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The Discontinuation of the EUR/CHF Minimum Exchange Rate in January 2015: Was it Expected?*

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Abstract

We derive risk-neutral probability densities for future euro/Swiss franc exchange rates as implied by option prices. We find that the credibility of the Swiss franc floor somewhat decreased as the spot exchange rate approached the lower bound of 1.20 CHF per euro. We also compare the forecasting performance of a random walk benchmark model with an error-correction model (ECM) augmented with option-implied break probabilities of breaching the currency floor. We find some evidence that the augmented ECM has an informational advantage over the random walk when using one-month break probabilities. But we find that one-month option-implied densities cannot predict the entire range of exchange rate realizations.

Keywords: Swiss franc, forecasting, options, risk-neutral probability densities **JEL-Classification:** C53, F31, F37

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1. Introduction

From September 2011 to January 2015 the SNB's official exchange rate policy was to maintain a minimum exchange rate of CHF 1.20 per euro. In order to do so, the central bank stated that it would be willing to buy foreign currency in unlimited quantities. On 15 January 2015, the Swiss National Bank (SNB) surprised markets by deciding to discontinue Switzerland's currency floor. The SNB justified discontinuation of the floor as follows, "Recently, divergences between the monetary policies of the major currency areas have increased significantly – a trend that is likely to become even more pronounced. The euro has depreciated considerably against the US dollar and this, in turn, has caused the Swiss franc to weaken against the US dollar." (SNB, 2015). A few days earlier, on 12 January 2015, the SNB's vice-chairman Jean-Pierre Danthine had said that "the minimum exchange rate must remain the cornerstone of our monetary policy" (Reuters, 2015). The currency floor was introduced in 2011 to halt the appreciation of the Swiss franc (see Figure 1) and protect the country's many exporters.¹ The currency floor had been effective in preventing an even larger erosion of the competitive position of Swiss exporters. The discontinuation by the SNB of its minimum exchange rate of CHF 1.20 per euro surprised market participants.

In this paper we quantify how unexpected the removal of the exchange rate floor was. We derive riskneutral probability density functions (pdfs) for euro/Swiss franc exchange rates as implied by the prices of options with maturities of 1 to 12 months, using both parametric and non-parametric approaches. We show the evolution of the probabilistic expectations, in the form of the probability of breaching the Swiss franc floor (break probability), and the associated higher-order moments (skewness and kurtosis), over the course of several important market events and exchange rate regime changes. We also study the forecasting performance of option-implied pdfs for euro/Swiss franc exchange rate returns during the time the Swiss franc floor was in place, considering the forecasting performance of an error-correction model augmented with break probabilities in comparison with a random walk benchmark model, as well as the forecasting performance of the entire option-implied pdfs.

Some economists argue that financial markets are an excellent window into future developments and thus help us to understand economic policy and politics – but markets are hardly omniscient. For example, they recently struggled to forecast the Brexit vote. In an influential paper, Meese and Rogoff (1983) tested how well the existing empirical exchange rate models of that time fit out of sample. They found, by using data from the 1970s, that a random walk model performed as well as any estimated structural or various time series models. These surprising findings are known as the "exchange rate disconnect puzzle". Since then studies that re-examined different currency pairs, different time periods, real time versus revised official macroeconomic data, and different linear

¹ The CHF has safe haven characteristics offering hedging against global risks. See, e.g., Grisse and Nitschka (2015).

structural models have failed to overcome this puzzle. Yet some studies such as Mark (1995), Engel et al. (2007), Gourinchas and Rey (2007) and Molodtsova and Papell (2009) found evidence in favour of structural models at short horizons. But as shown in Rogoff and Stavrakeva (2008), these findings are not robust to alternative time windows and tests. On the other hand, studies that seek nonlinear specifications to overcome the puzzle have some mixed findings. Diebold and Nason (1990) and Meese and Rose (1991) failed to outperform the linear random walk specification. By contrast, Wang and Wu (2012) found some promising results in favour of structural models. Yet after over 30 years of research since the publication of the Meese-Rogoff study, their findings remain very robust and the search for model specifications to beat the random walk null hypothesis is still ongoing.²

Malz (1996) employed various option price data to assess the credibility of commitments to the sterling - German mark exchange rate within the ERM exchange rate target zone, and to determine whether the bandwidths were consistent with market expectations. Malz (1997) used model-free option-implied densities to explore the forward rate bias. He found that option-implied volatility, skewness and kurtosis have considerable explanatory power for excess returns of forward exchange rates. Campa and Chang (1996), Malz (1996), and Haas, Mittnik, and Mizrach (2006) examined the information content of option-implied densities around ERM crises of 1992. All three studies find that option-implied measures provide useful information for policy makers. Haas et al. (2006) find that time-series based measures are also useful. Campa et al. (1998) studied the USD/DM and USD/JPY relationship. Campa et al. (1998) found a positive correlation between skewness and the spot rate, which means that the stronger the currency, the more expectations are skewed towards a further appreciation of that currency. Bates (1996) examined option-implied skewness and kurtosis of the USD/DM and USD/JPY exchange rates between 1984 and 1992, and finds that the option-implied higher moments contain significant information for the future USD/DM exchange rate, but not for the USD/JPY rate. Finally, Ivanova and Gutiérrez (2014) employed options to study probabilities assigned to future Libor rates, investigating the forecasting performance of such measures and using forecast biases to infer risk aversion. Our paper contributes to this literature by studying the forecasting performance of option-implied pdfs for the euro/Swiss franc exchange rate for exchange rate returns during the time the Swiss franc floor was in place.

Jermann (2016) and Hertrich and Zimmermann (2015) also estimate risk neutral break probabilities from options on the EUR/CHF exchange rate. Their approaches are not model-free, since both calculate the break probability from an option pricing model which imposes a hypothetical exchange rate for the case that the floor is abandoned. Jerman (2016) imposes a shadow exchange rate, which would be the spot rate without the floor, while Hertrich and Zimmermann (2015) impose a credible reflecting exchange rate barrier below the current floor, namely below the exercise prices for which

² For a thorough review of the exchange rate forecasting literature see Rossi (2013). Busch et al. (2011) showed that prices of options used by investors for hedge purposes can provide information about market perceptions of underlying exchange rate risks.

positive put option prices are available in the market. Jerman (2016) reports the smallest break probabilities during the time when the Swiss franc hit the floor in 2012, whereas Hertrich and Zimmermann (2015) find higher break probabilities at that time, which are closer to ours. The main difference between our results and the ones of Hertrich and Zimmermann (2015) is that they are reporting a zero break probability on several dates even for the three-month horizon. Neither of these papers report values for skewness and kurtosis. Mirkov et al. (2016) use a parametric method to derive the option-implied pdfs for the EUR/CHF exchange rate, assuming the pdfs to be a mixture of two log-normal distributions.

The paper is organized as follows. In Section 2 we derive option-implied pdfs for the Euro/Swiss franc exchange rate and present the evolution of break probabilities and higher-order moments over time. In Section 3 we evaluate the forecasting performance of break probabilities and the whole pdfs. Finally, Section 4 concludes.





Source: Bloomberg.

2. Option-Implied Probability Densities

Foreign exchange option prices provide a unique insight into investors' assessments of future payoffs and can thus be used as an indicator of broad market sentiment. This is due to the fact that an array of option contracts, based on different strike prices of the same underlying asset, is observed simultaneously on each trading date. The densities provide a description of the uncertainty associated with a prediction, and stands in contrast to point forecasts which do not contain any description of the associated uncertainty.³

³ FX option markets for safe haven currencies tend to be liquid since inadequate market depth and liquidity may result in volatile probability distribution functions for reasons other than changes in exchange rate expectations.

Breeden and Litzenberger (1978) have shown how the risk-neutral probability density function is related to the price of a European call option. Let c(t, X, T) be the observed market price of a European call option at time t with exercise price X, expiration date T and time to maturity $\tau = T - t$. Under the assumption of risk neutrality the instantaneous market value of a European call option at time t is given by its discounted expected value

$$c(t, X, T) = e^{-r\tau} \int_0^{+\infty} (S_T - X) \pi_t^{\tau}(S_T) dS_T,$$
(1)

where S_T denotes the exchange rate at the expiration date T, r the domestic risk free interest rate, and $\pi_t^{\tau}(x)$ the time t risk-neutral pdf of the exchange rate τ -months in the future. Taking the second derivative of equation (1) by applying the Leibniz rule twice yields

$$\frac{\partial^2 c(t, X, T)}{\partial X^2} = e^{-r\tau} \pi_t^{\tau}(X).$$
⁽²⁾

Hence, the risk-neutral pdf is equal to the second derivative of the call price function with respect to the strike price times $e^{r\tau}$.⁴

2.1 Parametric Density Estimation

To derive the pdf one would need a proper option pricing model and a continuum of strike prices in order to approximate equation (2). In over-the-counter (OTC) markets options are quoted in terms of their Black-Scholes (BS) implied volatility, where the Garman and Kohlagen (1983) formula is used

According to the BIS (2016), average daily turnover in April 2016 in the CHF was 4.8% of the global FX market turnover (since two currencies are involved in each transaction, the sum of shares in individual currencies sums to 200%), the 7th rank worldwide.

⁴ It should be clear that option implied risk neutral densities in general reflect only partly the market views about future outcomes of the underlying asset as they disregard risk aversion. The risk-neutrality assumption might have implications for the estimated distributional moments. If investors are more risk averse to higher (lower) relative to lower (higher) EUR/CHF exchange rates, then option payouts in the case of higher (lower) exchange rates will be priced above what the actual market-assigned probabilities would imply, resulting in a higher (lower) estimated probability distribution function mean and skewness. Analogously, if investors are relatively more risk averse to a more (less) extreme EUR/CHF exchange rate, estimates of kurtosis might be overstated (understated). However, in foreign exchange markets the risk premium is on average small (see, e.g., De Santis and Gèrard, 1997). As argued by Hanke et al. (2015), risk neutral and real world probabilities will therefore be close to each other. In accordance with this, Mirkov et al. (2016) estimate risk neutral and real world confidence bands of the CHF/EUR exchange rate from option implied densities and find that during the period of the SNB's currency floor both were nearly similar, which is further evidence that for exchange rates risk neutral densities seem to be sufficient for determining market views. In Appendix E, we estimate real-world densities (RWDs) from risk-neutral densities (RNDs), and find that during the minimum exchange rate of CHF 1.20 per euro the difference between RNDs and RWDs for the EUR/CHF exchange rate is negligible for a reasonable degree of risk aversion, in line with the literature.

for pricing. Given the foreign risk-free interest rate r_t^* , the standard normal cumulative distribution function $\Phi(\cdot)$ and the BS implied volatility σ_t , this formula is expressed by

$$\mathcal{C}_{BS}(S_t,\tau,X,\sigma_t,r_t,r_t^*) = e^{r^*\tau}S_t\Phi\left[\frac{\ln\left(\frac{S_t}{X}\right) + \left(r_t - r_t^* + \frac{\sigma_t}{2}\right)\tau}{\sigma_t\sqrt{\tau}}\right] - Xe^{-r\tau}\Phi\left[\frac{\ln\left(\frac{S_t}{X}\right) + \left(r_t - r_t^* - \frac{\sigma_t}{2}\right)\tau}{\sigma_t\sqrt{\tau}}\right].$$
(3)

Inserting the observed call price, $C_{BS,t}$, on the left side of equation (3) and solving numerically for σ_t would give the call options price in terms of implied volatilities.⁵ Depending on how far away the strike price X is from the actual value of the exchange rate the option has a different moneyness. FX-option markets measure the moneyness of an option in terms of the call options delta, which is given by⁶

$$\delta(S_t, \tau, X, \sigma_t, r_t, r_t^*) \equiv \frac{\partial C_{BS}}{\partial S_t} = e^{r_t^* \tau} \Phi \left[\frac{\ln\left(\frac{S_t}{X}\right) + \left(r_t - r_t^* + \frac{\sigma_t}{2}\right) \tau}{\sigma_t \sqrt{\tau}} \right].$$
(4)

For the parametric method, Appendix C describes the calculation of the BS implied volatility σ_t as a function of strike price X using prices of options that are ATM, and OTM with a δ of 25%, denoted by $\sigma_{25\delta,t}(X)$. The description follows Blake and Rule (2015). The resulting expression for $\sigma_{25\delta,t}$ of equation (C5) in Appendix C is then inserted into equation (3). To calculate the risk-neutral density, the second derivative in equation (2) for step-size *h* is approximated by the second order centralized difference quotient⁷

$$\frac{\partial^2 c(t,X,T)}{\partial X^2} \approx \frac{c(t,X+h,\tau) + c(t,X-h,\tau) - 2c(t,X,\tau)}{h^2}.$$
(5)

2.2 Non-Parametric Density Estimation

The parametric density approach assumes the volatility smile to be a quadratic function of the options delta.⁸ While the parametric approach offers parsimony, its main weakness is that the quadratic simile

⁵ Unlike realised volatility, implied volatility is a forward looking measure of the variability of the exchange rate. ⁶ The option delta, or the derivative of the Black-Scholes option value with respect to the spot rate, is a parameter showing how close the asset price is to the strike price. In other words, the options delta shows the moneyness of the option. An option is at-the-money (ATM) if the exercise price is equal to the current price of the underlying asset. A call option is out-of-the-money (OTM) if the exercise price is above the current price of the underlying asset. Interpolating implied volatilities over deltas was first performed by Malz (1997).

⁷ We use a MATLAB code provided by the Blake and Rule (2015) to solve equation (C5) for $\sigma_{25\delta,t}$. We revised the code to be able to calculate the difference quotient of equation (5) for different step sizes.

⁸ Another parametric method to derive the pdfs for the EUR/CHF exchange rate is used by Mirkov et al. (2016). They estimate the pdf to be a mixture of two log-normal distributions where they estimate the mixture weights plus the two means and variances from 25- and 10-delta option contracts described in equations (C1)-(C3) of

interpolation is only able to use three data points, while nowadays option prices with more delta values other than 25% are available. A more refined approach to estimating densities is to apply a non-parametric method of interpolation as suggested by Malz (2014). The non-parametric method uses a clamped cubic spline with flat ends to calculate the volatility smile.

For the non-parametric method, Appendix D describes the calculation of the BS implied volatility σ_t as a function of strike price *X*, $\sigma_t(X)$, following Blake and Rule (2015). The resulting function, $\sigma_t(X)$, calculated according to Appendix D, is then substituted into the Black-Scholes formula (3) to calculate the approximated second derivative by applying the second order centralized differential quotient as in equation (5).

2.3 Higher Order Moments

Once EUR/CHF pdfs have been calculated from option prices, computation of distributional moments provides useful summary statistics, particularly when these moments are viewed over a span of time. Of special interest are the skewness and the excess kurtosis, given by

$$sk_t^{\tau} = E\left[\left(\frac{X - E[X]}{\sigma}\right)^3\right] = \int_0^{+\infty} \frac{\left(x - E(X)\right)^3}{\sigma^3} \pi_t^{\tau}(x) dx \tag{6}$$

and

$$ex_t^{\tau} = E\left[\left(\frac{X - E[X]}{\sigma}\right)^4\right] - 3 = \left(\int_0^{+\infty} \frac{\left(x - E(X)\right)^4}{\sigma^4} \pi_t^{\tau}(x) dx\right) - 3$$
(7)

When the risk-neutral density has a positive skewness more probability mass is shifted towards its right tail, indicating a higher probability of a depreciation rather than of an appreciation of the Swiss franc against the euro. In contrast, a negative skewness coefficient indicates that the market expects the Swiss franc to be more likely to appreciate rather than to depreciate against the euro.⁹ The excess kurtosis measures market expectations of extreme changes in the EUR/CHF exchange rate. The higher the excess kurtosis, the higher the probability concentrated in the tails of the distribution.¹⁰

Appendix C. Because they assume the exchange rate to follow a mixture of log-normal distributions, the method is not model free.

⁹ We did robustness checks with different skewness measures such as the widely used skewness measure given by $[(25\% \text{ delta call implied volatility} - 25\% \text{ delta put implied volatility})/50\% \text{ delta implied volatility}, with largely similar results.}$

¹⁰ In general the boundaries of the integral of the skewness and excess kurtosis should range from minus infinity to plus infinity, but since the strike price would never fall below zero we have chosen the lower bound to be zero.

2.4 Data

For our analysis we use daily OTC market data for ATM-straddles, risk reversals and butterflies from Bloomberg with deltas of 10%, 25% and 35%. The maturities are one, three, six and twelve months. Euro Libor and Swiss franc Libor rates, which are provided by Datatream, are used as proxies for the foreign and domestic risk-free interest rates. The maturities for those are the same as for the option data. The EUR/CHF spot exchange rate is also on a daily basis and provided by Bloomberg. All data described range from 6 September 2011 to 31 August 2016.

2.5 Probability Densities During and After the Swiss Franc Floor and Market Expectations

The methodology laid out above is now applied to the EUR/CHF exchange rate. In other words, we employ the distributions implicit in option prices as a barometer of market sentiment and explore whether the various measures bring evidence to bear on the regime change in January 2015. Since we want to analyze financial market views about the behaviour of the exchange rate during the currency floor, we will take a closer look at the changes in the shape and the location of the densities over the time the Swiss franc floor was in place. Was the SNB's commitment and the sustainability of the policy questioned at the end of 2014?

Figure 2 shows plots of the one-month forward looking parametric pdfs during the time the Swiss franc floor was in place.¹¹ A first look at all panels reveals that the parametric densities are bellshaped, which is due to the quadratic smile approximation. Between September and December 2011 the densities shifted to the right, indicating an expected depreciation of the Swiss franc against the euro. In September 2011 the skewness of the density was negative together with a small kurtosis, but in the following months there was a notable increase to positive skewness values together with a slightly increasing kurtosis, indicating further that financial markets expected the Swiss franc to depreciate rather than appreciate against the euro within the next month. In January 2012 the density was not fat tailed but showed a considerably positive skewness. A shift in market beliefs becomes obvious from April to August 2012, when the Swiss franc came under huge appreciation pressure against the euro. The densities shifted to mean values close to 1.2 and the skewness turned negative, together with a rapid increase in the kurtosis. Hence there was some doubt about the future existence of the Swiss franc floor. At the end of 2012 the situation in the FX market calmed down as the EUR/CHF exchange rate moved away from its floor together with an increase of the mean and skewness of the density and a decrease in the kurtosis. From January to December 2013 the Swiss franc depreciated further against the euro and took values significantly above 1.2, which is shown by

¹¹ Plots of non-parametric densities for the same days can be found in Figure B1 in Appendix B. The qualitative behaviour is the same. The main difference is that the shape of the non-parametric densities is more flexible and therefore sometimes not bell-shaped.

rightward shifts of the densities away from mean levels of 1.2. The skewness was positive throughout 2013, while there was no strong indication of fat tails. This indicates that financial markets considered further depreciations as more likely than appreciations, while very large movements in either direction were considered as less likely. In 2014 the direction of market views changed again, as the Swiss franc appreciated against the euro, which was attended by leftward shifts of the densities towards mean values of 1.2. From January to December 2014 the skewness of the densities turned from positive to negative indicating a change in the directional views of financial markets. Moreover, at the end of 2014 the kurtosis increased, indicating that financial markets saw large movements of the exchange rate as more possible.



Figure 2: One-Month Parametric Option-Implied Probability Density Functions, 2011-2014

Next we look more closely at the days around the regime change in January 2015. Figure 3 shows how expectations changed for the probability distribution one month ahead for the days before and after the discontinuation of the currency floor in January 2015. In addition to the parametric densities, the non-parametric densities are also presented. Compared to the parametric densities the non-parametric ones can have a much more flexible shape, where even multiple modes are possible.¹² This is because of the

¹² We find that our estimated non-parametric risk-neutral densities and real-world densities are not bimodal, which provides evidence in favour of the credibility of the minimum exchange rate of CHF 1.20 per euro while it was in place.

flexible spline approximation of the volatility smile. The qualitative behavior around 15 January 2015 is the same. Both types show a distribution with a slightly negative skewness on 14 January 2015, with the variance of the non-parametric one a bit larger. The parametric density is leptokurtotic, while the excess kurtosis of the non-parametric one is close to zero. On 15 January 2015 the distribution of both types jumped from a mean of 1.2 to a mean of around 1.05, together with a large increase in the variance. The skewness of the parametric density does not change substantially, while the one of the non-parametric density increases. After 15 January 2015 the EUR/CHF exchange rate regime was a managed float, which was reflected in significant daily changes in the means of the risk-neutral densities.

Figure 3: One-Month Option-Implied Probability Density Functions, Parametric and Non-Parametric, Days Before and After Discontinuation of the Currency Floor



Break Probabilities

In order to evaluate the credibility of the currency floor, we calculate on each day the expected break probability for a given maturity, denoted by $Prob(S_t^{\tau} < 1.2)$. The estimated discontinuation probabilities are defined as the beliefs of market participants that the Swiss franc floor is abandoned at the end of a specified time horizon.¹³ We will use these time-varying break probabilities subsequently

¹³ Therefore, the probabilities are independent of the path of the exchange rate within the time horizon.

to forecast future exchange rates. The currency floor can be seen as credible if the break probability is significantly below 50%. By contrast, break probabilities above 50% indicate some expectation of an end to the minimum EUR/CHF exchange rate regime.



Figure 4: Implied Parametric and Non-Parametric Break Probabilities

Jul 2011 Dec 2011 May 2012 Oct 2012 Mar 2013 Aug 2013 Jan 2014 Jun 2014 Nov 2014 Apr 2015 Sep 2015 Feb 2016 Jul 2016 Dec 2016

Figure 4 shows the evolution of the implied parametric and non-parametric break probabilities for the four different maturities. A look at both figures suggests the same qualitative behaviour for both estimation methods, with the non-parametric estimates being a bit less volatile for maturities between three and twelve months. Between September 2011 and May 2012 the break probability for the twelve month maturity is significantly larger than for the one month maturity. For six and three months it is placed in between, with the expected ordering of higher break probabilities for longer maturities. In December 2011 the Swiss franc started appreciating against the euro and hit the floor of 1.2 from May to August 2012. During this time both the parametric and non-parametric break probabilities converge towards levels of 50%, or closely below, for all maturities, indicating that financial markets had significant doubts about the existence of the floor one to twelve months in the future. The large appreciation pressure ended in September 2012 and between October 2012 and November 2014 the Swiss franc depreciated significantly against the euro. During this period the one-month parametric and non-parametric break probability for all other

maturities the break probability was larger. For a maturity of twelve months the parametric break probability fluctuated between 35% and 45%, while for six and three months it fluctuated between 25% and 40% and 15% and 35%, respectively. The non-parametric break probabilities were less volatile for maturities of three to twelve months. For twelve months it fluctuated around 45%, for six months around 40% and for three months around 35%. Hence even for the period when the CHF/EUR exchange rate was far away from its floor, there was some doubt about its future existence. While the market was more or less sure about its existence within the next month, the story was different for longer time horizons of twelve or six months. Between December 2014 and the middle of January 2015, when the floor was abandoned, the Swiss franc again moved towards its floor, with the break probabilities again converging towards values between 45% and 50%. On 15 January 2015 the floor was discontinued, leading to a large appreciation of the Swiss franc against the euro, together with a big increase in the break probabilities for all maturities. Because the densities for longer maturities after mid-January 2015.

To demonstrate the strong negative relationship between the exchange rate and the break probabilities, scatter plots of the parametric break probabilities against the daily spot exchange rate for all maturities for the period 06/09/2011 to 14/01/2015 are presented in Figure 5. A similar pattern arises for all maturities as the point clouds suggest a negative and linear relationship. For the six-month horizon the point cloud scatters the least and shows the closest linear relationship, while for all other maturities the scatter is less tight. To quantify the strength of the linear relationship, Table 1 presents estimates of a simple linear regression model where the spot rate is regressed on an intercept and one of the break probabilities. The results show that for the six-month maturity the R^2 has the largest value, while for all other maturities the strength of the negative linear relationship is smaller than for the six-month maturity, but still large. This suggests that the credibility of the floor somewhat decreased as the spot exchange rate approached the lower bound of 1.20 CHF per euro.¹⁴

| | | 14/01/2015 | | |
|-------------|----------|------------|----------|-----------|
| | 1-M Prob | 3-M Prob | 6-M Prob | 12-M Prob |
| Coefficient | -0.0814 | -0.1137 | -0.1900 | -0.3054 |
| P-Value | (0.000) | (0.000) | (0.000) | (0.000) |
| R^2 | 0.8809 | 0.8871 | 0.9413 | 0.8280 |

 Table 1: Regression of Spot Exchange Rate on Parametric Break Probabilities 06/09/2011 to

<u>Notes:</u> OLS estimates of slope coefficient of: $S_t = \alpha + \beta x_t^{\tau} + \epsilon_t$. S_t is the time t spot exchange rate and x_t^{τ} is the τ -month forward looking parametric break probability.

To summarize the results for the break probability analysis, we conclude that financial markets

¹⁴ Scatter plots of the non-parametric break probabilities versus the daily spot exchange rate look similar and are therefore not shown. However, the plots scatter less compared to the parametric break probabilities, which is reflected by larger values of R^2 presented in Table A1 in Appendix A.

attached some credibility to the Swiss franc floor, since the break probabilities never significantly exceeded 50% while the floor was in place, but especially at longer maturities there was some doubt. Moreover we conclude that from January 2014 the credibility of the floor somewhat decreased, which can be seen from the break probabilities increasing since January 2014 for all maturities.



Figure 5: Parametric Break Probability vs Daily Spot Exchange Rate 06/09/2011 to 14/01/2015

Higher-order moments

Next, we turn to the skewness and excess kurtosis to try to identify possible signals of regime change. To analyze market views about the direction and magnitude of changes in the exchange rate, Figure 6 shows the parametric skewness and excess kurtosis coefficients for all maturities. We choose some threshold values for the skewness to make a clearer distinction between high and low levels. Although the particular choice of a threshold is always somewhat arbitrary, we choose the following ones: When the skewness is greater (less) than 1 (-1), we say that there is a substantial positive (negative) skewness. When it is between 0.5 and 1 (-0.5 and -1), we say that there is a moderate positive (negative) skewness. One can see from Figure 6 that the skewness has a similar pattern for the different maturities. At the beginning of the currency floor the skewness decreased, and the excess kurtosis coefficients indicated fat tails of the underlying distribution. After a short while the skewness for the one-month maturity turned positive, with moderate values of around 0.5, indicating a skewness towards depreciation. For the three-month maturity the skewness was between -0.5 and 0, while for all

other maturities it remained mostly negative with values fluctuating between -0.5 and -1, indicating a moderate asymmetry towards appreciation. During March and August 2012, when the Swiss franc appreciated strongly against the euro and hit the floor, the skewness decreased significantly for all maturities, while the excess kurtosis reached its maximum for all maturities. Hence, when the Swiss franc reached the floor, financial markets believed that a large move towards further appreciation became more likely. These results are consistent with the comparably large values of the break probabilities for all maturities at that time, shown in Figure 4. Between 2013 and mid-2014, when the Swiss franc depreciated significantly against the euro, a parallel movement of both coefficients was observable. At first the skewness coefficient turned positive for all maturities, and afterwards deceased to values close to zero. Simultaneously the excess kurtosis decreased from two to zero for all maturities between January 2013 and July 2014. The parallel movement of both the skewness and the excess kurtosis stopped at the time when the Swiss franc again moved close towards its floor between July 2014 and January 2015. This time period somehow mirrors the turbulent months in 2012 described above, but the values are much less extreme. The skewness for all maturities turned negative with values between -0.5 and -1 for one to six months, and 0 to -0.5 for twelve months. In contrast the excess kurtosis increased sharply to values between 0 and 2 for the one-month maturity, and to between 2 and 4 for all other maturities. Although there was a turnaround for both coefficients at the end of 2014, the skewness kept being negative for all maturities (except for one month), and the excess kurtosis kept being positive and comparably large for all maturities. After 15 January 2015, when the Swiss franc floor was finally removed, the excess kurtosis decreased to values below one for the one-month maturity, and to around one for all other maturities. The skewness fluctuated between 0.5 and -0.5 for the one-month maturity, and between 0 and -1 for the other maturities.¹⁵

To summarize our results for the density analysis, the Swiss franc floor had a higher credibility over shorter horizons. During the turbulent times in 2012 and at the end of 2014, the densities for all maturities tell basically the same story. Markets assessed the likelihood of discontinuing the Swiss franc floor within the option expiration to be about 50 percent (as shown in Figure 4), that is, there was some doubt about the SNB's commitment to the currency floor during those turbulent times.

¹⁵ The non-parametric skewness and excess kurtosis coefficients are presented in Figure B2 in Appendix B. Their qualitative behaviour is similar to the parametric coefficients. The main difference is that the extreme values are less pronounced and that their overall volatility is smaller. Mirkov et al. (2016), who also estimated risk neutral PDFs for the EUR/CHF exchange rate, approximated the three-month skewness of their pdfs by adding the differences between the 90th and 10th percentiles from the spot exchange rate, normalized by the difference between the 90th and 10th percentiles. They show three-day averages of this measure for the time the Swiss franc floor was in place. The results are quite similar. There was an increase until mid-2012, followed by negative values during the time the EUR/CHF exchange rate hit the floor. In 2013 they report positive skewness coefficients and in 2014 the skewness again turned negative.



Figure 6: Parametric Skewness and Excess-Kurtosis Coefficients

3. Forecasting Performance of the Option-Implied Break Probabilities

In an influential paper, Meese and Rogoff (1983) tested how well the existing empirical exchange rate models of that time fit out of sample. They found, by using data from the 1970s, that a random walk model performed as well as any estimated structural or various time series models. These surprising findings are known as the "Exchange rate disconnect puzzle". Since then studies that re-examined different currency pairs, different time periods, real time versus revised official macroeconomic data, and different linear structural models have failed to overcome this puzzle. As discussed in the introduction, there is some evidence in the literature that some measures based on option-implied pdfs contain significant information about future exchange rates. Next, we contribute to this literature by studying the forecasting performance of option-implied pdfs for the euro/Swiss franc exchange rate for exchange rate returns during the time the Swiss franc floor was in place, considering option-implied break probabilities, as well as the entire pdfs.

3.1 Point Forecasting Setup

The approach taken in this paper is as follows. In order to test the forecasting performance of the option variables, we will compare the out-of-sample forecasting performance of a benchmark model

against a model specification which is augmented by option variables. In an efficient market, the option price is equal to the discounted expectation of its value at maturity. Therefore, the benchmark model is chosen such that it matches the maturity of the different option contracts. Ignoring non-trading days, a month approximately has 22 days. Let $s_t = \ln(S_t)$, where S_t is the time t spot exchange rate, then the τ -step ahead ($\tau = 1,3$) benchmark random walk model is given by

$$E_{t-22\tau}(s_t - s_{t-22\tau}) = 0, (8)$$

where $E(\cdot)$ is the expectations operator. In a financial context, the τ -step ahead forecast is equivalent to one-step-ahead forecasting of τ -period exchange rate returns. According to the random walk without drift modelling approach, the best predictor of future exchange rates is the exchange rate today. In other words, exchange rate changes are unpredictable. Next, equation (8) is augmented with the density forecasts. We choose the break probability as an explanatory variable, since it aggregates the properties of the density in a single number, which enables us to conduct point forecasting with the densities. The augmented error-correction forecasting model (ECM) is given by

$$s_t - s_{t-22\tau} = \alpha + \beta (s_{t-22\tau} - \gamma_0 - \gamma_1 x_{t-22\tau}) + \epsilon_t$$
, (9)

where x_t are the option-implied break probabilities defined above. If the break probabilities help to forecast the exchange rate, then we should find that β is significantly different from zero.¹⁶ In the next section we provide details on how the forecasting power is evaluated.

3.2 The Choice of the Point and Density Evaluation Metrics

The predictive ability of the FX option variables can be evaluated according to the τ -step ahead outof-sample forecast performance of model (8) relative to the benchmark model (9). As a first step, we calculate and compare the mean errors (ME), the mean absolute errors (MAE), the root mean squared errors (RMSE) and the mean squared errors (MSE). Subsequently, we employ Clark and McCracken's forecast accuracy tests for nested models. The MSE-t test of Clark and McCracken (2001) is a onesided forecast accuracy test, which tests whether the difference between the MSEs of equations (8) and (9) is bigger than zero. The null hypothesis is that the difference in MSEs is below or equal to zero. The squared forecast errors for period t of equations (8) and (9) are denoted by $\widehat{u_{1,t}}^2$ and $\widehat{u_{2,t}}^2$, respectively. We define period t's difference in squared forecast errors as $d_t = \widehat{u_{1,t}}^2 - \widehat{u_{2,t}}^2$. Let MSE_1 and MSE_2 be the MSEs of equations (8) and (9). For a given number of forecasts, denoted by P,

¹⁶All time series are tested for unit roots. We find that all variables are I(1). Moreover the one and three-month break probabilities and the log exchange rate are cointegrated, based on a test using the Engle and Granger procedure.

the difference of MSEs is given by $\bar{d} = P^{-1} \sum_t d_t = MSE_1 - MSE_2$. With this notation the MSE-t test statistic can be written as

$$MSE - t = \sqrt{P} \frac{\bar{d}}{\sqrt{P^{-1} \sum_{t} \left(d_{t} - \bar{d}\right)^{2}}}.$$
(10)

The MSE-F test is another test for forecast accuracy. Its test statistic is given by

$$MSE - F = P \frac{\bar{d}}{P^{-1} \sum_{t} \hat{u_{2,t}}^2} = P \frac{MSE_1 - MSE_2}{MSE_2}.$$
 (11)

Instead of normalizing by the standard deviation of the difference in MSE's, the MSE-F test normalizes with the MSE of the augmented model itself. This is done because under the null hypothesis the prediction errors of the two models are the same, with a standard deviation of zero. This property may adversely affect the small sample properties of the tests.

The ENC-t test is an encompassing test, which evaluates whether the augmented model has an informational advantage. It tests whether the covariance of the errors of the restricted model of equation (8), and the difference of the errors between the restricted model and the unrestricted model of equation (9), is positive. The null hypothesis is that the estimated covariance is negative or equal to zero. When the null hypothesis is rejected, then the augmented model contains additional information. Let $c_t = \hat{u_{1,t}}^2 - \hat{u_{1,t}}\hat{u_{2,t}}$ and the covariance between $\hat{u_{1,t}}$ and $\hat{u_{1,t}} - \hat{u_{2,t}}$ be denoted by $\bar{c} = P^{-1}\sum_t (\hat{u_{1,t}}^2 - \hat{u_{1,t}}\hat{u_{2,t}})$. The ENC-t test statistic is then given by

$$ENC - t = \sqrt{P} \frac{\bar{c}}{\sqrt{P^{-1} \sum_{t} (c_t - \bar{c}\,)^2}}.$$
(12)

By analogy to the MSE-t and MSE-F tests, Clark and McCracken (2001) derive the ENC-F test, which normalizes equation (12) by using the MSE of the augmented model instead of the standard deviation of the estimated covariance, for the same reason the MSE-F test does. Therefore the ENC-F test statistic is given by

$$ENC - F = P \frac{\bar{c}}{P^{-1} \sum_{t} \hat{u_{2,t}}^2} = P \frac{\bar{c}}{MSE_2}$$
(13)

In addition to the evaluation methods described above, we conduct tests for the directional forecasting accuracy, first introduced by Pesaran and Timmermann (1992). This non-parametric test procedure does not depend on the MSE as it measures only whether the selected model is able to forecast the

right sign of the series to be predicted. While the MSE based tests seek to evaluate accuracy in general, this property might not always be appropriate when dealing with financial market data, because traders might be more interested in the general direction of the movement of a particular financial variable rather than an on average preferable forecast. Hence a forecasting model which is able to predict the right direction of a movement more often, but has a larger MSE than a model that does not do so, might be of more value for financial market participants when it comes to judging markets adjustments. To conduct the Pesaran-Timmermann-Test (*PT*) we compare the sign of the actual x-month return, $R_{t-22\tau}$, to a forecast of it, $\widehat{R_{t-22\tau}}$, from the following model

$$R_{t-22\tau} = \beta \Delta X_{t-22\tau} + \epsilon_t, \tag{14}$$

where $\Delta X_{t-22\tau}$ is the first difference of the break probability τ months earlier.¹⁷ Let the number of predictions be *n*. Under the null hypothesis that $R_{t-22\tau}$ and $\Delta X_{t-22\tau}$ are independently distributed, the number of predicted forecasts where the sign is correct, denoted by $n\hat{P}$, follows a binominal distribution. The *PT*-test then compares the fraction of right directional forecasts of equation (14), denoted by \hat{P} , with the fraction of co-movements of $\Delta X_{t-22\tau}$ and $R_{t-22\tau}$ denoted by \hat{P}_* . Under the null hypothesis that $R_{t-22\tau}$ is independent of $\Delta X_{t-22\tau}$, the number of right directional forecasts from equation (14) should not differ from the estimated fraction of co-movements. If the difference is significantly differs from zero, then $R_{t-22\tau}$ and $\Delta X_{t-22\tau}$ co-move and hence equation (14) has directional forecasting ability. The *PT* test statistic is given by¹⁸

$$PT = \frac{\hat{P} - \hat{P}_*}{var(\hat{P}) - var(\hat{P}_*)}.$$
(15)

The test procedures described above are evaluating τ -step ahead out-of-sample point forecasts. In what follows we shall complement the evaluation of the point forecasts by an evaluation of the density forecasts. Berkowitz (2001) established a procedure to test the forecast accuracy of density forecasts. The test procedure is based on the probability integral transformation (PIT) of Diebold et al. (1998). Given a time t density forecast of the exchange rate τ months in the future, $\pi_t^{\tau}(x)$, and the realized exchange rate after τ months, $S_{t+22\tau}$, the PIT can be written as

¹⁷ We use the first difference, because for the *PT* test all variables have to be stationary.

¹⁸ Pesaran and Timmermann (1992) derive $var(\hat{P})$ and $var(\hat{P}_*)$ to be $var(\hat{P}) = n^{-1}\hat{P}_*(1-\hat{P}_*)$ and $var(\hat{P}_*) = n^{-1}(2\hat{P}_y - 1)^2\hat{P}_x(1-\hat{P}_x) + n^{-1}(2\hat{P}_x - 1)^2\hat{P}_y(1-\hat{P}_y) + 4n^{-2}\hat{P}_y\hat{P}_x(1-\hat{P}_y)(1-\hat{P}_x)$. For ease of notation, the subscript *y* stands for $\Delta X_{t-22\tau}$ and the subscript *x* stands for $R_{t-22\tau}$. With these definitions \hat{P}_y and \hat{P}_x are the estimated probabilities that ΔX_{t-22*x} and R_{t-22*x} have a positive sign. They also show that under validity of the null hypothesis, the PT statistic converges in distribution towards a standard normal distribution.

$$z_t = \int_0^{S_{t+22\tau}} \pi_t^{\tau}(x) dx.$$
 (16)

Diebold et al. (2001) show that when the forecasting density at time t coincides with the realized density at $t + 22\tau$, then z_t is a realization of a uniformly and independent and identically distributed random variable, such that the sequence of z_t 's is given by $\{z_t\}_{t=1}^T \sim iid U(0,1)$.¹⁹ Given this property Berkowitz (2001) transforms z_t to be independent and identically normally distributed by using the inverse standard normal cumulative distribution function, such that $y_t = \Phi^{-1}(z_t)$ follows a standard normal distributed by Diebold et al. (2001), we test whether the y_t series is independently distributed by examining the autocorrelation function of the z_t sequence over several lags, and investigate whether the autocorrelations are significant. To test the joint hypothesis of an identical and independent normal distribution of the y_t series, Berkowitz (2001) first estimates the following regression,

$$y_t - \mu = \alpha(y_{t-1} - \mu) + \epsilon_t. \tag{17}$$

Second, given the log-likelihood of (17), denoted by $L(\hat{\mu}, \hat{\sigma}^2, \hat{\alpha})$, a likelihood ratio (LR) test with the joint hypothesis of $\mu = 0$, $\alpha = 0$ and $Var(\epsilon_t) = 1$, can be used for an i.i.d. normality test. The LR test statistic is given by²⁰

$$LR_3 = -2[L(0,1,0) - L(\hat{\mu}, \hat{\sigma}^2, \hat{\alpha})]$$
(18)

and under validity of the null hypothesis it is $\chi^2(3)$ distributed.²¹ Below we implement these methods.

3.3 Empirical Results

To evaluate the one-step ahead out-of-sample forecasting performance of the break probabilities, during the time the Swiss franc floor was in place, we focus on the one- and three-month maturities,

¹⁹ The test may be seen as an analogue of the Diebold and Mariano (1995) test for equal point forecast accuracy in the context of density forecasts. It is perhaps worth emphasizing that the test statistic measures the degree of association between expected and realised densities, and does not provide evidence for the rationality of expectations.

²⁰ It is important to mention that although we derive densities for every day over four years, it is only allowed to use densities derived from option data with non-overlapping maturities. Hence we can only use twelve per year for one-month forward looking densities, because otherwise we would face serially dependent $z_t s$ (see Ivanova and Puigvert Gutiérrez (2014) for more details).

²¹ One could also apply the non-parametric Kolmogorov-Smirnov test, but Diebold et al (1998) argue that it is only powerful in very large samples with more than 1000 observations. The test of Berkowitz (2001) has larger power in small samples, which is an advantage in our case since we have at most a sample size of n = 39 for the one-month maturity.

since these leave us with the highest number of observations. The number of observations in the insample periods for both maturities is chosen to be the same.²² The in-sample period for the one-month maturity is chosen to be between 6 October 2011 and 4 June 2012, and for the three-month maturity it is from 7 December 2011 to 3 August 2012. To check for robustness, results for different in-sample periods are presented in Appendix A. The in-sample periods start on 6 October 2011 for the onemonth maturity, and on 7 December 2011 for the three-month maturity.

| | ME | MAE | <u>RMSE</u> | MSE | MSE-F | MSE-t | ENC-F | ENC-t |
|-----------------|----------|---------|-------------|---------|---------|---------|---------|---------|
| <u>1 Month</u> | | | | | | | | |
| Benchmark | 0.00205 | 0.49413 | 0.75550 | 0.57078 | | | | |
| Model | | | | | | | | |
| Para | -0.19227 | 0.57418 | 0.78121 | 0.61030 | -44.099 | -1.958 | 37.617* | 3.0463* |
| Non-Para | -0.19836 | 0.58024 | 0.79012 | 0.62428 | -58.361 | -2.609 | 29.506* | 2.454* |
| <u>3 Months</u> | | | | | | | | |
| Benchmark | 0.01748 | 0.72141 | 0.93931 | 0.88229 | | | | |
| Model | | | | | | | | |
| Para | -0.23941 | 0.99949 | 1.17723 | 1.38587 | -231.46 | -12.930 | -46.276 | -5.5691 |
| Non-Para | -0.20703 | 0.94647 | 1.13585 | 1.29017 | -201.38 | -12.399 | -43.456 | -5.8043 |
| | | | | | | | | |

Table 2: MSE and ENC Tests of the Benchmark Model vs. ECM, 1 and 3-Month Maturities

<u>Notes:</u> 5%-critical values 1 month (3 months): MSE-F=0.966(1.045), MSE-t=0.241(0.268), ENC-F=3.418(3.384), ENC-t=1.386(1.392); In-sample period for 1 month and 3 months: 06/10/2011 to 04/06/2012 and 07/12/2011 to 03/08/2012, respectively; Out-of-sample period for 1 month and 3 months: 05/06/2012 to 14/01/2015 and 04/08/2012 to 14/01/2015, respectively.

The results for the forecasting comparison between the random walk and the ECM model, which includes the parametric and non-parametric break probabilities, are presented in Table 2 for the oneand three-month maturities.²³ For the ECM specification including the one-month parametric and nonparametric break probabilities, the ENC-F and ENC-t tests exceed the critical values, which indicates that the ECM has an informational advantage over the random walk. By contrast, the MSE-F and MSE-t test statistics are not significant at the one-month maturity. For the three-month maturity, the MSE-F, MSE-t, ENC-F and ENC-t test statistics are all insignificant, indicating that the ECM has no informational advantage over the random walk.

²² The in-sample size for the one and three-month maturities in the main part of this paper is 172 observations. Robustness tests with longer in-sample periods, which are presented in Appendix A, contain 238 and 370 observations.

²³ Since we are using overlapping returns with 22 lags in the point forecasting exercise, the partial autocorrelation functions for long lags may be large. Therefore, we have tested for significant partial autocorrelations of the residuals for every estimated ECM while performing the recursive forecasting exercise. We find that for both the parametric and non-parametric estimates almost all are not significant (the results are available on request).

As a robustness check, Tables A2 and A3 in Appendix A present results for different in-sample sizes. The results are strikingly similar. For both the parametric and non-parametric break probabilities at the one-month maturity, the ENC-F and ENC-t tests again exceed the critical values, indicating that the ECM has an informational advantage over the random walk, while the MSE-F and MSE-t test statistics remain insignificant. The ECMs for the three-month maturity also reveal the same pattern, with all four test statistics remaining insignificant, again indicating that the ECM has no informational advantage over the random walk.

| Paran | <u>netric</u> | <u>Non-Par</u> | rametric |
|----------------|-----------------|----------------|-----------------|
| <u>1 Month</u> | <u>3 Months</u> | <u>1 Month</u> | <u>3 Months</u> |
| 2.0422* | 1.0538 | 1.3235 | 1.4324 |

Table 3: PT-Test for Break Probabilities, One- and Three-Month Maturities

<u>Notes:</u> 5%-critical value: 1.96; In-sample period for 1 month and 3 months: 06/10/2011 to 04/06/2012 and 07/12/2011 to 03/08/2012, respectively; Out-of-sample period for 1 month and 3 months: 05/06/2012 to 14/01/2015 and 04/08/2012 to 14/01/2015, respectively.

The results of the *PT* test for the one and three-month break probabilities are presented in Table 3. The test indicates that the one-month parametric break probability is able to predict the sign of EUR/CHF exchange rate returns. With a value of 2.04 the test statistic clearly exceeds the 5%-critical value of 1.96. For all other specifications, there is no evidence for any ability of predicting the sign of exchange rate returns, with none of the test statistics having values greater than 1.96. When the in-sample size is increased, as presented in Tables A4 and A5 in Appendix A, the *PT* test indicates that also the one-month parametric break probability is no longer able to predict the right sign of exchange rate returns. Hence the results depend on the choice of the in-sample size and therefore are not robust. For all other specifications, increasing the in-sample size does not change the results of the tests, with all test statistics remaining below 1.96.

Next, we proceed with the analysis of the density distributions. To conduct the Berkowitz test we first investigate whether there is significant serial correlation left in the PIT values, z_t , by examining the correlogram. In addition we also check the correlogram of z_t^2 , z_t^3 and z_t^4 , to investigate whether there is significant serial correlation in the higher order moments of the z_t series. The quantitative assessment of the forecasting ability of the densities is presented in Table 4. The PIT series for the parametric and the non-parametric pdfs reject the null hypothesis of independent and identical normally distributed values. Thus, we conclude that the one-month option-implied densities cannot predict the entire range of exchange rate realizations.²⁴

²⁴ The empirical results in Figure B3 in Appendix B show that there is no significant serial correlation in any of the moments of the PIT series.

| | Parametric PDF | Non-Parametric PDF |
|---------------------------|-----------------------|---------------------------|
| LR ₃ Statistic | 42.01 | 60.27 |
| | (0.00) | (0.00) |

Table 4: Berkowitz Test, One-Month Maturity

<u>Notes:</u> $H_0: \{y_t\} \sim iid \ N(0,1)$; Under the validity of the null hypothesis the test statistic follows a $\chi^2(3)$ distribution; p-values are in parenthesis.

Our findings have a bearing on the interpretation of signals sent by financial markets. We find no strong evidence for the forecasting ability of risk-neutral pdfs for the EUR/CHF exchange rate. Although they may be used as a barometer of market sentiment, they do not produce superior forecasts compared with the benchmark random walk model over horizons of more than one month, and thus do not solve the Meeze and Rogoff puzzle.²⁵

4. Conclusion

In this paper we found that financial markets attached some credibility to the Swiss franc floor, since the break probabilities never significantly exceeded 50% while the floor was in place, but especially at longer maturities there was some doubt. Break probabilities increased from January 2014 for all maturities, suggesting that the credibility of the Swiss franc floor somewhat decreased. We also found that the credibility of the Swiss franc floor decreased to some degree as the spot exchange rate approached the lower bound of 1.20 CHF per euro.

Some economists argue that financial markets are an excellent window into future developments and thus help us to understand economic policy and politics – but markets are hardly omniscient. For example, they recently struggled to forecast the Brexit vote. It is well-known that high-frequency exchange rates are notoriously difficult to predict. The associated difficulties of such an undertaking are neatly expressed in the following still valid phrase, "In my judgement, a model that was consistently able to explain 10 percent of the actual quarter-to-quarter changes in exchange rates should either be rejected on the grounds that it is too good to be true or should be reported to the Vatican as a miracle justifying the canonization of a new saint" (Mussa, 1979, p. 50).

In this paper we compared the forecasting performance of a random walk benchmark model with an error-correction model augmented with option-implied break probabilities. For the one-month parametric and non-parametric break probabilities, we found some evidence that the ECM has an informational advantage over the random walk, while we found that the ECM has no informational

²⁵ The tractability and ease of implementation of our modelling framework should make the approach adaptable to other exchange rate target regimes.

advantage over the random walk for the three-month maturity. Considering the forecasting performance of the option-implied pdfs, we found that the one-month option-implied densities cannot predict the entire range of exchange rate realizations.

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Appendix A: Tables

| n of spot Enchange hat | | | |
|------------------------|--|---|---|
| 1-M Prob | 3-M Prob | 6-M Prob | 12-M Prob |
| -0.0933 | -0.1876 | -0.2678 | -0.4286 |
| (0.000) | (0.000) | (0.000) | (0.000) |
| 0.9271 | 0.9576 | 0.9665 | 0.9318 |
| - | <u><i>I-M Prob</i></u> -0.0933 (0.000) 0.9271 | I-M Prob 3-M Prob -0.0933 -0.1876 (0.000) (0.000) 0.9271 0.9576 | I-M Prob 3-M Prob 6-M Prob -0.0933 -0.1876 -0.2678 (0.000) (0.000) (0.000) 0.9271 0.9576 0.9665 |

Table A1: Regression of Spot Exchange Rate on Non-Parametric Break Probabilities 06/09/2011 to 14/01/2015

Notes: OLS estimates of slope coefficient of: $S_t = \alpha + \beta x_t^{\tau} + \epsilon_t$. S_t is the time t spot exchange rate and x_t^{τ} is the τ -month forward looking non-parametric break probability.

| | <u>ME</u> | MAE | <u>RMSE</u> | <u>MSE</u> | MSE-F | <u>MSE-t</u> | ENC-F | ENC-t |
|------------------|-----------|---------|-------------|------------|---------|--------------|---------|--------|
| <u>1 Month</u> | | | | | | | | |
| Benchmark | 0.00280 | 0.54463 | 0.79432 | 0.63094 | | | | |
| Model | | | | | | | | |
| Para | -0.21966 | 0.62647 | 0.82086 | 0.67381 | -39.191 | -1.922 | 33.998* | 3.042* |
| Non-Para | -0.22911 | 0.63121 | 0.82997 | 0.68886 | -51.792 | -2.556 | 26.667* | 2.448* |
| <u> 3 Months</u> | | | | | | | | |
| Benchmark | -0.03478 | 0.74774 | 0.96987 | 0.94065 | | | | |
| Model | | | | | | | | |
| Para | -0.37901 | 0.99935 | 1.18399 | 1.40185 | -188.18 | -10.970 | -33.735 | -4.196 |
| Non-Para | -0.35022 | 0.93414 | 1.14084 | 1.30152 | -158.60 | -10.172 | -34.153 | -4.660 |

Table A2: MSE and ENC Tests of the Benchmark Model vs. ECM, 1 and 3-Month Maturities

<u>Notes:</u> 5%-critical values for 1 month (3 months): MSE-F=1.441(1.524), MSE-t=0.396(0.422), ENC-F=3.128 (3.054), ENC-t=1.423(1.429); In-sample period for 1 month and 3 months: 06/10/2011 to 03/09/2012 and 07/12/2011 to 02/11/2012, respectively; Out-of-sample period for 1 month and 3 months: 05/09/2012 to 14/01/2015 and 05/11/2012 to 14/01/2015, respectively.

| | ME | MAE | RMSE | <u>MSE</u> | MSE-F | MSE-t | ENC-F | ENC-t |
|-----------------|----------|---------|---------|------------|---------|--------|---------|--------|
| <u>1 Month</u> | | | | | | | | |
| Benchmark | -0.10128 | 0.48154 | 0.68235 | 0.46507 | | | | |
| Model | | | | | | | | |
| Para | -0.39263 | 0.58963 | 0.72751 | 0.52927 | -58.218 | -2.292 | 36.214* | 2.590* |
| Non-Para | -0.39867 | 0.59403 | 0.74012 | 0.54778 | -72.607 | -2.963 | 26.089* | 1.970* |
| <u>3 Months</u> | | | | | | | | |
| Benchmark | -0.26877 | 0.63765 | 0.78947 | 0.62327 | | | | |
| Model | | | | | | | | |
| Para | -0.72094 | 0.90654 | 1.04262 | 1.08706 | -187.72 | -9.280 | -23.558 | -2.502 |
| Non-Para | -0.66152 | 0.84285 | 0.98116 | 0.96268 | -155.13 | -8.657 | -23.249 | -2.713 |

Table A3: MSE and ENC Tests of the Benchmark Model vs. ECM, 1 and 3-Month Maturities

<u>Notes:</u> 5%-critical values 1 month (3 months): MSE-F=1.782(1.818), MSE-t=0.608(0.626), ENC-F=2.549 (2.499), ENC-t=1.433(1.425); In-sample period for 1 month and 3 months: 06/10/2011 to 06/03/2013 and 07/12/2011 to 07/05/2013, respectively; Out-of-sample period for 1 month and 3 months: 07/03/2013 to 14/01/2015 and 08/05/2013 to 14/01/2015, respectively.

Table A4: PT-Test for Break Probabilities, 1 and 3-Month Maturities

| Parar | <u>netric</u> | Non-Pa | rametric |
|----------------|-----------------|----------------|-----------------|
| <u>1 Month</u> | <u>3 Months</u> | <u>1 Month</u> | <u>3 Months</u> |
| 1.9449 | 1.2895 | 1.3228 | 1.9449 |

<u>Notes:</u> 5%-Critical Value: 1.96; In-sample period for 1 month and 3 months: 06/10/2011 to 03/09/2012 and 07/12/2011 to 02/11/2012, respectively; Out-of-sample period for 1 month and 3 months: 05/09/2012 to 14/01/2015 and 05/11/2012 to 14/01/2015, respectively.

| Table A5 | : PT-Test | for Break | Probabilities, | 1 and 3 | 3-Month | Maturities |
|----------|-----------|-----------|----------------|---------|---------|------------|
|----------|-----------|-----------|----------------|---------|---------|------------|

| Paran | netric | Non-Par | rametric |
|----------------|-----------------|----------------|-----------------|
| <u>1 Month</u> | <u>3 Months</u> | <u>1 Month</u> | <u>3 Months</u> |
| 1.8909 | 1.2895 | 1.4431 | 1.9449 |

<u>Notes:</u> 5%-critical value: 1.96; In-sample period for 1 month and 3 months: 06/10/2011 to 06/03/2013 and 07/12/2011 to 07/05/2013, respectively; Out-of-sample period for 1 month and 3 months: 07/03/2013 to 14/01/2015 and 08/05/2013 to 14/01/2015, respectively.

Appendix B: Figures

Figure B1: One-Month Non-Parametric Option Implied Probability Density Functions, 2011-2014





Figure B2: Non-Parametric Skewness and Excess-Kurtosis Coefficients, One to Twelve Months



Figure B3: Autocorrelation functions of PIT Variables for the One-Month Maturity

Autocorrelation Functions of Non-Parametric PITs



Appendix C: Estimation of Black-Scholes implied volatility with non-parametric method

For the non-parametric method, Appendix C describes the calculation of the BS implied volatility σ_t as a function of strike price X using prices of options that are ATM, and OTM with a δ of 25%, denoted by $\sigma_{25\delta,t}(X)$, which can be inserted into equation (3). The description follows Blake and Rule (2015).

By put-call-parity the delta of a put option is equal to one minus the call delta, such that the call delta can be used as a general metric for moneyness. One standard Black-Scholes assumption is that the implied volatility σ_t is constant for all strike prices and hence all values of delta. This assumption is violated in reality, as implied volatility tends to increase for further out-of-the-money (OTM) options, resulting in the so called volatility smile. Often the volatility smile is referred to as a U-shaped function, which lies in (σ, δ) space. Its shape and level can be described by three option bundles, which are heavily traded in OTC markets. First the ATM straddle (atm_t) , which combines the purchase of an ATM call and ATM put option. The second one is the 25 δ risk reversal $(rr_{25\delta,t})$, which consists of an OTM put sell and an OTM call purchase, where both have the same delta.²⁶ Third there is the 25 δ butterfly $(bf_{25\delta,t})$, which consists of an OTM call purchase and an ATM straddle sale. In terms of implied volatilities the three combinations are given by²⁷:

$$atm_t = \sigma_{50\delta c,t} + \sigma_{50\delta p,t} \tag{C1}$$

$$rr_{25\delta,t} = \sigma_{25\delta c,t} - \sigma_{25\delta p,t} \tag{C2}$$

$$bf_{25\delta,t} = \frac{\sigma_{25\delta c,t} + \sigma_{25\delta p,t}}{2} - atm_t \tag{C3}$$

The ATM straddle determines the overall level of the smile, the risk-reversal the symmetry and the butterfly the curvature. The shape of the volatility smile is closely related to the shape of the underlying risk-neutral distribution. When the volatility smile is asymmetric due to a positive risk reversal, then the underlying risk-neutral distribution will have a positive skewness with more probability mass in the right tail. When talking about the EUR/CHF exchange rate this would indicate that the market considers a depreciation of the Swiss franc as more likely than an appreciation. Moreover, when the volatility smile has a higher curvature due to a higher butterfly, the underlying distribution will become leptokurtotic, indicating that the market considers large moves in either direction as more possible. To calculate the smile, Malz (1997) proposed to use the following quadratic approximation

$$\sigma_{25\delta,t}(\delta) = b_0 atm_t + b_1 rr_{25\delta,t}(\delta - 0.5) + b_2 bf_{25\delta,t}(\delta - 0.5)^2.$$
(C4)

Malz (1997) uses the property of the ATM straddle of having a delta equal to 50% and the definitions of risk-reversals and butterflies to derive $(b_0, b_1, b_2) = (1, -2, 16)$. To be able to take the second derivative of the call-price function as in equation (2), Malz (1997) transforms the volatility smile from (σ, δ) -space to (X, δ) -space, resulting in $\sigma_{25\delta,t}(X)$, which can be inserted into equation (3). The transformation is done by substituting equation (4) into equation (C4), resulting in

²⁶ Statistically, a risk-reversal is an indicator of the degree of skewness in the distribution. Risk reversal indices are widely used in theoretical as well as empirical research. See, for example, Beber et al. (2010) and Brunnermeier et al. (2009) for applications of the risk reversals in the foreign exchange markets.

²⁷ The bundles presented here are constructed from options with δ equal to 25%. The 25 δ options market is the most liquid one. Bloomberg provides those option bundles also for other delta values, for example 10% or 35%.

$$\begin{split} \sigma_{25\delta,t} &= atm_t - 2rr_{25\delta,t} \left(e^{r_t^* \tau} \Phi \left[\frac{\ln\left(\frac{S_t}{X}\right) + \left(r_t - r_t^* + \frac{\sigma_{25\delta,t}}{2}\right) \tau}{\sigma_{25\delta,t} \sqrt{\tau}} \right] - 0.5 \right) \\ &+ 16bf_{25\delta,t} \left(e^{r_t^* \tau} \Phi \left[\frac{\ln\left(\frac{S_t}{X}\right) + \left(r_t - r_t^* + \frac{\sigma_{25\delta,t}}{2}\right) \tau}{\sigma_{25\delta,t} \sqrt{\tau}} \right] - 0.5 \right)^2, \end{split}$$
(C5)

and solving for $\sigma_{25\delta,t}$ numerically.

Appendix D: Estimation of Black-Scholes implied volatility with non-parametric method

For the non-parametric method, Appendix D describes the calculation of the BS implied volatility σ_t as a function of strike price X, $\sigma_t(X)$, following Blake and Rule (2015). The resulting function, $\sigma_t(X)$, calculated according to Appendix D, is then substituted into the Black-Scholes formula of equation (3) to calculate the approximated second derivative by applying the second order centralized differential quotient as in equation (5).

The non-parametric method uses a clamped cubic spline with flat ends to calculate the volatility smile. A clamped cubic spline is a function which is used to inter- and extrapolate a set of data points, $\{(x_1, y_1), ..., (x_n, y_n)\}$, such that the resulting function is continuously differentiable at the nodes. At the boundary points (x_1, y_1) and (x_n, y_n) , Malz (2014) proposes to assume the interpolated function to have a derivative of zero, such that it has flat ends. The fitted function is defined piecewise and can be described by the general formula

$$y(x) = \begin{cases} x_1 \text{ for } x < x_1 \\ f(x) \text{ for } x_1 \le x < x_n \\ x_n \text{ for } x \ge x_n \end{cases}$$
(D1)

To apply the interpolation within the (σ, δ) -space we use seven data points for every day, namely the ATM-straddle, 10%, 25% and 35% delta put and call implied volatilities. According to Malz (1997) the OTM implied volatilities can be derived by using the formulas:

$$\sigma_{x\delta c,t} = atm_t + bf_{x\delta,t} + 0.5rr_{x\delta,t} \tag{D2}$$

$$\sigma_{x\delta p,t} = atm_t + bf_{x\delta,t} - 0.5rr_{x\delta,t} \tag{D3}$$

Since the spline function interpolates the smile between deltas of 10% and 90%, extrapolation has to be done to calculate the entire volatility smile. The flat end extrapolation is used in order to prevent the resulting volatility smile from violating no-arbitrage restrictions. Those impose that for strike prices becoming very large and very small the slope of the volatility smile converges to a value of zero.

To transform the volatility smile from (σ, δ) -space to (X, δ) -space, the resulting time t volatility smile, $\sigma_t(\delta)$, is inserted into the function for delta of equation

(4), which can be numerically solved for the strike price.²⁸ The resulting function, $\sigma_t(X)$, will be substituted into the Black-Scholes formula of equation (3) to calculate the approximated second derivative by applying the second order centralized differential quotient as in equation (5). Because the piecewise cubic polynomial function is very flexible, it might happen that for sufficiently small values of *h* the resulting density function is partly negative, due to interpolation imposed no-arbitrage violations. To overcome this issue, Malz (2014) suggests to set *h* large enough, such that there are no negative density values. In general, Malz (2014) argues that the propensity to generate negative densities is greatest when the general level of volatility is high. Hence a value for *h*, which produces a non-negative density at a date with very high ATM implied volatility, is likely to generate decent densities for all other dates.²⁹

²⁸ The solution method works by assigning for every strike price X a starting value to $\sigma_t(\delta)$ and then iterating between $\sigma_t(\delta)$ and the function for delta of equation (4), until a value for δ is found for which the resulting $\sigma_t(\delta)$ creates exactly that δ when inserted into equation (4). The MATLAB code to inter- and extrapolate and solving for $\sigma_t(X)$ is provided by Blake and Rule (2015).

²⁹ Malz (2014) argues that "...the estimated risk neutral probabilities are not terribly sensitive to variations in h...". Again we have adjusted the MATLAB code of Blake and Rule (2015), to adjust the step size in equation (5).

Appendix E: Recovering the Real-World EUR/CHF Densities

To model the relationship between risk-neutral densities (RNDs) and real-world densities (RWDs) one can make assumptions about risk preferences. Jackwerth (2000) argues that the risk neutral probability is equal to the real world probability times a risk aversion adjustment. Bearing this in mind, RWDs can be derived by considering specific utility functions to describe those preferences. Bliss and Panigirtzoglou (2004) have employed the CRRA utility function

$$u(x) = \frac{x^{1-\rho} - 1}{1-\rho},$$
(E1)

where ρ is the constant relative risk aversion parameter. As before let the RND be denoted by $\pi_t^{\tau}(x)$ and the new introduced RWD by $q_t^{\tau}(x)$. Bliss and Panigirtzoglou (2004) apply the CRRA transformation based on the general relationship between $q_t^{\tau}(x)$ and $\pi_t^{\tau}(x)$, derived by Ait-Sahalia and Lo (2000), as

$$q_t^{\tau}(x) = \frac{\frac{\pi_t^{\tau}(x)}{u'(x)}}{\int_0^{\infty} \frac{\pi_t^{\tau}(y)}{u'(y)} dy} = \frac{x^{\rho} \pi_t^{\tau}(x)}{\int_0^{\infty} y^{\rho} \pi_t^{\tau}(y) dy},$$
(E2)

where x and y stand for the potential realizations of the exchange rate at expiration. The term in the numerator of (E2) adjusts the RND to account for the risk preferences of the representative agent, while the term in the denominator rescales the transformed function, such that it integrates to unity. As can be seen from (E2), the degree of risk aversion will change the shape of the RWD compared to the RND over the full range of possible realizations. Liu et al. (2007) have also used this elegant transformation.

In what follows we are going to back out the RWDs for a range of values for the risk aversion parameter which has been found so far in the literature. A plausible assumption is that the relative risk aversion parameter is in the range of $3 < \rho < 10$. Several authors argued that $\rho = 3$ could serve well as a first order approximation for real-world risk taking behaviour in financial markets. Bliss and Panigirtzoglou (2004) and Liu et al. (2007) found relative risk aversion parameters between 2 and 4. Mehra and Prescott (1985) imposed an upper bound of 10 for the relative risk aversion parameter.³⁰

Figures (E1) and (E2) present parametric and non-parametric RNDs and RWDs for the one- and threemonth maturities on two specific dates between 6 September 2011 and 14 January 2015, which are representative for the particular time periods when the minimum exchange rate of CHF 1.20 per euro was in place. It is apparent that for the one- and three-month maturities the shape of the parametric and non-parametric RNDs and RWDs is almost the same for a reasonable range of ρ , with a small mean shift of the non-parametric densities for $\rho = 10$. Figures (E3) and (E4) present plots of the first, third and fourth moments of the parametric RWDs for $\rho = 3$ and $\rho = 10$. While there is almost no difference for $\rho = 3$, only a small difference in the skewness between 2012 and 2013 occurs for $\rho =$ 10 for the three-month maturity. A similar pattern can be observed for the non-parametric densities in Figures (E5) and (E6). For $\rho = 10$ there is a slight mean shift apparent and a small deviation in the skewness. Overall we conclude that during the minimum exchange rate of CHF 1.20 per euro the difference between RNDs and RWDs is negligible for a reasonable degree of risk aversion.³¹

³⁰ In the finance literature a few authors have argued that a risk aversion coefficient in the range of $20 < \rho < 30$ is needed to account for the equity premium puzzle. See, for example, Kandel and Stambaugh (1991). This, however, is a highly unplausible level of risk aversion. The relative risk aversion parameter $\rho = 25$, for example, implies that agents would rather accept a 17 percent reduction in consumption with certainty than a 50:50 chance of a 20 percent reduction (Romer, 2012, p. 388).

³¹ This finding is not unexpected. Mirkov et al. (2016) have estimated RNDs and RWDs for the future CHF/EUR exchange rate from option implied densities, by adjusting the mean of the RND by the currency risk premium. They find that both were nearly similar. Moreover De Santis and Gèrard (1997) find that in foreign exchange markets the risk premium is on average small.





0.004

0.002

0

1.16

1.18

1.2

1.22

1.24

1.26

1.28

1.3

1.32





Figure E2: Parametric and Non-Parametric RNDs and Calibrated RWDs, 3 Months Ahead

Figure E3: Time Series of Option-Implied RND and RWD Parametric Moments, $\rho = 3$





Figure E4: Time Series of Option-Implied RND and RWD Parametric Moments, $\rho = 10$

Figure E5: Time Series of Option-Implied RND and RWD Non-Parametric Moments, $\rho = 3$







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