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Stochastic Herding in Financial Markets
Evidence from Institutional Investor Equity Portfolios*

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Abstract

We estimate a structural model of herding behavior in which feedback arises due to mutual concerns of traders over the unobservable "true" level of market liquidity. In a herding regime, random shocks are exacerbated by endogenous feedback, producing a dampened power-law in the fluctuation of largest sales. The key to the fluctuation is that each trader responds not only to private information, but also to the aggregate behavior of others. Applying the model to the data on portfolios of institutional investors (fund managers), we find that the empirical distribution is consistent with model predictions. A stock's realized illiquidity propagates herding and raises the probability of observing a sell-off. The distribution function itself has desirable properties for evaluating "tail risk."

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1 Introduction

1.1 Motivation and Main Findings

Many apparent violations of the efficient market hypothesis, such as bubbles, crashes and "fat tails" in the distribution of financial market returns have been attributed to the tendency of investors to herd. Particularly, in a situation where traders have private information related to the pay-off of a financial asset their individual actions may trigger a cascade of similar actions by other traders. While the mechanism of a chain reaction through information revelation can explain a number of stylized facts in finance, such behavior remains difficult to identify empirically. This is partly because many theoretical underpinnings of herding, such as informational asymmetry, are unobservable and partly because the complex agent-based models of herding do not yield testable closed-form solutions. This paper attempts to bridge this gap.

We consider a stylized model of "synchronization risk" based on Abreu and Brunnermeier (2002, 2003).[1] A large number of traders simultaneously decide whether to remain invested in an asset or sell, knowing that selling an overvalued stock too early, before a critical mass of others sells, or too late, after a critical mass of others has sold, would lead to losses. The prospect of earning excess returns by riding the trend for an additional time period is weighed against the possibility that a large enough number of traders will dump the stock today, overwhelming market liquidity and forcing the price to drop, resulting in losses for those who remain. Each agent receives imperfect information about the market's ability to supply liquidity. The traders employ Bayesian learning and, in equilibrium, choose whether or not to continue holding the security based on their private information and the actions of others.[2] The equilibrium strategy exhibits complementarity, since each trader is more likely to sell when the aggregate number of sellers is higher. Herding in this

[1] In addition to funding risks, fund managers face what Abreu and Brunnermeier (2002, 2003) call "synchronization risk" – the risk of selling an overvalued stock too early, before a critical mass of other investors sells, or too late, after a critical mass of other investors has sold. Missing the timing of the price correction in either case would lead to losses and underperformance relative to other traders in the short-run. Such incentive to synchronize with other informed traders due to short time horizons and relative performance considerations (Shleifer and Vishny (1997)) can lead to herd behavior.

[2] The reliance on the actions of others for information rather than making decision based on prices alone implies that not all interactions between agents are mediated through the market and that these interactions are not anonymous, Cowan and Jonard (2003). For instance, Shiller and Pound (1989) find that word-of-mouth communications are important for the trading decisions of both individuals and institutional investors.

environment is stochastic because the information conveyed by the aggregate action may or may not dominate the private information available to each trader. The model predicts a non-trivial probability of "explosive" incidents of sell-offs due to herding.

Whereas the central limit theorem characterizes an outcome of a simple information aggregation process, choice correlation (e.g. herding) leads to fat tail effects. The equilibrium fraction of traders that herd on the same action is drawn from a probability distribution that exhibits exponential decay, which can be observed before the "explosive" sell-out takes place.[3]

Fitting the distribution implied by the model to the data on equity portfolios of institutional investment managers, we find that it is consistent with the empirical distribution of the number of institutional investment managers selling off their shares several quarters before the peak of the S&P 500 index in 2007 and that it outperforms a number of alternative distributions.[4] The parameter capturing the degree of herding behavior rises over time until the first quarter of major institutional sell-off of S&P 500 stocks. Consistent with model predictions, we find that exponential decay decreases in market illiquidity and in a stock's backward looking volatility measure. Both factors are important determinants of the degree of choice correlation among traders. Once the exponential truncation vanishes, an explosive synchronization occurs sooner or later. Then, through the information revealed by the actions of others, it becomes common knowledge among traders that the bubble has burst. Accordingly, all traders choose to sell. Since, at this stage, we only observe an aggregate of idiosyncratic variations in behavior, a normal distribution characterises the data due to the Central Limit Theorem. The analogous behavior is not found on the buy side in line with investors reacting differently to potential losses than to potential gains.[5]

[3]Morris and Shin (1999) also argue that choice interdependence among traders must be explicitly incorporated into estimates of "value at risk" and call for greater attention to game-theoretic issues since market outcomes depend on the actions of market participants. One such attempt is made by Nirei and Sushko (2011) who identify key features of foreign exchange speculation that make carry traders susceptible to stochastic herding. The ability of their approach to incorporate "rare" disasters as well as daily volatility in the same data generating process allows to use historical data to quantify the risk of currency crash even if such an event is not a part of the historical sample. However, unlike the present paper, their approach is less direct as they do not observe the *actions* of traders, but rather have to infer the impact of trades from prices.

[4]The data comes from 13F filings with the Securities and Exchange Commission (SEC) in which institutional investment managers report the number of shares under management for each individual security at quarterly frequency.

[5]We do, however, acknowledge that the mandate-based nature of the institutional asset management industry will constrain the buy and sell decisions taken by individual asset managers relative to what is assumed in our model (see, for example, CGFS (2003)), suggesting that any findings based on institutional equity holdings have to be interpreted

1.2 Stochastic Herding and Related Literature

Scharfstein and Stein (1990), Bikhchandani, Hirshleifer, and Welch (1992), Banerjee (1992), and Avery and Zemsky (1998) have formulated a theory of informational cascades, a type of herding that takes place when agents find it optimal to completely ignore their private information and follow the actions of others in a sequential move game.[6] Because players select their actions sequentially, the system will eventually, but unexpectedly, swing from one stable state to another. In contrast, in our framework herding is stochastic, with some foundation going back to probabilistic herding in the famous ant model of Kirman (1993).[7] Only a fraction of agents synchronize, the size of the fraction in turn depends on the realization of private signals. Stochastic herding emerges because strategic complementarity makes it optimal for some agents to place higher value on the informational content of the actions of others' relative to own private signals.[8] This setup differs from pure informational cascades similarly to Gul and Lundholm (1995) in that in our case, as in theirs, none of the information goes unused. As a result, strategic complementarity is not perfect and the transition between states for an agent is not deterministic even if some other agents herd, but rather happens with certain endogenous probability.

The probability distribution of herding agents is derived from the threshold rule governing their actions. This is similar to the threshold-based switching strategy in Global Games (see Morris and Shin (1998)). However, each agent's threshold is also affected by the observation of the actions of others because their actions aggregate private information and affect future payoff. In this sense,

with caution. For example, part of the uniform reduction in institutional asset holdings over the observation period may have been driven by the enactment of the Pension Protection Act of 2006 which, among other things, changed the accounting treatment of pension fund equity holdings, CGFS (2007).

[6] See Chari and Kehoe (2004) for the application of information cascades to financial markets.

[7] Alfarano, Lux, and Wagner (2005) and Alfarano and Lux (2007) extend the Kirman model in a different direction: they focus on the ability of the model with asymmetric transition probabilities of different types of traders to match higher moments in financial returns. In a related study Lux and Sornette (2002) illustrate the mechanics by which rational bubbles give rise to power-law tails in the distribution of returns. In contrast, we put greater emphasis on microfoundations employing the stochastic herding approach which focuses on the mapping of heterogeneous information onto the action space of rational agents.

[8] Related arbitrage literature includes Shleifer, DeLong, Summers, and Waldmann (1990) who show that rational traders will tend to ride the bubble because of risk aversion. Abreu and Brunnermeier (2003) model a continuous time coordination game in which the market finally crashes when a critical mass of arbitrageurs synchronizes their trades. In such a setting, it is futile for well-informed rational arbitrageurs to act on some piece of information unless a mass of other arbitrageurs will do so also. The coordination element coupled with information asymmetries create an incentive for fully rational investors to base their actions on the actions of others, i.e. herd.

the role of aggregate action is analogous to the role of endogenous public signals in the Tarashev (2007) extension of Global Games. Endogenously fluctuating thresholds can generate cascading behavior whereby agents continuously lower their threshold belief for liquidating an assets as they observe more and more liquidation around them. This leads to a non-trivial possibility of an "explosive" event in which the vast majority of traders liquidate simultaneously causing market liquidity to dry up. In this manner, we show that even if private signals about future market liquidity are normally distributed, the resulting aggregate action will follow a highly non-normal distribution implying stylized facts such as volatility clustering and fat tails in the distribution of financial returns.[9] Finally, agents are rational but myopic. This feature is particularly suitable for modelling trader behavior whose relative performance is often evaluated on a short-term basis.[10]

Empirical studies of herding have mostly focused on abnormal changes in institutional portfolio composition as evidence of herding (see Nofsinger and Sias (1999), Kim and Nofsinger (2005), and Jeon and Moffett (2010) for the ownership change portfolio approach).[11][12] Sias (2004) examines herding among institutional investors in NYSE and NASDAQ by using a more direct measure that looks at the correlation in the changes of an institution's holdings of a security with last period changes in holdings of other institutions. Our empirical approach is more closely related to Alfarano, Lux, and Wagner (2005) and Alfarano and Lux (2007). These authors focus on matching the empirical moments of asset returns with a model of switching trader sentiment. Similarly, we examine the goodness of fit of the empirical distribution of the number of selling agents to the theoretical distribution predicted by the threshold switching strategy with endogenous feedback.

We utilize two additional sources of variation in stock holdings not commonly found in data: the

[9]Our approach also bears some relationship to the studies of markets for information such as Veldkamp (2006) who identifies herding as an element of intrinsic instability because it makes markets respond disproportionally to seemingly trivial news.

[10]Our model is intended to explain fund manager choice of action at quarterly frequency so implicitly we assume that each manager optimizes with one quarter ahead horizon. Another class of investors whose behavior we do not model include individual investors and managers of funds with substantial restrictions on customer redemptions, access to a wider variety of investment instruments, and subject to less stringent regulations. These investors operate at a different performance horizon and have served as liquidity providers during such episodes as the 1987 stock market crash (Fung and Hsieh (2000)) to the more constrained institutional investors such as pension funds, endowment funds, and insurance companies that we focus on in this study.

[11]In related empirical studies McNichols and Trueman (1994) finds herding on earnings forecasts, Welch (2000) finds that security analysts herd, and Li and Yung (2004) finds evidence of institutional herding in the ADR market.

[12]Laboratory studies of herding in speculative attacks include Brunnermeier and Morgan (2004) and Cheung and Friedman (2009).

variation across individual investors and the variation across securities. This means that instead of observing one realization of the aggregate action during each period one can observe a sample of data points large enough to get an insight into the underlying data generating mechanism by looking at its distribution. Each observation in the sample is a group of institutional investors that fall within same class (e.g. banks, pension funds, etc.) holding the same stock. If investors are unsure about the accuracy of their private signal about future liquidity of a stock and are prone to follow the actions of others within the same stock-investor-type group, then, because of the complementarity of their market-timing strategies, the probability of observing large outliers is much higher compared to the case when investors act independently. Specifically, the distribution of their actions will exhibit a power law distribution with exponential decay.[13][14]

The paper is organized as follows. Section 2 presents the model of stochastic herding, derives the equilibrium distribution of herding agents, and conducts numerical simulations of the model. Section 3 discusses the data on institutional 13F filings and describes how the unit of observation is constructed. Section 4 examines the distribution of the actions by institutional investment managers from 2003:Q1 through 2008:Q1 covering both the run-up to and the collapse of the most recent U.S. equity bubble. In this section we compare the empirical distribution to the numerical simulations, evaluate the fit of the distribution implied by the model of stochastic herding against several alternatives and track the evolution of this behavior over time. Section 5 extends the theoretical and empirical exercise to account for the effect of market illiquidity and risk aversion on herding behavior. Section 6 concludes.

[13] Somewhat in line with traders' actions giving rise to a mixture of exponential and power-law distribution, Malevergne, Pisarenko, and Sornette (2005) find that the speed of probability decay of stock returns is bounded between stretched exponential and Pareto distributions.

[14] In a related work, Gabaix (2009) describes a number of data generating processes with feedback effects that have been known to produce power law distributions. However, we depart from their approach in several ways. Gabaix, Gopikrishnan, Plerou, and Stanley (2006) derive power-law scaling in trading activity from the power-law distribution in the size of the traders, while we obtain this result from the interactions of same-size traders. In other words, we obtain power-law scaling without imposing parametric assumptions on exogenous variables. Instead, it suffices that the signals about the true state are informative in the sense of satisfying the Monotone Likelihood Ratio Property (MLRP). For instance, as in this paper, the information and the true state can follow a bivariate normal distribution.

2 Model

2.1 Threshold Switching Strategy

In this section, we present a model of stochastic herding by informed traders. Our model setup is motivated by Abreu and Brunnermeier (2003) in which traders try to time their exit from a bubble market. In this setup, we apply an analytical tool shown by Nirei (2006, 2011) in order to obtain the distributional pattern of traders' herding. This distributional form then motivates our empirical investigation in the next section on the distributions of the herd size of traders before and during the sell-out period.

There are N informed traders indexed by $i = 1, 2, \ldots, N$, for conciseness we will refer to them simply as traders. Each trader is endowed with one unit of risky asset. The trader gains $(g-r)p$ by riding on bubbles and loses βp if the bubble bursts. Trader i can either sell ($a_i = 1$) or remain in the same position ($a_i = 0$). Each trader observes the aggregate number of selling traders $a \equiv \sum_{i=1}^{N} a_i$ and a private signal x_i. Let α denote the fraction of selling traders $\alpha = a/N$.

Market liquidity is denoted by θ.[15] The informed traders cannot observe θ, but only observe a noise-ridden proxy $x_i = \theta + \epsilon_i$. x_i is a private information and ϵ_i is independent across traders.

The bubble bursts if the selling pressure by the informed traders overwhelms the liquidity provided by the noise traders. The burst occurs if $\alpha > \theta$. Informed traders' expected utility of holding the asset is:

$$(g-r)p \Pr(\theta \geq \alpha \mid x_i, a, a_i = 0) - \beta p \Pr(\theta < \alpha \mid x_i, a, a_i = 0), \tag{1}$$

and the expected utility of selling is 0. Then the optimal strategy is to sell if:

$$\frac{g-r}{\beta} < \frac{\Pr(\theta < \alpha \mid x_i, a, a_i = 0)}{\Pr(\theta \geq \alpha \mid x_i, a, a_i = 0)}, \tag{2}$$

or, equivalently,

$$\frac{g-r}{\beta} < \frac{\Pr(x_i, a, a_i = 0, \theta < \alpha)}{\Pr(x_i, a, a_i = 0, \theta \geq \alpha)}, \tag{3}$$

and hold otherwise.

[15] θ represents the liquidity provided by noise traders – market participants that trade for reasons other than profiting from prices (e.g. liquidity reasons).

2.2 Equilibrium

We make a guess that all traders follow a threshold rule that trader i sells if $x_i \leq \bar{x}(\alpha)$ and holds otherwise. We will verify this guess later. We consider an equilibrium in which each trader does not have an incentive to deviate from the threshold rule at any observation (x_i, α), given that all the other traders obey the rule. When there are multiple equilibria for a realization of the private information (x_i), the outcome with the smallest a is selected. We denote the selected equilibrium by a^*. This equilibrium can be implemented by submission of supply schedule to a market maker. In this scheme, each trader submits their action of selling or holding conditional on α, and the market maker selects the smallest α such that it is equal to the aggregate supply conditional on α. The equilibrium can be interpreted as the outcome of a sequential trading where informed traders can sell immediately after observing the selling of other traders.

Define:

$$G(\bar{x}, a) = \Pr(x_j > \bar{x}(a) \mid \theta < a/N) \tag{4}$$

$$F(\bar{x}, a) = \Pr(x_j > \bar{x}(a) \mid \theta \geq a/N) \tag{5}$$

$$A(\bar{x}, a) = G(\bar{x}, a)/F(\bar{x}, a) \tag{6}$$

$$\delta(x_i, a) = \Pr(x_i, \theta < \alpha)/\Pr(x_i, \theta \geq \alpha) \tag{7}$$

$A(\bar{x}, a)$ represents the information revealed by a holding trader at the observed supply a. The information is expressed in the form of an odds ratio. $\delta(x_i, a)$ is the odds ratio obtained by the private information x_i.

Under the guessed threshold policy, the joint probability in (2) can be decomposed by the information revealed by the actions of traders. For example, when $a = 0$, the joint probability is written as:

$$\Pr(x_i, a = 0, a_i = 0, \theta < 0) = \Pr(x_i \mid \theta < 0) \Pr(x_j > \bar{x}(0) \mid \theta < 0)^{N-1} \Pr(\theta < 0) \tag{8}$$

Then, (3) is rewritten for $a = 0$ as:

$$\frac{g - r}{\beta} < A(\bar{x}(0), 0)^{N-1} \delta(x_i, 0) \tag{9}$$

Thus, $\bar{x}(0)$ is implicitly determined by:

$$\frac{g - r}{\beta} = A(\bar{x}(0), 0)^{N-1} \delta(\bar{x}(0), 0) \tag{10}$$

Now consider the case $a > 0$. If $a > 0$ were chosen to be an equilibrium, it reveals that no smaller supply $a' = 0, 1, \ldots, a-1$ is consistent with the supply schedule, since the market maker chooses the smallest α that is consistent with the supply schedule. Thus, the equilibrium reveals not only that there are a traders who sell conditional on a, but also that there are at least $a'+1$ traders who sell at a' for each $a' < a$.

Therefore, there are a traders with private information $x_i < \bar{x}(a)$, there are at least a traders with private information $x_i < \bar{x}(a-1)$, there are at least $a-1$ traders with $x_i < \bar{x}(a-2)$, and so forth up to that there is at least 1 trader with $x_i < \bar{x}(0)$. This set of conditions is equivalent to that there is one trader in each region $x_i < \bar{x}(a')$ for all $a' = 0, 1, \ldots, a-1$.

Consider the trader who would hold at $a'-1$ but sell at a'. Define the information revealed by such a trader at equilibrium a as follows:

$$B(\bar{x}(a'), a) = \frac{\Pr(x_j \le \bar{x}(a') \mid \theta < a/N)}{\Pr(x_j \le \bar{x}(a') \mid \theta \ge a/N)}. \tag{11}$$

Then, the selling condition (3) is rewritten for $a = 1$ as:

$$\frac{g-r}{\beta} < \delta(x_i, 1) A(\bar{x}(1), 1)^{N-2} B(\bar{x}(0), 1). \tag{12}$$

Then, $\bar{x}(1)$ is determined by $x_i = \bar{x}(1)$ that equates the both sides above. Generally, the threshold $\bar{x}$ is determined recursively by the equation:

$$\frac{g-r}{\beta} = \delta(\bar{x}(a), a) A(\bar{x}(a), a)^{N-1-a} \prod_{k=0}^{a-1} B(\bar{x}(k), a) \tag{13}$$

for $a = 0, 1, 2, \ldots, N-1$. We note that the posterior likelihood in (3) has three components: the private information x_i, the information revealed by holding actions of $N-1-a$ traders, and the information revealed by selling actions of a traders.

We assume that the prior belief on θ and the noise ϵ_i jointly follow a bivariate normal distribution with mean $(\theta_0, 0)$ and variance (σ_0^2, σ_e^2). Then (θ, x_i) also follows a bivariate normal distribution, since $x_i = \theta + \epsilon_i$. The normal distribution implies that:

$$\Pr(x_j > \bar{x} \mid \theta) < \Pr(x_j > \bar{x} \mid \theta'), \quad \text{for any } \theta < \theta'. \tag{14}$$

Thus,

$$A(\bar{x}, a) = \frac{\Pr(x_j > \bar{x} \mid \theta < a/N)}{\Pr(x_j > \bar{x} \mid \theta \ge a/N)} < 1 \tag{15}$$

for any a and $\bar{x}$. Likewise,

$$B(\bar{x}, a) = \frac{\Pr(x_j \le \bar{x} \mid \theta < a/N)}{\Pr(x_j \le \bar{x} \mid \theta \ge a/N)} > 1. \tag{16}$$

The threshold policy has the following property.

Proposition 1. *The increment of the threshold $\bar{x}(a+1) - \bar{x}(a)$ is positive and of order $1/N$ for a large N.*

Proof. See Appendix A. □

Next, we construct a fictitious tatonnement process that converges to the equilibrium a^* as a means to characterize the equilibrium. First, we define $-H'/H$ as the hazard rate for the traders who have remained holding the asset to sell upon observing a. Let θ_1 denote the realized value of the liquidity θ. Then $H(\bar{x}) = \int_{\bar{x}} e^{-\frac{(x_j-\theta_1)^2}{2\sigma_e^2}} dx_j / \sqrt{2\pi}\sigma_e$. We define $\mu(a)$ as the mean number of traders who do not sell upon observing $a-1$ but decide to sell upon observing a. Then:

$$\mu(a) = (H(\bar{x}(a-1)) - H(\bar{x}(a)))(N-a). \tag{17}$$

$\mu(a)$ is also expressed by the product of the increment in the threshold $\bar{x}(a+1) - \bar{x}(a)$, the hazard rate, and the number of traders who continue to hold the asset:

$$\mu(a) \sim \frac{H'}{H}(\bar{x}(a+1) - \bar{x}(a))(N-a) \to \frac{H'/H}{F_1/F} \frac{\log(B/A) + (\partial A/\partial \alpha)/A}{(G_1/F_1)/A - 1}, \tag{18}$$

where the limit is taken as $N \to \infty$.

Now, as a fictitious tatonnement process, we consider a best response dynamics $a_{u+1} = \Gamma(a_u)$ that starts from $a_0 = 0$, where a_{u+1} denotes the number of traders with private information $x_i < \bar{x}(a_u)$. We can show that the best response dynamics can be regarded as a tatonnement which converges to the selected equilibrium a^*.

Proposition 2. *For any realization of θ and (x_i), the best response dynamics a_u converges to the equilibrium selected by the market maker, a^*.*

Proof. Suppose that the best response dynamics did not stop at a^*. Then there exists a step s so that $a_s < a^* < a_{s+1}$. But, the definition of a^* prohibits that there is any $a' < a^*$ such that the number of traders with $x_i < \bar{x}(a')$ exceeds a^*. Hence, there is no such s. □

Unconditional on the realization of the private information, (a_u) can be regarded as a stochastic process. In the first step, a_1 follows a binomial distribution with population N and probability $\bar{x}(0)$. In the subsequent steps, the increment $a_{u+1} - a_u$ conditional on a_u follows a binomial distribution with population $N - a_u$ and probability $H(\bar{x}(a_{u-1})) - H(\bar{x}(a_u))$.

As $N \to \infty$, the binomial distribution asymptotically follows a Poisson distribution with mean $(N - a_u)(H(\bar{x}(a_{u-1})) - H(\bar{x}(a_u)))$. Now consider a special case where $\mu(a)$ defined in (17) is constant across a asymptotically as $N \to \infty$. If this holds, the asymptotic mean of the Poisson distribution above becomes $(a_u - a_{u-1})\mu$. A Poisson distribution with this mean is equivalent to $(a_u - a_{u-1})$-times convolution of a Poisson distribution with mean μ. Thus, in this particular case, the best response dynamics asymptotically follows a branching process with a Poisson distribution with mean μ, which is a population process that starts with the "founder" population with a_1 and each "parent" bears "children" whose number follow the Poisson with mean μ. The selected equilibrium a^* is the cumulated sum of the branching process. The following is known for the distribution function of the cumulated sum of a branching process.

Theorem 1. *Consider a branching process b_u, $u = 1, 2, \ldots$, in which the number of children born by a parent is a random variable with mean μ.*

1. *When $b_1 = 1$, the cumulated sum $Z = \sum_{u=1}^{\infty} b_u$ follows:*

$$\Pr(Z = z \mid b_1 = 1, z < \infty) \sim c^{-z} z^{-1.5} \tag{19}$$

for large z and for a constant $c \geq 1$ with the equality holding if and only if $\mu = 1$.

2. *The branching process converges to zero in a finite time u with probability one if and only if $\mu \leq 1$.*

3. *If $\mu > 1$, The cumulated sum Z is infinite with a non-zero probability.*

4. *If the number of children born by a parent follows a Poisson distribution with mean μ, then:*

$$\Pr(Z = z \mid b_1) = (b_1/z) e^{-\mu z} (\mu z)^{z - b_1} / (z - b_1)! \tag{20}$$

for $z = b_1, b_1 + 1, \ldots$.

5. *In addition to the previous assumption, if b_1 follows a Poisson distribution with mean μ_1, then:*

$$\Pr(Z = z) = \mu_1 e^{-(\mu z + \mu_1)} (\mu z + \mu_1)^{z-1} / z! \tag{21}$$

$$\sim (\mu e^{1-\mu})^z z^{-1.5} \tag{22}$$

The first item in this theorem implies that the number of traders in a herd follows a non-normal distribution function which exhibits a power-law decay with exponential truncation. Moreover, the distribution of the herd size exhibits a pure power law when $\mu = 1$, since then the exponential term

c disappears. The second item implies in our model that the best response dynamics converges with probability one if $\mu \leq 1$, and thus it verifies that the best response dynamics serves as a valid fictitious tatonnement in this case. The third item implies that there is a positive probability for an "explosive" event if $\mu > 1$. In our model, this event corresponds to the equilibrium in which all traders sell. The fourth and fifth items further characterize the herd size distribution, known as Borel-Tanner distribution in the queuing theory (Kingman (1993)). This particular distribution forms our preferred hypothesis in the empirical investigation of the herd size distribution in the next section.

2.3 Numerical Simulations

Before we move on to our empirical investigation, we numerically compute the model threshold $\bar{x}(a)$ and the equilibrium α^*. The purpose of this simulation is to show that the distribution of α^* (the equilibrium herd size) drawn from large number of simulations of the model follows the same distribution as the one obtained analytically. We set the parameter values as follows. The number of traders is $N = 160$, the return from riding the bubble is $g = 0.1$, the interest rate is $r = 0.04$, and the discount by the burst of the bubble is $\beta = 0.82$. The liquidity θ follows a normal distribution with mean 0.5 and standard deviation 0.3. The noise ϵ_i follows a normal distribution with zero mean and standard deviation 1.

Figure 1 plots the threshold function $\bar{x}(a)$. The plot is truncated at the point $a = 140$, since

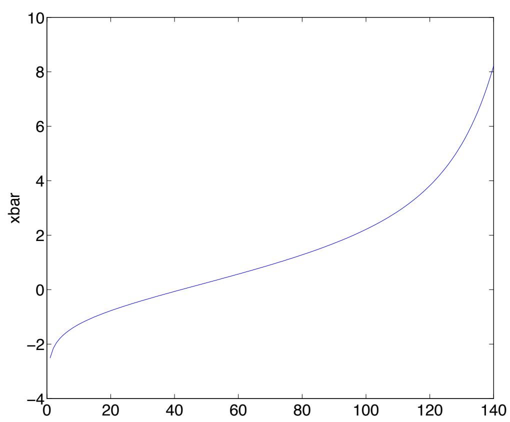

Figure 1: Threshold function $\bar{x}(a)$

for higher a we could not compute $\bar{x}$ because it is too large.

We then simulate the distribution of equilibrium a. We compute a for each draw of a random vector (ϵ_i), and iterate this for 100,000 times. We observe $a = 0$ for $72,908$ times, and observe $a = 140$ (the upper bound) for 1215 times. Figure 2 plots the histogram of the all $100,000$ observations. In Figure 2, it is clear that a is distributed similarly to an exponential distribution for $0 < a < 50$. There is no incident of $a > 50$ except for the 691 "explosive" incidents in which case basically all the traders decide to sell.

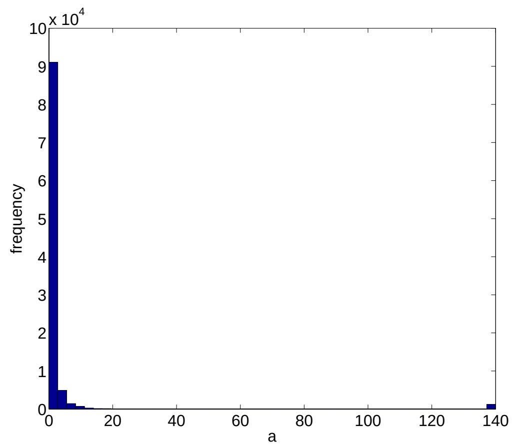

Figure 2: Histogram of a for $0 \leq a \leq 140$

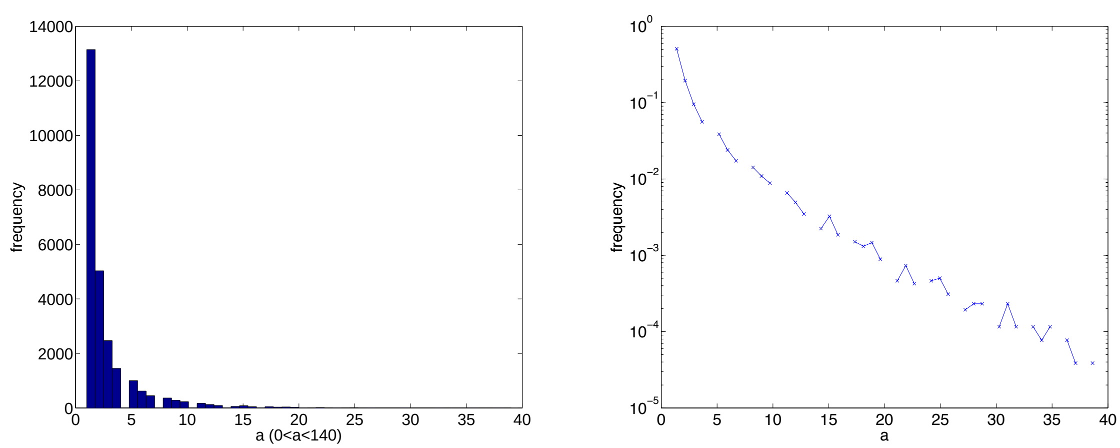

Figure 3: Histogram of a for $0 < a < 140$

Figure 3 shows the the blow-up plot of the histogram for $0 < a < 160$. The left panel plots the histogram on a linear scale and the right panel plots it on a semi-log scale. The tail distribution exhibits a straight line on semi-log scale, a characteristic of exponential decay. This is due to herding which is built into the model. It causes the frequency of the number of sellers in the tail to exceed predictions based on a random data generating process (such as normal dispersion of information or idiosyncratic shocks). Instead, the simulated distribution arises due to propagation effects in the underlying data generating process. The shape of the probability density function of the equilibrium distribution of a derived in the model (21) is illustrated in Figure 10 in Appendix B.

3 Data

3.1 Sources and the Unit of Observation

We use data on the institutional holdings of stocks included in the S&P500 index focusing on the time period around the latest run-up and the subsequent collapse of the U.S.stock market associated with the asset bubble of the 2000s. Institutional investors manage between 60 and 70 percent of outstanding U.S. stocks and are regarded as sophisticated investors whose rising importance in capital markets has been extensively documented by Gompers and Metrick (2001) among others.

Institutional investors increased their equity holdings markedly between 2003:Q1 and 2006:Q1 after which point the majority of them began reducing their stock portfolios to pre-2003 levels. In particular, managers of pension and endowments funds(who account for 48 percent of total market value of S&P 500 stocks or approximately 80 percent of total institutional holdings) began dumping S&P 500 stocks during 2006:Q2 and within four quarters virtually reverted their equity exposure to the pre-2003 level (Figure 4). This episode provides a unique opportunity to examine the role played by herding in the propagation of such massive adjustments. While some of this adjustment may have been driven by the enactment of the Pension Protection Act of 2006 (CGFS (2007)), herding behavior is especially suspect given the scale and the timing. It is noteworthy that this marked adjustment in institutional holdings preceded the subsequent stresses in the credit markets.[16]

We use data on institutional equity holdings from Spectrum database available through Thompson Financial.[17] The data is complied from quarterly 13F filings with SEC in which institutional investment managers with over $100 million under discretionary management are required to report

[16]See Brunnermeier (2009) for the timing of the 2007-2008 liquidity and credit crunch.

[17]Studies that utilize 13F data include Gompers and Metrick (2001), Brunnermeier and Nagel (2004), Sias (2004), Hardouvelis and Stamatiou (2011) and Campbell, Ramadorai, and Schwartz (2009)

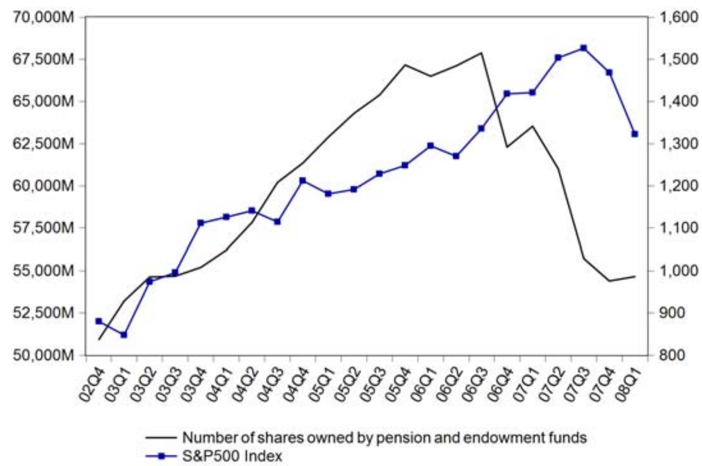

Figure 4: Number of shares (in millions) of S&P 500 stocks held by pension and endowment funds (the largest institutional investor category) and the S&P 500 index.

their long positions in exchange traded stocks, closed-end investment companies, equity options and warrants. In addition, we obtain daily data on stock prices, returns, and trading volume from the Center for Research in Security Prices (CRSP).

Table 1 shows the breakdown of institutional investment managers in our sample by type for each quarter from 2003:Q1 through 2008:Q1. Pension and endowment funds comprise the largest reporting category ranging between 71% and 80% of all institutional investment managers. Investment advisers comprise the second largest category followed by investment companies, insurance companies, and banks. In 2008:Q1 the dataset covers 2,119 pension and endowments funds, 521 investment advisers, 96 investment companies, 19 insurance companies, and 9 banks.

As Table 2 illustrates, institutional investors hold the majority of outstanding U.S. equities, as proxied by the S&P 500 stocks. The share of institutional holdings rose from 53% in 2003:Q1 to 67% in 2006:Q1 then declined steadily through 2008:Q1. Pension and endowment funds are the most dominant category accounting for more than four fifth of total institutional holdings of S&P 500 stocks.

The high degree of disaggregation in the Spectrum data allows to group institutional investment managers into stock-investor-type groups, $N(j,k)$, where j indicates an S&P500 stock and k indicates institutional investor type. For example, $N(\text{APPL, Banks and Trusts})$ is the number of banks and trusts that own Apple stock. Only groups with 10 managers or more are included in the sample. Table 3 shows the summary statistics. The number of quarterly observations for $N(j,k)$ ranges from 1,535 to 1,882. The size of the groups varies considerably, with quarterly mean ranging from 114 to 146, and quarterly maximum ranging from 1,046 and 1,222. Each quarter $a(j,k)$ out of $N(j,k)$ institutions in each group liquidate their holdings. Institutional managers dumping more than 80% of their holdings are counted into $a(j,k)$, but the results are generally robust to different cutoff levels.[18]

3.2 Summary Statistics

Table 4 shows quarterly summary statistics for $a(j,k)$. Note the stark difference between 2006:Q2 through 2007:Q1 and the surrounding quarters. During 2006:Q2 through 2007:Q1 the mean $a(j,k)$ is between 104 and 117 compared to 2 and 4 in other quarters and the maximum during this four quarter period ranges from 1057 to 1114 compared to 23 and 347 during other quarters. The corresponding *fraction* of institutions liquidating a stock, $a(j,k)/N(j,k)$, controls for any group size effect in the values of $a(j,k)$. Table 5 shows the summary statistics for $a(j,k)/N(j,k)$ confirming that during the period of 2006:Q2 through 2007:Q1 is associated with a large liquidation of stocks by institutional investment managers. The mean fraction of institutional managers liquidating a stock jumped to the 79% and 89% range from the earlier range of 3% to 4%. Moreover, during this four quarter period some stock-investor type groups experienced complete liquidation as seen from the maximum $\alpha(j,k)$'s of 100%. In sum, the summary statistics of $a(j,k)$ and $a(j,k)/N(j,k)$ in Tables 4 and 5 illustrate a regime change in institutional equity holdings during 2006:Q2 through 2007:Q1 when the vast majority were dumping their S&P 500 stocks. We refer to this period as the sell-out phase.

Focusing on the two quarters immediately preceding the sell-out phase, the summary statistics of $a(j,k)$ and $a(j,k)/N(j,k)$ show a rise in both mean and maximum values compared to previous quarters indicating a possible shift in institutional investment managers' behavior beginning to take place. The mean of $a(j,k)$ increased to 4 during 2005:Q4 and 2006:Q1 compared to 2 to 3 during all preceding quarters (Table 4) and the maximum $a(j,k)$ is 105, more than double the value

[18]The model of stochastic herding yields prediction regarding an "extreme" event, namely a complete liquidation of a position in a security. Realistically, large block holders, such as institutional investors, are restricted in their ability to unload a substantial number of shares at once, therefore we interpret the sale of 80% or greater share as an extreme event. The results are robust to different levels of cutoff, however, choosing the cutoff at 100% as stipulated by the model greatly reduces the number of observations while missing valuable information contained in extremely large sales approaching 100%.

during the four preceding quarters. The corresponding fraction, $\alpha(j,k)$, also rose during 2005:Q4 and 2006:Q1 compared to the preceding quarters (Table 5). This increase in the average and in the tail of the distribution of aggregate selling behavior may indicate greater degree of synchronization immediately before the regime change in 2006:Q2. In the remainder of the section we conduct distributional analysis motivated by the model of stochastic herding to examine whether the fat tail in the distribution of $a(j,k)$ during the run-up to the sell-out phase is a result of greater choice correlations and herding by institutional investment managers as opposed to being driven by uncorrelated events.

4 Analysis of Empirical Distribution

4.1 On Zipf's Law in the Distribution of Institutional Investor Holdings

Zipf's law, a Pareto distribution with exponent equal to 1 (Zipf (1949)), has been proposed as a possible explanation of the distribution of large trades in the stock market. Recently, Gabaix, Gopikrishnan, Plerou, and Stanley (2006) conclude from the CRSP data on mutual funds that their sizes follow a power law distribution with exponent equal to -1. They use this conclusion to derive power-law scaling in trading activity from the power-law distribution in the size of the traders. In contrast, we obtain a power-law result from trader interactions, irrespective of size distribution. In other words, we do not rely on the assumption of power-laws in exogenous variables.

In order to test if Zipf's law describes the behavior of institutional investors, we run "log rank-log size regressions" for the value of the change of each institutional investor's holdings in each stock as an independent variable (Table 6). For robustness, we also apply the Hill (1975) extreme value estimator.[19] In all our regressions, the size coefficient is negative and significantly different from zero (ranging from -0.372 to -0.445), but the Wald statistics reject the hypothesis that the

[19]The table presents results for the subsample of institutional investors that sold more than 80% of their stock holdings of a each stock in a particular quarter. We also tried three alternative specifications: a specification based on the total sample of the 13F investor's holdings and using the value of the total portfolio (rather than changes in shares under managements) of each institutional investor for both samples. The use of the full sample does not contradict the tail assumption of the power law distribution. By definition, 13F institutional investor holdings consist of the holdings of all managers with more than $100 million of assets under management (the cutoff for the power law distribution). While we correct for the bias in OLS esimator of Pareto exponent following Gabaix and Ibragimov (2011), we also acknowledge the pitfalls of the OLS and Hill estimators discussed in Clauset, Shalizi, and Newman (2009). Complete results are available upon request.

coefficient is equal to -1. Therefore our results do not support the hypothesis that the trades of the 13F institutional investor holdings are driven by Zipf's law (i.e. that the size coefficient in the "log rank-log size regressions" or the Hill's estimator are equal with -1).

4.2 Model Fit

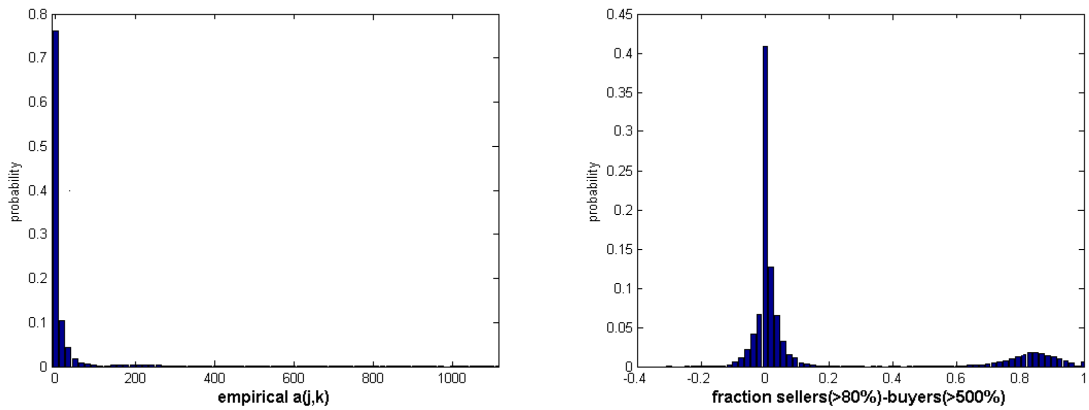

Figure 5: 2003:Q1 - 2008:Q1 (38,353 observations); *Left* histogram of empirical $a(j,k)$. *Right* histogram of $(a(j,k) - b(j,k))/N(j,k)$.

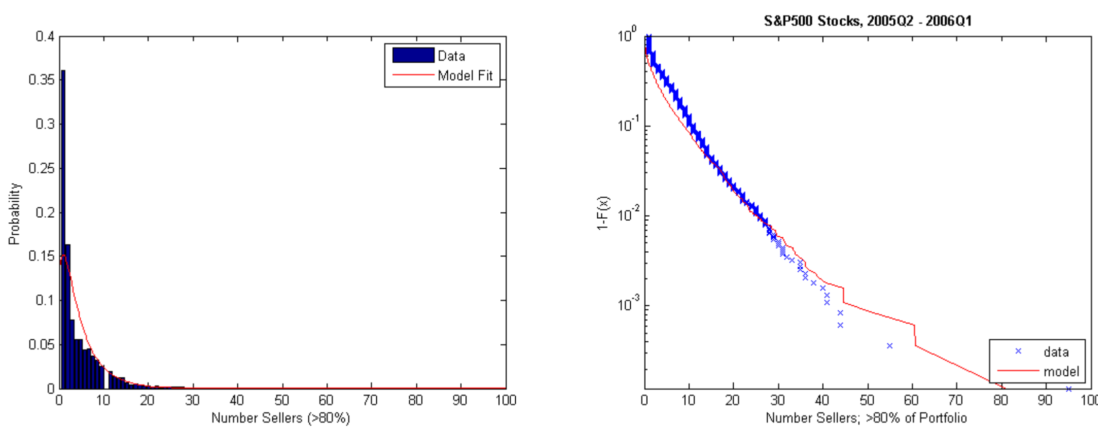

Figure 6: 2005:Q2 - 2006:Q1: *Left* histogram of empirical $a(j,k)$. *Right* semi-log probability plot of empirical $a(j,k)$ and the fitted model (red).

The left panel of Figure 5 shows the histogram of empirical $a(j,k)$ for the entire sample period (2003:Q1 through 2008:Q1). The histogram bears close similarity to the numerical simulations of the model in Figure 2. Like the distribution of simulated a, the empirical distribution of $a(j,k)$ exhibits a fat tail along with exponential decay in the high probability mass region. The mean number of institutional fund managers dumping a particular stock is 23, while the standard deviation is 79 and the maximum is 1114.

To control for rare events on the "buy side," we also examine a symmetric indicator to $a(j,k)$ for fund managers who increase their holdings of an S&P 500 stock by more than 5 times (inverse of 0.80) during a given quarter, $b(j,k)$. For each stock-investor-type group we then construct the net measure as $a(j,k) - b(j,k)$ and normalize it by group size $N(j,k)$. The right panel of Figure 5 shows the histogram of the corresponding fraction. The bimodality of the distribution indicates the presence of "explosive" sellout events, with virtually no observations in the intermediate range. Moreover, such extreme switching from low activity to high activity level is only present on the sell side, indicating that coordination on the same action characterizes sellouts but not purchases by institutional fund managers.

Independent rare events, such as portfolio liquidations due to idiosyncratic shocks, should be well approximated by a Poisson distribution. Recall that in Equation (22), μ_1 represents the Poisson mean of the number of agents taking extreme action at the beginning of the tatonnement process independently (responding only to their private signal), while μ represents the total number of agents induced to follow the actions of the first-mover until the system settles at a new equilibrium. In other words, μ quantifies the degree of herding. If $\mu = 0$ then Equation 22 reduces to a probability density function of a Poisson distribution with arrival rate μ_1, indicating the absence of herding (portfolio liquidations are independent of each other). On the other hand, $\mu = 1$ is a critical point with $\mu > 1$ corresponding to the phase where the explosive aggregate actions happen with a positive probability. In the intermediate range, the probability distribution of $a(j,k)$ will exhibit exponential decay, with the speed of the decay dictated by μ. We can also think of μ as a measure of length of the tail of the distribution – larger μ implies that an initial outlier (itself governed by Poisson arrival rate μ_1) attracts greater probability mass to itself, effectively stretching the tail.

The common benchmark distribution for rare independent events is Poisson. Table 7 shows the results of Kolmogorov-Smirnov goodness of fit test for Poisson distribution to $a(j,k)$. Poisson distribution is rejected at the 5% significance level with p-value=0 and the test statistic of 0.769 (three orders of magnitude larger than the critical value of 0.008). Apart from correlated arrivals, Poisson may also be rejected because the distribution of a discrete random variable with Poisson arrival rate asymptotes to normal in the limit. However, as Table 8 shows, the moments of $a(j,k)$

point at a highly non-normal distribution (consistent with the histograms in Figure 5). If the correlated arrival results from stochastic herding then Equation (22) should adequately characterize the probability distribution of emprical $a(j,k)$. Table 9 shows the associated maximum likelihood estimates (MLE) of the distribution parameters. The estimates for μ_1 and μ are 2.058 and 0.938 and are statistically significant at 1% level, indicating that stochastic herding is a plausible candidate for the underlying data generating mechanism of empirical $a(j,k)$.

Figure 6 focuses on the four quarter period before the sell-out phase (2005:Q2 through 2006:Q1). The left panel of Figure 6 shows the histogram of empirical $a(j,k)$ with distribution exhibiting exponential tail similar to the simulation in Figure 3. The largest value in the histogram corresponds to 95. The right panel of Figure 6 shows the corresponding semi-log probability plot. The straight line formed by the observations of $a(j,k)$ on the semi-log scale indicates an exponential distribution with persistent outliers, indicative of correlated arrivals in the underlying data generating process. The solid line shows the fit corresponding to the stochastic herding outcome (of Equation 22) to the empirical distribution of $a(j,k)$. The line was formed by sampling the data from the proportional theoretical probability density (Equation 22) with parameters first estimating using empirical $a(j,k)$ via MLE and the proportionality constant set equal to the theoretical prediction for the power exponent of 1.5.

4.3 The Sell-Out Phase in 2006:Q2-2007:Q1

Figure 7 plots $a(j,k)/N(j,k)$ against the cumulative distribution (log rank over the number of observations). The left panel corresponds to the 2005:Q2 through 2006:Q1 period, the four quarters preceding major institutional sales. The inverse of the slope of the semi-log plot provides an estimate of the mean parameter of an exponential distribution. A least squares regression for $a(j,k)/N(j,k)$ yields an estimate of the slope of -31.443 (standard error 0.055) with an R-squared 0.988. This examination of the semi-log plots favors a model that generates exponential rather than normal decay in $a(j,k)/N(j,k)$ during the final phase in the run-up to the shift in institutional behavior in 2006:Q2.

The probability plot in the left panel also shows a convex deviation from the exponential tail as the size of observations approaches zero. This is consistent with a Gamma-type distribution, such as the Borel-Tanner distribution (Equation (20)), which exhibits an exponential tail with a power decline near zero. Moreover, the small number of observations that lie very far from the probability mass are indicatibe of a Gamma-type distribution with a low value of the shape parameter. This

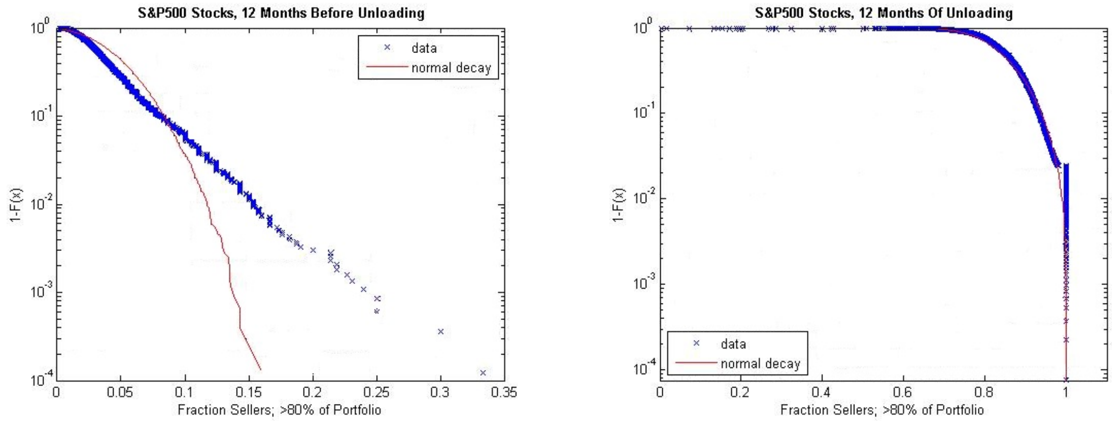

Figure 7: Semi-Log Probability Plots of $a(j,k)/N(j,k)$

is because, all observations drawn from a Gamma distribution will have the same expectation of the order $1/N$, but there is high probability that at least one observation will be several standard deviations greater than the average (Kingman (1993)).

The intuition behind semi-log plots is as follows. Suppose the average perception of the value of fundamentals is strong and the mean fraction of institutional investment managers liquidating a particular stock is small. In the absence of selling cascades within some stock-investor-type groups the probability of observing a given value of $a(j,k)/N(j,k)$ would be declining at an increasing rate as we move further away from the mean. This Gaussian decay would produce a concave line in the semi-log plot. On the other hand, suppose investors are attempting to time the market by basing their actions on the actions of others. For example, within stock investor-type group a fund manager having observed a small fraction of other fund managers liquidating their holdings in a particular stock interprets this as the beginning of a "correction" and is induced to sell herself. If the conditions are so fragile that even in the absence of major changes in the fundamentals a number of investors are inclined to act as this hypothetical fund manager, then we would observe selling cascades within some stock-investor-type groups, creating outliers. Hence, if investment managers are locked in a herding regime then, even though the mean of the aggregate liquidation may still be low, the probability of observing large deviations from the mean will be higher than predicted by Gaussian decay that characterizes random deviations.

The right panel of Figure 7 shows the semi-log probability plots of $a(j,k)/N(j,k))$ for 2006:Q2 through 2007:Q1. Consistent with transition from subcritical ($\mu < 1$) to supercritical phase ($\mu > 1$), this four quarter period is characterized by a state of "explosive" sell-outs. When the system is supercritical, there is a positive probability in which all the traders sell (explosion). Thus our model predicts a probability mass for fraction $a(j,k)/N(j,k) = 1$. If we allow for other exogenous randomness not considered in our model, then it is natural to think that the actual fraction is normally distributed around the mean close to 1.

The probability mass of $a(j,k)/N(j,k)$ is concentrated in the region between 0.8 and 1.0, indicating that the vast majority of institutional investors were dumping most of their S&P 500 stock. The relatively close fit of the normal distribution indicates that aggregate high mean value of $a(j,k)/N(j,k))$ is an informative summary statistic for the sell-out regime in the sense that the deviations from this high mean are random and the vast majority of institutions were liquidating their S&P 500 stocks during this period.

In sum, Figure 7 conveys two things. First, the sell-out ensued as early as 2006:Q2 and continued for approximately 4 quarters. Second, institutional investors in the stock market operated according to two different regimes during the duration of the bubble. During the run-up phase, the distribution of the aggregate action exhibits exponential decay, consistent with stochastic herding when the uncertainty over market timing actions of other institutional investment managers dominates. The exponential decay then vanishes during the sell-out phase. Such regime switching is consistent with transition from subcritical ($\mu < 1$) to supercritical phase ($\mu > 1$) with positive probability that all institutions act in unison (see *Theorem 1*).

Our hypothesis is that the process that generated empirical $a(j,k)$ shown in Figure 6 is best described by the probability density in Equation (22). We fit the model implied distribution against three alternatives: a truncated normal, Gamma, and Exponential. Table 10 shows the results.

The log likelihood values are higher for the model than any of the alternative distributions while truncated normal, which tests the possibility of Gaussian decay, has the smallest log likelihood value. In addition we conduct a non-nested goodness of fit test using Vuong's statistic.[20] The Vuong statistics for the model (H_1) against H_0 that data follows either Gaussian, Gamma, or Exponential distributions are 30.393, 21.785, and 28.140 respectively rejecting H_0 in favor of the model.

Recall that μ_1 is the Poisson mean of the number of traders induced to sell when there are

[20]It is based on Kullback-Leibler information criterion which tests if the hypothesized models are equally close to the true model against the alternative that one is closer. Defining $l_i = logL(i;H_1) - logL(i;H_0)$ as the log likelihood ratio for each observation i, Vuong's statistic, $V \equiv \sqrt{N}(l_i)/(Std(l_i))$, follows a standard normal distribution if the hypothesis H_0 and H_1 are equivalent (Vuong (1989)).

there are N traders. When there are $N - a$ traders, then the mean number induced traders is $\mu(N - a)/N$, which asymptotes to μ. Hence, μ quantifies that degree of herding which leads to a stretched tail in the distribution of $a(j, k)$. $\mu_1 = 2.068$ indicates approximately 2 managers within each group would have sold the stock even if no one else was selling. $\mu = 0.570$ indicates that on average during the 2005:Q2 through 2006:Q1 time period another fund manager would have chosen to follow the actions of these initial "random" sellers with a probability of $0.57/N$.

4.4 Exponential Decay and the Rise of μ Over Time

Figure 11 through Figure 13 show quarterly semi-log probability plots of empirical $a(j, k)$ against the data simulated from the model and the two benchmark alternatives, Poisson and normal distributions. The data was simulated with distribution parameters first obtained via MLE using empirical $a(j, k)$.[21] A concave line corresponds to an accelerating probability decay in the tail characteristic of a Gaussian distribution while a straight line indicates decelerating exponential decay. The model of stochastic herding predicts that due to choice correlations the distribution of the number of institutional investment managers liquidating a particular stock will exhibit exponential decay because of the persistence of outliers due to choice correlation.

During the early quarters (Figure 11), Poisson captures the probability decay close to the mean however misses the exponential decay in the tail. A normal distribution approximates the probability decline fairly well in 2003:Q2 when the probability mass in the empirical data follows a concave curve characteristic of the Gaussian decay. The fit of the model improves in 2003:Q4 and 2004:Q1, these are two quarters when mean and maximum of $a(j, k)$ temporarily increased (see Table 4). However, in both cases the empirical distribution exhibits bimodality and in both cases higher mean appears to have been driven by one outlier. It is nonetheless noteworthy that the tail of the distribution exhibits a rightward shift, as if pulled by the outliers but never lining up perfectly behind them.

The fit of the model improves substantially during 2006:Q1 (Figure 12), one quarter before the onset of the sell-off phase. The distribution of empirical $a(j, k)$ exhibits exponential decay, moreover the data points tend to from a more continuous line indicating higher instances of sell outs at intermediate values.

The following four quarters (2006:Q2 through 2007:Q1) the probability mass of $a(j, k)$ is concentrated around values an order of magnitude higher than in the previous period, indicating massive

[21] Note that for 2003:Q4, 2004:Q1, 2004:Q4 we show a second plot with estimation dropping one outlier.

institutional dumping of stocks. Moreover, a more dense empirical plot indicates much greater incidence of $a(j,k)$ across all stock-investor type groups. However, during this period the distribution of $a(j,k)$ also exhibits bimodality, likely driven by heterogeneity in group sizes. This is because, as indicated in the discussion of Figure 7 in previous section, when controlling for group size via $a(j,k)/N(j,k)$, the bimodality disappears in favor of Gaussian decay around the mean close to 1.

After the sellout period the herding signature virtually vanishes – the empirical distribution of $a(j,k)$ is similar to the earlier periods of 2003 and 2004, with bimodal features (in 2007:Q2 and 2007:Q4 in particular) and the decay in the probability mass region approximated fairly well by a normal distribution.

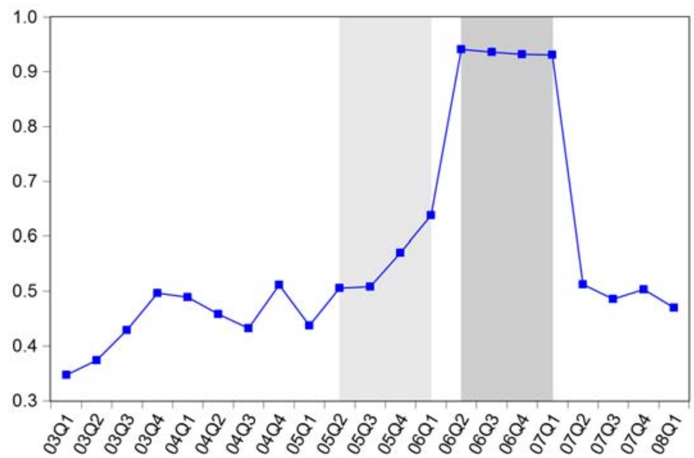

Figure 8: Herding – quarterly estimates of distribution parameter μ, where μ/N measures the probability of a "chain reaction" in response to a random liquidation by an investment manager. Initial independent liquidations occur with Poisson arrival rate of μ_1. The probability density of the aggregate action is then given by $\Pr(Z=z) = \mu_1 e^{-(\mu z+\mu_1)}(\mu z+\mu_1)^{z-1}/z!$

Table 11 supplements graphical simulation analysis with quarterly MLE parameter estimates for the model. The last column shows the results of a non-nested goodness of fit test based on Vuong's statistic. If $V > 1.96$ then H_0 of normal distribution is rejected in favor of H_1 of the model under 5% significance level. The goodness of fit test confirms the inference based on the semilog probability plots and shows that the empirical distribution rejects normal decay in favor of the model during all quarters except for 2006:Q2 through 2007:Q1. During the quarters when the model captures the empirical distribution the Poisson mean μ_1 is approximately 2 indicating that on average two investors in each stock-investor-type group, $N(j,k)$, chose to liquidate at random in the beginning of the tatonnement process. On the other hand, the estimates for μ, the degree of endogenous feedback, are rising from 0.347 in 2003:Q1 to 0.638 in 2006:Q1 indicating intensifying degree of herding up until sell-out phase.

The trend increase in μ, which accelerated during the last year before the sell-out phase, is shown in Figure 8. The rise of μ over time as the run-up on S&P 500 stocks continued is consistent with weakening fundamental anchors and a rising importance of market-timing considerations that make the system susceptible to herding. During the sell-out period the empirical data favors an alternate distribution, as seen by large negative Vuong's statistics. Note however that during the 2006:Q2 through 2007:Q1 period the estimates for μ range between 0.931 and 0.941 indicating that, although misspecified, the likelihood of a power-law with exponential truncation is maximized for μ close to 1, where $\mu = 1$ corresponds to the criticality at which exponential truncation vanished in favor of pure power law (consistent with semilog plots for this four quarter period shown in Figure 13). Finally, after the sellouts have subsided, exponential decay emerges once again but the estimates of μ remain below the 2006:Q1 level.

Overall, the rise of μ over time indicates that institutional investment manager actions increasingly exhibited contagious behavior intensifying the branching process until the sell-out phase. During the four quarters in 2003 the estimates of μ rise moderately after which point μ is approximately stationary until 2005:Q3, when μ begins to rise again until a sudden jump to the neighborhood of 1. This suggests that the population dynamics of fund manager behavior that we viewed as a the branching process with intensity μ transitioned from subcritical phase of $\mu < 1$ to a critical phase of $\mu = 1$ between 2006:Q1 and 2006:Q2. If in fact institutional fund managers learn about a stock's illiquidity, $\Pr(\theta < a/N \mid \cdot)/\Pr(\theta \geq a/N \mid \cdot)$, by accumulating private information and observing aggregate action, then over time Bayesian learning ensures that beliefs about θ converge and the triggering action eventually occurs with probability 1.[22] This is because as private information, which is jointly normally distributed with the true θ hence informative, accumulates over time the average belief decreases causing some managers to liquidate even if no one else is liquidating. Their actions affect the threshold of others triggering a chain of liquidations. If sufficient amount of private information has been accumulated over time such that the average belief is low enough, then the chain reaction becomes "explosive" in the sense of self-organized criticality put forth by Bak, Tang, and Wiesenfeld (1988). In Bak's sandpile model the distribution of the

[22] See Nirei (2011) for a more general dynamic extension to information aggregation problem in financial markets.

avalanche size depends on the slope of the sandpile. Our analog of the slope of the sandpile is the inverse of the average belief.

5 Model Extentions

5.1 Illiquidity

The key feature of the model is that a stock's illiquidity, $1/\theta$, propagates herding. Specifically, the model predicts a negative (positive) relationship between the intensity of the branching process, μ, and the realized liquidity (illiquidity) of a security.

We defined:

$$H(\bar{x}) = \int_{\bar{x}} e^{-\frac{(x_j-\theta_1)^2}{2\sigma_e^2}} dx_j/\sqrt{2\pi}\sigma_e. \tag{23}$$

Then, we obtain:

$$H'(\bar{x}) = -e^{-\frac{(\bar{x}-\theta_1)^2}{2\sigma_e^2}}/\sqrt{2\pi}\sigma_e < 0, \tag{24}$$

$$\frac{d}{d\theta_1}H'(\bar{x}) = H'(\bar{x})(\bar{x}-\theta_1)/\sigma_e^2. \tag{25}$$

Thus, $H'(\bar{x})$ is strictly increasing for any $\bar{x} < \theta_1$. From Equation (17), we have:

$$\mu(a) = (H(\bar{x}(a-1)) - H(\bar{x}(a)))(N-a). \tag{26}$$

Therefore, we obtain $d\mu(a)/d\theta_1 < 0$ for any a such that $\bar{x}(a) < \theta_1$.

This implies that, when the realized amount of liquidity θ_1 is decreased, μ increases for a range of a near 0. Thus, while the realized liquidity does not affect the strategy, it affects the outcome. Specifically, when the realized liquidity is lower, a large propagation is more likely to occur.

We confirm this point by numerical computations and simulations. The left panel of Figure 9 plots the numerically computed μ for various a using Equation (17). We computed for two cases in which the realized liquidity, θ_1, is equal to 0.8 and 0.2. (θ follows a normal distribution with mean 0.5 and standard deviation 0.3.) We observe that $\mu(a)$ is greater when the liquidity is low ($\theta_1 = 0.2$) for any a. The right panel of Figure 9 plots the histogram of simulated equilibrium outcome a for two cases, $\theta_1 = 0.8$ and 0.2. We observe that the two histograms start similarly near $a = 0$, but the histogram with the lower liquidity is exponentially truncated at the greater a and has the longer tail than the one with the higher liquidity.

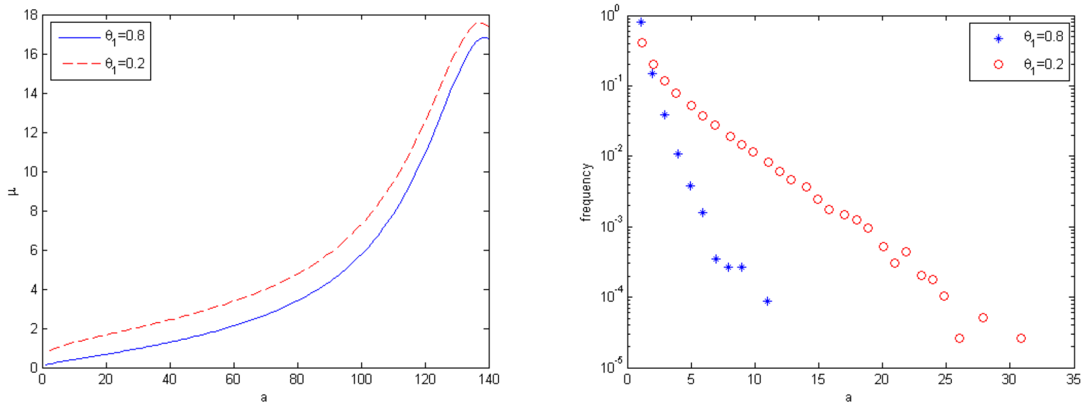

Figure 9: *Left:* μ as a function of a in Equation 17 for different realizations of liquidity θ. *Right:* simulated histograms of a for high and low realizations of θ

5.2 Risk Aversion

Risk aversion affects the herding intensity through a different channel. To see this, the expression for the Poisson mean of the number of traders induced to sell by trader a (Equation 26) can be rewritten as:

$$\mu(a) = \frac{H'(\bar{x})}{H(\bar{x})}\frac{d\bar{x}(a)}{da}(N-a). \tag{27}$$

While market illiquidity affects $\mu(a)$ through the hazard rate $H'(\bar{x})/H(\bar{x})$ (shown in the previous subsection), risk aversion affects $\mu(a)$ through the level shift in the threshold for a given a. To see why, note that a risk averse agent maximizes the certainty equivalent thereby setting the expected return equal to the risk premium (rather than equal zero). As a result, the threshold condition (13) modifies as follows:

$$\frac{g - r - \rho\sigma_r}{\beta} = \delta(\bar{x}(a), a)A(\bar{x}(a), a)^{N-1-a}\prod_{k=0}^{a-1} B(\bar{x}(k), a), \tag{28}$$

where ρ is the coefficient of risk aversion and σ_r denotes the volatility of returns of a particular stock. Intuitively, the agents' threshold value of signal for swtiching to selling is lower when risk aversion, ρ, in the ulitilty function is higher or when stock returns exhibited greater historical volatility, σ_r.

5.3 Empirical Specification

For the empirical implementation, we use Bayesian MCMC to fit Borel-Tanner distribution to the data treating the distribution parameter μ itself as a function of a stock j's realized illiquidity, $1/\theta_1(j)$, returns volatility, $\sigma_r(j)$, and risk aversion, ρ. We rely on the measure of Amihud (2002) to proxy for a stock's realized illiquidity, $1/\theta_1$: each quarter we calculate the average daily ratio of stock j's absolute return to its dollar trading volume. We then normalize the measure by the average market illiquidity (over all S&P 500 stocks) during that quarter to take out time variations in overall market liquidity. We proxy for historical volatility of each stock, $\sigma_r(j)$, using the quarterly standard deviation of a stock's daily returns. We use VIX to proxy for risk aversion, ρ.

We sample from the following hierarchical model using Metropolis-Hastings (MH) algorithm:[23][24]

$$\Pr(\alpha(j,k)) = \mu_1 e^{-(\mu_{j,k}\alpha(j,k)+\mu_1)}(\mu_{j,k}\alpha(j,k)+\mu_1)^{\alpha(j,k)-1}/\alpha(j,k)! \tag{29}$$

and

$$\mu_{j,k} = \gamma_0 + \gamma_1 1/\theta_1(j) + \gamma_2\sigma_r(j) + \gamma_3 VIX + \epsilon_{j,k}. \tag{30}$$

We pick trivial hyper-parameters for the priors:

$$\gamma_i \sim N(0,1),;\ i = 0,1 \tag{31}$$

$$\mu_1 \sim N(0,1) \tag{32}$$

$$\tau_\epsilon \sim \Gamma(\alpha_\epsilon, \beta_\epsilon), \tag{33}$$

where $N(\cdot)$ and $\Gamma(\cdot)$ denote Normal and Gamma distributions. We account for additional variability in $\mu_{j,k}$ via a random effects term, $\epsilon_{j,k}$, whose precision is measured by τ_ϵ.[25] We are interested in obtaining the coefficient on $1/\theta_1(j)$, $\sigma_r(j)$, and ρ.[26]

[23]The estimation was conducted with WinBUGS software following the Bayesian modeling framework outlined in Lunn, Thomas, Best, and Spiegelhalter (2000).

[24]See Chib and Greenberg (1995) for a comprehensive reference on Metropolis-Hastings algorithm. One major advantage of MH method is that, unlike Gibbs sampling, it does not require a conjugate prior for each distribution parameter, but samples from a proportional probability distribution to the density to be calculated.

[25]We use a diffuse $\Gamma(0.001, 0.001) prior for the precision of \tau_\epsilon$ of the random effects.

[26]The inference of coefficient significance and correlations rest on the assumption of unbiasedness of the estimates. Figure 14 and Figure 15 show Bayesian MCMC diagnostic plots for μ_1 and coefficients γ_0 through γ_3, with Figure 14 based on the baseline estimation where only illiquidity effect is considered. The left panels of Figure 14 and Figure

Table 12 shows the estimation results. The top panel shows the results for the reduced specification with illiquidity as the only control, while the bottom panel shows the results including a stock's volatility and the VIX. The economic magnitudes of the coefficients are difficult to interpret, but the positive and statistically significant coefficient on $1/\theta_1(j)$ is consistent with model predictions: greater illiquidity of a stock is associated with a greater degree of herding, higher μ. The coefficient on $\sigma_r(j)$ is also positive and significant (bottom panel), indicating that some fraction of the tail probability of large sell-outs of a stock is attributable to a stock average volatility during the quarter rather than illiquidity, possibly due to risk aversion. However, the coefficient on the VIX is negative, contrary to the risk aversion hypothesis. One possible explanation for the perverse coefficient on the VIX has to do with the time period under consideration: 2003:Q1 through 2006:Q1 is characterized by low overall financial market volatility and persistently low levels of overall risk aversion. Thus, as Figure 8 shows, to the extent that μ was rising during this period, the persistently low levels of VIX readings would in the end result in either zero or negative association between the two series. Furthermore, since VIX does not vary in the cross-section, it also effectively captures the time effect, further distorting the inference. Overall, the hypothesis of the central role of market illiquidity for herding is not rejected by the data while the results regarding risk aversion are mixed.

6 Conclusion

We considered a herding model in which each trader receives imperfect information about the market's ability to supply liquidity and chooses whether or not to sell the security based on her private information as well as the actions of others. Because of feedback effects, the equilibrium

15 plot the density of the samples. Symmetric bell curves indicate a good mixture and that a normal approximation to the standard errors is reasonable. The center panel shows the autocorrelation plots of the estimates, with $\gamma 0$ exhibiting a somewhat high persistence in the baseline specification depicted in Figure 14. The right panel shows a visual test for endogeneity via a scatter plot between sampled slope coefficients and the random effects component, τ_ϵ, which is bounded at zero from below. The scatter plots show a random spread consistent with exogeneity of the controls. Once the vector of controls is expanded to include σ_r and the VIX (coefficients γ_2 and γ_3 respectively), the persistence in the estimate of γ_0 is reduced. The sampled coefficients on the VIX (γ_3), however, exhibit very high persistence along with skewed density plot and correlation with the random effects terms (Figure 15) indicating that the estimate of γ_3 may be biased. Finally, Figure 16 shows Metropolis acceptance rates with acceptance rates broadly in the range of the commonly accepted level of 0.234 random walk MH algorithm. We discard the first 11,000 using the subsequent 89,000 for inference.

is stochastic and the "aggregate action" is characterized by a distribution exhibiting exponential decay embedding occasional "explosive" sell-outs. We obtain such "fat tail" distributions without imposing major parametric assumptions on exogenous variables. It suffices that the signals about the true state are informative in the sense of satisfying the MLRP.

The stochastic herding approach provides one plausible data generating mechanism for large adjustments in institutional equity holdings during our sample period. The distribution implied by the model matches the empirical distribution of the number of institutional investment managers selling off their shares each quarter before the peak of the SP 500 index in 2007. Moreover, in line with market-timing considerations, the distribution parameter reflecting the degree of herding is increasing in a stock's illiquidity, past volatility, as well as rises sharply prior to massive adjustments that began in earnest in 2006:Q2. The transition to the sell-out phase itself is consistent with the emergence of perfect strategic complementarity, which is predicted by the model when the fraction of herding agents reaches a critical level.

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A Proof of Proposition 1

We first transform $\delta(\bar{x}, a)$ as follows. By completing the square on θ we obtain:

$$e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} = e^{-\frac{(\theta-\mu_\theta(x_i))^2}{2\sigma_\theta^2}} \xi(x_i) \tag{34}$$

where,

$$\mu_\theta(x_i) \equiv \frac{x_i/\sigma_e^2 + \theta_0/\sigma_0^2}{1/\sigma_e^2 + 1/\sigma_0^2} \tag{35}$$

$$\sigma_\theta^2 \equiv (1/\sigma_e^2 + 1/\sigma_0^2)^{-1} \tag{36}$$

$$\xi(x_i) \equiv e^{\frac{\mu_\theta(x_i)^2}{2\sigma_\theta^2} - \frac{x_i^2}{2\sigma_e^2} - \frac{\theta_0^2}{2\sigma_0^2}}. \tag{37}$$

Then we have:

$$\delta(x_i, a) = \frac{\int^\alpha e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} d\theta}{\int_\alpha e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} d\theta} = \frac{\Phi(\alpha; x_i)}{1 - \Phi(\alpha; x_i)} \tag{38}$$

where $\Phi(\cdot; x_i)$ denotes the cumulative distribution function for a normal distribution with mean $\mu_\theta(x_i)$ and variance σ_θ^2. A and B are rewritten as:

$$\begin{aligned} A(\bar{x}, a) &= \frac{\int_{\bar{x}} \int^\alpha e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} d\theta dx_i}{\int_{\bar{x}} \int_\alpha e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} d\theta dx_i} \frac{\Pr(\theta \geq \alpha)}{\Pr(\theta < \alpha)} \\ &= \frac{\int_{\bar{x}} \Phi(\alpha; x_i)\xi(x_i) dx_i}{\int_{\bar{x}} (1 - \Phi(\alpha; x_i))\xi(x_i) dx_i} \frac{\Pr(\theta \geq \alpha)}{\Pr(\theta < \alpha)} \end{aligned} \tag{39}$$

$$\begin{aligned} B(\bar{x}(k), a) &= \frac{\int^{\bar{x}(k)} \int^\alpha e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} d\theta dx_i}{\int^{\bar{x}(k)} \int_\alpha e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} d\theta dx_i} \frac{\Pr(\theta \geq \alpha)}{\Pr(\theta < \alpha)} \\ &= \frac{\int^{\bar{x}(k)} \Phi(\alpha; x_i)\xi(x_i) dx_i}{\int^{\bar{x}(k)} (1 - \Phi(\alpha; x_i))\xi(x_i) dx_i} \frac{\Pr(\theta \geq \alpha)}{\Pr(\theta < \alpha)} \end{aligned} \tag{40}$$

By taking logarithm of (13) for a and $a + 1$ and subtracting each side, we obtain:

$$\begin{aligned} 0 &= \log \delta(\bar{x}(a+1), a+1) - \log \delta(\bar{x}(a), a) + (N-1-a)(\log A(\bar{x}(a+1), a+1) - \log A(\bar{x}(a), a)) \\ &\quad + \sum_{k=0}^{a-1} (\log B(\bar{x}(k), a+1) - \log B(\bar{x}(k), a)) + \log B(\bar{x}(a), a+1) - \log A(\bar{x}(a+1), a+1) \end{aligned} \tag{41}$$

The second argument a in δ and B affects the functions through $\alpha = a/N$ as in (38,40), and thus the direct effects of a on δ and B are of order $1/N$. Also, as we show shortly, the difference $\bar{x}(a+1) - \bar{x}(a)$ is of order $1/N$, and so are the effects of a on δ and B through $\bar{x}$. Hence, the difference terms in (41) on $\log \delta$ and $\log B$ are of order $1/N$ and tends to zero as N goes to infinity.

The difference term in $\log A$ is broken down as:

$$\frac{\log A(\bar{x}(a+1), a+1) - \log A(\bar{x}(a), a)}{1/N} \sim_{N\to\infty} \frac{\partial \log A(\bar{x}(a), a)}{\partial \bar{x}} \frac{\bar{x}(a+1) - \bar{x}(a)}{1/N} + \frac{\partial \log A(\bar{x}(a), a)/\partial a}{1/N} \tag{42}$$

Thus, as $N \to \infty$ for a fixed finite a, we have:

$$(N - 1 - a)\left(\bar{x}(a+1) - \bar{x}(a)\right) \to \frac{\log B(\bar{x}, a) - \log A(\bar{x}, a) + \partial \log A(\bar{x}(a), a)/\partial a}{-\partial \log A(\bar{x}, a)/\partial \bar{x}} \tag{43}$$

The right hand side is of order N^0, and hence it is shown that $\bar{x}(a+1) - \bar{x}(a)$ is of order $1/N$.

Next, we show that the right-hand side of (43) is positive. When a is increased by one, one trader switches sides from A to B, and this increases the right hand side of (13) because $A < B$. This effect appears as $\log B(\bar{x}, a) - \log A(\bar{x}, a)$ in (43).

Next, we show that $A(\bar{x}, a)$ increases in a for a fixed $\bar{x}$. We start by showing that $G(\bar{x}, a)$ is increasing in the second argument:

$$\begin{aligned}
\frac{\partial G(\bar{x}, a)}{\partial a} &= \frac{\partial}{\partial a}\left(\frac{\Pr(x_j > \bar{x}, \theta < a/N)}{\Pr(\theta < a/N)}\right) && (44)\\
&= \frac{\Pr(x_j > \bar{x}, \theta = a/N)\Pr(\theta < a/N) - \Pr(x_j > \bar{x}, \theta < a/N)\Pr(\theta = a/N)}{\Pr(\theta < a/N)^2} && (45)\\
&= \frac{\Pr(\theta = a/N)}{\Pr(\theta < a/N)}\left(\frac{\Pr(x_j > \bar{x}, \theta = a/N)}{\Pr(\theta = a/N)} - \frac{\Pr(x_j > \bar{x}, \theta < a/N)}{\Pr(\theta < a/N)}\right) && (46)\\
&= \frac{\Pr(\theta = a/N)}{\Pr(\theta < a/N)}\left(\Pr(x_j > \bar{x} \mid \theta = a/N) - \Pr(x_j > \bar{x} \mid \theta < a/N)\right) && (47)\\
&> 0 && (48)
\end{aligned}$$

where "Pr" denotes likelihood functions. The last inequality holds by the property (14). We show likewise that $F(\bar{x}, a)$ is decreasing in a. Since $A(\bar{x}, a) = G(\bar{x}, a)/F(\bar{x}, a)$, we obtain that A is increasing in a.

Finally, we show that $\partial A/\partial \bar{x} < 0$. Define F_1 and G_1 as the derivatives of F and G with respect to the first argument $\bar{x}$, respectively. Then:

$$\frac{\partial A(\bar{x}, a)}{\partial \bar{x}} = \frac{F_1(\bar{x}, a)}{F(\bar{x}, a)}\left(\frac{G_1(\bar{x}, a)}{F_1(\bar{x}, a)} - A\right). \tag{49}$$

G_1/F_1 can be rewritten as:

$$\frac{G_1(\bar{x}, a)}{F_1(\bar{x}, a)} = \frac{\Phi(\alpha; \bar{x})}{1 - \Phi(\alpha; \bar{x})} \frac{\Pr(\theta \geq \alpha)}{\Pr(\theta < \alpha)} \tag{50}$$

Then,

$$\frac{A}{G_1/F_1} = \int_{\bar{x}} \frac{\Phi(\alpha;x_i)}{\Phi(\alpha;\bar{x})}\xi(x_i)dx_i \bigg/ \int_{\bar{x}} \frac{1-\Phi(\alpha;x_i)}{1-\Phi(\alpha;\bar{x})}\xi(x_i)dx_i < 1 \tag{51}$$

where the inequality obtains by that $\Phi(\alpha;x_i) < \Phi(\alpha;\bar{x})$ for any $x_i > \bar{x}$. Noting that $F_1 < 0$, we obtain from (49) that $\partial A(\bar{x},a)/\partial\bar{x} < 0$.

Table 1: Number of managers in S&P 500 stocks, by institution type

Quarter	Banks		Insurance Companies		Investment Companies		Investment Advisors		Pension & Endowment Funds		Total
	Number	% tot.	Number	% tot.	Number	% tot.	Number	% tot.	Number	% tot.	Number
2003q1	11	0.92	18	1.8	128	6.39	135	14.16	1579	76.74	1871
2003q2	11	0.87	19	1.98	130	6.42	137	14.16	1593	76.57	1890
2003q3	11	0.88	18	1.88	132	6.63	137	14.3	1590	76.31	1888
2003q4	10	1.01	21	1.71	121	5.95	131	13.74	1699	77.59	1982
2004q1	10	1.16	20	1.82	129	6.49	133	13.44	1742	77.09	2034
2004q2	10	0.99	20	1.81	133	13.29	136	6.4	1742	77.51	2041
2004q3	9	1.11	20	1.8	125	12.59	136	6.09	1735	78.41	2025
2004q4	10	1.2	20	1.68	117	11.44	166	6.97	1869	78.71	2182
2005q1	9	1.16	20	1.65	121	11.76	169	6.87	1877	78.56	2196
2005q2	8	1.08	20	1.64	119	11.22	170	6.73	1901	79.32	2218
2005q3	7	1.11	19	1.83	114	11.35	167	6.67	1855	79.04	2162
2005q4	8	0.96	19	1.62	114	10.96	187	7.18	1973	79.28	2301
2006q1	9	1.06	19	1.61	111	10.27	189	6.65	2024	80.41	2352
2006q2	10	1.16	18	1.59	106	9.9	204	7.3	2046	80.05	2384
2006q3	9	0.94	18	1.52	103	9.66	240	8.57	2014	79.3	2384
2006q4	9	1.12	17	1.4	102	9.43	333	11.98	2087	76.08	2548
2007q1	8	0.99	17	1.39	102	9.31	337	11.53	2124	76.78	2588
2007q2	10	0.97	18	1.38	101	8.84	384	13.47	2096	75.34	2609
2007q3	9	0.88	18	1.41	97	8.72	405	14.97	2058	74.01	2587
2007q4	9	0.96	18	1.34	97	8.34	516	17.54	2124	71.82	2764
2008q1	9	1.07	19	1.33	96	8.54	521	17.6	2119	71.41	2764

Notes: The data is complied from quarterly 13F filings with SEC in which institutional investment managers with over $100 million under discretionary management are required to report their long positions in exchange traded stocks, closed-end investment companies, equity options and warrants. Source: Spectrum database available through Thompson Financial.

Table 2: Value of S&P 500 stocks; by institution type

Quarter	Banks			Insur. Comp.			Invest. Comp.			Invest. Advisors			Pension & Endowment Funds			Total	
	$ Mil.	% Tot.	% Mkt.	$ Mil.	% Tot.	% Mkt.	$ Mil.	% Tot.	% Mkt.	$ Mil.	% Tot.	% Mkt.	$ Mil.	% Tot.	% Mkt.	$ Mil.	% Mkt.
2003q1	95,900	0.92	1.20	209,000	1.8	2.61	466,000	6.39	5.81	95,900	14.16	1.20	3,350,000	76.74	41.77	8,020,000	52.58
2003q2	108,000	0.87	1.19	235,000	1.98	2.58	541,000	6.42	5.94	1,080,000	14.16	11.86	3,860,000	76.57	42.37	9,110,000	63.93
2003q3	111,000	0.88	1.21	258,000	1.88	2.82	588,000	6.63	6.43	1,090,000	14.3	11.93	3,970,000	76.31	43.44	9,140,000	65.83
2003q4	76,000	1.71	0.75	128,000	1.01	1.27	207,000	5.95	2.05	1,280,000	13.74	12.67	4,370,000	77.59	43.27	10,100,000	60.01
2004q1	130,000	1.16	1.25	231,000	1.82	2.22	648,000	6.49	6.23	1,270,000	13.44	12.21	4,700,000	77.09	45.19	10,400,000	67.11
2004q2	129,000	0.99	1.19	220,000	1.81	2.04	648,000	6.4	6.00	1,260,000	13.29	11.67	4,700,000	77.51	43.52	10,800,000	64.42
2004q3	126,000	1.11	1.17	222,000	1.8	2.06	642,000	6.09	5.94	1,200,000	12.59	11.11	4,740,000	78.41	43.89	10,800,000	64.17
2004q4	137,000	1.2	1.17	235,000	1.68	2.01	737,000	6.97	0.00	1,270,000	11.44	10.85	5,220,000	78.71	44.62	11,700,000	58.65
2005q1	131,000	1.16	1.21	231,000	1.65	2.14	716,000	6.87	6.63	1,200,000	11.76	11.11	4,780,000	78.56	44.26	10,800,000	65.35
2005q2	130,000	1.08	1.19	231,000	1.64	2.12	718,000	6.73	6.59	1,160,000	11.22	10.64	4,920,000	79.32	45.14	10,900,000	65.68
2005q3	124,000	1.11	1.11	242,000	1.83	2.16	694,000	6.67	6.20	1,200,000	11.35	10.71	5,090,000	79.04	45.45	11,200,000	65.63
2005q4	130,000	0.96	1.14	246,000	1.62	2.16	724,000	7.18	6.35	1,220,000	10.96	10.70	5,360,000	79.28	47.02	11,400,000	67.37
2006q1	137,000	1.06	1.15	262,000	1.61	2.20	752,000	6.71	6.32	1,150,000	10.27	9.66	5,680,000	80.36	47.73	11,900,000	67.07
2006q2	144,000	1.16	1.17	264,000	1.59	2.15	778,000	7.28	6.33	1,170,000	9.9	9.51	5,860,000	80.07	47.64	12,300,000	66.80
2006q3	117,000	0.94	0.91	283,000	1.52	2.19	962,000	8.56	7.46	1,210,000	9.66	9.38	6,020,000	79.32	46.67	12,900,000	66.60
2006q4	159,000	1.12	1.18	297,000	1.4	2.20	1,430,000	11.96	10.59	1,490,000	9.43	11.04	5,820,000	76.09	43.11	13,500,000	68.12
2007q1	155,000	0.99	1.14	303,000	1.39	2.23	1,260,000	9.31	9.26	1,440,000	11.55	10.59	6,030,000	76.76	44.34	13,600,000	67.56
2007q2	160,000	0.97	1.13	369,000	1.38	2.60	1,300,000	8.84	9.15	1,860,000	13.47	13.10	6,120,000	75.34	43.10	14,200,000	69.08
2007q3	158,000	0.88	1.11	371,000	1.41	2.61	1,700,000	8.72	11.97	2,080,000	14.9	14.65	5,750,000	74.01	40.49	14,200,000	70.84
2007q4	165,000	0.96	1.22	351,000	1.34	2.60	1,350,000	8.34	10.00	2,130,000	17.54	15.78	5,330,000	71.82	39.48	13,500,000	69.08
2008q1	141,000	1.07	1.18	302,000	1.33	2.52	1,200,000	8.54	10.00	1,870,000	17.64	15.58	4,710,000	71.41	39.25	12,000,000	68.53

Notes: The data is complied from quarterly 13F filings with SEC in which institutional investment managers with over $100 million under discretionary management are required to report their long positions in exchange traded stocks, closed-end investment companies, equity options and warrants. Source: Spectrum database available through Thompson Financial.

Table 3: Descriptive Statistics: $N(j,k)$

Quarter	Obs.	Mean	Std. Dev.	Skewness	Kurtosis	Min.	Max.
2003q1	1842	114.2921	149.1265	2.559299	11.37729	10	1046
2003q2	1882	114.1265	150.8741	2.613972	11.82006	10	1105
2003q3	1856	115.715	150.2376	2.560607	11.42463	10	1069
2003q4	1846	118.792	157.0115	2.541808	11.26566	10	1130
2004q1	1878	121.2758	159.8353	2.527476	11.15328	10	1157
2004q2	1878	122.1081	161.6158	2.501442	10.95575	10	1150
2004q3	1859	119.2883	160.4453	2.509865	11.01269	10	1123
2004q4	1833	125.7239	168.1973	2.471413	10.65422	10	1145
2005q1	1546	136.6177	179.9863	2.328704	9.529656	10	1161
2005q2	1537	138.2492	184.3018	2.320885	9.460284	10	1187
2005q3	1562	133.0205	179.1581	2.332849	9.483129	10	1141
2005q4	1535	140.6098	186.0956	2.315662	9.374518	10	1170
2006q1	1543	142.4543	191.4796	2.259286	8.978757	10	1195
2006q2	1792	133.4492	182.3782	2.403041	10.02165	10	1205
2006q3	1788	133.9636	180.7052	2.45352	10.33914	10	1199
2006q4	1749	140.0743	177.5046	2.464914	10.48713	10	1197
2007q1	1720	143.0715	181.7965	2.401363	10.01951	10	1220
2007q2	1741	146.2211	181.5936	2.361122	9.723414	10	1222
2007q3	1738	141.2819	174.153	2.425098	10.18531	10	1179
2007q4	1689	148.2587	175.7674	2.368556	9.921134	10	1179
2008q1	1697	144.0595	171.5843	2.405454	10.21005	10	1197

Notes: The Table shows the summary statistics for stock-investor-type groups, $N(j,k)$, where j indicates an S&P500 stock and k indicates institutional investor type. Only groups with 10 traders or more are included in the sample.

Table 4: Descriptive Statistics: $a(j,k)$

Quarter	Obs.	Mean	Std. Dev.	Skewness	Kurtosis	Min.	Max.
2003q1	1842	2.024973	2.913459	3.562305	31.30286	0	44
2003q2	1882	2.001063	2.899889	2.022484	7.914702	0	24
2003q3	1856	2.199353	3.3151	1.87077	6.450509	0	23
2003q4	1846	2.299025	6.832766	26.99445	969.2564	0	252
2004q1	1878	2.78967	8.817813	32.14487	1240.28	0	347
2004q2	1878	2.457934	3.83505	3.523452	22.88747	0	40
2004q3	1859	2.257127	3.583172	3.511261	22.64714	0	37
2004q4	1833	2.240589	5.00605	10.326	202.7108	0	123
2005q1	1546	2.982536	3.905065	4.128196	37.58728	0	57
2005q2	1537	2.688354	4.341368	3.362496	20.00555	0	43
2005q3	1562	2.744558	4.306713	3.373875	20.45202	0	42
2005q4	1535	3.730945	5.142803	2.502752	12.18058	0	47
2006q1	1543	4.04731	6.772548	4.319921	42.25052	0	105
2006q2	1792	103.9738	160.4168	2.646657	11.52395	0	1100
2006q3	1788	115.1141	159.1397	2.650348	11.76884	0	1114
2006q4	1749	112.6781	150.5121	2.685956	12.0627	0	1057
2007q1	1720	116.8035	152.82	2.664223	11.87025	0	1080
2007q2	1741	3.036186	5.233333	7.706638	123.9169	0	112
2007q3	1738	3.659379	4.685157	3.009861	24.03532	0	63
2007q4	1689	3.117229	4.644519	3.760667	30.52305	0	57
2008q1	1697	3.727166	4.535413	2.734147	18.2615	0	55

Notes: The table shows summary statistics of $a(j,k)$. Each quarter $a(j,k)$ out of $N(j,k)$ institutions in each group liquidate their holdings. Institutional managers dumping more than 80% of their holdings are counted into $a(j,k)$.

Table 5: Descriptive Statistics: $a(j,k)/N(j,k)$

Quarter	Obs.	Mean	Std. Dev.	Skewness	Kurtosis	Min.	Max.
2003q1	1203	0.0338277	0.0310166	2.800426	15.11922	0.0025974	0.2941177
2003q2	1090	0.03389	0.0274069	2.059337	8.368169	0.0026738	0.1904762
2003q3	985	0.032096	0.0296491	9.969194	182.9449	0.003861	0.6363636
2003q4	1088	0.0376633	0.0497596	8.49001	113.6885	0.000993	0.8113208
2004q1	1425	0.0398577	0.0462501	8.97107	134.2808	0.0027548	0.8202247
2004q2	1222	0.0392724	0.0350081	2.468958	13.05533	0.0023364	0.3636364
2004q3	1133	0.0378985	0.0329231	2.518009	14.81435	0.0027855	0.3571429
2004q4	961	0.0327997	0.0442431	8.801352	119.9139	0.0022422	0.7741935
2005q1	1308	0.0402617	0.031572	2.004466	8.523765	0.0023697	0.2666667
2005q2	947	0.0338203	0.0264428	2.175285	10.57713	0.0032626	0.2222222
2005q3	1088	0.034929	0.0301894	2.71143	14.75573	0.0022075	0.3043478
2005q4	1218	0.0407605	0.0281401	2.353076	13.41352	0.0033898	0.3
2006q1	1012	0.0416007	0.0283892	1.99926	8.37819	0.0035971	0.2
2006q2	1540	0.8455544	0.0936507	-1.105954	7.453066	0.137931	1
2006q3	1761	0.8789349	0.0704131	-1.133229	9.861254	0.1538462	1
2006q4	1731	0.7858961	0.0887199	-2.067533	13.76107	0.0185185	0.9473684
2007q1	1711	0.8308253	0.0812513	-3.080854	28.49091	0.0054054	1
2007q2	1180	0.0320758	0.0273514	2.807056	13.82265	0.0031315	0.2285714
2007q3	1253	0.0402934	0.0286195	1.766037	6.964115	0.0023256	0.2075472
2007q4	1110	0.030142	0.0215736	1.83541	7.408579	0.004329	0.1428571
2008q1	1280	0.0378383	0.0261516	1.951667	8.835462	0.0044643	0.2307692

Notes: The table shows quarterly summary statistics for the fraction of institutional investment managers dumping their stock within a stock-investor type group ($\alpha(j,k) \equiv a(j,k)/N(j,k)$).

Table 6: Estimates for Zipf's Law goodness of fit to quarterly changes in institutional equity holdings.

Quarter	Constant	t-stat	Pareto Exponent	t-stat	adjusted t-stat	Wald Stat. b=-1	R-sq.	Rbar-sq.	Hill estimator	adjusted t-stat	Wald Stat. b=-1
2003q1	12.634	419.7	-0.400	-154.0	-66.2	53535.4	0.730	0.730	-1.4E-01	464.8	3259.6
2003q2	13.230	435.6	-0.439	-167.6	-70.9	45937.7	0.737	0.737	-1.3E-01	469.2	4891.3
2003q3	13.016	437.8	-0.423	-165.6	-69.3	50906.7	0.741	0.741	-1.3E-01	469.3	4088.4
2003q4	13.219	425.4	-0.445	-166.5	-67.5	43174.4	0.753	0.753	-1.6E-01	474.1	2533.6
2004q1	13.041	484.4	-0.426	-181.6	-71.5	59617.1	0.764	0.764	-1.3E-01	483.6	4845.2
2004q2	13.069	450.0	-0.426	-169.9	-70.8	52180.5	0.742	0.742	-1.5E-01	484.8	3311.8
2004q3	13.065	427.5	-0.429	-163.0	-68.9	46963.9	0.737	0.737	-1.4E-01	477.2	3468.7
2004q4	12.759	441.9	-0.411	-162.9	-68.1	54559.0	0.742	0.742	-1.5E-01	486.6	2928.3
2005q1	13.050	441.1	-0.419	-165.2	-71.7	52649.1	0.726	0.726	-1.5E-01	465.4	3139.2
2005q2	12.488	432.8	-0.393	-155.0	-66.9	57133.6	0.729	0.729	-1.4E-01	466.7	3367.7
2005q3	12.513	403.4	-0.394	-147.6	-64.1	51389.4	0.726	0.726	-1.6E-01	461.6	2465.6
2005q4	12.313	473.9	-0.372	-165.3	-67.1	77793.2	0.752	0.752	-1.3E-01	470.0	4038.7
2006q1	12.585	468.3	-0.385	-167.0	-69.8	71332.7	0.742	0.742	-1.4E-01	474.5	3793.6
2006q2	12.648	421.7	-0.394	-155.2	-66.0	57199.8	0.735	0.735	-1.6E-01	495.2	2549.4
2006q3	12.821	484.1	-0.394	-175.5	-72.3	73124.2	0.747	0.747	-1.4E-01	495.0	3711.1
2006q4	12.679	435.5	-0.387	-157.2	-69.8	62164.3	0.718	0.718	-1.5E-01	501.0	3217.4
2007q1	13.230	532.2	-0.412	-197.7	-77.6	79869.0	0.765	0.765	-1.3E-01	501.8	4821.2
2007q2	12.756	482.2	-0.392	-171.0	-74.7	70450.6	0.726	0.726	-1.4E-01	510.2	3953.4
2007q3	12.999	575.4	-0.394	-200.6	-84.1	95587.8	0.741	0.741	-1.4E-01	500.8	4201.8
2007q4	12.706	507.8	-0.376	-175.4	-77.9	84629.4	0.718	0.718	-1.5E-01	505.9	3719.7
2008q1	12.872	555.4	-0.389	-192.8	-81.1	91593.0	0.739	0.738	-1.5E-01	500.0	3488.7

Notes: Columns 1-7 present the estimation results of the "log rank log size regression" $log(rank(\Delta_{shares}) = c + blog(\Delta_{shares})$. For the adjusted t-statistic of the OLS regression coefficient b, the assymetric standard error of the form $b(n/2)^{-1/2}$ was used. Columns 8-9 present the results of the Hill estimator. The assymetric standard error of the for $b(hill)n^{-1/2}$. For more information with regards to the assymetric standard error please refer to Gabaix and Ibragimov (2011)

Table 7: Kolmogorov-Smirnov test, Poisson distribution of $a(j,k)$ over the entire sample, 2003:Q1 - 2008:Q1

Variable	Obs.	Test Result	p-value	Test Stat.	Critical Value
$a(j,k)$	38,353	Reject	0.000	0.769	0.008

Notes: The table shows the results of Kolmogorov-Smirnov goodness of fit test for Poisson distribution to $a(j,k)$.

Table 8: Test for normality of $a(j,k)$ over the entire sample, 2003:Q1 - 2008:Q1

Variable	Obs.	Mean	Std. Dev.	Skewness	Kurtosis
$a(j,k)$	38,353	22.745	79.264	6.368	54.868

Notes: The table shows the estimates of the moments of $a(j,k)$, pointing at a highly non-normal distribution.

Table 9: Distribution parameter estimates for $a(j,k)$ for the entire sample, 2003:Q1 - 2008:Q1.

Variable	Obs.	μ_1	μ	Log Likelihood
$a(j,k)$	38,353	2.058	0.938	99728.410
		(0.006)	(0.001)	

Notes: The probability density for the hypothesized distribution is $\Pr(Z = z) = \mu_1 e^{-(\mu z+\mu_1)}(\mu z + \mu_1)^{z-1}/z!$

Table 10: Distribution parameter estimates for $a(j,k)$ for the 2005:Q2 - 2006:Q1 subsample.

	Distribution of $a(j,k)$							
	Model		Benchmark Distributions					
	Borel-Poisson		Trunc. Normal		Gamma		Exponential	
ML estimates	μ_1	2.058	mean	-97.461	α	1.103	β	4.781
		(0.029)		(7.152)		(0.021)		(0.072)
	μ	0.570	σ	20.000	β	4.335		
		(0.007)		(0.665)		(0.103)		
Log Likelihood		11148.789		10040.186		10925.596		10938.238
Vuong's statistic		H_1		30.393		21.785		28.140
Obs.		4,265						

Notes: The probability density for the hypothesized distribution is $\Pr(Z = z) = \mu_1 e^{-(\mu z+\mu_1)}(\mu z + \mu_1)^{z-1}/z!$. If $V > 1.96$ then H_0 of normal distribution is rejected in favor of H_1 of the model under 5% significance level.

Table 11: Quarterly distribution parameter estimates for $a(j,k)$.

Quarter	Obs.	μ_1	s.e.	μ	s.e.	Log Likelihood	Vuong's Statistic
2003q1	1203	2.023	(0.053)	0.347	(0.015)	2610.493	31.048
2003q2	1090	2.164	(0.060)	0.374	(0.016)	2479.326	28.628
2003q3	985	2.368	(0.069)	0.429	(0.016)	2412.327	25.413
2003q4	1088	1.963	(0.054)	0.497	(0.014)	2614.313	3.588
2004q1	1425	1.880	(0.045)	0.489	(0.012)	3343.738	–
2004q2	1222	2.046	(0.053)	0.458	(0.014)	2905.715	28.048
2004q3	1133	2.103	(0.057)	0.432	(0.014)	2661.898	28.686
2004q4	1833	2.087	(0.061)	0.512	(0.014)	2392.407	10.572
2005q1	1308	1.984	(0.050)	0.437	(0.013)	3024.613	23.576
2005q2	947	2.155	(0.064)	0.506	(0.015)	2384.011	22.919
2005q3	1088	1.938	(0.054)	0.508	(0.014)	2644.548	24.945
2005q4	1218	2.022	(0.054)	0.570	(0.012)	3170.367	25.764
2006q1	1012	2.234	(0.064)	0.638	(0.012)	2891.608	13.442
2006q2	1540	7.103	(0.138)	0.941	(0.002)	8660.686	-26.826
2006q3	1761	7.527	(0.136)	0.936	(0.002)	9866.110	-27.776
2006q4	1731	7.765	(0.142)	0.932	(0.002)	9696.680	-26.172
2007q1	1711	8.124	(0.149)	0.931	(0.002)	9645.581	-25.912
2007q2	1180	2.181	(0.057)	0.513	(0.013)	2991.997	15.100
2007q3	1253	2.606	(0.065)	0.486	(0.013)	3291.732	25.806
2007q4	1110	2.360	(0.064)	0.503	(0.013)	2867.704	24.548
2008q1	1280	2.619	(0.065)	0.470	(0.013)	3323.552	25.452

Notes: The table reports quarterly MLE parameter estimates for the probability density of the hypothesized distribution, $\Pr(Z = z) = \mu_1 e^{-(\mu z+\mu_1)}(\mu z + \mu_1)^{z-1}/z!$ If $V > 1.96$ then H_0 of normal distribution is rejected in favor of H_1 of the model under 5% significance level.

Table 12: Illiquidity, Risk Aversion, and the Empirical Proxy for Herding, μ

Dependent parameter: $\mu_{j,k}$							
node	mean	sd	MC error	2.50%	5.00%	95.00%	97.50%
Constant	-6.833	0.400	0.007	-7.661	-7.526	-6.208	-6.093
$1/\theta_1(j)$	0.646	0.139	0.001	0.359	0.407	0.863	0.904
μ_1	0.037	0.002	0.000	0.034	0.035	0.040	0.041
τ_ϵ	1.500	0.612	0.035	0.715	0.773	2.732	3.052
Constant	-0.270	0.999	0.009	-2.215	-1.924	1.375	1.711
$1/\theta_1(j)$	1.273	0.741	0.009	-0.116	0.103	2.546	2.824
$\sigma_r(j)$	2.003	1.006	0.008	0.042	0.335	3.653	3.979
VIX	-1.233	0.371	0.008	-2.079	-1.914	-0.718	-0.656
μ_1	0.104	0.003	0.000	0.098	0.099	0.109	0.110
τ_ϵ	0.568	0.470	0.027	0.052	0.065	1.478	1.633
Observations	12,236						

Notes: Results of Bayesian MCMC estimation of hierarchical model in Equation (29). $\Pr(Z = z) = \mu_1 e^{-(\mu z+\mu_1)}(\mu z + \mu_1)^{z-1}/z!$ with restriction $\mu = \gamma_0 + \gamma_1 1/\theta_1 +_2 \sigma_r + \gamma_3 VIX + \epsilon$. Higher μ indicates higher intensity of the branching process generating the Borel Tanner distribution. Results based on 89,000 samples after discarding the first 11,000 iterations as "burn-in". Confidence bounds computed under the normality assumption for the simulated parameter values. 2003:Q1 through 2006:Q1 time sample.

B Additional Figures

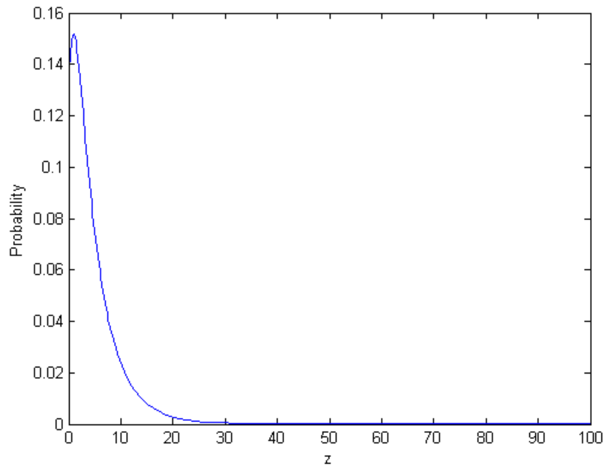

Figure 10: Probability density of the hypothesized distribution $\Pr(Z = z) = \mu_1 e^{-(\mu z+\mu_1)}(\mu z + \mu_1)^{z-1}/z!$. Parameters estimated using 2005:Q2-2006:Q1 data on institutional investor holdings of S&P 500 stocks: $\mu_1 = 2.060$, $\mu = 0.547$.

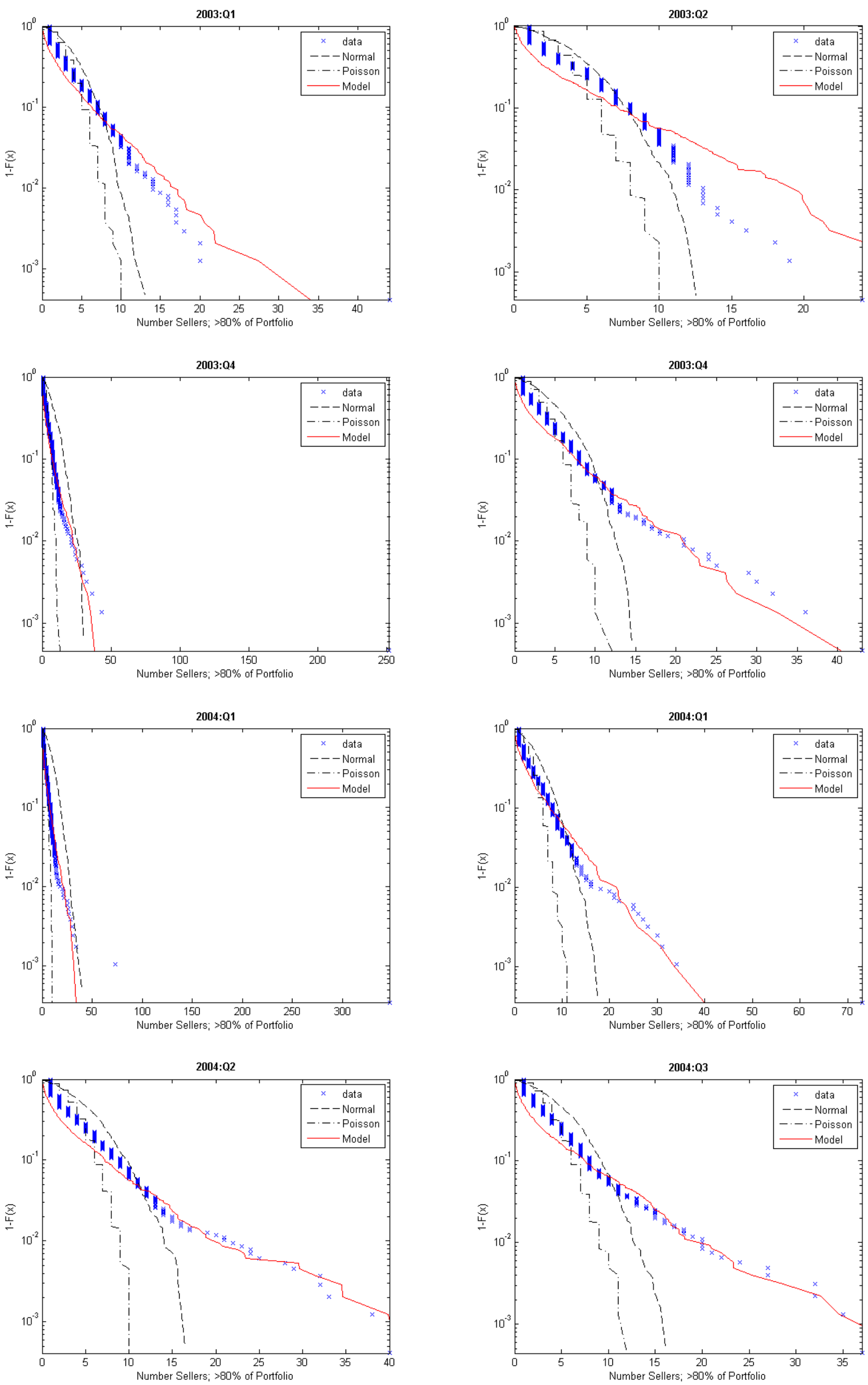

Figure 11: Semilog probability plot of $a(j, k)$ and comparison to data simulated using the model and the two alternatives, Normal and Poisson.

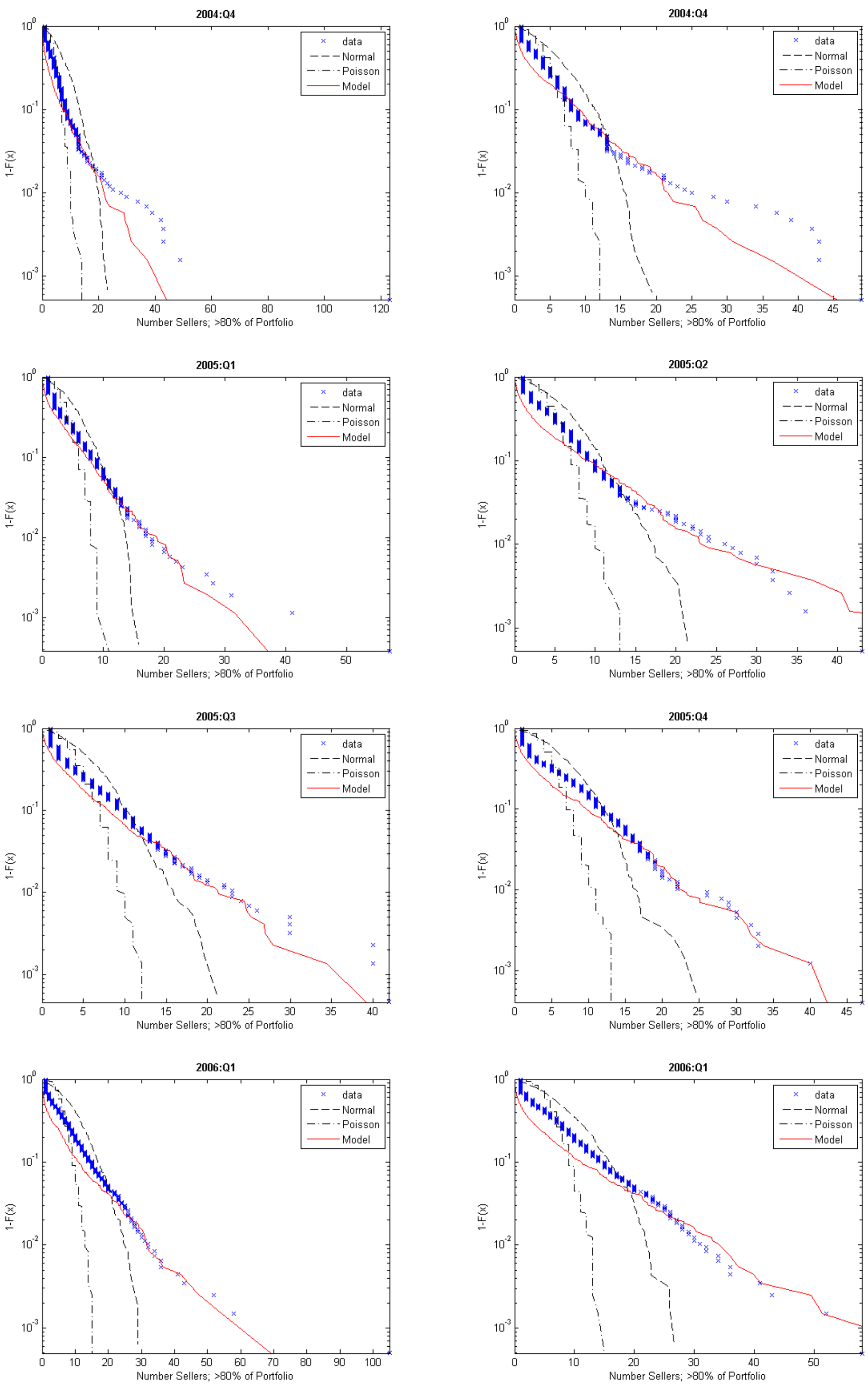

Figure 12: Semilog probability plot of $a(j,k)$ and comparison to data simulated using the model and the two alternatives, Normal and Poisson.

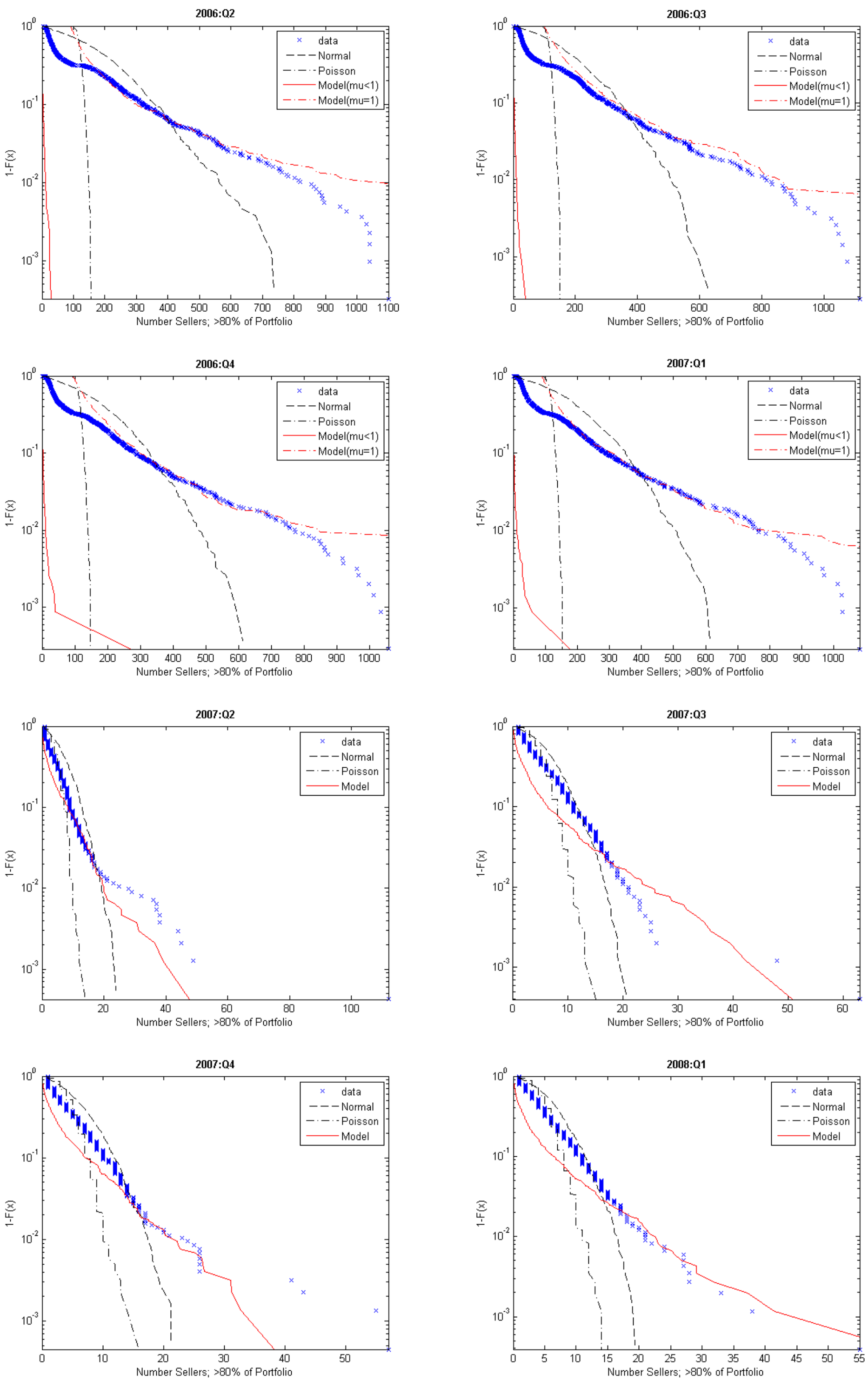

Figure 13: Semilog probability plot of $a(j,k)$ and comparison to data simulated using the model and the two alternatives, Normal and Poisson.

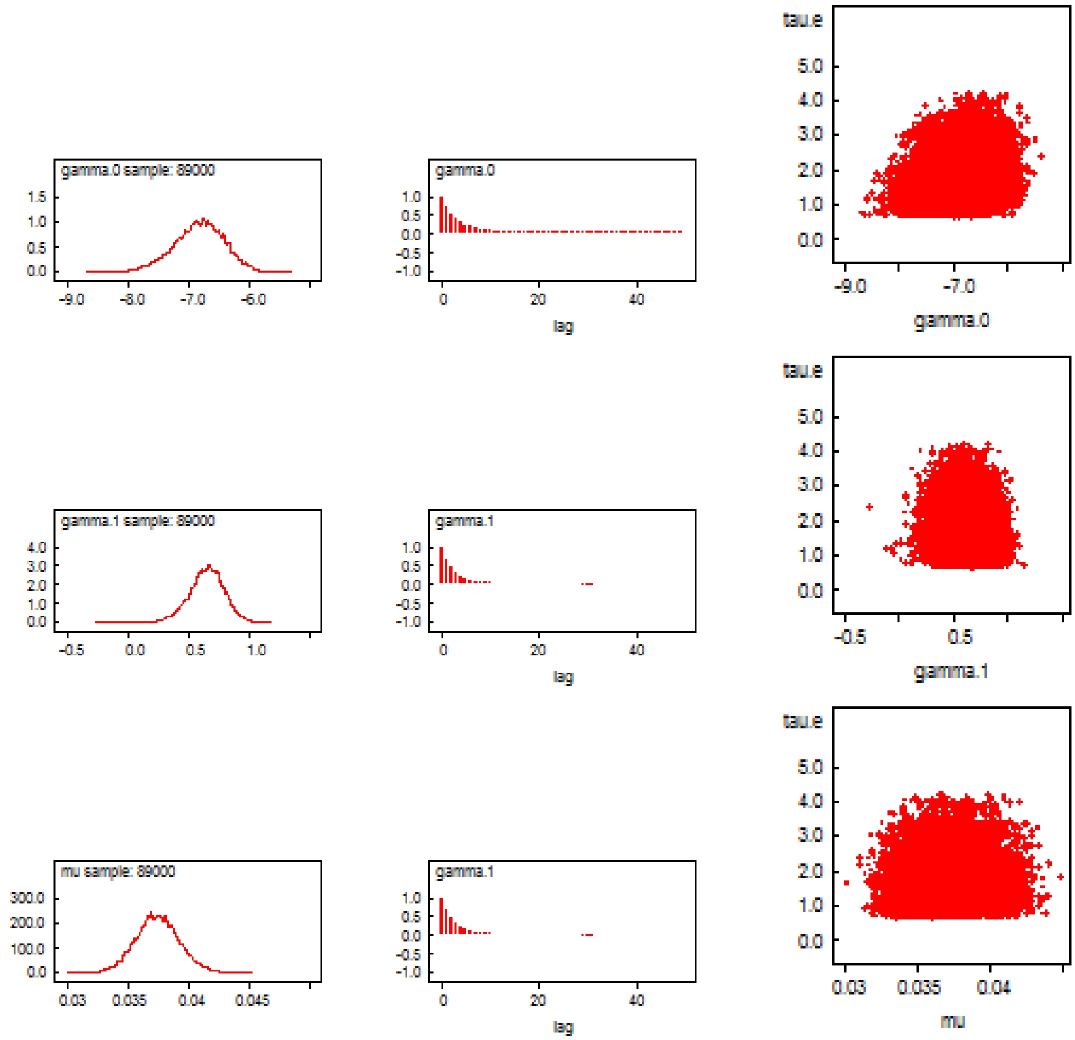

Figure 14: Bayesian MCMC diagnostics, baseline controls. *Left:* density plots; *center:* ACF plots, and *right:* scatter plots against τ_ϵ for γ_0, γ_1 and parameter μ_1.

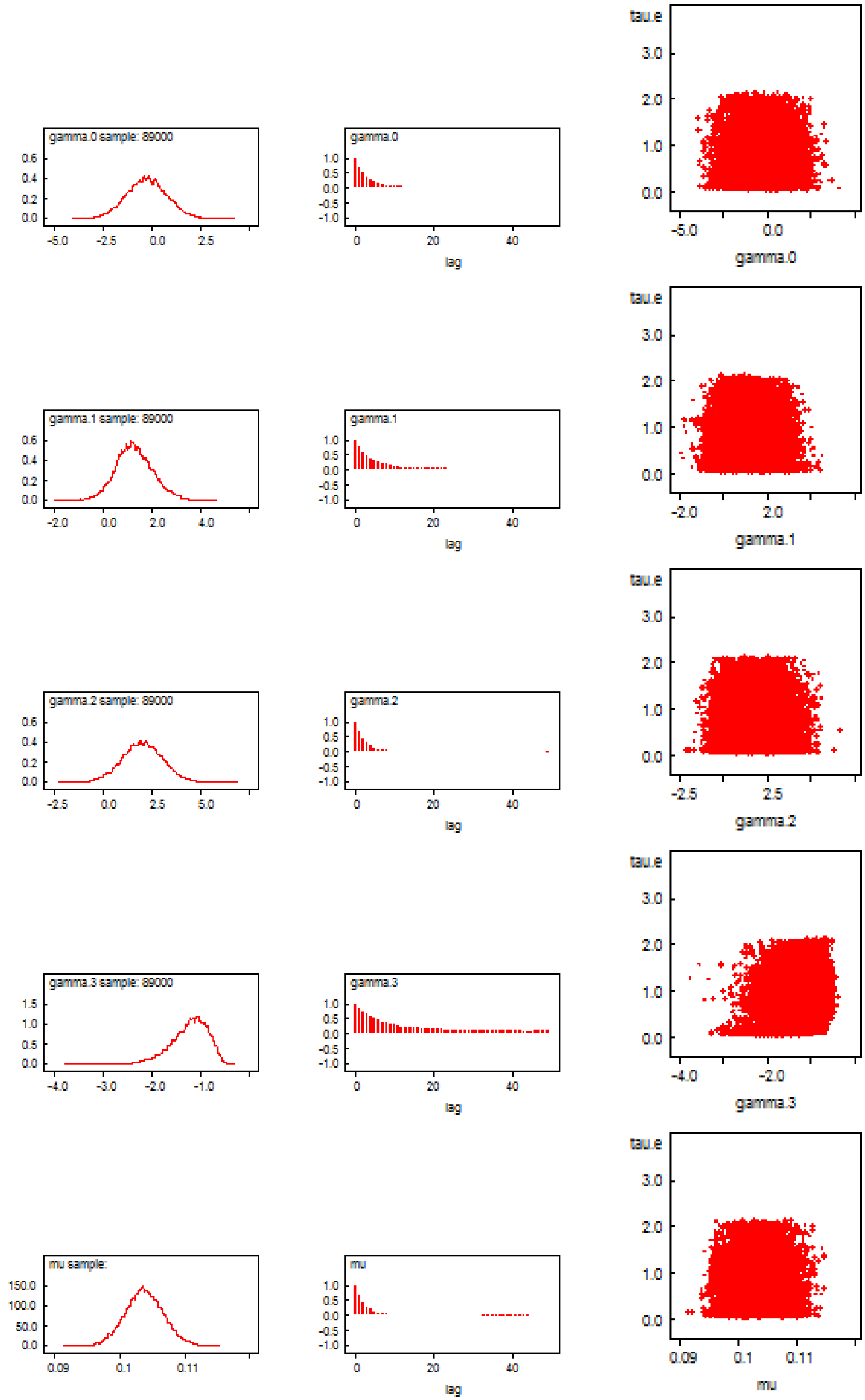

Figure 15: Bayesian MCMC diagnostics, expanded control vector. *Left:* density plots; *center:* ACF plots, and *right:* scatter plots against τ_ϵ for γ_0, γ_1 and parameter μ_1.

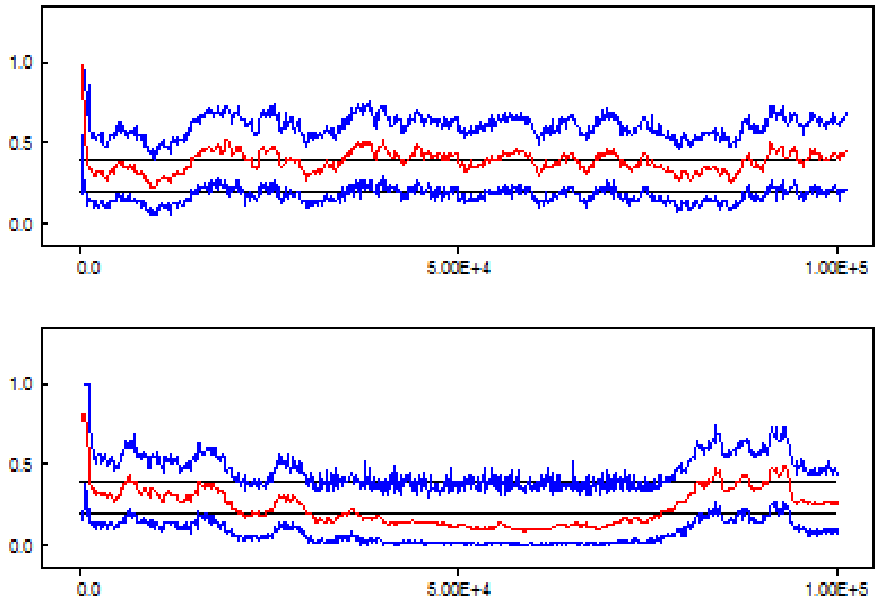

Figure 16: Bayesian MCMC diagnostics: metropolis acceptance rates. *Top:* baseline specification; *bottom:* expanded control vector

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