BIS

Working paper No. 34

MONETARY POLICY AND THE BEHAVIOUR OF INTEREST RATES: ARE LONG RATES EXCESSIVELY VOLATILE?

by

Stefan Gerlach

January 1996

BANK FOR INTERNATIONAL SETTLEMENTS
Monetary and Economic Department
BASLE

MONETARY POLICY AND THE BEHAVIOUR OF INTEREST RATES: ARE LONG RATES EXCESSIVELY VOLATILE?

by

Stefan Gerlach

January 1996

Abstract

This paper employs data on short and long interest rates for the G-10 countries, Australia, Austria and Spain to assess the expectations hypothesis (EH) of the term structure, using the Campbell-Shiller (1987, 1991) methodology. Although the EH is rejected in several countries, in all countries actual and theoretical long interest rates do move closely over time. This finding suggests that, at least from a monetary policy perspective, it is appropriate to view long interest rates as determined largely by expectations held by financial market participants concerning the future path of short term interest rates.

Contents

1. Introduction[1]

The relationship between short and longer-term interest rates plays an important role in the conduct of monetary policy. While central banks typically implement monetary policy by changing the availability and price of credit to the banking system in order to guide market-determined short-term rates, longer-term rates are likely to play a more important role in affecting households' and firms' spending decisions.[2] For instance, bank lending rates, in particular mortgage rates, may be linked formally or informally to long-term rates. Temporary movements in short-term rates may therefore have little impact on aggregate demand for goods and services. Long interest rates are also important because they are used by monetary policy-makers as informal indicators of inflation expectations in the financial markets. In addition, many central banks, in particular those which target inflation directly, use forward interest rates computed on the basis of the term structure of interest rates as indicators of expected future inflation rates.[3]

Although long-term rates rate play an important role in the design and implementation of monetary policy, there is a broad consensus between economists in and outside the central banking community that the determination of long-term rates is poorly understood. In particular, there is considerable evidence, both anecdotal and more formal, that long interest rates are "excessively" volatile in the sense that they seem to vary more than is warranted by economic fundamentals.[4] If sufficiently large, such excess volatility would reduce the information content of long interest rates and could render them of little value as information variables. Moreover, by weakening the link between short and long-term interest rates, excess volatility would make it more difficult for central banks to anticipate the responses of long rates to policy changes, and thus complicate the conduct of monetary policy.

A potential source of excess volatility is the existence of time-varying term premia. Thus, one way to assess whether long rates are excessively volatile is to test for the existence of such term premia. This can be done by testing whether the behaviour of long-term interest rates is compatible with the expectations hypothesis (EH) of the term structure, which states that long rates are determined by expected future levels of short-term interest rates plus, potentially, a *constant* term premium. Of course, the conventional wisdom is that the EH is easily rejected, and that time-varying term premia are pervasive in financial markets. Shiller (1990, p. 670), for instance, in his survey of

---

[1] I am very much indebted to Kostas Tsatsaronis for many useful discussions regarding the Campbell-Shiller methodology, and for showing me how to construct and calculate the Wald tests of the restrictions implied by the expectations hypothesis; and to Philippe Hainaut and Christian Dembiermont for assembling the data. Responsibility for remaining errors is my own.

[2] Goodfriend (1995) contains a clear discussion of the relationship between short and long interest rates and how these relationships are induced by systematic monetary policy.

[3] See, for instance, Bank of England (1995) or Sveriges Riksbank (1995).

[4] See Shiller (1990) for a survey of the empirical and theoretical literature on the term structure of interest rates.

the term structure literature in the *Handbook of Monetary Economics* concludes that "empirical work on the term structure has produced consensus on little more than that the rational expectations model ... can be rejected".

Recently, however, several authors have presented evidence that suggests that this conclusion may warrant reassessment. Using data from the far end of the term structure, Campbell and Shiller (1987) in their seminal paper show that while the restrictions imposed by the EH are easily rejected on data from the United States, spreads between counterfactual long rates (computed under the assumption that the EH is true) and short rates evolve over time in much the same way as actual spreads do. Thus, while the EH may be rejected on statistical grounds, it may nevertheless be the case that movements in expected future short interest rates explain a large fraction of movements in long interest rates. If so, the EH may, in this sense, have considerable economic content. Further evidence in support of the EH is provided by Hardouvelis (1994), who tests a number of different implications of the EH using data on three-month and ten-year rates for the G-7 countries.

A number of recent studies using data from the short end of the term structure have also found that it is easier to reject the EH hypothesis on recent data from the United States than on data for other time periods or other countries.[5] Mankiw and Miron (1986) use data on three and six-month interest rates to show that the EH does a much better job in accounting for the behaviour of the term structure of interest rates before the founding of the Federal Reserve in 1913. Mankiw and Miron argue that this finding is due to the fact that short-term rates were more predictable before the First World War. Further evidence in support of Mankiw and Miron's hypothesis is presented by Kugler (1988). Using short-term Euro-rates for the United States, Germany and Switzerland, Kugler shows that the spread between long and short interest rates does a much better job in predicting future short-term rates when central banks pursue money stock rules than when they smooth short-term interest rates. Using essentially the same data (but a different methodology), Kugler (1990) rejects the EH for the United States, but not for Germany and Switzerland, and interprets this as providing further evidence that central bank operating procedures play an important role in determining the predictive content of interest rate spreads. Gerlach and Smets (1995) test the EH using short-term Euro-rates for seventeen countries and find that, by and large, the EH does a good job in accounting for the behaviour of the term structure at the short end. The two most striking exceptions to this are the United States and Austria. They also show, as suggested by Mankiw and Miron (1986), the EH seems to fit the data better the more variable the expected changes in one-month interest rates are. Dahlquist and Jonsson (1995) also fail to reject the EH using Swedish data on short-term bills. In sum, there are many reasons for doubting whether the conventional wisdom that the EH is incompatible with the behaviour of short and long rates is right.

The purpose of this paper is to examine whether long-term interest rates in the G-10 countries, Australia, Austria and Spain appear to be largely determined by expectations about future short-term interest rates, or whether they display so much excess volatility that they are of little use as

---

5 See also Hardouvelis (1994).

indicators of interest rate expectations in financial markets for monetary policy purposes. Since the emphasis is thus not to formally test the EH, we pursue the analysis using the methodology proposed by Campbell and Shiller (1987) and compare actual long rates with the long rates one would observe if the EH is correct. However, for completeness, we also test the restrictions imposed by the expectations hypothesis.

The paper is organised as follows. In Section 2 we briefly review the expectations hypothesis and Campbell and Shiller's methodology for assessing the role of expectations in driving long interest rates. Section 3 contains a discussion of the empirical work. Using data beginning, depending on country, between the mid-1950s and the early 1980s and ending in 1991:4, we calculate counterfactual long rates under the assumption that the EH is correct. We show that the actual and theoretical long rates follow each other quite closely in all countries. However, formal tests reject the expectations hypothesis in several cases. We also calculate out-of-sample predictions of long rates using data spanning 1992:1-1995:2. The results suggest that actual and counterfactual long rates followed each other quite closely in this period despite the fact that interest rates displayed considerable movements in many countries . Conclusions are offered in Section 4.

2. The model

In this section we review the theoretical underpinnings for the empirical work that follows and present the econometric methodology, which is due to Campbell and Shiller (1987, 1991).[6] Since the empirical test is performed on data on yields to maturity for coupon-paying bonds, the presentation follows Hardouvelis (1994).

2.1 The expectations hypothesis

To see what the EH implies for the joint behaviour of long and short interest rates, let R_t denote the yield to maturity of a bond that matures in n periods, r_t the yield on a one-period instrument, ϕ_t a term premium, and E_t the expectations operator, conditional on information available at time t. In the empirical work below the long rate applies typically to ten-year bonds and the short rate to three-month securities. The yield on long bonds can be decomposed into a weighted average of expected future short-term rates and a term premium[7]

$$(1) \qquad R_t \equiv \sum_{i=0}^{n-1} w_i E_t r_{t+i} + E_t \phi_t,$$

where

[6] A number of authors have used this methodology to assess the EH: see MacDonald and Speight (1988), Kugler (1990), Taylor (1992), Hardouvelis (1994) and Engsted and Tanggaard (1995).

[7] See Shiller (1979, 1981) for a derivation of equation (1).

$$w_i \equiv g^i(1-g)/(1-g^n)$$

and $g \equiv (1+\bar{R})^{-1}$, where $\bar{R}$ denotes the mean level of the long interest rate in the sample. Equation (1) states that the long interest rate equals the weighted sum of the expected future one-period rates. As shown by Shiller (1979) and Shiller, Campbell and Shoenholtz (1983), the need for the weights arises from the fact that coupon-carrying bonds are used: since coupon bonds derive a large part of their value from payments made in the near future, it is appropriate to weight expected near-term one-period rates relatively heavily in equation (1).

Three aspects of the weighting scheme deserve comment. First, the weights follow a truncated Koyck distribution and sum to unity. Second, the weights are linked to $\bar{R}$: if the mean level of interest rates were to rise, the weights attached to one-period rates in the near term would rise and the weights attached to one-period rates further in the future would fall. To understand the reasons for this, it should be recalled that the linearisation underlying equation (1) assumes that new bonds are issued at par, so that the coupon rate equals the yield to maturity. An increase in the level of interest rates should thus be interpreted as an increase in the part of the returns that stems from coupon payments, which in turn shortens the duration of the bond. Third, if pure discount bonds were used, so that the coupon rate was zero, $g=1$ and $w_i = 1/n$.

Subtracting the short interest rate from both sides of equation (1) yields the following expression for the spread between long and short interest rates

(2) $$\sum_{i=0}^{n-1} w_i E_t r_{t+i} - r_t = R_t - r_t + E_t\phi_t,$$

which can be rearranged to yield

(3) $$(1-g^n)^{-1}\sum_{i=1}^{n-1}(g^i - g^n)E_t\Delta r_{t+i} = R_t - r_t + E_t\phi_t.$$

Equation (3) plays a critical role in what follows. To interpret it, recall that under the EH, $E_t\phi_t$ is constant. In that case, the term spread, $R_t - r_t$, is a direct measure of expected changes in short-term interest rates between time period t+1 and t+n-1.

2.2 Econometric analysis

Next we review the econometric methodology. Campbell and Shiller (1987, 1991) note that equation (3) imposes restrictions on the parameters in a bivariate VAR for the spread between long and short interest rates, $S_t \equiv R_t - r_t$, and the change in the short term rate, Δr_t. To see how Campbell and Shiller implement their test, consider the following first-order VAR

(4) $$Z_t = AZ_{t-1} + v_t,$$

where $Z_t \equiv [\Delta r_t \quad S_t]^T$, A is a matrix of VAR coefficients and v_t a vector of residuals. By measuring Z_t in deviations from its mean, constant terms do not appear in equation (4). Of course, since higher-order VAR systems can always be written in VAR(1) form, equation (4) imposes no restrictions on the order of the VAR.

The usefulness of the VAR representation stems from the fact that multi-period forecasts of future changes in the short-term rate can be constructed as

(5) $$E_t \Delta r_{t+j} = h_1^T A^j Z_t,$$

where $h_1^T \equiv [1 \quad 0]$ is a vector that selects the first element of the $A^j Z_t$-vector. Defining $h_2^T \equiv [0 \quad 1]$ (so that $h_2^T Z_t = S_t$), equation (4) can now be written as

(6) $$(1-g^n)^{-1} \sum_{i=1}^{n-1} (g^i - g^n) h_1^T A^i Z_t = h_2^T Z_t.$$

Before proceeding, it is useful to review equation (6) in some detail. First, note that the LHS captures the expectations, as embodied in the VAR, of changes in future short-term interest rates. By equation (3), the LHS can be thought of as the spread that would be observed if the EH was true. Campbell and Shiller (1987, 1991) refer to this as the theoretical spread, S_t^*, from which a theoretical long rate, R_t^*, can be calculated. It should be stressed that the appeal of Campbell and Shiller's econometric methodology is precisely that it provides an estimate of the spread or, alternatively, the long rate *under the hypothesis that the EH is true.* By comparing actual and theoretical spreads, the researcher can assess the economic, as opposed to statistical, significance of the hypothesis. In particular, it allows the researcher to determine, even if the restrictions implied by the EH are statistically rejected, how large a fraction of the movements in long interest rates is explained by movements in expected future short rates.

To proceed, recall that the RHS in equation (6) is simply the currently observed spread. Thus, if the EH was true, the LHS and the RHS of equation (6) should be equal, that is,

(7) $$(1-g^n)^{-1} \sum_{i=1}^{n-1} (g^i - g^n) h_1^T A^i = h_2^T.$$

As shown by Kugler (1990) and more clearly in the Technical Appendix, it is possible to formally test the restrictions imposed by the EH. For the time being, however, note that these restrictions involve solely the companion matrix, A, in equation (4). If the elements of A are estimated imprecisely, for instance because the VARs are overfitted, it will be difficult to reject the restrictions in equation (7). Thus, in order not to accept the hypothesis when it is false, it is important to have tight estimates of A. This, in turn, suggests that it is desirable to have a long sample period, and to select a relatively low-order VAR.

In sum, the first step of the Campbell and Shiller methodology involves the estimation of a VAR for the change in the short rate and the spread between long and short interest rates. The VAR is then used to forecast future short-term rates, and the predicted short-term rates are used to compute a counterfactual (or theoretical) long interest rate under the assumption that term premia are constant. Finally, the behaviour of actual and theoretical spreads -- or, equivalently, actual and theoretical long interest rates -- are compared in order to assess informally how well the EH explains movements in the term structure over time.

3. Empirical work

3.1 Preliminaries

In order to implement the above test, end-of-period quarterly data on three-month rates and long, usually ten-year, rates were collected for the G-10 countries, Australia, Austria and Spain. In view of the number of parameters that are required for the VAR models, it is desirable to have at least forty to fifty observations or about ten years of data. In several cases it did not prove possible to find ten-year yields going back as far and shorter interest rates had to be used. Thus, for Belgium a five-year rate was used, for the Netherlands a five to eight-year rate (treated as having a maturity of six years) and France a seven-year rate. For Austria a nine to ten-year rate (treated as a nine-year rate) was taken and for Sweden a four to five-year rate (treated as a four-year rate). In Switzerland, the rate on confederation bonds was used, with an assumed maturity of seven years.[8] In the case of Italy, the long yield is an average for bonds with a remaining maturity of more than one year. The average maturity of the bonds has since the mid-1980s been in the order of four to five years; for the calculations below the maturity was assumed to be four years. Finally, in Spain a five-year rate was used. The Data Appendix provides detailed information about the data series chosen.

There are large differences between countries with respect to the time period for which data, particularly on the long interest rate, are available. The work reported here strives to use all the data available: depending on country, the sample periods thus start as early as 1954 or as late as 1988. In several countries, however, while data are available for a considerable time period, regulatory barriers may have limited the role of market forces in determining interest rates in the early part of the sample. In these cases, the data from this part have been dropped.

The first step of the empirical analysis aims at establishing the appropriate lag length for the VARs. Since the study expands on Hardouvelis (1994), who estimates fourth-order VARs for all the G-7 countries, this lag length was a natural choice. However, it is difficult to believe that interest rates from as far back as four quarters ago would be useful in predicting future interest rates. To guide

---

8 Informal sensitivity checks suggested that the results were not materially affected by small changes in the assumptions regarding maturity.

the selection process, Akaike and Schwarz information criteria and Ljung-Box Q-statistics for whiteness of the residuals were calculated. Furthermore, sequential likelihood ratio tests were performed for the hypothesis of a third-order VAR against a fourth-order VAR; a second-order VAR against a fourth-order VAR; and a first-order VAR against a fourth-order VAR. Finally, the estimated parameters of the VAR were investigated. When a sufficient number of data points were available, a second-order VAR model was selected even when the tests suggested that a first-order model was appropriate.[9] Table 1 provides information about the selected order of the VAR and the sample period for the empirical analysis. Graph 1 contains time series plots of the short and long interest rates.

3.2 Tests of the expectations hypothesis

Next we turn to the results from the Campbell-Shiller analysis. Since monetary policy-makers typically focus their attention on the level of long interest rates rather than the spread, we present the results for the theoretical long rate, R_t^*, rather than the theoretical spread, $S_t^* \equiv R_t^* - r_t$, as is common in the literature. The usefulness of the expectations hypothesis is judged in three different ways.

First, we present time series plots of the actual and theoretical long rates, and the discrepancy between them. This provides an informal measure of how large a fraction of long interest rates is accounted for by expectations of future short-term rates.

Second, we examine the set of informal statistical measures of how closely the actual and the theoretical long rates move together typically used in the literature applying the Campbell-Shiller method. In particular, we present the slope coefficient in the regression

$$S_t^* = \delta + \gamma S_t + \varepsilon_t$$

the standard deviations of the theoretical and actual spreads (σ_{S^*} and σ_S), the ratio of the standard deviations (σ_{S^*} / σ_S), the correlations between the theoretical and actual spreads ($\rho_{S^*,S}$), the correlation between the changes in the theoretical and actual spreads ($\rho_{\Delta S^*,\Delta S}$). If movements in long interest rates are dominated by expectations of the future path of short interest rates, we would expect γ to be close to unity and the standard deviations of the actual and theoretical spreads to be similar, so that their ratio is close to unity, and the correlations to be close to unity. We also provide the standard deviation, measured in basis points, of the difference between the actual and theoretical long rates (σ_{R-R^*}). This measure gives an indication of how much actual and theoretical long rates deviated in the sample, and thus some idea of how closely we would expect them to differ out-of-sample. We also follow Kugler (1988) and provide the Chi-squared, and the associated marginal significance level (MSL), for formal Wald tests of the restriction imposed by the EH.

[9] In view of the limited number of data points for Spain, a first-order VAR model, as suggested by the different tests, was adopted.

Third, since the estimation period ends in 1991:4, in Section 3.6 we construct out-of-sample predictions of long interest rates for the period 1992:1-1995:2.[10] Given the close attention monetary policy makers have paid to the large falls in long bonds yields in late 1993 and the subsequent reversal that occurred in a number of markets in early 1994, it is particularly interesting to see how well the theoretical long rates calculated using the Campbell-Shiller method track actual long rates over this period.

3.3 Results for the United States

Since Campbell and Shiller's original analysis was performed on data for the United States, we review the results for this country in some detail.

Consider first Graph 2, which contains the actual and theoretical long interest rates and the difference between the two. Recall that the latter is computed under the assumption that the expectations hypothesis is true (that is, the long rate is the weighted sum of the predicted future short rates implied by the VAR). The graph illustrates that the theoretical and actual long rates evolve over time in broadly similar ways, which suggests that a large fraction of the movements in the long interest rate is due to shifting expectations about the future path of short interest rates. Despite this, however, there are some episodes during which there are large differences between the two rates. In particular, in 1973-74 actual long rates fell below the theoretical long rates. This was also the case in the 1978-82 period, when long rates rose dramatically.

Table 2 provides further information about how well the expectations hypothesis explains the behaviour of the long interest rates. Note first that the correlation between the levels ($\rho_{S^*,S}$) of as well as between the changes ($\rho_{\Delta S^*,\Delta S}$) in the actual and theoretical spreads is in both cases quite high. However, the variance of the theoretical spread, $R_t^* - r_t$, is about 40% of the variance of the actual spread, $R_t - r_t$. Thus, long rates appear considerably more variable than the predicted future path of short-term rates. Furthermore, the standard error of the difference between the actual and theoretical long rate, σ_{R-R^*}, is very large (78 basis points).

These statistics suggest that the EH is rejected by the data. To formally test the EH, we follow Kugler (1990) and calculate a Wald test of the restrictions in equation (7). The test statistic is 165.2, which is far beyond the 95% critical value of 12.6 for a $\chi^2(6)$. Thus, the marginal significance level is essentially zero, and we conclude that the observed differences between the actual and theoretical rates are statistically different. Despite this statistical rejection it appears, as stressed by Campbell and Shiller (1987), that movements in expected future short interest rates account for a very large fraction of the variance of long interest rates so that, in this sense, the EH does have considerable economic content.

---

[10] For data reasons, the VAR for Spain is estimated using data for 1989:4-1993:2, and the out-of-sample predictions are computed for the period 1993:3-1995:2.

3.4 Countries accepting the EH

Next we consider the results for the countries for which we do not reject the EH hypothesis, that is, Australia, Canada, France, Germany, Japan, the Netherlands, Switzerland and the United Kingdom. While we technically do not reject the EH hypothesis for Italy and Spain for the specific sample period for which the tests are reported, the results are sensitive to the choice of sample period. We therefore review the results for these countries together with the countries for which the restrictions are rejected outright.

As a first step, it is instructive to consider the plots of the actual and theoretical long rates in Canada, for which we have data for almost as long as the United States. Despite the fact that long interest rates evolve over time in a way very similar to those in the United States, the actual and theoretical long rates follow each other much more closely in Canada. The graphs for Switzerland and the United Kingdom (for which the sample period starts in the mid-1960s) and France and Germany (for which the sample period starts in the early 1970s) also display much smaller discrepancies between actual and theoretical long rates than does the graph for the United States. The graphs for Australia, Japan and the Netherlands similarly suggest that the EH does a good job in accounting for the behaviour of long interest rates in these countries.

To more formally assess the extent to which long interest rates reflect expectations of future short-term rates, consider Table 2. As indicated, the ratio of the variance of the theoretical to the actual spread is in most cases about 0.8-0.9. While this is higher than in the United States, in all countries the theoretical spread is less volatile than the actual spread, which suggests that time-varying risk premia may be present. Note also that the correlation of the levels of (and changes) the two spreads is typically about 0.9, and thus higher than in the United States, and that the standard deviation of the difference between actual and theoretical long rates is on average much smaller. Finally, and as already indicated, in no case are the restrictions implied by the EH rejected at the 5% level.[11]

3.5 Countries rejecting the EH

We conclude the review of the in-sample performance of the EH by considering the results for Austria, Belgium and Sweden, for which the hypothesis is rejected. While the EH is accepted by the Italian and Spanish data, the results for these countries are very sensitive to the exact choice of sample period. We therefore include Italy and Spain in this group.

While the test statistics in Table 2 clearly reject the EH, it is difficult to see a common cause of the rejections. For instance, the ratio of the standard deviation of the theoretical and actual spreads and the correlation coefficients do not appear fundamentally different from those in the previous group of countries. It may therefore be helpful to review the results for each country in more detail.

---

[11] However, the MSL for the Netherlands is 7%. Since, as argued below, the power of the test may be weak, the results should perhaps be interpreted as rejecting the EH.

As indicated by the different statistics in Table 2, the EH clearly does a very poor job in accounting for the behaviour of long rates in *Austria*. Since interest rates in Austria followed those in Germany quite closely in the estimation period, it is surprising that the EH is so easily rejected by the Austrian data. One potential explanation is that the sample period is simply too short. Indeed, the plots of the Austrian interest rates in Graph 1 do not suggest much of a relationship between the two rates in the estimation period (1983:4-1991:4). Perhaps the results would have been favourable to the EH if a longer data period had been used for estimation.

While the EH is clearly rejected by the Wald test, the results for *Belgium* in Table 2 are very similar to those for countries in which the model is accepted. It is therefore of interest to consider the time series plots of the short and long rates in Graph 1. As can be seen, the volatility of quarter-to-quarter changes in the short-term interest rates was much more pronounced before 1981. It may therefore be that the rejection of the EH stems from the fact that the data were generated by two distinct regimes rather than one as the VAR assumes.

The descriptive statistics for *Sweden* in Table 2 are also very similar to those for the countries in the previous group despite the fact that the EH is rejected. In particular, the ratio of the standard deviations of the two spreads, σ_{S^*} / σ_S, is close to unity, as are the estimated correlation coefficients for the levels, $\rho_{S^*,S}$, of and the changes, $\rho_{\Delta S^*,\Delta S}$, in the spreads. Also, the time series plots of the actual and theoretical long rates in Graph 1 do not point to any obvious break in the behaviour of the two rates. Thus, in the case of Sweden it is difficult to find an obvious explanation for the rejection of the EH.

Finally, consider the results for *Italy* and *Spain*. Technically, in both cases the EH is accepted by the data. However, the results are very sensitive to the choice of starting period. For instance, shortening the sample period for Italy by a few years leads to a rejection of the EH. Similarly, small changes in the sample period for Spain also cause the hypothesis to be rejected. In view of this sensitivity, the EH should be interpreted as being rejected by the data for the two countries.

One conclusion suggested by these results is that the Campbell-Shiller methodology is sensitive to the length of the sample period, potentially because of "peso problems". To see how these could arise, suppose that over a period of a few years market participants believe that the central bank will tighten monetary policy, but the central bank does not do so, for instance because an unexpected recession set in. If the sample period is short, so that there is no period of offsetting expectation errors in the other direction, tests of the EH are likely to reject. To phrase this differently, we may never be able to infer much about the extent to which interest rate expectations determine ten-year yields when the length of the sample period is only a few years. As this example suggests, it is inherently difficult to test the EH in short samples.

3.6 Out-of-sample predictions

It is hazardous to evaluate empirical models solely on the basis of in-sample performance. Since the estimation period ends in 1991:4, we next use out-of-sample data for the period 1992:1-1995:2 in order to assess how well the theoretical long rates implied by the fitted VARs match actual long rates in this period. These out-of-sample predictions are probably best interpreted as an informal test of the stability of the companion matrix in equation (4). Since the term structure of interest rates also experienced large changes during the turmoil in European foreign exchange markets in 1992-93, and long bond yields fell drastically in many countries in late 1993 only to rise abruptly in early 1994, the period used for the out-of-sample predictions is quite turbulent and should provide an interesting basis for assessing the model's predictive abilities.

In order to convey a sense of how well we would expect actual and theoretical long rates to follow each other, we have drawn a band around the theoretical rate with a width of two standard deviations of the discrepancy between the actual and theoretical long rate in the estimation period, that is, $\pm\, 2\, \sigma_{R-R^*}$. These bands do not take parameter uncertainty into account, and should therefore not be thought of as confidence bands.[12]

Briefly, the results in Graph 3 suggest that the actual long rates behaved very much as one might have expected in the out-of-sample period. Of course, the bands drawn in the graph are very broad, typically ± 60 basis points. Thus, actual long rates can move considerably relative to the theoretical rate without leaving the band.

4. Conclusions

Several conclusions follow from the results of the exercise above. The first of these is that the theoretical long rates explain a large fraction of the variance of observed long rates. Thus, as Campbell and Shiller (1987) concluded, the expectations hypothesis, even if formally rejected by statistical testing, does appear to have considerable economic content. One implication of this finding that is of considerable importance for central banks is that it seems sensible, at least for monetary policy purposes, to interpret movements in long interest rates as being largely determined by financial market expectations about the future path of short-term rates. In other words, long interest rates do not appear to be much more volatile, in this informal sense, than one would expect given the time series behaviour of short interest rates.

A second conclusion is that in a number of countries the EH is not rejected by the data. This conclusion must be qualified by the fact that the power of the Campbell-Shiller test is likely to be low. To see the reason for this, recall that the Campbell-Shiller methodology tests the EH restrictions essentially by asking whether the currently observed long interest rate is equal to the

[12] Out-of-sample prediction errors are associated with two sources of uncertainty: the first of these is the variance of the regression errors, and the second is the fact that the estimated parameters are subject to some uncertainty. This second source of uncertainty is disregarded here. See, for example, Pidyck and Rubinfeld (1991, Ch.8) for a discussion.

discounted future path of short interest rates as predicted by the VAR model. Since VARs involve a large number of parameters and thus exhaust degrees of freedom rapidly, the VAR parameters are likely to be relatively imprecisely estimated. The confidence bands associated with the predictions of future short-term rates are therefore likely to be very wide, and it may thus be difficult to reject the restrictions implied by the EH even when they are false. One reason to believe that the power of the test is low is the fact that the standard error of the discrepancy between actual and theoretical long rates are quite large, even when the restrictions are accepted.[13]

The third conclusion is that the behaviour of long interest rates in the United States does appear to be different from that of long rates elsewhere. It is particularly striking that while the EH is easily rejected in the United States, the same restrictions are not rejected by the Canadian data, despite the fact that the sample period and the time series plots of long and short rates are similar. It remains an important task for future research to explore further the possible explanation for this difference.

A fourth conclusion is that the Campbell-Shiller methodology appears to be sensitive to the length of the sample period. Not only in the model frequently rejected for the countries for which the sample period is short, but the results for Italy, Sweden and Spain are also sensitive to the choice of starting date, so that the model should probably be interpreted as rejected. One possible explanation for the tendency of the test to reject in short samples is the occurrence of "peso problems", that is, financial markets may have anticipated a very different path of short interest rates from the one that actually occurred in the sample period. Another possible explanation is that interest rate relationships may have shifted in the 1980s because of the continuing process of financial deregulation in many countries.

[13] The lack of power is also evident from Hardouvelis (1994), who fails to reject the EH using the Campbell-Shiller methodology with data for the G-7 countries, but easily rejects the implication of the EH that the slope parameter should be positive in a regression of the change in the long rate on the lagged long/short spread.

Table 1
Sample periods and order of VAR

	Sample	VAR
Australia	1981:1–1991:4	2
Austria	1983:4–1991:4	2
Belgium	1966:1–1991:4	3
Canada	1957:1–1991:4	4
France	1971:1–1991:4	2
Germany	1971:1–1991:4	2
Italy	1981:1–1991:4	3
Japan	1981:1–1991:4	3
Netherlands	1980:1–1991:4	3
Spain	1989:4–1993:2	1
Sweden	1985:2–1991:4	2
Switzerland	1963:4–1991:4	2
United Kingdom	1965:4–1991:4	2
United States	1954:2–1991:4	3

Table 2
Diagnostic statistics

	Australia	Austria	Belgium	Canada	France	Germany	Italy
	1981:1–1991:4	1983:4–1991:4	1966:1–1991:4	1957:1–1991:4	1971:1–1991:4	1971:1–1991:4	1981:1–1991:4
γ	0.799	–0.027	0.904	0.879	0.929	0.828	0.432
σ_{S*}	4.699	0.448	3.881	3.410	3.255	4.057	1.317
σ_S	5.838	3.100	4.159	3.822	3.411	4.795	1.998
σ_{S*} / σ_S	0.805	0.145	0.933	0.892	0.954	0.846	0.659
$\rho_{S^*,S}$	0.992	–0.183	0.969	0.985	0.973	0.978	0.655
$\rho_{\Delta S^*, \Delta S}$	0.956	0.304	0.910	0.859	0.943	0.890	0.910
σ_{R-R*}	0.523	1.285	0.414	0.299	0.313	0.471	0.604
Wald test	0.914	690.565	62.178	8.596	6.078	1.277	7.233
MSL	0.923	0.000	0.000	0.377	0.193	0.865	0.300

Notes: γ is the slope parameter in a regression of S* on S and a constant; σ_{S*} / σ_S is the standard deviation (multiplied by 1,000) of the actual (theoretical) spread; $\rho_{S^*,S}$ is the correlation between S and S*; $\rho_{\Delta S^*, \Delta S}$ is the correlation between ΔS and $\Delta S*$; σ_{R-R*} is the standard deviation of the difference between the actual, R, and theoretical, R*, long rates; the Wald test is distributed as a $\chi^2(2p)$, where p denotes the order of the VAR.

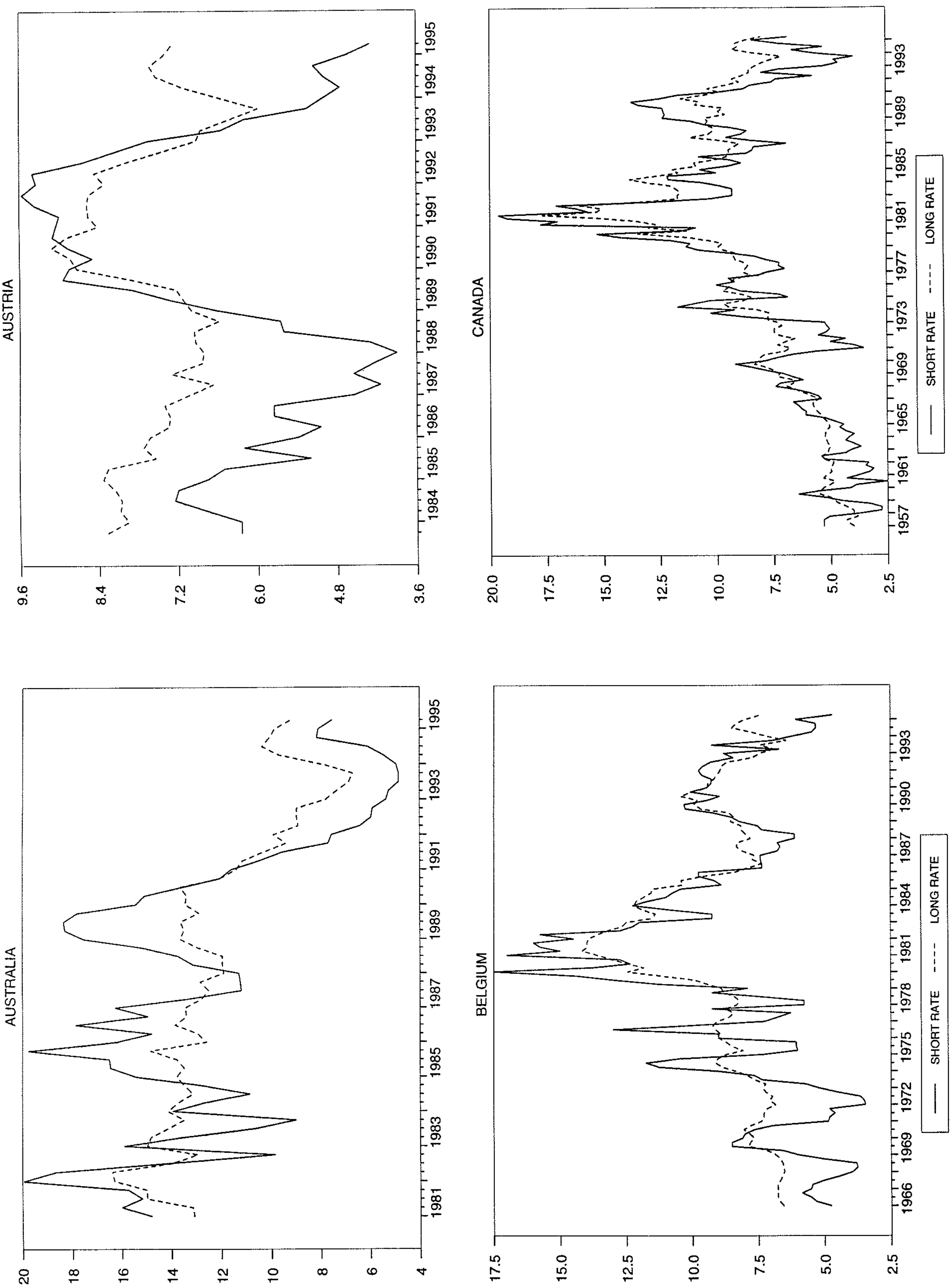
Table 2 (continued)

Diagnostic statistics

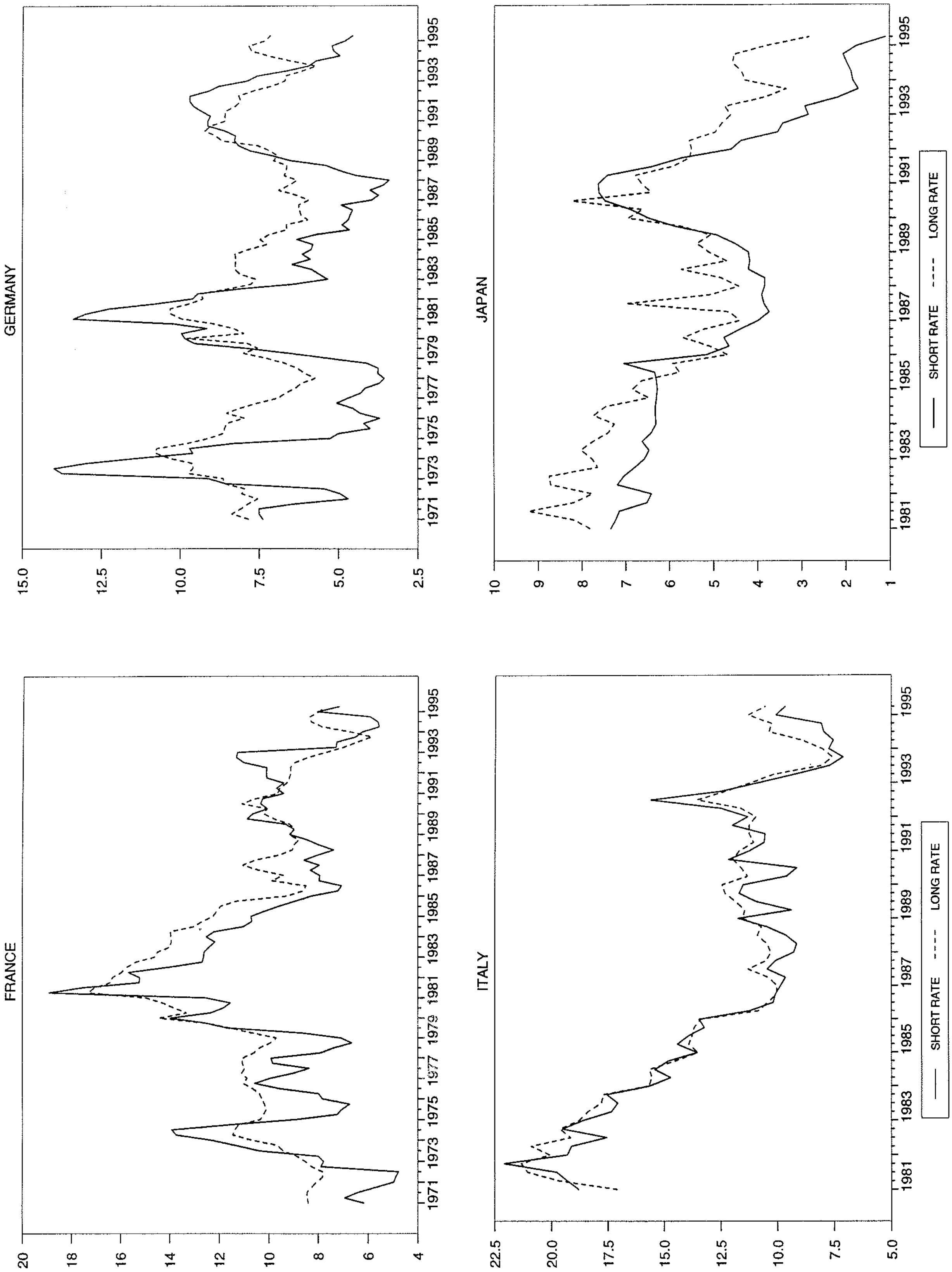
	Japan	Netherlands	Spain	Sweden	Switzerland	United Kingdom	United States
	1981:1–1991:4	1980:1–1991:4	1989:4–1993:2	1985:2–1991:4	1963:4–1991:4	1965:4–1991:4	1954:2–1991:4
γ	0.742	0.547	1.210	1.155	0.913	0.777	0.369
σ_{S*}	1.776	1.516	1.949	4.147	4.459	4.082	1.240
σ_S	2.200	2.653	1.604	3.490	4.831	5.231	2.949
σ_{S*} / σ_S	0.807	0.571	1.215	1.188	0.923	0.780	0.420
$\rho_{S^*,S}$	0.918	0.958	0.996	0.972	0.989	0.996	0.878
$\rho_{\Delta S^*,\Delta S}$	0.844	0.809	0.985	0.972	0.805	0.922	0.794
σ_{R-R*}	0.361	0.511	0.152	0.446	0.317	0.487	0.781
Wald test	8.819	11.577	1.981	12.076	1.223	5.153	165.192
MSL	0.184	0.072	0.371	0.017	0.874	0.272	0.000

Notes: γ is the slope parameter in a regression of S* on S and a constant; σ_{S*} / σ_S is the standard deviation (multiplied by 1,000) of the actual (theoretical) spread; $\rho_{S^*,S}$ is the correlation between S and S*; $\rho_{\Delta S^*,\Delta S}$ is the correlation between ΔS and ΔS^*; σ_{R-R*} is the standard deviation of the difference between the actual, R, and theoretical, R*, long rates; the Wald test is distributed as a $\chi^2(2p)$, where p denotes the order of the VAR.

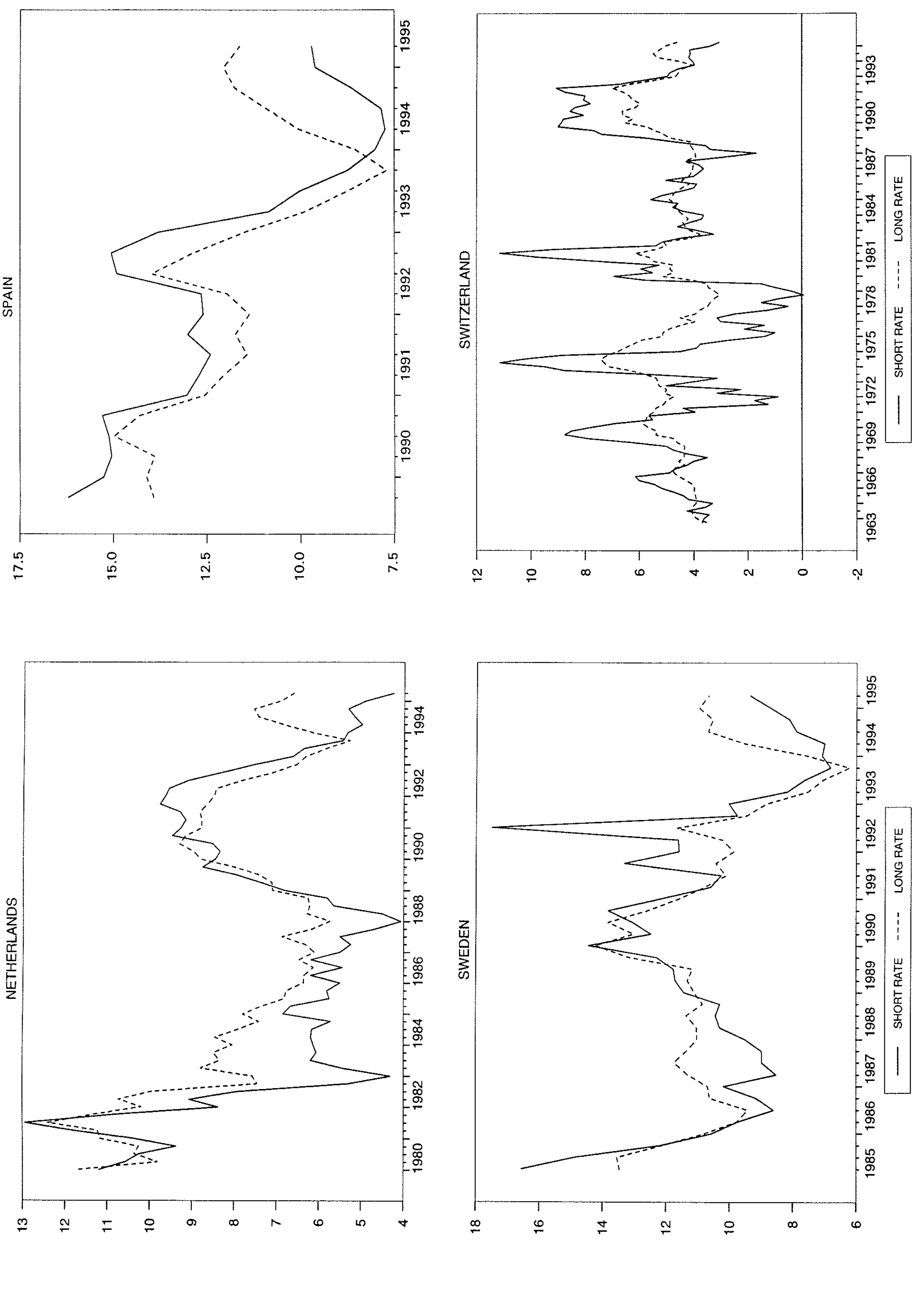
Graph 1
Short and long-term interest rates



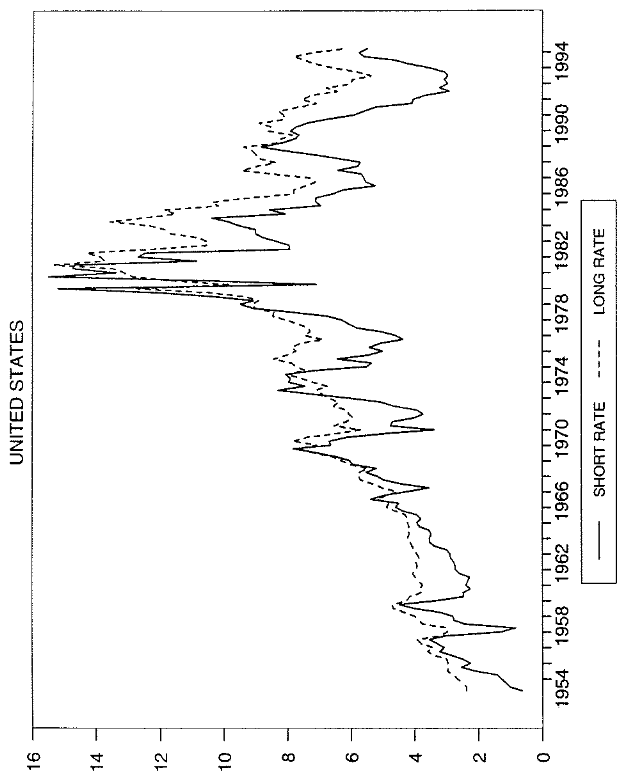
Graph 1 (cont.)

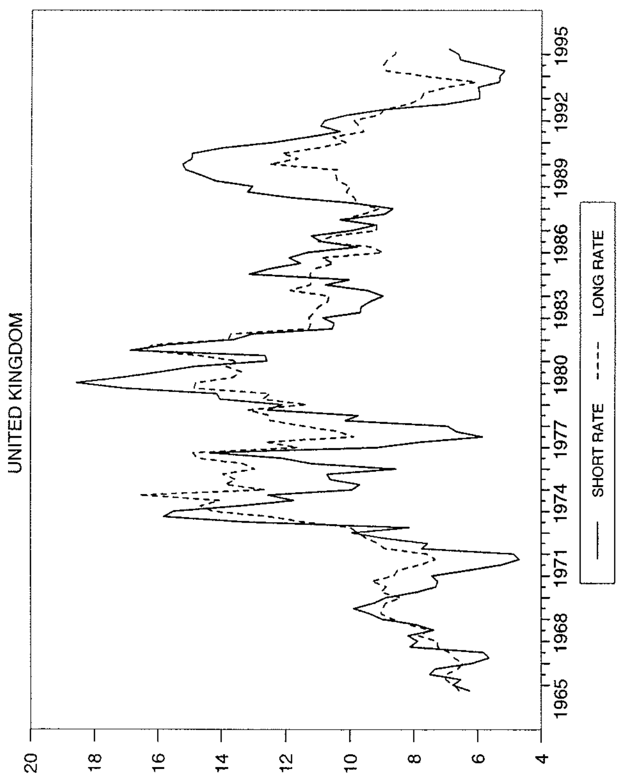


Graph 1 (cont.)

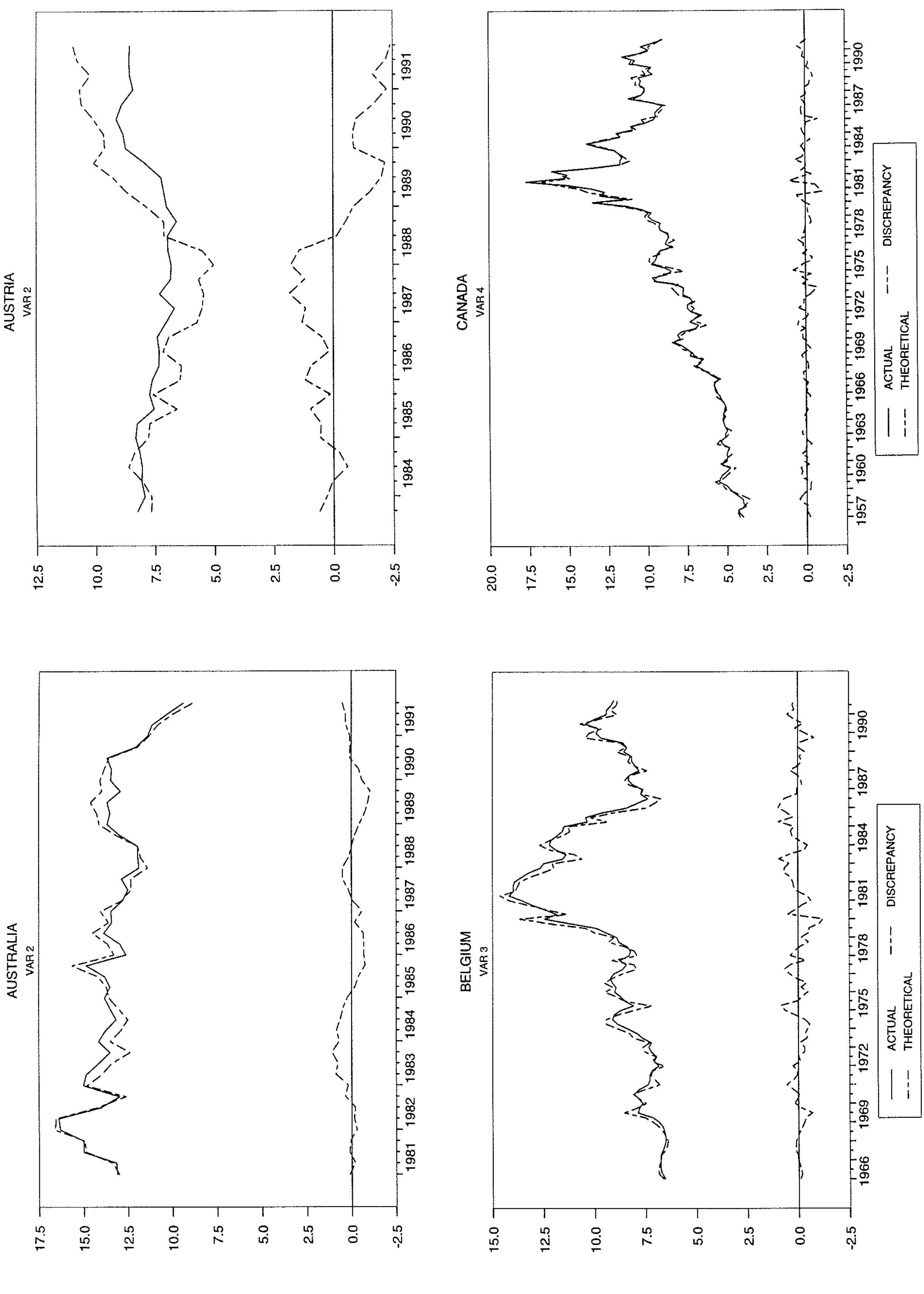

SPAIN
SWITZERLAND
NETHERLANDS
SWEDEN
SHORT RATE
LONG RATE

Graph 1 (cont.)

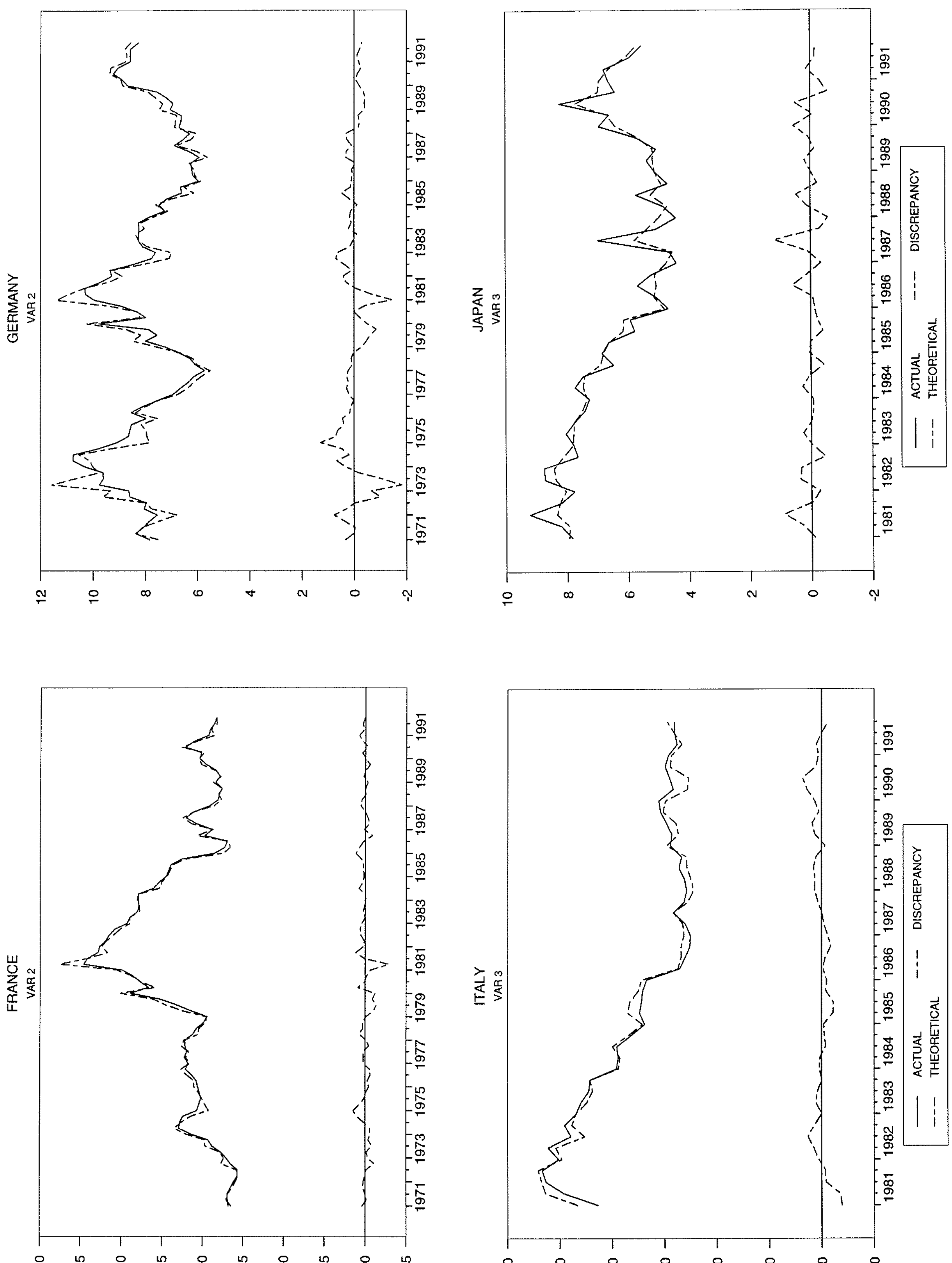

UNITED STATES
SHORT RATE
LONG RATE
1954 1958 1962 1966 1970 1974 1978 1982 1986 1990 1994
0 2 4 6 8 10 12 14 16


UNITED KINGDOM
SHORT RATE
LONG RATE
1965 1968 1971 1974 1977 1980 1983 1986 1989 1992 1995
4 6 8 10 12 14 16 18 20

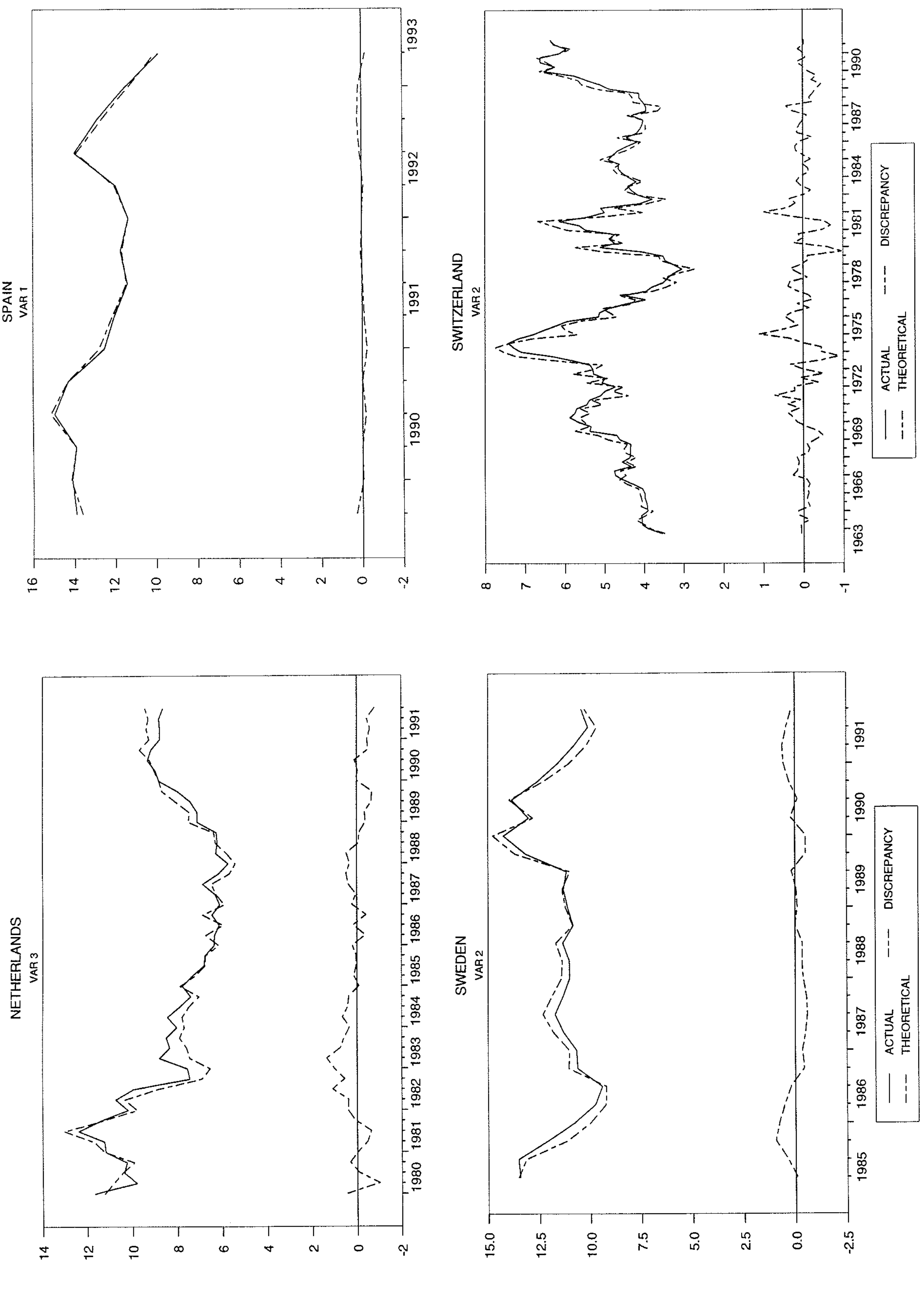
Graph 2
Actual and theoretical long-term interest rates, and the discrepancy – in sample



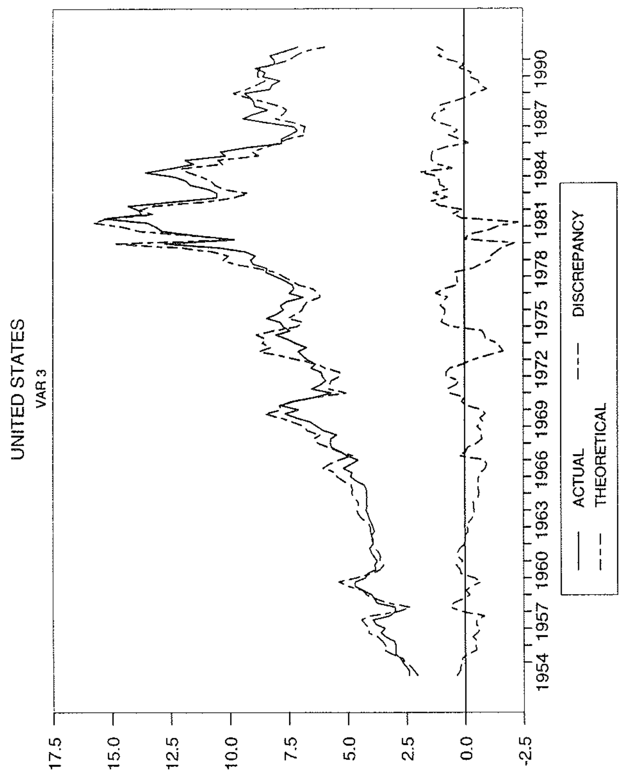
Graph 2 (cont.)

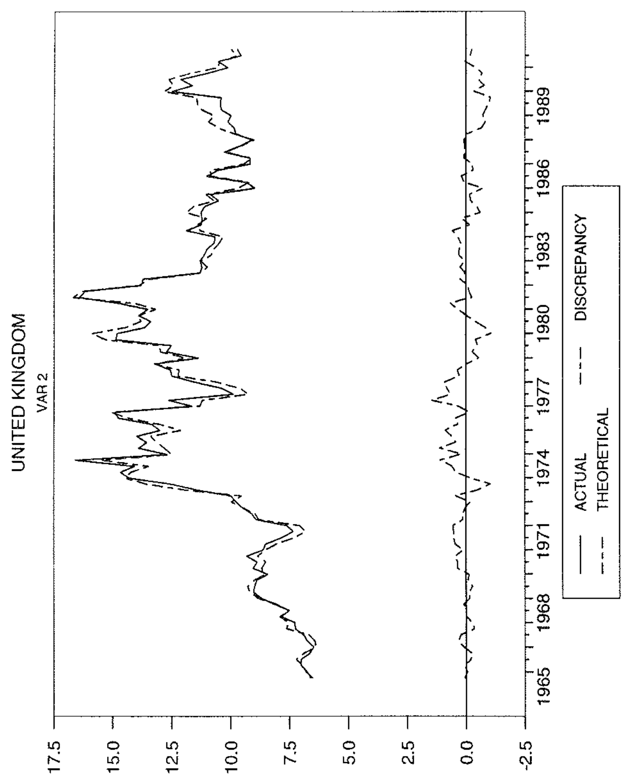

GERMANY
VAR 2
FRANCE
VAR 2
JAPAN
VAR 3
ITALY
VAR 3
ACTUAL
THEORETICAL
DISCREPANCY

Graph 2 (cont.)


NETHERLANDS
VAR 3
SPAIN
VAR 1
SWEDEN
VAR 2
SWITZERLAND
VAR 2
ACTUAL
THEORETICAL
DISCREPANCY

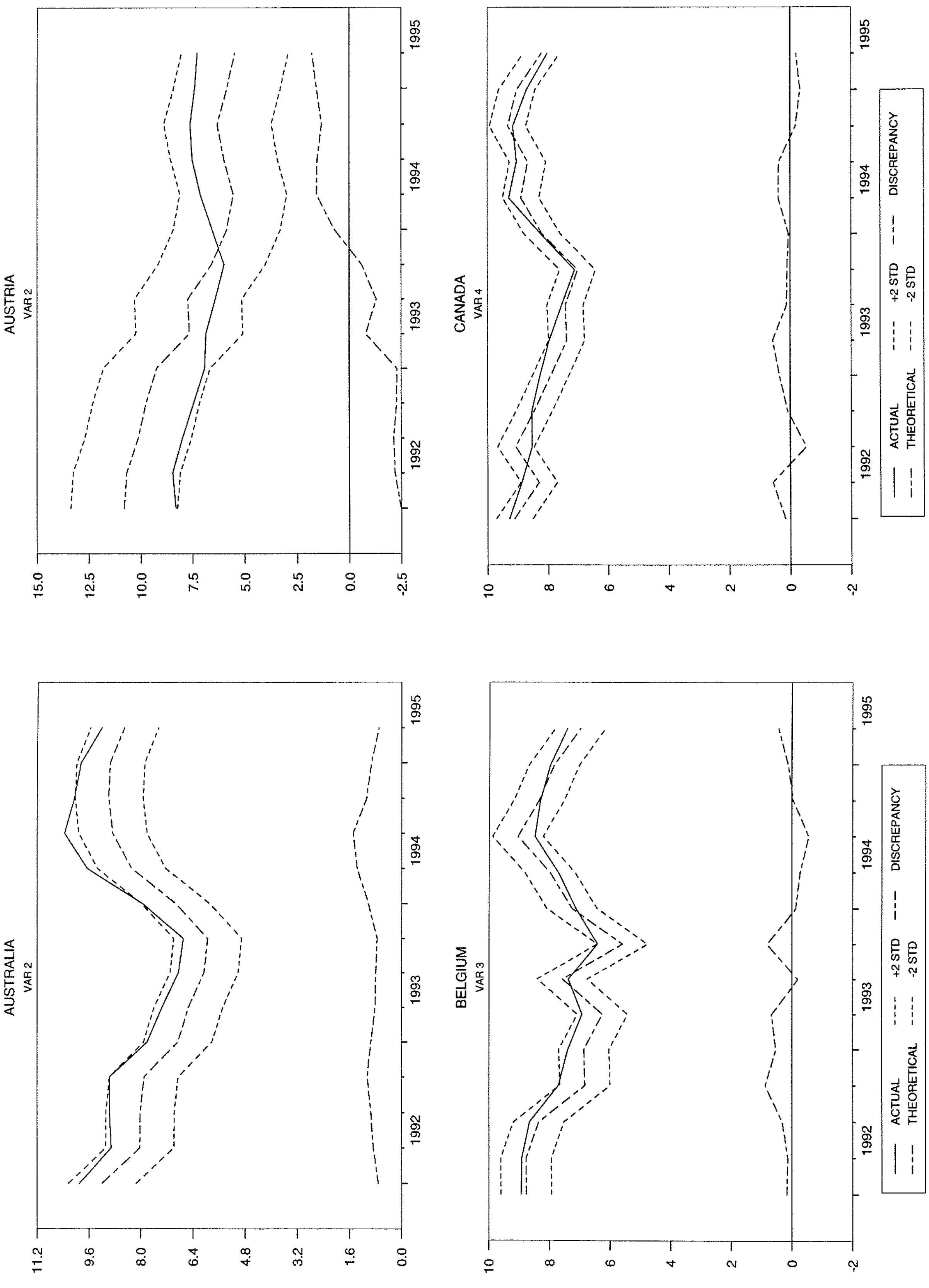
Graph 2 (cont.)


UNITED STATES
VAR 3
ACTUAL
THEORETICAL
DISCREPANCY
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
-2.5
1954 1957 1960 1963 1966 1969 1972 1975 1978 1981 1984 1987 1990

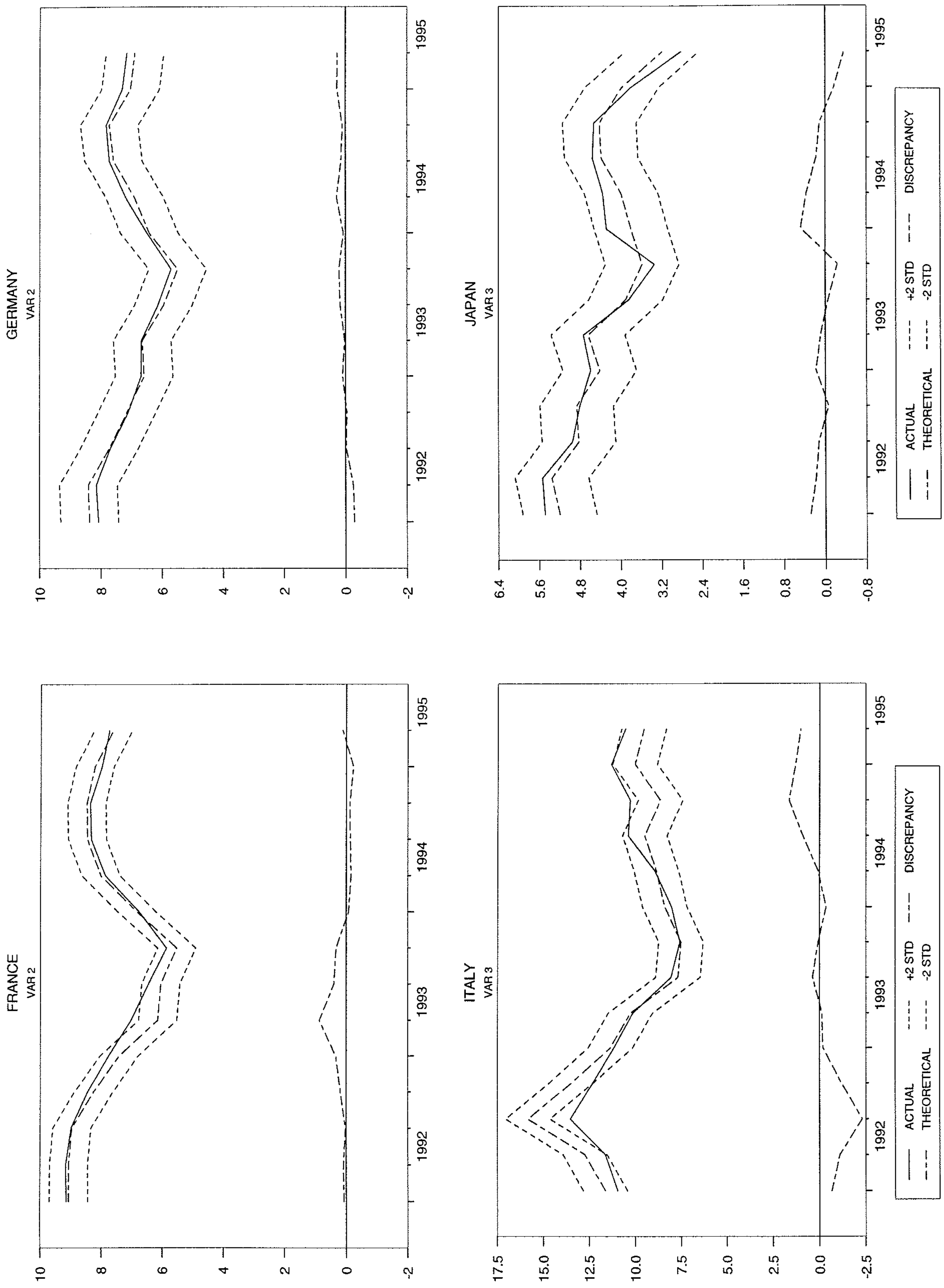

UNITED KINGDOM
VAR 2
ACTUAL
THEORETICAL
DISCREPANCY
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
-2.5
1965 1968 1971 1974 1977 1980 1983 1986 1989

Graph 3

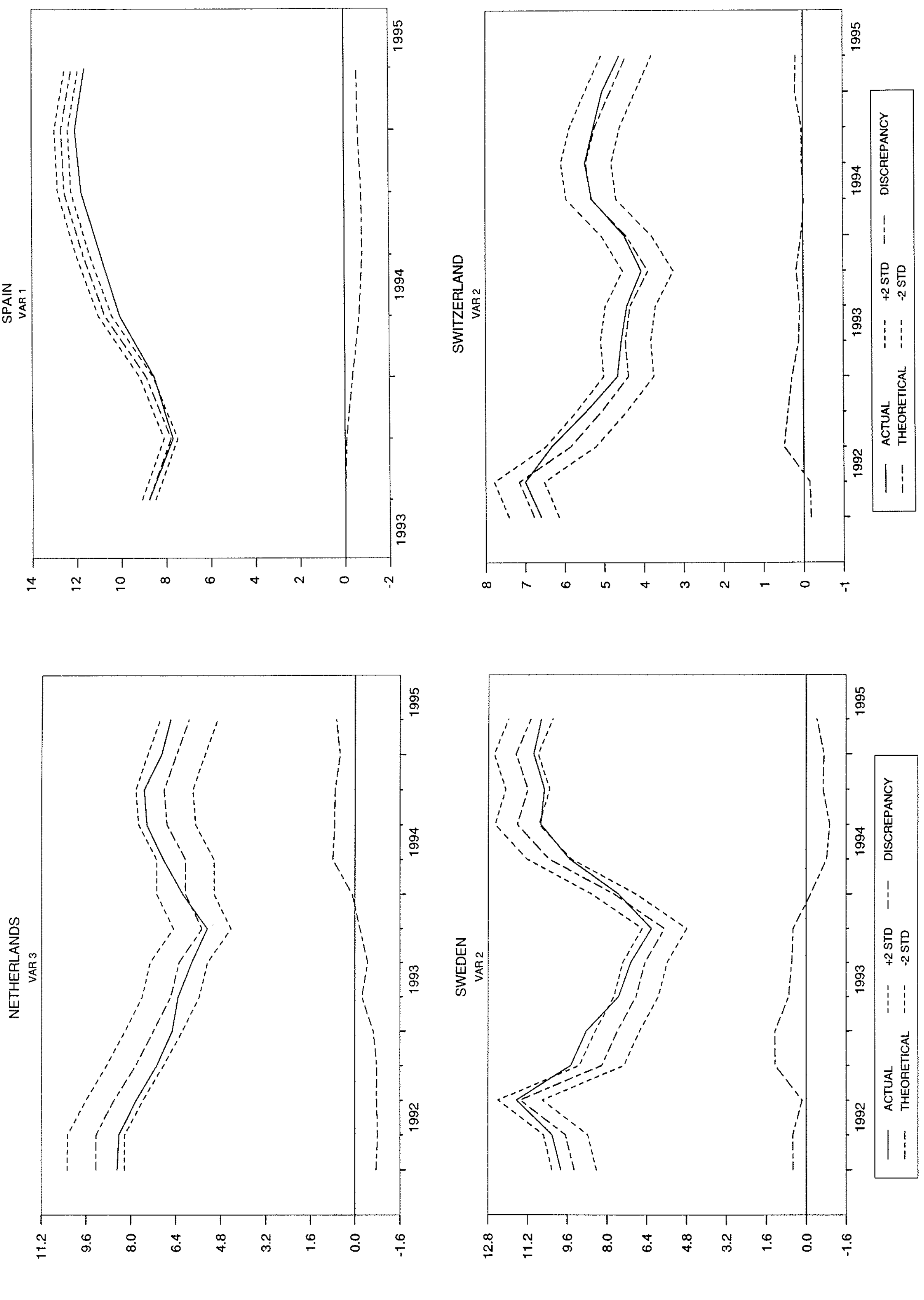
Actual and theoretical long-term interest rates, and the discrepancy – out of sample



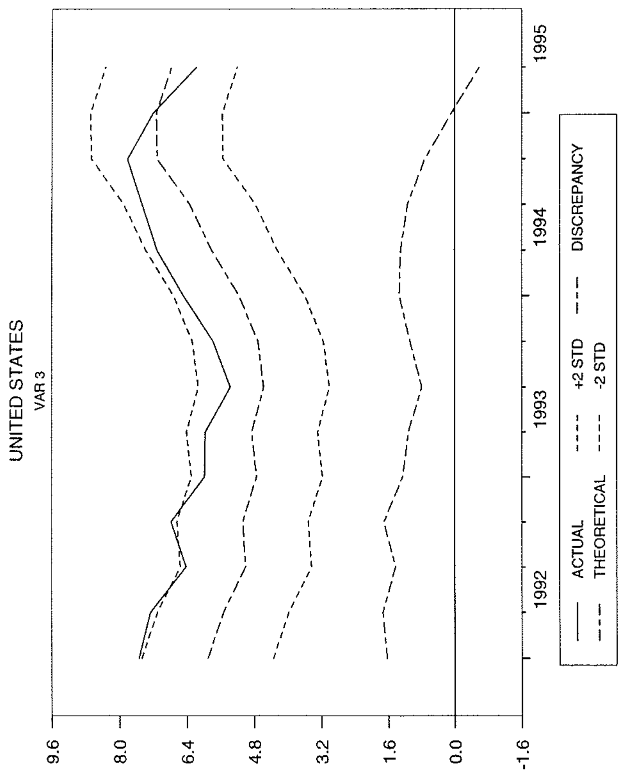
Graph 3 (cont.)

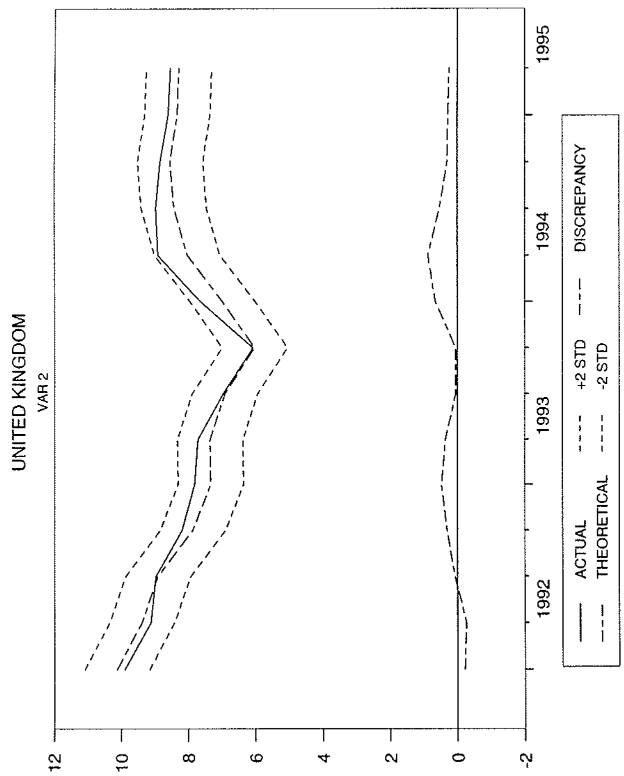

FRANCE
VAR 2
GERMANY
VAR 2
ITALY
VAR 3
JAPAN
VAR 3
ACTUAL
THEORETICAL
+2 STD
-2 STD
DISCREPANCY

Graph 3 (cont.)


NETHERLANDS
VAR 3
SPAIN
VAR 1
SWEDEN
VAR 2
SWITZERLAND
VAR 2
ACTUAL
THEORETICAL
+2 STD
-2 STD
DISCREPANCY
1992
1993
1994
1995

Graph 3 (cont.)


UNITED STATES
VAR 3
9.6
8.0
6.4
4.8
3.2
1.6
0.0
-1.6
1992
1993
1994
1995
ACTUAL
THEORETICAL
+2 STD
-2 STD
DISCREPANCY


UNITED KINGDOM
VAR 2
12
10
8
6
4
2
0
-2
1992
1993
1994
1995
ACTUAL
THEORETICAL
+2 STD
-2 STD
DISCREPANCY

TECHNICAL APPENDIX

This Appendix shows how to calculate the Wald test of the restrictions implied by the EH.\ The method follows Kugler (1990). As in Hardouvelis (1994), the discrepancy between the theoretical and estimated coefficient vectors is given by

$$r(\alpha)^T = h_2^T - \sum_{j=1}^{N-1} G^j h_1^T A^j,$$

where $r(\alpha)^T$ is a row vector, α is a vector of VAR parameters and $G^j = (g^j - g^N)/(1-g^N)$ for notational simplicity. Letting Ω denote the covariance matrix of the residuals, and $D^T = \frac{\partial r(\alpha)^T}{\partial \alpha}$, the Wald test is given by

(A2) $$W = r(\alpha)^T \left[D^T \Omega \otimes (Z^T Z)^{-1} D\right]^{-1} r(\alpha),$$

which is distributed χ^2_{2p}. In what follows the notation is simplified by writing $r(\alpha) = r$. To calculate the Wald statistic, D^T needs to be computed. To do so, note that $\frac{\partial r^T}{\partial \alpha} = \frac{\partial [Vec(r^T)]^T}{\partial [Vec(\alpha)]}$. The chain rule then implies that

(A3) $$\frac{\partial [Vec(r^T)]^T}{\partial Vec(\alpha)} = \frac{\partial [Vec(A)]^T}{\partial Vec(\alpha)} \times \frac{\partial [Vec(-\sum_{j=1}^{N-1} G^j h^T A^j)]^T}{\partial Vec(A)}$$

The first derivative is simple to calculate. Ruud (1987) states that

$$\frac{\partial Vec(CB)}{\partial x} = \frac{\partial}{\partial x}[Vec(B)]^T (I \otimes C^T) + \frac{\partial}{\partial x}[Vec(C)]^T (B \otimes I)$$

which can be used so calculate the second derivative in (A3). Setting $C = Gh^T$ and $B = A^j$ gives

(A4) $$\frac{\partial [Vec(A^j)]^T}{\partial Vec(A)}(I \otimes G^j h) + \frac{\partial G^j h^T}{\partial Vec(A)}(A^j \otimes I),$$

where the second term is zero (since $G^j h^T$ does not involve the parameters in α). The first term in (A4) can be calculated using the result, due to Schmidt (1974), that

$$\frac{\partial [Vec(A^j)]^T}{\partial Vec(A)} = \sum_{k=0}^{j-1} (A^T)^k \otimes A^{j-1-k}.$$

DATA APPENDIX

The following data series were used in the analysis:

Country	Starting date	Description of interest rates
Australia	1981:1	90-day bank accepted bills. Commonwealth government bonds, 10 years.
Austria	1983:4	3-month Euro-deposits. Federal government bonds, 9-10 years.
Belgium	1966:1	3-month Treasury certificates. Central government bonds, over 5 years.
Canada	1957:1	3-month prime corporate paper. Federal government bonds, over 10 years.
France	1971:1	3-month Paris interbank offered rate (PIBOR); prior to 1987:1, 3-month interbank loans against private bills. Public and semi-public sector bonds, over 7 years.
Germany	1971:1	3-month interbank loans (Frankfurt). Federal public bonds, 10 years.
Italy	1981:1	3-month Treasury bills, net of tax. Treasury bonds with a residual maturity of more than one year, net-of-tax.
Japan	1981:1	3-month Gensaki rate. Central government bonds, 10 years.
Netherlands	1981:1	3-month Amsterdam interbank offered rate (AIBOR); prior to 1985:4, 3-month interbank deposit rate. Central government bonds, 5-8 years.
Spain	1989:4	3-month interbank deposit rate. 5-year bond yield.
Sweden	1985:2	3-month Treasury discount notes. Central government bonds, 4-5 years.
Switzerland	1963:4	3-month Euro-deposits. Confederation bonds with at least 5 years to maturity.
United Kingdom	1965:4	3-month interbank deposits. Government bonds, 10 years.
United States	1954:2	3-month Treasury bills. 10-year Treasury bonds.

Note: All data are end-of-quarter except those for France (short-term rate), Italy and the United States (short and long-term rates), which are averages of daily rates of the last month of the quarter.

REFERENCES

Bank of England (1995): "Inflation report", March.

Campbell, J.Y. and R.J. Shiller (1987): "Cointegration and tests of present value models". *Journal of Political Economy*, 95, p. 1062-88.

Campbell, J.Y. and R.J. Shiller (1991): "Yield spreads and interest rate movements: a bird's eye view." *Review of Economic Studies*, 58, p. 495-514.

Dahlquist, M. and G. Jonsson (1995): "The information in Swedish short-maturity forward rates". *European Economic Review*, 39, p. 1115-31.

Engsted, T, and C. Tanggaard (1995): "The predictive power of yield spreads for future interest rates: Evidence from the Danish term structure." *Scandinavian Journal of Economics*, 97, p. 145-59.

Gerlach, S. and F. Smets (1995): "The term structure of Euro-rates: some evidence in support of the expectations hypothesis". *BIS working paper no. 28.*

Goodfriend, M. (1995): "Acquiring and maintaining credibility for low inflation: the US experience." In L. Leiderman and L.E.O. Svensson, *Inflation Targets*, CEPR, London.

Hardouvelis, G. (1994): "The term structure spread and future changes in long and short rates in the G7 countries". *Journal of Monetary Economics*, 33, p. 255-83.

Kugler, PP. (1988): "An empirical note on the term structure and interest rate stabilization policies." *Quarterly Journal of Economics*, 103, p. 789-92.

Kugler, P. (1990): "The term structure of Euro interest rates and rational expectations." *Journal of International Money and Finance*, 9, p. 234-44.

MacDonald, R. and A.E.H. Speight (1988): "The term structure of interest rates in the UK." *Bulletin of Economic Research*, 40, p. 287-99.

Mankiw, N.G. and J.A. Miron (1986): "The changing behavior of the term structure of interest rates." *Quarterly Journal of Economics*, 101, p. 211-28.

Pindyck, R.S. and D.I. Rubinfeld (1991): *Econometric models and economic forecasts.* Third ed., McGraw Hill, New York.

Ruud, P.A. (1987): "Economics 241A, Class Lecture Notes 2: Matrix Differentiation". Unpublished manuscript.

Schmidt, P. (1974): "The asymptotic distribution of forecast in the dynamic simulation of an econometric model." *Econometrica*, 42, p. 303-09.

Shiller, R.J. (1979): "The volatility of long-term interest rates and expectations models of the term structure". *Journal of Political Economy*, 87, p. 1190-1219.

Shiller, R.J. (1981): "Alternative tests of rational expectations models: the case of the term structure". *Journal of Econometrics*, 16, p. 71-87.

Shiller, R.J. (1990): "The term structure of interest rates", in B. M. Friedman and F.H. Hahn, Elsevier (ed.), *Handbook of Monetary Economics.*

Shiller, R.J. and J.Y. Campbell and K.L. Schoenholtz (1983): "Forward Rates and future policy: interpreting the term structure of interest rates". *Brookings Papers on Economic Activity*, 1, p. 173-217.

Sveriges Riksbank (1995): *Inflation and inflation expectations in Sweden*, March.

Taylor, M.P. (1992): "Modelling the yield curve." *Economic Journal*, 102, p. 524-37. (Abridged version of IMF Working Paper no. 134, 1991).