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Stochastic Volatility, Long Run Risks, and Aggregate Stock Market Fluctuations *

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October 2010

Abstract

What are the main drivers of fluctuations in the aggregate US stock market? In this paper, we attempt to resolve the long-lasting debate surrounding this question by designing and solving a consumption-based asset pricing model which incorporates stochastic volatility, long-run risks in consumption and dividends, and Epstein-Zin preferences. Utilizing Bayesian MCMC techniques, we estimate the model by fitting it to US data on the level of the aggregate US stock market, the short-term real risk-free interest rate, real consumption growth, and real dividend growth. Our results indicate that, over short and medium horizons, fluctuations in the level of the aggregate US stock market are mainly driven by changes in expected excess returns. Conversely, low frequency movements in the aggregate stock market are primarily driven by changes in the expected long-run growth rate of real dividends.

Keywords: Asset Pricing, Stochastic Volatility, Long-Run Risks, Bayesian MCMC Methods

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1 Introduction

From a market fundamentals point of view, the price of any asset is a function of two sets of expected values – the future dividends that the asset is expected to pay over its lifetime and the expected rates of return at which these dividends will be discounted back to their present values. One of the greatest challenges that asset pricing has been confronted with for the past several decades has been quantifying the contributions of each of the above components towards explaining the observed fluctuations in the aggregate stock market. The literature on the subject has gone back and forth between the two alternative explanations. Up until the early 1980's, it was widely assumed that expected equity returns did not vary over time. This meant that changes in expected future real dividend growth were considered to be the main drivers by default. However, Leroy and Porter (1981) and Shiller (1981) showed that the observed dividend series was too smooth to be able to explain the observed volatility in stock prices. There have been several papers since then (most notably Campbell and Shiller (1988, 1989), Campbell (1991), Cochrane (1991) and Campbell and Ammer (1999)) which have concluded, after using various assumptions and methodology, that stock price fluctuations are mainly caused by changes in expected returns.

One assumption that all of the above papers have in common is that market fundamentals are stationary and that none of them contains a permanent component in it. Barsky and DeLong (1993) have argued that if the dividend growth series was persistent enough, even small shocks to dividends could cause large swings in price-dividend ratios. More recently, Balke and Wohar (2002, 2006, and 2007, BW hereafter) have used a variety of approaches and methods to demonstrate that there is a fundamental problem with quantifying the relative importance of the two potential drivers. They have shown that inference is very sensitive to assumptions about the long-term properties of the two series. Namely, if one of the series is assumed to have a small permanent component, while the other one is not, the series with the permanent component is estimated to be the main driver. Moreover, when both series are assumed to have small non-stationary components, their model appears to be underidentified. Avdjiev and Balke (2009, AB hereafter) attempt to resolve that issue by adding consumption data to the benchmark model used in BW. There are a couple of ways in which consumption could help identify potential long-term trends in real dividend growth and expected returns. First, there are good theoretical reasons, which are backed by solid empirical evidence, to believe that the consumption growth series and the dividend growth series should have a common long-run component. Second, consumption growth and expected returns are related through the effect that consumption growth has on the stochastic discount factor. Nevertheless, AB conclude that consumption data alone is not sufficient to resolve the issue, mainly due to the well-known fact that the classic consumption-based asset pricing model is unable to simultaneously account for fluctuations in both stock prices and short-term real interest rates. They further argue that the main culprit for that failure of the consumption-based model is the fact that it implies that excess returns are constant over time.

Excess returns exist in order to compensate investors for the risk that is associated with a certain asset. Fundamentally, expected excess returns are a function of two variables - the quantity of risk that the asset carries and the price per unit of risk that investors demand to be compensated with for bearing that risk. Therefore, in order to have time-varying excess returns, a model must have time-varying quantity of risk, time-varying price of risk, or some combination of the two. The quantity of risk associated with an asset is related to the degree of uncertainty surrounding the returns of that asset and is usually measured by the conditional variance of returns. Therefore, time-varying quantity of risk would imply time-varying conditional variances of returns. In the standard consumption-based model, the conditional variances of returns are functions of the conditional variances of the macroeconomic fundamentals. That is why in such a model relaxing the assumption that the variances of macroeconomic fundamentals are constant through time would lead to time-varying quantity of risk, which would, in turn, imply that excess returns vary over time.

Intuitively, investors would require a high equity premium during times of high conditional volatility and a low equity premium during times of low conditional volatility.

In this paper, we enhance the model of AB by introducing time-variation in excess returns. We achieve that by allowing the conditional variances of macroeconomic fundamentals to be stochastic. While our paper is not the first one to do that, what sets it apart is the fact that, in contrast to the rest of the existing literature on the subject, we estimate our model by utilizing a full-information approach. More precisely, we use Bayesian Markov Chain Monte Carlo (MCMC) methods in order to estimate the unknown parameters and the unobserved states of our model, which we fit to actual data on real consumption growth, real dividend growth, the price-dividend ratio of the aggregate US stock market, and the real risk-free interest rate. As An and Schorfheide (2007) point out, this approach has three main advantages over the more conventional methods used by the rest of the literature. First, as already mentioned above, the analysis that we use is system-based and fits the model to actual observed time series. Second, it is based on the likelihood function generated by the underlying model rather than on some arbitrarily chosen measure such as the discrepancy between the impulse response functions implied by the model and those implied by an unrestricted VAR. Third, it allows us to use prior distributions in order to incorporate additional information (i.e. information not contained in the set of observable variables in the model) into the parameter estimation.

We use the results we obtain from the Bayesian MCMC estimation of our model in order to assess the importance of each of the model's fundamentals in triggering movements in the level of aggregate US stock market. We find that, over short and medium horizons, fluctuations in aggregate stock prices are mainly driven by changes in expected excess returns. Conversely, low frequency movements in the aggregate stock market are primarily caused by changes in expected dividend growth rates.

As mentioned above, several papers in the asset pricing literature have introduced time variation in expected excess returns by assuming time-varying volatility of consumption growth and dividend growth (Abel (1988), Kandel and Stambaugh (1991), Gennotte and Marsh (1993), and Bansal and Yaron (2006)).[1] Our paper is most closely related to the model presented in Bansal and Yaron (2006, BY hereafter). In their model, the level of economic uncertainty fluctuates over time and consumption growth and dividend growth are assumed to contain a common long-run predictable component. BY show that about 50% of the variability of equity prices is due to fluctuations in the expected growth rates of consumption and dividends. Their model attributes the other 50% to variation in economic uncertainty. In the context of our framework, these results could be interpreted as evidence that changes in expected dividend growth are roughly as important as changes in expected excess returns in driving the aggregate stock market.

Our work is also related to several papers that are centered on the assumption that consumption growth and dividend growth can be decomposed into permanent and transitory components. Croce (2009) builds a model of a production economy in which the productivity growth rate has a long-run risk component, which endogenously generates a common long-run risk component in consumption growth and dividend growth. Hansen, Heaton, and Li (2005) also build their model around the assumption that consumption growth and dividend growth have a common long-run component. They use empirical inputs from vector autoregressions (VARs) in order to quantify the long-run risk-return trade-off in the valuation of uncertain future cash flows that are exposed to fluctuations in macroeconomic growth. Bansal, Dittmar, and Kiku (2006) assume that consumption growth has permanent and transitory predictable components on their way to showing that the exposure of assets' dividends to permanent risks in consumption is a key determinant of the risk compensation in asset markets. They use the error-correction VAR (EC-VAR) framework to show that transitory price shocks dominate the variation of an asset's return at short horizons, whereas dividend shocks take over as the main source of asset return volatility as the time horizon

[1] Whitelaw (2000) is an example of a model in which expected excess returns vary as a results of time-varying correlation between marginal utility growth and returns.

increases. Bansal, Dittmar, and Lundblad (2005) explore the cross-sectional implications of an asset pricing model centered on a related assumption. Namely, they model the dividend growth process for any asset in the economy as a function of aggregate consumption growth in order to measure the covariance of innovations in consumption growth with innovations in current and expected future dividend growth. They show that aggregate consumption risks embodied in cash flows account for more that 60% of the observed cross-sectional variation in equity premia.

While the main assumptions about the time series properties of dividend growth and consumption growth in all of the above papers are similar to the ones we make, our methodology is significantly different. Bansal and Yaron (2006) and Croce (2009) calibrate their key parameters and rely on simulations, while Hansen, Heaton, and Li (2005), Bansal, Dittmar, and Kiku (2006), and Bansal, Dittmar, and Lundblad (2005) all use VARs, in one form or another, to estimate the key parameters and to derive the asset pricing implications of their models. In contrast, we utilize a full-information approach, which gives us the opportunity to explore features of the data that may not be captured by solely focusing on first and second moments of the observable variables.

The rest of the paper is organized as follows. In Section 2, we introduce the theoretical environment that we use as the foundation for our empirical investigation. In Section 3, we present the solution of our model. We outline our estimation approach in Section 4. In Section 5, we present and discuss our main results. We conclude in Section 6.

2 Theoretical Framework

The one-period gross real return of any asset i, R^i_{t+1}, can be expressed as:

$$R^i_{t+1} = \frac{P^i_{t+1} + D^i_{t+1}}{P^i_t}. \tag{1}$$

where P_t^i is the real price of the asset at the end of period t, and D_t^i is the real dividend that the asset pays during period t. Equation (1) can be rewritten as:

$$\frac{P_t^i}{D_t^i} = \frac{1}{R_{t+1}^i}\frac{D_{t+1}^i}{D_t^i}\left(1+\frac{P_{t+1}^i}{D_{t+1}^i}\right). \tag{2}$$

Using the log-linear approximation employed by Campbell and Shiller (1988, 1989), we obtain the following expression:

$$p_t^i - d_t^i \approx \triangle d_{t+1}^i - r_{t+1}^i + \rho\left(p_{t+1}^i - d_{t+1}^i\right) + k. \tag{3}$$

where $\rho \equiv \frac{\exp(\overline{p_t^i - d_t^i})}{1+\exp(\overline{p_t^i - d_t^i})}$, $\overline{p_t^i - d_t^i}$ is the average of the log price-dividend ratio over the sample,[2] $k \equiv -\log(\rho) - (1-\rho)\log(1/\rho - 1)$, and for any variable X_t: $x_t = \log(X_t)$ and $\triangle x_t = x_t - x_{t-1}$. Letting $pd_t^i \equiv p_t^i - d_t^i$ and taking expectations yields:

$$pd_t^i = \rho \underset{t}{E}\left(p_{t+1}^i - d_{t+1}^i\right) + \underset{t}{E}\triangle d_{t+1}^i - \underset{t}{E} r_{t+1}^i + k. \tag{4}$$

Recursively substituting in (4), we get:

$$pd_t^i = \frac{k}{1-\rho} + \sum_{j=0}^{\infty}\rho^j \underset{t}{E}\left(\triangle d_{t+1+j}^i - r_{t+1+j}^i\right) + \lim_{k\to\infty}\rho^k \underset{t}{E} p_{t+k}^i. \tag{5}$$

Ruling out explosive behavior for the log price-dividend ratio, we obtain:

$$pd_t^i = \frac{k}{1-\rho} + \sum_{j=0}^{\infty}\rho^j \underset{t}{E}\left(\triangle d_{t+1+j}^i - r_{t+1+j}^i\right). \tag{6}$$

According to (6), there are two possible sets of triggers of movements in the price-dividend ratio of any asset i: changes in its expected future real dividend growth and changes in its expected future returns.

Following Bansal and Yaron (2006), we assume that the model economy is populated by infinitely many identical agents and that there is a representative agent who maximizes the

[2] In our sample, $\overline{p_t^i - d_t^i} = 4.8625$, which implies that $\rho = 0.9923$.

recursive utility function originally introduced by Epstein and Zin (1989) and Weil (1989):

$$U_t = \left\{ (1-\delta)(C_t)^{\frac{1-\gamma}{\theta}} + \delta \left(\underset{t}{E} U_{t+1}^{1-\gamma} \right)^{1/\theta} \right\}^{\frac{\theta}{1-\gamma}}, \tag{7}$$

where $\delta \in (0,1)$ is the subjective time rate of preference, $\theta = \frac{1-\gamma}{1-1/\psi}$, $\gamma \geq 0$ is the coefficient of relative risk aversion, and $\psi \geq 0$ is the elasticity of intertemporal substitution (EIS). It is well-known that one of the main advantages of this type of utility function is that it allows for a separation of the parameters that control the representative agent's degree of relative risk aversion (γ) and the EIS (ψ). This feature, which is not available in the standard time-separable power utility function used in most consumption-based asset pricing models, is crucial for capturing several key aspects of the data. Economists using the standard power utility model have struggled with the fact that high levels of risk aversion (i.e. high γ) are required to match the observed equity premium in the data. These high levels of risk aversion, however, imply a relatively low level for the EIS (i.e. low ψ), which causes the model to predict that the real risk free rate is a lot more volatile than it is in the data. By contrast, the Epstein-Zin utility function separates the coefficient of relative risk aversion from the elasticity of intertemporal substitution, thus giving the model more flexibility to capture various properties of asset returns.

The logarithm of the of the stochastic discount factor (m_{t+1}) that is implied by the utility function given in (7) is:

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} (\triangle c_{t+1}) + (\theta - 1) \left(r_{t+1}^c \right), \tag{8}$$

where r_{t+1}^c is the gross return on an asset that delivers aggregate consumption as its dividend each period.

The Euler equation implied by the first order conditions of the utility-maximizing representative agent takes a familiar form:

$$\underset{t}{E} \left(M_{t+1} R_{t+1}^i \right) = 1. \tag{9}$$

Assuming log-normality for the stochastic discount factor and for returns, (9) can be re-written in logs:

$$\underset{t}{E}\left(m_{t+1}+r_{t+1}^{i}\right)+\frac{1}{2}\underset{t}{Var}\left(m_{t+1}+r_{t+1}^{i}\right)=0. \tag{10}$$

This implies that the expected value of the log-return of asset i can be written as:

$$\underset{t}{E}\left(r_{t+1}^{i}\right)=\underbrace{-\underset{t}{E}\left(m_{t+1}\right)-(0.5)\underset{t}{Var}\left(m_{t+1}\right)}_{r_{t+1}^{rf}}\underbrace{-\underset{t}{Cov}\left(m_{t+1},r_{t+1}^{i}\right)-(0.5)\underset{t}{Var}\left(r_{t+1}^{i}\right)}_{r_{t+1}^{e,i}}. \tag{11}$$

Note that the expected return on any asset i can be decomposed into two components - the one-period ahead real risk-free rate (r_{t+1}^{rf}) and the expected excess return ($r_{t+1}^{e,i}$) that asset i is required to deliver in order to compensate investors for the fact that its future payoffs are uncertain.

Following BW, we allow for the possibility that real dividend growth and real consumption growth can be decomposed into a non-stationary (permanent) component and a stationary (transitory) component:

$$x_t=x_t^{p}+x_t^{\tau}+w_t^{x} \qquad \text{for } x=\triangle c,\triangle d, \tag{12}$$

where x_t^p and x_t^τ are the permanent and transitory components, respectively, of x_t and $w_t^x \backsim N(0,\sigma_{w,x}^2)$ is an observation error. We assume that x_t^p follows a random walk:

$$x_t^{p}=x_{t-1}^{p}+\varepsilon_t^{x,p} \qquad \text{for } x=\triangle c,\triangle d, \tag{13}$$

where and $\varepsilon_t^{x,p} \backsim N\left(0,\left(\sigma_{x,p}^2\right)_t\right)$ is an exogenous structural shock. We also assume that all transitory components follow an autoregressive process of order 2:

$$x_t^{\tau}=\sum_{j=1}^{2}\phi_j x_{t-j}^{\tau}+\varepsilon_t^{x,\tau} \qquad \text{for } x=\triangle c,\triangle d. \tag{14}$$

where and $\varepsilon_t^{x,\tau} \backsim N\left(0,\left(\sigma_{x,\tau}^2\right)_t\right)$ is an exogenous structural shock.

Following AB, we impose a restriction on the non-stationary components of consumption growth and dividend growth:

$$\triangle d_t^p = \triangle c_t^p \qquad\qquad \text{for } \forall\ t, \tag{15}$$

As Campbell and Cochrane (1999) note, in the very long run, one can reasonably expect the correlation between consumption and dividend growth to approach 1.0 since the two series share the same long-run trends and should grow at the same pace as the overall economy. Bansal and Yaron (2006) and Hansen, Heaton, and Li (2005) also provide a similar argument on their way to assuming that consumption growth and dividend growth share a small, but persistent predictable component. Our assumption is slightly different from theirs, however, as they assume that the common component, although very persistent, is stationary, while we go a step further and assume that the joint component follows a random walk. Keeping in mind the caveat of Campbell and Cochrane(1999) that the correlation between consumption growth and dividend growth is notoriously difficult to measure because it is very sensitive to small changes in timing, we, nevertheless, interpret the empirical results of Campbell (2000), who reports that the contemporaneous correlation between consumption growth and dividend growth tends to increase as the measurement interval increases, as a confirmation of the hypothesis that the two series share a common long-run component. Furthermore, after analyzing international data from 11 developed countries, Campbell (2002) reports that, in general, correlations between consumption growth and dividend growth "increase strongly with the measurement horizon" and are "positive and often quite large in long-term annual data sets."

3 Model Solution

Given the assumptions described in the previous section, the state variables that are used to track the evolution of consumption growth and dividend growth can be summarized in the following state vector:

$$S_t^A = \left(\triangle c_t^p, \triangle c_t^\tau, \triangle d_t^\tau, \triangle c_{t-1}^\tau, \triangle d_{t-1}^\tau\right)', \tag{16}$$

The equation which describes the evolution of the unobserved states through time is:

$$S_t^A = F^A S_{t-1}^A + V_t^A, \tag{17}$$

where F^A is the transition matrix and is given by:

$$F^A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \phi_1^c & 0 & \phi_2^c & 0 \\ 0 & 0 & \phi_1^d & 0 & \phi_2^d \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \tag{18}$$

and V_t^A is a vector of exogenous structural shocks to the unobserved states:

$$V_t^A = (v_t^{cd}, v_t^c, v_t^d, 0, 0)', \tag{19}$$

$$V_t^A \sim N\left(0, Q_t^A\right), \tag{20}$$

$$Q_t^A = \begin{pmatrix} (\sigma_{cd}^2)_t & 0 & 0 & 0 & 0 \\ 0 & (\sigma_{c\tau}^2)_t & 0 & 0 & 0 \\ 0 & 0 & (\sigma_{c\tau}^2)_t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{21}$$

Note that in this state space model, unlike in a conventional one, the variance-covariance matrix, Q_t^A, is time-dependent. This is due to the fact that, as discussed in Section 2, we allow the conditional variances of the structural shocks to vary over time.[3] We assume that the evolution of the three time-varying variances is guided by the following stochastic processes:

$$diag(Q_t^A) = \begin{pmatrix} (\sigma_{cd}^2)_t \\ (\sigma_{c\tau}^2)_t \\ (\sigma_{d\tau}^2)_t \end{pmatrix} = \Lambda + S_t^B, \tag{22}$$

where

$$\Lambda = \begin{pmatrix} \Lambda_{\sigma,cd} \\ \Lambda_{\sigma,c\tau} \\ \Lambda_{\sigma,d\tau} \end{pmatrix},$$

$$S_t^B = \begin{pmatrix} \left(\widetilde{\sigma_{cd}^2}\right)_t \\ \left(\widetilde{\sigma_{c\tau}^2}\right)_t \\ \left(\widetilde{\sigma_{c\tau}^2}\right)_t \end{pmatrix},$$

$$S_t^B = F^B S_{t-1}^B + V_t^B, \tag{23}$$

$$F^B = \begin{pmatrix} \phi_{\sigma,cd} & 0 & 0 \\ 0 & \phi_{\sigma,c\tau} & 0 \\ 0 & 0 & \phi_{\sigma,d\tau} \end{pmatrix}$$

$$V_t^B \sim N\left(0, Q^b\right),$$

$$Q^b = \begin{pmatrix} \xi_{\sigma,cd}^2 & 0 & 0 \\ 0 & \xi_{\sigma,c\tau}^2 & 0 \\ 0 & 0 & \xi_{\sigma,d\tau}^2 \end{pmatrix},$$

and $\widetilde{\sigma_x^2} = (\sigma_x^2)_t - \Lambda_{\sigma,x}$.

[3]We assume that Q_t^A is diagonal (i.e. that the structural shocks in V_t^A are not correlated with each other) in order to decreases the number of estimated parameters in the benchmark model and thus reduce the computational burden on the MCMC algorithm. In work that is available upon request, we show that our main results are robust to relaxing that assumption.

We use the method of undetermined coefficients in order to solve for pd_t in terms of the relevant state variables. While the details of our solution are described in Appendix A, we use the rest of this section to go through the most important results. The log-price-dividend ratio is a function of future dividend growth rates and of future expected returns (6). The latter are, in turn, a function of expected consumption growth, of the conditional variances of the stochastic discount factor and returns, as well as of the covariance between the stochastic discount factor and returns (11). That is why we conjecture that the log-price-dividend ratio is ultimately a function of two sets of variables - the expected future values of the permanent and temporary components of real consumption growth and real dividend growth (summarized in S_t^A) and the conditional variances of these variables (summarized in S_t^B). Hence, our guess for the solution of the pd_t equation takes on the following form:

$$pd_t = A_p + H_p^A S_t^A + H_p^B S_t^B, \tag{24}$$

where H_p^A is a (1x5) row vector and H_p^B is a (1x3) row vector. After going through the steps described in Appendix A, the solution vectors in (24) turn out to be:

$$H_p^A = \left(H^d - \frac{1}{\psi} H^c \right) F^A \left(I - \rho F^A \right)^{-1}, \tag{25}$$

$$H_p^B = \left\{ (1-\theta) H_c^B \left(I - \rho_c F^B \right) + (0.5) \left[WPB \otimes WPB \right] H^{\Gamma} \right\} \left(I - \rho F^B \right)^{-1}, \tag{26}$$

where

$$H^c = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix}',$$

$$H^d = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix}',$$

$$WPB = \left[(\theta - 1 - \frac{\theta}{\psi}) H^c + (\theta - 1) \rho_c H_c^A + \rho H_p^A + H^d \right],$$

$$H_c^A = \left(1 - \frac{1}{\psi} \right) H^c F^A \left(I - \rho_c F^A \right)^{-1},$$

$$H_c^B = (0.5) \theta \left\{ \left[WCB \otimes WCB \right] H^{\Gamma} \right\} \left(I - \rho_c F^B \right)^{-1},$$

$$WCB = \left(1 - \frac{1}{\psi}\right) H^c F^A \left(I - \rho_c F^A\right)^{-1},$$

$$\rho_c \equiv \frac{\exp(\overline{p_t^c - c_t})}{1 + \exp(\overline{p_t^c - c_t})},$$

H^Γ is a (25x3) matrix whose entries are such that:

$$vec(Q_t^A) = H^\Gamma S_t^B,$$

and A_p is a constant.

There are a couple of important observations related to the above expressions that are worth mentioning at this point. First, the factor loading on the permanent component of consumption growth is $\left(1 - \frac{1}{\psi}\right)$. Therefore, when the EIS is greater than 1 (i.e. $\frac{1}{\psi} < 1$), an increase in the permanent component of consumption growth increases the price-dividend ratio. Conversely, when the EIS is smaller than 1 (i.e. $\frac{1}{\psi} > 1$), an increase in the permanent component of consumption growth decreases the price-dividend ratio. Intuitively, if the EIS is relatively high, agents are relatively more willing to shift consumption across time and the substitution effect (higher expected future dividends) dominates the income effect (lower expected future marginal utility). As a result, agents bid up the price of stocks relative to the current level of dividends. If, on the other hand, the EIS is relatively low, agents are relatively less willing to shift consumption across time and the income effect dominates the substitution effect. As a result, agents decrease their demand for stocks in order to finance an increase in their current consumption, thus bidding down stock prices. Second, the value of θ plays a crucial role in determining the signs of the factor loadings on the elements of S_t^B (i.e. the factor loadings on the time-varying variances of macroeconomic fundamentals). Just as in the Bansal and Yaron (2006) model, a negative θ implies that increases in volatility reduce the price-consumption ratio (i.e. the elements of H_c^B are negative). This, in turn, implies that the elements of the first term on the right-hand side of (26) are all negative, which is a necessary (but not a sufficient) condition for the elements of H_p^B to be negative.

Hence a negative value for θ is critical for replicating the empirically documented[4] and intuitively appealing fact that increases in the conditional volatility of consumption growth cause declines in the price-dividend ratio of the aggregate stock market. As we show in Section 5, our benchmark estimates are in line with that requirement.

The solution for the real risk-free interest rate turns out to be:

$$rfr_t = A_r + H_r^A S_t^A + H_r^B S_t^B, \tag{27}$$

where

$$H_r^A = \frac{1}{\psi} H^c F^A, \tag{28}$$

$$H_r^B = -(1-\theta) H_c^B \left(I - \rho_c F^B\right) - (0.5) \left\{ [WRB \otimes WRB] \, H^\Gamma \right\}, \tag{29}$$

$$WRB = \left[(\theta - 1 - \frac{\theta}{\psi}) H^c + (\theta - 1) \rho_c H_c^A \right].$$

and A_r is a constant.

Equation (28) expresses a relationship that is present in virtually all mainstream consumption-based asset pricing models. Namely, a rise in the expected growth rate of consumption causes an increase in the real risk-free interest rate. Meanwhile, according to (29), a negative θ implies that the non-zero elements of the first term on the right hand-side of (29) are all positive. Conversely, the elements of the second term on the right hand-side of (29) are all negative, regardless of the sign of θ. It turns out that, for virtually all plausible values of the estimated parameters of the model, the second term dominates the first one. That implies that a rise in the conditional volatility of consumption growth leads to a decline in the real risk-free interest rate. Intuitively, an increase in economic uncertainty raises economic agents' desired levels of precautionary saving. This increases the supply of loanable funds in the economy, which, in turn, causes real interest rates to decline.

[4] See Bansal and Yaron (2006).

Using a similar approach, we derive an expression that relates the expected excess returns on asset i, $\underset{t}{E}\left(r^{e,i}_{t+1}\right)$, to S^B_t:

$$\underset{t}{E}\left(r^{e,i}_{t+1}\right) = A_e + H^B_e S^B_t, \tag{30}$$

where

$$H^B_e = -\left[WRC \otimes WRD\right] H^\Gamma - (0.5)\left[WRD \otimes WRD\right] H^\Gamma \tag{31}$$

$$WRC = \left[(\theta - 1 - \frac{\theta}{\psi})H^c + (\theta - 1)\rho_c H^A_c\right],$$

$$WRD = \left[\rho H^A_p + H^d\right],$$

and A_e is a constant.

Note that the two terms on the right hand-side of (31) can be traced back to the third and fourth terms on the right hand-side of (11). Namely, $-\underset{t}{Cov}\left(m_{t+1}, r^i_{t+1}\right) = -\left[WRC \otimes WRD\right] H^\Gamma S^B_t$ and $-(0.5)\underset{t}{Var}\left(r^i_{t+1}\right) = -(0.5)\left[WRD \otimes WRD\right] S^B_t$.

4 Estimating the Model

In order to estimate the unknown parameters and the unobserved states of the model, we use data on four variables - the log price-dividend ration for the aggregate U.S. stock market (pd_t), log per-capita real consumption growth ($\triangle c_t$), log per-capita real dividend growth ($\triangle d_t$), and the log of the real risk-free interest rate (r^{rf}_{t+1}).[5] The vector of observed variables, Y_t, can therefore be written, as:

$$Y_t = \left(pd_t, \triangle c_t, \triangle d_t, r^{rf}_{t+1}\right)'. \tag{32}$$

Since our main goal in this paper is to decompose the fluctuations of asset prices around their means, rather than try to explain the levels of their means, we demean all of our observable variables. The equations relating pd_t and r^{rf}_{t+1} to the vectors of unobserved states (S^A_t and S^B_t) are given in (24) and (27), respectively. The equations relating the other two observable

[5] The exact data sources are described in Appendix B.

variables ($\triangle c_t$ and $\triangle d_t$,) to the two state vectors are trivial and can be easily derived by using (12). Combining the four equations relating the observable variables to the unobserved states allows us to write the observation equation:

$$Y_t = H^A S_t^A + H^B S_t^B + W_t, \tag{33}$$

where

$$H^A = \begin{bmatrix} & & H_p^A & & \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ & & H_r^A & & \end{bmatrix},$$

$$H^B = \begin{bmatrix} & & H_p^B & & \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ & & H_r^B & & \end{bmatrix},$$

and W_t is a vector of observation errors:

$$W_t = \left(w_t^p, w_t^c, w_t^d, w_t^r\right)', \tag{34}$$

$$W_t \sim N(0, R),$$

$$R = \begin{pmatrix} \sigma_{w,p}^2 & 0 & 0 & 0 \\ 0 & \sigma_{w,c}^2 & 0 & 0 \\ 0 & 0 & \sigma_{w,d}^2 & 0 \\ 0 & 0 & 0 & \sigma_{w,r}^2 \end{pmatrix}. \tag{35}$$

There are 19 unknown parameters ($\frac{1}{\psi}$, γ, ϕ_1^c, ϕ_2^c ϕ_1^d, ϕ_2^d, $\sigma_{w,p}^2$, $\sigma_{w,c}^2$, $\sigma_{w,d}^2$, $\sigma_{w,r}^2$, $\phi_{\sigma,cd}$, $\phi_{\sigma,c\tau}$, $\phi_{\sigma,d\tau}$, $\Lambda_{\sigma,cd}$, $\Lambda_{\sigma,c\tau}$, $\Lambda_{\sigma,d\tau}$, $\xi_{\sigma,cd}^2$, $\xi_{\sigma,c\tau}^2$, and $\xi_{\sigma,d\tau}^2$) and 1356 unobserved states (6 states for each of the 226 quarters of data that we use in the estimation of the model) to be estimated.[6] We use Bayesian Markov Chain Monte Carlo (MCMC) methods in order to obtain estimates of the posterior distributions of the unknown parameters and the unobserved states. The details of the estimation procedure are described in Appendix C.

5 Results

5.1 Estimated Parameters and States

The second and third columns of Table 1 summarize the prior distributions that we have selected for the estimation of the model. It is well-known that one of the main advantages of using Bayesian MCMC estimation methods is that they allow for flexible ways of incorporating prior information into the estimation of the model. In an attempt to let the data "tell us" as much as possible about the model's parameters and states, we use relatively flat prior distributions for most of the estimated variables unless we have a solid economic reason for not doing so. For example, we restrict each of the variances of the observation errors (i.e. each of the diagonal elements of R) to be at most 20% of the conditional variance of its respective observable variable. That restriction is forcing the model to explain at least 80% of the fluctuations in each of the observable variables with fluctuations in the relevant state variables rather than with errors in the observation equations. In order to truly incorporate all relevant prior information in the estimation, where appropriate, we impose prior restrictions on the relationships between certain variables rather than on the individual variables themselves. For instance, following the claim of virtually every paper in the long-run risks literature (e.g. Balke and Wohar (2002), Bansal and Yaron (2006), and Croce (2009)) that if the long-run components of consumption growth and dividend growth do exist, they are

[6] Note that θ does not need to be explicitly estimated since the estimates for $\frac{1}{\psi}$ and γ imply an estimate for θ ($\widehat{\theta} = \frac{1-\widehat{\gamma}}{1-\widehat{1/\psi}}$,).

orders of magnitude smaller than their respective observed series, we incorporate a prior distribution over the ratio of the average variances of the permanent and the temporary components of consumption. Namely, we impose an inverse-gamma prior distribution over the ratio of the variance of the temporary component of consumption and the variance of the permanent component of consumption with a mode of 100. We are thus "asking" the model to place higher probabilities on outcomes in which the variance of permanent consumption is significantly smaller than the variance of temporary consumption.

Our estimates of the posterior distributions of the unknown parameters are summarized in the last three columns of Table 1. There are several estimates that deserve attention. Our estimates of the intertemporal elasticity of substitution (ψ) and the coefficient of relative risk aversion (γ) are within the range of estimates that have been reported in the literature. Unlike many asset pricing models, which require values for γ of 10 and above to match certain moments in the data, ours manages to fit the observed stock price and interest rates series reasonably well at levels of risk aversion that are much closer to the estimates obtained in most micro studies on the issue. Our estimate of the EIS is higher than the estimates of Hall (1988) and Campbell (1999), who estimate it to be lower than 1. They are, however, in line with the results of Hansen and Singleton (1982) and Attanasio and Weber (1989), both of whom conclude that EIS is above 1. More recently, several authors (e.g. Guvenen (2001) and Vissing-Jorgensen (2002)) have argued that ψ is well over 1. Furthermore, Bansal and Yaron (2006) argue that the results of Hall (1988) and Campbell (1999), which are obtained by regressing date t+1 consumption growth on the date t real risk-free rate, suffer from a downward bias since they do not account for time variation in the conditional volatility of consumption growth.

The estimates of $\widehat{\psi}$ and $\widehat{\gamma}$ that we obtain have an important implication for the effect of macroeconomic volatility on stock prices. The combination of $\widehat{\psi} > 1$ and $\widehat{\gamma} > 1$ implies that $\widehat{\theta} < 0$, which, as argued in Section 3, is critical for allowing the model to capture the empirical fact that an increase in macroeconomic uncertainty causes the price-dividend ratio of the aggregate stock market to decline. Intuitively, a rise in the conditional volatility of fundamentals affects the price-dividend ratio via two distinct effects that it has on expected returns. First, it decreases the real risk-free rate through the precautionary savings effect discussed in Section 3. Second, it increases the expected excess returns that investors require in order to be compensated for the additional risk that they bear when holding an asset whose payoffs are uncertain. In our model, as well as in the data, the latter effect dominates former, thus causing a negative relationship between macroeconomic volatility and the level of the aggregate stock market.

The three autoregressive processes that govern the evolution of the variances of fundamentals over time appear to be quite persistent, as their AR(1) coefficients ($\phi_{\sigma,cd}$, $\phi_{\sigma,c\tau}$, and $\phi_{\sigma,d\tau}$) are all estimated to be greater than 0.9. This explains the important role that these processes play in driving fluctuations in stock prices and in the risk-free interest rate (discussed in greater detail in Section 5.2) As expected, the mean of the variance of the non-stationary component of consumption growth and dividend growth ($\Lambda_{\sigma,cd}$) is orders of magnitude smaller than its respective stationary counterparts ($\Lambda_{\sigma,c\tau}$ and $\Lambda_{\sigma,d\tau}$). The estimates of the variances of the shocks to the conditional volatilities (ξ_{cd}^2, $\xi_{c\tau}^2$, and $\xi_{d\tau}^2$) indicate that the average shock to the variance of temporary dividend growth is significantly larger than the average shocks to the variances of the temporary and permanent consumption growth components. Finally, the variances of all four observation errors are significantly lower than the upper bounds imposed in the prior distributions, indicating that the model does quite well against the data. The good overall fit of the model is confirmed by figures 1 through 4, which plot the four observable series versus the model's predictions.

5.2 Historical Decompositions and Variance Decompositions

Tables 2 through 5 present the variance decompositions implied by the estimated model for the four observable variables at various horizons. According to Table 2, fluctuations in the log-price dividend ratio at short and medium horizons tend to be driven largely by fluctuations in the conditional variance of the non-stationary component of consumption growth (63% and 59%, respectively). Changes in the conditional mean of the permanent component of consumption growth come at a distant second place (with 13% and 25%, respectively), followed by changes in the volatility of the stationary component of consumption growth (12% and 11%, respectively). In contrast, low frequency movements in the aggregate stock market tend to be driven by changes in the expected mean of permanent dividend growth (72%). While the contribution of time-varying variance of the long-run component of consumption growth declines substantially at long horizons, it is still fairly significant (22%). Figure 5 displays a visual confirmation of the results from Table 2. It shows a historical decomposition for the log price-dividend ratio of the aggregate US stock market. The low frequency movements in the log price-dividend ratio are closely matched by swings in the expected value of the long-run component of consumption growth. Medium and high frequency fluctuations in the aggregate stock market level appear to be closely matched by fluctuations in the conditional volatility of the long-run component of consumption growth. Both of these observations are in line with the results presented in Table 2.

Table 6 presents a different angle. Rather than decomposing fluctuations in the log price-dividend ratio by the contributions of each of the structural shocks in the model (as in Table 2), it breaks down the variance of pd_t by the contributions of each of the three potential drivers discussed in the Section 1. At short and medium horizons, the variance of pd_t tends to be dominated by fluctuations in expected excess returns. Given the results from Table 2, this should not be surprising since, as discussed in Section 2, fluctuations in conditional volatilities are the exclusive triggers of fluctuations in expected excess returns in the model. In the long run, changes in the conditional mean of dividend growth explain the largest share of the variance of aggregate stock price fluctuations. Once again, this is in line with the results of the decomposition presented in Table 2 since the shared long-run component of consumption and dividend growth is, by construction, what is driving both of these series at low frequencies.

Figure 9 offers an intriguing historical perspective on the relative importance of the main drivers of fluctuations in the aggregate US stock market over the past several decades. As discussed above, changes in expected dividend growth were the main drivers at lower frequencies. For example, the stock market run-ups of the 1950's, 1980's, and 1990's were mainly the result of gradually-improving expectations about future dividend growth. Similarly, the decline of the US stock market during the 1970's could largely be attributed to deteriorating long-run dividend growth expectations. Note that the fluctuations in the aggregate price-dividend ratio during all of those episodes would have been even larger had it not been for the offsetting effect of changes in the expectations of future real risk-free interest rates. Recall that, in our model, consumption growth and dividend growth share the same permanent component. Therefore, an increase in expected long-run dividend growth is equivalent to an increase in expected long-run consumption growth. The latter causes a rise the expectations of future real risk-free interest rates (28), which, in turn, causes the price-dividend ratio to decline (6), thus partially offsetting the positive effect that an increase in expected dividend growth has on the price-dividend ratio.

Despite the dominance of changes in expected dividend growth at low frequencies, it is changes in expected excess returns that appear to be responsible for most of the large high-frequency swings in the price-dividend ratio. For instance, our results indicate that the stock market crash of October 1987 was caused by a sharp spike in macroeconomic uncertainty rather than by a dramatic downward revision of expectations about future aggregate dividends. Similarly, our model attributes most of the 2007-08 decline in the US stock market to rising conditional volatility of macroeconomic fundamentals. Expectations of future dividend growth also declined during that episode. However, a large part of that effect was offset by the impact of lower expected future real interest rates, triggered by lower expected macroeconomic growth.

Figures 10 and 14 plot the two sets of variance decompositions discussed above as functions of horizon. Figure 10 shows that fluctuations in the conditional mean of permanent consumption growth gradually take over as the main drivers at the expense of fluctuations in the conditional volatility of the same component.[7] These results imply that short-term and medium-term investors (i.e. those with investment horizons of up to 10-15 years) should be more concerned about sharp increases in macroeconomic uncertainty than about bad news about the economy's growth prospects. The opposite appears to be true for long-term investors (i.e. those with investment horizons that are longer than 10-15 years).

Figure 14 tells the same story, but from the perspective of dividend growth and expected returns. Namely, volatility in expected excess returns is what is driving the stock market in the short and medium run, while news about dividends is the main driver in the long-run. This result manages to reconcile the two competing views about the main sources of aggregate stock market fluctuations described in Section 1. On one hand, as first pointed out by Leroy and Porter (1981) and Shiller (1981), the observed fluctuations in aggregate stock prices over short and medium horizons are way too large to be justified by changes in expected dividends. Therefore, it must be changes in expected excess returns that are the main drivers at these frequencies. On the other hand, as pointed out by Barsky and DeLong (1993) and by Balke and Wohar (2002 and 2006) changes in expected dividend growth could be the main drivers if the dividend growth series is assumed to be sufficiently persistent. Our results indicate that both of those groups of economists were right, albeit at different frequencies. We show that the point made by the former group of economists holds at short and medium term horizons, whereas the conjecture made by the latter group is indeed valid if one focuses on long term horizons. Intuitively, if the level of the aggregate stock market declined by 50% over the course of a recession, it is more plausible to argue that this happened because investors demanded to be compensated for the rise in macroeconomic uncertainty rather than because they suddenly dramatically revised downwards their expectations of

[7] The actual change of positions between the two series takes place at a horizon of approximately 14 years.

future aggregate dividends in the economy. If on the other hand, the price-dividend ratio of the aggregate stock market gradually increased over the course of a couple of decades, it would be easier to make the case that this occurred due to gradually-revised long-term dividend growth expectations rather than as a result of a sustained twenty-year decline in macroeconomic volatility.

Summaries of variance decomposition for the short-term real risk-free rate at various horizons are presented in Table 3. At all horizons, the variance of this variable appears to be dominated by movements in the conditional volatility of the non-stationary component of consumption growth. Fluctuations in that component explain more than 90% of the total variance of the real risk-free rate at all horizons. The results from Table 3 are confirmed by the historical decomposition of the real interest rate in Figure 8 and by Figure 13, which plots the variances decompositions of the real risk-free rate, implied by our estimates of the model, as a function of the horizon.

Finally, Tables 4 and 5 put the results obtained from the variance decompositions of the price-dividend ratio into perspective. Some might argue that the fact that the joint permanent component of consumption growth and dividend growth is the only variable that is assumed to have a unit root might somehow influence the result that this particular component is the main driver of the price-dividend ration in the long-run. A necessary (but not a sufficient) condition for that potential criticism to have any merit is that the joint permanent component of consumption growth and dividend growth is the main long-run driver of these two variables themselves. However, the results presented in Tables 4 and 5 allow us to categorically dismiss all suspicions of that kind. Namely, the third columns of these tables reveal that the joint permanent component of consumption growth and dividend growth is far from being the main driver of these two variables at any horizon. These conclusions are visually confirmed by the results displayed in Figures 11 and 12.

6 Conclusion

In this paper, we attempt to resolve the long-lasting debate about the main sources of fluctuations in the aggregate US stock market. In order to accomplish that, we design and solve a consumption-based asset pricing model which incorporates stochastic volatility, long-run risks in consumption and dividends, and Epstein-Zin preferences. Utilizing Bayesian MCMC techniques, we estimate the model by using US data on the level of the aggregate stock market, short-term real risk-free rates, consumption growth, and dividend growth. We conclude that, over short and medium horizons, fluctuations in the aggregate US stock market are mainly driven by changes in expected excess returns. Conversely, low-frequency movements in aggregate stock prices are primarily caused by changes in expected dividend growth rates.

The paper offers several possible directions for further research. An intriguing extension would be to add time-varying risk aversion to the time-varying variances model presented here. This feature would introduce fluctuations in the price of risk over time in addition to the fluctuations in the quantity of risk introduced in this paper. It would be also interesting to extend the theoretical environment of the model into a general equilibrium setting. Justiniano and Primiceri (2008) have already explored the effects of time-varying volatility on macroeconomic fluctuations. As the title of their paper suggests, however, they were mostly concerned with the business cycle implications of their model. We believe that it would be extremely informative to include data on asset prices (particularly data on stock prices) in the estimation of a DSGE model with time-varying volatility and time-varying risk aversion and study the model's asset pricing implications.

Appendix A. Solution Methodology

We start by solving for r^c_{t+1}, the return on the hypothetical asset that delivers aggregate consumption as its dividend each period. Let P^c_t be the price of that asset at time t. Since its dividend is equal to aggregate consumption, its log-price-dividend ratio is given by:

$$pc_t \equiv p^c_t - c_t = \log(P^c_t / C_t). \tag{A.1}$$

Using the same line of reasoning as the one described in the justification of the guess for the expression for pd_t in Section 3, we make the following guess for pc_t:

$$pc_t = A^{TVV}_c + H^A_c S^A_t + H^B_c S^B_t. \tag{A.2}$$

Note that (2) can be modified to obtain an expression for pc_t:

$$pc_t = \rho_c \left(pc_{t+1}\right) + \triangle c_{t+1} - r^c_{t+1} + k_c \tag{A.3}$$

Rearranging terms in (A.3), we solve for r^c_{t+1}:

$$r^c_{t+1} = \rho_c \left(pc_{t+1}\right) + \triangle c_{t+1} - pc_t + k_c. \tag{A.4}$$

Using our guess for pc_t (A.2) and the fact that $\triangle c_{t+1} = H^c S^A_{t+1}$, we can rewrite (A.4) as:

$$r^c_{t+1} = \rho \left(A^{TVV}_c + H^A_c S^A_{t+1} + H^B_c S^B_{t+1}\right) + H^c S^A_{t+1} - \left(A^{TVV}_c + H^A_c S^A_t + H^B_c S^B_t\right) + k_c. \tag{A.5}$$

Note that the log-version of the Euler equation (10) must hold for the returns of all assets in the economy. Therefore, we can write:

$$\underset{t}{E}\left(m_{t+1} + r^c_{t+1}\right) + \frac{1}{2} \underset{t}{Var}\left(m_{t+1} + r^c_{t+1}\right) = 0. \tag{A.6}$$

Using (8), we can write:

$$\begin{aligned} m_{t+1} + r^c_{t+1} &= \theta \log \delta - \frac{\theta}{\psi}\left(\triangle c_{t+1}\right) + (\theta)\left(r^c_{t+1}\right) = \\ & \theta \log \delta - \frac{\theta}{\psi}\left(H^c S^A_{t+1}\right) + \theta \left[\begin{array}{c} \rho_c \left(A^{TVV}_c + H^A_c S^A_{t+1} + H^B_c S^B_{t+1}\right) + \\ H^c S^A_{t+1} - \left(A^{TVV}_c + H^A_c S^A_t + H^B_c S^B_t\right) + k_c \end{array} \right] \end{aligned} \tag{A.7}$$

Combining (A.7) with equations (18) and (23), we can write:[8]

$$\underset{t}{E}\left[m_{t+1}+r_{t+1}^{c}\right]=\left\{-\frac{\theta}{\psi}H^{c}F^{A}+\theta\left[\left(\rho_{c}H_{c}^{A}+H^{c}\right)F^{A}-H_{c}^{A}\right]\right\}S_{t}^{A}+\theta\left(\rho_{c}F^{B}-I\right)H_{C}^{B}S_{t}^{B} \tag{A.8}$$

$$\underset{t}{Var}\left(m_{t+1}+r_{t+1}^{c}\right)=\left\{\left[\left(\theta-\frac{\theta}{\psi}\right)H^{c}+\theta\rho_{c}H_{c}^{A}\right]\Gamma\right\}\otimes\left\{\left[\left(\theta-\frac{\theta}{\psi}\right)H^{c}+\theta\rho_{c}H_{c}^{A}\right]\Gamma\right\}H^{\Gamma}S_{t}^{B}, \tag{A.9}$$

Plugging (A.8) and (A.9) into (A.6), we get:

$$\begin{aligned} 0 \;=\; & \left\{-\frac{\theta}{\psi}H^{c}F^{A}+\theta\left[\left(\rho_{c}H_{c}^{A}+H^{c}\right)F^{A}-H_{c}^{A}\right]\right\}S_{t}^{A}+ \\ & +\theta\left(\rho_{c}F^{B}-I\right)H_{c}^{B}S_{t}^{B}+ \\ & +\frac{1}{2}\left\{\left[\left(\theta-\frac{\theta}{\psi}\right)H^{c}+\theta\rho_{c}H_{c}^{A}\right]\Gamma\right\}\otimes\left\{\left[\left(\theta-\frac{\theta}{\psi}\right)H^{c}+\theta\rho_{c}H_{c}^{A}\right]\Gamma\right\}H^{\Gamma}S_{t}^{B}. \end{aligned} \tag{A.10}$$

Since (A.10) has to hold for all values of the state variables, combining all the S_t^A and S_t^B terms and setting them equal to each other yields the following two equations:

$$-\frac{\theta}{\psi}H^{c}F^{A}+\theta\left[\left(\rho_{c}H_{c}^{A}+H^{c}\right)F^{A}-H_{c}^{A}\right]=0, \tag{A.11}$$

$$\theta\left(\rho_{c}F^{B}-I\right)H_{c}^{B}+\frac{1}{2}\left\{\left[\left(\theta-\frac{\theta}{\psi}\right)H^{c}+\theta\rho_{c}H_{c}^{A}\right]\Gamma\right\}\otimes\left\{\left[\left(\theta-\frac{\theta}{\psi}\right)H^{c}+\theta\rho_{c}H_{c}^{A}\right]\Gamma\right\}H^{\Gamma}=0. \tag{A.12}$$

Solving the above two equations yields:

$$H_{c}^{A}=\left[\left(1-\frac{1}{\psi}\right)H^{c}\right]F^{A}\left(I-\rho_{c}F^{A}\right)^{-1}, \tag{A.13}$$

$$H_{c}^{B}=(0.5)\theta\left\{\left[WCB\otimes WCB\right]H^{\Gamma}\right\}\left(I-\rho_{c}F^{B}\right)^{-1}, \tag{A.14}$$

where

$$WCB=\left(1-\frac{1}{\psi}\right)H^{c}F^{A}\left(I-\rho_{c}F^{A}\right)^{-1}\Gamma.$$

[8] Throughout the solution derivations, we use the fact that, for any three matrices, A, B, and C: $vec(ABC)=(C'\otimes A)\,vec(B)$.

Having obtained the expressions for H_c^A and H_c^B, we can now plug (A.13) and (A.14) into (A.5) and then use the resulting expressions for r_{t+1}^c in (8) in order to obtain the following the expression for the stochastic discount factor:

$$\begin{aligned} m_{t+1} &= A_m - \left(\frac{1}{\psi} H^c F^A\right) S_t^A + (1-\theta) H_c^B \left(I - \rho_c F^B\right) S_t^B + \\ & \left[\left(\theta - 1 - \frac{\theta}{\psi}\right) H^c + \rho_c (\theta - 1) H_c^A\right] \left(V_{t+1}^A\right) + \rho_c (\theta - 1) H_c^B \left(V_{t+1}^B\right). \end{aligned} \tag{A.15}$$

Having solved for the stochastic discount factor in terms of the state variables, we are now ready to turn our attention to the price-dividend ratio for the aggregate stock market (pd_t). Note that we can use (3) in order to solve for the return of the aggregate stock market:

$$r_{t+1} = \rho \left(pd_{t+1}\right) + \triangle d_{t+1} - pd_t + k. \tag{A.16}$$

Using the fact that $\triangle d_{t+1} = H^d S_{t+1}^A$ and plugging our guess for pd_t (24) in (A.16), we get:

$$r_{t+1} = \rho \left(A_p^{TVV} + H_p^A S_{t+1}^A + H_p^B S_{t+1}^B\right) + H^d S_{t+1}^A - \left(A_p^{TVV} + H_p^A S_t^A + H_p^B S_t^B\right) + k. \tag{A.17}$$

Note that:

$$\begin{aligned} \underset{t}{E} \left[m_{t+1} + r_{t+1}\right] &= \left\{-\frac{1}{\psi} H^c F^A + H_p^A \left(\rho F^A - I\right) + H^d F^A\right\} S_t^A + \\ & \left[(\theta - 1) H_C^B \left(\rho_c F^B - I\right) + H_p^B \left(\rho F^B - I\right)\right] S_t^B \end{aligned} \tag{A.18}$$

$$\underset{t}{Var} \left(m_{t+1} + r_{t+1}\right) = \left[WPB \otimes WPB\right] H^\Gamma S_t^B + (WPS) \Sigma^B (WPS)', \tag{A.19}$$

where

$$WPB = \left[\left(\theta - 1 - \frac{\theta}{\psi}\right) H^c + (\theta - 1) \rho_c H_c^A + \rho H_p^A + H^d\right] \Gamma,$$

$$WPS = (\theta - 1) \rho_c H_C^B + \rho H_p^B.$$

Next, we plug (A.18) and (A.19) into the log-version of the Euler equation (10) and get:

$$
\begin{aligned}
0 \;=\; & (WPS)\,\Sigma^B\,(WPS)' + \\
& \left\{-\frac{1}{\psi}H^c F^A + H_p^A\left(\rho F^A - I\right) + H^d F^A\right\} S_t^A + \\
& + \left[(\theta-1)\,H_C^B\left(\rho_c F^B - I\right) + H_P^B\left(\rho F^B - I\right)\right] S_t^B + \\
& + \left[WPB \otimes WPB\right] H^\Gamma S_t^B.
\end{aligned} \tag{A.20}
$$

Since (A.20) has to hold for all values of the state variables, we combine all the S_t^A and S_t^B terms and set them equal to each other. This yields the following two equations:

$$
-\frac{1}{\psi}H^c F^A + H_p^A\left(\rho F^A - I\right) + H^d F^A = 0
$$

$$
\left[(\theta-1)\,H_c^B\left(\rho_c F^B - I\right) + H_p^B\left(\rho F^B - I\right)\right] + \left\{[WPB]\,\Gamma\right\} \otimes \left\{[WPB]\,\Gamma\right\} H^\Gamma = 0
$$

Solving the above two equations yields:

$$
H_p^A = \left(H^d - \frac{1}{\psi}H^c\right) F^A \left(I - \rho F^A\right)^{-1}, \tag{A.21}
$$

$$
H_p^B = \left\{(1-\theta)\,H_c^B\left(I - \rho_c F^B\right) + \frac{1}{2}\left[WPB \otimes WPB\right] H^\Gamma\right\} \left(I - \rho F^B\right)^{-1}. \tag{A.22}
$$

Note that (A.15) in combination with (11) allows us to express the one-period ahead real risk-free rate as a function of the two state vectors:

$$
\begin{aligned}
r_{t+1}^{rf} \;=\; & -\underset{t}{E}\left(m_{t+1}\right) - \frac{1}{2}\underset{t}{Var}\left(m_{t+1}\right) = \\
& -A_m - \left[(\theta-1)\rho_c H_C^B\right]\left[\Sigma^B\right]\left[(\theta-1)\rho_c H_c^B\right] \\
& + \left(\frac{1}{\psi}H^c F^A\right) S_t^A - (1-\theta)\,H_c^B\left(I - \rho_c F^B\right) S_t^B \\
& - \frac{1}{2}\left\{(WRB) \otimes (WRB)\right\} H^\Gamma S_t^B,
\end{aligned} \tag{A.23}
$$

where

$$
WRB = \left[(\theta - 1 - \frac{\theta}{\psi})H^c + (\theta-1)\rho_c H_c^A\right]\Gamma.
$$

Appendix B. Data Sources

The data that we use to estimate the unknown parameters and the unobserved states of our model is quarterly and runs from 1952:01 to 2008:02. For our stock market variables, we use CRSP data, which includes all stocks listed on the NYSE, the AMEX, and the NASDAQ, in order to construct the aggregate log-price-dividend ratio and log-dividend-growth series. Following the approach taken by Bansal, Dittmar, and Lundblad (2005), we define the total return per dollar invested, R_{t+1}, to be:

$$R_{t+1} = h_{t+1} + y_{t+1},$$

where h_{t+1} is the gross price appreciation from t to $t+1$ and y_{t+1} is the dividend yield during period $t+1$. Since we observe R_{t+1} (RET in CRSP terminology) and h_{t+1} (RETX in CRSP terminology), we can easily obtain the dividend yield, y_{t+1}, by taking the difference between the two series:

$$y_{t+1} = R_{t+1} - h_{t+1.}$$

Having obtained the above series, it is straightforward to generate the actual price (P_t) and dividend (D_t) series.

$$\begin{aligned} P_{t+1} &= h_t P_t, \qquad & t = 2, ..., T-1, \quad P_1 = 1, \\ D_t &= y_t P_t, \qquad & t = 1, ..., T. \end{aligned}$$

Note that we have constructed the P_t series in such a way that it corresponds to total market valuation, not price per share as in some other studies. Similarly D_t corresponds to total dividends paid by all companies in the market, not dividends per share. We use the yield on the three-month Treasury bill to measure the short-term nominal risk-free interest rate. We use data on consumption of non-durables and services, obtained from the National Income and Product Accounts (NIPA) tables, in order to construct our nominal consumption growth series. We convert all nominal series into real by using the price index that is implied by the data on nominal and real consumption of non-durables and services (also obtained from the NIPA tables).

Appendix C. Bayesian Estimation of the State-Space Model

We use Bayesian Markov Chain Monte Carlo (MCMC) methods in order to obtain estimates of the joint posterior distribution of the structural parameters and the unobserved states. We modify the approach used in Avdjiev and Balke (2009) in order to account for the fact that the state space model we derive in this paper differs from a conventional state space model due to the fact that the variances of the shocks to state equation A (17) are functions of the shocks to state equation B (23). We split the 19 unknown parameters into two vectors. The first one ($\Theta^A = [\frac{1}{\psi}, \gamma, \phi_1^c, \phi_2^c, \phi_1^d, \phi_2^d, \sigma^2_{w,p}, \sigma^2_{w,c}, \sigma^2_{w,d}, \sigma^2_{w,r}]$) contains the parameters that are directly related to state equation A. The second vector ($\Theta^B = [\phi_{\sigma,cd}, \phi_{\sigma,c\tau}, \phi_{\sigma,d\tau}, \Lambda_{\sigma,cd}, \Lambda_{\sigma,c\tau}, \Lambda_{\sigma,d\tau}, \xi^2_{\sigma,cd}, \xi^2_{\sigma,c\tau}, \xi^2_{\sigma,d\tau}]$) consists of the parameters that are directly related to state equation B.

We use a combination of the Random Walk Metropolis-Hastings (RWMH) and the Gibbs Sampling (GS) algorithms in the following way:

1. We choose arbitrary initial values for the structural parameters, $\left\{\Theta^A\right\}^0$ and $\left\{\Theta^B\right\}^0$, and for the unobserved states, $\left\{S_T^A\right\}^0$ and $\left\{S_T^B\right\}^0$.

2. For $i = 1, ..., n_{sim}$, we iterate the following four steps:

2.1. Given $\left\{\Theta^A\right\}^{i-1}$, $\left\{\Theta^B\right\}^{i-1}$, and $\left\{S_T^B\right\}^{i-1}$, we use the Kalman Filter to obtain the conditional distribution of S_T^A: $P(S_T^A | \left\{\Theta^A\right\}^{i-1}, \left\{\Theta^B\right\}^{i-1}, \left\{S_T^B\right\}^{i-1}, Y_T)$. We obtain a draw, $\left\{S_T^A\right\}^i$, from $P(S_T^A | \left\{\Theta^A\right\}^{i-1}, \left\{\Theta^B\right\}^{i-1}, \left\{S_T^B\right\}^{i-1}, Y_T)$. In this step, we use the "filter forward, sample backward" approach proposed by Carter and Kohn (1994) and discussed in Kim and Nelson (1999).

2.2. Given $\left\{\Theta^A\right\}^{i-1}$, we draw a candidate set of parameters, $\left\{\Theta^A\right\}^c$, from a previously specified distribution: $g_a(\left\{\Theta^A\right\}^c | \left\{\Theta^A\right\}^{i-1})$. Our choice of $g_a(\cdot)$ is such that, $\left\{\Theta^A\right\}^c = \left\{\Theta^A\right\}^{i-1} + v_a$, where v_a is drawn from a multivariate t-distribution with five degrees of freedom and a covariance matrix Σ^A. We set Σ^A to be a scaled version of the Hessian matrix of the log posterior probability, evaluated at the posterior mode. We choose the scale so that $20\% - 30\%$ of the candidate draws are accepted. We determine the acceptance probability, α_a, for the candidate draw:

$$\alpha_a(\left\{\Theta^A\right\}^c, \left\{\Theta^A\right\}^{i-1}) = \min\left\{\widetilde{\alpha_a}, 1\right\},$$

where

$$\widetilde{\alpha_a} = \frac{\exp(l(Y_T, \left\{\Theta^A\right\}^c, \left\{\Theta^B\right\}^{i-1}, \left\{S_T^B\right\}^{i-1}))\pi(\left\{\Theta^A\right\}^c)}{\exp(l(Y_T, \left\{\Theta^A\right\}^{i-1}, \left\{\Theta^B\right\}^{i-1}, \left\{S_T^B\right\}^{i-1}))\pi(\left\{\Theta^A\right\}^{i-1})},$$

$l(Y_T, \Theta^A, \Theta^B, S_T^B)$ is the log-likelihood and $\pi(\Theta^A)$ is the prior probability of Θ^A. We select $\left\{\Theta^A\right\}^i$ according to the following rule:

$$\left\{\Theta^A\right\}^i = \left\{\Theta^A\right\}^c \text{ with probability } \alpha_a;$$
$$\left\{\Theta^A\right\}^i = \left\{\Theta^A\right\}^{i-1} \text{ with probability } 1 - \alpha_a.$$

2.3. Given $\left\{S_T^B\right\}^{i-1}$, we use the RWMH algorithm to draw a candidate $\left\{S_t^B\right\}^c$ for each point in time t ($t = 1...T$) from a previously specified distribution: $g_s(\left\{S_t^B\right\}^c \mid \left\{S_t^B\right\}^{i-1})$. Our choice of $g_s(\cdot)$ is such that, $\left\{S_t^B\right\}^c = \left\{S_t^B\right\}^{i-1} + v_s$, where v_s is drawn from a multivariate t-distribution with five degrees of freedom and a covariance matrix Σ^{S_B}. We choose the scale so that $20\% - 30\%$ of the candidate draws are accepted. We determine the acceptance probability, α_s, for the candidate draw:

$$\alpha_s(\left\{S_t^B\right\}^c, \left\{S_t^B\right\}^{i-1}) = \min\left\{\widetilde{\alpha_s}, 1\right\},$$

where

$$\widetilde{\alpha_s} = \frac{p(\left\{S_t^B\right\}^c \mid \left\{S_{t-1}^B\right\}^i, \left\{S_{t+1}^B\right\}^{i-1}, \left\{\Theta^B\right\}^{i-1}) \exp(l(Y_T, \left\{\Theta^A\right\}^i, \left\{\Theta^B\right\}^{i-1}, \left\{S_T^B\right\}^c))}{p(\left\{S_t^B\right\}^{i-1} \mid \left\{S_{t-1}^B\right\}^i, \left\{S_{t+1}^B\right\}^{i-1}, \left\{\Theta^B\right\}^{i-1}) \exp(l(Y_T, \left\{\Theta^A\right\}^i, \left\{\Theta^B\right\}^{i-1}, \left\{S_T^B\right\}^{i-1}))}.$$

We select $\left\{S_t^B\right\}^i$ according to the following rule:

$$\left\{S_t^B\right\}^i = \left\{S_t^B\right\}^c \text{ with probability } \alpha_s;$$
$$\left\{S_t^B\right\}^i = \left\{S_t^B\right\}^{i-1} \text{ with probability } 1-\alpha_s.$$

2.4. Given $\left\{\Theta^B\right\}^{i-1}$, we draw a candidate set of parameters, $\left\{\Theta^B\right\}^c$, from a previously specified distribution: $g_b(\left\{\Theta^B\right\}^c \mid \left\{\Theta^B\right\}^{i-1})$. Our choice of $g_b(\cdot)$ is such that, $\left\{\Theta^B\right\}^c = \left\{\Theta^B\right\}^{i-1} + v_b$, where v_b is drawn from a multivariate t-distribution with five degrees of freedom and a covariance matrix Σ^B. We set Σ^B to be a scaled version of the Hessian matrix of the log posterior probability, evaluated at the posterior mode. We choose the scale so that $20\% - 30\%$ of the candidate draws are accepted. We determine the acceptance probability, α_b, for the candidate draw:

$$\alpha_b(\left\{\Theta^B\right\}^c, \left\{\Theta^B\right\}^{i-1}) = \min\left\{\widetilde{\alpha_b}, 1\right\},$$

where

$$\widetilde{\alpha_b} = \frac{\exp(l(Y_T, \left\{\Theta^A\right\}^i, \left\{\Theta^B\right\}^c, \left\{S_T^B\right\}^i))p(\left\{\Theta^B\right\}^c \mid \left\{S_T^B\right\}^i)\pi(\left\{\Theta^B\right\}^c)}{\exp(l(Y_T, \left\{\Theta^A\right\}^i, \left\{\Theta^B\right\}^{i-1}, \left\{S_T^B\right\}^i))p(\left\{\Theta^B\right\}^{i-1} \mid \left\{S_T^B\right\}^i)\pi(\left\{\Theta^B\right\}^{i-1})}.$$

We select $\left\{\Theta^B\right\}^i$ according to the following rule:

$$\left\{\Theta^B\right\}^i = \left\{\Theta^B\right\}^c \text{ with probability } \alpha_b;$$
$$\left\{\Theta^B\right\}^i = \left\{\Theta^B\right\}^{i-1} \text{ with probability } 1-\alpha_b.$$

If $i < n_{sim}$, we return to step 2.1. If $i = n_{sim}$, we move on to step 3.

3. We discard the first m draws ($m < n_{sim}$) in order to ensure that the initial conditions do not influence our estimates in any way. We approximate the expected value of any function of interest, $f(\Theta)$, by using the following formula:

$$\widehat{f(\Theta)} = \left(\frac{1}{n_{sim} - m}\right) \sum_{i=m+1}^{n_{sim}} f(\Theta^{(i)}).$$

In this particular application, we run 200,000 iterations of the sampling procedure (i.e. we set $n_{sim} = 200,000$) and we use only the last 10,000 draws (i.e. $m = 190,000$) to make inference about the posterior distributions of the structural parameters and the unobserved states.

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Table 1. Prior and Posterior Parameter Distributions

	Prior		Posterior		
Parameter	Mean	Variance	Median	5th Percentile	95th Percentile
$\frac{1}{\psi}$	0.5	10	0.4704	0.4587	0.4856
γ	1.5	10	2.3141	2.2857	2.3295
ϕ_1^c	0	2	0.4019	0.3958	0.4085
ϕ_2^c	0	2	0.2229	0.2191	0.2301
ϕ_1^d	0	2	0.4223	0.4085	0.4374
ϕ_2^d	0	2	0.1836	0.1802	0.1886
$\phi_{\sigma,cd}$	0.5	10	0.9499	0.9442	0.9541
$\phi_{\sigma,c\tau}$	0.5	10	0.9520	0.9475	0.9594
$\phi_{\sigma,d\tau}$	0.5	10	0.9092	0.9013	0.9168
$\sigma_{w,p}^2$	$0.15\sigma_p^2$	$0.5\sigma_p^2$	$0.1280\sigma_p^2$	$0.1229\sigma_p^2$	$0.1315\sigma_p^2$
$\sigma_{w,c}^2$	$0.15\sigma_c^2$	$0.5\sigma_c^2$	$0.0989\sigma_c^2$	$0.0927\sigma_c^2$	$0.1054\sigma_c^2$
$\sigma_{w,d}^2$	$0.15\sigma_d^2$	$0.5\sigma_d^2$	$0.0254\sigma_d^2$	$0.0214\sigma_d^2$	$0.0298\sigma_d^2$
$\sigma_{w,r}^2$	$0.15\sigma_r^2$	$0.5\sigma_r^2$	$0.1262\sigma_r^2$	$0.1205\sigma_r^2$	$0.1310\sigma_r^2$
$\Lambda_{\sigma,cd}$	0.005	2	0.0017	0.0015	0.0018
$\Lambda_{\sigma,c\tau}$	0.5	1	0.3707	0.3651	0.3758
$\Lambda_{\sigma,d\tau}$	3	1	2.9478	2.8979	2.9894
$\xi_{\sigma,cd}^2$	0.001	1	0.0024	0.0020	0.0029
$\xi_{\sigma,c\tau}^2$	0.01	1	0.0058	0.0051	0.0068
$\xi_{\sigma,d\tau}^2$	0.03	1	0.2098	0.1712	0.2415

Table 2. Price Dividend Ratio Variance Decompositions

	Short Run	Medium Run	Long Run
$\triangle c_t^p$	13.2	24.7	72.3
$\triangle c_t^\tau$	0.3	0.1	0.0
$\triangle d_t^\tau$	7.6	2.8	0.7
$\sigma_{cd,p}^2$	63.3	58.9	21.9
$\sigma_{c,\tau}^2$	12.0	11.4	4.3
$\sigma_{d,\tau}^2$	3.6	2.4	0.8

Note: Short Run = 1 to 6 quarters ahead FEVD,

Medium Run = 7 to 32 quarters ahead FEVD,

Long Run = unconditional FEVD.

Table 3. Real Risk-Free Rate Variance Decompositions

	Short Run	Medium Run	Long Run
$\triangle c_t^p$	0.1	0.3	2.2
$\triangle c_t^\tau$	4.0	1.6	1.2
$\triangle d_t^\tau$	-	-	-
$\sigma_{cd,p}^2$	93.2	95.4	93.9
$\sigma_{c,\tau}^2$	2.6	2.8	2.8
$\sigma_{d,\tau}^2$	-	-	-

Note: Short Run = 1 to 6 quarters ahead FEVD,

Medium Run = 7 to 32 quarters ahead FEVD,

Long Run = unconditional FEVD.

Table 4. Consumption Growth Variance Decompositions

	Short Run	Medium Run	Long Run
$\triangle c_t^p$	0.9	5.6	35.3
$\triangle c_t^\tau$	99.1	94.4	64.7
$\triangle d_t^\tau$	-	-	-
$\sigma^2_{cd,p}$	-	-	-
$\sigma^2_{c,\tau}$	-	-	-
$\sigma^2_{d,\tau}$	-	-	-

Note: Short Run = 1 to 6 quarters ahead FEVD,

Medium Run = 7 to 32 quarters ahead FEVD,

Long Run = unconditional FEVD.

Table 5. Dividend Growth Variance Decompositions

	Short Run	Medium Run	Long Run
$\triangle c_t^p$	0.1	0.8	6.7
$\triangle c_t^\tau$	-	-	-
$\triangle d_t^\tau$	99.9	99.2	93.3
$\sigma^2_{cd,p}$	-	-	-
$\sigma^2_{c,\tau}$	-	-	-
$\sigma^2_{d,\tau}$	-	-	-

Note: Short Run = 1 to 6 quarters ahead FEVD,

Medium Run = 7 to 32 quarters ahead FEVD,

Long Run = unconditional FEVD.

Table 6. Price Dividend Ratio Variance Decompositions, by Component

	Short Run	Medium Run	Long Run
$\triangle d_t$	54.8	90.5	258.4
rfr_t	142.7	142.7	102.8
r_t^e	404.8	376.8	140.1
$\sigma_{\triangle d_t, rfr_t}$	-44.4	-82.9	-242.5
σ_{rfr_t, r_t^e}	-457.8	-427.0	-158.9

Note: Short Run = 1 to 6 quarters ahead FEVD,

Medium Run = 7 to 32 quarters ahead FEVD,

Long Run = unconditional FEVD.

Figure 1: Price-Dividend Ratio Fit

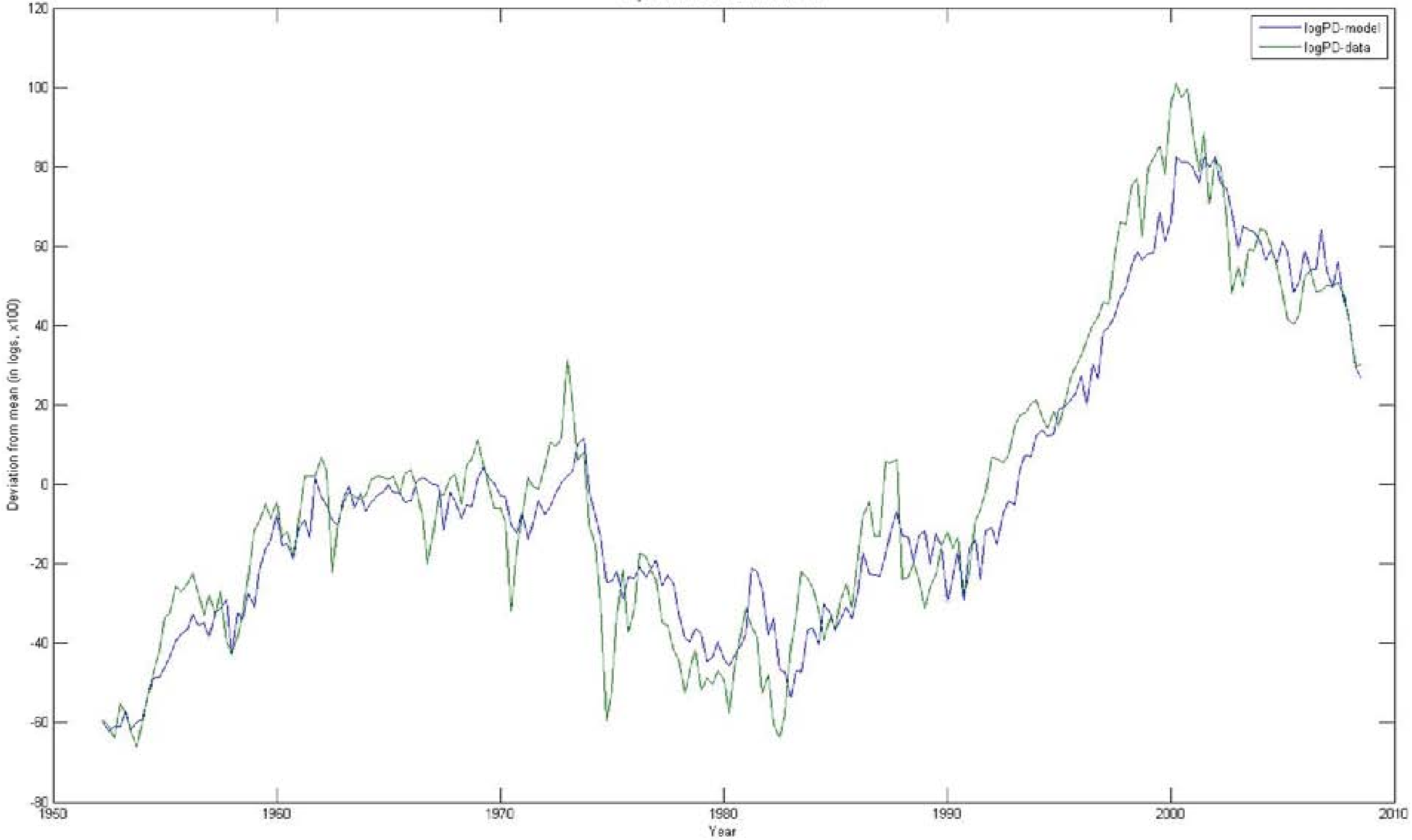

Figure 2: Consumption Growth Fit

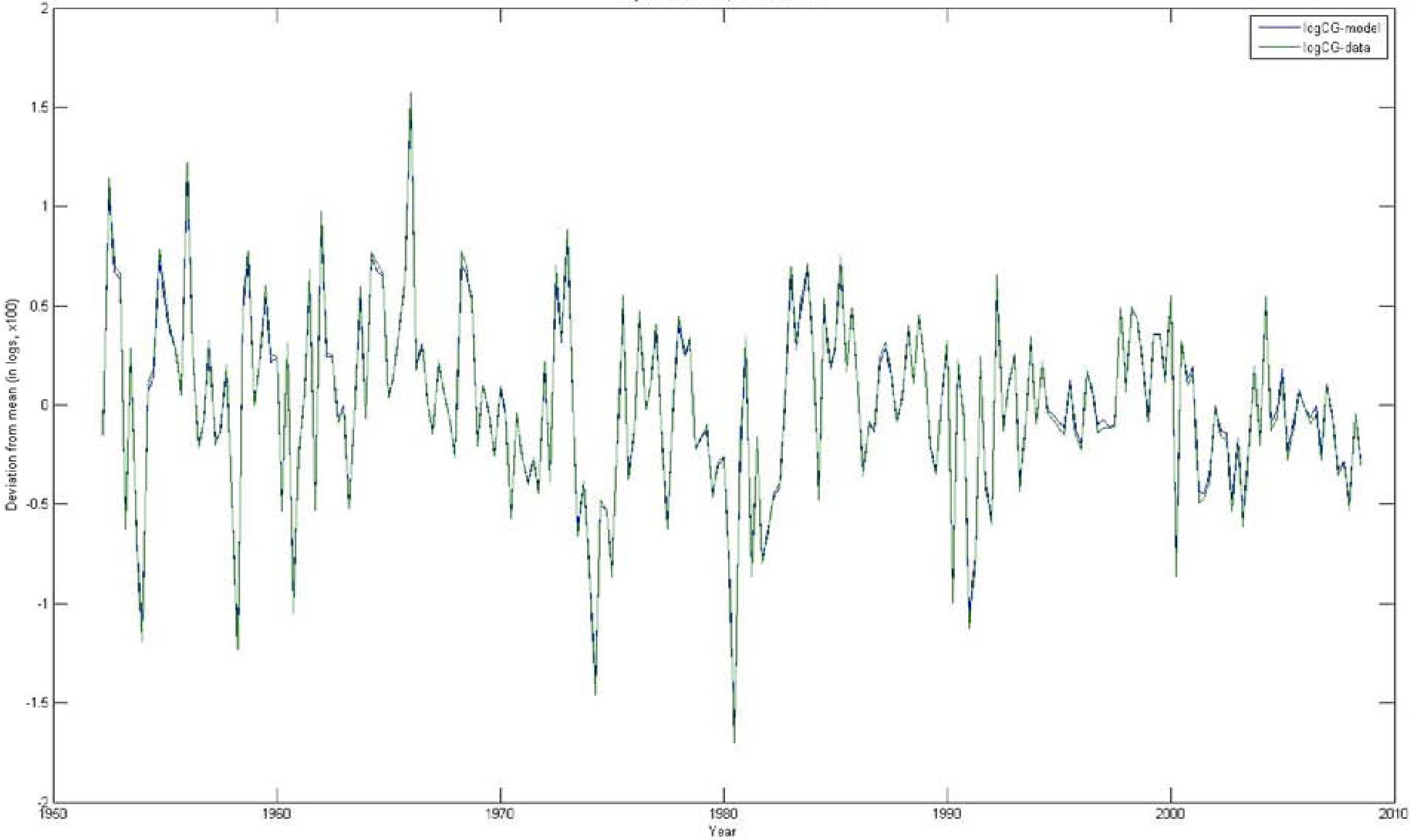

Figure 3: Dividend Growth Fit

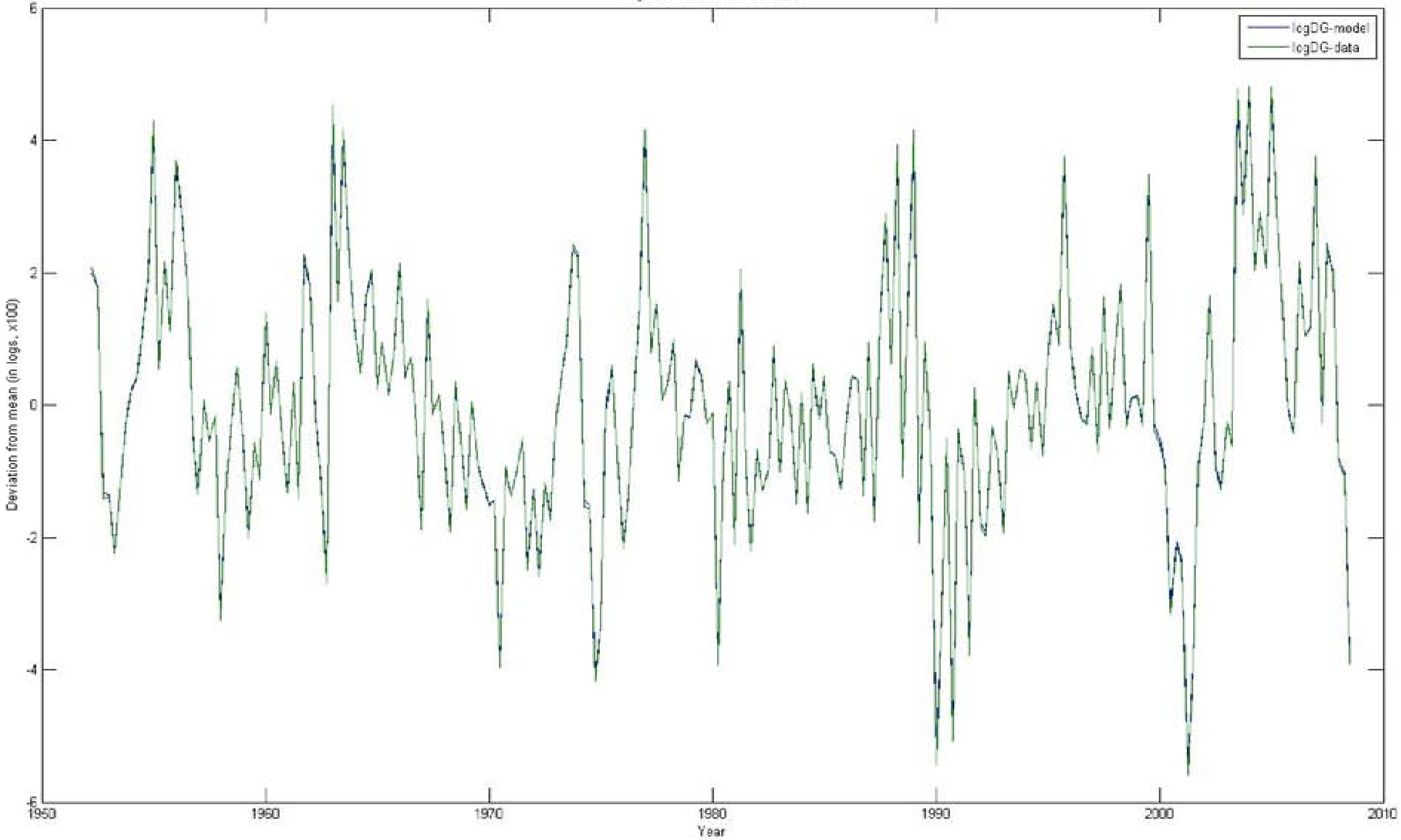

Figure 4: Real Risk-Free Rate Fit

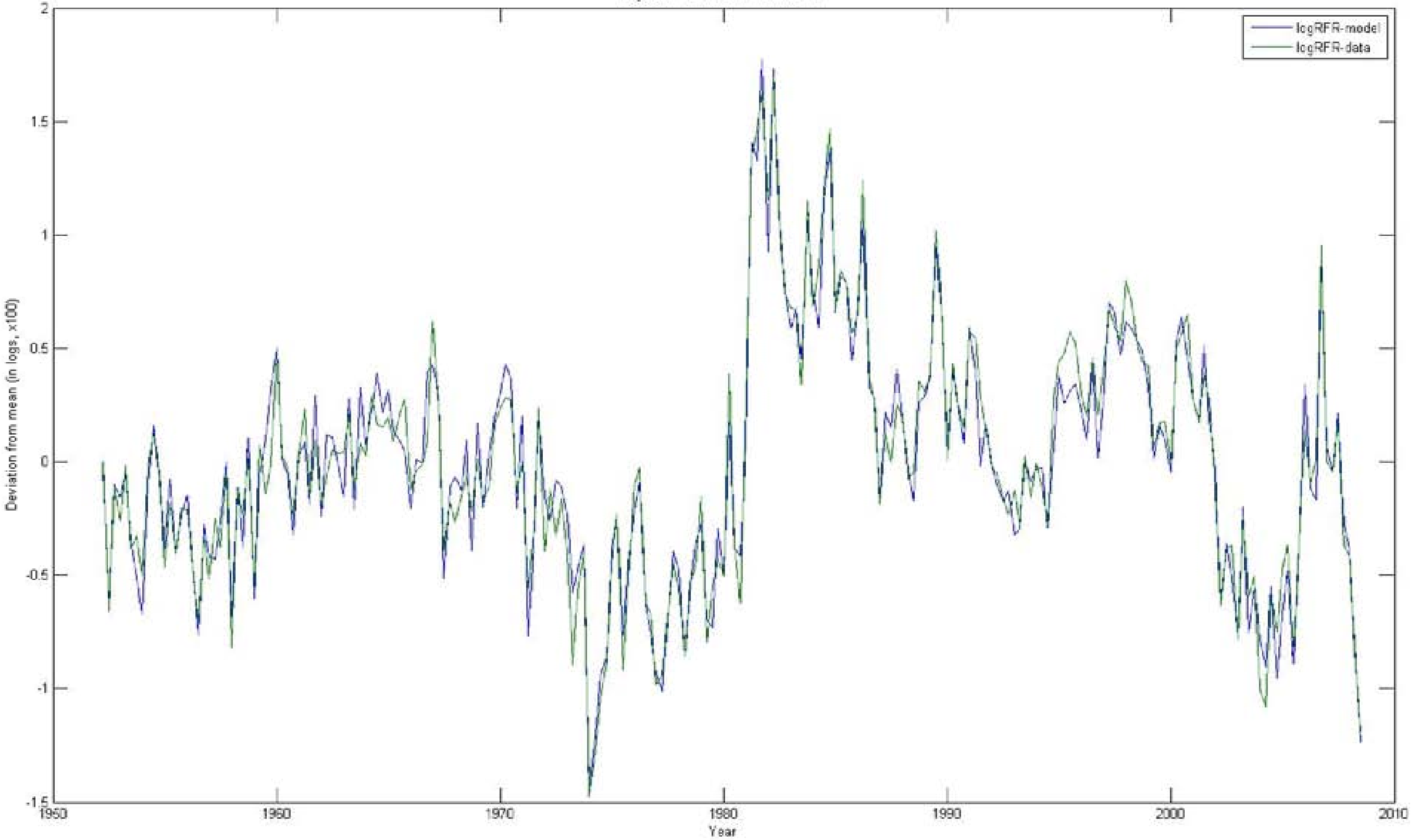

Figure 5: Price-Dividend Ratio Historical Decomposition

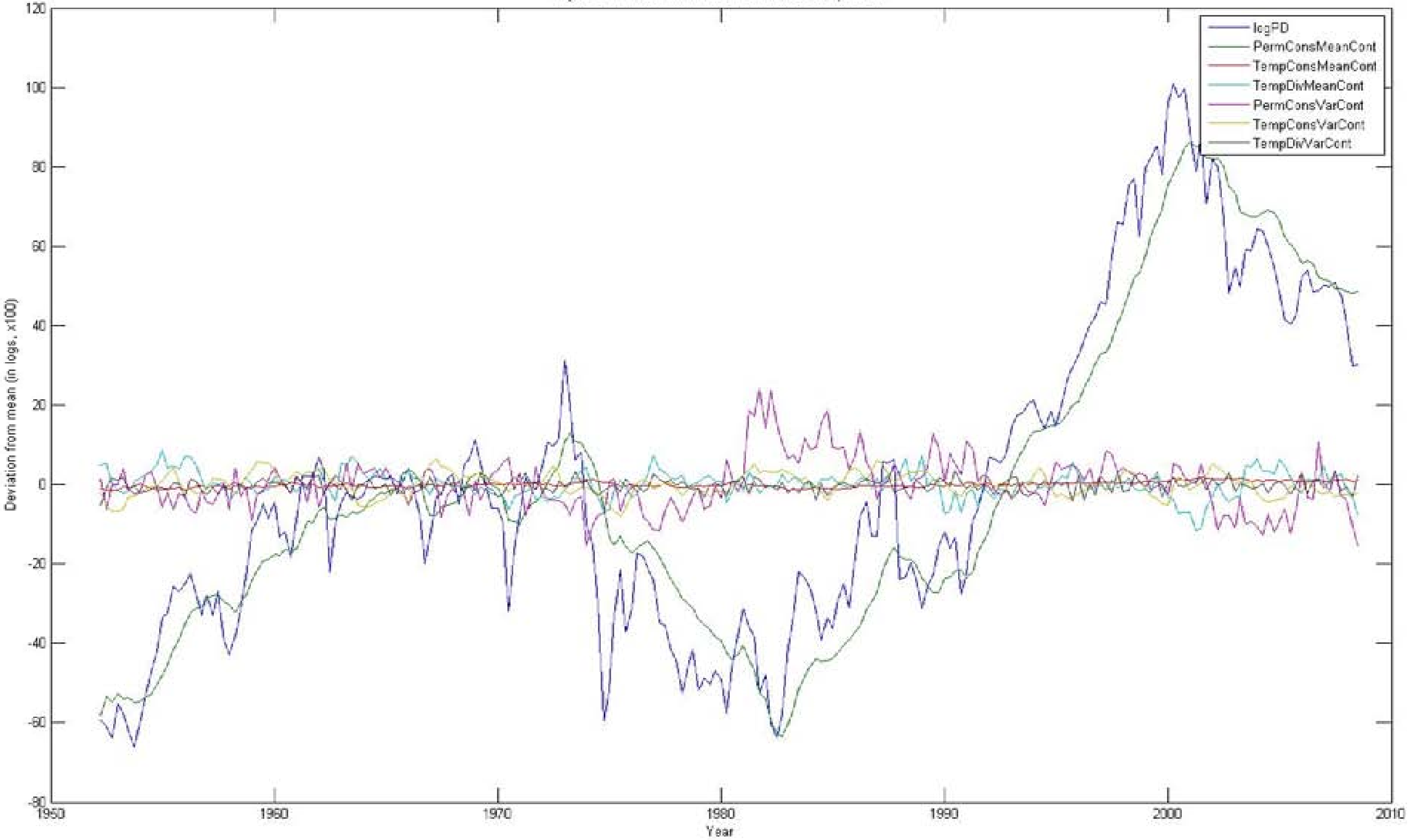

Figure 6: Consumption Growth Historical Decomposition

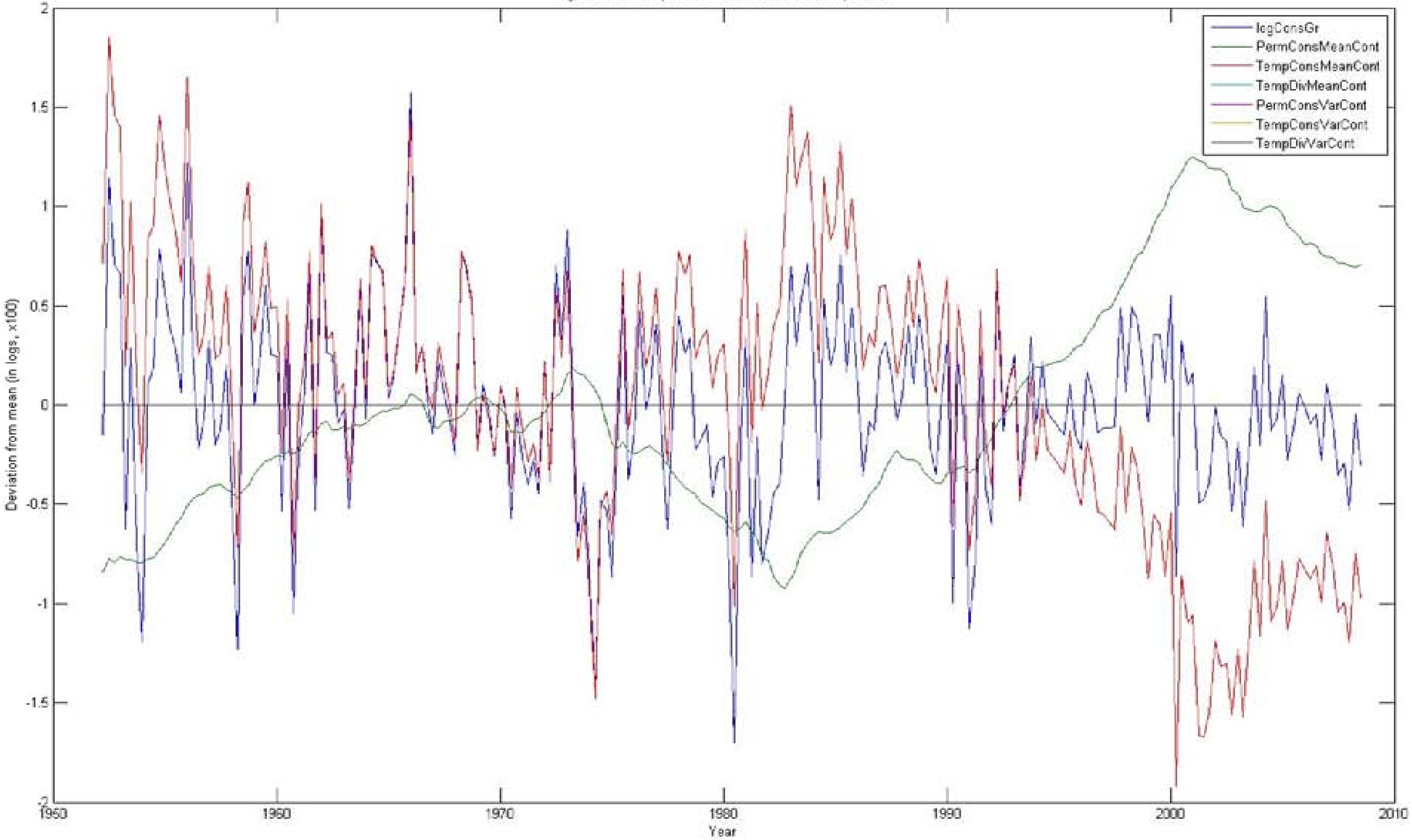

Figure 7: Dividend Growth Historical Decomposition

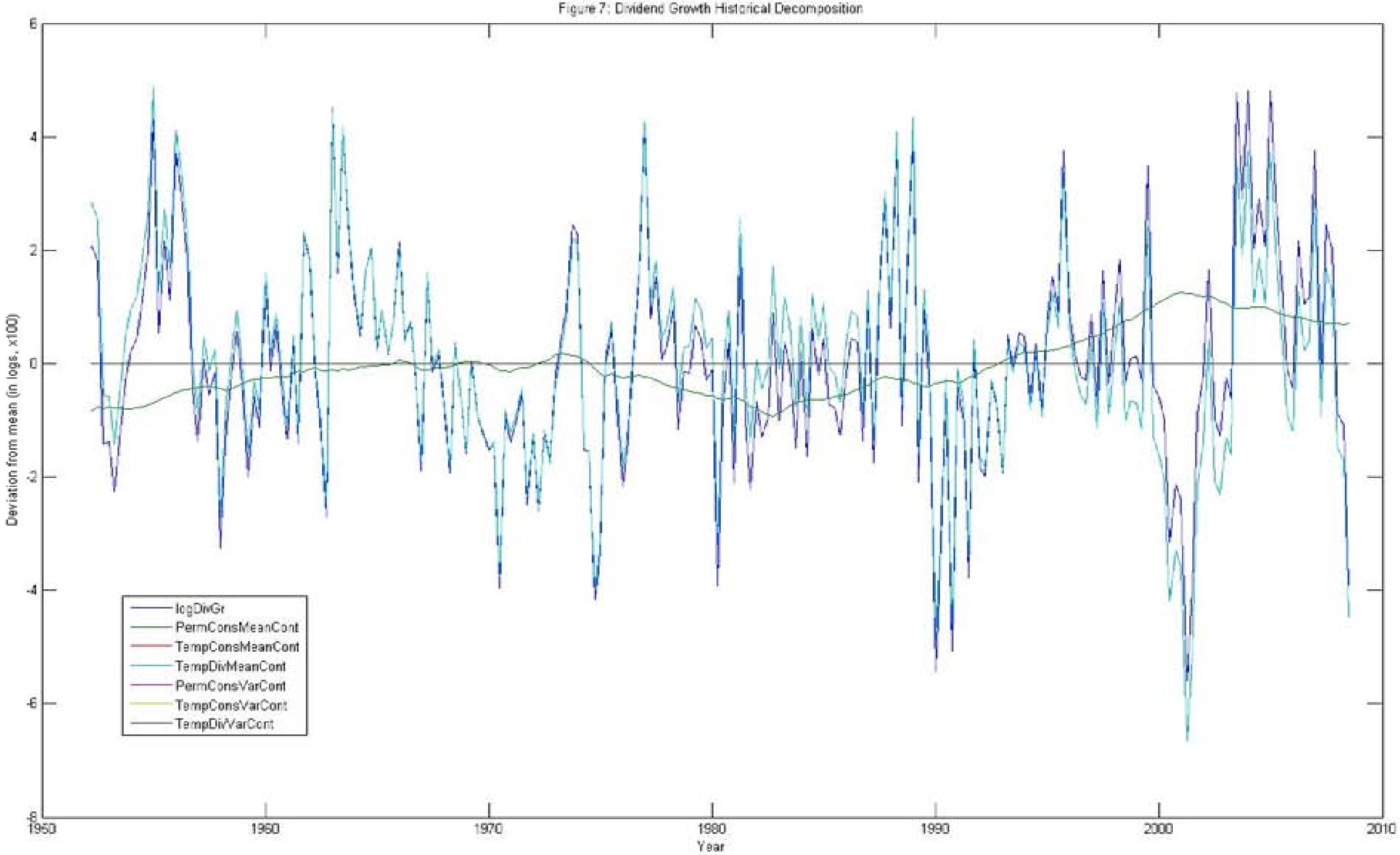

Figure 8: Real Risk-Free Rate Historical Decomposition

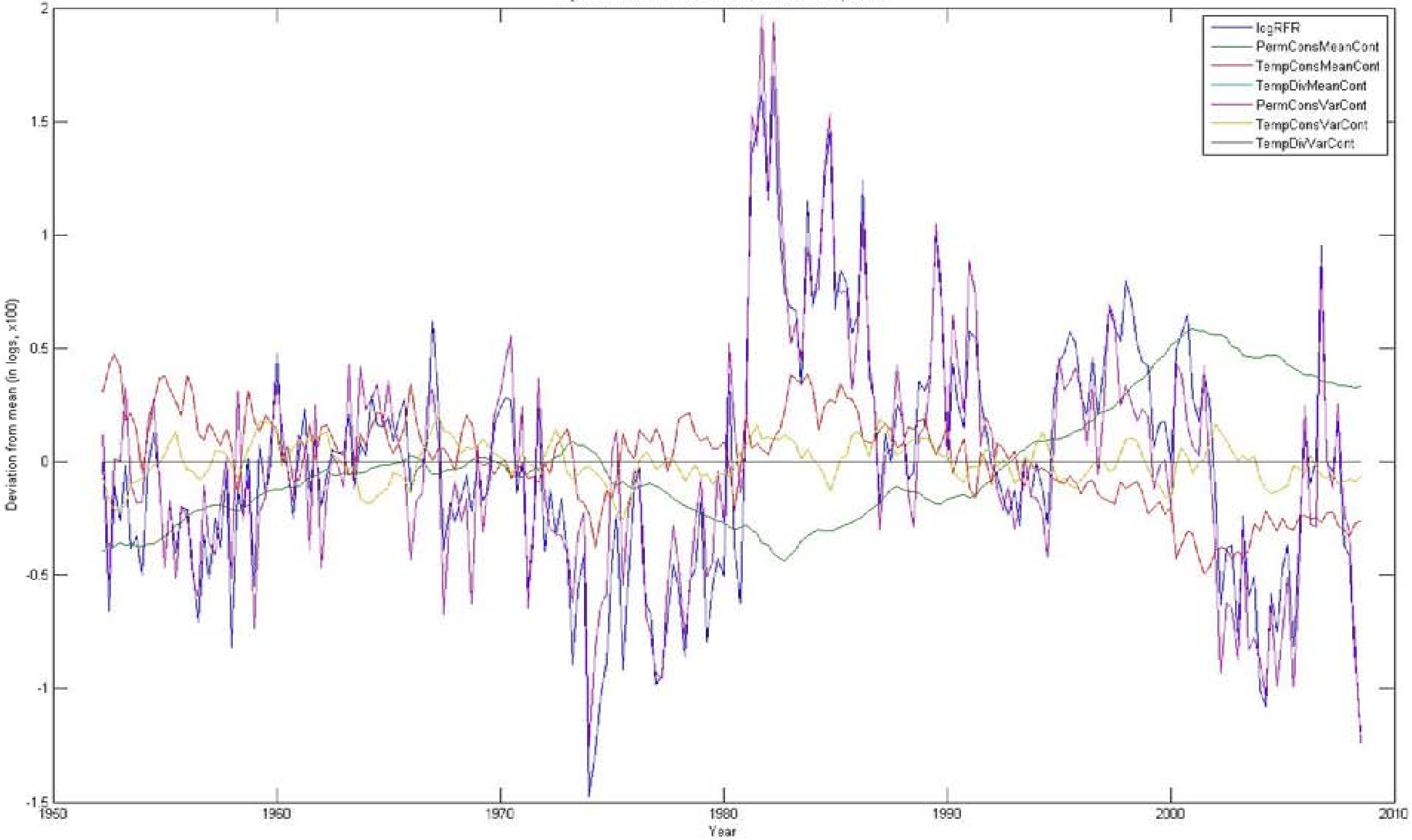

Figure 9: Price-Dividend Ratio Historical Decomposition, by Component

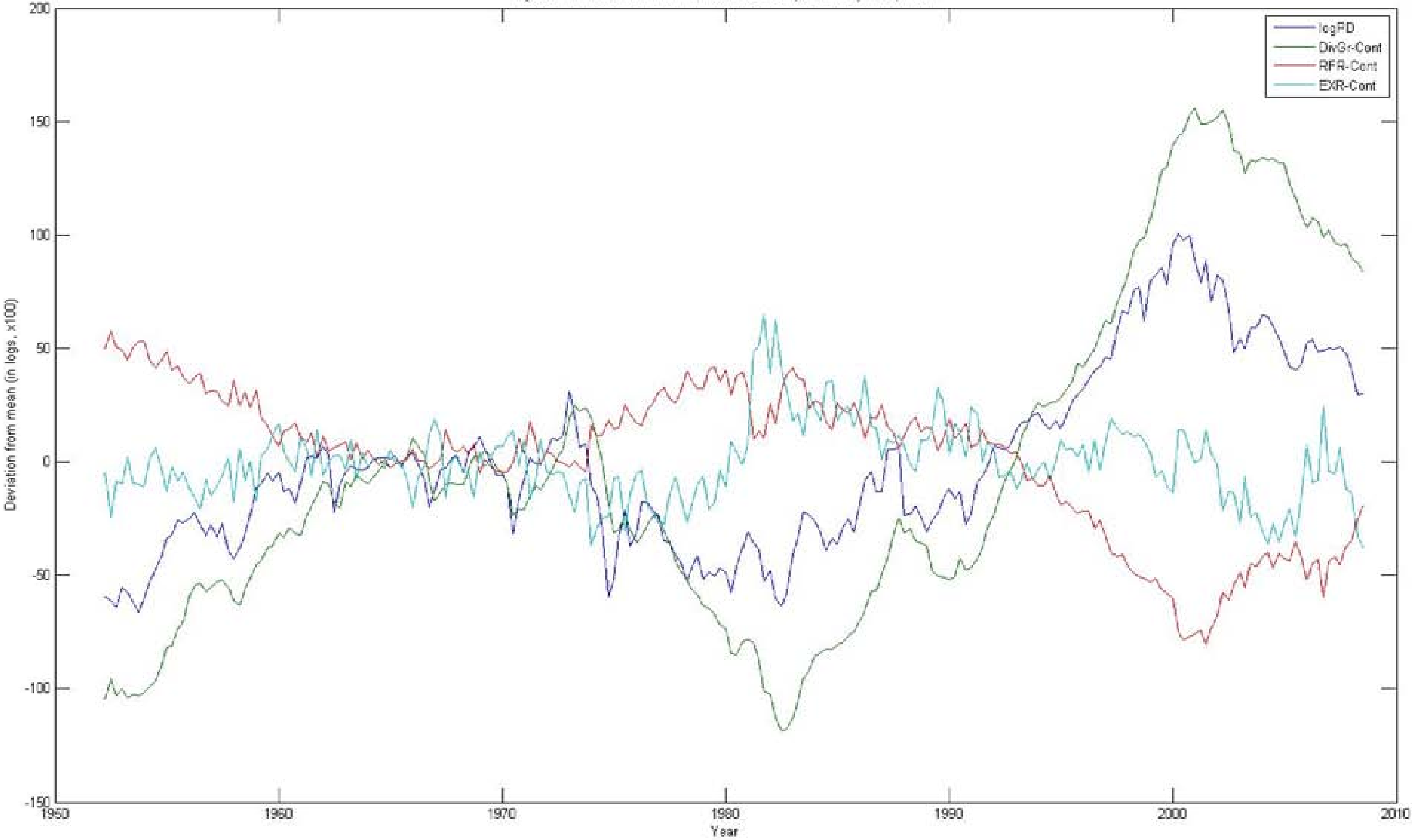

Figure 10: Variance Decompositions for the Price-Dividend Ratio

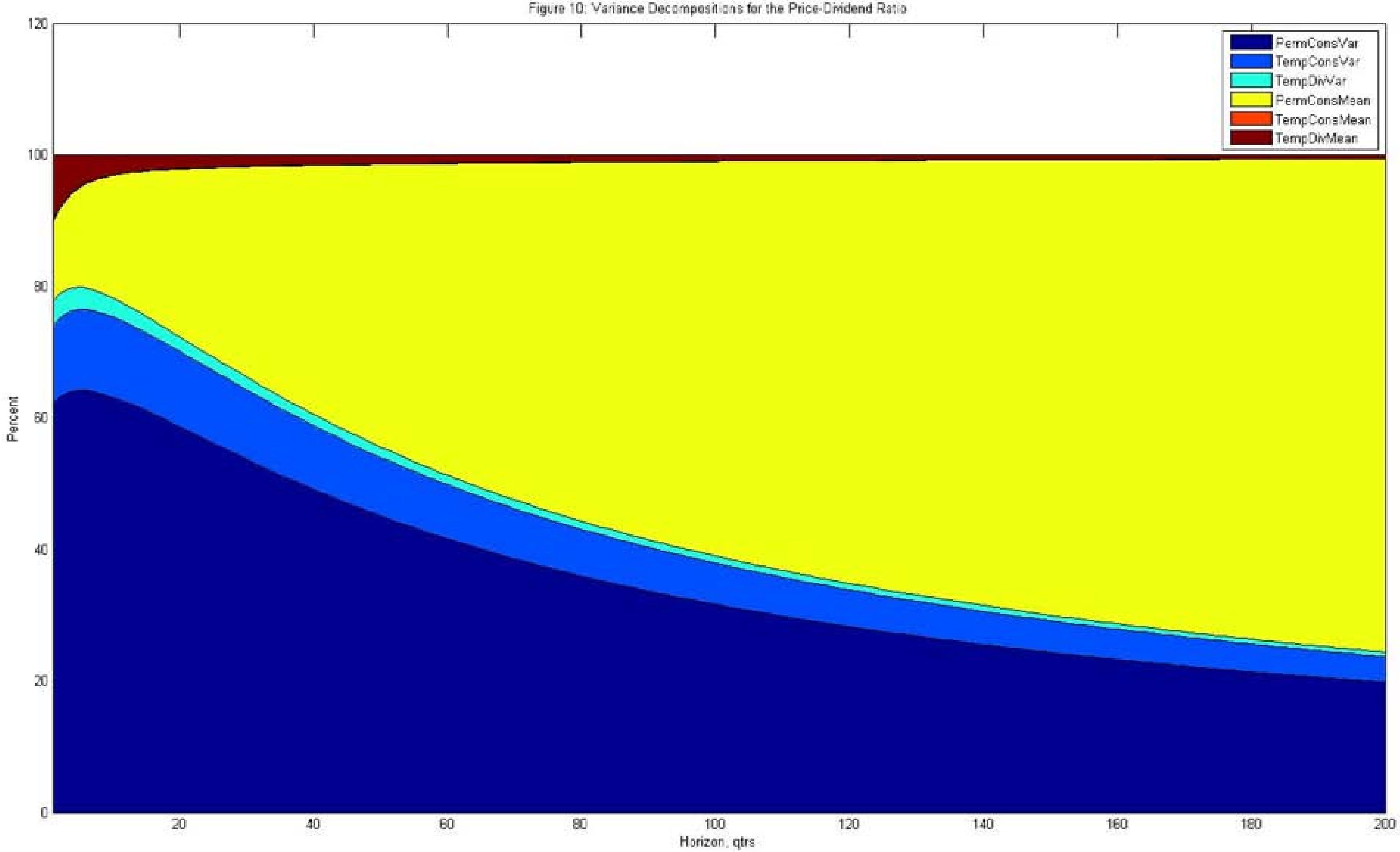

Figure 11: Variance Decompositions for Consumption Growth

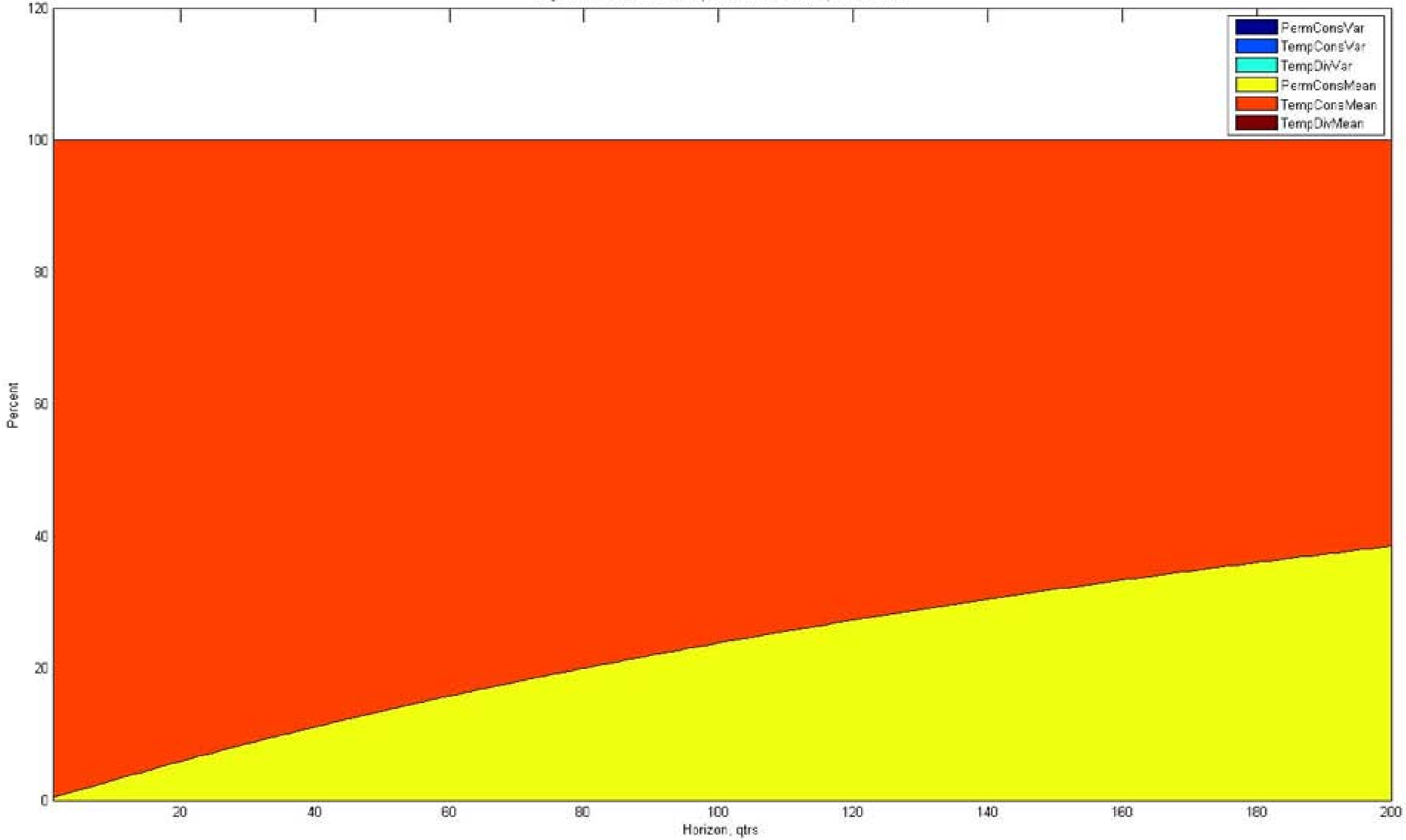

Figure 12: Variance Decompositions for Dividend Growth

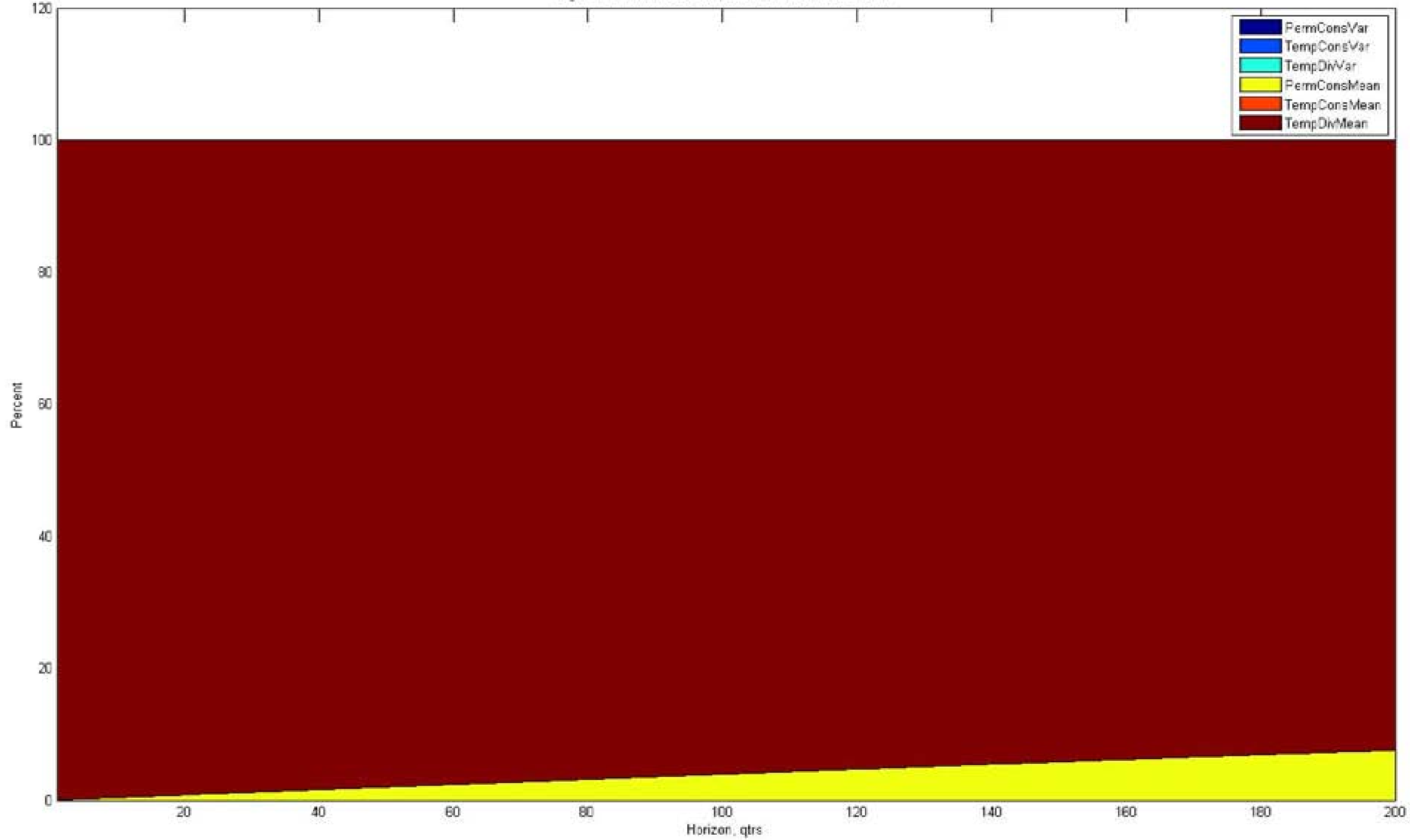

Figure 13: Variance Decompositions for the Real Risk-Free Rate

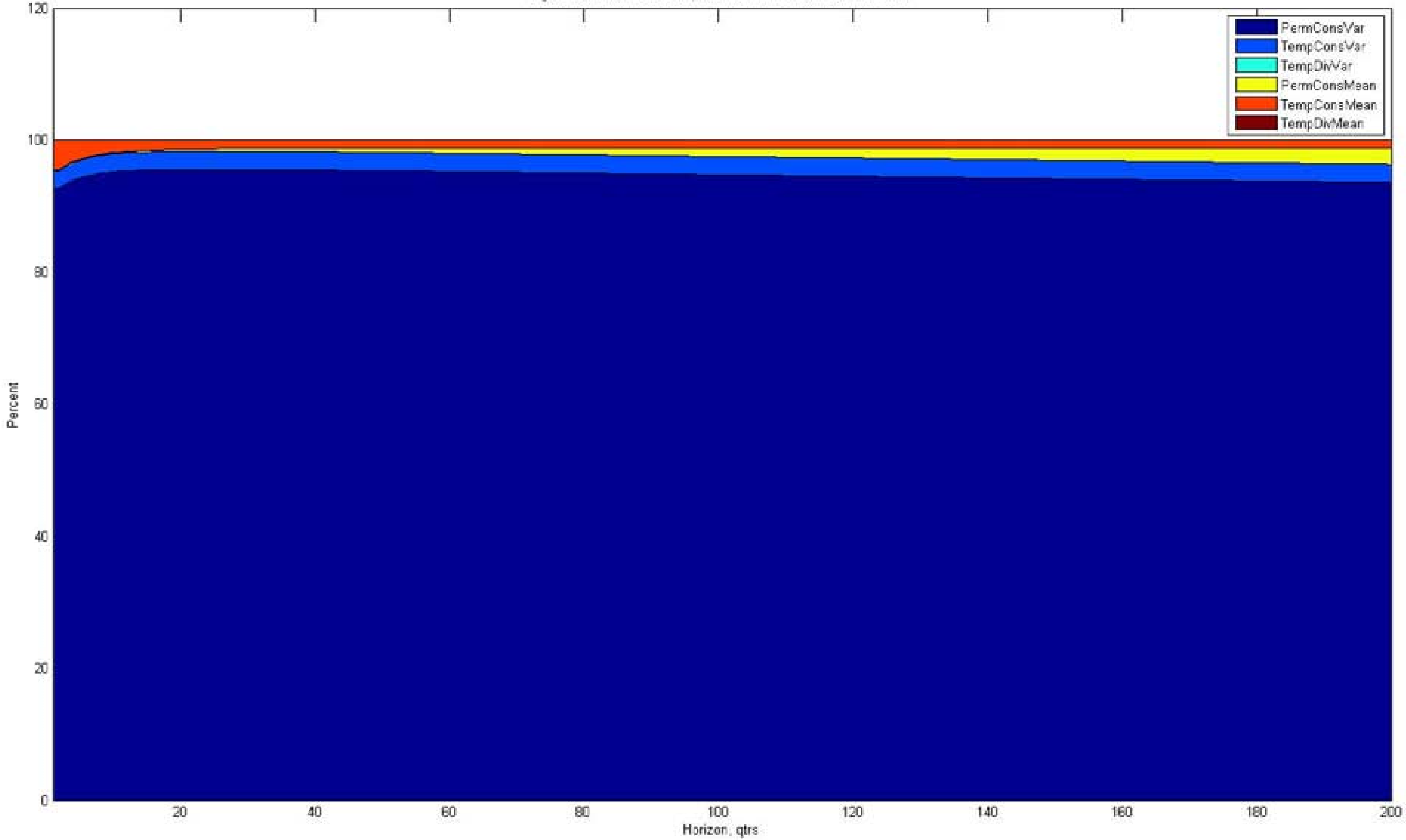

Figure 14: Variance Decompositions for Price-Dividend Ratio, by Component