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by Marco Lombardi and Francesco Ravazzolo

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# On the correlation between commodity and equity returns: implications for portfolio allocation*

Marco J. Lombardi[†] Francesco Ravazzolo[‡]

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**Abstract**

In the recent years several commentators hinted at an increase of the correlation between equity and commodity prices, and blamed investment in commodity-related products for this. First, this paper investigates such claims by looking at various measures of correlation. Next, we assess what are the implications of higher correlations between oil and equity prices for asset allocation. We develop a time-varying Bayesian Dynamic Conditional Correlation model for volatilities and correlations and find that joint modelling commodity and equity prices produces more accurate point and density forecasts, which lead to substantial benefits in portfolio allocation. This, however, comes at the price of higher portfolio volatility. Therefore, the popular view that commodities are to be included in one's portfolio as a hedging device is not grounded.

**Keywords:** Commodity prices, equity prices, density forecasting, correlation, Bayesian DCC.

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[†]Bank for International Settlements, Centralbahnplatz 2, CH-4002 Basel (Switzerland). Email: marco.lombardi@bis.org

[‡]Norges Bank and BI Norwegian Business School, Bankplassen 2, NO-0107 Oslo (Norway). Email: francesco.ravazzolo@norges-bank.no

# 1 Introduction

The past decade has witnessed a broad-based surge in commodity prices, with oil a frontrunner. The upward trend in prices has been ascribed to booming demand at the global level, but fluctuations around it have been substantial, especially after the onset of the Great Recession. Investing in commodities has generated hefty returns and has become increasingly popular, in spite of the high risks associated with this type of investment, due to the inherent volatility of commodity prices. Indeed, most fund managers have started advising their customers to devote a share of their portfolios to commodity-related products as part of long-term diversification strategy. This is often motivated by the fact that, over the long run, commodities are believed to display low correlation with other asset classes, most notably with equities (Gorton and Rouwenhorst [2006]).

At the same time, substantial inflows into commodity-related investment products have led many commentators to speculate on whether commodities are increasingly behaving as an asset class. The empirical evidence on a lasting impact of financial investment on commodity prices is, at best, scant (see Fattouh et al. [2012] for a recent survey). In the segment of commodities, however, financial investors may have less commodity-specific knowledge and a different attitude compared to commercial players, and hence enter or exit trades based on their overall perceptions of the macroeconomic situation rather than market-specific factors.

If this is the case, it would suggest that commodity prices are increasingly influenced by shocks coming from the demand side. This is a well-established fact in the literature on oil, following the seminal work by Kilian [2009]. The fact that equity prices are also likely to have been largely driven by shocks related to the global economic activity, especially in the aftermath of the financial crisis, could then explain the increase in correlations.[1]

[1] Tang and Xiong [2010], on a similar note, report evidence of increased correlations between the returns of commodities that are included in indexes tracked by investment funds.

In this paper, we try to shed some light on these issues. If one looks at the most recent years, commodity and equity prices appeared to be increasingly correlated, both being apparently more sensitive to news concerning the global macroeconomic environment rather than to idiosyncratic and market-specific shocks. We first provide a complete characterization of this phenomenon, by computing correlations using different methodologies and trying to identify relevant turning points. Our results suggest that correlations, after having hovered around zero for more than a decade, have indeed increased markedly since mid-2008.

The co-movements between commodity and equity prices have already been examined in the literature: Gorton and Rouwenhorst [2006] report that commodity returns display negative correlation with equity returns over a long sample, running from 1959 to 2004. Büyüşahin et al. [2010] explore the correlations on the pre-crisis period, failing to find evidence of significant changes. Using post-2008 data, Büyüşahin and Robe [2012] find that correlations between equities and commodities increased amid greater participation by speculators. Other papers focused more specifically on oil: Kilian and Park [2009] report that the response of equity prices to oil price shocks depends on the nature of the shocks; Cassassus and Higuera [2011] show that oil price changes are good predictors of equity returns; Chang et al. [2011] report evidence of volatility spillovers between oil and equity prices. To the best of our knowledge, no studies have examined yet the joint predictability of commodity and equity prices, nor implications of such predictability for asset allocation.

To investigate to what extent co-movements between commodity and equity prices could be exploited to improve forecasts in either direction, we use constant parameter univariate and bivariate models for commodity and equity prices, and derive a time-varying Bayesian Dynamic Conditional Correlation (DCC) model (Della Corte et al. [2010]), which can account for the changes in the relationship between commodity and equity prices observed after 2008. We conduct a density forecasting exercise and find that the DCC provides statistically superior density forecasts compared to a plain random walk model.

We then assess the economic value of such forecasting gains by considering an asset-allocation framework with commodities and equities. The use of a Bayesian estimation framework allows us to explicitly account for the fact that higher order moments and the full predictive densities of commodity and equity prices are uncertain and vary over time. Neglecting this and just focusing on mean and variances estimated on a fixed sample can lead to suboptimal allocation. The estimated predictive densities depend on the data and the prior, and account for estimation risk in the portfolio allocation; see Kandel and Stambaugh [1996], Barberis [2000], Avramov [2002], Cremers [2002], Kan and Zhou [2007] and Jacquier and Polson [2012]. Ravazzolo et al. [2008], Guidolin and Na [2008] and Pettenuzzo and Timmermann [2011] extend their analysis to account for instability uncertainty. Della Corte et al. [2011] and Della Corte et al. [2010] assess the economic value of volatility and correlation timing. We extend this literature by including in the asset allocation problem a new asset class, i.e. commodities. Our results indicate that using a time-varying joint model for commodity and equity prices leads to economic gains relative to passive strategies, especially at times of large price swings. At the same time, an investment strategy which also includes commodities in a portfolio produces substantially higher volatility than portfolios which do not include them. For short investment horizons this is evident for the whole sample and not only from September 2008 onwards, when correlations between equities and commodities increase. This is at odds with the common notion that commodities can serve as a hedge.

The remainder of the paper is organized as follows. Section 2 describes the data and documents changes in correlation over our sample. Section 3 presents bivariate models for commodity and equity prices and investigates their forecast accuracy. We then move to apply these findings to active asset allocation strategies in Section 4; Section 5 concludes.

## 2 Commodity and equity markets

The idea that commodity and equity prices should display a negative correlation can be deducted from expressing equity prices as the discounted value of future dividends. If prices of inputs in the production process (energy, metals, raw materials) increase, firms will see their profits shrink, other things equal, and will therefore have less dividend to distribute.[2] In a sense, however, this suggests that commodity prices are driven exogenously. It is now widely acknowledged that this is not the case. Commodity price increases often come on the back of buoyant demand due to booming economic activity (Kilian [2009]). Therefore, the sign of the correlation becomes less clear-cut: both equity and commodity prices could increase on positive news about the global macroeconomic outlook materialize. Over the past years, market commentators have indeed often pointed at commodity and equity prices moving in the same direction as a consequence of more optimistic or more pessimistic expectations about the global economy. Kilian and Vega [2011] investigate formally the impact of various macroeconomic announcements on the oil price over a sample running from 1983 to 2008, but do not find a systematic relationship.

In this section, we will gauge correlations between oil and equity returns using a number of different methods. To start with, we collected weekly returns generated by the Morgan Stanley Capital International global equity index (MSCI) and the Standard & Poor's Goldman Sachs commodity index (SPGSCI), starting from January 1980 until December 2012, see Figure 1.[3] Commodity prices are substantially more volatile as Figure 2 indicates. Figure 3 reports sample correlations computed on moving windows of different length; this is indeed what most commentators refer to when discussing co-movements of commodity and equity prices. It emerges clearly that correlations have been sometimes positive and sometimes negative, but on average have hovered around zero. In the recent

[2]While this is likely to be the case for each individual firm, at the aggregate level things may be less clear cut, since increases in input prices can be passed through customers.

[3]Data was obtained from Bloomberg.

episode, however, positive correlations appear to have been stronger and more persistent compared to the past: it started with the bursting of the financial crisis in 2008, and although correlations have recently declined, they are still positive. Using different windows alters somewhat the size of correlation, but the pattern remains.

Instead of using rolling windows of different length, which spread the influence of extreme episodes or periods of market turbulence over time, one can use model-based approaches to estimating correlations. A popular approach is the Dynamic Conditional Correlation (DCC) model proposed by Engle [2002]:

$$\begin{aligned}
y_t &= \Phi(L)y_{t-1} + v_t \\
v_t &= H_t^{0.5}\varepsilon_t,\ \varepsilon_t \sim N(0, I_N),\ H_t = D_t R_t D_t \\
D_t^2 &= \mathrm{diag}\{\omega_i\} + \mathrm{diag}\{\kappa_i\} v_{t-1} v_{t-1}' + \mathrm{diag}\{\lambda_i\} D_{t-1}^2,\ i = 1, ..., N \\
Q_t &= S(\imath\imath' - A - B) + A\varepsilon_{t-1}\varepsilon_{t-1}' + BQ_{t-1} \\
R_t &= \mathrm{diag}\{Q_t\}^{-1} Q_t \mathrm{diag}\{Q_t\}^{-1}
\end{aligned} \tag{1}$$

where $y_t$ is a $(N \times 1)$ vector of dependent variables, $S$ is the unconditional correlation matrix of $\varepsilon_t$, $A$, $B$ and $S(\imath\imath' - A - B)$ are positive semidefinite matrices; see Appendix for model details. In a nutshell, the DCC is a multivariate GARCH model in which correlations are time-varying according to an autoregressive specification. As such, the DCC accounts for both the time-varying features of volatilities and correlations.

The dynamic conditional correlations are reported in Figure 4. Although correlations appear to be smaller compared to those estimated using rolling windows, they still look persistently positive towards the end of the sample, in line with the findings of Büyüşahin and Robe [2012].[4]

[4] The dynamics of the USD exchange rate could alter the correlation pattern, since non-US equities in the MSCI index are priced in US dollars. To control for this, we repeated our analysis with the S&P500 index in place of the MSCI index. The results in terms of correlation (see Figure 8 in the appendix), statistical predictability and economic gains (available upon request) are similar to those discussed in the remainder of the paper.

Since visual inspection reveals a persistent increase in correlation towards the end of the sample, we formally tested for a breakpoint in the correlation pattern using the methodology proposed by Andrews [1993] and Andrews and Ploberger [1994]. The results of the test point to a significant breakpoint on 5 September 2008, just before the bankruptcy of Lehmann Bros.[5] Interestingly, the same breakdate is found on window-based and model-based correlations.

Again, this comes at no surprise if one takes into account that, in the downturn and the subsequent economic recovery, both commodity and equity prices are likely to have been driven by the same common shocks, i.e. news on the shape of the global macroeconomic outlook.

# 3 Joint forecasts of commodity and equity prices

The debate on the predictability of equity prices is still an open issue in empirical research, see for example Welch and Goyal [2008]. Market efficiency theories imply not predictability, whether market friction theories imply predictability. Evidence of predictability of oil prices has, on contrary, been subject to a break in recent years: from the mid-90's, research evidence suggests not predictability where future prices contain all the relevant information and alternative models cannot improve forecast accuracy. However, very recent studies such as Baumeister and Kilian [2012] find that several (Bayesian) reduced-form Vector Autoregressive models outperform forecasts based on future prices in a real-time setting. Baumeister and Kilian's models apply macroeconomic data to forecast oil prices, but they do not explore the linkage with equity prices. Furthermore, their analysis refers mainly to point forecasting. Kandel and Stambaugh [1996] and Barberis

[5]The test was conducted using the average, sup and LM statistics, with the p-values tabulated by Hansen [1997]. All test statistics are significant at the 1% level. As recommended, we cut off the first and last 10% of the observations.

[2000], among others, discuss the role of parameter uncertainty and Bayesian analysis as tool to cope with for return predictability and for asset allocation.

We produce weekly point and density forecasts for commodity and equity returns over the sample from 2005W1 to 2012W52 for a total of 417 weeks. We compute $h = 1, 2, ..., 24$ steps ahead forecast for each vintage using a bivariate Bayesian Vector Autoregressive model with Minnesota type prior (VAR), see Clark and Ravazzolo [2012] for details, and a bivariate Bayesian DCC model.[6] The main advantage of density forecasting over a plain point forecast is that it can take into account higher moments, which are key ingredients for the asset allocation exercise and can lead to substantial economic gains.

We compare these forecasts to a random walk (RW), i.e. the point forecast is taken to be today's price and the predicted volatility is the sample standard deviation, as well as a Bayesian autoregressive model (AR). Bayesian inference on the listed models allows to derive complete predictive densities, whose statistical accuracy is evaluated in the forecasting exercise. More specifically, we consider several evaluation statistics for point and density forecasts previously proposed in the literature, see Billio et al. [2012] for a recent application. We evaluate commodity and equity forecasts separately in this section, and use marginal densities from bivariate models.

We compare point forecasts in terms of Mean Square Prediction Errors (MSPE)

$$MSPE_k = \frac{1}{t^*} \sum_{t=\underline{t}}^{\overline{t}} e^2_{k,t+1},$$

where $t^* = \overline{t} - \underline{t} + 1$ and $e^2_{k,t+1}$ is the square prediction error of model $k$. We evaluate the predictive densities using two relative measures. First, we consider a Kullback-Leibler Information Criterion (KLIC) based measure; see for example Kitamura [2002], Mitchell and Hall [2005], Hall and Mitchell [2007], Amisano and Giacomini [2007], Kascha and

[6]Further details on the model and the estimation algorithm are reported in the appendix. Della Corte et al. [2010] present an application to assess the economic value of time-varying correlation timing.

Ravazzolo [2010]. The KLIC distance between the true density $p(y_{t+1}|y_{1:t})$ of a random variable $y_{t+1}$ and some candidate density $p(\tilde{y}_{k,t+1}|y_{1:t})$ obtained from model $k$ is defined as

$$\begin{aligned} \text{KLIC}_{k,t+1} &= \int p(y_{t+1}|y_{1:t}) \ln \frac{p(y_{t+1}|y_{1:t})}{p(\tilde{y}_{k,t+1}|y_{1:t})} dy_{t+1}, \\ &= \mathbb{E}_t[\ln p(y_{t+1}|y_{1:t}) - \ln p(\tilde{y}_{k,t+1}|y_{1:t}))]. \end{aligned} \tag{2}$$

where $\mathbb{E}_t(\cdot) = \mathbb{E}(\cdot|\mathcal{F}_t)$ is the conditional expectation given information set $\mathcal{F}_t$ at time $t$. An estimate can be obtained from the average of the sample information, $y_{\underline{t}+1}, \ldots, y_{\bar{t}+1}$, on $p(y_{t+1}|y_{1:t})$ and $p(\tilde{y}_{k,t+1}|y_{1:t})$:

$$\overline{KLIC}_{k+1} = \frac{1}{t^*} \sum_{t=\underline{t}}^{\bar{t}} [\ln p(y_{t+1}|y_{1:t}) - \ln p(\tilde{y}_{k,t+1}|y_{1:t})]. \tag{3}$$

The KLIC chooses the model which on average gives higher probability to events that have actually occurred. In reality we do not know the true density, but for the comparison of two competing models, it is sufficient to consider the Logarithmic Score (LS), which corresponds to the latter term in the above sum,

$$LS_k = -\frac{1}{t^*} \sum_{t=\underline{t}}^{\bar{t}} \ln p(\tilde{y}_{k,t+1}|y_{1:t}), \tag{4}$$

for all $k$ and to choose the model for which the expression in (4) is minimal, or as we report in our tables, the opposite of the expression in (4) is maximal.

Since the distribution properties of a statistical test to compare density accuracy performances, measured in terms of LS, are not derived when working with nested models and expanding estimation windows, as is our case, we follow Groen et al. [2012] and test the null of equal finite sample forecast accuracy, *against* the alternative that a model outperformed the RW benchmark using the Harvey et al. [1997] small sample correction of the Diebold and Mariano [1995] and West [1996] statistic to standard normal critical values.[7] Following evidence in Clark and McCracken [2012] for point forecasting, we apply the same test to investigate superiority in square prediction errors.

Table 1 reports point and density forecast results. Absolute predictability for commodity returns is substantially lower than absolute predictability for equity returns: MSPEs are higher and LS lower for all horizons. Data characteristics discussed in section 2 can explain the result. However, the evidence is somewhat different for relative predictability and point toward density forecasting. The models produce very similar point forecasts; but the DCC model gives the highest LS for all the horizons in forecasting commodities, but not in forecasting equity returns. Therefore, equities seem to contain relevant information to forecast commodities, whether the opposite is not supported by our analysis. Ferraro et al. [2012] find opposite evidence when investigating exchange rate and oil price predictability: oil prices forecast exchange rates, but exchange rates do not forecast oil prices. However, reported improvements are often very small.

Moreover, the DCC model gives more accurate forecasts relative to the RW benchmark for all the horizons up to 24 weeks and improvements in density forecasting are always statistically significant. So, a time-varying covariance matrix which can model instability in volatility and correlations between the two variables, as discussed in Section 2 is an important ingredient to predict higher moments of the joint commodity and equity predictive density. Figure 5 indicates that most of the gains are in September 2008; but for several horizons gains persist also in the aftermath of the Lehman bankrupcy.

[7] Given that we maximise the LS, we use right-tail p-values for the LS test.

# 4 Asset allocation exercise

How should an investor read the results reported in the last two sections? On one hand, the finding that correlation has become positive in the aftermath of the crisis may suggest that commodities can no longer work as a hedge in one's portfolio. On the other hand, the fact that commodity returns are predicted by equity returns may bring benefits if one follows a dynamic asset allocation strategy.

To answer these questions, we need to investigate the economic value of jointly modelling commodities and equities in the setting of an investment strategy which allows both assets to be included in one's portfolio. To this end, we develop an active short-term investment exercise. The investor's portfolio consists of the equity index, the commodity index and risk free bonds only.[8]

At the end of each week $t$, the investor decides upon the fraction $\alpha_{s,t+h}$ of her portfolio to be held in stocks, $\alpha_{c,t+h}$ in the commodity index and the remaining part in the risk free asset for the period $t+h$, based on the forecast of the commodity and stock index returns. We constrain $\alpha_{s,t+h}$, $\alpha_{c,t+h}$ not allowing for short-sales or leveraging (see Barberis [2000]) and to have at least 50% of the invested portfolio in risky asset. We assume that the investor maximizes a power utility function:

$$u(R_{t+h}) = \frac{R_{t+h}^{1-\gamma}}{1-\gamma}, \qquad \gamma > 1, \tag{5}$$

where $\gamma$ is the coefficient of relative risk aversion and $R_{t+h}$ is the wealth at time $t+h$, which is equal to

$$R_{t+h} = R_t \left( (1 - \alpha_{s,t+h} - \alpha_{c,t+h}) \exp(y_{f,t+h}) + \alpha_{t+h} \exp(y_{f,t+h} + \tilde{y}_{t+h}) \right), \tag{6}$$

[8]The risk free asset is approximated by using the weekly federal funds rate; data was obtained from the Fred database at the Federal Reserve Bank of St Louis.

where $R_t$ denotes initial wealth, $\alpha_{t+h} = (\alpha_{s,t+h}, \alpha_{c,t+h})$, $y_{f,t+h}$ the h-step ahead risk free rate and $\tilde{y}_{t+h}$ the h-step ahead bivariate forecast of the commodity and stock index returns in excess of the risk free made at time $t$. Our investor does not rebalance the portfolios in the period from $t$ to $t+h$, but keeps positions on the three assets constant.[9]

When the initial wealth is set equal to one, i.e. $R_0 = 1$, the investor solves the following problem:

$$\max_{\alpha_{t+h}\in[0,1]^2 \sum \alpha_{t+h}\leq 1} \mathbb{E}_t \left( \frac{((1-\alpha_{s,t+h}-\alpha_{c,t+h})\exp(y_{f,t+h}) + \alpha_{t+h}\exp(y_{f,t+h}+\tilde{y}_{t+h}))^{1-\gamma}}{1-\gamma} \right).$$

The expectation $\mathbb{E}_t(\cdot)$ depends on the predictive density for the commodity and stock excess returns, $p(\tilde{y}_{t+h}|y_{1:t})$ and the problem can be rewritten as:

$$\max_{\alpha_{t+h}\in[0,1]^2 \sum \alpha_{t+h}\leq 1} \int u(R_{t+h}) p(\tilde{y}_{t+h}|y_{1:t}) d\tilde{y}_{t+h}. \tag{7}$$

We approximate the integral in (7) by generating $G$ independent draws from the predictive density $p(\tilde{y}_{t+h}|y_{1:t})$, $\tilde{y}^g_{t+h}$, and then use a numerical optimization method to find:

$$\max_{\alpha_{t+h}\in[0,1]} \frac{1}{G} \sum_{g=1}^{G} \left( \frac{((1-\alpha_{t+h})\exp(y_{f,t+h}) + \alpha_{t+h}\exp(y_{f,t+h}+\tilde{y}^g_{t+h}))^{1-\gamma}}{1-\gamma} \right). \tag{8}$$

We consider an investor who can choose between different forecast densities of the (excess) commodity and equity returns $y_{t+h}$ to solve the optimal allocation problem described above. We include three cases in the empirical analysis below and assume the investor uses alternatively the density from the RW and AR univariate models for each

[9] In the case of dynamic asset allocation the long-run investor is allowed to rebalance her portfolio during the investment period, adjusting the portfolio weights to reflect new information that arrives. Solving the resulting dynamic programming problem is complicated due to the large number of state variables that enter the problem in a highly nonlinear way, see Barberis [2000] and Guidolin and Timmermann [2007].

series, the bivariate BVAR and the bivariate BVAR-DCC. Moreover, since the portfolio weights in the active investment strategies change every period, we include transaction costs of $c = 0.05\%$, i.e. 5 basis points.

We evaluate the different investment strategies for an investor with a risk aversion parameter $\gamma = 5$, as in Barberis [2000], by computing the *ex post* annualized mean portfolio return, the annualized standard deviation and the Sharpe ratio.[10] We compare the wealth provided at time $t + h$ by two resulting portfolios by determining the value of multiplication factor of wealth $\Delta$ which equates their average utilities. For example, suppose we compare two strategies A and B.

$$\sum_{t=\underline{t}}^{\bar{t}} u(R_{A,t+h}) = \sum_{t=\underline{t}}^{\bar{t}} u(R_{B,t+h}/\exp(r)), \tag{9}$$

where $u(R_{A,t+h})$ and $u(R_{B,t+h})$ are the wealth provided at time $T + h$ by the two resulting portfolios A and B, respectively. Following West et al. [1993], we interpret $\Delta$ as the maximum performance fee the investor would be willing to pay to switch from strategy A to strategy B.[11] We infer the value added of strategies based on individual models and the combination scheme by computing $\Delta$ with respect to three static benchmark strategies: holding only stock ($\Delta_s$), holding only commodities ($\Delta_c$), and holding 70% in equities and 30% in commodities ($\Delta_{70/30}$).

Finally we compute the certainty equivalent return (CER) for each strategy, in formula:

$$CER_{t+h} = u^{-1}(\mathbb{E}_t(u(R_{t+h}))), \tag{10}$$

where $u^{1-}$ is the inverse of the power utility function defined in (5). Strategy with maximum CER is preferred. We do not report the final value as for other alternative measures, but plot how cumulative difference between any strategy A and investing 100%

[10] Results are qualitatively similar for $\gamma = 2, \dots, 6$.

[11] See, for example, Fleming et al. [2001] for an application with stock returns.

of the portfolio in the stock market:

$$CERD_{k,t+1} = \sum_{s=\underline{t}}^{t} (CER_{A,t+h} - CER_{s,t+h}), \quad (11)$$

where $k = RW, AR, ..., FRR$. If $CERD_{k,t+h}$ increases at observation $t+h$, this indicates that the strategy $k$ gives higher CER than the benchmark strategy.

Results in Table 2 strengthen the evidence reported in the section on point and density forecasting. The VAR and DCC models give higher SR than the RW and have positive 'entrance fees' relative to passive strategies for all the horizons. Gains are robust to reasonable transaction costs. The DCC provides the highest gains for horizons longer than two and up to eight weeks. Gains are substantial compared to alternative models for 2- and 4-weeks horizons.

So, a joint modelling of commodity and equity returns with time-varying volatility can produce statistically and economically significant gains. To shed light how such gains are made, Figure 5 plots the CER differential relative to a passive strategy of investing 100% of the portfolio in stock prices over the sample period for a two weeks horizons. For all the six investment horizons we consider, the gains arise mainly in the second part of 2008, i.e. during the most turbulent time of the recent financial crisis, when equity and commodity prices are likely to have been driven by the same common shocks. However, the whole period from the beginning of the Great Financial Crisis from August 2007 to the end of the sample shows that the active strategies lead to economic gains. Those yielded by the DCC are marginally higher compared to RW models and substantially higher than AR and VAR models.

We now turn to the question of what are the relative advantages of a portfolio including commodities in addition to equities and a risk-free asset. To answer this, we compare the results of Table 2 with those coming from a similar portfolio allocation exercise, in which the weight associated to commodities is set to zero. Results, reported in Table 3, suggest that the inclusion of commodities in one's portfolio has boosted returns when using a DCC model, in particular for horizons of 2 to 4 weeks. However, this comes at the price of a substantially higher volatility, irrespective of the model used. Therefore, the common lore that commodities should serve as a hedge does not seem to be solidly grounded. We compare the realized 1-year moving window standard deviation of the portfolio based on DCC forecasts which actively invests in both stock and commodity indexes and the risk free to an active portfolio that invests only on stock and risk free using a forecast from the RW model (since it produced more accurate predictions that the AR model). Figure 7 shows that the large increase in the volatility of the stock/commodity portfolio is attained in September 2008. Nevertheless, for horizons shorter than 8 weeks this portfolio has higher volatility than the portfolio without commodities for the whole sample.

## 5 Concluding remarks

This paper has shown that the correlation between commodity and equity returns has substantially increased after the onset of the recent financial crisis. We have then investigated the joint predictability of commodity and equity returns, and its benefits in a dynamic asset allocation exercise. Relative to a benchmark random-walk model, a bivariate Bayesian DCC model, which can account for time variation in the correlation pattern, produces statistically more accurate density forecasts and gives large economic gains in an asset allocation exercise. The value of an active strategy based on DCC forecasts is large compared to passive strategies during turbulent times. At the same time, an investment strategy which also includes commodities in a portfolio produces substantially higher volatility and not always produces higher Sharpe ratios. This is at odds with the common notion that commodities serve as a hedge.

Private and institutional investors have displayed an increasing appetite for commodities over the past years. Our findings have far-fetching policy implications in this respect. On one side, our results provide empirical support for the inclusion of commodities in a portfolio. At the same time, however, we have also found that this comes at the cost of an increase in volatility. Therefore, the growing appetite for commodities is likely to produce more volatile portfolios. Digging further into the financial stability implications of the increasing correlation of commodity and equity returns is a relevant subject for future research.

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# Appendix A

## DCC: a Bayesian estimation algorithm

The DCC model for the $(N \times 1)$ vector $y_t$ is formulated as:

$$
\begin{aligned}
y_t &= \Phi(L)y_{t-1} + v_t \\
v_t &= H_t^{0.5}\varepsilon_t,\ \varepsilon_t \sim N(0, I_N),\ H_t = D_t R_t D_t \\
D_t^2 &= \text{diag}\{\omega_i\} + \text{diag}\{\kappa_i\} v_{t-1}v_{t-1}^{'} + \text{diag}\{\lambda_i\} D_{t-1}^2,\ i = 1, ..., N \\
Q_t &= S(\imath\imath^{'} - A - B) + A\varepsilon_{t-1}\varepsilon_{t-1}^{'} + BQ_{t-1} \\
R_t &= \text{diag}\{Q_t\}^{-1} Q_t \text{diag}\{Q_t\}^{-1}
\end{aligned}
\tag{12}
$$

where $S$ is the unconditional correlation matrix of $\varepsilon_t$, $A$, $B$ and $S(\imath\imath^{'} - A - B)$ are positive semidefinite matrices. In our exercise $N = 2$, therefore the parameters $A$ and $B$ reduce to scalar and above conditions to $A > 0$, $B > 0$, $A + B < 1$. Following Engle [2002], the log likelihood can be expressed as:

$$
lnL = -\frac{1}{2}\sum_{t=1}^{T}(N\ln(2\pi) + 2\ln|D_t| + v_t^{'}D_t^{-1}D_t^{-1}v_t - \varepsilon_t\varepsilon_t^{'} + \ln|R_t| + \varepsilon_{t-1}^{'}R_t^{-1}\varepsilon_{t-1}) \tag{13}
$$

We estimate the DCC model using a Metropolis-Hastings algorithm. Define the vector $\alpha_i = (\Phi_0, ..., \Phi_L, \omega_i, \kappa_i, \lambda_i, A, B)^{'}$, with $\Phi(L) = (\Phi_0, ..., \Phi_L)$, $i = 1, .., N$, and $\alpha_j$ the j-th element of it. The sampling scheme consists of the following iterative steps.

Step 1: At iteration $s$, generate a point $\alpha_j^*$ from the random walk kernel

$$
\alpha_j^* = \alpha_j^{i-1} + \epsilon_j,\ \epsilon \sim N(0, Q), \tag{14}
$$

where $Q$ is a diagonal matrix and $\sigma_j^2$ is its j-th diagonal element, and $\alpha_j^{s-1}$ is the $(s-1)$th iterate of $\alpha_j$. Therefore, we draw row elements of $\Phi_0, ..., \Phi_L$ and $\omega_i, \kappa_i, \lambda_i, A, B$

independently. Then accept $\alpha_j^*$ as $\alpha_j^s$ with probability $p = \min\left[1, f(\alpha_j^*)/f(\alpha_j^{s-1})\right]$, where $f()$ is the likelihood of model (12) times priors. Otherwise, set $\alpha_j^* = \alpha_j^{s-1}$. The elements of $Q$ are tuned by monitoring the acceptance rate to lie between 25% and 50%.

Step 2: After $M$ iterations, we apply the following independent kernel MH algorithm. Generate $\alpha_j^*$ from

$$\alpha_j^* = \mu_{\alpha_j}^{i-1} + \epsilon_j, \quad \epsilon \sim N(0, Q_{\alpha_j}), \tag{15}$$

where $\mu_{\alpha_j}$ and $Q_{\alpha_j}$ are, respectively, the sample mean and the sample covariance of the first $M$ iterates for $\alpha_j$. Then accept $\alpha_j^*$ as $\alpha_j^i$ with probability

$$p = \min\left[1, \frac{f(\alpha_j^*)g(\alpha_j^{s-1})}{f(\alpha_j^{s-1})g(\alpha_j^*)}\right], \tag{16}$$

where $g()$ is a Gaussian proposal density (15).

Priors

We set normal priors for $\Phi(L)$ with mean and variance equal to OLS estimates. The priors for $\omega_i, \kappa_i, \lambda_i, A, B$ are uniform distributed and satisfy the restrictions $\omega_i > 0$, $\kappa_i > 0, \lambda_i >$, $\kappa_i + \lambda_i < 1$, $A > 0$, $B > 0$, $A + B < 1$. We note that different priors for the coefficients $A$ and $B$ of the correlation matrix should be considered if the dimension of the model is larger than two, see discussion in Tokuda et al. [2012].

Table 1: **Forecast accuracy for commodity and equity returns**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Commodity index** | | | | | | | | | | | | |
| | h=1 | | | | h=2 | | | | h=4 | | | |
| | RW | AR | VAR | DCC | RW | AR | VAR | DCC | RW | AR | VAR | DCC |
| MSPE | 12.864 | 13.002 | 12.965 | 12.978 | 12.890 | 12.991 | 12.836 | 12.980 | 12.944 | 13.048 | 13.165 | 13.184 |
| LS | -3.351 | -3.341 | -3.220 | -2.669* | -3.300 | -3.495 | -3.279 | -2.766* | -3.259 | -3.567 | -3.315 | -2.645* |
| | h=8 | | | | h=12 | | | | h=24 | | | |
| | RW | AR | VAR | DCC | RW | AR | VAR | DCC | RW | AR | VAR | DCC |
| MSPE | 12.988 | 13.003 | 12.899 | 12.922 | 13.028 | 13.063 | 13.196 | 13.149 | 12.905 | 12.968 | 12.950 | 12.957 |
| LS | -3.269 | -3.383 | -3.398 | -2.661* | -3.266 | -3.479 | -3.593 | -2.880* | -3.263 | -3.699 | -3.734 | -2.972* |
| **Equity index** | | | | | | | | | | | | |
| | h=1 | | | | h=2 | | | | h=4 | | | |
| | RW | AR | VAR | DCC | RW | AR | VAR | DCC | RW | AR | VAR | DCC |
| MSPE | 8.081 | 8.256 | 8.536 | 8.515 | 8.088 | 8.116 | 8.345 | 8.355 | 12.905 | 12.968 | 12.950 | 12.968 |
| LS | -3.573 | -3.991 | -4.124 | -4.432 | -3.522 | -4.174 | -4.206 | -4.095 | -3.263 | -3.699 | -3.734 | -3.239 |
| | h=8 | | | | h=12 | | | | h=24 | | | |
| | RW | AR | VAR | DCC | RW | AR | VAR | DCC | RW | AR | VAR | DCC |
| MSPE | 8.201 | 8.191 | 8.294 | 8.348 | 8.280 | 8.251 | 8.274 | 8.267 | 8.454 | 8.437 | 8.426 | 8.478 |
| LS | -3.560 | -3.976 | -4.316 | -4.181 | -3.666 | -4.319 | -4.301 | -4.003 | -3.473 | -4.347 | -4.397 | -4.275 |

Notes: $RW$, $AR$, $VAR$, $DCC$ denote models as defined in Section 3; MSPE is the Mean Square Prediction Error; LS is the average Logarithmic Score. Lower MSPE and higher LS imply more accurate forecasts. One $^*$ represent rejections of the null hypothesis of equal predictability following the Harvey et al. [1997] type of test at 10%.

Table 2: **Economic value of portfolios with commodities**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No transaction costs | | | | | | | | | | | | |
| | h=1 | | | | h=2 | | | | h=4 | | | |
| | RW | AR | VAR | DCC | RW | AR | VAR | DCC | RW | AR | VAR | DCC |
| Mean Ret | 4.33 | 2.78 | 2.82 | 2.05 | 2.95 | 2.98 | 2.69 | 4.09 | 2.48 | 2.61 | 2.96 | 3.30 |
| St dev | 16.61 | 15.87 | 16.43 | 16.51 | 16.75 | 16.71 | 17.07 | 16.43 | 17.39 | 17.36 | 17.57 | 17.73 |
| SR | **0.15** | 0.05 | 0.05 | 0.01 | 0.06 | 0.06 | 0.04 | **0.13** | 0.03 | 0.04 | 0.06 | **0.08** |
| Δ_$s$ | **94.90** | 78.84 | 70.90 | 58.10 | 90.07 | 90.52 | 77.03 | **116.03** | 110.25 | 106.63 | 103.65 | **110.79** |
| Δ_$c$ | **60.25** | 44.18 | 36.24 | 23.45 | 36.02 | 36.46 | 22.97 | **61.97** | 109.74 | 106.12 | 103.13 | **110.27** |
| Δ_$70-30$ | **50.02** | 33.95 | 26.01 | 13.22 | 28.83 | 29.28 | 15.79 | **54.79** | 42.08 | 38.46 | 35.47 | **42.61** |
| | h=8 | | | | h=12 | | | | h=24 | | | |
| | RW | AR | VAR | DCC | RW | AR | VAR | DCC | RW | AR | VAR | DCC |
| Mean Ret | 4.63 | 3.55 | 5.29 | 5.24 | 3.05 | 0.33 | 1.81 | 2.54 | 3.49 | 0.96 | 2.15 | 2.37 |
| St dev | 22.12 | 23.05 | 22.89 | 22.87 | 18.03 | 20.60 | 18.90 | 19.36 | 19.35 | 20.77 | 18.02 | 19.82 |
| SR | 0.12 | 0.07 | 0.15 | **0.16** | **0.06** | -0.08 | -0.01 | 0.03 | **0.08** | -0.06 | 0.01 | 0.02 |
| Δ_$s$ | 101.14 | 15.62 | 103.73 | **104.39** | **317.98** | -106.26 | 86.98 | 107.12 | **680.53** | 158.93 | 500.64 | 362.04 |
| Δ_$c$ | 87.11 | 0.59 | 88.70 | **89.36** | **920.78** | 496.55 | 689.79 | 709.93 | **2440.69** | 1919.08 | 2260.79 | 2122.20 |
| Δ_$70-30$ | 26.45 | -61.07 | 27.04 | **27.70** | **353.53** | -70.71 | 122.53 | 142.67 | **910.50** | 388.90 | 730.61 | 592.01 |
| 5 bp transaction costs | | | | | | | | | | | | |
| | h=1 | | | | h=2 | | | | h=4 | | | |
| | RW | AR | VAR | DCC | RW | AR | VAR | DCC | RW | AR | VAR | DCC |
| Mean Ret | 2.68 | 1.19 | 2.41 | 2.87 | 2.39 | 2.19 | 1.93 | 3.36 | 2.27 | 2.35 | 2.74 | 3.04 |
| St dev | 17.61 | 19.18 | 18.37 | 18.74 | 16.76 | 16.71 | 17.08 | 16.43 | 17.39 | 17.36 | 17.57 | 17.72 |
| SR | 0.04 | -0.04 | 0.02 | **0.05** | 0.03 | 0.01 | 0.00 | **0.09** | 0.02 | 0.02 | 0.05 | **0.06** |
| Δ_$s$ | **221.04** | 23.96 | 111.36 | 120.01 | 78.57 | 74.53 | 61.45 | **101.06** | 103.29 | 99.24 | 97.29 | **103.39** |
| Δ_$c$ | **379.60** | 182.52 | 269.93 | 278.58 | 24.52 | 36.46 | 7.40 | **47.00** | 102.77 | 101.12 | 96.77 | **102.87** |
| Δ_$70-30$ | **168.94** | -28.13 | 59.27 | 67.92 | 17.33 | 13.29 | 0.21 | **39.82** | 35.11 | 31.07 | 29.11 | **35.21** |
| | h=8 | | | | h=12 | | | | h=24 | | | |
| | RW | AR | VAR | DCC | RW | AR | VAR | DCC | RW | AR | VAR | DCC |
| Mean Ret | 2.60 | 1.09 | 2.32 | 2.74 | 3.01 | 0.27 | 1.76 | 2.45 | 3.48 | 0.94 | 2.13 | 2.33 |
| St dev | 17.61 | 19.18 | 18.37 | 18.73 | 18.03 | 20.59 | 18.90 | 19.35 | 19.35 | 20.77 | 18.02 | 19.82 |
| SR | **0.04** | -0.05 | 0.02 | **0.04** | **0.06** | -0.09 | -0.01 | 0.02 | **0.08** | -0.06 | 0.00 | 0.01 |
| Δ_$s$ | **217.91** | 20.11 | 107.75 | 115.10 | **316.00** | -109.00 | 84.16 | 102.99 | **679.46** | 157.47 | 499.03 | 359.16 |
| Δ_$c$ | **376.47** | 182.52 | 266.32 | 273.66 | **918.81** | 496.55 | 686.97 | 705.80 | **2439.61** | 1919.08 | 2259.18 | 2119.32 |
| Δ_$70-30$ | **165.81** | -31.98 | 55.66 | 63.01 | **351.55** | -73.45 | 119.71 | 138.54 | **909.43** | 387.44 | 729.00 | 589.13 |

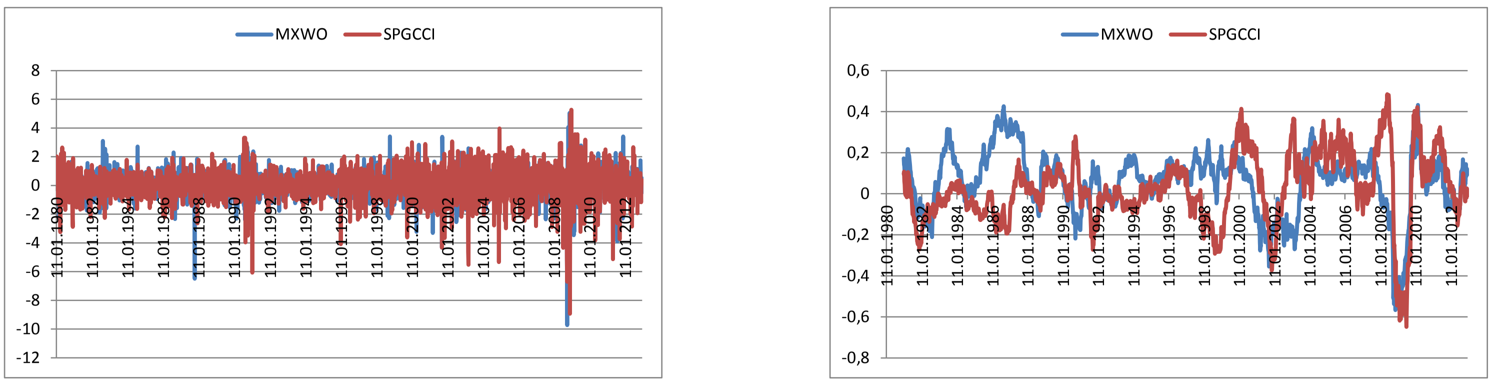
*Note*: $WN$, $AR$, $VAR$, $DCC$ denote individual models defined in Section 3; Mean Ret is the annualized mean portfolio return; St dev is the annualized standard deviation; SR is the Sharpe ratio. $\Delta_s$, $\Delta_o$, $\Delta_f$ are the performance fees for switching from an active strategy to passive strategies holding only stock ($r_s$), only commodity ($r_c$), and the 70/30 passive strategy ($r_m$).

Table 3: **Economic value of portfolios without commodities**

| | h=1 | | h=2 | | h=4 | |
|---|---|---|---|---|---|---|
| | RW | AR | RW | AR | RW | AR |
| Mean Ret | 2.92 | 0.83 | 2.87 | 1.18 | 2.90 | 1.68 |
| St dev | 10.31 | 14.43 | 10.07 | 14.79 | 10.24 | 14.73 |
| SR | 0.10 | -0.08 | 0.09 | -0.05 | 0.08 | -0.02 |
| | h=8 | | h=12 | | h=24 | |
| | RW | AR | RW | AR | RW | AR |
| Mean Ret | 2.80 | 1.51 | 2.78 | 1.56 | 2.88 | 2.75 |
| St dev | 10.51 | 14.77 | 10.67 | 13.70 | 11.73 | 12.05 |
| SR | 0.08 | -0.03 | 0.07 | -0.03 | 0.07 | 0.06 |

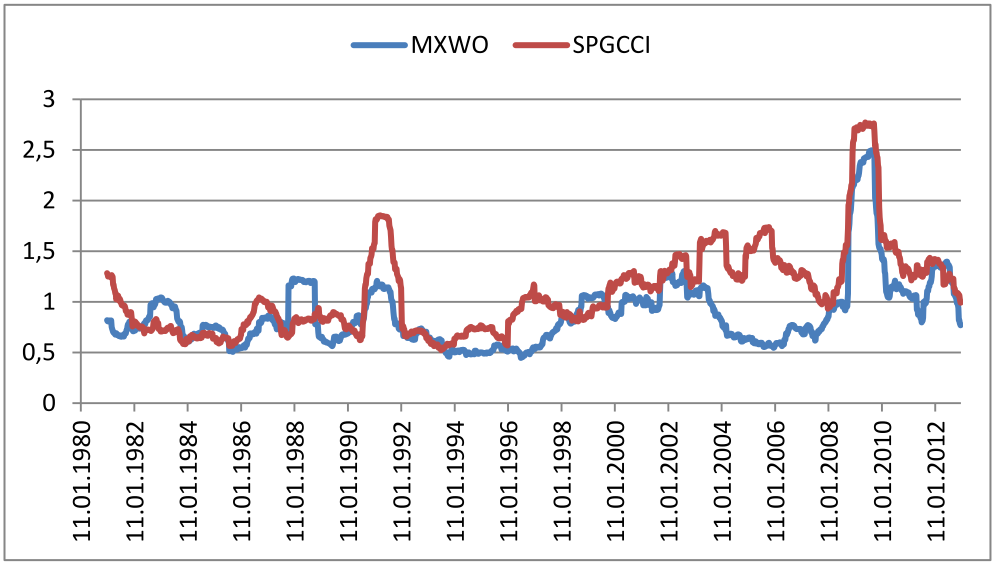
*Note*: $WN$, $AR$: denote individual models defined in Section 3; Mean Ret is the annualized mean portfolio return; St dev is the annualized standard deviation; SR is the Sharpe ratio.

Figure 1: **Commodity and equity returns**



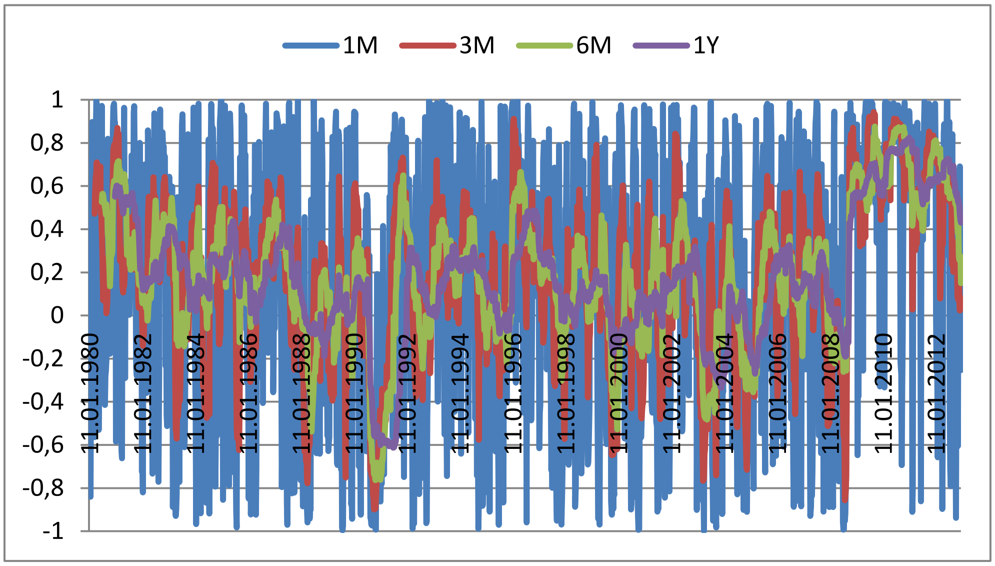

*Note*: Weekly returns of commodity and equity indexes (left panel) and average returns over a 1-year moving window (right panel).

Figure 2: **Commodity and equity volatilities**



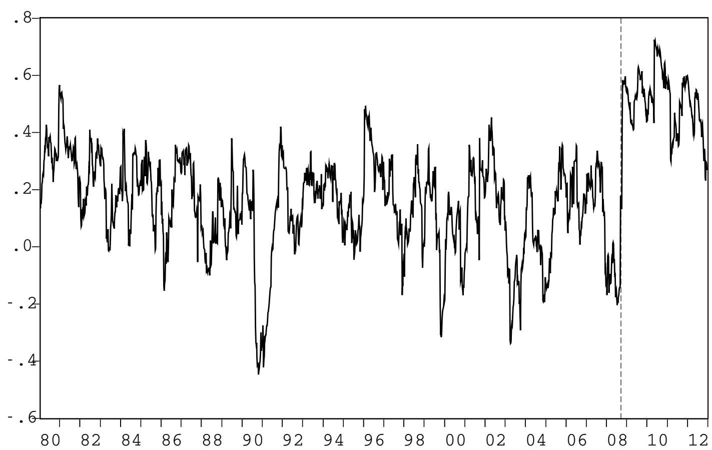

*Note*: Standard deviations over a 1-year moving window of commodity and equity returns.

Figure 3: **Commodity and equity correlations**



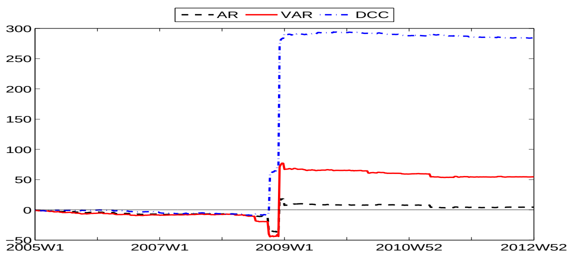

*Note*: Sample correlations between commodity and equity returns computed on moving windows of different length: 3-months (3M); 6 months (6M); and 1-year (1Y).

Figure 4: **Dynamic Conditional Correlations between the MSCI and the SPGSCI**

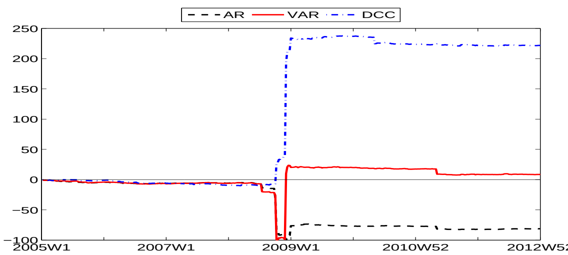



*Note*: Correlations between commodity and equity returns estimated using a Dynamic Conditional Correlation model. The vertical dashed line marks the estimated breakpoint (5 September 2008).

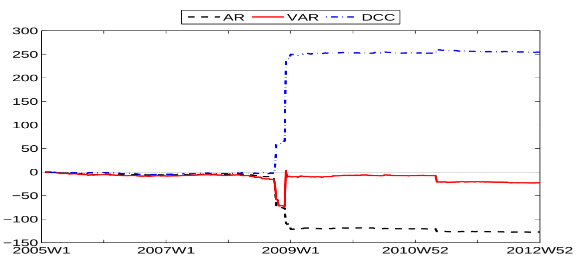
Figure 5: **Log-Score Differentials**



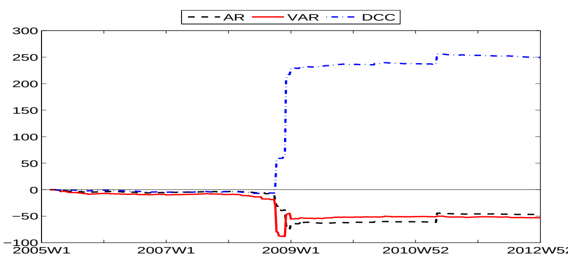

$h = 1$



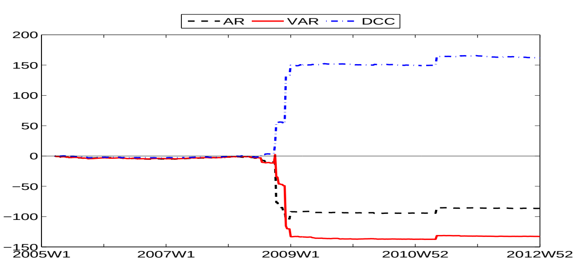

$h = 2$



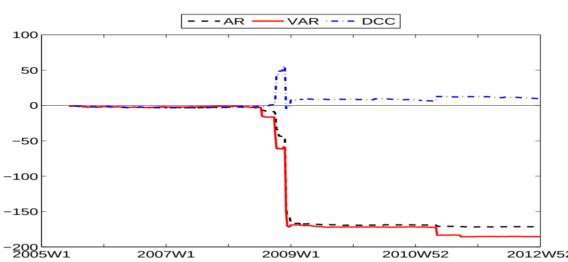

$h = 4$




$h = 8$



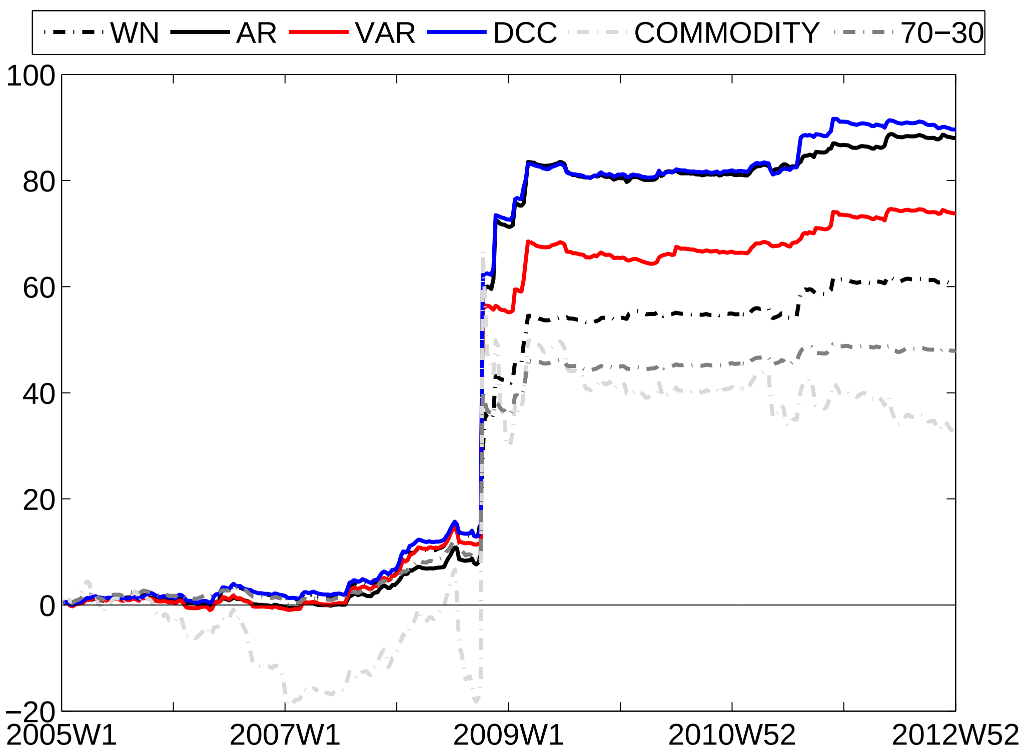

$h = 12$




$h = 24$

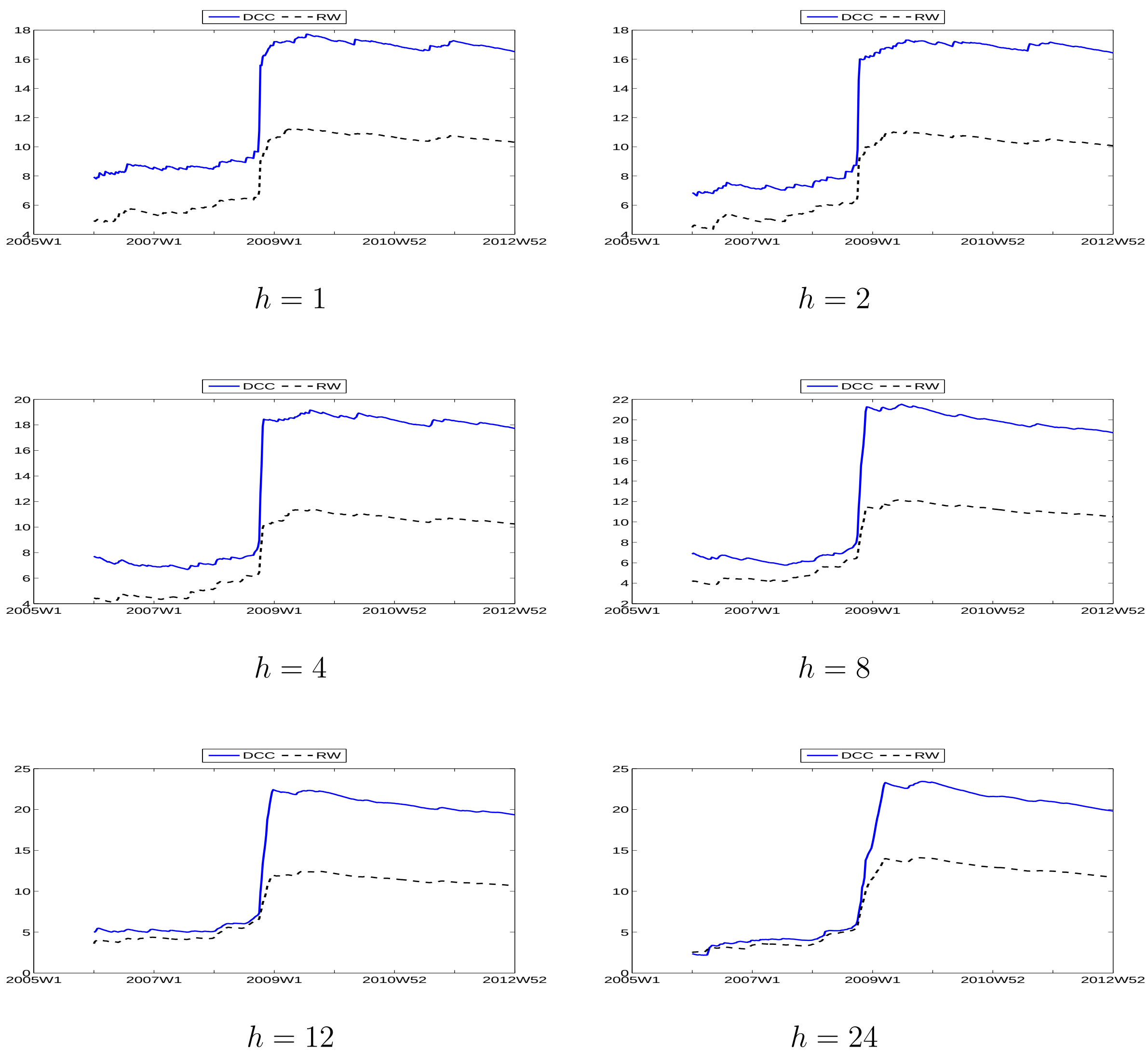
*Note*: Log-Score (CER) Differentials versus the Log-Score of the RW benchmark.

Figure 6: **CER Differentials**



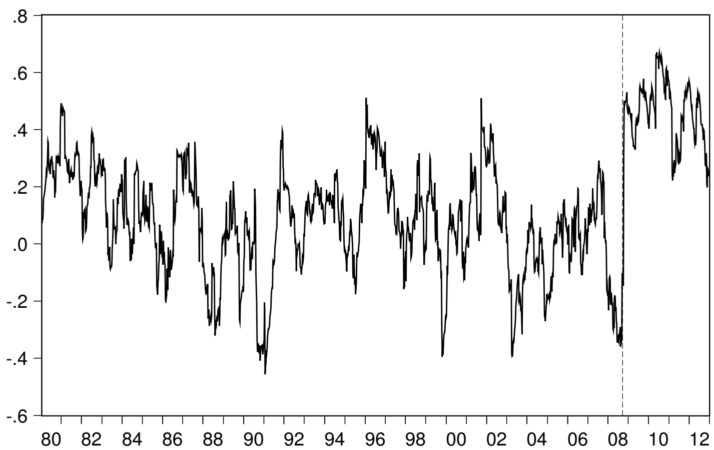

*Note*: Certainty Equivalent Return (CER) Differential at an horizon of two weeks, with respect to the CER of the passive strategy holding only equities, with transaction costs set at 5 basis points.

Figure 7: **Volatility for portfolios with and without commodities**




*Note*: Standard deviation of portfolio returns including commodities (using a DCC model for the dynamic asset allocation) and excluding commodities (using a RW model for the dynamic asset allocation).

Figure 8: **Dynamic Conditional Correlations between the S&P500 and the SPGSCI**




*Note*: Correlations between commodity and equity returns estimated using a Dynamic Conditional Correlation model. The vertical dashed line marks the estimated breakpoint (5 September 2008).