TESTING THE QUANTITY THEORY USING LONG-RUN AVERAGED CROSS-COUNTRY DATA

by

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December 1995

BANK FOR INTERNATIONAL SETTLEMENTS
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Abstract

Using data from Barro (1990), Dwyer and Hafer (1988), Duck (1993) and Vogel (1974), we revisit the finding that cross-sectional regressions of long-run average inflation on money growth and real income growth support the quantity theory, and conclude that, as is frequently argued, this depends on the inclusion in the sample of a few countries with very high money growth. The most likely reason for the rejection of the theory when these data points are excluded is simultaneity bias, the importance of which is mitigated when high-inflation countries are included in the sample. Omitted variables bias may also play a role, but measurement errors are unlikely to do so.
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1. Introduction

There is much evidence that the two central implications of the quantity theory of money — that increases in money growth lead to equal increases in the rate of inflation, and that increases in real income growth lead to approximately equal reductions in the rate of inflation — are supported by data from a broad range of countries and for different time periods. A common way of presenting the evidence for the quantity theory is to compute average inflation, money growth and income growth for a large number of countries for extended time periods, and to present scatterplots of average inflation against average money growth or to present regressions of average inflation on average money growth and average real income growth. Lucas (1980, p. 1006), for instance, argues that "the ideal experiment for testing [the implications of the quantity theory] would be a comparison of long-term average behaviour across economies with different monetary policies but similar in other respects". He uses data reported by Vogel (1974) to provide a scatterplot of average inflation against money growth for sixteen Latin American economies for the period 1950-69 and refers to the plot as a "particularly clean example" of a test of the quantity theory. The quantity theory has been tested using such long-run averaged data by a number of other authors, including Schwartz (1973), Vogel (1974), Saidi (1981), Lothian (1985), Dwyer and Hafer (1988) and Duck (1988 and 1993). Furthermore, several recent intermediate-level textbooks in macroeconomics use such averaged data as evidence in support of the quantity theory: see Barro (1990), Abel and Bernanke (1992), Sachs and Larrain (1992) and Burda and Wyplosz (1993).

It is commonly argued that the results in the above studies depend on the inclusion of a few high-inflation countries in the data set. Somewhat surprisingly, this both simple and plausible hypothesis has apparently never been formally tested in the literature. The first purpose of this paper is to document, using data from four published studies on the neutrality of money, that the finding of a one-to-one relationship between money growth and inflation and a one-to-minus-one relationship between real income growth and inflation does indeed hinge on the inclusion of a few data points. This lack of stability appears to different degrees in the data sets used by Barro (1990), Dwyer and Hafer (1988), Duck (1993) and Vogel (1974). For instance, using Barro's data (1990, pp. 153-4), we show that if 5 of the 79 observations were dropped, the hypothesis of a unit coefficient on money growth in the inflation equation could be rejected. If 19 observations, or less than a quarter of the sample, were dropped the point estimate of the coefficient on money growth in the inflation equation would be 0.27, which is significantly different from 0.4. We also show that the same pattern repeats itself to a lesser degree for the income coefficient. For instance, if we drop 11 observations from Barro's data set we can reject the hypothesis that the income coefficient is minus unity, and if we drop 23 observations, or less than a third of the sample, we cannot reject the hypothesis that it is zero.

The finding that the evidence in support of quantity theory does depend on data from a few high-inflation countries could be interpreted as suggesting that the theory does not hold for countries experiencing low inflation and monetary volatility. The second purpose of the paper is to
argue that this conclusion is unwarranted, and to show that measurement error in the money stock series, the omission of relevant explanatory variables such as interest rates and expected inflation rates and simultaneity bias provide plausible explanations for the failure of the quantity theory when only countries with low inflation and money growth are included in the sample. Thus, the detected parameter instability does not necessarily constitute evidence against the quantity theory.

The paper is structured as follows. In the next section we provide descriptive statistics for the data sets reported in (listed in order of declining size) Barro (1990), Dwyer and Hafer (1988), Duck (1993) and Vogel (1974). We also present scatterplots of average inflation against money growth and regress average inflation on average money and real income growth. While these cross-sectional regressions are strongly supportive of the quantity theory, the scatterplots do suggest that the results may be heavily influenced by the presence in the sample of a few observations with very high money growth and inflation rates. In Section 3 we formally explore this hypothesis. Since the data are cross-sectional, the order in which they enter the regression is arbitrary, so that we can sort the data in ascending order of inflation, money growth and income growth and estimate the parameters using recursive least squares.\(^1\) Plots of the estimated parameters display substantial instability, and indicate that when a few observations are excluded from the sample, the estimated parameters on money growth and real income growth are frequently significantly different from the values implied by the quantity theory of money. This instability is also documented more formally using Chow-tests. In Section 4 we review the effect of (i) measurement errors on the money stock series, (ii) omitted variables and (iii) simultaneity bias for the point estimates of the parameters. While all these econometric problems are possible sources of the observed parameter non-constancy, simultaneity bias is a particularly plausible explanation. Section 5 offers some conclusions.

2. The data

This study uses four published data sets to examine the robustness of the tests of the quantity theory. Barro (1990) tabulates average inflation and money and income growth for a cross-section of 79 countries for various periods after 1950 and provides scatterplots of inflation against money growth.\(^2\) Dwyer and Hafer (1988) provide the same data for 62 countries for the 1961-78 period.\(^3\) Duck (1993) reports equivalent data for 33 countries for the period 1962-84.\(^4\) Finally, Vogel (1974) offers data from 16 Latin American countries for the period 1950-69.\(^5\) Table 1 contains

---

\(^1\) This sorting strategy underlies the test for heteroscedasticity proposed by Goldfeld and Quandt (1965).

\(^2\) Barro's data set contains 83 observations, but we dropped the four observations for which income growth data are missing. Inflation is measured by the CPI, money by currency and income by real GDP.

\(^3\) Dwyer and Hafer measure inflation by the GDP/GNP deflator and real income by GDP/GNP. The definition of money is not given in the paper.

\(^4\) Using data from the IMF's International Financial Statistics, Duck measures inflation by the GDP/GNP deflator, money as "money plus quasi money" and real income by GDP/GNP.

\(^5\) Vogel measures inflation by the CPI, money by currency plus demand deposits and real income by nominal GNP divided by the CPI.
descriptive statistics for the four data sets. The table indicates that there are large cross-country differences in inflation and money growth: average annual inflation and money growth rates vary between a few percent and over 200%. Average annual income growth rates also range from about -4% to 9%.

<table>
<thead>
<tr>
<th></th>
<th>Barro's Data Set (79 observations)</th>
<th>Dwyer and Hafer's Data Set (62 Observations)</th>
<th>Duck's Data Set (33 Observations)</th>
<th>Vogef's Data Set (16 Observations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prices</td>
<td>3.1</td>
<td>1.6</td>
<td>2.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Money</td>
<td>4.2</td>
<td>2.1</td>
<td>6.4</td>
<td>3.5</td>
</tr>
<tr>
<td>Income</td>
<td>0.1</td>
<td>-4.0</td>
<td>-0.6</td>
<td>0.7</td>
</tr>
<tr>
<td>Minimum</td>
<td>93.9</td>
<td>205.9</td>
<td>68.1</td>
<td>43.0</td>
</tr>
<tr>
<td>Maximum</td>
<td>91.6</td>
<td>220.3</td>
<td>71.1</td>
<td>41.6</td>
</tr>
<tr>
<td>Mean</td>
<td>15.5</td>
<td>22.7</td>
<td>20.5</td>
<td>18.0</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>14.0</td>
<td>35.2</td>
<td>16.3</td>
<td>13.4</td>
</tr>
</tbody>
</table>

Graph 1 contains scatterplots of average inflation against average money growth for the four data sets. These scatterplots constitute the simplest test of the quantity theory. As can be seen, the relationship is almost exactly one-to-one, as suggested by the theory. Furthermore, with the exception of Vogel's data set, most of the observations fall in the lower left-hand corner of the plots, indicating that extreme inflation rates are rare, and that the results may be sensitive to the presence of a few high-inflation countries.

A more formal test is offered in Table 2, which presents the results for cross-sectional regressions using the four data sets. We regress the average inflation rate, \( \pi_i \), on a constant, average money growth, \( \mu_i \), and average real income growth, \( \gamma_i \), that is

\[
(1) \quad \pi_i = \alpha + \beta \mu_i + \delta \gamma_i + \epsilon_i
\]
where $e_i$ denotes the regression residual. We interpret the quantity theory as stating that $\beta = 1$ and $\delta = -1$. Note that the money coefficient, $\beta$, is statistically insignificantly different from unity and that in the first three samples a 95% confidence band for the parameter is about $0.95 - 1.05$ (in the case of Vogel's data, which only comprise 16 observations, it is about $0.95 - 1.15$). Furthermore, the income coefficient, $\delta$, is significantly different from zero, but insignificantly different from minus one. Note also that a White (1980) test fails to reject the hypothesis of homoscedasticity for all but Dwyer and Hafer's data. These scatterplots and regression results are typically interpreted as providing striking support for the quantity theory of money.

Graph 1

Scatterplots of inflation against money growth

DATA FROM BARRO (1990)

DATA FROM DUCK (1993)

DATA FROM VOGEL (1974)

DATA FROM DWYER AND HAFER (1986)
Table 2
Cross-sectional regressions*

\[ \pi_i = \alpha + \beta \mu_i + \delta \gamma_i + \varepsilon_i \]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Constant</th>
<th>Money</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.50</td>
<td>1.00</td>
<td>-0.98</td>
</tr>
<tr>
<td>Standard error</td>
<td>1.09</td>
<td>0.03</td>
<td>0.24</td>
</tr>
<tr>
<td>MSL (in %)</td>
<td>64.8</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Barro's Data Set (79 observations)

Note: R-squared = 0.96, SE = 2.93, White test: MSL = 9.0 %.

| Estimate   | 0.29     | 1.02  | -0.63  |
| Standard error | 1.40     | 0.07  | 0.27   |
| MSL (in %) | 83.6     | 0.0   | 2.5    |

Dwyer and Hafer's Data Set (62 Observations)

Note: R-squared = 0.93, SE = 2.24, White test: MSL = 0.0 %.

| Estimate   | -1.81    | 1.00  | -0.77  |
| Standard error | 0.96     | 0.02  | 0.24   |
| MSL (in %) | 6.9      | 0.0   | 0.3    |

Duck's Data Set (33 Observations)

Note: R-squared = 0.99, SE = 1.93, White test: MSL = 26.0 %.

| Estimate   | 1.35     | 1.04  | -1.38  |
| Standard error | 2.52     | 0.05  | 0.40   |
| MSL (in %) | 60.1     | 0.0   | 0.4    |

Vogel's Data Set (16 Observations)

Note: R-squared = 0.99, SE = 1.96, White test: MSL = 10.9 %.

* Heteroscedasticity consistent standard errors.

3. Recursive estimates and stability tests

In this section we explore the hypothesis that the finding of a coefficient of unity for money growth and a coefficient of minus unity for real income growth depends on the inclusion of a few observations with very high money and income growth rates. In order to test this hypothesis, we exploit the fact that since the data are cross-sectional we are free to change the order in which they enter the regression. Given the claim that the results of the neutrality tests depend on the inclusion of a few high-inflation countries in the data, it is natural to sort the data in ascending order of inflation and to estimate the parameters using recursive least squares. We also sort the data in ascending order of money growth and real income growth and redo the recursive estimates. The results for the four data sets are shown in Graphs 2-5.

Before turning to the results of the recursive estimates, note that, on the assumption that the quantity theory holds for all countries, the point estimates of the parameters should be independent
of the sub-sample used. Of course, data from countries with low inflation, money growth and/or income growth may not be very informative. Thus, the point estimates of the parameters may be numerically quite different from 1 (or -1), but should not be significantly different from 1 (or -1).

3.1 Results using Barro’s data

Consider first the results for Barro’s (1990) data set, which is the largest of the four data sets studied here. Panel A in Graph 2 provides recursive estimates of the money parameter, $\beta$, when the data are sorted in ascending order of inflation. The plot indicates that if the last 5 of Barro’s 79 data points were dropped we would reject the hypothesis of a unit coefficient on money growth. Indeed, had we dropped the last 20 observations, or 25% of the data, we would have obtained a point estimate for $\beta$ of about 0.25, with a 95% confidence interval of about 0.15-0.35. What is striking is that it is so easy to reject the hypothesis that $\beta$ is unity, not that the point estimate of $\beta$ is below unity. Panel B provides recursive estimates of the income coefficient, $\delta$, for the inflation-sorted data. Again we see that $\delta = -1$ only results when the ten or so countries with the highest inflation rates are included in the sample. Dropping the last 10 observations, we can reject the hypothesis that $\delta = -1$.

Next, we sort the data in ascending order of money growth. The recursive estimates of $\beta$ in Panel C show the same pattern as the inflation-sorted data: if we dropped the last 10 observations we could reject the hypothesis that the money coefficient is unity. The recursive estimates of the income coefficient, however, are in this case more stable (Panel D).

Finally, we sort the data in ascending order of real income growth. The coefficient on money presented in Panel E is in this case insignificantly different from unity through the entire sample period. The estimate of the income coefficient, however, would be insignificantly different from zero if the last 20 observations were dropped.

In sum, the recursive least squares estimates indicate that the estimates of $\beta$ and $\delta$ are sensitive to the inclusion of a few countries in the sample. When these countries are excluded from the sample, $\beta$ and $\delta$ are frequently significantly different from the values implied by the quantity theory of money. This instability is also present in the other three data sets, as discussed below.

3.2 Results using Dwyer and Hafer’s data

The results using Dwyer and Hafer’s data are presented in Graph 3. This data set comprises averages for 62 countries for the period 1979-84. Perhaps not surprisingly, the results are very similar to those for Barro’s data. In particular, when the data are sorted in ascending order of inflation, the quantity theory’s proposition that $\beta = 1$ and $\delta = -1$ can easily be rejected unless the ten countries with the highest inflation rates are included in the sample. Sorting by money growth reveals less instability. However, it is interesting to note that the confidence band on the money parameter shrinks very quickly when additional observations are added. Sorting by income growth also indicates that the money coefficient may not be stable.
Graph 2
Data from Barro (1990)

Panel A: Sorted by Inflation

Panel B: Sorted by Inflation

Panel C: Sorted by Money Growth

Panel D: Sorted by Money Growth

Panel E: Sorted by Income Growth

Panel F: Sorted by Income Growth
Graph 3
Data from Dwyer and Hafer (1988)
3.3 Results using Duck's data

Graph 4 contains recursive estimates of \( \beta \) and \( \delta \) using Duck's data for 33 countries for the period 1962-84. In this case the results appear much more in line with the quantity theory: while the parameter estimates vary as countries are added to the sample, the confidence bands for \( \beta \) and \( \delta \) typically contain the values implied by the quantity theory. The one exception is the point estimate of \( \delta \) when the data set is sorted in ascending order of real income growth (Panel F): the plot illustrates that the point estimate of the income coefficient would be zero (and significantly different from -1) if data from the five countries with highest income growth were excluded.

3.4 Results using Vogel's data

Finally we consider the data set for the 16 Latin American countries reported by Vogel (1974). The results in Graph 5 are quite similar to those for Barro's data in Graph 2: \( \beta \) and \( \delta \) are about 0.5 and -0.5 in the "early" part of the sample when the data are sorted according to ascending inflation or money growth rates, and only as the "high inflation" observations are added do the point estimates of the parameters tend to 1 and -1 (see Panels A-D). In this case, the parameters appear stable when the data are sorted according to income growth.

3.5 Stability tests

The above analysis suggests that the point estimates of the parameters are indeed sensitive to the inclusion of a few observations: in particular, the finding of a unit coefficient on money growth appears sensitive to the inclusion of a few "high inflation" countries. To test this hypothesis more formally we divide each sample into two groups of equal size and perform Chow tests for structural stability.\(^6\) Since there is evidence of heteroscedasticity in the data, we also estimate the equation with three break-dummies and performe heteroscedasticity-consistent F-tests of the hypothesis that the dummies are zero. The Marginal Significant Levels for the standard Chow test and for the F-test for the four different data sets and the three different sorting methods are shown in Table 3.

Consider the results for Barro's data set when the data points are sorted according to the inflation rate. The results indicate that the probability that the parameters are the same in the first and second halves of the sample is zero. This also holds when the data are sorted in ascending order of money growth. However, we cannot reject the hypothesis of constant parameters using the Chow test and the F-test on the dummies when the data are sorted according to income growth rate. For brevity we do not review the test results for the other data sets: the reader will see that they also reject the hypothesis of constant parameters whenever the data are sorted according to the inflation or money growth rate.

\(^6\) The number of observations in the first/second group are as follows: Barro's data 39/40, Dwyer and Hafer's data 31/31, Duck's data 17/16, and Vogel's data 8/8.
Graph 5
Data from Vogel (1974)
Table 3
Marginal Significance Levels for Chow tests for parameter stability
and heteroscedasticity-consistent F-tests for break-dummies
(in percentages)

<table>
<thead>
<tr>
<th>Ordered according to</th>
<th>MSL for Chow</th>
<th>MSL for F-tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>increasing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Money growth</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Income growth</td>
<td>55.4</td>
<td>48.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>16.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Money growth</td>
<td>67.0</td>
<td>35.9</td>
</tr>
<tr>
<td>Income growth</td>
<td>3.1</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>4.2</td>
<td>3.1</td>
</tr>
<tr>
<td>Money growth</td>
<td>16.3</td>
<td>4.2</td>
</tr>
<tr>
<td>Income growth</td>
<td>2.4</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>3.6</td>
<td>0.3</td>
</tr>
<tr>
<td>Money growth</td>
<td>3.6</td>
<td>0.3</td>
</tr>
<tr>
<td>Income growth</td>
<td>19.5</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Dwyer and Hafer’s Data Set (62 Observations)

Duck’s Data Set (33 Observations)

Vogel’s Data Set (16 Observations)

Notes: For the F-tests, we estimated \( \pi_i = \alpha + \beta \mu_i + \delta \gamma_i + \alpha' D + \beta' \mu_i'D + \delta' \gamma_i + \varepsilon_i \) where \( D \) is a break-dummy taking the value 0 in the first half of the sample and 1 in the second half. The F-test is for the hypothesis that \( \alpha' = \beta' = \delta' = 0 \) and is computed using White (1980) standard errors.

This section has formally documented that support for the quantity theory in cross-sectional regressions does indeed hinge on the inclusion of a few observations in the data. We next ask how to interpret this finding.

4. Interpretation

One interpretation of the econometric results presented above is that the quantity theory is rejected by the data. In our view, this interpretation is not warranted by the findings. First, the theory gives no guidance regarding precisely what monetary aggregate is the appropriate explanatory variable in the inflation regression. The money growth rates used here may therefore be thought of as being subject to measurement error. Second, the inflation equation is extremely simple, and disregards a number of factors such as interest rates and expected inflation rates that have explanatory power in velocity equations. There is thus some omitted variables bias that needs to be borne in mind in interpreting the results. Third, it is difficult to believe that money growth, which is a policy variable, is not set, at least partially, in response to inflation. Simultaneity bias is thus likely to be present. In
sum, it is likely that the rejection of the quantity theory documented above may be spurious and due to econometric problems

Next we review the consequences of measurement errors, omitted variables and simultaneity bias. In order to keep the analysis as clear as possible, we disregard the effects of real income on money demand.\(^7\)

4.1 Measurement errors

We first review the effects of measurement errors on the money stock.\(^8\) Assume that the true model is

\[
\pi_i = \beta \mu_i^R + \epsilon_i
\]

(2)

where under the null hypothesis we have \(\beta = 1\) and where \(\mu_i^R\) denotes the average growth rate of the (unknown) relevant monetary aggregate in country \(i\). Instead of using data on \(\mu_i^R\), we use data on some other monetary aggregate \(\mu_i = \mu_i^R + \zeta_i\), where \(\zeta_i\) can be thought of as a measurement error which for simplicity we assume is distributed \(N(0, \sigma^2_\zeta)\).\(^9\) It is well known (e.g. Johnston (1984, p. 428)) that

\[
\hat{\beta} = \frac{\beta}{1 + \sigma^2_\zeta / \sigma^2_{\mu^R}}
\]

(3)

where \(\sigma^2_{\mu^R}\) denotes the variance of \(\mu_i^R\). Equation (3) illustrates that measurement errors will bias the estimate of the money growth coefficient towards zero.\(^10\)

To see how measurement error may explain the observed instability, suppose, as seems plausible, that as we include countries with increasingly high inflation and money growth rates in the sample the relative importance of the measurement error is reduced in the sense that the ratio of the variances, \(\sigma^2_\zeta / \sigma^2_{\mu^R}\), falls.\(^11\) In that case, the estimated coefficient on money growth will approach unity as observations are added. In sum, the absence of a one-to-one relationship between money growth and inflation at low inflation rates may be a result of the fact that in countries with low

\(^7\) Alternatively, the reader may reinterpret \(\mu_i\) as nominal money growth minus real income growth.

\(^8\) Of course, inflation and real income growth may also be measured with error. However, it is unlikely that this could explain the low estimates of the money growth coefficient.

\(^9\) Of course, by the relevant aggregate we do not necessarily mean the monetary base, M1, M2 or M3, but perhaps one of these minus one or more of its components, or some even larger aggregate.

\(^10\) They will also increase the variance of the estimated coefficient.

\(^11\) This captures the intuitive argument that for countries with very high money growth it is relatively unimportant exactly how money is measured.
inflation and money growth the exact choice of monetary aggregate used in the empirical analysis is
important.

However, while measurement errors on the money stock provide a possible explanation for the low estimates of \( \beta \), it is implausible that the errors are sufficiently large to explain the size of the bias. For instance, if \( \hat{\beta} = 0.25 \), the implied value for \( \sigma^2_i / \sigma^2_{\mu_i} \) is 3. It is difficult to believe that the variance of the measurement error is three times the variance of true money growth.

4.2 Omitted variables bias

An alternative explanation for the instability of the estimates \( \beta \)'s is that the regression results are subject to omitted variables bias. To see this, assume that the true model is

\[
\pi_i = \beta' \mu_i^t + \theta \nu_i + \varepsilon_i
\]

where \( \beta' = 1, \theta \neq 0 \) and \( \nu_i \) is the omitted variable. Suppose next that we estimate

\[
\pi_i = \beta \mu_i + \varepsilon_i
\]

It is then easy to show (e.g. Johnston (1984, p. 430)) that

\[
\hat{\beta} = \beta' + \theta \rho_{\mu \nu} \frac{\sigma_{\nu}}{\sigma_{\mu}}
\]

Thus, the estimate of the money growth parameter will be biased if the omitted variable is correlated with the money growth rate. Suppose, furthermore, that the correlation between money growth and the omitted variable, \( \rho_{\mu \nu} \), is independent of the rate of money growth and, as seems plausible, that as countries with increasingly higher money growth are included in the sample, \( \sigma_{\mu} \) increases at a faster rate than \( \sigma_{\nu} \). The omitted variable bias is then mitigated as we add high-inflation observations to the sample. This formalises the idea that the effects of money growth are clearer in high-inflation countries because the variance of the money variable increasingly dominates the variance of the omitted variable(s).

Of course, the omission of variables not only affects the point estimate of \( \hat{\beta} \) but also affects the precision with which it is estimated. Next we demonstrate that the use of the misspecified model (5) leads the econometrician to reject the hypothesis of a unit money growth coefficient too often. Recall that the estimate of the variance of \( \hat{\beta} \) in the misspecified model (5) is given by

\[
\sigma^2_{\hat{\beta}} = \frac{\sigma^2_{\varepsilon}}{\sum \mu^2}
\]
Rao and Miller (1971, p. 63) show that the theoretical variance of $\hat{\beta}'$ in the true relation (4) is given by

$$\sigma_p^2 = \frac{\sigma_e^2}{\sum \mu^2 (1 - \rho_{\mu v}^2)}$$

so that $\sigma_p^2 \leq \sigma_e^2$. The omission of variables in the inflation equation therefore tends to make the estimate of the standard error of the money growth coefficient in the misspecified equation too small. The econometrician is thus overly likely to reject the null hypothesis that the money growth coefficient is unity.

However, one problem with the omitted variables explanation is that it does not explain why the estimates of $\hat{\beta}$ are systematically below unity for low-inflation countries (since there is no reason to presume that $\theta_{p_{\mu v}} < 0$).

### 4.3 Simultaneity bias

Estimating (1) using OLS presumes that average money growth is exogenous. Since the supply of money is influenced by the monetary authority, and since monetary authorities typically attempt to limit inflationary pressures by their conduct of policy, the assumption that money growth is determined independently of inflation is clearly unrealistic. We next demonstrate that the resulting endogeneity of money growth is a possible explanation for the finding that $\hat{\beta}$ is smaller than unity when only countries with low money growth and inflation are included in the sample.

To see how the endogeneity of money may matter, suppose that the monetary authority adjusts money growth in response to the rate of inflation. Formally, we let

$$\mu_i = \omega + \lambda \pi_i + z_i$$

where we expect $\lambda < 0$. According to (9), money growth is determined by "systematic monetary policy", as captured by $\lambda \pi_i$, and by other exogenous factors, as captured by $z_i$. One prominent such exogenous factor is central bank financing of fiscal deficits.

In what follows we will refer to (9) as the "supply schedule" (and (1) as the "demand schedule"). Since there is considerable evidence that real output growth is at least approximately independent of inflation, excluding it from the supply schedule seems appropriate for this long-run model.\(^{12}\) Note, however, that although real income growth does not enter (9) directly, increases in real income growth and thus in the demand for money will nevertheless affect money growth by reducing inflation.

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\(^{12}\) Of course, in a model that focused on the short-run behavior of output and money growth, it would probably be more appropriate to let income growth enter independently into (9).
It is helpful to ask, given eqs. (1) and (9), how the estimate of $\beta$ depends on the parameters in the supply function and the variance of the errors in the two equations. It is easy to show that

$$\hat{\beta} = \beta + (1 - \beta \lambda) \lambda / (\lambda^2 + \sigma^2)$$

where $\sigma^2 = \sigma_x^2 / \sigma_e^2$. To interpret equation (10), suppose that $\beta = 1$. Graph 6 presents plots of $\hat{\beta}$ against $\sigma^2$ for $\lambda$ ranging from -0.25 to -1.00. As can be seen, estimates of $\beta$ in the order of 0.25-0.50 are likely for this range of $\lambda$. Note also that since episodes of extreme inflation are typically caused by central bank financing of fiscal deficits, it seems likely that the variance of $z_i$ will increase as countries with increasingly higher inflation and money growth are included in the sample. If so, $\sigma^2$ will increase and $\hat{\beta}$ will tend to unity as additional data points are added. Simultaneity bias thus provides a possible explanation of why $\beta$ is frequently estimated to be significantly below unity for low-inflation countries, but unity when countries with high money growth are also considered.

Taken together, these results suggest that the finding that the point estimate of the money growth coefficient tends to be below, and significantly different from, unity should not necessarily be taken as evidence for the hypothesis that the quantity theory does not hold for low-inflation countries.
5. Conclusions

This paper has shown that cross-sectional regressions of long-run averages of inflation on money and real income growth typically display substantial parameter instability. In particular, the finding of a one-to-one relationship between money growth and inflation is sensitive to the inclusion of data from a few countries with very high inflation rates. While this lack of stability may not be surprising, it has apparently never been demonstrated in the literature before. However, while the quantity theory is rejected if data from only moderate and low-inflation countries are considered, we argue that the likely explanation is simply that the parameter estimates are affected by simultaneity bias and omitted variables bias, which are mitigated as countries with higher inflation and money growth rates are included in the sample.
References


