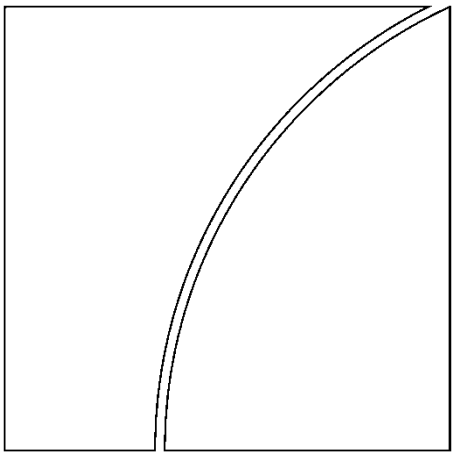
BANK FOR INTERNATIONAL SETTLEMENTS



BIS Working Papers

No 372

Inflation Dynamics in the Presence of Informal Labour Markets

by Paul Castillo and Carlos Montoro

Monetary and Economic Department

February 2012

JEL classification: E32, E50, J64, O17.

Keywords: Monetary Policy, New Keynesian Model, Informal Economy, Labour Market Frictions.

BIS Working Papers are written by members of the Monetary and Economic Department of the Bank for International Settlements, and from time to time by other economists, and are published by the Bank. The papers are on subjects of topical interest and are technical in character. The views expressed in them are those of their authors and not necessarily the views of the BIS.

This publication is available on the BIS website (www.bis.org).

ISSN 1020-0959 (print)

ISSN 1682-7678 (online)

Inflation Dynamics in the Presence of Informal Labour Markets*,♣

Paul Castillo♦ and Carlos Montoro♠

Abstract

In this paper we analyse the effects of informal labour markets on the dynamics of inflation and on the transmission of aggregate demand and supply shocks. In doing so, we incorporate the informal sector in a modified New Keynesian model with labour market frictions as in the Diamond-Mortensen-Pissarides model. Our main results show that the informal economy generates a "buffer" effect that diminishes the pressure of demand shocks on inflation. This finding is consistent with the empirical literature on the effects of informal labour markets in business cycle fluctuations. This result implies that, in economies with large informal labour markets, changes in interest rates are more effective in stimulating real output and there is less impact on inflation. Furthermore, the model produces cyclical flows from informal to formal employment, consistent with the data.

JEL Classification: E32, E50, J64, O17

Keywords: Monetary Policy, New Keynesian Model, Informal Economy, Labour Market Frictions.

---

* We would like to thank Philip Turner, Lars Ljungqvist, Randal Wright, Paul Levine, Gabriel Rodriguez, Hugo Vega, Alexandre Janiak, participants at the "Third Monetary Policy Research Workshop in Latin America and the Caribbean" organised by Banco de la Republica and CCBS-Bank of England, participants at the 2007 Research Meeting organized by Banco Central de Reserva del Perú, participants at the University of Surrey Economic Workshop 2008, participants at the 2008 LACEA Meeting held in Rio de Janeiro, participants at the 2010 16th International Conference on Computing in Economics and Finance and two anonymous referees for useful suggestions and comments. Alan Villegas provided excellent research assistance. The views expressed herein are those of the authors and do not necessarily reflect those of the Banco Central de Reserva del Perú nor those of the Bank for International Settlements. Any errors are our own responsibility.

♣ A previous version of this paper was circulated under the title: “Monetary policy in the presence of informal labour markets”. This paper has been accepted for publication and will appear in a revised form, translated to Spanish, subsequent to editorial input by Banco Central de Chile, in *Economía Chilena*.

♦ Banco Central de Reserva del Perú and Universidad del Pacifico, E-mail: paul.castillo@bcrp.gob.pe.

♠ Bank for International Settlements, Address correspondence to: Carlos Montoro, Office for the Americas, Bank for International Settlements, Torre Chapultepec - Rubén Darío 281 - 1703, Col. Bosque de Chapultepec - 11580, México DF -- México; tel: +52 55 9138 0294; fax: +52 55 9138 0299; e-mail: carlos.montoro@bis.org.

1. Introduction

The New Keynesian model has become a useful tool for both academics and policymakers to analyse monetary policy design. However, this strand of the literature typically ignores labour market frictions. In particular, it assumes that labour markets are perfectly competitive and consequently aggregate fluctuations only adjust at the intensive labour margin. Nevertheless, empirical studies show that at business cycle frequencies labour usage adjusts not only at the intensive margin but also at the extensive margin, which generates fluctuations in unemployment. Therefore, this model is not suited to study the link between inflation and unemployment and it is limited in explaining some stylised facts of the data.[1]

Recently, some authors have extended the New Keynesian model to include labour market frictions and unemployment along the lines of the Diamond-Mortensen-Pissarides (DMP) model.[2] The DMP framework includes labour market frictions, such as costs of matching vacancies and workers searching for a job. These kinds of frictions generate dynamics in the unemployment rate that are closer to the data and have implications for monetary policy. Chéron and Langot (2000) show that the combination of search frictions and sticky prices can reproduce the negative correlation of inflation with employment and vacancies, reported in the literature for developed economies, when money supply shocks are included in the model. Christoffel and Linzert (2005) show how real wage rigidities can generate endogenous inflation persistence within the New Keynesian model incorporating search frictions. Trigari (2006), using an estimated general equilibrium model that integrates a theory of equilibrium unemployment into a monetary model with nominal price rigidities, shows that search frictions generate a lower elasticity of marginal costs with respect to output, which explains the sluggishness of inflation and the persistence of output that are observed in the data.

The study of the flows between employment and unemployment is important for developed economies, since they capture most of the labour market fluctuations. However, in developing economies, where labour markets are characterised by having a large proportion of the labour force employed in semi-illegal irregular jobs – so-called *informal* employment[3] – the study of the flows between the *formal* and *informal* sectors becomes more relevant.

There is empirical evidence that the presence of informal labour markets affects the business cycle dynamics of an economy. More precisely, this evidence shows that informal labour markets act as a buffer stock for regulated formal employment, increasing labour market flexibility and affecting the transmission mechanisms of shocks to the economy. For instance, Bovi (2007), using labour market data for Italy, finds that informal employment is pro-cyclical, whereas formal employment is almost acyclical. Other authors have also found similar evidence. Carrillo and Pugno (2004) and Bowler and Morisi (2006) report a cyclical pattern for informal employment in a set of emerging economies.

Given the importance of the informal economy for developing countries, the design of monetary policy should carefully consider its effects on the labour market and inflation dynamics. In particular, from the monetary policy point of view it is important to answer the following questions: how does the presence of the informal sector affect inflation dynamics

---

[1] For instance, the basic New Keynesian model is not suitable for explaining the procyclicality of the job destruction rate and the well-documented negative correlation between unemployment and inflation.

[2] Among those authors are Walsh (2003, 2005), Alexopoulus (2004), Trigari (2006, 2009), Blanchard and Galí (2010), Krause and Lubik (2007), Thomas (2008), Gertler and Trigari (2009), and Ravenna and Walsh (2008). For a reference to the DMP model see Pissarides (2000).

[3] Djankov, et al. (2002) and Schneider (2004) estimate that informal employment is between 40% and 80% of the total labor force in developing economies.

and the transmission mechanism of monetary policy, and what determines the flows between formal and informal employment?

To address these questions, in this paper we extend a standard closed economy New Keynesian model adding labour market frictions, as in Blanchard and Galí (2010). Differently from them, however, we model a dual labour market economy considering the existence of formal and informal labour contracts. The model economy is composed of households, retailers, firms and the central bank. Households receive utility from the consumption of a continuum of differentiated goods and supply labour in a decentralised labour market subject to search and hiring costs. Retailers, on the other hand, produce under monopolistic competition differentiated consumption goods and set prices according to a Calvo-type price setting rule. Retailers use as production input a wholesale good, which is produced by firms under perfect competition using labour. Finally, the central bank implements monetary policy by setting the short-term interest rate according to a Taylor-type feedback policy rule.

To the best of our knowledge, this is the first paper that analyses the implications for inflation dynamics of the presence of an informal labour market. Previous papers have studied how the informal jobs in the labour market are generated; see, for example, Bosch (2004, 2006), Fugazza and Jaques (2002), Kolm and Larsen (2003), and Boeri and Garibaldi (2006). However, those models focus on the real economy and do not analyse the interaction between the informal sector and monetary policy.

We introduce labour market frictions considering that firms face hiring costs, which depend on the degree of labour market tightness, defined by the ratio of hires to unemployment. These hiring costs generate a friction in the labour market similar to the cost of posting a vacancy in the standard DMP model. Furthermore, we introduce informality within the model by assuming firms in the wholesale sector can choose between two types of production processes: formal and informal. The process labelled as formal has higher productivity and larger hiring costs. In contrast, in the informal process workers are less productive but hiring costs are smaller. We focus on an equilibrium where firms use both production technologies, and thus informal and formal workers coexist.

The key implication of this dual-production technology is that firms' marginal costs depend on the level of informality, measured by the proportion of informal employment in the total labour force. During periods of high aggregate demand, firms find it optimal to use the informal technology more intensively because marginal costs associated with this technology are lower than those of the formal one. Accordingly, firms' behaviour optimally lessens the impact of aggregate demand on their marginal costs. Conversely, when demand is low and therefore hiring costs are lower, firms optimally increase their relative use of formal labour.

This result is similar to the one reported by Trigari (2006). In her model, the possibility that firms adjust output using both the intensive and extensive margin of labour reduces the response of marginal costs to output fluctuations. In our model, the informal sector allows firms to reduce marginal costs and to expand output more flexibly because informal labour has lower hiring costs. Differently from Trigari (2006), however, in our set-up firms choose two different types of labour in the extensive margin. This additional degree of flexibility is the crucial mechanism behind the lower response of inflation to output in our model. As Galí (2010) points out, in the case of Trigari (2006) her results are sensitive to the assumption that marginal disutility of labour is constant in the model with search frictions but increasing in labour effort in her base model without search frictions. Our set-up does not have this assumption, since marginal disutility of labour is comparable across models with and without informal labour markets in this paper.

Furthermore, informality makes formal employment less cyclical, which is in line with the stylised facts reported by Bovi (2007). When a worker receives an offer to sign a formal labour contract, he has two options: either to accept the offer and receive the corresponding wage rate, or to wait for another one in the hope of obtaining a higher wage rate. When the economy includes informal labour markets, the cost of waiting is larger since the probability

of receiving a new offer of a formal labour contract is much lower. This possibility of waiting for a longer period induces workers to accept job offers more frequently, particularly in the informal sector, inducing firms to hire informal workers, which dampens the response of employment in the formal sector. Hence, firms in economies with informal labour markets have more flexibility to expand output, thus demand shocks generate lower inflation and larger output expansions. In this case, the positive response of informal employment to demand shocks is larger than the one observed in the formal sector.

At the aggregate level, the model shows that informality affects the dynamics of domestic inflation along several dimensions. First, it generates a link between unemployment flows and inflation dynamics. Second, through its relationship with firms' marginal costs, it reduces the impact of aggregate demand on domestic inflation.

The paper is organised as follows: the next section presents the model of an economy with monopolistic competition, nominal rigidities and dual labour market rigidities. Section 3 shows the implications of the informal economy in the equilibrium of the model. Section 4 presents, in some quantitative exercises, the effects of the informal economy in the steady state, the inflation dynamics and the transmission mechanism of monetary policy. The last section concludes.

2 The model

The economy is populated by a continuum of households that consume final goods and supply labour in a decentralised labour market subject to search and hiring costs. Firms produce a wholesale good, which is used as input to produce differentiated final consumption goods by retailers and the central bank that sets the nominal interest rate through a Taylor rule. Retailers operate in monopolistic competitive markets, where prices are sticky.

2.1 Preferences

The representative household is made up of a continuum of members represented by the unit interval. Each household maximises the following utility function,

$$\mathbf{U}_t = E_o\left\{\sum_{t=0}^{\infty}\beta^t\left[\log\left(C_t\right)-\chi\frac{N_t^{1+\varphi}}{1+\varphi}\right]\right\}, \tag{2.1}$$

where C_t is a composite basket defined over a continuum of differentiated goods that have an elasticity of substitution $\varepsilon>1$, φ is the inverse of the labour supply elasticity, χ is a positive preference parameter and $\beta<1$ is the discount factor.

$$C_t=\left[\int_0^1 C_t(z)^{\frac{\varepsilon-1}{\varepsilon}}dz\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.2}$$

and N_t stands for the fraction of household members that are employed, that satisfies the constraint $0\le N_t\le 1$. At the beginning of each period a fraction N_{t-1} of the family members are employed and the difference is unemployed. From this pool of employed household members, each period a fraction δ lose their jobs, and among those unemployed, a measure H_t is hired. Thus, employment evolves according the following condition,

$$N_t=\left(1-\delta\right)N_{t-1}+H_t. \tag{2.3}$$

Household members, when unemployed, receive a constant income associated to home production, W^u, whereas when they are employed they can either work under a formal

contract and receive a wage rate W_t^F, or they can work under an informal contract, where the wage rate is W_t^I. Informal contracts differ from formal ones mainly because firms face lower hiring costs under informal contracts.[4] Total employment in the economy is defined as follows,

$$N_t = N_t^F + N_t^I, \tag{2.4}$$

where N_t^F and N_t^I, represent the stock of employed workers under formal and informal contracts. We introduce an index that measures the tightness of the labour market, denoted by X_t and defined as the ratio of hires to the level of unemployment before the hiring decision has taken place, that is $X_t = \frac{H_t}{U_t}$ where $U_t = 1-(1-\delta)N_{t-1}$. Alternatively, labour market tightness can be interpreted as the probability that a worker has of being hired, that is the job finding rate. We further assume that the job finding rate is different for formal and informal contracts, in particular we define, $X_t^F = \frac{H_t^F}{U_t}$, as the job finding rate in the formal labour market and as $X_t^I = \frac{H_t^I}{U_t}$, the corresponding rate in the informal market. It follows that:

$$X_t = X_t^F + X_t^I. \tag{2.5}$$

Households can smooth consumption using a nominal one-period discount bond, B_t which pays a nominal interest rate, i_t every period. Therefore, the households' budget constraint is given by:

$$P_t C_t + B_t = \left[W_t^F N_t^F + W_t^I N_t^I + P_t W^u \left(1-N_t\right)\right] + B_{t-1}(1+i_t) + P_t \Gamma_t^R, \tag{2.6}$$

where Γ_t^R stands for firm's profits in the retail sector and P_t is the consumer price index. The first order condition that determines the optimum level of consumption and savings is given by the following Euler equation that equalises the cost of postponing consumption with its expected marginal benefit,

$$1 = \beta E_t\left[\frac{P_t C_t}{P_{t+1} C_{t+1}}\left(1+i_t\right)\right]. \tag{2.7}$$

Optimal intratemporal consumption allocation determines the demand for each variety of consumption good as follows,

$$C_t(z) = \left[\frac{P_t(z)}{P_t}\right]^{-\varepsilon} C_t. \tag{2.8}$$

2.2 Technology and Labour Market Dynamics

2.2.1 Wholesale Producers

There is a continuum of wholesale good producers indexed by $z \in [0,1]$. All of them use two different constant returns to scale technologies, $Y_t^F(z)$ and $Y_t^I(z)$, such that,

$$Y_t^W(z) = Y_t^F(z) + Y_t^I(z). \tag{2.9}$$

[4] See Batini and Levine (2010) for a detailed discussion on the definition and implications of informality.

The first of these technologies, $Y_t^F(z)$, uses formal labour for production whereas $Y_t^I(z)$ uses workers hired under informal contracts. These two production functions are presented next,

$$Y_t^F(z) = A_t N_t^F(z), \tag{2.10}$$

$$Y_t^I(z) = \gamma A_t N_t^I(z), \tag{2.11}$$

where $\gamma \leq 1$ and A_t stands for the level of productivity.[5] Hiring costs capture the fact that formal and informal jobs are subject to different regulation costs. Formal jobs usually require that firms pay benefits to workers, which is not usually the case for informal jobs. Following Blanchard and Galí (2010) we assume that hiring costs are increasing on each type of labour market tightness,[6] as follows,

$$G_t^F = B^F A_t \left(X_t^F\right)^{\alpha_F}, \tag{2.12}$$

$$G_t^I = B^I A_t \left(X_t^I\right)^{\alpha_I}, \tag{2.13}$$

where we restrict $B^F > B^I$ and $\alpha_F > \alpha_I$. These assumptions capture the fact that for formal jobs, given the same level of market tightness, hiring costs are larger and more sensitive to labour market conditions due to regulation. According to this, formal labour contracts are only offered when it becomes profitable to pay the higher hiring costs. Firms hire $H_t^F(z)$ and $H_t^I(z)$ workers of each type every period. Therefore, the laws of motion of both types of labour are determined by,

$$N_t^j(z) = (1-\delta) N_{t-1}^j(z) + H_t^j(z), \quad \text{for } j = \{F, I\}. \tag{2.14}$$

Under these conditions, the firms' problem consists in choosing sequences of $\{N_t^I(z)\}$, $\{H_t^I(z)\}$, $\{N_t^F(z)\}$, $\{H_t^F(z)\}$ to maximise the following expected discounted profit function,

$$E_t\left(\sum_{j=0}^{\infty} Q_{t,t+j} \Gamma_{t+j}(z)\right), \tag{2.15}$$

where $Q_{t,t+j} \equiv \beta^j \frac{C_t}{C_{t+j}}$ is the stochastic discount factor and

$$\Gamma_t(z) = \frac{P_t^w}{P_t}\left[A_t N_t^F(z) + \gamma A_t N_t^I(z)\right] - W_t^I N_t^I(z) - W_t^F N_t^F(z) - G_t^I H_t^I(z) - G_t^F H_t^F(z), \tag{2.16}$$

where P_t^w is the price of the wholesale good. The corresponding first order conditions are given by:

---

[5] An alternative way to rationalize this assumption is consider that firms that hire formal workers can have access to public services, which can enhance their productivity levels. See Loayza (1997) for a model that uses this type of setup.

[6] Galí (2010) shows that this hiring cost formulation is equivalent to the matching function approach adopted by the DMP search literature. More precisely, under the assumption of homogeneity of degree one of the matching function $M(V_t, U_t)$, which depends on the aggregate level of vacancies (V_t) and unemployment (U_t), the cost per hire is also function of the job finding rate.

$$N_t^F(z):\ \frac{P_t^w}{P_t}A_t - W_t^F - G_t^F + (1-\delta)\beta E_t\left(Q_{t,t+1}G_{t+1}^I\right) = 0, \tag{2.17}$$

$$N_t^I(z):\ \gamma\frac{P_t^w}{P_t}A_t - W_t^I - G_t^I + (1-\delta)\beta E_t\left(Q_{t,t+1}G_{t+1}^I\right) = 0. \tag{2.18}$$

The intuition of these two equations is simple. Firms hire workers of each type of labour up to the point where the value of their marginal productivity equals the cost of hiring an additional worker. In this case, this marginal cost is not given only by real wages as in the case of perfectly competitive labour markets, but also by the costs generated by hiring. Also, from the cost minimisation problem of the firms, it holds that,

$$\frac{P_t^w}{P_t} = MC_t, \tag{2.19}$$

where MC_t is the real marginal cost respect to output, equal to

$$MC_t = \frac{W_t^F + G_t^F - (1-\delta)\beta E_t\left(Q_{t,t+1}G_{t+1}^F\right)}{A_t} \tag{2.20}$$

$$= \frac{W_t^I + G_t^I - (1-\delta)\beta E_t\left(Q_{t,t+1}G_{t+1}^I\right)}{\gamma A_t}. \tag{2.21}$$

According to this expression, in equilibrium labour moves from one sector to the other (and from or to unemployment) in such a way that real marginal costs are the same in each sector. Also, aggregate labour in each sector equals:

$$N_t^j = \int_0^1 N_t^j(z)dz,\ \text{for } j = \{F, I\}. \tag{2.22}$$

2.2.2 Wage determination

We assume that wages are set in a Nash bargaining process. Let's denote by λ the bargaining power of workers and by V_t^F, V_t^I, V_t^U the value functions of a representative household that has a marginal member employed in the formal and informal sector, respectively.

$$V_t^F = W_t^F - \chi C_t N_t^{\varphi} + \beta E_t\left\{Q_{t,t+1}\left[\left(1-\delta+\delta X_{t+1}^F\right)V_{t+1}^F + \delta X_{t+1}^I V_{t+1}^I + \delta\left(1-X_{t+1}\right)V_{t+1}^U\right]\right\}, \tag{2.23}$$

$$V_t^I = W_t^I - \chi C_t N_t^{\varphi} + \beta E_t\left\{Q_{t,t+1}\left[\left(1-\delta+\delta X_{t+1}^I\right)V_{t+1}^I + \delta X_{t+1}^F V_{t+1}^F + \delta\left(1-X_{t+1}\right)V_{t+1}^U\right]\right\}. \tag{2.24}$$

A worker that signs a formal contract enjoys in period t his wage net of the marginal rate of substitution. Also, he faces the probability δ of loosing his job at the end of period t and a probability $(1-\delta)$ of maintaining his formal job in $t+1$ and enjoy V_{t+1}^F. Given that he looses his job, he can enjoy V_{t+1}^F, V_{t+1}^I and V_{t+1}^U with probability X_{t+1}^F, X_{t+1}^I and $(1-X_{t+1})$, respectively. A similar interpretation applies for the value function of informal workers.

Similarly for the case of unemployed household members, the corresponding value function is determined by,

$$V_t^U = W^u + \beta E_t\left\{Q_{t,t+1}\left[X_{t+1}^F V_{t+1}^F + X_{t+1}^I V_{t+1}^I + \left(1-X_{t+1}\right)V_{t+1}^U\right]\right\}. \tag{2.25}$$

An unemployed worker receives the current payoff of W^u from home production and in the next period he can become either formally employed, informally employed or stay unemployed with probability X^F_{t+1}, X^I_{t+1} and $\left(1-X_{t+1}\right)$, respectively.

As the firm's surplus from an established employment relationship is given by the hiring cost,[7] from the Nash bargain the workers' surplus has to be determined by:

$$V^F_t - V^U_t = \lambda G^F_t, \tag{2.26}$$

$$V^I_t - V^U_t = \lambda G^I_t. \tag{2.27}$$

Using this condition, we can transform equations (2.23) and (2.24), such that wages in the formal and informal sector are determined by:

$$\lambda G^F_t = W^F_t - \left(W^u + \chi C_t N^{\varphi}_t\right) + \beta\left(1-\delta\right)\lambda E_t Q_{t,t+1}\left[\left(1-X^F_{t+1}\right)G^F_{t+1} - X^I_{t+1}G^I_{t+1}\right], \tag{2.28}$$

$$\lambda G^I_t = W^F_t - \left(W^u + \chi C_t N^{\varphi}_t\right) + \beta\left(1-\delta\right)\lambda E_t Q_{t,t+1}\left[\left(1-X^I_{t+1}\right)G^I{}_{t+1} - X^F_{t+1}G^F_{t+1}\right]. \tag{2.29}$$

These two conditions together with (2.20) and (2.21) characterise the labour market equilibrium.

2.3 Retail Firms

Each retail firm uses wholesale goods to produce differentiated final consumption goods using a one to one technology. This in turn implies that the marginal cost retailers face (MC^R_t) is exactly equal to the relative price of the wholesale good, that is

$$MC^R_t = \frac{P^W_t}{P_t} = MC_t. \tag{2.30}$$

As we can see from (2.20) and (2.21), marginal costs depend on real wages from both the formal and the informal labour market. Furthermore, we assume that each retailer sets prices following a staggered pricing mechanism *a la* Calvo. Each firm faces an exogenous probability of changing prices given by $\left(1-\theta\right)$. The optimal price that solves the firm's problem is given by

$$\left(\frac{P^*_t(z)}{P_t}\right) = \frac{\mu E_t\left(\sum_{k=0}^{\infty}\theta^k Q_{t,t+k} MC_{t+k} F^{\varepsilon}_{t,t+k} Y_{t+k}\right)}{E_t\left(\sum_{k=0}^{\infty}\theta^k Q_{t,t+k} F^{\varepsilon-1}_{t,t+k} Y_{t+k}\right)}, \tag{2.31}$$

where $\mu = \frac{\varepsilon}{\varepsilon-1}$ is the price mark-up, $P^*_t\left(z\right)$ is the optimal price level chosen by the firm, $F_{t,t+k} = \frac{P_{t+k}}{P_t}$ is the cumulative level of inflation and Y_{t+k} is the aggregate level of output.

Since only a fraction $\left(1-\theta\right)$ of firms changes prices every period and the remaining fraction keeps its price fixed, the aggregate price level is given by the following equation:

[7] This is because any current worker can be immediately replaced with someone who is unemployed by paying the hiring costs.

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left[P_t^*(z)\right]^{1-\varepsilon} \tag{2.32}$$

Following Benigno and Woodford (2005), equations (2.31) and (2.32) can be written recursively introducing the auxiliary variables NN_t and DD_t:

$$\theta(\Pi_t)^{\varepsilon-1} = 1-(1-\theta)\left(\frac{NN_t}{DD_t}\right)^{1-\varepsilon}, \tag{2.33}$$

$$DD_t = Y_t(C_t)^{-1} + \theta\beta E_t\left[(\Pi_{t+1})^{\varepsilon-1} DD_{t+1}\right], \tag{2.34}$$

$$NN_t = \mu Y_t(C_t)^{-1} MC_t + \theta\beta E_t\left[(\Pi_{t+1})^{\varepsilon} NN_{t+1}\right], \tag{2.35}$$

where $\Pi_t = P_t / P_{t-1}$ is the gross inflation rate. Equation (2.33) comes from the aggregation of individual firms' prices. The ratio NN_t / DD_t represents the optimal relative price $P_t^*(z)/P_t$. Equations (2.33), (2.34) and (2.35) summarise the recursive representation of the non- linear Phillips curve.

2.4 The labour market and the marginal costs

After subtracting the labour demand equations (2.20) and (2.21) and the wage setting equations (2.28) and (2.29) for both sectors, we obtain, respectively:

$$W_t^F - W_t^I = (1-\gamma) A_t MC_t - \left(\tilde{G}_t^F - \tilde{G}_t^I\right) \tag{2.36}$$

$$W_t^F - W_t^I = \lambda\left(\tilde{G}_t^F - \tilde{G}_t^I\right) \tag{2.37}$$

where $\tilde{G}_t^j \equiv G_t^j - (1-\delta) E_t Q_{t,t+1} G_{t+1}^j$ for $j = \{I, F\}$ is the cost of hiring a worker in sector j in period t net of the expected saving on hiring costs next period. After equating both equations we get:

$$(1-\gamma) A_t MC_t = (1+\lambda)\left(\tilde{G}_t^F - \tilde{G}_t^I\right) \tag{2.38}$$

In this case of a dual labour market, marginal costs become a function of productivity and the hiring costs. In contrast, in the case that only the formal labour market is present, marginal costs would be:

$$A_t MC_t = \chi C_t N_t^{\varphi} + W^U + (1+\lambda)\tilde{G}_t^F + \lambda(1-\delta) E_t Q_{t,t+1} G_{t+1}^F X_{t+1}^F \tag{2.39}$$

Condition (2.38) establishes that firm's marginal costs are determined by the difference between the hiring cost of a formal versus an informal worker, and the relatively gain in productivity of hiring a formal worker. Notice that the marginal rate of substitution does not affect marginal costs directly, but only indirectly through its impact on hiring costs.

Since the link between firm's marginal costs and the marginal rate of substitution is the main channel through which output gap affects marginal costs in the new Keynesian model, this directly channel through is not present in our set-up. Demand shocks, which increase consumption and labour supply, only affect marginal cost through its indirect impact on hiring costs.

A demand shock generates an increase in hiring but since at the margin informal workers are relatively cheaper for firms, hiring of informal workers increase more than for formal workers.

This substitution effect reduces firm's marginal costs and it generates the buffer effects of informal labour markets. Moreover, as marginal costs depend on hiring costs, in turn they depend on the tightness of each sector. Then, the relationship between inflation and marginal costs, becomes a function of inflation, aggregate unemployment and employment in the formal and informal sector.

2.5 Market Clearing

Aggregating the budget constraint over all households we obtain,

$$C_t = W_t^I N_t^I + W_t^F N_t^F + W^u \left(1 - N_t\right) + \Gamma_t^R \tag{2.40}$$

Since the wholesale sector is in perfect competition, profits are zero for each firm, thus we have that,

$$\frac{P_t^w}{P_t} Y_t^w = W_t^I N_t^I + W_t^F N_t^F + G_t^I H_t^I + G_t^F H_t^F \tag{2.41}$$

and also since profits in the retail sector are $\Gamma_t^R = Y_t - \frac{P_t^w}{P_t} Y_t^w$, we have that,

$$Y_t = C_t + G_t^I H_t^I + G_t^F H_t^F - W^u \left(1 - N_t\right) \tag{2.42}$$

and

$$Y_t^w = Y_t \Delta_t \tag{2.43}$$

where $\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} dz$ is a measure of price dispersion.

2.6 Monetary Policy

We assume that the central bank conducts monetary policy by targeting the nominal interest rate in the following way:

$$\left(1 + i_t\right) = \left(1 + i\right) \left(\frac{\Pi_t}{\Pi}\right)^{\varphi_\pi} \left(Y_t^{gap}\right)^{\varphi_y} \tag{2.44}$$

where, $\varphi_\pi > 1$ and $\varphi_y > 0$ measure the response of the nominal interest rate to inflation and output gap (Y_t^{gap}), respectively. The output gap is measured by the difference of output with respect of its natural level. This latter is calculated at the equilibrium of flexible prices, in which the probability of not changing prices for retail firms $\theta = 0$. Moreover, we express variables in the steady state values without time subscript (i.e. X for variable X_t).

3. The role of the informal economy

3.1 The efficient allocation

The efficient equilibrium is equivalent to the social planner problem of maximizing the utility of the representative agent (2.1), subject to: the production function for wholesale goods (2.9), (2.10) and (2.11), and retail goods, the resources constraint (2.42), the law of motion for both types of labour (2.14), and the aggregation conditions for both types of labour (2.22) and for total labour (2.4), considering also the constraint $0 \leq N_t \leq 1$.

The optimality conditions for the social planner's problem can be written as:

$$\chi C_t N_t^{\varphi} \le A_t - \left(1+\alpha^F\right)G_t^F - W^u + \left(1-\delta\right)E_t\left\{Q_{t,t+1}G_{t+1}^F\left[1+\alpha^F\left(1-X_{t+1}^F\right)\right]\right\}, \tag{3.1}$$

$$\chi C_t N_t^{\varphi} \le \gamma A_t - \left(1+\alpha^I\right)G_t^I - W^u + \left(1-\delta\right)E_t\left\{Q_{t,t+1}G_{t+1}^I\left[1+\alpha^I\left(1-X_{t+1}^I\right)\right]\right\}, \tag{3.2}$$

which must hold with equality if $N_t < 1$. These conditions determine, respectively, employment in both the formal and the informal sectors. Note that in the absence of hiring costs the second condition is irrelevant, since it is possible to produce more with the same costs using only the formal technology. Moreover, if we also exclude the unemployment benefits W^u when $B^F = B^I = 0$, the optimal solution is $N_t^F = N_t$ and

$$\chi N_t^{1+\varphi} \le 1, \tag{3.3}$$

which comes from replacing the resources constraint in (3.1). In this case, a nec essary condition for full employment is to have $\chi = 1$. Moreover, with unemployment benefits this condition becomes:

$$\chi\left[N_t + \frac{W^u}{A_t - W^u}\right]N_t^{\varphi} \le 1 \tag{3.4}$$

By simple inspection of conditions (3.3) and (3.4) we can note that the inclusion of unemployment benefits would reduce the employment level and consequently increase unemployment.

Moreover, in the general case with hiring costs, a condition for both formal and informal employment to coexist in the efficient allocation is that the RHS of equations (3.1) and (3.2) be the same. A necessary condition for having both types of labour is that in equilibrium hiring costs in the informal sector are lower than in the formal sector, which compensates for the lower productivity in the informal sector.

3.2 The labour market equilibrium in steady state

We can analyse the steady state of the model as the intersection of labour demand with labour supply for each sector. The complete system of equations is shown in appendix B. The labour demand for each sector equalises the real wage with its respective marginal rate of transformation, that is:

$$W^F = A\frac{1}{\mu} - G^F\left[1-\beta\left(1-\delta\right)\right] \tag{3.5}$$

$$W^I = \gamma A\frac{1}{\mu} - G^I\left[1-\beta\left(1-\delta\right)\right] \tag{3.6}$$

where G^F and G^I are both functions of N^F and N^I.

The labour supply consists on the wage curve for each sector:

$$W^F = \chi C N^{\varphi} + W^U + \lambda\left[G^F - \beta\left(1-\delta\right)\left(\left(1-X^F\right)G^F - X^I G^I\right)\right] \tag{3.7}$$

$$W^I = \chi C N^{\varphi} + W^U + \lambda\left[G^I - \beta\left(1-\delta\right)\left(\left(1-X^I\right)G^I - X^F G^F\right)\right] \tag{3.8}$$

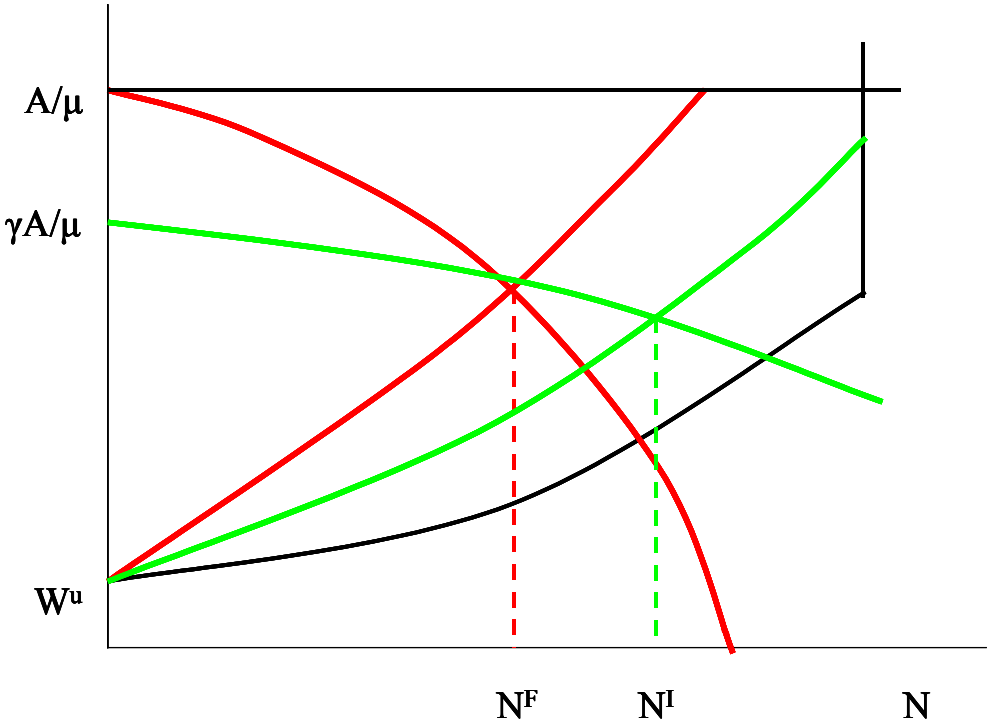
where C, N, X^F and X^I are also functions of N^F and N^I. The intersection of these two sets of equations gives the solution for real wages and labour in each sector.

In figure 3.1 we show a graphical representation of the labour market equilibrium in steady state.[8] In the case without labour market frictions, labour demand is given by a horizontal line at A/μ and the wage curve is an upward sloping curve with intercept at W^u when $N<1$ and a vertical line at the value of full employment. When introducing labour market frictions in a dual market, labour demand in the formal sector is a downward sloping curve that starts from the intercept at A/μ and the wage curve is an upward sloping curve that also starts in W^u, but is steeper than in the case without labour market frictions. The intersection of these two curves defines N^F. For the case of the informal economy, labour demand is a downward curve that starts at $\gamma A/\mu$ and the wage curve is an upward curve that starts at W^u. Both curves for the informal economy are less steep than those of the formal economy, which indicates that labour in the informal economy is more elastic.

Let's analyse for example the effects in steady state of an increase in the parameter of rigidity in the formal sector. In figure 3.2 we show that an increase in B^F generates in the formal sector a downward movement of labour demand curve and an upward shift of the wage setting curve, which reduces formal labour. As unemployment increases, this reduces tightness in the informal sector, moving the labour demand curve upwards and the wage setting curve downwards, increasing employment in the informal sector.

Figure 3.1

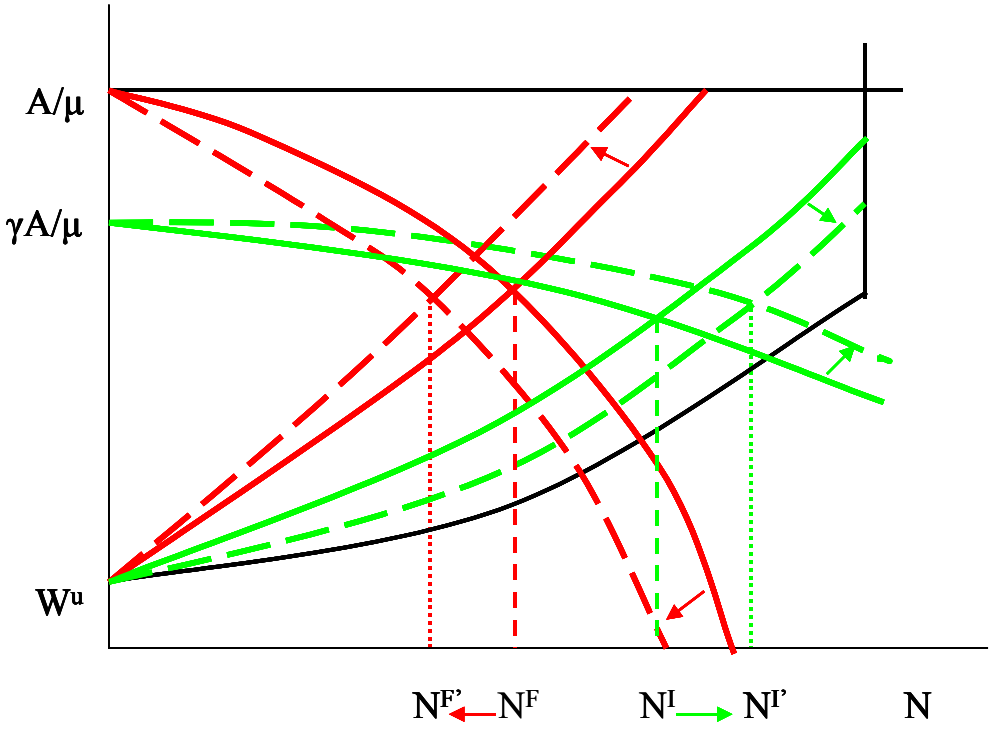
Labour market equilibrium in steady state



[8] Figures 3.1 and 3.2 are not drawn to scale and are presented only for didactic purposes.

Figure 3.2

Labour market equilibrium in steady state.
The effects of an increase in hiring costs of the formal sector



4. Quantitative analysis

4.1 Benchmark Parameters

We parameterise the model using values commonly used in the in the New-Keynesian literature:

Table 1

Standard Parameters of the Model

$\beta = 0.99$	$\varphi_\pi = 1.5$	$\varphi_y = 0.5$
$\theta = 2/3$	$\eta = 4$	$\varepsilon = 6$
$\chi = 1$		

We consider the reservation wage as a proportion of the value added of the informal sector in steady state, that is: $W^u = \psi\left(\frac{\gamma A}{\mu}\right)$ for $\psi = 0.75.$ For the technology parameters we take $A = 1$ and $\gamma = 0.95$. For the hiring costs functions we use the following: $\alpha_F = 1.5 > \alpha_I = 0.75$ and $B_F = 2.5 > B_I = 0.5$ to characterise the flexibility of the informal labour market in comparison with the formal one. The separation rate $\delta = 0.20$ is parameterised a bit higher than in Blanchard and Galí (2010) to produce less endogenous persistency in the dynamics of the model. The workers' bargaining power is set at $\lambda = 0.5$. The autoregressive coefficient

of both demand and supply shocks is set at 0.9 and their standard deviations are normalised at 1.

4.2 The steady state

Given this parameterisation, we show in Table 2 the implied steady state of the model for the case when no labour market rigidities are present ($B^F = B^I = 0$), the case with informal economy and the case when informality is not present ($\gamma = 0$).

Table 2

Implied steady state of the model

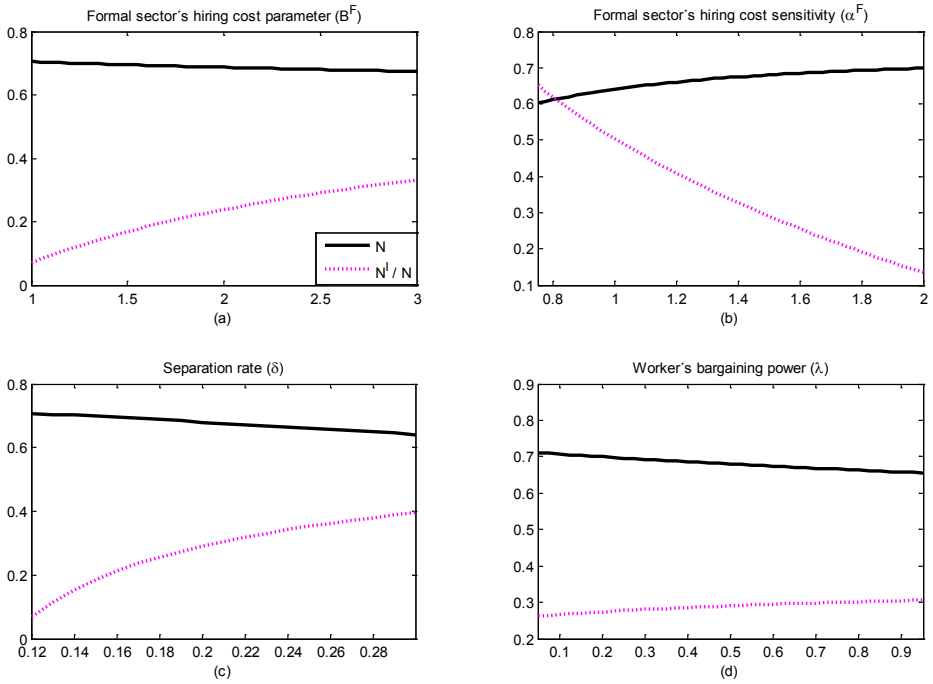
	Without labour market rigidities	With informality	Without Informality
Y	1	0.670	0.640
N	1	0.680	0.640
N^F	1	0.483	0.640
N^I	0	0.198	0.000

In the case where labour market frictions are absent, labour is at full employment and output is normalised at 1, labour is hired completely in the formal sector because of the lower productivity of the informal sector. When introducing hiring costs in both sectors, informal production arises. However, it is important to note that total production is higher in the economy with informality than in the case without it, because the informal sector becomes an optimal second best alternative to larger hiring costs in the formal sector. Moreover, total employment is higher in the economy with an informal sector.

In figure 4.1 we show some sensitivity of the steady state to some parameters associated to the labour market equilibrium. In panel (a) we show that an increase in the parameter B^F of the formal sector's hiring cost function, maintaining the other parameters constant, raises the proportion of informal labour in total unemployment and reduces marginally total employment. In contrast, the elasticity of the formal sector's hiring cost to the tightening ratio (α^F) in panel (b) has the opposite effect, since an increase on this parameter also reduces the value of the hiring cost in the formal sector. In panel (c) we show that an increase in the separation rate reduces employment and raises the participation of the informal sector in the labour market. As shown in equations (3.5) to (3.8), an increase in the separation rate reduces the expected saving on hiring costs next period, which in turn increases net hiring costs. The share of informal employment increases, because this sector is proportionally less affected by changes in this parameter. Finally, panel (d) shows that a larger worker's bargaining power also reduces total employment, because it pushes-up real wages, which also moves some workers from the formal to the informal sector. However, as shown, the sensitivity of the equilibrium to this parameter is very small.

Figure 4.1

Steady state – sensitivity analysis



4.3 The buffer effect of informal labour markets

The empirical evidence reported in the introduction shows that informal labour markets act as a buffer stock of labour, increasing the flexibility of the labour market and affecting the transmission mechanism of shocks to the economy. The micro-founded model developed in this paper delivers this result and shows how the presence of an informal economy affects the transmission mechanism of monetary policy.

As figure 4.2 shows, inflation response to a demand shock is more than double in an economy where all labour contracts are formal than in an economy where informal employment exists. Consistently, output increases more in this latter case, since informal employment helps to reduce the pressure on marginal costs in formal labour markets, generating a larger incentive for firms to increase production.

When a worker receives an offer to sign a formal labour contract, she has two options: either to accept the offer and receive the corresponding wage rate or wait for another one hoping to obtain a larger wage rate. When in the economy there are informal labour markets, the cost of waiting is larger since the probability of receiving a new offer of a formal labour contract is much lower in this case. This possibility of waiting for a longer period induces workers to accept more frequently job offers. Hence, firms in economies with informal labour markets have more flexibility to expand output, thus demand shocks generate lower inflation and larger output expansions. The impulse response functions depicted on the three panels at the bottom of figure 4.1 show this buffer effect in terms unemployment and employment flows. As these pictures shows, in the presence of an informal sector, formal employment increases less in response to demand shocks, but total employment increases by more because of the reaction of informal employment. This lower response in formal employment also imply less pressure on labour market tightening, since a fraction of labour demand is being provided by informal jobs. As result, hiring costs increase by less and, consequently, also marginal costs and inflation.

When productivity shocks hit the economy, informal labour markets amplify the effects of these shocks on inflation and output. As figure 4.2 shows, output increase and inflation falls by more when informal labour markets operate in responses to productivity shocks. Informal labour markets in this case also allow firms more flexibility when hiring workers, since they can hire informal workers. This flexibility lessens the impact of firm's hiring on the formal labour market tightening, allowing a larger expansion of output at a lower marginal cost. Inflation falls in response to an expected fall in future marginal costs. On impact marginal costs rises to reflect higher hiring costs but it became negative in the future in response to expected persistent gains in productivity. Although, at the margin the improvement in productivity is larger in formal labour contracts, firms still have incentives to hire workers under informal labour contracts since this type of contracts are relatively cheaper than formal ones. Similarly to the case of demand shocks, the buffer effect generates flows of employment from the formal to the informal sector in response to productivity shocks.

It is interesting to note as well that informal labour markets render formal employment less procyclical and informal employment more procyclical, an implication that is in line with the stylized facts reported by Bovi (2007).

The key implication for inflation dynamics that informal labour markets generate is that the Phillips curve depends, not only on the level of aggregate unemployment, but also on the flows of unemployment in the formal and informal labour markets. Furthermore, this result implies that in economies with large informal labour markets, the correlation between inflation and the output gap conditional on demand shocks is lower, thus changes in interest rates have relatively less impact on inflation.

Another interesting result is that unemployment becomes more counter-cyclical in the presence of informal labour markets, a result that has proven to be difficult to obtain in the New Keynesian model with flexible wages. As Galí (2010) shows, labour market frictions cannot deliver a large negative correlation between inflation and unemployment unless nominal wage rigidities are append to the analysis.

In the specification of the model we have assumed that some parameters associated to the labour market are the same for the formal and the informal sector, such as the bargaining power of workers and the job separation rate. Since other characteristics of the informal sector are the less power of workers to negotiate their wage and the lack of job security, which consequently imply a larger probability of being fired under a negative shock; it is natural to think that parameter λ is lower whilst δ is larger in the informal sector. In figures 4.4 and 4.5 we present some sensitivity analysis to the responses to demand and supply shocks, considering that a different parameterisation for λ and δ in the informal sector. More precisely, we consider $\lambda^I = 0.1$ and $\delta^I = 0.3$, keeping fixed $\lambda^F = 0.5$ and $\delta^F = 0.2$.

As we can see from these figures, those changes in the parameterisation of the model do not have an important quantitative effect on the dynamic of aggregate variables, such as inflation, output and total employment. However, there are some significant changes in the dynamics of the formal and informal employment. More precisely, the reduction in the bargaining power of informal workers and the increase in the separation rate in that sector, increase the responsiveness of informal labour to demand and productivity shocks, but this is compensated by a lower initial response in formal employment. Yet, these changes do not have an important impact on neither on total employment nor on real marginal costs and therefore the dynamics of inflation and output are not affected.

Figure 4.2

Impulse responses to a demand shock

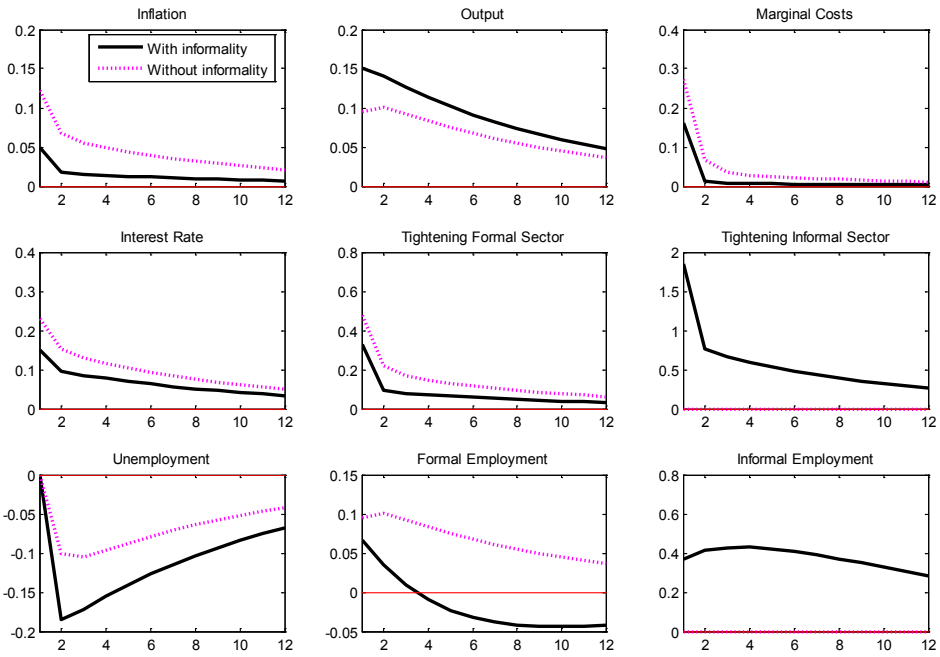


Figure 4.3

Impulse responses to a productivity shock

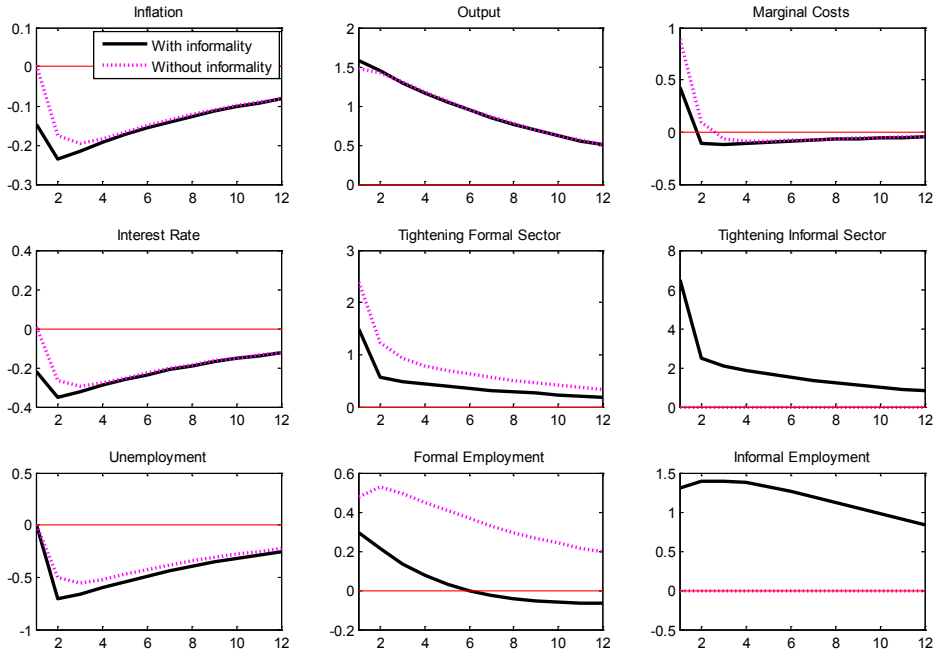


Figure 4.4

Impulse responses to a demand shock – sensitivity analysis

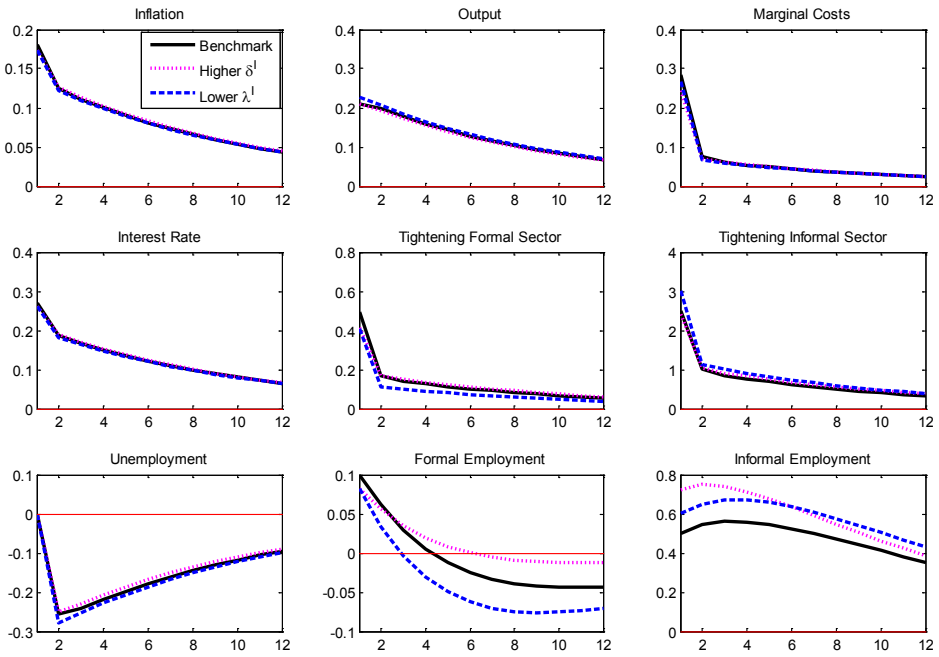
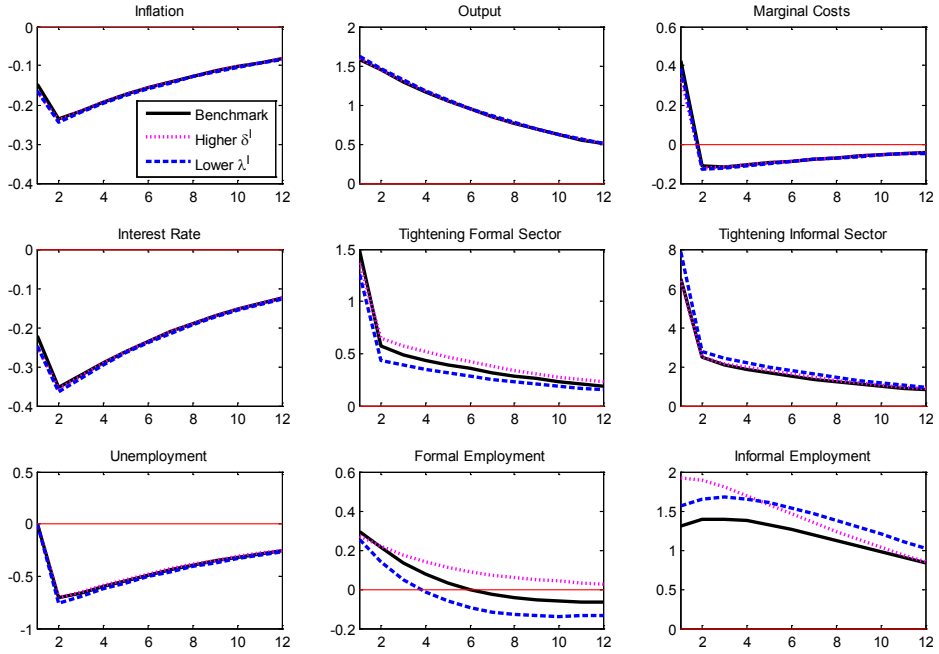


Figure 4.5

Impulse responses to a productivity shock – sensitivity analysis



5. Concluding remarks

Informal labour markets are widespread in emerging economies. This paper has shown that this feature of labour markets has a profound impact on the dynamics of inflation and the transmission mechanism of demand and productivity shocks. A large pool of informal workers is a buffer stock of labour that allows firms to expand output in a more flexible manner without putting pressure on wages. In particular, firms at the margin can substitute formal jobs with informal ones and expand output without raising their marginal costs. In this case, inflation depends not only on the level of unemployment but also on the flows of unemployment from formal to informal labour markets. Consequently, inflation also becomes less responsive to demand shocks.

Furthermore, the buffer stock effect on labour markets that this model generates is consistent with empirical evidence that formal employment is less procyclical than informal employment. This result has important implications for the costs of stabilisation policies. In particular, since inflation is less responsive to demand shocks, larger contractions in output would be required to stabilise inflation. Therefore, in economies of this type it becomes even more important to act pre-emptively to avoid deviations of inflation expectations.

The model presented in this paper is highly stylised, mainly to keep it tractable. However, it can be extended in many directions; for instance, alternative frictions to generate informal labour markets in equilibrium can be considered besides hiring costs to examine the interaction between monetary policy and labour market policies. Also, this framework can be used to analyse optimal monetary policy, following the work of Thomas (2008).

References

Alexopoulus, M (2004): "Unemployment and the business cycle", Journal of Monetary Economics, 51(2), 277-298.

Batini, N, Y-B Kim, P Levine and E Lotti (2010): "Informal labour and credit markets: A survey", IMF Working Papers 10/42.

Blanchard, O and J Galí (2010): "Labor markets and monetary policy: A new Keynesian model with unemployment", American Economic Journal: Macroeconomics, 2(2), 1-30.

Benigno, P and M Woodford (2005): "Inflation stabilization and welfare: The case of a distorted steady state", *Journal of the European Economic Association*, 3(6), 1185-1236.

Boeri, T and P Garibaldi (2006): "Shadow sorting", CEPR Discussion Paper Series 5487.

Bosch, M (2004): "Start up costs, informality and policy complentarities", mimeo LSE.

Bosch, M (2006): "Job creation and job destruction in the presence of informal labour markets", CEP-LSE Discussion Papers 761.

Bovi, M (2007): "Shadow employment and the labour productivity dynamics", LABOUR, 21(4-5), 735-761.

Bowler, M and T L Morisi (2006): "Understanding the employment measures from the CPS and CES survey", Monthly Labor Review, 129(2), 23-38.

Carrillo, M R and M Pugno (2004): "The underground economy and underdevelopment", Economic Systems, 28(3), 257-279.

Chéron, A and F Langot (2000): "The Phillips and Beveridge curves revisited", Economic Letters, 69(3), 371-376.

Christoffel, K and T Linzert (2005): "The role of real wage ridigities and labor market frictions for unemployment and inflation dynamics", ECB Working Paper Series 556.

Djankov, S, R La Porta, F Lopez-de-Silanes and A Shleifer (2002): “The regulation of entry”, Quarterly Journal of Economics, 117(1), 1-37.

Fugazza, M and J-F Jaques (2003): “Labour market institutions, taxations and the underground economy”, Journal of Public Economics, 88(1-2), 395-418.

Gertler, M and A Trigari (2009): “Unemployment fluctuations with staggered Nash wage bargaining”, Journal of Political Economy, 117(1), 38-86.

Galí, J (2010): “Monetary policy and unemployment”, in Friedman, B and M Woodford (eds), Handbook of Monetary Economics, 3, 487-546.

Kolm, A-S and B Larsen (2003): “Does tax evasion affect unemployment and educational choice?”, Copenhagen Business School Working paper 12-2003.

Krause, M U and T A Lubik (2007): “The (ir)relevance of real wage rigidity in the new Keynesian model with search frictions”, Journal of Monetary Economics, 54(3), 706-727.

Pissarides, C (2000): Equilibrium Unemployment Theory, MIT Press.

Loayza, N (1997):“The Economics of the informal sector: A simple model and some empirical evidence from Latin America”, World Bank Policy Research Working Paper 1727.

Ravenna, F and C E Walsh (2008): “Vacancies, unemployment and the Phillips curve”, European Economic Review, 52(8), 1494-1521.

Schneider, F (2004): “The size of the shadow economies of 145 countries all over the world: First results over the period 1999 to 2003”, IZA Discussion Paper 1431.

Thomas, C (2008): “Search and matching frictions and optimal monetary policy”, Journal of Monetary Economics, 55(5), 936-956.

Trigari, A (2009): “Equilibrium unemployment, job flows and inflation dynamics”, Journal of Money, Credit and Banking, 41(1), 1-33.

Trigari, A (2006): “The role of search frictions and bargaining of inflation dynamics”, IGIER Working Paper 304.

Walsh, C E (2003): “Labor market search and monetary shocks”, in Altuğ, S, J Chadha and C Nolan (eds), Elements of Dynamic Macroeconomic Analysis, Cambridge University Press, 451-486.

Walsh, C E (2005): “Labor market search, sticky prices, and interest rate policies”, Review of Economic Dynamics, 8(4), 829-849.

A. The non-linear system of equations

The dynamic equilibrium of this economy is given by the following set of 19 equations with 19 endogenous variables[9]:

A.1 Aggregate demand

$$1 = \beta E_t\left[\frac{C_t}{C_{t+1}}\frac{(1+i_t)}{(1+\Pi_{t+1})}\right] \tag{A.1}$$

$$(1+i_t) = (1+i)\left(\frac{\Pi_t}{\Pi}\right)^{\varphi_\pi}\left(Y_t^{gap}\right)^{\varphi_y} \tag{A.2}$$

A.2 Aggregate Supply

Price setting in the retail sector gives the Phillips curve:

$$\theta\left(\Pi_t\right)^{\varepsilon-1} = 1-(1-\theta)\left(\frac{NN_t}{DD_t}\right)^{1-\varepsilon} \tag{A.3}$$

$$DD_t = Y_t\left(C_t\right)^{-1} + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\varepsilon-1} DD_{t+1}\right] \tag{A.4}$$

$$NN_t = \mu Y_t\left(C_t\right)^{-1} MC_t + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\varepsilon} NN_{t+1}\right] \tag{A.5}$$

The production function, which determines marginal costs:

$$Y_t^W = Y^F + Y^I \tag{A.6}$$

$$Y_t^F = A_t N_t^F \tag{A.7}$$

$$Y_t^I = \gamma A_t N_t^I \tag{A.8}$$

A.3 Labour Market

Labour demands:

$$W_t^F = A_t MC_t - \left[G_t^F - (1-\delta)E_t Q_{t,t+1} G_{t+1}^F\right] \tag{A.9}$$

$$W_t^I = \gamma A_t MC_t - \left[G_t^I - (1-\delta)E_t Q_{t,t+1} G_{t+1}^I\right] \tag{A.10}$$

The wage setting curves are:

$$W_t^F = \chi C_t N_t^{\varphi} + W^U + \lambda\left\{G_t^F - (1-\delta)E_t Q_{t,t+1}\left[G_{t+1}^F\left(1-X_{t+1}^F\right) - G_{t+1}^I X_{t+1}^I\right]\right\} \tag{A.11}$$

$$W_t^I = \chi C_t N_t^{\varphi} + W^U + \lambda\left\{G_t^I - (1-\delta)E_t Q_{t,t+1}\left[G_{t+1}^I\left(1-X_{t+1}^I\right) - G_{t+1}^F X_{t+1}^F\right]\right\} \tag{A.12}$$

---

[9] A similar set $\theta = 0$ is given for the flexible price equilibrium. The output gap is measured by deviations of output from its flexible price equilibrium value.

where:

$$Q_{t,t+1} = \beta \frac{C_t}{C_{t+1}} \tag{A.13}$$

Hiring costs are given by:

$$G_t^F = B^F A_t \left(X_t^F \right)^{\alpha_F} \tag{A.14}$$

$$G_t^I = B^I A_t \left(X_t^F \right)^{\alpha_I} \tag{A.15}$$

Labour market tightness evolves as:

$$X_t^F = \frac{H_t^F}{1-(1-\delta) N_{t-1}} \tag{A.16}$$

$$X_t^I = \frac{H_t^I}{1-(1-\delta) N_{t-1}} \tag{A.17}$$

The evolution of labour in the formal and informal sector:

$$N_t^F = (1-\delta) N_{t-1}^F + H_t^F \tag{A.18}$$

$$N_t^I = (1-\delta) N_{t-1}^I + H_t^I \tag{A.19}$$

A.4 Aggregation

The aggregate resource constraint:

$$Y_t = C_t + G_t^F H_t^F + G_t^I H_t^I - W^u \left(1 - N_t\right) \tag{A.20}$$

Aggregate production for wholesale goods:

$$Y_t^W = Y_t \Delta_t \tag{A.21}$$

where:

$$\Delta_t = \int_0^1 \left[\frac{P_t(z)}{P_t} \right]^{-\varepsilon} dz \tag{A.22}$$

Also, we have total labour as:

$$N_t = N_t^I + N_t^F \tag{A.23}$$

B. Solving the steady-state

When solving the steady state, we have 13 equations for the same number of variables $\left\{Y, N, W^F, W^I, N^F, N^I, C, Y^F, Y^I, X^F, X^I, G^F, G^I\right\}$:

Aggregate conditions:

$$Y = Y^F + Y^I \tag{B.1}$$

$$N = N^F + N^I \tag{B.2}$$

Consider each labour demand:

$$W^F = A\frac{1}{\mu} - G^F\left[1-\beta\left(1-\delta\right)\right] \tag{B.3}$$

$$W^I = \gamma A\frac{1}{\mu} - G^I\left[1-\beta\left(1-\delta\right)\right] \tag{B.4}$$

Labour supply:

$$W^F = \chi C N^{\varphi} + W^U + \lambda\left\{G^F - \beta\left(1-\delta\right)\left[\left(1-X^F\right)G^F - X^I G^I\right]\right\} \tag{B.5}$$

$$W^I = \chi C N^{\varphi} + W^U + \lambda\left\{G^I - \beta\left(1-\delta\right)\left[\left(1-X^I\right)G^I - X^F G^F\right]\right\} \tag{B.6}$$

The aggregate budget constraint:

$$Y = C + \delta G^F N^F + \delta G^I N^I - W^u\left(1-N\right) \tag{B.7}$$

The production function:

$$Y^F = AN^F \tag{B.8}$$

$$Y^I = \gamma AN^I \tag{B.9}$$

Labour tightness:

$$X^F = \frac{\delta N^F}{1-\left(1-\delta\right)N} \tag{B.10}$$

$$X^I = \frac{\delta N^I}{1-\left(1-\delta\right)N} \tag{B.11}$$

Hiring costs:

$$G^F = B^I A\left(X\right)^{\alpha_F} \tag{B.12}$$

$$G^I = B^F A\left(X\right)^{\alpha_I} \tag{B.13}$$

We can replace the aggregate production function and labour equation (equations (B.1) and (B.2)), the aggregate budget constraint (equation (B.7)), the production function for each sector (equations (B.8) and (B.9)), the definition of labour tightness (equations (B.10) and (B.11)) and the hiring costs functions (equations (B.12) and (B.13)) in the labour demand and supply curve equations, to obtain a system of 4 equations for the real wage and labour in each sector

C. The log-linear equilibrium dynamics

We denote variables in their log deviations around the steady state with lower case letters [i.e., $x_t = \ln\left(X_t / X\right)$].The dynamics of the model are given by the set of equations for 19 endogenous variables $\left\{c_t, i_t, \pi_t, mc_t, y_t^F, y_t^I, w_t^F, w_t^I, n_t^F, n_t^I, q_t, g_t^F, g_t^I, x_t^F, x_t^I, h_t^F, h_t^I, y_t, n_t\right\}$ and 2 exogenous variables $\left\{d_t, a_t\right\}$.

The aggregate demand is determined from the aggregate resources constraint:

$$y_t = \frac{C}{Y}c_t + \frac{G^F H^F}{Y}\left(g_t^F + h_t^F\right) + \frac{G^I H^I}{Y}\left(g_t^I + h_t^I\right) + \frac{W^u N}{Y}n_t + d_t, \tag{C.1}$$

where we have included an exogenous demand shock, d_t,which follows an AR(1) process. This exogenous demand shock can be interpreted as a shock in government expenditures, when including the public sector into the model. In this model aggregate demand equals the sum of consumption, total hiring costs and demand shocks. Consumption is determined by the Euler equation:

$$c_t = E_t c_{t+1} - \left(i_t - E_t \pi_{t+1}\right), \tag{C.2}$$

and hiring costs are equal to $g_t^j = a_t + \alpha_j x_t$ for $j = \{F, I\}$ and the measure of workers hired is determined from the evolution of labour in each sector, $n_t^j = (1-\delta) n_{t-1}^j + \delta h_t^j$ for $j = \{F, I\}$. The labour market tightness is defined by: $x_t^j = h_t^j + \frac{(1-\delta)N}{1-(1-\delta)N} n_{t-1}$ for $j = \{F, I\}$.

Aggregate supply in this model with nominal rigidities and dual labour market rigidities is equal to traditional New-Keynesian Phillips curve:

$$\pi_t = \kappa mc_t + E_t \pi_{t+1}, \tag{C.3}$$

where $\kappa = (1-\theta)(1-\theta\beta)/\theta$. The informal economy affects inflation through the effects on marginal costs. Since the economy produces using two different types of technology, total production is $y_t = \frac{Y^F}{Y} y_t^F + \frac{Y^I}{Y} y_t^I$, where the production of each sector is given by: $y_t^j = a_t + n_t^j$ for $j = \{F, I\}$, where the technology shock (a_t) is also assumed to follow an AR(1) process.

Labour demand in each sector is equal to

$$w_t^j = \Phi^j \left(a_t + mc_t\right) + \frac{\tilde{G}^j}{W^j} \tilde{g}_t^j, \tag{C.4}$$

for $j = \{F, I\}$, $\tilde{g}_t^j = g_t^j - (1-\delta)\beta E_t \left(q_{t+1,t} + g_{t+1}^j\right)$ and $E_t q_{t,t+1} = c_t - E_t c_{t+1}$ is the stochastic discount factor. These relative weights in the labour demand of productivity and marginal costs are respectively $\Phi^F \equiv \frac{A}{W^F \mu}$ and $\Phi^I \equiv \gamma \frac{A}{W^I \mu}$.

On the other hand, the labour supply of each sector is given by the respective wage curve

$$\begin{aligned} w_t^j = {} & \Omega^j \left(c_t + \eta n_t\right) + \lambda \frac{\tilde{G}^j}{W^j} \tilde{g}_t^j + \\ & \frac{(1-\delta)\beta\lambda}{W^j} \left[G^F X^F \left(q_{t,t+1} + g_{t+1}^F + x_{t+1}^F\right) + G^I X^I \left(q_{t,t+1} + g_{t+1}^I + x_{t+1}^I\right) \right], \end{aligned} \tag{C.5}$$

for $j = \{F, I\}$. The weights are given respectively by $\Omega^j \equiv \chi C N^\eta / W^j$ Total labour equals: $n_t = \frac{N^I}{N} n_t^I + \frac{N^F}{N} n_t^F$.

Also, note that we can use the labour market equilibrium equations to solve for the real marginal costs, as in equation (2.38)

$$mc_t = \frac{\mu}{A} \left(\frac{1+\lambda}{1-\gamma} \right) \left(\tilde{G}^F \tilde{g}_t^F - \tilde{G}^I \tilde{g}_t^I \right) - a_t, \tag{C.6}$$

which, in turn, depends on productivity, unemployment and employment en each sector. Finally, monetary policy is determined under a standard Taylor rule:

$$i_t = \varphi_\pi \pi_t + \varphi_y y_t^{gap}. \tag{C.7}$$

Previous volumes in this series

No	Title	Author
371 February 2012	Stochastic Herding in Financial Markets Evidence from Institutional Investor Equity Portfolios	Makoto Nirei, Theodoros Stamatiou and Vladyslav Sushko
370 February 2012	Credit at times of stress: Latin American lessons from the global financial crisis	Carlos Montoro and Liliana Rojas-Suarez
369 January 2012	Stress-testing macro stress testing: does it live up to expectations?	Claudio Borio, Mathias Drehmann and Kostas Tsatsaronis
368 January 2012	The sustainability of pension schemes	Srichander Ramaswamy
367 December 2011	Is the long-term interest rate a policy victim, a policy variable or a policy lodestar?	Philip Turner
366 December 2011	Currency Momentum Strategies	Lukas Menkhoff, Lucio Sarno, Maik Schmeling and Andreas Schrimpf
365 December 2011	Was This Time Different?: Fiscal Policy in Commodity Republics	Luis Felipe Céspedes and Andrés Velasco
364 December 2011	Perceptions and misperceptions of fiscal inflation	Eric M Leeper and Todd B Walker
363 December 2011	The Liquidation of Government Debt	Carmen M Reinhart and M Belen Sbrancia
362 December 2011	The “Austerity Myth”: Gain Without Pain?	Roberto Perotti
361 December 2011	Long-term fiscal sustainability in major economies	Alan J Auerbach
360 November 2011	China’s evolving reserve requirements	Guonan Ma, Yan Xiandong and Liu Xi
359 November 2011	Bank heterogeneity and interest rate setting: What lessons have we learned since Lehman Brothers?	Leonardo Gambacorta and Paolo Emilio Mistrulli
358 November 2011	News driven business cycles and data on asset prices in estimated DSGE models	Stefan Avdjiev
357 November 2011	Rescue packages and bank lending	Michael Brei, Leonardo Gambacorta and Goetz von Peter
356 November 2011	The Impact of the International Financial Crisis on Asia and the Pacific: Highlighting Monetary Policy Challenges from a Negative Asset Price Bubble Perspective	Andrew Filardo

All volumes are available on our website www.bis.org.