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Global Value Chains and Effective Exchange Rates at the Country-Sector Level*

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Abstract

The real effective exchange rate (REER) is one of the most cited statistical constructs in open-economy macroeconomics. We show that the models used to compute these numbers are not rich enough to allow for the rising importance of global value chains. Moreover, because different sectors within a country participate in international production sharing at different stages, sector level variations are also important for determining competitiveness. Incorporating these features, we develop a framework to compute REER at both the sector and country level and apply it on inter-country input-output tables to study the properties of the new measures of REER for 35 sectors in 40 countries.

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1 Introduction

The Real Effective Exchange Rate (REER) is one of the the most quoted indices in international economics. It measures competitiveness by quantifying the sensitivity of demand for output originating from a particular country as a function of changes in world prices.[1] Leading organizations like the International Monetary Fund (IMF), Bank of International Settlements (BIS), OECD as well as various central banks around the world devote substantial time, effort and resources into computing and analysing these REER indices. However, most of these standard REER measures make a number of simplifying assumptions which are increasingly becoming questionable in a world characterized by global value chains. For instance, they assume that every country exports only final goods which are produced without using imported intermediate goods, and work with single sector models, implicitly imposing that all sectors within a country are identical in terms of their interaction with each other and the rest of the world. The pitfall associated with these assumptions can be illustrated in the context of a stylized world with three countries involved in a global value chain–China, Japan and the US. Suppose Japan manufactures raw materials for the production of a mobile phone and ships it to China which acts as an assembly point. China in turn exports the finished product to the US, where it is consumed by US consumers. Not recognizing this global value chain structure, traditional models would assume that Japan exports final goods to China, and hence a depreciation of the Japanese currency hurts Chinese competitiveness. In reality however, a decrease in the price of Japanese components could very well lead to an increase in demand for China's output and hence an improvement of its competitiveness, i.e a depreciation of its REER. This example shows that REER computed using the standard frameworks is not only inaccurate in terms of magnitude, but may also have the wrong sign. Accounting for such influences is becoming increasingly more important given the rise in intermediate goods trade in the last two decades as shown by Wang et al. (2013).[2]

Addressing these issues through the lens of a macroeconomic model designed to capture global value chains and sectoral heterogeneity within and across countries, the paper makes four novel contributions to the literature on REER.

Firstly, we depart from the single sector framework and allow for multiple sectors within each country. Sectors differ in the nature and extent of their interaction with each other and

[1] See Chinn (2006) for a primer on the concept of REER and Rogoff (2005) for an application and discussion.

[2] For OECD countries Miroudot et al. (2009) find the share of trade in intermediate goods and services to be 56% and 73% respectively. As emphasized in Baldwin and Lopez-Gonzalez (2012), intermediate goods trade and vertical specialization have grown many-fold in developing countries starting in the 1980s (see also Wang et al., 2013) Also important is the import content of exports, epitomized by the prevalence of processing trade involving Asian economies, especially China. Koopman et al. (2014) find that the import content of exports is as high as 90 percent for some sectors in China.

with sectors in the rest of the world, both in terms of sourcing inputs as well as selling their output. We build on the literature (in particular Bems and Johnson (2017)) that has already shown the importance of distinguishing intermediate and final goods trade in measuring REER, and find that allowing for sectoral heterogeneity provides an even bigger improvement in the quantification of REER as compared to incorporating imported inputs in a single sector framework. Sectoral heterogeneity affects both components of REER-weights as well as prices. The intuition comes from the fact that sectors that are typically prominent exporters are also simultaneously large importers, and hence models which aggregate them with sectors that non-traded, overstate the domestic value added content in exports and hence the impact of changes in foreign prices on competitiveness as measured by the REER.

Secondly, acknowledging that value added and gross output (as well as exports) become delinked in the presence of imported inputs, we develop separate REER measures for value added (GVC-REER) and gross output (Q-REER). While value added competitiveness is the primary object of interest, gross output competitiveness may also be useful in certain contexts. For instance, productivity gains from exports may be linked to gross exports rather than their value added counterparts, due to positive spillovers from imported inputs and technology.

Thirdly, apart from country-level REERs, we also develop REER measures at the sectoral level. With increasing specialization and trade in intermediate inputs, inter-sectoral linkages between countries differ substantially from aggregate country level relationships. Wang et al. (2013) have documented and explored various dimensions of this heterogeneity and have found that different sectors in a country tend to participate in cross-border production sharing by different extents and in different ways. For example, according to Wang et al. (2013) , some sectors mostly engage in regional value chains (i.e., buying or selling intermediate inputs with neighboring countries), whereas others engage in truly global value chains (i.e., sourcing and selling a significant amount of inputs to countries on different continents). This implies substantial heterogeneity in changes in competitiveness across sectors to a given change in foreign price vector. An aggregate country level measure is incapable of capturing these. Indeed, in section 8 we document several instances where the REERs move in opposite directions for different sectors within a country. This makes sectoral REERs a useful addition to the information set of policy makers.

Fourthly, we apply the framework to a bilateral context and develop a new measure of bilateral real exchange rate (GVC-RER) that provides an improvement upon the current measures by acknowledging the presence of sectoral heterogeneity and trade in inputs.

We take the framework to the data by parametrizing the model using global input output tables from the world input-output database (WIOD), and create a database of REER weights and REER indices for 40 countries and 1435 country-sector pairs for the period 1995-2009. The results highlight the importance of the novel features introduced in the modeling part. For instance, we find that while allowing for the distinction between intermediate and final goods flows does lead to differences in REER weights (as shown by Bems and Johnson (2017)) the difference is much starker once we allow for sectoral heterogeneity as well. With regard to the much discussed case of China's REER, our results reveal two important findings over the period 1995-2009. Firstly, we find that the exchange rate appreciated more during this time when viewed in value added terms (GVC-REER) as opposed to gross output terms (Q-REER), signaling that a depreciation in countries from which China sourced inputs lead to an increase in the competitiveness of its exports which was greater than that of its own domestic factors of production. Secondly, when viewed in a bilateral context against the US, the appreciation of the Chinese exchange rate from 1995-2009 was much more pronounced (i.e the case for the exchange rate being "undervalued" during this time is much weaker) under our GVC-RER scheme that correctly accounts for China's complex participation in global value chains, compared to the standard measures which ignore these features. On the debate on imbalances within the eurozone, our results based on the bilateral GVC-RER between Greece and Germany indicate that the fall in competitiveness for Greece (i.e an appreciation) vis-a-vis Germany from 1995-2009 was much larger when measured through GVC-RER as opposed to the conventional real exchange rate measure, indicating that the standard bilateral REER measure underestimated the extent to which lack of inflation in Germany was hurting competitiveness in Greece.

It is worth emphasizing that our objective is to model relatively short-term and small-scale movements in competitiveness. We therefore take the nature of GVC and trade patterns across countries and sectors as given and do not consider the issue of endogenous off-shoring and production sharing decisions.[3]

The remainder of the paper is structured as follows: We begin with a discussion of the literature on REER in section 2, followed by a brief introduction to the concept of REER in section 3. Section 4 presents the main model which is then used in section 5 to develop new measures of REER at the sector level. Section 6 uses the sector-level framework to develop measures of REER at the country level. Section 7 provides a stylized example to illustrate the properties of the new REER measure. We then proceed to the empirical part of the paper in section 8 which focusses on highlighting properties of the new REER measures

[3]There is a growing literature on organization of global value chains that looks into these questions. See for instance Antràs and Chor (2013), Antràs (2014), Costinot et al. (2013) and Johnson and Moxnes (2012). Moreover, due to the complex nature of the model we solve it using log linearization techniques. This further reinforces the view that our REER indices are best suited for short-term movements resulting from shocks that are not too large so as to affect organization of Global value chains (GVCs).

computed from the data, both at the country and at the sector level, drawing comparisons with the existing literature in the process. Section 9 discusses the extension of the analysis to a bilateral setting, and section 10 concludes.

2 Related Literature

This paper is a contribution to the literature on international trade and price competitiveness. As mentioned above, the most prominent and commonly cited REER measures today do not distinguish between trade in intermediate and final goods and consider all trade flows to be in the latter category. A few recent papers have recognized this drawback and have made attempts to address them. Bems and Johnson (2017) allow for trade in intermediates and compute the REER weighting matrix at the country level. They however do not allow for sectoral heterogeneity which we find to be a critical feature in determining patterns in competitiveness, and do not consider REERs at the sector level. Our attempt to incorporate sector level price indices and build sector level exchange rates has a precedent in the work of Bennett and Zarnic (2009). However, their work does not incorporate trade in intermediate goods and uses an IMF-like weighting matrix. In developing measures designed specifically to measure external (export) competitiveness, Lommatzsch et al. (2016) also construct REER indices in which they adjust the price index by putting weight on sectors in accordance with their export shares. However, they do not allow for sectoral heterogeneity in weights, which in turn also influences the relevant foreign price index used in the REER computation and hence biases the REER measure as we show in the paper. With regard to gross output competitiveness, Bayoumi et al. (2013) propose a measure in which they borrow the weighting matrix from the IMF but adjust the price indices to acknowledge the presence of imported inputs. As we show however, this only ends up being a partial adjustment as the original weighting matrix is not adjusted.

More broadly, the paper is motivated by and is linked to the relatively new but rapidly expanding literature on global value chains and their implications for macroeconomic variables (see Hummels et al., 2001, Baldwin and Lopez-Gonzalez, 2012 and Auer et al. (2016)) as well as the literature on trade statistics and export accounting in the presence of intermediate goods trade (Koopman et al., 2012 and Wang et al., 2013).

3 The Concept of REER as a Measure of Competitiveness

The real effective exchange rate measures change in competitiveness by quantifying changes in the demand for goods produced by a country as a function of changes in relative prices.[4] To be more precise, if V_J is the demand for the goods produced (or alternatively, value added) by country J, then the effective exchange rate of country J is defined as:

$$\triangle REER_J = \triangle V_J = G_J\left(\{\triangle p\}_{i=1}^{n}\right) \tag{3.1}$$

where $\{\triangle p\}_{i=1}^{n}$ is a vector of price changes in all countries including the home country. Note that by assumption no other variables except the prices explicitly enter the function $G(.)$. Hence by construction REER is a partial equilibrium construct where the primitive shocks that lead to the observed price changes are not modeled. Moreover the demand side of the economy is assumed to be exogenous and the aggregate final demand is assumed to be constant (although demand is allowed to switch between different goods in response to changes in prices).

The function $G_J(.)$ is homogenous of degree zero, so that the model satisfies neutrality in the sense that if all prices (including the home price) double, then the relative demands remain unchanged, and since by construction aggregate demand is held fixed, the absolute demand for each good also remains unaffected.

It is important to note that REER models like ours do not assume balanced trade or any restrictions on the trade balance. In our empirical framework, trade balances are allowed to be non-zero in the steady state and are calibrated to their observed counterparts in the data. This is in line with the partial equilibrium setup common in the literature in which the demand side is exogenous. The motivation for doing so comes from the acknowledgment that price competitiveness is a supply side concept and measures how changes in the cost structure of a producer makes its product more competitive by enabling it to capture demand from other producers. In order to isolate the role of competitiveness it is therefore critical to abstract from the influences coming from the demand side.

4 The Model

Consider a world economy comprising n countries. There are m sectors within each country. Each country-sector is called a "production entity" and there are a total number nm of these production entities in the world economy. Each entity uses a production function with its own value added and a composite intermediate input which can contain intermediate inputs from all mn entities including itself. The output of each entity can be used either as a final good (consumed in any of the n countries) or as an input by another entity. Hence there

[4] In this literature the use of the word "competitiveness" is appropriate only in the context of price competitiveness, and does not necessarily extend to the notion of competitiveness measured in terms of profits, market shares etc.

are a total of nm producers and $nm+n$ consumers (nm entities plus n final goods consumers) in the economy. Both the production function and final goods consumption aggregators are nested CES (constant elasticity of substitution) aggregators which are described in detail next.

4.1 Production

The production function for entity (c, l) is given the expression:[5]

$$Q_l^c = \left[(w_l^{vc})^{1/\sigma^3}(V_l^c)^{\frac{\sigma^3-1}{\sigma^3}} + (w_l^{Xc})^{1/\sigma^3}(X_l^c)^{\frac{\sigma^3-1}{\sigma^3}}\right]^{\frac{\sigma^3}{\sigma^3-1}} \tag{4.1}$$

Here, V_l^c is the value added by the entity, X_l^c is an aggregate bundle of intermediate inputs, and σ^3 is the elasticity of substitution between the two inputs. X_l^c is in turn a CES aggregate, combining inputs from each entity (i, s), denoted by X_{sl}^{ic}, in a three stage nested CES aggregate with elasticities of substitution σ^2, σ^{1h} and σ^1 as follows:

The intermediate input X_l^c is and aggregate of m sectoral components,

$$X_l^c = \left[\sum_{s=1}^{m}(w_{sl}^c)^{1/\sigma^2}(X_{sl}^c)^{\frac{\sigma^2-1}{\sigma^2}}\right]^{\frac{\sigma^2}{\sigma^2-1}} \tag{4.2}$$

each of which is in turn a combination of the sector s input from the domestic sector and an aggregate sector s input form foreign sectors. The elasticity of substitution between these two inputs is σ_s^{1h}. [6]

$$X_{sl}^c = \left[(w_{sl}^{cc})^{1/\sigma_s^{1h}}(X_{sl}^{cc})^{\frac{\sigma_s^{1h}-1}{\sigma_s^{1h}}} + (w(f)_{sl}^c)^{1/\sigma_s^{1h}}(X(f)_{sl}^c)^{\frac{\sigma_s^{1h}-1}{\sigma_s^{1h}}}\right]^{\frac{\sigma_s^{1h}}{\sigma_s^{1h}-1}} \tag{4.3}$$

For the sector s foreign input bundle, inputs from all foreign countries from that sector are aggregated to form sectoral intermediate inputs $\{X(f)_{sl}^c\}_{s=1}^m$. In other words, $X(f)_{sl}^c$ is the aggregate sector s foreign intermediate input used in production by country c sector l

$$X(f)_{sl}^c = \left[\sum_{i=1, i\neq c}^{n}(w_{sl}^{ic})^{1/\sigma_s^1}(X_{sl}^{ic})^{\frac{\sigma_s^1-1}{\sigma_s^1}}\right]^{\frac{\sigma_s^1}{\sigma_s^1-1}}, s = 1, 2, ..m \tag{4.4}$$

[5]Notation: We use superscripts to denote countries and subscripts to denote sectors. When 2 scripts are present, the first one denotes the source country and the second denotes the destination country. For example, X_{sl}^{ic} denotes output produced by (source) country i sector s that is used by (destination) country c sector l.

[6]With this two step framework we are allowing for a distinction between "macro" (σ^{1h}) and "micro" (σ^1) elasticities for each sector, which is a feature of the data documented in the literature–see Feenstra et al. (2014).

Here X_{sl}^{ic} denotes inputs from country i sector s used in production by country c sector l, the w's are aggregation weights and σ^1 is the (constant) elasticity of substitution between different foreign varieties of the sector s output in the production function of entity (c, l)

4.2 Preferences

A country specific final good is obtained by aggregating goods from all nm production entities in two stages.

$$F_s^c(f) = \left[\sum_{i=1, i\neq c}^{n} (\kappa_s^{ic})^{1/\theta_s^1(c)} (F_s^{ic})^{\frac{\theta_s^1(c)-1}{\theta_s^1(c)}} \right]^{\frac{\theta_s^1(c)}{\theta_s^1(c)-1}} \tag{4.5}$$

$$F_s{}^c = \left[(\kappa_s^{cc})^{1/\theta_s^{1h}(c)} (F_s^{cc})^{\frac{\theta_s^1(c)-1}{\theta_s^1(c)}} + (\kappa(f)_s^c)^{1/\theta_s^{1h}(c)} (F(f)_s^c)^{\frac{\theta_s^{1h}(c)-1}{\theta_s^{1h}(c)}} \right]^{\frac{\theta_s^{1h}(c)}{\theta_s^{1h}(c)-1}} \tag{4.6}$$

$$F^c = \left[\sum_{s=1}^{m} (\kappa_s^c)^{1/\theta^2(c)} (F_s^c)^{\frac{\theta^2(c)-1}{\theta^2(c)}} \right]^{\frac{\theta^2(c)}{\theta^2(c)-1}} \tag{4.7}$$

4.3 Market clearing

Gross output from an entity is absorbed either as an intermediate input or a final good (we do not allow for inventory accumulation or any inter-temporal effects). Thus the following market clearing condition holds $\forall (c, l)$

$$Q_l^c = \sum_{i=1}^{n} F_l^{ci} + \sum_{j=1}^{m} \sum_{k=1}^{n} X_{lj}^{ck} \tag{4.8}$$

5 Computation of Effective Exchange Rate Weighting Matrices

In order to define the exchange rates we take prices and final demands in all countries as exogenous and compute the demand for value added and gross output of different entities as functions of prices.

5.1 Demand for value added as a function of price of value added: (GVC-REER)

The appendix shows that the demand for value added can be written as

$$vec\left(\hat{V}_l^c\right) = W_V vec\left(\hat{p}(V)_l^c\right) + W_{FV} vec\left(\hat{F}^c\right) \tag{5.1}$$

Here $\left(vec\left(\hat{V}_l^c\right)\right)_{nmX1}$ is the vector of changes in value added stacked across all countries and sectors, and W_V and W_F are nm by nm matrices derived in the appendix. Assuming the change in final demand $vec\left(\hat{F}^c\right)$ to be zero, the nm by nm matrix premultiplying $vec\left(\hat{p}(V)_l^c\right)$ can be interpreted as a matrix of weights for the real effective exchange rate, as it measures how the demand for value added originating in a country-sector changes when price of value added changes in any other entity.

Interpretation in the case with constant elasticity:

Appendix C shows that under the constant elasticity assumption the weight assignment by country sector (h, l) to country-sector (c, s) where $(h, l) \neq (c, s)$ can be written as follows:

$$w_{ls}^{hc} = \sum_{k=1}^{n}\left[\frac{\left(p(V)_l^h V_l^{hk}\right)\left(p(V)_s^c V_s^{ck}\right)}{\left(p(V)_l^h V_l^h\right)\left(P^K F^k\right)}\right], (h, l) \neq (c, s) \tag{5.2}$$

where we use lower case w to denote constant elasticity weights. The weight assigned by country sector (h, l) to country-sector (c, s) where $(h, l) \neq (c, s)$ is a weighted sum of the value added created by country-sector (c, s) and absorbed by each of the countries $k(= 1, .., n)$, where the weights are given by the value added created by (h, l) that is absorbed in the same country k. This captures both mutual and third country competition, because the weight is high if both $\left(p(V)_l^h V_l^{hk}\right)$ and $\left(p(V)_s^c V_s^{ck}\right)$ are high, which happens when both (h, l) and (c, s) have a high share of value added exports to country k.

Relaxing the Uniform Elasticity Assumption

In this paper, we focus primarily on the constant elasticity case in order to focus on our main contribution, which is to highlight the role of sectoral heterogeneity in computing REER indices. Here we provide a brief flavor of the results when the constant elasticity assumption is relaxed, and further results, with examples are developed in the appendix.

Proposition 5.1. *Suppose all production and consumption elasticities are constant and equal* *to σ and θ respectively.*[7] *Then starting at the uniform elasticity equilibrium, the effect of a change in elasticity on the weight assigned by entity (h, l) to entity (c, s) is given by:*

$$\frac{\partial w_{ls}^{hc}}{\partial \theta} = w_{ls}^{hc} - \frac{v_l^h v_s^c \sum_{c_1=1}^{n} \sum_{c_2=1}^{n} \sum_{k=1}^{m} b_{lk}^{hc_1} b_{sk}^{cc_1} (p(Q)_k^{c_1} F_k^{c_1 c_2})}{p(V)_l^h V_l^h}, (h, l) \neq (c, s) \tag{5.3}$$

Proof(sketch): See appendix C.

Here b is used to denote elements of the global Leontief inverse matrix and $p(Q)$ is used to denote price of gross output.(5.3) shows that an increase in elasticity of substitution of consumption holding everything else constant (including the production elasticity) has two opposing effects on the weight assigned by home entity (h, l) to the foreign entity (c, s). The two terms correspond to the expenditure switching and complementarity effect. In particular, the first effect (expenditure switching) is positive and is given by the constant elasticity weight w_{ls}^{hc}, which is always positive in the constant elasticity case. In addition, there is the countervailing complementarity effect which comes from the second term on the right hand side. This term is high when the products $b_{lk}^{hc_1} b_{sk}^{cc_1}$ are high for various entities indexed by (c_1, k), which in turn happens if the outputs of the two entities are used together in production (i, e entities such as (c_1,k) which use the output of (c, s) as an input, also uses the output of (h, l) as an input).

Intuitively, when the price of (c, s) decreases, its quantity demanded increases. This effect is greater the greater is the elasticity of substitution between goods (θ). Moreover, an increase in demand for (c, s) will end up increasing the output of (h, l) if it is highly complementary with (c, s).

5.2 Gross Output Competitiveness

We also derive the demand for aggregate output as a function of the price of value added (this is analogous to the "goods" REER measure proposed in Bayoumi et al. (2013) (See appendix for steps of proof).

$$vec\left(\hat{Q}_l^c\right) = W_Q vec\left(p(\hat{V})_l^c\right) + W_{FQ} vec\left(\hat{F}^c\right) \tag{5.4}$$

Here W_Q is an nm by nm weighting matrix derived in appendix B. Again putting the change in final demand $vec\left(\hat{F}^c\right)$ to be zero, the nm by nm matrix premultiplying $vec\left(\hat{p}_l^{vc}\right)$ can be interpreted as a matrix of weights for the real effective exchange rate with regard to

[7] This is equivalent to assumption (A2) in section 6

gross competitiveness, i.e it measures how the demand for output of a country-sector changes with changes in prices of other country-sectors. This is in contrast to the first measure defined above, which looks at changes in demand for value added. (As is shown in 6.2, the two are the same in the special case where gross output is the same as value added)

6 Building Country-level REER From Ground Up

Value added weights at the country level

This section provides a method to aggregate the country-sector level weights derived above and defines country level weights analogous to the ones commonly discussed in the literature. We show that the aggregated weights so derived in general do not correspond to any of the ones proposed in the literature except in knife-edge cases. This is attributable to the fact that our measure exploits inter-sectoral linkages between countries to provide a more comprehensive measure of competitiveness than what can be obtained by using just country level data.

To derive the expression for country-level value added weights, we start with the following decomposition of the nominal GDP of country c into its different sectoral components:

$$p(V)^c V^c = \sum_{l=1}^{m} p(V)_l^c V_l^c \tag{6.1}$$

log linearizing this equation we get:

$$\hat{p}(V)^c + \hat{V}^c = \sum_{l=1}^{m} \left(\frac{p(V)_l^c V_l^c}{p(V)^c V^c} \right) \left[\hat{p}(V)_l^c + \hat{V}_l^c \right] \tag{6.2}$$

Stacking the n equations in (6.2) we can write the system in matrix notation as:

$$vec\left(\hat{p}(V)^c\right)_{nX1} + vec\left(\hat{V}^c\right)_{nX1} = R_V \left[vec\left(\hat{p}(V)_l^c\right)_{nmX1} + vec\left(\hat{V}_l^c\right)_{nmX1} \right] \tag{6.3}$$

where

$$(R_V)_{nXnm} = \begin{pmatrix} S_1^V & 0_m^{'} & .. & 0_m^{'} \\ 0_m^{'} & S_2^V & & \vdots \\ \vdots & & .. & \vdots \\ 0_m^{'} & 0_m^{'} & .. & S_n^V \end{pmatrix} \tag{6.4}$$

and $\left(S_i^V\right)_{1Xm} = \left(\frac{p(V)_1^i V_1^i}{p(V)^i V^i}, \frac{p(V)_2^i V_2^i}{p(V)^i V^i}, .., \frac{p(V)_m^i V_m^i}{p(V)^i V^i} \right)$ and 0_m is an m by 1 matrix of zeros. By definition the change in the GDP deflator is the weighted sum of change in its components and hence (6.3) reduces to

$$vec\left(\hat{V}^{c}\right)_{nX1}=R_{V}\left[vec\left(\hat{V}_{l}^{c}\right)_{nmX1}\right] \tag{6.5}$$

using (5.1) in (6.5) and imposing $vec\left(\hat{F}^{c}\right)=0$ as before we get:

$$vec\left(\hat{V}_{l}^{c}\right)=R_{V}W_{V}vec\left(\hat{p}(V)_{l}^{c}\right) \tag{6.6}$$

Defining the two measures of country level value added exchange rates:

When sector level price indices are available, (6.6) defines the change in the country level GVC-REER, i.e

$$\triangle log(GVC-REER)=W_{V}(C)vec\left(\hat{p}(V)_{l}^{c}\right) \tag{6.7}$$

where the n by nm matrix $W_{V}=R_{V}W_{V}$ is the weighting matrix which can be interpreted as follows: the weight assigned by country i to country j sector l is itself a weighted sum of the weights assigned by each sector of country i to (j,l), with the weights being proportional to the country i sector's share of value added as a fraction of total value added by country i

$$W_{V}{}_{l}^{ij}=\sum_{s=1}^{m}\left(\frac{p(V)_{s}^{i}V_{s}^{i}}{p(V)^{i}V^{i}}\right)(W_{V})_{sl}^{ij} \tag{6.8}$$

If sector level prices are not available, then we need a further approximation. In particular, we need to assume a mapping between sector level prices and GDP deflator, i.e between $\hat{p}^{vc}$ and $\{\hat{p}_{l}^{vc}\}_{l=1}^{M}$. We make the relatively uninformed assumption that all sectoral level prices change in the same proportion as the aggregate GDP deflator, i.e we make the following assumption[8].

Assumption (AP):

$$p(\hat{V})^{j}=\hat{p}(V)_{l}^{j}\forall l\forall j \tag{6.9}$$

Using this assumption we can define our second measure of country level value added exchange rate, GVC-REER(GDPdef) as follows:

[8]Note that in a world with price rigidity and producer currency pricing this assumption is satisfied automatically.

$$\triangle log(GVC - REER(GDPdef)) = W_V(CG)vec\left(\hat{p}(V)^c\right) \tag{6.10}$$

where $W_V(CG) = R_V W_V R_g$ is an n by n matrix of weights and $R_g = I_n \otimes 1_m$

Link to other measures in the literature:

Our second measure of country level exchange rates which uses only the GDP deflator (GVC-REER(GDPdef)) has an n by n weighting matrix as all other measures in the literature and we can hence make a comparison with them.

Given the country-sector level weights (W_V), the country level weights ($W_V(CG)$) have an intuitive interpretation. The weight assigned by country i to country j is a weighted sum of the weights assigned by each sector of country i to each sector of country j, with the weights being proportional to the home sector's share of value added as a fraction of total home value added.

$$W_V(GDPdef)^{ij} = \sum_{s=1}^{m} \left(\frac{p(V)^i_s V^i_s}{p(V)^i V^i}\right)\left(\sum_{k=1}^{m}(W_V)^{ij}_{sk}\right) \tag{6.11}$$

These country level weights defined here are different from others proposed in the literature in several respects. The closest to our measure is the one by Bems and Johnson (2017) who also take into account the input-output linkages in their measure and define weights in terms of value added, but do not exploit sector level linkages across countries. Because of the greater information used in our measure, it is in general different from their VAREER and IOREER, even under the assumption of all elasticities being the same. The following proposition shows that even under the uniform elasticity assumption, GVC-REER and VAREER differ from each other except in special cases.

Condition 7.1

$$v^i \sum_{c=1}^{n} b^{ic} F^{cj} = \sum_{l=1}^{m} v^i_s \sum_{c=1}^{n} \sum_{s=1}^{m} b^{ic}_{ls} F^{cj}_s \forall i, j \tag{6.12}$$

where $v^i_l = \frac{p(V)^i_l V^i_l}{p(Q)^i_l Q^i_l}$ is the value added share for entity (i, l) and b denotes a generic element of the global inter-country Leontief inverse matrix.

Proposition 6.1.

The country level weights ($W_V(GDPdef)$) defined above reduces to VAREER (and

IOREER) weights defined in Bems and Johnson (2017) if either of the two conditions below are satisfied.

1. (A2) and condition 6.12

2. (A2) and (A3) , i.e no trade in intermediates

The proof is given in appendix D. The first part of the proposition shows that outside of the knife-edge case in which the above condition is satisfied, the GVC-REER(GDPdef) weights which exploit inter-sectoral linkages between countries will dominate the VAREER measure. Intuitively, condition (6.12) is satisfied if different sectors within a country are "symmetric" with regard to their input-output linkages with the rest of the world, for in that case aggregation across sectors within a country will be a closer approximation to the behavior of each individual sector. The next section will provide an example to illustrate the role played by the condition in aggregating weights at the country level.

The second part of the proposition shows that differences between GVC-REER and VAREER vanish when there is no trade in intermediates. This shows that if there is no trade in intermediates, then aggregating trade flows across sectors within a country does not lead to any loss of information as far as computation of real effective exchange rate is concerned. Intuitively, if all production by all entities involves only own value added and no intermediates, then there is no asymmetry between sectors within a country and hence aggregation does not lead to any loss of relevant information.

To summarize, both sector level prices (the more accurate and our preferred measure) and country level prices (as in the literature) can be used to compute REER indices using the sector level weighting matrices discussed above, yielding the following two expressions for country level GVC-REERs.

$$\triangle log(GVC - REER) = W_V(C) vec\left(\hat{p}(V)^c_l\right) \tag{6.13}$$

$$\triangle log(GVC - REER(GDPdef)) = W_V(CG) vec\left(\hat{p}(V)^c\right) \tag{6.14}$$

Where $W_V(C)$ and $W_V(CG)$ are weighting matrices of dimension $n$by nm and n by $n$respectively. The two measures of Q-REER (for gross output competitiveness are defined analogously).[9] In general, weighting matrices using sector level information (W_V^{cs}) as well as REER indices incorporating sector level price variations provide a more accurate measure of competitiveness compared to the measures which use only country level aggregate trade

[9]The detailed derivations are provided in appendix

flows and price indices.[10] This is illustrated in the example that follows.

Gross output exchange rate at country level

Following a similar procedure to the one used for GVC-REER, we can define weights and exchange rates for gross output at the country level:

$$\triangle log(QREER) = W_Q(C)vec\left(\hat{p}(V)_l^c\right) \tag{6.15}$$

$$\triangle log(QREER(GDPdef)) = W_Q(CG)vec\left(\hat{p}(V)^c\right) \tag{6.16}$$

Relationship to other REER Weighting Matrices in the Literature

We now link the two REER measures proposed in the previous section and some common REER measures in the literature with particular emphasis on whether and under what conditions the different measures in the literature can be recovered from the more general measures proposed here.

Assumptions:

(A1)$m = 1$. i.e, each country has only one sector
(A2) Constant elasticity:$\sigma_1 = \sigma_2 = \sigma_3 = \theta_1 = \theta_2$=1
(A3) No intermediates in production and only final goods are traded.

Proposition 6.2.

1. *Under (A1) :*

   *GVC-REER =IOREER (*Bems and Johnson (2017)*)*

2. *Under (A1) and (A2) :*

   *GVC-REER=IOREER=VAREER (*Bems and Johnson (2017)*)*

3. *Under (A1), (A2), (A3):*

   *GVC-REER=Q-REER=IOREER=VAREER=Goods-REER (*Bayoumi et al. (2013) *) =IMF-REER (*Bayoumi et al. (2005) *)* [11]

[10] Appendix D discusses the set of restrictive conditions under which the two approaches give identical expressions

[11] *As mentioned before, note that the IMF uses the CPI to compute REER, but in this section we will use IMF-REER to denote total effective exchange rates computed with IMF weights but using the GDP deflator, to make the measure comparable with other measures proposed here and in BJ*

proof: see appendix.

The main conclusion from these results is that the GVC-REER measure does not reduce to the common measures currently in use, such as those of the Federal Reserve (Loretan, 2005), BIS (Klau et al., 2008) or IMF (Bayoumi et al., 2005), as well as the more sophisticated ones, outside of very strong assumptions.

7 Illustrative Example: A three country global value chain

Consider a world economy comprising three countries (J, C and U) and two sectors within each country indexed by subscript $i \in \{1, 2\}$. The input-output linkages are given in table 1. The example mimics a stylized global value chain of the following form:

$$J_1 \to C_2 \to (C, U) \tag{7.1}$$

Sector J_1 in country J exports raw materials to sector C_2 in country C, which combines them with its own value added input to produce final goods which are then subsequently consumed in C and exported to U. The rest of the sectors only use their own value added input and sell domestically. Next, we illustrate how sectoral heterogeneity affects the measurement of REER along two dimensions-the REER weights themselves and their interaction with sectoral price changes.

Sectoral heterogeneity and GVC-REER weights

The key advantage in our REER weighting scheme compared to the literature is that the framework is flexible enough to incorporate information from input output tables like Table 1a that are at the country-sector level, whereas the other REER schemes in the literature work with input-output linkages at the country level (analogous to table 1b).

To see the consequences of the loss of information on sectoral heterogeneity that this aggregation entails, table 2 shows the weight assigned by country U to country C and J under different REER weighting schemes. Consider first a comparison between GVC-REER and the VAREER measure of Bems and Johnson (2017). Both recognize that part of the trade between J and C comprises of intermediate goods, and hence assign a lower value to W_{UC} than the IMF weight. However, GVC-REER has a lower value of W_{UC} compared to VAREER. This is attributable to the fact that unlike VAREER, the GVC-REER recognizes that the sector in C which actually competes with U is C_2, and C_2 has a lower value added share than C_1. Hence,

Table 1 – *Input output table for example 7*

(a) Sector level input-output table

		J		C		U		JFinal	CFinal	Ufinal	total output
		J_1	J_2	C_1	C_2	U_1	U_2				
J	J_1	0	0	0	2	0	0	1	0	0	3
	J_2	0	0	0	0	0	0	1	0	0	1
C	C_1	0	0	0	0	0	0	0	2	0	2
	C_2	0	0	0	0	0	0	0	1	2.5	3.5
U	U_1	0	0	0	0	0	0	0	0	2	2
	U_2	0	0	0	0	0	0	0	0	1	1
VA		3	1	2	1.5	2	1				
total output		3	1	2	3.5	2	1				

(b) Country level input output table

	J	C	U	J final	C final	U final	Total output
J	0	2	0	2	0	0	4
C	0	0	0	0.5	2.5	2.5	5.5
U	0	0	0	0	0	3	3
Value added	4	3.5	3				
Total output	4	5.5	3				

the VAREER weight, which essentially treats U as competing with C as a whole (which is an average of C_1 and C_2) puts a higher weight on C as it overestimates the value added originating in C that is competing with U in $U's$ final goods market. Similarly, GVC-REER assigns a higher value to W_{UJ} compared to VAREER, recognizing the importance of J in determining the competitiveness of U through the input it supplies to C_2. The benchmark IMF measure not only fails to recognize this sectoral heterogeneity, but also implicitly assumes that all trade is in final goods, and hence no part of production carried out in J is consumed in U. This is evident from a value of zero assigned to W_{UJ} under IMF framework.

Note that since neither sector in U uses intermediate inputs and consequently there is no distinction between value added and gross output, and Q-REER and GVC-REER weights coincide. This equivalence breaks down only when a country imports intermediate inputs, as is the case with C in this example. In this case W_{CJ} is higher in GVC-REER (W_{CJ}=0.56) than it is under the Q-REER (W_{CJ}=-1.33). In fact, in the latter case W_{CJ} is actually negative, implying that a fall in J's prices leads to an increase in the demand for C's output (although not of its value added) as it embodies the output of J_1 which becomes more competitive with the fall in its price.

Table 2 *– REER weight assigned by J to C (W_{JC}) under different weighting schemes for example 7*

	GVC-REER	VAREER (BJ)	IMF	Q-REER
W_{UJ}	0.57	0.36	0	0.57
W_{UC}	0.43	0.67	1	0.43

Notes: GVC-REER and Q-REER are measures proposed in this paper. "VAREER (BJ)" stands for the value added exchange rate measure proposed in Bems and Johnson (2017) and "IMF" stands for the IMF REER measure based on Bayoumi et al. (2005).

Sectoral heterogeneity and REER indices

The second dimension of sectoral heterogeneity which leads to an improvement in our REER measure compared to others in the literature is the flexibility to incorporate heterogeneity in sectoral prices. As an example, consider the implications for the REER of U when the price of C_1 increases, and all other prices remain unchanged (i.e $\hat{p_{c_1}} = 1$, $\hat{p_{c_2}} = 0$, $\hat{p_{J_1}} = \hat{p_{J_2}} = \hat{p_{U_1}} = \hat{p_{U_2}} = 0$). Since the value added share of C_1 and C_2 in C are 0.57 and 0.43 respectively, the computed change in aggregate price index in C is given by $\hat{p_c} = 0.57$. Given a value of $W_{UC} = 0.43$, a measure like VAREER which uses aggregate (country level) price indices and weights would compute the change in REER for U that is different from $GVC - REER$, which recognizes that the entire weight of W_{UC} is concentrated on C_2, and since $\hat{p_{c_2}} = 0$, the $GVC - \hat{REER}_U = 0$ as well. In particular:

$$
\begin{aligned}
VA\hat{REER}_U &= W_{UC}\hat{p_c} = 0.57 * 0.43 = 0.25 \\
GVC - \hat{REER}_U &= W_{UC_1}\hat{p_{c_1}} + W_{UC_2}\hat{p_{c_2}} = 0 * 1 + 0.43 * 0 = 0
\end{aligned}
$$

Since the only price change concerns the sector in C which is entirely domestically oriented, competitiveness of U should not be affected, as rightly concluded by the GVC-REER measure. Indeed, as evident in Table 3 which shows the full sector level GVC-REER weighting matrix, the weight assigned by both sectors on U on C_1 is zero.

On the other hand, the aggregation implicit in VAREER and all other measures in the literature limits their flexibility in properly deciphering such distinctions.

8 Taking the model to the data

Our main source of data is the World Input-Output Database (WIOD), which provides a time series of input-output tables covering 40 countries and 35 sectors from 1995-2011. The data is available in both current and previous year prices which enables us to compute price

Table 3 – *Country-Sector level weighting matrix for 7*

		J		C		U	
		J_1	J_2	C_1	C_2	U_1	U_2
J	J_1	-1	0.25	0.19	0.18	0.26	0.13
	J_2	1	-1	0	0	0	0
C	C_1	0.57	0	-1	0.43	0	0
	C_2	0.29	0	0.23	-1	0.32	0.16
U	U_1	0.41	0	0	0.31	-1	0.29
	U_2	0.32	0	0	0.24	0.44	-1

indices for different entries in the input-output table. A detailed description of this database can be found in Timmer and Erumban (2012).

In order to focus on the role of sectoral heterogeneity, in this section we focus exclusively on the case with constant elasticities. Appendix F provides a detailed discussion for the heterogenous elasticity case, including the estimation of elasticities and their incorporation in GVC-REERs.

8.1 REER Weights

Table 4 summarizes the different REER weighting matrices that are generated by our framework. In addition to the country level weights which have been studied in the literature, our framework also provides weights at various levels of disaggregation. Each of these could be of interest to policy makers depending on the question at hand, and provide information which cannot be captured in the aggregated country level weights. For instance, if a price shock (whether through tariffs, productivity or the nominal exchange rate) has a heterogenous impact across different sectors in the country of origin, then the country by country-sector weighting matrix (row 3) would provide a more accurate estimate of the impact on a foreign country as opposed to the country by country weighting matrix.

As an example to illustrate the difference in REER weights in our measure compared to others in the literature, Table 5 shows the ranking of 5 largest competitor countries for Japan in the year 2007 based on the different country-level REER schemes discussed above. The GVC-REER scheme, which acknowledges the prominence of supply chain trade between China and Japan, assigns a lower weight to China, recognizing that the final destination of exports from Japan to China is not always China itself, whereas the standard IMF weighting scheme which fails to make this distinction assigns a higher weight to China than the US, due to the high volume of gross trade between China and Japan.

With the prominence of global value chains and the accompanying divergence between

Table 4 – Summary of REER weights

	Dimension
Country by country	n by n by T
Country by country-sector	n by nm by T
Country-sector by country	nm by n by T
Country-sector by country-sector	nm by nm by T

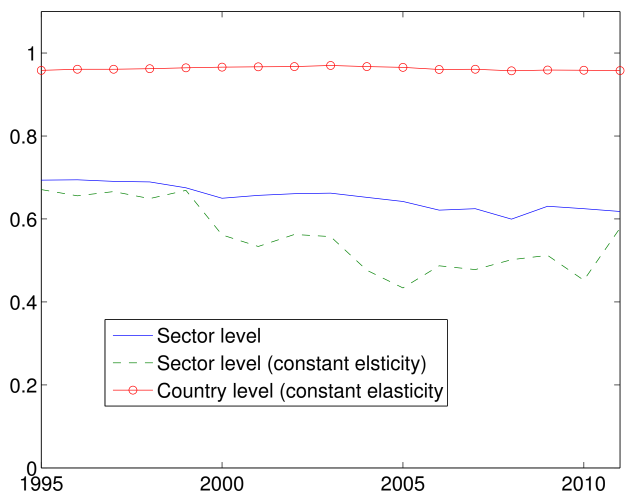
Note: n= number of countries, m = number of sectors, T = number of time periods in the sample.

Table 5 – Ranking of Countries According to REER Weight Assigned by Japan

Rank	**GVC-REER(CE)**	Goods-REER	IMF
1	**'ROW'**	'ROW'	'ROW'
2	**'United States'**	'China'	'China'
3	**'China'**	'United States'	'United States'
4	**'Germany'**	'Korea'	'Korea'
5	**'Korea'**	'Taiwan'	'Taiwan'

Notes: "ROW" stands for the rest of the world region in WIOD. "IMF" and "Goods-REER" correspond to weighting schemes proposed in Bayoumi et al. (2005) and Bayoumi et al. (2013) respectively.

Figure 8.1 – *Correlation between Value added and Gross Output REER Weights*



gross and value added exports that has been documented in the literature, our analysis allows us to see how the relationship between gross output and value added competitiveness has evolved over time. Figure 8.1 plots the cross-sectional correlation between value added (GVC-REER) and gross output (Q-REER) REER weights for the 17 years in the sample (1995-2011). While no discernible pattern emerges when we look at the country level weights, a declining trend is visible at the sectoral level, pointing towards a divergence between value added and gross output competitiveness.

These results show that as far as REER weights are concerned, the effect of global value chains is more visible at the sector level than at the country level.

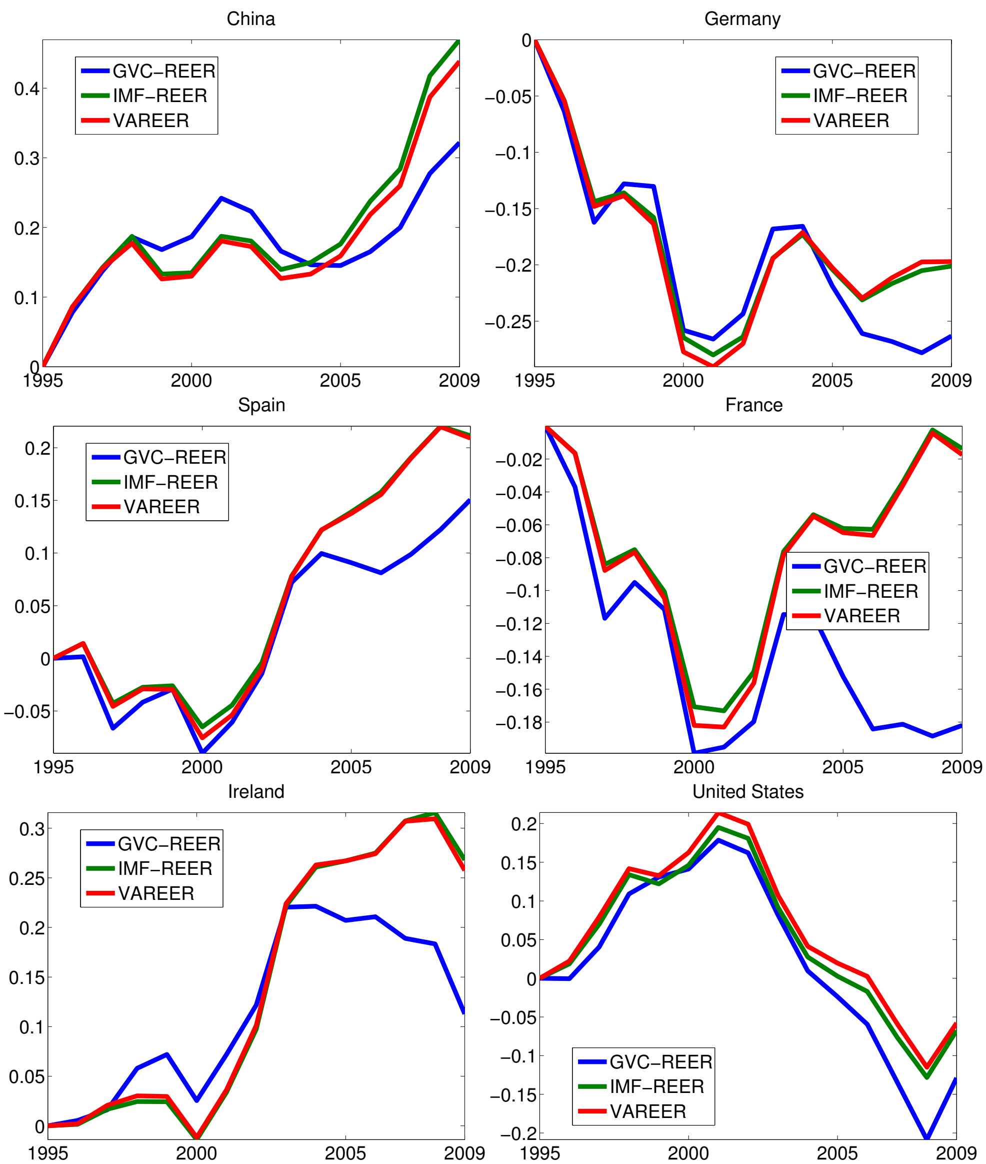
8.2 Multilateral Exchange rates

Figure 8.2 illustrates a comparison between three different REER indices. The GVC-REER is our proposed measure of REER which incorporates input-output linkages as well as sectoral heterogeneity (in both REER weights as well as price indices). The VAREER corresponds to the value-added REER measure proposed in Bems and Johnson (2017) which accounts for input-output linkages but not sectoral heterogeneity. IMF-REER is a measure which uses the IMF weighting scheme that ignores both input-output linkages as well as sectoral heterogeneity.[12]

As is evident in the figure, although the three measures move broadly in line with one

[12]To make the comparison more transparent, we use the GDP deflator in constructing the IMF-REER, instead of CPI or unit labor cost which are the other commonly used alternatives.

Figure 8.2 – *Comparison of three REER indices for select countries*



Comparison of three REER indices for select countries.The GVC-REER is our proposed measure of REER which incorporates input-output linkages as well as sectoral heterogeneity (in both REER weights as well as price indices). The VAREER corresponds to the value-added REER measure proposed in Bems and Johnson (2017) which accounts for input-output linkages but not sectoral heterogeneity. IMF-REER is a measure which uses the IMF weighting scheme that ignores both input-output linkages as well as sectoral heterogeneity.

Table 6 – Comparison of correlations across three REER indices

	GVC-REER	VAREER	IMF-REER
GVC-REER	1		
VAREER	0.92	1	
IMF-REER	0.92	0.99	1

The table displays the mean time series correlations between the indicated pairs of REERs across the sample of 41 countries. GVC-REER is the measure in developed in this paper, whereas VAREER is the measure proposed by Bems and Johnson (2015). Th IMF REER is constructed using a method similar to that in Bems and Johnson (2015).

another through the sample, differences do emerge and are particularly pronounced for certain countries in certain time periods. It is also pertinent to note that the the IMR-REER and VAREER tend to have a higher comovement with one another compared to that with the GVC-REER, an observation which is confirmed by looking more systematically at the correlations in table 6. This suggests that accounting for sectoral heterogeneity is even more important than merely accounting for the presence of input-output linkages at the aggregate country level as is done by Bems and Johnson (2017).

As shown in the preceding sections, the GVC-REER measures improve upon VAREER by incorporating sectoral heterogeneity, which enters the former in two forms (i) REER weights and (ii) price indices. To further investigate the nature and source of differences between the two measures, we decompose the difference into a term that captures the contribution of weights, and the remaining term which captures the contribution of differences in prices using the following framework.

$$\begin{aligned} REER\_GAP_{it} &= VAREER_{it} - (GVC - REER_{it}) \\ &= \sum_{j=n}^{n} W_t^{i,j,VAREER} \hat{p}_t^j - \sum_{j=1}^{n}\sum_{s=1}^{m} W_{s,t}^{i,j,GVC-REER} \hat{p_{s,t}}^j \\ &= \underbrace{\sum_{j=1}^{n} \left(W_t^{i,j,VAREER} - W_t^{i,j,GVC} \right) \hat{p}_t^j}_{Term1} \\ &\quad \underbrace{+ W_t^{i,j,GVC} \hat{p}_t^j - \sum_{j=1}^{n}\sum_{s=1}^{m} W_{s,t}^{i,j,GVC-REER} \hat{p_{s,t}}^j}_{Term2} \end{aligned}$$

Here, $W_t^{i,j,VAREER}$ is the VAREER weighting matrix, $W_{s,t}^{i,j,GVC-REER}$ is the n by nm GVC-REER weighing matrix, and $W_t^{i,j,GVC}$ is the n by n GVC-REER weighting matrix

Table 7 – *Decomposing the difference between GVC-REER and VAREER*

Country	REER GAP (%)	Contributions (share in %)	
		Weights	Prices
'Australia'	11.94	5.73	6.21
'Brazil'	1.79	7.64	-5.84
'China'	-6.64	3.72	-10.35
'Germany'	-0.41	6.29	-6.7
'Spain'	-0.85	4.93	-5.79
'France'	-10.48	5.81	-16.29
'Ireland'	-9.95	4.28	-14.23
'Taiwan'	8.49	5.72	2.77
'United States'	-7.1	8.51	-15.61

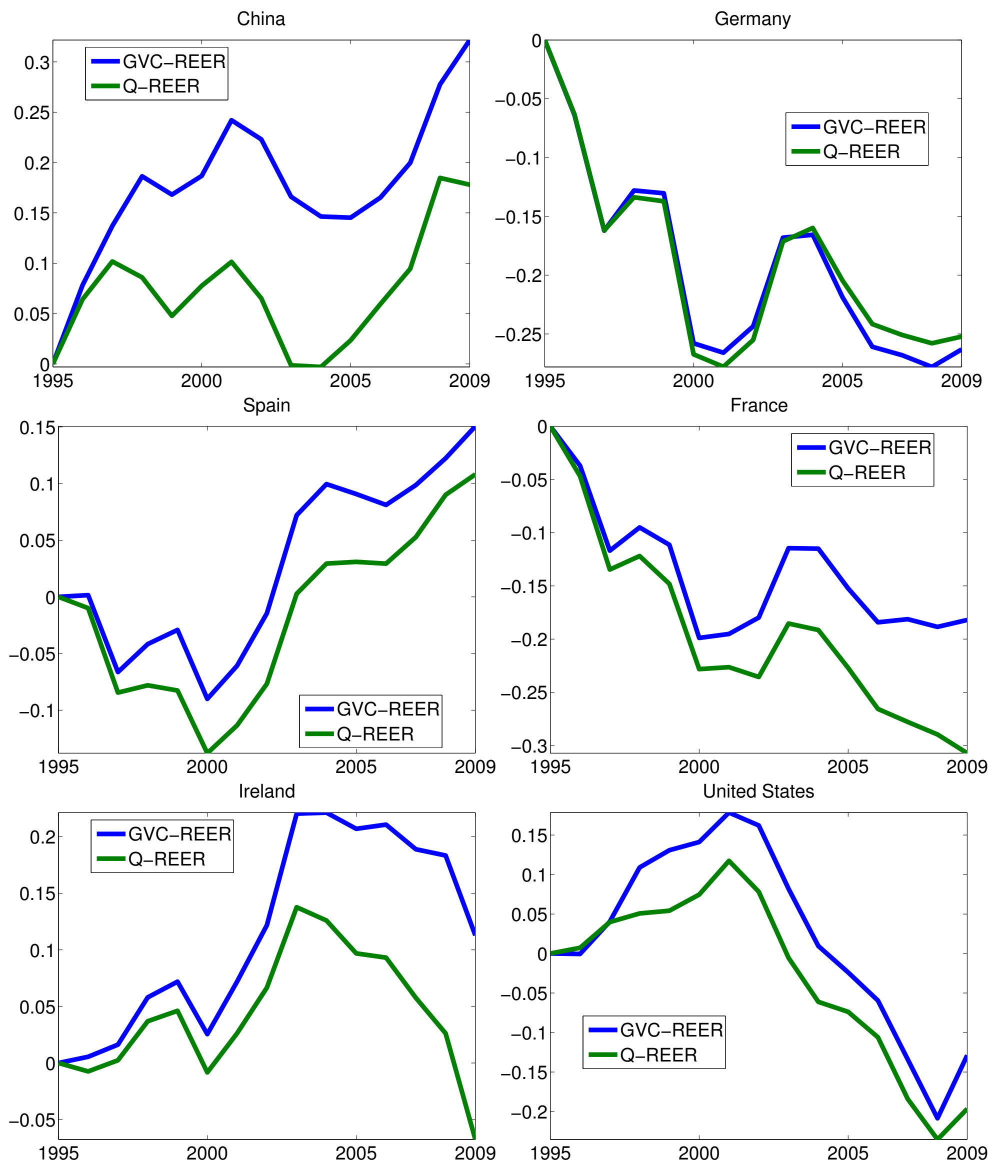
REER GAP denotes the cumulative (from 1995-2009) difference between VAREER and GVC-REER in percentage points.

designed to measure the departure of weights from VAREER. $\hat{p_t^j}$ is the $n$dimensional vector of changes in the GDP deflator and $\hat{p_{s,t}}^j$ is the nm dimensional vector of sectoral price changes.

Table 7 shows a decomposition of the cumulative difference (between 1995 and 2009) between the VAREER and GVC-REER for select countries. For Australia for instance, the table shows that the difference between the VAREER and GVC-REER between 1995 and 2009 is 11.94%, implying that the Australian dollar has appreciated more by 11.9% according to VAREER as opposed to GVC-REER. Of this difference of 11.9%, 5.73% is attributable to differences in weighs between the two measures, and the remainder (6.2%) is attributable to differences in prices. As is evident based on the numbers in the table, both the REER weights as well as price indices make significant contributions to the overall differences attributable to sectoral heterogeneity.

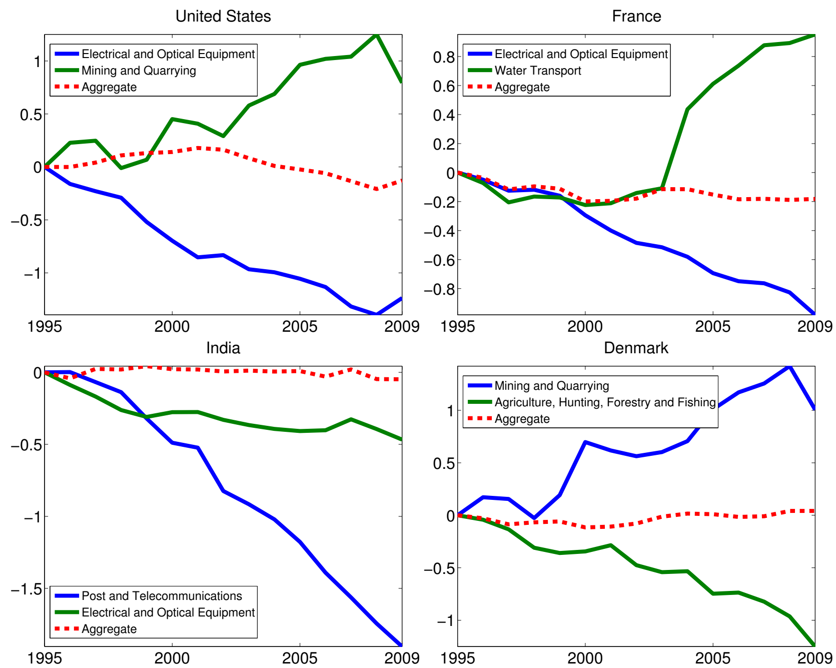
An advantage of our framework is that it allows us to study the competitiveness of value added separately from that of gross output. Figure 8.3 shows a comparison of value added competitiveness (GVC-REER) and gross output competitiveness (Q-REER) for select countries. It is interesting to see that for China, the GVC-REER has appreciated in a much more pronounced fashion than the Q-REER. This reflects the fact that even as Chinese labor has become more expensive over these years, the fall in imported input costs has been responsible for cushioning the impact of this rise in labor costs on output and gross output and exports. This patten also shows how ignoring value added components and looking at gross output competitiveness can lead to a false inference of the Chinese currency being undervalued. On the other hand, for the US, which operates relatively upstream in the global value chain compared to China, the differences between the two measures are more modest.

Figure 8.3 – *Comparison of GVC-REER and Q-REER for select countries*



Comparison of GVC-REER and Q-REER for select countries. GVC-REER measures value added competitiveness, whereas Q-REER measures competitiveness of gross output produced by the country.

Figure 8.4 – *Sector level Exchange rates along with Aggregate country REER for select countries*



Notes: All indices are in logs and normalized to zero at the start of the sample period so the value on the y axis can be read as the percentage deviation from the start of the sample. In this figure the IMF convention is adopted so that an increase corresponds to an appreciation.

8.3 Sectoral REERs

As emphasized earlier, one of the main contributions of the paper is to provide a framework to assess competitiveness at the sector level. Figure 8.4 shows some sector level exchange rates for select countries, along with the aggregate country level exchange rate. As can be seen in the figure, we find evidence of substantial heterogeneity across movements in competitiveness for sectors within countries. In the case of the US for instance, the heterogeneity in competitiveness between sectors is evident in the divergence between the electrical and optical equipment sector which has made massive gains in competitiveness (as evidenced by the large decline/depreciation in REER), whereas the construction sector has shown the opposite pattern.

To further quantify the extent of divergence in competitiveness across sectors within a country, table 8 lists countries with the highest and lowest dispersion across sectoral GVC-REERs, as measured by the average standard deviation of cross-sectional REER movements across sectors within a country in a year. The numbers show that the cross-sectional dispersion is substantial, and even for the country with the lowest dispersion. In particular, on average, a standard deviation of the yearly changes in GVC-REER between different sectors is between

Table 8 – *Countries with highest and lowest divergence of REERs across sectors*

High Dispersion		Low Dispersion	
Czech Republic	0.19	Malta	0.02
Slovak Republic	0.14	China	0.03
Russia	0.13	Ireland	0.04
Bulgaria	0.12	Taiwan	0.04
Sweden	0.11	Spain	0.05

Notes: This table shows 5 countries with the highest and lowest dispersion of REER movements across sectors. The dispersion is computed as the average standard deviation of REER movements within a country (i.e an average of 14 observations on the standard deviation for each time period).

20% (Czech Republic) and 2% (Malta).

8.4 Stability of GVC-REER weights across time

Sectoral input-output data required to compute our benchmark GVC-REER indices is often only available at low frequency (annual or less) and with significant lags. This potentially challenges the usefulness of the GVC-REER framework developed in this paper to compute and update REERs in a timely manner. In order to check the stability of GVC-REER weights over time, we recompute them using fixed weights (calibrated to the year 2005). Table 9 shows that these GVC-REER indices computed with fixed weights exhibit a high degree of correlation with the original ones with time-varying weights, both at the country and sector level. This indicates that time variation weights is minimal and is not a significant source of variation in the GVC-REER measures, and hence using lagged input-output data to compute the GVC-REER weights is unlikely to be a major source of bias in the measurement of REER.

9 Application: Bilateral Real Exchange Rates

The bilateral real exchange rate (RER) between countries h and f is defined as follows:

$$RER^{hf} = \hat{p}(V)^f - \hat{p}(V)^h \quad (9.1)$$

where $\hat{p}^f$ and $\hat{p}^h$ are changes in aggregate (country wide) price indices measured in a common currency.

Based on the the insights gained from the previous sections we argue that to measure competitiveness of one country against another in a bilateral context, a modified version

Table 9 – *Stability of GVC_REER weights over time: Correlation between time varying and fixed weight GVC-REERs*

Level of aggregation	Country level GVC-REER	Sector level GVC-REER
Correlations		
mean	0.99	0.99
maximum	0.99	1.00
minimum	0.95 (India)	0.69 (Malta, Hotels and Restaurants) (WIOD sector 22)
No. Of observations	41	1435

Notes: This table summarises correlations between year to year percentage changes in GVC-REERs computed using time varying and fixed weights. For GVC-REER with time varying weights, weights are allowed to change in every time period and are calibrated using current year input-output tables. For the fixed weight GVC-REER, weights for the year 2005 are applied to all years in the sample (1995-2009)

of the GVC framework proposed in this paper provides an improvement over the standard RER measures computed using an aggregate price index (such as those in Chinn, 2006) like the GDP deflator. Once again, the key insight is that ignoring trade in intermediates and/or sectoral heterogeneity can lead to incorrect inferences regarding movements in price competitiveness. For instance, an overall increase in GDP deflator in country A relative to country B could indicate a depreciation or an increase in competitiveness for A, even if the sectors in B which compete more intensively with A experience an increase in relative prices.[13]

We define our bilateral RER measure, the "GVC-RER" as follows

$$GVC - RER^{hf} = \sum_{i=1}^{m} v_i^h \left[\sum_{j=1}^{m} w_{ij}^{hh} \hat{p}(V)_j^h + \sum_{j=1}^{m} w_{ij}^{hf} \hat{p}(V)_j^f \right] \tag{9.2}$$

Here,$v_i^h = \frac{p(V)_i^h V_i^h}{\sum_{j=1}^{m} p(V)_j^h V_j^h}$ is the share of sector i in country h's total value added, so that $\sum_{i=1}^{m} v_i^h = 1$.[14] The ws are weights that are analogous to the GVC-REER weights.

Figure 9.1 shows the comparison of the two RER measures for select country pairs that highlight differences between the two measures. The left panel shows that while China's real exchange rate vis-a vis the US depreciated substantially between 1998 and 2004 when measured using the standard RER measure, GVC-RER measure displays a more secular appreciation through the sample period.[15] The right panel shows that the loss in competitiveness for Greece

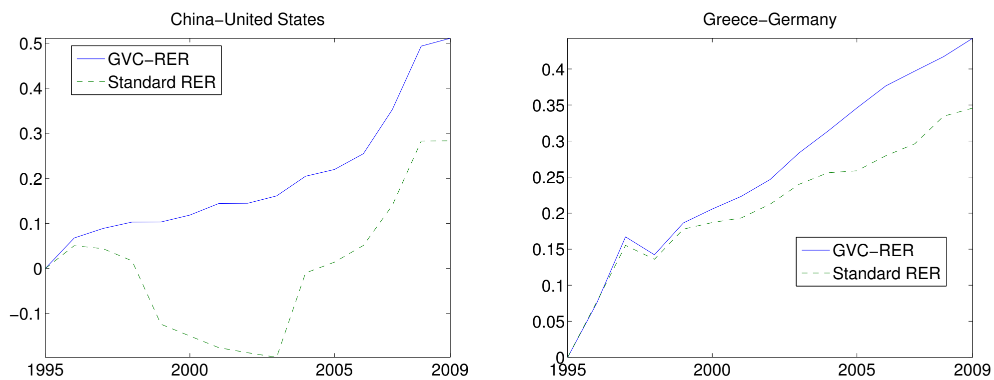
[13] Appendix G provides a numerical example to illustrate this point

[14] Bilateral gross output effective exchange rates (Q-RER) can be defined analogously.

[15] Electrical and optical equipment (WIOD sector 14), a sector which has seen a substantial decline in price

(i.e an appreciation) vis a vis Germany is much larger when measured through GVC-RER as opposed to the conventional real exchange rate measure, indicating that the standard bilateral REER measure is underestimating the extent to which lack of inflation in Germany has hurt competitiveness in Greece.

Figure 9.1 *– Comparison of GVC-RER and standard RER bilateral exchange rates*



Notes: All indices are in logs and normalized to zero at the start of the sample period so the reading on the value on the y axis can be read as the percentage deviation from the start of the sample. In this figure the IMF convention is adopted so that an increase corresponds to an appreciation. GVC-REER uses weights computed under the uniform elasticity assumption. In computing these indices, the weights are normalized so that the sum of the home country and foreign country weights are equal in magnitude, as is the case with the standard RER measure. Unlike in the GVC-REER effective exchange rate computation, here the normalization of weights cannot be avoided, since otherwise the GVC-RER measure would be dominated by home prices because home sectors (especially the own sector) on average carry much higher GVC-REER weights.

10 Conclusion

With the rising prominence of global value chains in international trade, standard models typically used in the literature to measure price competitiveness via the real effective exchange rate (REER) are increasingly becoming obsolete. In this paper, we provide a framework for computing the REER which allows for the complex patterns of international trade (including imported inputs and global value chains) and sectoral heterogeneity in terms of how different sectors in a country differ in sourcing their inputs as well as selling their output to different entities. In doing so, the paper makes four novel contributions to the literature on REER.

(over 100%) in the US through the sample period, provides a useful illustration of this divergence. While the standard RER measure gives a small weight to this sector in line with its value added share in the US economy, the GVC-RER measure gives a much higher weight, recognizing the importance of this sector as far as competitiveness with China is concerned.

Firstly, as mentioned above, we allow for sectoral heterogeneity within countries. We show that while allowing for imported inputs (i.e a distinction between value added and gross trade flows), as has already been done in the literature[16], does provide an improvement over classical REER measures, a major further improvement comes from allowing for sectoral heterogeneity, a feature that is a unique to our framework in this literature. Sectoral heterogeneity affects both components of REER-weights as well as prices. Secondly, we separately quantify and develop REER measures for value added and gross output, and highlight how we improve upon the previous attempts in the literature to accomplish this. Thirdly, apart from country-level REERs, we develop REER measures at the sectoral level. This allows us to study behaviors in competitiveness across sectors within a country, as well as across countries within a sector. Fourthly, we apply the framework to a bilateral context and develop a new measure of bilateral real exchange rate (GVC-RER) that provides an improvement upon the current measures by acknowledging the presence of sectoral heterogeneity and trade in inputs.

To illustrate the importance of these improvements made on the modeling front, we parametrize the model using global input output data from the world input output database (WIOD). We create a database of REER weights and indices for 40 countries and 1435 country-sector pairs for the period 1995-2009. The results highlight that the role of sectoral heterogeneity in both REER weights as well as indices is quantitatively important in measuring competitiveness through the REER.

This is the first paper to develop REER measures and leaves several avenues for further development and exploration. In order to focus on the role of sectoral heterogeneity, we have largely abstracted from the issue of heterogeneity in the elasticity of substitution across product categories (intermediate vs final goods) as well as across countries, especially in the empirical part. The model indeed allows for these features, and in the appendix we discuss estimation of these elasticities which can further improve the measures of REER.

The paper constructs a new and novel database of sectoral REERs which can be used to study competitiveness of sectors and how they have evolved over time. It can be used to address many questions, including understanding the sources and implications of the dispersion in competitiveness within countries and within sectors across countries.

Following the literature to clearly illustrate our marginal contributions, we have also restricted the analysis to a partial equilibrium, and an essentially static framework. Extending this analysis to general equilibrium and understanding the role played by sectoral heterogeneity in determining competitiveness in a dynamic setting remains a promising avenue for future exploration.

[16] most notably by Bems and Johnson (2017)

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Online Appendix

A Solution of the general model <For online publication>

A.1 First order conditions

A.1.1 first order conditions for production:

$$V_l^c = w_l^{vc} \left(\frac{p(V)_l^c}{p(Q)_l^c}\right)^{-\sigma^3(c,l)} Q_l^c \tag{A.1}$$

$$X_l^c = w_l^{Xc} \left(\frac{p(X)_l^c}{p(Q)_l^c}\right)^{-\sigma^3(c,l)} Q_l^c \tag{A.2}$$

$$X_{sl}^c = w_{sl}^c \left(\frac{p(X)_{sl}^c}{p(X)_l^c}\right)^{-\sigma^2(c,l)} X_l^c \tag{A.3}$$

$$X_{sl}^{ic} = w_{sl}^{ic} \left(\frac{p(Q)_s^i}{p(X)_{sl}^{(f)c}}\right)^{-\sigma_s^1(c,l)} X(f)_{sl}^c \tag{A.4}$$

$$X_{sl}^{cc} = w_{sl}^{cc} \left(\frac{p(Q)_s^c}{p(X)_{sl}^{(f)c}}\right)^{-\sigma_s^{1h}(c,l)} X_{sl}^c \tag{A.5}$$

$$X_{sl}^c(f) = w(f)_{sl}^c \left(\frac{p(X)_{sl}^{(f)c}}{p(X)_{sl}^c}\right)^{-\sigma_s^{1h}(c,l)} X_{sl}^c \tag{A.6}$$

Here q_l^c and q_{sl}^c are price indices corresponding to X_l^c and X_{sl}^c respectively and are given by:

$$p(X)_l^c = \left[\sum_{s=1}^{m} (w_{sl}^c)(p(X)_{sl}^c)^{1-\sigma^2(c,l)}\right]^{\frac{1}{1-\sigma^2(c,l)}} \tag{A.7}$$

$$p(X)_{sl}^{(f)c} = \left[\sum_{i=1, i\neq c}^{n} (w_{sl}^{ic})(p(Q)_s^i)^{1-\sigma_s^1(c,l)}\right]^{\frac{1}{1-\sigma_s^1(c,l)}} \tag{A.8}$$

$$p(X)_{sl}^c = \left[(w_{sl}^{cc})(p(Q)_s^c)^{1-\sigma^{1h}(c,l)} + (w_l^{Xc})(p(X)_{sl}^{(f)c})^{1-\sigma^{1h}(c,l)}\right]^{\frac{1}{1-\sigma^{1h}(c,l)}} \tag{A.9}$$

and price of gross output is given by:

$$p(Q)_l^c = \left[(w_l^{vc})(p(V)_l^c)^{1-\sigma^3(c,l)} + (w_l^{Xc})(p(X)_l^c)^{1-\sigma^3(c,l)}\right]^{\frac{1}{1-\sigma^3(c,l)}} \tag{A.10}$$

where $p(V)_l^c$ is the price of value added(i.e price of factor of production) of country c sector l

A.1.2 First order conditions for final consumption:

$$F_s^{ic} = \kappa_s^{ic}\left(\frac{p(Q)_s^i}{P(f)_s^c}\right)^{-\theta_s^1(c)} F(f)_s^c \tag{A.11}$$

$$F_s^{cc} = \kappa_s^{cc}\left(\frac{p(Q)_s^c}{P_s^c}\right)^{-\theta_s^{1h}(c)} F_s^c \tag{A.12}$$

$$F(f)_s^c = \kappa(f)_s^c\left(\frac{P(f)_s^c}{P_s^c}\right)^{-\theta_s^{1h}(c)} F_s^c \tag{A.13}$$

$$F_s^c = \kappa_s^c\left(\frac{P_s^c}{P^c}\right)^{-\theta^2(c)} F^c \tag{A.14}$$

Here P_s^c and P^c are price indices for sector s good and aggregate good consumed by country c, respectively and are given by

$$P_s^c(f) = \left[\sum_{i=1, i\neq c}^{n}(\kappa_s^{ic})(p(Q)_s^i)^{1-\theta_s^1(c)}\right]^{\frac{1}{1-\theta_s^1(c)}} \tag{A.15}$$

$$P_s^c = \left[(\kappa_s^{cc})(p(Q)_s^c)^{1-\theta_s^{1h}(c)} + (\kappa(f)_l^c)(P(f)_s^c)^{1-\theta_s^{1h}(c)}\right]^{\frac{1}{1-\theta_s^{1h}(c)}} \tag{A.16}$$

$$P^c = \left[\sum_{s=1}^{m}(\kappa_s^c)(P_s^c)^{1-\theta^2(c)}\right]^{\frac{1}{1-\theta^2(c)}} \tag{A.17}$$

Let $[A]_{nmXnm}$ be the input-output coefficient matrix at the country-sector level, i.e the $(i,j)^{th}$ block which has dimension mXm is given by

$$[A]_{mXm}^{ij} = \begin{pmatrix} a_{11}^{ij} & a_{12}^{ij} & .. & a_{1m}^{ij} \\ a_{21}^{ij} & a_{22}^{ij} & .. & a_{2m}^{ij} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1}^{ij} & a_{m2}^{ij} & .. & a_{mm}^{ij} \end{pmatrix} \tag{A.18}$$

where a_{sl}^{ij} denotes the output of (i,s) used in the production of one unit of (j,l), i.e

$$a^{ij}_{sl} = \frac{p(Q)^i_s X^{ij}_{sl}}{p(Q)^j_l Q^j_l} \tag{A.19}$$

Let $[B]_{nmXnm}$ be the corresponding total requirement matrix given by

$$[B]_{nmXnm} \quad = \quad (I_{nm} - [A])^{-1} \tag{A.20}$$

Also, define the matrix $[D_Q]_{nmxnm}$ to be a diagonal matrix with the $(cl)^{th}$ diagonal entry given by $\frac{1}{p^c_l Q^c_l}$

Log Linearization:

A note on notation:

- for any variable Y^{ab}_{cd}, $vec\left(Y^{ab}_{cd}\right)$ denotes a vector with all components of Y^{ab}_{cd} stacked together
- The stacking order is as follows: d, c, b, d. i, e first the home sector index changes, followed by foreign sector, followed by home country and finally foreign country
  - $vec\left(Y^{b}_{cd}\right), vec\left(Y^{ab}_{c}\right)$ etc are defined accordingly.
- Examples in a 2 by 2 case $(m = n = 2)$
  - $vec\left(Y^{ab}_{cd}\right) = (Y^{11}_{11}, Y^{11}_{12}, Y^{11}_{21}, Y^{11}_{22}, Y^{12}_{11}, Y^{12}_{12}, Y^{12}_{21}, Y^{12}_{22}, Y^{21}_{11}, Y^{21}_{12}, Y^{21}_{21}, Y^{21}_{22}, Y^{22}_{11}, Y^{22}_{12}, Y^{22}_{21}, Y^{22}_{22})'$
  - $vec\left(Y^{b}_{cd}\right) = (Y^1_{11}, Y^1_{12}, Y^1_{21}, Y^1_{22}, Y^2_{11}, Y^2_{12}, Y^2_{21}, Y^2_{22})'$

This appendix contains the log linearized first order and market clearing conditions and organizes them in stacked matrix notation which will be useful in deriving the results that follow. A variable with a "^" denotes log deviation from steady state.

Log linearizing and stacking components of production function and price indices:(to simplify notation further, we omit the parenthesis for gross output prices, i.e the parenthesis containing "Q" is omitted)

Raw expression $\quad X(f)^c_{sl} = \left[\sum_{i=1,i\neq c}^{n} (w^{ic}_{sl})^{1/\sigma^1_s(c,l)} (X^{ic}_{sl})^{\frac{\sigma^1_s(c,l)-1}{\sigma^1_s(c,l)}}\right]^{\frac{\sigma^1_s(c,l)}{\sigma^1_s(c,l)-1}}$

Log linearized expression $\quad \hat{X(f)}^c_{sl} = \sum_{i=1,i\neq c}^{n} \left(\frac{p^i_s X^{ic}_{sl}}{P(X)^{(f)}{}^c_{sl} X(f)^c_{sl}}\right) \hat{X}^{ic}_{sl}$

Stacked vector:

$$\left(vec(\hat{X}(f)^c_{sl})\right) = \underbrace{W_{1XXH}}_{nm^2Xn^2m^2} vec(\hat{X}^{ic}_{sl}) \tag{A.21}$$

Raw expression $X^c_{sl} = \left[(w^c_{sl})^{1/\sigma^{1h}_s(c,l)}(X^{cc}_{sl})^{\frac{\sigma^{1h}_s(c,l)-1}{\sigma^{1h}_s(c,l)}} + (w(f)^c_{sl})^{1/\sigma^{1h}_s(c,l)}(X(f)^c_{sl})^{\frac{\sigma^{1h}_s(c,l)-1}{\sigma^{1h}_s(c,l)}}\right]^{\frac{\sigma^{1h}_s(c,l)}{\sigma^{1h}_s(c,l)-1}}$

Log linearized expression $\hat{X}^c_{sl} = \sum_{i=1}^n \left(\frac{p^i_s X^{ic}_{sl}}{p(X)^c_{sl} X^c_{sl}}\right) \hat{X}^{ic}_{sl}$

Stacked Vector:

$$\left(vec(\hat{X}^c_{sl})\right) = \underbrace{W_{1XX}}_{nm^2Xn^2m^2} vec(\hat{X}^{ic}_{sl}) \tag{A.22}$$

Raw expression $X^c_l = \left[\sum_{s=1}^m (w^c_{sl})^{1/\sigma^2(c,l)}(X^c_{sl})^{\frac{\sigma^2(c,l)-1}{\sigma^2(c,l)}}\right]^{\frac{\sigma^2(c,l)}{\sigma^2(c,l)-1}}$

Log linearized expression $\hat{X}^c_l = \sum_{s=1}^m \left(\frac{p(X)q^c_{sl}X^c_{sl}}{p(X)^c_l X^c_l}\right) \hat{X}^c_{sl}$

Stacked vector:

$$vec(\hat{X}^c_l) = (W_{2XX})_{nmXnm^2} vec(\hat{X}^c_{sl}) \tag{A.23}$$

Raw expression $q(f)^c_{sl} = \left[\sum_{i=1, i\neq c}^n (w^{ic}_{sl})(p^i_s)^{1-\sigma^1_s(c,l)}\right]^{\frac{1}{1-\sigma^1_s(c,l)}}$

Log linearized expression $\hat{q}^c_{sl}(f) = \sum_{i=1, i\neq c}^n \left(\frac{p^i_s X^{ic}_{sl}}{p(X)^c_{sl} X^c_{sl}}\right) \hat{p^i_s}$

Stacked vector:

$$vec(\hat{q}^c_{sl}(f)) = (W_{1XPH})_{nm^2Xnm} vec(\hat{p}^i_s) \tag{A.24}$$

Raw expression $q_{sl}^{c}=\left[(w_{sl}^{cc})(p^{c}{}_{s})^{1-\sigma^{1h}(c,l)}+(w_{l}^{Xc})(p(X)^{(f)}{}_{sl}^{c})^{1-\sigma^{1h}(c,l)}\right]^{\frac{1}{1-\sigma^{1h}(c,l)}}$

Log linearized expression $\hat{p(X)}_{sl}^{c}=\sum_{i=1}^{n}\left(\frac{p_s^i X_{sl}^{ic}}{p(X)_{sl}^{c}X_{sl}^{c}}\right)\hat{p_s^i}$

Stacked vector:

$$vec(\hat{p(X)}_{sl}^{c})=(W_{1XP})_{nm^2Xnm}\,vec(\hat{p}_s^i) \tag{A.25}$$

Raw expression $p(X)_l^c=\left[\sum_{s=1}^{m}(w_{sl}^{c})(p(X)_{sl}^{c})^{1-\sigma^2(c,l)}\right]^{\frac{1}{1-\sigma^2(c,l)}}$

Log linearized expression $\hat{q}_l^c=\sum_{s=1}^{m}\left(\frac{p(X)_{sl}^{c}X_{sl}^{c}}{p(X)_l^cX_l^c}\right)\hat{q_{sl}^{c}}$

Stacked vector:

$$vec(\hat{q}_l^c)=(W_{2Xp})_{nmXnm^2}\,vec(\hat{p(X)}_{sl}^{c}) \tag{A.26}$$

Raw expression $P_s^c(f)=\left[\sum_{i=1,i\neq c}^{n}(\kappa_s^{ic})(p_s^i)^{1-\theta_s^1(c)}\right]^{\frac{1}{1-\theta_s^1(c)}}$

Log linearized expression $\hat{P(f)}_s^c=\sum_{i=1,i\neq c}^{n}\left(\frac{p_s^iF_s^{ic}}{P(f)_s^cF(f)_s^c}\right)\hat{p}_s^i$

Stacked vector:

$$vec\left(\hat{P}_s^c\right)_{nmX1}=(W_{1FPH})_{nmXnm}vec\left(p_s^i\right)_{nmX1} \tag{A.27}$$

Raw expression $P_s^c=\left[(\kappa_s^{cc})(p_s^{cc})^{1-\theta_s^{1h}(c)}+(\kappa(f)_l^c)(P(f)_s^c)^{1-\theta_s^{1h}(c)}\right]^{\frac{1}{1-\theta_s^{1h}(c)}}$

Log linearized expression $\hat{P}_s^c=\sum_{i=1}^{n}\left(\frac{p_s^iF_s^{ic}}{P_s^cF_s^c}\right)\hat{p}_s^i$

Stacked vector:

$$vec\left(\hat{P}_s^c\right)_{nmX1}=(W_{1FP})_{nmXnm}vec\left(p_s^i\right)_{nmX1} \tag{A.28}$$

Raw expression $P^c = \left[\sum_{s=1}^{m} (\kappa_s^c)(P_s^c)^{1-\theta^2(c)}\right]^{\frac{1}{1-\theta^2(c)}}$

Log linearized expression $\hat{P}^c = \sum_{s=1}^{m} \left(\frac{P_s^c F_s^c}{P^c F^c}\right) \hat{P}_s^c$

Stacked vector:

$$vec\left(\hat{P}^c\right)_{nX1} = (W_{2FP})_{nXnm} Vec\left(\hat{P}_s^c\right)_{nmX1} \tag{A.29}$$

Final goods consumption first order conditions:

$$\begin{aligned}
\hat{F_s^{ic}} &= -\theta_s^1(c)(\hat{p}_s^i - P(\hat{f})_s{}^c) + F(\hat{f})_s{}^c && \text{(A.30)}\\
\hat{F_s^{cc}} &= -\theta_s^{1h}(c)(\hat{p}_s^c - \hat{P}_s{}^c) + \hat{F}_s{}^c && \text{(A.31)}\\
F(\hat{f})_s^c &= -\theta_s^{1h}(c)(\hat{P}(f)_s^c - \hat{P}_s{}^c) + \hat{F}_s{}^c && \text{(A.32)}\\
\hat{F}_s^c &= -\theta^2(c)(\hat{P}_s^c - \hat{P}^c) + \hat{F}^c && \text{(A.33)}
\end{aligned}$$

We can combine these 4 conditions to write:

$$\begin{aligned}
\hat{F}_s{}^{ic} &= -\theta_s^1(c)\hat{p}_s^i + \left(\theta_s^1(c) - \theta_s^{1h}(c)\right) P(\hat{f})_s^c + \left(\theta_s^{1h}(c) - \theta^2(c)\right) \hat{P}_s^c + \theta^2(c)\hat{P}^c + \hat{F}^c\\
\hat{F}_s{}^{cc} &= -\theta_s^{1h}(c)\hat{p}_s^c + \left(\theta_s^{1h}(c) - \theta^2(c)\right) \hat{P}_s^c + \theta^2(c)\hat{P}^c + \hat{F}^c
\end{aligned}$$

We can now stack the above n^2m equations to write a single matrix equation as follows:

$$\begin{aligned}
vec\left(\hat{F_s^{ic}}\right)_{n^2mX1} &= J_F(i \neq c)\left[(Y_1)_{n^2mXnm} vec(\theta_s^1(c))_{nmX1}\right] \odot \left[(Y_2)_{n^2mXnm} vec(\hat{p}_s^i)_{nmX1}\right]\\
&- -J_F(i = c)\left[(Y_1)_{n^2mXnm} vec(\theta_s^{1h}(c))_{nmX1}\right] \odot \left[(Y_2)_{n^2mXnm} vec(\hat{p}_s^i)_{nmX1}\right]\\
&+ J_F(i \neq c)\left[Y_1\left(vec(\theta_s^1(c))_{nmX1} - vec(\theta_s^{1h}(c))_{nmX1}\right)\right] \odot \left[Y_1 vec(\hat{P}(f)_s^c)_{nmX1}\right]\\
&+ \left(Y_1 vec(\theta_s^{1h}(c))_{nmX1} - (Y_3)_{n^2mXn} vec(\theta^2(c))_{nmX1}\right) \odot \left(Y_1 vec(\hat{P}_s^c)_{nmX1}\right)\\
&+ \left[Y_3 vec(\theta^2(c))_{nmX1}\right] \odot \left[Y_3 vec(\hat{P}^c)_{nX1}\right] + Y_3\hat{F}^c
\end{aligned}$$

where $Y_1 = 1_n \otimes I_{nm}$, $Y_2 = I_n \otimes 1_n \otimes I_m$, $Y_3 = 1_n \otimes I_n \otimes 1_m$, $\odot$ is the element by element multiplication operator for two vectors and $J_F(x)$ is an n^2m by 1 vector with ones in all indices that satisfy the condition x and zero elsewhere.

Combining this with (A.29) and (A.28),

$$vec\left(\hat{F_s^{ic}}\right)_{n^2mX1} = Z_F vec(\hat{p}_s^i)_{nmX1} + Y_3\hat{F}^c \tag{A.34}$$

where

$$\begin{aligned}
(Z_F)_{n^2mXnm} &= J_F(i \neq c)\left[(Y_1)_{n^2mXnm} vec(\theta_s^1(c))_{nmX1}\right] \odot \left[(Y_2)_{n^2mXnm}\right] \\
&- -J_F(i = c)\left[(Y_1)_{n^2mXnm} vec(\theta_s^{1h}(c))_{nmX1}\right] \odot \left[(Y_2)_{n^2mXnm}\right] \\
&+ J_F(i \neq c)\left[Y_1\left(vec(\theta_s^1(c))_{nmX1} - vec(\theta_s^{1h}(c))_{nmX1}\right)\right] \odot \left[Y_1 W_{FH}\right] \\
&+ \left(Y_1 vec(\theta_s^{1h}(c))_{nmX1} - (Y_3)_{n^2mXn} vec(\theta^2(c))_{nmX1}\right) \odot (Y_1 W_{1FP}) \\
&+ \left[Y_3 vec(\theta^2(c))_{nmX1}\right] \odot \left[Y_3 W_{2FP} W_{1FP}\right]
\end{aligned} \tag{A.35}$$

Log linearizing Production first order conditions:

$$V_l^c = w_l^{vc}\left(\frac{p_l^{vc}}{p_l^c}\right)^{-\sigma^3(c,l)} Q_l^c \tag{A.36}$$

$$X_l^c = w_l^{Xc}\left(\frac{q_l^c}{p_l^c}\right)^{-\sigma^3(c,l)} Q_l^c \tag{A.37}$$

$$X_{sl}^c = w_{sl}^c\left(\frac{q_{sl}^c}{q_l^c}\right)^{-\sigma^2(c,l)} X_l^c \tag{A.38}$$

$$X_{sl}^{ic} = w_{sl}^{ic}\left(\frac{p_s^i}{q(f)_{sl}^c}\right)^{-\sigma_s^1(c,l)} X(f)_{sl}^c \tag{A.39}$$

$$X_{sl}^{cc} = w_{sl}^{cc}\left(\frac{p_s^c}{q_{sl}^c}\right)^{-\sigma_s^{1h}(c,l)} X_{sl}^c \tag{A.40}$$

$$X_{sl}^c(f) = w(f)_{sl}^c\left(\frac{q(f)_{sl}^c}{q_{sl}^c}\right)^{-\sigma_s^{1h}(c,l)} X_{sl}^c \tag{A.41}$$

$$\begin{aligned}
\hat{X_{sl}^{ic}} &= -\sigma_s^1(c,l)\hat{p}_s^i + \sigma_s^1(c,l)p(\hat{X})_{sl}^{(f)c} + \hat{X}(f)_{sl}^c \\
\hat{X_{sl}^{cc}} &= -\sigma_s^{1h}(c,l)\hat{p}_s^c + \sigma_s^{1h}(c,l)p(\hat{X})_{sl}^c + \hat{X}_{sl}^c \\
\hat{X_{sl}^c(f)} &= -\sigma_s^{1h}(c,l)p(\hat{X})_{sl}^{(f)c} + \sigma_s^{1h}(c,l)p(\hat{X})_{sl}^c + \hat{X}_{sl}^c \\
\hat{X}_{sl}^c &= -\sigma^2(c,l)p(\hat{X})_{sl}^c + \sigma^{2h}(c,l)p(\hat{X})_l^c + \hat{X}_l^c
\end{aligned}$$

$$
\begin{aligned}
\hat{X}_{sl}^{ic} &= -\sigma_s^1(c,l)\hat{p}_s^i + \left(\sigma_s^1(c,l) - \sigma_s^{1h}(c,l)\right) p(\hat{X}) + \left(\sigma_s^{1h}(c,l) - \sigma^2(c,l)\right) p(\hat{X})_{sl}^{c} \\
&+ \left(\sigma^2(c,l) - \sigma^3(c,l)\right) p(\hat{X})_l^{c} + \sigma^3(c,l)\hat{p}_l^c + \hat{Q}_l^c \\
\hat{X}_{sl}^{cc} &= -\sigma_s^{1h}(c,l)\hat{p}_s^c + \left(\sigma_s^{1h}(c,l) - \sigma^2(c,l)\right) p(\hat{X})_{sl}^{c} \\
&+ \left(\sigma^2(c,l) - \sigma^3(c,l)\right) \hat{q}_l^c + \sigma^3(c,l)\hat{p}_l^c + \hat{Q}_l^c
\end{aligned}
$$

These n^2m^2 equations can be stacked to write

$$
\begin{aligned}
vec\left(\hat{X}_{sl}^{ic}\right)_{n^2m^2} &= -J_X(i \neq c)\left[C_1 vec\left(\sigma_s^1(c,l)\right)_{nm^2X1}\right] \odot \left[C_3 vec(\hat{p}_s^i)_{nmX1}\right] \\
&- J_X(i = c)\left[C_1 vec\left(\sigma_s^{1h}(c,l)\right)_{nm^2X1}\right] \odot \left[C_3 vec(\hat{p}_s^i)_{nmX1}\right] \\
&+ J_X(i \neq c)\left[C_1\left(vec\left(\sigma_s^1(c,l)\right)_{nm^2X1} - vec\left(\sigma_s^{1h}(c,l)\right)_{nm^2X1}\right)\right] \odot \left[C_1 p(\hat{X})_{sl}^{(f)c}\right] \\
&+ \left[C_2\left(vec\left(\sigma^2(c,l)\right)_{nmX1} - vec\left(\sigma^3(c,l)\right)_{nmX1}\right)\right] \odot \left[C_2 p(\hat{X})_l^c\right] \\
&+ \left[C_1 vec\left(\sigma_s^{1h}(c,l)\right)_{nm^2X1} - C_2 vec\left(\sigma^2(c,l)\right)_{nmX1}\right] \odot \left[C_1 p(\hat{X})_{sl}^c\right] \\
&+ \left[C_2 vec\left(\sigma^3(c,l)\right)_{nmX1}\right] \odot \left[C_2 vec(\hat{p}_s^i)_{nmX1}\right] + C_2\hat{Q}_l^c
\end{aligned}
$$

where $C_1 = 1_n \otimes I_{nm^2}$, $C_2 = 1_n \otimes I_n \otimes 1_m \otimes I_m$, $C_3 = I_n \otimes 1_n \otimes I_m \otimes 1_m$. $J_X(y)$ is an n^2m by 1 vector with ones in all indices that satisfy the condition y and zero elsewhere.

Combining this with (A.22) - (A.26) we get:

$$
vec\left(\hat{X}_{sl}^{ic}\right)_{n^2m^2} = Z_X vec(\hat{p}_s^i)_{nmX1} + C_2\hat{Q}_l^{\,c} \tag{A.42}
$$

where

$$
\begin{aligned}
Z_X &= -J_X(i \neq c)\left[C_1 vec\left(\sigma_s^1(c,l)\right)_{nm^2X1}\right] \odot [C_3] \\
&- J_X(i = c)\left[C_1 vec\left(\sigma_s^{1h}(c,l)\right)_{nm^2X1}\right] \odot [C_3] \\
&+ J_X(i \neq c)\left[C_1\left(vec\left(\sigma_s^1(c,l)\right)_{nm^2X1} - vec\left(\sigma_s^{1h}(c,l)\right)_{nm^2X1}\right)\right] \odot [C_1 W_{XH}] \\
&+ \left[C_2\left(vec\left(\sigma^2(c,l)\right)_{nmX1} - vec\left(\sigma^3(c,l)\right)_{nmX1}\right)\right] \odot [C_2 W_{2XP} W_{1XP}] \\
&+ \left[C_1 vec\left(\sigma_s^{1h}(c,l)\right)_{nm^2X1} - C_2 vec\left(\sigma^2(c,l)\right)_{nmX1}\right] \odot [C_1 W_{1XP}] \\
&+ \left[C_2 vec\left(\sigma^3(c,l)\right)_{nmX1}\right] \odot [C_2]
\end{aligned} \tag{A.43}
$$

Next, linearizing the production function we have:

$$vec\left(\hat{Q}_l^c\right) = (D_v)_{nmXnm}\left(vec\left(\hat{V}_l^c\right)\right)_{nmX1} + (D_X)_{nmXnm}\, vec\left(\hat{X}_l^c\right) \tag{A.44}$$

$$vec\left(\hat{p_l^c}\right) = D_v vec\left(\hat{p}(V)_l^c\right) + D_X vec\left(p(\hat{X})_l^c\right) \tag{A.45}$$

(here D_v and D_X are $nmXnm$ diagonal matrices denoting the shares of value added and intermediate inputs in the production of different goods , i.e the lc^{th} diagonal entry of D_v is $\frac{p(V)_l^c V_l^c}{p(Q)_l^c Q_l^c}$ and that of D_X is $\frac{p(X)_l^c X_l^c}{p(Q)_l^c Q_l^c}$. We can use (A.25) and (A.26) in (A.45)to obtain the following expression linking price of gross output and price of value added:

$$vec\left(\hat{p_l^c}\right) = \left(I - D_X W_{2XP} W_{1XP}\right)^{-1} D_V vec\left(p(\hat{V})_l^c\right) \tag{A.46}$$

The market clearing conditions (4.8) can be linearized as:

$$\hat{Q}_j^i = \sum_{h=1}^{n}\sum_{l=1}^{m} \frac{X_{jl}^{ih}}{Q_j^i}\hat{X}_{jl}^{ih} + \sum_{h=1}^{n}\frac{F_j^{ih}}{Q_j^i}\hat{F}_j^{ih} \tag{A.47}$$

As before, these can be written in stacked form by creating matrices S_X and S_F from the above equations to yield:

$$vec\left(\hat{Q}_l^c\right) = (S_F)_{nmXn^2m}\, vec\left(\hat{F_s^{fc}}\right) + (S_X)_{nmXn^2m^2}\, vec\left(\hat{X}_{sl}^{fc}\right) \tag{A.48}$$

B Derivations of the expressions for GVC-REER and Q-REER((5.1) and (5.4))<For online publication>

From (A.48) and (A.42) we get

$$vec\left(\hat{Q}_l^c\right)\left[I_{nm} - S_X C_2\right] = \left(S_X Z_X + S_F Z_F\right) vec\left(\hat{p}_l^c\right) + S_F Y_3 vec\left(\hat{F}^c\right) \tag{B.1}$$

Using (A.46) in (B.1)and rearranging we get:

$$\begin{aligned} vec\left(\hat{Q}_l^c\right) &= \left[I_{nm} - S_X C_2\right]^{-1}\left(S_X Z_X + S_F Z_F\right)\left(I - D_X W_{2XP} W_{1XP}\right)^{-1} D_V vec\left(p(\hat{V})_l^c\right) \\ &\quad + \left[I_{nm} - S_X C_2\right]^{-1} S_F Y_3 vec\left(\hat{F}^i\right) \end{aligned} \tag{B.2}$$

This is equation (5.4) in the main text.

Next, starting from the linearized production function $vec\left(\hat{Q}_l^c\right) = D_v vec\left(\hat{V}_l^c\right) + D_X vec\left(\hat{X}_l^c\right)$ we first use (A.23) and (A.22) to get:

$$vec\left(\hat{Q}_l^c\right) = D_v vec\left(\hat{V}_l^c\right) + D_X W_{2XX} W_{1XX} vec\left(\hat{X}_{sl}^{ic}\right) \tag{B.3}$$

substituting (A.42) in (B.3) and rearranging we get:

$$vec\left(\hat{Q}_l^c\right)\left[I - D_X W_{2XX} W_{1XX} C_2\right] = D_v vec\left(\hat{V}_l^c\right) + D_X W_{2XX} W_{1XX} Z_X vec\left(\hat{p}_l^c\right) \tag{B.4}$$

It can be shown that $W_{2XX}W_{1XX}C_2 = I$ and hence $[I - D_X W_{2XX} W_{1XX} Z_4 Z_6] = D_v$ so that the above expression simplifies to:

$$vec\left(\hat{Q}_l^c\right) = vec\left(\hat{V}_l^c\right) + D_V^{-1} D_X W_{2XX} W_{1XX} Z_X \left(I - D_X W_{2XP} W_{1XP}\right)^{-1} D_V vec\left(p(\hat{V})_l^c\right) \tag{B.5}$$

eliminating $vec\left(\hat{Q}_l^c\right)$ from (B.2) and (B.5) we get:

$$\begin{aligned} vec\left(\hat{V}_l^c\right) &= \left\{(I_{nm} - S_X C_2)^{-1}(S_F Z_F + S_X Z_X) - D_v^{-1} D_X W_{2XX} W_{1XX} Z_X\right\}\left(I - D_X W_{2XP} W_{1XP}\right)^{-1} D_V vec\left(\hat{p}_l^{vc}\right) \\ &+ \left(I - S_X Z_4 Z_6\right)^{-1} S_F Y_3 vec\left(\hat{F}^c\right) \end{aligned} \tag{B.6}$$

It is easy to show the following identities:

$$\left(I_{nm} - S_X C_2\right)^{-1} = D_Q^{-1} B D_Q \tag{B.7}$$

$$\left(I - D_X W_{2XP} W_{1XP}\right)^{-1} = B' \tag{B.8}$$

Substituting (B.7) and (B.8) in (B.6) we get (5.1) in the main text, with:

$$W_V = \left[D_Q^{-1} B D_Q (S_F Z_F + S_X Z_X) - D_V^{-1} D_X W_{2XX} W_{1XX} Z_X\right] B' D_V \tag{B.9}$$

C Proofs of Propositions<For online publication>

C.1 Sketch of Proof of Proposition 5.1

In this appendix we sketch the proof of proposition 5.1. Since the underlying intuition is preserved in the case with $m = 1$, we will sketch the proof for this simplified case.

The expression for the weighting matrix is given by:

$$w = \left\{D_Q^{-1} B D_Q \left(S_F Z_F + S_X Z_X\right) - D_v^{-1} D_X W_{2XX} W_{1XX} Z_X\right\} B' D_V \tag{C.1}$$

As shown in proposition (6.2), under the constant elasticity assumption and $m = 1$, the GVC-REER weighting matrix reduces to VAREER weighting matrix defined in Bems and Johnson (2012), which according to equation (18) in that paper is given by

$$w = -I + D_Q^{-1} B D_Q S_F M_2 B' D_v \tag{C.2}$$

define the matrices
$Z_1 = Z_4 = 1_n \otimes I_n \equiv M_2$
$Z_2 = Z_5 = I_n \otimes 1_n \equiv M_1$
Under the constant elasticity assumption, from (A.43) and (A.35) we have:

$$Z_X = \sigma(M_2 - M_1) \tag{C.3}$$

$$Z_F = \theta(M_2 W_{FP} - M_1) \tag{C.4}$$

Taking the partial derivative of (C.1) wrt θ

$$\frac{\partial w}{\partial \theta} = D_Q^{-1} B D_Q S_F (M_2 W_{FP} - M_1) B' D_V \tag{C.5}$$

using (C.2) in (C.5) , the following relationship holds for the off diagonal elements of w

$$\frac{\partial w^{ij}}{\partial \theta} = w^{ij} - \left[D_Q^{-1} B D_Q S_F M_1 B' D_V\right]_{ij}, i \neq j \tag{C.6}$$

Simplifying the last term in the above expression gives (5.3) in the main text.

C.2 Proof of Proposition (6.2):

Part 1.

the GVC-REER weighting matrix under (A2) is given by:

$$W_V = \left\{(I - S_X Z_4 Z_6)^{-1} - D_v^{-1} D_X W_{2XX} W_{1XX} Z_X\right\} (I - D_X W_{2Xp} W_{1Xp})^{-1} D_V \tag{C.7}$$

where
$Z_X = \sigma_1(Z_4 W_{1XP} - Z_5) + \sigma_2(Z_4 Z_6 W_{2XP} W_{1XP} - Z_4 W_{1XP}) + \sigma_3(Z_4 Z_6 - Z_4 Z_6 W_{2XP} W_{1XP})$
with
$Z_1 = 1_n \otimes I_{nm}$, $Z_2 = I_n \otimes (1_n \otimes I_m)$, $Z_3 = I_n \otimes 1_m$, $(Z_4)_{n^2m^2Xnm^2} = 1_n \otimes I_{nm^2}$, $(Z_5)_{n^2m^2Xnm} = I_n \otimes 1_n \otimes I_m \otimes 1_m$ and $Z_6 = (I_n \otimes 1_m) \otimes I_m$

for $m = 1$, the different matrices in the above equation simplify as:
$Z_1 = Z_4 = 1_n \otimes I_n \equiv M_2$[17]
$Z_2 = Z_5 = I_n \otimes 1_n \equiv M_1$
$Z_3 = Z_6 = I_n$

$W_{2FP} = W_{2XX} = W_{2XP} = I_n$
$D_X W_{1Xp} = \Omega'$, where Ω is the country level input output matrix with $\Omega_{ij} = \frac{p^i X_{ij}}{p^j Q_j}$

$$\begin{aligned} Z_X &= \sigma_1(Z_4 W_{1XP} - Z_5) + \sigma_2(Z_4 Z_6 W_{2XP} W_{1XP} - Z_4 W_{1XP}) + \sigma_3(Z_4 Z_6 - Z_4 Z_6 W_{2XP} W_{1XP}) \\ &= \sigma_1(M_2 W_{1XP} - M_1) + \sigma_2(M_2 W_{1XP} - M_2 W_{1XP}) + \sigma_3(M_2 - M_2 W_{1XP}) \\ &= \sigma_1(M_2 W_{1XP} - M_1) + \sigma_3(M_2 - M_2 W_{1XP}) \end{aligned} \tag{C.8}$$

$$\begin{aligned} Z_F &= \theta_1(Z_1 W_{1FP} - Z_2) + \theta_2 \left(Z_1 Z_3 W_{2FP} W_{1FP} - Z_1 W_{1FP}\right) \\ &= \theta_1(M_2 W_{FP} - M_1) \end{aligned}$$

Substituting all these in the expression for Z_{Vclp} we get

$$\begin{aligned} W_V &= -\theta_1 \left(I - S_X M_2\right)^{-1} S_F (M_1 - M_2 W_{FP})(I - \Omega')^{-1} D_V \\ &+ \left(I - S_X M_2\right)^{-1} S_X \left[\sigma_1(M_2 W_{1XP} - M_1) + \sigma_3(M_2 - M_2 W_{1XP})\right] (I - \Omega')^{-1} D_V \\ &- D_V^{-1} D_X W_X \left[\sigma_1(M_2 W_{1XP} - M_1) + \sigma_3(M_2 - M_2 W_{1XP})\right] (I - \Omega')^{-1} D_V \end{aligned}$$

This is the same as equation (33) in section 5 of Bems and Johnson (2012) IOREER-BJ. Part 2 and 3 follow directly from Bems and Johnson (2012).

Part 4:

The IMF manufacturing weights are given by (Bayoumi et al. (2005))

$$W^{ij}_{imfm} = \frac{\sum_k w^{ik} s^{jk}}{\sum_k w^{ik}(1 - s^{ik})} \tag{C.9}$$

[17] In this section the matrices M_1 and M_2 are as defined in Bems and Johnson (2012) and are different from the ones defined earlier in this paper.

where $s^{jk} = \frac{sales^{jk}}{\sum_l sales^{lk}}$ and $w^{ik} = \frac{sales^{ik}}{\sum_n sales^{in}}$($sales^{ij}$ denotes gross sales from country i to country j)

Substituting the expressions for s^{jk} and w^{ik} in W^{ij} and simplifying we get:

$$W^{ij}_{imfm} = \frac{1}{T^{imfm}_i} \sum_k \left(\frac{sales^{ik}}{\sum_n sales^{in}} \right) \left(\frac{sales^{jk}}{\sum_l sales^{lk}} \right) \tag{C.10}$$

where

$$T^{imfm}_i = 1 - \sum_k \left(\frac{sales^{ik}}{\sum_n sales^{in}} \right) \left(\frac{sales^{ik}}{\sum_l sales^{lk}} \right) \tag{C.11}$$

From parts 1-3 we know that under (A1), (A2) TEER and VAREER-BJ are equivalent and given by equation (24) in BJ which is reproduced below.

$$W^{ij}_{BJ} = \frac{1}{T^{BJ}_i} \sum_k \left(\frac{p^{iv} V^{ik}}{P^{iv} V^i} \right) \left(\frac{p^{jv} V^{jk}}{P^k F^k} \right) \tag{C.12}$$

with $T^{BJ}_i = \sum_k \left(\frac{p^{iv} V^{ik}}{P^{iv} V^i} \right) \left(\frac{p^{iv} V^{ik}}{P^k F^k} \right)$

Under the assumption of no intermediates (A3)we have:

- $p^{iv} = p^i$, $Q^i = V^i$, $V^{ik} = F^{ik}$
- $sales^{ik} = p^{iv} V^{ik} = p^i V^{ik}$
- $\sum_n sales^{in} = \sum_n p^{iv} V^{in} = p^{iv} V^i$
- $\sum_l sales^{lk} = \sum_l p^{lv} V^{lk} = P^k F^k$

Substituting these in (C.10) and (C.11)

$W^{ij}_{imfm} = W^{ij}_{BJ}$

Finally, using $\alpha_c = \alpha_T = 0$ we have

$W^{ij}_{imf} = W^{ij}_{BJ}$

The equivalence of IMF-REER to GOOD-SREER and IRER follows in a straightforward manner from the respective papers (Bayoumi et al. (2013) and Thorbecke (2011))

Interpretation in the case of constant elasticity

Under the assumption that all elasticities (both in production and consumption) are the same, we can interpret the country-sector level weights purely in terms of value added trade flows. Suppose the common elasticity is η . Without loss of generality we can assume η to be unity since it factors out. Then the weighting matrix W can be written as above:

$$W_V = -I_{nm} + M_1 M_2 \tag{C.13}$$

The matrix M_1 is an nm by n matrix with each row corresponding to a unique production entity. Along this row, the n columns give the value added created by the production entity that is finally absorbed by each country. As an example, the entry corresponding to row (i,l) and column j gives the value added created by production entity (i,l) that is eventually absorbed in country j as a fraction of total value added created by the production entity (i,l). Entries in this matrix can thus be interpreted as export shares in value added terms. The corresponding mathematical expression is[18]

$$M_1((i,l),j) = \frac{v_l^i \sum_{c=1}^{n} \sum_{s=1}^{m} b_{ls}^{ic} \left(p(Q)_s^c F_s^{cj}\right)}{p(V)_l^i V_l^i} \tag{C.14}$$

where $v_l^i = \frac{p_l^{vi} V_l^i}{p_l^i Q_l^i}$. For later, it is convenient to write this expression compactly as:

$$M_1((i,l),j) = \frac{p(V)_l^i V_l^{ij}}{p(V)_l^i V_l^i} \tag{C.15}$$

where $p_l^{vi} V_l^{ij}$ is the value added created by production entity (i,l) that is finally absorbed in country j.

Matrix M_2 is an n by nm matrix with each column corresponding to a unique production entity and each row containing the value added created by the entity corresponding to the column that is absorbed in each country, as a fraction of the total final demand in that country. As an example, the entry corresponding to column (i,l) and row j gives the value added created by production entity (i,l) that is ultimately absorbed in country j as a fraction of total final demand of country j. The corresponding mathematical expression is :

$$M_2(j,(i,l)) = \frac{v_l^i \sum_{c=1}^{n} \sum_{s=1}^{m} b_{ls}^{ic} \left(p(Q)_s^c F_s^{cj}\right)}{P^j F^j} \tag{C.16}$$

As above, it turns out to be more convenient to rewrite the above expression in short-hand notation as follows:

$$M_2(j,(i,l)) = \frac{p(V)_l^i V_l^{ij}}{P^j F^j} \tag{C.17}$$

Using the generic terms from (C.15) and (C.17)we can write the weight assignment by country sector (h,l) to country-sector (c,s) where $(h,l) \neq (c,s)$ as follows:

[18] The raw expression of the matrix M_1 is $\frac{\sum_{c=1}^{n} \sum_{s=1}^{m} b_{ls}^{ic} p(Q)_s^c F_s^{cj}}{p(Q)_l^i Q_l^i}$. Multiplying and dividing by $v_l^i = \frac{p(V)_l^i V_l^i}{p(Q)_l^i Q_l^i}$ yields the expression below.

$$w_{ls}^{hc} = \sum_{k=1}^{n} \left[\frac{\left(p(V)_l^h V_l^{hk}\right)\left(p(V)_s^c V_s^{ck}\right)}{\left(p(V)_l^h V_l^h\right)\left(P^K F^k\right)} \right], (h,l) \neq (c,s) \tag{C.18}$$

where we use lower case w to denote constant elasticity weights. In particular, the weight assigned by country sector (h,l) to country-sector (c,s) where $(h,l) \neq (c,s)$ is a weighted sum of the value added created by country-sector (c,s) and absorbed by each of the countries $k(=1,..,n)$, where the weights are given by the value added created by (h,l) that is absorbed in the same country k. This captures both mutual and third country competition, because the weight is high if both $\left(p(V)_l^h V_l^{hk}\right)$ and $\left(p(V)_s^c V_s^{ck}\right)$ are high, which happens when both (h,l) and (c,s) have a high share of value added exports to country k.

(C.19)

D Equivalence of GVC-REER weighting matrices with Bems and Johnson (2017)

Condition 7.1

$$v^i \sum_{c=1}^{n} b^{ic} F^{cj} = \sum_{l=1}^{m} v_s^i \sum_{c=1}^{n} \sum_{s=1}^{m} b_{ls}^{ic} F_s^{cj} \forall i,j \tag{D.1}$$

where $v_l^i = \frac{p(V)_l^i V_l^i}{p(Q)_l^i Q_l^i}$ is the value added share for entity (i,l) and b denotes a generic element of the global inter-country Leontief inverse matrix.

Proposition D.1.

The country level weights $(W_V(GDPdef))$ defined above reduces to VAREER (and IOREER) weights defined in Bems and Johnson (2017) if either of the two conditions below are satisfied.

1. (A2), (A3) and condition 5. 1

2. (A3), (A4) and $\theta_1 = \theta_1^h = \theta_2$

Proof:

Part 1

We start with the following expression for GVC-REER weights at the country-sector level (C.7).

under the constant elasticity assumption:

$$\begin{aligned} Z_X &= -Z_5 + Z_4 Z_6 && \text{(D.2)} \\ Z_F &= -Z_2 + Z_1 Z_3 W_{2FP} W_{1FP} && \text{(D.3)} \end{aligned}$$

Here, without loss of generality we can assume that the elasticity is 1.

$$\begin{aligned} (I - S_X Z_4 Z_6)^{-1} &= D_Q^{-1} B D_Q \equiv \lambda && \text{(D.4)} \\ (I - D_X W_{2XP} W_{1XP})^{-1} &= B' && \text{(D.5)} \end{aligned}$$

Substituting (D.2), (D.3), (D.4) and (D.5) in (C.7)

$$\begin{aligned} W_V &= \left[(\lambda(S_F Z_F + S_X Z_X) - D_V^{-1} D_X W_{2XX} W_{1XX} Z_X\right] B' D_V \\ &= \lambda S_F Z_1 Z_3 W_{2FP} W_{1FP} B' D_v + \left[\lambda S_X Z_4 Z_6 - \lambda\left(S_F Z_2 + S_X Z_5\right) - D_V^{-1} D_X W_{2XX} W_{1XX} Z_X\right] B' D_V && \text{(D.6)} \end{aligned}$$

Using the identities $S_F Z_2 + S_X Z_5 = I$ and $D_V - D_X W_{2XX} W_{1XX} Z_X = (I - A)' = B'^{-1}$, we can show that the second term in (D.6) is the identity matrix, so that (D.6) reduces to:

$$\begin{aligned} W_V &= -I_{nm} + \lambda S_F Z_1 Z_3 W_{2FP} W_{1FP} [B_l^c]' D_v && \text{(D.7)} \\ &= -I_{nm} + M_{1m} M_{2m} \end{aligned}$$

where

$$\begin{aligned} M_{1m} &= \lambda S_F Z_1 Z_3 \\ M_{2m} &= W_{2FP} W_{1FP} [B_l^c]' D_v \end{aligned}$$

Next, the country level weights (which correspond to VAREER in Bems and Johnson (2017)) are given by:

$$W_V^1 = \left\{ \left(I - S_X^1 Z_4^1 Z_6^1 \right)^{-1} \left(S_F^1 Z_F^1 + S_X^1 Z_X^1 \right) - (D_v^1)^{-1} D_X^1 W_{1XX} Z_X^1 \right\} \left(I - D_X^1 W_{1Xp} \right)^{-1} D_V^1 \tag{D.8}$$

(where the superscript 1 on the matrices on the RHS of (D.8) indicates that the matrix corresponds to the case where $m = 1$)

Following steps similar to those used to derive (D.7) we can get an analogous expression:

$$\begin{aligned} W_V^1 &= -I_n + \lambda^1 S_F^1 Z_1^1 Z_3^1 W_{1FP}^1 [B^c]' D_v \\ &= -I_n + M_1 M_2 \end{aligned} \tag{D.9}$$

where

$$\begin{aligned} M_1 &= \lambda^1 S_F^1 Z_1^1 Z_3^1 \\ M_2 &= W_{1FP}^1 [B^c]' D_v^1 \end{aligned} \tag{D.10}$$

The 2 country level weights are equal iff

$$R_V W_V R_g = W_V(CG) \tag{D.11}$$

'

Since $R_V R_g = I_n$, a necessary and sufficient condition for (D.11) to hold is :

$$(R_V M_{1m})(M_{2m} R_g) = M_1 M_2 \tag{D.12}$$

$$
\begin{aligned}
(R_V M_{1m})_{ij} &= \sum_{c=1}^{n}\sum_{l=1}^{m}\sum_{s=1}^{m}\left(\frac{v_s^i b_{sl}^{ic} F_l^{cj}}{p^{vi}V^i}\right)\\
(M_{2m}R_g)_{ij} &= \sum_{c=1}^{n}\sum_{l=1}^{m}\sum_{s=1}^{m}\left(\frac{v_s^j b_{sl}^{jc} F_l^{ci}}{P^i F^i}\right)\\
(M_1)_{ij} &= \sum_{c=1}^{n}\left(\frac{v^i b^{ic} F^{cj}}{p^{vi}V^i}\right)\\
(M_2)_{ij} &= \sum_{c=1}^{n}\left(\frac{v^j b^{jc} F^{ci}}{P^i F^i}\right)
\end{aligned}
$$

here $v_s^i = \left(\frac{p_s^{vi} V_s^i}{p_s^i Q_s^i}\right)$.

From these expressions it is clear that the condition (D.12) is satisfied for all values if and only if

$$v^i \sum_{c=1}^{n} b^{ic} f^{cj} = \sum_{l=1}^{m} v_s^i \sum_{c=1}^{n}\sum_{s=1}^{m} b_{ls}^{ic} F_s^{cj} \forall i,j \tag{D.13}$$

or stacking these conditions in matrix notation:

$$diag[v^c]_{nXn}[B^c]_{nXn}[F^C]_{nXn} = (M_V)_{nXnm} diag[v_l^c]_{nmXnm}[B_l^c]_{nmXnm}[F_l^c]_{nmXn} \tag{D.14}$$

Part 2:

Under (A3), (A4) and $\theta_1 = \theta_2$ (=1(wlog)) we have,

$diag[v^c]_{nXn} = [B^c]_{nXn} = I_n$,

$diag[v_l^c]_{nmXnm} = [B_l^c]_{nmXnm} = I_{nm}$

$(M_V)_{nXnm}[F_l^c]_{nmXn} = [F^C]_{nXn}$

With these simplifications condition (D.14) is automatically satisfied and hence GVC-REER(CG) is equivalent to VAREER.

E Price Indices from WIOD

In order to check the behavior of sector level price indices constructed from WIOD which are used in our benchmark REER estimates, we construct a synthetic GDP deflator by weighting sectoral price indices in accordance with their shares in GDP, and compare it to the aggregate GDP deflator obtained from a national income accounts database.[19] The latter is multiplied by the nominal exchange rate so that both price indices are in a common currency (the US dollar). In theory, both these price indices should be identical, but this is unlikely to be the case in practice given the possibility of not only measurement error, but also differences in methodology (including differences in the actual underlying data used to compute these price indices). In our sample, however, we find a high correlation between the two GDP deflators, with both the mean and median correlation across the sample of 41 countries being in excess of 0.98 (although the correlation is as low as 0.64 for Taiwan). Figure E.1 illustrates a comparison of the two GDP deflator indices for select countries.

F Heterogenous Elasticity of Substitution

As shown in the paper, differences in the elasticity of substitution, both across product categories (intermediate vs final) as well as across countries and sectors can have an important bearing on the measurement of REER. This section begins by highlighting this with an illustrative example. We then discuss the estimation of elasticities using data from the WIOD. These elasticities are then used in the parametrisation of the model to create new GVC-REER estimates. On comparing these with the constant elasticity estimtes, we find that the differences are much more stark for sectoral REERs than for country level REERs.

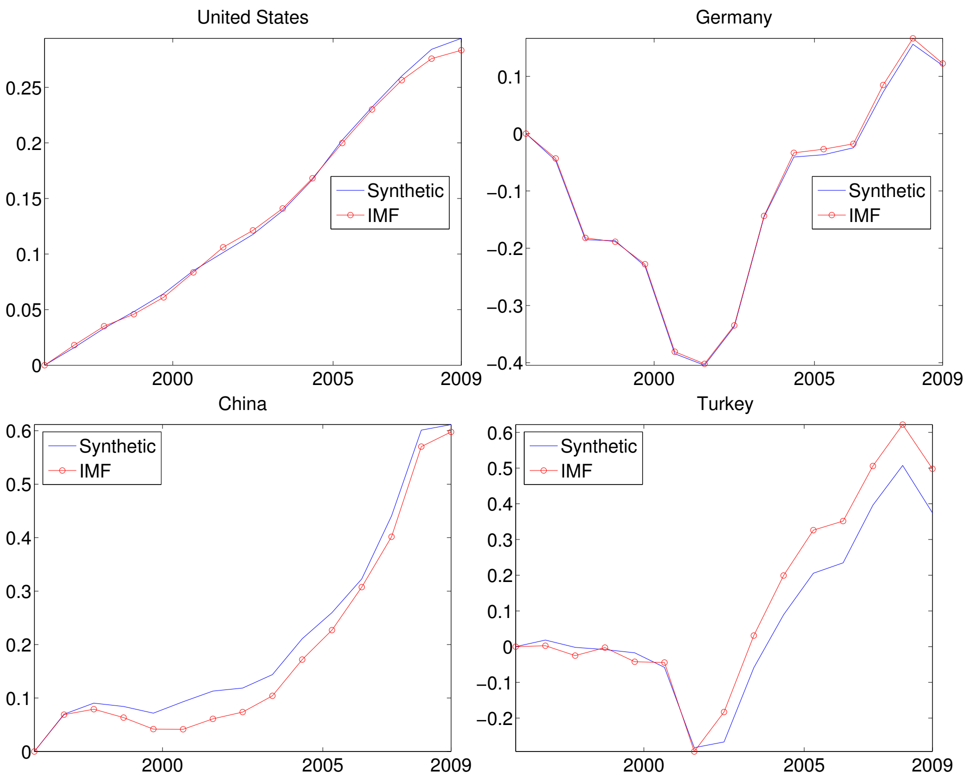
Example F.1. Three country world with limited trade in intermediate inputs:

Consider the following 3 country one sector example where the input output linkages are restricted to just one non-zero entry. Country C imports intermediates from country J, puts in own value added and sells the output to all the three countries as final output. Table 10 displays the associated input-output table.

In this simplified example only two elasticities are relevant, namely σ_3 (elasticity of substitution between C's value added and intermediate input from J in C's gross output)

[19] We use the GDP deflator time series from the IMF International Financial Statistics (IFS) database

Figure E.1



Notes: Comparison of GDP deflators. "IMF" indicates the aggregate GDP deflator series in US dollars (obtained from the IMF IFS database). "Synthetic" denotes a GDP deflator constructed using sector level price indices and value added shares from the WIOD.

Table 10 *– Input output table for F.1*

	J	C	U	J final	C final	U final	Total output
J	0	1	0	1	0	0	2
C	0	0	0	0.1	0.1	1	1. 2
U	0	0	0	0	0	1	1
Value added	2	0.2	1				
Total output	2	1. 2	1				

and θ_1 (elasticity of substitution between final goods in the final consumption basket of all countries. For simplicity, this elasticity is assumed to be common across countries).

Consider the weight assigned by country C to country J, W_{CJ}, which measures the change in demand for value added by C when price of value added by J changes. A decrease in $p(V)^J$ affects the demand for C's value added via two channels. Firstly, with regard to final goods consumption, a decrease in $p(V)^J$ leads to a shift towards J's value added (and goods containing value added by J, namely the gross output of C) in the final goods consumption bundle of all countries. The strength of this effect depends on θ_1. A higher θ_1 means that goods are more substitutable in the final goods consumption bundle of countries and hence the shift towards J's value added will be more pronounced when its price decreases. Secondly, with regard to intermediate goods and production mix, a decrease in the price of J's value added leads to a shift towards J's value added and a shift away from C's value added in the production function of C. The strength of this effect depends on σ_3. The higher is this elasticity, the higher is the shift towards J's value added in C's production (at the expense of C's own value added) and hence higher is the fall in demand for C's value added.

Table 11 presents weights based on different schemes for this example when $\sigma_3 = 1.5$, $\theta_1 = 5$. (as is done by the IMF and others, weights are normalized so that own weight is -1 and is not reported). Several aspects of the differences in the weighting schemes are noteworthy. Firstly, note that there are no negative weights in the IMF and the VAREER weighting matrix. In fact it can be easily shown that these weighting schemes are not flexible enough to accommodate negative weights under any circumstances. Next, note from column 1 that W_{JC} and W_{CJ} are negative in the GVC-REER measure. As discussed above, this is a consequence of the input output structure and a combination of a relatively high θ_1 (=5) and low σ_3 (=1. 5). Column 3 illustrates that as far as gross output is concerned, the magnitude of the negative weight assigned by country C to country J is much larger. This is because only the first effect discussed above (i.e shift in final demand) affects gross output, whereas the second effect (shift towards intermediate composition) does not affect the gross output measure.

That the uniform elasticity assumption is overly restrictive can also be noted from the observation that the VAREER(BJ) weight which does take into account trade in intermediates, does worse than the IMF weight which ignores it, although both have the wrong sign.

Column 4 shows that the Goods-REER measure of Bayoumi et al. (2013) falls somewhere in between the GVC-REER and the Q-REER measures (columns 1 and 3) so that it measures neither gross output competitiveness nor value added competitiveness. Although the aim in Bayoumi et al. (2013) is to capture gross competitiveness, they fall short of doing so because their measure uses the IMF weighting scheme which does not account for trade in

Table 11 – *Comparison of weights under different measures for example F.1*

	GVC-REER (PWW)	VAREER (BJ)	Q-REER (PWW)	GOODS-REER (BST)	IMF Weights (BLS)
W_{JC}	**-0.04**	**0.19**	**-0.04**	**1.0**	**1.00**
W_{JU}	1. 04	0.80	1. 04	0	0
W_{CJ}	**-0.25**	**0.54**	**-4.07**	**-3.40**	**0.26**
W_{CU}	1.25	0.45	5.07	4.40	0.73
W_{UJ}	0.83	0.83	0.83	0.83	0
W_{UC}	0.16	0.16	0.16	0.16	1

key	
PWW	This paper
BJ	Bems and Johnson (2012)
BST	Bayoumi et al. (2013)
BLS	Bayoumi et al. (2005)

intermediates. This aspect is further illustrated by the fact that the GOODS-REER measure (which in turn inherits this property from the IMF measure) assigns a value of 0 to W_{JU} because there is no direct trade between J and U. However, J's value added does reach U via C and so the correct weighting matrix must have $W_{JU} \neq 0$.

Lastly, note from the last two rows of table 11 that the weights assigned by country U to the remaining two countries are the same in all the measures except IMF. This is a consequence of the fact that the in the example the US trades in only final goods and all its production comprises entirely of its own value added.

F.1 Estimation of elasticities <For online publication>

F.1.1 Framework

The approach used here will be based on recent work by Soderbery (2013) which outlines certain drawbacks in the preceding two papers and proposes an estimator which outperforms them. Consider a generic CES Armington aggregator defined as follows:

$$D_t = \left[\sum_{k \in K} (w_k)^{1/\eta} (D_{kt})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{F.1}$$

The objective is to estimate the demand elasticity η. The double differenced demand equation in terms of expenditure shares is given by[20]:

[20] See Soderbery (2013) , Broda and Weinstein (2006) or Feenstra (1994) for further details including the actual derivation

$$\triangle^r ln(s_{kt}) = -(\eta - 1)\triangle^r ln(p_{kt}) + \epsilon^r_{kt} \tag{F.2}$$

where $\triangle^r ln(x_{kt}) = \triangle ln(x_{kt}) - \triangle ln(x_{rt})$ and $\triangle ln(x_{jt}) = ln(x_{jt}) - ln(x_{j(t-1)}), x = s, p$ r is called a reference variety and is typically chosen to be the one with the largest share . s_{kt} is the expenditure share of the k^{th} variety and is given by:

$$s_{kt} = \frac{p_{kt}D_{kt}}{\sum_{k\in K} p_{kt}D_{kt}} \tag{F.3}$$

Next, given a supply curve with elasticity ρ, the supply curve in terms of differenced shares and prices can be written as:

$$\triangle^r ln(p_{kt}) = \left(\frac{\rho}{1+\rho}\right)\triangle^r ln(s_{kt}) + \delta^r_{kt} \tag{F.4}$$

If the demand and supply disturbances are independent across time, then the 2 equations can be multiplied and scaled to yield:

$$Y_{kt} = \theta_1 Z_{1kt} + \theta_2 Z_{2kt} + u_{kt} \tag{F.5}$$

where $Y_{kt} = (\triangle^r ln(p_{kt}))^2$, $Z_{1kt} = (\triangle^r ln(s_{kt}))^2$, $Z_{2kt} = (\triangle^r ln(p_{kt}))(\triangle^r ln(s_{kt}))$, and $u_{kt} = \frac{\epsilon^r_{kt}\delta^r_{kt}}{1-\phi}$.

Further, the parameters of this regression model can be mapped to the primitive parameters of the demand and supply system as follows:

$\phi = \frac{\rho(\eta-1)}{1+\rho\eta} \in [0, \frac{\sigma-1}{\sigma})$

$\theta_1 = \frac{\phi}{(\eta-1)^2(1-\phi)}$ $\theta_2 = \frac{2\phi-1}{(\eta-1)(1-\phi)}$

Consistent estimates of θ_1 can be obtained by using the moment condition $E(u_{kt}) = 0$, where consistency relies on $T \to \infty$. [21] If standard procedures (2SLS or LIML) yield a value of θ_1 that gives imaginary values for η and ρ or values with the wrong sign, then the grid search or the non-linear search method of Soderbery (2013) can be used.

F.1.2 Results

We construct sectoral price indices for all cells in the WIOD input output table using the tables in previous year prices. For a fixed production entity (identified by the country-sector pair (c, l)) and a fixed sector s, table 12 shows how the estimation of the different elasticities in the model maps onto the procedure outlined above.

[21] Given the nature of the data, the value of T is typically very small. For example Soderbery (2013) uses an unbalanced panel with 15 years of data

Table 12 – *Elasticity Estimation*

	D	D_k	p_k
Production elasticities			
$\sigma_s^1(c,l)$	$(X(f)_{sl}^c)$	(X_{sl}^{kc})	$p(Q)_s^k$
$\sigma_s^{1h}(c,l)$	(X_{sl}^c)	$(X_{sl}^{cc}),(X(f)_{sl}^c)$	$p(Q)_s^k$
$\sigma^2(c,l)$	(X_l^c)	(X_{kl}^c)	$p(X)_{kl}^c$
$\sigma^3(c,l)$	(Q_l^c)	(X_l^c,V_l^c)	$(p(Q)_l^c,p(V)_l^c)$
Consumption elasticities			
$\theta_s^1(c)$	$(F(f)_s^c)$	(F_s^{kc})	$p(Q)_s^k$
$\theta_s^{1h}(c)$	(F_s^c)	$(F_s^c),(F(f)_s^c)$	$p(Q)_s^k$
$\theta^1(c)$	(F^c)	(F_k^c)	P_k^c

This table shows how the model in section 4 maps into the general framework for estimation of elasticities discussed in section F.1

Table 13 provides median estimates for each elasticity computed using the WIOD sample form 1995-2009. The medians are computed across all estimates, whose number differs according to the degree of aggregation in the nested CES framework. Although slightly on the higher side, these numbers are broadly in the range of estimates obtained in the trade literature (Broda and Weinstein (2006)), but fairly high compared to the estimates obtained in the macro literature (Justiniano and Preston (2010))[22]Our goal in this paper is to illustrate the extent to which relaxing the constant elasticity assumption impacts the measurement of REER. To that end, we take a first step by only introducing a single estimate for each of the seven structural elasticities in the model. The role of heterogeneity in elasticities of substitution across different sector and country groups would also be of interest and can be easily incorporated and studied in our framework. Table 13 in appendix F.1 provides some estimates for different sector and country groups.

F.2 Comparing GVC-REERs: Constant elasticity vs heterogenous elasticity

In order to assess the importance of incorporating heterogeneity in the elasticity of substitution emphasized above, we define a statistic to qualitatively capture the differences in REER based on uniform and heterogenous elasticity. For each entity (e) and for each year, we create a variable d_t^e which takes the value one if the GVC-REER uniform elasticity and heterogenous elasticity indices move in opposite directions and zero otherwise.

$$d_t^e = 1\left(sign(\triangle GVC - REER(BM)_t) \neq sign(\triangle GVC - REER(CE)_t)\right) \tag{F.6}$$

[22]As shown in Imbs and Méjean (2012), the macro estimates suffer from a downward bias due to aggregation.

Table 13 – *Estimates of elasticities of substitution by sector (for production) and by country (for consumption elasticity) groups*

	Median Consumption Elasticities		
	θ^1	θ^{1h}	θ^2
Full Sample	**16.05**	**6.06**	**1.93**
OECD(28)	14.84	5.46	2.13
Non-OECD(13)	21.49	9.00	1.32
Asia (7)	22.32	2.40	2.155
Europe (29)	15.088	6.06	1.16
Americas (4)	14.23	7.31	1.65

	Median Production Elasticities			
	σ^1	σ^{1h}	σ^2	σ^3
Full Sample	**16.94**	**8.93**	**4.26**	**0.93**
primary(2)	15.96	13.20	6.13	0.84
secondary(15)	16.19	5.33	5.08	0.94
tertiary(18)	17.74	10.59	3.79	1.02

We then compute the mean of d_t^e for each e across all time periods and to define the "Divergence index" for entity e as follows:

$$d^e = \frac{\sum_{t=2}^{T} d_t^e}{T-1} \tag{F.7}$$

Note that d^e takes the value zero if the two REER measures always agree in their direction of movement and takes the value of 1 if they never agree, i.e always move in opposite directions.

Table 14 summarizes the distribution of the divergence index for GVC-REER.[23] At the country level, the maximum number of times the measures move in opposite directions is 4 out of 14 years (29%). This happens for the Netherlands. At the country-sector level, the divergence index is much larger on average and reaches as high as 79% (11 out of 14 years) for certain country-sectors in the sample. The main takeaway from these results is that the consequences of heterogeneity in elasticities, at least qualitatively, are more evident in the case of REERs at the country-sector rather than at the aggregate country level.

G Bilateral GVC-RER: Example

Consider a two country world where each country has two sectors. There is no trade in intermediate goods and production comprises entirely of own value added. Table 15 shows how the final demand is distributed across sectors.

Suppose in addition, $\hat{p}(V)_1^C = -0.01, \hat{p}(V)_2^C = 0.02, \hat{p}(V)_1^U = 0, \hat{p}(V)_2^U = 0$ (all prices are

[23]The results for other REERs including Q-REER are qualitatively similar

Table 14 *– Divergence index for Countries and sectors*

level of Aggregation	Country	Country-Sector
Sample size	41	1435
$Mean(d^e)$	0.10	0.24
$Median(d^e)$	0.07	0.21
$Stdev(d^e)$	0.07	0.14
$min(d^e)$	0	0
$max(d^e)$	0.29	0.79

Table 15 *– IO table for bilateral RER*

		C		U		CFinal	Ufinal	total output
		C1	C2	U1	U2			
C	C1	0	0	0	0	1	1	2
	C2	0	0	0	0	3	0	3
U	U1	0	0	0	0	0	1	1
	U2	0	0	0	0	0	1	1
VA		2	3	1	1			
total output		2	3	1	1			

in a common currency, so nominal exchange rate is already incorporated)

Based on the conventional RER definition using an aggregate country level price index,

$$\hat{RER}^{US-CH} = \hat{p}(V)^C - \hat{p}(V)^U = 0.008 \tag{G.1}$$

and hence the conventional RER measure would indicate an increase in competitiveness of the US. This however is misleading since the entire price increase comes from China's sector 2 which does not compete with any of the US sectors. Moreover, the Chinese sector which does compete with the US is C1, which actually experiences a decrease in its price, so the correct measure of competitiveness must signal an appreciation of the US exchange rate against China, not a depreciation as measured by the standard RER in 9.1.

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