequality (17) holds for $x = l$, $X = L$, $y = m$ and $Y = M$ if and only if

$$g_1 > f_1. \tag{22}$$

This result is fully in line with the drivers of relative efficiency under different binding constraints. Greater curvatures of $G(\cdot)$ and $F(\cdot)$ – i.e. higher $|g_2|$ and $|f_2|$, respectively – imply higher sensitivities of marginal returns to size but, per condition (20), also reduce the optimal sizes of the respective units. In the specific case at hand, the two counteracting effects offset each other exactly. By contrast, higher linear coefficients, g_1 and f_1, raise the optimal unit sizes without affecting the sensitivity of marginal returns to these sizes. Thus, the relative values of these coefficients drive relative efficiency when LR binds for the two units.

4.4 Response to a tighter constraint on date 0

The date-0 problem incorporates a *non-linear* transformation of date-1 cash flows – recall (15) and (16). That said, when a contemporaneously binding constraint tightens, the date-1 capital re-allocation rule results extend to the date-0 problem.[9]

Importantly, even a contemporaneously non-binding constraint can be influencing on date 0. If the bank perceives such a constraint as binding for certain date-1 states – i.e. for certain realisations of date-1 shocks – then it would re-allocate capital in response to a tightening of the constraint. Following the same rule, the bank would re-allocate capital towards the business unit that is more efficient in those date-1 states in which the tightened constraint binds. We confirm this numerically in the next section, where we calibrate the model.

5 Numerical Illustrations

To illustrate the above analytic takeaways, we calibrate our model to US bank data. Section 5.1 discusses these data and the calibration approach. The calibration is simplified by the fact that the initial capital endowment, K_0, is a scale variable, which we set to 1 without loss of generality. Section 5.2 works with the calibrated model, focuses on date 1 and portrays how the bank re-adjusts its balance sheet in response to a tightening of a binding constraint. Maintaining the focus on date 1, Section 5.3 reports corresponding results for a bank-wide application of the LR constraint. In Section 5.4, we focus on date 0 and examine how the bank responds to a tightening of an influencing constraint.

5.1 Data and Calibration

Data We calibrate the model to data on US bank holding companies (BHCs) with significant exposures to both credit and market risk. Concretely, we select all banks that are subject to the Dodd-Frank stress-testing exercise and for which the relevant variables are available from SNL. This results in 28 BHCs in total, for which we collect quarterly financial statements from 2012

[9] The bank also takes into consideration the state-contingent date-1 tightness of constraints that bind on date 0. However, our numerical exercise reveals that such considerations are of second order relative to the constraints' tightness on date 0.

to 2015. In addition, we obtain US corporate-bond bid-ask spread estimates from the Federal Reserve Bank of New York for the period from the first quarter of 2012 to the second quarter of 2015. These estimates are based on TRACE securities trading data.

We transform the data to represent a bank with a two-period planning horizon. For one, we compute bank-specific means and standard deviations across time, which we then average across banks. In addition, we annualise the quarterly data, so that one period in the model corresponds to one year.

Calibration The calibration builds on several sets of parameters. The first set comprises parameters that we derive directly from the data or the literature (Table 1, first block). First, we set the discount factor to a standard value. Then, we derive the borrowing cost R from the average interest expense in the data. And we set the LR constraint to be consistent with the US supplementary leverage ratio (SLR).

The last parameters in the first set are the percentiles defining the loan and market-making units' VaR constraints, a_l and a_m. For these parameters, we match the minimal capital requirements that internationally active banks need to satisfy in order to comply with the BCBS's credit- and market-risk frameworks. We thus set a_l to be consistent with a 99.9% VaR at a 1-year horizon. For the market-making unit, we refer to the 99% VaR at the 40-day horizon.[10] This corresponds to the requirement for exposures to investment-grade corporate bonds, which is broadly consistent with the fixed-income instruments in our data. We then transform the 40-day 99% VaR percentile into its one-year counterpart. Concretely, denoting the standard normal CDF by Φ and assuming 250 business days in a year, we set

$$a_m = 1 - \Phi\left(\sqrt{40/250}\,\Phi^{-1}(0.99)\right). \tag{23}$$

Next, we derive six model-implied moments and match them with their data-implied values (Table 1, second block). We obtain closed-form expressions for the model-implied moments on the basis of the solution to the bank's date-1 problem. These are six equations comprising the nine unknown parameters shown in the third block of Table 1.

To solve these equations, we need to reduce the number of unknown parameters so that it is not higher than the number of equations. To this end, we impose the following three restrictions. First, we assume that $(\alpha + \gamma)/2 = R$, which implies that the bank's borrowing cost lies half way between the intercepts of the supply and demand schedules for the fixed-income security. Second, we assume that $\beta = \delta$, which equalises the absolute values of the price elasticities of the same supply and demand. Together, these two assumptions imply that the expected capital gain on bonds is equal to R. Third, we assume that the bank is unconstrained at date-1 with close to zero probability. This imposes a restriction on the ratio of the actual to the unconstrained (i.e. counterfactual) size of the bank's bond inventory. Given these assumptions, the model is exactly identified. This means that the values of the parameters in the third block of Table 1 deliver model-implied moments that are exactly equal to their data-implied values in the second block. For completeness, we also report the marginal return from market-making (f_1) and the associated curvature (f_2) which depend on $\alpha, \beta, \gamma, \delta$ and λ, as shown in (5) (fourth block).

[10] Recent revisions to regulatory standards for market risk are cast in terms of expected shortfall, as opposed to VaR (BCBS [2016]). Given that we consider Gaussian shocks, the two risk metrics deliver identical results.

Parameter/Moment	Name	Value
Δ	Discount factor	0.99
R	Borrowing cost	1.0209
χ	LR	0.03
a_m	Market-making VaR	0.176
a_l	Lending VaR	0.001
L/M	Lending to market-making volume ratio	5.0089
$E[s/p]$	Expected spread as a ratio of price	0.0077
$K/(L+M)$	Leverage ratio	0.0732
K_m/M	Leverage ratio of market-making unit	0.0382
$G(L)/L$	Expected gross return from lending unit	1.0424
$F(M)/M$	Expected gross return from market-making unit	1.0333
α	Demand schedule intercept	1.0260
β	Demand schedule slope	0.0003
γ	Supply schedule intercept	1.0157
δ	Supply schedule slope	0.0003
λ	Market-making technology	1.6103
σ_ϵ	Stdev. of supply schedule	0.0497
g_1	Marginal return from loans	1.0472
g_2	Curvature in loan return	–0.0004
σ_Z	Stdev. of loan payoff	0.0334
f_1	Marginal return from market-making	1.0374
f_2	Curvature in market-making return	–0.0018

Table 1: Parameters and moments. To be consistent with the International Financial Reporting Standards (IFRS), the GAAP based leverage ratio computed from bank balance sheet data in SNL is adjusted downwards by a factor of 0.22. This adjustment factor is estimated from data on US banks that report both GAAP and IFRS-based leverage ratios.

The model calibration is consistent with some commonly held views on the characteristics of bank lending and market-making. First, lending has a higher expected gross return (Table 1). Second, the market-making unit is more leveraged, as implied by its leverage ratio being lower than that of the bank as a whole. Third, given the fixed capital endowment, the higher gross expected returns and the lower leverage of the lending unit imply that this unit also generates higher expected *net* returns.

The calibration does not provide an immediate answer as to which of the two units is more efficient and, thus, would attract capital if a constraint tightens. On the one hand, the loan unit is roughly five times larger. We argued in Section 4.3.1 that, all else equal, this would imply that the loan unit is more efficient. On the other hand, however, the marginal gross cash flow of the market-making unit is more sensitive to the unit's size (i.e. $|f_2| > |g_2|$ in Table 1). On its own, this would imply that the market-making unit is more efficient. In the next subsection, we check which of the two drivers dominates.

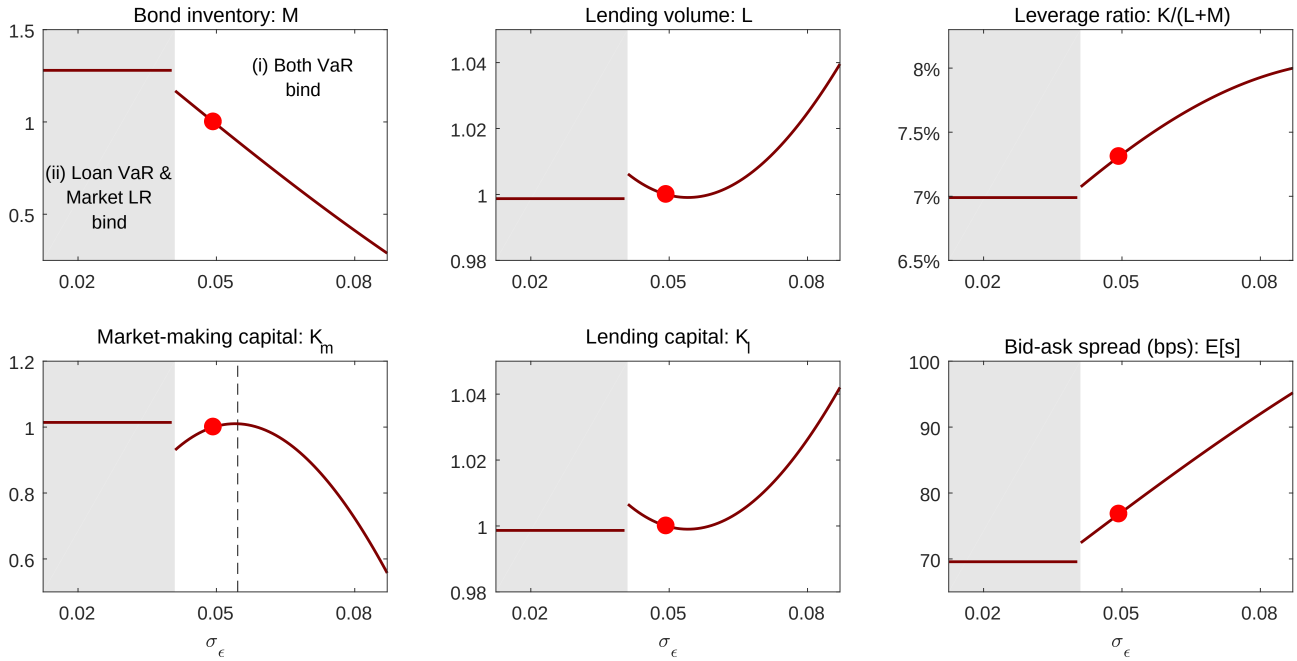

Figure 4: Effect of a change in market risk, σ_ϵ, on bank choices. The vertical axes in left-hand and centre panels express quantities relative to their benchmark values (red dot). The bid-ask spread (bottom right-hand panel) is expressed in basis points relative to the bond price. The shaded areas indicate regions underpinned by different binding constraints.

5.2 Date-1 comparative statics

In this subsection, we illustrate date-1 capital re-allocations in response to changes in financial conditions. For each of the examples, we start from our baseline calibration, deviate from it by changing the value of one model parameter, and record the implications of the capital re-allocation rule that we derived in Section 4.

The first comparative statics exercise has to do with changes in market risk. Its results are portrayed in Figure 4, where the benchmark calibration outcomes are shown with red dots. As we move from left to right in each panel, market risk increases.

Looking at how the bank adjusts its market-making capital (bottom left-hand panel), we identify three phases. For a sufficiently low level of market risk, the LR constraint binds for the market-making unit, whereas this unit's VaR constraint is slack (grey-shaded area). Changes in market risk within this phase are thus inconsequential. As market risks increase, the market-making VaR constraint starts to bind (white area).[11] In this second phase, which includes the calibration benchmark (red dot), the bank responds to higher market risk by reducing the size of the market-making unit (top left-hand panel), as implied by Proposition 1.1. In this phase, the market-making unit is more efficient because of the higher sensitivity of its marginal return and despite its small relative size (recall Section 4.3.1 and Table 1). Thus, in line with the rule developed in Proposition 1.2, it attracts more capital.

[11] The discontinuity in each of the panels of Figure 4 is due to the different optimality conditions in the grey-shaded and white regions. The loan unit's VaR-based CRAR is equalised to the market-making unit's LR-based CRAR in the former region but to the corresponding VaR-based CRAR in the latter region. Similar discontinuities are observed in subsequent figures for similar reasons.

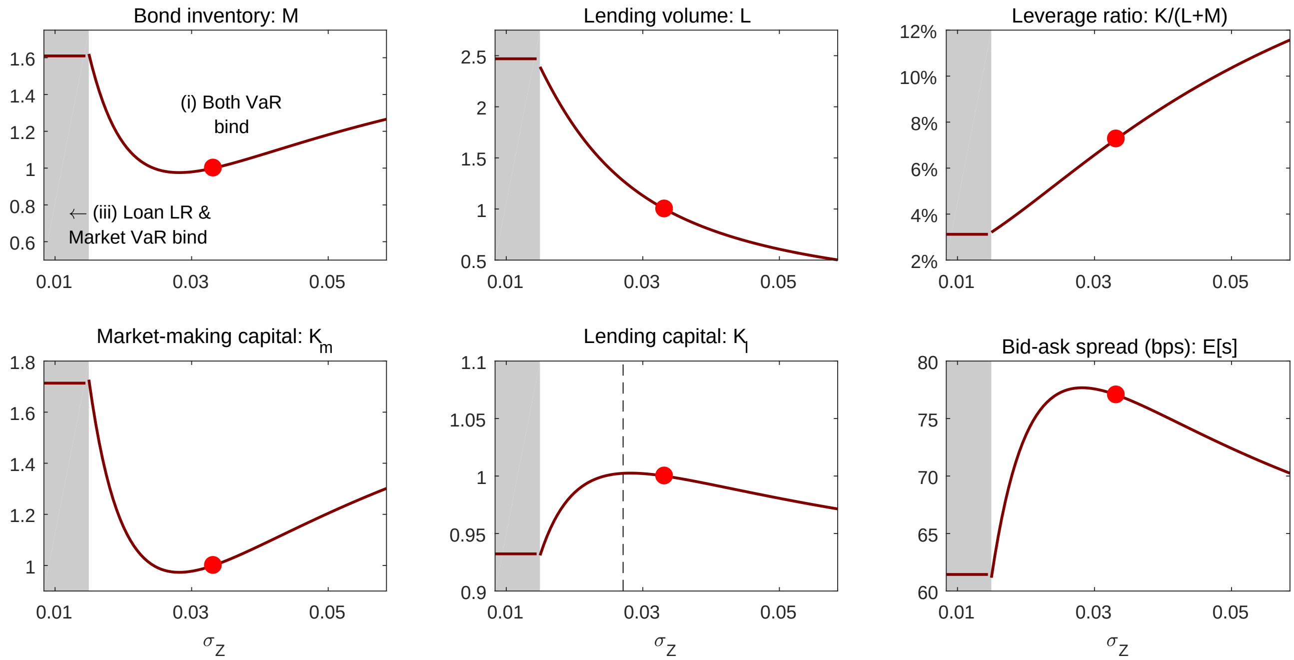

Figure 5: Effect of a change in credit risk, σ_Z, on bank choices. The vertical axes in left-hand and centre panels express quantities relative to their benchmark values (red dot). The bid-ask spread (bottom right-hand panel) is expressed in basis points relative to the bond price. The shaded areas indicate regions underpinned by different binding constraints.

As we move to even higher levels of market risk, we enter a third phase, in which market-making has contracted to such an extent that it is now the less efficient unit. Thus, following again the capital re-allocation rule, the bank draws capital from the market-making unit and allocates it to the loan unit (bottom centre panel). The inverted U-shaped curve for the market-making capital reflects the change in relative efficiency due to changes in market risk.

Figure 4 portrays two direct outcomes of the reduction in market-making on the back of a binding VaR constraint and rising market risk. First, the expected bid-ask spread increases (bottom right-hand panel), in line with equation (2). Second, as higher risk means scarcer capital, the overall balance sheet contracts which – given the fixed capital stock – implies a higher leverage ratio (top right-hand panel).

Changes in credit risk lead to qualitatively similar outcomes (Figure 5). The loan unit's VaR constraint does not bind for low levels of credit risk, implying that changes in this risk are inconsequential (grey-shaded areas). As credit risk increases, this VaR constraint starts to bind (white area) and, per Proposition 1.1, the loan unit contracts (top centre panel). Again, the capital allocation reflects the units' relative efficiency. At intermediate levels of credit risk, the loan unit is more efficient owing to its sufficiently larger size. Thus, per Proposition 1.2, a rise in credit risk from these levels induces the bank to reallocate capital towards the loan unit (bottom centre panel) and away from the market-making unit (bottom left-hand panel). This leads to a contraction of the bond inventory (top left-hand panel). For sufficiently high credit risk, such as at the calibration benchmark (red dot), the loan unit becomes relatively less efficient because of its reduced size. Here, an increase in credit risk prompts a re-allocation of capital towards market-making (bottom left-hand panel), which results in an expansion of the bond inventory (top left-hand panel) and a decline in the expected bid-ask spread (bottom right-hand panel). These re-allocation patterns surface as an inverted U-shaped curve of the loan unit's capital (bottom centre panel).

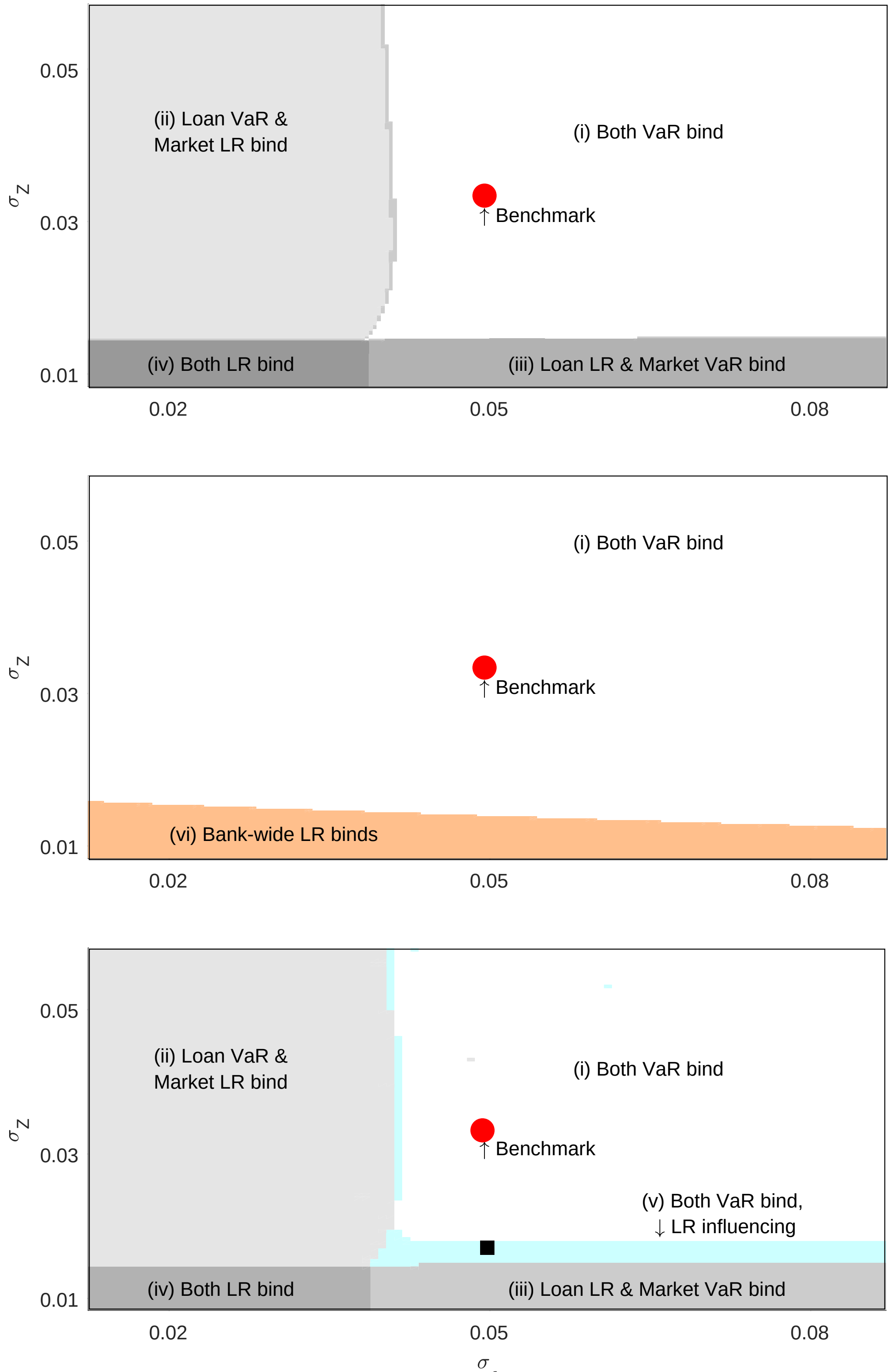

Figure 6: Changing relevance of constraints.

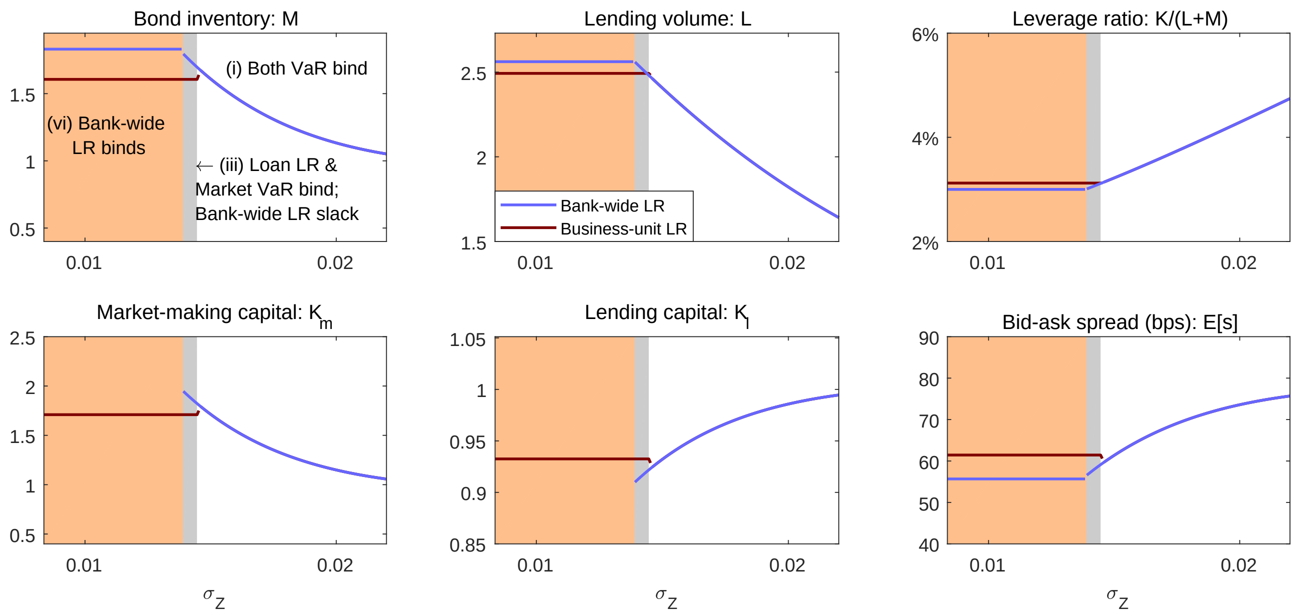

Figure 7: Effect of a change in credit risk, σ_Z, on bank choices in the bank-wide LR and business-unit LR cases. The vertical axes in left-hand and centre panels express quantities relative to their benchmark values (benchmark not shown). The bid-ask spread (bottom right-hand panel) is expressed in basis points relative to the bond price. The shaded areas indicate regions underpinned by different binding constraints in the two cases.

Overall, changes in market or credit risk give rise to four binding-constraint combinations at date 1 (Figure 6, top panel). First, for sufficiently high levels of both risks – such as at the calibration benchmark (red dot) – the two units' VaR constraints bind (region (i)). Second, lowering only market risk (i.e. moving to the left in the panel) implies that the market-making unit is bound by the LR constraint, while the VaR constraint binds for the loan unit (region (ii)); Symmetrically, lowering only credit risk (e.g. moving to the bottom in the panel from the red dot) implies that the loan unit is bound by the LR constraint, while the VaR constraint binds for the market-making unit (region (iii)). Finally, low levels of both market and credit risk make the LR constraints bind for both units (region (iv)).

5.3 Date-1 comparative statics, bank-wide LR constraint

Applying the LR constraint at the overall bank level alleviates the scarcity of capital and increases the bank's value in those cases where the LR binds for only one business unit in the business-unit LR regime.[12] This is consistent with (i) the proof in Appendix A.4 that at least one business unit must expand when the bank switches from business-unit to bank-wide LR constraint; and (ii) our specific parameterisation. To illustrate this point, we zoom in on relatively low levels of credit risk where LR constraints come into play (Figure 7). We plot as blue lines comparative statics results from a bank-wide application of the LR constraint. For comparison, we also plot (with red lines) the corresponding results from Figure 5, which are based on LR constraints at the business-unit level.

The comparison of the two scenarios delivers four takeaways. First, for sufficiently high levels

[12]In the business-unit LR regime, if the LR binds for both units, changing the application of the LR constraint to the overall bank level would have no effect on the bank's choices.

of credit risk (white areas), switching to a bank-wide application of the LR constraint is inconsequential as this constraint is irrelevant. Concretely, the blue and red lines coincide for high σ_Z in Figure 7. Second, relative to a business-unit LR constraint, a bank-wide LR constraint starts binding only at lower levels of risk. In Figure 7, this is seen in that the bank-wide LR constraint does not bind in the grey region whereas the loan unit's LR constraint does bind. The same takeaway emerges for both credit and market risk in Figure 6. This figure shows that, applied to each business unit separately (top panel), the LR constraint binds for wider parameter regions than if applied to the bank as a whole (centre panel).

Third, for intermediate levels of credit risk (grey-shaded regions in Figure 7), switching from a business-unit to a bank-wide LR constraint alters the binding constraint for the loan unit. Namely this unit's LR constraint is replaced by the less demanding VaR constraint. This allows the bank to re-allocate capital from the loan to the market-making unit (bottom, left-hand and centre panels) while expanding both business units (top, left-hand and centre panels). It also translates into a lower leverage ratio (top, right-hand panel) and a lower bid-ask spread (bottom, right-hand panel).

Fourth, for low levels of credit risk (amber regions in Figure 7), the switch to a bank-wide LR constraint alleviates capital scarcity. Thus, the bank expands both units (top left-hand and centre panels), lowers its leverage ratio (top right-hand panel) and lowers the bid-ask spread (bottom right-hand panel). When a bank-wide LR binds, the level of capital allocated to a specific business unit is indeterminate. To signal this, we do not plot blue lines in the amber regions of the bottom left-hand and centre panels.

5.4 Date-0 comparative statics

We now consider the date-0 implications of the calibration on the basis of LR constraints at the business-unit level. The bottom panel of Figure 6 provides a comprehensive view as regards which constraint binds on date 0 for different levels of risk. The benchmark calibration (red dot) belongs again to the region in which the VaR constraints bind for each of the two business units. And, similar to the date-1 case, the market-making unit continues to be the more efficient one in a neighbourhood around the benchmark calibration. Thus, deviations from the benchmark that are due to changes in market or credit risk have similar implications as those portrayed above in Figures 4 and 5.[13]

The main novelty on date 0 is the possibility that a constraint influences the bank without binding contemporaneously. In particular, our calibration implies that LR constraints are non-binding but influencing for intermediate levels of credit or market risk (Figure 6, bottom panel, region (v)). To examine the implications of an influencing constraint, we select a parameter configuration that differs from the calibration benchmark only in that credit risk is lower (black square).[14]

To verify relative efficiency in this case, we focus on those future, i.e. date-1, states for which an LR constraint binds. And we confirm that – given a tightening of an LR constraint – downsizing the two units to satisfy this constraint at the initial capital allocation results in a higher CRAR for the loan unit. Thus, in terms of the definition in Section 4.2, the loan unit is more efficient.

Relative efficiency governs the bank's responses when an influencing LR constraint tightens, i.e. χ increases (Figure 8). Namely, a tightening of the constraint leads to a re-allocation of

[13]We verify that the tightening of any contemporaneously binding constraint has qualitatively similar implications on dates 1 and 0. These results are available upon request.

[14]We focus on an influencing LR constraint, but note that the same reasoning applies to the case of an influencing VaR constraint.

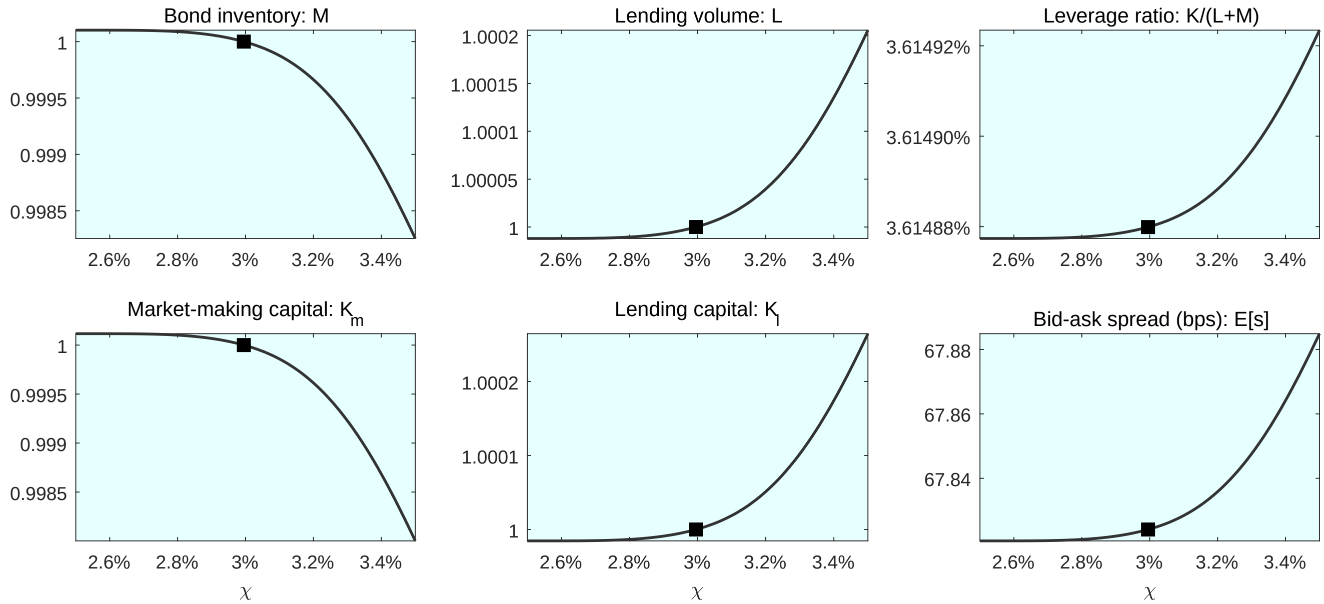

Figure 8: An influencing LR constraint for low credit risk. The vertical axes in left-hand and centre panels express quantities relative to the starting point (black square). The bid-ask spread (bottom right-hand panel) is expressed in basis points relative to the bond price.

capital from the market-making to the loan unit (bottom left-hand and centre panels). Since the contemporaneously binding VaR constraints have not changed, the capital re-allocation translates into a smaller market-making unit and a larger loan unit (top left-hand and centre panels). And since a tightened constraint makes capital scarcer, the overall balance sheet size declines and the leverage ratio rises (top right-hand panel). Finally, the smaller bond inventory triggers an increase in the bid-ask spread (bottom right-hand panel).

6 Conclusion

The above analysis reveals that bank capital allocation creates potentially important spillovers across different business units and over time. These spillovers emerge even when successive shocks are uncorrelated and each unit is subject to independent sources of risk and faces separate capital constraints. They are the result of centralised decision-making that optimises the overall value of the bank.

The spillovers imply that the same outcome may have very different root causes. For instance, a reduction in market-making could be due to stricter regulation or risk management. But it could also stem from a change in measured credit risk that leads the bank to optimally re-allocate capital away from market-making. Importantly, the outcome could have been the opposite if the change had occurred for a different initial level of measured risk. Furthermore, a constraint that does not bind contemporaneously could still influence decisions if financial conditions evolve and imply that this constraint is more or less likely to bind in the future. All this calls for extra caution in empirical analyses of the drivers of observable bank behaviour.

References

Adrian, T., Boyarchenko, N., Shachar, O., 2017. Dealer balance sheets and bond liquidity provision. Journal of Monetary Economics 89, 92–109.

Adrian, T., Shin, H. S., 2011. Financial intermediary balance sheet management. Annual Review of Financial Economics 3, 289–307.

Armstrong, M., Vickers, J., 1991. Welfare effects of price discrimination by a regulated monopolist. The Rand journal of economics, 571–580.

Bank of England, July 2016. Financial stability report. Issue no 39.

Bao, J., O'Hara, M., Zhou, A., 2016. The Volcker rule and market-making in times of stress. Finance and Economics Discussion Series 2016-102, Board of Governors of the Federal Reserve System.

Baranova, Y., Liu, Z., Shakir, T., July 2017. Dealer intermediation, market liquidity and the impact of regulatory reform. Staff Working Paper 665, Bank of England.

Baud, N., Frachot, A., Igigabel, P., Martineu, P., Roncalli, T., February 2000. An analysis framework for bank capital allocation. Discussion paper, Crédit Lyonnais Research.

BCBS, November 2005. International convergence of capital measurement and capital standards.

BCBS, July 2009. Revisions to the Basel II market risk framework.

BCBS, June 2011. Basel III: A global regulatory framework for more resilient banks and banking systems.

BCBS, January 2014. Basel III leverage ratio framework and disclosure requirements.

BCBS, January 2016. Minimum capital requirements for market risk.

Begley, T. A., Purnanandam, A., Zheng, K., 2017. The strategic underreporting of bank risk. Review of Financial Studies 30 (10), 3376–3415.

Behn, M., Haselmann, R. F., Vig, V., July 2016. The limits of model-based regulation. ECB Working Paper 1928, European Central Bank.

Cecchetti, S. G., Kashyap, A. K., November 2016. What binds? Interactions between bank capital and liquidity regulations. Mimeo.

Chami, R., Cosimano, T. F., Ma, J., Rochon, C., February 2017. What's different about bank holding companies? IMF Working Paper WP/17/26, International Monetary Fund.

Erel, I., Myers, S. C., Read, J. A., 2015. A theory of risk capital. Journal of Financial Economics 118 (3), 620–635.

Fender, I., Lewrick, U., December 2015. Calibrating the leverage ratio. BIS Quarterly Review, 43–58.

Froot, K. A., Stein, J. C., 1998. Risk management, capital budgeting, and capital structure policy for financial institutions: an integrated approach. Journal of Financial Economics 47 (1), 55–82.

Garman, M. B., 1976. Market microstructure. Journal of Financial Economics 3 (3), 257–275.

Glosten, L. R., Milgrom, P. R., 1985. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. Journal of Financial Economics 14 (1), 71–100.

Gorton, G., Metrick, A., 2012. Securitized banking and the run on repo. Journal of Financial Economics 104, 425–451.

Guill, G. D., 2016. Bankers trust and the birth of modern risk management. Journal of Applied Corporate Finance 28 (1), 19–29.

Ita, A., February 2016. Capital allocation in large banks – a renewed look at practice. SSRN Working Paper.

James, C. M., 1996. RAROC based capital budgeting and performance evaluation: a case study of bank capital allocation. Financial Institutions Center, The Wharton School, University of Pennsylvania.

Kyle, A. S., 1989. Informed speculation with imperfect competition. The Review of Economic Studies 56 (3), 317–355.

Nishiguchi, K., Hiroshi, K., Takanori, S., October 1998. Capital allocation and bank management based on the quantification of credit risk. Economic policy review, Federal Reserve Bank of New York.

O'Hara, M., Oldfield, G. S., 1986. The microeconomics of market making. Journal of Financial and Quantitative analysis 21 (04), 361–376.

Perold, A. F., 2005. Capital allocation in financial firms. Journal of Applied Corporate Finance 17 (3), 110–118.

Schmalensee, R., 1981. Output and welfare implications of monopolistic third-degree price discrimination. The American Economic Review 71 (1), 242–247.

Stein, J. C., 1997. Internal capital markets and the competition for corporate resources. The Journal of Finance 52 (1), 111–133.

Stoughton, N. M., Zechner, J., 2007. Optimal capital allocation using RAROC™ and EVA®. Journal of Financial Intermediation 16 (3), 312–342.

Tarashev, N., 2010. Measuring portfolio credit risk correctly: Why parameter uncertainty matters. Journal of Banking & Finance 34 (9), 2065–2076.

Wall, L. D., Peterson, D. R., 1987. The effect of capital adequacy guidelines on large bank holding companies. Journal of Banking & Finance 11, 581–600.

Wells Fargo, 1974. Annual report – letter to shareholders. Wells Fargo & Company.

Zaik, E., Kelling, W. G., Retting, G., James, C., 1996. RAROC at Bank of America: from theory to practice. Journal of Applied Corporate Finance 9, 83–93.

A Appendix

A.1 Business-unit LR constraint

The optimisation problem of the bank at date $t \in \{0, 1\}$ is as follows:

$$U_t(K_t) = \max_{L_t, M_t, K_{mt}, K_{lt}} \Delta \Bigg\{ E \Big[U_{t+1} \Big(V_{mt+1}(M_t, K_{mt}; \epsilon_{t+1}) + V_{lt+1}(L_t, K_{lt}; Z_{t+1}) \Big) \Big] \Bigg\}$$

subject to

$$\begin{aligned}
MRC^{LR}_{mt} &\equiv \chi M_t \leq K_{mt} \quad [\zeta^{LR}_{mt}] && (24)\\
MRC^{LR}_{lt} &\equiv \chi L_t \leq K_{lt} \quad [\zeta^{LR}_{lt}] && (25)\\
MRC^{VaR}_{mt} &\equiv \frac{RM_t + \Omega M_t - F(M_t)}{R} \leq K_{mt} \quad [\zeta^{VaR}_{mt}] && (26)\\
MRC^{VaR}_{lt} &\equiv \frac{RL_t + \Theta L_t - G(L_t)}{R} \leq K_{lt} \quad [\zeta^{VaR}_{lt}] && (27)\\
K_{mt} + K_{lt} &= K_t \quad [\eta_t]
\end{aligned}$$

where U_t is the value function, the bank's capital stock, K_t, is the state variable and Lagrange multipliers are in square brackets. Since the bank is risk-neutral, U_2 is the identity function.

Suppose that the binding constraints for the loan and market-making units are i and j, respectively. We then obtain the feasibility condition

$$MRC^i_{mt} + MRC^j_{lt} \leq K_t \qquad i, j \in \{VaR, LR\}$$

and the first-order optimality conditions for the two units:

$$[M_t] : E\left[U'_{t+1}(.) \frac{\partial V_{mt+1}}{\partial M_t} \right] - \zeta^i_{mt} \frac{\partial MRC^i_{mt}}{\partial M_t} = 0$$

$$[L_t] : E\left[U'_{t+1}(.) \frac{\partial V_{lt+1}}{\partial L_t} \right] - \zeta^j_{lt} \frac{\partial MRC^j_{lt}}{\partial L_t} = 0.$$

Since capital is fungible between the two units, the Lagrange multipliers of the two binding constraints are equal in equilibrium, $\zeta^i_{mt} = \zeta^j_{lt}$. Defining CRARs as in (16), it follows that they should be equal across units:

$$\underbrace{E\left[U'_{t+1}(.) \frac{\partial V_{mt+1}}{\partial M_t} \right] \Big/ \frac{\partial MRC^i_{mt}}{\partial M_t}}_{CRAR^i_{mt}} = \underbrace{E\left[U'_{t+1}(.) \frac{\partial V_{lt+1}}{\partial L_t} \right] \Big/ \frac{\partial MRC^j_{lt}}{\partial L_t}}_{CRAR^j_{lt}}.$$

In the special case of no binding constraint, $\zeta^i_{mt} = \zeta^j_{lt} = 0$ and the bank equalises RARs, as defined in (15).

Note that MRCs increase in the size of the underlying unit: $dMRC^i_{xt}/dX > 0$. For binding LR constraints, this is seen immediately in expressions (24) and (25). For binding VaR constraints, expressions (26) and (27) reveal that MRC is a convex function of the unit size. Since this function also passes through the $(0, 0)$ point, it is guaranteed to be increasing wherever size and MRC are both positive (which is the only relevant case).

Note also that the date-1 CRAR decreases in the underlying unit's size: $dCRAR^i_{xt}/dX < 0$. This is the result of diminishing marginal returns, which make (i) marginal cash flows decrease in the unit's size – recall expressions (5) and (6) – and (ii) marginal MRC weakly increase in this size – see expressions (24) to (27).

A.2 Proof of Proposition 1.2

Let x stand for the business unit whose binding constraint is tightened and let X stand for this unit's size. Conversely, let y and Y stand for the other business unit and its size. By proposition (1.1), the tightened constraint causes unit x to shrink: $\dot{X} < 0$. In this appendix, we prove under what conditions the capital allocated to unit x, i.e. K_x, rises (drops), thus necessitating a drop (rise) in K_y and Y.

Consider two hypothetical responses of unit x to a tightening of its binding constraint. Namely, we denote by $\dot{X}|_{\overline{MRC^i_x}} < 0$ and $\dot{X}|_{\overline{CRAR^i_x}} < 0$ the change in X that maintains the unit's MRC – respectively, CRAR – constant. Each of these changes is negative because, as proved in Appendix A.1 – $dMRC^i_x/dX > 0$ and $dCRAR^i_x/dX < 0$.

Next, we show that the *actual* response, $\dot{X}$, can be neither smaller nor larger than *both* of these hypothetical responses.

- $\dot{X} < min(\dot{X}|_{\overline{MRC^i_x}}, \dot{X}|_{\overline{CRAR^i_x}}) < 0$ leads to a contradiction. If this ordering held in equilibrium: (i) $\dot{X} < \dot{X}|_{\overline{MRC^i_x}}$ would imply a drop in K_x; (ii) $\dot{X} < \dot{X}|_{\overline{CRAR^i_x}}$ and $dCRAR^i_x/dX < 0$ would imply a rise in the CRAR of unit x. When K_x declines, K_y increases because the overall capital is fixed and unit y was initially facing a binding constraint. Since this constraint did not change, an increase in K_y implies a higher Y. But a higher Y and $dCRAR^i_y/dY < 0$ imply a drop in the CRAR of unit y. In conjunction with (ii), this implies that the CRARs are not equal across business units. This cannot be an equilibrium.
- $max(\dot{X}|_{\overline{MRC^i_x}}, \dot{X}|_{\overline{CRAR^i_x}}) < \dot{X} < 0$ also leads to a contradiction. The argument is symmetric to the previous one, with all signs reversed and the conclusion still being that the CRARs are not equal across business units.

There are thus two possible orderings of the actual and hypothetical responses.

- $\dot{X}|_{\overline{MRC^i_x}} < \dot{X} < \dot{X}|_{\overline{CRAR^i_x}} < 0$. In this case, X decreases by less than what is needed to maintain the original MRC. Thus, the MRC increases and so must K_x for the tightened constraint to be satisfied.
- $\dot{X}|_{\overline{CRAR^i_x}} < \dot{X} < \dot{X}|_{\overline{MRC^i_x}} < 0$. In this case, a symmetric argument leads to the conclusion that K_x declines.

The first ordering is a necessary and sufficient condition for unit x to be the more efficient one. When this ordering is in place, $dCRAR^i_x/dX < 0$ implies that the shrinkage of unit x that maintains the initial MRC also raises the CRAR above its initial level. Symmetrically, if the second of the two possible orderings is in place, the shrinkage of unit x that maintains the initial MRC also suppresses the CRAR below its initial level. Or, in terms of the notation introduced in Section 4.2, $\dot{X}|_{\overline{MRC^i_x}} < \dot{X}|_{\overline{CRAR^i_x}}$ is equivalent to $\partial CRAR^i_x/\partial i + (\partial CRAR^i_x/\partial X)\,\dot{X}|_{\overline{MRC^i_x}} > 0$. And since constraint i does not bind for unit y, it is trivially the case that $\partial CRAR^j_y\big/\partial i + \left(\partial CRAR^j_y\big/\partial Y\right)\dot{Y}|_{\overline{MRC^j_y}} = 0$. Thus, unit x satisfies the efficiency condition (17) if and only if $\dot{X}|_{\overline{MRC^i_x}} < \dot{X}|_{\overline{CRAR^i_x}}$.

Finally, we showed above that capital is re-allocated to unit x if and only if $\dot{X}|_{\overline{MRC^i_x}} < \dot{X}|_{\overline{CRAR^i_x}}$. This proves Proposition 1.2.

A.3 Proof of Proposition 2.2

Recall the equilibrium conditions:

$$\frac{G'(L) - R}{\chi} = \frac{F'(M) - R}{\chi} \tag{28}$$

$$\chi M + \chi L = K \tag{29}$$

Without loss of generality, let the lending unit be more efficient. Applying the definition of efficiency (17) to equations (28) and (29) leads to:

$$- MF''(M) < -G''(L)L. \tag{30}$$

The goal is to show that K_l increases in response to a higher χ if and only if the latter inequality holds. Total differentiation of equations (28) and (29) implies:

$$G''(L)\dot{L} = F''(M)\dot{M}, \tag{31}$$

$$\chi\dot{L} + L + \chi\dot{M} + M = 0. \tag{32}$$

And the solution of this system of equation delivers:

$$\dot{K_l} = \chi\dot{L} + L = -\frac{MF''(M) - LG''(L)}{F''(M) + G''(L)},$$

where the first equality follows from the definition $K_l \equiv \chi L$. Indeed, the last expression is positive if and only if inequality (30) holds.

A.4 Bank-wide LR constraint

In this appendix, we show that both business units cannot shrink if the bank switches from a business-unit to a bank-wide LR constraint. In the first case, the constraint set is the intersection of allocations that are feasible given the two VaR constraints and the two LR constraints:

$$\mathcal{C}_1 = \Bigg\{(L, M, K_l, K_m) \in \mathcal{R}^4_+ \quad | \quad K_m + K_l = K;\ \chi L \leq K_l;\ \chi M \leq K_m; \\ \frac{RL + \Theta L - G(L)}{R} \leq K_l;\ \frac{RM + \Omega M - F(M)}{R} \leq K_m\Bigg\}$$

In the second case, the constraint set is the intersection of allocations that are feasible given the two VaR constraints and the single bank-wide LR constraint:

$$\mathcal{C}_2 = \Bigg\{(L, M, K_l, K_m) \in \mathcal{R}^4_+ \quad | \quad K_m + K_l = K;\ \chi(L + M) \leq K; \\ \frac{RL + \Theta L - G(L)}{R} \leq K_l;\ \frac{RM + \Omega M - F(M)}{R} \leq K_m\Bigg\}$$

We will first show that $\mathcal{C}_1 \subset \mathcal{C}_2$. To this end, consider an arbitrary point $c = (L', M', K_l', K_m') \in \mathcal{C}_1$. First, note that c satisfies the two VaR constraints, as they are common determinants of $\mathcal{C}_1$ and $\mathcal{C}_2$. Second, note that since c satisfies $\chi L' \leq K_m'$ and $\chi M' \leq K_l'$, it trivially satisfies $\chi(L' + M') \leq K = K_m' + K_l'$. Therefore, any point in $\mathcal{C}_1$ belongs to $\mathcal{C}_2$ as well.

$\mathcal{C}_1 \subset \mathcal{C}_2$ implies that the maximum of the value function achieved under $\mathcal{C}_2$ is at least as high as that achieved under $\mathcal{C}_1$. Since this function is monotonically increasing in L and M, it then follows that L and M cannot be both smaller in the bank-wide LR regime.

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