BANK FOR INTERNATIONAL SETTLEMENTS

BIS Working Papers

No 603

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by Iñaki Aldasoro, Iván Alves

Monetary and Economic Department

January 2017

JEL classification: G21, D85, C67

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This publication is available on the BIS website (www.bis.org).

ISSN 1020-0959 (print)
ISSN 1682-7678 (online)

Multiplex interbank networks and systemic importance
An application to European data☆

Iñaki Aldasoro[1], Iván Alves[2]

Abstract

Research on interbank networks and systemic importance is starting to recognise that the web of exposures linking banks' balance sheets is more complex than the single-layer-of-exposure approach suggests. We use data on exposures between large European banks, broken down by both maturity and instrument type, to characterise the main features of the multiplex (or multi-layered) structure of the network of large European banks. Banks that are well connected or important in one network, tend to also be well connected in other networks (i.e. the network features positively correlated multiplexity). The different layers exhibit a high degree of similarity, stemming both from standard similarity analyses as well as a core-periphery analyses at the layer level. We propose measures of systemic importance that fit the case in which banks are connected through an arbitrary number of layers (be it by instrument, maturity or a combination of both). Such measures allow for a decomposition of the global systemic importance index for any bank into the contributions of each of the sub-networks, providing a potentially useful tool for banking regulators and supervisors in identifying tailored policy responses. We use the dataset of exposures between large European banks to illustrate that both the methodology and the specific level of network aggregation may matter both in the determination of *interconnectedness* and in the policy making process.

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1. Introduction

Growing interest in the analysis of financial interconnectedness and the assessment of systemic risk reflects policy concerns extending well beyond traditional micro-prudential supervision. Risk externalities of bank behaviour, which are not taken into account by micro-prudential policies, call for a macroprudential approach (see Crockett (2000)). The recent financial crisis

☆Forthcoming in the *Journal of Financial Stability.* The views expressed here are those of the authors and do not necessarily reflect those of the BIS, the ECB or the ESCB. We thank Ben Craig for sharing code for the computation of the core-periphery structure and two anonymous referees, Robert Beyer, Marco D'Errico, Ingo Fender, Anne-Caroline Hüser and participants in the X Seminar on Risk, Financial Stability and Banking of the Banco Central do Brasil and the RiskLab/ESRB/Bank of Finland conference on Systemic Risk Analytics for helpful comments and suggestions. For the early stages of this work Aldasoro gratefully acknowledges financial support from DFG grant FA 1022/1-2 and research support from the Research Center SAFE, funded by the State of Hessen initiative for research LOEWE. Iván passed away in December 2016. His friendship and kindness will be deeply missed.

[1]Bank for International Settlements. *Corresponding author.* Contact address: Bank for International Settlements, Centralbahnplatz 2, 4051 Basel, Switzerland. Phone: +41 61 280 8683. E-mail: Inaki.Aldasoro@bis.org.

[2]European Central Bank.

and the stress suffered in interbank markets brought to the fore the relevance of bank interconnectedness and the importance of higher-order feedback loops embedded in the reciprocal web of exposures linking financial institutions.

Of critical importance in macroprudential policy is the identification of key players in the financial network, as exemplified by recently introduced *Basel Committee on Banking Supervision* (BCBS) requirements for global systemically important banks (G-SIBs, see Basel Committee on Banking Supervision (2011)). While early contributions on interbank contagion and networks have focused on aggregated exposures, it is now increasingly recognised that the web of credit relationships linking banks' balance sheets is generally more intricate and complex. The empirical literature thus far has either disregarded heterogeneity in credit relationships or worked with only one layer (typically the overnight unsecured market), resting on the tenet that it is representative of the whole web of exposures.[3] Of course, there is a very good reason why most of the extant literature on interbank networks has worked with the simplification of a single layer of exposures, namely data availability. The feature present in many networks whereby the edges or links connecting nodes can be of multiple types has been termed *multiplexity*, in contrast to the *monoplex* nature of networks represented by a single set of connections.

The main contribution of our paper is the proposal of a decomposition of systemic importance into the contribution of different constituent layers. The starting point is a holistic accounting representation of the balance sheet of the banking system, that allows for a consistent decomposition of systemic importance. Building on a dataset that provides the necessary granularity, this paper employs the framework by Aldasoro and Angeloni (2015) and expands two of their systemic importance measures to the case in which banks are connected through different layers. The goal is to attribute to each subnetwork its contribution to the systemic importance index for any given bank. Our dataset of exposures between large European banks features a high level of disaggregation in terms of instruments and maturity, and it was originally introduced into the literature in Alves et al. (2013). We analyse its multiplex structure and use it to illustrate the proposed measures.

Macro prudential policy addressing banks' systemic importance could indeed benefit from considering sub-networks and the aggregated network separately from each other. To the extent that transmission channels' magnitude and speed differ across layers, institutions' systemic importance may differ at the aggregate and more granular levels. Furthermore, systemic importance may depend on which activity is at the time more critical or more directly addressed by the specific policy being considered. This is indeed the logic that drives the policy process of assessing banks' importance at the *Basel Committee on Banking Supervision*, whereby global importance is constructed by an aggregation of the presence of the banks in relevant activities (see Basel Committee on Banking Supervision (2013)). The resulting scores are then (exogeneously) weighted to derive a unique ranking of systemic importance. Yet, for policy making purposes, it can be useful to refine the measurement of systemic importance in such a way that individual banks' importance relative to a given fragility can be identified. This requires granular information on banks' centrality in the interconnectivity of the relevant activities. Our approach builds on the logic that drives the existing processes for assessing banks' systemic importance at both the national and international levels, particularly as far as interconnectedness is concerned. However, it delves deeper into this aspect by considering the different layers in an integrated accounting framework.

The remainder of the paper is structured as follows. Section 2 briefly outlines the relation to the literature, whereas in Section 3 we develop the logic behind multilayer networks, present

[3]Exceptions to this are the recent contributions by Bargigli et al. (2015) and Langfield et al. (2014) among others. See the literature review section below for more details.

the approach to systemic importance in single layer networks and its extension to the multiplex case. Section 4 presents the analysis of the multiplex structure of the network of large European banks, while Section 5 uses the data to illustrate our proposed measures and their decomposition. Finally, Section 6 concludes.

2. Related Literature

Our paper is related to strands of literature which can be traced back to the seminal contributions by Allen and Gale (2000) and Freixas et al. (2000). These theoretical papers were pivotal in their recognition of the importance of the structure of interconnections between financial institutions. In the wake of these contributions, distinct approaches emerged in the literature, ranging from a static understanding of financial relationships to dynamic approaches built on assumed interactive frameworks.[4] Whereas interactive models include those dealing with contagion simulations (with specific assumptions on the reaction function of banks), the static types look at empirical data of some form of link between banks and remain mute on the mechanisms characterising contagion. Our paper relates to the empirical approach to systemic importance on the basis of static networks, and highlights the important policy content of the granularity of information in the analysis of systemic importance. The method used in this paper derives interconnectedness measures from traditional network models based on sufficiently granular data on financial institutions' balance sheets.

In recent years, a number of papers on real-world interbank networks and payment systems have built on specific datasets by constructing indicators with a financial focus.[5] Starting with the contribution by Boss et al. (2004) based on the Austrian interbank market, other studies made use of alternative country-specific datasets: Craig and von Peter (2014) for Germany, Soramäki et al. (2007) and Bech and Atalay (2008) for the U.S.A., Degryse and Nguyen (2007) for Belgium, van Lelyveld and In't Veld (2012) for the Netherlands, Fricke and Lux (2012) for Italy, Langfield et al. (2014) for the U.K., and Alves et al. (2013) for large European banks, among others. Taken together, these and similar studies provide a series of "stylised facts" of real-world interbank networks: a tiered banking network structure (with a core of highly connected institutions to which other periphery banks are linked), low density, low average path length, a scale-free degree distribution, high clustering relative to random networks and disassortative behaviour.

Our paper builds on the dataset of Alves et al. (2013) (more on the dataset in Appendix A) and the recent contribution by Aldasoro and Angeloni (2015) that uses measures of the traditional input-output literature translated to a banking context.

While it is usually recognised that the web of exposures linking financial institutions is much more complex than a single matrix of exposures would suggest, data limitations have typically hindered more holistic analyses of interbank networks in which financial institutions' balance sheets are intertwined through a variety of layers. Recent contributions are starting to fill this gap. Bargigli et al. (2015) study the multiplex structure of interbank networks using Italian data broken down by maturity and by the nature of the contract involved (secured versus unsecured). They find that different layers present several topological and metric properties which are layer-specific, whereas other properties are of a more universal nature. Using granular U.K. interbank

[4] For a good overview of simulation studies and methods applied to interbank contagion see Upper (2011). For recent overviews of the literature on interbank exposure networks and interbank networks at large see Langfield and Soramäki (2014) and Hüser (2015) respectively.

[5] Examples of these are *DebtRank* by Battiston et al. (2012), *SinkRank* by Soramäki and Cook (2013) or the contributions by Denbee et al. (2016) and Greenwood et al. (2015) among others.

data, Langfield et al. (2014) present an analysis of different layers of the U.K. interbank exposure and funding networks. Their findings, again, point to the importance of considering different layers, as structure typically differs among them. For instance, how close the network resembles a core-periphery structure depends on the asset class considered. León et al. (2014) study the network of Colombian sovereign securities settlements by combining data on transactions from three different individual networks. Interestingly, they find that the most important layer in terms of market value transacted does not transfer its properties to the multiplex (or aggregated) network. Molina-Borboa et al. (2015) present an analysis of the persistence and overlap of relationships between banks in a decomposition of the Mexican banking system's exposures network.

Even when detailed data is available, the papers addressing different layers of exposures between banks typically perform separate analyses for each layer and the aggregated network. We add to this literature by providing two holistic accounting-based measures that allow for decomposing systemic importance into layer-specific contributions. Two closely related papers are the contributions by Montagna and Kok (2013) and Poledna et al. (2015), which show non-linearities that emerge when risks from different layers sum up together, when there are dynamics and reaction functions embedded in the system. The former present a multi-layered network consisting of three different subnetworks: short term interbank loans, longer term bilateral exposures and common exposures in banks' securities portfolios. They embed an agent-based model on top of the multi-layered network, thereby providing dynamic mechanisms for shocks transmission via balance sheet adjustment. Their model is calibrated to European data and further balance sheet items are considered for a more comprehensive assessment of systemic risk. Contrary to them, in our paper the different layers are directly linked by a self-contained accounting framework which includes bilateral data, from which the measures are derived and decomposed. Our paper differs also in that it does not include endogenous mechanisms for balance sheet adjustment. Poledna et al. (2015) use a rich dataset for the Mexican interbank system, study its multiplex structure and show the existence of non-linearity in the way risks are aggregated: systemic risk for the aggregated network is larger than for the sum of the component subnetworks. This important insight is obtained by using the *DebtRank* measure developed by Battiston et al. (2012), applied to the different layers and the aggregated network separately. The framework of the DebtRank algorithm to identify systemic banks is formally related to the one presented here, as both build on representations based on eigen-systems, that take into account all possible interconnections. Two important differences in approach are worth noting, however. First, at a technical level, DebtRank focuses on the first two elements in the infinite series which summarises the interactions in the network, under the premise that otherwise cycles are considered.[6] While we do not question the wisdom of this approach, our framework instead considers all indirect connections as they provide for a benchmark of potential impact (akin to an impulse-response function). Second, our framework is constructed directly from an accounting representation of the balance sheet of the banking system, so centrality is embedded within a consistent system larger than the interbank network itself. Our framework provides a mapping between balance sheet characteristics, with the interbank matrix playing an important, but not exclusive, role. Poledna et al. (2015) look at the different layers one at a time and then at the aggregate layer, whereas we take a different route instead by starting from the aggregate and then decomposing it.

[6] In a sense, this introduces a behavioral assumption on the reaction of banks. Our framework, on the other hand, does not venture into such assumptions. For more details on the infinite series summarizing interactions in the network, see section 3 below.

3. Analysing complexity in multilayered banking networks

Following a brief review of the convenient input-output technique adopted by Aldasoro and Angeloni (2015) in approaching financial networks of a single layer (or aggregate) network (i.e. a *monoplex* interbank network) and an outline of the notion of systemic importance in section 3.1, section 3.2 develops the analytical foundation for connections through different "layers" (i.e. a *multiplex* interbank network).

3.1. The Input-Output approach to banking and the notion of systemic importance

We consider a banking system composed of n banks, each of which collects deposits and equity, lends to non bank customers and lends to and borrows from other banks. The aggregated balance sheet in matrix notation can be expressed as follows:[7]

$$\mathbf{e} + \mathbf{d} + \mathbf{X}'\mathbf{i} = \mathbf{X}\mathbf{i} + \mathbf{l} \tag{1}$$

where $\mathbf{e}$, $\mathbf{d}$, $\mathbf{l}$ are column vectors denoting respectively, equity, deposits and total non-interbank lending (composed of loans, net securities holdings and lending to (reserves at) the central bank), $\mathbf{i}$ is a unit (i.e. summation) vector of appropriate size, and $\mathbf{X}$ is the matrix of interbank gross bilateral positions,[8] where an element x_{ij} represents lending from bank i to bank j and where by construction $x_{jj} = 0$, $\forall j = 1, ..., n$. All magnitudes are expressed in monetary terms, say euros.

Let $\mathbf{q}$ be a vector with total bank assets/liabilities and $\hat{\mathbf{q}}$ be a corresponding diagonal matrix, such that $\hat{\mathbf{q}}\mathbf{i} = \mathbf{q}$. Then the right hand side of Equation 1 can be written in the following form:

$$\mathbf{q} = \mathbf{X}\hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}\mathbf{i} + \mathbf{l} = \mathbf{A}\mathbf{q} + \mathbf{l} \tag{2}$$

where $\mathbf{A} \equiv \mathbf{X}\hat{\mathbf{q}}^{-1}$ is the matrix of interbank positions in which each *column* is divided by the total assets of the *borrowing* bank. Hence, the columns of $\mathbf{A}$ are fractions of unity and express, for each bank, the share of funding from other banks as a ratio to total funding. Equation 2 is similar in form and interpretation to the familiar input-output system.[9] For a given matrix $\mathbf{A}$, the relation between loans and total assets is given by the well-known Leontief inverse $\mathbf{B} \equiv (\mathbf{I} - \mathbf{A})^{-1}$:

$$\mathbf{q} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{l} \equiv \mathbf{B}\mathbf{l} \tag{3}$$

The Leontief inverse $\mathbf{B}$ captures all direct and indirect connections between banks,[10] a feature that relates it to a standard result in graph theory: matrix $\mathbf{B}$ can be expressed as an infinite series as $\mathbf{B} = \mathbf{I} + \sum_{k=1}^{\infty} \mathbf{A}^k$ (we use this property later in the paper). Self impact is captured by the identity matrix, direct impact by $\mathbf{A}$ and second and higher rounds of indirect interconnections

[7] Unless otherwise specified, we use standard notation from matrix algebra. By capital bold fonts (e.g. $\mathbf{X}$) we denote an $n \times n$ matrix with generic elements x_{ij}, whereas lower case bold fonts (e.g. $\mathbf{x}$) represent $n \times 1$ column vectors with generic elements x_i. The transpose of a matrix or vector is indicated with a prime (as in $\mathbf{X}'$ or $\mathbf{x}'$). The vector $\mathbf{x}_j$ denotes de j^{th} column of matrix $\mathbf{X}$, whereas $\mathbf{x}'_i$ stands for the i^{th} row of matrix $\mathbf{X}$. The identity matrix is indicated by $\mathbf{I}$, the unit (column) vector is indicated by $\mathbf{i}$, and $\mathbf{i}_j$ stands for the j^{th} column of $\mathbf{I}$. Finally, a lower case bold letter with a "hat" on it (e.g. $\hat{\mathbf{x}}$) denotes an $n \times n$ diagonal matrix with the vector $\mathbf{x}$ on its main diagonal.

[8] We can think of $\mathbf{X}$ as aggregating different types of exposures between banks. Such a matrix has been the focus of analysis of much of the work on financial networks and we refer to it as the *monoplex* interbank network.

[9] For a complete treatment of input-output analysis including several different applications see the manual by Miller and Blair (2009) or the classic work by Pasinetti (1977).

[10] See Aldasoro and Angeloni (2015) for the conditions needed to express total assets in terms of non-interbank lending and the Leontief inverse, and for a discussion on the stability of matrix $\mathbf{X}$.

are captured by $\mathbf{A}^k$, with $k \geq 2$. The (i, j)-th element of matrix $\mathbf{A}^k$ will be positive if there exists a path of length k between banks i and j. In our context this implies that banks i and j are linked indirectly in terms of borrowing/lending relationships via $k - 1$ intermediaries. The Leontief inverse captures in a unique way the magnifying and distributive role shocks on lending or investment have on the interbank system.

Given this setting, we can study how distress in non-interbank loans for one bank (l_i) affects all banks in the system, in a way that depends on the matrix of bank interconnections $\mathbf{B}$. Additionally, we may assess the effect that distress in the primary sources of funding (equity and deposits) has on the total size of banks' balance sheets. To this end we need a slight transformation of the framework just presented, by focusing on the liability side instead of the asset side:

$$\mathbf{q} = \underbrace{\mathbf{e} + \mathbf{d}}_{\equiv \mathbf{v}} + \mathbf{X}'\mathbf{i} = \mathbf{v} + \mathbf{X}'\mathbf{i} \tag{4}$$

$$\iff \mathbf{q}' = \mathbf{v}' + \mathbf{i}'\hat{\mathbf{q}}\hat{\mathbf{q}}^{-\mathbf{1}}\mathbf{X} = \mathbf{v}' + \mathbf{q}'\mathbf{O} \tag{5}$$

where $\hat{\mathbf{q}}^{-\mathbf{1}}\mathbf{X} \equiv \mathbf{O}$ is the matrix of output coefficients. This is the matrix of interbank positions in which each *row* is divided by the total assets of the *lending* bank. Hence the *rows* of $\mathbf{O}$ are fractions of unity that express, for each bank, the share of funding provided to other banks as a share of total funding provided.

It is then straightforward to see that Equation 5 yields the following:

$$\mathbf{q}' = \mathbf{v}'\mathbf{G}, \tag{6}$$

where $\mathbf{G} \equiv (\mathbf{I} - \mathbf{O})^{-1}$. Equation 6 represents the supply-side version of the input-output scheme (known as the "Ghosh inverse").

There are different ways in which the importance of nodes in a network can be characterised, ranging from simple degree metrics (measuring the number of a node's linkages), to more elaborate metrics trying to ascertain the specific role a node may play within the distribution function served by the network. Although this issue is also central in the context of multiplex networks, we seek here the simplest tractable decomposition of standard measures of importance that illustrate the lessons to be drawn from the decomposition itself.

Aldasoro and Angeloni (2015) present several measures to assess the systemic importance of banks in the interbank system, adapted from the input-output literature. Of particular interest here are two measures: namely "backward" and "forward" linkages related to Equation 3 and Equation 6, respectively. These measures are related to interconnectedness risks stemming from credit extension and funding requirements, and they build on the mappings established in Equation 3 and Equation 6.

Backward linkages are useful in illustrating the transmission mechanism associated to distress in non-interbank assets. Assume, for example, that the banking system suffers a shock to the non-interbank lending portion of one of its member banks, say bank j. On impact, the balance sheet of bank j is obviously negatively affected, and this would show up in the j^{th} element of vectors $\mathbf{q}$ and $\mathbf{l}$. Subsequently, however, banks other than j may be affected as bank j starts curtailing credit demand from other banks in the system: as the fraction of interbank borrowing relative to total assets/liabilities remains constant, bank j reduces its interbank market activity. Such second round effects are going to be captured by the elements in the j^{th} column of matrix $\mathbf{A}$. Following this logic, third and subsequent round effects will be captured by the different powers of matrix $\mathbf{A}$ (starting from $\mathbf{A}^2$, ad infinitum). Ultimately, a unitary drawdown in non-interbank lending for bank j would affect the system to an extent given by the j^{th} column of matrix $\mathbf{B}$. The Rasmussen-Hirschman (RH) backward linkage index for bank j, which we denote as h_{b_j}, can therefore be computed as the sum of all elements in column j of matrix $\mathbf{B}$:

$$h_{b_j} \equiv \mathbf{i}'\mathbf{B}\mathbf{i}_j \tag{7}$$

The notion of a *backward* linkage stems from the fact that the hypothetical shock originates in non-interbank lending and the index traces *back* its effect through the entire system, with interbank linkages playing a crucial role in the process.[11] The backward index can be normalised by, for instance, relating it to the mean of the system (which we set equal to one):

$$\bar{h}_{b_j} \equiv n\frac{\mathbf{i}'\mathbf{B}\mathbf{i}_j}{\mathbf{i}'\mathbf{B}\mathbf{i}} \tag{8}$$

Similarly, the forward index captures the transmission of shocks on equity and deposits. It builds on Equation 6 by summing along the row dimension of the so-called "Ghosh" inverse $\mathbf{G}$ (based on the output matrix $\mathbf{O}$):

$$h_{f_j} \equiv \mathbf{i}_j'\mathbf{G}\mathbf{i} \tag{9}$$

This equation establishes an alternative mapping, that goes from primary (non-interbank) sources of financing to total assets/liabilities. In this context it is useful to think about a hypothetical shock that may come from the non-interbank funding side of the balance sheet of bank j. As opposed to the backward linkage indicator, this index traces what happens with *uses* of funds in the hypothetical event of a unitary shock hitting the *sources* of funds. As before, the matrix of interconnections plays a critical role in this transmission process. It should be noted though that the matrices $\mathbf{A}$ and $\mathbf{O}$ represent different transformations of the original matrix of interbank exposures. Each of these transformations underpins the logic of the two indicators: matrix $\mathbf{A}$ starts at the end of the "banking process" with a change in non-interbank lending and traces its effect *backward* through the system,[12] by focusing on the matrix of interconnections from a borrower's perspective; matrix $\mathbf{O}$, on the other hand, starts at the beginning with a change in primary sources of funding and traces the effects *forward* through the system, by focusing on the matrix of exposures from a lender's perspective.

As with the backward index, the forward index can also be normalised relative to the mean of system:

$$\bar{h}_{f_j} \equiv n\frac{\mathbf{i}_j'\mathbf{G}\mathbf{i}}{\mathbf{i}'\mathbf{G}\mathbf{i}} \tag{10}$$

Based on the normalised version of the indices one can construct a taxonomy of systemic importance as seen from either banks' credit or funding activities. By aligning the indicators with the mean of the system (normalised at 1 here), a bank showing an indicator above 1 will present an above average score of systemic importance, i.e. a shock to this bank will affect the system more than the same shock to the average of all banks. This taxonomy is summarized in Table 1 and we use it in Section 4 in order to identify the set of systemically important banks as those for which both normalised indicators are above one, i.e. banks having above average institutional levels of interconnectedness.[13] On the other hand, for the purposes of illustrating

[11] It is important to stress that this indicator, as well as the other considered in this paper, do not necessitate the existence of a shock to be computed. In other terms, there is no actual contagion process taking place.

[12] It is implicitly assumed that the final goal of banking is the provision of credit to the non-financial sector.

[13] Battiston et al. (2015) present a taxonomy in a similar spirit by classifying banks as *impactful* or *vulnerable*, i.e. those that cost most equity depletion to others and those that suffer the most in terms of equity depletion because of others, respectively. Parallels can be drawn between the categorizations, as for instance banks that

the multiplex extension of the two measures, we stick to the unnormalised version of the indices as they allow for better comparability.

Table 1: Classification based on backward and forward linkages

		$\bar{h}_{f_j}$	
		<1	>1
$\bar{h}_{b_j}$	<1	Generally independent	Important provider of funds
	>1	Dependent on funds from others	Key bank

Before moving to the multiplex extension of our measures, a few remarks are in order regarding the nature and limitations of the approach. As noted earlier, the framework used here relates to the empirical approach that uses static representations of financial networks to draw implications for systemic importance. Our indicators are not measures of *systemic risk*, as there is no actual *risk* being measured. Even when we motivate the measures by referring to hypothetical shocks, we don't observe actual shocks and hence there is no shock propagation process.[14]

The approach takes the balance sheet of the banking system as given and, in the spirit of the classic input-output literature, uses the interbank matrix as a quantifiable accounting representation of general interdependence in the banking system. It is incontestable as any accounting relationship can be and agnostic in terms of theory, i.e. a *data framework*. This is in contrast to *theoretical frameworks*, which require a theory or model to account for the observed reality and, hence, come with assumptions that may or may not be contestable on different grounds. In any case, it is important to stress that these assumptions are inherent to the *model* and are not a necessary feature of the *data*.

Some recent works bridge the distance between model and data frameworks by micro-founding the observed interbank network (see Cohen-Cole et al. (2015) or Denbee et al. (2016), who closely follow the seminal contribution by Ballester et al. (2006) originally applied to social networks). By assuming certain functional forms for the costs of forming links, and focusing exclusively on the lending aspect of interbank interactions, this literature presents an important result: the equilibrium outcome of the lending game between banks is proportional to the well-known Katz-Bonacich centrality measure (see Bonacich (1987)). It is straightforward to see that, if total assets are exactly the same for each and every bank in our system, the measures discussed here are virtually identical to Katz-Bonacich centrality as used in this literature.[15] However, the advantage of our framework is that we do not need to assume any functional form for any hypothetical utility or cost function. Furthermore, our data framework is flexible, allowing us to alternate the focus between the lending and borrowing sides of the market. Hence, it is arguably more general and illustrates that centrality-like measures are in fact embedded in accounting relationships.[16] Our measures are meant to be *ex-ante* measures: *given* a snapshot

are more *dependent on funds* by definition are high borrowers and through their distress can therefore severely impair other actors in the network (i.e. they are more *impactful*).

[14] In a similar fashion, for example, degree centrality is an indicator of how many counterparties would be affected *in the event* a shock takes place, without the need to have an actual shock in order to derive the measure.

[15] In this case, the so-called decay factor used in these papers would equal the inverse of total assets for each bank.

[16] Aldasoro and Angeloni (2015) show how the backward and forward linkage measures mathematically relate

of the banking system as captured by its balance sheet in matrix form, the measures will rank banks according to different criteria that potentially drive systemic importance. The measures are silent as to what could be the change in the configuration of the network and the effect at large given, say, a default of one bank. But it should be noted that when we try to ascertain the (hypothetical) potential reaction to changes in one or more elements in our system, "the character of such reactions depends upon the *initial structural properties of the empirically given system*" (Leontief (1937), emphasis added).

On the downside, by virtue of not incorporating explicit micro-foundations, we cannot study how to affect the incentives of players in the market to achieve a specific outcome. Our approach does not incorporate reaction functions of financial institutions, which arguably play a critical role in distress propagation in times of stress. To the extent that the interbank matrix is substantially rewired under stress, measures of systemic risk and systemic importance will tend to diverge and potentially deliver different results. The evidence points, however, to a relative stability and persistence of interbank matrices (see the discussion in Aldasoro and Angeloni (2015) or more recent evidence in Roukny et al. (2014) and Bluhm et al. (2016)).

3.2. Systemic importance in a multiplex context

Interconnectedness analysis generally focuses on a single network. However, in many, if not most, complex systems, the web of links connecting the actors of interest is more intricate and involved. In social networks, this is so because the links connecting different actors can take place at distinct levels: for instance people might be connected in the "workplace" network and not in the "gym" network. If one were to aggregate all connections into a single matrix of interconnections, the implications for, say, the spread of rumours, could be non-trivially altered. In transportation networks the multi-level nature of connections is even more evident: think for example of the bus, tram, subway and suburban train networks in any modern city. These networks share many nodes (in many stations one can commute from one network to the other in order to reach the final destination) and serve the overall purpose of taking people from point A to point B in a cost-effective manner. Inoperability of one node might be quite consequential if, for example, the node is an important nexus between different network layersx.

Similarly, the relationship between financial institutions can be based on many forms of links, including ownership, common interest on third parties, but also different types and maturities of financial contracts per se. Aggregation of these relationships also affects non-trivially the behaviour of lending relationships and, importantly, the flow and transmission of risk.

The fact that in many networks the edges or links connecting nodes can be of multiple types has been termed *multiplexity*,[17] as opposed to a single-layer type network which is referred to as a *monoplex* network.

A single-layer graph is typically characterised as a tuple $G = (E, V)$ where V is the set of vertices or nodes and $E \subseteq V \times V$ is the set of edges or links. The characterisation of multi-layer networks requires the specification of levels or layers of connectivity between the nodes.[18]

to in/out degree and strength centrality and appendix 3 of that paper also shows how generalized versions of the backward and forward linkages converge to, respectively, the left and right eigenvectors of the input and output matrices (what we define as $\mathbf{A}$ and $\mathbf{O}$ in the paper). The connection to the "key player" problem (or as Denbee et al. (2016) put it, the "level key player") is also discussed in that paper. We refer the reader to Aldasoro and Angeloni (2015) for further details on these issues, as well as for two simple numerical examples used for comparison of the measures.

[17]See De Domenico et al. (2013), D'Agostino and Scala (2014), Kivelä et al. (2014) and references therein. As noted by these authors, the engineering and in particular the sociology literature have pioneered the study of multilayer networks.

[18]Regarding the terminology and technical notation that we use to discuss multi-layer networks, we follow the review article by Kivelä et al. (2014).

To keep a high level of generality, one can note that a multi-layer network can have different *aspects*. A clear example of what an *aspect* represents is given by the case that concerns us in this paper, namely interbank networks. In this context, different aspects would help characterise different types of exposures. Thus: one *aspect* of banks' exposures depends on the "instrument type", whereas another *aspect* on the "maturity type" of the exposure (for a stylised example see Figure 1 below). Each *aspect* can have one or more *elementary layers*. In the interbank market example the *aspect* "maturity type" can for instance have three elementary layers: "short term", "medium term" and "long term"; while the *aspect* "instrument type" could be divided in, say, "credit claims" and "derivatives". A network can have any number of aspects d and one can define the sequence of aspects $\mathbf{L} = \{L_a\}_{a=1}^{d}$, such that to each *aspect* a corresponds a set of elementary layers L_a. The term *layer* is hence reserved for the combination of all *elementary layers* corresponding to all *aspects* of the network.

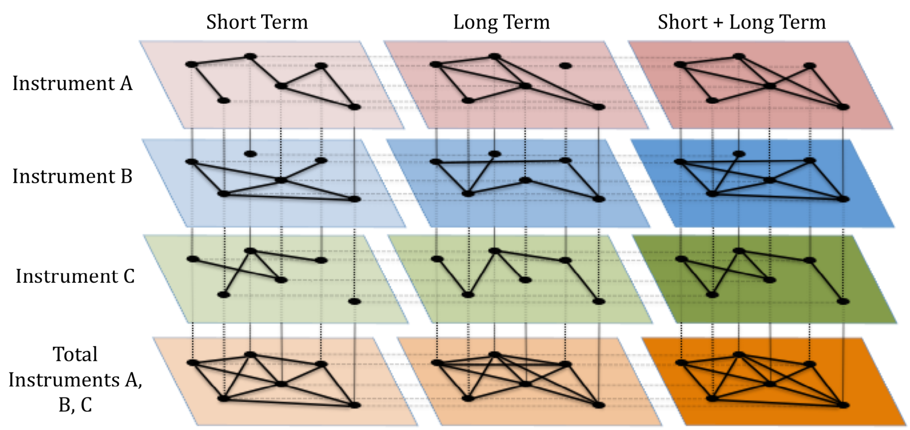

Figure 1: A stylised representation of a multiplex interbank network, where directionality and weights of links have been omitted. Black dots indicate node-aligned banks, and thick lines indicate connections within a given layer. This network displays two aspects, i.e. maturity type and instrument type. The former aspect is composed of two elementary layers (short term and long term), whereas the latter aspect presents three elementary layers (instruments A, B and C). The multiplex network is therefore composed of $2 \times 3 = 6$ layers.

The network in Figure 1 generates 6 aggregated networks depending on the aspect one wishes to emphasise. If the focus is on maturity, one can aggregate the networks across instruments to get the overall short term and overall long term networks. Whereas, if the focus is instead on instrument types, one can aggregate across maturity in order to get the network for instrument i (short + long term). Finally, either way one chooses to aggregate, one can always get the overall aggregation of exposures (the lower right network).

Generally speaking, in a multi-layer network nodes can be present in all or a subset of the layers, and links can not only exist between a given node and its counterpart in another layer, but also between different nodes in different layers. In this regard, our setting is more simple and we are therefore able to abstract from some complications that would require further notation: *(i)* every layer is composed of the exact same set of nodes, and *(ii)* inter-layer links are implicit and given between a node and its counterpart (i.e. a copy of the same node) in another layer. The first feature implies that our network of interest is *node-aligned* and one can define the set of vertices as $V_M = V \times L_1 \times \ldots \times L_d$ (implying an edge set given by $E_M \subseteq V_M \times V_M$), the second feature implies that the network is *diagonal*.[19] We therefore have that multiplex networks are a subset of multi-layer networks, and we can characterise them by the quadruplet $M = (V_M, E_M, V, \mathbf{L})$.

Having briefly provided some terminology for multiplex networks, we now exemplify the measures presented above with banks' exposures being composed by a variety of instruments or maturities. Of particular interest is the decomposition of the systemic importance index for the aggregated network for a given bank, i.e. a decomposition of the vectors $\mathbf{h}_{(b)}$ and $\mathbf{h}_{(f)}$ introduced above. That is, instead of calculating systemic importance measures for the different layers and for the overall network separately, ideally one would like to see how much of the overall systemic importance score of a given bank can be attributed to each of the different subnetworks that together constitute the aggregated network of connections.

To make the discussion general, let us assume that there are $\alpha = 1, \ldots, L$ different layers, such that $\mathbf{X} = \sum_{\alpha=1}^{L} \mathbf{X}_\alpha$. Without loss of generality these could represent the combination of *elementary layers* of different *aspects*: for example one aspect could be instrument type whereas another could be maturity type, as shown in Figure 1 and discussed above. The balance sheet of the banking system would read as:

$$\mathbf{e} + \mathbf{d} + \left(\sum_{\alpha=1}^{L} \mathbf{X}'_\alpha \right) \mathbf{i} = \left(\sum_{\alpha=1}^{L} \mathbf{X}_\alpha \right) \mathbf{i} + \mathbf{l} \tag{11}$$

Focusing on the asset side of the banking system we can perform transformations analogous to those used to arrive to Equation 2 and Equation 3, and we obtain the same expression involving the Leontief inverse, namely $\mathbf{q} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{l} \equiv \mathbf{Bl}$, with $\mathbf{A} = \sum_{\alpha=1}^{L} \mathbf{A}_\alpha$ and $\mathbf{A}_\alpha \equiv \mathbf{X}_\alpha \hat{\mathbf{q}}^{-1}$, where as before each matrix $\mathbf{A}_\alpha$ represents the interbank network for layer α where each *column* is divided by the total assets of the *borrowing* bank.

A useful property of the Leontief inverse is that can be expressed as an infinite series:

$$\begin{aligned}
\mathbf{B} &= (\mathbf{I} - \mathbf{A})^{-1} \\
&= \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots \\
&= \mathbf{I} + \mathbf{A}\left(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \cdots\right) \\
&= \mathbf{I} + \mathbf{AB}
\end{aligned}$$

Using this and noting that $\mathbf{A} = \sum_{\alpha=1}^{L} \mathbf{A}_\alpha$, Equation 3 can be expressed as:

$$\mathbf{q} = \mathbf{Bl} = (\mathbf{I} + \mathbf{AB})\,\mathbf{l} = \left(\mathbf{I} + \sum_{\alpha=1}^{L} \mathbf{H}_\alpha \right) \mathbf{l} \tag{12}$$

where $\mathbf{H}_\alpha \equiv \mathbf{A}_\alpha \mathbf{B}$, $\alpha = 1, \ldots, L$.[20] It is apparent from Equation 12 that the balance sheet equation can be decomposed into layers, such that each layer's role in magnifying shocks on the asset's side is evident.

[19] Alternatively, one can define the multilayer network as a multigraph that can be represented by a weighted adjacency tensor with 3 dimensions (or 4 if we also consider maturity on top of instrument type). Using tensor decompositions one can assign scores to different layers, thereby providing an alternative way of decomposing multiplexity. See Kolda et al. (2005) for a proposal on network centrality using tensor decompositions and Bonacina et al. (2015) for an application to networks of corporate governance in italian firms. While these applications focus on the networks themselves, the approach used here takes a broader look at the balance sheet of the banking system and derives measures of systemic importance within this accounting framework. We thank an anonymous referee for noting the connection to tensor decompositions.

[20] An alternative way of arriving at Equation 12 is by noting that $\mathbf{q} = \mathbf{Aq} + \mathbf{l}$ and $\mathbf{q} = \mathbf{Bl}$, replacing the second equation in the right-hand side of the first one and using $\mathbf{A} = \sum_{\alpha=1}^{L} \mathbf{A}_\alpha$.

The backward linkage index is still calculated as in Equation 7, but now we are able to attribute to each layer α its contribution to the overall systemic importance index, as measured by the column sum of the $\mathbf{H}_\alpha$ matrices.

To see what the column sums of matrix $\mathbf{H}_\alpha$ (i.e. for an instrument α) look like we can re-express the matrix in vector notation as follows:

$$\mathbf{H}_\alpha = \begin{bmatrix} \mathbf{a}'_{\alpha 1}\mathbf{b}_1 & \cdots & \mathbf{a}'_{\alpha 1}\mathbf{b}_n \\ \vdots & \ddots & \vdots \\ \mathbf{a}'_{\alpha n}\mathbf{b}_1 & \cdots & \mathbf{a}'_{\alpha n}\mathbf{b}_n \end{bmatrix} \tag{13}$$

where $\mathbf{a}'_{\alpha i}$ indicates the i^{th} row of matrix $\mathbf{A}_\alpha$ and $\mathbf{b}_j$ denotes the j^{th} column of matrix $\mathbf{B}$.

Now consider the backward linkage indicator of bank j, which is given by the sum of the elements in column j of matrix $\mathbf{B}$ as shown in Equation 7. The share of this index that can be attributed to layer α is given in turn by the sum of the elements in column j of matrix $\mathbf{H}_\alpha$:

$$\mathbf{i}'\mathbf{H}_\alpha\mathbf{i}_j = \mathbf{a}'_{\alpha 1}\mathbf{b}_j + \cdots + \mathbf{a}'_{\alpha n}\mathbf{b}_j = \left(\mathbf{a}'_{\alpha 1} + \cdots + \mathbf{a}'_{\alpha n}\right)\mathbf{b}_j \tag{14}$$

Note that the overall effect behind the logic of the backward linkage index is captured by the vector $\mathbf{b}_j$, i.e. the j^{th} column of the Leontief inverse $\mathbf{B}$. In the case of a monoplex system the sum of the elements of $\mathbf{b}_j$ would constitute the index of interest. Here we are concerned with the disentanglement of how each layer connecting banks contributes to the overall systemic importance index of banks. This is captured by the $\mathbf{a}_{\alpha i}$, $i = 1, \ldots, n$.

If bank j has exposures on layer α, it follows that this layer will have a bearing on the final index of systemic importance for bank j. On the other hand, bank j might not be exposed to another bank i on layer α, so distress in bank i on layer α will not affect bank j directly. In a monoplex network there exists the possibility that j is nonetheless affected through a third bank k which is exposed to i and to which bank j is itself exposed. The incorporation of this type of channel is indeed a major asset of network analysis. A multiplex structure expands these indirect channels, and these channels can help in assessing the nature and extent of the policy impact.

For the sake of argument, let us assume that bank j has no exposures on layer α, then it will be the case that $\mathbf{a}'_{\alpha j} = \mathbf{0}'$. By Equation 14 it is obvious that this does not imply that layer α is not relevant to account for the overall systemic importance index of bank j, since bank j is exposed to other banks on other layers $\boldsymbol{\beta} \neq \alpha, \boldsymbol{\beta} = 1, \ldots, L$ (as captured by $\mathbf{b}_j$) and these other banks might themselves be exposed in layer α (as captured by $\mathbf{a}'_{\alpha k} \neq \mathbf{0}'$, $k \neq j, k = 1, \ldots, n$).

Likewise, shocks originating in the liabilities are magnified via the different layers via a similar decomposition. As noted above, in the case of forward linkages, interest lies in tracing forward the effect of unitary declines in the primary sources of funding (deposits and equity) of any given financial institution, using the supply-side version of the input output model (see Equation 6). The forward index involves summation along the row dimension of matrix $\mathbf{G} = (\mathbf{I} - \mathbf{O})^{-1}$. In the context of a multiplex network we note that the output matrix $\mathbf{O}$ can be expressed as $\mathbf{O} = \sum_{\alpha=1}^{L} \mathbf{O}_\alpha$, where the output matrix for each layer α ($\alpha = 1, \ldots, L$) is in turn given by $\mathbf{O}_\alpha = \hat{\mathbf{q}}^{-1}\mathbf{X}_\alpha$.

Using the same logic for infinite series as above, we can re-express the Ghosh inverse as:

$$\mathbf{G} = (\mathbf{I} - \mathbf{O})^{-1} = \mathbf{I} + \mathbf{GO} = \mathbf{I} + \sum_{\alpha=1}^{L} \mathbf{K}_\alpha \tag{15}$$

where $\mathbf{K}_\alpha = \mathbf{G}\mathbf{O}_\alpha$, $\alpha = 1, \dots, L$. In vector notation such $\mathbf{K}_\alpha$ matrices can be expressed as:

$$\mathbf{K}_\alpha = \begin{bmatrix} \mathbf{g}_1'\mathbf{o}_{\alpha 1} & \cdots & \mathbf{g}_1'\mathbf{o}_{\alpha n} \\ \vdots & \ddots & \vdots \\ \mathbf{g}_n'\mathbf{o}_{\alpha 1} & \cdots & \mathbf{g}_n'\mathbf{o}_{\alpha n} \end{bmatrix} \tag{16}$$

where $\mathbf{g}_i'$ stands for the i^{th} row of $\mathbf{G}$, whereas $\mathbf{o}_{\alpha j}$ denotes the j^{th} column of $\mathbf{O}_\alpha$.

Since the forward linkage indicator is constructed by summing along the row dimension of matrix $\mathbf{G}$, that part of the index for bank i that can be attributed to layer α can be expressed as:

$$\mathbf{i}_i'\mathbf{K}_\alpha\mathbf{i} = \mathbf{g}_i'\mathbf{o}_{\alpha 1} + \cdots + \mathbf{g}_i'\mathbf{o}_{\alpha n} \tag{17}$$

As before, lack of exposure by bank i in layer α implies that $\mathbf{o}_{\alpha i} = \mathbf{0}$, but layer α can still contribute to systemic importance as measured by the forward index via the vectors $\mathbf{o}_{\alpha k}$ $(k \neq i, k = 1, \dots, L)$ and their interactions with the total effect as measured by $\mathbf{g}_i'$.

The focus here is on measures of systemic importance which are decomposable into layer-specific contributions. We note, however, that further variations of systemic importance measures in a multiplex context can be derived from the framework. For instance, Aldasoro and Angeloni (2015) present the total linkage effect for bank j, defined as $t_j = \mathbf{i}'\mathbf{q} - \mathbf{i}'\mathbf{q}^{-j} = \mathbf{i}'\mathbf{B} - \mathbf{i}'\mathbf{B}^{-j}\mathbf{l}$, where $\mathbf{B}^{-j} = \left(\mathbf{I} - \mathbf{A}^{-j}\right)^{-1}$ and $\mathbf{A}^{-j}$ is the input matrix in which the j^{th} row and column have been set to zero. This captures the cost, in terms of total system assets lost, of eliminating bank j from the interbank system.[21] In a multiplex context, one could think of at least three possibilities. First, it is possible to evaluate which is the "key layer" by computing the total linkage after eliminating a layer α, say, derivatives (and picking the layer that maximizes this): $t_\alpha = \mathbf{i}'\mathbf{B}\mathbf{U}_\alpha \left(\mathbf{I} + \mathbf{D}_\alpha\mathbf{V}_\alpha'\mathbf{B}\mathbf{U}_\alpha\right)^{-1} \mathbf{D}_\alpha\mathbf{V}_\alpha'\mathbf{B}\mathbf{l}$, where we have used the singular value decomposition of the matrix corresponding to layer α, $\mathbf{A}_\alpha = \mathbf{U}_\alpha\mathbf{D}_\alpha\mathbf{V}_\alpha'$.[22] A second option could be to evaluate the total linkage effect when eliminating bank j from all layers, $t_j = \frac{1}{1-\mathbf{i}_j'\mathbf{A}\mathbf{B}\mathbf{i}_j}\mathbf{i}'\mathbf{B}\mathbf{i}_j\mathbf{i}_j'\mathbf{A}\mathbf{B}\mathbf{l}$, provided $\mathbf{i}_j'\mathbf{A}\mathbf{B}\mathbf{i}_j \neq 1$. Finally, one could evaluate the total linkage effect stemming from the elimination of bank j only from layer α: $t_{j,\alpha} = \frac{1}{1-\mathbf{i}_j'\mathbf{A}_\alpha\mathbf{B}\mathbf{i}_j}\mathbf{i}'\mathbf{B}\mathbf{i}_j\mathbf{i}_j'\mathbf{A}_\alpha\mathbf{B}\mathbf{l}$.

4. The multi-layered network of large European banks

This section motivates the analysis of multiplex networks by zooming in on different characteristics of the multi-layered network of large European banks. In particular, we quantify the extent of similarity between the different layers, we evaluate the correlation of layer-specific measures of systemic importance across layers, and we compute the core-periphery structure of the different layers and the aggregated network separately. This sets the stage for the decomposition of systemic importance along the lines proposed in 3, which we undertake in the following section.

[21] It is formally very similar to the problem of identifying the key bank in Cohen-Cole et al. (2015) and level key player in Denbee et al. (2016). More precisely, finding the key bank is equivalent to finding the bank with the maximum total linkage. While the approach in these papers focuses on the interbank network itself, the total linkage has a broader focus. As noted earlier, these papers heavily draw from the seminal contribution by Ballester et al. (2006). The development of the total linkage effect indicator in the input-output literature pre-dates that of the key player as defined in Ballester et al. (2006) (see for instance Miller and Blair (2009)).

[22] See for instance Meyer (2000).

Due to space considerations, we relegate the detailed description of the data used to Appendix A, which also presents a network plot. We only shall note at this stage a couple of relevant features. In particular, the dataset presents two *aspects*, namely *instrument* and *maturity* type. The partition of exposures according to instrument type is given by: *(i) assets (A)*, further subdivided into *credit claims (CC)*, *debt securities (DS)* and *other assets (OA)*, *(ii) derivatives (D)*, and *(iii) off-balance-sheet (OffBS)* exposures. In addition, exposures according to maturity type are divided into: *(i)* less than one year including on sight ("short term (S)"), *(ii)* more than one year ("long term (L)"), and *(iii)* a residual of unspecified maturity (U).

4.1. Layer similarity analysis

Network layers' *similarity* or *proximity* can be assessed in a variety of ways, which in essence boil down to measuring how similar the layer's structures are. At a very basic level of comparison, as noted by Bargigli et al. (2015), it is important to distinguish between topological similarity and point-wise similarity, as one does not necessarily imply the other. For instance, two networks may be very similar in terms of density, degree distribution, etc., but the existence of a link between two nodes in the first network may be irrelevant to explain the existence of an analogous link in the second network. For interbank exposures such differences are particularly relevant, since for two identical distributions of a given characteristic across layers, point-wise dissimilarity would indicate institutional specialisation in the trade of an instrument or within a maturity type, or changes of interbank relations when carrying out the analysis in time.[23]

The focus here is on point-wise similarity,[24] in particular using measures designed for binary and weighted networks.[25] A distance metric useful for binary representations of the network layers is the *Jaccard similarity index* (J), capturing the probability of observing a given connection in a network conditional on observing the same link in the other network. For a given pair of vectors $\mathbf{x}$ and $\mathbf{y}$, the index is computed as the quotient between the size of the intersection and size of the union of the two ordered vectors:

$$J(\mathbf{x}, \mathbf{y}) = \frac{|\mathbf{x} \cap \mathbf{y}|}{|\mathbf{x} \cup \mathbf{y}|} \tag{18}$$

For networks with weighted links the *Cosine similarity index* (C) can be used as a proximity metric. As indicated by its name, C measures the cosine of the angle formed by the two vectors by means of a normalised dot product between them:[26]

$$C(\mathbf{x}, \mathbf{y}) = \frac{\mathbf{x} \cdot \mathbf{y}}{||\mathbf{x}|| \times ||\mathbf{y}||} \tag{19}$$

Table 2 presents results for Jaccard (lower triangle) and Cosine (upper triangle) similarity for European banks, according to instrument type.[27] While numbers are rarely above 50% (in

[23]Given the lack of a time series of interbank exposures for large European banks we are only able to perform the first type of comparison in the present paper.

[24]Alves et al. (2013) present some measures on topological similarity so we shall not repeat them here. Appendix A presents degree distributions as an additional means of comparing the structure of the different layers.

[25]Binary networks are those indicating only whether ties do or do not exist, in a 0-1 fashion. Weighted networks assign some value to the relationship being modelled (for instance, a monetary value). Both types of measures considered here represent the matrices as ordered vectors which are then compared.

[26]See the discussion in Bargigli et al. (2015) regarding the preference for this measure as opposed to the Pearson correlation coefficient for weighted networks. In the case of both similarity metrics used here, we have that $S \in [0, 1]$, $S = J, C$. In principle the Cosine similarity index ranges from -1 to 1, but since the network in our application presents non-negative values only, this index only takes non-negative values.

[27]Some of the indices need to be interpreted with caution. For instance, the index comparing the *credit claims* network with the *total assets* network will be necessarily high as one is a subset of the other. While still

particular for non-overlapping networks), they are relatively big (compared to, for instance, *Table 7* in Bargigli et al. (2015)), pointing to a relative lack of complementarity between instruments. This is particularly true when comparing *derivatives* with *assets*. *Off-balance-sheet* exposures present lower indices when compared to *assets* and *derivatives*, though values above 40% can still be considered relatively large. When comparing overlapping networks one can see that roughly 80% of the connections that are present in the *total* network are also present in the *assets* network, whereas this percentage drops below 50% when the comparison is between *off-balance-sheet* and *total* exposures. This last number implies, for instance, that almost half of the connections present in the *total* aggregated network are also present in the *off-balance-sheet* network.

As can be seen in the upper triangle of Table 2, similarity computed based on weights rather than on a binary indicator of existence/absence of relationship delivers lower values for the index, but the overall distribution remains unchanged.

	A-CC	**A-DS**	**A-Other**	**A-Total**	**Derivatives**	**Off BS**	**Total**
A-CC		**0.32**	**0.29**	0.80	**0.33**	**0.18**	0.70
A-DS	**0.50**		**0.08**	0.82	**0.26**	**0.24**	0.71
A-Other	**0.18**	**0.15**		0.29	**0.10**	**0.12**	0.26
A-Total	0.70	0.78	0.16		**0.36**	**0.26**	0.88
Derivatives	**0.50**	**0.46**	**0.15**	**0.53**		**0.13**	0.66
Off BS	**0.44**	**0.37**	**0.16**	**0.41**	**0.41**		0.54
Total	0.57	0.63	0.13	0.81	0.61	0.48	

Table 2: *Jaccard* (lower triangle) and *Cosine* (upper triangle) Similarity Indices, by instrument type. Non-overlapping networks are highlighted in bold fonts. A=Assets, CC=Credit Claims, DS=Debt Securities, OffBS=Off Balance Sheet. Higher index values indicate greater point-wise similarity.

A relative lack of complementarity between different maturities can also be appreciated from the high values reported in Table 3: the long and short term networks share 62% of connections. This number drops to 43% when evaluating Cosine similarity (upper triangle of Table 3, still a high number for a comparison of weighted matrices).[28] Table B.4 in Appendix B delves more deeply into the combination of instrument and maturity and reinforces the message: higher values of similarity are typically between different maturities for the same type of instrument.

	Long	**Short**	**Total**	**Unclassified**
Long		**0.43**	0.75	**0.03**
Short	**0.62**		0.81	**0.23**
Total	0.69	0.73		0.50
Unclassified	**0.04**	**0.03**	0.16	

Table 3: *Jaccard* (lower triangle) and *Cosine* (upper triangle) Similarity Indices, by maturity type. Higher index values indicate greater point-wise similarity.

informative, it should be read in a different light as the index comparing, say, *credit claims* and *derivatives*. We highlight with bold fonts the indices corresponding to "non-overlapping" networks in Tables 2 and 3.

[28] Some matrix comparisons between overlapping networks present a Cosine above the Jaccard similarity: this indicates that when taking monetary values into account the proximity between the matrices is higher than when only considering the existence/absence of relationships.

Layers, therefore, appear *closer* to the extent they represent the same type of business, and to some extent maturity, underlying the relation. The differences, however, can be non-negligible across layers of activity types. This is a first indication of the importance of explicitly addressing the multilayer structure of financial networks.

4.2. Systemic importance across layers of the banking network

In this subsection, we evaluate the robustness of institutions' relative systemic importance across layers, as well as gauging the appropriateness of using of an aggregate network representation for supporting policy. We then contrast the core-periphery structure of the different layers.

Banks that are well connected or important in a network might also be well connected in other networks. When such importance in connectedness is persistent across layers, the network is said to feature positively *correlated multiplexity*.[29] Such feature has been suggested to be central for the unfolding of failure cascades through the system (see Buldyrev et al. (2010)).

In order to evaluate the extent of correlated multiplexity, we first look at a number of measures of centrality not allowing for an analytic decomposition between layers, which capture alternative notions of systemic importance, and test whether there is evidence of correlated multiplexity for any or all of these measures. When the centrality rankings of banks across layers is *resilient*, then one can speak of the presence of correlated multiplexity. In this case, independently of the specific structure of the transmission involved, the *central* institutions are likely to be the same. When this is not the case, then multiplexity is not correlated and this centrality is layer-specific.

Correlation of systemic importance across layers. We consider degree and strength measures of bank centrality, which for a directed network measure the number of relationships and the weight of such links, respectively. These represent local measures of centrality, as the focus is on the immediate neighborhood of banks. A directed version of *PageRank*, the algorithm used by Google to rank webpages within their search engine, is used for connectivity at the global level.[30]

The correlation of the measures across layers (see tables B.5, B.6 and B.7) overwhelmingly points to the existence of positively correlated multiplexity. Banks that are well connected or important in a network tend also to be well connected in other networks. This is particularly strong when looking at degree centrality for both its incoming and outgoing versions (though in general out-degree centrality tends to present slightly lower correlations than in-degree centrality). When looking at strength centrality, the message remains unaltered, though the magnitudes are reduced, in particular for out-strength in the short term derivatives network and the two off-balance-sheet networks. Focusing on node-specific global measures of importance does not alter

[29]The correlation in nodes' degrees across layers has been termed *correlated multiplexity* by Lee et al. (2014). For instance, it can be expected that a person with many links in the "friendship" layer of a social multiplex network also has a relatively high number of links in other networks by virtue of her being a friendly person. Parshani et al. (2010) show how well connected ports tend to couple with well connected airports. Barigozzi et al. (2010) present similar evidence for the international trade network by comparing commodity-specific subnetworks and the aggregate.

[30]*PageRank* builds on the simplest global measure of centrality, namely eigenvector centrality, but improves upon some of its shortcomings. Eigenvector centrality is a global measure of centrality, as the score of a given node depends upon the score of nodes to which it is connected, in a recursive manner. The most notable shortcoming of classic eigenvector centrality arises in the case in which a node presents a zero score for incoming or outgoing links, which can translate into other nodes which are not part of a strongly-connected component presenting also a score of zero (see Newman (2010)).

the takeaway from the analysis: the correlation of PageRank centrality across networks delivers similar results to strength centrality, though slightly stronger than the latter (Table B.7).[31]

Taken together, these results suggest that, for the network considered, centrality is strongly permanent across layers, underlining the usefulness of ranking under limited information in identifying critical institutions. In contrast, using data on the Mexican interbank system Molina-Borboa et al. (2015) find that important actors in the different layers differ and take this as a significant reason to motivate the use of multiplex networks instead of just a single aggregated network. We agree, but argue further that even in the case in which the importance of actors is strongly persistent across layers (as it is the case for the network considered here), there is still significant value in considering all possible layers (i.e. the multiplex network), especially if one is able to decompose systemic importance into layer-specific contributions, as our method illustrates. Before substantiating this point in Section 5, we look into differences in the core-periphery structure across layers to illustrate further differences.

Core-periphery structure. An important avenue for characterising structural features of networks is the core-periphery paradigm, which describes a structure that is ubiquitous in several types of networks.[32] To identify the core-periphery structure, we focus here on the *block-modeling*-based approach of Craig and von Peter (2014).[33]

The results are shown in Figure C.12, which suggests that core size is large, with "marginal networks" being an exception (i.e. *other assets* and *unspecified maturity*).[34] Important in reading these large figures is noting that the sample of banks in the study is already the core of the European financial system, and thus we are looking at the "core of the core". In particular, banks in the sample are already "national champions", and tend to be well connected and thereby display strong interconnectivity density. Each of the different layers feature more than 20 banks in the core, whereas the overall network presents 31 banks in the core. Within the instrument type division, the off-balance sheet network features the best fit to the core-periphery model (lowest error score), whereas short term exposures fit the core-periphery model slightly better than long term exposures. In terms of core composition, the instrument dimension shows a bit more heterogeneity: there are 11 banks which are part of the core in all subnetworks (excluding *other assets*). For the maturity dimension, this is not the case: out of the 23 banks present in the core of the long term network, 20 are also part of the short term core. Results from a continuous measure of coreness that allows to establish a ranking of banks *within* the core (see Appendix C) show that we never find the same bank at the top of the coreness ranking, regardless of whether the focus is on instrument or maturity type.

Even though the stability of these results through time cannot be asserted with the information available, they already indicate the importance of some banks across and within layers may not be as universal as suggested by the monoplex approach. In order to better understand what differences may be evident in the data that escape the broader correlation analysis and which may motivate differences in the core-periphery boundary across measures, we now look

[31] Two additional centrality measures, *closeness* (in its *in* and *out* versions) and *betweenness* centrality, show similar results. The results on correlated multiplexity are even stronger when considering Spearman rank correlation instead of the simple Pearson correlation. Results are available upon request from the authors.

[32] For applications to interbank networks see: Craig and von Peter (2014) who propose an algorithm for the identification of the core set of banks based on the idea of intermediation and use it with German data, Fricke and Lux (2012) for the case of Italy, van Lelyveld and In't Veld (2012) for the Netherlands, Soramäki et al. (2007) for the FedWire payment system in the U.S. and Langfield et al. (2014) for the U.K., among others. For applications in other sciences see references in Borgatti and Everett (1999).

[33] Due to space considerations we relegate the details to Appendix C, where we also also present results for the core-periphery *profile* approach of Della Rossa et al. (2013).

[34] We thank Ben Craig for making available the code needed for the computation.

in more detail at the alternative and more detailed specification of decomposition of the layer structures.

5. Identifying systemic importance across layers numerically

The decomposition of global (monoplex) systemic importance into layer-specific contributions developed in section 3.2 provides an estimate of the contribution of a bank's activity within each layer to the overall importance of that bank. The decomposition of backward and forward indices for the top 10 banks according to instrument and maturity type is illustrated in figures 2 and 3. There is no reason to expect that the ranking of top 10 banks for each indicator would coincide, and indeed this is not the case. That said, 5 out of the top 10 banks are shared by the two indicators.

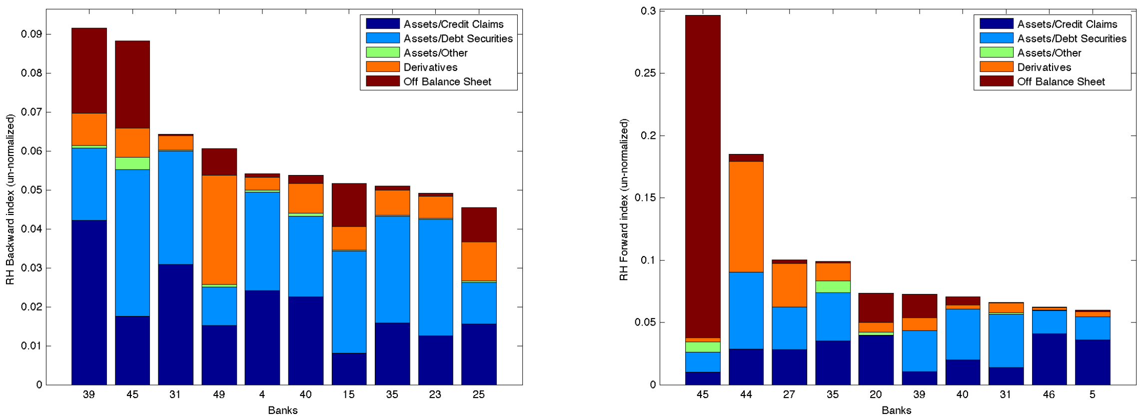

Figure 2: RH backward (left) and forward (right) indices, for top 10 banks by instrument type.

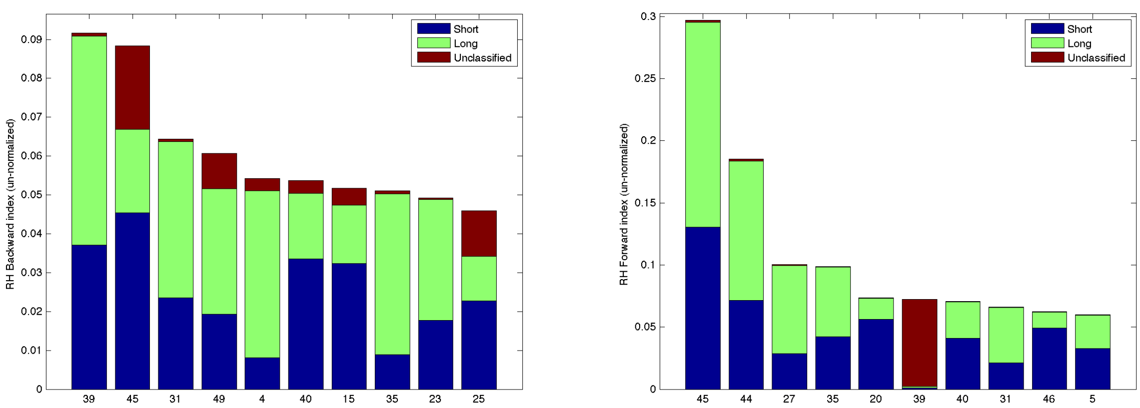

Figure 3: RH backward (left) and forward (right) indices, for top 10 banks by maturity type.

For the analysis of the decomposition of systemic importance indicators into the contributions by each layer, we make use of the classification outlined in Table 1 and focus on the set of systemic banks, i.e. those that present both normalised backward and forward indices above one. Figure 4 presents the normalised indices for all banks for the two indicators considered; systemic banks are those in the upper right quadrant of the figure (i.e. those that show a score above the mean of the system in both indicators). Some banks highly ranked on one of the indicators may not be considered systemically important because they rank poorly on the other indicator: this is the case most notably for the third ranking bank according to the forward index (bank 27). Additionally, there is one bank that is not in the top 10 banks in either indicator, but still is systemically important. Not surprisingly, five banks feature in both lists of top ten banks.

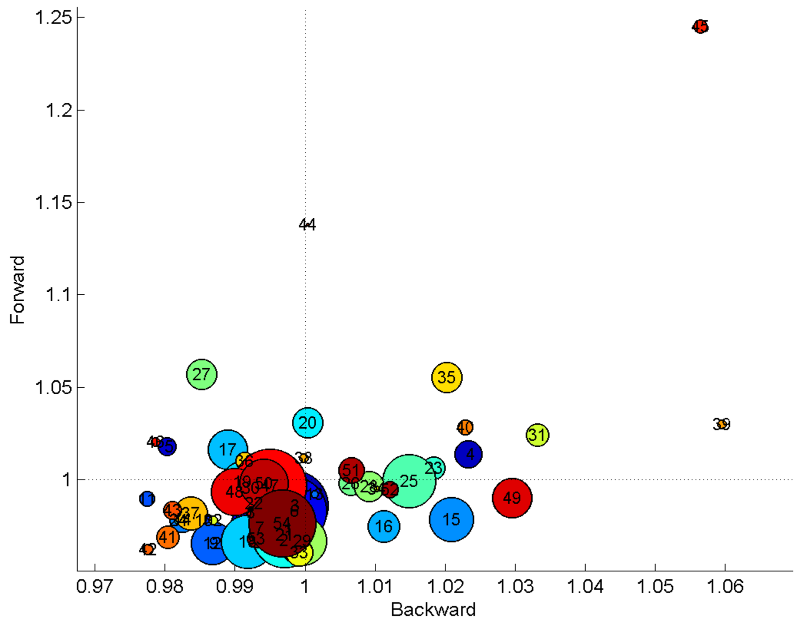

Figure 4: Normalized backward and forward indices. Bubble size indicates (scaled) total assets.

Figure 5 presents the backward and forward indices for systemically important banks by instrument type. For the backward index, a significant portion of the contribution to the overall systemic importance of the most important banks is given by the two main *asset* exposure subcategories, namely *credit claims* and *debt securities*. This is broadly in line with the composition of exposures by instrument (see Figure A.7), in particular for the two top banks for which also *off balance sheet* exposures account for a non-negligible share of their critical transmission role.

Despite accounting for slightly more than 25% of exposures, the *derivatives* network does not contribute much to the ranking of systemically important banks, pointing to one important insight of our analysis: importance in terms of interconnectivity is driven by more than size.

The backward index focuses on the importance of the transmission of shocks to the borrowing side of the balance sheet, by summing along the column dimension of the transformed interbank matrix. Conversely, by summing along the row dimension, the forward index illustrates the transmission of shocks on the lending side of the balance sheet. In the case of such asset shocks, the outline of importance is quite different: the top-ranked bank (bank 45) stands out notably against the rest, and an overwhelming majority of this difference is attributable to the *off-balance sheet* network. Naturally, bank 45 is an important player in terms of interconnectedness in this particular network,[35] but still it only accounts for roughly 1/5 of all off balance sheet exposures

[35] In fact, this bank belongs to the core of only two networks: the *other assets* (a rather "marginal" network) and the *off-balance sheet* networks.

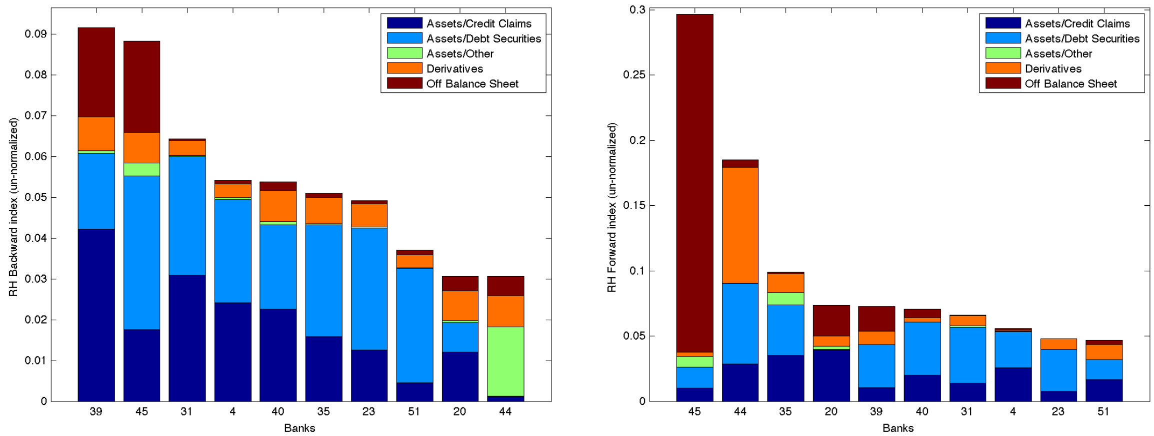

Figure 5: RH backward (left) and forward (right) indices, for systemic banks by instrument type.

and it is not in fact the top-ranked bank in this regard. A network that has a rather minor share of overall exposures (the *off-balance sheet* network accounts for 1/7 of exposures, see Appendix A) can nonetheless be a major driver of the systemic importance score of specific banks. While such a result might seem obvious at the intuitive level, our measures provide a clear-cut way of quantifying this. Note that such insights would go unnoticed in a layer-specific analysis of centrality.

The second-ranked bank (bank 44), in contrast, has on the other hand a significant share of its score accounted for by the derivatives network. It is worth noting that this bank is not the largest derivatives holder, i.e. it does not account for a large share of exposures in this network.[36] This critically emphasizes the importance of multi-layered network analysis. Interestingly, four out of the ten systemically important banks do not form part of the core of any network (according to Craig and von Peter (2014)'s algorithm). That is, systemic importance measured in terms of a bank's ability to transmit shocks also needs to be qualified by the nature of the shock affecting the system, in addition to the market through which the transmission takes place.

The comparison of a bank's forward and backward transmission role indicates that the drivers of systemic importance can vary substantially depending on the criterion of emphasis, and there is value in the consideration of both dimensions simultaneously.[37] Figure D.15 presents a comparison of the contribution of the different layers to the systemic importance score of all banks, by instrument type. It can be seen that while the contribution of the *off balance sheet* network to the forward score of bank 45 is substantial, this is not so for the backward index (nor for other layers like derivatives for the same bank). In these charts, a layer contributes more to systemic importance as captured by one type of indicator vis-à-vis the other to the extent that more bubbles lie on one side of the 45 degree line.

Figure 6 displays the partition of systemic importance indices by maturity type for the 10 more systemic banks. *Long* and *short term* exposures account for roughly 4/5 of the total

[36] It in fact ranks in position 35 in terms of share of exposures in this network, and even lower in terms of borrowing.

[37] Furthermore, the methodology presented in Section 3.2 allows for further decompositions, though for ease of exposition we do not pursue this here. For instance, we could partition the indices according to *both* instrument and maturity type, allowing for a finer distinction of the sources of systemic importance. For illustrative purposes, we focus on a specific partition by instrument and maturity type, and a specific subset of banks.

(see Figure A.7), in almost equal proportions. For the backward index, we see that, with a couple of exceptions, *long term* exposures account for the largest share. Most notable within the exceptions is the second highest ranking bank, for which half of the index is explained by *short* term exposures with the other half shared equally between *long term* exposures and those of *unspecified* maturity.

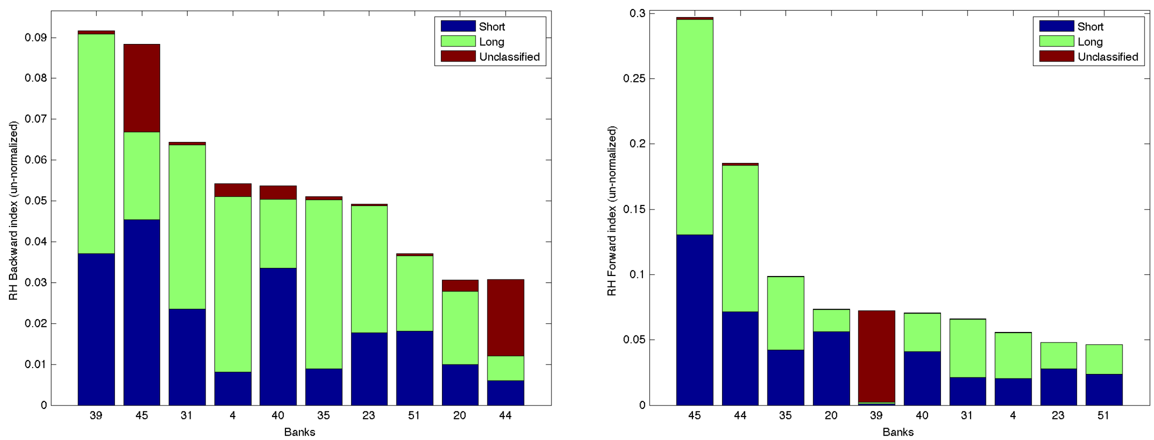

Figure 6: RH backward (left) and forward (right) indices, for systemic banks by maturity type.

When it comes to the forward index, we see that for the three top systemically important banks at least half of the score is accounted for by the *long term* network. With the exception of one bank, the network of *unspecified* maturity does not play a role (the entire forward transmission score of the exceptional bank is attributable to this network). It could in principle be the case that a bank has a great deal of its systemic importance score explained by *off-balance sheet* instruments of *unspecified* maturity. The ability to detect such opacity as the driver of the systemic importance ranking of a bank is another potentially interesting insight from the analysis we present. Similar to Figure D.15, Figure D.16 presents a comparison of contributions by the different layers to backward and forward indices by maturity type.

A key finding is that identification of systemic importance is far from being a synonym with being in the core, neither for the aggregate network nor for its different layers, and the choice among the different implementations matters for identification. Based on the algorithm by Craig and von Peter (2014), five out of the ten systemically important banks belong to the core of the *short term* network, whereas three out of this five also belong to the core of the *long term* network. Based on the alternative algorithm for identification of a core-periphery profile by Della Rossa et al. (2013) , only one systemic bank belongs to the core of the *long term* network (bank 35) and none to the *short term*. There are no banks that are present in the core of most networks regardless of the method used to identify the core. When the method used is that of Craig and von Peter (2014), bank 35 is present in all cores except that of the *off-balance sheet* network.

Overall, as apparent from the figures presented above, the method proposed here allows for a decomposition and description of banks' systemic importance from a holistic perspective, starting from a matrix representation of the balance sheet of the entire banking system. Even though the derivation of the measures is grounded in theory and involves some matrix algebra, the ultimate motivation of our endeavour is practical in nature. We posit that such measures can be of potential use for bank regulators and supervisors, provided that sufficiently granular data are available.

6. Concluding remarks

The recent financial crisis brought to the fore the relevance of interconnectedness in general, and in particular in interbank markets. Of critical importance in this context is the identification of the key players in the financial network. While early contributions on the topic have focused on aggregated exposures, it is now increasingly recognised that the web of reciprocal exposures linking bank balance sheets is more intricate and complex. Interbank networks are better characterised as multiplex networks, featuring connections at multiple levels.

In the present paper, we analyse the multiplex structure of the network of large European banks, making use of a detailed dataset presenting exposures partitioned according to maturity and instrument type. We find a high level of similarity between the different layers (both by instrument and maturity), a core periphery structure which comprises a large core (relative to studies using country-specific datasets), and positively correlated multiplexity. The results suggest that an institution's role in the channel of transmission is critical in determining the global importance of such institution, and that the notion of importance may not be related to its traditionally studied core-periphery role. Nevertheless, key forms of non-decomposable (across layers) centrality measures are resilient across layers, indicating that centrality at the layer-level can be robust to the absence of granular data availability.

We develop two measures of systemic importance suited to the case in which banks are connected through an arbitrary number of different layers. This allows us to compute systemic importance indicators and decompose them into the contributions of the different layers, providing a holistic analysis that truly incorporates the multiplex structure of the network (instead of doing separate analyses for the different layers and the aggregate network) and is built directly from a consistent accounting representation of the system. Previous literature has justified the need to use granular data by noting that banks can rank differently in different layers in terms of systemic importance and thereby the focus on a single layer can be misleading. We confirm that, even when centrality is persistent across layers, there is still important information to be obtained from granular data, in particular if one is able to decompose global systemic importance into layer-specific contributions. Nevertheless, we also show that when such granular data is not available, simple measures can be a good second best. We illustrate our measures with the dataset on exposures between large European banks, delving deeply into issues of interconnectedness across the various layers of an integrated accounting framework. The results suggest that our proposed measures can be useful tools for practical policy analysis.

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Appendix A. Overview of the data

We make use of a unique dataset of anonymised interbank exposures between large European banks, originally presented in Alves et al. (2013). The preparation of the dataset was part of a collective effort undertaken by the *European Systemic Risk Board*, the *European Banking Authority* and national supervisory authorities. In particular, the dataset presents bilateral exposures between 53 large European banks as of end 2011, with breakdown according to both maturity and instrument type.[38] Banks report their exposures at the level of the banking group, the implication being that non-financial subsidiaries and insurance are not included in the reporting exercise, and exposures to counterparties were aggregated using an accounting scope of consolidation (hence including all subsidiaries of that counterparty). The exposure data represents a directed and weighted network, in which each node is represented by a banking group and each link going from node A to node B accounts for an exposure of the former to the latter. The multiplex structure of the dataset stems from the level of disaggregation that the dataset contains, namely along the instrument and maturity dimensions.

The partition of exposures according to instrument type is as follows: *(i)* assets, further subdivided into credit claims, debt securities and other assets, *(ii)* derivatives, and *(iii)* off-balance-sheet exposures. On the other hand, exposures according to maturity type are structured according to: *(i)* less than one year including on sight ("short term"), *(ii)* more than one year ("long term"), and *(iii)* a residual of unspecified maturity.[39] Coming back to the terminology introduced in section 3, the interbank exposure dataset used presents two *aspects*, namely *instrument* type and *maturity* type. The former has either 3 or 5 *elementary layers* (depending on whether we consider the subdivision of assets or not) whereas the latter presents 3 *elementary layers*. As a consequence we can have either 9 or 15 *layers*.[40]

The combination of the three types of asset exposures (namely credit claims, debt securities and other assets) accounts for almost 2/3 of overall exposures (see Figure A.7). Derivatives explain 27% of exposures, whereas off-balance-sheet items account for the smallest share at 17%. Furthermore, almost all banks report exposures on credit claims and debt securities and therefore the shares are more evenly distributed among banks: to reach 80% of total exposures for each of these categories one needs 18 banks. While also a significant amount of banks report derivatives exposures, these are more concentrated as it takes 12 banks to reach the 80% share of total exposures in this category. Off-balance-sheet exposures, on the other hand, are characterised by fewer banks reporting and much more concentration: the first 6 banks account for 80% of exposures in this category. Regarding maturity type, the distinction between exposures is not different as it is for instruments: short and long exposures account for roughly the same share of overall exposures and both present a similar number of reporting banks.[41]

[38] There are 53 reporting banks, while there is a bank which did not report exposures to other institutions. Hence, there are 53 banks reporting exposures to 54 banks, and the matrices we work with are of dimension 54-by-54.

[39] The original dataset presents further subdivisions of interbank credit claims and off-balance-sheet items. We keep the partition mentioned above as it is enough for our illustrative purposes. But it should be noted that the method presented here can be applied to any arbitrary number of sub-divisions.

[40] As noted earlier, one can focus attention on a given aspect of the network by aggregating across the other aspect , i.e. in order to focus on maturity type, aggregate across instrument type to have the "short term", "long term" and "unspecified maturity" networks.

[41] It should be noted that in the case of long term exposures there is one bank which stands far above the rest, accounting for almost 20% of exposures in this category.

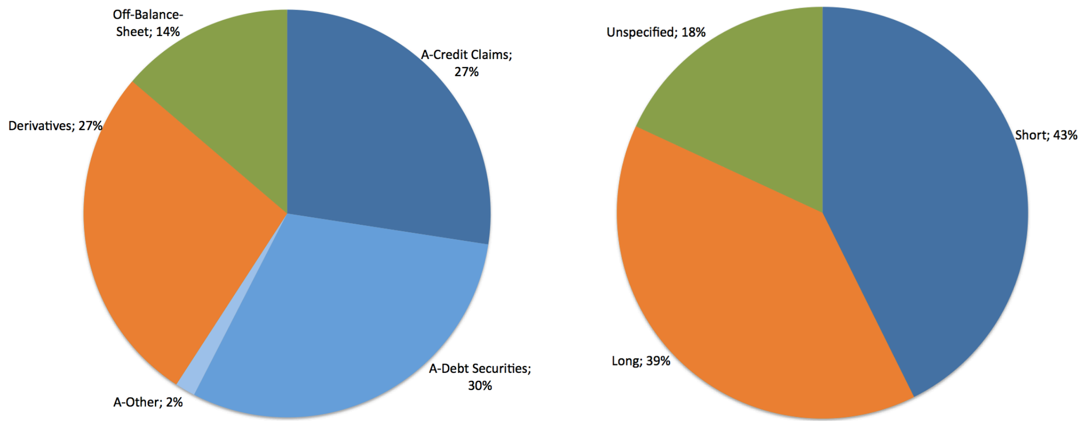

Figure A.7: Composition of exposures by instrument and maturity (left and right panel respectively).

Exposures of unspecified maturity behave quite differently, with just 7 banks reporting and with the first 3 accounting for 75% of exposures in this category. When combining the maturity and instrument type dimensions, one sees that each instrument presents its own nuances in terms of maturity breakdown (see Figure A.8). While asset exposures are marginally tilted toward shorter maturities, derivatives present more diversity and are slightly leaning towards longer term exposures. Off-balance-sheet exposures, on the other hand present a good deal of dispersion for short term but not so much for long term exposures, which present a very small median. Furthermore, for this instrument type there is a non-negligible share of exposures of unspecified maturity, which speaks to the well-known opacity of these types of instruments.

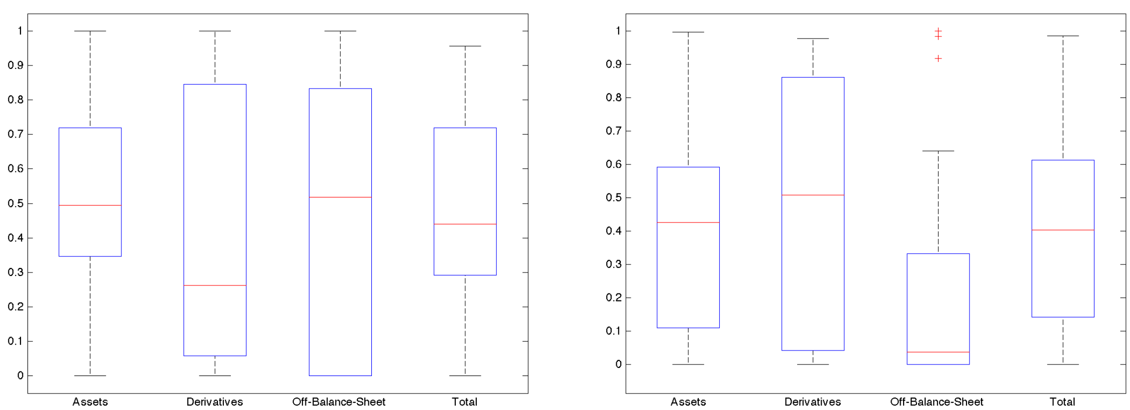

Figure A.8: Share of short and long term exposures (left and right panels respectively) in total exposure by instrument.

It should be noted that the network built with the data comprises the exposures *between* the group of banks which participated in the exercise, hence exposures of this group of banks to other EU banks are not considered in the analysis. That said, the bilateral exposures reported by banks represent slightly more than half of total exposures to EU banks, thereby capturing a good piece of the action, in particular between the larger banks.[42]

[42]The share of exposures to large EU banks relative to total exposure to EU banks varies depending on

The level of connectivity of the network of large European banks is large relative to other studies that focus on national banking systems, as reflected in the relatively high density of the network: 60% of all possible connections are actually present for total exposures. While the density is naturally lower in composing layers (48%, 36% and 29% for assets, derivatives and off-balance-sheet respectively), values still remain well above those encountered in previous studies focusing on national banking systems.[43] This can be appreciated visually on Figure A.9, which provides the real data counterpart to the stylized multilayer chart presented in the main body of the paper.

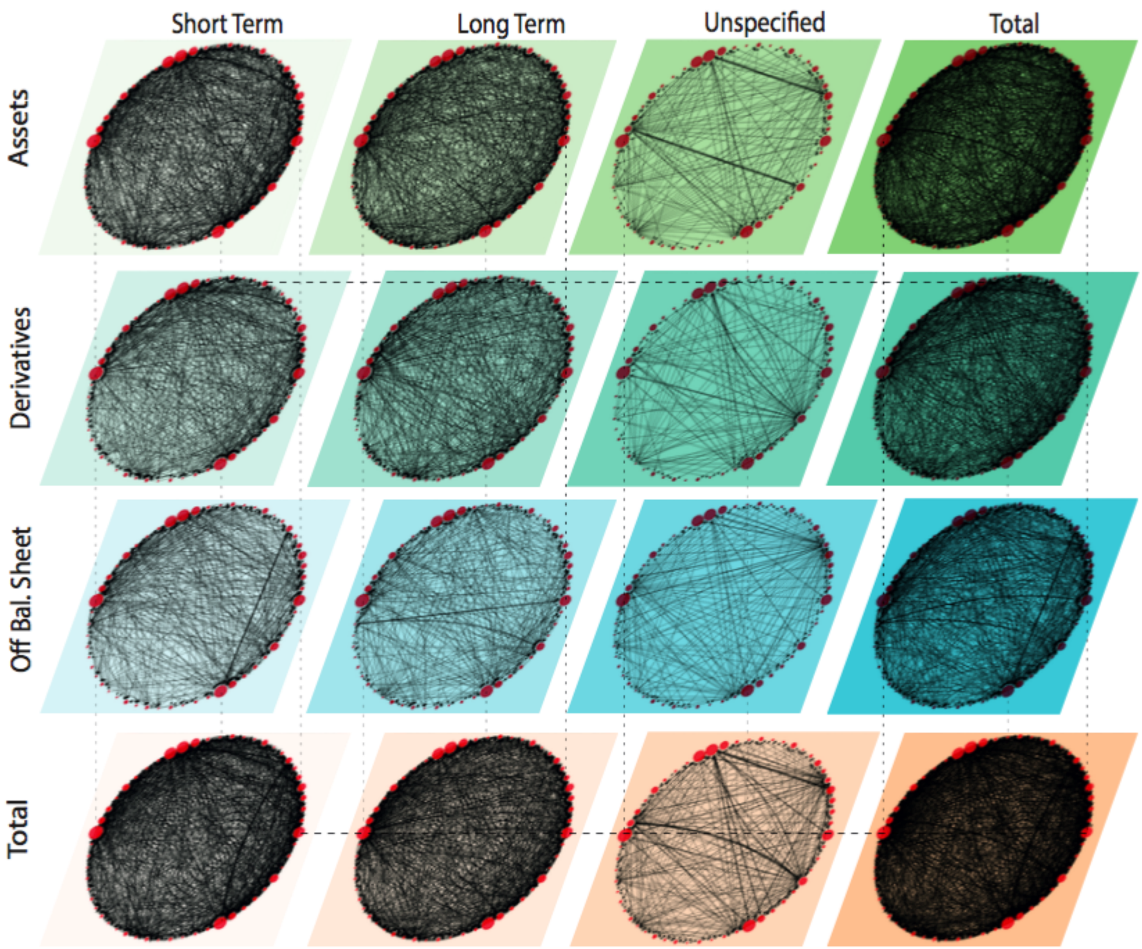

Figure A.9: The multilayer network of large European banks. Node size indicates total assets, links width indicate size of exposure, whereas link direction goes from lender to borrower bank.

Figure A.10 and Figure A.11 present the degree distributions according to instrument and

instrument type and bank, with the median for total exposures being 60%. For details see Alves et al. (2013), in particular *Chart 2*.

[43]See among others Craig and von Peter (2014), van Lelyveld and In't Veld (2012), Fricke and Lux (2012), Soramäki et al. (2007) for the cases of Germany, the Netherlands, Italy and the U.S. respectively.

maturity type respectively. While there is not much of a difference between the degree distributions of the long and short term networks, the layers corresponding to different instruments present more diversity. The layer corresponding to the assets network is closer to the overall network for both in- and out-degree distributions, in line with the share of these type of exposures in overall exposures (see Figure A.7). On the other hand, the off-balance-sheet layer is farthest from the total network, with the derivatives layer lying somewhere in between. The distinction between layers becomes less clear-cut for the out-degree distribution.

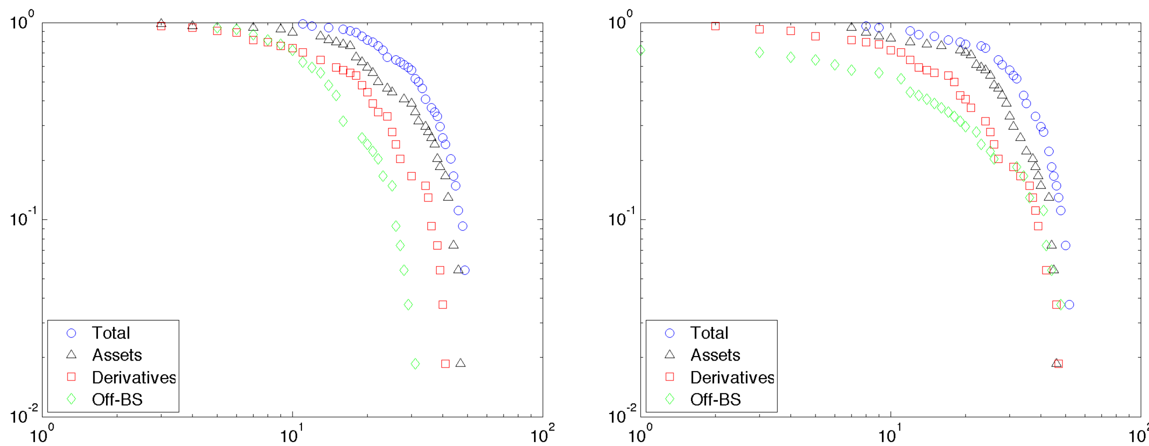

Figure A.10: In- and out-degree distributions - left and right panel respectively - by instrument type (in log-log scale).

Further properties of the network are its high level of reciprocity (especially for higher levels of aggregation of exposures) and its disassortative behaviour. Higher reciprocity implies that a high share of exposures are reciprocated by a corresponding counter-exposure, suggesting that systemic risk might be lower for high levels of aggregation since some exposures might be netted. A network is said to be assortative (with respect to its nodes' degrees) if high (low) degree nodes tend to be connected to other high (low) degree nodes. If instead high degree nodes tend to be connected to low degree nodes the network is said to be disassortative. Interbank networks are typically found to be disassortative, and the network of large European banks is no exception. This feature is closely associated to the core-periphery structure of interbank networks found in the extant literature, and it reflects efficient specialisation between the actors in the network. In the European interbank network this specialisation is more apparent in granular instrument and maturity types. For further details on the network and some of its topological properties we refer the interested reader to Section 3 in Alves et al. (2013).

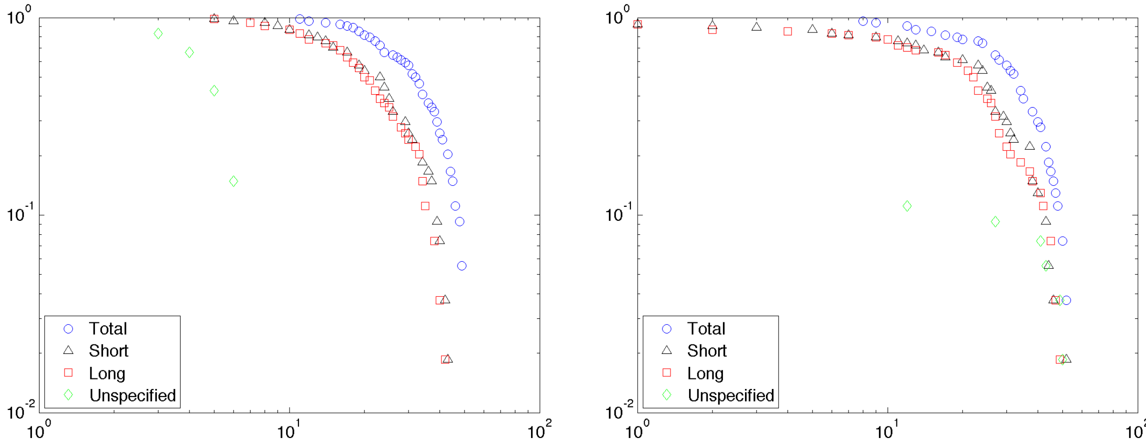

Figure A.11: In- and out-degree distributions - left and right panel respectively - by maturity type (in log-log scale).

Appendix B. Additional Tables

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*
***(1)* Assets-CC L**		0.23	0.33	0.14	0.30	0.14	0.35	0.06	0.08	0.11
***(2)* Assets-CC S**	0.33		0.22	0.13	0.08	0.15	0.24	0.15	0.04	0.12
***(3)* Assets-DS L**	0.31	0.43		0.31	0.07	0.07	0.24	0.09	0.04	0.23
***(4)* Assets-DS S**	0.25	0.37	0.42		0.01	0.05	0.11	0.04	0.05	0.28
***(5)* Assets-Other L**	0.16	0.09	0.09	0.08		0.09	0.13	0.02	0.01	0.01
***(6)* Assets-Other S**	0.13	0.11	0.10	0.11	0.18		0.07	0.03	0.15	0.12
***(7)* Derivatives L**	0.27	0.35	0.35	0.27	0.11	0.11		0.18	0.07	0.11
***(8)* Derivatives S**	0.27	0.40	0.33	0.24	0.09	0.09	0.45		0.02	0.06
***(9)* OffBS L**	0.35	0.25	0.26	0.23	0.11	0.09	0.23	0.23		0.36
***(10)* OffBS S**	0.32	0.38	0.27	0.26	0.11	0.10	0.29	0.34	0.31	

Table B.4: *Jaccard* (lower triangle) and *Cosine* (upper triangle) Similarity Indices, by instrument and maturity type. CC stands for Credit Claims, DS stands for Debt Securities, and L (S) stands for Long (Short) Term.

	Assets-L	**Assets-S**	**Deriv.-L**	**Deriv.-S**	**OffBS-L**	**OffBS-S**
Assets-L		0.77***	0.64***	0.52***	0.57***	0.44***
Assets-S	0.88***		0.71***	0.58***	0.39***	0.60***
Deriv.-L	0.69***	0.80***		0.72***	0.44***	0.53***
Deriv.-S	0.78***	0.89***	0.90***		0.40***	0.63***
OffBS-L	0.83***	0.87***	0.73***	0.79***		0.52***
OffBS-S	0.84***	0.91***	0.79***	0.86***	0.92***	

Table B.5: Correlation indices for *in-degree* (lower triangle) and *out-degree* (upper triangle) centrality. L (S) stands for Long (Short) Term. *** (**,*) denotes statistical significance at the 1% (5%, 10%) level.

	Assets-L	**Assets-S**	**Deriv.-L**	**Deriv.-S**	**OffBS-L**	**OffBS-S**
Assets-L		0.62***	0.83***	0.15	0.14	0.25*
Assets-S	0.50***		0.49***	0.18	0.08	0.26*
Deriv.-L	0.46***	0.64***		0.11	0.17	0.17
Deriv.-S	0.37***	0.70***	0.85***		-0.00	0.09
OffBS-L	0.26*	0.67***	0.48***	0.55***		0.69***
OffBS-S	0.40***	0.79***	0.55***	0.55***	0.52***	

Table B.6: Correlation indices for *in-strength* (lower triangle) and *out-strength* (upper triangle) centrality. L (S) stands for Long (Short) Term. *** (**,*) denotes statistical significance at the 1% (5%, 10%) level.

	Assets-L	**Assets-S**	**Deriv.-L**	**Deriv.-S**	**OffBS-L**	**OffBS-S**
Assets-L		0.62***	0.78***	0.14	0.18	0.39***
Assets-S	0.60***		0.45***	0.18	0.07	0.22
Deriv.-L	0.52***	0.71***		0.15	0.16	0.19
Deriv.-S	0.46***	0.75***	0.87***		-0.00	0.16
OffBS-L	0.33**	0.65***	0.52***	0.53***		0.73***
OffBS-S	0.50***	0.80***	0.61***	0.61***	0.56***	

Table B.7: Correlation indices for *PageRank in* (lower triangle) and *out* (upper triangle) centrality. L (S) stands for Long (Short) Term. *** (**,*) denotes statistical significance at the 1% (5%, 10%) level.

Appendix C. Core-periphery Analysis and Figures

In the wake of the seminal contribution by Borgatti and Everett (1999), different methods to assess the core-periphery structure of networks have been proposed. Borgatti and Everett (1999) propose two types of core-periphery analyses, namely a discrete and a continuous version. The former builds on block-modelling and proposes a two-class partition of nodes, in which some nodes form part of the densely connected core (a *1-block* in block-modelling terminology) while the remaining nodes make up the periphery, loosely connected to the core and not connected within itself (a *0-block*). A stylised representation in matrix notation would be as follows:

$$\mathbf{M} = \begin{bmatrix} \text{Core-Core} & \text{Core-Periphery} \\ \text{Periphery-Core} & \text{Periphery-Periphery} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & ? \\ ? & \mathbf{0} \end{bmatrix} \tag{C.1}$$

where the off-diagonal blocks are left to be specified by the researcher depending on the application at hand. The idea is then to a find a fit (optimal in some sense) between a given empirical network and this idealised structure, such that the optimal core size is found in the process.

The continuous model of core-periphery analysis assigns a level of “coreness” to each node, without the need to partition the network into two (or more) classes of nodes ex ante. The notion of *closeness* plays an important role in this method, as the strength of the connection between a pair of nodes will be a function of their closeness to the centre.

For the case of interbank networks, the seminal contribution by Craig and von Peter (2014) builds on the block-modelling approach and motivates the choice of a specific structure for the off-diagonal elements of the block matrix. In particular, building on the concept of intermediation, they suggest that core banks should lend to and borrow from at least one bank in the periphery. This translates into two restrictions: the *Core-Periphery* block should be row regular (i.e. each row has at least one 1) and the *Periphery-Core* block should be column regular (i.e. each column has at least one 1). According to this approach, core banks are a strict subset of intermediaries.

A more flexible definition of the core used by Della Rossa et al. (2013) avoids the explicit (and often artificial) partition in subnetworks of block-modelling and is more natural for weighted networks, in contrast to block-modelling which is better suited for binary networks.

Della Rossa et al. (2013) associate to each network a *core-periphery profile*, which is a discrete and non-decreasing function $\alpha_1, \alpha_2, \ldots, \alpha_n$ that: (i) assigns a *coreness* value to each node, (ii) provides a graphical representation of the network structure, (iii) and allows for the computation of a centralisation indicator which measures the distance to an idealised core-periphery structure (the "star" network). Furthermore, the method allows for the definition of the α-periphery which collects all nodes with a coreness value below a given threshold α,[44] and identifies the set of p-nodes, which are those that constitute the periphery in a strict sense, i.e. those nodes that together form a *0-block*.[45]

Below we present figures summarizing the results from the two approaches as applied to our data. Since we comment on the Craig and von Peter (2014) approach in the main text, we comment here only on the second approach.

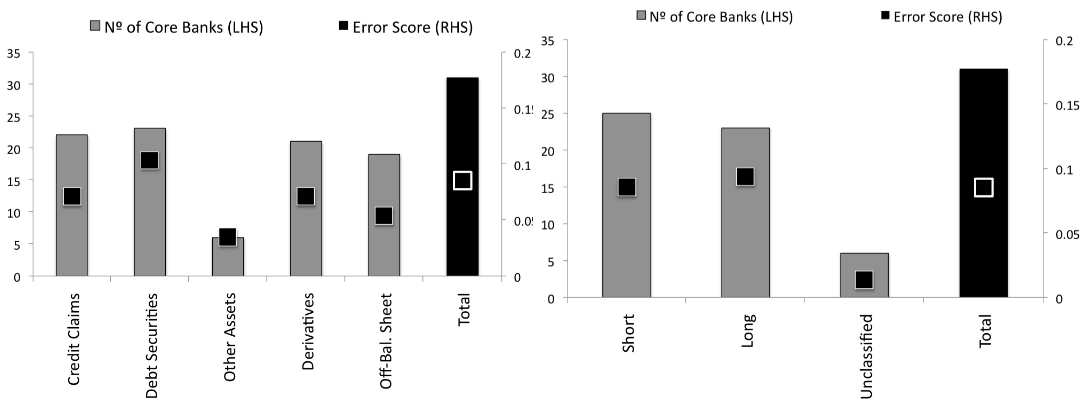

Figure C.12: Core banks and error score based on Craig and von Peter (2014) algorithm, by instrument and maturity (left and right panel respectively). The error score is expressed as a share of all possible connections.

Figure C.13 presents the core-periphery profile by instrument and maturity type against two extreme benchmarks, namely the complete network and the star network. There are some similarities with respect to the insights gained from the block-modelling approach, but also some differences. In particular, the partition according to the instruments' maturity shows more homogeneous results than that of the instruments' type, where differences are more clear.

In addition, the short term network is closer to an idealised core-periphery structure (in the block-modelling analysis this manifested itself in a better fit, see Figure C.12), whereas in the second analysis the core-periphery profile of the long term network is very similar to that of the total network (same number of core banks[46] and very similar centralisation index, see also Figure C.14). There are three banks which are simultaneously present in the core of the short term, long term and total networks.

Regarding results of the type partition, *off-balance sheet* and *derivatives* exposures show more proximity to the star-network ideal (Figure C.13) and a higher centralisation index (just as before they showed a lower error score, see Figure C.12). Different from the the block-modelling

[44] Alternatively, one can think of the α-core as the complement of the α-periphery.

[45] The code for the computation of the *core-periphery profile* is available from the authors' website.

[46] We take as core banks those that have an α-coreness above 0.5. See Appendix C for details.

approach, the *debt securities* network shows the most proximity to the total aggregated network in terms of its core-periphery profile. There is only one bank that belongs to the core in all networks according to instrument type (excluding *other assets*). At the same time, all of the banks belonging to the core in the *derivatives* and *off-balance sheet* networks are part of the core in the aggregated network, while for the *debt securities* network this number is only 4 out of 8. Similar core-periphery profiles are therefore no guarantee of similar composition of the core.

Finally, we note that at the very top of the coreness ranking we never find the same bank for any pair of networks when the focus is on either instrument or maturity type. That said, the bank with the highest coreness in the *short term* network is also the same bank sitting at the top of the *debt securities* coreness ranking (also for the aggregated network), whereas the top bank in the *long term* network coincides with that of the *derivatives* network.

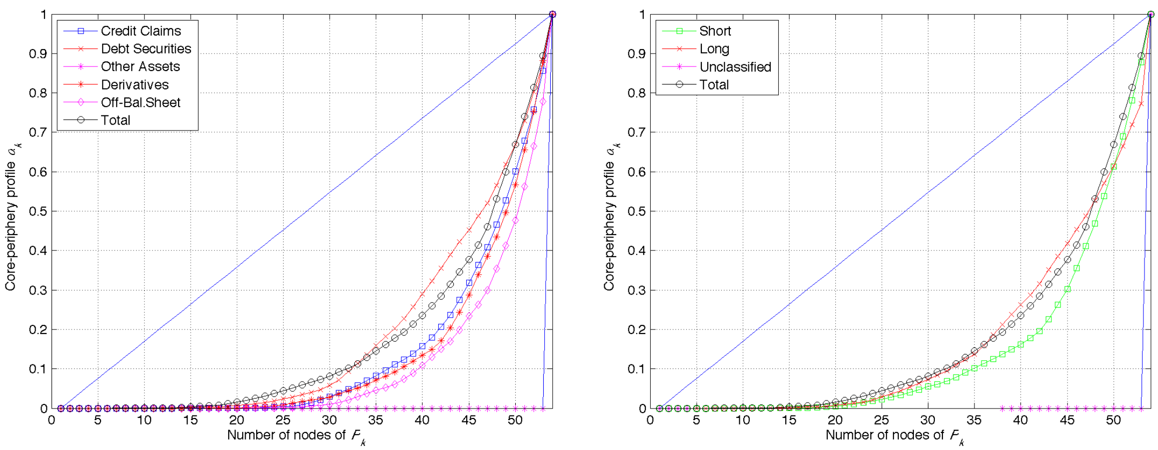

Figure C.13: Core-periphery profile by instrument and maturity (left and right panel respectively), based on the method by Della Rossa et al. (2013). Blue straight lines indicate the complete (diagonal) and star ("inverted L") networks as benchmarks.

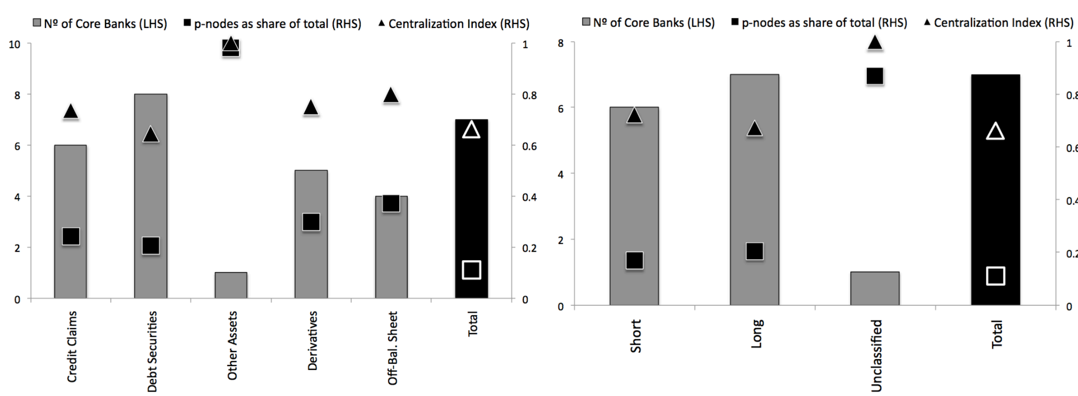

Figure C.14: Core banks, p-nodes and centralisation by instrument and maturity (left and right panel respectively), based on the method by Della Rossa et al. (2013). Core banks are those with $\alpha_k > 0.5$; p-nodes are periphery nodes in the strict sense ($\alpha_k = 0$).

Appendix D. Additional Figures

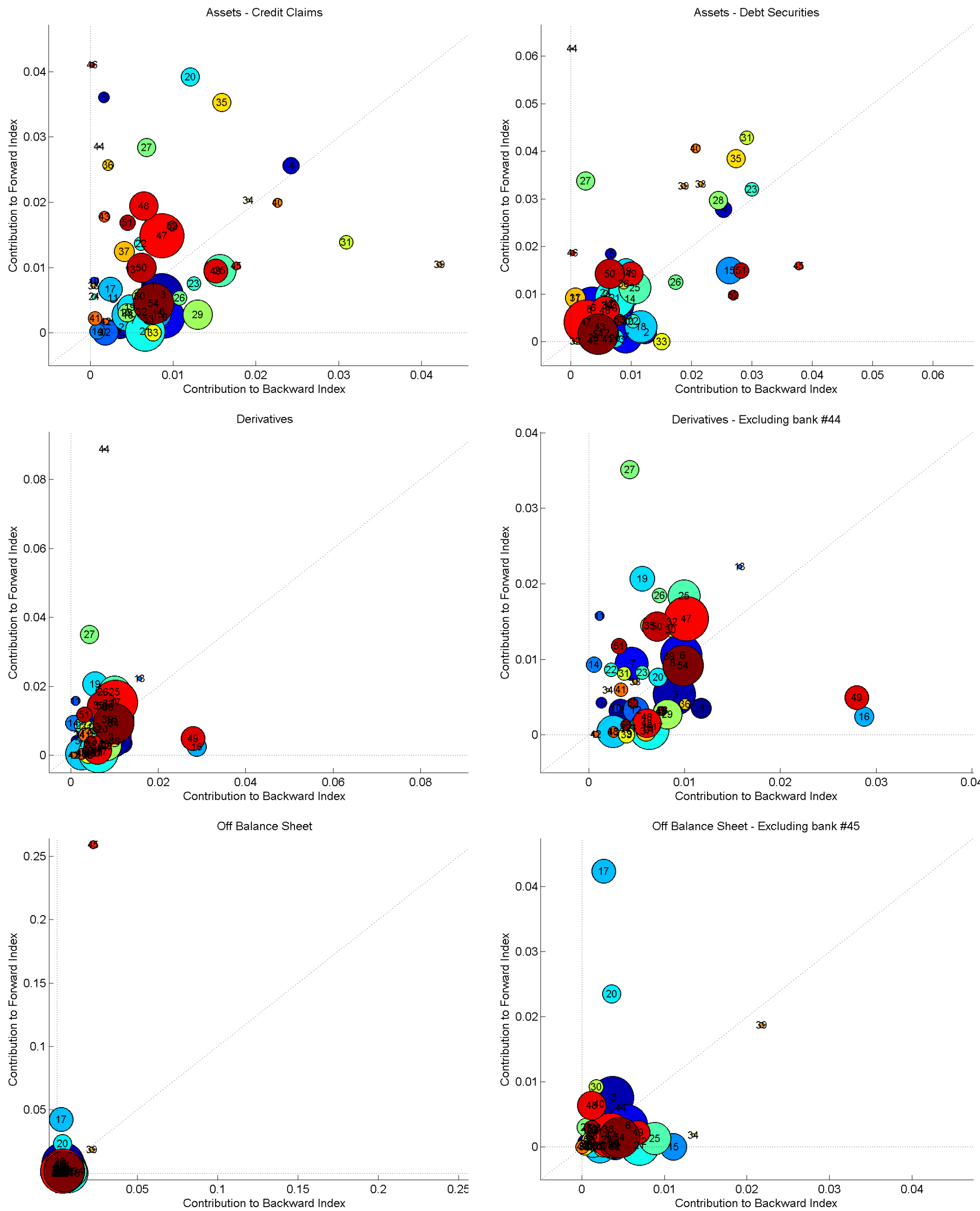

Figure D.15: Contribution to backward and forward indices by layer of instrument type. Bubble size indicates (scaled) total assets.

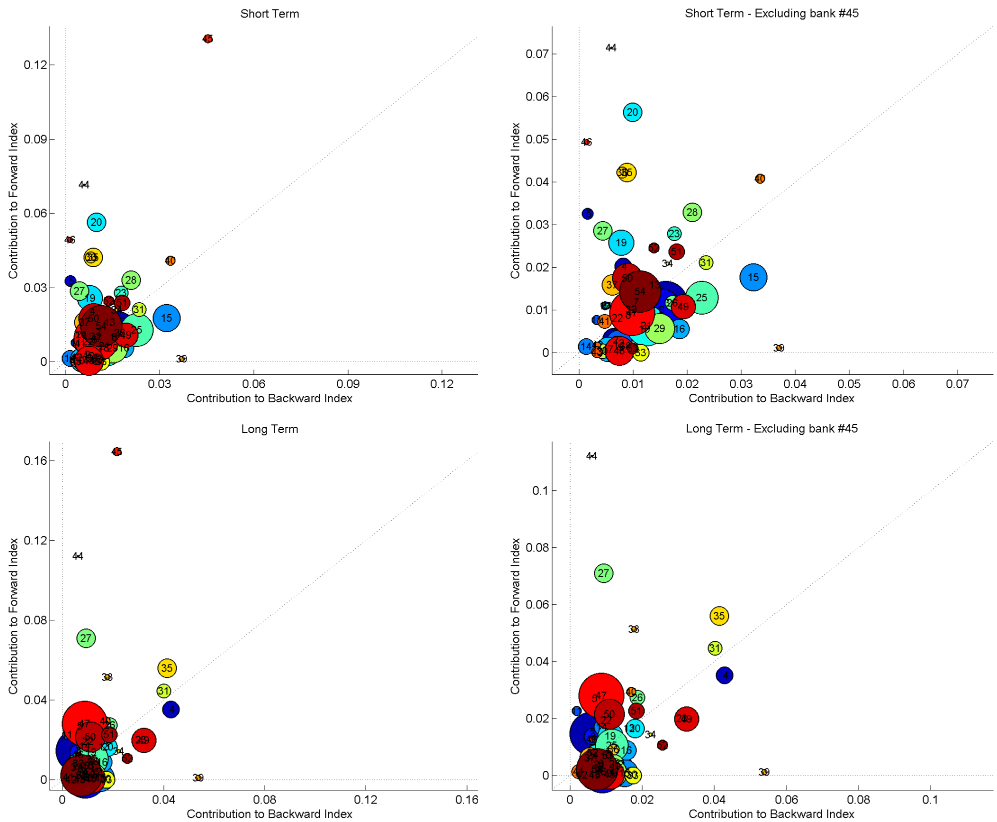

Figure D.16: Contribution to backward and forward indices by layer of maturity type. Bubble size indicates (scaled) total assets.

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