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Bank Networks: Contagion, Systemic Risk and Prudential Policy[☆]

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Abstract

We present a network model of the interbank market in which optimizing risk averse banks lend to each other and invest in non-liquid assets. Market clearing takes place through a tâtonnement process which yields the equilibrium price, while traded quantities are determined by means of an assortative matching process. Contagion occurs through liquidity hoarding, interbank interlinkages and fire sale externalities. The resulting network configuration exhibits a core-periphery structure, dis-assortative behavior and low density. Within this framework we analyze the effects of a stylized set of prudential policies on the stability/efficiency trade-off. Liquidity requirements unequivocally decrease systemic risk, but at the cost of lower efficiency (measured by aggregate investment in non-liquid assets). Equity requirements also tend to reduce risk (hence increase stability), though without reducing significantly overall investment. On this basis, our results provide general support for the Basel III approach based on complementary regulatory metrics.

Keywords: banking networks, systemic risk, contagion, fire sales, prudential regulation
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1. Introduction

The propagation of bank losses which turned a shock to a small segment of the US financial system (the sub-prime mortgage market) into a large global banking crisis in 2007-2008 was due to multiple channels of contagion: liquidity hoarding due to banks' precautionary behavior, direct cross-exposures in interbank markets and fire sale externalities. In the face of shocks to one segment of the financial markets and increasing uncertainty, banks start to hoard liquidity. As a result of the market freeze,[4] many banks found themselves unable to honor their debt obligations in interbank markets. To cope with liquidity shocks and to fulfill equity requirements, most banks were forced to sell non-liquid assets: the ensuing fall in asset prices[5] produced, under mark-to-market accounting, indirect losses to the balance sheet of banks exposed to those assets. Liquidity spirals turned then into insolvency.

Several papers have shown that credit interlinkages and fire sale externalities are not able to produce large contagion effects if taken in isolation (see for instance Caccioli et al. (2014) or Glasserman & Young (2014)). Our model embeds both channels and envisages a third crucial channel, namely liquidity hoarding (see also Afonso & Shin (2011)). To the best of our knowledge, so far no theoretical model has jointly examined these channels of contagion to assess their impact on systemic risk. After dissecting the qualitative and quantitative aspects of risk transmission, we use the model to determine which prudential policy requirements can strike the best balance between reducing systemic risk and fostering investment in long term assets.

To examine the above channels of contagion and to assess the efficacy of various types of prudential constraints, we build a banking network model. The model consists of N *risk averse* heterogeneous banks which perform optimizing portfolio decisions constrained by equity and liquidity requirements. Our framework integrates the micro-foundations of optimizing banks' decisions within a network structure with interacting agents. Indeed, we do not adopt the convention often used in network models according to which links among nodes are exogenous (and probabilistic) and nodes' behavior is best described by heuristic rules. On the contrary, we adopt the well established economic methodology according to which agents are optimizing, decisions are micro-founded and the price mechanism is endogenous. Once prices are determined in our model, trading partners in the interbank market are obtained through an assortative matching process (a complementary alternative to our approach is pursued in Anand et al. (2015)).

The convexity in the optimization problem has two implications. First, a bank can be both a borrower and a lender at the same time: this is a realistic feature of interbank markets. Second, coupled with convex marginal objectives in profits, it generates precautionary liquidity hoarding in the face of large shocks. The emerging liquidity freeze contributes to exacerbate loss propagation. Banks invest in non-liquid assets, which trade at common prices, hence fire sale externalities emerge. Our banks also trade debt contracts with each other in the interbank market, hence defaults and debt interlinkages contribute to loss propagation. Markets are defined by a price vector and a procedure to match trading partners. The equilibrium price vector (in both the interbank and non-liquid asset markets) is reached through a tâtonnement process,[6] in which prices are endogenously determined by sequential convergence of excess demand and sup-

[4] The increase in the LIBOR rate was a clear sign of liquidity hoarding. After the sub-prime financial shock the spread between the LIBOR and the U.S. Treasury went up 2% points and remained so for about nine months. As a mean of comparison during the Saving and Loans crisis the spread went up 1% point and remained so for nearly a month.

[5] Fire sales are akin to pecuniary externalities as they work through changes in market prices and operate in the presence of equity constraints. See Greenwood et al. (2015) and Mas-Colell et al. (1995), chapter 11.

[6] See also Cifuentes et al. (2005), Bluhm et al. (2014), Duffie & Zhu (2011).

ply. Once prices are determined, actual trading among heterogeneous banks takes place through an assortative matching process.[7] Before examining the contagion channels in our model we assess its empirical performance and find that it can replicate important structural/topological features of real world interbank networks (core-periphery structure, low density, dis-assortative behavior).

In order to evaluate policy alternatives and the interplay of contagion channels, we expose the model to shocks to the non-liquid asset portion of banks' balance sheets. This generates a reduction in the price of such assets, which can be self-reinforcing, and which also triggers the contagion channels outlined above.

In assessing the contagion channels we find a strong connection between the contribution of banks to systemic risk and their total assets.[8] When considering specific balance sheet items, we find that both high interbank borrowing as well as high investment in non-liquid assets are important in explaining the contribution of banks to systemic risk generation. High interbank borrowing increases the scope of risk transmission through direct debt linkages. Investment in non-liquid assets enlarges the scope of fire sale externalities. Both channels are amplified if we take into account risk averse banks. When we analyze the impact of regulatory policy interestingly we find that an increase in the liquidity requirement reduces systemic risk more sharply and more rapidly than an increase in equity requirements. As banks are required to hold more liquidity, they reduce their exposure in the interbank market as well as their investment in non-liquid assets in absolute terms. The fall in interbank supply produces an increase in the interbank interest rate, which, due to asset substitution, induces a fall in non-liquid asset investment relative to interbank lending. Banks become less interconnected in the interbank market and less exposed to swings in the price of non-liquid assets. Both channels of contagion (cross-exposures and fire sale externalities) become less active. With an increase in the equity requirement instead the demand of interbank borrowing falls and so does the interbank rate. Banks substitute interbank lending, which has become less profitable, with investment in non-liquid assets. While the scope of network externalities and cascades in debt defaults falls, the scope of pecuniary externalities increases. On balance, systemic risk, and the contribution of each bank to it, declines, but less than with an increase in liquidity requirements.

The rest of the paper is structured as follows. Section 2 relates our paper to the literature. Section 3 describes the model. Section 4 presents the baseline network topology and discusses the empirical matching. Section 5 analyzes the response of the network model to shocks to non-liquid assets and the contribution of each bank to systemic risk. Section 6 focuses on the policy analysis. Section 7 concludes. Appendices with figures and tables follow.

2. Related Literature

There has been a recent surge in interest in the analysis of contagion, particularly using network models. Three main channels have been explored in the analysis of contagion. The first is the direct interconnection channel. The transmission mechanism which generates cas-

[7] This is inspired by Becker (1973). The numerical algorithm designed to implement the equilibrium obtained through assortative matching is an iterative minimum distance algorithm along the lines indicated by Gale & Shapley (1962) and Shapley & Shubik (1972).

[8] Systemic risk is measured by the share of assets of defaulting banks to total assets in the system and banks' contribution to it by means of the Shapley value. The latter has been borrowed from the literature on both cooperative and non-cooperative games. See Shapley (1953) and Gul (1989) respectively for the seminal contributions, and Drehmann & Tarashev (2013) and Bluhm et al. (2014) for applications to banking. In particular, we follow closely the latter. Other centrality measures for systemic importance are considered in one of the appendices.

cading defaults via direct interconnections is typically modeled using lattice-theoretic models and solving for the unique fixed-point of the equilibrium mapping (see among others Eisenberg & Noe (2001), Afonso & Shin (2011) or Elliott et al. (2014)). A second contagion channel is due to fire-sale externalities (often referred to as pecuniary externality, see also Greenwood et al. (2015)) which emerge in presence of asset commonality and mark-to-market accounting (see among others Cifuentes et al. (2005)): as one bank is hit by a shock, it tries to sell assets to meet VaR or capital constraints. Under mark-to-market accounting, the endogenous fall in market prices negatively affects other banks' balance sheets. Cifuentes et al. (2005) formalized this mechanism, which was subsequently used by Bluhm et al. (2014) among others. In particular, our paper builds on the latter contribution. There are also recent works that embed both channels within micro-founded models of banks (see Bluhm et al. (2014) and Halaj & Kok (2015)).

Our model encompasses both channels and shows that both are important to account for risk propagation. Moreover, we bring to the fore a third mechanism based on liquidity hoarding: once financial distress has emerged banks become more cautious and hoard liquidity. The ensuing liquidity freeze amplifies risk propagation. A similar channel is present also in Afonso & Shin (2011) and Acharya & Merrouche (2013).

Our paper also speaks about the tension between risk-sharing and risk-contagion in networks. While on the one side increasing connectivity might foster risk-sharing and liquidity, on the other side it increases the exposure of each bank to shocks, particularly so if clusters are not evenly spread. An early contribution emphasizing the risk-sharing role of networks is Allen & Gale (2000) which shows the existence of a monotonically decreasing relation between systemic risk and the degree of connectivity. In their model each bank is linked only to one neighbor along a ring. They show that the probability of a bankruptcy avalanche is equal to 1 in the credit chain, but that, as the number of partners of each bank increases (namely when the credit network becomes complete), the risk of individual default goes asymptotically to zero due to the improved risk-sharing possibilities. More recent views instead emphasize the role of contagion and show that a trade-off emerges between decreasing individual risk due to risk sharing and increasing systemic risk due to the amplification of financial distress. Battiston et al. (2012) show for instance that the relation between connectivity and systemic risk is hump shaped: at relatively low levels of connectivity, the risk of individual default goes down with density thanks to risk-sharing while at high levels of connectivity, a positive feedback loop makes a bank under distress more prone to default as the number of partners under distress increases. Gai et al. (2011) also derive a non-monotonic relationship between connectivity and systemic risk. The paper by Elliott et al. (2014) also studies how the network structure affects the balance between risk-sharing and contagion risk. Finally, the trade-off is examined in **?**, who explore how market segmentation can improve it.

Our paper is related to the literature analyzing metrics of systemic risk and measuring the contribution of each bank to it (namely metrics of systemic importance). A connection can also be established with the literature analyzing matching mechanisms in markets along the lines indicated by Becker and Shapley and Shubik (see for instance Becker (1973) and Shapley & Shubik (1972)). Finally, our paper is related to an emerging literature studying prudential regulation in financial networks (see for instance Gai et al. (2011) among many others).

3. The Banking Network

At a general level, a network can be represented by a list of nodes and the links connecting them. When applied to banking, it is straightforward to identify the nodes with banks and the links with the borrowing and lending relationships between the banks. In this spirit, the interbank system can be succinctly summarized by a matrix $\mathbf{X}$ with element x_{ij} representing the exposure (through lending) of bank i to bank j. We consider a financial system consisting of N banks, hence the matrix $\mathbf{X}$ will be of dimension $n \times n$. Two important features of our network are worth noting: (i) it is a weighted network, i.e. a link between banks i and j is indicated by the element $x_{ij} \in \mathbb{R}_{\geq 0}$ and represents the amount (in money) lent by bank i to bank j; (ii) it is a directed network, i.e. the existence of a link in one direction does not imply the existence of a link going in the opposite direction and therefore the matrix is not necessarily symmetric ($x_{ij} \neq x_{ji}$, $i \neq j$). Notice that each bank can be both a borrower and a lender vis-à-vis different counterparties. An important aspect is that cross-lending positions (hence the network links) result endogenously from the banks' optimizing decisions (see next section) and the markets' tâtonnement processes. Banks in our model are characterized also by external (non interbank) assets (cash and non-liquid assets) and liabilities (deposits). As usual, equity or net worth is defined as the difference between total assets and total liabilities. By assumption, banks are heterogeneous due to different returns on non-liquid assets and the levels of calibrated equity and deposits.

Prices in the interbank market and the market for non-liquid assets are determined by tâtonnement processes. In setting up the benchmark banking system the interbank tâtonnement process is instrumental in delivering interbank market equilibrium, whereas after setting the system and in the aftermath of a shock the tâtonnement process in the market for non-liquid assets captures the unfolding of fire sales and is instrumental in the amplification of the shock transmission process. The logic of the tâtonnement processes implies the introduction of fictitious Walrasian auctioneers (see also Cifuentes et al. (2005) or Duffie & Zhu (2011)) which collect individual notional quantities, aggregate them and adjust the relevant price in order to bring the notional aggregate demand and supply in line with each other.[9] Once a clearing price has been achieved, actual trade takes place. Traded quantities in the interbank market are determined according to a closest matching algorithm which operationalizes an assortative matching mechanism along the lines of Becker (1973) (see Section 3.2 for details). A general overview of the model and the channels which operate in it are described visually in Figure 1.

3.1. The banking problem

Our network consists of optimizing banks which solve portfolio optimization problems subject to regulatory and balance sheet constraints. Banks are risk averse and have convex marginal utilities.[10] The convex optimization problem allows us to account for interior solutions for both borrowing and lending. Banks are therefore on both sides of the interbank market vis-à-vis different counterparties: this is a realistic feature of interbank markets and is a necessary condition for a core-periphery configuration to emerge (see Craig & von Peter (2014)). Furthermore we assume that banks have convex marginal utilities with respect to profits.[11] Empirical observation shows that banks tend to adopt precautionary behavior in an uncertain environment.[12] Convex marginal utilities allow us to account for this fact, since in this case banks' expected marginal utility, hence banks' precautionary savings, tends to increase with the degree of uncertainty.

[9] Banks in our model are risk averse, hence have concave objective functions and linear constraints. The convexity of the optimization problem and the assumption of an exponential aggregate supply function guarantees that individual and aggregate excess demand and supply behave in both markets according to Liapunov convergence.

[10] See Halaj & Kok (2015) for a similar approach with risk averse banks and for a discussion between risk averse versus risk neutral modeling of banks' optimization problem.

[11] This amounts to assuming a positive third derivative.

[12] See also Afonso & Shin (2011).

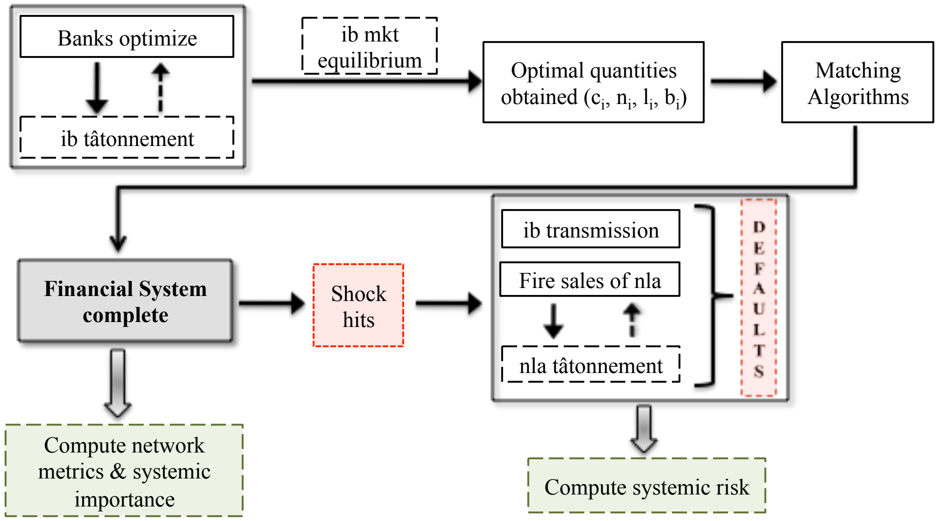


Figure 1: A bird's eye view of the model.

Banks' portfolios are made up of cash, non-liquid assets and interbank lending. Moreover, banks are funded by means of deposits and interbank loans. Hence, the balance sheet of bank i is given by:

$$c_i + pn_i + \underbrace{l_{i1} + l_{i2} + ... + l_{ik}}_{\equiv l_i} = d_i + \underbrace{b_{i1} + b_{i2} + ... + b_{ik'}}_{\equiv b_i} + e_i \tag{1}$$

where c_i represents cash holdings, n_i denotes the volume and p the price of non liquid assets (so that pn_i is the market value of the non liquid portion of the bank's portfolio), d_i stands for deposits and e_i for equity. l_{ij} is the amount lent to bank j where $j = 1, 2, ..., k$ and k is the cardinality of the set of borrowers from the bank in question; b_{ij} is the amount borrowed from bank j where $j = 1, 2, ..., k'$ and k' is the cardinality of the set of lenders to the bank in question. Hence $l_i = \sum_{j=1}^{k} l_{ij}$ stands for total interbank lending and $b_i = \sum_{j=1}^{k'} b_{ij}$ stands for total interbank borrowing.[13]

The bank's optimization decisions are subject to two stylized regulatory requirements:

$$c_i \geq \alpha d_i \tag{2}$$

$$\frac{c_i + pn_i + l_i - d_i - b_i}{\omega_n pn_i + \omega_l l_i} \geq \eta \tag{3}$$

Equation 2 is a liquidity requirement according to which banks must hold at least a fraction α of their deposits in cash.[14] Equation 3 is an equity requirement (which could also be rationalized as resulting from a VaR internal model). It states that the ratio of equity at market prices (at the

[13]Note that since banks cannot lend to nor borrow from themselves, we set $l_{ii} = b_{ii} = 0 \ \forall \ i = 1, ..., N$.

[14]Basel III proposes the liquidity coverage ratio (LCR), which is somewhat more involved than Equation 2. Given the stylized nature of our model the LCR is not easy to capture, yet we consider that the liquidity requirement in Equation 2 provides a good approximation to the constraints faced by the bank in terms of liquidity management.

numerator) over risk weighted assets (at the denominator) must not fall below a certain threshold η.[15] Cash enters the constraint with zero risk weight since it is risk-less in our model, while ω_n and ω_l represent the risk weights on non-liquid assets and interbank lending respectively.

The bank's preferences are represented by a CRRA utility function:

$$U(\pi_i) = \frac{(\pi_i)^{1-\sigma}}{1-\sigma} \tag{4}$$

where π_i stands for bank i's profits and σ stands for the bank's risk aversion. As explained above the convex maximization problem serves a dual purpose. First, it allows us to obtain interior solutions for borrowing and lending. Second, since the CRRA utility function is characterized by convex marginal utilities (positive third derivatives), this gives rise to banks' precautionary behavior in the model. As the variance of shocks increases, banks become more cautious and hoard liquidity in anticipation of higher profit uncertainty.[16]

Another important aspect of concave optimization is that in non-linear set-ups, the variance in assets' returns affects the bank's decision. Higher variance in assets' returns reduces expected banks' utility, thereby reducing the extent of their involvement both in lending as well non-liquid assets investment. This is also the sense in which higher uncertainty in assets' returns (interbank lending as well as non-liquid assets) produces liquidity hoarding and credit crunches. In this set up it is convenient to take a second order Taylor approximation of the expected utility of profits.

The second order approximation of Equation 4 in the neighborhood of the expected value of profits $E[\pi]$ reads as follows:[17]

$$U(\pi_i) \approx U(E[\pi_i]) + U_\pi(\pi_i - E[\pi_i]) + \frac{1}{2}U_{\pi\pi}(\pi_i - E[\pi_i])^2 \tag{5}$$

Taking expectations on both sides of equation 5 and yields:

$$\begin{aligned} E\left[U(\pi_i)\right] &\approx \underbrace{E\left[U(E[\pi_i])\right]}_{=U(E[\pi_i])\text{ by LIE}} + U_\pi \underbrace{E\left[(\pi_i - E[\pi_i])\right]}_{=0\text{ by LIE}} + \frac{1}{2}U_{\pi\pi}\underbrace{E\left[(\pi_i - E[\pi_i])^2\right]}_{=\mathrm{Var}(\pi_i)=\sigma_\pi^2} \\ &\approx U(E[\pi_i]) + \frac{1}{2}U_{\pi\pi}\sigma_\pi^2 \end{aligned} \tag{6}$$

where we have used the law of iterated expectations and where σ_π^2 stands for the variance of profits.

Given the CRRA function $U(\pi_i) = \frac{(\pi_i)^{1-\sigma}}{1-\sigma}$, where σ is the coefficient of risk aversion, we can compute the second derivative as $U_{\pi\pi} = -\sigma E[\pi_i]^{-(1+\sigma)}$. Notice that under certainty equivalence (namely when $E[U'''(\pi)] = 0$) the equality $E\left[U(\pi_i)\right] = U(E[\pi_i])$ holds at all states. With CRRA utility, the third derivative with respect to profits is positive, which in turn implies that the expected marginal utility grows with the variability of profits. Furthermore since, $U'' < 0$, expected utility is equal to the utility of expected profits minus a term that depends on the volatility of bank profits and the risk aversion parameter. This is a direct consequence of

[15] This threshold is composed of two parts: $\eta = \gamma + \tau$. The first component (γ) is the policy-chosen capital requirement, whereas the second (τ) is an exogenous buffer introduced for technical reasons and which can also be seen as a buffer that markets require on top of supervisory capital requirements. Note that Equation 3 will typically not be binding, given banks' risk aversion. For more details see Table 1 below.

[16] Technically, when the third derivative is positive this means that the utility becomes more concave on the tails.

[17] Note that all partial derivatives are also evaluated at $E[\pi]$.

Jensen's inequality and provides the standard rationale for precautionary saving. Using the expression derived above for $U_{\pi\pi}$, the expected utility of profits can be written as:

$$E\left[U(\pi_i)\right] \approx \frac{E[\pi_i]^{1-\sigma}}{1-\sigma} - \frac{\sigma}{2}E[\pi_i]^{-(1+\sigma)}\sigma_\pi^2 \tag{7}$$

Equation 7 represents the objective function that bank i maximizes subject to the constraints introduced above. With these elements in mind the problem of bank i can be summarized as follows:[18]

$$\begin{aligned} &\underset{\{c_i,n_i,l_i,b_i\}}{\text{Max}} \quad E[U(\pi_i)] \\ &s.t. \;\; \textit{Equation 2}, \textit{Equation 3}, \textit{Equation 1} \\ &c_i, n_i, l_i, b_i \geq 0 \end{aligned} \tag{P}$$

Before moving forward and for the sake of completeness we derive next the precise form of profits, as well as their variance.

The bank's profits are given by the returns on lending in the interbank market (at the interest rate r^l) plus returns from investments in non-liquid assets (whose rate of return is r_i^n) minus the expected costs from interbank borrowing.[19] The rate of return on non-liquid assets is exogenous and heterogeneous across banks: we assume that banks have access to investment opportunities with different degrees of profitability. The interest rates on borrowed funds are also heterogeneous across banks due to a risk premium.[20] In lending to j, bank i charges a premium r_j^p over the risk-free interest rate (i.e. the interest rate on interbank loans r^l), which depends on the probability of default of j, δ_j. The premium can be derived through an arbitrage condition. By lending l_{ij} to j, bank i expects to earn an amount given by the following equation:

$$\underbrace{(1-\delta_j)\left(r^l + r_j^p\right)l_{ij}}_{\text{with no default}} + \underbrace{\delta_j\left(r^l + r_j^p\right)(1-\xi)\,l_{ij}}_{\text{with default}} \tag{8}$$

where ξ is the loss given default parameter. If bank j cannot default, bank i gets:

$$l_{ij}r^l \tag{9}$$

By equating 8 and 9 we can solve for the fair risk premium charged to counterparty j:

$$r_j^p = \frac{\xi\delta_j}{1-\xi\delta_j}r^l \tag{10}$$

[18] The demand for equity in our model arises as residual from banks' asset and liability optimal choice, while we assume that the supply of equity is exogenous and elastic. It should be noted though that raising equity might entail adjustment costs as investors' supply might not be fully elastic. This would be an interesting future extension of our model, which we believe could further amplify the fire sale externalities. As in the face of shocks banks rebalance their portfolio to meet the equity requirement, the asset adjustment might be larger when raising equity is made stickier.

[19] For simplicity it is assumed that deposits and cash/reserves are not remunerated. Note that since these would be a fixed number if calibrated they would only shift up or down the responses that we see from the model. Furthermore, such shifts would be indeed hard to even perceive.

[20] In what follows for the derivation of the premium we draw on Bluhm et al. (2014).

It is immediate to verify that the premium is calculated so that, by lending to j, bank i expects to get $r^l l_{ij}$ (to obtain this, substitute the premium back into equation 8). We can interpret condition 8 also as a participation constraint: bank i will lend to bank j only if it gets an expected return from lending equal to the risk free rate, i.e. the opportunity cost of lending. By summing up over all possible counterparties of bank i, and recalling that $l_i = \sum_{j=1}^{k} l_{ij}$, we retrieve the overall gain that bank i expects to achieve by lending to all the borrowers: $r^l l_i$. On the other hand, as a borrower, bank i must also pay the premium associated to its own default probability. Since banks charge a fair risk premium, the returns that banks obtain from non-defaulting borrowers offset the losses resulting from contracts with defaulting borrowers. Borrowing banks, on the other hand, must always pay the premium. Therefore the cost of borrowing is given by: $r_i^b b_i = (r^l + r_i^p) b_i = \frac{1}{1-\xi\delta_i} r^l b_i$.

Finally, the gains from investment in non-liquid assets are given by: $r_i^n \frac{n_i}{p}$. Given these assumptions, the profits of bank i read as follows:

$$\pi_i = r_i^n \frac{n_i}{p} + r^l l_i - (r^l + r_i^p) b_i = r_i^n \frac{n_i}{p} + r^l l_i - \frac{1}{1-\xi\delta_i} r^l b_i \tag{11}$$

Having obtained an expression for profits, we now compute their variance. Notice that volatility only derives from uncertainty in non-liquid asset returns and from default premia on borrowing. These are cross-sectional variances and they are the only which can be considered in our setting, which is static and hence does not allow for the consideration of time series variances. The return on interbank lending as well as the price of non-liquid assets are endogenous and therefore will ultimately depend on exogenous elements of the model and of the shocks assumed.[21] Finally, it should be noted that in setting up the system the price of non-liquid assets is set to 1, which is a status-quo scenario in which aggregate sales of non-liquid assets are zero and therefore no fire sales are present. Given the sources of uncertainty we obtain the following volatility of profits:

$$\begin{aligned} \sigma_\pi^2 &= \mathrm{Var}\left(r_i^n \frac{n_i}{p} + r^l l_i - \frac{1}{1-\xi\delta_i} r^l b_i \right) && (12) \\ &= \left(\frac{n_i}{p}\right)^2 \sigma_{r_i^n}^2 - (b_i r^l)^2 \mathrm{Var}\left(\frac{1}{1-\xi\delta_i}\right) + 2\frac{n_i}{p} r^l b_i \mathrm{cov}\left(r_i^n, \frac{1}{1-\xi\delta_i}\right) && (13) \end{aligned}$$

We know that $\delta_i \in [0,1]$. Furthermore, even when $f(\delta_i) = \frac{1}{1-\xi\delta_i}$ is a a convex function, over a realistic range of δ_i it is essentially linear and it is therefore sensible to obtain the variance of $f(\delta_i)$ through a first order Taylor approximation around the expected value of δ_i, which yields:

$$\mathrm{Var}\left(\frac{1}{1-\xi\delta_i}\right) = \xi^2 (1 - \xi E[\delta_i])^{-4} \sigma_{\delta_i}^2 \tag{14}$$

We assume that the ex ante correlation between return on non-liquid assets and costs of borrowing is zero, hence we can set the covariance term in Equation 12 to zero. This leaves us with the following expression for the variance of profits:

$$\sigma_\pi^2 = \left(\frac{n_i}{p}\right)^2 \sigma_{r_i^n}^2 - (b_i r^l)^2 \xi^2 (1 - \xi E[\delta_i])^{-4} \sigma_{\delta_i}^2 \tag{15}$$

[21] Furthermore, given the nature of the fire sales externalities, it is virtually impossible for banks to form an expectation about them, as they would need to know the entire balance sheet of the banking system in every state of the world. For a similar argument see Caballero & Simsek (2013).

3.2. Interbank Market Clearing

The interbank market clears in two stages. In the first stage a standard tâtonnement process is applied and the interbank interest rate is obtained by clearing excess demand/supply. Individual demands and supplies (as obtained from banks' optimization) are summed up to obtain market demand and supply. If excess demand or supply occurs at the market level, the interbank rate is adjusted sequentially to eliminate the discrepancy. In the second stage, after the equilibrium interbank rate has been determined, a matching algorithm determines the actual pairs of banks involved into bilateral trading (at market prices). We aim to capture here the behavior of centralized interbank markets as opposed to markets in which bilateral bargaining is the main mechanism driving the matching of banks. Additionally, as noted by Glasserman & Young (2014), to assess the potential damage that can come from interbank connections the precise shape of the network is not as important as some balance sheet ratios that better capture this potential damage, like for instance total interbank borrowing or total assets/liabilities. These are precisely the quantities on which banks focus in our model, as we aim to assess how banks navigate the trade-offs between the different types of externalities and their investment in long term assets.

Price Tâtonnement in the Interbank Market. For a given calibration of the model, which includes an initial level of the interbank interest rate, the bank chooses the optimal demand (b_i) and supply (l_i) of interbank debt trading. These are submitted to a Walrasian auctioneer who sums them up and obtains the market demand $B = \sum_{i=1}^{N} b_i$ and supply $L = \sum_{i=1}^{N} l_i$. If $B > L$ there is excess notional demand in the market and therefore r^l is increased, whereas the opposite happens if $B < L$.[22]. Changes in the interbank rates are bounded within intervals which guarantee the existence of an equilibrium see Mas-Colell et al. (1995)).

The clearing price process delivers an equilibrium interest rate as well as two vectors, $\mathbf{l} = [l_1\ l_2\ ...\ l_N]$ and $\mathbf{b} = [b_1\ b_2\ ...\ b_N]$, which correspond to optimal lending and borrowing of all banks for given equilibrium prices.

Matching Trading Partners. Once the equilibrium interest rate has been obtained, actual bilateral trading relations among banks need to be determined. In other words, given the vectors $\mathbf{l}$ and $\mathbf{b}$ obtained during the price clearing process we need to match pairs of banks for the actual trading to take place. We match partners by relying on the concept of assortative matching (see Becker (1973)) described below. Practically, we need to determine how bank i distributes its lending ($l_i = \sum_{i=1}^{k} l_{ij}$) and/or borrowing ($b_i = \sum_{i=1}^{k'} b_{ij}$) among its potential counterparties to deliver the matrix of interbank positions $\mathbf{X}$.

Let us start by defining the surplus generated by the trading as $S(l_i, b_j)$. Notice that l_i and b_j, namely the lending and borrowing positions of each bank, are scalars that identify a characteristic of each bank. Following Becker (1973) we can order the banks according to the size of the trading position, namely the defining characteristic through which we wish to match them. It is possible to assume that the surplus from trading will increase with respect to the characteristics of banks on both sides of the market:

$$\frac{\partial^2 S(l_i, b_j)}{\partial l_i \partial b_j} \geq 0 \tag{16}$$

[22]This iteration takes place in fictitious time as in standard tâtonnement processes. After the interest rate is adjusted, banks re-optimize their balance sheet. Banks, however, are only matched with other banks (i.e. trade with each other) once the equilibrium interest rate has been determined.

The condition in equation 16 corresponds to the assumption of positive complementarity in Becker (1973). Intuitively, the trading value for each pair is larger when partners are matched whose combined absolute excess demand is minimal. This allows banks to satisfy their excess demand within one single trading round and avoid further search costs.[23]

When positive complementarities are in place it is possible to show that perfect positive assortative matching is the efficient allocation. Indeed imagine that banks are matched so that the one with the highest borrowing, $\bar{b}$, pairs with the one with the lowest lending, $\underline{\mathrm{l}}$, and let's assume that the total surplus in this case is larger than total surplus under perfect positive assortative matching. This situation corresponds to the following condition:

$$S(\underline{\mathrm{l}}, \bar{b}) + S(\bar{l}, \underline{\mathrm{b}}) > S(\bar{l}, \bar{b}) + S(\underline{\mathrm{l}}, \underline{\mathrm{b}}) \tag{17}$$

The above condition can also be written as follows:

$$S(\underline{\mathrm{l}}, \bar{b}) - S(\bar{l}, \bar{b}) - S(\underline{\mathrm{l}}, \underline{\mathrm{b}}) + S(\bar{l}, \underline{\mathrm{b}}) > 0 \tag{18}$$

We can then sum up over all borrowing banks the change in surplus due to a change in the lending partner:

$$\sum_{b_j} \frac{\partial S(\underline{\mathrm{l}}, b_j)}{\partial b_j} - \sum_{b_j} \frac{\partial S(\bar{l}, b_j)}{\partial b_j} > 0 \tag{19}$$

The last condition is equivalent to:

$$-\sum_{l_i} \frac{\partial^2 S(l_i, b_j)}{\partial l_i \partial b_j} > 0 \tag{20}$$

The condition in 20 contradicts the positive complementarity assumption in 16 proving that matching pairs differently would not deliver a higher surplus.

Numerically we will implement the positive assortative matching condition detailed above through an algorithm based on *closest matching*, or *minimum distance.* The vectors of lending and borrowing are ordered in descending order and transactions are assigned. For the sake of argument, say banks i and j are the largest lender and borrower respectively, then the element (i, j) of the interbank matrix will be given by $x_{ij} = \min\{l_i, b_j\}$. This process goes over all pairs of banks and whatever residual desired amount that remains after every transaction is stored for the next round of the algorithm. Since in our setting, as in the real world, banks are on both sides of the market, some complications may arise. In particular, an issue which can emerge is that, because of the order in which the transactions are ordered, a bank will eventually be "matched against itself" at the last stage of the algorithm. Of course this cannot be the case since, as mentioned earlier, we assume that banks do not trade with themselves. When we encounter such issue, the algorithm starts again from scratch but introduces a random swapping in the ordering of banks. The achievement of a solution is in this way guaranteed. In this case matching takes place sequentially following the notion of deferred-acceptance established in Gale & Shapley (1962). The interbank trading matrix obtained by this method delivers a low level of connectivity, providing in fact a minimum density matrix. This low level of density or connectivity is in line with the one observed in the data. The CMA is also based on a stability rationale, as it is generally compatible with pair-wise efficiency and has been proposed

[23] One could also assume that banks have a convex cost from trading with each additional partner. Hence for a given value of the surplus this cost is reflected in the fact that the total value from trading can be written as follows: $S(l_i, b_j) = f(l_i, b_j) - c\sum_{i=1}^{k} l_{ij}$ for a lending bank and as $S(l_i, b_j) = f(l_i, b_j) - c\sum_{i=1}^{k} b_{ij}$ for a borrowing bank.

in the seminal treaty of Shubik (1999) as most apt to capture clearing in borrowing and lending relations.[24]

3.3. Price Tâtonnement in the Market for Non-Liquid Assets

In this section we briefly describe the clearing process used for the non-liquid asset market, which is modeled along the lines of Cifuentes et al. (2005) and operates once a shock has hit the system. As mentioned earlier, the price of non-liquid assets is set to 1 when the financial system is set up. This is the price corresponding to zero aggregate sales and banks fulfilling regulatory requirements (i.e. the "status quo" price). The occurrence of shocks to banks' non-liquid asset holdings may force them to put some of their stock of assets on the market in order to fulfill regulatory requirements. This increases the supply of assets above demand. As a result the price adjusts to clear the market.

The logic of the mechanism can be described as follows. Consider the situation in which bank i is forced to sell non-liquid assets for an amount s_i in order to fulfill the equity requirement. An expression for s_i can be obtained by replacing n_i with $n_i - s_i$ in the denominator of Equation 3 and solving for s_i. From that it is straightforward to see that s_i will be decreasing in prices p, implying in turn that the aggregate sales function $S(p) = \sum_i s_i(p)$ is also decreasing in p. Defining the aggregate demand function as $\Theta(p) : [p, 1] \to [p, 1]$, an equilibrium price solves the following fixed point problem: $\Theta(p) = d^{-1}(S(p))$.

The price at which total aggregate sales are zero, namely $p = 1$, can certainly be considered one equilibrium price. But a key insight from Cifuentes et al. (2005) is that a second (stable) equilibrium price exists, to the extent that the supply curve $S(p)$ lies above the demand curve $D(p)$ for some range of values. The convergence to the second equilibrium price is guaranteed by using the following inverse demand function[25]:

$$p = exp(-\beta \sum_i s_i), \tag{21}$$

where β is a positive constant to scale the price responsiveness with respect to non-liquid assets sold, and s_i is the amount of bank i's non-liquid assets sold on the market.

For an initial decline in prices to, say, p_0, banks will respond by putting an amount $S(p_0)$ on the market. But given Equation 21, this will in turn push the price down to $p_1 = d^{-1}(S(p_0))$. This generates further sales to the tune of $S(p_1)$. This process goes on until a new equilibrium price p^* is reached. For further details on the mechanism we refer the reader to the seminal contribution by Cifuentes et al. (2005).

3.4. Equilibrium Definition

Definition. A competitive equilibrium in our model is defined as follows:

(i) A quadruple (l_i, b_i, n_i, c_i) for each bank i that solves the optimization problem P.

(ii) A clearing price in the interbank market, r^l, which satisfies $B = L$, with $B = \sum_{i=1}^N b_i$ and $L = \sum_{i=1}^N l_i$.

(iii) A trading-matching algorithm for the interbank market.

(iv) A clearing price for the market of non-liquid assets, p, that solves the fixed point: $\Theta(p) = d^{-1}(s(p))$.

[24] In a previous version of this paper we also considered two alternative matching mechanisms, namely the maximum entropy algorithm and a random matching algorithm with a loading factor calibrated to obtain a density in between the extremes of CMA and maximum entropy. These two alternatives deliver networks with a significantly different topology. Results are available upon request.

[25] This function can be rationalized by assuming the existence of some noise traders in the market.

3.5. Risk Transmission Channels in the Model

Before proceeding with the simulation results, it is useful to highlight the main channels of risk transmission in this model. There are three channels which operate simultaneously; to fix ideas we start by describing the effects of real interlinkages.[26]

First, a direct channel goes through the lending exposure in the interbank market. When bank i is hit by a shock which makes it unable to repay interbank debt, default losses are transmitted to all the banks exposed to i through interbank loans. Depending on the size of losses, these banks, in turn, might find themselves unable to fulfill their obligations in the interbank market.

The increase of default losses and in the uncertainty of debt repayment makes risk averse banks more cautious. They therefore hoard liquidity. The ensuing fall in the supply of liquidity increases the likelihood that banks will not honor their debts, reduces banks' resiliency to shocks and amplifies the cascading effects of losses. Notice that convex marginal objectives with respect to returns are also crucial in determining an increase in precautionary savings in the face of increasing uncertainty.

Liquidity shortage quickly turns into insolvency. Moreover, it reduces banks' exposure to non liquid assets. Eventually banks are forced to sell non-liquid assets if they do not meet regulatory requirements. If the sale of the assets is large enough, the market experiences a collapse of the asset price. This is the essence of pecuniary externalities, namely the fact that liquidity scarcity and the ensuing individual banks' decisions have an impact on market prices. In an environment in which banks' balance sheets are measured with mark-to-market accounting, the fall in the asset price induces accounting losses to all banks which have invested in the same asset. Accounting losses force other banks to sell non-liquid assets under distress. This vicious circle also contributes to turn a small shock into a spiraling chain of sales and losses. Three elements are crucial in determining the existence of fire sale externalities in our model. First, the presence of equity requirements affects market demand elasticities in a way that individual banks' decisions about asset sales do end up affecting market prices. Second, the tâtonnement process described above produces falls in asset prices whenever supply exceeds demand. Third, banks' balance sheet items are evaluated with a mark-to-market accounting procedure.

All the above-mentioned channels (credit interconnections among banks, liquidity hoarding and fire sales) have played an important role during the 2007 crisis. Caballero & Simsek (2013) for instance describe the origin of fire sale externalities in a model in which the complex financial architecture also induces uncertainty, which amplifies financial panic. Afonso & Shin (2011) instead focus on loss transmission due to direct exposure of banks in the money market and through liquidity hoarding. Our model merges those approaches and gains a full picture of the extent of the cascade following shocks to individual banks[27].

Notice that the mechanisms just described are in place even if the shock hits a single bank. However to produce a more realistic picture in the simulations presented below we assume a multivariate normal distribution of shocks to non-liquid assets: initial losses can therefore hit all banks and can also in principle be correlated. Therefore our numerical exercise will account for the quantitative relevance of contagion by assuming also asset risk commonality.

At this stage, it is instructive to discuss the impact of the various channels also through analytical derivations. Specifically, using the banks' first order conditions to the optimal problem outlined in P we can derive expressions for the various risk-premia characterizing our model. Those risk premia provide an extent of the size and evolution of systemic risk and can be put

[26] It is important to note though that in the simulations the shock transmission process is kickstarted by means of shocks to non-liquid assets.

[27] A short description of the shock transmission process is given in Appendix A.

in relation to margins which proxy the contagion channels operating in our model. Of course, due to the presence of many constraints and choice variables, the optimization problem itself is too complex to be solved analytically. This is the reason why we resort to simulations. The derivations below, therefore, aim at guiding our thoughts in thinking about contagion and interpreting the results of the simulations.

Merging together the bank's first order conditions with respect to interbank lending, l_i, and borrowing, b_i, we obtain a metric for the interbank risk premium. The latter read as follows:

$$IR = r^l \left(\frac{\xi \delta_i}{\xi \delta_i - 1} \right) = \frac{\frac{\sigma}{2} E[\pi_i]^{-(1+\sigma)} \frac{\partial \sigma_\pi^2}{\partial b_i} + \lambda_2 \eta \omega_l}{E[\pi_i]^{-\sigma} + \frac{\sigma}{2}(1+\sigma) E[\pi_i]^{-\sigma-2} \sigma_\pi^2} \tag{22}$$

where λ_2 is the Lagrange multiplier on the equity requirement. This premium provides the extent to which interbank network externalities impact risk through the propagation of debt defaults. Notice that $\frac{\partial \sigma_\pi^2}{\partial b_i}$ can be either positive or negative thereby contributing to decrease or increase the interbank risk premium. Whether the term $\frac{\partial \sigma_\pi^2}{\partial b_i}$ is positive or negative depends upon whether $Var\left(\frac{1}{1-\xi\delta_i}\right)$ is larger or smaller than the term $2\frac{n_i}{p}\text{cov}\left(r_i^n, \frac{1}{1-\xi\delta_i}\right)$. The variance of the interbank default premium, $Var\left(\frac{1}{1-\xi\delta_i}\right)$, captures the risk of interbank debt default, while the term $2\frac{n_i}{p}\text{cov}\left(r_i^n, \frac{1}{1-\xi\delta_i}\right)$ determines whether interbank default losses are compensated by returns on non-liquid assets. If the first term is larger than the second this means that interbank network externalities are large, $\frac{\partial \sigma_\pi^2}{\partial b_i} \leq 0$, and this raises the interbank risk premium.

We now merge the banks' first order conditions with respect to non-liquid assets and interbank lending. This leads to an asset risk premium which reads as follows:

$$AR = \left(\frac{r_i^n}{p^2} - r^l \right) = \frac{\frac{\sigma}{2} E[\pi_i]^{-(1+\sigma)} \frac{\partial \sigma_\pi^2}{\partial n_i} \frac{1}{p} + \lambda_2 \eta (\omega_n - \omega_l)}{E[\pi_i]^{-\sigma} + \frac{\sigma}{2}(1+\sigma) E[\pi_i]^{-\sigma-2} \sigma_\pi^2} \tag{23}$$

The above asset risk premium captures the role of asset substitution for risk. Banks have always the option to invest either in non-liquid assets or in interbank lending. If the spread between the two is large banks will prefer to invest in non-liquid assets and this raises the scope for fire sale externalities. Indeed the terms on the right hand side of equation 23 all depend upon the transmission channels linked to fire sale externalities. The term $\frac{\partial \sigma_\pi^2}{\partial n_i}$, which is positive, captures the fact that higher banks' exposure to non-liquid asset increase profits' risk, σ_π^2. A higher $\frac{\partial \sigma_\pi^2}{\partial n_i}$ contributes to increase the overall asset risk premium. Furthermore, as is well known, fire sale externalities are larger when equity constraints bind: indeed $\lambda_2 \geq 0$ contributes to increase the asset risk premium, as the risk weight on non-liquid assets is larger than the one for interbank lending.

At last, merging the banks' first order conditions for non-liquid assets and interbank borrowing we obtain the following banks' external finance premium:

$$EF = \left(\frac{r_i^n}{p^2} - \frac{r^l}{1 - \xi \delta_i} \right) = \frac{\frac{\sigma}{2} E[\pi_i]^{-(1+\sigma)} \left[\frac{1}{p} \frac{\partial \sigma_\pi^2}{\partial n_i} + \frac{\partial \sigma_\pi^2}{\partial b_i} \right] + \lambda_2 \eta \omega_n}{E[\pi_i]^{-\sigma} + \frac{\sigma}{2}(1+\sigma) E[\pi_i]^{-\sigma-2} \sigma_\pi^2} \tag{24}$$

The above external finance premium measures the risk induced by fire sale externalities, net of the risk induced by interbank debt defaults. This premium increases when the equity constraint is binding. As with the asset risk premium, the external finance premium positively depends on $\frac{\partial \sigma_\pi^2}{\partial n_i}$, whereas the effect of $\frac{\partial \sigma_\pi^2}{\partial b_i}$ on the finance premium will depend on whether this expression is positive or negative, as discussed for the interbank risk premium.

3.6. Systemic Risk

The 2007-8 crisis moved the attention of supervisory authorities from the too-big-to fail to the too-interconnected-to fail banks. In the past, systemic importance was primarily seen in the context of size, pointing to measures such as concentration indices such as the Herfindahl index. Following the crisis experience, the concept of systemic importance was expanded to also include those banks that are highly interconnected with others. To measure the relevance of interconnections, an important distinction arises between ex ante and ex post metrics. Ex ante measures determine the contribution of each bank to systemic risk based on a time-t static configuration of the network. These measures are useful as they identify banks/nodes which can potentially be risk spreaders, but they have little predictive power, as they do not consider the transformations in the network topology following shocks. On the contrary ex post measures do so, hence they can be fruitfully used in stress tests. Overall ex ante measures can be used for preemptive actions, while ex post measures can be used to predict the possible extent of contagion in the aftermath of shocks, an information crucial to establish the correct implementation of post-crisis remedies.

Our focus here is on one ex post metric, namely the Shapley value[28]. In Appendix B we report the performance in the numerical analysis of a set of ex ante metrics, namely network centrality measures, as well as their comparison with the Shapley value. The Shapley value comes from the literature on cooperative and non-cooperative game theory, and provides the contribution (through permutations) of each bank to an aggregate value. The latter in our case is computed via the ratio of assets from all defaulting banks to total assets, $\Phi = \frac{\sum_\Omega assets_\Omega}{\sum_i assets_i}$, where $\Omega \in i$ identifies the set of defaulting banks. One desirable property of the Shapley value is additivity, which in our case implies that the marginal contribution of each bank adds up to the aggregate default probability.

Formally the Shapley value is defined as follows. Define first C as a coalition of players which is a subset of the set defining all possible coalitions with N players (the latter denoted by C_N). In this spirit, C_{-i} stands for a coalition which does not include player/bank i. Next, define v^Ψ a function which maps subsets of players to the real numbers (i.e. $v^\Psi : 2^N \to \mathbb{R}$, where by convention it is assumed that $v(\emptyset) = 0$). This so called characteristic function will generate a value $v^\Psi(C)$ for every possible coalition C: in our case this value is systemic risk when the coalition C of banks is being shocked. Similarly, $v^\Psi(C_{-i})$ will indicate the value generated by a coalition which does not include bank i (i.e. the systemic risk generated when the group of banks C, excluding i, is being shocked). With these elements in mind, the Shapley value for bank i can be expressed in the following way:

$$\Xi_i(v^\Psi) = \frac{1}{N!} \sum_{C \in C_N} \left(v^\Psi(C_{-i} \cup i) - v^\Psi(C_{-i}) \right) \tag{25}$$

where $v^\Psi(C_{-i} \cup i)$ is the value obtained by coalition C_{-i} but when also including bank i. That is, $\Xi_i(v^\Psi)$ gives the average marginal contribution of player i over all possible coalitions of player set N. In our context the Shapley value is thus a way of assigning to each bank its average marginal contribution to systemic risk. Note that the index Ψ denotes different possible shock scenarios, hence banks' contribution to systemic risk is computed conditional on a shock vector to the banking system.[29]

[28] See Shapley (1953) for the formal problem. Drehmann & Tarashev (2013) applied this concept to banking for the first time, and it was subsequently used by several authors.

[29] As can be seen by the fact that the possible coalitions which can be formed with player set N is given by 2^N, the computation of the Shapley value is usually subject to the curse of dimensionality. For

4. Baseline Scenario Results and Empirical Matching

In this section we present the baseline network configuration, which we characterize using synthetic metrics, namely density, average path length, assortativity, clustering, betweenness and eigenvector centrality. Additionally, we consider other features of the final configuration of the network which are useful in assessing its realism. In particular we consider the ratio of interbank assets to total assets, the equilibrium interest rate achieved through the interbank market tâtonnement process, the number of intermediaries in the system (i.e. banks which both borrow and lend), and the subset of intermediaries which form the core of the system.[30]

Our primary goal is to verify that our banking network shares topological properties with the empirical counterparts. We indeed find that our model is able to replicate a number of stylized facts characterizing real world interbank networks (core-periphery structure, low density and dis-assortative behavior).

Before presenting the simulation results for the baseline structure, we describe the model calibration, which is largely based on banking and regulatory data. Table 1 summarizes calibrated values and shock distributions.

Following Drehmann & Tarashev (2013), the number of banks is set to 20. This keeps the system manageable in terms of size (allowing us to track the behavior of different banks) and in terms of computation time. All policy related parameters are taken from the implementation of Basel III in Europe (see the *Regulation No 575/2013 of the European Parliament and of the Council* of 26 June 2013). The liquidity requirement (α), equity requirement (γ), risk weights on non-liquid assets (ω_n) and interbank lending (ω_l) are set respectively to 10%, 8%, 0.2 and 1.[31] We use data from Bureau van Dijk's Bankscope database to calibrate deposits and equity. We take the average of total assets for the period 2011-2013 for Euro Area (EA) banks, and use deposits and equity (again averaged over 2011-2013) of the top 20 banks in terms of assets. The return on non-liquid assets is randomly drawn from a uniform distribution over the range $0-15\%$ (the variance is computed accordingly), whereas the vector of shocks to non-liquid assets, which is the starting point of the shock transmission process, is drawn from a multivariate normal distribution with a mean of 5, a variance of 25 and zero covariance (we draw 1000 shocks to evaluate the model). We set the loss given default parameter ξ to 0.5 (see for instance Memmel & Sachs (2013)), whereas we assign values of 0.5% and 0.3% to the expected probability of default and its variance respectively. Finally, the banks' risk aversion parameter σ is set equal to 2. For precautionary saving to arise such parameter must be larger than 1. Note also that the parameter β capturing the price responsiveness to the sale of non-liquid assets is endogenous and calculated as the number necessary to achieve a 10% drop if all non-liquid assets optimally

this reason it is normally approximated in numerical simulations by the average marginal contribution of players to the aggregate value over M randomly sampled permutations or coalitions, $\Xi_i(v^\Psi) \approx \hat{\Xi}_i(v^\Psi) = \frac{1}{M}\sum_{C\in C_M}\left(v^\Psi(C_i\cup i) - v^\Psi(C_{-i})\right)$.

[30] As noted by Craig & von Peter (2014), interbank markets typically present a tiered structure, and intermediation plays a key role in determining that structure. In particular, an interbank market is tiered when there are banks which intermediate between other banks that are not directly connected. The two tiers thus form a core of densely connected banks and a periphery of banks unconnected to each other but connected to the core. Core banks are therefore a strict subset of intermediaries: those intermediaries that serve to connect peripheral banks that would otherwise be disconnected from each other.

[31] We view γ as a floor regulatory value. The banks' capital buffer τ (on top of the equity requirement) is set to 1%. As noted earlier, this can be seen as market requirement on top of regulatory requirements, and it also serves a technical purpose by guaranteeing that a shock will not push banks automatically into fire-sales (in case the equity requirement is binding pre-shock). Note that this does not imply that all banks will choose the same equity requirement as a results of their optimization. Furthermore, given banks' risk aversion it will typically be the case that the equity constraint is not binding.

chosen by banks are sold on the market (see Greenwood et al. (2015) and references therein for price responsiveness in fire sales processes).

Par./Var.	Description	Value
N	Number of banks in the system	20
α	Liquidity requirement ratio	0.10
ω_n	Risk weight on non-liquid assets	1
ω_l	Risk weight on interbank lending	0.20
γ	Equity requirement ratio	0.08
τ	Equity buffer	0.01
d_i	Bank deposits	Top20 EA
e_i	Bank equity	Top20 EA
σ	Bank risk aversion	2
ξ	Loss given default	0.5
$E[\delta]$	Expected default probability	0.005
σ^2_δ	Variance of default probability	0.003
r_i^n	Return on non-liquid assets	$U(0, 0.15)$
$\sigma^2_{r_i^b}$	Variance of r_i^n	$\frac{1}{12}(\max(r_i^n) - \min(r_i^n))^2$
Ψ	Shocks to non-liquid assets	$\aleph(\mathbf{5}, 25 * \mathbf{I})$

Table 1: Baseline calibration

We start by describing the partitions of banks into borrowers and lenders, the share of interbank assets over total assets and the equilibrium interbank rate (see also Table 2 below). Given the above calibration, the equilibrium interbank rate is 2.98%, in line with the pre-crisis average of EONIA. Interbank assets as a share of total assets stand at 23.7%, also in line with real world counterparts. There are 5 banks that only lend (banks $6, 10, 16, 17$ and 19), 6 that only borrow $(2, 5, 7, 8, 14$ and $15)$ and 9 that both borrow and lend $(1, 3, 4, 9, 11, 12, 13, 18$ and $20)$. Generally speaking banks which borrow are those whose returns on non-liquid assets are high (and higher than returns on interbank lending). Since those have good investment opportunities they wish to invest and require liquidity beyond the one present in their portfolio. On the contrary banks decide to lend when the rate that they receive on bank lending is higher than the rate of return on non-liquid assets. The convexity of the optimization problem implies that internal solutions exist and banks can be on both sides of the market, i.e., they can be borrowers and lenders at the same time. Few large banks enter both sides of the market and act as central nodes: those banks have high returns on non-liquid assets, hence they wish to obtain liquidity for investment, but they also have large cash balances and are willing to lend to acquire a diversified portfolio.

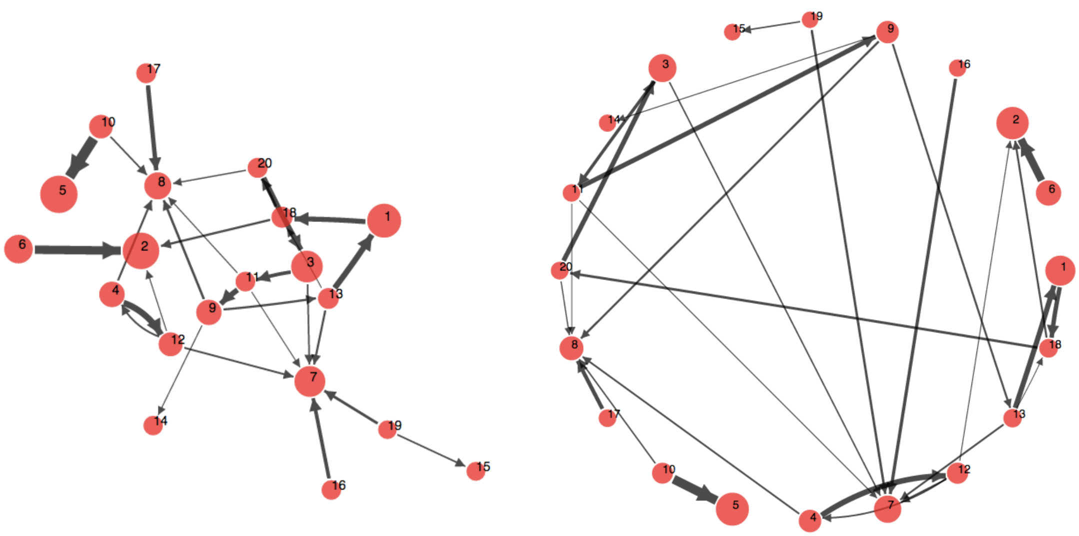
4.1. Synthetic Measures of Network Architecture and Empirical Matching

Our next step is to describe the network topology by using synthetic network indicators.[32] Notice that synthetic metrics describing the network largely depend upon the banks' optimization problem and upon the matching algorithm. On the other hand, for the static network configuration the three contagion channels described previously do not play a role since they

[32]To compute some of the network indicators we made use of the Brain Connectivity Toolbox and the MatlabBGL library.

become operative only when banks are hit by shocks. The network response to shocks and the role of the contagion channels for systemic risk will be analyzed in Section 5.

Figure 2 presents the baseline configuration with an interbank matrix computed via the closest matching algorithm, given the parameters from Table 1. Different nodes represent banks and their size is given by total assets. The width of arrows indicates the amounts transacted and an arrow going from i to j indicates that i is exposed to j through lending. The number of links is not particularly high; in network parlance, the network exhibits low density. In fact the density of the network is 7.37%, in line with the evidence from country-specific studies of interbank markets.[33]



(a) Standard representation

(b) Circle representation

Figure 2: Baseline network configuration

Table 2 shows results for the other synthetic metrics considered, given the baseline parameterization.

The first two network metrics are closely related. The density of the network is the fraction of existing links over the total amount of possible links, whereas the average degree is the average number of connections per bank. Both metrics proxy the extent of diversification in the network. By construction, the CMA network presents low density and hence a low average degree: a bank is connected on average to 1.4 other banks.

The average path length is the mean shortest path between pairs of nodes. It gives an idea of the ease with which one can expect to get from a given node to any other given node. In our case this number is 2.6, implying that the average bank is almost 3 connections away. The average path length is small, in line with real-world interbank networks (see Alves et al. (2013) or Boss et al. (2004) among others). This implies that exposure is not far away for the average bank in the network.

Betweenness and eigenvector centrality are computed as averages for all nodes in the network. The CMA network features high betweenness and eigenvector centrality since a few banks act

[33]See for instance van Lelyveld & In't Veld (2012) for the Dutch case. Regardless of the specific number, a general finding from the literature is that interbank markets present low density.

Density (%)	7.37
Average Degree	1.40
Average Path Length	2.60
Betweenness Centrality (Av.)	7.10
Eigenvector Centrality (Av.)	0.13
Clustering Coefficient (Av.)	0.03
Assortativity	
out-in degree	-0.15
in-out degree	0.26
out-out degree	-0.31
in-in degree	-0.44
# Intermediaries	9
# Core Banks	3
Interbank Assets/Total Assets (%)	23.68
Equilibrium Interbank Rate (%)	2.98

Table 2: Network characteristics - Baseline setting

as gatekeepers.

The clustering coefficient measures the tendency of neighbors of a given node to connect to each other, thereby generating a cluster of connections. For our network configuration the average clustering coefficient is low, especially in relation to other types of networks (for instance, trade networks), and in line with evidence on real-world interbank networks (see Hüser (2015) for a summary of stylized facts).

The assortativity coefficient aims at capturing the tendency of high-degree nodes to be linked to other high-degree nodes. As noted by Bargigli et al. (2015), interbank networks tend to be dis-assortative, implying that high-degree nodes tend to connect to other high-degree nodes less frequently than would be expected under the assumption of a random rewiring of the network that preserves the nodes' degrees. With the exception of the *in-out* coefficient, which presents positive assortativity, our network presents in fact dis-assortative behavior. These results are in line with those observed in the data (see for instance Bargigli et al. (2015) or Alves et al. (2013) among others). Notice that dis-assortative behavior is associated with core-periphery structures; this is true both in the data and in our model. As already mentioned above, a necessary condition for the presence of a core-periphery structure is to have banks which both borrow and lend, i.e. to have intermediaries. Out of the 20 banks in our model, 9 are intermediaries. Furthermore, from these 9 banks, 3 constitute the core of the network.[34]

To sum up topological properties of our network as captured by most synthetic indicators, are in line with their empirical counterparts. In particular the network is characterized by low density, low clustering, low average path length, dis-assortative behavior and a core-periphery structure in which the core is a strict subset of all intermediaries. Further results for the simulation of the baseline network can be found in Appendix B.

[34]Our conception of the core follows that of the seminal work of Craig & von Peter (2014). We thank Ben Craig for sharing the code for the computation of the core-periphery structure.

5. Model Response to Shocks

An essential prerequisite of prudential regulation consists in measuring systemic risk and identifying systemically important banks. Assessing the contribution of each bank to risk propagation is indeed a crucial aspect of the inspecting activity that supervisors conduct to prevent crises. To this aim and prior to the analysis of the prudential policy we present some metrics that measure the contribution of each bank to systemic risk and that allow the supervisor to detect systemically important intermediaries. In this section we focus specifically on the Shapley value. Given the system-wide default probability following a distribution of banks' shocks, the Shapley value determines the contribution of each bank to it.

Figure 3 presents each bank's contribution to systemic risk, based on the Shapley value methodology. The clearing algorithm for the interbank market used is that of Eisenberg & Noe (2001). We simulate shocks to the value of non-liquid assets with multivariate normal distributions. In response to those shocks all channels of contagion are activated. First and foremost, banks become more cautious and start to hoard liquidity thereby producing a credit crunch in the interbank market. The fall in the supply of liquidity together with the adverse shocks on some banks' assets produces many de-stabilizing effects: some banks stop honoring their debt obligations, most banks de-leverage and some banks sell their non-liquid assets to meet equity and liquidity requirements. All those actions trigger further losses. Liquidity hoarding reduces the system's resiliency to shocks: banks which do not repay their debt transmit direct losses to exposed lenders; fire sales of non-liquid assets, by triggering falls in assets prices, transmit indirect losses to the balance sheets of other banks because of asset commonality.

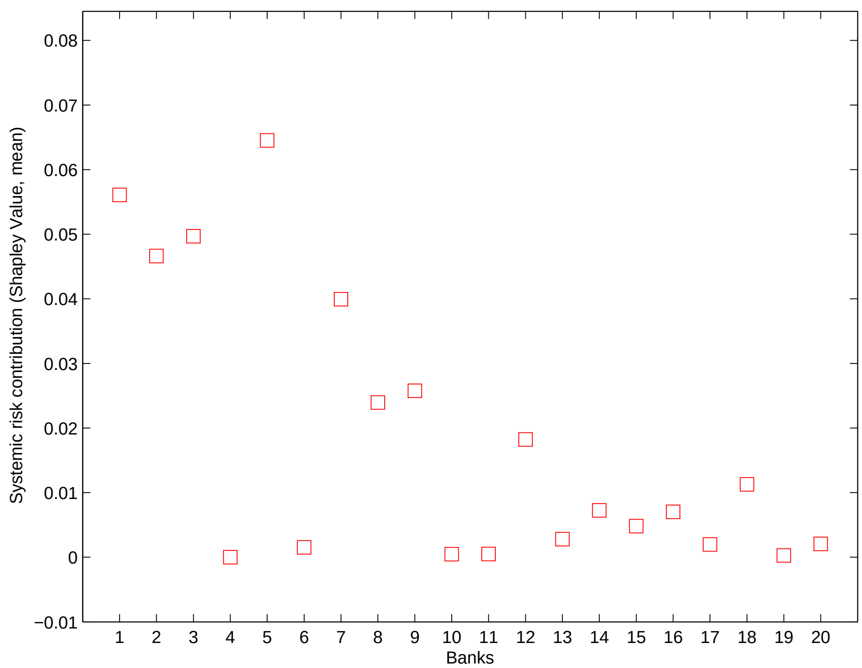


Figure 3: Contribution to systemic risk (mean Shapley Value) by bank

By jointly analyzing the data in Figure 3 and the banks' optimal portfolio allocations as reported in Table B.3 in Appendix B we find that the banks which contribute the most to systemic risk are the ones which both borrow in the interbank market and invest highly in non-liquid assets.[35] Generally speaking we find a strong connection between Shapley value and total assets.[36] Interbank borrowing increases the extent of risk transmission through direct interconnections, while investment in non-liquid assets increases the extent of risk transmission via fire sale externalities. The more banks borrow and the more banks invest in non-liquid assets, the larger is their contribution to cascading defaults and to systemic risk. The rationale behind this is as follows. Banks which leverage more in the interbank market are clearly more exposed to the risk of default on interbank debts. The larger is the size of debt default the larger are the losses that banks transmit to their counterparts. Borrowing banks therefore contribute to systemic risk since they are the vehicle of network/interconnection externalities. On the other hand, banks which invest more in non-liquid assets transmit risks since they are the vehicle of pecuniary externalities. The higher is the fraction of non-liquid asset investment, the higher is the negative impact that banks' fire sales have on market prices. The higher is the collapse in market prices, the higher are the accounting losses experienced by all other banks due to asset commonality and mark-to-market accounting. Notice that banks which invest and borrow a lot are also those with the highest returns on non-liquid assets investment. As banks invest more they also grow in size, consequently there is also a positive correlation between banks' size and systemic risk. By jointly observing Figure 3 and total assets as from Table B.3 (which presents the optimal balance sheet structure in the baseline setting) we can infer, for instance, that smaller banks tend to contribute less to systemic risk. While the Shapley value shows a strong connection to total assets, the connection to other balance sheet items or relevant balance sheet ratios is not particularly strong (see Figure B.6 in Appendix B.2).[37] To assess the role of banks' risk aversion and precautionary savings on the transmission of risk we present the main results for systemic risk by comparing the models with and without risk averse banks: see subsection 6.1.

To test the robustness of the Shapley value we compute the ranking of systemically important banks also using alternative metrics, namely network centrality indicators. Due to space considerations, simulation results for those are presented and discussed in Appendix B.

6. Policy Analysis: Stability versus Efficiency

The new Basel III regulatory framework is an approach centered on multiple, complementary prudential standards, with minimum requirements for both capital and liquidity. A crucial policy question is whether adjustments to these requirements should be expected to affect systemic risk and banks' individual contributions to it. This will involve trade-offs. For instance, higher equity requirements are likely to be beneficial because they reduce the extent of banks' leverage and increase their ability to absorb losses. Yet, this may have to be traded-off against an impaired ability to invest in illiquid assets, as tighter equity requirements typically increase the extent of pecuniary externalities (via a higher elasticity of excess asset demand). Intuitively, the tighter the constraint, the higher may be the need to sell assets to meet the regulatory requirements in the face of adverse shocks. Note that, in our model, the investment in external non-liquid assets is a proxy for the integration of the banking system with the real economy so that we can take it as a measure of efficiency and as a crude substitute for welfare. Hence an increase in equity

[35] Usually those are also the banks with the higher returns on non-liquid assets investment.

[36] That said, it is important to note that "size" alone does not capture the role of too-interconnected-to-fail, whereas the Shapley value does. This is a way of incorporating the interconnectedness metric used by the Financial Stability Board to designate systemic institutions.

[37] This holds irrespective of the matching algorithm used: in exercises not reported here we have computed the interbank matrix using other matching algorithms (which deliver a different network topology) and the qualitative message stays unaltered.

requirements generates a trade off between financial stability (mitigation of systemic risk) and efficiency (maximization of investment in external assets). Similar trade-offs apply to liquidity requirements.

We inspect the variations in systemic risk and in the optimal allocation for different values of the liquidity requirement α and of the equity requirement γ. As in the baseline setting, we evaluate the model subject to 1000 shocks.

Figure 4 summarizes the main results from the policy experiments. We consider six indicators. From left to right in the first half (first and second row) of the table: systemic risk (measured as the ratio of assets of defaulting banks to total assets of the system), interbank lending as a share of total assets, non-liquid assets as a share of equity; in the second half (third and fourth row): equilibrium interbank interest rate, aggregate leverage (solid line, left axis) and simple average leverage of the 5 most leveraged banks (dashed line, right axis) and network density. For each indicator there are two panels: the upper (lower) panel shows the dynamic pattern of the indicator as the liquidity (equity) requirement increases.

We start by examining how overall systemic risk and the contribution of each bank to it change when altering the two policy parameters. Let's consider first the effects of changes in the liquidity requirement (first and third row of the figure). At first glance, overall systemic risk shows a downward trend when the liquidity parameter α goes up (see the first panel in the first row of Figure 4). That said, as is obvious from the chart, starting from values around 0.2 systemic risk exhibits a jig-saw behavior within this general downward trend. Such behavior poses a challenge for prudential regulators as even small changes in the liquidity ratio can have significant effects on systemic risk. It is therefore important to understand the origins of this pattern and its implications. First of all, by simulating the model with risk-neutral bankers we observe a much smoother pattern. Hence convex optimization problems are a primary reason for this behavior. Indeed in this case the dynamic of risk has first order effects on banks' optimal decisions, which in turn become much more sensitive to all channels responsible for risk amplification (fire sale and network externalities primarily). This has important consequences for the prudential authority. It highlights the fact that the design of regulation in complex environments that feature significant risk amplification channels is a much more demanding task than the one envisaged in simple banking models featuring linear optimization and no market interactions. Notice that in our simulations we have included error bands around the dynamic of systemic risk in response to changes in regulation. Those are instructive particularly in the context of a complex and uncertain policy environment: error bands indeed provide a metric for the possible variation in the target (systemic risk) for a given policy action. There are some banks that always contribute to systemic risk (mostly banks 1, 2, 3, 5, 12 and 16, see Figure C.9). The rationale for these results is as follows. As banks must hold more liquidity for precautionary motives, their exposure in the interbank market declines, though this is not reflected in interbank assets as a share of total assets (second panel in the first row) since the reduction in non-liquid assets is quite substantial (third panel in the first row). The interbank interest rate increases due to the scarce supply of liquidity (first panel in the third row) and banks' investment in non-liquid assets declines as available liquidity falls. Overall, there is a strong reduction in the scope for fire sale externalities and a relatively milder increase in the scope for network externalities. Since both systemic risk and the investment in non-liquid assets go down for the range of values of α under consideration, we detect a trade off between the mitigation of financial instability (as proxied by systemic risk) and efficiency (as proxied by aggregate investment in non-liquid assets).

Results are somehow more complex when we increase the equity requirement. As γ increases, overall systemic risk declines over an initial range, but it stays flat after roughly 0.13 (first panel in the second row). Banks de-leverage (second panel in the fourth row) and the interbank interest

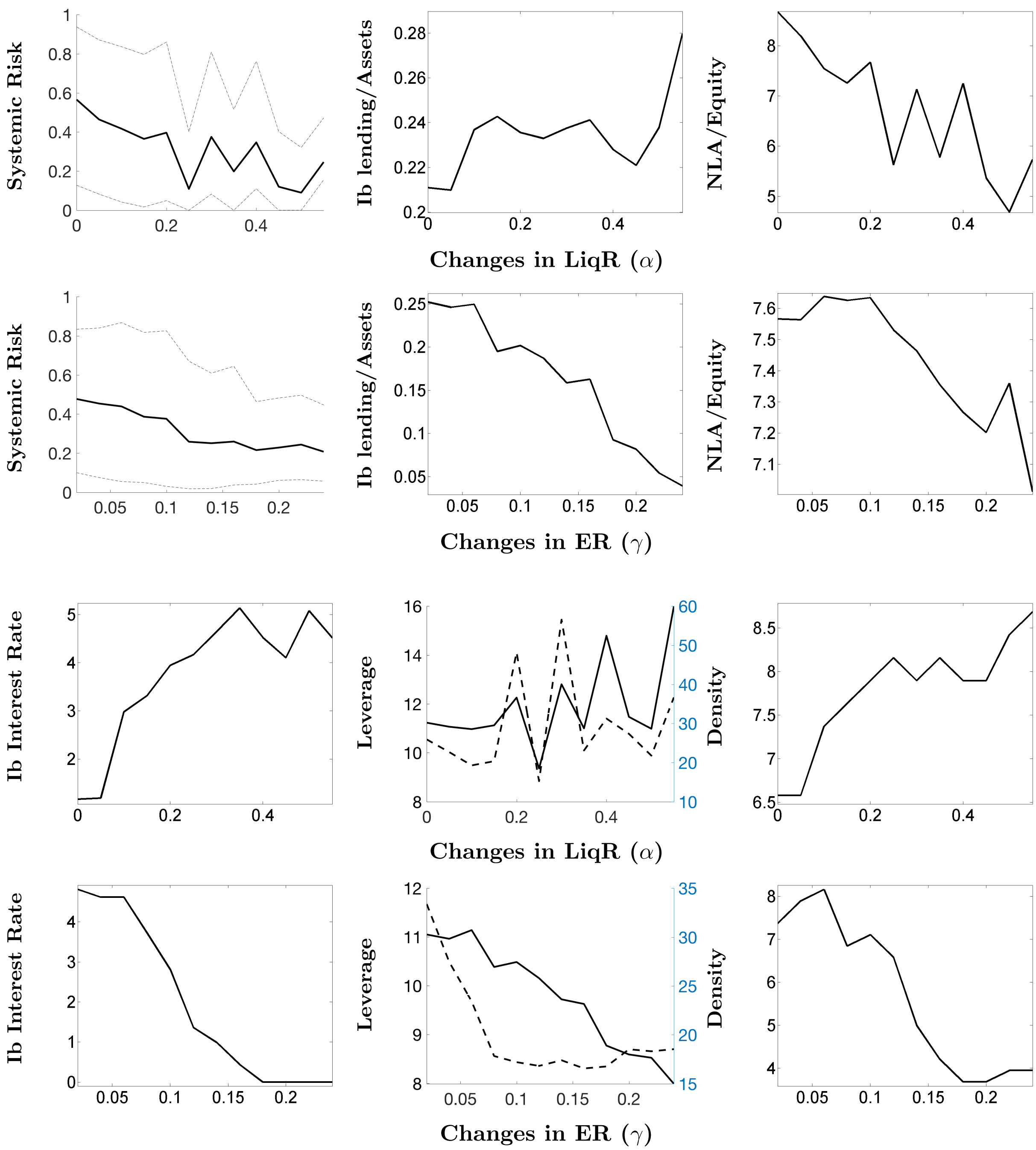


Figure 4: Main results of the policy analysis. On the y-axis, in the panels of the first half of the table (first and second row), from left to right we measure: systemic risk, interbank lending over total assets, non-liquid assets over equity; in the panels of the second half (third and fourth row): equilibrium interbank interest rate, aggregate leverage (solid, left-hand-side) and simple average leverage of 5 most leveraged banks (dashed, right-hand-side), and network density.

rate declines (first panel in the fourth row) as the demand of liquid funds has gone down. This reduces the overall scope for transmitting default losses, and in fact interbank lending as a percentage of assets reaches very low values (second panel in the second row). However, banks also reduce the amount of liquid assets (not shown here), while keeping the amount of non-liquid asset investment roughly unchanged (as a fraction of equity) for an initial range and then only reducing it slightly (third panel in the second row).[38] The scope of risk transmission through fire sales is therefore only slightly reduced. Increasing the equity requirement above 10% seems to have a non-negligible impact on systemic risk, while at the same time not reducing efficiency as strongly as with increases in the liquidity requirement. This improvement is concentrated in the range 10-13%, as beyond that point systemic risk is largely unresponsive to a higher equity requirement.

As for the contribution of each bank to overall systemic risk (see Shapley values in Figure C.10) we observe that, while most banks tend to transmit less risk as γ increases, a few instead tend to contribute more. Since all banks are less exposed to the interbank market the scope for loss cascades through network linkages is reduced. On the other hand some banks invest more in non-liquid assets. This exposes the latter to the swings in the market price for non-liquid assets and increases the probability that they will engage in fire sales.

The sixth panel of the third (fourth) row describes the evolution of network density for increasing levels of the liquidity (equity) requirement.[39] Network density is increasing with the liquidity requirement parameter α and decreasing with capital requirement γ. In the latter case, density is roughly halved over the range of values of γ considered. While the upper limit for network density is roughly the same for the two policy exercises, it is worth noting that in the case of changes in the liquidity requirement, density never falls below the starting value of approximately 6.5%, whereas it falls to almost 3.5% when increasing the equity requirement. When changing the equity requirement there is a noticeable drop starting at around $\gamma = 0.12$. The reason for this can be seen in Figure C.8b in Appendix C. The number of active banks in the interbank market drops substantially. This is true, in particular, for those banks that both borrow and lend. If we take the number of banks on both sides of the market as a proxy for intermediation activity, Figure C.8b shows that intermediation reaches a peak when $\gamma = 0.12$. As the equity requirement increases, fewer banks are active in the market and the ones that are actually active demand less liquidity relative to existing supply, forcing the continuous downward trend in the interbank rate that we see in the first panel, fourth row of Figure 4.

As Figure C.8a shows, no such development occurs when increasing the liquidity requirement. This essentially leaves the number of active banks unchanged. When the liquidity requirement increases two countervailing forces seem at work that balance each other. As the liquidity requirement goes up, banks supply less liquidity in the interbank market and this has a depressing effect on density and other measures such as closeness (not shown here). On the other hand, some banks increase their demand of liquid funds driving the interbank rate up and inducing other banks to substitute investment in non-liquid assets with interbank lending. This asset substitution effect increases the available liquidity in the interbank market (as shown in the second panel, first row of Figure 4), which in turn has a positive impact on density and related measures.

[38] Notice that in our model issuing equities does not entail adjustment costs. In reality, depending on the degree of financial market development, some adjustment costs might make equity adjustment stickier. If so, it is possible that, in the face of increases in equity requirements, banks might decide to partly increase equities and partly reduce their assets in order to re-balance the ratio. In any case we would observe a stronger fall in non-liquid assets under an increase in equity requirements than under an increase in liquidity requirements.

[39] Average degree, path length and clustering coefficients paint a very similar picture so we left them out for the sake of space.

While we do not evaluate the joint optimality of the regulatory requirements in our model, the policy exercise provides support for the notion that the different requirements can be combined to complement each other. In Figure 4, whenever we evaluate the change in a policy parameter (say, liquidity), the other policy parameter is kept at its benchmark value. Thereby, changes in the liquidity (equity) requirement in the context of a fixed equity (liquidity) requirement lead to improvements in systemic risk. Furthermore, the path towards reaching the joint benchmark scenario (i.e. when moving from $\alpha = 0$ to $\alpha = 0.1$ in the case of liquidity and from $\gamma = 0$ to $\gamma = 0.08$ for equity) is associated with reduced systemic risk and minimal negative adverse effects in terms of non-liquid asset investment and interbank market activity.

To sum up, within the confines of our model,[40] increasing the liquidity requirement unambiguously reduces systemic risk as it reduces the investment in non-liquid assets while only marginally increasing the scope for network externalities. The fall in the overall non-liquid asset investment shows however that an increase in the liquidity requirement reduces system efficiency. An increase in the equity requirement also decreases systemic risk (though the latter remains flat after $\gamma = 0.13$), but without a substantial decrease in efficiency.

6.1. Systemic Risk and Contagion Channels

To assess the contribution of each of the channels considered (liquidity hoarding, interconnections and fire sales) we compare the evolution of systemic risk (under different values for α and γ) under four alternative models:

- **Model 1**: this model is the benchmark considered so far, featuring risk averse banks and the interaction of fire sales and network externalities.
- **Model 2**: this model has risk neutral instead of risk averse banks, hence the objective function is linear and simply given by utility of expected profits, which in this case is equal to expected utility of profits. The constraints remain the same, and fire sales and interbank contagion are also kept. It is worth noting that in this model there are no banks that participate on both sides of the market simultaneously, i.e. they are either borrowers or lenders.
- **Model 3**: this model is similar to *Model 1* but it eliminates the fire sales channel. Non-liquid assets are no longer a choice variable of banks and are instead calibrated by the values banks would have chosen if given the chance. Once a shock hits banks cannot sell the assets and the transmission of distress takes place only through the interbank channel.
- **Model 4**: this model is a small variation of *Model 3*, in which we set the risk aversion parameter to $\sigma = 0$ (i.e. risk-neutral banks).

Results from the comparison exercise are presented in Figure 5, which shows the effects of changes in the liquidity and equity requirements on systemic risk, interbank lending over total assets and non-liquid assets over equity.

We can summarize the difference in results as follows. First, the benchmark model (with all contagion channels) shows larger swings in the changes of systemic risk with respect to α and γ. This is due to the fact that the presence of risk averse agents features higher non-linearities by triggering precautionary saving. Second, in *Model 4* systemic risk increases with respect

[40] It is important to note that ours is a stylized model, not encompassing general equilibrium effects for the economy at large. Importantly, it does not feature a central bank, which could mitigate the stress in the banking system following a shock. Such interventions by a central bank can be critical, in particular in terms of liquidity.

to increases in α. This is empirically puzzling, although it is internally consistent with the assumptions of model 4, namely the absence of other investment opportunities beyond those in non-liquid assets and the assumption of $\sigma = 0$. As the liquidity requirement increases, banks which are short of funds increase their demand of interbank borrowing. This raises the interbank rate and makes interbank lending attractive for banks which have excess liquidity. Overall network linkages in the interbank market increase and so does contagion of default risk.

Our benchmark model has two important appealing features. First, it generates realistic amplifications of risk and features non-linearity in transmission channels: both are realistic features of banking panics triggered by contagion channels. Second, and contrary to alternative models considered, it provides reasonable predictions for the response of the network to changes in policy regulations.

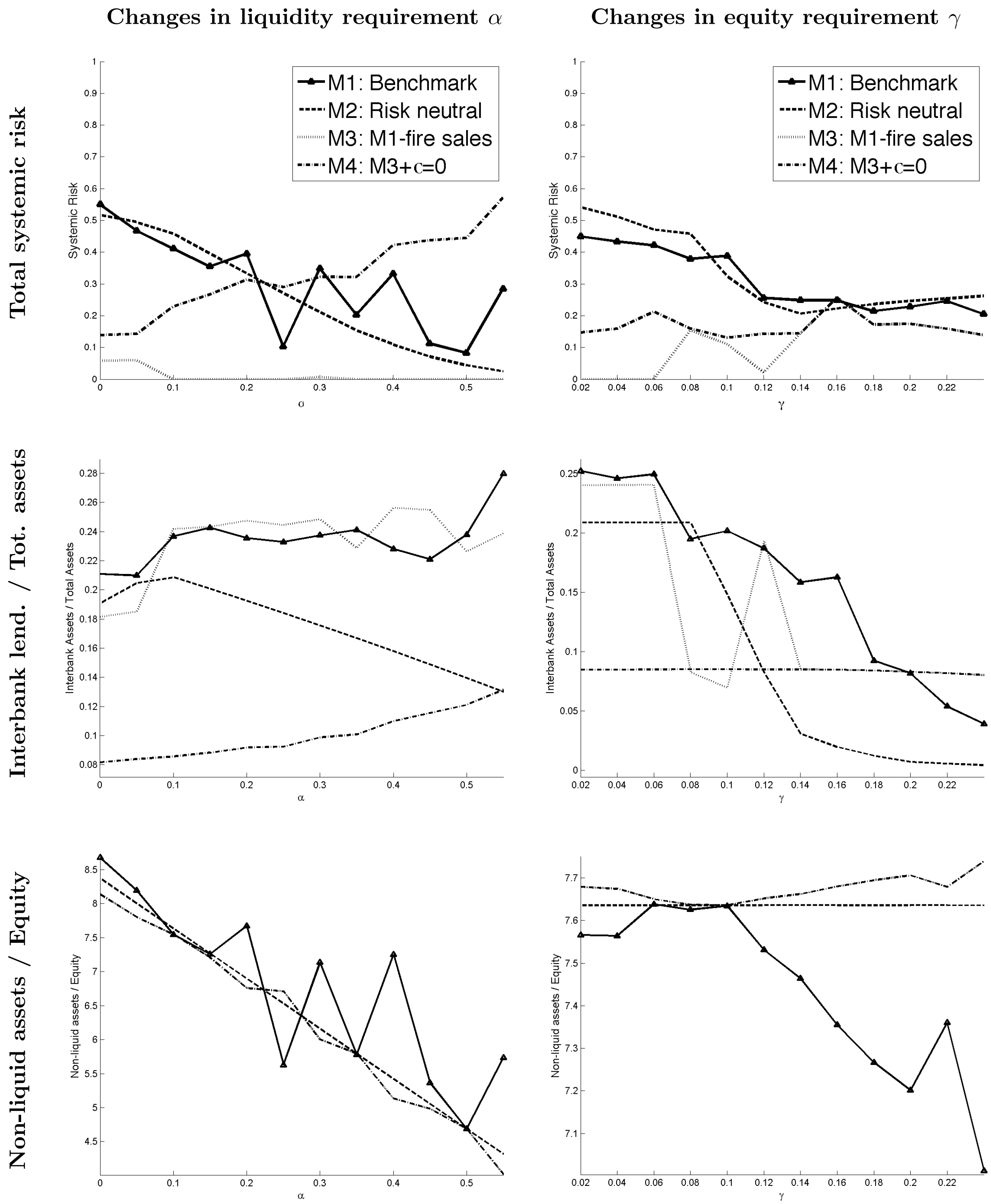


Figure 5: Model Comparison

7. Concluding Remarks

We have analyzed a banking network model featuring risk transmission via different channels. Banks in our model are risk averse and solve an optimal portfolio problem. The individual optimization problems and the market clearing processes deliver a matrix of network links in the interbank market. Each bank can be both borrower and lender vis-à-vis different counterparties. Shocks to one bank are transmitted through defaults on interbank debt, through price collapses of non-liquid assets triggered by fire sales or through liquidity hoarding. Clearing in the market takes place through a price tâtonnement iterative process and through a trading matching algorithm, namely closest matching (or minimum distance). The network thus obtained replicates some characteristics from the empirical counterparts. In particular, it presents low density, low average degree, dis-assortative behavior and a core-periphery structure.

We use our banking network to assess the role of prudential regulation in reducing systemic risk. We find that increasing the liquidity requirement unequivocally reduces systemic risk and the contribution of each bank to it. As banks must hold more liquidity for precautionary motives, their exposure in the interbank market declines, though this is not reflected in interbank assets as a share of total assets as the reduction in non-liquid assets is quite substantial. The former limits somewhat the scope for network externalities, whereas the latter substantially reduces the scope for pecuniary externalities. The reduction in non-liquid assets is so strong that there is an associated cost to it in terms of efficiency of the system, highlighting the existing trade-off between stability and efficiency. An increase in the equity requirement instead does not present this strong trade-off. Systemic risk decreases, in particular for an initial range of values of γ. The scope for network externalities is persistently reduced as the share of interbank assets over total assets steadily declines to reach very low values in the upper range of γ. While there is also a slight reduction in the scope for fire sales externalities, the reduction in non-liquid assets is relatively minor. The system becomes more homogeneous and the potential damage from interbank market collapses is markedly reduced. This comes at the expense of having less banks trade in the interbank market, with an associated reduction in its density.

In light of recent policy discussions, our model highlights some important trade-offs to bear in mind. First, the model suggests that increasing the equity requirement relative to the benchmark can yield benefits in terms of more stability without any cost in terms of a substantial reduction in banks' non-liquid assets investments. At the same time, there are limits to this strategy. Too high a regulatory burden can make the requirements close to binding, which can increase the likelihood of fire sales in the aftermath of large-scale shocks. On the other side, the results of our model give strong support for introducing liquidity requirements in terms of their potential for systemic risk reduction, though with the caveat of costs in terms of reduced non-liquid asset investment. Finally, our results provide general support for the Basel III approach based on complementary regulatory metrics.

We have explored the effects of contagion and risk transmission stemming from the asset side of banks' balance sheets. Incorporating risk originating from the liability side would take our model one step further in the direction of realism. We leave this avenue for future research.

References

Acharya, V. V. & Merrouche, O. (2013). Precautionary hoarding of liquidity and inter-bank markets : Evidence from the sub-prime crisis. *Review of Finance*, *17*(1), 107–160.

Afonso, G. & Shin, H. S. (2011). Precautionary demand and liquidity in payment systems. *Journal of Money, Credit and Banking*, *43*, 589–619.

Allen, F. & Gale, D. (2000). Financial contagion. *Journal of Political Economy*, *108*(1), 1–33.

Alves, I., Ferrari, S., Franchini, P., Heam, J.-C., Jurca, P., Langfield, S., Laviola, S., Liedorp, F., Sánchez, A., Tavolaro, S., & Vuillemey, G. (2013). Structure and resilience of the european interbank market. Occasional Papers 3, European Systemic Risk Board.

Anand, K., Craig, B., & von Peter, G. (2015). Filling in the blanks: network structure and interbank contagion. *Quantitative Finance*, *15*(4).

Bargigli, L., di Iasio, G., Infante, L., Lillo, F., & Pierobon, F. (2015). The multiplex structure of interbank networks. *Quantitative Finance*, *15*(4).

Battiston, S., Delli Gatti, D., Gallegati, M., Greenwald, B., & Stiglitz, J. E. (2012). Liaisons dangereuses: Increasing connectivity, risk sharing, and systemic risk. *Journal of Economic Dynamics and Control*, *36*(36), 1121–1141.

Becker, G. S. (1973). A theroy of the marriage market: Part i. *The Journal of Political Economy*, *81*(4), 813–846.

Bluhm, M., Faia, E., & Krahnen, J. P. (2014). Endogenous banks' networks, cascades and systemic risk. Working Paper 12, SAFE.

Boss, M., Elsinger, H., Summer, M., & Thurner, S. (2004). Network topology of the interbank market. *Quantitative Finance*, *4*, 677–684.

Caballero, R. J. & Simsek, A. (2013). Fire sales in a model of complexity. *Journal of Finance*, *68*(6), 2549–2587.

Caccioli, F., Farmer, J. D., Foti, N., & Rockmore, D. (2014). Overlapping portfolios, contagion, and financial stability. *Journal of Economic Dynamics and Control*, (http://dx.doi.org/10.1016/j.jedc.2014.09.041).

Cifuentes, R., Ferrucci, G., & Shin, H. S. (2005). Liquidity risk and contagion. *Journal of the European Economic Association*, *3*(2-3), 556–566.

Craig, B. & von Peter, G. (2014). Interbank tiering and money center banks. *Journal of Financial Intermediation*, *23*(3), 322–347.

Drehmann, M. & Tarashev, N. (2013). Measuring the systemic importance of interconnected banks. *Journal of Financial Intermediation*, *22*(4), 586–607.

Duffie, D. & Zhu, H. (2011). Does a central clearing counterparty reduce counterparty risk? *Review of Asset Pricing Studies*, *1*(1), 74–95.

Eisenberg, L. & Noe, T. H. (2001). Systemic risk in financial networks. *Management Science*, *47*(2), 236–249.

Elliott, M. L., Golub, B., & Jackson, M. O. (2014). Financial networks and contagion. *American Economic Review*, *104*(10), 3115–53.

Gai, P., Haldane, A., & Kapadia, S. (2011). Complexity, concentration and contagio. *Journal of Monetary Economics*, *58*(5).

Gale, D. & Shapley, L. (1962). College admissions and the stability of marriage. *American Mathematical Monthly*, *69*, 9–15.

Glasserman, P. & Young, P. (2014). How likely is contagion in financial networks? *Journal of Banking & Finance*, (http://dx.doi.org/10.1016/j.jbankfin.2014.02.006).

Greenwood, R., Landier, A., & Thesmar, D. (2015). Vulnerable banks. *Journal of Financial Economics*, *115*(3), 471–485.

Gul, F. (1989). Bargaining foundations of shapley value. *Econometrica*, *57*(1), 81–95.

Halaj, G. & Kok, C. (2015). Modeling emergence of the interbank networks. *Quantitative Finance*, *15*(4).

Hüser, A.-C. (2015). Too interconnected to fail: A survey of the interbank networks literature. *Journal of Network Theory in Finance*, *1*(3), 1–50.

Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). *Microeconomic Theory*. Oxford University Press.

Memmel, C. & Sachs, A. (2013). Contagion in the interbank market and its determinants. *Journal of Financial Stability*, *9*(1), 46–54.

Shapley, L. (1953). A value for n-person games. In H. Kuhn & A. Tucker (Eds.), *Contributions to the Theory of Games*, volume II of *Annals of Mathematical Studies* (pp. 307–317). Princeton University Press.

Shapley, L. S. & Shubik, M. (1972). The assignment game i: The core. *International Journal of Game Theory*, *1*, 111–130.

Shubik, M. (1999). *The Theory of Money and Financial Institutions*, volume Volume 1. The MIT Press.

van Lelyveld, I. & In't Veld, D. (2012). Finding the core: Network structure in interbank markets. DNB Working Papers 348, Netherlands Central Bank.

Appendix A. Shock Transmission

The shock transmission process can be succinctly summarized as follows. After the vector of shocks is drawn the supply of non-liquid assets will be affected and therefore the price will have to be adjusted. Following such adjustment, some banks may not be able to fulfill their interbank commitments. Such banks will liquidate their entire non-liquid asset holdings, pay as much as they can to interbank creditors and be added to the default set. The interbank adjustment is done following the now classic algorithm outlined in Eisenberg & Noe (2001). Note that, at this stage, interbank connections are taken as given and banks are not re-optimizing; changes to the interbank market structure are at this point the result of applying the clearing mechanism of Eisenberg & Noe (2001). At the same time, many banks may not be able to fulfill the equity requirement. Within this group, two sub-groups may be distinguished. First there are those banks that after selling part of their non-liquid asset holdings will be able to fulfill the equity requirement; the second group cannot fulfill the requirement even after selling all their non-liquid assets. The former group will just liquidate what it needs in order to comply with requirements, whereas the latter group will liquidate all and be added to the default set. All the non-liquid assets put on the market by all banks will be used for a recalculation of the price p and start a new round of the transmission process. When no more defaults occur the algorithm stops and systemic risk is computed as set out in the main text.

Appendix B. Additional results for baseline scenario

Appendix B.1. Balance sheet characteristics and systemic importance ranking

	Banks																			
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Cash	56.9	53.6	48.7	30.5	57.0	46.6	36.4	20.5	22.9	25.8	3.9	14.3	12.7	10.7	6.8	12.1	12.9	6.7	7.3	7.7
Non-liquid assets	606.0	807.3	529.2	211.9	838.3	296.6	250.8	428.7	284.9	40.7	8.2	252.8	24.0	111.1	88.9	51.8	63.1	111.1	100.0	11.6
Interbank lend.	92.3	0.0	74.8	144.7	0.0	175.8	153.6	0.0	64.6	234.5	46.9	44.7	138.0	0.0	0.0	62.1	59.0	0.0	0.0	66.7
Interbank borr.	130.7	234.9	117.2	29.1	244.3	0.0	19.8	196.2	117.5	0.0	0.0	145.8	31.7	4.8	19.7	0.0	0.0	40.8	25.3	0.0
Total Assets (A)	755.2	860.9	652.7	387.1	895.3	519.0	440.8	449.2	372.5	301.0	59.0	311.8	174.7	121.8	95.7	126.0	135.0	117.8	107.3	86.0
Equity	55.6	90.0	48.5	53.0	81.0	53.0	57.0	48.0	26.0	43.0	20.0	23.0	16.0	10.0	8.0	5.0	6.0	10.0	9.0	9.0
Leverage	13.6	9.6	13.5	7.3	11.1	9.8	7.7	9.4	14.3	7.0	3.0	13.6	10.9	12.2	12.0	25.2	22.5	11.8	11.9	9.6
Liquid assets/A (%)	19.8	6.2	18.9	45.3	6.4	42.9	43.1	4.6	23.5	86.5	86.2	18.9	86.3	8.8	7.1	58.9	53.3	5.7	6.8	86.5
Interbank lend./A (%)	12.2	0.0	11.5	37.4	0.0	33.9	34.8	0.0	17.4	77.9	79.6	14.3	79.0	0.0	0.0	49.3	43.7	0.0	0.0	77.6
Interb. borr./Liab. (%)	18.7	30.5	19.4	8.7	30.0	0.0	5.1	48.9	33.9	0.0	0.0	50.5	20.0	4.3	22.5	0.0	0.0	37.9	25.7	0.0
Nla/Dep. (%)	106.5	150.6	108.7	69.5	147.1	63.6	68.9	209.1	124.4	15.8	20.9	176.8	18.9	103.8	130.7	42.8	48.9	165.8	137.0	15.0
Nla/Equity (%)	1090.7	897.0	1092.1	399.9	1035.0	559.6	440.0	893.1	1095.9	94.7	40.8	1099.3	149.9	1111.1	1111.1	1035.2	1051.0	1111.1	1111.1	128.8
Interb. lend./Equity (%)	166.1	0.0	154.3	273.0	0.0	331.7	269.4	0.0	248.6	545.3	234.7	194.3	862.8	0.0	0.0	1242.8	984.0	0.0	0.0	741.2

Table B.3: Optimal balance sheet items - Baseline setting

Centrality	**Banks**																			
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
In-degree	5	3	6	7	8	16	1	2	9	17	10	11	12	13	14	18	19	4	20	15
Out-degree	11	15	5	6	16	12	17	18	1	7	2	3	4	19	20	13	14	8	9	10
In-out degree	12	8	9	10	15	16	1	2	3	13	4	5	6	17	18	19	20	7	14	11
Closeness-in	13	4	11	9	15	16	2	3	12	17	10	14	8	5	7	18	19	1	20	6
Closeness-out	11	15	6	10	16	14	17	18	3	9	2	4	1	19	20	12	13	7	8	5
Closeness-in-out	14	8	11	12	20	19	4	5	6	15	3	7	1	9	16	17	18	2	13	10
Betweenness	9	10	3	8	11	12	13	14	2	15	4	7	6	16	17	18	19	1	20	5
Eigenvector (left)	5	9	3	12	13	14	10	6	1	15	4	16	8	11	17	18	19	2	20	7
Eigenvector (right)	3	8	5	9	10	11	12	13	7	14	4	15	1	16	17	18	19	6	20	2

Table B.4: Systemic importance ranking by network centrality measures - Baseline setting

Appendix B.2. Additional results on Shapley value and systemic importance

Figure B.6 plots the Shapley value versus bank characteristics. Results point to a strong connection with total assets as discussed in the main body of the paper. The connection to other balance sheet items is rather weak.

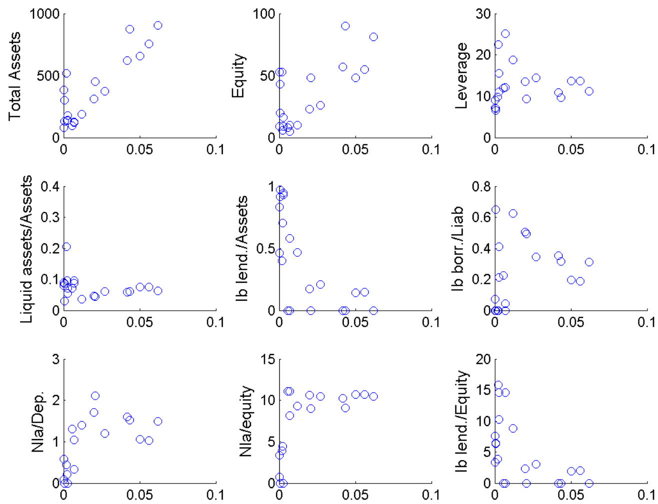


Figure B.6: SV vs. bank characteristics

For systemic importance measures we consider network centrality indicators. In graph theory and network analysis the centrality of a vertex or node measures its relative importance within the graph. In particular, we consider the following measures: degree, closeness, betweenness and eigenvector centrality.[41] Degree centrality captures the number of connections that a bank has. In networks in which the direction of links matter, like ours, it can be divided into in- and out-degree. The former accounts for the number of links "arriving" to a node, whereas the latter quantifies the number of links "leaving" a node. Closeness centrality assesses the importance of nodes based on how reachable they are from all other nodes (i.e. how "close" they are). Betweenness centrality gauges the relative importance of nodes based on how often they lie in paths connecting other nodes (i.e. how important they are as "gatekeepers"). Finally, eigenvector centrality is a generalization of degree centrality which captures the idea that connections to other nodes which are themselves well connected should carry more weight.[42]

Table B.4 above presents the ranking of systemic importance for the baseline setting and for all the measures considered. Depending on the measure one chooses to focus on, the assessment differs substantially for many banks. At one extreme we have for instance bank 7, which can be ranked first according to one measure, and up to seventeenth by another. There are some banks that are consistently ranked high or low (see for instance bank 18 for the former and bank 17 for the latter).

Another interesting question is whether systemic importance measures (i.e. centrality indicators) and systemic risk measures (i.e. Shapley value) deliver a consistent ranking. Figure B.7

[41] With this choice we cover the range of possible measures based on standard taxonomy (see for instance Alves et al. (2013)).

[42] In directed networks one can also subdivide closeness and eigenvector centrality, the former into *in* and *out* versions, the latter into *left* and *right* eigenvectors.

sheds light on this issue by plotting the Shapley value versus the different network centrality measures considered.[43] The bottom line is that there is no apparent connection between the ranking provided by the two types of measures. While this may seem disappointing at first glance, one should bear in mind that these measures are not only different algebraically, but also conceptually. Systemic importance measures are of an ex-ante nature in the sense that all that is needed for their computation is a matrix representing the connections between banks. Importantly, to construct these measures there is no need for a shock to hit the system and thereby no need either for the specification of behavioral responses. They are in this sense also static. For systemic risk indicators to be computed one needs indeed to measure risk, and to that end assume some kind of shock to the system.[44] Furthermore, behavioral responses of some sort are needed for the shock process to converge. In this respect this type of measures have a more dynamic flavor.

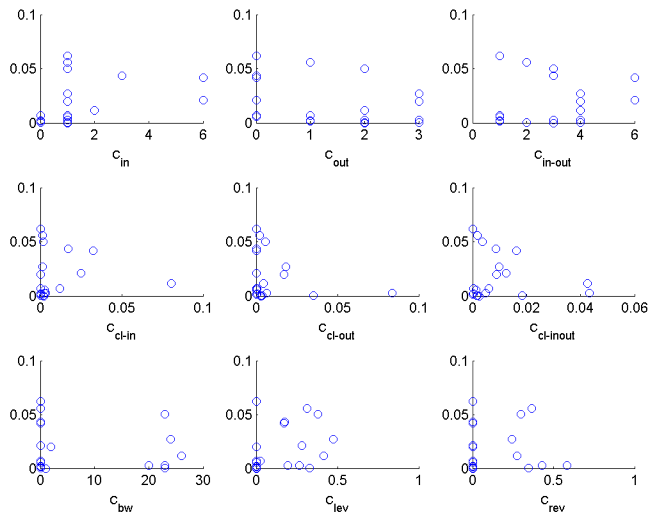
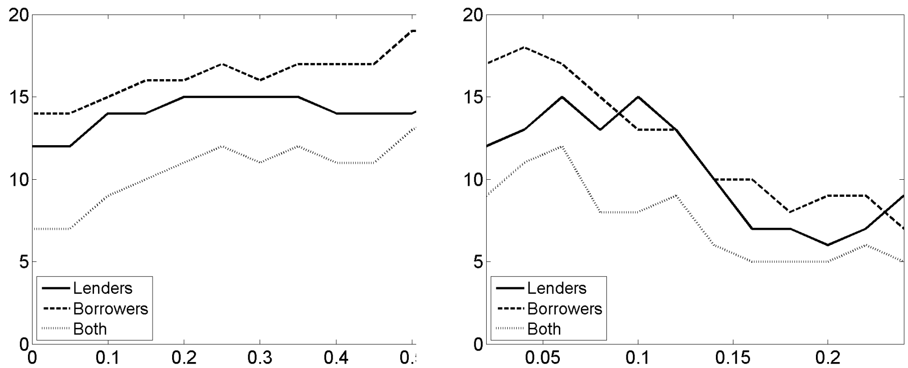


Figure B.7: SV vs. centrality measures

Appendix C. Additional results for policy analysis

[43]In the working paper version of this paper we also perform the comparison with other family of systemic importance indicators, namely input-output-based measures, and the message remains unaltered.

[44]This can be for example the targeted exogenous failure of a given institution, the sequential exogenous failure of all institutions, or as we explore in this paper, multivariate shocks to all banks simultaneously.



(a) Changes in $LiqR$ (α)

(b) Changes in ER (γ)

Figure C.8: Number of active banks in interbank market for different values of α and γ

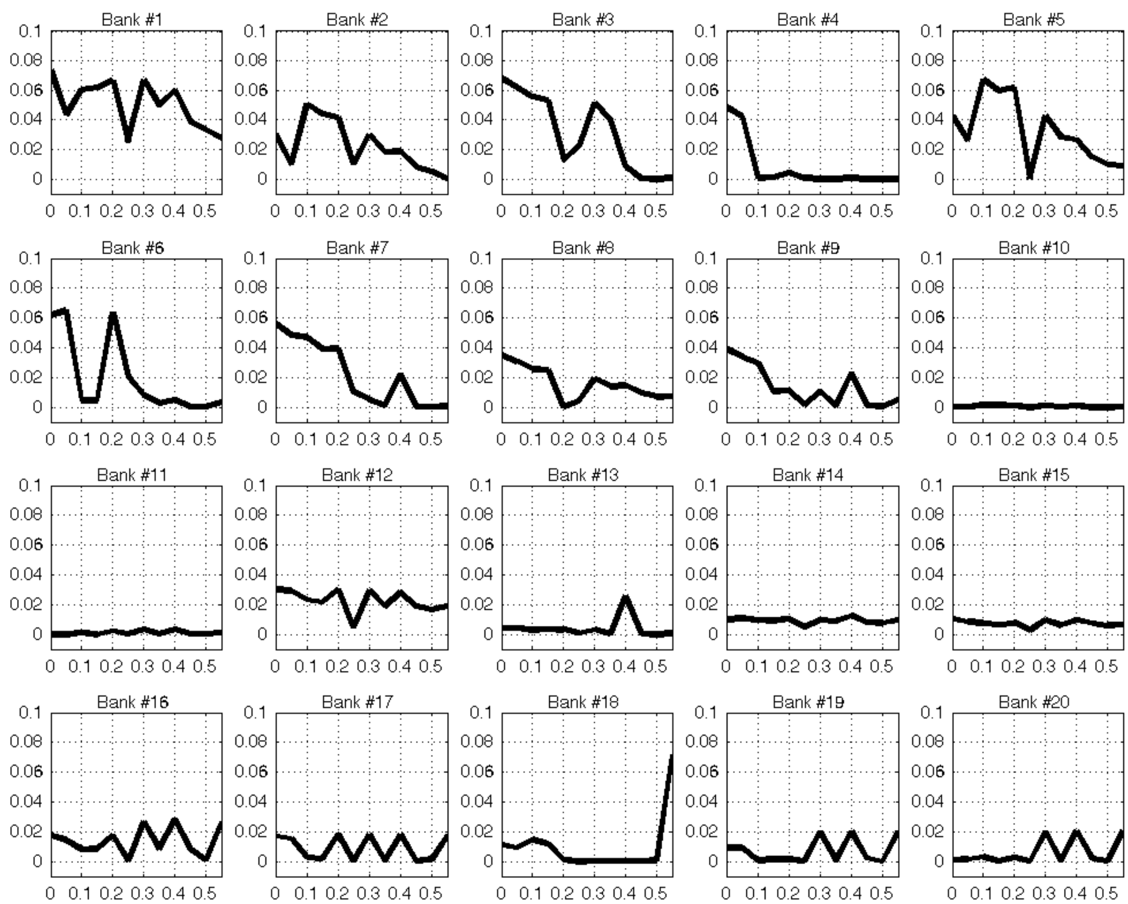


Figure C.9: Contribution to systemic risk (Shapley Value, y axis) by bank for different values of α (x axis)

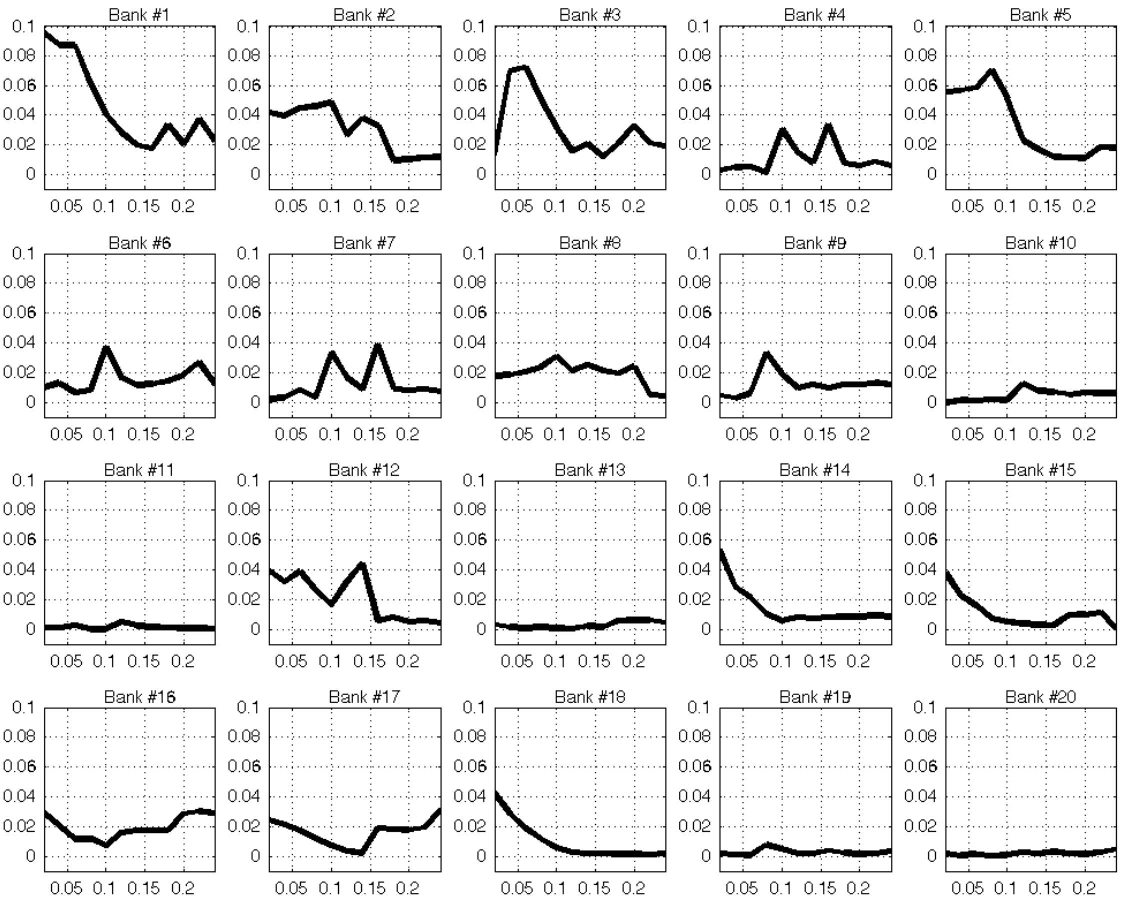


Figure C.10: Contribution to systemic risk (Shapley Value, y axis) by bank for different values of γ (x axis)

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