BANK FOR INTERNATIONAL SETTLEMENTS

BIS Working Papers
No 613

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Monetary and Economic Department

February 2017

JEL classification: E32, F41, F44

Keywords: Trends Shocks, Risk Sharing, Real Exchange Rates

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This publication is available on the BIS website (www.bis.org).

ISSN 1020-0959 (print)
ISSN 1682-7678 (online)

Risk Sharing and Real Exchange Rates: The Role of Non-tradable Sector and Trend Shocks

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February 2017

Abstract

Most of the international macro models, in contrast to the data, imply a very high level of risk sharing across countries and very low real exchange rate (RER) volatility relative to output. In this paper we show that a standard two-country two-good model augmented with conintegrated TFP processes comes closer to matching the data.We first show that the tradable and non-tradable total factor productivity (TFP) processes of the US and Europe have unit roots and can be modelled by a vector error correction model (VECM). Then, we develop a standard two-country and two-good (tradable and non-tradable) DSGE model and study the quantitative implications. Cointegrated TFP shocks, or trend shocks, generate significant income effects and amplify the mechanisms that produce high RER volatility. Moreover, trend shocks can break the tight link between relative consumption and RER for low and high values of trade elasticity parameters.

Key Words: Trends Shocks, Risk Sharing, Real Exchange Rates

Jel Classification: E32, F41, F44

[1]We thank Juan F. Rubio-Ramirez, Fatih Ekinci, Hande Küçük and seminar participants at the CEE 2012 Annual Conference at Bogazici University, Central Bank of the Republic of Turkey and Fall 2012 Midwest Macroeconomics Meetings at the University of Colorado Boulder for valuable comments and suggestions.

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1 Introduction

Risk sharing and the RER volatility are the two well known puzzles in the international real business cycle literature. The standard international real business cycle (IRBC) model introduced by Backus, Kehoe and Kydland (1992,1995) (the so called BKK model) feature transitory shocks to the TFP process and generates nearly perfect correlation between relative consumption and RER, which implies a very high level of risk sharing. However, Backus and Smith (1993) and Kollmann (1995) document that the correlation between relative consumption and RER is negative in the data, which means that a country consumes more when its consumption basket is relatively more expensive. Another puzzling feature of the data that IRBC models fail to mimic is the relative volatility of RER with respect to output. Models generate very low RER volatility at odds with data, which is referred as the exchange rate volatility puzzle.

The main reason why standard IRBC models cannot match the data is as follows. When there is a positive transitory shock to the TFP of tradable goods in one country, implying an increase in the supply, the price of tradable goods decrease and the terms of trade (ToT) depreciate. Cheaper inputs in the one country help the producers in the other country to increase their production. Consequently, there is a high level of risk sharing between countries.

However, if TFP processes are contintegrated than the mechanism changes significantly. Cointegrated TFP shocks, or trend shocks, generate significant income effects and amplify the mechanisms that produce high volatility in the RER. Moreover, the mechanism modeled in this paper breaks the tight link between relative consumption and RER for low and high values of trade elasticity parameters. For low values of trade elasticity, a positive trend shock to the TFP in one country increases the demand for tradable goods in that country due to the income effect. Since tradable goods are not easily substitutable, the prices of tradable and non-tradable goods increase, and in turn, the RER appreciates. As the RER appreciates together with a higher consumption in that country, the model generates a negative correlation between relative consumption and RER, implying low risk sharing as in the data. For high values of the trade elasticity parameter, a positive trend shock increases the demand for both sectors, but the price change in tradable sector is limited since tradable goods are now substitutes. Although the tradable price does not change much, non-tradable prices increase since tradable and non-tradable goods are complements. In turn, the RER appreciates and this again breaks the risk sharing.

From the above arguments it becomes clear that the nature of the TFP processes are important for the dynamics of the model. Therefore, in this paper, we first show that the tradable and non-tradable

TFP processes of the US and Europe have unit roots and the relation between these processes can be well captured by a VECM. Then, we develop an international macro model with two countries, two goods (tradable and non-tradable) and a non-contingent single bond. Our model with cointegrated TFP processes improves the model's performance and our results come closer to match real exchange rate (RER) volatility and risk sharing in the data. In our setting, the source of improvement is cointegrated TFP processes that act similar to very persistent transitory shocks.

The stubborn nature of the risk sharing and RER volatility puzzles have attracted widespread interest in the literature. Several papers analyse risk sharing within different frameworks. For example, Heathcote and Perri (2002) loosens the complete market assumption in BKK and introduces a non-contingent single asset. They conclude that trade related statistics improve in the financial autarky case. Chari, Kehoe and Mcgrattan (2002) show that in a world with sticky prices and a non-contingent bond, the risk sharing puzzle still exists while their model generates high RER volatility. Among others, two studies that find low with risk sharing are Corsetti, Dedola and Leduc (2008), hereafter CDL, and Benigno and Thoenissen (2008), hereafter BT[1]. CDL set up an IRBC model with tradable and non-tradable sectors. With highly persistent TFP shocks, their model imply high RER volatility and low risk sharing for low and high trade elasticity parameters. BT use a two-country two-sector, tradable and non-tradable, model with an explicit role of monetary policy. They obtain the Backus-Smith relation and argue that Balassa-Samuelson effect produces negative cross-correlation between relative consumption and RER. However, the relative volatility of RER is small for low and high trade elasticities. A more recent paper by Benigno and Küçük (2011) investigates the consumption-RER anomaly in an environment with different asset market structures. For the single bond case, they exhaustively explain the movements in the relative consumption and RER for different values of the trade elasticity parameter. However, they find that if the asset markets consist of two bonds, the single bond case results disappear and the risk sharing puzzle re-emerges[2].

The studies discussed above assume that TFP processes are stationary. However, Aguiar and Gopinath (2007), in their seminal work, allow the trend component of the TFP process to follow a stochastic path. Using this innovation, their small open economy model can explain some characteristic features of the

[1] For some other papers on risk sharing and real exchange rates, see Baxter and Crucini (1995), Benigno and Küçük (2011), Burstein et al. (2005), Cole and Obstfeld (1991), Dellas and Stockman (1989), Dimitriev and Roberts (2012), Dotsey and Duarte (2008), Engel and Matsumoto (2009), Fitzgerald (2011), Heathcote and Perri (2009), Kehoe and Perri (2002), Kollmann (1996), Kose et al. (2009), Palacios-Huerta (2001), Rabanal and Tuesta (2010) .

[2] However, Arslan et al. (2012) shows that in the two bond-case, trend shocks breaks the risk sharing in the model.

emerging economies. Rabanal, Rubio-Ramirez and Tuesta (2011), hereafter RRT, embed the trend shocks into a two-country framework and show that TFP processes for the US and the rest of the world are cointegrated[3]. Their model generates higher RER volatility compared to similar models in which TFP processes are assumed to be transitory. In a similar paper, Ireland (2011) compares the sources of growth in the US and Euro Area, where technology, preference and investment-specific technology shocks are cointegrated in an estimated two country model.

Our model is closely related to those of CDL and RRT. The main difference from the CDL model is that the TFPs are modeled as cointegrated processes instead of transitory processes. CDL obtains negative correlation between relative consumption and RER for high values of trade elasticity parameter only with persistent transitory shocks, which is not supported by the data. However, our model can address the Backus-Smith puzzle with cointegrated TFP processes. On the other hand, our main difference from RRT is the inclusion of the non-tradable sector in the model. This extra feature improves the results for the Backus-Smith puzzle significantly whereas the improvement in RER volatility is limited.

The rest of the paper is as follows. Section 2 presents the model and calibration of parameters Section 3 shows our results and Section 4 implements robustness analysis. Finally, Section 5 concludes the paper.

2 Model

We setup a two-country two-good production economy model with a single tradable bond. Our model includes tradable and non-tradable sectors similar to the models in CDL and BT. However, unlike them, we assume that the TFP processes are cointegrated as in RRT and Ireland (2011)[4].

Our world economy consists of two countries, one representing the home country (H) and the other representing the foreign country (F). Sectors are indexed as $i = HT, HN, FT, FN$ representing the home tradable and non-tradable sectors and foreign tradable and non-tradable sectors, respectively and the time period is denoted with $t = 0, 1, 2, ...$ subscripts. In each country, firms use capital, labor and sector specific labor technology to produce tradable inputs and non-tradable inputs. Production sharing takes place in intermediate goods, so countries use both home and foreign tradable inputs to produce their respective intermediate goods. Then, they combine this intermediate good with non-tradable input to produce their distinctive final goods, which are later to be consumed or invested by the representative households of each country.

[3] The rest of the world consists of the Euro area, Japan, Canada, the United Kingdom and Australia.

[4] These studies have only the tradable sector.

2.1 Tradable and Non-tradable Goods-Producing Firms

The perfectly competitive tradable and non-tradable good producer firms combine capital and labor with their sector specific technology through a Cobb-Douglas production function to obtain the tradable goods (Y_t^{HT}, Y_t^{FT}) and the non-tradable goods (Y_t^{HN}, Y_t^{FN}). The production function is:

$$Y_t^i = (K_t^i)^{\alpha}(Z_t^i L_t^i)^{1-\alpha} \tag{1}$$

where $i \in \{HT, HN, FT, FN\}$. The firm rents K_t^i units of capital and hires L_t^i units of labor in the production with technology Z_t^i. Capital share in production is α, which lies between 0 and 1. Tradable and non-tradable firms in both countries maximize their profits (2) by taking all prices as given:

$$\max_{K_t^i \geq 0, L_t^i \geq 0} P_t^i Y_t^i - Q_t^i K_t^i - W_t^i L_t^i \tag{2}$$

where P_t^i is the price of tradable or non-tradable goods, Q_t^i is the rental price for capital, and W_t^i is the wage for $i \in \{HT, HN, FT, FN\}$.

2.2 Intermediate Goods Producing Firms

There is production sharing between countries, i.e. both countries use each other's tradable inputs to produce their respective tradable intermediate goods. The competitive intermediate goods producer firms of the home country use A_t^H units of home tradable good and B_t^H units of foreign tradable good to produce $Y_t^{H,int}$ units of intermediate good which is used by final good firms of home country. Home country intermediate good producers use a constant elasticity of substitution production technology as follows:

$$Y_t^{H,int} = \left[(1-\omega_1)^{\frac{1}{\theta}}(A_t^H)^{\frac{\theta-1}{\theta}} + (\omega_1)^{\frac{1}{\theta}}(B_t^H)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{3}$$

where θ measures the elasticity of substitution between the home tradable input (A_t^H) and foreign tradable input (B_t^H), and ω_1 is the share of foreign tradable input in the home country's intermediate good production. Firms maximize their profits (equation 4) by taking all prices as given:

$$\max_{A_t^{HT} \geq 0, B_t^{HT} \geq 0} P_t^{H,int} Y_t^{H,int} - P_t^A A_t^H - P_t^B B_t^H \tag{4}$$

where $P_t^{H,int}$ is the price of home intermediate good.

Similarly, intermediate goods firms of the foreign country use A_t^F units of home tradable good and B_t^F units of foreign tradable good to produce $Y_t^{F,int}$ units of intermediate good that are used by the final good producer firms of the foreign country. Foreign country intermediate good producers use a constant elasticity of substitution production technology as follows:

$$Y_t^{F,int} = \left[(\omega_1)^{\frac{1}{\theta}} (A_t^F)^{\frac{\theta-1}{\theta}} + (1-\omega_1)^{\frac{1}{\theta}} (B_t^F)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{5}$$

where A_t^F is the home tradable input, B_t^F is the foreign tradable input, and ω_1 is the share of home tradable input in the foreign country's intermediate good production. Firms maximize their profits (equation 6) by taking all prices as given:

$$\max_{A_t^{FT} \geq 0, B_t^{FT} \geq 0} P_t^{F,int} Y_t^{H,int} - P_t^A A_t^F - P_t^B B_t^F \tag{6}$$

where $P_t^{F,int}$ is the price of foreign intermediate good.

2.3 Final Goods-Producing Firms

Home final goods producing firms combine $Y_t^{H,int}$ units of home intermediate good and Y_t^{HN} units of home non-tradable goods through a production function with constant elasticity of substitution in order to produce Y_t^H units of final home good with price of P_t^H:

$$Y_t^H = \left[(1-\omega_2)^{\frac{1}{\theta_2}} (Y_t^{H,int})^{\frac{\theta_2-1}{\theta_2}} + (\omega_2)^{\frac{1}{\theta_2}} (Y_t^{HN})^{\frac{\theta_2-1}{\theta_2}} \right]^{\frac{\theta_2}{\theta_2-1}} \tag{7}$$

where θ_2 is the elasticity of substitution between intermediate good and non-tradable good, and ω_2 is the share of non-tradable input in the home country's final good production.

The foreign final goods-producing firms use $Y_t^{F,int}$ units of foreign intermediate good and Y_t^{FN} units of foreign non-tradable goods in order to produce Y_t^F units of final home good with price of P_t^F, either to consume or invest. Foreign final goods-producing firms use a constant elasticity of substitution production technology as follows:

$$Y_t^F = \left[(1-\omega_2)^{\frac{1}{\theta_2}} (Y_t^{F,int})^{\frac{\theta_2-1}{\theta_2}} + (\omega_2)^{\frac{1}{\theta_2}} (Y_t^{FN})^{\frac{\theta_2-1}{\theta_2}} \right]^{\frac{\theta_2}{\theta_2-1}}. \tag{8}$$

Both final good producers maximize their profits by taking the prices as given.

2.4 Representative Households

The expected life time utility of the representative home consumer is described by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ \frac{[(C_t^H)^{\mu}(1 - L_t^{HT} - L_t^{HN})^{(1-\mu)}]^{(1-\gamma)}}{1-\gamma} \} \tag{9}$$

where C_t^H is the consumption, β is the discount factor, γ is the risk aversion parameter, and μ is the share parameter between consumption and leisure. Households provide labor services L_t^{HT} to tradable and L_t^{HN} to non-tradable firms in their countries at the wage rates of W_t^{HT} and W_t^{HN}. They also own the capital stock in both sectors (K_t^{HT} and K_t^{HN}) and rent it to the firms at rates Q_t^{HT} and Q_t^{HN}. Both labor and capital are mobile across sectors within the country, but they are immobile across countries. Households can trade an international bond D_t^H. In total, the income of households is composed of wage income from labor supply, rent income from capital supply and the interest income from the international bond. Households use their income to finance their consumption C_t^H and investment in tradable (I_t^{HT}) and non-tradable sectors (I_t^{HN}), and to buy new bonds D_{t+1}^H. They also pay adjustment costs for capital and bond changes. Then, the budget constraint for households in the home country is:

$$\begin{aligned} & P_t^H C_t^H + P_t^H I_t^{HT} + P_t^H I_t^{HN} + \frac{P_t^H D_{t+1}^H}{R_t} + P_t^H \frac{\phi_d}{2} U_t^H \left(\frac{D_{t+1}^H}{U_t^H} \right)^2 \\ \leq \quad & W_t^{HT} L_t^{HT} + W_t^{HN} L_t^{HN} + Q_t^{HT} K_t^{HT} + Q_t^{HN} K_t^{HN} + P_t^H D_t^H \end{aligned} \tag{10}$$

where $1/R_t$ is the price of one unit bond at time t that matures at period $t+1$. $\phi_d > 0$ is the adjustment cost parameter for bond holdings that closes small open economy models as discussed in Schmitt-Grohe and Uribe (2003). Bond adjustment cost is scaled by a factor $U_t^H = Z_t^{HT}$ as in Ireland (2011) to achieve consistency in the model and ensure a zero adjustment cost along the steady-state growth path. The law of motion for the capital of both tradable and non-tradable sectors are:

$$K_{t+1}^{HT} \leq (1-\delta) K_t^{HT} + I_t^{HT} - \frac{\phi_k}{2} \left(\frac{I_t^{HT}}{K_t^{HT}} - \eta^{HT} \right)^2 K_t^{HT} \tag{11}$$

$$K_{t+1}^{HN} \leq (1-\delta) K_t^{HN} + I_t^{HN} - \frac{\phi_k}{2} \left(\frac{I_t^{HN}}{K_t^{HN}} - \eta^{HN} \right)^2 K_t^{HN} \tag{12}$$

where $\phi_k > 0$ is the adjustment cost parameter for capital, δ is the depreciation rate ($1 > \delta > 0$), η^{HT} and η^{HN} are the parameters that provide zero cost along the steady-state growth path.

The representative home consumer maximizes the expected utility (9) subject to the constraints (10), (11) and (12) where the choice variables are $(C_t^H, L_t^{HT}, L_t^{HN}, I_t^{HT}, I_t^{HN}, K_{t+1}^{HT}, K_{t+1}^{HN}, D_{t+1}^H)$. The problem of the representative foreign consumer is symmetric, where C_t^F is the foreign consumption, L_t^{FT} and L_t^{FN} are hours worked, I_t^{FT} and I_t^{FN} are investment, K_{t+1}^{FT} and K_{t+1}^{FN} are capital stocks in the tradable and non-tradable sectors respectively, and D_{t+1}^F is the bond holdings.

2.5 Trade Variables and Equilibrium Conditions

The home country's net exports are expressed in the units of home final good:

$$N_t^H = (P_t^{HT} A_t^F - P_t^{FT} B_t^H)/P_t^H \tag{13}$$

For both countries there are two more relevant prices, i.e. terms of trade and real exchange rates. We define the terms of trade, ToT_t as the ratio of its import prices to its export prices, $ToT_t = P_t^{FT}/P_t^{HT}$. We define the real exchange rate, ReR_t, as the ratio of foreign final goods prices to home final goods prices, $ReR_t = P_t^F/P_t^H$. An increase in the ToT means a depreciation of the terms of trade of the home country by making its export prices less expensive or import prices more expensive. An increase in the ReR implies a depreciation of real exchange rate for the home country and an appreciation for the foreign country.

Market clearing conditions for tradable good sectors in home and foreign countries are given as:

$$Y_t^{HT} = A_t^H + A_t^F \tag{14}$$

$$Y_t^{FT} = B_t^H + B_t^F \tag{15}$$

Since there is no trade in final goods, final goods in the country is used for consumption, investment and adjustment costs and we have the following resource constraints:

$$Y_t^H = C_t^H + \frac{\phi_d}{2} U_t^H \left(\frac{D_{t+1}^H}{U_t^H} \right)^2 + \ I_t^{HT} + I_t^{HN} \tag{16}$$

$$Y_t^F = C_t^F + \frac{\phi_d}{2} U_t^F \left(\frac{D_{t+1}^F}{U_t^F} \right)^2 + \ I_t^{FT} + I_t^{FN} \tag{17}$$

There is only one bond in the international financial markets and the net supply is zero, giving us:

$$D_t^H + D_t^F = 0 \tag{18}$$

3 Vector Error Correction Model (VECM)

This section documents the cointegration relationship between the TFP levels of the tradable and non-tradable sectors in both countries. First, we describe the data and show that all four TFP processes include unit root. Second, all six pairs of TFP processes are cointegrated. Finally, we estimate the TFP system with three cointegrating vectors.

3.1 Data and Cointegration

We calculate the TFP processes for tradable and non-tradable sectors by using the 60 industry database of Groningen Growth and Development Centre as in BT. The data spans the period between 1979 and 2004. We take the US as the home country and EU-15 as the foreign country. Fifty seven sectors in each country are grouped into two as tradable and non-tradable sectors[5]. The TFP process for each group is equal to the logarithm of the weighted sum of sub-sector-level TFP processes. We denote the TFP processes of home tradable, home non-tradable, foreign tradable and foreign non-tradable sectors with $\ln(Z_t^{HT}), \ln(Z_t^{HN})$, $\ln(Z_t^{FT})$, $\ln(Z_t^{FN})$, respectively.

First, we provide evidence for the presence of one unit root in the TFP processes. Table 1 presents the test results for the augmented Dicky-Fuller test (ADF), the Elliot, Rothenberg and Stock (1996) detrended residual test, and the Phillips and Perron (1988) test (PP). The lag length is chosen according to Schwarz information criterion. We assume a constant and a trend in each specification. Also, tests reject the null hypothesis for first-difference series. So, TFP processes are non-stationary and include one unit root.

[5] A detailed list is availbale upon request.

Table 1: Unit Root Tests

		ADF	ERS	PP
$\ln(Z_t^{HT})$	Level	-2.13*	-2.42*	-2.38*
	First-diff.	-4.57	-4.44	-4.58
$\ln(Z_t^{HN})$	Level	-1.19*	-2.15*	-2.24*
	First-diff.	-8.50	-8.50	-10.81
$\ln(Z_t^{FT})$	Level	-2.56*	-2.79*	-2.63*
	First-diff.	-4.68	-4.89	-11.85
$\ln(Z_t^{FN})$	Level	-1.67*	-1.82*	-1.74*
	First-diff.	-4.67	-4.81	-4.67

* denotes null hypothesis of unit root is not rejected at 5% level

Second, we investigate the cointegration relationship between the six possible pairs of these four TFP processes[6]. For this purpose, we use the Johansen (1991) trace and maximum eigenvalue cointegration tests. Table 2 reports the results for both tests under the "intercept without deterministic trend" specification and lag the length is set to zero. The tests for all pairs indicate one cointegrating equation at 5% level[7].

[6] Since our data is annual and we have a small number of years, we prefer to do pairwise cointegration tests in Table 2. As the number of parameters we try to estimate grows exponentially when we do cointegration analysis for all four series, we would get very loose confidence intervals.

[7] As a robustness test, we also repeat cointegration analysis with the Groningen Growth and Development Centre (GGDC) 10-Sector Database. This data covers a much longer period of 1951-2009 but does not have TFP variable. So, we can construct labor productivities for the estimates. We do pairwise cointegration analysis for US-UK pair of sectors and get very similar results. Furthermore, with such a long dataset, we do cointegration test for all four variables and find a cointegrating equation as well.

Table 2: Pairwise Cointegration

	Vectors	Eigenvalue	Trace	p-value	Max-Eigen.	p-value
(Z_t^{HT}, Z_t^{HN})	0	0.60	26.24	0.01	21.76	0.01
	1	0.17	4.47	0.35	4.47	0.35
(Z_t^{HT}, Z_t^{FT})	0	0.79	41.47	0	37.18	0
	1	0.16	4.29	0.37	4.29	0.37
(Z_t^{HT}, Z_t^{FN})	0	0.88	57.37	0	51.16	0
	1	0.23	6.21	0.18	6.21	0.18
(Z_t^{HN}, Z_t^{FT})	0	0.81	46.01	0	39.27	0
	1	0.24	6.74	0.14	6.74	0.14
(Z_t^{HN}, Z_t^{FN})	0	0.88	59.10	0	51.80	0
	1	0.26	7.30	0.11	7.30	0.11
(Z_t^{FT}, Z_t^{FN})	0	0.90	62.92	0	55.90	0
	1	0.25	7.02	0.13	7.02	0.13

3.2 The VECM Model

The above analysis show that each pair of TFP processes are cointegrated. However, this is not adequate to ensure a balanced growth path. For balanced growth, $\ln(Z_t^i)$ and $\ln(Z_t^j)$ should be cointegrated with a cointegrating vector (1,-1). Formally, the two variable VECM system can be written for $i, j \in \{HT, HN, FT, FN\}$ and $i \neq j$:

$$\Delta \log(Z_t^i) = c^i + \kappa^i(\log(Z_{t-1}^i) - \gamma \log(Z_{t-1}^j) - \log \xi) + \varepsilon_t^i \tag{19}$$

$$\Delta \log(Z_t^j) = c^j + \kappa^j(\log(Z_{t-1}^i) - \gamma \log(Z_{t-1}^j) - \log \xi) + \varepsilon_t^j \tag{20}$$

where Δ is the first-difference operator, $(1, -\gamma)$ is the cointegrating vector, $\varepsilon_t^i \sim N(0, \sigma^i)$ and $\varepsilon_t^j \sim N(0, \sigma^j)$ are noise terms. As stated in RRT, if the hypothesis $\gamma = 1$ cannot be rejected in this system, then balance growth cannot be rejected. Cointegrated vector (1,-1) implies that any positive deviation in the growth rate difference, $\log(Z_{t-1}^i) - \log(Z_{t-1}^j)$, decreases the growth of $\Delta \log(Z_t^i)$ and increases $\Delta \log(Z_t^j)$ for $\kappa^i > 0$ and $\kappa^j < 0$. This relationship ensures a balanced growth path for the existence of the steady state of the system. Table 3 reports the likelihood ratio test statistics for $\gamma = 1$ under the specification "intercept without deterministic trend" and the lag length is set to zero. Test statistics show that $\gamma = 1$ cannot be rejected for all the pairs at a significance level of 1%[8]. Thus, the likelihood test indicates that a balanced growth path cannot be rejected[9].

Table 3: Likelihood Ratio Tests

	Restriction	Likelihood	d.o.f.	p-value
(Z_t^{HT}, Z_t^{HN})	None	118.20	-	-
	$\gamma = 1$	116.59	1	0.07
(Z_t^{HT}, Z_t^{FT})	None	122.44	-	-
	$\gamma = 1$	122.41	1	0.83
(Z_t^{HT}, Z_t^{FN})	None	149.32	-	-
	$\gamma = 1$	149.21	1	0.65
(Z_t^{HN}, Z_t^{FT})	None	128.60	-	-
	$\gamma = 1$	127.99	1	0.27
(Z_t^{HN}, Z_t^{FN})	None	154.44	-	-
	$\gamma = 1$	154.26	1	0.36
(Z_t^{FT}, Z_t^{FN})	None	165.64	-	-
	$\gamma = 1$	163.60	1	0.04

Our purpose is to estimate a VECM model for the four-variable system $(\ln(Z_t^{HT}), \ln(Z_t^{HN}), \ln(Z_t^{FT}), \ln(Z_t^{FN}))$ that ensures balanced growth. We expand the two variable system estimated in Ireland (2011) to a four variable system:

[8] In Table 3, we reject the cointegrating vectors of (1,-1) for home T and NT pair at 10 percent and for foreign T and NT pair at 5 percent. However, we do not reject the cointegrating vector at 1 percent level. We interpret these results in favor of the balanced growth path in our model. As a robustness test, we do the same estimations with GGDC 10-sector database as well. We do not reject the cointegrating vector of (1,-1) at 10 percent for all pairs in this longer dataset.

[9] The ideal test in Table 3 should be checking whether we can reject or not reject cointegrating vectors of type (3,-1,-1,-1). However, we cannot do that properly with current data set since existing tests only give accurate results with large enough sample size and therefore we proceed with pairwise cointegrating vectors. However, with the GGDC 10-sector database we can perform such a test. The estimation with this dataset does not reject cointegrating vector of (3,-1,-1,-1). This supports our hypothesis of balanced growth path in the model.

$$\ln(Z_t^{HT}/Z_{t-1}^{HT}) = (1-\rho^{HT})\ln(z^{HT}) + \rho^{HT}\ln(Z_{t-1}^{HN}/Z_{t-2}^{HN}) + \kappa^{HTHN}\ln(Z_{t-1}^{HT}/Z_{t-1}^{HN}) + \kappa^{HTFT}\ln(Z_{t-1}^{HT}/Z_{t-1}^{FT}) + \kappa^{HTFN}\ln(Z_{t-1}^{HT}/Z_{t-1}^{FN}) + \epsilon_t^{HT} \quad (21)$$

$$\ln(Z_t^{HN}/Z_{t-1}^{HN}) = (1-\rho^{HN})\ln(z^{HN}) + \rho^{HN} ln(Z_{t-1}^{HN}/Z_{t-2}^{HN}) + \kappa^{HNHT}\ln(Z_{t-1}^{HN}/Z_{t-1}^{HT}) + \kappa^{HNFT}\ln(Z_{t-1}^{HN}/Z_{t-1}^{FT}) + \kappa^{HNFN}\ln(Z_{t-1}^{HN}/Z_{t-1}^{FN}) + \epsilon_t^{HN} \quad (22)$$

$$\ln(Z_t^{FT}/Z_{t-1}^{FT}) = (1-\rho^{FT})\ln(z^{FT}) + \rho^{FT}\ln(Z_{t-1}^{FT}/Z_{t-2}^{FT}) + \kappa^{FTHT}\ln(Z_{t-1}^{FT}/Z_{t-1}^{HT}) + \kappa^{FTHN}\ln(Z_{t-1}^{FT}/Z_{t-1}^{HN}) + \kappa^{FTFN}\ln(Z_{t-1}^{FT}/Z_{t-1}^{HN}) + \epsilon_t^{FT} \quad (23)$$

$$\ln(Z_t^{FN}/Z_{t-1}^{FN}) = (1-\rho^{FN})\ln(z^{FN}) + \rho^{FN}\ln(Z_{t-1}^{FN}/Z_{t-2}^{FN}) + \kappa^{FNHT}\ln(Z_{t-1}^{FN}/Z_{t-1}^{HT}) + \kappa^{FNHN}\ln(Z_{t-1}^{FN}/Z_{t-1}^{HN}) + \kappa^{FNFT}\ln(Z_{t-1}^{FN}/Z_{t-1}^{FT}) + \epsilon_t^{FN} \quad (24)$$

where $z^{HT}, z^{HN}, z^{FT}, z^{FN}$ are the long-run average steady-state growth rates of $Z_t^{HT}, Z_t^{HN}, Z_t^{FT}, Z_t^{FN}$; $\rho^{HT}, \rho^{HN}, \rho^{FT}, \rho^{FN}$ are the persistence parameters for growth rates, κ^{ij}'s are the negative correction parameters for shocks that determine the convergence speed across sectors, where $i, j \in \{HT, HN, FN, FT\}$ and $i \neq j$; and $\epsilon_t^{HT}, \epsilon_t^{HN}, \epsilon_t^{FT}, \epsilon_t^{FN}$ are Gaussian random processes with zero mean and standard deviations $\sigma^{HT}, \sigma^{HN}, \sigma^{FT}, \sigma^{FN}$, respectively. Equation (21) implies that the growth rate of Z_t^{HT} depends on the its own growth in the previous period through persistence parameter. In the equation, we use one lag of the dependent variable, and since we are working with annual data, this lag length assumption is enough to catch the convergence. Z_t^{HT} also depends on the relative growth rates $Z_{t-1}^{HT}/Z_{t-1}^{HN}, Z_{t-1}^{HT}/Z_{t-1}^{FT} Z_{t-1}^{HT}/Z_{t-1}^{FN}$. If the level of technology in the home tradable sector is higher than the remaining three sectors at time t-1, the growth rate of Z_t^{HT} decreases while the growth rate of remaining sectors increase through equations (22)-(24) at time t to ensure balanced growth rates in the long run.

Actually, the model defined with equations (21)-(24) is a VECM model with three cointegrating vectors under the restriction $\kappa^{ij} = \kappa$ for all $i, j \in \{HT, HN, FN, FT\}$ and $i \neq j$. Otherwise, the system cannot be represented by 3 cointegrating vectors. The cointegrating vectors for the system are:

$$v_t^{HT} = 3\ln(Z_t^{HT}) - \ln(Z_t^{HN}) - \ln(Z_t^{FT}) - \ln(Z_t^{FN}) \quad (25)$$

$$v_t^{HN} = 3\ln(Z_t^{HN}) - \ln(Z_t^{HT}) - \ln(Z_t^{FT}) - \ln(Z_t^{FN}) \tag{26}$$

$$v_t^{FT} = 3\ln(Z_t^{FT}) - \ln(Z_t^{HT}) - \ln(Z_t^{HN}) - \ln(Z_t^{FN}) \tag{27}$$

Thus, we can rewrite the system in equations (21)-(24) by using the cointegrating vectors:

$$\Delta Z_t^{HT} = (1-\rho^{HT})\ln(z^{HT}) + \rho^{HT}\Delta Z_{t-1}^{HT} + \kappa v_{t-1}^{HT} + \epsilon_t^{HT} \tag{28}$$

$$\Delta Z_t^{HN} = (1-\rho^{HN})\ln(z^{HN}) + \rho^{HN}\Delta Z_{t-1}^{HN} + \kappa v_{t-1}^{HN} + \epsilon_t^{HN} \tag{29}$$

$$\Delta Z_t^{FT} = (1-\rho^{FT})\ln(z^{FT}) + \rho^{FT}\Delta Z_{t-1}^{FT} + \kappa v_{t-1}^{FT} + \epsilon_t^{FT} \tag{30}$$

$$\Delta Z_t^{FN} = (1-\rho^{FN})\ln(z^{FN}) + \rho^{FN}\Delta Z_{t-1}^{FN} - \kappa(v_{t-1}^{HT} + v_{t-1}^{HN} + v_{t-1}^{FT}) + \epsilon_t^{FN} \tag{31}$$

3.3 Calibration

Most of the parameter values are standard and taken from the literature, as presented in Table 4. Since we use annual data in our analysis, we set the discount factor β to 0.96 and the depreciation rate δ to 0.1. We follow CDL and set the consumption share in the household's utility function μ to 0.34. We calibrate the capital share in production α to 0.36 as in Backus et al. (1995). We use γ =2 as the risk aversion parameter.

There are a number of estimates of the elasticity between tradable inputs (θ) in the literature. CDL points out that the estimates of θ range from 0.1 to 2 and they show that trade elasticity values of around 0.5 and 4 solve the Backus-Smith puzzle[10]. RRT simulate their model for $\theta = 0.62$ and $\theta = 0.85$. Ireland (2011) estimates the elasticity parameter and finds $\theta = 1.47$ for the non-stationary model with additional shocks to investment and preferences. Thus, given the large range of parameters used in the literature, we find it instructive to study with different values of elasticity parameter, $\theta = 0.5, 0.62, 0.85, 1.47, 2, 4$. The share of non-tradable goods in the final good production is $\omega_2 = 0.5$, following Stockman and Tesar (1995). The import share in intermediate good production is calibrated as $\omega_1 = 0.2$ by combining the value of the share parameter in Ireland (2011) with the share of non-tradable goods in final good

[10] CDL includes a distribution sector in their model, so, their trade elasticity parameters correspond to $0.85/2 \simeq 0.43$ and $8/2 = 4$.

production. The elasticity between intermediate goods and non-tradable goods is set at $\theta_2 = 0.5$ implying that they are complements.

The steady state growth rate of TFP shocks $(z^{HT}, z^{HN}, z^{FT}, z^{FN})$ are estimated by using the annual data series covering 1979:2004 period and set to 1.004. In particular, this is the average growth rate of tradable and non-tradable sectors in the US and Europe by using 60 industry database of Groningen Growth and Development Centre. The growth rates of TFP shocks are set as equal to achieve a balanced growth path. The parameters $\eta^{HT}, \eta^{HN}, \eta^{FT}, \eta^{FN}$ are set to $\eta^i = z^i - (1-\delta)$, $i \in \{HT, HN, FN, FT\}$, that provide no capital adjustment costs along the balanced growth path as discussed earlier. Moreover, $\phi_d = 0.001$ is chosen as in Ireland (2011) to have a unique balanced growth path. The capital adjustment cost parameter $\phi_k = 0.001$ is calibrated to achieve a plausible standard deviation ratio between investment and output.

The persistence parameters, correction parameter and standard deviations of shocks are estimated separately by assuming symmetry between home and foreign countries as in CDL by using the VECM. Table 5 presents the results. The maximum likelihood estimates of persistence parameters for tradable sector $\rho^{HT} = \rho^{FT} = 0.46$ and non-tradable sector $\rho^{HN} = \rho^{FN} = 0.2$ are in line with the previous findings in the literature[11]. The estimate of the correction parameter is $\kappa = -0.015$. This value is close to the estimation in RRT and Ireland (2011), which guarantees the balanced growth path but also permits the series to deverge enough so that trend shocks generate important wealth dynamics. In the robustness section, we also simulate our model for a set of persistence parameters .

[11] Aguiar and Gopinath (2007) utilize GMM to estimate the persistence parameter of trend growth for a small open economy model with one sector. Their estimation for Canada ranges between 0.03 and 0.29 for the period 1981Q1 to 2003Q2. Ireland (2011) uses Bayesian methods for a two-country model without a non-tradable sector and estimates US and EU persistency parameters as 0.1519 and 0.3845 respectively for the period 1970Q1 to 2007Q4.

Table 4: Calibrated Parameters

Definition	Parameter	Value
Discount factor	β	0.96
Depreciation rate	δ	0.1
Risk aversion	γ	2
Consumption share in utility	μ	0.34
Capital share in production	α	0.36
Elasticity between home and foreign tradable goods	θ	$0.5, 0.62, 0.85, 1.47, 2, 4$
Share of imports in intermediate goods	ω_1	0.5
Elasticity between intermediate goods and non-tradable goods	θ_2	0.5
Share of non-tradables in final goods	ω_2	0.2
Steady state growth rates of TFP shocks	$z^{HT}, z^{HN}, z^{FT}, z^{FN}$	1.004
Bond adjustment cost	ϕ_d	0.001
Capital adjustment cost	ϕ_k	0.001

Table 5: Estimated Parameters

Definition	Parameter	Value
Correction parameter	κ	-0.015
Persistence parameter for tradable sector	ρ^{HT}, ρ^{FT}	0.46
persistence parameter for non-tradable sector	ρ^{HN}, ρ^{FN}	0.2
Standard deviation of tradable shocks	σ^{HT}, σ^{FT}	0.023
Standard deviation of non-tradable shocks	σ^{HN}, σ^{FN}	0.013

4 Results

4.1 Non-stationary Model

We stationarize our model by dividing all variables with their growth rates along the steady state growth path.[12] The resulting model, which includes stationarized variables, is linearized around the steady state. We simulate the model 100 times for 1250 periods and reconstruct non-stationary variables by utilizing growth rates. Before calculating the statistics we remove the first 1000 periods. The HP-filtered statistics

[12] Appendix D includes the normalization process and gives full system of equations.

for the non-stationary model are given in Table 6, which also includes the data statistics taken from BT.[13] We display the statistics for different values of the trade elasticity parameter $\theta = 0.5$, 0.62, 0.85, 1.47, 2, 4.

Table 6: Statistics

	Data	Non-stationary Model					
		$\theta = 0.5$	$\theta = 0.62$	$\theta = 0.85$	$\theta = 1.47$	$\theta = 2$	$\theta = 4$
$\sigma(Y)$	1.57	1.34	1.25	1.37	1.43	1.49	1.59
$\sigma(C)/\sigma(Y)$	0.76	0.88	0.75	0.72	0.70	0.68	0.62
$\sigma(I)/\sigma(Y)$	4.33	1.88	2.01	2.03	2.11	2.20	2.57
$\sigma(N)/\sigma(Y)$	0.31	0.24	0.22	0.23	0.30	0.34	0.46
$\sigma(ReR)/\sigma(Y)$	6.16	5.05	2.56	1.19	0.66	0.67	0.84
$\sigma(ToT)/\sigma(Y)$	2.12[14]	7.67	4.62	2.77	1.70	1.37	0.92
$cor(Y,Y^*)$	0.35	0.54	0.37	0.18	0.04	-0.02	-0.18
$cor(C,C^*)$	0.06	0.59	0.59	0.43	0.34	0.33	0.36
$cor(C/C^*, ReR)$	-0.45	-0.11	0.78	0.95	0.67	0.39	0.05
$cor(Y, ReR)$	-0.09	-0.43	0.59	0.58	0.37	0.11	-0.30
$\rho(ReR)$	0.82	0.62	0.66	0.75	0.77	0.68	0.57

The first main result of our model is the success in breaking risk sharing for low ($\theta = 0.5$) and high ($\theta = 4$) trade elasticities. The main mechanism generating the Backus-Smith relation for $\theta = 0.5$ is as follows: A trend shock to technology increases the demand for home tradable goods due to the strong wealth effect coming from the future expected growth. Since tradable goods are not substitutes across countries and there is home-bias in production, the wealth effect dominates the substitution effect and in turn the price of home tradables increases substantially. This results in an appreciation of RER following the appreciation in TOT. On the other hand, the foreign country experiences a negative wealth effect due to the increase in home tradable prices. Thus, the home country with a relatively more expensive consumption basket consumes more and we observe a negative correlation, -0.11, between relative consumption and RER.

[13] We use the same data source as BT for the estimation of TFP processes.

[14] CDL states that if U.S. import prices is replaced by the trade-weighted export deflators, this value becomes 3.02.

The mechanism for the high trade elasticity parameter $\theta = 4$ works through the non-tradable sector. A positive trend shock increases the demand for both tradable and non-tradable goods. Since tradable goods are substitutes in this case, the income effect cannot dominate substitution effect for tradables. However, we observe an increase in non-tradable prices because tradable and non-tradable goods in final good production are complements. The increase in non-tradable prices leads to an appreciation in RER and model improves the consumption-RER anomaly.

The mechanisms discussed in the previous two paragraphs are closely related to those of CDL. However, CDL needs high persistence in transitory shocks to solve the Backus-Smith puzzle, which is supported by data. On the other hand, as we model TFP processes as cointegrated processes instead of transitory processes, our model is able to address the puzzle for high trade elasticity without the need for high persistence.

Our model also generates high RER volatility for low trade elasticities ($\theta = 0.5$ and $\theta = 0.62$) stemming from the volatility in TOT. As the elasticity parameter increases, TOT becomes less volatile since the substitution effect becomes relatively more dominant with respect to income effect generated by trend shocks. However, after a certain degree of trade elasticity, variation in non-tradable prices improves the RER volatility, though not much, through the aforementioned mechanism for $\theta = 4$.

Although our model performs well in terms of the Backus-Smith puzzle and RER puzzles, investment volatility is low compared to the other RBC models. This issue is present also in RRT and the main reason behind this is the low persistence of trend shocks together with the adjustment costs. Since the persistence in trend shocks is low, consumers do not vary their investment decision much. As the persistence increases, the volatility of investment increases.[15] We can say that there is a trade-off between investment volatility and RER volatility.

4.2 Separating the Effects of Trend Shocks and Non-tradables

In order to see the effects of trend shocks and non-tradables separately, we close each channel of our model one by one and repeat our simulation exercise. First, we assume that there are transitory TFP shocks instead of trend shocks. We call this specification as the stationary model, which is very close to the model of CDL except that we do not have a distribution sector. In the second exercise, we remove the non-tradable sector from our basic model, but we still assume that TFP shocks are cointegrated and call this model as the non-stationary model with the tradable sector. This version of the model is exactly

[15] See the robustness section for results with higher persistency parameters.

the same as the model in RRT.[16] However, our calibration differs from CDL and RRT since we use the data set from Groningen Growth and Development Centre. Afterwards, we calculate the statistics by the same methodology in Table 6 and report the results in Table 7 and Table 8.

Table 7 illustrates that using trend shocks instead of transitory shocks to TFP significantly improves the performance of the model in terms of the Backus-Smith and the RER volatility puzzles. As discussed before, trend shocks acts like persistent transitory shocks. If the home country receives a positive shock to its trend, it generates a strong wealth effect in the country and domestic households increase consumption more compared to the case in which the home country receives transitory shocks. Moreover, tradable good price movement is higher in the non-stationary model relative to the stationary model if the tradable goods are complements, where elasticity parameter is low $\theta = 0.50$ Thus, the Backus-Smith correlation is negative, -0.11, in the non-stationary model whereas the same correlation is highly positive, 0.76, for the stationary model.

The improvement resulting from the trend shock diminishes as the substitution between tradable goods increases. However, for high values of the trade elasticity parameter the effect of trend shocks becomes evident again. For the trade elasticity parameter $\theta = 4$, fluctuations in tradable good prices are limited and most of the variation in RER comes from non-tradable good prices. Moreover, the non-stationary model generates almost zero correlation between relative consumption and RER, whereas stationary model generates positive correlation 0.57.

[16] The details of the calibrations of the stationary model and the nonstationary models with only tradable sector are in Appendix A and B.

Table 7: The Effects of Trend Shocks

	$\sigma(ReR)/\sigma(Y)$	$cor(C/C^{*}, ReR)$
Data	6.16	-0.45
$Stationary\ Model\ (\theta = 0.50)$	1.25	0.76
$Non-stationary\ Model\ (\theta = 0.50)$	5.05	-0.11
$Stationary\ Model\ (\theta = 0.62)$	0.74	0.92
$Non-stationary\ Model\ (\theta = 0.62)$	2.56	0.78
$Stationary\ Model\ (\theta = 0.85)$	0.48	0.96
$Non-stationary\ Model\ (\theta = 0.85)$	1.19	0.75
$Stationary\ Model\ (\theta = 1.47)$	0.32	0.92
$Non-stationary\ Model\ (\theta = 1.47)$	0.66	0.67
$Stationary\ Model\ (\theta = 2)$	0.28	0.84
$Non-stationary\ Model\ (\theta = 2)$	0.67	0.39
$Stationary\ Model\ (\theta = 4)$	0.24	0.57
$Non-stationary\ Model\ (\theta = 4)$	0.84	0.05

Table 8 reports the results of our basic model and the non-stationary model with tradable sector. For the given values of trade elasticity parameter, our basic model exhibits a higher RER volatility with respect to non-stationary model with tradable sector. For the Backus-Smith puzzle, the existence of non-tradable sector provides limited improvement; but for high values of trade elasticity the gap between the Backus-Smith correlation of the two models widens. Thus, we can conclude that the addition of the non-tradable sector to RRT shows its effect at higher trade elasticity parameters where the fluctuations in the non-tradable good prices dominate the movements in RER.

Table 8: The Effects of the Non-Tradable Sector

	$\sigma(RER)/\sigma(Y)$	$cor(C/C^*, RER)$
Data	6.16	-0.45
$Non-stationary\ Model\ with\ tradable\ sector$ ($\theta = 0.62$)	2.12	0.93
$Non-stationary\ Model$ ($\theta = 0.62$)	2.56	0.78
$Non-stationary\ Model\ with\ tradable\ sector$ ($\theta = 0.85$)	1.05	0.99
$Non-stationary\ Model$ ($\theta = 0.85$)	1.19	0.95
$Non-stationary\ Model\ with\ tradable\ sector$ ($\theta = 0.1.47$)	0.54	0.94
$Non-stationary\ Model$ ($\theta = 1.47$)	0.66	0.67
$Non-stationary\ Model\ with\ tradable\ sector$ ($\theta = 2$)	0.43	0.85
$Non-stationary\ Model$ ($\theta = 2$)	0.67	0.39
$Non-stationary\ Model\ with\ tradable\ sector$ ($\theta = 4$)	0.32	0.58
$Non-stationary\ Model$ ($\theta = 4$)	0.84	0.05

Figure 1 displays the Backus-Smith performance of four models and summarizes Tables 7 and 8. Three of the four models are the ones presented above: non-stationary model, stationary model and non-stationary model with tradable sector. The last model is stationary model with tradable sector where we remove non-tradable sector and assume transitory shocks for TFP.[17] Figure 1 shows that using trend shocks and non-tradable sector significantly improves the performance in terms of the Backus-Smith puzzle. The performance of our basic model becomes more evident in the corner regions for the trade elasticity parameter. However, models with transitory shocks generate high correlation between relative consumption and risk sharing for all elasticities because of goods trade as discussed by Cole and Obstfeld (1991).[18] Figure 2 summarizes the relative volatility of real exchange rates for different values of trade

[17] The details for the transitory model with tradable sector are in Appendix C.

[18] Corsetti et al. (2008) is able to produce negative correlation for low θ. However, the correlation that we obtained from transitory model is close to unity. The high persistency of TFP shocks and the presence of the distribution sector are the main sources of difference.

elasticity and for different models. Relative volatility improves significantly with non-stationary models only for the low values of trade elasticities. For the high values of trade elasticity, non-stationary model with nontradable sector performs best but the absolute value is still very low compared to the data moment. In sum, low values of trade elasticity seem to be necessary to account for both the low level of risk sharing and the high level of relative volatility in real exchange rates.

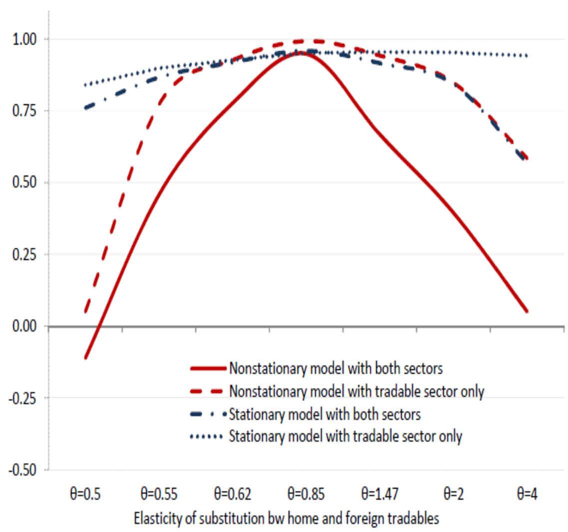

Figure 1: Backus-Smith Correlations

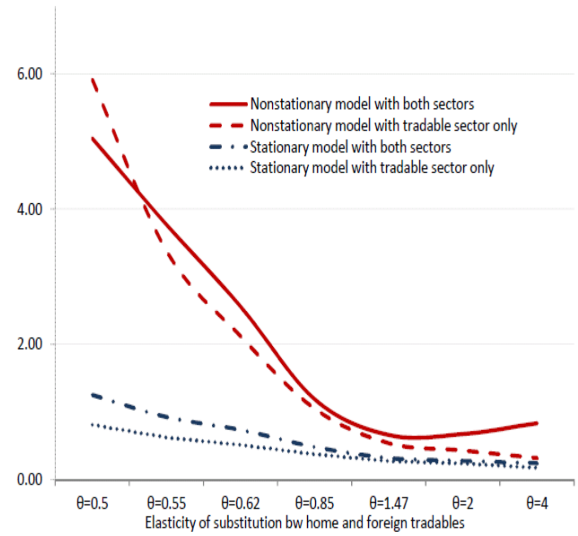

Figure 2: Relative Volatility of Real Exchange Rates

5 Robustness

The statistics of the non-stationary model for different values of persistence and correction parameters are presented in Table 9. Regarding the robustness of the results to the persistence parameter, all sectoral persistence parameters of trend shocks are set equal to each other for simplicity ($\rho = \rho^{HT} = \rho^{FT} = \rho^{HN} = \rho^{FN}$) except the baseline calibration, which is taken from Table 5. The results show that the correlation between relative consumptions and RER weakens as persistence increases. The increase in persistence increases the investment of households in the presence of positive technology shocks and weakens the instantaneous demand.

The right columns in Table 9 presents the robustness analysis with respect to the correction parameter (κ), which determines the spillover of shocks across the sectors and countries. In our benchmark case, we have $\kappa = -0.015$, which implies a low spillover. We also simulate our model with a lower correction parameter $\kappa = -0.008$, implying a lower spillover. A technology shock to the home country results in an increase in consumption due to the expectation of a high growth period. However, consumption in the foreign country does not respond to this shock because of the low spillover. As a result, the level of risk sharing worsens and the correlation between RER and relative consumption declines. As expected, if the level of spillover increases, benefit of foreign country from a positive shock in home country increases and risk sharing also increases as shown for parameter $\kappa = -0.03$. With a very strong spillover implied by $\kappa = -0.1$, positive risk sharing comes back.

Table 9: Robustness

		Persistence of Shocks			Correction Parameter		
		$\theta = 0.5,\ \kappa^{ij} = -0.015$			$\theta = 0.5,\ \rho^{HT}, \rho^{FT} = 0.46,\ \ \rho^{HN}, \rho^{FN} = 0.2$		
	Data	*Baseline*	$\rho = 0.2$	$\rho = 0.7$	$\kappa = -0.008$	$\kappa = -0.03$	$\kappa = -0.1$
$\sigma(Y)$	1.57	1.34	1.05	2.22	1.75	1.26	1.38
$\sigma(C)/\sigma(Y)$	0.76	0.85	0.85	0.83	0.89	0.78	0.61
$\sigma(I)/\sigma(Y)$	4.33	1.88	1.59	2.33	1.68	2.18	2.54
$\sigma(RER)/\sigma(Y)$	6.16	5.05	5.54	3.79	5.56	3.72	1.61
cor(Y,Y*)	0.35	0.54	0.6	0.05	0.32	0.40	0.07
cor(C,C*)	0.06	0.59	0.6	0.40	0.39	0.65	0.74
cor(C/C*,RER)	-0.45	-0.11	-0.12	0.05	-0.43	0.41	0.75

6 Conclusion

This paper focuses on the Backus-Smith and the real exchange rate volatility puzzles in international macroeconomics. First, we show that the TFP processes of tradable and non-tradable production in the US and Europe can be modelled by VECM. Then, we construct an international real business cycle model in which the TFP shocks are cointegrated. These cointegrated TFP shocks, or trend shocks, behave like highly persistent transitory shocks and strengthen the wealth effect. This strong wealth effect helps the model to generate the Backus-Smith relationship and the real exchange rate volatility for low and high values of elasticity parameters between tradable goods. In addition to the trend shocks, modeling the non-tradable sector extends the parameter space that the model can match the Backus-Smith relationship.

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7 Appendices

7.1 Appendix A: Stationary Model

We also run our model by assuming stationary TFP processes to illustrate the importance of introduction of trend shocks. For this purpose, the production function of tradable and non-tradable sectors are modified as:

$$Y_t^i = e^{\zeta_t^i}(K_t^i)^{\alpha}(Z_t^i L_t^i)^{1-\alpha}, i \in \{HT, HN, FN, FT\} \tag{32}$$

where ζ_t is transitory technology shock and it is assumed that $Z_t^i = 1$ $\forall t = 0, 1, ...$ to obtain the stationary model. Then the shock processes are assumed to follow AR(1) processes:

$$\zeta_t^i = \widetilde{\rho}^i \zeta_{t-1}^i + \varepsilon_t^i \text{ for } i \in \{HT, HN, FN, FT\} \tag{33}$$

where $\widetilde{\rho}^{HT}, \widetilde{\rho}^{HN}, \widetilde{\rho}^{FT}, \widetilde{\rho}^{FN}$ are persistence parameters, and $\widetilde{\varepsilon}_t^{HT}, \widetilde{\varepsilon}_t^{HN}, \widetilde{\varepsilon}_t^{FT}, \widetilde{\varepsilon}_t^{FN}$ are Gaussian random processes with zero mean and standard deviations $\widetilde{\sigma}^{HT}, \widetilde{\sigma}^{HN}, \widetilde{\sigma}^{FT}, \widetilde{\sigma}^{FN}$.

We use cycles obtained by HP-filter ($\lambda = 400$) in maximum likelihood estimation. Persistence parameters ($\widetilde{\rho}^{HT} = \widetilde{\rho}^{FT} = 0.38, \widetilde{\rho}^{HN} = \widetilde{\rho}^{FN} = 0.2$) and standard deviations ($\widetilde{\sigma}^{HT} = \widetilde{\sigma}^{FT} = 0.017, \widetilde{\sigma}^{HN} = \widetilde{\sigma}^{FN} = 0.009$) for the stationary version of the model are estimated by allowing a symmetry between tradable and non-tradable sectors using the same data. The persistence parameters are small when compared with the literature. However, note that BT also obtains small persistence parameters for non-tradable sector. Also, the correlation matrix of the $\widetilde{\varepsilon}_t^{HT}, \widetilde{\varepsilon}_t^{HN}, \widetilde{\varepsilon}_t^{FT}, \widetilde{\varepsilon}_t^{FN}$:

$$\begin{vmatrix} 1 & 0 & 0.04 & 0.24 \\ 0 & 1 & 0.04 & -0.08 \\ 0.04 & 0.04 & 1 & 0.37 \\ 0.24 & -0.08 & 0.37 & 1 \end{vmatrix}$$

7.2 Appendix B: Non-stationary Model with Tradable Sector

We obtain TFP series for both countries by combining the TFP processes for tradable and non-tradable sector in the 60 industry database of Groningen Growth and Development Centre For this purpose, the production function of tradable and non-tradable sectors are modified as:

$$Y_t^i = (K_t^i)^\alpha (Z_t^i L_t^i)^{1-\alpha}, i \in \{H, F\} \tag{34}$$

where H represents home country and F represents foreign country. Then we estimate the following processes

$$\ln(Z_t^H / Z_{t-1}^H) = (1 - \rho^H) \ln(z^H) + \rho^H \ln(Z/Z_{t-2}^H) + \kappa \ln(Z_{t-1}^H / Z_{t-1}^F) + \varepsilon_t^H \tag{35}$$

$$\ln(Z_t^F / Z_{t-1}^F) = (1 - \rho^F) \ln(z^F) + \rho^F \ln(Z/Z_{t-2}^H) + \kappa \ln(Z_{t-1}^F / Z_{t-1}^H) + \varepsilon_t^F \tag{36}$$

where Z_t^H, Z_t^F are the TFP processes for home and foreign country, ρ^H, ρ^F are persistence parameters, and $\varepsilon_t^H, \varepsilon_t^F$ are Gaussian random processes with zero mean and standard deviations $\widetilde{\sigma}^H, \widetilde{\sigma}^F$.

Maximum likelihood estimates of persistence parameters are $\rho^H = \rho^F = 0.46$, correction parameter is $\kappa = -0.021$ and standard deviations are $\sigma^H = 0.018$ and $\sigma^F = 0.008$ by assuming a symmetry between persistence parameters of home and foreign country.

7.3 Appendix C: Stationary Model with Tradable Sector

Similar to Appendix A we modify the production function $Y_t^i = e^{\zeta_t^i} (K_t^i)^\alpha (Z_t^i L_t^i)^{1-\alpha}$ for $i \in \{H, F\}$ where H represents home country and F represents foreign country. We estimate the following TFP processes by using the HP-filtered ($\lambda = 400$) log TFP data used in non-stationary model with one sector:

$$\zeta_t^H = \widetilde{\rho}^H \zeta_{t-1}^H + \tilde{\varepsilon}_t^H \tag{37}$$

$$\zeta_t^F = \widetilde{\rho}^F \zeta_{t-1}^F + \tilde{\varepsilon}_t^F \tag{38}$$

where ζ_t^H, ζ_t^F are the TFP processes for home and foreign country, $\widetilde{\rho}^H, \widetilde{\rho}^F$ are persistence parameters, and $\tilde{\varepsilon}_t^H, \tilde{\varepsilon}_t^F$ are Gaussian random processes with zero mean and standard deviations $\widetilde{\sigma}^H, \widetilde{\sigma}^F$. The maximum likelihood estimation results are AR terms $\widetilde{\rho}^H = 0.2, \widetilde{\rho}^F = 0.2$ and standard deviations $\widetilde{\sigma}^H = 0.011, \widetilde{\sigma}^F = 0.006$. The correlation matrix of shocks is very close to identity matrix.

7.4 Appendix D: System of Equations

Most of the variables listed above demonstrate non-stationary characteristics. Non-stationary variables are transformed into stationary versions by removing non-stationary terms. Stationary variables constitutes the stationary system that can be linearized around its steady state. The scaled variables are defined below.

Home variables (24): $c_t^H = C_t^H/U_{t-1}^H$, $i_t^{HT} = I_t^{HT}/U_{t-1}^H$, $i_t^{HN} = I_t^{HN}/U_{t-1}^H$, $l_t^{HT} = L_t^{HT}$, $l_t^{HN} = L_t^{HN}$, $k_t^{HT} = K_t^{HT}/U_{t-1}^H$, $k_t^{HN} = K_t^{HN}/U_{t-1}^H$, $d_t^H = D_t^H/U_{t-1}^H$, $\lambda_t^H = (U_{t-1}^H)^{1-\mu(1-\gamma)}\Lambda_t^H$, $\xi_t^{HT} = (U_{t-1}^H)^{1-\mu(1-\gamma)}\Xi_t^{HT}$, $\xi_t^{HN} = (U_{t-1}^H)^{1-\mu(1-\gamma)}\Xi_t^{HN}$, $y_t^{HT} = Y_t^{HT}/U_{t-1}^H$, $y_t^{HN} = Y_t^{HN}/U_{t-1}^H$, $a_t^H = A_t^H/U_{t-1}^H$, $b_t^H = B_t^H/U_{t-1}^H$, $p_t^H = 1$ (numeraire), $y_t^{H,int} = Y_t^{H,int}/U_{t-1}^H$, $y_t^H = Y_t^H/U_{t-1}^H$, $w_t^{HT} = W_t^{HT}/(U_{t-1}^H P_t^H)$, $q_t^{HT} = Q_t^{HT}/P_t^H$, $w_t^{HN} = W_t^{HN}/(U_{t-1}^H P_t^H)$, $q_t^{HN} = Q_t^{HN}/P_t^H$, $z_t^{HT} = Z_t^{HT}/Z_{t-1}^{HT}$, $z_t^{HN} = Z_t^{HN}/Z_{t-1}^{HN}$.

Foreign variables (24): $c_t^F = C_t^F/U_{t-1}^F$, $i_t^{FT} = I_t^{FT}/U_{t-1}^F$, $i_t^{FN} = I_t^{FN}/U_{t-1}^F$, $l_t^{FT} = L_t^{FT}$, $l_t^{FN} = L_t^{FN}$, $k_t^{FT} = K_t^{FT}/U_{t-1}^F$, $k_t^{FN} = K_t^{FN}/U_{t-1}^F$, $d_t^F = D_t^F/U_{t-1}^F$, $\lambda_t^F = (U_{t-1}^F)^{1-\mu(1-\gamma)}\Lambda_t^F$, $\xi_t^{FT} = (U_{t-1}^F)^{1-\mu(1-\gamma)}\Xi_t^{FT}$, $\xi_t^{FN} = (U_{t-1}^F)^{1-\mu(1-\gamma)}\Xi_t^{FN}$, $y_t^{FT} = Y_t^{FT}/U_{t-1}^F$, $y_t^{FN} = Y_t^{FN}/U_{t-1}^F$, $a_t^F = A_t^F/U_{t-1}^F$, $b_t^F = B_t^F/U_{t-1}^F$, $p_t^F = P_t^F/P_t^H$, $y_t^{F,int} = Y_t^{F,int}/U_{t-1}^F$, $y_t^F = Y_t^F/U_{t-1}^F$, $w_t^{FT} = W_t^{FT}/(U_{t-1}^F P_t^F)$, $q_t^{FT} = Q_t^{FT}/P_t^F$, $w_t^{FN} = W_t^{FN}/(U_{t-1}^F P_t^F)$, $q_t^{FN} = Q_t^{FN}/P_t^F$, $z_t^{FT} = Z_t^{FT}/Z_{t-1}^{FT}$, $z_t^{FN} = Z_t^{FN}/Z_{t-1}^{FN}$.

Other variables (11): $r_t = R_t$, $p_t^{HT} = P_t^{HT}/P_t^H$, $p_t^{FT} = P_t^{FT}/P_t^H$, $p_t^{HN} = P_t^{HN}/P_t^H$, $p_t^{FN} = P_t^{FN}/P_t^H$, $p_t^{H,int} = P_t^{H,int}/P_t^H$, $p_t^{F,int} = P_t^{F,int}/P_t^H$,$.rer_t = RER_t$,$.tot_t = TOT_t$,$.n_t^H = N_t^H/U_{t-1}^H$,$.n_t^F = N_t^F/U_{t-1}^F$.

Shock ratios (12): $z_t^{HTHN} = Z_t^{HT}/Z_t^{HN}$, $z_t^{HTHFT} = Z_t^{HT}/Z_t^{FT}$, $z_t^{HTFN} = Z_t^{HT}/Z_t^{FN}$, $z_t^{HNHT} = Z_t^{HN}/Z_t^{HT}$, $z_t^{HNFT} = Z_t^{HN}/Z_t^{FT}$, $z_t^{HNFN} = Z_t^{HN}/Z_t^{FN}$, $z_t^{FTHT} = Z_t^{FT}/Z_t^{HT}$, $z_t^{FTHN} = Z_t^{FT}/Z_t^{HN}$, $z_t^{FTFN} = Z_t^{FT}/Z_t^{FN}$, $z_t^{FNHT} = Z_t^{FN}/Z_t^{HT}$, $z_t^{FNHN} = Z_t^{FN}/Z_t^{HN}$, $z_t^{FNFT} = Z_t^{FN}/Z_t^{FT}$.

The stationary system in terms of stationary variables are written below.

Tradable and non-tradable sectors:

$$y_t^{HT} = (k_t^{HT})^{\alpha}(z_t^{HT} l_t^{HT})^{(1-\alpha)} \tag{39}$$

$$\alpha p_t^{HT} y_t^{HT} = q_t^{HT} k_t^{HT} \tag{40}$$

$$(1-\alpha) p_t^{HT} y_t^{HT} = w_t^{HT} l_t^{HT} \tag{41}$$

$$y_t^{FT} = (k_t^{FT})^{\alpha}(z_t^{FT} l_t^{FT})^{(1-\alpha)} \tag{42}$$

$$\alpha p_t^{FT} y_t^{FT} = p_t^F q_t^{FT} k_t^{FT} \tag{43}$$

$$(1-\alpha) p_t^{FT} y_t^{FT} = p_t^F w_t^{FT} l_t^{FT} \tag{44}$$

$$y_t^{HN} = (k_t^{HN})^{\alpha}(z_t^{HN} z_t^{HNHT} l_t^{HN})^{(1-\alpha)} \tag{45}$$

$$\alpha p_t^{HN} y_t^{HN} = q_t^{HN} k_t^{HN} \tag{46}$$

$$(1-\alpha) p_t^{HN} y_t^{HN} = w_t^{HN} l_t^{HN} \tag{47}$$

$$y_t^{FN} = (k_t^{FN})^{\alpha}(z_t^{FN} z_t^{FNFT} l_t^{FN})^{(1-\alpha)} \tag{48}$$

$$\alpha p_t^{FN} y_t^{FN} = p_t^F q_t^{FN} k_t^{FN} \tag{49}$$

$$(1-\alpha) p_t^{FN} y_t^{FN} = p_t^F w_t^{FN} l_t^{FN} \tag{50}$$

Intermediate goods producers and final goods producers:

$$y_t^{H,int} = \left[(1-\omega_1)^{\frac{1}{\theta}} (a_t^H)^{\frac{\theta-1}{\theta}} + (\omega_1)^{\frac{1}{\theta}} (b_t^H)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{51}$$

$$p_t^{HT} = p_t^{H,int}(y_t^{H,int})^{(1/\theta)}(1-\omega_1)^{(1/\theta)}(a_t^H)^{(-1/\theta)} \tag{52}$$

$$p_t^{FT} = p_t^{H,int}(y_t^{H,int})^{(1/\theta)}\omega_1{}^{(1/\theta)}(b_t^H)^{(-1/\theta)} \tag{53}$$

$$y_t^{F,int} = \left[(\omega_1)^{\frac{1}{\theta}}(a_t^F)^{\frac{\theta-1}{\theta}} + (1-\omega_1)^{\frac{1}{\theta}}(b_t^F)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{54}$$

$$p_t^{HT} = p_t^{F,int}(y_t^{F,int})^{(1/\theta)}(\omega_1)^{(1/\theta)}(a_t^F)^{(-1/\theta)} \tag{55}$$

$$p_t^{FT} = p_t^{F,int}(y_t^{F,int})^{(1/\theta)}(1-\omega_1)^{(1/\theta)}(b_t^F)^{(-1/\theta)} \tag{56}$$

$$y_t^H = \left[(1-\omega_2)^{\frac{1}{\theta_2}}(y_t^{H,int})^{\frac{\theta_2-1}{\theta_2}} + (\omega_2)^{\frac{1}{\theta_2}}(y_t^{HN})^{\frac{\theta_2-1}{\theta_2}}\right]^{\frac{\theta_2}{\theta_2-1}} \tag{57}$$

$$p_t^{H,int} = (y_t^H)^{(1/\theta_2)}(1-\omega_2)^{(1/\theta_2)}(y_t^{H,int})^{(-1/\theta_2)} \tag{58}$$

$$p_t^{HN} = (y_t^H)^{(1/\theta_2)}\omega_2{}^{(1/\theta_2)}(y_t^{HN})^{(-1/\theta_2)} \tag{59}$$

$$y_t^F = \left[(1-\omega_2)^{\frac{1}{\theta_2}}(y_t^{F,int})^{\frac{\theta_2-1}{\theta_2}} + (\omega_2)^{\frac{1}{\theta_2}}(y_t^{FN})^{\frac{\theta_2-1}{\theta_2}}\right]^{\frac{\theta_2}{\theta_2-1}} \tag{60}$$

$$p_t^{F,int} = p_t^F(y_t^F)^{(1/\theta_2)}(1-\omega_2)^{(1/\theta_2)}(y_t^{F,int})^{(-1/\theta_2)} \tag{61}$$

$$p_t^{FN} = p_t^F(y_t^F)^{(1/\theta_2)}(\omega_2)^{(1/\theta_2)}(y_t^{FN})^{(-1/\theta_2)} \tag{62}$$

Representative households:

$$w_t^{HT}l_t^{HT} + q_t^{HT}k_t^{HT} + w_t^{HN}l_t^{HN} + q_t^{HN}k_t^{HN} + d_t^H = c_t^H + \frac{\phi_d}{2}z_t^{HT}(d_{t+1}^H)^2 + i_t^{HT} + i_t^{HN} \tag{63}$$

$$(1-\delta)k_t^{HT} + i_t^{HT} - \frac{\phi_k}{2}(\frac{i_t^{HT}}{k_t^{HT}} - \eta^{HT})^2 k_t^{HT} = z_t^{HT}k_{t+1}^{HT} \tag{64}$$

$$(1-\delta)k_t^{HN} + i_t^{HN} - \frac{\phi_k}{2}(\frac{i_t^{HN}}{k_t^{HN}} - \eta^{HN})^2 k_t^{HN} = z_t^{HT} k_{t+1}^{HN} \tag{65}$$

$$\mu((c_t^H)^\mu (1 - l_t^{HT} - l_t^{HN})^{(1-\mu)})^{1-\gamma} = \lambda_t^H c_t^H \tag{66}$$

$$(1-\mu)((c_t^H)^\mu (1 - l_t^{HT} - l_t^{HN})^{(1-\mu)})^{1-\gamma} = \lambda_t^H w_t^{HT} (1 - l_t^{HT} - l_t^{HN}) \tag{67}$$

$$(1-\mu)((c_t^H)^\mu (1 - l_t^{HT} - l_t^{HN})^{(1-\mu)})^{1-\gamma} = \lambda_t^H w_t^{HN} (1 - l_t^{HT} - l_t^{HN}) \tag{68}$$

$$\lambda_t^H = \xi_t^{HT}(1 - \phi_k(\frac{i_t^{HT}}{k_t^{HT}} - \eta^{HT})) \tag{69}$$

$$\lambda_t^H = \xi_t^{HN}(1 - \phi_k(\frac{i_t^{HN}}{k_t^{HN}} - \eta^{HN})) \tag{70}$$

$$(z_t^{HT})^{(1-\mu(1-\gamma))} \xi_t^{HT} = \beta \lambda_{t+1}^H q_{t+1}^{HT} + \beta \xi_{t+1}^{HT} \left[1 - \delta + \phi_k(\frac{i_{t+1}^{HT}}{k_{t+1}^{HT}} - \eta^{HT}) \frac{i_{t+1}^{HT}}{k_{t+1}^{HT}} - \frac{\phi_k}{2}(\frac{i_{t+1}^{HT}}{k_{t+1}^{HT}} - \eta^{HT})^2 \right] \tag{71}$$

$$(z_t^{HT})^{(1-\mu(1-\gamma))} \xi_t^{HN} = \beta \lambda_{t+1}^H q_{t+1}^{HN} + \beta \xi_{t+1}^{HN} \left[1 - \delta + \phi_k(\frac{i_{t+1}^{HN}}{k_{t+1}^{HN}} - \eta^{HN}) \frac{i_{t+1}^{HN}}{k_{t+1}^{HN}} - \frac{\phi_k}{2}(\frac{i_{t+1}^{HN}}{k_{t+1}^{HN}} - \eta^{HN})^2 \right] \tag{72}$$

$$(z_t^{HT})^{(1-\mu(1-\gamma))} \lambda_t^H (\frac{1}{r_t} + \phi_d d_t^H) = \beta \lambda_{t+1}^H \tag{73}$$

$$(1-\delta)k_t^{FT} + i_t^{FT} - \frac{\phi_k}{2}(\frac{i_t^{FT}}{k_t^{FT}} - \eta^{FT})^2 k_t^{FT} = z_t^{FT} k_{t+1}^{FT} \tag{74}$$

$$(1-\delta)k_t^{FN} + i_t^{FN} - \frac{\phi_k}{2}(\frac{i_t^{FN}}{k_t^{FN}} - \eta^{FN})^2 k_t^{FN} = z_t^{FT} k_{t+1}^{FN} \tag{75}$$

$$\mu((c_t^F)^\mu (1 - l_t^{FT} - l_t^{FN})^{(1-\mu)})^{1-\gamma} = \lambda_t^F c_t^F \tag{76}$$

$$(1-\mu)((c_t^F)^\mu (1 - l_t^{FT} - l_t^{FN})^{(1-\mu)})^{1-\gamma} = \lambda_t^F w_t^{FT} (1 - l_t^{FT} - l_t^{FN}) \tag{77}$$

$$(1-\mu)((c_t^F)^{\mu}(1-l_t^{FT}-l_t^{FN})^{(1-\mu)})^{1-\gamma}=\lambda_t^F w_t^{FN}(1-l_t^{FT}-l_t^{FN}) \tag{78}$$

$$\lambda_t^F=\xi_t^{FT}(1-\phi_k(\frac{i_t^{FT}}{k_t^{FT}}-\eta^{FT})) \tag{79}$$

$$\lambda_t^F=\xi_t^{FN}(1-\phi_k(\frac{i_t^{FN}}{k_t^{FN}}-\eta^{FN})) \tag{80}$$

$$(z_t^{FT})^{(1-\mu(1-\gamma))}\xi_t^{FT}=\beta\lambda_{t+1}^F q_{t+1}^{FT}+\beta\xi_{t+1}^{FT}\left[1-\delta+\phi_k(\frac{i_{t+1}^{FT}}{k_{t+1}^{FT}}-\eta^{FT})\frac{i_{t+1}^{FT}}{k_{t+1}^{FT}}-\frac{\phi_k}{2}(\frac{i_{t+1}^{FT}}{k_{t+1}^{FT}}-\eta^{FT})^2\right] \tag{81}$$

$$(z_t^{FT})^{(1-\mu(1-\gamma))}\xi_t^{FN}=\beta\lambda_{t+1}^F q_{t+1}^{FN}+\beta\xi_{t+1}^{FN}\left[1-\delta+\phi_k(\frac{i_{t+1}^{FN}}{k_{t+1}^{FN}}-\eta^{FN})\frac{i_{t+1}^{FN}}{k_{t+1}^{FN}}-\frac{\phi_k}{2}(\frac{i_{t+1}^{FN}}{k_{t+1}^{FN}}-\eta^{FN})^2\right] \tag{82}$$

$$(z_t^{FT})^{(1-\mu(1-\gamma))}\lambda_t^F(\frac{1}{r_t p_t^F}+\phi_d d_t^F)=\frac{\beta\lambda_{t+1}^F}{p_{t+1}^F} \tag{83}$$

Trade variables and equilibrium conditions

$$n_t^H=(z_{t-1}^{HTFT})^{-1}p_t^{HT}a_t^F-p_t^{FT}b_t^H \tag{84}$$

$$p_t^F n_t^F=z_{t-1}^{HTFT}p_t^{FT}b_t^H-p_t^{HT}a_t^F \tag{85}$$

$$rer_t=p_t^F \tag{86}$$

$$tot_t=p_t^{FT}/p_t^{HT} \tag{87}$$

$$y_t^{HT}=a_t^H+a_t^F(z_{t-1}^{HTFT})^{-1} \tag{88}$$

$$y_t^{FT}=b_t^H z_{t-1}^{HTFT}+b_t^F \tag{89}$$

$$d_t^H z_{t-1}^{HTFT}+d_t^F=0 \tag{90}$$

$$y_t^H = c_t^H + \frac{\phi_d}{2} z_t^{HT} (d_{t+1}^H)^2 + i_t^{HT} + i_t^{HN} \tag{91}$$

$$y_t^F = c_t^F + \frac{\phi_d}{2} z_t^{FT} (d_{t+1}^F)^2 + i_t^{FT} + i_t^{FN} \tag{92}$$

$$p_t^H = 1 \tag{93}$$

Exogenous shock process:

$$\begin{aligned} \ln(z_t^{HT}) &= (1-\rho^{HT})\ln(z^{HT}) + \rho^{HT} ln(z_{t-1}^{HT}) + \kappa^{HTHN} \ln(z_{t-1}^{HTHN}) \\ &+ \kappa^{HTFT} \ln(z_{t-1}^{HTFT}) + \kappa^{HTFN} \ln(z_{t-1}^{HTFN}) + \epsilon_t^{HT} \end{aligned} \tag{94}$$

$$\begin{aligned} \ln(z_t^{HN}) &= (1-\rho^{HN})\ln(z^{HN}) + \rho^{HN} ln(z_{t-1}^{HN}) + \kappa^{HNHT} \ln(z_{t-1}^{HNHT}) \\ &+ \kappa^{HNFT} \ln(z_{t-1}^{HNFT}) + \kappa^{HNFN} \ln(z_{t-1}^{HNFN}) + \epsilon_t^{HN} \end{aligned} \tag{95}$$

$$\begin{aligned} \ln(z_t^{FT}) &= (1-\rho^{FT})\ln(z^{FT}) + \rho^{FT} ln(z_{t-1}^{FT}) + \kappa^{FTHT} \ln(z_{t-1}^{FTHT}) \\ &+ \kappa^{FTHN} \ln(z_{t-1}^{FTHN}) + \kappa^{FTFN} \ln(z_{t-1}^{FTHN}) + \epsilon_t^{FT} \end{aligned} \tag{96}$$

$$\begin{aligned} \ln(z_t^{FN}) &= (1-\rho^{FN})\ln(z^{FN}) + \rho^{FN} \ln(z_{t-1}^{FN}) + \kappa^{FNHT} \ln(z_{t-1}^{FNHT}) \\ &+ \kappa^{FNHN} \ln(z_{t-1}^{FNHN}) + \kappa^{FNFT} \ln(z_{t-1}^{FNFT}) + \epsilon_t^{FN} \end{aligned} \tag{97}$$

Shock ratios:

$$z_t^{HTHN} = \frac{z_t^{HT}}{z_t^{HN}} z_{t-1}^{HTHN} \tag{98}$$

$$z_t^{HTFT} = \frac{z_t^{HT}}{z_t^{FT}} z_{t-1}^{HTFT} \tag{99}$$

$$z_t^{HTFN} = \frac{z_t^{HT}}{z_t^{FN}} z_{t-1}^{HTFN} \tag{100}$$

$$z_t^{HNHT} = \frac{z_t^{HN}}{z_t^{HT}} z_{t-1}^{HNHT} \tag{101}$$

$$z_t^{HNFT} = \frac{z_t^{HN}}{z_t^{FT}} z_{t-1}^{HNFT} \tag{102}$$

$$z_t^{HNFN} = \frac{z_t^{HN}}{z_t^{FN}} z_{t-1}^{HNFN} \tag{103}$$

$$z_t^{FTHT} = \frac{z_t^{FT}}{z_t^{HT}} z_{t-1}^{FTHT} \tag{104}$$

$$z_t^{FTHN} = \frac{z_t^{FT}}{z_t^{HN}} z_{t-1}^{FTHN} \tag{105}$$

$$z_t^{FTFN} = \frac{z_t^{FT}}{z_t^{FN}} z_{t-1}^{FTFN} \tag{106}$$

$$z_t^{FNHT} = \frac{z_t^{FN}}{z_t^{HT}} z_{t-1}^{FNHT} \tag{107}$$

$$z_t^{FNHN} = \frac{z_t^{FN}}{z_t^{HN}} z_{t-1}^{FNHN} \tag{108}$$

$$z_t^{FNFT} = \frac{z_t^{FN}}{z_t^{FT}} z_{t-1}^{FNFT} \tag{109}$$

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