BANK FOR INTERNATIONAL SETTLEMENTS

BIS Working Papers

No 547

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March 2016

JEL classification: D82, D84, F31.

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This publication is available on the BIS website (www.bis.org).

ISSN 1020-0959 (print)
ISSN 1682-7678 (online)

When Pegging Ties Your Hands

Intertemporal Considerations in Currency Crises*

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First version: March 2015
This version: February 2016

Abstract

Could a less conservative central bank – one that faces a more severe time inconsistency problem – be less likely to succumb to an attack on a currency peg? Traditional currency-crisis models provide a firm answer: No. We argue that the answer stems from these models' narrow focus on how a central bank's response to a speculative attack affects output and inflation in the short run. The answer may reverse if we recognize that a credible currency peg solves time consistency issues in the long run. As a less conservative central bank stands to benefit more from tying its own hands, it should find a peg more valuable.

Keywords: currency crises, strategic uncertainty, global games, time inconsistency.

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*The views expressed in this paper are our own, not necessarily those of the BIS. For useful comments and suggestions we thank Michael Devereux, Andy Filardo, Giovanni Lombardo, Stephen Morris, Jochen Schanz, Hyun Shin, Kostas Tsatsaronis, Marco Visentini-Scarzanella and Chen Zhou.

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1 Introduction

In this paper, we revisit the seminal contribution of Rogoff (1985), who defines the conservatism of a central bank as its aversion to inflation. In a small open economy with a flexible exchange rate, conservatism determines the central bank's capacity to control currency devaluation. The less conservative the central bank, the greater its temptation to outsmart markets with a surprise devaluation in order to boost output. However, the central bank's surprises cannot be systematic. The greater is the central bank's temptation, the more forceful are the preemptive actions of rational private agents, thus leading to higher devaluation without output gains. In other words, a less conservative central bank faces greater time inconsistency problems à la Barro and Gordon (1983).

Textbook models of currency crises have shown that central bank conservatism plays a key role also when the currency regime is an *adjustable peg*.[1] Under such a regime, the central bank needs to take two decisions: first, whether to abandon the peg and, if it does, by how much to devalue. These models confine the two choices within a single period, featuring a one-shot game between the central bank and private agents (see for example Morris and Shin, 1998; Obstfeld and Rogoff, 1996; Obstfeld, 1994). In a one shot game, the impact of time inconsistency on the size of devaluation (outside the peg) goes hand in hand with that on the likelihood of abandoning the peg. Confronted with stronger pressure from the private sector, a less conservative central bank is more likely to abandon the peg and then devalue by more.

Despite its intuitive appeal, this conclusion rests on an incomplete argument. While textbook models account for short-run considerations of the central bank – to tame current inflation by pegging or boost current output by devaluing – they abstract from long-run considerations. Such considerations will be relevant, however, if the central bank's exchange-rate decision has long-lasting implications. By standing ready to peg even under pressure, a central bank can earn the right to join a currency union, as stipulated for example by the Maastricht criteria for entering the third stage of EMU and adopting the euro. Joining a currency union institutionalizes the peg and – by tying the central bank's hands – addresses its time inconsistency problems.

Since a less conservative central bank faces greater time inconsistency issues, it would benefit more from addressing these issues by pegging in the short run. The question then becomes whether the one-shot game conclusions of textbook models are robust to allowing for intertemporal considerations. Such considerations call for examining separately (i) the

[1] For the sake of conciseness, we refer to second-generation currency crises models as "textbook models". These models were first introduced by Obstfeld (1996). They distinguish themselves from the so-called first generation models (Krugman, 1979; Flood and Garber, 1984) by allowing market beliefs to drive speculative currency attacks (together with economic fundamentals).

binary decision (peg or no peg) and (ii) the devaluation size, conditional on the peg being abandoned. If two central banks have decided to devalue, the textbook models have shown that the less conservative one – which has smaller aversion to inflation – would devalue by more. But could the drive to address greater time inconsistency issues lead the less conservative central bank to stick to the peg even when the more conservative one would not?

Framework. In this paper, we contribute to the currency crisis literature by departing from the traditional one-shot game setup in order to study the intertemporal trade-offs that a central bank faces in managing the nominal value of its currency. In designing the central bank's short-run optimization problem, we stick as closely as possible to the textbook model. As far as the long run is concerned, the central bank in our model can solve its time inconsistency problems by joining a currency union.

We start with a model of imperfect competition in the labour market, which provides microfoundations for the central bank's time inconsistency problem. This is a model of a small open economy, in which the domestic price level pins down the nominal exchange rate. Each period, workers set nominal wages in anticipation of the price level, targeting a real wage that maximizes the return from working. Importantly, workers exploit their market power to set their wages at a mark-up above the socially optimal wage. Acting after nominal wages have been set, the central bank picks the price level. The higher is the nominal wage mark-up, the greater is the pressure on the central bank to raise output by inflating prices, i.e. by devaluing. In each period, the central bank weighs this pressure against its own dislike for inflation/devaluation. The central bank's weight on inflation-driven – relative to output-driven – losses captures its conservatism, i.e. its resolve to resist labour-market pressure.

Our model implies that (i) a credible peg is the socially optimal equilibrium consistent with rational decision making but (ii) the central bank cannot attain this equilibrium if, period by period, it focuses exclusively on its current output- and inflation-driven losses. A credible peg is optimal because it eliminates inflation-driven losses, while output-driven losses are independent of the exchange rate regime in equilibrium. But labour market distortions create incentives for the central bank to abandon the peg after workers have set their wages. As workers anticipate these incentives, we obtain the classic time inconsistency problem.

We explore two different ways of attaining partial credibility for the peg. For the first, we follow textbook models of currency crises and assume an ad hoc fixed cost of abandoning the peg, which could be interpreted as a reputational cost to the central banker. This leads to a one-shot game, with all actions and payoffs occurring within a single period: the short run. The second approach is to consider two repeated games: one in the short run and one in the long run. If the central bank preserves the peg in the short run, it can join a currency union, where it has no choice but to peg. By contrast, abandoning the peg in the short run eliminates the option of joining the currency union and leads endogenously to a managed float in the long run. In effect, the second approach – which is an innovation of our paper – replaces the ad hoc commitment device with endogenous long-run (net) benefits of pegging in the short run.[2]

Results. To determine how the likelihood of abandoning the peg depends on the central bank's conservatism, it is necessary to consider the relationship between conservatism and the short-run (net) costs of pegging. In line with textbook models, this relationship is negative. All else the same, workers expect greater devaluation from a less conservative central bank and set higher nominal wages, thus raising the spectre of greater output losses if the peg survives. The short-run costs of withstanding workers' pressure – i.e. the short-run costs of pegging – are thus higher for a less conservative central bank.

In the one-shot game, the short-run cost of pegging is the only channel through which the central bank's conservatism affects the likelihood of devaluation. Facing higher short-run costs, a less conservative central bank is less likely to preserve the peg. This is the case under perfect foresight – which gives rise to multiple equilibria – and under dispersed information – which leads to a unique threshold equilibrium (Morris and Shin, 1998). As this is a standard result, our main question is whether it reverses in a repeated game setting.

In our repeated game setting, the central bank's decision about the currency regime is based both on the short-run (net) costs and the long-run (net) benefits of pegging. Like the short-run costs, the long-run benefits are now also endogenous and are also negatively related to the central bank's conservatism. Facing greater time-inconsistency issues on its own, a less conservative central bank stands to gain more from upholding the peg and joining a currency union.

The analysis of the repeated games setting boils down to comparing two (negative) elasticities. These are: (i) the elasticity of the short-run costs of pegging with respect to the central bank's conservatism (henceforth, the "elasticity of short-run costs" or the "short-run elasticity"); and (ii) the corresponding elasticity of the long-run benefits of pegging in the short run (the "elasticity of long-run benefits" or "the long-run elasticity"). *If the elasticity of long-run benefits is higher (in absolute value) than the elasticity of short-run costs, then a less conservative central bank is more likely to uphold a peg, all else the same.* Conversely, if the long-run elasticity is lower (in absolute value) than the short-run elasticity, we obtain the opposite, textbook result.

In comparing these two elasticities in the repeated games setting, we first assume perfect

[2] The long term benefits we derive are similar in spirit to those analysed in different settings by Clerc et al. (2011), Cooley and Quadrini (2003) and Alesina and Barro (2002). These articles do not consider speculative attacks but study instead the flexibility vs. credibility tradeoffs of giving up monetary independence for a currency union.

foresight. As in the one-shot game, workers' perceptions underpin multiple equilibria in the short run: for an intermediate region of the state space, workers' perceptions are self-fulfilling and can lead to either a peg or a devaluation. In contrast to the one-shot game, however, perceptions now determine also the sign of the relationship between the central bank's conservatism and the likelihood of upholding the peg in the short run.

Suppose first that, in the short run, workers perceive a devaluation whenever it is supported in equilibrium. In this case, they anticipate a higher devaluation rate from a less conservative central bank. This feeds into short-run nominal wages, thus raising the elasticity of the short-run costs of pegging (with respect to the central bank's conservatism) above the corresponding elasticity of long-run benefits. As a result, a less conservative central bank is more likely to abandon the peg.

The relationship between the central bank's conservatism and the devaluation probability reverses if workers perceive a peg whenever it is supported in equilibrium. In this case, workers perceive the central bank's hands as tied. As a result, the central bank's conservatism is irrelevant for workers' wages in the short run and, thus, does not influence the short-run pressure on the peg. This brings the short-run elasticity below the long-run one, implying that a less conservative central bank is more likely to uphold the peg.

We then study repeated games under dispersed information in the private sector. Dispersed information introduces strategic uncertainty: each worker is unsure about the actions and beliefs of others. Strategic uncertainty matters only in the short run, as only then is the fate of the peg still unknown and depends on workers' collective action (the aggregate wage bill in our model). Thus, in forming beliefs about the exchange rate, each worker needs to form beliefs about other workers' actions. In line with standard lessons from global games, strategic uncertainty gives rise to a unique equilibrium by impairing workers' coordination capacity (Morris and Shin, 1998).

For our analysis, it is important that – by impairing coordination – strategic uncertainty dampens the impact of the central bank's conservatism on workers' collective action.[3] In particular, we find that the elasticity of the short-run costs of pegging with respect to the central bank's conservatism is smaller (in absolute value) under dispersed information than under the perfect-foresight scenario in which workers perceive a devaluation. At the same time, strategic uncertainty does not affect the corresponding elasticity of long-run benefits, as the currency regime is common knowledge in the long run.

On its own, however, strategic uncertainty does not lower the short-run elasticity enough to reverse the relationship between the central bank's conservatism and devaluation probability. Even when strategic uncertainty is at its highest level – i.e. even when there is only

[3] Angeletos et al. (2007) and Chamley (2003) study how strategic uncertainty and the capacity to coordinate evolve over time as private agents learn from past outcomes in a dynamic setting.

private information about the economy's exogenous fundamentals – the short-run elasticity remains higher that the long-run elasticity. Thus, we still obtain that a less conservative central bank is more likely to abandon the peg.

In looking for a mechanism that would further dampen the elasticity of short run costs, we let the wages of *some* of the workers be predetermined in the short run, e.g. as a result of rigid contracts. Workers with predetermined wages cannot engage in a preemptive action even if they perceive a high likelihood of devaluation. In the spirit of Corsetti et al. (2004), this also reduces the aggressiveness of the preemptive actions by the rest of the workers, those with flexible wages. We thus find that pre-determined wages reduce not only the short-run pressure on the currency regime but also the elasticity of the short-run costs of pegging with respect to the central bank's conservatism. By contrast, the corresponding long-run elasticity is unchanged, as wages are flexible in the long run. This opens the door to reversing the textbook result on the relationship between the central bank's conservatism and the likelihood of abandoning the peg. Indeed, we show that a sufficiently large fraction of predetermined wages drives the elasticity of short-run costs below the elasticity of long-run benefits, implying that the probability of maintaining the peg is higher for a less conservative central bank.

Roadmap. The rest of the paper is organized as follows. We introduce a macro model with labour-market distortions in the next section. Then, in Section 3, we present a linear-quadratic reduced form of this model, which gives rise to micro-founded social welfare. Combining social welfare with a term capturing the central bank's aversion to inflation delivers the central bank's period loss, which is a key object of our analysis. In the subsequent sections, we analyze four variants of the model. In Section 4, we study a one-shot game, first under common knowledge and then under dispersed information. In Section 5, we switch to repeated games, again under common knowledge and then dispersed information. The repeated games setting allows us to also study the implications of predetermined wages in the short run.

2 The model

We present our model in this section.

Agents. The economy comprises a mainland and a continuum of islands. It is populated by a household and a firm, which operates on the mainland and also runs a continuum of subsidiaries, each on a different island. Islands are indexed by $i \in [0, 1]$. There is also a monetary authority, i.e. a central bank.

The household has a continuum of workers, each represented by a point in the square $[0, 1]^2$. At the beginning of each period, a measure-one continuum of workers is matched with a specific island, and leaves the mainland to go to work. Once on the island, the workers obey a set of rules (spelled-out below) and in exchange they all receive the same amount of consumption.

There is a single consumption good and it is produced by the firm on the mainland. The production of that good requires island-specific intermediary goods, which are produced on the islands by the local subsidiaries. Imperfect substitutability of intermediate goods endows workers with market power.

Preferences and production technology. The household has standard preferences over consumption and leisure,

$$\mathbb{E}_{i,0} \sum_{t=1}^{\tau} \beta^{t-1} \left(\frac{C_t^{1-\varsigma}}{1-\varsigma} - \int \frac{N_{i,t}^{1+\varphi}}{1+\varphi} di \right), \tag{1}$$

where C_t denotes final good consumption, which is the same across labour types; $N_{i,t}$ the quantity of labour supplied by the workers on island i; and $\beta \in (0,1)$ is the discount factor. We study below a one-shot game ($\tau = 1$) and a repeated-games setting ($\tau = 2$).

The household's budget constraint is given by

$$P_t C_t \leq \int W_{i,t} N_{i,t} di + T_t, \tag{2}$$

where P_t denotes the price level; $W_{i,t}$ is the nominal wage paid to workers on island i and T_t represents nominal transfers from the firm to the household.

On the mainland, the firm produces the consumption good according to the production technology

$$Y_t = A N_t^{1-\alpha}, \tag{3}$$

where A denotes total factor productivity and $N_t \equiv \left(\int N_{i,t}^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}}$, with $\nu > 1$, is a CES index of intermediate goods, produced according to subsidiaries' linear technology for transforming island-specific labour input into an island-specific intermediate good. Defining the aggregate wage index as $W_t \equiv \left(\int W_{i,t}^{1-\nu} di \right)^{\frac{1}{\nu-1}}$, the firm's profits are:

$$P_t Y_t - W_t N_t. \tag{4}$$

The firm chooses the labour index N_t and the cross-island labour allocation $\{N_{i,t}\}_{i \in [0,1]}$ to maximize profits, taking the price level P_t and the wage index W_t as given.[4] Maximizing (4) accordingly yields demand for the labour index

$$N_t = A^{\frac{1}{\alpha}} \left(\frac{W_t}{P_t} \right)^{-\frac{1}{\alpha}} (1-\alpha)^{\frac{1}{\alpha}}, \tag{5}$$

[4] The mainland firm's equilibrium profits, which are rebated lump sum to the household, equal $\alpha P_t Y_t = T_t$.

as well as for island-specific labour,

$$N_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\nu} N_t. \tag{6}$$

Workers are assumed to adhere to a set of rules in order to receive the same consumption. The workers on any particular island i have to accept the wage set by a local trade union and then stand ready to supply as much labour as the local subsidiary demands at that wage. The trade union on island i comprises the continuum of identical workers sent by the household to that island, i.e. a coalition. This union chooses the nominal wage $W_{i,t}$ at the beginning of period t to maximize the expected return from labour of its coalition:

$$\mathbb{E}_{i,t} \left[C_t^{-\varsigma} \frac{W_{i,t}}{P_t} N_{i,t} - \frac{N_{i,t}^{1+\varphi}}{1+\varphi} \right],$$

subject to demand for island-specific labour $N_{i,t}$, equation (6).[5] This gives rise to the following first order condition for the nominal wage:

$$\mathbb{E}_{i,t} \left[C_t^{-\varsigma} \frac{W_{i,t}}{P_t} N_{i,t} - \frac{\nu}{\nu - 1} N_{i,t}^{1+\varphi} \right] = 0, \tag{7}$$

We refer to this equation as the optimal wage-setting condition. The first term in the square bracket represents the marginal return from working, converted in utility terms by applying the marginal utility of consumption, $C_t^{-\varsigma}$, while the second is the marginal cost of supplying labour, augmented by the wage markup, $\frac{\nu}{\nu-1}$, which varies inversely with the elasticity of substitution across intermediate inputs ν. Intuitively, the smaller the substitutability across inputs and their corresponding labour types, the larger the market power of workers and the higher the individually optimal wage. Importantly, the individually optimal wage is higher than the socially optimal wage and converges to it as the elasticity of substitution across inputs becomes arbitrarily large, $\nu \to \infty$.

Monetary authority. We ignore the microfoundations of money and close the model by imposing an ad-hoc cash-in-advance constraint on total expenditure as in Angeletos and La'O (2011),

$$P_t Y_t = M_t \tag{8}$$

where M_t can either be interpreted as money supply or as nominal aggregate demand. By selecting M_t, the central bank pins down P_t in accordance with the currency regime it is in (see also Woodford, 2003, Chapter 3.2.1).

[5] Introducing labour market frictions through unions which optimise expected return from labour is a standard modeling device in the structural macro (i.e. DSGE) literature. For a full treatment, see Christiano et al. (2010).

Currency regime. We also assume that absolute purchasing power parity holds. Thus, the real exchange rate is $\text{RER}_t = P_t/e_t P^* = 1$, where P^* denotes the foreign price level and e_t is the nominal exchange rate: the domestic currency price of the foreign currency. Assuming further that the foreign price level is fixed, $P^* = 1$, we obtain $P_t = e_t$. Domestic inflation is thus equivalent to devaluation.

The price level depends on the currency regime that the central bank operates in. If the regime is a currency union, the central bank *has to* peg the exchange rate: i.e. it sets the price at its level in the previous period. If the regime is a managed float, the central bank is free to choose the price level.

3 Reduced-form of the model

We now present a linear-quadratic reduced-form of the model above. Lower-case letters denote log deviations from the deterministic steady state. Because unions' market power raises the privately optimal wage above the socially optimal one, and there are no corrective taxes, the steady state is inefficient. Since unions are made of workers, in the remainder of the paper we will use "unions" and "workers" interchangeably.

Workers. In Appendix A.1, we show that the wage-setting condition equation (7) log-linearizes to $\mathbb{E}_{i,t}\left[-\varsigma c_t + w_{i,t} - p_t - \varphi n_{i,t}\right] = 0$. Here, $-\varsigma c_t + w_{i,t} - p_t$ is the return from working (in terms of utility), and $-\varphi n_{i,t}$ is the cost of working. The optimal wage equates the two. Using the production function for the final good (3), the optimality condition for average labour (5) and the expression for island-specific labour demand (6), we can rewrite the log-linear wage-setting condition as:

$$w_{i,t} = \mathbb{E}_{i,t}\left[\delta_p p_t + (1-\delta_p)\, w_t\right], \tag{9}$$

where w_t denotes the linearized version of the wage index, $w_t = \int w_{i,t} di$, and $\delta_p = \frac{\varsigma(1-\alpha)+\varphi+\alpha}{\alpha(1+\nu\varphi)} > 0$. In the remainder of the paper, we refer to w_t as the "average wage".

For tractability we consider the limits in which the household is risk neutral, $\varsigma \to 0$ and preferences are linear in labour, $\varphi \to 0$. As shown by Hansen (1985), assuming linear preferences over labour is equivalent to considering a framework with indivisible labour and lotteries. Given such a framework, for any preferences over labour, workers behave as if these preferences were linear. In this special case, the wage-setting condition is given by

$$w_{i,t} = \mathbb{E}_{i,t}\left[p_t\right], \tag{10}$$

which allows us to focus on workers' incentives to stabilize their real wage.

Central bank. In each period the central bank has two options: (i) peg the currency, i.e. set $p_t = p_{t-1}$; or (ii) abandon the peg and manage a float. If it abandons the peg, the central bank manages the float by picking p_t to minimize the following period loss,

$$L_t^A = \underbrace{\frac{1}{2}\frac{\alpha}{1-\alpha}\left(\hat{y}_t - \hat{y}^*\right)^2}_{\text{align w/ steady state}} \quad \underbrace{- \theta_t \left(\hat{y}_t - \hat{y}^*\right)}_{\text{correct steady state inefficiency}} \quad \underbrace{+ \frac{\chi}{2} p_t^2}_{\text{minimise inflation}} . \tag{11}$$

where the difference $\hat{y}_t - \hat{y}^*$ represents the deviation of the welfare-relevant output gap $\hat{y}_t$ from its steady state value $\hat{y}^*$. Each gap is relative to the level of output that would prevail in the absence of the labour market distortions outlined in Section 2. The first term in the period loss indicates a preference of the central bank to align the output gap with its steady-state value. However, the steady-state output is inefficiently low because of the sub-optimally high wages stemming from the labour market distortions: i.e. $\hat{y}^* < 0$. The second term in the period loss, with $\theta_t > 0$, captures the central bank's additional preference to raise output above its steady-state value. Finally, after normalizing p_{t-1} to 0, the third term captures the central bank's aversion to inflation.[6]

We can relate the loss function (11) to the microfoundations presented in Section 2. If θ_t were constant and equal to the inverse of the elasticity of substitution across intermediate inputs $1/\nu$, the first two terms in (11) would be equal to a second-order approximation of the aggregate welfare losses accruing to households (see Appendix A.2). Hence, the central bank objective (11) departs from social welfare in two ways. First, the central bank places a different, time-varying, weight on negative output gaps – and thus on labour market distortions – than households. Second, it has a stronger aversion to inflation than households: hence $\chi > 0$.[7] We will see below that, in order for equilibrium inflation to be bounded, it is

[6] The assumption $p_{t-1} = 0$ is without loss of generality. Relaxing it amounts to treating all endogenous variables as *deviations* from p_{t-1}, which does not affect any of the derivations or conclusions.

[7] Our model does not generate a trade-off between inflation and output stabilisation. Woodford (2003, Chapter 6) shows that in New Classical models with imperfect competition and price/wage rigidities, social welfare can be approximated as a linear combination of the (square of the) output gap and a price/wage dispersion term. The exact way in which changes in the price level affect price dispersion depends on the details of price/wage-setting. In our model, the approximation yields a wage dispersion term that is positive only under dispersed information. Wage dispersion arises because of an information friction that is not affected by changes in the price level. As an alternative modelling strategy, we could have set up the reduced form model as a game between the monetary authority and *price-setting firms* who choose prices given their expectations about the nominal value of aggregate demand. However, such a model would not have delivered a microfounded output/inflation trade-off, because the output gap is proportional to inflation surprises, which are the source of price dispersion. Woodford also shows that a sufficient assumption to microfound a quadratic *price* inflation term in the welfare criterion is Calvo price-setting. We did experiment with Calvo pricing for final good producers. There is a trade-off, however, between generating an endogenous inflation term in the central bank's objective and obtaining a tractable expression for the linearised wage-setting condition. Under monopolistic competition amongst final producers, we could not have conveniently exploited the optimality condition for final producers to write aggregate labour N_t as an explicit function of the real wage, $W_t - P_t$ (see Appendix A.1). With Calvo pricing, the optimality condition for final producers would have been a complicated forward-looking expression involving expectations of future marginal costs, and hence of future real wages.

necessary that the central bank's differ from the household's.

Using the production function for the final good (3), the optimality condition for the labour index (5) we can write the "aggregate supply" curve of this economy as $y_t = -\frac{1-\alpha}{\alpha}(w_t - p_t)$, where y_t is the log-deviation of output from steady state. Observing that $y_t = \hat{y}_t - \hat{y}^*$ and substituting in (11) we finally obtain a version of the period loss in which the only endogenous magnitudes stem from the households' and the central bank's choice variables, w_t and p_t, respectively:

$$L_t^A = \frac{1}{2}(w_t - p_t)^2 + \theta_t (w_t - p_t) + \frac{\chi}{2} p_t^2. \tag{12}$$

If the central bank abandons the peg, it chooses the price that minimizes these losses, taking the average wage as given:

$$p_t = \frac{1}{1+\chi}(w_t + \theta_t). \tag{13}$$

This price reflects various trade-offs faced by the central bank. First, the monetary authority seeks to stabilize the welfare-relevant output gap, that is, to minimize $\frac{1}{2}(w_t - p_t)^2$. If this were the central bank's only concern, it would set the price level p_t equal to the average wage w_t. Second, in order to mitigate the negative output gap, the authority targets a negative real wage: $\theta_t (w_t - p_t) < 0$. Added to the first consideration, this increases the price level to $w_t + \theta_t$. In this sense, $\theta_t > 0$ leads to an inflation bias in price setting under a managed float. Third, the central bank dislikes inflation. This lowers the optimal price level to $\frac{1}{1+\chi}(w_t + \theta_t)$, where $\frac{1}{1+\chi} < 1$.

The loss-minimizing price level given by (13) increases with the inflation bias θ_t and increases with the aggregate nominal wage w_t. Thus, the higher are nominal wages, the wider is the wedge between (i) the price level minimizing the central bank's period loss and (ii) the price level consistent with a peg, $p_t = 0$. In this sense, the level of nominal wages represents workers' pressure on the currency peg. However, the greater is the central bank's aversion to inflation, the stronger is its resistance to this pressure: the wedge between p_t in equation (13) and $p_t = 0$ decreases in χ for a given w_t. Following Rogoff (1985), we refer to the inflation aversion parameter, χ, as the central bank's "conservatism".

Time inconsistency of monetary policy. In choosing whether to peg or set the price level according to (13), the central bank compares the associated losses. As regards the period loss in (12), we have:

$$\left. L_t^A \right| \text{ abandon peg} = \frac{1}{2}\frac{\chi}{1+\chi}(w_t + \theta_t)^2 - \frac{1}{2}\theta_t^2$$

$$\left. L_t^A \right| \text{ peg} = \frac{1}{2}(w_t + \theta_t)^2 - \frac{1}{2}\theta_t^2$$

Accordingly, the *net* cost of pegging is given by

$$c\left(w_t, \theta, \chi\right) \equiv L_t^A\Big| \text{ peg} - L_t^A\Big| \text{ abandon peg} = \frac{1}{2}\frac{\left(w_t + \theta_t\right)^2}{1+\chi} > 0. \tag{14}$$

Since the net cost of pegging is positive, devaluing is a dominant strategy for a central bank that only cares about output losses. Intuitively, for any given average wage, a devaluation sufficiently reduces the output gap to more than compensate for higher inflation in the central bank's period loss. *Thus, pegging is time inconsistent.*

As a result, there is only one way for a peg to be partially credible. Abandoning the peg must generate central bank losses over and above the period loss in (12). We consider two alternative ways of introducing such losses.

1. We suppose that workers and the central bank are engaged in a one-shot game ($\tau = 1$ in equation (1)). In this case, the central bank faces a fixed ad hoc cost $\bar{v}$ if and only if it abandons the peg. This is the textbook setup of e.g. Obstfeld and Rogoff (1996).

2. A key modelling innovation of this paper is to introduce an *endogenous* alternative to the above ad hoc cost of abandoning the peg. Namely, we consider sequential games over two periods: the short run and the long run ($\tau = 2$).

   (a) If the central bank pegs in the short run, it has the option to join a currency union. If it does join, it pegs unconditionally in the long run as well.

   (b) If the central bank abandons the peg in the short run, then it cannot join the currency union and faces only its period loss (12) in the long run. Since abandoning the peg is a dominant strategy in this case, the central bank floats the currency.

Fundamentals. The only source of uncertainty in this model is the dislike of the central bank for negative output gaps, θ_t. We refer to θ_t as the "fundamentals" of the economy. A higher value of θ_t leads to a higher inflationary bias and thus corresponds to worse fundamentals. At the beginning of each period, nature draws θ_t from a uniform distribution with support on $[\theta_L, \theta_H]$, where $\theta_L > 0$.[8] The fundamentals are i.i.d. over time.

Workers' information. In setting the wage rate, the workers on each island make use of the information they have at the beginning of the period. We consider two different information structures. The first is perfect foresight, whereby workers observe the fundamentals θ_t before choosing the optimal nominal wage. The second structure features uncertainty about θ. All workers start with a common prior that is different from the true distribution

[8] Throughout the paper, we consider only positive values of the fundamentals, which are associated with a devaluation of the currency under a managed float. Because the cost of pegging is quadratic, results are mirror symmetric for negative values, which lead to revaluations. See also Appendix C.2.

of θ_t. Under the prior, θ_t is a Gaussian random variable, $\theta_t \sim N(\theta_0, \sigma_0^2)$, and i.i.d. across periods. Then the workers on each island i observe an island-specific private signal:

$$x_{i,t} = \theta_t + \sigma_x \xi_{i,t}, \tag{15}$$

where the noise $\xi_{i,t} \sim N(0,1)$ is i.i.d. across periods and in the cross-section and is independent of θ. The posterior distributions of the fundamentals, which are given by $\frac{\frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_x} \exp\left[-\frac{1}{2\sigma_x^2}(x-\theta)^2\right]}{\int_{\theta_L}^{\theta_H} \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_x} \exp\left[-\frac{1}{2\sigma_x^2}(x-\theta)^2\right] d\theta}$, differ across islands. Moreover, in the limit $\sigma_x \to 0$, they converge to a Gaussian:

$$\theta_t | x_{i,t} \sim N\left(\psi x_{i,t} + (1-\psi)\theta_0, \sigma_{\theta|x}^2\right), \tag{16}$$

where $\psi \equiv \frac{\sigma_0^2}{\sigma_0^2 + \sigma_x^2}$ and $\sigma_{\theta|x}^2 \equiv \frac{\sigma_0^2 \sigma_x^2}{\sigma_0^2 + \sigma_x^2}$ (see Appendix B). In the remainder of the paper, we focus on this special case.

4 One-shot game

We now study the reduced-form model introduced in Section 3 when workers and the central bank are engaged in a one-shot game. In this game, the central bank abandons the peg for a managed float, if and only if the net payoff to defending the peg is negative:

$$\bar{v} \leq c(w, \theta, \chi), \tag{17}$$

where the right-hand side is as defined in equation (14) and we have suppressed time subscripts.

4.1 Perfect foresight

Consider first the case in which workers possess perfect foresight. In this case, all wages are identical and equal to the aggregate wage, $w_i = w$. By the wage-setting equation (10) we then obtain simply

$$w = p. \tag{18}$$

Then, if a peg is sustained in equilibrium, $p(\theta, \chi) = 0$, equations (10) and (12) deliver the average wage and the period loss:

$$p(\theta, \chi) = 0,\ w(\theta, \chi) = 0,\ L^A(\theta, \chi) = 0. \tag{19}$$

Alternatively, for a managed float to be an equilibrium outcome, the same two equations together with (13) imply the following price, wage and loss:

$$p(\theta,\chi)=\frac{\theta}{\chi},\ w(\theta,\chi)=\frac{\theta}{\chi},\ L^{A}(\theta,\chi)=\frac{1}{2}\frac{\theta^{2}}{\chi}. \tag{20}$$

A difference between the central bank's and the household's aversion to inflation, i.e. $\chi\neq 0$, is necessary for a bounded equilibrium.

Because workers act before the central bank, there are multiple equilibria for intermediate values of the fundamentals. This is a common feature of second-generation models of currency crises.

Proposition 1 (equilibrium with common knowledge)**.** *There are two threshold values of the fundamentals. The first one, denoted by $\underline{\theta}$, is the positive solution to $\bar{v}-c\left(\frac{\theta}{\chi},\theta,\chi\right)=0$:*

$$\underline{\theta}=\sqrt{2\bar{v}\frac{\chi^{2}}{1+\chi}}. \tag{21}$$

The second one, denoted by $\overline{\theta}$, is the positive solution to $\bar{v}-c(0,\theta,\chi)=0$:

$$\overline{\theta}=\sqrt{2\bar{v}(1+\chi)} \tag{22}$$

For low values of the fundamentals, $\theta\leq\underline{\theta}$, there exists a unique equilibrium such that condition (17) leads the central bank to sustain the peg. The equilibrium is fully characterised by (19)*. For high values of the fundamentals, $\theta>\overline{\theta}$, there exists a unique equilibrium such that condition* (17) *leads the central bank to abandon the peg. The equilibrium is fully characterised by* (20)*. For intermediate values of the fundamentals, $\theta\in\left(\underline{\theta},\overline{\theta}\right]$, there are multiple equilibria: the central bank pegs or devalues, depending on whether workers anticipate a peg or a devaluation.*

There are two important aspects of the equilibrium under perfect foresight in the one-shot game. First, under a peg, central bank conservatism does not matter for equilibrium outcomes: as shown in (19), the price level, average wage and period loss do not depend on χ. Second, these variables *do* depend on χ in a managed float regime, as shown by (20). In particular, the equilibrium price, wage and period loss all decrease in χ, reflecting the greater aversion to inflation of a more conservative central bank.

The multiplicity region of the state space, $\left(\underline{\theta},\overline{\theta}\right]$, shares important properties with the multiplicity region in the reduced form model of Morris and Shin (1998, henceforth MS). In the common knowledge benchmark of the MS model, the worst fundamentals supporting a peg are such that the benefit of maintaining the peg equal the cost of pegging in the absence of pressure from the private sector. In our case, the corresponding benefit is on the left-hand side of expression (17): $\bar{v}$. And the corresponding cost is on the right-hand side for $w = p = 0$. In the MS framework, a downward shift of the cost-of-pegging schedule strengthens the peg. Corollary 1 below shows that this is the case in our model, too. An increase in χ – corresponding to a more conservative central bank – results in such a shift and also strengthens the peg.

The parallels between the MS and our model extend also to the best fundamentals that can witness a devaluation. In the MS model, these fundamentals worsen (the peg strengthens) as the size of the potential devaluation declines. Similarly, equation (20) shows that, under a managed float, the devaluation size declines in χ while the upper threshold, $\overline{\theta}$, increases in χ.

Corollary 1. *Both the lower threshold, $\underline{\theta}$, and the upper threshold, $\overline{\theta}$, of the multiplicity region are increasing in central bank conservatism, χ.*

It is useful to step back and consider carefully the drivers of the relationship between the central bank's conservatism and a threshold value of the fundamentals. Denoting such a value with the generic θ^t, we know that it solves the indifference condition $\bar{v} = c\left(w, \theta^t, \chi\right)$. Totally differentiating this condition with respect to χ, we obtain

$$\begin{aligned} \frac{d\theta^t}{d\chi} &= \frac{\frac{d\bar{v}}{d\chi}\frac{\chi}{\bar{v}} - \frac{dc}{d\chi}\frac{\chi}{c}}{\frac{dc}{d\theta}\frac{\chi}{c}}, \qquad (23) \\ &= \frac{\frac{d\bar{v}}{d\chi}\frac{\chi}{\bar{v}} - \left(\frac{\partial}{\partial\chi}c\left(w,\theta^t,\chi\right) + \frac{\partial}{\partial w}c\left(w,\theta^t,\chi\right)\frac{\partial}{\partial\chi}w\left(\theta^t,\chi\right)\right)\frac{\chi}{c(w,\theta^t,\chi)}}{\left[\frac{\partial}{\partial w}c\left(w,\theta^t,\chi\right)\frac{\partial}{\partial\theta}w\left(\theta^t,\chi\right) + \frac{\partial}{\partial\theta}c\left(w,\theta^t,\chi\right)\right]\frac{\chi}{c(w,\theta^t,\chi)}}, \end{aligned}$$

where we have used the facts that at the critical level of fundamentals $\bar{v} = c\left(w, \theta, \chi\right)$, and that $\frac{dc}{d\chi} = \frac{\partial c}{\partial w}\frac{\partial w}{\partial\chi} + \frac{\partial c}{\partial\chi}$. Quite intuitively – and as confirmed by (14), (19) and (20) – the denominator of this expression is positive. The three underlying drivers operate in the same direction. First, the cost of pegging increases with the pressure from the private sector: $\frac{\partial}{\partial w}c\left(w,\theta^t,\chi\right) > 0$. Second, this pressure builds up as the fundamentals worsen: $\frac{\partial}{\partial\theta}w\left(\theta^t,\chi\right) > 0$ (recall that a higher fundamental θ means higher distortions). Third, a deterioration in the fundamentals also increases the cost of pegging directly: $\frac{\partial}{\partial\theta}c\left(w,\theta^t,\chi\right) > 0$. These effects will remain qualitatively the same in all the specifications we study.

Thus, turning to the numerator in expression (23), it is the relative size of two elasticities that determines the impact of central bank conservatism on the critical values of the fundamentals. The first is the elasticity of the benefit of pegging: $\frac{d\bar{v}}{d\chi}\frac{\chi}{\bar{v}}$. In the present setting, this is zero. The second is the sensitivity of the cost of pegging: $\frac{dc}{d\chi}\frac{\chi}{c}$. Intuitively – and as confirmed by (14), (19) and (20) – this elasticity is negative: a more conservative central bank (i) perceives a lower cost of pegging, $\frac{\partial c}{\partial\chi} < 0$, and (ii) is better at managing the devaluation pressure from the private sector: $\frac{\partial c}{\partial w}\frac{\partial w}{\partial\chi} < 0$.

In the current setting, a more conservative central bank faces higher fundamental thresholds because (in absolute terms) the elasticity of the benefit of pegging with respect to conservatism χ is smaller than the elasticity of the cost. By Proposition 1, the peg is abandoned when the fundamentals rise above the relevant threshold. Thus, facing a higher threshold, a more conservative central bank is *less* likely to abandon the peg.

4.2 Dispersed information

We now assume that each worker does not observe the fundamentals θ directly but observes a private signal x, as defined in equation (15).

This setting delivers two main takeaways. First, in line with the global games literature, the dispersed information leads to a unique equilibrium. Second, similar to the multiple equilibria in the perfect-foresight setting, the unique equilibrium implies that a more conservative central bank is less likely to abandon the peg.

We focus on an equilibrium in which the central bank follows a threshold devaluation strategy. We conjecture and verify that there exists a $\theta^* > 0$ such that p is given by equation (13) if $\theta \geq \theta^*$, and $p = 0$ otherwise. The wage-setting condition (10) returns an expression for the individually optimal wage as a function of private information x, the threshold θ^* and the conservatism of the central bank χ,

$$w_x\left(x, \theta^*, \chi\right) = \mathbb{E}\left[p\left(\theta, \theta^*, \chi\right)|x\right], \tag{24}$$

where the expectation is taken with respect to the posterior distribution of θ given the private signal x. Equation (24) highlights the conjecture that the individually optimal wage and the equilibrium price both depend on the threshold θ^* and the conservatism of the central bank χ. However, the individual wage reflects a worker's private signal x, while the price reflects the fundamentals θ. Integrating across households we obtain an expression characterizing the average wage

$$w\left(\theta, \theta^*, \chi\right) = \mathbb{E}\left[\mathbb{E}\left[p\left(\theta, \theta^*, \chi\right)|x\right]|\theta\right], \tag{25}$$

where the outermost expectation is now taken with respect to the distribution of the private signal x conditional on the fundamental θ. In Appendix C we show that the average wage $w\left(\theta, \theta^*, \chi\right)$ is the fixed point of an operator,

$$w\left(\theta, \theta^*, \chi\right) = T\left[w\right]\left(\theta, \theta^*, \chi\right). \tag{26}$$

The operator T is defined as

$$T\left[w\right]\left(\theta, \theta^*, \chi\right) \equiv \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(w\left(\zeta, \theta^*, \chi\right) - \zeta\right)\phi\left(\zeta, \theta, \sigma\right)d\zeta, \tag{27}$$

with $\sigma^2 \equiv 2\sigma_x^2$. The function $\phi(\cdot, \mu, s)$ denotes the density of a Gaussian random variable with mean μ and standard deviation s. We show in Appendix C.1 that the operator T has a unique fixed point, so the candidate equilibrium aggregate wage function is well-defined for any $\theta > 0$. We also show that this function is differentiable in all its three arguments.

Lemma 2 (Existence and uniqueness of the wage function). *If workers believe the central bank follows a threshold strategy such that $p = \frac{1}{1+\chi}(w+\theta)$ if $\theta \geq \theta^* > 0$ and $p = 0$ otherwise, the average wage $w(\theta, \theta^*, \chi)$ is the unique fixed point of the operator T defined by equation* (27)*.*

Sketch of the proof. Although the operator T is not a contraction, the term $\frac{1}{1+\chi} \in (0,1)$. As a result, we can show that, starting from an appropriately selected function w_0, the sequence $\{T^n[w_0]\}$ is a Cauchy sequence in $\mathbb{R}$, and so for $n \to +\infty$ it converges to some w^* which is a fixed point of T by the continuity of T. We show uniqueness by contradiction.

We next verify that it is indeed optimal for the central bank to abandon the peg when $\theta \geq \theta^*$. By equation (17), the central bank switches to a managed float if and only if the cost of pegging is sufficiently high relative to the benefit:

$$\bar{v} \leq c\left(w\left(\theta, \theta^*, \chi\right), \theta^*, \chi\right), \tag{28}$$

with θ^* implicitly defined by the indifference condition $\bar{v} = c\left(w\left(\theta^*, \theta^*, \chi\right), \theta^*, \chi\right)$. Hence, to verify the conjecture that a threshold equilibrium exists, it suffices to show that the right-hand side of (28) is increasing in θ. We show this in Appendix C.4 and thus obtain the following proposition:

Proposition 3. *There exists a threshold equilibrium such that the central bank abandons the peg if fundamentals are sufficiently bad, $\theta \geq \theta^*$. The threshold θ^* is sandwiched between its common knowledge counterparts, $\underline{\theta} < \theta^* < \overline{\theta}$.*

In order to solve for the equilibrium average wage, we need to resort to numerical methods. Figure 1 provides an example. In a related setting, Guimaraes and Morris (2007) solve for the equilibrium in closed form. We cannot apply the same techniques because in our setting the private sector payoffs are affected both by the likelihood of a devaluation and by its size.This implies a global game with continuous actions for *both* the private sector and the central bank.

[FIG. 1 ABOUT HERE]

That said, we prove analytically that the more conservative is the central bank, i.e. the higher is χ, the higher is the threshold θ^*. We start with the analogue of (23) under dispersed information:

$$\begin{aligned}\frac{d\theta^*}{d\chi} &= \frac{\frac{d\bar{v}}{d\chi}\frac{\chi}{\bar{v}} - \frac{dc}{d\chi}\frac{\chi}{c}}{\frac{dc}{d\theta}\frac{\chi}{c}}, \qquad (29)\\ &= \left.\frac{\frac{d\bar{v}}{d\chi}\frac{\chi}{\bar{v}} - \left(\frac{\partial}{\partial w}c\left(w,\theta,\chi\right)\frac{\partial}{\partial\chi}w\left(\theta,\theta^*,\chi\right) + \frac{\partial}{\partial\chi}c\left(w,\theta,\chi\right)\right)\frac{\chi}{c(w,\theta,\chi)}}{\left[\frac{\partial}{\partial w}c\left(w,\theta,\chi\right)\left(\frac{\partial}{\partial\theta}w\left(\theta,\theta^*,\chi\right) + \frac{\partial}{\partial\theta^*}w\left(\theta,\theta^*,\chi\right)\right) + \frac{\partial}{\partial\theta}c\left(w,\theta,\chi\right)\right]\frac{\chi}{c(w,\theta,\chi)}}\right|_{\theta=\theta^*}.\end{aligned}$$

Now, we need to consider separately the dependence of the average wage on (i) the fundamentals and (ii) the threshold value of the fundamentals. Nevertheless, we show in Appendix C.4 that the denominator is still negative and for the same reasons as in the perfect-foresight setting: the cost of pegging declines as the fundamentals improve.

So, it again boils down to comparing the elasticity of the benefit of pegging to central bank conservatism, $\frac{d\bar{v}}{d\chi}\frac{\chi}{\bar{v}}$, with the corresponding elasticity of the cost of pegging, $\frac{dc}{d\chi}\frac{\chi}{c}$. Since the benefits of pegging in the one-shot game stem from an ad hoc constant, the former elasticity is zero. And, as in the perfect-foresight case, the cost of pegging is decreasing in χ (see (14) and results in Appendix C.3). Thus, the cost of pegging is more sensitive to χ than the benefit of pegging, implying that the critical threshold increases in χ: $\frac{d\theta^*}{d\chi} > 0$. In other words, a more conservative central bank is less likely to abandon the peg.

Figure 2 illustrates this result as well as the statement in Proposition 3 that the critical threshold under dispersed information lies between the common knowledge thresholds.

[FIG. 2 ABOUT HERE]

5 Repeated games

In this section, the mechanism for attaining partial credibility of the peg stems from allowing the central bank and workers to play two subsequent games: one in the short run (SR) and one in the long run (LR). Each game features the same (net) cost of pegging, conditional on the aggregate wage: as given by expression (14).

5.1 The long run

The central bank's options in the long run depend on its actions in the short run. If the central bank pegs in the short run, then it has the *option* to join a currency union that enforces the peg in the long run. Once in the currency union, there is no uncertainty about the exchange rate regime as the peg is assumed to be perfectly credible. This means that equation (19) applies and the central bank's losses are zero.

Alternatively, the central bank may choose not enter the currency union in the long run. This could happen for either one of two reasons. First, the central may choose to not exercise its option to join after pegging in the short run. Second, it would forego this option altogether by devaluing in the short run. In either case, the central bank would face only the period loss (12) in the long run, which implies that pegging is time inconsistent.

This time inconsistency result implies that, as in the one-shot game setting, there is no uncertainty about the exchange rate regime. If the central bank devalues in the short run, there is common knowledge that the regime is a managed float in the long run. In turn, common knowledge about the regime implies that the average wage is the same irrespective of whether workers (i) observe θ_{LR} directly, as in Section 4.1; or (ii) observe infinitely precise private signals about θ_{LR}, as in Section 4.2. In Appendix E, we show that, under either information structure, the long-term average wage is $\frac{1}{2}\frac{1}{\chi}\theta_{LR}$ under a managed float. Then, as in equation (20), the central bank's costs long-run costs under a managed float are equal to $\frac{1}{2}\frac{\theta_{LR}^2}{\chi}$.

The central bank's long-run costs under a (perfectly credible) peg are unambiguously lower than those under a (perfectly credible) float: $0 < \frac{1}{2}\frac{\theta_{LR}^2}{\chi}$. It follows that, if the central bank has the option of joining the currency union in the long run, it always exercises it. In comparison to a managed float, a currency union delivers a net benefit: $\frac{1}{2}\frac{\theta_{LR}^2}{\chi} > 0$. This is the long-run benefit of pegging in the short run.

5.2 The short run

The long-run benefit of joining the currency union gives rise to partial credibility of the peg in the short run. In particular, the central bank abandons the peg in the short run if and only if:

$$\underbrace{\beta\frac{1}{2}\frac{1}{\chi}\mathbb{E}\left[\theta_{LR}^2\right]}_{\text{long-run benefit}} \equiv v(\chi) \leq c(w,\theta_{SR},\chi) \equiv \underbrace{\frac{1}{2}\frac{1}{1+\chi}(w(\theta_{SR},\chi)+\theta_{SR})^2}_{\text{short-run cost}} \tag{30}$$

where $\beta \in (0,1)$ is the discount factor and the expectation operator incorporates the unconditional distribution of θ_{LR}. Inequality (30) is the repeated-games analogue of inequality (17) from the one shot-game setting. On the left-hand side, the endogenous long-term benefit of pegging, $v(\chi)$, replaces the ad hoc fixed benefit in the one-shot game. On the right-hand side, the short-run cost of pegging is as given by expression (14).

We now turn to studying how central bank conservatism, χ, affects the likelihood of a devaluation, that is, the value of fundamental threshold(s). As in the one-shot game, this boils down to comparing the elasticity of the short-run cost of pegging to χ with the corresponding elasticity of the long-run benefit. Relative the one-shot setting, the short-run cost has not changed, and it is decreasing in χ. However, the long-run benefit is now different, and importantly, it is also decreasing in χ (see (30)). To see the intuition, recall that the long-run benefit of pegging is the benefit of solving the time inconsistency problem by tying the central bank's hands within a currency union. Since a more conservative central bank is better at managing inflation/devaluation on its own (in the sense that the devaluation rate decreases with χ, as per (20)), it faces a less severe time inconsistency problem and thus stands to benefit less from joining the currency union.

In the remaining three subsections, we dissect the conditions under which the short-run elasticity is smaller than the long-run elasticity. When this is the case, a more conservative central bank is more likely to abandon the peg in the short run.

5.2.1 Perfect foresight

First, let agents set their wages under perfect knowledge of the current fundamentals. As in the one-shot game, there are again multiple equilibria characterized by the respective short-run thresholds, $\underline{\theta}$ and $\overline{\theta}$. In other words, Proposition 1 still applies, with the caveat that the thresholds need to be replaced by

$$\underline{\theta} = \sqrt{v(\chi)\,\frac{\chi^2}{1+\chi}}, \tag{31}$$

and

$$\overline{\theta} = \sqrt{v(\chi)\,(1+\chi)}, \tag{32}$$

respectively.[9]

The impact of central bank conservatism, χ, on either threshold can again be summarized in the expression

$$\begin{aligned}\frac{d\theta^t}{d\chi} &= \frac{\frac{dv}{d\chi}\frac{\chi}{v} - \frac{dc}{d\chi}\frac{\chi}{c}}{\frac{dc}{d\theta}\frac{\chi}{c}}, \\ &= \frac{\frac{dv(\chi)}{d\chi}\frac{\chi}{v(\chi)} - \left(\frac{\partial}{\partial w}c\left(w,\theta^t,\chi\right)\frac{\partial}{\partial\chi}w\left(\theta^t,\chi\right) + \frac{\partial}{\partial\chi}c\left(w,\theta^t,\chi\right)\right)\frac{\chi}{c(w,\theta^t,\chi)}}{\left[\frac{\partial}{\partial w}c\left(w,\theta^t,\chi\right)\frac{\partial}{\partial\theta}w\left(\theta^t,\chi\right) + \frac{\partial}{\partial\theta}c\left(w,\theta^t,\chi\right)\right]\frac{\chi}{c(w,\theta^t,\chi)}},\end{aligned} \tag{33}$$

where θ^t stands for either $\underline{\theta}$ or $\overline{\theta}$, and w denotes the short-run average wage. This is the repeated-game equivalent of 23. As in the one-shot game, the short-run cost of pegging decreases as the fundamentals improve (the denominator is still positive, $\frac{\partial c}{\partial w}\frac{\partial w}{\partial\theta} + \frac{\partial c}{\partial\theta} > 0$) and as the central bank becomes more conservative (the term in parentheses in the numerator is still negative, $\frac{\partial c}{\partial w}\frac{\partial w}{\partial\chi} + \frac{\partial c}{\partial\chi} < 0$).

[9] To simplify notation, we do not explicitly index short-run objects by the subscript SR. Note that the long-run value of the fundamentals θ_{LR} only enters the equilibrium analysis indirectly through its second moment $\mathbb{E}\left[\theta_{LR}^2\right]$, which is embedded in the long-run benefit $v(\chi)$.

However, unlike in the one-shot setting, the long-run benefit of pegging, $v(\chi)$, now depends on conservatism χ. This property of the repeated-games setup leads to the result that changes in χ have opposite effects on the two thresholds.

Corollary 2. *The lower threshold of the multiplicity region, $\underline{\theta}$, is increasing in central bank's conservatism, χ. The upper threshold, $\overline{\theta}$, is decreasing in χ.*

Proof. See Appendix F.

Corollary 2 is the repeated-game analogue of Corollary 1. The two have similar implications for the lower threshold $\underline{\theta}$, which is computed under the assumption that workers expect a devaluation. Since the devaluation rate is sensitive to central bank conservatism χ, so is the average wage w. This makes the short-run cost of pegging more elastic with respect to χ than the long-run benefit of pegging, both in the one-shot and the repeated-games setting, $\left|\frac{dc}{d\chi}\frac{\chi}{c}\right| > \left|\frac{dv}{d\chi}\frac{\chi}{v}\right|$. The upshot is that, if workers anticipate a devaluation and it is rational for them to do so, a more conservative central bank abandons the peg at weaker fundamentals. In other words, the threshold $\bar{\theta}$ decreases with χ.

By contrast, Corollaries 1 and 2 differ because of the upper threshold $\overline{\theta}$. This threshold is computed under the assumption that workers expect the peg to be upheld. Under a peg, central bank conservatism χ does not matter for equilibrium outcomes, so the average wage is perfectly inelastic with respect to χ, $\frac{\partial w}{\partial \chi} = 0$. The elasticity of the short-run cost of pegging with respect to χ , $\frac{dc}{d\chi}\frac{\chi}{c}$, thus becomes smaller in absolute value. However, since a more conservative central bank is by definition more averse to inflation so $\frac{\partial c}{\partial \chi} < 0$, and the elasticity is still negative.

In the one-shot game, the ad-hoc long-run benefit of pegging is inelastic with respect to χ, so the short-run elasticity $\left|\frac{dc}{d\chi}\frac{\chi}{c}\right|$ still dominates. The story is reversed in the repeated-game setting. In this case, the endogenous long-term benefits of pegging are sensitive to χ, $\frac{dv}{d\chi}\frac{\chi}{v} < 0$. Moreover, $\left|\frac{dc}{d\chi}\frac{\chi}{c}\right| = \left|\frac{\partial c}{\partial \chi}\frac{\chi}{c}\right| < \left|\frac{dv}{d\chi}\frac{\chi}{v}\right|$, implying that a *less conservative* central bank abandons the peg for worse fundamentals, so $\underline{\theta}$ is increasing in χ. As a result, in a repeated-games setting, a less conservative central bank is less likely to abandon the peg when workers expect it to do otherwise.

5.2.2 Dispersed information

Suppose that workers observe noisy private signals about the fundamentals, as defined in equation (15). As in the one-shot game, there is a threshold equilibrium in the repeated-games setting. Proposition 3 still applies, with the threshold θ^* being equal to the positive root of $v(\chi) = c(w(\theta^*, \theta^*, \chi), \theta^*, \chi)$. Implicitly differentiating the indifference condition for the monetary authority with respect to χ, we get:

$$\begin{aligned}
\frac{d\theta^*}{d\chi} &= \frac{\frac{dv}{d\chi}\frac{\chi}{v} - \frac{dc}{d\chi}\frac{\chi}{c}}{\frac{dc}{d\theta}\frac{\chi}{c}}, \qquad (34)\\
&= \left.\frac{\frac{dv(\chi)}{d\chi}\frac{\chi}{v(\chi)} - \left(\frac{\partial}{\partial w}c\left(w,\theta,\chi\right)\frac{\partial}{\partial \chi}w\left(\theta,\theta^*,\chi,\lambda\right) + \frac{\partial}{\partial \chi}c\left(w,\theta,\chi\right)\right)\frac{\chi}{c(w,\theta,\chi)}}{\left[\frac{\partial}{\partial w}c\left(w,\theta,\chi\right)\left(\frac{\partial}{\partial \theta}w\left(\theta,\theta^*,\chi,\lambda\right) + \frac{\partial}{\partial \theta^*}w\left(\theta,\theta^*,\chi,\lambda\right)\right) + \frac{\partial}{\partial \theta}c\left(w,\theta,\chi\right)\right]\frac{\chi}{c(w,\theta,\chi)}}\right|_{\theta=\theta^*}.
\end{aligned}$$

We obtain again that the short-run cost of pegging declines as the fundamentals improve (implying that $\frac{\partial c}{\partial w}\left(\frac{\partial w}{\partial \theta} + \frac{\partial w}{\partial \theta^*}\right) + \frac{\partial c}{\partial \theta} > 0$) and as central bank conservatism increases ($\frac{\partial c}{\partial w}\frac{\partial w}{\partial \chi} + \frac{\partial c}{\partial \chi} < 0$). In contrast to the one-shot setting, the long-run benefits of a devaluation decrease in conservatism, $\frac{dv}{d\chi} < 0$.

Therefore, in order to determine the direction of the impact of χ on the threshold θ^*, we once again need to compare the short-run elasticity $\left|\frac{dc}{d\chi}\frac{\chi}{c}\right|$ to the long-run elasticity $\left|\frac{dv}{d\chi}\frac{\chi}{\nu}\right|$. Note that the introduction of dispersed information does not affect the latter elasticity because the currency regime is common knowledge in the long run. We thus turn to the short-run elasticity. Since dispersed information creates strategic uncertainty among workers in the short run, it impairs their coordination capacity. As a result, the short-run elasticity under dispersed information is *smaller* (in absolute value) than the corresponding short-run elasticity when workers have perfect foresight and coordinate their wages given the shared belief that the central bank is going to *devalue*.[10] But it is also *larger* than the corresponding short-run elasticity when workers have perfect foresight and coordinate their wages given the shared belief that the central bank is sticking to the *peg*.

Whether the short-run elasticity is larger than the long-run elasticity under dispersed information is an open question. We answer this question by resorting to numerical simulations, which reveal that, in qualitative terms, the short-run cost of pegging is still more elastic with respect to χ than is the long-run benefit. Thus, as portrayed in Figure 3 and Figure 4, θ^* is an increasing function of χ. In other words, a more conservative central bank is less likely to abandon the peg.

[FIG. 3 AND 4 ABOUT HERE]

5.2.3 Predetermined wages

In this subsection we explore a mechanism that can reverse the relationship between the conservatism of a central bank and the likelihood of a devaluation. This mechanism affects

[10] This stems from (i) the smaller elasticity of the average wage to χ under dispersed information than under perfect foresight and (ii) the fact that all workers believe the central bank is going to devalue. Concretely, on the basis of results in Appendix C.2 and Appendix C.4), we obtain that $-\frac{\partial}{\partial \chi}w\left(\theta,\theta^*,\chi\right) < \frac{\chi}{\kappa+\chi}\left(-\frac{\kappa}{\chi^2}\theta\right) < -\frac{\kappa}{\chi^2}\theta$.

only the elasticity of the short-run cost of pegging to conservatism, and it ensures that this elasticity is smaller than its long-run counterpart, the elasticity of the long-run benefit of pegging. As it hinges on the endogenous intertemporal trade-off of the repeated-games setting, the mechanism does not work in the one-shot game.

Let a share $\lambda \in [0,1)$ of the workers have predetermined wages in *the short run* but let all workers have fully flexible wages in the long run. A higher λ means that more workers cannot respond to the perceived likelihood or magnitude of devaluations. Thus, the higher is λ, the less elastic is the short-run average wage to central bank conservatism, χ, and to the fundamentals, θ. Relative to the specifications studied above, this lowers the elasticity of the short-run cost of pegging to changes in χ. And because of the flexibility of long-run wages, the value of λ plays no role in the long run.

Perfect foresight. Under perfect foresight, Proposition 1 continues to apply and there are multiple equilibria. The upper dominance threshold $\bar{\theta}$ is unaffected by the existence of passive workers because it stems from active workers believing the peg is going to be upheld, and thus behaving as if they were passive themselves.[11] Thus, we again obtain $\frac{d\bar{\theta}}{d\chi} < 0$.

In Appendix G, we derive the lower threshold, $\underline{\theta}$, which reflects the share of predetermined wages:

$$\underline{\theta} = \sqrt{\frac{v(\chi)}{\frac{1+\chi}{(\lambda+\chi)^2}}}, \tag{35}$$

where $v(\chi)$ is as defined in expression (30). We thus get the following result.

Corollary 3. *The lower threshold of the multiplicity region decreases in the central bank's conservatism, χ, for a sufficiently high fraction of predetermined wages, $\lambda \in \left(\frac{\chi}{1+2\chi}, 1\right]$, and increases otherwise. The upper threshold, $\bar{\theta}$, is unaffected by λ and decreases in χ.*

To see the parallel with the corresponding result in Corollary 2 of Section 5.2.1, note that the impact of the central bank's conservatism on the lower threshold is now given by:

$$\begin{aligned} \frac{d\underline{\theta}}{d\chi} &= \frac{\frac{dv}{d\chi}\frac{\chi}{v} - \frac{dc}{d\chi}\frac{\chi}{c}}{\frac{dc}{d\theta}\frac{\chi}{c}}, \\ &= \frac{\frac{dv(\chi)}{d\chi}\frac{\chi}{v(\chi)} - \left(\frac{\partial}{\partial w}c(w,\underline{\theta},\chi,\lambda)\frac{\partial}{\partial\chi}w(\underline{\theta},\chi,\lambda) + \frac{\partial}{\partial\chi}c(w,\underline{\theta},\chi,\lambda)\right)\frac{\chi}{c(w,\underline{\theta},\chi,\lambda)}}{\left[\frac{\partial}{\partial w}c(w,\underline{\theta},\chi,\lambda)\frac{\partial}{\partial\theta}w(\underline{\theta},\chi,\lambda) + \frac{\partial}{\partial\theta}c(w,\underline{\theta},\chi,\lambda)\right]\frac{\chi}{c(w,\underline{\theta},\chi,\lambda)}}. \end{aligned} \tag{36}$$

Compared to equation (33), the long-run benefit of pegging, $v(\chi)$, is the same. In addition, the short-run cost of pegging still decreases in conservatism, $\frac{\partial c}{\partial w}\frac{\partial w}{\partial\chi} + \frac{\partial c}{\partial\chi} < 0$, and as the fundamentals improve, $\frac{\partial c}{\partial w}\frac{\partial w}{\partial\theta} + \frac{\partial c}{\partial\theta} > 0$. That said, as the share of predetermined wages rises

[11] This is without loss of generality. Even if we had allowed the predetermined wage to be poisitive, the sensitivity of the lower threshold $\underline{\theta}$ with respect to χ would be independent of both the preset wage level and λ.

and $\lambda \to 1$, the equilibrium converges to one such that all workers expect the peg to be maintained. In this equilibrium, the short-run elasticity is lower than the long-run elasticity, $\left|\frac{dv}{d\chi}\frac{\chi}{v}\right| > \left|\frac{dc}{d\chi}\frac{\chi}{c}\right|$ (recall Section 5.2.1). By continuity, the last inequality holds for $\lambda < 1$ and we thus obtain Corollary 3.

Dispersed information. Suppose now that there is dispersed information about θ. Then Proposition 3, which established the existence of a threshold equilibrium under dispersed information, still applies, with the caveat that the aggregate wage now satisfies

$$w(\theta, \theta^*, \chi, \lambda) = (1-\lambda)\,\mathbb{E}_x\left[w_1(x, \theta^*, \chi, \lambda)\,|\theta\right], \tag{37}$$

where $w_1(x, \theta^*, \chi, \lambda)$ denotes the wage set by active workers, and is as given in equation (10). The devaluation condition of the central bank is still given by (17), with the average wage w now given by (37). Implicitly differentiating this condition, we generalize equation (34) to obtain the impact of conservatism of the on the critical value of fundamentals in the presence of passive workers:

$$\begin{aligned}
\frac{d\theta^*}{d\chi} &= \frac{\frac{dv}{d\chi}\frac{\chi}{v} - \frac{dc}{d\chi}\frac{\chi}{c}}{\frac{dc}{d\theta}\frac{\chi}{c}}, \qquad (38)\\
&= \left.\frac{\frac{dv(\chi)}{d\chi}\frac{\chi}{v(\chi)} - \left(\frac{\partial}{\partial w}c(w,\theta,\chi,\lambda)\frac{\partial}{\partial \chi}w(\theta,\theta^*,\chi,\lambda) + \frac{\partial}{\partial \chi}c(w,\theta,\chi)\right)\frac{\chi}{c(w,\theta,\chi)}}{\left[\frac{\partial}{\partial w}c(w,\theta,\chi)\left(\frac{\partial}{\partial \theta}w(\theta,\theta^*,\chi,\lambda) + \frac{\partial}{\partial \theta^*}w(\theta,\theta^*,\chi,\lambda)\right) + \frac{\partial}{\partial \theta}c(w,\theta,\chi)\right]\frac{\chi}{c(w,\theta,\chi)}}\right|_{\theta=\theta^*(\lambda)}.
\end{aligned}$$

The long-run benefit of pegging, $v(\chi)$, is the same as in equation (34). In addition, the short-run cost of pegging still decreases in the central bank's conservatism, $\frac{\partial c}{\partial w}\frac{\partial w}{\partial \chi} + \frac{\partial c}{\partial \chi} < 0$, and as the fundamentals improve, $\frac{\partial c}{\partial w}\frac{\partial w}{\partial \theta} + \frac{\partial c}{\partial \theta} > 0$. However, a higher λ dampens the elasticity of the average wage bill with respect to χ, $\left|\frac{\partial w}{\partial \chi}\right|$, thereby dampening the short-run elasticity too, $\left|\frac{dc}{d\chi}\frac{\chi}{c}\right|$.

We resort to numerical simulations to explore whether dampening the elasticity of the average wage is sufficient to ensure that the long-run elasticity dominates the short-run elasticity, $\left|\frac{dv}{d\chi}\frac{\chi}{v}\right| > \left|\frac{dc}{d\chi}\frac{\chi}{c}\right|$. We find that this can indeed be the case even for very small λ, as illustrated in Figure 5 and Figure 6 where we obtain that the dispersed information threshold is decreasing in χ.

[FIG. 5 AND 6 ABOUT HERE]

Thus, provided that a sufficiently large fraction of the wages are predetermined, conventional wisdom gets reversed. A *less conservative* central bank is *less* likely to abandon the peg. This is the case when active workers have perfect foresight and are inclined to coordinate their actions on the basis of the belief that the central bank is going to uphold the peg. And it is also the case under dispersed information.

6 Conclusion

In this paper, we built on microfoundations and developed a repeated-games version of the textbook second-generation currency crises model, in which the central bank faces time inconsistency issues. In our model, both the short-run cost and the long-run benefit of pegging decrease with the conservatism of the central bank, that is, the central bank's aversion to inflation. Provided that the long-run benefit is more elastic than the short-run cost to changes in conservatism (in absolute value), a less conservative central bank is more likely to preserve the peg in order to solve its time inconsistency issues. This result overturns conventional wisdom about currency crises, which maintains instead that a more conservative central bank is more likely to float. The condition about elasticities would indeed hold for labour markets that are sufficiently rigid in the sense that a large enough share of wages are predetermined.

Appendices

A The linearized model

In this section we present a log linear approximation of the wage-setting equation and of social welfare. Together, these two blocks constitute the "reduced form" of our setup. Throughout this appendix we use lowercase to denote log of variables, and hats to denote deviations of log variables from their log values in the approximating equilibrium. So for any variable X_t, $x_t = \ln(X_t)$ and $\hat{x}_t \equiv \ln(X_t) - \ln(X^*)$.

We linearize around a symmetric two-period $t = 1, 2$ equilibrium of the non-linear model with no uncertainty. Given (a^*, p^*), this equilibrium is a vector (c^*, n^*, w^*). We consider the same process for productivity and in the approximated and approximating equilibrium, so $\hat{a}_t = a^* - a^* = 0$. We also assume that in the approximating equilibrium the wage markup is the same as in the equilibrium to be approximated, so $\hat{\mu}_t = \mu_t - \mu = 0$. We assume that the approximating equilibrium is not efficient. In particular, labour market distortions drive a wedge between the marginal rate of substitution between consumption and leisure MRS^* and the marginal product of labour MPN^*. From the wage-setting equation, (7), we obtain that $\frac{W^*}{P^*} = \mathcal{M}^* MRS^*$, where $\mathcal{M}^*_t \equiv \frac{\nu}{\nu-1}$ and $MRS^*_t \equiv (C^*)^{\varsigma} (N^*)^{\varphi}$. From the optimality condition for the final good producer, (5), we obtain that $\frac{W^*}{P^*} = MPN^*$, where $MPN^* \equiv (1-\alpha)(N^*_t)^{-\alpha}$. The equilibrium is inefficient since $\frac{MPN^*}{MRS^*} = \mathcal{M}^* > 1$. Since $MPN^* > MRS^*$ and MPN^* is decreasing in N^* while MRS^* is increasing, we have that employment is too low, and output is accordingly also too low.

A.1 Approximating the wage-setting condition

We next log-linearize the wage-setting condition (7),

$$\mathbb{E}\left[C_t^{-\varsigma} \frac{W_{i,t}}{P_t} N_{i,t}\right] = \mathbb{E}\left[\mu_t N_{i,t}^{1+\varphi}\right],$$

with $\mu \equiv \ln\{\mathcal{M}\} = \ln\left(\frac{\nu}{\nu-1}\right)$. Consider first the term within the expectation on the left-hand side. Taking a first order Taylor approximation around (c^*, w^*, p^*, n^*) returns

$$C_t^{-\varsigma} \frac{W_{i,t}}{P_t} N_{i,t} = \exp\{-\varsigma c^* - p^* + w^* + n^*\}\left[1 - \varsigma(c_t - c^*) - (p_t - p^*) + (w_{i,t} - w^*) + (n_{i,t} - n^*)\right]. \tag{A.1}$$

A first order Taylor approximation of the term within the expectation on the right-hand side around n^* instead returns

$$\mu N_{i,t}^{1+\varphi} = \exp\{\mu + (1+\varphi) n^*\}\left[1 + (1+\varphi)(n_{i,t} - n^*)\right]. \tag{A.2}$$

The wage-setting condition evaluated in the approximation point returns $w^* - p^* = \mu + \varsigma c^* + \varphi n^*$. As a result, we can write (A.1) as

$$C_t^{-\varsigma} \frac{W_{i,t}}{P_t} N_{i,t} = \exp\left\{\mu + (1+\varphi) n^*\right\} \left[1 - \varsigma (c_t - c^*) - (p_t - p^*) + (w_{i,t} - w^*) + (n_{i,t} - n^*)\right].$$

Taking expectations and using again the result that $w_t^* - p^* = \mu + \varsigma c^* + \varphi n_t^*$, we finally obtain

$$w_{i,t} = \mathbb{E}[\mu + p_t + \varsigma c_t + \varphi n_{i,t}]. \tag{A.3}$$

Log-linearizing the labour demand equation, (6), we obtain

$$n_{i,t} = -\nu (w_{i,t} - w_t) + n_t, \tag{A.4}$$

so then substituting back into expression (A.5) and collecting terms, we end up with

$$(1 + \varphi\nu) w_{i,t} = \mathbb{E}[\mu + p_t + \varsigma c_t + \varphi\nu w_t + \varphi n_t] \tag{A.5}$$

We now re-write equation (A.5) so the right hand side depends only on TFP a_t, the log aggregate nominal wage w_t and the log price level, p_t. Using the goods market clearing condition $c_t = y_t$, the (log) production function $y_t = a_t + (1-\alpha) n_t$ (see (3)), and the (log) optimality condition for firms, $\ln(1-\alpha) + a_t - \alpha n_t = w_t - p_t$ (see (5)) we obtain

$$\begin{aligned} w_{i,t} \;=\; & \mathbb{E}\left[\frac{\varphi + \varsigma}{\alpha(1+\nu\varphi)} a_t + \frac{\varphi + \varsigma(1-\alpha)}{\alpha(1+\nu\varphi)} \ln(1-\alpha) \right. \\ & \left. + \frac{\varsigma(1-\alpha) + \alpha + \varphi}{\alpha(1+\nu\varphi)} p_t - \frac{\varsigma(1-\alpha) + \varphi(1-\alpha\nu)}{\alpha(1+\nu\varphi)} w_t \right]. \end{aligned}$$

And now in deviations from the approximating equilibrium we write the wage-setting equation more compactly as

$$\hat{w}_{i,t} = \mathbb{E}\left[\delta_a \hat{a}_t + \delta_{p,t} \hat{p}_t + \delta_{w,t} \hat{w}_t\right], \tag{A.6}$$

where hats denote deviations from the approximating equilibrium and the coefficients are given as

$$\delta_{a,t} \equiv \tfrac{\varphi+\varsigma}{\alpha(1+\nu\varphi)}, \quad \delta_p \equiv \tfrac{\varsigma(1-\alpha)+\alpha+\varphi}{\alpha(1+\nu\varphi)}, \quad \delta_w \equiv -\tfrac{\varsigma(1-\alpha)+\varphi(1-\alpha\nu)}{\alpha(1+\nu\varphi)},$$

Under the assumption that $(\varsigma, \varphi) = (0, 0)$, we finally end up with

$$\hat{w}_{i,t} = \mathbb{E}\left[\hat{p}_t\right].$$

A.2 Approximating social welfare

Taking a second order Taylor approximation of social welfare, we obtain

$$\int_0^1 (U_{i,t} - U^*)\, di = (C^*)^{-\varsigma} C^* \left(\hat{y}_t + \frac{1-\varsigma}{2}\hat{y}_t^2\right) - (N^*)^{\varphi} N^* \left(\int_0^1 \hat{n}_{i,t} di + \frac{1+\varphi}{2}\int_0^1 \hat{n}_{i,t}^2 di\right), \tag{A.7}$$

Our objective is to write the second term in the sum $\int_0^1 \hat{n}_{i,t}di + \frac{1+\varphi}{2}\int_0^1 \hat{n}_{i,t}^2 di$ so it depends on $\hat{y}_t$ and $\hat{y}_t^2$ only.

To that end, recall the definition of the intermediate good index, $N_t = \left(\int (N_{i,t})^{\frac{\nu-1}{\nu}} di\right)^{\frac{\nu}{\nu-1}}$. It implies that

$$1 \equiv \int \left(\frac{N_{i,t}}{N_t}\right)^{\frac{\nu-1}{\nu}} di.$$

Taking a second order Taylor approximation of $\left(\frac{N_{i,t}}{N_t}\right)^{\frac{\nu-1}{\nu}}$ returns

$$\left(\frac{N_{i,t}}{N_t}\right)^{\frac{\nu-1}{\nu}} = 1 + \frac{\nu-1}{\nu}\left(\hat{n}_{i,t} - \hat{n}_t\right) + \frac{1}{2}\left(\frac{\nu-1}{\nu}\right)^2 \left(\hat{n}_{i,t} - \hat{n}_t\right)^2.$$

Hence, we obtain

$$1 \equiv \int \left(\frac{N_{i,t}}{N_t}\right)^{\frac{\nu-1}{\nu}} di = 1 + \frac{\nu-1}{\nu}\int \left(\hat{n}_{i,t} - \hat{n}_t\right) di + \frac{1}{2}\left(\frac{\nu-1}{\nu}\right)^2 \int \left(\hat{n}_{i,t} - \hat{n}_t\right)^2 di.$$

Re-arranging we get

$$\begin{aligned} \int \hat{n}_{i,t} di &= \hat{n}_t - \frac{1}{2}\frac{\left(\frac{\nu-1}{\nu}\right)^2}{\frac{\nu-1}{\nu}} \int \left(\hat{n}_{i,t} - \hat{n}_t\right)^2 di \\ &= \hat{n}_t - \frac{1}{2}(\nu - 1)\,\nu \int \left(\hat{w}_{i,t} - \hat{w}_t\right)^2 di \end{aligned} \tag{A.8}$$

where the second line uses the expression for labour demand, $\hat{n}_{i,t} = -\nu\left(\hat{w}_{i,t} - \hat{w}_t\right) + \hat{n}_t$. This takes care of the first integral in $\int_0^1 \hat{n}_{i,t}di + \frac{1+\varphi}{2}\int_0^1 \hat{n}_{i,t}^2 di$. We still need to get rid of the second integral,

$$\begin{aligned} \int_0^1 \hat{n}_{i,t}^2 di &= \int \left(-\nu\left(\hat{w}_{i,t} - \hat{w}_t\right) + \hat{n}_t\right)^2 di \\ &= \hat{n}_t^2 + \nu^2 \int \left(\hat{w}_{i,t} - \hat{w}_t\right)^2 di - 2\nu\hat{n}_t \int \left(\hat{w}_{i,t} - \hat{w}_t\right) di. \end{aligned}$$

Consider the third term, $\int \left(\hat{w}_{i,t} - \hat{w}_t\right) di$. Taking a first order Taylor approximation of the average wage $\left(\frac{W_{i,t}}{W_t}\right)^{1-\nu}$ returns

$$1 \equiv \int \left(\frac{W_{i,t}}{W_t}\right)^{1-\nu} di = 1 + (1-\nu) \int (\hat{w}_{i,t} - \hat{w}_t)\, di + \frac{1}{2}(1-\nu)^2 \int (\hat{w}_{i,t} - \hat{w}_t)^2\, di,$$

that is,

$$\int (\hat{w}_{i,t} - \hat{w}_t)\, di = \frac{1}{2}(\nu - 1) \int (\hat{w}_{i,t} - \hat{w}_t)^2\, di.$$

Substituting back in the expression for $\int_0^1 \hat{n}_{i,t}^2 di$ we obtain

$$\int_0^1 \hat{n}_{i,t}^2 di = \hat{n}_t^2 + \nu^2 \int (\hat{w}_{i,t} - \hat{w}_t)^2\, di,$$

since $-\nu \hat{n}_t (\nu - 1) \int (\hat{w}_{i,t} - \hat{w}_t)^2$ is a term of order higher than 2. Letting $var_i \{w_{i,t}\} \equiv \int (\hat{w}_{i,t} - \hat{w}_t)^2\, di$, we can finally write $\int_0^1 \hat{n}_{i,t} di + \frac{1+\varphi}{2} \int_0^1 \hat{n}_{i,t}^2 di$ as

$$\int \hat{n}_{i,t} di + \frac{1+\varphi}{2} \int_0^1 \hat{n}_{i,t}^2 di = \hat{n}_t + \frac{1+\varphi}{2} \hat{n}_t^2 + \frac{1}{2} \nu (\nu\varphi + 1)\, var_i \{w_{i,t}\}.$$

Substituting back into equation equation (A.7), we are left with

$$\int_0^1 \frac{(U_{i,t} - U^*)}{(C^*)^{-\varsigma} C^*} di = \hat{y}_t + \frac{1-\varsigma}{2} \hat{y}_t^2 - \frac{(N^*)^{\varphi} N^*}{(C^*)^{-\varsigma} C^*} \left(\hat{n}_t + \frac{1+\varphi}{2} \hat{n}_t^2 + \frac{1}{2} \nu (\nu\varphi + 1)\, var_i \{w_{i,t}\} \right).$$

Now using the production function $\hat{y}_t = \hat{a}_t + (1-\alpha)\hat{n}_t$, substituting in and ignoring terms independent of policy we obtain

$$\begin{aligned} \int_0^1 \frac{U_{i,t} - U^*}{(C^*)^{-\varsigma} C^*} di =& \hat{y}_t + \frac{1-\varsigma}{2} \hat{y}_t^2 \\ & - \frac{(N^*)^{\varphi} N^*}{(C_t^*)^{-\varsigma} C^*} \left(\frac{\hat{y}_t - \hat{a}_t}{1-\alpha} + \frac{1+\varphi}{2} \frac{(\hat{y}_t - \hat{a}_t)^2}{(1-\alpha)^2} + \frac{\nu(1+\varphi\nu)}{2} var_i \{w_{i,t}\} \right). \end{aligned}$$

Now consider now the term $\frac{(N^*)^{\varphi} N^*}{(C^*)^{-\varsigma} C^*}$,

$$\begin{aligned} \frac{(N^*)^{\varphi} N^*}{(C_t^*)^{-\varsigma} C^*} &= MRS^* \frac{N^*}{C^*} = \frac{MRS^*}{MPN^*}(1-\alpha) \text{ since } MPN^* = (1-\alpha) C_t^*/N^* \\ &= \left(1 - \frac{1}{\nu}\right)(1-\alpha) \end{aligned}$$

where the second line follows since $MRS^* = \frac{\frac{W_t^*}{P^*}}{\mathcal{M}}$ and $MPN^* = \frac{W^*}{P^*}$, and recalling that $\mathcal{M} = \frac{\nu}{\nu-1}$. The term $\frac{1}{\nu}$ can thus be interpreted as steady state distortions. We assume that

$\frac{1}{\nu}$ is small enough that its product with a second order term can be ignored. As a result,

$$\begin{aligned}\int_0^1 \frac{U_{i,t}-U^*}{(C^*)^{-\varsigma} C^*} di \approx & \hat{y}_t + \frac{1-\varsigma}{2}\hat{y}_t^2 - \left(1-\frac{1}{\nu}\right)(1-\alpha)\left(\frac{\hat{y}_t-\hat{a}_t}{1-\alpha} + \frac{1+\varphi}{2}\frac{(\hat{y}_t-\hat{a}_t)^2}{(1-\alpha)^2}\right) - \frac{\Upsilon}{2} var_i\{w_{i,t}\} \\ = & -\frac{1}{2}\left(\varsigma + \frac{\alpha+\varphi}{1-\alpha}\right)\left(\hat{y}_t - \frac{\varphi+1}{\varsigma+\alpha+\varphi-\varsigma\alpha}\hat{a}_t\right)^2 + \frac{1}{\nu}\hat{y}_t - \frac{\Upsilon}{2} var_i\{w_{i,t}\},\end{aligned}$$

where $\Upsilon \equiv (1-\alpha)\nu(1+\varphi\nu)$. Note that

$$\begin{aligned}\hat{y}_t - \frac{\varphi+1}{\varsigma+\alpha+\varphi-\varsigma\alpha}\hat{a}_t &= y_t - y^* \text{ since } \hat{a}_t = 0 \\ &= y_t - y_t^e + y_t^e - y^* \\ &= y_t - y_t^e - (y^* - y_t^e)\end{aligned}$$

Hence we can write the losses as

$$L = \frac{1}{2}\left(\varsigma + \frac{\alpha+\varphi}{1-\alpha}\right)((y_t-y_t^e)-(y^*-y_t^e))^2 - \frac{1}{\nu}((y_t-y_t^e)-(y^*-y_t^e)) + \frac{1}{2}\Upsilon var_i\{w_{i,t}\}.$$

This is useful for interpretation. Moreover, observe that since $\hat{y}_t = \hat{a}_t + (1-\alpha)(\hat{a}_t - (\hat{w}-\hat{p}))$ and $\hat{a}_t = 0$, we can write this as

$$\begin{aligned}L &= \frac{1}{2}\left(\varsigma + \frac{\alpha+\varphi}{1-\alpha}\right)\hat{y}_t^2 - \frac{1}{\nu}\hat{y}_t + \frac{\Upsilon}{2} var_i\{w_{i,t}\}, \\ &= \frac{1}{2}\left(\varsigma + \frac{\alpha+\varphi}{1-\alpha}\right)\left(\frac{1-\alpha}{\alpha}\right)^2(\hat{w}-\hat{p})^2 + \frac{1}{\nu}\left(\frac{1-\alpha}{\alpha}\right)(\hat{w}-\hat{p}) + \frac{\Upsilon}{2} var_i\{w_{i,t}\}.\end{aligned}$$

Now consider the term $\frac{\Upsilon}{2} var_i\{w_{i,t}\}$. Under common knowledge about θ, all workers set the same wage, so this term is zero because $var_i\{w_{i,t}\} = 0$. Under dispersed information about θ, on the other hand, $var_i\{w_{i,t}\} > 0$. The variance of wages, however, arises purely because of heterogenous beliefs, not because of fluctuations in the price level. Hence, in the case of heterogenous information, we can treat $\frac{\Upsilon}{2} var_i\{w_{i,t}\}$ as a term independent of monetary policy in the approximation of social welfare. Hence, we finally obtain

$$L = \frac{1}{2}(\hat{w}-\hat{p})^2 + \frac{1}{\nu}\frac{1}{\left(\varsigma + \frac{\alpha+\varphi}{1-\alpha}\right)\left(\frac{1-\alpha}{\alpha}\right)}(\hat{w}-\hat{p}).$$

B Aggregate wage with dispersed info

Workers have a prior about θ which is uniform over $[\theta_L, \theta_H]$ with $\theta_L > 0$. They observe a signal x given by $x = \theta + \sigma_x \varepsilon$, where ε is $N(0,1)$. We begin by showing that $\theta | x$ converges in probability to a Gaussian with mean x and variance σ_x^2 as $\sigma_x^2 \to 0$.

It suffices to show that $U(\sigma_x) = \frac{\theta|x - x}{\sigma_x}$ converges to a $N(0,1)$ as $\sigma_x^2 \to 0$. The r.v. $\theta|x$ has a density function given by:

$$\frac{\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_x}\exp\left[-\frac{1}{2\sigma_x^2}(x-\theta)^2\right]}{\int_{\theta_L}^{\theta_H}\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_x}\exp\left[-\frac{1}{2\sigma_x^2}(x-\theta)^2\right]d\theta}. \tag{B.9}$$

It is clear that the integral of this function on the interval $[\theta_L, \theta_H]$ is one, so this function is indeed a density.

Then, we work with $U(\sigma_x) = \frac{\theta|x - x}{\sigma_x}$, which is a linear transformation of $\theta|x$. One can apply the Jacobian method to find its density. Accordingly, substitute θ by $\sigma_x u + x$ in the above density, and multiply it with the first order derivative which is equal to σ_x. We obtain:

$$\frac{\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}u^2\right]}{\int_{\frac{\theta_L - x}{\sigma_x}}^{\frac{\theta_H - x}{\sigma_x}}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}u^2\right]du} \text{ for } u \in \left[\frac{\theta_L - x}{\sigma_x}, \frac{\theta_H - x}{\sigma_x}\right], \tag{B.10}$$

which is also a density.

Next we consider the limit of the above density as $\sigma_x^2 \to 0$. There are three scenarios: (i) $x \in (\theta_L, \theta_H)$; (ii) $x \leq \theta_L$ and (iii) $x \geq \theta_H$.

As $\sigma_x^2 \to 0$, the joint probability of the two latter scenarios tends to zero. So, in terms of convergence in probability, we only need to consider the first situation. In this case, the range of the density characterised by equation (B.10) tends to $(-\infty, +\infty)$. As a result, the integral in the denominator of the right-hand side of (B.10) tends to 1. To conclude, U converges to a standard normal distribution.

Dropping hats and time subscripts, the optimal wage equation for a worker who has observed signal x therefore becomes

$$w_i(x, \theta^*, \chi) = \int_{-\infty}^{+\infty} p(\zeta, \theta^*, \chi)\phi(\zeta, x, \sigma_x)\, d\zeta,$$

Next, aggregate over x to obtain

$$w(\theta, \theta^*, \chi) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p(\zeta, \theta^*, \chi)\,\phi(\zeta, x, \sigma_x)\,\phi(x, \theta, \sigma_x)\, d\zeta dx,$$

We next show that we can integrate out x, since

$$\phi(\zeta, x, \sigma_x)\,\phi(x, \theta, \sigma_x) = \phi(\zeta, \theta, \sigma)\,\phi\left(x, \frac{\zeta + \theta}{2}, \frac{\sigma}{2}\right)$$

where $\sigma = \sqrt{2}\sigma_x$. Consider then the product of the two Gaussian densities,

$$\frac{1}{2\pi}\frac{1}{\sigma_x^2}\exp\left\{-\frac{1}{2\sigma_x^2}\left[(z-x)^2+(\theta-x)^2\right]\right\}.$$

After some manipulations of the term in curly braces we obtain:

$$\frac{1}{2\pi}\frac{1}{\sigma_x^2}\exp\left\{-\frac{1}{2\left(\frac{\sigma_x}{\sqrt{2}}\right)^2}\left(x-\frac{\zeta+\theta}{2}\right)^2\right\}\exp\left\{-\frac{1}{2\left(\sqrt{2}\sigma_x\right)^2}(\zeta-\theta)^2\right\}$$

that is, the result above.

Integrating out x, the aggregate wage equation can be written as

$$w(\theta,\theta^*,\chi) = \int_{-\infty}^{+\infty} p(\zeta,\theta^*,\chi)\phi(\zeta,\theta,\sigma)\,d\zeta. \tag{B.11}$$

C Results about the wage equation

We now prove existence, uniqueness and monotonicity of the aggregate wage. Suppose workers believe that the central bank follows a threshold strategy such that

$$p(\zeta,\theta^*,\chi) = \begin{cases} \frac{1}{1+\chi}(w+\zeta) & \text{if } \zeta \le \theta^* \\ 0 & \zeta > \theta^* \end{cases},$$

for some $\theta^* > 0$. Then we can write the wage equation as

$$w(\theta,\theta^*,\chi) = \int_{-\infty}^{+\infty} a(\zeta,\theta^*,\chi)\left(w(\zeta,\theta^*,\chi)+\zeta\right)\phi(\zeta,\theta,\sigma)\,d\zeta, \tag{C.12}$$

where the function $a(\zeta,\theta^*,\chi)$ is defined as

$$a(\zeta,\theta^*,\chi) = \begin{cases} 0 & \text{if } \zeta \ge \theta^* \\ \frac{1}{1+\chi} & \zeta \ge \theta^* \end{cases}. \tag{C.13}$$

Note that since $\chi > 0$, $0 < a \le \frac{1}{1+\chi} < 1$.

Consider the operator associated with the right hand side of equation (C.12),

$$T[x](\theta,\theta^*,\chi) \equiv \int_{-\infty}^{+\infty} a(\zeta,\theta^*,\chi)\left(w(\zeta,\theta^*,\chi)+\zeta\right)\phi(\zeta,\theta,\sigma)\,d\zeta. \tag{C.14}$$

In principle, the operator T maps the space of continuous functions on $\mathbb{R}\times\mathbb{R}_+^2$ such that the integral $\int a(\zeta,\theta^*,\chi)\left(w(\zeta,\theta^*,\chi)+\zeta\right)\phi(\zeta,\theta,\sigma)\,d\zeta$ exists onto itself. We consider however only positive values of θ^*, $\theta^* > 0$, and only values of $\chi > 0$. We also make enough assumptions on the information structure to ensure that negative values of θ can be safely ignored.

C.1 Proof of Lemma 2

Existence part. We next show that there exists a fixed point of T. The proof is by induction. We start from some x_0 and we iterate. We show that the iteration procedure converges.

Fix (θ, θ^*, χ). Consider a particular starting point, $x_0(\theta, \theta^*, \chi) = 0$. Letting $x_1 = T[x_0]$, we can write

$$\begin{aligned}
|x_1 - x_0| &= \frac{1}{1+\chi}\left|\int_{\theta^*}^{+\infty} \zeta\phi(\zeta, \theta, \sigma)\, d\zeta\right| \\
&\leq \frac{1}{1+\chi}\left|\theta\Phi\left(\frac{\theta-\theta^*}{\sigma}\right) + \sigma\phi\left(\frac{\theta^*-\theta}{\sigma}\right)\right| \\
&\leq \frac{1}{1+\chi}Q(\theta)
\end{aligned}$$

with $Q(\theta) \equiv \left|\theta\Phi\left(\frac{\theta-\theta^*}{\sigma}\right) + \sigma\phi\left(\frac{\theta^*-\theta}{\sigma}\right)\right|$. Iterating again, $x_2 = T^2[x_0]$, we have that

$$\begin{aligned}
|x_2 - x_1| &= \left|\int_{\theta^*}^{+\infty} \frac{1}{1+\chi}\left(x_1(\zeta, \theta^*, \chi) - x_0(\zeta, \theta^*, \chi)\right)\phi(\zeta, \theta, \sigma)\, d\zeta\right| \\
&\leq \frac{1}{1+\chi}\left|\int_{\theta^*}^{+\infty} \left(x_1(\zeta, \theta^*, \chi) - x_0(\zeta, \theta^*, \chi)\right)\phi(\zeta, \theta, \sigma)\, d\zeta\right| \\
&\leq \frac{1}{1+\chi}\int_{\theta^*}^{+\infty} \left|x_1(\zeta, \theta^*, \chi) - x_0(\zeta, \theta^*, \chi)\right|\phi(\zeta, \theta, \sigma)\, d\zeta \\
&\leq \left(\frac{1}{1+\chi}\right)^2 Q(\theta).
\end{aligned}$$

By induction, then,

$$|x_n - x_{n-1}| \leq \left(\frac{1}{1+\chi}\right)^n Q(\theta).$$

Observe that for fixed θ, $\lim_{n\to+\infty}\left(\frac{1}{1+\chi}\right)^n Q(\theta) = 0$ since $\left|\frac{1}{1+\chi}\right| < 1$. Given that the right hand side converges to zero as $n \to \infty$, for every $\varepsilon > 0$ there exists an N such that for all $n, m \geq N$ we have that

$$|x_n - x_m| \leq \varepsilon,$$

which means that $\{x_n\}$ is a sequence on a complete metric space ($\mathbb{R}$ with the absolute value norm). By completeness of $\mathbb{R}$, there exists some x^* such that $x_n \to x^*$. By continuity of the operator T, we get that x^* is a fixed point, since

$$T[x^*] = T\left[\lim_{n\to+\infty} x_n\right] = \lim_{n\to+\infty} T(x_n) = \lim_{n\to+\infty} x_{n+1} = x^*.$$

This established convergence to a fixed point (existence), but there may be more than one.

Uniqueness part. For uniqueness, consider first x^a and x^b. Define $x_n^a \equiv T^n[x^a]$ and

$x_n^b \equiv T^n\left[x^b\right]$. Observe that $\left|x_n^a - x_n^b\right|$ is bounded by

$$\left|x_n^a - x_n^b\right| \leq \left(\frac{1}{1+\chi}\right)^n M,$$

for some $M > 0$. It follows that $\lim_{n\to+\infty} \sum_{j=n}^{+\infty} \left|x_j^a - x_j^b\right| = 0$, so

$$\sum_{j=0}^{+\infty} \left|x_j^a - x_j^b\right| < +\infty.$$

Suppose that there are two fixed points, x^* and x^{**}. Then we have that

$$\sum_{j=0}^{+\infty} \left|T^j\left[x^*\right] - T^j\left[x^{**}\right]\right| = \sum_{j=0}^{+\infty} \left|x^* - x^{**}\right| < +\infty,$$

which implies that $x^* = x^{**}$.

Differentiability. Observe that while the function $x_0(\theta, \theta^*, \chi)$ is not differentiable with respect to θ , the function $T[x_0](\theta, \theta^*, \chi)$ and all its higher-order iterates $\{T^n[x_0]\}_{n\geq 2}$ instead are differentiable with respect to all three arguments θ, θ^* and χ. It follows that the fixed point of the T operator x^* is also differentiable.

C.2 Wage sandwiching

To verify that it is indeed optimal for the monetary authority to abandon the peg when $\theta \geq \theta^*$, we need to establish that the candidate equilibrium wage function is sandwiched between its common knowledge counterparts. For this, we need to be able to abstract from negative values of the fundamentals, which we formalize with the following assumption.

Assumption 1. *Assume that θ_0, σ_0 and σ_x are such that $\left|\int_{-\infty}^{0} \zeta\phi\left(\zeta, \theta, \sigma\right) d\zeta\right| < \varepsilon$, for ε arbitrarily small and $\theta > 0$.*

Lemma 4. *If $\theta > 0$ and $\theta^* > 0$ the candidate equilibrium aggregate nominal wage $w\left(\theta, \theta^*, \chi\right)$ is sandwiched between (i) the aggregate nominal wage under common knowledge when workers expect the central bank to uphold the peg and (ii) the aggregate nominal wage under common knowledge when they expect the bank to abandon the peg, $w(\theta, \theta^*, \chi) \in \left(0, \frac{\theta}{\chi}\right)$.*

The proof follows. Recall that

$$T\left[x\right] = \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} \left(x\left(\zeta, \theta^*, \chi\right) + \zeta\right) \phi\left(\zeta, \theta, \sigma\right) d\zeta.$$

Now define $\underline{T}\left[x\right]$ as the operator associated with the right hand side of the wage equation when workers expect the central bank to never abandon the peg,

$$\underline{T}[x] \equiv \frac{1}{1+\chi} \int_{-\infty}^{+\infty} 0 \phi(\zeta, \theta, \sigma) \, d\zeta. \tag{C.15}$$

This operator has a unique fixed point,

$$\underline{x}(\theta, \theta^*, \chi) = 0.$$

Observe that for $\theta \to -\infty$, applying the T operator is equivalent to applying the $\underline{T}$ operator. Hence, for θ arbitrarily large and negative the wage rate under dispersed information converges to the wage rate under common knowledge when workers expect the peg to be upheld.

Otherwise, we obtain that

$$T[\underline{x}] - \underline{T}[\underline{x}] = T[\underline{x}] = \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} (\underline{x}(\theta, \theta^*, \chi) + \zeta) \phi(\zeta, \theta, \sigma) \, d\zeta > 0$$

since $\theta^* > 0$ and $\theta > 0$. Moreover,

$$T^2[\underline{x}] - \underline{T}^2[\underline{x}] = \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} (T[\underline{x}](\zeta, \theta^*, \chi) + \zeta) \phi(\zeta, \theta, \sigma) \, d\zeta > 0,$$

since $T[\underline{x}] > 0$ and $\frac{1}{1+\chi} \int_{\theta^*}^{+\infty} \zeta \phi(\zeta, \theta, \sigma) \, d\zeta > 0$.
By induction, we obtain that $\lim_{n\to\infty} (T^n[\underline{x}] - \underline{T}^n[\underline{x}]) > 0$. Since $\lim_{n\to\infty} T^n[\underline{x}] = x^*$, and $\lim_{n\to\infty} T^n[\underline{x}] = \underline{x}$, it follows that $x^* - \underline{x} > 0$.

Now, define $\overline{T}[x]$ as the operator associated with the right hand side of the wage equation when workers expect the central bank to always abandon the peg,

$$\overline{T}[x] \equiv \frac{1}{1+\chi} \int_{-\infty}^{+\infty} (x(\zeta, \theta^*, \chi) + \zeta) \phi(\zeta, \theta, \sigma) \, d\zeta. \tag{C.16}$$

For $\theta \to +\infty$, applying the T operator is equivalent to applying the $\overline{T}$ operator. For θ arbitrarily large and positive the wage rate under dispersed information converges to the wage rate under common knowledge when workers expect a devaluation. Like $\underline{T}[x]$, the operator $\overline{T}[x]$ also has a unique fixed point,

$$\overline{x}(\theta, \chi) = \frac{1}{\chi} \theta.$$

Assume that $\sigma_x \to 0$ so $\psi \to 1$ and $\theta \to \theta$. Now consider

$$T[\overline{x}] - \bar{T}[\overline{x}] \quad = \quad \frac{1}{(1+\chi)^2} \int_{\theta^*}^{+\infty} \left(\frac{1}{\chi}\zeta + \zeta \right) \phi(\zeta, \theta, \sigma) \, d\zeta - \frac{1}{(1+\chi)^2} \int_{-\infty}^{\infty} \zeta \phi(\zeta, \theta, \sigma) \, d\zeta$$

$$= -\frac{1}{(1+\chi)^2}\int_{-\infty}^{\theta^*}\zeta\phi(\zeta,\theta,\sigma)\,d\zeta < 0$$

since $\theta^* > 0$ and by Assumption 1 $\int_{-\infty}^{\theta^*}\zeta\phi\phi(\zeta,\theta,\sigma)\,d\zeta = \int_0^{\theta^*}\zeta\phi(\zeta,\theta,\sigma)\,d\zeta > 0$. As for the second iteration round,

$$\begin{aligned}
T^2[\overline{x}] - \bar{T}^2[\overline{x}] &= \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}(T[\bar{x}](\zeta,\theta^*,\chi)+\zeta)\,\phi(\zeta,\theta,\sigma)\,d\zeta \\
&\quad -\frac{1}{1+\chi}\int_{-\infty}^{\infty}\left(\bar{T}[\bar{x}](\zeta,\theta^*,\chi)+\zeta\right)\phi(\zeta,\theta,\sigma)\,d\zeta \\
&= \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(T[\bar{x}](\zeta,\theta^*,\chi)-\bar{T}[\bar{x}](\zeta,\theta^*,\chi)\right)\phi(\zeta,\theta,\sigma)\,d\zeta \\
&\quad -\frac{1}{1+\chi}\int_{-\infty}^{\theta^*}\left(\bar{T}[\bar{x}](\zeta,\theta^*,\chi)+\zeta\right)\phi(\zeta,\theta,\sigma)\,d\zeta,
\end{aligned}$$

that is,

$$\begin{aligned}
T^2[\overline{x}] - \bar{T}^2[\overline{x}] &= \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(T[\bar{x}](\zeta,\theta^*,\chi)-\bar{T}[\bar{x}](\zeta,\theta^*,\chi)\right)\phi(\zeta,\theta,\sigma)\,d\zeta \\
&\quad -\frac{1}{(1+\chi)^2}\int_{-\infty}^{\theta^*}\zeta\phi(\zeta,\theta,\sigma)\,d\zeta, \\
&< 0
\end{aligned}$$

where the last inequality follows from the fact that $T[\overline{x}]-\bar{T}[\overline{x}] < 0$ and that $\int_{-\infty}^{\theta^*}\zeta\phi(\zeta,\theta,\sigma)\,d\zeta > 0$. Again by induction, then, $x^* - \overline{x} < 0$.

C.3 Wage monotonicity

Lemma 5. *The candidate wage function $w(\theta,\theta^*,\chi)$ is (i) increasing in θ; (ii) increasing in θ^* and (iii) decreasing in χ.*

The proof follows. **Part (i).** We now show that the fixed point of T is increasing in θ. Using the change of variable $z = \frac{\zeta-\theta}{\sigma}$we write the T operator as

$$T[x] = \frac{1}{1+\chi}\int_{\frac{\theta^*-\bar{\theta}}{\sigma}}^{\infty}(x(\sigma z+\theta,\theta^*,\chi)+\sigma z+\theta)\,\phi(z,0,1)\,dz.$$

Now suppose that x is differentiable with respect to its first argument. Further assume that $x(\theta^*,\theta^*,\chi) > 0$ and $\frac{\partial}{\partial\theta}x(\theta,\theta^*,\chi) > 0$, where the notation $\frac{\partial}{\partial\theta}x$ denotes the partial derivative of x with respect to its first argument. Then we obtain

$$\frac{\partial}{\partial\theta}T[x] = \frac{1}{1+\chi}\psi(x(\theta^*,\theta^*,\chi)+\theta^*)\,\phi(\theta^*,\theta,\sigma)$$

$$+\frac{1}{1+\chi}\int_{\frac{\theta^*-\bar{\theta}}{\sigma}}^{\infty}\left(\frac{\partial}{\partial\theta}x\left(\sigma z+\theta,\theta^*,\chi\right)+\psi\right)\phi\left(z,0,1\right)dz>0.$$

Consider now $x_0\left(\theta,\theta^*,\chi\right)=0$. Then we have that

$$T\left[x_0\right]=\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\zeta\phi\left(\zeta,\theta,\sigma\right)d\zeta>0,$$

since $\theta^*>0$. Now using the change of variable $z=\frac{\zeta-\theta}{\sigma}$, we write

$$T\left[x_0\right]=\frac{1}{1+\chi}\int_{\frac{\theta^*-\bar{\theta}}{\sigma}}^{+\infty}\left(\sigma z+\theta\right)\phi\left(\zeta,\theta,\sigma\right)\sigma dz.$$

Differentiating with respect to θ we obtain

$$\frac{\partial}{\partial\theta}T\left[x_0\right]=\frac{1}{1+\chi}\psi\theta^*\phi\left(\theta^*,\theta,\sigma\right)+\frac{1}{1+\chi}\psi\int_{-\infty}^{\frac{\theta^*-\bar{\theta}}{\sigma}}\phi\left(z,0,1\right)dz>0,$$

since $\theta^*>0$ and $\phi\left(z,0,1\right)$ is a density. This establishes that $x_1=T\left[x_0\right]$ satisfies $x_1\left(\theta^*,\theta^*,\chi\right)>0$ and $\frac{\partial}{\partial\theta}x_1\left(\theta,\theta^*,\chi\right)>0$. Now consider the second iterate. We have that

$$T\left[x_1\right]=\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(x_1\left(\zeta,\theta^*,\chi\right)+\zeta\right)\phi\left(\zeta,\theta,\sigma\right)d\zeta>0,$$

since $x_1\left(\zeta,\theta^*,\chi\right)>0$ for $\theta^*>0$ and $\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\zeta\phi\left(\zeta,\theta,\sigma\right)d\zeta>0$ for $\theta^*>0$. Again differentiating with respect to θ we obtain

$$\begin{aligned}\frac{\partial}{\partial\theta}T^2\left[x_0\right]=&\frac{1}{1+\chi}\psi\left(x_1\left(\theta^*,\theta^*,\chi\right)+\theta^*\right)\phi\left(\theta^*,\theta,\sigma\right)\\&+\frac{1}{1+\chi}\int_{\frac{\theta^*-\bar{\theta}}{\sigma}}^{\infty}\left(\frac{\partial}{\partial\theta}x_1\left(\sigma z+\theta,\theta^*,\chi\right)+\psi\right)\phi\left(z,0,1\right)dz>0,\end{aligned}$$

where the inequality follows from $\theta^*>0$, $x_1\left(\theta^*,\theta^*,\chi\right)>0$, $\frac{\partial}{\partial\theta}x_1>0$ and the fact that $\phi\left(z,0,1\right)$ is a density. We have thus shown that $x_2=T\left[x_1\right]$ satisfies $x_2\left(\theta^*,\theta^*,\chi\right)>0$ and $\frac{\partial}{\partial\theta}x_2\left(\theta,\theta^*,\chi\right)>0$. By induction, we obtain that $\lim_{n\to+\infty}x_n=x^*=T\left[x^*\right]$ is also increasing. Since x^* is the unique fixed point of T, the wage function is monotonically increasing.

Part (ii). We now show that the fixed point of T is decreasing in θ^*. Again, fix $x_0\left(\theta,\theta^*,\chi\right)=0$. The first iterate $T\left[x_0\right]=\frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\zeta\phi\left(\zeta,\theta,\sigma\right)d\zeta$ is decreasing in θ^*,

$$\frac{\partial T\left[x_0\right]}{\partial\theta^*}=-\frac{1}{1+\chi}\theta^*\phi\left(\theta^*,\theta,\sigma\right)<0,$$

since $\theta^* > 0$ and $\phi(\theta^*, \theta, \sigma) > 0$. Then consider the second iterate,

$$T^2[x_0] = \frac{1}{1+\chi}\int_{\theta^*}^{+\infty} (x_1(\zeta, \theta^*, \chi) + \zeta)\,\phi(\zeta, \theta, \sigma)\, d\zeta,$$

where $x_1 = T[x_0]$. Differentiating with respect to θ^* we obtain

$$\begin{aligned}\frac{\partial T^2[x_0]}{\partial \theta^*} &= -\frac{1}{1+\chi}(x_1(\theta^*, \theta^*, \chi) + \theta^*)\,\phi(\theta^*, \theta, \sigma) \\ &\quad + \frac{1}{1+\chi}\int_{\theta^*}^{+\infty} \frac{\partial}{\partial \theta^*} x_1(\zeta, \theta^*, \chi)\,\phi(\zeta, \theta, \sigma)\, d\zeta \\ &< 0,\end{aligned}$$

where the inequality follows from $x_1(\theta^*, \theta^*, \chi) > 0$ (see part (i) of the proof), $\theta^* > 0$ and $\frac{\partial}{\partial \theta^*} x_1 < 0$. We have thus established that $x_2(\theta, \theta^*, \chi) = T^2[x_0](\theta, \theta^*, \chi)$ is decreasing in θ^*. By induction, we obtain that $\lim_{n\to+\infty} x_n(\theta, \theta^*, \chi) = x^*(\theta, \theta^*, \chi) = T[x^*](\theta, \theta^*, \chi)$ is also decreasing in θ^*. Since x^* is the unique fixed point of T, x^* is monotonically decreasing in θ^*.

Part (iii). We now show that the fixed point of T is decreasing in χ. Fix $x_0(\theta, \theta^*, \chi) = 0$. Differentiating $T[x_0] = \frac{1}{1+\chi}\int_{\theta^*}^{+\infty} \zeta\phi(\zeta, \theta, \sigma)\, d\zeta$ with respect to χ we obtain

$$\frac{\partial T[x_0]}{\partial \chi} = -\frac{1}{(1+\chi)^2}\int_{\theta^*}^{+\infty} \zeta\phi(\zeta, \theta, \sigma)\, d\zeta < 0,$$

since $\int_{\theta^*}^{+\infty} \zeta\phi(\zeta, \theta, \sigma)\, d\zeta > 0$ for $\theta^* > 0$. Differentiating the second iterate with respect to χ we obtain

$$\begin{aligned}\frac{\partial T^2[x_0]}{\partial \chi} &= -\frac{1}{(1+\chi)^2}\int_{\theta^*}^{+\infty} (x_1(\zeta, \theta^*, \chi) + \zeta)\,\phi(\zeta, \theta, \sigma)\, d\zeta \\ &\quad + \frac{1}{1+\chi}\int_{\theta^*}^{+\infty} \frac{\partial}{\partial \chi} x_1(\zeta, \theta^*, \chi)\,\phi(\zeta, \theta, \sigma)\, d\zeta.\end{aligned}$$

Since $x_1 = \frac{1}{1+\chi}\int_{\theta^*}^{+\infty} \zeta\phi(\zeta, \theta, \sigma)\, d\zeta > 0$ and $\frac{\partial}{\partial \chi} x_1 < 0$, it follows that $x_2(\theta, \theta^*, \chi) = T^2[x_0](\theta, \theta^*, \chi)$ is decreasing in χ. By induction, we obtain that $\lim_{n\to+\infty} x_n(\theta, \theta^*, \chi) = x^*(\theta, \theta^*, \chi) = T[x^*](\theta, \theta^*, \chi)$ is also decreasing in χ. Since x^* is the unique fixed point of T, x^* is monotonically decreasing in χ.

C.4 Proof of Proposition 3

First, we verify the conjecture. The central bank switches to a managed float if and only if the cost of pegging $c(w, \theta, \chi)$ is sufficiently high relative to the benefit v,

$$v \leq c\left(w\left(\theta, \theta^*, \chi\right), \theta^*, \chi\right).$$

for all $\theta \geq \theta^* > 0$, with θ^* implicitly defined as a positive root of

$$v = \frac{1}{2}\frac{1}{1+\chi}\left(w\left(\theta, \theta, \chi\right) + \theta\right)^2. \tag{C.17}$$

Fix χ and let $\Omega(\theta)$ denote the right hand side of equation (C.17). By Lemma 4, $\theta < w(\theta, \theta, \chi) + \theta < \left(1 + \frac{1}{\chi}\right)\theta$ for all $\theta > 0$. Moreover, we have argued that as $\theta \to 0$, $w(\theta, \theta, \chi) + \theta \to \theta$. On the other hand, $w(\theta, \theta, \chi) + \theta \to \frac{\theta}{\chi}$ as $\theta \to +\infty$. Hence it must be the case that for $\theta \to 0$, $\Omega(\theta) \to 0$. For $\theta \to +\infty$, on the other hand, $\Omega(\theta) \to +\infty$. Since $\Omega(\theta)$ is a continuous function on $(0, +\infty)$, to verify the conjecture that a threshold equilibrium exists, it suffices to show that that $\Omega(\theta)$ is increasing in θ for $\theta > 0$. We have that

$$\frac{d\Omega(\theta)}{d\theta} = \frac{\left(w\left(\theta, \theta^*, \chi\right) + \theta\right)}{1+\chi}\left(\frac{\partial}{\partial\theta}w\left(\theta, \theta^*, \chi\right) + \frac{\partial}{\partial\theta^*}w\left(\theta, \theta^*, \chi\right) + 1\right)\bigg|_{\theta^*=\theta>0}.$$

First, observe that by Lemma 4 $w(\theta, \theta, \chi) + \theta > \theta$. We now worry about the second term, which is potentially a problem because $\frac{\partial}{\partial\theta}w > 0$ but $\frac{\partial}{\partial\theta^*}w < 0$. However, the first derivative dominates for any $\theta > 0$ and $\theta^* > 0$.

Again, let $\sigma_x \to 0$ so $\psi = 1$. Let $x_0(\theta, \theta^*, \chi) = 0$. By previous arguments, we have that

$$\frac{\partial}{\partial\theta}T[x_0] + \frac{\partial}{\partial\theta^*}T[x_0] = \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\phi(\zeta, \theta, \sigma)\,d\zeta > 0.$$

Consider now the second iterate. Then we have that

$$\begin{aligned}\frac{\partial}{\partial\theta}T^2[x_0] + \frac{\partial}{\partial\theta^*}T^2[x_0] &= \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\left(\frac{\partial}{\partial\theta}x_1\left(\theta, \theta^*, \chi\right) + 1\right)\phi(\zeta, \theta, \sigma)\,dz \\ &\quad + \frac{1}{1+\chi}\int_{\theta^*}^{+\infty}\frac{\partial}{\partial\theta^*}x_1\left(\zeta, \theta^*, \chi\right)\phi(\zeta, \theta, \sigma)\,d\zeta \\ &> 0\end{aligned}$$

since $x_1 = T[x_0]$ and we have shown that $\frac{\partial}{\partial\theta}T[x_0] + \frac{\partial}{\partial\theta^*}T[x_0] > 0$. By induction, then, $\lim_{n\to+\infty}\left(\frac{\partial}{\partial\theta}T^n[x_0] + \frac{\partial}{\partial\theta^*}T^n[x_0]\right) = \frac{\partial}{\partial\theta}\lim_{n\to+\infty}T^n[x_0] + \frac{\partial}{\partial\theta^*}\lim_{n\to+\infty}T^n[x_0] = \frac{\partial}{\partial\theta}x^* + \frac{\partial}{\partial\theta^*}x^* > 0$. Since x^* is the unique fixed point of T, it follows that the aggregate wage equation satisfies the condition that $\frac{\partial}{\partial\theta}w + \frac{\partial}{\partial\theta^*}w > 0$.

There remains to check that $\theta^* \in \left(\underline{\theta}, \overline{\theta}\right)$. This follows immediately from the fact that

$\theta < w(\theta, \theta, \chi) < \frac{\theta}{\chi}$ and the definitions of $\underline{\theta}$ and $\overline{\theta}$.

D Proof of Corollary 1

Recall the expressions for the thresholds in the main body, (21) and (22). Differentiating with respect to χ we obtain that both thresholds are increasing in χ,

$$\begin{aligned} sign\left(\frac{\partial \underline{\theta}}{\partial \chi}\right) &= sign\left(\frac{d}{d\chi}\frac{\chi^2}{1+\chi}\right) = sign\left(\frac{\chi}{(\chi+1)^2}(\chi+2)\right) = +, \\ sign\left(\frac{\partial \overline{\theta}}{\partial \chi}\right) &= sign\left(\frac{d}{d\chi}\chi\right) = sign\,(1) = +. \end{aligned}$$

E The long-run average wage bill

Consider perfect foresight. Then if the central bank has maintained the peg in the short-run, workers expect the peg to the maintained in the long-run as well, and by equation (10) the average wage bill is $w_{LR}(\theta_{LR}, \chi) = 0$. If, on the other hand, the central bank has not maintained the peg, workers expect a devaluation, in which case the average long run bill is $w_{LR}(\theta_{LR}, \chi) = \theta_{LR}/\chi$.

Consider dispersed information. If workers expect the peg to be maintained, the average wage bill solves

$$w_{LR}(\theta_{LR}, \chi) = \int_{-\infty}^{+\infty} 0\phi(\zeta, \theta, \sigma)\, d\zeta,$$

and so again, $_{LR}(\theta_{LR}, \chi) = 0$. If workers expect a devaluation, the average wage bill solves

$$w_{LR}(\theta_{LR}, \chi) = \overline{T}[w_{LR}](\theta_{LR}, \chi),$$

where the operator $\overline{T}[x]$ is defined as $\overline{T}[x](\theta_{LR}, \chi) = \frac{1}{1+\chi}\int_{-\infty}^{+\infty}(x(\zeta, \chi) + \zeta)\,\phi\left(\zeta, \overline{\theta}(\theta_{LR}), \sigma\right) d\zeta$. It is easy to verify that $w_{LR}(\theta_{LR}, \chi) = \theta_{LR}/\chi$ is the fixed point of $\overline{T}$ when $\sigma_x \to 0$.

F Proof of Corollary 2

By implicit differentiation, we obtain

$$\begin{aligned} sign\left(\frac{\partial \underline{\theta}}{\partial \chi}\right) &= sign\left(\frac{d}{d\chi}\frac{\chi^2}{1+\chi}v(\chi)\right) = sign\left(\frac{\chi}{(\chi+1)^2}(\chi+2)\,v(\chi) + \frac{\chi^2}{1+\chi}\frac{dv}{d\chi}\right) \\ &= sign\left(\frac{\chi}{(\chi+1)^2}(\chi+2)\,v - \frac{\chi^2}{1+\chi}\frac{1}{\chi}v\right) = sign\left(v\frac{\chi}{(\chi+1)^2}\right) = +, \end{aligned}$$

where the second line uses the result that $\frac{dv}{d\chi} = -\frac{v}{\chi}$. As for the upper threshold, we have that

$$sign\left(\frac{\partial\overline{\theta}}{\partial\chi}\right) = sign\left(\frac{d}{d\chi}(1+\chi)\, v(\chi)\right) = sign\left(v - (1+\chi)\frac{1}{\chi}v\right) = sign\left(-\frac{v}{\chi}\right) = -.$$

Hence, the lower threshold is still increasing but there is a reversal for the upper threshold.

G Proof of Corollary 3

Suppose that active agents expect a devaluation. By equation (10), recalling that the devaluation price is given by (13) and using the fact that $w = (1-\lambda)\, w_1 + \lambda w_2$, we obtain that for $w_2 \geq 0$ active wages are given by

$$w_1 = \frac{\theta + \lambda w_2}{\lambda + \chi}$$

with the corresponding average wage given by

$$w = \frac{1-\lambda}{\lambda+\chi}\theta + \lambda\frac{\chi+1}{\lambda+\chi}w_2.$$

If the authority were to indeed devalue, it would set a price equal to

$$p = \frac{1}{\lambda+\chi}\theta + \frac{\lambda}{\lambda+\chi}w_2.$$

The central bank then devalues if and only if

$$\frac{\beta}{\chi}\mathbb{E}\left[\theta_{LR}^2\right] \leq \frac{1+\chi}{(\lambda+\chi)^2}\left(\theta_{SR} + \lambda w_2\right)^2$$

which enables us to characterize the lower dominance threshold as

$$\underline{\theta} = \sqrt{v(\chi)\frac{(\lambda+\chi)^2}{1+\chi}} - \lambda w_2.$$

Now consider $\underline{\theta}$. Differentiating with respect to χ, we obtain

$$\begin{aligned} sign\left(\frac{\partial\underline{\theta}}{\partial\chi}\right) &= sign\left(\frac{\partial}{\partial\chi}\frac{(\lambda+\chi)^2}{\chi}\frac{1}{1+\chi}\right) \\ &= sign\left(P(\lambda)\right) \end{aligned}$$

The sign of this expression depends on the term $P(\lambda) = -(2\chi+1)\lambda + \chi$. Observe that the function P is continuous, with

$$\begin{aligned}
\lim_{\lambda\to 0} P(\lambda) &= \chi > 0, \\
\lim_{\lambda\to 1} P(\lambda) &= -(1+\chi) < 0.
\end{aligned}$$

Moreover, P is decreasing in λ,

$$\frac{dP}{d\lambda} = -(2\chi+1) < 0.$$

So there exists a range of values of λ, $\left(\frac{\chi}{1+2\chi}, 1\right]$ such that $P(\lambda) < 0$. As a result, we have that

$$\frac{\partial\theta}{\partial\chi} < 0 \text{ for } \lambda \in \left(\frac{\chi}{1+2\chi}, 1\right],$$

and $\frac{\partial\theta}{\partial\chi} > 0$ otherwise. In addition, we have that as λ increases, the sensitivity of $\underline{\theta}$ to χ also decreases,

$$\begin{aligned}
sign\left(\frac{\partial}{\partial\lambda}\left(\frac{\partial\overline{\theta}}{\partial\chi}\right)\right) &= sign\left(\frac{\partial}{\partial\lambda}\left(-(\lambda-\chi+2\lambda\chi)\frac{\lambda+\chi}{\chi^2(\chi+1)^2}\right)\right). \\
&= -sign(\frac{\partial}{\partial\lambda}(\lambda-\chi+2\lambda\chi)(\lambda+\chi)) \\
&= -sign(2\left(\lambda+\chi^2+2\lambda\chi\right)) \\
&= -
\end{aligned}$$

Now suppose active workers expect a peg. Since active wages are given by $w_1 = 0$, the corresponding average wage is given by

$$w = \lambda w_2.$$

The central bank then devalues if and only if

$$v(\chi) \leq \frac{1}{1+\chi}(\lambda w_2 + \theta_{SR})^2$$

which enables us to characterize the upper dominance threshold as

$$\overline{\theta} = \sqrt{v(\chi)(1+\chi)} - \lambda w_2.$$

Since $\overline{\theta}$ is computed under the assumption that active workers expect the peg to be upheld, the sensitivity of $\overline{\theta}$ to λ is unaffected by λ, $\frac{\partial}{\partial\lambda}\left(\frac{\partial\overline{\theta}}{\partial\chi}\right) = 0$.

To lighten the notation in the body of the paper we set $w_2 = 0$. This assumption is without loss of generality since w_2 is just a constant and it does not affect the sensitivity of the critical thresholds with respect to the conservatism of the central bank, χ.

H Results about $w(\theta, \theta^*, \chi)$ when $\lambda > 0$

Consider the variant of the model in which a fraction λ of workers cannot readjust their wages. The individually optimal wage for workers who can optimize is $w_{1,i} = \mathbb{E}_\theta [p|x]$. Averaging across these workers, we obtain

$$\begin{aligned} w_1(x, \theta^*, \chi, \lambda) &= \mathbb{E}_x [\mathbb{E}_\theta [p|x] |\theta] \\ &= \int_{-\infty}^{\infty} p(\zeta, \theta^*, \chi) \phi(\zeta, \theta, \sigma) d\zeta \\ &= \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} (w(\zeta, \theta^*, \chi, \lambda) + \zeta) \phi(\zeta, \theta, \sigma) d\zeta \\ &= \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} ((1-\lambda) w_1(\zeta, \theta^*, \chi, \lambda) + \lambda w_2 + \zeta) \phi(\zeta, \theta, \sigma) d\zeta. \end{aligned}$$

Setting $w_2 = 0$ and observing that the average wage $w = (1-\lambda) w_1$, we can write the equation for the average wage as

$$w_\lambda(\theta, \theta^*, \chi, \lambda) = (1-\lambda) \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} (w_\lambda(\zeta, \theta^*, \chi, \lambda) + \zeta) \phi(\zeta, \theta, \sigma) d\zeta.$$

where we have used the notation w_λ to denote the average wage bill function in the presence of predetermined wages to avoid any confusion with the benchmark average bill function w that applies when $\lambda = 0$. Let T_λ denote the operator associated with the right hand side of this equation, so $T_\lambda = (1-\lambda) T$. Since $1 - \lambda < 1$, all the results that apply to the average wage defined using T also apply to the average wage defined using T_λ.

Next, we show that the average wage is less sensitive to both χ and θ. We start by showing that a positive λ reduces the aggregate wage. Fix $x_0 = 0$. Then $T[x_0] > T_\lambda [x_0]$ since $\frac{\lambda}{1+\chi} \int_{\theta^*}^{+\infty} (w_\lambda(\zeta, \theta^*, \chi, \lambda) + \zeta) \phi(\zeta, \theta, \sigma) d\zeta > 0$ for $\theta^* > 0$. Moreover,

$$\begin{aligned} T^2[x_0] - T_\lambda^2[x_0] =& \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} (T[x_0](\zeta, \theta^*, \chi) + \zeta) \phi(\zeta, \theta, \sigma) d\zeta \\ &- \frac{1-\lambda}{1+\chi} \int_{\theta^*}^{+\infty} (T_\lambda[x_0](\zeta, \theta^*, \chi, \lambda) + \zeta\phi)(\zeta, \theta, \sigma) d\zeta \\ =& \frac{1}{1+\chi} \int_{\theta^*}^{+\infty} (T[x_0](\zeta, \theta^*, \chi) - T_\lambda[x_0](\zeta, \theta^*, \chi, \lambda)) \phi(\zeta, \theta, \sigma) d\zeta \\ &+ \frac{\lambda}{1+\chi} \int_{\theta^*}^{+\infty} \zeta\phi(\zeta, \theta, \sigma) d\zeta > 0. \end{aligned}$$

By induction and using previous results about existence and uniqueness of a fixed point of the two operators, introducing passive workers reduces the average wage, $w(\theta, \theta^*, \chi) > w_\lambda(\theta, \theta^*, \chi, \lambda)$. Observe that as a result, it must be the case that $\theta^*_\lambda > \theta^*$. This is because the critical threshold solves $v = c(w, \theta, \chi)$, and the cost of pegging c is increasing in w.

We now show that the wage is less sensitive to χ when $\lambda > 0$. The argument relies on the proof that the fixed point of T is decreasing in χ. Again, fix $x_0(\theta, \theta^*, \chi) = 0$. Differentiating with respect to χ we obtain

$$\left|\frac{\partial T[x_0]}{\partial \chi}\right| - \left|\frac{\partial T_\lambda[x_0]}{\partial \chi}\right| = \frac{\lambda}{(1+\chi)^2}\int_{\theta^*}^{+\infty} \zeta\phi(\zeta, \theta, \sigma)\, d\zeta > 0,$$

since $\int_{\theta^*}^{+\infty} \zeta\phi(\zeta, \theta, \sigma)\, d\zeta > 0$ for $\theta^* > 0$. Differentiating the second iterate with respect to χ we obtain

$$\begin{aligned}
&\left|\frac{\partial T^2[x_0]}{\partial \chi}\right| - \left|\frac{\partial T^2_\lambda[x_0]}{\partial \chi}\right| \\
&= \frac{1}{(1+\chi)^2}\int_{\theta^*}^{+\infty} \left(\left(T[x_0](\zeta, \theta^*, \chi) - T_\lambda[x_0](\zeta, \theta^*, \chi, \lambda)\right)\right)\phi(\zeta, \theta, \sigma)\, d\zeta \\
&+ \frac{1}{1+\chi}\int_{\theta^*}^{+\infty} \left(\left|\frac{\partial}{\partial \chi}T[x_0](\zeta, \theta^*, \chi)\right| - \left|\frac{\partial}{\partial \chi}T_\lambda[x_0](\zeta, \theta^*, \chi, \lambda)\right|\right)\phi(\zeta, \theta, \sigma)\, d\zeta > 0
\end{aligned}$$

where the last inequality follows from the fact that $T[x_0](\zeta, \theta^*, \chi) > T_\lambda[x_0](\zeta, \theta^*, \chi) > 0$ for $\lambda > 0$ and that $\left|\frac{\partial T[x_0]}{\partial \chi}\right| - \left|\frac{\partial T_\lambda[x_0]}{\partial \chi}\right| > 0$. Again by induction and using previous results about existence and uniqueness of a fixed point of the two operators, introducing passive workers makes the average wage less sensitive to χ, $0 > \frac{\partial w_\lambda}{\partial \chi} > \frac{\partial w}{\partial \chi}$.

We now show that the wage is less sensitive to θ and θ^* when $\lambda > 0$. Let $x_0(\theta, \theta^*, \chi) = 0$. By previous arguments, we have that

$$\frac{\partial}{\partial \theta}T[x_0] + \frac{\partial}{\partial \theta^*}T[x_0] - \frac{\partial}{\partial \theta}T_\lambda[x_0] - \frac{\partial}{\partial \theta^*}T_\lambda[x_0] = \frac{\lambda}{1+\chi}\int_{\theta^*}^{+\infty} \phi(\zeta, \theta, \sigma)\, d\zeta > 0.$$

Consider now the second iterate. Then we have that

$$\begin{aligned}
&\frac{\partial}{\partial \theta}T[x_0] + \frac{\partial}{\partial \theta^*}T[x_0] - \frac{\partial}{\partial \theta}T_\lambda[x_0] - \frac{\partial}{\partial \theta^*}T_\lambda[x_0] \\
&= \frac{1}{1+\chi}\int_{\theta^*}^{+\infty} \left[\frac{\partial}{\partial \theta}T[x_0](\theta, \theta^*, \chi) + \frac{\partial}{\partial \theta^*}T[x_0](\zeta, \theta^*, \chi)\right]\phi(\zeta, \theta, \sigma)\, dz \\
&- \frac{1}{1+\chi}\int_{\theta^*}^{+\infty} \left[\frac{\partial}{\partial \theta}T_\lambda[x_0](\theta, \theta^*, \chi, \lambda) + \frac{\partial}{\partial \theta^*}T_\lambda[x_0](\theta, \theta^*, \chi, \lambda)\right]\phi(\zeta, \theta, \sigma)\, dz \\
&+ \frac{\lambda}{1+\chi}\int_{\theta^*}^{+\infty} \phi(\zeta, \theta, \sigma)\, dz \\
&+ \frac{\lambda}{1+\chi}\int_{\theta^*}^{+\infty} \frac{\partial}{\partial \theta}T_\lambda[x_0](\theta, \theta^*, \chi, \lambda)\,\phi(\zeta, \theta, \sigma)\, dz
\end{aligned}$$

$$+\frac{\lambda}{1+\chi}\int_{\theta^*}^{+\infty}\frac{\partial}{\partial\theta^*}T_\lambda\left[x_0\right]\left(\theta,\theta^*,\chi,\lambda\right)\phi\left(\zeta,\theta,\sigma\right)d\zeta>0$$

where the last inequality follows because $\frac{\partial T[x_0]}{\partial\theta}+\frac{\partial T[x_0]}{\partial\theta^*}-\frac{\partial T_\lambda[x_0]}{\partial\theta}-\frac{\partial T_\lambda[x_0]}{\partial\theta^*}>0$ and $\frac{\partial T_\lambda[x_0]}{\partial\theta}+\frac{\partial T_\lambda[x_0]}{\partial\theta^*}>0$. Again by induction and using previous results about existence and uniqueness of a fixed point of the two operators, introducing passive workers results in $\frac{\partial w}{\partial\theta}+\frac{\partial w}{\partial\theta^*}>\frac{\partial w_\lambda}{\partial\theta}+\frac{\partial w_\lambda}{\partial\theta^*}>0$.

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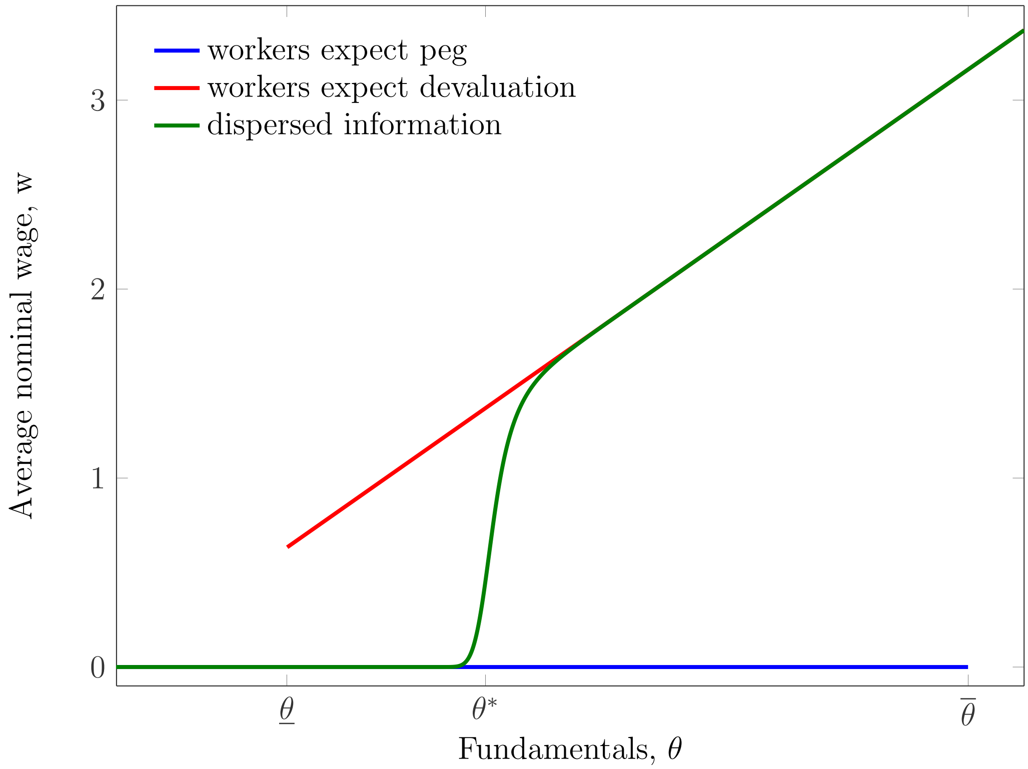

Figure 1: Equilibria under perfect foresight (red and blue lines) and dispersed information (green line). Figure drawn for $\chi = .25$, $\sigma = 0.01$.

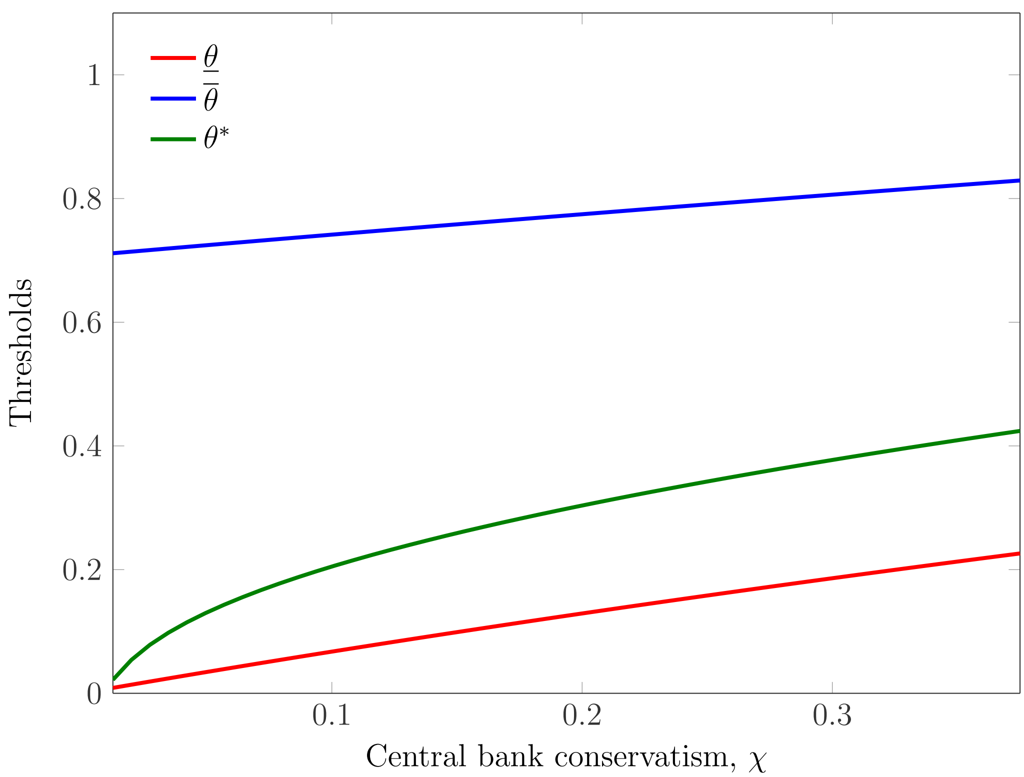

Figure 2: One-shot game. All thresholds rise with χ. Figure drawn for $\sigma = 0.01$.

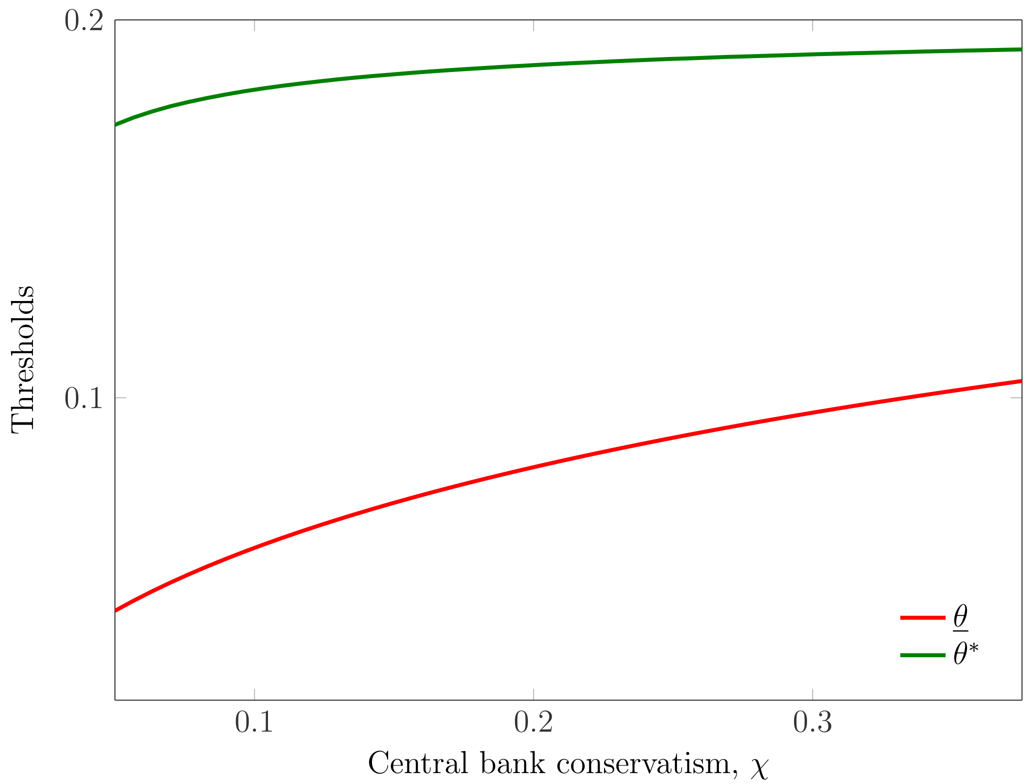

Figure 3: Repeated games. Strategic uncertainty makes θ^* less sensitive to χ than $\underline{\theta}$. Figure drawn for $\sigma = 0.01$, $\beta\mathbb{E}[\theta_{LR}^2] = 0.04$.

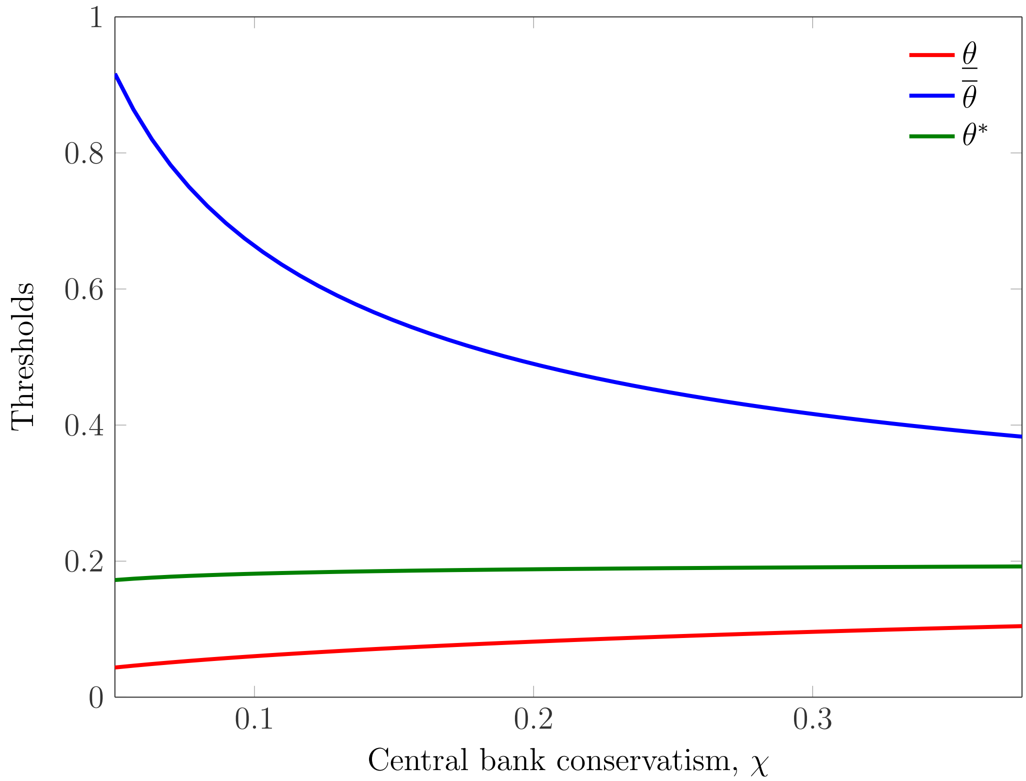

Figure 4: Repeated games. The upper threshold decreases in χ. Figure drawn for $\sigma = 0.01$, $\beta\mathbb{E}[\theta_{LR}^2] = 0.04$.

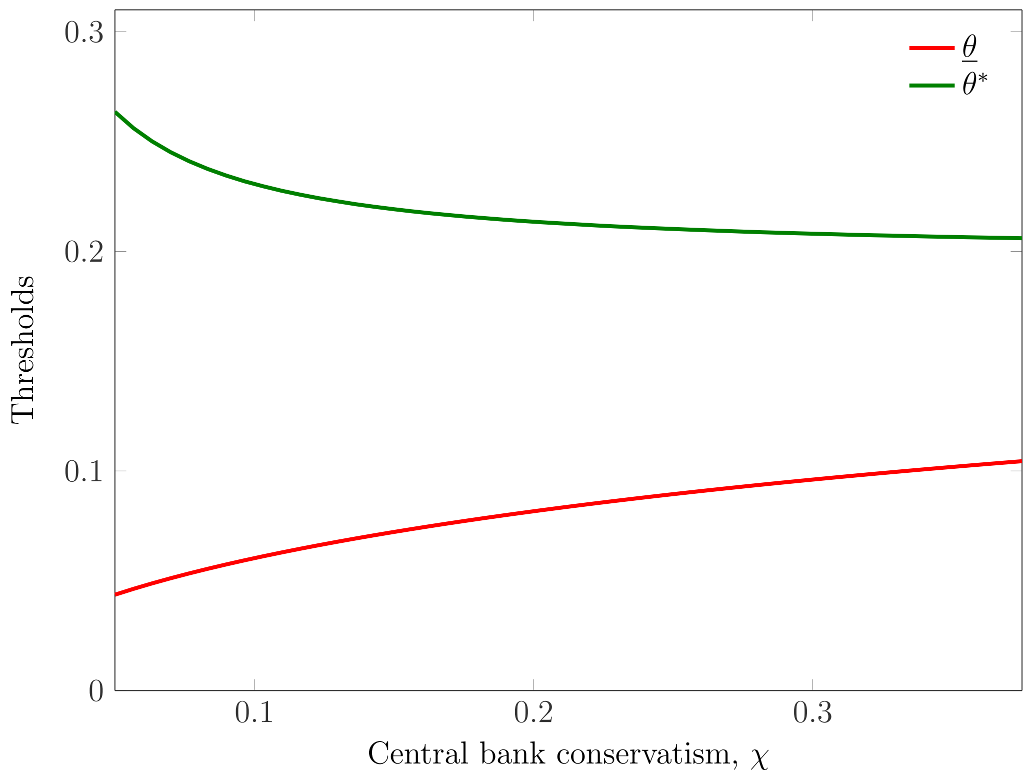

Figure 5: : Repeated games and pre-determined wages. The dispersed-information threshold decreases in χ. Figure drawn for $\sigma = 0.01$, $\beta\mathbb{E}[\theta_{LR}^2] = 0.04$, $\lambda = 0.05$.

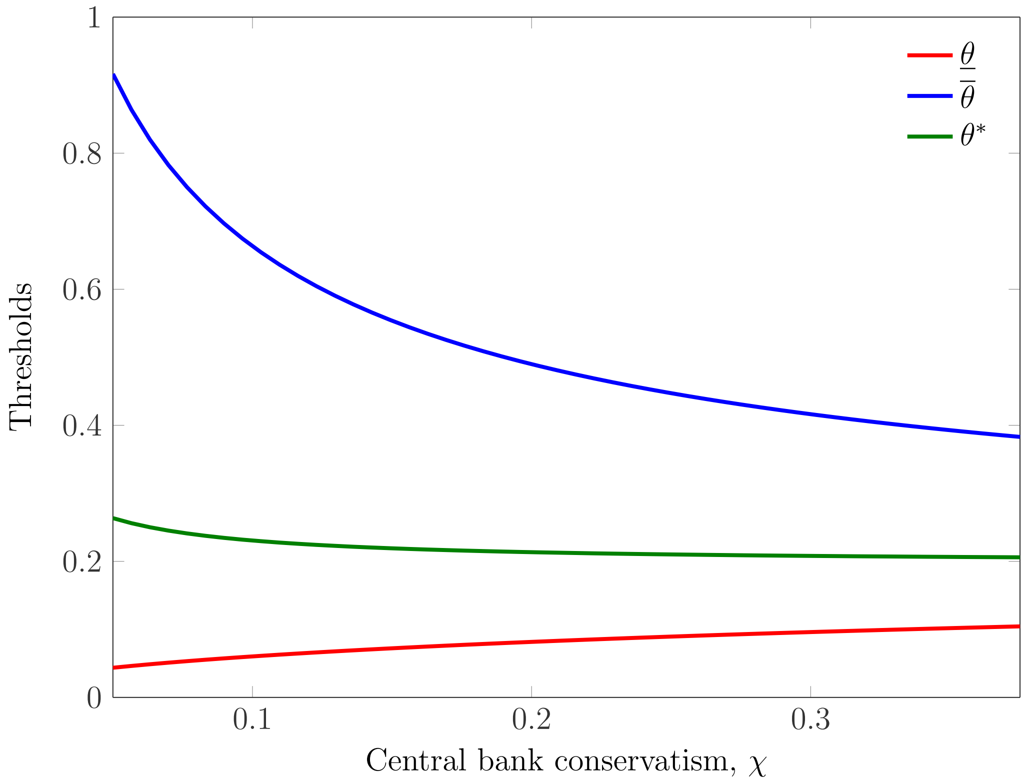

Figure 6: Repeated games and pre-determined wages. The upper perfect-foresight threshold and the dispersed-information threshold both decrease in χ. Figure drawn for $\sigma = 0.01$, $\beta\mathbb{E}[\theta_{LR}^2] = 0.04$, $\lambda = 0.05$.

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