ONLINE APPENDIX*

BUSINESS CYCLES IN AN OIL ECONOMY

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This appendix contains supplementary material not included in the paper. More details about the model are provided in Section A, a description of the data used for estimation in Section B, a robustness analysis using an alternative OECD dataset in Section C, and a set of additional figures and tables referred to in the main text in Section D.

A THE MODEL

The model consists of two country blocks, referred to as home and foreign. With the exception of oil demand, the home block is a small scale version of its foreign counterpart, which represents the rest of the world. Our focus is on the limiting case where the home economy's relative size goes to zero, implying that the foreign block approximates a closed economy. Here, we focus on the part of the home economy that is abstracted from in the main text.

A.1 HOUSEHOLDS

Consider household member $h \in [0, 1]$ working in sector $j \in \{c, m, s\}$ (manufacturing, services or oil supply). He maximizes expected lifetime utility, given in period t by

$$\mathcal{W}_{j,t}(h) = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} Z_{U,s} \left[(1 - \chi_C) \ln \left(C_t(h) - \chi_C C_{t-1} \right) - \chi_N \frac{L_{j,t}(h)^{1+\varphi}}{1+\varphi} \right].$$

$C_t(h)$ denotes period t consumption while $L_{j,t}(h)$ denotes hours worked for household member h. β is the subjective time discount factor. $Z_{U,t}$ is an intertemporal preference shock. We assume the existence of a complete set of tradable Arrow securities within each economy. This makes consumption independent of the wage history, and household

*The views expressed are solely those of the authors and do not necessarily reflect those of Norges Bank, or the Bank of England or its committees.

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member h's consumption equals aggregate consumption. We drop the h-subscript whenever possible from now on. Maximization of lifetime utility is subject to a sequence of budget constraints. The period t constraint takes the following form when measured in consumption units:

$$\begin{aligned} & C_t + P^i_{r,t} I_t + B_{rH,t+1} + \mathcal{S}_t B^*_{rF,t+1} \\ & \quad \leq \Omega_t L_t + R^k_t K_t + \mathcal{D}_t - T_t + R_{t-1} B_{rH,t} \Pi_t^{-1} + R^*_{t-1} \Upsilon_{t-1} \mathcal{S}_t B^*_{rF,t} \Pi_t^{*-1} \end{aligned} \tag{A.1}$$

Aggregate consumption, investment, and domestic and foreign bond savings are denoted by C_t, I_t, and $B_{rH,t+1} + \mathcal{S}_t B^*_{rF,t+1}$ respectively. $P^i_{r,t}$ and $\mathcal{S}_t$ represent the real investment price and the real exchange rate. The income side consists of aggregate labor income $\Omega_t L_t$, capital income $R^k_t K_t$, dividends from firms $\mathcal{D}_t$, lump-sum taxes, and bond returns. The aggregate real wage rate, capital rental rate, and nominal rates on domestic and foreign savings are denoted by Ω_t, R^k_t, R_t and R^*_t respectively. Π_t and Π^*_t represent domestic and foreign (gross) consumer price inflation. Households face a premium in foreign bond markets given by $\Upsilon_{t-1} = \exp\left(-\epsilon_B \frac{NFA_t - NFA}{VA}\right) Z^*_{B,t}$, where $\frac{NFA_t}{VA} = \mathcal{S}_t \frac{NFA^*_t}{VA}$ is the net foreign asset position (the sum of privately held foreign bonds and the sovereign wealth fund) in real terms and as a share of steady state value added. $Z^*_{B,t}$ captures deviations from uncovered interest rate parity, referred to as risk premium shocks. Investments are subject to a law of motion for aggregate capital. It involves investment adjustment costs and an investment efficiency shock $Z_{I,t}$:

$$K_{t+1} = (1-\delta) K_t + Z_{I,t} \left[1 - \Psi\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \tag{A.2}$$

The adjustment cost is specified as $\Psi\left(\frac{I_t}{I_{t-1}}\right) = \frac{\epsilon_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$. Optimality conditions with respect to C_t, $B_{rH,t+1}$, $B^*_{rF,t+1}$, K_{t+1}, and I_t, follow below:

$$\Lambda_t = Z_{U,t} (1-\chi_C)^{\sigma} (C_t - \chi_C C_{t-1})^{-\sigma} \tag{A.3}$$

$$\mathbb{E}_t\left(R_t^{-1}\right) = \beta \mathbb{E}_t\left(\frac{\Lambda_{t+1}}{\Lambda_t} \Pi_{t+1}^{-1}\right) \tag{A.4}$$

$$\mathbb{E}_t\left(R_t^{*-1}\right) = \beta \mathbb{E}_t\left(\frac{\Lambda_{t+1}}{\Lambda_t} \Pi_{t+1}^{*-1} \frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} \Upsilon_t\right) \tag{A.5}$$

$$\mathcal{Q}_t = \beta \mathbb{E}_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\left[R^k_{t+1} + (1-\delta)\mathcal{Q}_{t+1}\right]\right) \tag{A.6}$$

$$\begin{aligned} P^i_{r,t} = {} & \mathcal{Q}_t Z_{I,t} \left[1 - \Psi\left(\frac{I_t}{I_{t-1}}\right) - \Psi'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} \right] \\ & + \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \mathcal{Q}_{t+1} Z_{I,t+1} \Psi'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 \right] \end{aligned} \tag{A.7}$$

Equation (A.3) equates the marginal utility of consumption with the shadow value of the budget constraint. Equation (A.4) defines the optimal intertemporal consumption path while foreign bond savings are defined by (A.5). Equation (A.6) determines $\mathcal{Q}_t$, the present marginal value of capital. Finally, (A.7) equates the relative price of investments with the gain of more capital tomorrow. The optimality conditions above are intertemporal and aggregate.

Sectoral (intratemporal) consumption and investment demand follow from the CES aggregators

$$C_t = \left[\xi^{\frac{1}{\nu}} C_{m,t}^{\frac{\nu-1}{\nu}} + (1-\xi)^{\frac{1}{\nu}} C_{s,t}^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \tag{A.8}$$

and

$$I_t = \left[\varpi^{\frac{1}{\nu_i}} I_{m,t}^{\frac{\nu_i-1}{\nu_i}} + (1-\varpi)^{\frac{1}{\nu_i}} I_{s,t}^{\frac{\nu_i-1}{\nu_i}}\right]^{\frac{\nu_i}{\nu_i-1}}, \tag{A.9}$$

respectively:

$$\frac{C_{m,t}}{C_{s,t}} = \frac{\xi}{1-\xi}\left(\frac{P_{s,t}}{P_{m,t}}\right)^{\nu}, \qquad \frac{I_{m,t}}{I_{s,t}} = \frac{\varpi}{1-\varpi}\left(\frac{P_{s,t}}{P_{m,t}}\right)^{\nu_i} \tag{A.10}$$

Thus, relative sector demand depends on the sector prices $P_{m,t}$ and $P_{s,t}$. We exploit input-output data from Statistics Norway to parameterize the expenditure weights on manufactured goods; ξ and ϖ.

Sectoral labor markets are similar to the labor market in Erceg, Henderson, and Levin (2000), but we assume that workers cannot move freely across sectors within the business cycle. Denote by $\mu_j \in (0,1)$ the measure of household members working in sector j ($\sum_{j=\{m,s,c\}} \mu_j = 1$). A competitive labor bundler buys hours from all the household members employed in the sector in order to construct an aggregate labor service $N_{j,t}$.

$$N_{j,t} = \left[\left(\frac{1}{\mu_j}\right)^{\frac{\epsilon_{w,t}}{1+\epsilon_{w,t}}} \int_{\mu_j} L_{j,t}(h)^{\frac{1}{1+\epsilon_{w,t}}} dh\right]^{1+\epsilon_{w,t}},$$

where $\epsilon_{w,t}$ is referred to as a wage markup shock. Optimal demand for worker h-hours is given by

$$L_{j,t}(h) = \frac{1}{\mu_j}\left(\frac{W_{j,t}(h)}{W_{j,t}}\right)^{-\frac{1+\epsilon_{w,t}}{\epsilon_{w,t}}} N_{j,t} = \left(\frac{W_{j,t}(h)}{W_{j,t}}\right)^{-\frac{1+\epsilon_{w,t}}{\epsilon_{w,t}}} L_{j,t},$$

where $W_{j,t}(h)$ ($W_{j,t}$) is the nominal worker-specific (sectoral) wage rate. $L_{j,t} = \frac{N_{j,t}}{\mu_j}$ is defined as average effective labor hours per worker in the sector. Each period, only a fraction $1-\theta_w$ of the workers can re-optimize wages. The remaining $1-\theta_w$ workers index wages according to the indexation rule $W_{j,t}(h) = W_{j,t-1}(h)\ddot{\Pi}_{w,t}$ ($\ddot{\Pi}_{w,t} = \Pi_{t-1}^{\iota_w}\Pi^{1-\iota_w}$). Let $\bar{W}_{j,t}$ denote the optimal wage for those that re-optimize. It is common across workers and satisfies

$$0 = \mathbb{E}_t \sum_{s=t}^{\infty} (\beta\theta_w)^{s-t} \Lambda_s \frac{L_{j,s}(h)}{P_s}\left[\bar{W}_{j,t}\prod_{i=1}^{s-t}\ddot{\Pi}_{w,s-i} - (1+\epsilon_{w,t})\, MRS_{j,s|t}(h) P_s\right]. \tag{A.11}$$

$MRS_{j,s|t}(h)$ is the marginal rate of substitution (between consumption and labor) in period s, given a wage last set in period t. Finally, sectoral wages follow the following law of motion:

$$W_{j,t} = \left[\theta_w \left(W_{j,t-1} \ddot{\Pi}_{w,t} \right)^{-\frac{1}{\epsilon_{w,t}}} + (1-\theta_w) \bar{W}_{j,t}^{-\frac{1}{\epsilon_{w,t}}} \right]^{-\epsilon_{w,t}}.$$

This equation, combined with the wage setting equation, can be used to derive a sectoral New Keynesian wage Phillips curve.

A.2 Firms

Output by domestic firm f in sector $j \in \{c, m, s\}$ is given by

$$Y_{j,t}(f) = Z_{Aj,t} X_{j,t}(f)^{\phi_j} N_{j,t}(f)^{\psi_j} K_{j,t}(f)^{1-\phi_j-\psi_j} - \Phi_j. \tag{A.12}$$

Here, $X_{j,t}(f)$, $N_{j,t}(f)$ and $K_{j,t}(f)$ are firm f's use of materials, labor and capital respectively. Φ_j is a fixed production cost that will be calibrated to ensure zero profit in steady state. $Z_{Aj,t}$ represents sector-specific productivity shocks. Intermediate trade is modeled as in Bouakez, Cardia, and Ruge-Murcia (2009) and Bergholt (2015), where $X_{j,t}$ is a composite of inputs produced in manufacturing and services:

$$X_{j,t} = \left[\zeta_j^{\frac{1}{\nu_j}} X_{mj,t}^{\frac{\nu_j-1}{\nu_j}} + (1-\zeta_j)^{\frac{1}{\nu_j}} X_{sj,t}^{\frac{\nu_j-1}{\nu_j}} \right]^{\frac{\nu_j}{\nu_j-1}} \tag{A.13}$$

Firm f maximizes an expected stream of dividends given by $\mathbb{E}_t \sum_{s=t}^{\infty} \mathcal{Z}_{t,s} P_s \mathcal{D}_{j,s}(f)$, where $\mathcal{Z}_{t,s} = \beta^{s-t} \frac{\Lambda_s}{\Lambda_t} \frac{P_t}{P_s}$ is the stochastic discount factor. Dividends and total costs are:

$$\begin{aligned} \mathcal{D}_{j,s}(f) &= P_{rHj,s}(f) A_{Hj,s}(f) + P^*_{rHj,s}(f) A^*_{Hj,s}(f) - TC_{rj,s}(f) \\ TC_{rj,s}(f) &= P^x_{rj,s} X_{j,s}(f) + \Omega_{j,s} N_{j,s}(f) + R^k_s K_{j,s}(f), \end{aligned}$$

$A_{Hj,s}(f)$ is total domestic absorption of the firm's output, while $A^*_{Hj,s}(f)$ is the exported counterpart. $P_{rHj,s}(f) = \frac{P_{Hj,s}(f)}{P_s}$ and $P^*_{rHj,s}(f) = \frac{\mathcal{E}_s P^*_{Hj,s}(f)}{P_s}$ are the associated real prices in domestic currency, $\mathcal{E}_s$ is the nominal exchange rate. Optimal factor demand follows:

$$\frac{N_{j,t}(f)}{X_{j,t}(f)} = \frac{\psi_j}{\phi_j} \frac{P^x_{rj,t}}{\Omega_{j,t}}, \tag{A.14}$$

$$\frac{K_{j,t}(f)}{X_{j,t}(f)} = \frac{1-\phi_j-\psi_j}{\phi_j} \frac{P^x_{rj,t}}{R^k_t}, \tag{A.15}$$

$$\frac{X_{mj,t}(f)}{X_{sj,t}(f)} = \frac{\zeta_j}{1-\zeta_j} \left(\frac{P_{s,t}}{P_{m,t}} \right)^{\nu_j} \tag{A.16}$$

Price setting is subject to monopolistic competition and sticky prices in a way analogous to the labor market. Firms set prices à la Calvo (1983), but exported goods are priced in local currency (LCP). We denote by $1-\theta_{pj}$ the probability of an optimal price change. Non-optimizing firms index prices according to rules $P_{Hj,t}(f) = P_{Hj,t-1}(f)\ddot{\Pi}_{Hj,t}$ ($\ddot{\Pi}_{Hj,t} = \Pi^{\iota_p}_{Hj,t-1}\Pi^{1-\iota_p}_{Hj}$) and $P^*_{Hj,t}(f) = P^*_{Hj,t-1}(f)\ddot{\Pi}^*_{Hj,t}$ ($\ddot{\Pi}^*_{Hj,t} = \Pi^{*\iota_p}_{Hj,t-1}\Pi^{*1-\iota_p}_{Hj}$). Let $\bar{P}_{jH,t}$ and $\bar{P}^*_{Hj,t}$ be the optimal new prices. They are common across firms, and satisfy:

$$
\begin{aligned}
0 &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta_{pj}^{s-t} \mathcal{Z}_{t,s} X_{Hj,s}(f) \left[\bar{P}_{Hj,t} \prod_{i=1}^{s-t} \ddot{\Pi}_{Hj,s-i} - (1+\epsilon_{p,s}) P_s RMC_{j,s}(f) \right] \\
0 &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta_{pj}^{s-t} \mathcal{Z}_{t,s} X^*_{Hj,s}(f) \mathcal{E}_s \left[\bar{P}^*_{Hj,t} \prod_{i=1}^{s-t} \ddot{\Pi}^*_{Hj,s-i} - (1+\epsilon_{p,s}) P_s RMC_{j,s}(f) \right]
\end{aligned}
\tag{A.17}
$$

Our production technology implies that all firms face the same real marginal cost $RMC_{j,t}$:

$$
RMC_{j,t} = \frac{1}{Z_{Aj,t}} \left(\frac{P^x_{rj,t}}{\phi_j} \right)^{\phi_j} \left(\frac{\Omega_{j,t}}{\psi_j} \right)^{\psi_j} \left(\frac{R^k_t}{1-\phi_j-\psi_j} \right)^{1-\phi_j-\psi_j} \tag{A.18}
$$

The staggered price setting structure combined with partial indexation implies the following price dynamics:

$$
\begin{aligned}
P_{Hj,t} &= \left[\theta_{pj} \left(P_{Hj,t-1} \ddot{\Pi}_{Hj,t} \right)^{-\frac{1}{\epsilon_{p,t}}} + (1-\theta_{pj}) \bar{P}_{Hj,t}^{-\frac{1}{\epsilon_{p,t}}} \right]^{-\epsilon_{p,t}} \\
P^*_{Hj,t} &= \left[\theta_{pj} \left(P^*_{Hj,t-1} \ddot{\Pi}^*_{Hj,t} \right)^{-\frac{1}{\epsilon_{p,t}}} + (1-\theta_{pj}) \bar{P}^{*\,-\frac{1}{\epsilon_{p,t}}}_{Hj,t} \right]^{-\epsilon_{p,t}}
\end{aligned}
$$

One can combine these with the optimal pricing equations in order to derive New Keynesian price Phillips curves for domestic goods and exports. Import prices ($P_{Fj,t}$) are set similarly, except that they are chosen by foreign firms with costs in foreign currency.

A.3 Aggregation

Each sector $j \in \{m, s, c\}$ has a competitive bundler who combines individual goods into a final aggregate good $A_{j,t}$. Aggregation is subject to a nested CES structure (exports are aggregated in the same way abroad):

$$
\begin{aligned}
A_{Hj,t} &= \left(\int_0^1 A_{Hj,t}(f)^{\frac{1}{1+\epsilon_{p,t}}} df \right)^{1+\epsilon_{p,t}} \\
A_{Fj,t} &= \left(\int_0^1 A_{Fj,t}(f)^{\frac{1}{1+\epsilon^*_{p,t}}} df \right)^{1+\epsilon^*_{p,t}} \\
A_{j,t} &= \left[\bar{\alpha}_j^{\frac{1}{\eta}} A_{Hj,t}^{\frac{\eta-1}{\eta}} + (1-\bar{\alpha}_j)^{\frac{1}{\eta}} A_{Fj,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}
\end{aligned}
\tag{A.19}
$$

Domestically produced goods and imports at the sector level are denoted by $A_{Hj,t}$ and $A_{Fj,t}$, respectively. The expenditure weight $\bar{\alpha}_j$ is defined as $\bar{\alpha}_j = 1 - (1-\varsigma)(1-\alpha_j)$, where $\varsigma \in [0,1]$ represents the relative population size of home and $\alpha_j \in [0,1]$ is the degree of home bias. Cost minimization gives rise to a set of optimal demand schedules, expressed below in the limit as $\varsigma \to 0$:

$$\begin{aligned} A_{Hj,t}(f) &= \left(\frac{P_{Hj,t}(f)}{P_{Hj,t}}\right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} A_{Hj,t}, \quad & A_{Fj,t}(f) &= \left(\frac{P_{Fj,t}(f)}{P_{Fj,t}}\right)^{-\frac{1+\epsilon^*_{p,t}}{\epsilon^*_{p,t}}} A_{Fj,t}, \\ A_{Hj,t} &= \alpha_j \left(\frac{P_{H,t}}{P_{j,t}}\right)^{-\eta} A_{j,t}, & A_{Fj,t} &= (1-\alpha_j)\left(\frac{P_{Fj,t}}{P_{j,t}}\right)^{-\eta} A_{j,t} \end{aligned} \tag{A.20}$$

The final good $A_{j,t}$ is used to cover demand for private and public consumption, investments, and intermediate inputs:

$$A_{j,t} = C_{j,t} + I_{j,t} + G_{j,t} + X_{jm,t} + X_{js,t} + X_{jc,t}$$

Our setup implies equal import shares across goods within sectors (e.g. $C_{j,t}$ and $I_{j,t}$), but different import shares across aggregate goods (such as C_t and I_t).

Market clearing at the firm level dictates that $Y_{j,t}(f) = A_{Hj,t}(f) + A^*_{Hj,t}(f)$. Sectoral market clearing follows:[1]

$$Y_{j,t} = \int_0^1 \left(A_{Hj,t}(f) + A^*_{Hj,t}(f)\right) df = A_{Hj,t}\Delta_{Hj,t} + A^*_{Hj,t}\Delta^*_{Hj,t},$$

where sectoral output is defined as

$$Y_{j,t} = \int_0^1 Y_{j,t}(f)\, df = Z_{Aj,t} X_{j,t}^{\phi_j} N_{j,t}^{\psi_j} K_{j,t}^{1-\phi_j-\psi_j} - \Phi_j.$$

$X_{j,t} = \int_0^1 X_{j,t}(f)\, df$, $N_{j,t} = \int_0^1 N_{j,t}(f)\, df$, and $K_{j,t} = \int_0^1 K_{j,t}(f)\, df$ represent total factor use. Total hours worked, in contrast, is $\int_{\mu_j} L_{j,t}(h)\, dh = \frac{1}{\mu_j} N_{j,t}\Delta_{wj,t} = L_{j,t}\Delta_{wj,t}$.[2] Hours worked per person in the entire economy is $L_t = \sum_{j=1}^{\mathcal{J}} \mu_j L_{j,t}$.

We note, for completeness, that all variables are measured per home capita. Firm f's exports per foreign capita, denoted by $\tilde{A}^*_{Hj,t}(f)$, is linked to $A^*_{Hj,t}(f)$ by the identity $A^*_{Hj,t}(f) = \frac{1-\varsigma}{\varsigma}\tilde{A}^*_{Hj,t}(f)$. It follows that per capita absorption abroad is given by

$$\tilde{A}^*_{Hj,t} = 0, \qquad \tilde{A}^*_{Fj,t} = 0, \qquad \text{and} \qquad A^*_{Fj,t} = A^*_{j,t}$$

when $\varsigma \to 0$. This is the sense in which the foreign block approximates a closed economy.

Finally we define real value added at the sector level in consumption units. It can be

[1] $\Delta_{Hj,t} = \int_0^1 \left(\frac{P_{Hj,t}(f)}{P_{Hj,t}}\right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} df = 1$ and $\Delta^*_{Hj,t} = \int_0^1 \left(\frac{P^*_{Hj,t}(f)}{P^*_{Hj,t}}\right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} df = 1$ hold up to first order.

[2] $\Delta_{wj,t} = \int_{\bar{\mu}_{j-1}}^{\bar{\mu}_j} \left(\frac{W_{j,t}(h)}{W_{j,t}}\right)^{-\frac{1+\epsilon_{w,t}}{\epsilon_{w,t}}} dh = \mu_j$ holds up to first order.

written in three different, but model consistent ways:

$$
\begin{aligned}
VA_{j,t} &= P_{rHj,t}A_{Hj,t} + P^*_{rHj,t}A^*_{Hj,t} - P^x_{rj,t}X_{j,t} \\
&= \Omega_{j,t}N_{j,t} + R^k_t K_{j,t} + \mathcal{D}_{j,t} \\
&= P_{rj,t}\left(C_{j,t} + I_{j,t} + G_{j,t}\right) + TB_{j,t} + P_{rj,t}\sum_{l=\{m,s,c\}} X_{jl,t} - P^x_{rj,t}X_{j,t}
\end{aligned}
\tag{A.21}
$$

The first line defines sectoral value added according to the output approach, i.e. as the value of gross output minus the value of intermediate inputs. The second line measures value added according to the income approach, where $\mathcal{D}_{j,t} = \int_0^1 \mathcal{D}_{j,t}(f)\, df$ is the sum of sectoral dividends. The last line uses the expenditure approach, where $TB_{j,t} = P^*_{rHj,t}A^*_{Hj,t} - P_{rFj,t}A_{Fj,t}$ is the trade balance. The cross-sectoral balance for intermediate inputs cancels out in the aggregate, non-oil economy. Thus, aggregate non-oil GDP is

$$
GDP_t = \sum_{j\in\{m,s\}} VA_{j,t} = C_t + P^i_{r,t}I_t + P^g_{r,t}G_t + TB_t + P^x_{rc,t}X_{c,t},
$$

as in the main text. The private economy's aggregate trade balance, $TB_t = TB_{m,t} + TB_{s,t}$, contributes to the accumulation of privately held foreign bonds:

$$
\mathcal{S}_t B^*_{rF,t+1} = R^*_{t-1}\Upsilon_{t-1}\mathcal{S}_t B^*_{rF,t}\Pi^{*-1}_t + TB_t
$$

The total net foreign asset position of home follows as the sum of private and public savings in international assets:

$$
NFA_{t+1} = \mathcal{S}_t B^*_{rF,t+1} + \mathcal{S}_t SWF^*_{t+1}
$$

This completes our model description.

B Data

This section describes the dataset used for estimation of the DSGE model. All variables are quarterly and cover the period 1995Q1-2015Q4. We use Norwegian data as observables for the commodity exporter, and data from EU28 as observables for the foreign economy. The latter is substituted with the OECD in robustness checks. All raw data are publicly available from EuroStat, OECD, Statistics Norway, the Federal Reserve Bank of St. Louis, and Norges Bank. A detailed overview of the data sources is provided in Table B.1.

The joint posterior distribution of our baseline model is guided by 19 observable time series. For each country we observe value added in services and manufacturing, private and public consumption, private investments, wage inflation, core consumer price inflation (adjusted for taxes and energy), and the interest rate. Additional observables for the Norwegian economy are the import weighted exchange rate, petroleum output, and investments in the petroleum sector. Our final observable variable is the Brent oil price. EU data are measured in Euro while OECD data are measured in PPP adjusted U.S. dollars. We use aggregate value added data for the OECD. For Norway and EU28 we use the 3-month interbank offered rate as the observable interest rate. For OECD we use a GDP weighted average of the interbank rates in the Euro area and the U.S. The wage rate in all countries is the hourly earnings rate in manufacturing, published by the OECD as part of the Main Economic Indicators. The observed exchange rate is the import weighted effective exchange rate I44, published by Norges Bank.

EuroStat decomposes aggregate GDP into sectoral value added in 10 sectors: (i) agriculture, forestry and fishing, (ii) manufacturing, (iii) construction, (iv) wholesale and retail trade, transport, accommodation and food service activities, (v) information and communication, (vi) financial and insurance activities, (vii) real estate, (viii) professional, scientific and technical activities; administrative and support service activities, (ix) public administration, defense, education, human health and social work, and (x) arts, entertainment and recreation; other service activities. We aggregate (i)-(iii) into a goods (manufacturing) sector and (iv)-(viii) plus (x) into services, respectively. (ix) is used to calibrate the public sector.

The model variables are defined per capita as well as in terms of consumption goods. They are also measured in log deviation from trend. We accommodate these definitions by constructing the observable domestic variables as follows (see Table B.1 for notation):

- $VA_{j,t} = \frac{VA^{\text{Data}}_{j,t}}{P^{\text{Data}}_{\text{cpi},t} LF^{\text{Data}}_t}$
- $C_t = \frac{C^{\text{Data}}_t}{P^{\text{Data}}_{\text{cpi},t} LF^{\text{Data}}_t}$
- $G_t = \frac{G^{\text{Data}}_t}{P^{\text{Data}}_{\text{cpi},t} LF^{\text{Data}}_t}$
- $P^i_{r,t} I_t = \frac{I^{\text{Data}}_t}{P^{\text{Data}}_{\text{cpi},t} LF^{\text{Data}}_t}$
- $\Pi_t = \frac{P^{\text{Data}}_t}{P^{\text{Data}}_{t-4}}$
- $\Pi_{w,t} = \frac{W^{\text{Data}}_t}{W^{\text{Data}}_{t-4}}$
- $R_t = \frac{R^{\text{Data}}_t}{400}$
- $\mathcal{S}_t = \frac{\mathcal{E}^{\text{Data}}_t P^{\text{Data}*}_{\text{cpi},t}}{P^{\text{Data}}_{\text{cpi},t}}$
- $Y_{o,t} = \frac{Y^{\text{Data}}_{\text{oil},t}}{LF^{\text{Data}}_t}$
- $P^i_{rc,t} I_{o,t} = \frac{I^{\text{Data}}_{\text{oil},t}}{P^{\text{Data}}_{\text{cpi},t} LF^{\text{Data}}_t}$
- $P^*_{o,t} = \frac{P^{\text{Data}}_{\text{oil},t}}{P^{\text{Data}}_{\text{cpi},t}}$

Table B.1: Data sources

Variable	*Description*	*Source*		
		Norway	*EU28*	*OECD*
$VA_{j,t}^{\text{Data}}$	GDP	EuroStat	EuroStat	OECD
C_t^{Data}	Private consumption expenditures	EuroStat	EuroStat	OECD
I_t^{Data}	Gross fixed capital formation	EuroStat	EuroStat	OECD
G_t^{Data}	Government consumption expenditures	EuroStat	–	–
$P_{\text{cpi},t}^{\text{Data}}$	Implicit CPI deflator	Statistics Norway	EuroStat	OECD
R_t^{Data}	Interest rate	Norges Bank	OECD	OECD
W_t^{Data}	Wage index	OECD	OECD	OECD
LF_t^{Data}	Population	OECD	OECD	OECD
$\mathcal{E}_t^{\text{Data}}$	Nominal exchange rate (I44)	Norges Bank	–	–
$P_{\text{oil},t}^{\text{Data}}$	Nominal oil price (Brent spot)	–	FRED	FRED
$Y_{\text{oil},t}^{\text{Data}}$	Oil output	Statistics Norway	–	–
$I_{\text{oil},t}^{\text{Data}}$	Oil investments	Statistics Norway	–	–

Foreign variables are defined similarly. We take logs of all variables. Most raw data are seasonally adjusted at the source, but we use the X13 ARIMA-SEATS program (available from the U.S. Census Bureau) in order to identify and filter out additional seasonal patterns. Quantity variables are also de-trended. To this end we use a backward-looking HP-filter ($\lambda = 1600$) which, consistent with expectations formation in the model, does not exploit current information about future realizations.

C Robustness analysis using OECD data

The main analysis uses data from EU28 as proxy for the international economy. EU28 accounts for 2/3 of Norway's international trade, and harmonized data on sectoral value added are available for all EU28 members in addition to Norway. The main drawback of the EU data is that they cover less than a quarter of world GDP. This section, therefore, redo the analysis using OECD as proxy for the global economy. OECD data do not include sectoral variables, but they cover about 2/3 of global GDP. Details about the data are reported in the data appendix.

C.1 VAR results

First, we re-estimate the VAR model in the paper. Impulse responses are shown in Figure C.1 and Figure C.2, respectively. They are based on the same identification scheme as in the main paper. Qualitatively, we note similar responses as with EU28 data. That is, both the oil price shock and the international activity shock are associated with rising sectoral value added in all three sectors in Norway. The oil price shock is also associated with exchange rate appreciation. Quantitatively, we observe more persistent responses to the oil price shock in the domestic economy when OECD data are used.

C.2 Estimated DSGE parameters

Next, we estimate the DSGE model using OECD data. The model itself is left unchanged, as well as priors and the calibration. Prior and posterior parameters are reported in Table C.1. We emphasize a couple of observations: first, both the estimated oil demand elasticity and the volatility of international oil productivity shocks are higher than in the main analysis. Higher substitution elasticity implies smaller effects of oil price shocks in the international economy (due to more substitution towards other goods). More volatile oil supply shocks imply the opposite. Second, most of the international shocks are less persistent when OECD data are used. This suggests that international shocks might play a smaller role for the domestic economy at long forecasting horizons, compared with the results based on data from EU28. Most other parameters are fairly similar to those in the main text.

C.3 Filtered predictions

Figure C.3 compares OECD data with one step ahead predictions from the model. The fit is largely comparable with that using EU data, although it deteriorates somewhat for public demand and the exchange rate. Measurement errors play, once again, only a minor role.

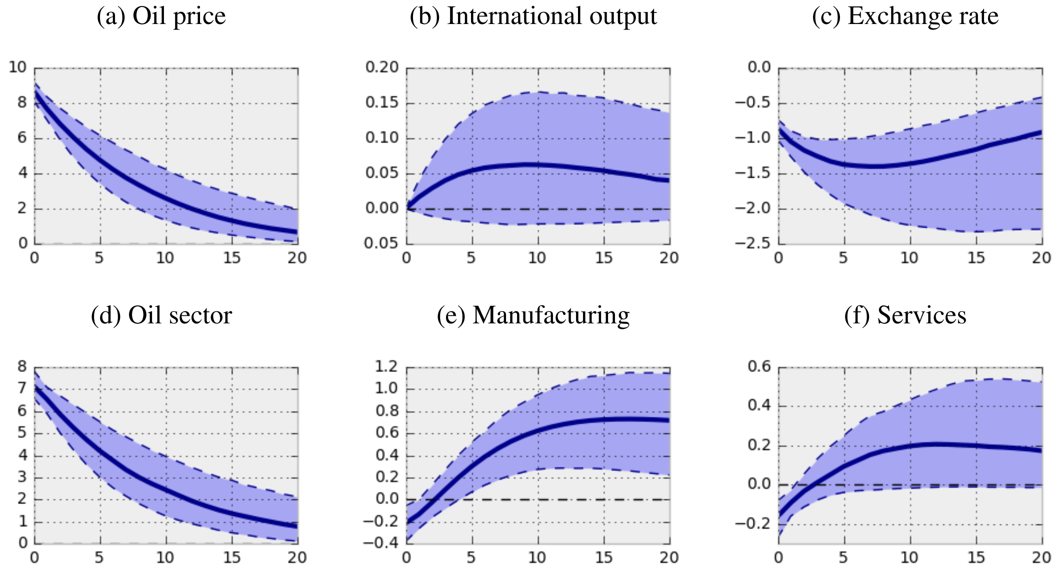
C.4 Forecast error variance decomposition

Figure C.4 reports the forecast error variance decomposition of mainland GDP. Compared with the baseline, we find that foreign shocks are more important in the short run. However, both foreign macro shocks and the foreign oil supply shock are less dominant at long horizons. At the 10 years horizon, the variance share accounted for by oil supply drops from 33% to 19%. This number is still consistent with the view that oil price volatility is important for mainland GDP.

C.5 Impulse responses

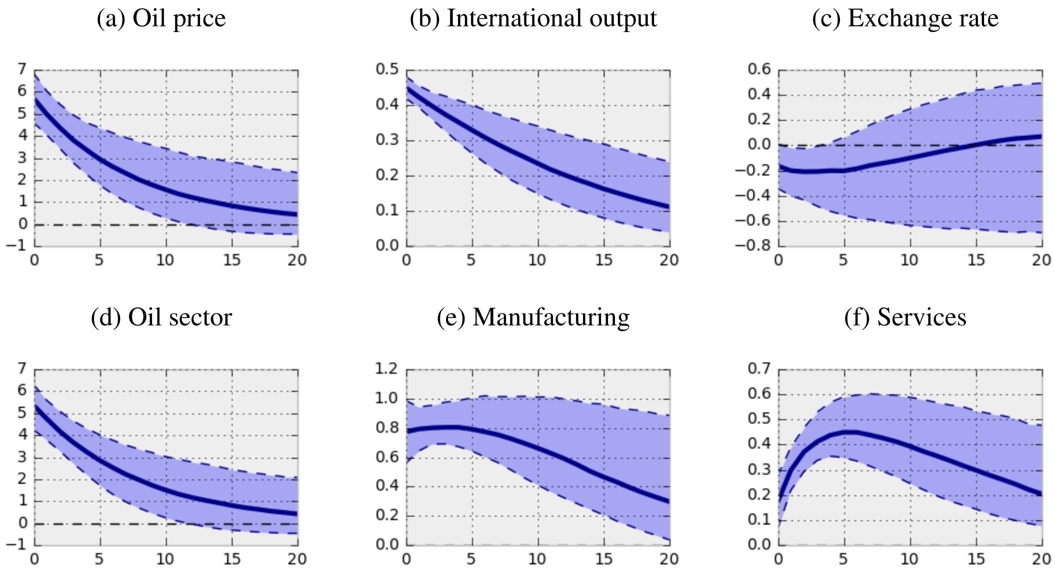
Posterior impulse responses to an oil price shock are reported in Figure C.5 and Figure C.6, respectively. They are largely comparable with those based on EU data. Qualitatively, an oil price boom leads to lower activity abroad and higher activity in Norway. The shapes of the responses resemble closely those in the main analysis. Quantitatively, we see that most effects in OECD are slightly smaller, although the oil price dynamics are largely unchanged. Also the Norwegian variables display somewhat smaller effects, consistent with the variance decomposition described above.

Figure C.1: International oil price shock



Note: Impulse responses to a one standard deviation shock to the real oil price. Calculations are based on 1000 draws from the posterior distribution. Median and 68 % credible bands.

Figure C.2: International activity shock



Note: Impulse responses to a one standard deviation shock to international activity. Calculations are based on 1000 draws from the posterior distribution. Median and 68 % credible bands.

Table C.1: Prior and posterior distributions

		Prior	Posterior domestic and oil			Posterior foreign		
		Prior(P1,P2)	Mode	Mean	5%-95%	Mode	Mean	5%-95%
χ_C	Habit	B(0.70,0.10)	0.86	0.74	0.64-0.91	0.82	0.80	0.75-0.86
ϵ_I	Inv. adj. cost	G(5.00,1.00)	5.83	6.18	5.20-7.16	4.96	5.23	4.43-6.00
θ_w	Calvo wages	B(0.65,0.07)	0.80	0.75	0.68-0.81	0.84	0.83	0.79-0.87
ι_w	Indexation, π_w	B(0.30,0.15)	0.42	0.48	0.31-0.63	0.37	0.42	0.28-0.56
θ_{pm}	Calvo manu.	B(0.45,0.07)	0.55	0.57	0.51-0.62	0.57	0.53	0.47-0.58
θ_{ps}	Calvo serv.	B(0.65,0.07)	0.69	0.72	0.67-0.77	0.92	0.92	0.89-0.94
ι_p	Indexation, π_p	B(0.30,0.15)	0.64	0.59	0.44-0.72	0.20	0.22	0.12-0.34
ρ_r	Smoothing, r	B(0.50,0.10)	0.85	0.84	0.81-0.87	0.69	0.70	0.64-0.77
ρ_π	Taylor, π	N(2.00,0.20)	2.13	2.22	2.08-2.36	1.70	1.61	1.51-1.70
ρ_{de}	Taylor, Δe	N(0.10,0.05)	0.01	0.07	0.00-0.12	–	–	–
ρ_y	Taylor, gdp	N(0.13,0.05)	0.13	0.17	0.11-0.23	0.12	0.10	0.05-0.15
η	H-F elasticity	G(1.00,0.15)	0.62	0.63	0.57-0.69	–	–	–
τ_y	Fiscal, gdp	N(0.00,0.15)	0.15	0.14	0.02-0.28	–	–	–
τ_o	Fiscal, swf	N(0.00,0.15)	0.05	0.08	0.03-0.14	–	–	–
ρ_g	Fiscal, g	B(0.50,0.15)	0.55	0.47	0.34-0.64	–	–	–
ϵ_O	Inv. adj. cost oil	G(5.00,1.00)	6.00	5.69	4.65-6.55	–	–	–
ς_d^*	Oil demand elast.	G(0.30,0.15)	–	–	–	0.55	0.53	0.43-0.64
ς_s^*	Oil supply elast.	G(0.30,0.15)	–	–	–	0.01	0.01	0.00-0.02
ρ_A	Technology	B(0.35,0.15)	0.37	0.42	0.31-0.55	0.71	0.74	0.66-0.83
ρ_I	Investment	B(0.35,0.15)	0.43	0.32	0.17-0.47	0.47	0.50	0.41-0.59
ρ_U	Preferences	B(0.35,0.15)	0.46	0.69	0.39-0.88	0.89	0.82	0.70-0.93
ρ_W	Wage markup	B(0.35,0.15)	0.53	0.64	0.53-0.74	0.53	0.47	0.33-0.66
ρ_M	Price markup	B(0.35,0.15)	0.87	0.88	0.83-0.94	0.74	0.71	0.65-0.78
ρ_B	UIP	B(0.50,0.15)	0.95	0.94	0.92-0.96	–	–	–
ρ_F	Oil investment	B(0.50,0.15)	0.71	0.80	0.70-0.89	–	–	–
ρ_{Ao}	Oil supply	B(0.50,0.15)	0.71	0.74	0.66-0.83	0.83	0.89	0.83-0.94
σ_{Am}	Sd tech. manu.	IG(0.50,2.00)	1.22	1.26	1.01-1.49	0.52	0.53	0.33-0.72
σ_{As}	Sd tech. serv.	IG(0.50,2.00)	1.28	1.39	1.01-1.77	7.23	7.41	6.33-8.46
σ_I	Sd investment	IG(0.50,2.00)	10.97	13.11	9.48-16.32	0.65	0.85	0.48-1.22
σ_U	Sd preferences	IG(0.50,2.00)	5.97	4.22	2.32-7.61	1.14	1.12	0.77-1.47
σ_G	Sd government	IG(0.50,2.00)	0.95	0.87	0.75-0.99	–	–	–
σ_W	Sd labor supply	IG(0.10,2.00)	0.26	0.24	0.18-0.29	0.09	0.11	0.07-0.15
σ_{Mm}	Sd markup manu.	IG(0.10,2.00)	0.45	0.46	0.30-0.63	0.11	0.14	0.07-0.21
σ_{Ms}	Sd markup serv.	IG(0.10,2.00)	0.10	0.13	0.08-0.17	0.07	0.10	0.07-0.13
σ_R	Sd mon. pol.	IG(0.02,2.00)	0.14	0.16	0.13-0.18	0.13	0.14	0.11-0.17
σ_B	Sd UIP	IG(0.50,2.00)	0.24	0.25	0.20-0.29	–	–	–
σ_F	Sd oil inv.	IG(0.50,2.00)	10.83	9.34	7.14-11.53	–	–	–
σ_{Ao}	Sd oil supply	IG(0.50,2.00)	2.69	2.89	2.50-3.26	7.24	6.31	4.53-8.07

Note: Posterior moments are computed from 5,000,000 draws generated by the Random Walk Metropolis-Hastings algorithm, where the first 4,000,000 are used as burn-in. B denotes the beta distribution, N normal, G gamma, and IG inverse gamma. P1 and P2 denote the prior mean and standard deviation. For IG, P1 and P2 denote the prior mode and degrees of freedom, respectively. Shock volatilities are multiplied by 100 relative to the text.

Figure C.3: Data versus one step ahead filtered estimates (median) – OECD

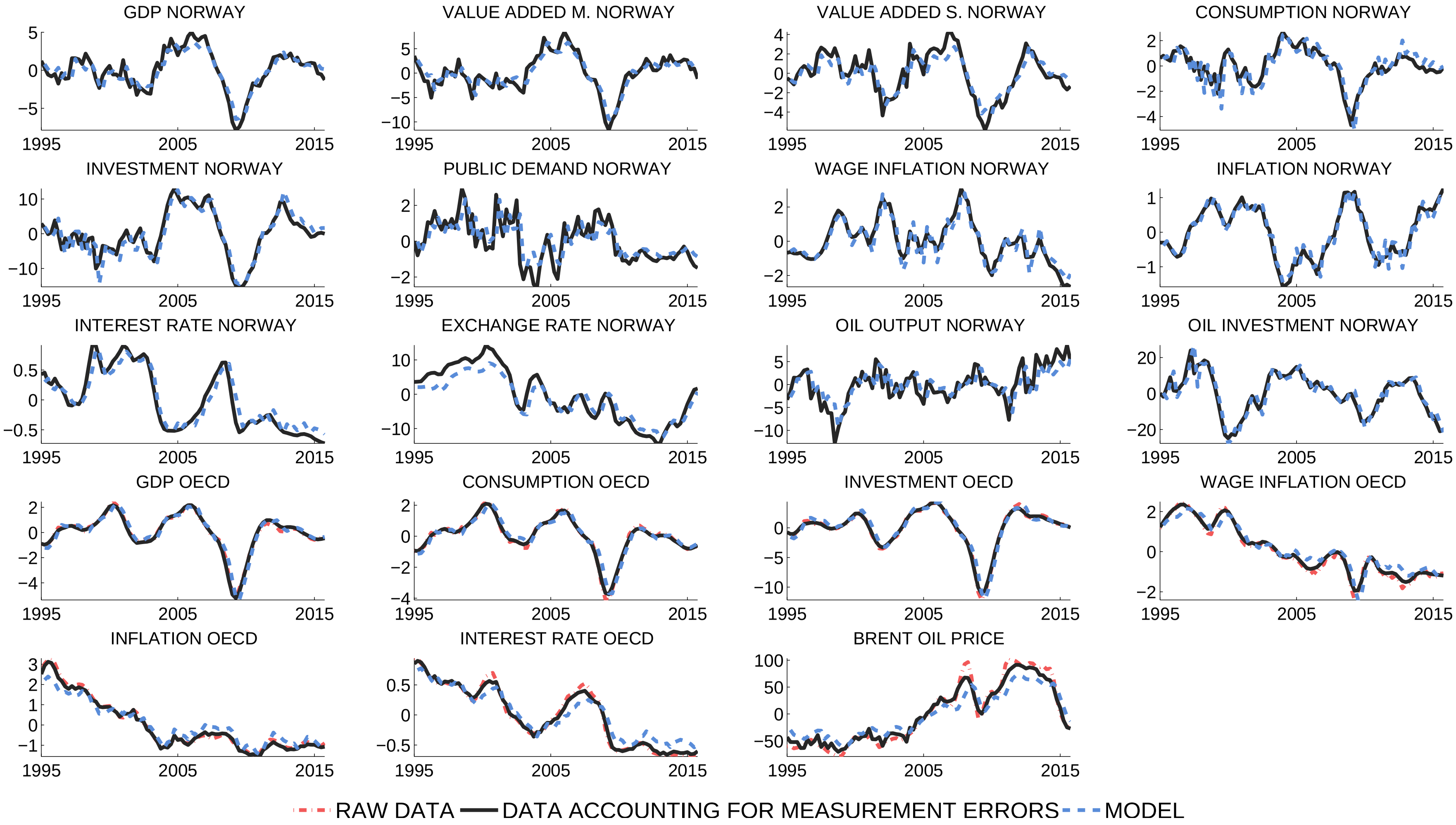
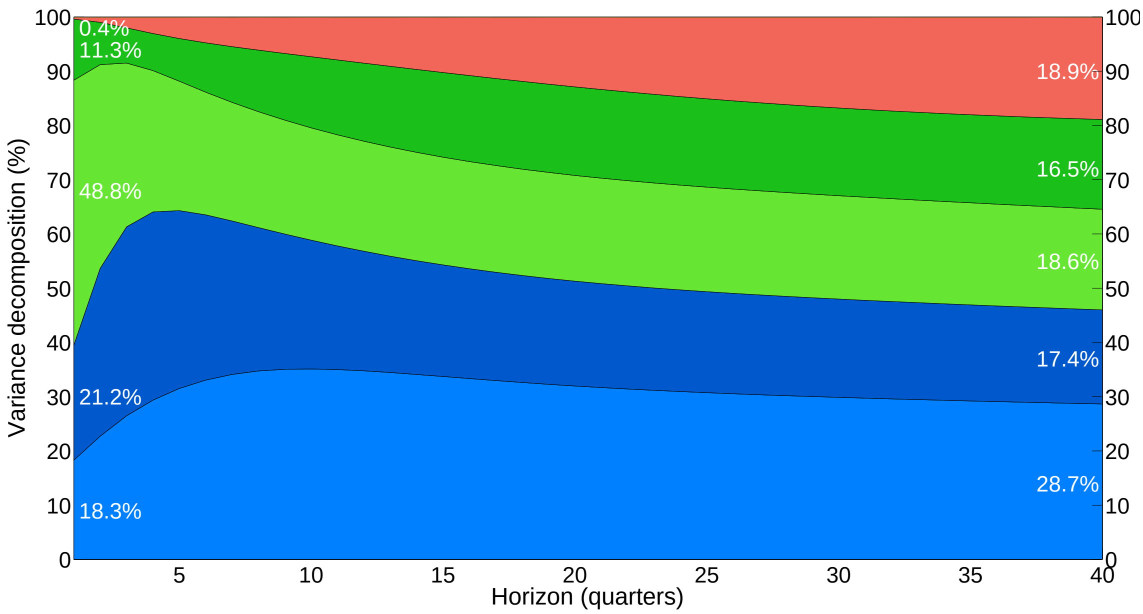
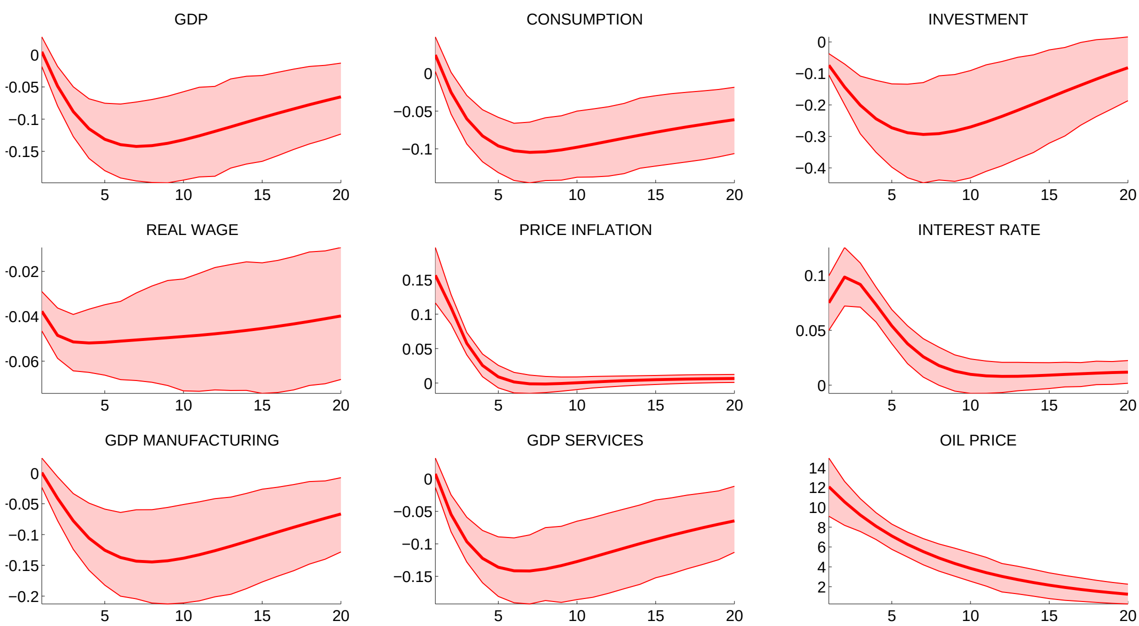


Figure C.4: Forecast error variance decomposition of mainland GDP



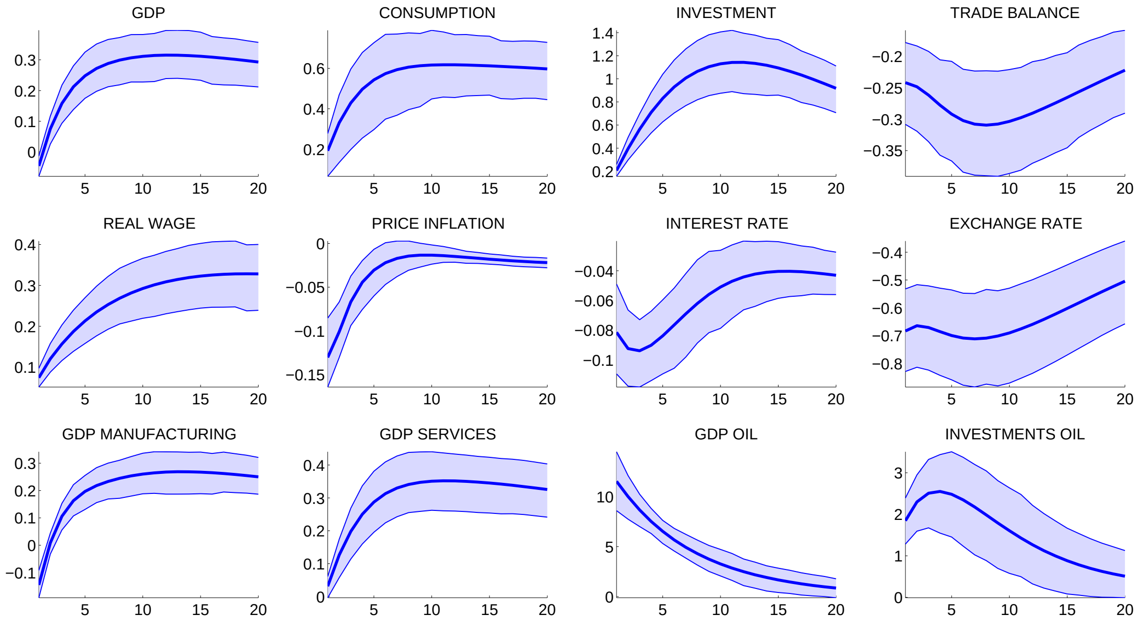
Note: Forecast error variance decomposition of GDP in mainland Norway. Calculated at the posterior mean. Shocks are decomposed as follows: Domestic supply shocks (light blue), domestic demand shocks (dark blue), international supply shocks (light green), international demand shocks (dark green), and shocks in oil markets (light red). Numbers in white at the left and right hand side are decompositions at the 1- and 40-quarter horizons, respectively.

Figure C.5: International responses to an international oil price shock – OECD data



Note: Bayesian impulse responses of international variables to an international oil price shock (one standard deviation). Mean (solid line) and 90% highest probability intervals (shaded area) based on every 1000th draw from the posterior MCMC chain. Inflation and the interest rate are expressed in annual terms.

Figure C.6: Domestic responses to an international oil price shock – OECD data



Note: Bayesian impulse responses of domestic variables to an international oil price shock (one standard deviation). Mean (solid line) and 90% highest probability intervals (shaded area) based on every 1000th draw from the posterior MCMC chain. All variables except value added and investments in oil are from the mainland economy. Inflation and the interest rate are expressed in annual terms.

D Additional figures and tables

Table D.1: Steady state ratios in the benchmark model

	Description	*Data*	*Model*
C/VA	Consumption share in aggregate GDP	0.38	0.39
I/VA	Investment share in aggregate GDP	0.21	0.21
G/VA	Public spending share in aggregate GDP	0.21	0.20
$(A_H^* + O)/\text{VA}$	Export share in aggregate GDP	0.48	0.48
A_F/VA	Import share in aggregate GDP	0.28	0.28
GDP_o/VA	Oil share in aggregate GDP	0.22	0.21
GDP_m/VA	Manufacturing share in aggregate GDP	0.29	0.33
GDP_s/VA	Service sector share in aggregate GDP	0.49	0.46
I_o/I	Oil share in aggregate investments	0.25	0.24
$O/(A_H^* + O)$	Oil share in aggregate exports	0.47	0.45
μ_m	Share of labor force in manufacturing	–	0.41
μ_s	Share of labor force in services	–	0.57
μ_o	Share of labor force in oil sector	–	0.02

Note: This table presents ratios in the non-stochastic steady state as implied by the baseline calibration. Data refers to corresponding sample averages in the data.

Figure D.1: Smoothed shocks (90% highest probability densities)

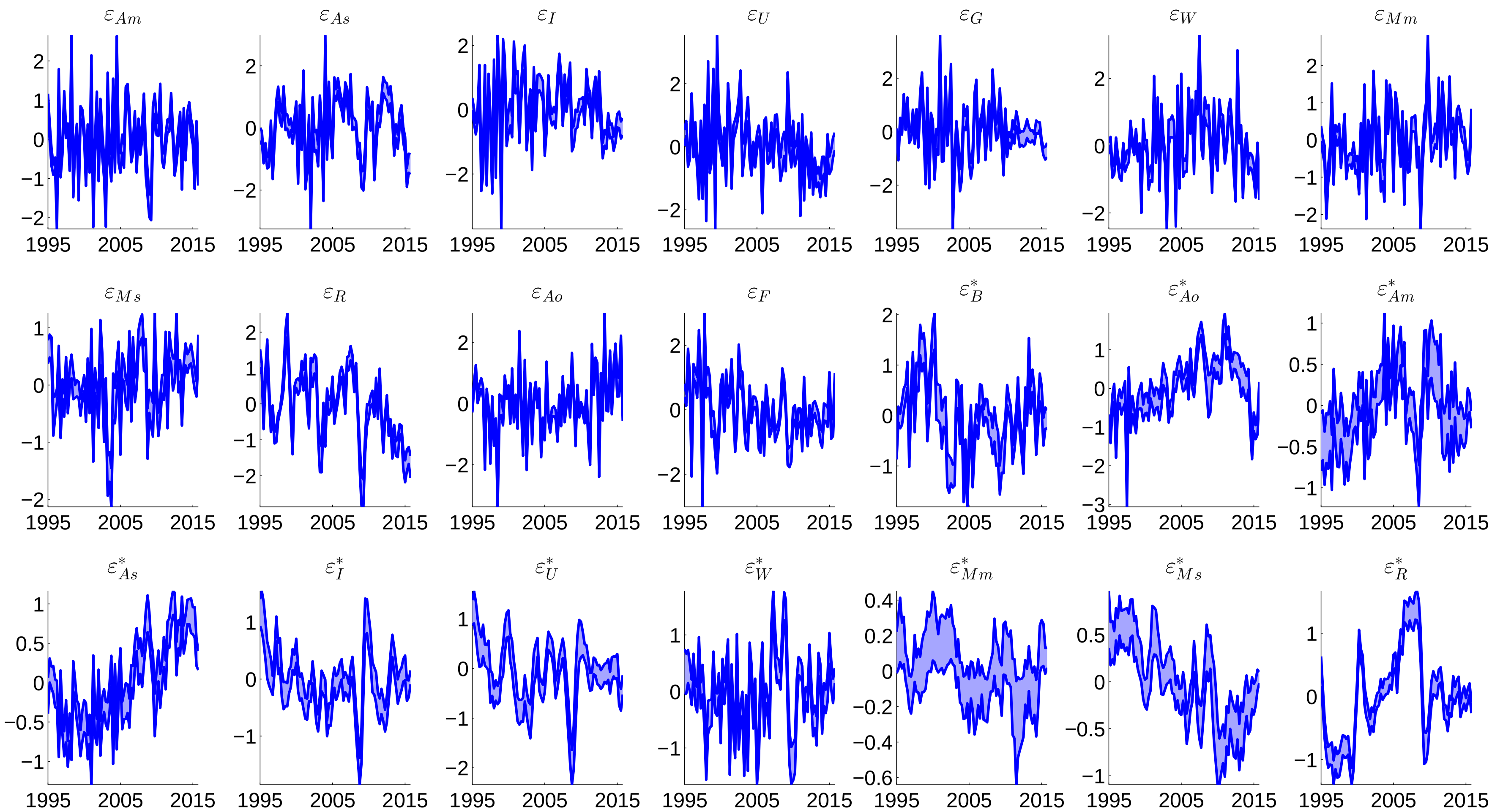


Figure D.2: Empirical (black) and theoretical (blue) second moments, domestic economy

Figure D.3: Empirical (black) and theoretical (blue) second moments, domestic and foreign economy

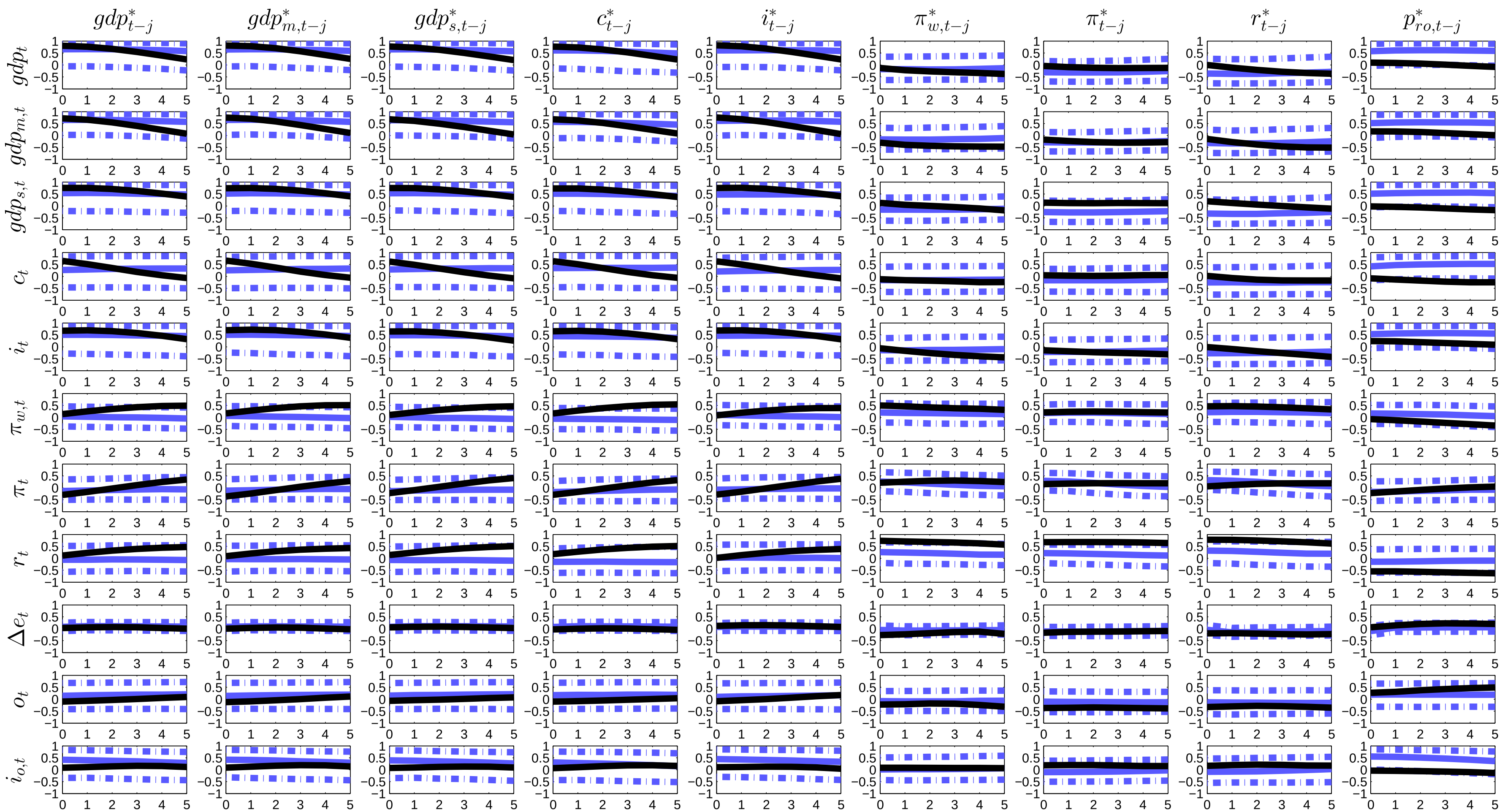


Figure D.4: Historical variance decomposition, mainland GDP

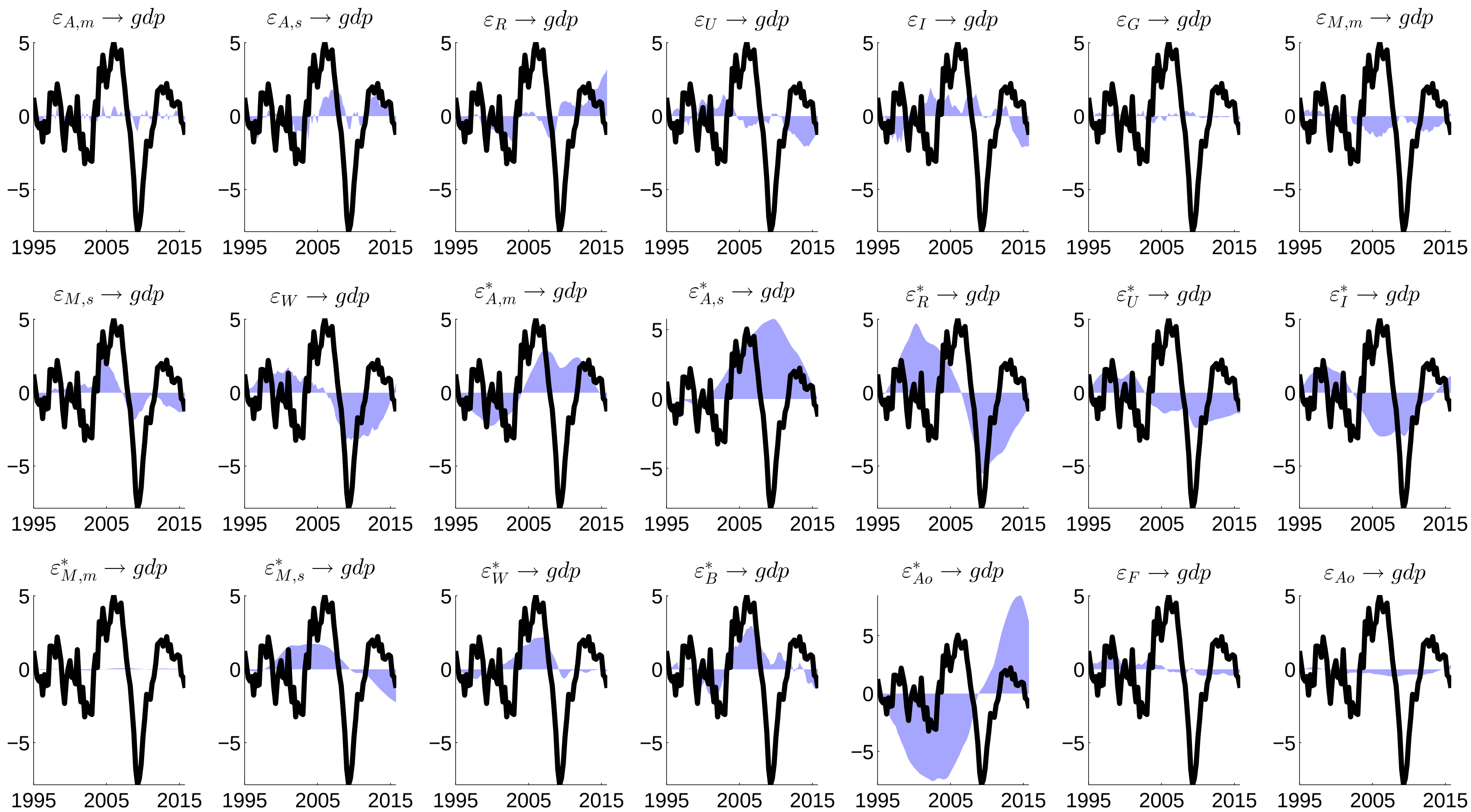
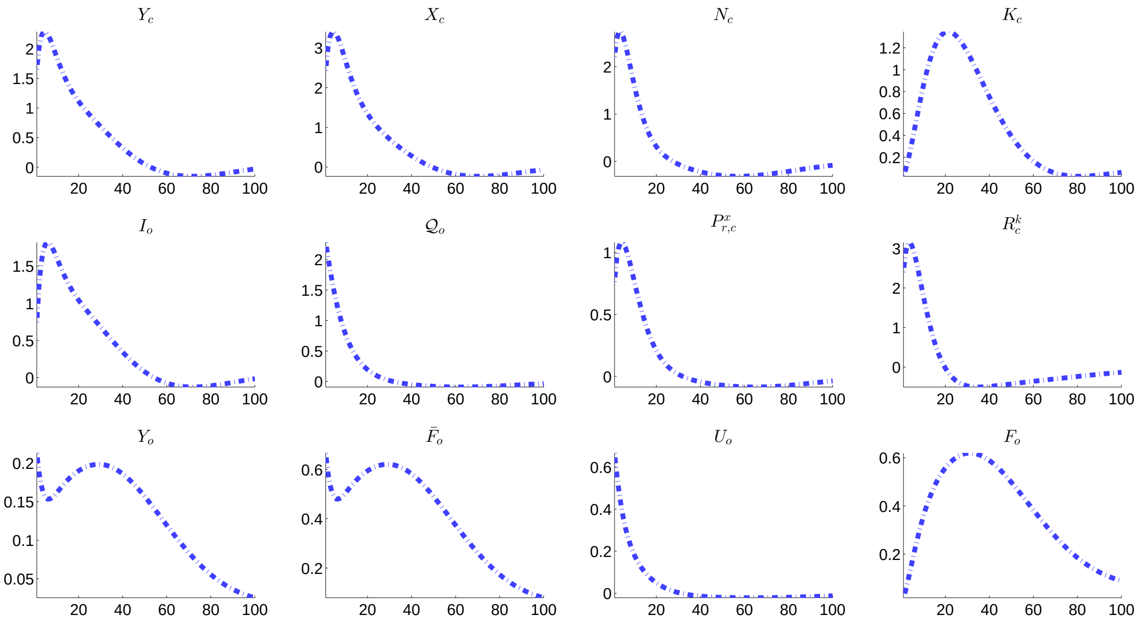
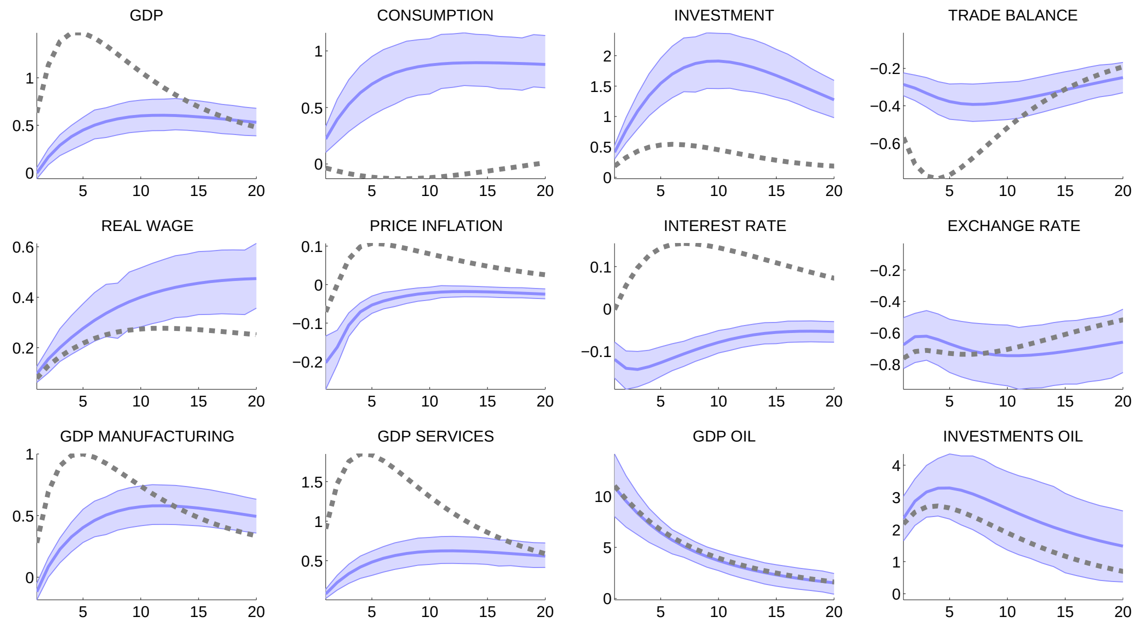


Figure D.5: Dynamics in domestic oil industry after an international oil price shock



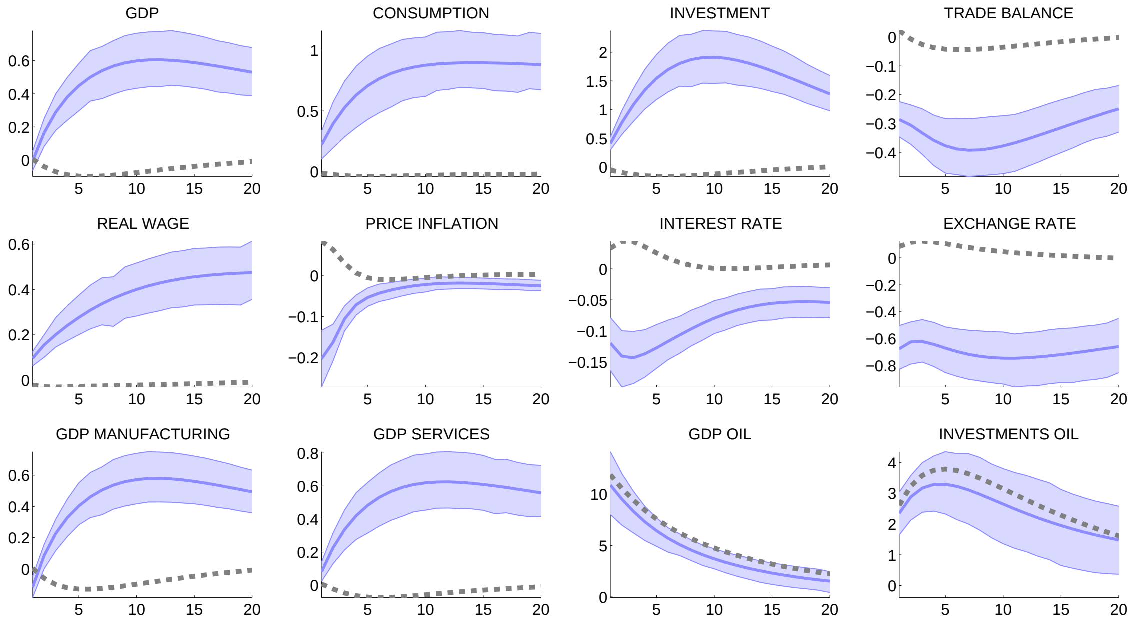
Note: Posterior mean impulse responses to an international oil price shock (one standard deviation). See Figure D.6 for details.

Figure D.6: An international oil price shock without the sovereign wealth fund



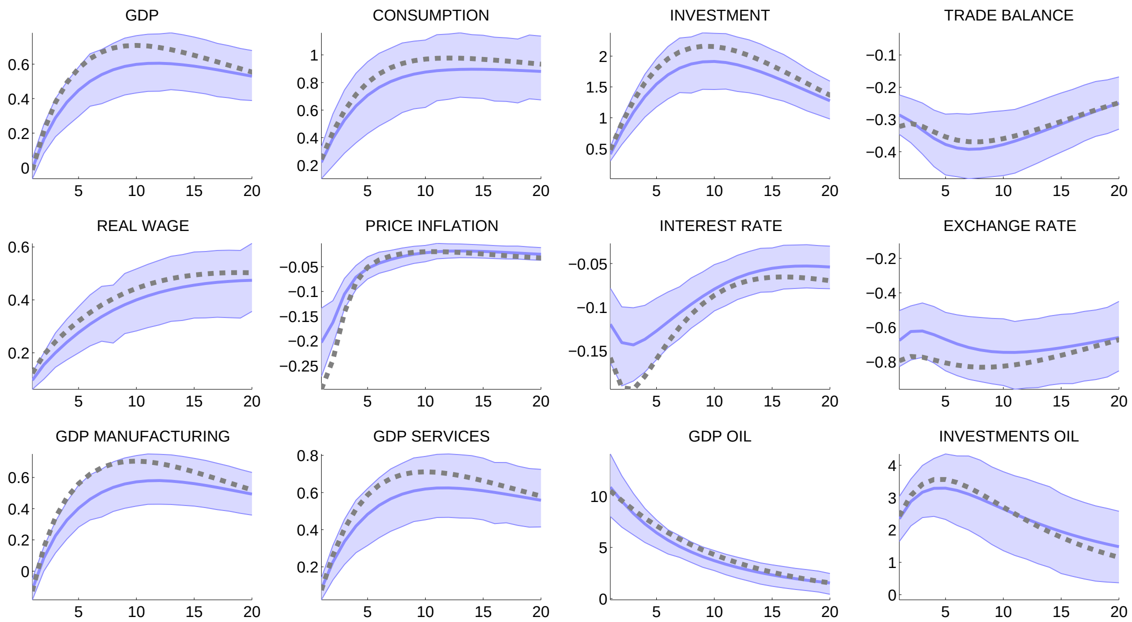
Note: Bayesian impulse responses to an international oil price shock (one standard deviation). Blue areas represent the baseline responses while gray dotted lines represent the counterfactual at the posterior mean.

Figure D.7: An international oil price shock without the supply chain



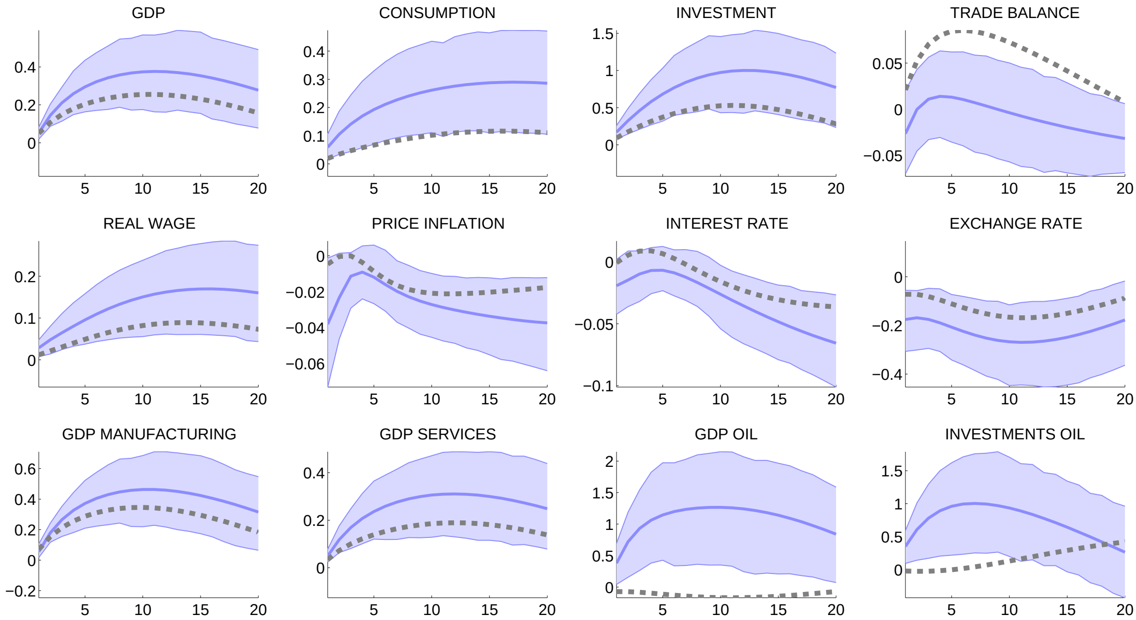
Note: Bayesian impulse responses to an international oil price shock (one standard deviation). Blue areas represent the baseline responses while gray dotted lines represent the counterfactual at the posterior mean.

Figure D.8: An international oil price shock without feedback to macro



Note: Bayesian impulse responses to an international oil price shock (one standard deviation). Blue areas represent the baseline responses while gray dotted lines represent the counterfactual at the posterior mean.

Figure D.9: An international investment shock without feedback to oil



Note: Bayesian impulse responses to an international oil price shock (one standard deviation). Blue areas represent the baseline responses while gray dotted lines represent the counterfactual at the posterior mean.

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