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Effects of Capital Controls on Foreign Exchange Liquidity

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Abstract

The literature on capital controls has focused on their use as tools to manage capital and improve macroeconomic and financial stability. However, there is a lack of analysis of their effect on foreign exchange (FX) market liquidity. In particular, technological and regulatory changes in FX markets over the past decade have had an influence on the effect of capital controls on alternative indicators of FX liquidity. In this paper, we introduce a theoretical model showing that, if capital controls are modelled as entry costs, then fewer investors will enter an economy. This will reduce the market's ability to accommodate large order flows without a significant change in the exchange rate (a market depth measure of liquidity). On the other hand, if capital controls are modelled as transaction costs, they can reduce the effective spread (a cost-based measure of liquidity). Using a panel of 20 emerging market economies and a novel measure of capital account restrictiveness, we provide empirical evidence showing that capital controls can reduce cost-based measures of FX market liquidity. The results imply that capital controls are effective in reducing the implicit cost component of FX market liquidity but can also have a negative structural effect on the FX market by making it more vulnerable to order flow imbalances.

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1 Introduction

Capital controls are policies that have in the recent decade been implemented with the purpose of shielding an economy from the risks associated with sudden changes in capital flows (Ostry et al (2010) and Engel (2012)). In most of the literature, capital controls are treated as second-best measures that reduce financial system distortions caused by excessive inflows of foreign capital. There has been no recent study on the effect of capital controls on foreign exchange (FX) market liquidity.[1] Illiquidity in the FX market can have adverse effects on cross-border investment and on the ability of market participants to finance transactions. In extreme cases, the lack of FX market liquidity has been associated with market closure and with balance of payments crises in emerging economies (BIS (2017)). On the one hand, by restricting flows, capital controls could reduce exchange rate volatility and help tighten bid-ask spreads, which would imply an improvement in market liquidity. On the other hand, capital controls could have the undesired side effect of increasing the sensitivity of the exchange rate to large fluctuations in flows, reducing market liquidity. This implies that the particular indicator used to measure liquidity is crucial in understanding the effects of capital controls on the FX market.

There are two main strands of theoretical models on capital controls. In the first, capital controls address an overborrowing externality in financially constrained economies (Korinek (2010, 2017), Bianchi (2011), Benigno et al (2013), Bengui and Bianchi (2014), and Korinek and Sandri (2014)). Agents are vulnerable to pecuniary externalities when their borrowing capacity is limited by the quantity and/or quality of collateralised assets. In the event of a shock, there is an amplification effect whereby capital outflows and a depreciation of the exchange rate further tighten financial constraints. As a second-best solution, taxes on capital inflows lead to an increase in the domestic interest rate, which then curbs domestic borrowing and encourages domestic saving.[2] The second branch of the literature focuses on the use of capital controls as tools to correct aggregate demand externalities in models with nominal rigidities and limits to the use of monetary policy (Farhi and Werning (2012, 2013, 2014), Farhi, Gopinath and Itskhoki (2011), and Schmitt-Grohe and Uribe (2012)). These rigidities include fixed exchange rates and stickiness in both

[1] One reason is that advances in technology and new regulations have altered the structure of FX markets and made it harder to analyse the effects of capital controls. In particular, different indicators of liquidity can only give a partial picture of the state of the FX market.

[2] Costinot et al (2011) also propose a theory of capital controls that emphasises interest rate manipulation but in a model without externalities.

prices and wages. In models of small open economies with a flexible exchange rate and free capital mobility, a sudden stop puts downward pressure on the exchange rate, which ultimately helps the economy recover. These models provide a solution when alternative mechanisms are not available. Capital controls can prevent large fluctuations in aggregate demand and smooth the terms of trade, which weaken the amplification effect generated by the nominal rigidities of the model. The role of capital controls is to tame the outflow of capital, not because of the flow itself, but because of the negative effects of depreciation on financially constrained firms.

In the first part of the paper, we introduce a theoretical model that captures the effects of capital controls on FX market liquidity. In contrast to the traditional literature on capital controls, the model is based on market microstructure theory. It follows Vayanos and Wang (2012), who analyse how asymmetric information and imperfect competition affect liquidity and asset prices. In their framework, agent heterogeneity is introduced through different endowments and information. In our model, there are two segmented markets in which a risky asset with the same expected return is traded at different prices. Every agent receives the same endowment in each market and there is perfect information. However, heterogeneity arises from the probability that agents will face a liquidity shock forcing them to reduce their holdings of the risky asset in exchange for liquidity. Traders base their selling decisions on how their trades affect prices in both markets. In this way, the model allows us to define two different indicators of FX liquidity. The first indicator is market depth, which measures the ability of the market to sustain large order flows without a significant change in price.[3] The second indicator is the effective spread, which captures the transaction cost component of market liquidity.

Capital controls are modelled in two distinct ways to analyse their effects on our two measures of FX liquidity. First, they are introduced as a fixed entry cost to the market, which deters the entrance of potential new investors. A smaller number of investors negatively affects liquidity supply and reduces market depth. Second, capital controls are modelled as a transaction cost that has to be paid by investors wanting to sell an asset. In this case, capital controls reduce excess order flows, which causes the spread between the actual transaction price and the price in a perfectly liquid market to be lower. The

[3] Kyle (1985), Pagano (1989), Gromb and Vayanos (2003), and Foucault, Pagano and Roell (2014) propose models in which market depth is determined endogenously. However, all these models tend to focus on how privileged information can affect price volatility.

results from the model show that capital controls can have contrasting effects on different dimensions of FX market liquidity.

The second part of the paper studies the effects of capital controls on FX liquidity from an empirical perspective. We construct a monthly database of capital control and FX liquidity indicators for 20 emerging market economies (EMEs) during the period ranging from 1989 to 2015. We focus on EMEs because they have been more active in their use of capital controls than advanced economies. In particular, EMEs have implemented capital controls in the past with three main goals: 1) to control the exchange rate; 2) to curb capital inflows; and 3) to ensure financial stability (Fratzscher (2012), Fernández et al (2013) and Pasricha et al (2015)).[4]

To measure capital controls, we propose a new monthly index constructed using information from the International Monetary Fund's Annual Report on Exchange Arrangements and Exchange Restrictions. The innovative feature is that the index is an intensive measure, in the sense that it captures subtle changes in a country's capital control policy. For example, the index distinguishes between the implementation of a tax on financial operations and an increase in the tax. The indices most commonly used in the literature only capture the introduction of a given tax and not any subsequent increase or reduction.[5] Finally, we disaggregate the index into three different sub-indices: an index that measures controls on capital transactions; one that measures controls on capital inflows; and another one that measures controls on capital outflows.

The measure of FX liquidity used in our empirical analysis is based on the implicit cost of liquidity. The most widely used cost-based measure is the difference between the best ask (buy) and the best bid (sell) price: the bid-ask spread. It measures the implicit cost of trading in the market because it represents deviations from the hypothetical price that could be obtained in a perfectly liquid market. We use such a cost-based measures of FX liquidity, following Karnaukh et al (2015). Their measure takes information from bid-ask spreads and high-low transaction prices. It has been shown to have a high degree of correlation with the effective bid-ask spreads calculated from high-

[4]Empirical papers on the effect of capital controls on capital flows and other macroeconomic variables include: Magud, Reinhart and Rogoff (2011), Forbes and Warnock (2012), Forbes, Fratzscher, Kostka et al (2012), Olaberría (2012), Pasricha (2012), Bluedorn et al (2013), Contessi et al (2013) and Giordani et al (2014).

[5]An exception is the index proposed by Pasricha et al (2015)

frequency data. The estimation method is a fixed-effects dynamic panel of 20 countries with the liquidity measures as dependent variables and the indices of capital controls as one of the explanatory variables. The empirical results show that capital controls have a negative and significant effect on the cost-based measure used in the analysis. Such results are in line with the outcome of theoretical models that introduce capital controls as transaction costs. Empirical evidence on the effects of capital controls on market depth could not be verified because price impact measures require access to high-frequency data and this information was not readily available.[6]

The results from the empirical and theoretical models stress the link between capital flow management policies and FX market liquidity. In particular, the results highlight the importance of the particular indicator used, since each measure characterises a different dimension of liquidity. Capital controls are effective in reducing the implicit cost component of liquidity but can also have a negative effect on the structure of the FX market by making it more vulnerable to order flow imbalances. This means that policymakers who are considering imposing capital controls should not only consider their direct effect on flows but also the adverse and unintended effects they can have on market liquidity. The paper is structured as follows: section 2 presents the theoretical entry model with inventory risk; section 3 presents the empirical model on the effects of capital controls on cost-based measures of liquidity; and section 4 concludes.

2 Entry model with inventory risk

The model is based on Vayanos and Wang (2012). There are three periods, $t = 0, 1, 2$ and two segmented financial markets $j \in A, B$. In each financial market a riskless and a risky asset are traded, both paying off in period 2 in terms of a consumption good. In each individual market, both assets are in the same fixed supply. The riskless asset pays off one unit of the consumption good with certainty and the risky asset pays off V units, where V has a normal distribution with mean μ and variance σ^2. Taking the riskless asset as the numeraire, the price of the risky asset in period t is P_t, where $P_2 = v$.

There are M traders who in period 0 choose first whether to enter one or both of the markets. Traders who decided to enter one market then decide which of the two markets to enter. In terms of the FX market, we can think

[6]See for example Mancini et al (2013).

about it as two counties where the currencies have similar intrinsic value but trade at a different exchange rate. Intuitively, a disconnect occurs because economies with identical fundamentals feature different equilibrium exchange rates depending on the incentives of the financiers to hold the resulting global imbalances (Gabaix and Maggiori (2014)). We will denote as N the number of traders who entered both markets and as N_j the traders who only enter market $j \in \{A, B\}$. In each market traders receive an endowment of the per capita supply of the risky asset in that market (e_{0j}) and an endowment of the total per capita supply of the riskless asset.[7] The difference between each market is that in market B there are capital controls. Capital controls will enter the model in two ways. First, as a fixed cost of entry to market B, which needs to be paid in period 0. This fixed cost can represent quantitative limits on the amount of trades that can be performed, or the perceived cost of future imposition of controls to outflows. The reason why capital controls are modelled first as a fixed cost is to abstract on any direct effect that the controls might have on flows. In this case, we are assuming that controls influence the decision of traders on whether to enter the market or not, but not the amount they trade in each market. The second way capital controls are modelled follows the more traditional approach of assuming they can be analysed as per-unit transaction costs that have to be paid by dealers wanting to sell the asset. In this case, the interpretation is that capital controls represent a tax on financial operations that affect the amount that the dealers trade in each market. Since the model is solved using backwards induction, the effect of controls is analysed in the next section.

Wealth of agents type i in market j during period t will be denoted as W^i_{tj}. Where i denotes whether the agent trades in both markets, trades only in A or trades only in B. Wealth in period 2 is equal to consumption:

$$C^i_2 = W^i_{2A} + W^i_{2B} \tag{1}$$

In period 1, traders maximize the expected utility of their wealth in each market in period 2 (W^i_{2j}). The assumption that traders do not maximize the expected utility of their total wealth is to emphasise that markets are segmented. If this was not the case, traders would trade until the price in both markets were the same. With this assumption it is possible to obtain different prices in each market in equilibrium. Another justification of this assumption is that the implementation of capital controls in one of the markets prevents dealers from perfectly adjusting their holdings of the asset between markets,

[7]The riskless asset can be thought of as a vehicle currency, like the US dollar.

then the best they can do is to maximize their wealth in each market.

Traders are risk averse and preferences can be represented by a *CARA* utility function with common risk aversion coefficient ρ. Given that the return of the asset has a normal distribution, the utility function can be expressed as a mean-variance function:

$$E[U(W_{2j}^i)] = E[W_{2j}^i] - \frac{\rho}{2} Var[W_{2j}^i] \tag{2}$$

We introduce heterogeneity by assuming that with probability π traders who entered both markets are subject to a liquidity shock in period 0. This shock forces them to sell from both markets a total value of L of the risky asset. Since these traders exchange the risky asset for liquidity we will denote them as liquidity demanders. On the other hand, agents who only trade in one market do not face the liquidity shock and instead provide liquidity. Hence we will denote them as liquidity suppliers. Traders who enter both markets can be thought of as large hedge funds, who allocate funds based on the market characteristics rather than on specific characteristics of each country. This type of traders are more prone to change their investment strategy when there are changes in the global environment. That means that to invest in other markets, they require liquidity obtained by exiting some other market.

The trading mechanism through which liquidity suppliers and demanders trade works through an auctioneer in each market who receives the desired net asset holdings of all dealers. The auctioneer then determines the equilibrium price that clears the market. The desired asset holdings that the traders send to the auctioneer can take two forms. The first type are *price schedules*, which are a function of the price and specify a quantity depending on the clearing price in each market. The second type are *market orders*, which are fixed quantities independent of the price. Whether traders decide to send price schedules or market orders to the auctioneer in each market is determined by the functional form of their equilibrium net asset holdings.

2.1 Liquidity suppliers' problem

Traders behave competitively and the market is organized as a call auction. In period 1, liquidity suppliers choose net asset holdings s_{1j} of the risky asset in market j to maximize their expected wealth in each market:

$$W_{2j}^s = W_{1j}^s + P_{1j}s_{1j} + V[e_{0j} - s_{1j}] \tag{3}$$

where e_{j0} is the endowment of the risky asset the agent received in market j in period 0.

Liquidity suppliers solve the following problem:

$$\max_{s_{1j}} \quad \sum_{j\in\{A,B\}} W_{1j}^{s} + P_{1j}s_{1j} + \mu[e_{0j} - s_{1j}] - \frac{\rho\sigma^2}{2}[e_{0j} - s_{1j}]^2 \tag{4}$$

It is not necessary to distinguish between traders that only have holdings of the asset in one market or traders who have assets in both markets but are not subject to the liquidity constraint since the utility specification is separable in wealth of each market. Suppliers who trade only in one market would have a wealth of zero in the other market. Taking first order conditions, the optimal price schedule is:

$$s_{1j}^{*} = \frac{P_{1j} - \mu}{\rho\sigma^2} + e_{0j} \tag{5}$$

Traders want to sell their period 0 endowment of the asset in the first period because its return involves risk. To compensate for the risk, the price has to be lower than the risk adjusted mean of the return. If s_{1j} is negative, it indicates that the trader is buying the asset and supplying liquidity. To persuade the trader to buy an amount of the asset even greater than his initial endowment, the price has to be low enough not only to absorb the risk from his endowment but it also needs to include a risk premium (a discount) from the extra liquidity he is providing.

2.2 Liquidity demanders' problem

A liquidity demander's objective function is the same as the one for liquidity suppliers; however, their problem includes two constraints. The first constraint is the *liquidity constraint*, which forces traders to reduce their holdings of the asset in both markets to exchange liquidity with value L. This constraint represents any type of situation where investors need to liquidate assets in some markets either for liquidity needs, credit shocks, or reversal of capital flows. The second constraint is the *no-short constraint*, which indicates that traders cannot take a short position in either market. This constraint is necessary to prevent that traders engage in arbitrage to satisfy the liquidity constraint. Otherwise they could take a short position in the market with the higher price to avoid having to exit the market with the lower price. This constraint, in addition to the utility specification, is what keeps market segmented and allows that in equilibrium the same asset is traded at different prices.

Liquidity demanders choose net asset holdings m_{1j} of the risky asset in market j to solve the following problem:

$$\begin{aligned} \max_{m_{1j}} \quad & \sum_{j \in \{A,B\}} W_{1j}^D + P_{1j} m_{1j} + \mu[e_{0j} - m_{1j}] - \frac{\rho\sigma^2}{2}[e_{0j} - m_{1j}]^2 \\ \text{subject to} \quad & P_{1A} m_{1A} + P_{1B} m_{1B} \geq L \\ & m_{1j} \leq e_{1j}, \; j \in \{A,B\}. \end{aligned} \tag{6}$$

Taking first order conditions, the optimal net asset holdings of the liquidity demander is:

$$m_{1j}^* = \frac{(1+\lambda)P_{1j} - \mu - \eta_j}{\rho\sigma^2} + e_{0j} \tag{7}$$

where λ is the Lagrange multiplier associated with the liquidity constraint and η_j is the Lagrange multiplier associated with the no-short constraint in market j.

When the liquidity constraint is binding, the optimal net asset holdings of the liquidity demanders and suppliers have different characteristics. Unlike liquidity suppliers, who send price schedules to the market-clearing auctioneer, liquidity demanders send market orders, that is, quantities that are not a function of the market clearing price. The difference between net asset holdings of the two traders arises from the Bertrand competition nature of the market clearing mechanism. Traders who need to sell the asset would want to sell in the market with the higher price. By sending a price schedule they would be dragging the price down as they compete against themselves to obtain the best price.

Lemma 2.1. *The liquidity constraint is binding and liquidity demanders send market orders to the market-clearing auctioneers.*

The proof of the lemma can be found in the appendix, but the result is intuitive. If the liquidity constraint is not binding, a liquidity demander's desired net asset holdings would be the same as the liquidity supplier. Net asset holdings also represent the excess supply of each of the traders, which in equilibrium have to add up to zero. All traders want to sell their initial endowment of the risky asset, which implies that the equilibrium price will compensate them just enough to cover the risk from holding that endowment.

In equilibrium, the individual net asset holdings of any trader would be zero and there would be no trade in equilibrium. Therefore, the liquidity constraint would not be satisfied and this would lead to a contradiction.

The distinction that liquidity demanders send market orders instead of price schedules is crucial in determining the depth of the market. Liquidity suppliers will absorb the excess supply of the asset put forth by liquidity demanders. Then, how the equilibrium price responds to the excess supply of market orders characterizes market liquidity.

2.3 The liquidity supply and measures of FX market liquidity

An auctioneer in each market ($j \in \{A, B\}$) receives the price schedules (s_{i1j}) of the $(1-\pi)N + N_j$ liquidity suppliers and parcels out the market orders (m_{k1j}) of the πN liquidity demanders at the price that clears the market. To clear the market, the net asset holdings of all traders have to be equal to zero:

$$\sum_{i=1}^{(1-\pi)N+N_j} s_{i1j} + \sum_{k=1}^{\pi N} m_{k1j} = 0 \tag{8}$$

Since only the price schedules of liquidity suppliers depend on the price, we can substitute their equilibrium net asset holdings in the market clearing condition to obtain an equation that determines the market's supply of liquidity. Then, by solving for the price as a function of the market orders and the initial endowment we obtain the inverse liquidity supply:[8]

$$P_{1j} = \mu - \rho\sigma^2 e_{0j} - \frac{\rho\sigma^2 \pi N}{(1-\pi)N + N_j} m_{1j} \tag{9}$$

To better understand the inverse liquidity supply, we can decompose it in two parts. The first two terms represent the equilibrium *midquote*. The midquote reflects the asset's expected fundamental value (μ) and the inventory risk adjustment ($\rho\sigma^2 e_{0j}$). At this price the liquidity suppliers are willing to hold precisely their initial endowment. If the market orders (excess supply) of the asset m_{1j} is positive, the price includes a discount to compensate liquidity suppliers from adding more stock to their initial holdings of the risky asset. The equation for the liquidity supply allows us to define two different measures of liquidity:

[8] There is no heterogeneity between liquidity demanders, so we can factor out their number when adding their market orders.

- **Market depth** (Δ_j) is defined as the sensitivity of the price to the market orders of liquidity demanders. In this case, market depth is the inverse of the coefficient that multiplies the market orders in the inverse liquidity supply:

$$\Delta_j = \frac{(1-\pi)N + N_j}{\rho\sigma^2\pi N} \tag{10}$$

- The **effective spread** measures implicit trading cost by using the prices paid by dealers and is gauged by the difference between the midquote price and the price at which the order is executed:

$$S_j = P_{1j} - \left(\mu - \rho\sigma^2 e_{0j}\right) \tag{11}$$

Both measures depend on the same parameter values but they describe different dimensions of FX market liquidity. An increase in market depth will reduce how prices react to order flow imbalances, which would also reduce the effective spread. However, there can be transaction costs charged to the final price that would affect the spread but would not change market depth.

Market liquidity as characterized by these two measures is decreasing in the risk aversion coefficient of the traders (ρ), the variance of the return of the risky asset (σ^2), the probability of the liquidity shock (π) and the number of traders who hold stock of the asset in both markets (N). These relationships are consistent with the literature of market microstructure (Kyle (1985), Pagano (1989)). However, there is a main difference in the economic interpretation of the measures of liquidity in Kyle (1985) and in our model. The price impact that arises from the order flow in the model by Kyle results from the strategic decision of the informed trader to disguise his private signal and from the attempt of risk-neutral liquidity providers to anticipate the behaviour of the informed trader. In our model, the resulting difference in market liquidity arises from the disconnect that can occur because markets with identical fundamental feature different equilibrium prices depending on the incentives of liquidity suppliers to hold the resulting order flow imbalances. The more risk averse the traders and the riskier the stock, implies a greater risk premium included in the price of the asset, reducing market liquidity. Moreover, an increase in the probability of a trader being subject to the liquidity shock and a larger number of traders who hold stock of the asset in both assets increases the relative number of liquidity demanders to liquidity suppliers. Each supplier would have to take on more of the market orders of the liquidity demanders, which involves a higher discount on the price of the asset, reducing market liquidity.

2.4 Equilibrium market orders and price

To solve for the equilibrium market orders we substitute the inverse liquidity supply into the optimal net asset holdings of liquidity demanders. From Lemma 2.1, the liquidity constraint is always binding and assuming, without loss of generality, that the no-short constraint is not binding, i.e. the Lagrange multiplier associated with the constraint is zero, $\eta_j = 0$ for $j \in A, B$), we obtain a closed form solution for the market orders as a function of market depth.[9]

Proposition 2.1. In response to a liquidity shock that forces traders to sell a total value of (L) from their stock of the risky asset in two segmented markets, liquidity demanders sell proportionally more assets in the deeper market:

$$m_{1j}^{*} = \frac{\lambda}{1 + \dfrac{(1+\lambda)}{\Delta_j}} (\mu - \rho\sigma^2 e_{0j}) \tag{12}$$

where Δ_j is the depth of market j, μ the mean return of the asset, λ is the Lagrange multiplier associated with the liquidity constraint and e_{0j} the initial endowment in market j.

Traders that require immediate liquidity are forced to trade the asset at a discount. Therefore, they will start to withdraw from the market where their trades will have a smaller impact, that is, the deeper market. The assumptions of the separability in wealth of the utility function and the no-short constraint, segment the markets, which allows for the price of an asset with the same stochastic return to be different in each market. The following result characterizes prices in each market with respect to their midquote price:

Proposition 2.2. The price discount from the midquote as a result of the liquidity shock, is lower in the deeper market:

$$P_{1j}^{*} = \frac{1}{1 + \dfrac{\pi N \lambda}{N_j + N}} (\mu - \rho\sigma^2 e_{0j}) \tag{13}$$

[9]The importance of the no-short constraint is that it prevents arbitrage, but for a large enough initial endowment we can assume that it is not binding. The complete proof can be found in the appendix.

The difference in market depth between markets is completely determined by the number of traders who have a stock of the asset in only one market (N_j). As their number increases, the depth of the market is greater and the discount at which the price is traded compared to the midquote is smaller. In equilibrium, the price in both markets adjust to incorporate the extra demand for liquidity. Even though more funds are withdrawn from the deeper market, the price is lower in the shallow market. Why wouldn't traders withdraw funds from both markets until the price in both of them are the same? The answer lies on the assumptions on market segmentation. The utility specification of traders implies that they perceive risk independently from each market. When deciding on which market to sell, they not only take into account the discount at which they are selling the asset, but also that they will be reducing their risky inventory of the asset. That is, by selling the asset in the deeper market there is a lower discount, but by selling in the shallow market they are getting rid of some risk. The result would be different if, instead, traders maximized the expected utility of their total wealth. In this case, the effect on the relative position in each market would not be present and the price in both markets would be the same.

With the equilibrium price and net asset holdings of liquidity suppliers and demanders we can calculate their expected wealth. Going back one period, traders consider the advantages and disadvantages of entering either both markets or only one. With this setup we can introduce capital controls in two different way to analyse how they affect the two measures of liquidity. In the next section, we consider the case were capital controls are assumed to be a fixed cost that deters the entrance of potential investors to the economy, which affects market depth. Then, we consider the case were capital controls can be modelled as an explicit trading cost, which will have a direct effect on the order flows and hence on the effective spread.

2.5 Effects of capital controls on market depth

In period 0 there are M traders choosing to enter both markets or only one market. If the trader enters both markets, he receives an endowment of the risky asset in each market. If the trader decides to enter only one of them, he then decides which of the two markets to enter ($j \in \{A, B\}$), receiving an endowment of the risky asset only in the market he entered. The supply of the risky asset is the same in both markets (e) and is divided equally among all the traders that entered that market. Then, if N traders entered both markets and N_j traders only entered market j, each receives an endowment of the risky asset of: $e_{0j} = \frac{e}{N+N_j}$.

As mentioned before, the return of the risky asset is the same in both markets. The difference between markets is that in one of them (from now on market B) there are capital controls. Capital controls will be modelled as a fixed cost (C) of entering market B. In this section, capital controls will not have a direct effect on flows into the country, rather, they will affect the decision of traders on whether to enter the market or not, which will affect how the endowment of the asset is distributed among the remaining traders. In equilibrium, a trader must be indifferent between choosing to enter both markets, entering market A or entering market B. That is, the equilibrium N and N_j, $j \in \{A, B\}$ must satisfy:

$$\begin{aligned}
&E[U(W_{2A}^s)] = E[U(W_{2B}^s)] - C \\
&= \pi\left(E[U(W_{2A}^s)] + E[U(W_{2B}^s)] - C\right) + (1-\pi)\left(E[U(W_{2A}^d)] + E[U(W_{2B}^d)] - C\right)
\end{aligned}$$

where W_{2j}^i is the period 2 wealth of the trader in market $j \in \{A, B\}$ and the superscript s denotes a liquidity supplier and d a liquidity demander.

The safe asset is also in fixed supply in both markets, but unlike the risky asset, each trader receives a share equal to the total per capita supply of the asset. This means that every trader starts with the same endowment of the safe asset across markets. The benefit from entering both markets is that in each market they will receive an endowment of the risky asset, if they only enter one, they will only receive an endowment in that market. The risk from entering both markets is that with probability π they will be subject to the liquidity shock that will require them to sell some of their assets at a discounted price. By entering only one market, traders get rid of this risk and with certainty form part of the liquidity supply.

The type of traders that enter both markets can be thought of as large hedge or investment funds that allocate their funds based on the overall characteristics of a market (return of the asset) and not on the particular characteristics of each country. This type of traders are more prone to face liquidity needs when wanting to reallocate their investment to other markets after unexpected shocks to the global economy or changes in the tide of flows. In contrast, traders that only enter one market can be thought as smaller investors that prefer to choose one market based on its specific characteristics. An example can be domestic firms that borrow foreign capital to fulfil their investment needs. These traders will always receive benefits from being part of the liquidity supply but at a lower overall return. The fixed cost of entering market B encompasses the indirect effects of capital controls. This fixed cost includes initial costs of entry (controls on inflows) and potential restrictions on exiting the market (controls on outflows). To abstract from any effect that capital controls may have on the initial supply of the asset the fixed cost will not have an effect on the total flow of foreign assets that enter the economy (e), but only in the amount that each trader receives as endowment.

In the literature of market microstructure (Pagano (1989), Foucault, Pagano and Roell (2014)) liquidity bequests liquidity in the sense that traders want to enter deeper markets because prices are less volatile. In contrast, in our model, if we assume a starting point where the midquote is the same across markets, a trader would prefer to enter the shallow market. The reason is that the trader already knows that he will form part of the liquidity supply, hence he will receive higher returns in the market where the asset is sold at a higher discount, that is, the shallow market. However, there is an endowment effect that goes in the opposite direction to the price effect. By entering the market, the trader receives a lower share of the risky asset, which would lower his overall wealth. Without capital controls, these effects will balance out.[10]

Lemma 2.2. *In absence of capital controls, traders who only enter one market will divide equally among markets.*

The implementation of capital controls in one market will deter the entrance of investors to that market. In equilibrium, the magnitude of the controls will balance out the price effect and the endowment effect. As capital controls increase, less traders will enter that market, making it shallower. These traders will receive a larger share of the endowment and trade it at a higher discount, but the returns from these benefits will cancel out by the cost of the controls. Given a positive capital control policy, in equilibrium, there is a positive number of traders in each of the categories.

Proposition 2.3. For a given cost (C) of entering market B, if M is the total number of traders:

$$0 < N < M \text{ Not all traders enter both markets.}$$

$$0 < N_B < \frac{N}{2} \text{ At least some traders enter the market with controls.}$$

where N is decreasing in L (liquidity shock) and π (probability of a liquidity shock), and N_B is decreasing in the cost of entry C.

[10]The complete proof can be found in the appendix.

The number of investors in each country affects market depth. In this way, the transmission mechanism of capital controls proposed in this model differentiates between the direct price effect and the effect on price sensitivity. The country with a larger number of investors will have a deeper market, where the exchange rate will react less to changes in order flows. This means that if we measure FX liquidity by how sensitive prices are or how deep the market is, then the market with no controls will be more liquid. Investors that enter both markets withdraw more funds from the deeper marker when facing a liquidity shock. However, the price will drop less in the deeper market since it is better suited to absorb the extra demand for liquidity.

2.6 Effects of capital controls on the effective spread

In this section we model capital controls as an explicit cost that liquidity demanders have to pay in order to trade the asset. This interpretation is more in line with the traditional view of the mechanism through which capital controls affect capital flows. In this case capital controls will not have any effect on the number of investors that enter the economy on period 0, but they will have an effect on how liquidity demanders chose to allocate their orders in both markets. We assume that in market B there is per-unit transaction cost C of trading the asset. This cost does not have to be paid by liquidity suppliers, that is, the cost is one-sided. Then, liquidity demanders choose net asset holdings m_{1j} of the risky asset in market j to solve the following problem:

$$\begin{aligned}
\max_{m_{1j}} \quad & \sum_{j\in\{A,B\}} W_{1j}^D + (P_{1j} - C_j)m_{1j} + \mu[e_{0j} - m_{1j}] - \frac{\rho\sigma^2}{2}[e_{0j} - m_{1j}]^2 \\
\text{subject to} \quad & P_{1A}m_{1A} + (P_{1B} - C)m_{1B} \geq L \\
& m_{1j} \leq e_{1j}, \ j \in \{A, B\} \\
& C_A = 0, \quad C_B = C
\end{aligned} \tag{14}$$

To solve for the equilibrium market orders we substitute the inverse liquidity supply into the optimal net asset holdings of liquidity demanders obtained from the first order conditions of the problem above.[11]

[11] Since liquidity suppliers do not pay the transaction cost then the inverse liquidity supply is the same as given by equation (9). The complete proof can be found in the appendix.

Proposition 2.4. In response to a liquidity shock that forces traders to sell a total value of (L) from their stock of the risky asset in two segmented markets, one in which there are capital controls of C, liquidity demanders sell more assets in the market without capital controls:

$$m_{1j}^{*} = \frac{\lambda(\mu - \rho\sigma^2 e_0) - (1+\lambda)C_j}{1 + \dfrac{(1+\lambda)}{\Delta}} \tag{15}$$

where $C_A = 0$, $C_B = C$, Δ is the depth of market, μ the mean return of the asset, λ is the Lagrange multiplier associated with the liquidity constraint and e_0 the initial endowment.

Note that since there are no entry cost, then by Lemma 2.2 we have the same number of traders and the same initial endowment in both markets. This implies that market depth is the same in both markets. Traders that require immediate liquidity are not only forced to trade the asset at a discount, but also need to pay the transaction cost in market B. Therefore they will start to withdraw from the market without capital controls until the price discount is such that it covers the transaction cost. The assumptions on the separability in wealth of the utility function and the no-short constraint, allows for the price to be different in each market. The following result describes the effective spread in each market.[12]

Proposition 2.5. The effective spread between the transaction price and the price in a hypothetical perfectly liquid market is lower in the market with capital controls:

$$S_{1j} = \frac{-\lambda(\mu - \rho\sigma^2 e_0) + (1+\lambda)C_j}{1 + \Delta + \lambda} \tag{16}$$

where $C_A = 0$, $C_B = C$, Δ is the depth of market, μ the mean return of the asset, λ is the Lagrange multiplier associated with the liquidity constraint and e_0 the initial endowment.

Intuitively, liquidity demanders send less market orders in the market with controls, which means that these orders have to be sold at a lower discount compared to the market without transaction costs. By restricting flows, the effective spread is lower in the market with capital controls.

The theoretical model presented in this section stresses the importance of the indicator used to measure market liquidity and the contrasting effects

[12]The complete proof can be found in the appendix.

that capital controls can have on these measures. If capital controls have the effect of deterring the entrance of potential investors into the market, then this market will have a lower depth and hence lower market liquidity. On the other hand, if capital controls are effective in reducing order flow imbalances and spreads, then market liquidity as characterized by the effective spread will be higher.

3 Empirical evidence of the effects of capital controls on FX liquidity

In this section we present empirical evidence on the effects of capital controls on FX liquidity. Several researchers have innovated in creating different indices of capital controls or of financial and capital account openness. However, there is no consensus on a universal measure and each has its own advantages and disadvantages.[13] There are two main drawbacks from existing measures. First is that most have an annual periodicity and second is that most consider capital controls as a binary variable.[14]

The importance of constructing an index that is intensive and has a higher than annual frequency can be exemplified by considering the case of Brazil. In 2010 Brazil implemented a tax on financial operations (Imposto sobre Operações Financeiras IOF) in response to a surge in foreign flows and risks of currency appreciation (Pereira da Silva and Harris (2012)). A debate initiated on the effectiveness of these controls to curb the flow of capital into Brazil and the possible spillover effects they could have on other countries.[15] During the fourth quarter of 2009 the IOF tax related to non-resident fixed income was set to 2 percent and then it was increased to 6 percent during

[13]Measures of capital account restrictiveness can be found in Quinn and Toyoda (2008), Montiel and Reinhart (1999), Chinn and Ito (2008), Pasricha et al (2015).

[14]One exception is Pasricha et al (2015) who calculate daily changes in regulation affecting capital account transactions. They consider the number of policy measures and have two aggregation methods. One treats each policy homogeneously and the other assigns a weight based on the weight the asset/liability targeted by the policy has on the total financial assets/liabilities of the country.

[15]Forbes, Fratzscher, Kostka and Straub (2012) study the effects of the implementation of Brazil's tax on financial operations on the composition of investors' portfolios using data from the Emerging Portfolio Fund Research database and investors interviews. They find that an increase in the tax reduces portfolio flows, and they argue that the effects of capital controls operate through a change in investors' expectations about future policies, rather than from the cost of controls.

the fourth quarter of 2010. Annual measures reflected an increase in capital controls during the whole year of 2009 and then again during the whole year of 2010. If the dependent variable has a higher periodicity (eg quarterly for capital flows, monthly for portfolio flows, etc) then the conclusions obtained from the annual measures can be deceiving. Moreover, most measures do not distinguish between different intensities of the controls. This implies that they indicate only if the country has in effect controls or not, but they do not differentiate in their magnitude. For example, an increase in the IOF tax from 2 percent to 6 percent would only show as a constant imposition of controls in Brazil through 2009 and 2010, but would not show that the policy was altered and was more restrictive.

We propose a new intensive measure of capital account restrictiveness (IMCAR) that is a *de jure*[16] measure based on information from the International Monetary Fund's Annual Report on Exchange Arrangements and Exchange Restrictions (*AREAER*).[17] The IMCAR index is the standardized first principal component of sub-indices of each of the nine categories in the *AREAER* (Table 1). It was constructed as an intensity measure to account for the change in tightening or loosening of restrictions. For every category, a sub-index is normalized to ten in December 1998.[18] This would represent the initial condition of the restrictiveness of the capital account. Using the information on the monthly changes in regulations provided in the *AREAER*, for each category, the index increases one unit if the restriction was tightened or decreases one unit if the restriction was loosened.[19] The coding rule changes for subtle adjustments in the policies. For example, for the case of the Brazilian IOF tax, the index increased in one unit during the month it was first implemented, but the subsequent increase from 2 to 6 percent accounted for half a unit increase in the index. Moreover, changes in the categories of transactions that were subject to the tax were counted as a change in half a unit in the index. If more transactions were subject to the tax, the index increased by half a unit and if less transactions were subject to the tax the index decreased in half a

[16] *De jure* indices are based on rules or legal restrictions, while *de facto* indices use an instrumental variable or data on capital flows to measure the degree of capital mobility.

[17] The countries considered in the analysis are: Brazil, Chile, China, Colombia, Egypt, Hungary, India, Indonesia, Malaysia, Mexico, Peru, Philippines, Poland, Romania, Russia, Saudi Arabia, South Africa, Thailand, Turkey, and Uruguay.

[18] For the two categories that include subcategories (exchange arrangements and capital transactions) the index is normalized to ten for each of the individual subcategories.

[19] Examples of tightening of controls include: regulations that require capital transactions to be made through authorized banks or exchange houses or an increase in quantitative limits, such as limited ownership or limited amount of transferring.

unit. This coding procedure creates an index that is easy to update since the change in restrictions does not depend on the overall history of controls but only on the change in regulation from the previous month.

Table 1: Categories in IMF's AREAER

Categories	Weights:Brazil
A) Exchange arrangements	0.33
A.1) Exchange rate structure	*0.71*
A.2) Exchange tax	*0.71*
A.3) Exchange subsidy	-
A.4) Forward exchange market	-
B) Arrangements for payments and receipts	-
C) Resident Accounts	0.54
D) Nonresident accounts	-
E) Imports and imports payments	0.05
F) Export and export proceeds	0.17
G) Payments for invisible transactions	0.04
H) Proceeds from invisible transactions	0.02
I) Capital transactions	0.74
I.1) Capital or money market instruments	*0.48*
I.2) Derivatives and other instruments	*0.53*
I.3) Credit operations	*0.27*
I.4) Direct investment	*0.25*
I.5) Liquidation of direct investment	-
I.6) Real estate transactions	-
I.7) Capital movements	*0.21*
I.8) Provisions specific to commercial banks	*0.53*
I.9) Provisions specific to institutional investors	*0.01*
I.10) Other controls imposed by securities laws	-
Total Variance Explained	83.26

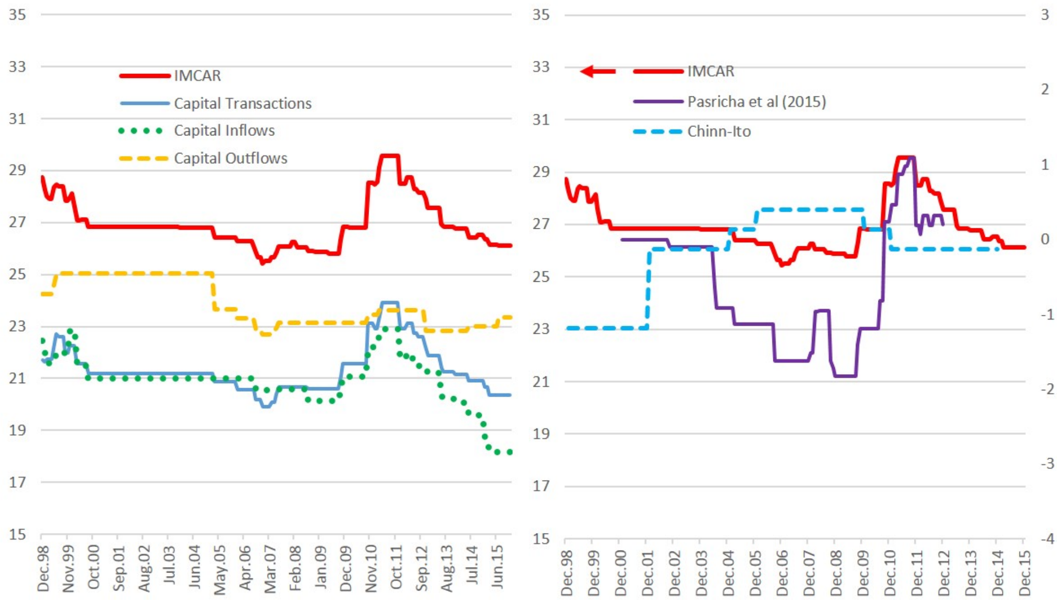
Once the index is constructed for each of the subcategories, we use principal component analysis in two steps to calculate the IMCAR. The initial step is to calculate the first principal component of the categories that are disaggregated into subcategories (exchange arrangements and capital transactions). Then, their individual first principal component is the main index for those two categories. Finally, the IMCAR corresponds to the first principal component of the standardized indices of the nine main categories. The weights obtained from the principal component analysis determine how each of the categories contribute to explaining the variance of the IMCAR. Table 1 shows the example of the IMCAR for Brazil. The numbers correspond to the coefficients in the linear combination of the first principal component.[20] In this example, the sub-index of capital transactions plays a major role since it has the highest weight. Since the principal components were obtained from the standardized variables, the weight (divided by the number of categories) is the correlation between the IMCAR and each of the indices of the categories.

Capital flow management polices also differ on whether they are targeting capital inflows or capital outflows. Controls on capital inflows are policies that restrict flows that increase residents' liabilities to non-residents. Capital controls on outflows are policies that restrict the increase of residents' assets (or claims on non-residents). To establish the different effect that controls on inflows and outflows can have on FX liquidity, we construct two more sub-indices. Each policy change in the subcategories of capital transactions was classified as a policy that affected inflows or outflows. Then, we use the same coding procedure as with the aggregate index to construct individual sub-indices of inflows and outflows for each category. Finally, the sub-indices on controls on capital inflows and controls on capital outflows correspond to the standardized first principal component of each of the subcategories.

The left-hand side of graph 2 shows the IMCAR, the sub-index of controls on capital transactions, the sub-index of controls on capital inflows and the sub-index of controls on capital outflows for Brazil. As can be seen from the graph, the IMCAR for Brasil follows closely the sub-index on capital transactions, however the more general index includes more information and shows greater variance. The right-hand side of graph 2 compares the IMCAR with the capital account openness index of Chinn and Ito (2013), the most commonly used measure of capital controls, and the weighted net inflow tightening index (NIT) of Pasricha et al (2015), the most similar index to the IMCAR in terms of how the changes in policies are incorporated. As can be seen from the graph, both the IMCAR and NIT index capture more comprehensibly the capital controls policies of Brazil. In particular, both indices describe the evolution of the implementation of the Brazilian tax on financial operations during 2009-2013. Three advantages of the IMCAR compared to the NIT index are that it covers a longer period (1998-2015), it considers more countries

[20]The coefficients in the subcategories correspond to the weight in the category index. Some categories were dropped from the index since there was no change in regulation.

(20 emerging economies) and it includes more categories (NIT only includes capital transactions and excludes foreign direct investment).



(a) Subindices of IMCAR

(b) Comparisson with other indices

Figure 1: IMCAR for Brazil

3.1 Cost-based measures of FX Liquidity

The cost-based indicator of FX liquidity used in the analysis was calculated following Karnaukh, Ranaldo and Söderlind (2015). The indicator is a combination of two readily available measures: the bid-ask spread (BA) and an estimator proposed by Corwin and Schultz (2012) (CS). In a perfectly liquid market, transaction cost would be zero and sellers and buyers would transact at the same price, hence the bid-ask spread captures the implicit cost of trading in the market. The midquote, the average between the best bid and ask price, is often used as a proxy of the price in a hypothetical perfectly liquid market. We obtain the monthly average of the bid-ask spread from Bloomberg and the quote corresponds to the closing quote of the market for the currencies analysed.[21] Corwin and Schultz (2012) develop a bid-ask spread

[21] The countries considered are: Brazil, Chile, China, Colombia, Egypt, Hungary, India, Indonesia, Malaysia, Mexico, Peru, Philippines, Poland, Romania, Russia, Saudi Arabia, South Africa, Thailand, Turkey, and Uruguay. For the Mexican peso we use the 5 pm EST

estimator from daily high and low prices. They assume that the high price is buyer-initiated and the low price is seller-initiated. The high-low price ratio then reflects both the true variance of the underlying stock and the bid-ask spread. The variance component of the ratio grows proportionately with time, while the spread component remains the same. Then, using two equations, the first a function of the high-low ratios on two consecutive single days and the second a function of the high-low ratio from a single two-day period, they can solve for both the spread and the variance. To construct the Corwin and Schultz estimator we use data on the high and low prices of the exchange rate from Datastream. Karnaukh, Ranaldo and Söderlind (2015) find that a combination of the Corwin and Schultz estimator and the daily bid-ask quotes has a high correlation with an intraday (high-frequency) indicator of the effective bid-ask spread. The correlation has been attributed to the fact that the indicator aggregates from a larger set of information. We calculate the three FX liquidity indicators for the 20 emerging market economies from December 1998 to December 2015. Graph 2 shows an aggregate across countries of the three indicators. Notice that since the indicators are cots-based measures, an increase represents lower liquidity in the FX market. The three indicators show a similar trend, however each can capture different components of the implicit cost of trading in the FX market.

3.2 Estimation Method

The estimation method is a fixed-effects dynamic panel of 20 countries with the liquidity measure as dependent variables and the indices of capital controls as one of the explanatory variables. The database consists of monthly observations from December 1998 to December 2015 and the equation estimated is the following:

$$Illiq_{i,t} = CC_{i,t-1} + VIX_t + Comm_t + R^*s_t + R^*m_t + R_t + \varepsilon_{i,t} \quad (17)$$

The dependant variable ($Illiq_{i,t}$) corresponds to the three illiquidity measures described in the previous section (BA, CS, KRS).[22] The first independent variable is the index of capital controls ($CC_{i,t-1}$) lagged one month to address endogeneity concerns. The main index of capital controls (IMCAR),

spread since this currency trades 24 hours.

[22]We show the results using the BA and CS measures as robustness checks in the next section.

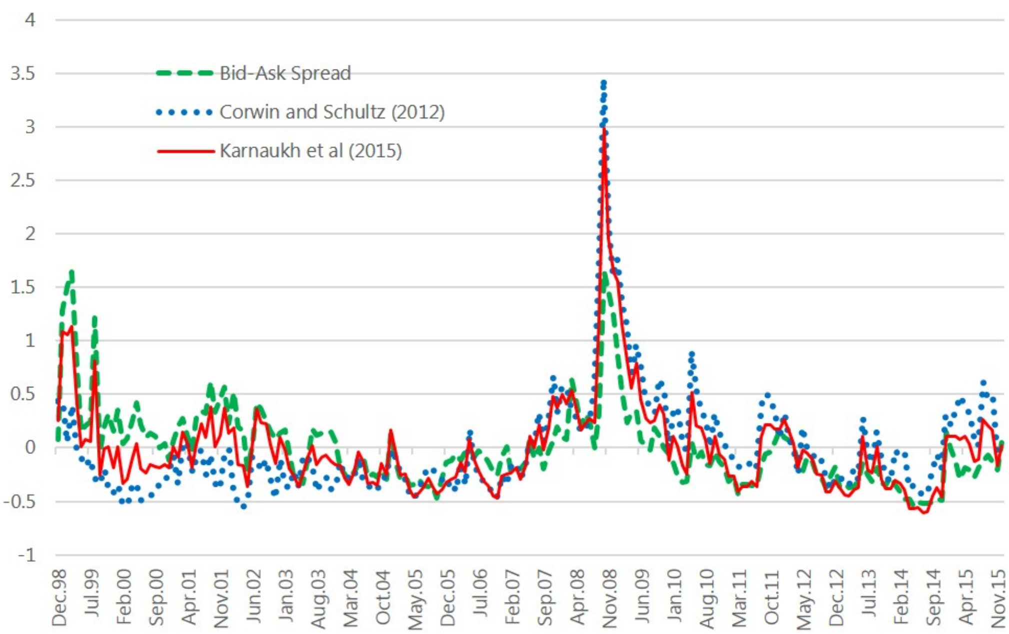


Figure 2: Cost-based measures of FX liquidity

the sub-index of controls on capital transactions (Cap Trans), the sub-index of controls on capital inflows (Cap Inflows) and the sub-index of controls on capital outflows (Cap Outflows) alternate in the specification to determine if each one has a different effect. In the baseline specification I include five additional control variables. To control for risk, global risk is measured by the VIX index calculated by the Chicago Board Options Exchange.[23] The next control variable is an index of commodity prices ($Comm_t$) calculated by the Commodity Research Bureau.[24] The last set of control variables are related to international and domestic interest rates. The Libor overnight interest rate corresponds to the international short-term interest rate ($R^{*}s$), the Libor 6 month interest rate correspond to the international medium-term interest rate ($R^{*}m$), and each country's interbank interest rate corresponds to the domestic interest rate (R).[25] Table 2 present the results of the panel regression. Standard errors are clustered by country and are adjusted for presence of both heteroskedasticity and autocorrelation of unknown form (HAC-Newey-West).

[23]The VIX is a volatility index based on trading of S&P 100 (OEX) options.

[24]The commodity research bureau index spot is an unweighted geometric mean of the individual commodity price relatives, i.e. of the ratios of the current prices to the base period prices. Data are obtained from Bloomberg.

[25]We obtain the data on interest rates from Bloomberg.

Table 2: Panel Regression Results

	KRS	KRS	KRS	KRS
IMCAR	-0.163***			
	(0.049)			
Cap Trans		-0.126**		
		(0.055)		
Cap Inflows			-0.118*	
			(0.061)	
Cap Outflows				-0.103*
				(0.054)
VIX	0.360 ***	0.358***	0.360***	0.357***
	(0.047)	(0.046)	(0.046)	(0.045)
Comm	0.475**	0.493*	0.530**	0.484*
	(0.227)	(0.238)	(0.237)	(0.244)
R*s	-0.291*	-0.307*	-0.309	-0.307**
	(0.147)	(0.144)	(0.144)	(0.142)
R*m	0.328**	0.345**	0.349**	0.342**
	(0.144)	(0.0140)	(0.141)	(0.138)
R	0.257**	0.248**	0.251**	0.249**
	(0.090)	(0.089)	(0.090)	(0.091)
R^2	0.196	0.191	190	0.188
No. Obs	4,045	4,045	4,045	4,045
No. Gps	20	20	20	20

Robust standard errors (clustered by country) in parentheses.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

All regressions include country and time trend fixed effects.

The first result is that all indices of capital controls have a negative and statistically significant effect on illiquidity. This means that an increase in capital controls increases FX liquidity, at least for the dimension captured by this specific measure. Intuitively, an increase in capital controls has the direct effect of restricting flows, which stabilizes the exchange rate and compresses spreads. The results are consistent with the conclusions from the theoretical model on the effect of capital controls on the effective spread. However, given the lack of readily available data to construct a price impact indicator of FX liquidity, we could not verify the effects of capital controls on FX market depth.

The volatility index has a positive and statistically significant effect on FX market illiquidity. The strong relationship between FX Markets and stock, bond, and money markets explain these results. Investors that own assets denominated in foreign currencies are vulnerable to FX illiquidity risk, then in periods of high stress they rebalance their portfolios towards more liquid and less risky securities. Moreover, changes in the structure of the FX market driven by technology, such as the rise of algorithmic and high-frequency trading, can also explain this result. Evidence suggests that algorithmic trading stops during periods of perceived high volatility, which generates episodes of illiquidity in the FX market. As investors perceive more risk, there is a reduction in FX liquidity.

Changes in commodities prices affect FX liquidity through their effect on an economy's exports. The results show that the commodity price index has a positive and statistically significant effect the cost-based measure. As commodity prices increase there is a lower demand for commodities which would result in a reduction in export revenues for commodity exporters. This in turn leads to a lower supply of foreign exchange.[26]

As mentioned previously, FX liquidity is strongly related with movements in bonds and money markets. This relation can also be observed by the results on the impact of international interest rates and domestic interest rates on FX liquidity. With respect to international interest rates, there are two different effects that need to be considered. First, the international short-term interest rate has a negative and statistically significant effect on FX illiquidity. Intuitively, if there is an increase in the international short-term interest

[26]Kohlscheen, Avalos and Schrimpf (2016) show that variations in commodity prices have an effect on nominal exchange rates that goes beyond the impact of global risk appetite. In particular commodity prices explain a significant part of the variation of the exchange rate that is orthogonal to risk.

rate then the cost of borrowing in the international market is higher than in the domestic market, which will lead to higher liquidity in the domestic market. On the other hand, the medium-term international interest rate has a positive and statistically significant effect on FX illiquidity. Medium-term interest rates are associated closely with international capital flows and the search for yield. When there is an increase in the medium-term international interest rate, foreign assets offer a better return than domestic assets, triggering capital outflows from the domestic economy and reducing FX liquidity. This explains the different signs in the coefficients of the international interest rates. Finally, the interpretation of the positive sign of the domestic interest rate on FX illiquidity is the same as with the international short-term interest rate. As domestic rates increase, domestic banks will borrow less domestically, reducing liquidity in the FX market.

3.3 Robustness Check

As a robustness check we estimate the same panel regression as in equation (17), but using the BA and the CS indicators as the measures of FX illiquidity. Table 3 shows that the results from the baseline model remain the same. An increase in capital controls reduces the implicit-cost component of FX liquidity. The only exceptions are the coefficients for the sub-indices of capital controls using the CS indicator, which are not significant. One possible reason can be that the IMCAR considers a broader set of categories that could affect the CS measure and that are not captured by the sub-indices. These results have the same explanation as with the baseline model, by restricting flows, capital controls can have the effect of compressing spreads, which increases FX liquidity. Most of the sign and significance of the rest of the control variables remain the same.

Table 3: Panel Regression Results

	BA	CS	BA	CS	BA	CS	BA	CS
IMCAR	-0.106**	-0.172*						
	(0.038)	(0.087)						
Cap Trans			-0.115**	-0.098				
			(0.044)	(0.088)				
Inflows					-0.118**	-0.091		
					(0.044)	(0.095)		
Outflows							-0.082*	-0.082
							(0.044)	(0.084)
VIX	0.238***	0.391***	0.236***	0.390***	0.237***	0.391***	0.235***	0.389***
	(0.045)	(0.053)	(0.054)	(0.051)	(0.044)	(0.051)	(0.045)	(0.051)
Comm	0.859**	0.341	0.886*	0.344	0.929**	0.372	0.872**	0.337
	(0.323)	(0.391)	(0.330)	(0.399)	(0.325)	(0.392)	(0.327)	(0.402)
R*s	-0.149	-0.311**	-0.156	-0.332**	-0.157	-0.334**	-0.158	-0.332**
	(0.131)	(0.128)	(0.129)	(0.126)	(0.129)	(0.127)	(0.128)	(0.125)
R*m	0.175	0.362***	0.183	0.383***	0.185	0.386***	0.182	0.381**
	(0.132)	(0.122)	(0.130)	(0.119)	(0.131)	(0.119)	(0.129)	(0.117)
R	0.223**	0.162*	0.218*	0.152*	0.221*	0.154*	0.218*	0.152*
	(0.107)	(0.083)	(0.105)	(0.079)	(0.105)	(0.081)	(0.107)	(0.079)
R^2	0.128	0.218	0.130	0.205	0.131	0.204	0.126	0.203
No. Obs	4,045	4,045	4,045	4,045	4,045	4,045	4,045	4,045
No. Gps	20	20	20	20	20	20	20	20

Robust standard errors (clustered by country) in parentheses.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

All regressions include country fixed effects and time trend fixed effects.

As a second robustness exercise we add more global control variables to the model. In particular, we add three more control variables: the price of oil (Oil), the JP Morgan Emerging Market Bond index ($EMBI$) and an index on global production ($Global$).[27] Table 4 present the results using the IMCAR index. The sign and significance of the capital controls index remains the same. However, none of the additional control variables have a statistically significant effect. Regarding the price of oil, by performing individual country regressions, there is a statistically significant effect for those countries that export oil (eg Saudi Arabia, Russia, Mexico and Colombia). The mechanism is the same as with the more general commodity price index. An increase in the price of oil would reduce export revenues in these countries and reduce FX market liquidity. However, in the full sample, there is no statistically significant effect. Regarding the EMBI, which is used as a measure of risk in emerging markets, it could be the case that the VIX is a better variable to capture risk in the model and the EMBI is not statistically significant. Finally, with respect to the global production index, the intuition would be that higher global production would increase capital flows, which would increase FX market liquidity. However, the coefficient is not statistically significant.

The third robustness exercise consist on adding the credit default swap (CDS) spread for each country. However, given data limitations, three countries had to be dropped from the exercise and the number of observations was reduced given the dates for which the CDS spreads were available.[28] The sign and significance of the impact of controls on FX liquidity remains the same as in the baseline specification. The CDS spread has a positive and statistically significant effect for the BA and KRS measures of illiquidity. The effect of an increase in the CDS spread on FX market liquidity has the same interpretation as the effect of the VIX index. As CDS spreads rise, investors perceive that there is more risk associated with that country, which would push investors to rebalance their portfolios towards assets in less risky countries. This would reduce FX liquidity in the country perceived as riskier. By including the CDS spreads, the effects on the international interest rates are no longer significant, which could means that the CDS spread is better able to capture the effect of international investors' decisions.

[27] The price of oil corresponds to the price of the Brent blend. The global production index corresponds to the Global OECD leading economic indicator. All series were obtained from Bloomberg.

[28] India, South Africa and Saudi Arabia were dropped from the panel. The CDS spreads were obtained from Bloomberg.

Table 4: Panel Regression Results

	BA	CS	KRS	BA	CS	KRS
IMCAR	-0.127**	-0.158*	-0.163*	-0.080**	-0.099*	-0.104**
	(0.035)	(0.092)	(0.055)	(0.029)	(0.029)	(0.056)
VIX	0.184***	0.434***	0.360 ***	0.260***	0.466***	0.391***
	(0.058)	(0.070)	(0.063)	(0.045)	(0.073)	(0.068)
Comm	1.132**	0.390	0.629**	0.708**	0.290	0.438**
	(0.356)	(0.484)	(0.259)	(0.327)	(0.415)	(0.182)
R*s	-0.190	-0.263**	-0.299**	0.024	0.027	0.020
	(0.140)	(0.121)	(0.149)	(0.128)	(0.176)	(0.172)
R*m	0.229	0.314**	0.339**	-0.008	0.027	0.020
	(0.134)	(0.115)	(0.142)	(0.129)	(0.167)	(0.0164)
R	0.201*	0.176*	0.255**	0.351***	0.206	0.355***
	(0.114)	(0.088)	(0.091)	(0.107)	(0.166)	(0.068)
Oil	-0.206	-0.239	-0.236			
	(0.82)	(0.173)	(0.144)			
EMBI	0.506	-0.513	-0.002			
	(0.406)	(0.387)	(0.424)			
Global	0.111	-0.230	0.047			
	(0.260)	(0.244)	(0.263)			
CDS				0.305**	0.061	0.349*
				(0.128)	(0.320)	(0.182)
R^2	0.136	0.222	0.198	0.241	0.218	0.302
No. Obs	4,045	4,045	4,045	2,932	2,932	2,932
No. Gps	20	20	20	17	17	17

Robust standard errors (clustered by country) in parentheses.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

All regressions include country fixed effects and time trend fixed effects.

4 Conclusions

Due to the risks relating to volatile waves of capital flows, policymakers have reconsidered the use of capital flow management policies (IMF (2010)). These policies are intended to shield the economy against imbalances generated by episodes of extremely large capital flows, with a view to strengthening the financial system. The economic literature has not come to a consensus on the effectiveness and consequences of capital controls.

This paper contributes to the debate by looking at the effects of capital controls on FX market liquidity. It does so, first, by proposing a theoretical model that analyses the effect of capital controls on two different measures of market liquidity: market depth and effective spread. Second, it introduces a new measure of capital account restrictiveness and estimates the effect of capital controls on cost-based measures of FX liquidity by using a panel of 20 EMEs.

The theoretical model shows that, if the imposition of controls deters the entrance of investors and affects liquidity supply, then these policies can reduce market depth. During a period when traders in risky assets are subject to a liquidity shock, they reduce their stockholdings from deep markets. However, a deep market is able to sustain a large flight of capital with prices reacting less strongly. Related to FX market liquidity, the introduction of controls would increase the price impact of order flow imbalances, that is, it would reduce market liquidity as characterised by market depth. On the other hand, if capital controls are modelled as transaction costs, the model shows that, by reducing excess order flows, there is a reduction in the effective spread. In this case, capital controls would increase market liquidity as characterised by implicit costs. The results highlight the importance of the particular indicator used to measure market liquidity. If capital controls are intended to reduce price sensitivity to the potential hazards they generate in the financial system, the proposed theoretical model indicates that their implementation could be exacerbating the problem they are intended to prevent.

In the second part of the paper, we introduce a new measure of capital controls: an intensive index with monthly periodicity that can account for subtle changes in capital flow management policies. Furthermore, we construct three sub-indices of capital controls: on capital transactions, on capital inflows and on capital outflows. These sub-indices, and additional control variables, are used to analyse the determinants of three implicit cost measures of FX market illiquidity for a panel of 20 EMEs. The results show that capital controls have a negative and statistically significant effect on cost-based measured of market illiquidity. This means that capital controls are effective in reducing the implicit cost component of liquidity. However, we could not obtain evidence of the indirect mechanism through which capital controls affect market depth. Since cost-based and price-impact measures are not perfectly correlated, a reduction in spreads does not necessarily imply an increase in depth. Future research would involve the calculation of price-impact measures to assess the effect of capital controls on the market depth dimension of liquidity.

The results from the two models complement each other. They imply that, in the short run, capital controls would be effective in increasing FX market liquidity by reducing spreads. However, the theoretical model predicts that economies that previously introduced controls will observe greater exchange rate volatility owing to reduced market depth. In countries with balance sheet effects and debt denominated in foreign currency, sharp exchange rate depreciations can be detrimental to the stability of the financial system. If the objective is to have a stronger financial system and lower price sensitivity, the unintended adverse second round effects of capital controls indicate that they might not be the right policy choice.

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Appendix A Proofs of Lemmas and Propositions

Lemma 2.1 *The liquidity supply is always binding and liquidity demanders send market orders to the market-clearing auctioneers.*

Proof. The liquidity demander's problem is the following:

$$\begin{aligned} \max_{m_{1j}} \quad & \sum_{j\in\{A,B\}} W_{1j}^D + P_{1j}m_{1j} + \mu[e_{0j} - m_{1j}] - \frac{\rho\sigma^2}{2}[e_{0j} - m_{1j}]^2 \\ \text{subject to} \quad & P_{1A}m_{1A} + P_{1B}m_{1B} \geq L \\ & m_{1j} \leq e_{1j},\ j \in \{A, B\}. \end{aligned}$$

The associated Lagrangian is:
$\mathcal{L}(m_{1A}, m_{1B}, \lambda, \eta A, \eta_B) = \sum_{j\in\{A,B\}} W_{1j}^D + P_{1j}m_{1j} + \mu[e_{0j} - m_{1j}] - \frac{\rho\sigma^2}{2}[e_{0j} - m_{1j}]^2 - \lambda\left(L - P_{1A}m_{1A} - P_{1B}m_{1B}\right) - \sum_{j\in\{A,B\}} \eta_j\left(m_{1j} - e_{1j}\right)$

and the first order conditions are:

$$P_{1A} - \mu + \rho\sigma^2[e_{0A} - m_{1A}] + \lambda P_{1A} - \eta_A = 0 \quad (18)$$
$$P_{1B} - \mu + \rho\sigma^2[e_{0B} - m_{1B}] + \lambda P_{1B} - \eta_B = 0 \quad (19)$$
$$L - P_{1A}m_{1A} - P_{1B}m_{1B} = 0 \quad (20)$$
$$m_{1A} - e_{1A} = 0 \quad (21)$$
$$m_{1B} - e_{1B} = 0 \quad (22)$$
$$\lambda\left(L - P_{1A}m_{1A} - P_{1B}m_{1B}\right) \quad \lambda \geq 0 \quad (23)$$
$$\eta_A\left(m_{1A} - e_{1A}\right) \quad \eta_A \geq 0 \quad (24)$$
$$\eta_B\left(m_{1B} - e_{1B}\right) \quad \eta_B \geq 0 \quad (25)$$
$$(26)$$

If the liquidity constraint is not binding, and without loss of generality we assume that the no-short constraint is not binding either, then we have that the optimal net asset holdings of liquidity demanders are:

$$m_{1j}^* = \frac{P_{1j} - \mu}{\rho\sigma^2} + e_{0j}$$

To clear the market, the sum of the optimal net asset holdings of liquidity suppliers and demanders have to add up to 0:

$$\sum_{i=1}^{(1-\pi)N+N_j} s_{i1j} + \sum_{k=1}^{\pi N} m_{k1j} = 0$$

$$\sum_{i=1}^{(1-\pi)N+N_j} \left(\frac{P_{1j}-\mu}{\rho\sigma^2} + e_{0j}\right) + \sum_{k=1}^{\pi N} \left(\frac{P_{1j}-\mu}{\rho\sigma^2} + e_{0j}\right) = 0$$

$$(N+N_j)\frac{P_{1j}-\mu}{\rho\sigma^2} + e_{0j} = 0$$

$$P_{1j} = \mu - \rho\sigma^2 e_{0j}$$

Substituting the equilibrium price into the net asset holdings of liquidity demanders we obtain $m_{1j} = 0$

$$m_{1j}^* = \frac{P_{1j}-\mu}{\rho\sigma^2} + e_{0j} = \frac{\mu - \rho\sigma^2 e_{0j} - \mu}{\rho\sigma^2} + e_{0j} = 0$$

but this clearly violates the liquidity constraint. Hence the liquidity supply is binding.

□

Proposition 2.1 In response to a liquidity shock (L) liquidity demanders sell more assets in deeper markets:

$$m_j^* = \frac{\lambda}{1 + \dfrac{(1+\lambda)}{\Delta_j}}(\mu - e_j)$$

where Δ_j is the depth of market j, μ the mean return of the asset, λ is the Lagrange multiplier associated with the liquidity constraint and e_j the initial endowment in market j.

Proof. Using Lemma 2.1 $\lambda > 0$, and liquidity demanders send market orders to the auctioneer. This means we can treat the net asset holdings of liquidity demanders as constants in the market clearing condition to solve for the liquidity supply of the market:

$$\sum_{i=1}^{(1-\pi)N+N_j} s_{i1j} + \sum_{k=1}^{\pi N} m_{k1j} = 0$$

$$((1-\pi)N + N_j)\left(\frac{P_{1j} - \mu}{\rho\sigma^2} + e_{0j}\right) + (\pi N)m_{1j} = 0$$

$$P_{1j} = \mu - \rho\sigma^2 e_{0j} - \frac{\rho\sigma^2 \pi N}{(1-\pi)N + N_j} m_{1j}$$

is the inverse liquidity supply of the market, then if we define market depth as:

$$\Delta_j = \frac{(1-\pi)N + N_j}{\rho\sigma^2 \pi N}$$

We can substitute the optimal price schedule into the first order condition of liquidity demanders:

$$m^*_{1j} = \frac{(1+\lambda)P_{1j} - \mu}{\rho\sigma^2} + e_{0j} =$$

$$\frac{(1+\lambda)\left(\mu - \rho\sigma^2 e_{0j} - \frac{\rho\sigma^2\pi N}{(1-\pi)N+N_j} m_{1j}\right) - \mu}{\rho\sigma^2} + e_{0j} \implies$$

$$m^*_{1j} = \frac{\lambda}{1 + \dfrac{(1+\lambda)}{\Delta_j}}(\mu - \rho\sigma^2 e_{0j})$$

□

Proposition 2.2 The price discount from the midquote as a result of the liquidity shock, is lower in the deeper market:

$$P^*_{1j} = \frac{1}{1 + \dfrac{\pi N \lambda}{N_j + N}}(\mu - \rho\sigma^2 e_{0j})$$

Proof. Substituting the result from Proposition 2.1 and the definition of market depth into the inverse liquidity supply we obtain the result:

$$P_{1j} = \mu - \rho\sigma^2 e_{0j} - \frac{\rho\sigma^2 \pi N}{(1-\pi)N + N_j} m_{1j} =$$

$$\mu - \rho\sigma^2 e_{0j} - \frac{\rho\sigma^2 \pi N}{(1-\pi)N + N_j} \left(\frac{\lambda}{1 + \frac{(1+\lambda)}{\frac{(1-\pi)N+N_j}{\rho\sigma^2\pi N}}} (\mu - \rho\sigma^2 e_{0j}) \right) =$$

$$\frac{1}{1 + \frac{\pi N \lambda}{N_j + N}} (\mu - \rho\sigma^2 e_{0j})$$

□

Lemma 2.2 In absence of capital controls, traders who only enter one market will divide equally among markets.

Proof. In absence of controls the equilibrium condition requires:

$$E[U(W_{2A}^s)] = E[U(W_{2B}^s)] \iff$$

$$W_{1A}^s + P_{1A}s_{1A} + \mu[e_{0A} - s_{1A}] - \frac{\rho\sigma^2}{2}[e_{0A} - s_{1A}]^2 =$$

$$W_{1B}^s + P_{1B}s_{1B} + \mu[e_{0A} - s_{1B}] - \frac{\rho\sigma^2}{2}[e_{0B} - s_{1B}]^2 \iff$$

$$P_{1A}\left(\frac{P_{1A}-\mu}{\rho\sigma^2} + e_{0A}\right) + \mu\left[-\frac{P_{1A}-\mu}{\rho\sigma^2}\right] - \frac{\rho\sigma^2}{2}\left[-\frac{P_{1A}-\mu}{\rho\sigma^2}\right]^2 =$$

$$P_{1B}\left(\frac{P_{1B}-\mu}{\rho\sigma^2} + e_{0B}\right) + \mu\left[-\frac{P_{1B}-\mu}{\rho\sigma^2}\right] - \frac{\rho\sigma^2}{2}\left[-\frac{P_{1B}-\mu}{\rho\sigma^2}\right]^2 \iff$$

$$P_{1A} = P_{1B} \iff$$

$$\frac{1}{1 + \frac{\pi N \lambda}{N_A + N}}(\mu - \rho\sigma^2 \frac{e}{N_A + N}) = \frac{1}{1 + \frac{\pi N \lambda}{N_B + N}}(\mu - \rho\sigma^2 \frac{e}{N_B + N}) \iff$$

$$N_A = N_B$$

Where in the third line their initial wealth in period 1 is the same because the riskless asset is divided equally among all traders independent on the market they choose. □

Proposition 2.3 For a given cost (C) of entering market B, if M is the total number of traders in the market:

$0 < N < M$ Not all traders enter both markets.

$0 < N_B < \frac{N}{2}$ At least some traders enter the market with controls.

where N is decreasing in L (liquidity shock) and π (probability of a liquidity shock), and N_B is decreasing in the cost of entry C.

Proof. Assume that all traders enter both markets, then there would be no difference in the price in both markets. A trader could obtain higher profits by only entering market A, not paying the fixed cost of entry to market B and receiving positive profits with certainty in that market. Therefore not all traders enter both markets. Now assume that there is a fixed number of traders who enter both markets (N), then to show that there is a lower number of traders we can follow the inequalities from lemma 3.1 to obtain that $N_A > N_B$, where the fixed cost C counteracts the benefits the traders receive by selling the asset at a higher discount in the shallow market. □

Proposition 2.4 In response to a liquidity shock that forces traders to sell a total value of (L) from their stock of the risky asset in two segmented markets, one in which there are transaction costs of C, liquidity demanders sell more assets in the market without capital controls:

$$m_{1j}^* = \frac{\lambda(\mu - \rho\sigma^2 e_0) - (1+\lambda)C_j}{1 + \frac{(1+\lambda)}{\Delta}} \tag{27}$$

where $C_A = 0$, $C_B = C$, Δ is the depth of market, μ the mean return of the asset, λ is the Lagrange multiplier associated with the liquidity constraint and e_0 the initial endowment.

Proof. From the first order conditions of the liquidity demander's problem we obtain:

$$m_{1j} = \frac{(1+\lambda)(P_{1j} - C_j) - \mu - \eta_j}{\rho\sigma^2} + e_{0j}$$

From Lemma 2.1, the liquidity constraint is binding ($\lambda > 0$) and we assume that the no-short constraint is not binding ($\eta_j = 0$) Since the transaction cost only has to be paid by liquidity demanders then the liquidity supply is the same as in equation (9). We substitute the inverse liquidity supply into the equation obtained from the first order conditions. Also note that since there are no entry costs then market depth and the initial endowments are the same in both markets:

$$m_{1j}^* = \frac{(1+\lambda)\left(\mu - \rho\sigma^2 e_0 - \dfrac{1}{\Delta}m_{1j}\right) - \mu - (1+\lambda)C_j}{\rho\sigma^2} + e_0 \implies$$

$$m_{1j}^* = \frac{\lambda(\mu - \rho\sigma^2 e_0) - (1+\lambda)C_j}{1 + \dfrac{(1+\lambda)}{\Delta}}$$

Then, since $C_A = 0$ and $C_B = C$:

$$m_{1A}^* = \frac{\lambda(\mu - \rho\sigma^2 e_0)}{1 + \dfrac{(1+\lambda)}{\Delta}} > \frac{\lambda(\mu - \rho\sigma^2 e_0) - (1+\lambda)C}{1 + \dfrac{(1+\lambda)}{\Delta}} = m_{1B}^*$$

Then liquidity demanders sell less assets in the market with capital controls.

□

Proposition 2.5 The effective spread between the transaction price and the price in a hypothetical perfectly liquid market is lower in the market with capital controls:

$$S_{1j} = \frac{-\lambda(\mu - \rho\sigma^2 e_0) + (1+\lambda)C_j}{1 + \Delta + \lambda} \tag{28}$$

where $C_A = 0$, $C_B = C$, Δ is the depth of market, μ the mean return of the asset, λ is the Lagrange multiplier associated with the liquidity constraint and e_0 the initial endowment.

Proof. To obtain the transaction price we substitute the optimal market orders obtained from proposition 2.4 into the inverse liquidity supply in equation (9).

$$P_{1j} = \mu - \rho\sigma^2 e_0 - \frac{1}{\Delta}m_{1j} =$$

$$\mu - \rho\sigma^2 e_{0j} - \frac{\lambda(\mu - \rho\sigma^2 e_0) - (1+\lambda)C_j}{1 + \Delta + \lambda}$$

Then if we define the effective spread as the difference between the transaction price and the midquote we obtain:

$$|S_{1A}| = \frac{\lambda(\mu - \rho\sigma^2 e_0)}{1 + \Delta + \lambda} > \frac{\lambda(\mu - \rho\sigma^2 e_0) - (1+\lambda)C}{1 + \Delta + \lambda} = |S_{1B}|$$

Hence in the market without controls the effective spread is higher. □

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