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Ageing, property prices and money demand

Kiyohiko G Nishimura and Előd Takáts[1]

Abstract

When the baby boomers joined the workforce and started saving, money supply and property prices entered a rising trajectory. We conclude that demography was the long-run driver of this process, basing our argument on data from 22 advanced economies for the 1950–2010 period. According to our lifecycle model, large working-age populations saved for their old age by investing in property and broad money instruments, such as deposits. In the past, savings activity by baby boomers drove up property prices and also increased demand for money. As baby boomers retire, these dynamics will go into reverse. Falling demand for savings, including money and deposits, might hinder banks in their efforts to collect deposits and thereby bring down excessively high loan-to-deposit ratios. Our model also confirms that monetary stability contributes to long-run property price stability.

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Contents

1. Motivation

The populations of most advanced economies grew rapidly after World War II. When the postwar baby boomer generation entered the labour force and began saving, property prices and money supply started to rise strongly in many countries. Yet inflation remained at around targeted levels. This is somewhat puzzling. How is it that inflation affected property prices but not those of goods and services? And this in spite of the steadily rising money supply? Linking these three observations, we argue that baby boomers and their saving decisions pushed up property prices but simultaneously kept a lid on inflation.

Our main contribution is to investigate money in conjunction with demography and property prices. To do this, we use an overlapping generation model with lifecycle, and test the results empirically on a large dataset. In sharp contrast to the quantity theory of money, our model predicts that the effect of demography on money holding goes beyond its impact on nominal output. We use data from 22 advanced economies over the 1950–2010 period to confirm that the size of the working-age population significantly and robustly affects the Marshallian K (ie the ratio of M2 to nominal GDP).

Based on the lifecycle theory, we build a standard stylised overlapping generation model dating back to Brumberg and Modigliani (1954) and Ando and Modigliani (1963). In this model, during their working-age career people save (ie buy assets) and in old age dissave (ie sell these assets). Besides real assets (such as land or property), we add to the model nominal assets (such as money or deposits).

In this model, when a larger cohort, such as the baby boomers in the post-war United States, enters the workforce, they increase the demand for saving assets, both property and money. The implications for the fixed supply of property are straightforward: stronger demand from the baby boomers drives up real property prices – and, when they retire and sell their property prices decline.

The model's monetary implications follow less directly because these will depend on the money supply. We investigate the extreme cases: for a fixed and a fully elastic money supply. First, with a fixed money supply, increased demand for money increases the price of real money holdings, which means that the price of goods expressed in money declines. In other words, when the baby boomers enter the workforce they have a deflationary impact. Conversely, the retirement of the boomers is inflationary.

The second case is where the financial system supplies money elastically and the central bank targets inflation. There, the money supply adjusts in step with the demographic changes, not the price level. Thus, the baby boomers' increased demand for money translates into higher money holdings without changing the price level. And the falling demand for money after the baby boomers retire has the effect of shrinking the monetary aggregates.

Importantly, as baby boomers enter the labour force, money demand increases faster than the economy grows. Intuitively, nominal money with its stable real value is a very good investment compared to property. While property will fall in value in real terms when baby boomers retire, nominal money will keep its real value. Consequently, the demographic transition from the baby boom to an ageing population affects the Marshallian K, ie the ratio of money supply to economic output. More precisely, the size of the working-age population is positively associated with the Marshallian K. The positive relationship between demography and the Marshallian K differs distinctly from the predictions of the quantity theory of money.

Furthermore, the model also shows that a flexible money supply combined with inflation targeting will reduce the long-run volatility of real property prices to a lower level than would be experienced under a fixed money supply. This is because the increased money supply provides additional saving vehicles for baby boomers, who therefore do not need to invest as much in property as they would under a fixed money supply regime. Similarly, when baby boomers retire, they feel less compelled to sell their property. Thus, the existence of a flexible money supply acts as a stable store of value and hence reduces the demographic shifts in demand for property and thereby damps the long-run real volatility of property prices.

When linking the theoretical model to data, we focus mostly on the implications of the elastic money supply system. Of course, real monetary conditions are more complex than those of our stylised model but, as discussed in Borio and Disyatat (2011), the financial system can and does create money very elastically to accommodate money demand. Furthermore, many central banks aim to keep inflation in check even if they have not formally adopted an inflation targeting regime.

We show that the empirical evidence is consistent with the theoretical implications of our model. We focus on the relationship between working-age populations and the Marshallian K. In our database encompassing 22 advanced economies over the 1951–2010 period, the size of the working-age population is significantly and robustly correlated with the Marshallian K. Each additional percentage point increase in the share of the working-age population is associated with an equivalent increase in the Marshallian K. The effect remains robust and significant in various subperiods, with time or country fixed effects, and if certain countries are excluded from the sample, or if interest rates, inflation and asset prices are added as additional explanatory variables. Importantly, we consider that our main contribution is to show the consistently robust relationship between demography and money; we do not argue for the precise size of the estimated impact.

Turning to the empirical relationship between demography and property prices, we confirm in our database the earlier findings of Nishimura (2011) and Takáts (2012), who found significant empirical links between demography and property prices – supporting the original insight from Mankiw and Weil (1989).[2] We also find a similar, although weaker, relationship between demography and financial asset prices. That the relationship between demography and financial assets is weaker is not entirely surprising because arbitrage and international diversification is much more straightforward with financial assets than with property.

Our results have far-reaching implications for monetary policy. First, the shrinking of working-age populations in many advanced economies will create inflationary pressures that will need to be countered. Second, the choice of monetary regime might affect property price volatility. In particular, moves to stabilise prices might also lend stability to property prices during a demographic transition – a factor relevant for authorities that are considering the adoption of an inflation targeting regime. Third, ageing will reduce broad money demand, especially in rapidly ageing Europe and advanced Asia. Thus, ageing might hinder banks in their efforts to collect deposits and hence bring down excessively high loan-to-deposit ratios. Of course, demographic forces take effect over the long run, so that short- or even medium-term effects may be overshadowed by other developments. In particular, the continuing crisis might raise precautionary saving demand for safe assets, which might dominate demographic dis-saving.[3]

Our research adds to a small but growing literature that investigates how changing demographic structures can affect nominal variables such as money or inflation. The existence of this relationship has significant implications: even if ageing affects real macroeconomic variables, nominal change does not necessarily follow as central banks play

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[2] Interestingly, some earlier studies such as Engelhardt and Poterba (1991) or Berg (1996) found no significant demographic impact. However, these studies used much a smaller sample than Takáts (2012), which drew on data from 22 advanced economies over the 1970–2010 period. Sample size matters because identification is fraught with difficulty in single country studies or in small samples, as Hendershott (1991) pointed in out in his critique of the Mankiw and Weil (1989) paper.

[3] We believe that such effects can explain the strong demand for money and safe nominal assets in Japan over the past decade.

a large role in setting nominal variables. Using data from the United States, Fair and Dominguez (1990) demonstrate that the share of prime saving-age people in the population is positively associated with money holding, ie that demography does affect money holding. This is consistent with our theoretical and empirical results. In a similar vein, Mayor and Pearl (1984) demonstrate, again from US data, that a higher share of prime saving-age people in the population reduces the velocity of money and thereby increases money holdings. Studies focusing on inflation also show results that are consistent with our approach. Based on US data, McMillan and Baesel (1990) argue that, as baby boomers enter their prime saving age, they will put downward pressure on inflation. Expanding the data set for 20 OECD countries, Lindh and Malmberg (2000) also confirm empirically that larger working-age populations are associated with lower inflation. Our results on financial asset prices are also consistent with the growing literature on this topic.[4]

Our paper could also be linked to the literature on optimal long-term monetary policy arrangements. One avenue for extending our work would be to follow Alchian and Klein (1973) and explicitly broaden monetary stability targets to include property prices. Thus, central banks could choose to “lean against the wind” over longer demographic cycles. In this respect, our results suggest that demographic pressures influence consumer and property price inflation in opposite directions. Thus, optimal policy arrangements might require additional instruments, such as macroprudential tools.

The remainder of the paper is organised as follows. The second section introduces the theoretical model. The third analyses the data empirically. The fourth discusses the economic impact and the final one concludes with policy implications.

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[4] One stream of literature used data from the United States. Yoo (1994) found a relationship between demography and treasury yields. However, Bergantino (1998) showed that demography explains a substantial part of the postwar equity return variation. Poterba (2001) revisited this evidence, but only found a weak demographic impact for treasury bills and no impact for other financial assets. Brooks (2002) and Geanakoplos et al (2004) also found that ageing affects asset prices. Most recently, Favero et al (2009) showed that the ratio of the middle-aged to the young population explains the dividend-price ratio, ie equity valuations. Another stream of studies used international data to improve identification. Davis and Li (2003) found that the size of the prime saving-age cohort significantly correlates with asset prices in a seven-country sample. Brooks (2006) and Ang and Maddaloni (2005) used international data to revisit the identification of earlier single-country studies and in general found strong demographic effects. Recently, Liu and Spiegel (2011) found that baby boomer retirement would have a substantial negative effect on equity valuations.

2. Theoretical model

2.1 Model setup

A small overlapping generation model, following Allais (1947), Samuelson (1958) and Diamond (1965), is set up with a lifecycle. Identical agents live for two periods, which we call young and old age. Young agents work for an income and save to consume in old age. Saving is done through a divisible utility-bearing real asset called property and through utility-bearing money. Old agents do not work; they sell their accumulated assets (property and money) and consume. At time *t*, there are n_t young agents; hence, at time *t+1* there are n_t old agents. Formally, individual agents' utility function (*U*) can be written as follows:

$$U = \ln(c_t^Y) + \ln(h_t) + \ln\left(\frac{M_t}{P_t}\right) + \beta \ln(c_{t+1}^O) \tag{1}$$

where *ln(.)* is the natural logarithm, c^Y is consumption when young, and c^O is consumption when old, $0<\beta<1$ is the discount factor and *t* is the time period index.

Individual agents maximise their utility function (1) subject to young and old age resource constraints, described by equations (2) and (3), respectively. In period *t*, the young age consumption ($c^Y{}_t$) is limited by young age exogenous income (y^Y) and reduced by property investment, ie real property purchase (h_t) multiplied by price (q_t) and by real money holding, ie nominal money held (M_t) divided by the price level (P_t). Formally:

$$c_t^Y \leq y^Y - h_t q_t - \frac{M_t}{P_t} \tag{2}$$

In period, *t+1* the young generation of period *t* turns old, the old age population of period t dies and a new young age population is born. The consumption of the old at *t+1* ($c^O{}_{t+1}$) is constrained by the value of their savings. This is the sum of the value of their property, ie real property purchases (h_t) in the previous period, multiplied by the current property price (q_{t+1}), and of their real money holding, ie nominal money acquired in the previous period (M_t) divided by the current price level (P_{t+1}):

$$c_{t+1}^O \leq h_t q_{t+1} + \frac{M_t}{P_{t+1}} \tag{3}$$

We assume that the property supply is exogenously set at a constant stock ($0<H^*$). This feature would, in fact, imply that our property variable should be interpreted more like land than somewhat elastic housing. Thus, in equilibrium, as agents are identical, the property price is related to property stocks and the young age population as follows:

$$h_t = \frac{H^*}{n_t} \tag{4}$$

We will explore two versions of the model depending on the money supply: one with fixed money supply and the other with elastic money supply.

In our solution we will specifically explore a stylised demographic transition, which captures the experience of many advanced economies in the post-war period. Table 1 summarises the stages of this stylised demographic transition. The economy starts in a steady state (*t=0*) with population size at $n+\gamma$. Then, unexpectedly, the population increases to $n+\Delta$ (*t=1*, baby boom, where $0<\gamma<\Delta$). In the baby boom period, there are more young productive workers than old people, which can be thought of as a demographic dividend. However, the next generation is assumed to be smaller at size *n* (*t=2*, ageing period), which implies that old people now outnumber the working-age population. In the following period, the system stabilises at this new, lower population steady state (*t=3,4,...*).

Table 1

Demographic transition model

Time	Young population size	Old population size	Name of period
t=0	n+γ	n+γ	old steady state
t=1	n+Δ	n+γ	baby boom
t=2	n	n+Δ	ageing
t=3, 4, ...	n	n	new steady state

2.2 Fixed money supply

The fixed money supply version is similar to how a strict gold standard regime works, ie even if paper money exists it is fully backed by gold. The financial system does not create "inside" money, all the money is "outside" like gold. This also implies that the general price level is determined by the marginal utility of this money. Formally, we set aggregate money supply at constant M^* in the model.

Given that in the log utility framework, both constraints would bind in equilibrium, it is possible to substitute the constraints (2) and (3) into the utility maximisation problem (1) as follows:

$$U = \ln\left(y^Y - h_t q_t - \frac{M_t}{P_t}\right) + \ln(h_t) + \ln\left(\frac{M_t}{P_t}\right) + \beta \ln\left(h_t q_{t+1} + \frac{M_t}{P_{t+1}}\right) \tag{5}$$

The solution is characterised by the first-order conditions of h_t and M_t, respectively:

$$(h_t) \qquad \frac{-q_t}{y^Y - h_t q_t - \frac{M_t}{P_t}} + \frac{1}{h_t} + \frac{\beta q_{t+1}}{h_t q_{t+1} + \frac{M_t}{P_{t+1}}} = 0 \tag{6}$$

$$(M_t) \qquad \frac{-\frac{1}{P_t}}{y^Y - h_t q_t - \frac{M_t}{P_t}} + \frac{1}{M_t} + \frac{\frac{\beta}{P_{t+1}}}{h_t q_{t+1} + \frac{M_t}{P_{t+1}}} = 0 \tag{7}$$

Furthermore, as agents are identical, we can also write up the equilibrium values of M_t using the constant aggregate money supply (M^*) along with the known generation size:

$$M_t = \frac{M^*}{n_t} \tag{8}$$

We proceed to verify that the solution takes the following form, where K is some constant (The appendix shows that our guess was indeed correct by solving the model formally using the system of difference equations.):

$$q_t = K\frac{y^Y}{h_t} \qquad \text{and} \qquad \frac{1}{P_t} = K\frac{y^Y}{M_t}$$

Substituting back to (6) and using (7) and (8) yields:

$$\frac{-K\frac{y^Y n_t}{H^*}}{y_t^Y - 2Ky^Y} + \frac{n_t}{H^*} + \frac{\beta K \frac{y^Y n_t}{H^*}(1+g_t)(1+d_t)}{Ky^Y(1+g_t)(1+d_t) + Ky^Y(1+g_t)(1+d_t)} = 0$$

$$\frac{-K}{1-2K}+1+\frac{\beta}{2}=0$$

which further yields that

$$K=\frac{2+\beta}{6+2\beta}$$

Substituting back to (8) and using (9) and (10) yields the same K:

$$\frac{-K\frac{y^{Y}n_{t}}{M^{*}}}{y_{t}^{Y}-2Ky^{Y}}+\frac{n_{t}}{M^{*}}+\frac{\beta K\frac{y^{Y}n_{t}}{M^{*}}(1+g_{t})(1+d_{t})}{Ky^{Y}(1+g_{t})(1+d_{t})+Ky^{Y}(1+g_{t})(1+d_{t})}=0$$

$$\frac{-K}{1-2K}+1+\frac{\beta}{2}=0$$

Thus, the solution is:

$$q_{t}=\frac{\left(2+\beta\right)y_{t}^{Y}n_{t}}{\left(6+2\beta\right)H^{*}} \tag{9}$$

$$\frac{1}{P_{t}}=\frac{\left(2+\beta\right)y^{Y}n_{t}}{\left(6+2\beta\right)M^{*}} \tag{10}$$

This further implies that both the real value of property and the real value of money (ie the inverse of the price level) depend on both economic and demographic factors. However, the evolution of property prices exactly follows the evolution of the general aggregate economy. Note that these conclusions hold generally and thereby also for the demographic transitions depicted in Table 1. Thus, with a fixed money supply, demography and economic growth do not affect the Marshallian K, ie he ratio of money to nominal economic output. Formally:

$$MK_{t}^{FMS}=\frac{M^{*}}{y^{Y}P_{t}n_{t}}=\frac{\left(2+\beta\right)}{\left(6+2\beta\right)} \tag{11}$$

2.3 Elastic money supply with inflation targeting

Second, we consider a fully elastic money supply under inflation targeting. The financial system could be thought of providing the money supply, for instance in the forms of deposits, to meet money demand. We assume that this money supply is infinitely elastic for analytic simplicity. In order to keep the exposition simple, we do not model the asset side of the financial system and we are content to focus on the liability side, ie on broad money.[5]

Furthermore, we also assume that there is an inflation targeting central bank that stabilises the price level at unity. We do not formally model how this targeted inflation is reached.

Inflation targeting can be written formally as follows:

[5] In the interests of simplicity, we do not explicitly model the asset side of the financial system and how it repays the money or deposit. In this respect, our model is partial. Returns on the asset side are not necessarily stable in real terms and could mean profits during the baby boom and losses during the baby bust. The losses can be seen as a financial crisis, for instance. We also developed a general equilibrium model, which is available upon request. In the general equilibrium model, the whole financial system, including the central bank, is owned by old agents. When the money supply expands these receive transfers and when it shrinks they cover the losses. The conclusions for the Marshallian K or for property prices do not change.

$$P_t = P^* = 1 \tag{12}$$

Hence, the utility maximisation problem for individual agents can be written as follows:

$$U = \ln(c_t^Y) + \ln(h_t) + \ln(M_t) + \beta \ln(c_{t+1}^O) \tag{13}$$

subject to the budget constraint at time *t*

$$c_t^Y \leq y^Y - h_t q_t - M_t \tag{14}$$

and at time *t+1*

$$c_{t+1}^O \leq h_t q_{t+1} + M_t \tag{15}$$

We solve the model in two steps. First, we obtain the solution for the steady state. Second, we consider the demographic transition as shown in Table 1 explicitly. In particular, we look at the shock of the baby boom and then that of ageing.

2.3.1 Steady state

In the steady state, we can again solve the problem without explicitly considering difference equations. As for notations, we write all equations without time indices for the sake of simplicity. First, we substitute in constraints (14) and (15) into (13) and derive the first-order conditions, formally:

$$(h_t) \qquad \frac{-q}{y^Y - hq - M} + \frac{1}{h} + \frac{\beta q}{hq + M} = 0 \tag{16}$$

$$(M_t) \qquad \frac{-1}{y^Y - hq - M} + \frac{1}{M} + \frac{\beta}{hq + M} = 0 \tag{17}$$

Multiplying (17) by q and subtracting (16) from it yields

$$\frac{q}{M} - \frac{1}{h} = 0$$

which is equivalent to

$$M = hq \tag{18}$$

Rewriting (17) by using (18) gives the following expression:

$$\frac{-1}{y^Y - 2M} + \frac{1}{M} + \frac{\beta}{2M} = 0$$

$$2M = (2+\beta)\left(y^Y - 2M\right)$$

$$(6+2\beta)M = (2+\beta)y^Y$$

which straightforwardly yields the steady state solution for M:

$$M^{SS} = \frac{2+\beta}{6+2\beta} y^Y \tag{19}$$

and using equilibrium property supply constraint (4) yields the steady state solution for q:

$$q^{SS} = \frac{2+\beta}{6+2\beta} y^Y \frac{n}{H^*} \tag{20}$$

This implies that the steady state nominal money demand per working-age population *(M)* depends on the per working-age population nominal economic output (y^Y as the price level is at 1). Hence, aggregate money demand depends on economic and demographic factors.

However, the steady state Marshallian K does not depend on the size of the working-age population, ie all steady states, irrespective of population size have the same Marshallian K. We can write the Marshallian K as the ratio of aggregate money and aggregate nominal output as follows:

$$MK^{ITSS} = \frac{Mn}{y^{Y}n} = \frac{2+\beta}{6+2\beta} \tag{21}$$

In fact, the steady state Marshallian K is not only constant but it also takes the same value as the Marshallian K under a fixed money supply regime.

Demographic transition

As the previous subsection provides a solution for the two steady states, we focus on solving the model for baby boom period (*t=1*) and the ageing period (*t=2*) depicted on Table 1. We first exploit the loglinear characteristics of utility and show that the model is boiled down to the dynamics of the real (or here equivalently nominal) property price q_t. To see this, we combine the first-order conditions into one difference equation governing the dynamics of the real property price.

We obtain the first-order conditions by combining the utility maximisation problem (13) with the two constraints (14) and (15) that are binding in equilibrium and condition (12):

$$(h_t) \qquad \frac{-q_t}{y^{Y} - h_t q_t - M_t} + \frac{1}{h_t} + \frac{\beta q_{t+1}}{h_t q_{t+1} + M_t} = 0 \tag{22}$$

$$(M_t) \qquad \frac{-1}{y^{Y} - h_t q_t - M_t} + \frac{1}{M_t} + \frac{\beta}{h_t q_{t+1} + M_t} = 0 \tag{23}$$

Combining (22) divided by q_t and (23) in order to eliminate the left-hand side of both equations yields:

$$\frac{1}{M_t} + \frac{\beta}{h_t q_{t+1} + M_t} = \frac{1}{h_t q_t} + \frac{\beta q_{t+1} / q_t}{h_t q_{t+1} + M_t}$$

Rearranging terms yields

$$\frac{h_t q_{t+1} + M_t}{M_t} + \beta = \frac{h_t q_{t+1} + M_t}{h_t q_t} + \beta \frac{q_{t+1}}{q_t}$$

and

$$\frac{h_t q_{t+1}}{M_t} + (1+\beta) = \frac{M_t}{h_t q_t} + (1+\beta)\frac{q_{t+1}}{q_t}$$

And then we have the following difference equation for equilibrium property prices, taking account of the equilibrium conditions of the property market (4):

$$q_{t+1} = \left\{ \frac{1+\beta - \dfrac{M_t}{q_t}\dfrac{n_t}{H^*}}{1+\beta - \dfrac{q_t}{M_t}\dfrac{H^*}{n_t}} \right\} q_t \tag{24}$$

At the same time, (22) coupled with (4) yields the other difference equation that drives equilibrium property prices:

$$\frac{1}{y^Y - \frac{H^*}{n_t} q_t - M_t} = \frac{1}{\frac{H^*}{n_t} q_t} + \frac{\beta q_{t+1} / q_t}{\frac{H^*}{n_t} q_{t+1} + M_t} \tag{25}$$

Here y_t^Y and n_t are exogenous variables and H^* is constant. The system of difference equations (24) and (25) has a distinctive property: namely, if q_{t+1} is determined, then q_t and M_t are determined backward. Note that q_t is the price variable, ie it is a jumping variable. Thus, we make a standard assumption that q_t converges to the steady state value q^{SS}.

Then we proceed to characterise the solution backwards from the known steady state. Notice, that in periods $t=2$ and afterwards, all exogenous variables are constant, so that price stability is achieved by putting the economy immediately into the steady state. Thus $q_2=q^{SS}$.

At period $t=1$ then combining equations (24) and (20) yields:

$$\frac{2+\beta}{6+2\beta} y^Y \frac{n}{H^*} = q^{SS} = \frac{(1+\beta) q_1 - \frac{n+\Delta}{H^*} M_1}{1+\beta - \frac{H^*}{n+\Delta} \frac{q_1}{M_1}}$$

This allows us to express q_1 as a function of model parameters and M_1:

$$q_1 = \frac{\frac{n+\Delta}{H^*} M_1 + (1+\beta) \frac{2+\beta}{6+2\beta} y^Y \frac{n}{H^*}}{1+\beta + \frac{2+\beta}{6+2\beta} y^Y \frac{n}{n+\Delta} \frac{1}{M_1}} \tag{26}$$

Similarly, from (25) at time $t=1$ and (20) we have

$$\frac{1}{y^Y - \frac{H^*}{n+\Delta} q_1 - M_1} = \frac{1}{\frac{H^*}{n+\Delta} q_1} + \frac{\beta \frac{2+\beta}{6+2\beta} y^Y \frac{n}{H^*} \frac{1}{q_1}}{\frac{n}{n+\Delta} \frac{2+\beta}{6+2\beta} y^Y + M_1}$$

This allows us to express q_1 again as a function of model parameters and M_1:

$$q_1 = \frac{n+\Delta}{H^*} (y^Y - M_1) \frac{(1+\beta) \frac{2+\beta}{6+2\beta} y^Y \frac{n}{n+\Delta} + M_1}{(2+\beta) \frac{2+\beta}{6+2\beta} \frac{n}{n+\Delta} y^Y + 2M_1} \tag{27}$$

Combining (26) and (27) yields:

$$\frac{1}{1+\beta + \frac{2+\beta}{6+2\beta} y^Y \frac{n}{n+\Delta} \frac{1}{M_1}} = (y^Y - M_1) \frac{1}{(2+\beta) \frac{2+\beta}{6+2\beta} \frac{n}{n+\Delta} y^Y + 2M_1}$$

This can be expressed in a quadratic form after some straightforward simplifications:

$$(3+\beta)(M_1)^2 + \left[(3+\beta) \frac{2+\beta}{6+2\beta} \frac{n}{n+\Delta} - (1+\beta) \right] y^Y M_1 - \frac{2+\beta}{6+2\beta} \frac{n}{n+\Delta} \left(y^Y\right)^2 = 0$$

As in the above equation, the first term is positive and last term is negative so that there is a unique positive solution to the quadratic equation:

$$M_1 = y^Y \frac{-\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]+\sqrt{\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]^2+\frac{n(4+2\beta)}{n+\Delta}}}{6+2\beta} \tag{28}$$

As we obtained the closed-form solution for M1, we can also obtain the closed-form solution q1 by combining (28) with either from (26) or (27). Formally, the solution using (26) is:

$$q_1 = \frac{y^Y n}{(6+2\beta)H^*} \frac{\frac{n+\Delta}{n}\left\{-\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]+\sqrt{\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]^2+\frac{n(4+2\beta)}{n+\Delta}}\right\}+(1+\beta)(2+\beta)}{1+\beta+\frac{(2+\beta)\frac{n}{n+\Delta}}{-\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]+\sqrt{\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]^2+\frac{n(4+2\beta)}{n+\Delta}}}}$$

2.3.2 Marshallian K in demographic transition

We have already shown that, both under the fixed money supply regime and the steady state of the elastic money supply regime, the Marshallian K is independent of demography. However, the Marshallian K will increase in the first "baby boom" stage of demographic transition. To see that, first consider that the Marshallian K at *t=1* can be written as:

$$MK_1^{EMS} = \frac{M_1 n_1}{y^Y n_1} \tag{29}$$

Thus, the Marshallian K increases in the first phase of the demographic transition if and only if the expression in (29) is higher than the expression in (21). Formally, this is equivalent with the expression below:

$$\frac{-\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]+\sqrt{\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]^2+\frac{n(4+2\beta)}{n+\Delta}}}{6+2\beta} > \frac{2+\beta}{6+2\beta}$$

The expression can be simplified further as

$$\sqrt{\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]^2+\frac{n(4+2\beta)}{n+\Delta}} > 2+\beta+\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]$$

$$\frac{n(4+2\beta)}{n+\Delta} > (2+\beta)^2+(4+2\beta)\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]$$

And then to

$$1 > \frac{n}{n+\Delta}$$

which always holds.

In sum, the Marshallian K increases in the first ("baby boom") phase (*t=1)* of the demographic transition under an elastic money supply. And it declines back to a steady state value in the second ("ageing") phase of the transition at *t=2*.

2.3.3 Property prices in demographic transition

In order to discuss real property price volatility, it is useful to recognise that real property prices are the same under fixed and elastic money supply regimes in all periods, except in period $t=1$ as Table 2 summarises. Consequently, in order to demonstrate that property price volatility is lower under an elastic money supply regime, it is sufficient to show that real property prices are lower under an elastic money supply than under a fixed money supply regime at time t=1.

Table 2

Real property prices in demographic transition

Time period	t=0	t=1	t=2	t=3	t=4…
Fixed money supply	$q^*(n+\gamma)$	$q^*(n+\Delta)$	q^*n	q^*n	q^*n
Elastic money supply	$q^*(n+\gamma)$	q_1	q^*n	q^*n	q^*n

Where $q^* = \frac{2+\beta}{6+2\beta}\frac{y^Y}{H^*}$ and q_1 is determined by equation (26), (27) and (28).

Thus, we only need to prove that, by the notation of Table 2:

$$q^*(n+\Delta) > q_1$$

This inequality can be written, after dividing both sides by n+Δ, as follows:

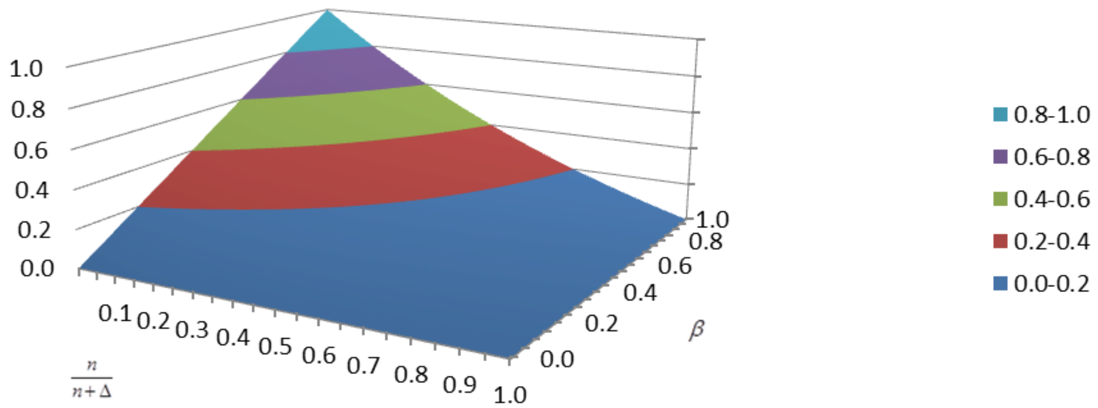
$$(2+\beta) > \frac{-\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]+\sqrt{\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]^2+\frac{n}{n+\Delta}(4+2\beta)}+(1+\beta)(2+\beta)\frac{n}{n+\Delta}}{1+\beta+\sqrt{\frac{(2+\beta)\frac{n}{n+\Delta}}{-\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]+\sqrt{\left[\frac{2+\beta}{2}\frac{n}{n+\Delta}-(1+\beta)\right]^2+\frac{n}{n+\Delta}(4+2\beta)}}}} \tag{30}$$

As both β and $n/n+\Delta$ are on the unit interval and the function is continuous we can show that the inequality holds by numerically calculating the two sides of equation (30). Graph 1 shows that the difference between the left-hand side and the right-hand side is always positive, ie property price volatility is always higher in the fixed money supply regime than in the elastic money supply regime. Furthermore, the volatility difference is increasing with the size of the demographic transition (inverse of $n/n+\Delta$) and in the strength of time preference (β).

Graph 1

Property price volatility difference between fixed and elastic money supply[1]

Higher positive values denote higher property price volatility under fixed money supply regime



[1] Difference between the left-hand side and the right-hand side of equation (30).

2.3.4 *Empirical implications*

The model's most important empirical implications concern the time of the demographic transition. In the postwar period, the birth of the baby boomers has provided such a demographic transition in most advanced economies. Unfortunately, we are unable to test the model's predictions on property price volatility as we have no comparable set of countries with both fixed money and elastic money supply regimes in this period. Most advanced economies followed monetary policies where inflationary concerns played a key role, while the financial system created money (in the form of deposits, for instance) to accommodate saving needs. Therefore most advanced economies remained close to our inflation targeting model – even if they did not officially adopt inflation targeting.

Consequently, we focus on the predictions on the evolution of the Marshallian K, the ratio of money to nominal economic output; and compare our model's predictions with historical data. The elastic money supply model with inflation targeting predicts that the expansion of the working-age population is positively associated with the Marshallian K. When baby boomers enter the workforce, they drive the Marshallian K higher. Conversely, when the subsequent, smaller generation replaces the baby boomers in an ageing society, they drive the Marshallian K lower. This result is in sharp contrast with the quantity theory of money, which would predict that the Marshallian K is independent of demography, and that demography should affect money demand only through output or interest rates. Given that the model is highly stylised, the predicted elasticities are unlikely to be exactly reproduced in the data. However, we do expect from the model a significant and robust positive correlation between the working-age population and the Marshallian K.

The model also confirms earlier works, such as Nishimura (2011) and Takáts (2012), arguing that demography affects real asset prices more than economic growth would imply. Although this result is not entirely new, we also test our model's predictions on real asset prices.

3. Empirical investigations

The empirical analysis is divided into three major steps. First we describe our data and sources. Second, we establish that there is robust correlation between monetary aggregates and demography. Third, we confirm the correlation between demography and asset prices.

3.1 Data

We use the longest possible data for the postwar period, starting in many countries as early as 1951 and ending in 2010. We cover 22 advanced economies: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Ireland, Italy, Japan, Korea, the Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States.

As for data sources, we extensively use the Global Financial Data database for historical money (M2), nominal GDP, stock indices and short-term interest rates. For more recent data, we rely on OECD Economic Outlook data for nominal GDP and IMF International Financial Statistics (IFS) data for M2. As for the post-euro M2 series, we use the euro area M2 breakdown published by the ECB and national central banks on their websites. BIS property price index data are used for property prices. For inflation rates, we use the IMF IFS database and whenever possible we extend it backwards with the help of national statistical sources. Yearly demographic data on population distribution come from the UN Population Projections (2011) database. All data are available upon request.

We merge different data series to create long-run time series. When data from two series overlap, we rebase the starting point of the newer data to eliminate data breaks with the older data. This does not affect the results, as we focus on changes not levels in the analysis. Unfortunately, in the case of the euro's introduction, there are no overlapping series for M2 and we need to drop the euro area accession years from our analysis.

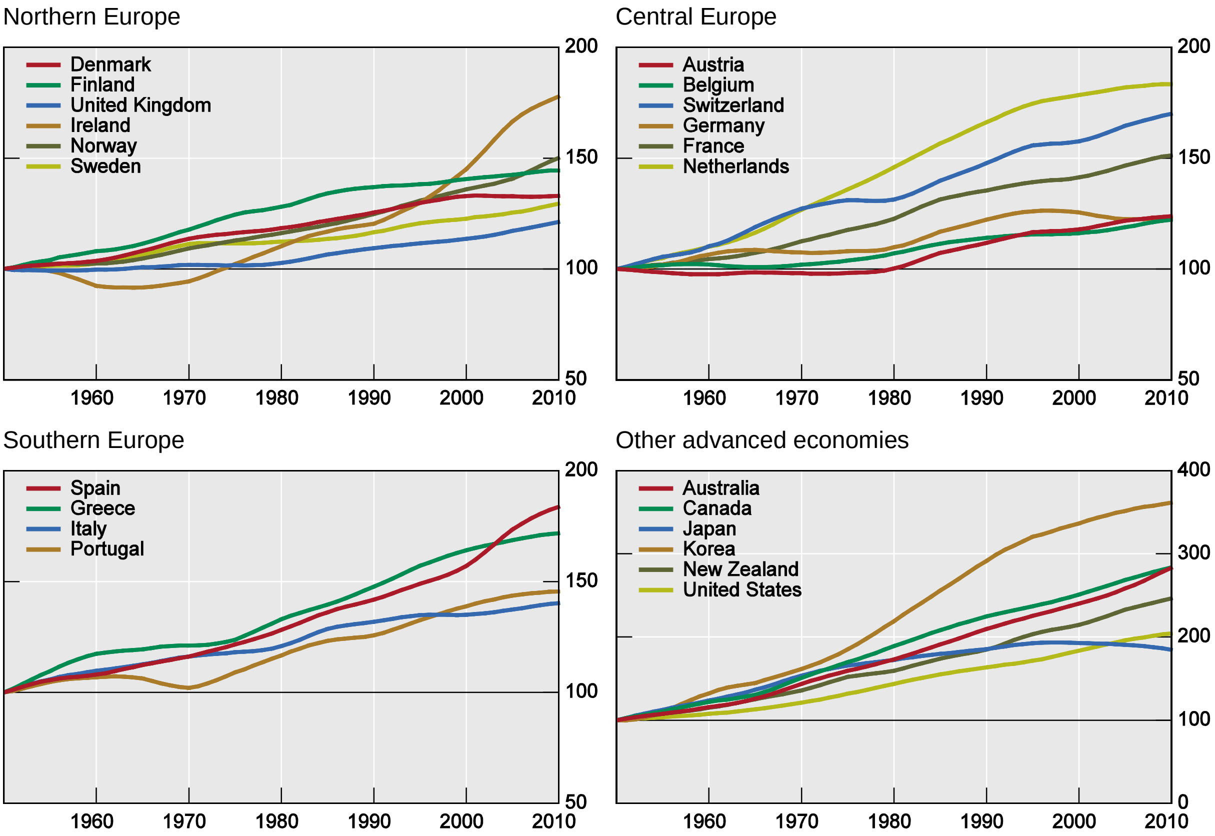
Following the model, for demographic data we focus on working-age populations, ie the number of people between the age of 20 and 64, both for monetary aggregates and for asset prices. Graph 2 shows the evolution of working-age populations in four geographic regions. In general, the working-age population growth was much slower in European countries (upper two panels, and lower left-hand panel) than in non-European countries (lower right-hand panel). To highlight this, consider the example of Japan. Though working-age population growth in Japan was the slowest among non-European countries in our sample and the working population has actually declined there over the past 12 years, growth in the 1950–2010 period was still higher than in any European country.

Graph 2 also highlights the variety of experiences, which is useful for identifying the impact of demography on monetary aggregates. Not only do growth rates differ substantially, but many countries have also experienced episodes of decline in their working-age populations. Ireland, for instance, experienced declines in the 1950s and 1960s, although later rapid population growth resulted in one of the highest overall growth rates in Europe. Portugal experienced declines in the late 1960s and Germany in the 1950s, 1960s and also in the 2000s. And, of course, Japan has experienced declines since the late 1990s. These declines are especially relevant if we want to understand the future impact of demography on money holdings and inflation, given that the UN Population Projections (2011) predict that working-age populations will soon start declining in many advanced economies.

Graph 2

Working-age population[1]

1950 = 100



[1] Population in age range 20–64. Source: UN Population Projections (2011).

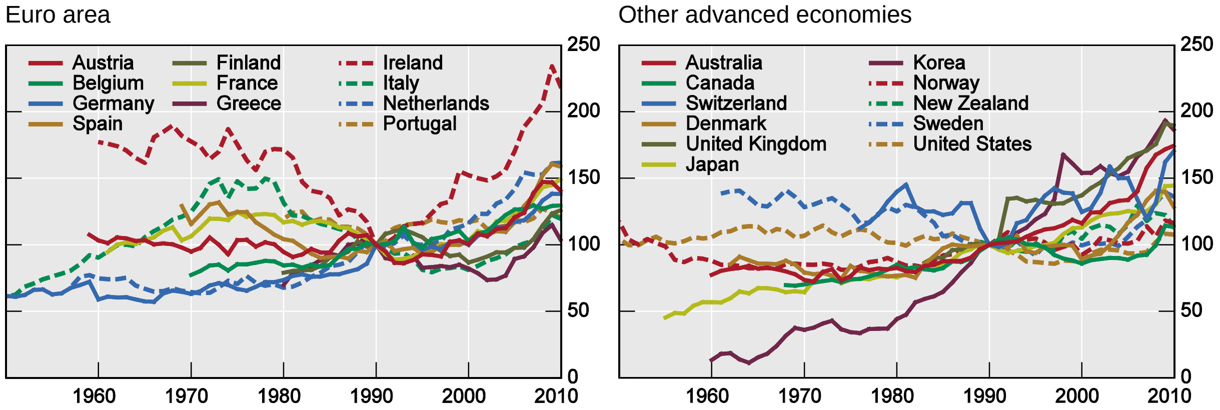
3.2 Demography and money

In the empirical analysis on money, we focus on the Marshallian K, ie the ratio of money supply and economic output. For the money supply we use M2 and for economic output the nominal GDP, as these are consistently available over a longer time horizon. Graph 3 shows the evolution of the Marshallian K in our sample separately for the euro area (left-hand panel) and other advanced economies (right-hand panel).

Graph 3

Marshallian K[1]

1990 = 100



[1] M2 as a percentage of the nominal GDP. Break in the time series, M2 contribution to the euro area M2, starting in 1998 for Spain and Netherlands; in 2001 for Greece and in 1999 for the other countries. Smoothed.

Sources: IMF, *International Financial Statistics*; OECD, *Economic Outlook*; Datastream; Global Financial Data; national data.

As is evident from Graph 3, the evolution of Marshallian K varies enormously. While it increased steadily in fast-developing Japan and especially in Korea, it remained largely stable in Sweden and in the United States. Remarkably, the Marshallian K increased rapidly in most advanced economies during the decade leading up to the financial crisis, as a consequence of the ongoing credit boom.

3.2.1 Benchmark regression

To assess the impact of demography on the money supply, we first run the following regression on the full panel, which we will use as a benchmark for future investigations:

$$d\log K = c + \alpha d\log(workage) + \varepsilon \qquad \text{(A)}$$

where d*log* denotes yearly changes in the natural logarithm of the variable in question, *K* is the Marshallian K, *workage* is the size of the working-age population (ie the population between 20 and 64 years old), *c* is a constant, and parameter α is the focus of our interest as it shows the relationship between *K* and *workage*. As the setup is loglinear, the α estimate captures elasticity.

Table 3 shows the results of the benchmark regression in detail. In particular, the coefficient of our interest α is around unity and significant at a 1% level. Given the loglinear setup, this implies that a 1% increase in the working-age population is associated with a 1% rise in the Marshallian K, ie there is unit elasticity. Hence, demography affects the money supply significantly beyond its impact on economic output.

As we focus on yearly changes, the results can be seen as strong. We can dispel concerns about trending variables. Furthermore, we can also confidently exclude reverse causality, as changes in economic output or money supply today could not have affected birth rates 20 years ago.

Table 3

Benchmark regression

α coefficient	standard error	t-statistic	R-squared	Durbin Watson
1.0395***	0.1484	7.00	0.0031	1.4988

Estimating regression (A) for the full sample over the 1950–2010 time period, ***=significant at 1%

3.2.2 Robustness

These results are new and, to ensure the robustness of our findings, we have undertaken extensive robustness checks in several dimensions. First, adding time and country fixed effects to the regression do not materially change the coefficient estimate α (Table 4). Importantly, the coefficients remain very strongly significant with fixed effects, although the results suggest that controlling for fixed effects would result in slightly lower estimates of α than in our benchmark model.

Table 4

Fixed effects

	α coefficient	Standard error	t-statistic	Observations
M1	**1.0395*** **	**0.1484**	**7.00**	**980**
Time fixed effects	0.8848***	0.2388	3.70	980
Time and country fixed effects	0.6993**	0.3390	2.06	980

Estimating regression (A) with time and country fixed effects for the full sample over the 1950–2010 period.

***=significant at 1%

Second, we investigate whether the chosen time period affects the estimates of α. Table 5 confirms that the results obtained in the benchmark model (M1) remain very robust to the time period chosen. Restricting the sample to the 1960–2010 period (M2), when postwar reconstruction was largely complete, does not materially affect the estimates. Neither does further restricting the sample to the post-Bretton Woods period (M3). Splitting the sample into two halves also does not affect materially the estimates of α (M4 and M5). Similarly, focusing on the period prior to the euro's introduction does not seem to affect the results (M6), even when we exclude the postwar reconstruction period (M7). Finally, splitting the sample into three 20-year periods (M8, M9 and M10), where each corresponds roughly to a generation, also confirms the robustness of the estimates.

Third, we repeat the analysis by excluding countries in our sample one by one. The detailed results are shown in Table A1 in the appendix. The results are extremely robust for the exclusion of any individual countries. When Korea is excluded, the point estimate of α drops slightly, but it still remains very significant. In all other cases, the estimates are indistinguishable from one another.

Fourth, we control for interest rates, inflation and asset prices (Table 6). According to the quantity theory of money, nominal interest rates could negatively affect money holding. We find a negative but insignificant impact for short-term interest rates (M11) but a very significant impact for the change in the short-term interest rate (M12) even after controlling for the levels (M13). Most importantly, incorporating short-term interest rates and their changes leaves the α coefficient robustly close to the benchmark estimates and highly significant.

Table 5

Robustness for time period choice

Model	α coefficient	standard error	t-statistic	observations
M1 - benchmark	**1.0395*** **	**0.1484**	**7.00**	**980**
M2 (1960–2010)	0.9932***	0.1513	6.57	938
M3 (1973–2010)	0.9428***	0.1620	5.82	770
M4 (1950–1980)	0.7956***	0.2464	3.23	342
M5 (1980–2010)	1.2273***	0.1831	6.70	655
M6 (1950–1998)	0.8794***	0.1658	5.30	724
M7 (1960–1998)	0.8169***	0.1699	4.81	682
M8 (1950–1970)	1.1696***	0.4027	2.90	178
M9 (1970–1990)	0.6921***	0.1893	3.66	387
M10 (1990–2010)	1.2404***	0.2530	4.90	452

Estimating regression (A) for the above time periods. ***=significant at 1%

Turning to inflation, it is clear that, in practice, central banks do not perfectly control inflation. This might affect the results, as inflationary shocks increase the cost of holding money and thus could reduce the Marshallian K. Indeed, the data show that higher inflation (M14), accelerating inflation (M15) and the combination of the two (M16) are all associated with a significantly lower Marshallian K. This expected result, however, does not materially affect the size and the statistical significance of α. Thus, if anything, it suggests that after controlling for inflation, the impact of demography is slightly larger on the Marshallian K than in our benchmark model.

Regarding asset prices, asset price booms might drive credit booms and thus could indirectly drive the Marshallian K. However, when we control for asset prices, the coefficient estimates for the working-age population remain stable (M17–M19). The coefficient estimates also remain stable when we control for asset prices and inflation simultaneously (M20–M21). Furthermore, even controlling simultaneously for interest rates, inflation and asset prices does not affect the coefficient estimates or its significance materially (M22–M24). Given that our inflation targeting model predicts that the working-age population drives asset prices and the Marshallian K simultaneously, these joint estimations could in theory suffer from multicollinearity, but this does not seem to be the case.

Finally, we re-estimate the impact of smaller cohorts, and we examine five-year cohorts (Table A2 in the appendix). The results clearly show that prime savers, ie populations between 45 and 64 years of age, have a consistently significant impact on the evolution of the Marshallian K. This remains true even after controlling for short-term rates, inflation, changes in inflation and equity prices. Building on these results, we re-estimate equation (1) with the prime saving-age population and find them consistently significant (Table A3 in the Appendix). This is not entirely surprising: according to the lifecycle theory and some empirical estimates (such as those of Davis and Li (2003)) prime saving-age populations are expected to have the largest impact on asset prices. Thus, we could expect that their savings would have the largest impact on money holdings. In addition, we also estimate the benchmark regression by adding the old age (65+ years) population, the young (0–19 years) population and the old age dependency ratio (ie the ratio of old to working-age population).

All of these variables are insignificant and controlling for them does not affect the significance or the size of the α estimates.

Table 6

Controls for interest rates, inflation and asset prices

Model	α	Int rate	d(int rate)	Inflation	d(inflation)	dln(stock)	dln(prop)
M1	**1.0395*** **						
M11	1.2224***	–0.0355					
M12	1.0011***		–0.4703***				
M13	1.0810***	–0.0151	–0.4642***				
M14	1.6862***			–0.1627***			
M15	1.0014***				–0.6559***		
M16	1.3764***			–0.0940**	–0.6157***		
M17	0.9816***					–0.0026	
M18	1.0193***						–0.0004
M19	1.0282***					–0.0153	0.0107
M20	1.3696***			–0.1023***	–0.6464***	–0.0169**	
M21	1.2645***			–0.0992*	–0.5164***	–0.0216**	0.0238
M22	1.2464***	0.0115		–0.1110	–0.5114***	–0.0218**	0.0235
M23	1.1416***	0.0245	–0.3229**	–0.1124	–0.3714***	–0.0221**	0.0289
M24	0.8924***		–0.3382**		–0.4049***	–0.0201**	0.0346

Estimating regression (A) by adding an additional control variable in each model as specified in the columns. Int rate is the short-term interest rate, d(int rate) is the change in the short-run interest rate, inflation is the inflation rate, d(inflation) the yearly change in the inflation rate, dln means the natural logarithmic difference, stock is the real stock price index and prop is the BIS real property price index. Statistics for coefficient □□

***=significant at 1%, **=significant at 5%, *=significant at 10%

In sum, we find the results shown in the benchmark model extremely robust. Introducing fixed effects could argue for a slightly lower α coefficient while some controls could imply a slightly higher coefficient. All in all, however, the results are extremely robust and seem to be close to unit elasticity. We can confirm that demography, or more precisely the size of the working-age population, does significantly and robustly affect the Marshallian K.

3.3 Demography and asset prices

The model also has implications for property prices. The property described in the model has an exogenously fixed supply so that it would correspond most closely to land. When translating the model to available economic data, we focus on housing and land prices and stock prices. These assets represent substantial values in saving decisions and are used as tools for long-run wealth accumulation. Of course, their supply is elastic, which suggests that we should expect demographic effects that are weaker than what would prevail under a fixed supply of assets.

The theoretical model predicts that real property prices depend on both how large and how wealthy the population is, ie asset prices should depend on the economic output of the working-age population and the size of the working-age population. The following regression formalises this relationship:

$$d \log A = c + \gamma d \log(workage) + \delta d \log(rGDPpw) + \varepsilon \qquad \text{(B)}$$

where A denotes the real asset price (equity or property), rGDPpw is the real GDP per working-age population and the other variables are as in regression (A). Parameter γ is the focus of our interest as it shows the relationship between asset price *A* and *workage*. As country- and time-specific supply elasticity is crucial, we also use period and country fixed effects in the benchmark setup.

The results in Table 7 below confirm the main thrust of our model. Both equity (centre column) and property prices (right-hand column) react positively and significantly to changes in the working-age population in the benchmark model run on the full panel over the full 1950–2010 period (M25). The coefficient estimates of γ are larger than unity and, in general, larger and more significant for property prices than for equities. Given the loglinear setup, this implies that a 1% increase in the working-age population is associated with a rise of more than 1% in asset prices. This result is even stronger than what would be expected based on the model, especially in view of the somewhat elastic supply of these assets.

Table 7

Demography and asset prices

Model	Stocks	Property
M25 (equation 2)	**1.5613***	**3.3878*****
M26 (1970–2010)	2.2428*	3.2924***
M27 (1980–2010)	2.3981	3.5822***
M28 (1970–2000)	2.4322	3.7805***
M29 (old age dependency ratio)	2.0185**	3.2268***
M30 (young age population)	1.5542*	3.3935***
M31 (old age population)	1.6066*	3.3559***
M32 (inflation)	2.1811**	3.4382***
M33 (changes in inflation)	1.5567*	3.3775***
M34 (Marshallian K)	1.7966*	3.2487***

Estimating regression (B) with real stock and property prices for asset prices as indicated by the columns. Statistics for coefficient [illegible] restricts the time period as specified. M29–34 adds an additional control variable. M29: adding the old age dependency ratio, ie the ratio of population over 64 and those between 20 and 64, M30: adding the natural logarithmic difference in the size of young age populations, ie those below age 20, M31: adding the natural logarithmic difference in the size of old age populations, ie those aged 65 and above, M32: adding inflation, M33: adding yearly changes in inflation, M34: adding the Marshallian K.

***=significant at 1%, **=significant at 5%, *=significant at 10%

The results are also robust for the relationship between demography and asset prices. Restricting the sample period does not materially affect the estimates (Table 6: M26–M28). However, the estimated coefficients for equity prices, which are generally less significant than for property prices, can become insignificant in some restricted periods. Adding controls for demographic variables, such as the old age dependency ratio (M29), the size of the young (M30) and old age populations (M31), does not materially affect the coefficient estimates. Similarly, inflation levels (M32) or changes in inflation (M33) do not affect the coefficient estimates. Finally, the results also remain robust after controlling for changes in the Marshallian K (M34). As in the case of the regressions on the Marshallian K (Table 3: M17–M24), M34 also raises the potential for multicollinearity, but this does not seem to materialise.

In sum, the results confirm that asset prices are linked to demographic developments, especially to the evolution of working-age populations. We do not pursue further tests for robustness here, as substantial evidence has already been accumulated in the literature, including in Nishimura (2011) and Takáts (2012), which confirms the relationship between demography and asset prices.

4. Economic impact of ageing

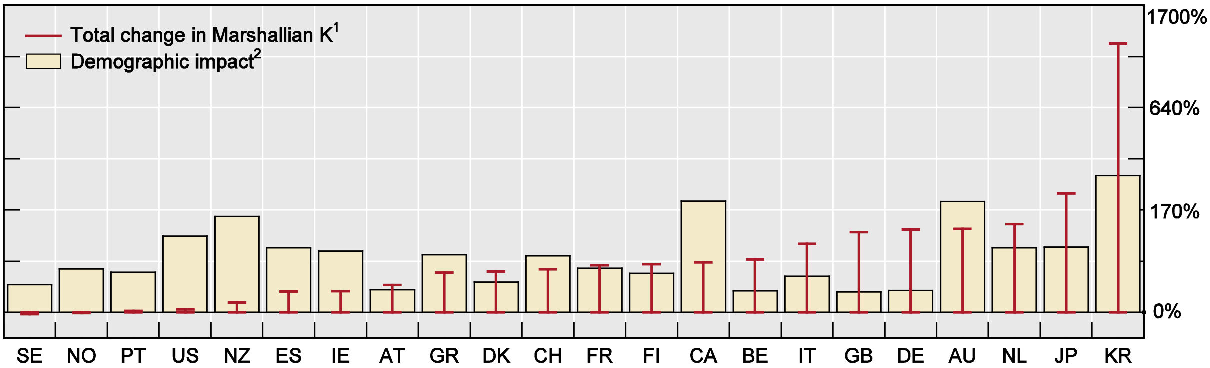
In the empirical sections, we have shown that demography affects the Marshallian K in a statistically significant and robust way. We demonstrate here that these effects are also significant economically. Economic significance cannot be taken for granted because changes in the working-age population explain by themselves less than 1% of the yearly variation in the Marshallian K (Table 2). That demography is not the key driver of year-on-year changes in the money supply is not entirely surprising as the money stocks are notoriously volatile and many factors could affect them.

More importantly, demography does explain a substantial part of the long-run variation in the Marshallian K. This is possible, because the individually small short-run changes push the Marshallian K in the same direction and these small changes accumulate over time. The beige bars on Graph 4 show how large this accumulated estimated demographic impact was in our sample based on the benchmark regression coefficient of Table 1. In fact, demography explains a large share of the long-run change in the Marshallian K (thin vertical lines).

Graph 4

Demographic impact on Marshallian K

1950–2010



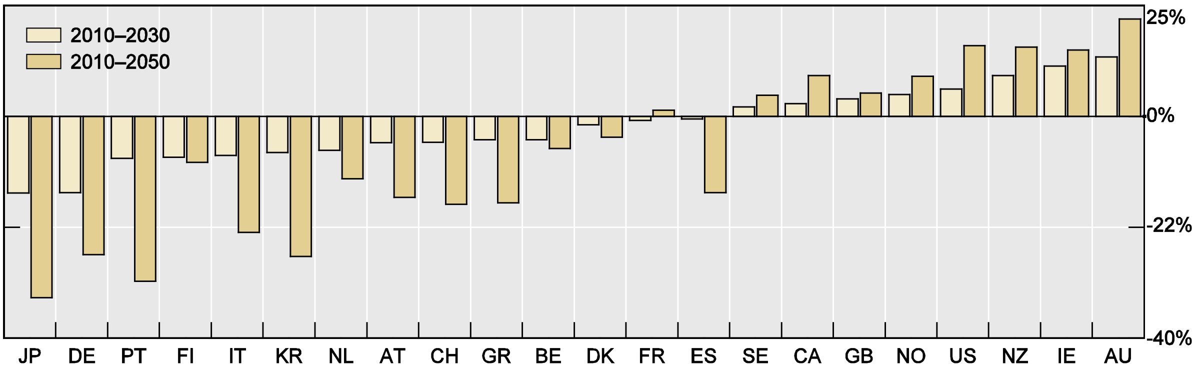
[1] Logarithmic difference in M2 as a percentage of the nominal GDP over the full period. Full period: data from 1950 or from the first available data point. [2] Benchmark regression α coefficient multiplied with the logarithmic difference in the size of working-age population (between 20–64) over the full period.

Perhaps most interestingly, we can use the results to think further about the future impact of demographic change on money demand. This is not to say that we believe it possible to reliably forecast complex future social and economic trends. In fact, the track record of demographic projections, from Malthus onwards, is quite discouraging. Both Keynes (1937) and Schumpeter (1943) discussed population decline as a likely trend – just a few years before the baby boom took off. Fortunately, we do not have to rely on demographic forecasts. We know with certainty today, absent major catastrophes or wars, the size of the working-age populations until 2030 because everybody who will enter the labour force over the next 20 years has already been born. Thus, with the above proviso, we already have the relevant figures. For the reader's interest, we also use projections for working-age populations up to 2050 to illustrate longer-term challenges. However, we caution that these longer-term figures are subject to unpredictable changes in fertility and migration.

Graph 5 shows the demographic impact in the coming decades, which we obtain by combining figures on working-age populations from the UN Population Projections (2011) with our benchmark model coefficient to estimate the demographic impact. The beige bars show the impact until 2030 using the benchmark regression coefficient. Remember that this part of the exercise does not rely on demographic projections, as all these labour force entrants have already been born. The emerging picture for the future stands in sharp contrast with the past depicted in Graph 4: in many advanced economies, demographic factors will stop contributing to money supply growth and will start to reduce the Marshallian K. The impact is largest in Japan and Germany, but it is prevalent in many European countries and in Korea. Reflecting their more youthful demographics, Scandinavia and many English-speaking countries (Australia, Ireland, New Zealand, the United States, the United Kingdom and Canada) will see an increase in money demand.

Graph 5

Demographic impact[1]



[1] Benchmark regression α coefficient multiplied with the logarithmic difference in the size of working-age populations (between 20–64).

Using the UN Population Projections (2011) and their assumptions for fertility and migration for the next 20 years, we extend our time frame to 2050 (orange bars). In most cases, the same countries remain at the top and the bottom ranges of the estimated demographic impact. Interestingly, the figures suggest that the negative demographic impact on the Marshallian K will accelerate in Italy, Portugal and Spain as the ageing process in southern Europe accelerates and catches up with that of Germany and Japan.

5. Conclusion

We have presented a model which shows that demographic change affects monetary aggregates. In particular, the size of the working-age population during a demographic transition raises the Marshallian K, the ratio of money such as M2 to nominal GDP. This prediction differentiates our model from the standard quantity theory of money. In data covering 22 advanced economies over the 1951–2010 period, we found strong and robust empirical support for the theory: on average, a 1% increase in the working-age population is associated with a 1% rise in the Marshallian K. This is a long-term relation: year-to-year changes in the working-age population have little effect on broad money. Our model also suggests that a flexible money supply regime together with inflation targeting is likely to reduce the long-run volatility of real property prices.

These findings are potentially relevant to policymakers for three main reasons. First, the results suggest that the imminent ageing of most advanced and many emerging economies would imply stronger inflationary pressures. These demographic headwinds are in marked contrast with the deflationary demographic effects seen over the past few decades. Of course, central banks can and should resist these inflationary pressures, but our results suggest that central banks might find inflation more difficult to control in the future than in the past.

Second, monetary stability helps to stabilise real property prices. Our results suggest that money with a stable value, if provided elastically by the financial system, can both absorb excess saving demands in the early phase of demographic transition and then shrink in line with the falling demand for money as the population ages. This flexibility dampens the pressures for both real appreciation and depreciation of property prices. Furthermore, an elastic supply of government bonds would have the same effect as an elastic supply of broad money. This result might be especially relevant to emerging economies that face a rapid demographic transition and, at the same time, are reviewing the design of their monetary system.

Third, ageing implies a lower demand for money, thereby less household demand for deposits, which might complicate the deleveraging of high loan-to-deposit ratios. In many advanced economies, the financial system has created an even greater volume of financial assets than would have been justified by money demand stemming from the demographic boom. This resulted in high loan-to-deposit ratios. While collecting more deposits would be a straightforward way to facilitate the necessary deleveraging, ageing would seem to lower demand for money from households and thus for deposits. This implies that deleveraging would have to be effected more on assets than on the liability side in an ageing economy. This tendency could be especially relevant for ageing Europe.

Of course, these policy implications come with strong caveats. Most importantly, we have identified demography as a long-run driver of money demand but, in the short or even the medium run, several other factors might prevail over demography. In Japan, for instance, ageing is at a more advanced stage than in other advanced economies. And while real financial asset and property prices have started to decline, demand for money has remained strong and inflationary pressures low because the Japanese crisis and subsequent low growth have provided a strong incentive for precautionary saving. If a similar scenario were to unfold in Europe, therefore, money demand would not necessarily fall and inflationary pressures might remain subdued.

In summary, our paper has raised the question of how ageing might affect the monetary system. It contributes to a wider literature that seeks to assess the full impact of ageing on the economy by adding a monetary perspective. We hope that our efforts in this direction will serve as a basis for further research on the monetary consequences of ageing.

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Appendix

Formal solution for the fixed money supply model

The formal solution uses the same steps as in the elastic money supply model. Notice first, that M_t=M*/ n_t. Then, using the first-order conditions (6) and (7), we obtain:

$$\frac{1}{M_t}+\frac{\beta\frac{P_t}{P_{t+1}}}{h_t q_{t+1}+\frac{M_t}{P_{t+1}}}=\frac{1}{h_t q_t}+\frac{\beta\frac{q_{t+1}}{q_t}}{h_t q_{t+1}+M_t}$$

Rearranging terms yields

$$\frac{h_t q_{t+1}+\frac{M_t}{P_{t+1}}}{\frac{M_t}{P_t}}+\beta\frac{P_t}{P_{t+1}}=\frac{h_t q_{t+1}+\frac{M_t}{P_{t+1}}}{h_t q_t}+\beta\frac{q_{t+1}}{q_t}$$

and

$$\frac{h_t q_{t+1}}{\frac{M_t}{P_t}}+(1+\beta)\frac{P_t}{P_{t+1}}=\frac{\frac{M_t}{P_{t+1}}}{h_t q_t}+(1+\beta)\frac{q_{t+1}}{q_t}$$

Then, multiplying both sides by P_t/P_{t+1} yields

$$\frac{h_t q_{t+1}P_{t+1}}{M_t}+(1+\beta)=\frac{M_t}{h_t q_t P_t}+(1+\beta)\frac{q_{t+1}P_{t+1}}{q_t P_t}$$

Then, by using condition (8), the law of motion for nominal property prices can be written as:

$$z_{t+1}=\left\{\frac{1+\beta-\frac{M^*}{H^*}\frac{1}{z_t}}{1+\beta-\frac{M^*}{H^*}z_t}\right\}z_t$$

It is remarkable that the nominal price dynamics do not depend on the demographic factor. Technically speaking, the difference equation is an autonomous first-order difference equation, so that one boundary condition determines the complete path of z_t.

In order to solve this difference equation of z_t for t ≧ 1, we need one boundary condition. Note that z_t is the price of property stock, which is a jumping variable having no natural initial condition. Instead of the initial condition, we make a standard assumption of the convergence of z_{t+1} to the steady state value z_∞. It can immediately be shown that

$$z_\infty=\frac{M^*}{H^*}$$

Next, we show that the solution in the fixed money supply case is locally unstable.

$$\left[\frac{dz_{t+1}}{dz_t}\right]_{z_t=\frac{M^*}{H^*}} = \left\{\frac{1+\beta-\frac{M^*}{H^*}\frac{1}{z_t}}{1+\beta-\frac{M^*}{H^*}z_t}\right\} + z_t\left\{\frac{\frac{M^*}{H^*}\frac{1}{{z_t}^2}\left(1+\beta-\frac{M^*}{H^*}\frac{1}{z_t}\right)-\left(1+\beta-\frac{M^*}{H^*}\frac{1}{z_t}\right)\frac{M^*}{H^*}}{\left(1+\beta-\frac{M^*}{H^*}z_t\right)^2}\right\}_{z_t=\frac{M^*}{H^*}}$$

$$\left[\frac{dz_{t+1}}{dz_t}\right]_{z_t=\frac{M^*}{H^*}} = 1+\frac{\frac{M^*}{H^*}+\frac{H^*}{M^*}}{\beta} > 1$$

Therefore, in order to converge to the steady state z_t needs to jump to z_∞ immediately, thus:

$$z_t = z_\infty = \frac{M^*}{H^*} : \forall t$$

Note, that this also implies when we break z_t into its components (q_t and P_t) that

$$\frac{q_{t+1}}{q_t} = \frac{P_t}{P_{t+1}}$$

Consequently, we have from the first-order conditions (8) divided by q_t that

$$\frac{-1}{y_t^Y - \frac{2M_t}{P_t}} + \frac{1}{\frac{M_t}{P_t}} + \frac{\beta\frac{q_{t+1}}{q_t}}{\frac{q_{t+1}}{q_t}h_t q_{t+1} + \frac{P_t}{P_{t+1}}\frac{M_t}{P_t}} = 0$$

which yields

$$\frac{M_t}{P_t} = \frac{(2+\beta)}{(6+2\beta)}y_t^Y$$

and then the same solution obtained by guess and verify follows trivially.

Appendix Tables

Table A1

Robustness to country exclusion

	α coefficient	standard error	t-statistic	Observations
M1 (full sample)	**1.0395*** **	**0.1484**	**7.00**	**980**
- Austria	1.0482***	0.1502	6.98	930
- Australia	1.0859***	0.1613	6.74	930
- Belgium	1.0322***	0.1503	6.87	941
- Canada	1.0819***	0.1584	6.83	938
- Switzerland	1.0564***	0.1463	7.22	946
- Germany	1.0445***	0.1504	6.94	921
- Denmark	1.0518***	0.1492	7.05	932
- Spain	1.0588***	0.1525	6.94	940
- Finland	1.0274***	0.1496	6.86	951
- France	1.0435***	0.1522	6.85	932
- United Kingdom	1.0123***	0.1487	6.81	953
- Greece	1.0412***	0.1488	7.00	951
- Ireland	1.0937***	0.1550	7.06	930
- Italy	1.0301***	0.1508	6.83	921
- Japan	1.0328***	0.1560	6.62	925
- Korea	0.6959***	0.1479	4.71	930
- Netherlands	1.0503***	0.1539	6.83	928
- Norway	1.0521***	0.1521	6.91	920
- New Zealand	1.0704***	0.1507	7.10	959
- Portugal	1.0500***	0.1508	6.96	951
- Sweden	1.0451***	0.1506	6.94	931
- United States	1.1204***	0.1572	7.13	920

Estimating regression (A) excluding countries listed one by one in the first column for the 1950–2010 period. Period fixed effect. ***=significant at 1%, **=significant at 5%, *=significant at 10%

Table A2

Granular age cohorts

cohort	AM1	AM2	AM3	AM4	AM5
0–4	0.1039	0.0707	0.0344	0.1274	0.0966
5–9	–0.1329	–0.1462	–0.1574*	–0.1149	–0.1561*
10–14	–0.0116	–0.0177	–0.0156	0.0507	–0.0495
15–19	0.1321	0.1379	0.1859*	0.1407	0.1029
20–24	–0.1558	–0.1453	–0.1081	–0.1470	–0.1535
25–29	0.0394	0.0740	0.1618	0.0606	–0.0165
30–34	0.0722	0.1042	0.1757**	0.0643	0.0633
35–39	0.0657	0.1068	0.1711**	0.0330	0.0633
40–44	0.1062	0.1334*	0.1704**	0.0917	0.0947
45–49	**0.2194*** **	**0.2432*** **	**0.2509*** **	**0.1866*** **	**0.2113*** **
50–54	**0.1667** **	**0.1824*** **	**0.1856*** **	**0.1836*** **	**0.1822*** **
55–59	**0.1189** **	**0.1368** **	**0.1445** **	**0.1002* **	**0.1125* **
60–64	**0.2554*** **	**0.2792*** **	**0.2865*** **	**0.2424*** **	**0.2661*** **
65–69	0.0768	0.0815	0.0761	0.1103*	0.0841
70–74	–0.0484	–0.0367	0.0063	–0.0404	–0.0477
75–79	–0.0364	–0.0309	0.0241	–0.0238	–0.0420
interest rates		–0.0516			
inflation			–0.1769***		
d(inflation)				–0.6554***	
dln(stock)					–0.0148***

Estimating regression (A) by using age cohort figures instead of aggregate working-age population for the 1950–2010 period and using time fixed effect. ***=significant at 1%, **=significant at 5%, *=significant at 10%

Table A3

Prime saving age (45–64 years) regressions

variables	AM6	AM7	AM8	AM9
dlog(primeage)	0.7779***	0.8777***	0.7500***	0.7612***
inflation		–0.0567*		
d(inflation)			–0.6468***	
dln(stock)				–0.0046

Estimating regression (A) by using prime saving-age (45–64 years old population) instead of working-age population for the 1950–2010 period. Inflation 1% denoted as 0.01. ***=significant at 1%, *=significant at 10%