BANK FOR INTERNATIONAL SETTLEMENTS

BIS Working Papers

No 307

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Monetary and Economic Department

April 2010

JEL classification: D61, E61.

Keywords: Optimal Monetary Policy, Welfare, Second Order Solution, Oil Price Shocks, Endogenous Trade-off.

Copies of publications are available from:

Bank for International Settlements
Communications
CH-4002 Basel, Switzerland

E-mail: publications@bis.org

Fax: +41 61 280 9100 and +41 61 280 8100

This publication is available on the BIS website (www.bis.org).

ISSN 1020-0959 (print)

ISBN 1682-7678 (online)

Oil Shocks and Optimal Monetary Policy[*†]

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First version: November 2005.
This version: March 2010.

Abstract

In practice, central banks have been confronted with a trade-off between stabilising inflation and output when dealing with rising oil prices. This contrasts with the result in the standard New Keynesian model that ensuring complete price stability is the optimal thing to do, even when an oil shock leads to large output drops. To reconcile this apparent contradiction, this paper investigates how monetary policy should react to oil shocks in a microfounded model with staggered price-setting and with oil as an input in a CES production function. In particular, we extend Benigno and Woodford (2005) to obtain a second order approximation to the expected utility of the representative household when the steady state is distorted and the economy is hit by oil price shocks.

The main result is that oil price shocks generate an endogenous trade-off between inflation and output stabilisation when oil has low substitutability in production. Therefore, it becomes optimal for the monetary authority to stabilise partially the effects of oil shocks on inflation and some inflation is desirable. We also find, in contrast to Benigno and Woodford (2005), that this trade-off is reduced, but not eliminated, when we get rid of the effects of monopolistic distortions in the steady state. Moreover, the size of the endogenous "cost-push" shock generated by fluctuations in the oil price increases when oil is more difficult to substitute by other factors.

JEL Classification: D61, E61.

Keywords: Optimal Monetary Policy, Welfare, Second Order Solution, Oil Price Shocks, Endogenous Trade-off.

[*]This paper was started while I was working at Banco Central de Reserva del Perú. I would like to thank Chris Pissarides, Gianluca Benigno, Pierpaolo Benigno, John Driffill, Paul Castillo, Marco Vega, Vicente Tuesta, Jean-Marc Natal, participants at the Macroeconomics Student Seminar at LSE and the BCRP, and two anonymous referees for their valuable comments and suggestions. I also thank Sandra Gonzalez and Louisa Wagner for editorial support. The views expressed herein are those of the author and do not necessarily reflect those of the Banco Central de Reserva del Perú nor those of the Bank for International Settlements. Any errors are my own responsibility. Address correspondence to: Carlos Montoro, Office for the Americas, Bank for International Settlements, Torre Chapultepec - Rubén Darío 281 - 1703, Col. Bosque de Chapultepec - 11580, México DF – México; tel: +52 55 9138 0294; fax: +52 55 9138 0299; e-mail: carlos.montoro@bis.org.

[†]This paper has been accepted for publication and will appear in a revised form, subsequent to editorial input by Cambridge University Press, in Macroeconomic Dynamics.

1 Introduction

Oil is an important production factor in economic activity because every industry uses it to some extent. Moreover, since oil cannot be easily substituted by other production factors, economic activity is heavily dependent on its use. Furthermore, the oil price is determined in a weakly competitive market; there are few large oil producers dominating the world market, setting its price above a perfect competition level. Also, its price fluctuates considerably due to the effects of supply and demand shocks in this market[1].

The heavy dependence on oil and the high volatility of its price generates a concern among the policymakers on how to react to oil shocks. Oil shocks have serious effects on the economy because they raise prices for an important production input and for important consumer goods (gasoline and heating oil). This causes an increase in inflation and subsequently a decrease in output, generating also a dilemma for policymaking. On one hand, if monetary policy makers focus exclusively on the recessive effects of oil shocks and try to stabilise output, this would generate inflation. On the other hand, if monetary policy makers focus exclusively on neutralising the impact of the shock on inflation through a contractive monetary policy, some sluggishness in the response of prices to changes in output would imply large reductions in output. In practice, when dealing with rising oil prices, policymakers have been confronted with a trade-off between stabilising inflation and output. But, what exactly should be the optimal stabilisation of inflation and output? Which factors affect this trade-off? To our knowledge the formal study of this topic is limited[2].

However, the behaviour of central banks in practice contrasts with the result in the standard new Keynesian model that ensuring complete price stability is the optimal thing to do, even when an oil shock leads to large drops in output. To deal with this apparent contradiction and to answer the questions presented above, we extend the literature on optimal monetary policy including oil in the production process in a standard New Keynesian model. In doing so, we extend Benigno and Woodford (2005) to obtain a second-order approximation to the expected utility of the representative household, when the steady state is distorted and the economy is hit by oil price shocks. We include oil as a non-produced input as in Blanchard and Galí (2007), but differently from those authors we use a constant-elasticity-of-substitution (CES) production function to capture the low substitutability of oil. Then, a low elasticity of substitution between labour and oil indicates a high dependence on oil[3].

The analysis of optimal monetary policy in microfounded models with staggered price set-

[1] For example during the 1970s and through the 1990s most of the oil shocks seemed to be on the international supply side, either because of attempts to gain more oil revenue or because of supply interruptions, such as the Iranian Revolution and the first Gulf war. In contrast, in the 2000s the high price of oil is more related to demand growth in the USA, China, India, and other countries. On the other hand, Kilian (2009) found that all major real oil price increases since the mid-1970s can be traced to increased global aggregate demand and/or increases in oil-specific demand.

[2] There are a few exceptions. For instance, Natal (2009) showed that extending our work, including oil in the consumption goods bundle in a CES form, amplifies the trade-off between stabilizing inflation and the welfare output gap. In a different approach, Nakov and Pescaroti (2009) also find a trade-off when modeling explicitaly the oil production in the global economy, which is generated by a dynamic distortion due to imperfect competition in the oil market.

[3] In contrast, Blanchard and Gali (2007) use a Cobb-Douglas production function.

ting using a quadratic welfare approximation was first introduced by Rotemberg and Woodford (1997) and expounded by Woodford (2003) and Benigno and Woodford (2005). This method allows us to obtain a linear policy rule derived from maximising the quadratic approximation of the welfare objective subject to the linear constraints that are first-order approximations of the structural equations. This methodology is called linear-quadratic (LQ). The advantage of this approach is that it allows to characterise analytically how changes in the production function and in the oil shock process affect the monetary policy problem. Moreover, in contrast to the Ramsey policy methodology, which also allows a correct calculation of a linear approximation of the optimal policy rule, the LQ approach is useful to evaluate not only the optimal rules, but also to evaluate and rank sub-optimal monetary policy rules.

A property of standard New Keynesian models is that stabilising inflation is equivalent to stabilising output around some desired level, unless some exogenous cost-push shock disturbances are taken into account. Blanchard and Galí (2007) called this feature the "divine coincidence". These authors argue that this special feature comes from the absence of non-trivial real imperfections, such as real wage rigidities. Similarly, Benigno and Woodford (2004, 2005) show that this trade-off also arises when the steady state of the model is distorted and there are government purchases in the model.

We found that, when oil is introduced as a low-substitutable input in a New Keynesian model, a trade-off arises between stabilising inflation and the gap between output and some desired level. We call this desired level the “target level”. In this case, because output at the target level fluctuates less than it does at the natural level, it becomes optimal to the monetary authority to react partially to oil shocks and therefore, some inflation is desirable.

The intuition of this result is that when oil is considered a gross complement to labour in production in a CES technology, the divine coincidence disappears. This result is similar to the case of real wage rigidities explained in Blanchard and Galí (2007), where stabilizing inflation is no longer equivalent to stabilizing the welfare-relevant output gap. However, the mechanism here is different. This trade-off is generated by the convexity of real marginal costs with respect the real oil price, which produce a time varying wedge between the marginal rate of substitution and the marginal productivity of labour that impede to replicate the first best equilibrium. Moreover, eliminating the distortions in steady state reduces the trade-off, because this wedge becomes less sensitive with respect to the oil price. However, in contrast to Benigno and Woodford (2005), making the steady state efficient cannot eliminate this trade-off.

Also, the substitutability among production factors affects both the weights on the two stabilisation objectives and the definition of the welfare-relevant output gap. The lower the elasticity of substitution, the higher the cost-push shock generated by oil shocks and the lower the weight on output stabilisation relative to inflation stabilisation. Moreover, when the share of oil in the production function is higher, or the steady-state oil price is higher, the size of the cost-push shock increases.

Section 2 presents our New Keynesian model with oil prices in the production function. Section 3 includes a linear quadratic approximation to the policy problem. Section 4 uses the linear quadratic approximation to the problem to solve for the different rules of monetary policy and make some comparative statics to the parameters related to oil. The last section concludes.

2 A New Keynesian model with oil prices

The model economy corresponds to the standard New Keynesian Model in the line of Clarida et al (2000). In order to capture oil shocks we follow Blanchard and Galí (2007) by introducing a non-produced input M, represented in this case by oil. Q will be the real price of oil which is assumed to be exogenous. This model is similar to the one used by Castillo et al (2007), except that we additionally include taxes on sales of intermediate goods to analyse the distortions in steady state.

2.1 Households

We assume the following utility function on consumption and labour of the representative consumer

$$U_{t_o} = E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+v}}{1+v} \right], \tag{2.1}$$

where σ represents the coefficient of risk aversion and v captures the inverse of the elasticity of labour supply. The optimiser consumer takes decisions subject to a standard budget constraint which is given by

$$C_t = \frac{W_t L_t}{P_t} + \frac{B_{t-1}}{P_t} - \frac{1}{R_t} \frac{B_t}{P_t} + \frac{\Gamma_t}{P_t} + \frac{T_t}{P_t}, \tag{2.2}$$

where W_t is the nominal wage, P_t is the price of the consumption good, B_t is the end of period nominal bond holdings, R_t is the riskless nominal gross interest rate , Γ_t is the share of the representative household on total nominal profits, and T_t are net transfers from the government. The first order conditions for the optimising consumer's problem are:

$$1 = \beta E_t \left[R_t \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right], \tag{2.3}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} L_t^{v} = MRS_t. \tag{2.4}$$

Equation (2.3) is the standard Euler equation that determines the optimal path of consumption. At the optimum the representative consumer is indifferent between consuming today or tomorrow, whereas equation (2.4) describes the optimal labour supply decision. MRS_t denotes for the marginal rate of substitution between labour and consumption. We assume that labour markets are competitive and also that individuals work in each sector $z \in [0, 1]$. Therefore, L_t corresponds to the aggregate labour supply:

$$L_t = \int_0^1 L_t(z) dz. \tag{2.5}$$

2.2 Firms

2.2.1 Final good producers

There is a continuum of final good producers of mass one, indexed by $f \in [0,1]$ that operate in an environment of perfect competition. They use intermediate goods as inputs, indexed by $z \in [0,1]$ to produce final consumption goods using the following technology:

$$Y_t^f = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.6}$$

where ε is the elasticity of substitution between intermediate goods. The demand function of each type of differentiated good is obtained by aggregating the input demand of final good producers:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t, \tag{2.7}$$

where the price level is equal to the marginal cost of the final good producers and is given by:

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}. \tag{2.8}$$

and Y_t represents the aggregate level of output.

$$Y_t = \int_0^1 Y_t^f df. \tag{2.9}$$

2.2.2 Intermediate goods producers

There is a continuum of intermediate good producers indexed by $z \in [0,1]$. All of them have the following CES production function

$$Y_t(z) = \left[(1-\alpha) \left(L_t(z) \right)^{\frac{\psi-1}{\psi}} + \alpha \left(M_t(z) \right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}, \tag{2.10}$$

where M is oil which enters as a non-produced input, ψ represents the intratemporal elasticity of substitution between labour-input and oil and α denotes the quasi-share of oil in the production function. We use this generic production function in order to capture the fact that oil has few substitutes. In general we assume that ψ is lower than one. The real oil price, Q_t, is assumed to follow an $AR(1)$ process in logs,

$$\log Q_t = (1-\rho) \log \overline{Q} + \rho \log Q_{t-1} + \xi_t, \tag{2.11}$$

where $\overline{Q}$ is the steady state level of oil price and ξ_t is an *i.i.d.* shock. From the cost minimisation problem of the firm we obtain an expression for the real marginal cost given by:

$$MC_t(z) = \left[(1-\alpha)^{\psi} \left(\frac{W_t}{P_t} \right)^{1-\psi} + \alpha^{\psi} \left(Q_t \right)^{1-\psi} \right]^{\frac{1}{1-\psi}}, \tag{2.12}$$

where $MC_t(z)$ represents the real marginal cost and W_t nominal wages. Notice that marginal costs are the same for all intermediate firms, since technology has constant returns to scale and factor markets are competitive, i e $MC_t(z) = MC_t$. On the other hand, the first order condition for intermediate goods producers with respect to labour imply that the marginal product of labor, MPL_t, satisfy:

$$MPL_t(z) = (1-\alpha)\left(\frac{Y_t(z)}{L_t(z)}\right)^{1/\psi} = \frac{W_t/P_t}{MC_t}. \tag{2.13}$$

Equation (2.13) imply the following labour demand for the individual firm:

$$L_t^d(z) = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} Y_t(z). \tag{2.14}$$

Intermediate producers set prices following a staggered pricing mechanism *a la Calvo*. Each firm faces an exogenous probability of changing prices given by $(1-\theta)$. A firm that changes its price in period t chooses its new price $P_t(z)$ to maximise:

$$E_t \sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} \Gamma\left(P_t(z), P_{t+k}, MC_{t+k}, Y_{t+k}\right),$$

where $\zeta_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the stochastic discount factor. The function: $\Gamma\left(P_t(z), P_t, MC_t, Y_t\right) \equiv \left[(1-\tau)P_t(z) - P_t\, MC_t\right]\left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t$ is the after-tax nominal profits of the supplier of good z with price $P_t(z)$, where the aggregate demand and aggregate marginal costs are equal to Y_t and MC_t, respectively. τ is the proportional tax on sale revenues which we assume constant. The optimal price that solves the firm's problem is given by

$$\left(\frac{P_t^*(z)}{P_t}\right) = \frac{\mu E_t\left[\sum\limits_{k=0}^{\infty} \theta^k \zeta_{t,t+k} MC_{t,t+k} F_{t+k}^{\varepsilon+1} Y_{t+k}\right]}{E_t\left[\sum\limits_{k=0}^{\infty} \theta^k \zeta_{t,t+k} F_{t+k}^{\varepsilon} Y_{t+k}\right]}, \tag{2.15}$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon-1}/(1-\tau)$ is the price markup net of taxes, $P_t^*(z)$ is the optimal price level chosen by the firm and $F_{t+k} = \frac{P_{t+k}}{P_t}$ the cumulative level of inflation. The optimal price solves equation (2.15) and is determined by the average of expected future marginal costs as follows:

$$\left(\frac{P_t^*(z)}{P_t}\right) = \mu E_t\left[\sum_{k=0}^{\infty} \varphi_{t,t+k} MC_{t,t+k}\right], \tag{2.16}$$

where $\varphi_{t,t+k} \equiv \frac{\theta^k \zeta_{t,t+k} F_{t+k}^{\varepsilon+1} Y_{t+k}}{E_t\left[\sum\limits_{k=0}^{\infty} \theta^k \zeta_{t,t+k} F_{t+k}^{\varepsilon} Y_{t+k}\right]}$.

Since only a fraction $(1-\theta)$ of firms changes prices every period and the remaining one keeps its price fixed, the aggregate price level, defined as the price of the final good that minimise the cost of the final goods producers, is given by the following equation:

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*(z)\right)^{1-\varepsilon}. \tag{2.17}$$

Following Benigno and Woodford (2005), equations (2.15) and (2.17) can be written recursively introducing the auxiliary variables N_t and D_t (see appendix B for details on the derivation):

$$\theta\left(\Pi_t\right)^{\varepsilon-1}=1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\epsilon}, \tag{2.18}$$

$$D_t=Y_t\left(C_t\right)^{-\sigma}+\theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\epsilon-1}D_{t+1}\right], \tag{2.19}$$

$$N_t=\mu Y_t\left(C_t\right)^{-\sigma}MC_t+\theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\epsilon}N_{t+1}\right], \tag{2.20}$$

where $\Pi_t=P_t/P_{t-1}$ is the gross inflation rate. Equation (2.18) comes from the aggregation of individual firms prices. The ratio N_t/D_t represents the optimal relative price $P_t^*\left(z\right)/P_t$. These three last equations summarise the recursive representation of the non linear Phillips curve.

2.3 Government and monetary policy

In the model we assume that the government owns the oil endowment. Oil is produced in the economy at zero cost and sold to the firms at the exogenous price Q_t. The government transfers all the revenues generated by oil to consumers represented by $P_tQ_tM_t$. There is also a proportional tax on sale revenues (τ). We abstract from any other role for the government and assume that it runs a balanced budget every period. Then, the budget constraint implies that total net transfers in real terms are:

$$\frac{T_t}{P_t}=Q_tM_t+\tau Y_t.$$

Moreover, we abstract from any monetary frictions assuming that the central bank can control directly the risk-less short-term interest rate R_t.

2.4 Market clearing

In equilibrium labour, intermediate and final goods markets clear. Because of the assumption on the government transfers, the economy-wide resource constraint is given by

$$Y_t=C_t. \tag{2.21}$$

The labour market clearing condition is given by:

$$L_t=L_t^d, \tag{2.22}$$

where the demand for labour comes from the aggregation of individual intermediate producers in the same way as for the labour supply:

$$\begin{aligned} L_t^d &= \int_0^1 L_t^d(z)dz=\left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi}\int_0^1 Y_t(z)dz \\ &= \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi}Y_t\Delta_t, \end{aligned} \tag{2.23}$$

where $\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} dz$ is a measure of price dispersion. Since relative prices differ across firms due to staggered price setting, input usage will differ as well, implying that is not possible to use the usual representative firm assumption. Therefore, the price dispersion factor, Δ_t appears in the aggregate labour demand equation. We can also use (2.17) to derive the law of motion of Δ_t

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\left(\Pi_t\right)^{\varepsilon-1}}{1-\theta}\right)^{\varepsilon/(\varepsilon-1)} + \theta\Delta_{t-1}\left(\Pi_t\right)^{\varepsilon}. \tag{2.24}$$

Note that inflation affects welfare of the representative agent through the labour market. We can see, from (2.24), that higher inflation increases price dispersion and, from (2.23), that higher price dispersion increases the labour amount necessary to produce a certain level of output, implying more disutility on (2.1).

2.5 The steady state

Variables in the steady state are denoted overlined (i.e. $\overline{X}$). The details of the steady state of the variables are in appendix A. We depart from a steady state where gross inflation $\overline{\Pi} = 1$. Output in steady state is given by: $\overline{Y} = \left((1-\overline{\alpha})\overline{MC}\right)^{\frac{1}{\sigma+v}}\left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1+\psi v}{\sigma+v}\frac{1}{1-\psi}}$, where real marginal costs in steady state are:

$$\overline{MC} = \frac{1-\tau}{\varepsilon/\left(\varepsilon-1\right)} \leq 1, \tag{2.25}$$

where $\overline{\alpha} \equiv \alpha^{\psi}\left(\frac{\overline{Q}}{\overline{MC}}\right)^{1-\psi}$ is the share of oil on total costs in steady state. Note that, from the definition of $\overline{\alpha}$, the steady state value of output depends on the steady state ratio of the real oil price with respect to real marginal costs. This implies that a permanent increase in the real oil price will generate a permanent increase in $\overline{\alpha}$, given $\psi < 1$. Also, as in standard New Keynesian models, the real marginal costs in steady state are equal to the inverse of the mark-up. Since monopolistic competition and taxes affect the steady state of the model, output in steady state can be below the efficient level (the steady state is distorted). In the special case that $\tau = -1/\left(\varepsilon-1\right) < 0$, distortions are eliminated and the steady state is efficient. Let's denote the steady state distortion by

$$\Phi = 1 - \frac{1-\tau}{\varepsilon/\left(\varepsilon-1\right)}.$$

We have that $\Phi = 0$ when a subsidy on sales makes the steady state undistorted.

2.6 The log linear economy and the natural equilibrium

To illustrate the effects of oil in the dynamic equilibrium of the economy, we take a log linear approximation of equations (2.3), (2.4),(2.11),(2.12),(2.18),(2.19),(2.20) and (2.23) around the deterministic steady-state. We denote variables in their log deviations around the steady state with lower case letters (i.e. $x_t = \log(X_t/\overline{X})$). After, imposing the goods and labour market clearing conditions to eliminate real wages from the system, the dynamics of the economy are determined by the following equations:

$$l_t = y_t - \delta \left[(\sigma + v) y_t - q_t \right], \tag{2.26}$$

$$mc_t = \chi (v + \sigma) y_t + (1 - \chi) q_t, \tag{2.27}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t, \tag{2.28}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{t+1}), \tag{2.29}$$

$$q_t = \rho q_{t-1} + \xi_t, \tag{2.30}$$

where $\delta \equiv \psi \chi \frac{\overline{\alpha}}{1-\overline{\alpha}}$, $\chi \equiv \frac{1-\overline{\alpha}}{1+v\psi\overline{\alpha}}$ and $\kappa \equiv \frac{1-\theta}{\theta} (1 - \theta\beta)$. δ and $(1 - \chi)$ account for the effects of oil prices in labour and marginal costs, respectively. κ is the elasticity of inflation respect to marginal costs.

Interestingly, the effects of oil prices on marginal costs, given by $(1 - \chi)$ in equation (2.27), depend crucially on the quasi-share of oil in the production function, α, and on the elasticity of substitution between oil and labour, ψ. Thus, when α is larger χ is smaller, making marginal costs more responsive to oil prices. Also, when ψ is lower, the impact of oil on marginal costs is larger. It is important to note that even though the quasi-share of oil in the production function, α, can be small, its impact on marginal cost, $\overline{\alpha}$, can be magnified when oil has few substitutes (that is when ψ is low). Moreover, a permanent increase in real oil price or in the distortions in steady state (that is an increase in $\overline{Q}$ or a decrease on $\overline{MC}$), would make the marginal costs of firms more sensitive to oil price shocks since it increases $\overline{\alpha}$. In the case that $\alpha = 0$, the model collapses to a standard closed economy New Keynesian model without oil.

The natural equilibrium corresponds to the case that nominal rigidities are absent and prices are flexible. We denote variables in this equilibrium with the supra-index "n". Under flexible prices real marginal costs satisfy $mc_t^n = 0$ and the equilibrium can be expressed as:

$$(y_t - y_t^n) = E_t \left(y_{t+1} - y_{t+1}^n \right) - \frac{1}{\sigma} (r_t - E_t \pi_{t+1}), \tag{2.31}$$

$$\pi_t = \kappa_y (y_t - y_t^n) + \beta E_t \pi_{t+1}, \tag{2.32}$$

where $\kappa_y \equiv \kappa \chi (v + \sigma)$. Equations (2.31) and (2.32) are the dynamic IS and the Phillips curve, respectively, in terms of the output gap $(y_t - y_t^n)$. The natural level of output depends negatively on deviations of the oil price from its steady state:

$$y_t^n = - \left(\frac{1 + \psi v}{\sigma + v} \right) \left(\frac{\overline{\alpha}}{1 - \overline{\alpha}} \right) q_t. \tag{2.33}$$

The natural output depends, among other parameters, on the share of oil on total costs in steady state. The higher $\overline{\alpha}$ the more important the impact of oil price shocks on the natural level. Also, note from equation (2.33) that the response of the natural output to oil shocks is qualitatively similar to the reaction to productivity shocks in the standard New Keynesian model with the opposite sign. However, as we will see in the next section, the assumption of low substitutability of oil has important effects on the design of optimal monetary policy.

2.7 Calibration

As benchmark calibration we set a quarterly discount factor, β, equal to 0.99 which implies an annualised rate of interest of 4%. For the coefficient of risk aversion parameter, σ, we choose a value of 1 and the inverse of the elasticity of labor supply, v, is calibrated to be equal to 0.5, similar to those values used in the *RBC* literature. The probability of the Calvo lottery is set equal to 0.66 which implies that firms adjust prices, on average, every three quarters. We choose a degree of monopolistic competition, ε, equal to 7.88, which implies a firm mark-up of 15% over the marginal cost considering $\tau = 0$. We set the value of the elasticity of substitution between oil and labour in $\psi = 0.2$, equal to the average value reported by Hamilton (2009). We calibrate $\overline{\alpha} = 0.02895$ using information from the National Income Product accounts for the US[4]. Finally, we assume a persistent AR(1) process for the logarithm of the real oil price ($\rho = 0.95$).

3 A linear-quadratic approximate problem

In this section we characterize the sources of the trade-off between stabilising inflation and economic activity that arise in this economy. Also, we present a second order approximation of the welfare function of the representative household as function of purely quadratic terms. This representation allows us to characterise the policy problem using only a linear approximation of the structural equations of the model and also to rank sub-optimal monetary policy rules.

Since the model has an additional production input different from labour, a standard second order Taylor approximation of the welfare function will include linear terms, which would lead to an inaccurate approximation of the optimal policy in a linear-quadratic approach. To deal with this issue, we use the methodology proposed by Benigno and Woodford (2005), which consists on eliminating the linear terms of the policy objective using a second order approximation of the aggregate supply.

3.1 Sources of the trade-off

The efficient equilibrium is equivalent to the social planner problem of maximizing the utility of the representative agent, subject to: the production function for final goods and intermediate goods, the resources constraint and the aggregation conditions for both production inputs. The efficiency conditions for this problem imply that the marginal rate of substitution is equal to the marginal productivity of labour:

$$MRS_t = MPL_t\left(z\right), \tag{3.1}$$

and a symmetric allocation in equilibrium, $C_t\left(z\right) = C_t$ and $L_t\left(z\right) = L_t$, for every z.

In the decentralised equilibrium of the model, the ratio between the marginal rate of substitution and the marginal productivity of labour equals the real marginal costs:

[4] In particular, using the demand for oil in steady state we have: $\overline{\alpha} \equiv \mu\frac{\overline{QM}}{\overline{Y}}$. $\overline{QM}/\overline{Y}$ is estimated as the ratio of: (oil and other fuels used for production) / (value added), from the National Income Product accounts (www.bea.gov). The average value of $\overline{QM}/\overline{Y}$ is 2.5% for the period 1972-2006 and $\mu = 1.15$ in our calibration, then $\overline{\alpha} = 1.15 \times 2.5\% = 2.895\%$

$$\frac{MRS_t}{MPL_t(z)} = MC_t \equiv 1 - \Phi_t, \tag{3.2}$$

where Φ_t is the measure of the wedge between them. The optimality condition (3.1) implies that this wedge must be constant and equal to zero, that is $\Phi_t = 0$, to be socially optimal. A second order Taylor expansion of equation (3.2) in logarithms is:

$$\begin{aligned} \Phi_t &= \Phi - \chi(\sigma + v)(y_t - y_t^n) - \chi v \widehat{\Delta}_t \\ &\quad - \frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}}\chi^2(1-\chi)\chi(\sigma+v)\left[y_t + \frac{\chi}{1-\chi}y_t^n\right]^2 + O\left(\|\xi_t\|^3\right), \end{aligned} \tag{3.3}$$

where $\|\xi_t\|$ denotes a bound on the size of the oil price shock. If monetary policy can be used to replicate the natural equilibrium, this wedge becomes:

$$\Phi_t^{flex} = \Phi - \frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}}\frac{1}{\sigma+v}(q_t)^2 + O\left(\|\xi_t\|^3\right), \tag{3.4}$$

where we have used the definition of the natural output and evaluated the price dispersion term at zero. Note from equation (3.4), that when replicating the flexible price allocation in the decentralized equilibrium, the wedge is time varying and depends on the oil price. Because of this, a trade-off arises: it means that it is not possible at the same time to stabilise inflation and to replicate the social planner equilibrium under the presence of oil shocks, unless $\psi = 1$ as in the Cobb-Douglas case.

As shown above, when oil is considered a gross complement to labour in production in a CES technology, the divine coincidence disappears. This result is similar to the case of real wage rigidities explained in Blanchard and Galí (2007), where stabilising inflation is no longer equivalent to stabilising the welfare-relevant output gap. However, the mechanism here is different. In this case, the flexible price allocation cannot replicate the social planner allocation because of the second order effects of oil shocks in the wedge between the marginal rate of substitution and the marginal product of labour. When oil is difficult to substitute in production, real marginal costs become a convex function of the real oil price, because the participation of this input in marginal costs also increases with its price.

Interestingly, eliminating the distortions in steady state cannot eliminate the trade-off. In this case, after making $\Phi = 0$, the wedge becomes:

$$\Phi_t^{flex,efss} = -\frac{1}{2}\frac{1-\psi}{1-\widetilde{\alpha}}\frac{1}{\sigma+v}(q_t)^2 + O\left(\|\xi_t\|^3\right) \tag{3.5}$$

for $\widetilde{\alpha} \equiv \alpha^{\psi}\left(\overline{Q}\right)^{1-\psi} \leq \overline{\alpha}$. In this case, eliminating the distortion in steady state eliminates the constant and reduces the variability of the wedge with respect to the oil price. However, it is still not possible to replicate the social planner equilibrium under the presence of oil shocks. The intuition of this result is that when oil is considered a gross complement to labor in production in a CES technology, the share of oil in total costs in steady state depends also on the steady state distortion, when eliminating the distortion (a more competitive economy) makes the wedge less sensitive to increases in the real oil price. Though, making the steady state efficient cannot eliminate completely this sensitivity.

To measure this trade-off, in the next sub-section we derive a quadratic loss function from the second order Taylor expansion of the welfare function of the representative agent. We obtain a expression in terms of inflation and the deviations of output from a target level (the welfare-relevant output gap). This target level accounts for the effects of oil shocks in the wedge and maximises welfare of the representative agent when inflation is zero.

3.2 A second-order approximation to utility

A second order Taylor-series approximation to the utility function, expanding around the non-stochastic steady-state allocation, is:

$$U_{t_o} = \overline{Y}\overline{u}_c \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left(\Phi_L y_t + \frac{1}{2} u_{yy} y_t^2 + u_{yq} y_t q_t + u_{\Delta} \widehat{\Delta}_t \right) + t.i.p. + O\left(\|\xi_t\|^3 \right), \tag{3.6}$$

where $y_t \equiv \log \left(Y_t / \overline{Y} \right)$ and $\widehat{\Delta}_t \equiv \log \Delta_t$ measure deviations of aggregate output and the price dispersion measure from their steady state levels, respectively. The term $t.i.p.$ collects terms that are independent of policy (constants and functions of exogenous disturbances) and hence irrelevant for ranking alternative policies. The coefficients: u_{yy}, u_{yq} and u_{Δ} are defined in the appendix B. Φ_L is the wedge in steady state between consumption and labour in the utility function, defined by:

$$\begin{aligned} \Phi_L &= 1 - \frac{\overline{V}_L}{\overline{U}_C} \frac{d\overline{L}}{d\overline{Y}} \\ &= 1 - (1 - \overline{\alpha})(1 - \Phi)(1 - \delta(\sigma + v)). \end{aligned} \tag{3.7}$$

Note that in an economy with labour as the only input in the production function, as in Benigno and Woodford (2005), the wedge between consumption and labour in the utility function is equal to the distortion in steady state Φ. In those models, a tax rate that eliminates this distortion also eliminates the linear term in the second order Taylor expansion of the utility function. However, in an economy with other inputs different than labour we have in general that $\Phi_L \neq \Phi$, and eliminating the monopolistic distortion doesn't eliminate the linear term in equation (3.6).

We use the second order Taylor expansion of the price dispersion equation to substitute $\widehat{\Delta}_t$ as a function of quadratic terms of inflation in our welfare approximation. Also, we use the second order approximation of the Phillips curve to solve for the infinite discounted sum of the expected level of output as function of purely quadratic terms. Then, as in Benigno and Woodford (2005) we replace this last expression in (3.6) and rewrite it as:

$$U_{t_o} = -\Omega \left[E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left(\frac{1}{2} \lambda (y_t - y_t^*)^2 + \frac{1}{2} \pi_t^2 \right) - T_{t_o} \right] + t.i.p. + O\left(\|\xi_t\|^3 \right), \tag{3.8}$$

where $\Omega = \overline{Y}\overline{u}_c \lambda_\pi$ and $T_{t_o} = \frac{\Phi_L}{\kappa_y} v_{t_o}$, λ_π and v_{t_o} are defined in the appendix. λ measures the relative weight between a welfare-relevant output gap and inflation. y_t^* is the target output, the level of output that maximises our measure of welfare when inflation is zero. The values of λ and y_t^* are given by:

$$\lambda = \frac{\kappa_y}{\varepsilon}(1-\sigma\psi\overline{\alpha})\gamma, \quad (3.9)$$

$$y_t^* = -\left(\frac{1+\psi v}{\sigma+v}\right)\left(\frac{\alpha^*}{1-\alpha^*}\right)q_t, \quad (3.10)$$

where α^* accounts for the share of oil on total costs in steady state that replicates the target level of output, given by:

$$\alpha^* = \frac{\overline{\alpha}}{1+\eta}. \quad (3.11)$$

Both γ and η are a function of the deep parameters of the model, they are defined in the appendix and characterised in the next section. Note that the target level of output is written in a similar way as the natural level of output in equation (2.33), for a different share of oil on total costs in steady state.

3.3 The linear-quadratic policy problem

The policy objective U_{t_o} can be written in terms of inflation and the welfare-relevant output gap defined by x_t:

$$x_t \equiv y_t - y_t^*.$$

Benigno and Woodford (2005) showed that maximisation of U_{t_o} is equivalent to minimise the following loss function L_{t_o}

$$L_{t_o} \equiv E_{t_o}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\left(\frac{1}{2}\lambda x_t^2 + \frac{1}{2}\pi_t^2\right), \quad (3.12)$$

subject to a predetermined value of π_{t_o}[5] and the Phillips curve for any date from t_o onwards:

$$\pi_t = \kappa_y x_t + \beta E_t \pi_{t+1} + u_t. \quad (3.13)$$

Note that we have expressed (3.13) in terms of the welfare relevant output gap, x_t. u_t is a "cost-push" shock, which is proportional to the deviations in the real oil price:

$$\begin{aligned} u_t &\equiv \kappa_y\left(y_t^* - y_t^n\right) \quad (3.14)\\ &= \varpi q_t, \end{aligned}$$

where

$$\varpi \equiv \kappa_y\left(\frac{1+\psi v}{\sigma+v}\right)\left[\frac{\overline{\alpha}}{1-\overline{\alpha}} - \frac{\alpha^*}{1-\alpha^*}\right].$$

[5] Maximising equation (3.8) implies minimising (3.12) subject to a predeterminated value of v_{t_o}. Moreover, because the objective function is purely quadratic, a linear approximation of v_{t_o} suffices to describe the initial commitments, given by $v_{t_o} = \pi_{t_o}$.

In this model a "cost-push" shock arises endogenously since oil generates a trade-off between stabilising inflation and deviations of output from a target level, different from the natural level. In the next section we characterise the conditions under which oil shocks preclude simultaneous stabilisation of inflation and the welfare-relevant output gap.

If we are interested in evaluating monetary policy from a timeless perspective, that is optimising without regard of possible short run effects and avoiding possible time inconsistency problems. In this case, the predetermined value of π_{t_o} must equal $\pi^*_{t_o}$, the optimal value of inflation at t_o consistent with the policy problem. Thus, the policy objective consists on minimising (3.12) subject to the initial inflation rate:

$$\pi_{t_o} = \pi^*_{t_o}. \tag{3.15}$$

4 Optimal monetary response to oil shocks

In this section we use the linear-quadratic policy problem defined in the previous section to evaluate optimal and sub-optimal monetary policy rules under oil shocks. This policy problem can be summarised to maximise the following Lagrangian:

$$\mathcal{L}_{t_o} \equiv -E_{t_o} \left\{ \begin{array}{c} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left[\frac{1}{2}\lambda x_t^2 + \frac{1}{2}\pi_t^2 - \varphi_t \left(\pi_t - \kappa_y x_t - \beta E_t \pi_{t+1} - u_t \right) \right] \\ +\varphi_{t_o-1} \left(\pi_{t_o} - \pi^*_{t_o} \right) \end{array} \right\} \tag{4.1}$$

where $\beta^{t-t_o}\varphi_t$ is the Lagrange multiplier at period t.

The second order conditions for this problem are well defined for $\lambda \geq 0$, which is the case for plausible parameters of the model[6]. Then, as Benigno and Woodford (2005) show, since the loss function is convex, randomisation of monetary policy is welfare reducing and there are welfare gains when using monetary policy rules.

Under certain circumstances the optimal policy involves complete stabilisation of the inflation rate at zero for every period, that is complete price stability. These conditions are related to how oil enters in the production function and are summarised in the following proposition:

Proposition 1 *When the production function is Cobb-Douglas the efficient level of output is equivalent to the natural level of output.*

In the case of a Cobb-Douglas production function, the elasticity of substitution between labour and oil is unity (i.e. $\psi = 1$). In this case $\eta = 0$ and the share of oil on the marginal costs in the efficient level is equal to the share in the distorted steady state, equal to α (that is $\alpha^* = \overline{\alpha} = \alpha$) Then, the efficient level of output is equal to the natural level of output.

In this special case of the CES production function, fluctuations in output caused by oil shocks at the target level equals the fluctuations in the natural level. Then, stabilisation of output around the natural level also implies stabilisation around the target level. This is a

[6] More precisely, we are interested on studying the model when $0 < \psi \leq 1$ and σ not too high. Since λ is positive for $\psi \leq 1$ and $\sigma < (\overline{\alpha}\psi)^{-1}$, which is a very high value for the threshold since $\overline{\alpha}$ is lower than one and small.

special case in which the "divine coincidence" appears. Therefore, setting output equal to the target level also implies complete stabilisation of inflation at zero.

In this particular case there is no trade-off between stabilising output and inflation. However, in a more general specification of the CES production function this trade-off appears, as it is established in the next proposition:

Proposition 2 *When oil is difficult to substitute in production the efficient output responds less to oil shocks than the natural level, which generates a trade-off.*

When oil is difficult to substitute the elasticity of substitution between inputs is lower than one (that is $\psi < 1$). In this case $\eta > 0$ and the share of oil on total costs in steady state that replicates the target level of output is lower than in the steady state (that is $\alpha^* < \overline{\alpha}$), which causes that the target output fluctuates less than the natural level (that is $|y_t^*| < |y_t^n|$). Then, in this case it is not possible to have both inflation zero and output at the target level at all periods. In this case a "cost-push" shock arises endogenously which generates a trade-off between stabilising inflation and the welfare-relevant output gap. This "cost-push" is proportional to the difference between y_t^* and y_t^n, as shown in equation (3.14).

As mentioned in the previous section, this trade-off is generated by the convexity of real marginal costs with respect the real oil price, which produces a time varying wedge between the marginal rate of substitution and the marginal productivity of labour. Moreover, eliminating the distortions in steady state reduces the trade-off, because this wedge becomes less sensitive with respect to the oil price. However, making the steady state efficient cannot eliminate this trade-off.

Figure 4.1 shows the effect of the elasticity of substitution on α^* and $\overline{\alpha}$ and on y^* and y^n . As mentioned in proposition 1, when $\psi = 1$ then $\alpha^* = \overline{\alpha} = \alpha$. Similarly, as in proposition 2, lower ψ increases both α^* and $\overline{\alpha}$, but α^* is always lower than $\overline{\alpha}$. Also in this case, the efficient output fluctuates less than the natural level of output for a 1 percent increase in the real oil price. Because of this difference between y^* and y^n, the endogenous "cost-push" shock also increases when the elasticity of substitution ψ is lower. Moreover, this figure also shows the effects when distortions in steady state are eliminated. In this case, both α^* and $\overline{\alpha}$ decrease and y^* and y^n become less sensitive to an oil price shock.

It is also important to analyse how the production function affects λ, the weight between stabilising the welfare relevant output-gap and inflation. In the special case of a Cobb-Douglas production function, the coefficient γ defined in the previous section equals 1 and the relative weight in the loss function between welfare-relevant output gap and inflation stabilisation (λ) becomes $\frac{\kappa_y}{\varepsilon}\left(1-\sigma\alpha\right)$. This is similar to the coefficient found for many authors for the case of a closed economy[7], which is the ratio of the effect of output on inflation in the Phillips curve and the elasticity of substitution among goods, but multiplied by the additional term $(1 - \sigma\alpha)$.

The relative weight in the loss function between welfare-relevant output gap and inflation stabilisation is decreasing with the degree of price stickiness (θ) and the elasticity of substitution among goods (ε). When prices are more sticky (larger θ), κ_y is lower and price dispersion is

[7] See for example Woodford (2003) and Benigno and Woodford (2005).

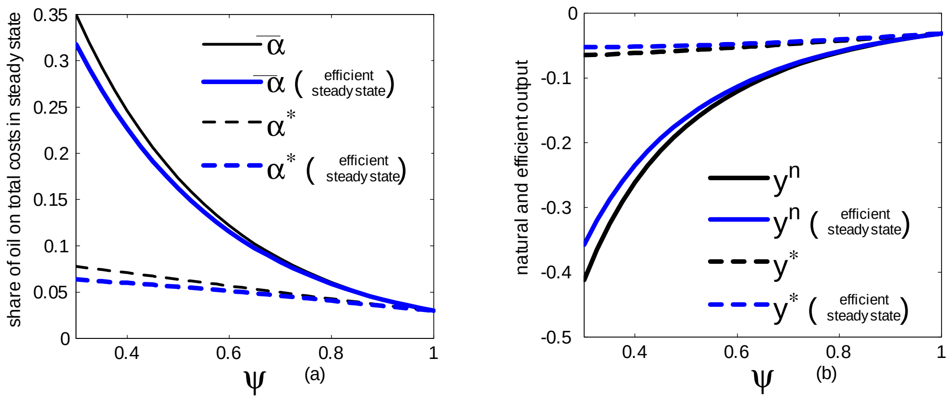

Figure 4.1: (a) Steady state and efficient share of oil on marginal costs. (b) Natural and efficient level of output.

higher. Similarly, a larger elasticity of substitution among goods (ε) amplifies the welfare losses caused by any given price dispersion. In both cases, the costs of inflation are more important and output stabilization has a lower weight relative to inflation stabilisation.

The term $(1 - \sigma\alpha)$ captures the effects of oil shocks on inflation through costs. When the weight of oil in the production function (α) is higher, the effects of oil shocks in marginal costs and inflation are more important. Then, the more important it becomes to stabilise inflation in respect to output.

The next proposition describes the behaviour of λ with respect to the elasticity of substitution ψ.

Proposition 3 *The lower the elasticity of substitution between oil and labour, the lower the weight in the loss function between welfare-relevant output gap and inflation stabilisation (λ).*

When the elasticity of substitution ψ is lower, the effect of output fluctuations on inflation becomes smaller (κ_y). This implies a higher relative effect on inflation respect to output, and therefore lower λ. This also implies a higher sacrifice ratio, since you need relatively larger interest rate changes in order to stabilise inflation.

The next graph shows the effects on λ of the elasticity of substitution for three different values of α. λ takes its highest value when $\psi = 1$ and decreases exponentially for lower ψ. Also, higher α reduces λ , which means a higher weight of inflation relative to output fluctuations in the welfare function.

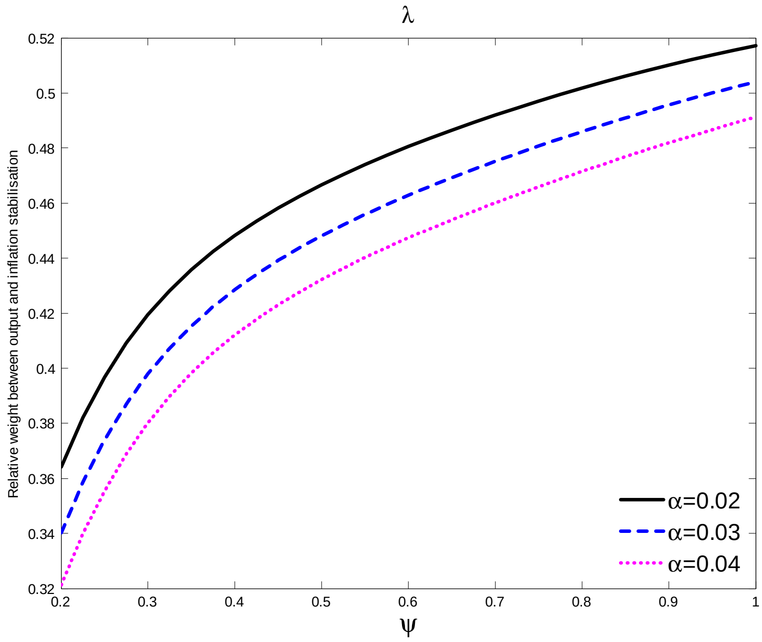

Figure 4.2: Relative weight between output and inflation stabilisation (λ).

4.1 Optimal unconstrained response to oil shocks from a timeless perspective

When we solve for the Lagrangian (4.1), we obtain the following first order conditions that characterise the solution of the optimal path of inflation and the welfare-relevant output gap in terms of the Lagrange multipliers:

Proposition 4 *The optimal unconstrained response to oil shocks is given by the following conditions:*

$$\begin{aligned} \pi_t &= \varphi_{t-1} - \varphi_t, \\ x_t &= \frac{\kappa_y}{\lambda}\varphi_t, \end{aligned}$$

where φ_t is the Lagrange multiplier of the optimisation problem, that has the following law of motion :

$$\varphi_t = \tau_\varphi \varphi_{t-1} - \phi q_t,$$

for $\phi \equiv \frac{\tau_\varphi}{1-\beta\tau_\varphi\rho}\varpi$, and satisfies the initial condition:

$$\varphi_{t_o-1} = -\phi \sum_{k=0}^{\infty} \tau_\varphi^k q_{t-1-k},$$

where $\tau_\varphi \equiv Z - \sqrt{Z^2 - \frac{1}{\beta}} < 1$ and $Z \equiv \left((1+\beta) + \frac{\kappa_y^2}{\lambda}\right)/(2\beta)$.

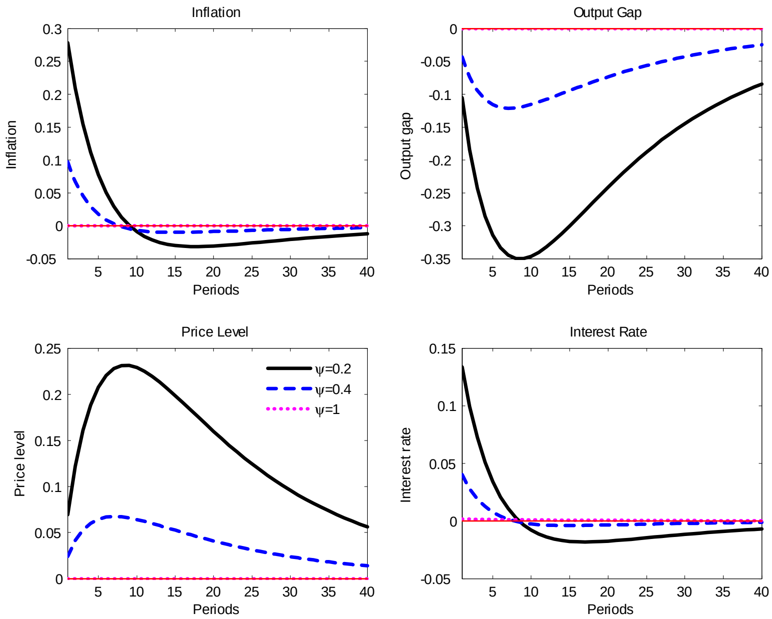

Figure 4.3: Impulse response to an oil shock under optimal unconstrained monetary policy.

The proof is in the appendix. From a timeless perspective the initial condition for φ_{t_o-1} depends on the past realisations of the oil prices and it is time-consistent with the policy problem.

Also, we define the impulse response of a shock in the oil price in period t (ξ_t) in a variable z in $t+j$ as the unexpected change in its transition path. Then the impulse is calculated by:

$$I_t\left(z_{t+j}\right) = E_t\left[z_{t+j}\right] - E_{t-1}\left[z_{t+j}\right],$$

and the impulse response for inflation, the price level and the welfare-relevant output gap for the optimal policy is:

$$I_t^{opt}\left(\pi_{t+j}\right) = \left(\frac{\rho^{j+1}-\tau_\varphi^{j+1}}{\rho-\tau_\varphi} - \frac{\rho^{j}-\tau_\varphi^{j}}{\rho-\tau_\varphi}\right)\phi\xi_t, \tag{4.2}$$

$$I_t^{opt}\left(p_{t+j}\right) = \left(\frac{\rho^{j+1}-\tau_\varphi^{j+1}}{\rho-\tau_\varphi}\right)\phi\xi_t, \tag{4.3}$$

$$I_t^{opt}\left(x_{t+j}\right) = -\frac{\kappa_y}{\lambda}\left(\frac{\rho^{j+1}-\tau_\varphi^{j+1}}{\rho-\tau_\varphi}\right)\phi\xi_t, \tag{4.4}$$

See appendix B.3 for details on the derivation.

Figure 4.3 shows the optimal unconstrained impulse response functions of inflation, the welfare-relevant output gap, the price level and the nominal interest rate to an oil price shock of size one for different values of the elasticity of substitution (ψ). Inflation and the nominal interest rate are in yearly terms. The benchmark case is a value of $\psi = 0.2$. In these graphs we can see that after an oil shock the optimal response is an increase of inflation and a reduction of the welfare-relevant output gap. The nominal interest rate also increases to partially offset the effects of the oil shock on inflation. Inflation after 8 quarters become negative as the optimal unconstrained plan is associated with price stability. That is, after some time, the price level returns to its initial level. To summarise, the optimal response to an oil shock implies an effect on impact on inflation that dies out very rapidly and a more persistent effect on output.

An increase in the elasticity of substitution from 0.2 to 0.4 reduces the size of the cost push shock, diminishes $\overline{\alpha}$ but increases λ. Then, the impact on all the variables is reduced, inflation being initially the more affected variable. Also, the higher impact on welfare-relevant output gap is after 8 quarters. In contrast, when the elasticity of substitution is unity, since there is no such a trade-off, both inflation and welfare-relevant output gap are zero in every period.

4.2 Evaluation of suboptimal rules - the non-inertial plan

We can use our linear-quadratic policy problem for ranking alternative sub-optimal policies. One example of such policies is the optimal non-inertial plan. By a non-inertial policy we mean a monetary policy rule that depends only on the current state of the economy. In this case, if the policy results in a determinate equilibrium, then the endogenous variables depend also on the current state.

If the current state of the economy is given by the cost push shock, which has the following law of motion:

$$u_t = \rho u_{t-1} + \varpi \xi_t,$$

where ξ_t is the oil price shock and ϖ is defined in the previous section. A first order general description of the possible equilibrium dynamics can be written in the form [8]:

$$\begin{aligned} \pi_t &= \overline{\pi} + f_\pi u_t, && (4.5)\\ x_t &= \overline{x} + f_x u_t, && (4.6)\\ \varphi_t &= \overline{\varphi} + f_\varphi u_t, && (4.7) \end{aligned}$$

where we need to determine the coefficients: $\overline{\pi}, \overline{x}, \overline{\varphi}, f_\pi, f_x$ and f_φ. To solve for the optimal non-inertial plan we need to replace (4.5),(4.6) and (4.7) in the Lagrangian (4.1) and solve for the coefficients that maximise the objective function. The results are summarised in the following proposition:

Proposition 5 *The optimal non-inertial plan is given by* $\pi_t = \overline{\pi} + f_\pi u_t$ *and* $x_t = \overline{x} + f_x u_t$*,*

[8] Note that in this sub-section we focus on the simplest case of the non-inertial plan, in which all endogenous variables depends only the current state of the economy. In contrast, Benigno and Woodford (2005) work with a different non-inertial plan, in which the lagrange multipliers satisfy the first order conditions of the unconstrained problem

where

$$\overline{\pi} = 0, \quad f_{\pi} = \frac{\lambda(1-\rho)}{\kappa_y^2 + \lambda(1-\beta\rho)(1-\rho)}.$$
$$\overline{x} = 0, \quad f_{x} = \frac{\kappa_y}{\kappa_y^2 + \lambda(1-\beta\rho)(1-\rho)}.$$

Note that in the optimal non-inertial plan the ratio of inflation/output gap is constant and equal to $\frac{\lambda(1-\rho)}{\kappa_y}$. The higher the weight in the loss function for output fluctuations relative to inflation fluctuations, the higher the inflation rate. Also, the more persistent the oil shocks, the lower the weight on inflation relative to the welfare-relevant output-gap.

Similar to the optimal case, the impulse response functions for inflation and output are defined by:

$$\begin{aligned} I_t^{ni}(\pi_{t+j}) &= f_{\pi}\varpi\rho^j\xi_t, \\ I_t^{ni}(x_{t+j}) &= f_{\pi}\varpi\rho^j\xi_t. \end{aligned}$$

Figure 4.4 shows the responses in the optimal non-inertial plan to an unitary oil price shock. As shown, the main difference in respect to the previous plan is that in the optimal non-inertial plan inflation returns to its initial level after some time, but in the optimal unconstrained plan the price level is the one that converges. This implies that inflation must be negative after some quarters in the optimal unconstrained plan. Also, the reduction in the welfare-relevant output gap is much lower on impact in the case of the optimal unconstrained plan in comparison with the optimal non inertial plan. In the latter, the reduction in the welfare-relevance is proportional to the increase in inflation.

Both exercises, the optimal unconstrained plan and the optimal non-inertial plan, show that to the extent that economies are more dependent on oil, in the sense that oil is difficult to substitute, the impact of oil shocks on both inflation and output is greater. Also, in this case, monetary policy should react by raising more the nominal interest rate and allowing relatively more fluctuations in inflation than in output.

Furthermore, figure 4.4 shows the responses under the optimal non-inertial plan when ψ increases from 0.2 to 0.4. As shown, the impact on all the variables is reduced, because an increase of ψ diminishes the size of the cost-push shock. Also, the increase of ψ makes λ larger, which makes the impact on inflation relatively higher with respect to the response of the welfare-relevant output gap. As in the unconstrained case, when $\psi = 1$ the trade-off disappears. In that case, inflation is zero in every period and output equals its target level.

After analysing the optimal plans, in figure 4.5 we plot the welfare losses for these two type of policies for different elasticities of substitution ψ. The welfare losses are normalized with respect to the variance of oil shocks. As shown, the welfare loss under both regimes are the same, equal to zero, when the production function is Cobb-Douglas. Moreover, when the elasticity of substitution ψ decreases, the difference in welfare losses under both policy plans increases exponentially, which is consistent with the increase in the size of the "cost-push" shock.

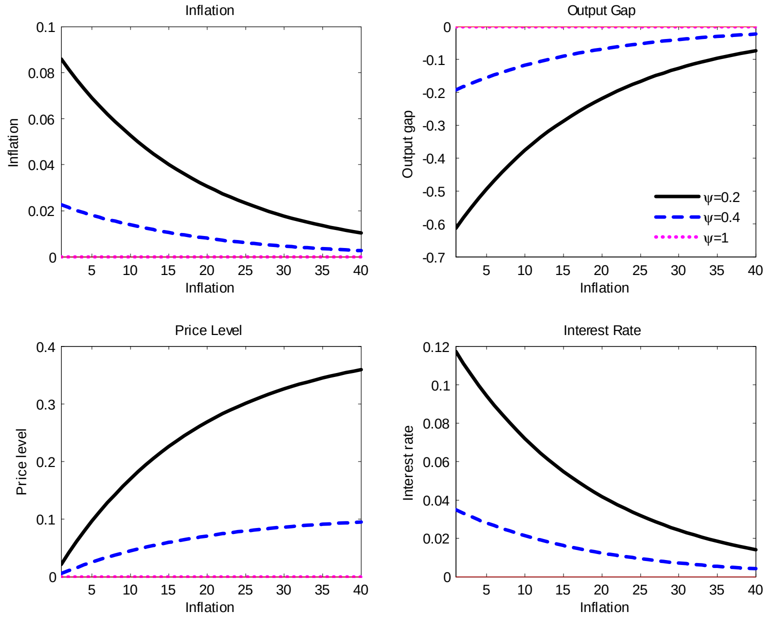

Figure 4.4: Impulse response to an oil shock under the optimal non-inertial plan.

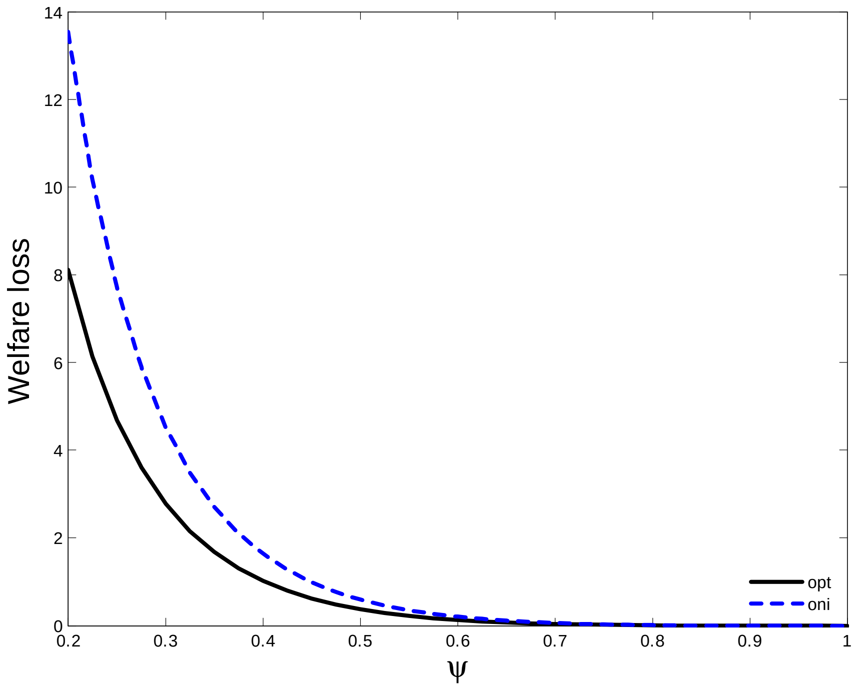

Figure 4.5: Welfare losses under both plans.

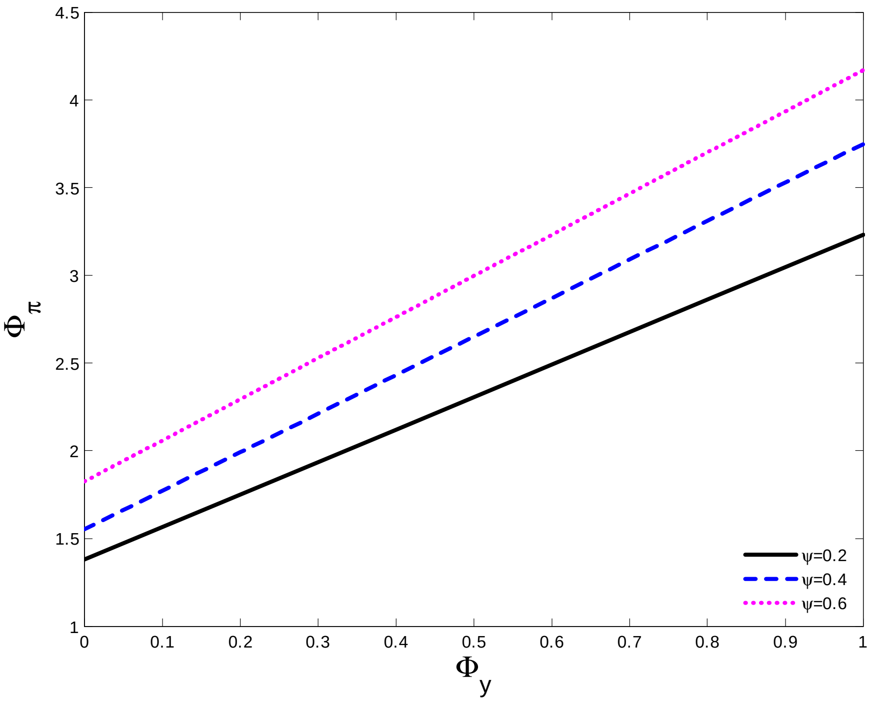

Figure 4.6: Simple rule coefficients that implement the optimal non-inertial plan.

4.3 A simple rule that implements the optimal non inertial plan

Optimal monetary plans can be difficult to communicate and implement, because they rely on real-time calculations of the welfare-relevant output gap and the size of the "cost-push" shock, which are unobservable variables. Because of this, in this subsection we estimate a simple interest rate rule that implements the optimal non-inertial plan that is based only on observable variables, such as inflation and output. This rule has the following form:

$$r_t = \phi_\pi \pi_t + \phi_y y_t. \tag{4.8}$$

An advantage of using a specification such as (4.8) is that we can compare it with feedback rules that have been estimated for different economies. To estimate (4.8), we replace this policy rule in dynamic IS equation (2.29) and use the solution from the optimal non-inertial plan for inflation (4.5) and output gap (4.6) and the output target-level (3.1), to solve for the coefficients ϕ_π and ϕ_y that solve the equilibrium. The solution for these coefficients is exact because there is only one shock in the economy. Also, there is not only one set, but a continuous combination of parameters ϕ_π and ϕ_y that implement this optimal plan.

In figure 4.6 we show the combination of parameters of the simple rule that implement the optimal non-inertial plan for different values of the elasticity of substitution ψ. A first thing to note is that there is that there is a positive relationship between ϕ_π and ϕ_y, which is consistent with the fact that an oil shock implies a trade-off. That is, if the response in the feedback rule to inflation is higher, then the response to output fluctuations must also be higher to compensate for the effects of oil shocks on economic activity. Moreover, when the elasticity of substitution is lower, the trade-off increases and the intercept in figure 4.6 is lower. This implies that an economy with inflation targeting where oil is more difficult to substitute should have a less agressive response to inflation than in an economy that is less dependant on oil.

Also, consistent with a larger trade-off for lower elasticity of substitution, the response to output fluctuations must increase more for a given increase in the response to inflation fluctuations. That is, the slope in figure 4.6 becomes flatter. This implies that in a flexible inflation targeting regime, due to oil shocks considerations, a more agressive response to inflation fluctuations must be accompained with stronger response to output fluctuations.

5 Conclusions

This paper characterises the utility-based loss function for a closed economy in which oil is used in the production process, there is staggered price setting and monopolistic competition. As in Benigno and Woodford (2005), our utility based-loss function is a quadratic on inflation and the deviations of output from a target level, which is the welfare-relevant output gap.

We found that this target level differs from the natural level of output when the elasticity of substitution between labour and oil is different from one. This generates a trade-off between stabilising inflation and output in the presence of oil shocks. Also, the cost-push shocks involved in this trade-off are proportional to oil shocks. The lower this elasticity of substitution, the higher the size of the cost-push shock. This trade-off is generated by the convexity of real marginal costs with respect to the real oil price, which produces a time varying wedge between the marginal rate of substitution and the marginal productivity of labour. We also find that eliminating the distortions in steady state reduces the trade-off, because this wedge becomes less sensitive with respect to the oil price. However, in contrast to Benigno and Woodford (2005), making the steady state efficient cannot eliminate this trade-off.

Furthermore, the relative weight between the welfare-relevant output gap and inflation on the utility-based loss function depends directly on this elasticity of substitution. On the contrary, the higher the share of oil in the production function, the smaller the relative weight.

These results show that to the extent that economies are more dependent on oil, in the sense that oil is difficult to substitute in production, the impact of oil shocks on both inflation and output is higher. Also, in this case the central bank should allow less fluctuation on inflation relative to output due to oil shocks. Moreover, these results shed light on how technological improvements which reduces the dependence on oil, also reduce the impact of oil shocks on the economy.

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A Appendix: the deterministic steady state

The non-stochastic steady state of the endogenous variables for $\overline{\Pi} = 1$ is given by:

Interest rate	$\overline{R} = \beta^{-1}$
Marginal costs	$\overline{MC} = \left(\frac{\varepsilon-1}{\varepsilon}\right)(1-\tau)$
Real wages	$\overline{W/P} = \frac{1-\overline{\alpha}}{\overline{MC}}\left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1}{1-\psi}}$
Output	$\overline{Y} = \left(\frac{1-\overline{\alpha}}{\overline{MC}}\right)^{\frac{1}{\sigma+\nu}}\left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1+\psi\nu}{\sigma+\nu}\frac{1}{1-\psi}}$
Labor	$\overline{L} = \left(\frac{1-\overline{\alpha}}{\overline{MC}}\right)^{\frac{1}{\sigma+\nu}}\left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1-\sigma\psi}{\sigma+\nu}\frac{1}{1-\psi}}$

Table A.1: The deterministic steady state

where $\overline{\alpha} \equiv \alpha^{\psi}\left(\overline{Q}/\overline{MC}\right)^{1-\psi}\overline{\alpha}$ is the share of oil in the total costs in steady state. Notice that the steady state values of real wages, output and labour depend on the steady state ratio of oil prices with respect to the marginal cost. This implies that permanent changes in oil prices would generate changes in the steady state of these variables. Also, as the standard New-Keynesian models, the marginal cost in steady state is equal to the inverse of the mark-up.

Since monopolistic competition affects the steady state of the model, output in steady state is below the efficient level. We call to this feature a distorted steady state and $\Phi \equiv 1 - \overline{MC}$ accounts for effects of the monopolistic distortions in steady state.

Since the technology has constant returns to scale, we have that:

$$\begin{aligned} \frac{\overline{V}_L}{\overline{U}_C}\frac{\overline{L}}{\overline{Y}} &= \left(\frac{\overline{W/P}}{\overline{MC}}\frac{\overline{L}}{\overline{Y}}\right)\overline{MC} \\ &= (1-\overline{\alpha})(1-\Phi) \end{aligned}$$

the ratio of the marginal rate of substitution multiplied by the ratio labour/output is a proportion $(1-\overline{\alpha})$ of the marginal costs. This expression helps us to obtain the wedge between the consumption and labor in the utility function in steady state:

$$\begin{aligned} \frac{\overline{V}_L}{\overline{U}_C}\frac{d\overline{L}}{d\overline{Y}} &= \left(\frac{\overline{V}_L}{\overline{U}_C}\frac{\overline{L}}{\overline{Y}}\right)\left(\frac{d\overline{L}/\overline{L}}{d\overline{Y}/\overline{Y}}\right) \\ &= (1-\overline{\alpha})(1-\Phi)(1-\delta(\sigma+v)) \\ &\equiv 1-\Phi_L \end{aligned}$$

B Appendix: The second order solution of the model

B.1 The recursive AS equation

We divide the equation for the aggregate price level (2.17) by $P_t^{1-\varepsilon}$ and make $P_t/P_{t-1} = \Pi_t$

$$1 = \theta\left(\Pi_t\right)^{-(1-\varepsilon)} + (1-\theta)\left(\frac{P_t^*(z)}{P_t}\right)^{1-\varepsilon} \tag{B-1}$$

Aggregate inflation is a function of the optimal price level of firm z. Also, from equation (2.15) the optimal price of a typical firm can be written as:

$$\frac{P_t^*(z)}{P_t} = \frac{N_t}{D_t}$$

where, after using the definition for the stochastic discount factor: $\zeta_{t,t+k} = \beta^k\left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+k}}$, we define N_t and D_t as follows:

$$N_t = E_t\left[\sum_{k=0}^{\infty}\mu\left(\theta\beta\right)^k F_{t,t+k}^{\varepsilon}Y_{t+k}C_{t+k}^{-\sigma}MC_{t+k}\right] \tag{B-2}$$

$$D_t = E_t\left[\sum_{k=0}^{\infty}\left(\theta\beta\right)^k F_{t,t+k}^{\varepsilon-1}Y_{t+k}C_{t+k}^{-\sigma}\right] \tag{B-3}$$

N_t and D_t can be expanded as:

$$N_t = \mu Y_t C_t^{-\sigma}MC_t + E_t\left[\Pi_{t+1}^{\varepsilon}\sum_{k=0}^{\infty}\mu\left(\theta\beta\right)^{k+1}F_{t+1,t+1+k}^{\varepsilon}Y_{t+1+k}C_{t+1+k}^{-\sigma}MC_{t+1+k}\right] \tag{B-4}$$

$$D_t = Y_t C_t^{-\sigma} + E_t\left[\Pi_{t+1}^{\varepsilon-1}\sum_{k=0}^{\infty}\left(\theta\beta\right)^{k+1}F_{t+1,t+1+k}^{\varepsilon-1}C_{t+1+k}^{-\sigma}Y_{t+1+k}\right] \tag{B-5}$$

where we have used the definition for $F_{t,t+k} = P_{t+k}/P_t$.

The Phillips curve with oil prices is given by the following three equations:

$$\theta\left(\Pi_t\right)^{\varepsilon-1} = 1-(1-\theta)\left(\frac{P_t^*(z)}{P_t}\right)^{1-\varepsilon} \tag{B-6}$$

$$N_t = \mu Y_t^{1-\sigma}MC_t + \theta\beta E_t\left(\Pi_{t+1}\right)^{\varepsilon}N_{t+1} \tag{B-7}$$

$$D_t = Y_t^{1-\sigma} + \theta\beta E_t\left(\Pi_{t+1}\right)^{\varepsilon-1}D_{t+1} \tag{B-8}$$

where we have reordered equation (B-1) and we have used equations (B-2) and (B-3), evaluated one period forward, to replace N_{t+1} and D_{t+1} in equations (B-4) and (B-5), and use the law of iterated expectations.

B.2 The second order approximation of the model

In this sub-section we present a log-quadratic (Taylor-series) approximation of the fundamental equations of the model around the steady state, a detailed derivation is provided in Appendix B. The second-order Taylor-series expansion serves to compute the equilibrium fluctuations of the endogenous variables of the model up to a residual of order $O\left(\|\xi\|^2\right)$, where $\|\xi_t\|$ is a bound on the size of the oil price shock. Up to second order, equations (2.26) to (2.29) are replaced by the following set of log-quadratic equations:

Labour Market	
$l_t = y_t - \delta\left[(v+\sigma) y_t - q_t\right] + \frac{\chi}{1-\alpha}\widehat{\Delta}_t + \frac{1}{2}\frac{1-\psi}{1-\alpha}\delta\chi^2\left[(v+\sigma) y_t - q_t\right]^2 + O\left(\|\xi\|^3\right)$	$B-i$
Aggregate Supply	
Marginal Costs	
$mc_t = \chi (v+\sigma) y_t + (1-\chi) q_t + \frac{1}{2}\frac{1-\psi}{1-\alpha}(1-\chi)\chi^2\left[(v+\sigma) y_t - q_t\right]^2 + \chi v\widehat{\Delta}_t + O\left(\|\xi\|^3\right)$	$B-ii$
Price dispersion	
$\widehat{\Delta}_t = \theta\widehat{\Delta}_t + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\pi_t^2 + O\left(\|\xi\|^3\right)$	$B-iii$
Phillips Curve	
$v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t\left(2(1-\sigma) y_t + mc_t\right) + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\|\xi\|^3\right)$	$B-iv$
where we have defined the auxiliary variables:	
$v_t \equiv \pi_t + \left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)\pi_t^2 + \frac{1}{2}(1-\theta\beta)\pi_t z_t$	$B-v$
$z_t \equiv 2(1-\sigma) y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon-1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\|\xi_t\|^2\right)$	$B-vi$
Aggregate Demand	
$y_t = E_t y_{t+1} - \frac{1}{\sigma}(r_t - E_t\pi_{t+1}) - \frac{1}{2}\sigma E_t\left[(y_t - y_{t+1}) - \frac{1}{\sigma}(r_t - \pi_{t+1})\right]^2 + O\left(\|\xi\|^3\right)$	$B-vii$

Table A.1: Second order Taylor expansion of the equations of the model

Equations (B-i) and (B-ii) are obtained taking a second-order Taylor-series expansion of the aggregate labour and the real marginal cost equation, after using the labour market equilibrium to eliminate real wages. $\widehat{\Delta}_t$ is the log-deviation of the price dispersion measure Δ_t, which is a second order function of inflation and its dynamics, which is represented with equation (B-iii).

We replace the equation for the marginal costs (B-ii) in the second order expansion of the Philips curve and iterate forward. Then, replace recursively the price dispersion terms from equation (B-iii) to obtain the infinite sum of the Phillips curve only as a function of output, inflation and the oil shock:

$$v_{t_o} = \sum_{t=t_o}^{\infty}\beta^{t-t_o}\left\{\begin{array}{c}\kappa_y y_t + \kappa_q q_t + \frac{1}{2}\varepsilon(1+\chi v)\pi_t^2 \\ +\frac{1}{2}\kappa\left[c_{yy}y_t^2 + 2c_{yq}y_t q_t + c_{qq}q_t^2\right]\end{array}\right\} + (1-\theta)\chi v\widehat{\Delta}_{t_o-1} + \left(\|\xi_t\|^3\right) \quad \text{(B-9)}$$

where c_{yy}, c_{yq} and c_{qq} are defined in the appendix.

B.2.1 The MC equation and the labour market equilibrium

The real marginal cost (2.12) and the labour market equations (2.4 and 2.23) have the following second order expansion:

$$mc_t = (1-\overline{\alpha})\, w_t + \overline{\alpha} q_t + \frac{1}{2}\overline{\alpha}\,(1-\overline{\alpha})\,(1-\psi)\,(w_t - q_t)^2 + O\left(\|\xi_t\|^3\right) \tag{B-10}$$

$$w_t = v l_t + \sigma y_t \tag{B-11}$$

$$l_t = y_t - \psi\,(w_t - mc_t) + \widehat{\Delta}_t \tag{B-12}$$

Where w_t and $\widehat{\Delta}_t$ are, respectively, the log of the deviation of the real wage and the price dispersion measure from their respective steady state. Notice that equations $(B-11)$ and $(B-12)$ are not approximations, but exact expressions. Solving equations $(B-11)$ and $(B-12)$ for the equilibrium real wage:

$$w_t = \frac{1}{1+v\psi}\left[(v+\sigma)\, y_t + v\psi mc_t + v\widehat{\Delta}_t\right] \tag{B-13}$$

Plugging the real wage in equation $(B-10)$ and simplifying:

$$\begin{aligned} mc_t \;=\; & \chi\,(\sigma+v)\, y_t + (1-\chi)\,(q_t) + \chi v \widehat{\Delta}_t \\ & + \frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}}\chi^2\,(1-\chi)\,[(\sigma+v)\, y_t - q_t]^2 + O\left(\|\xi_t\|^3\right) \end{aligned} \tag{B-14}$$

where $\chi \equiv (1-\overline{\alpha})\,/\,(1+v\psi\overline{\alpha})$. This is the equation $(B-ii)$ in the previous section. This expression is the second order expansion of the real marginal cost as a function of output and the oil prices. Similarly, we can express labour in equilibrium as a function of output and oil prices:

$$l_t = y_t - \delta\,[(v+\sigma)\, y_t - q_t] + \frac{\chi}{1-\overline{\alpha}}\widehat{\Delta}_t + \frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}}\delta\chi^2\,[(v+\sigma)\, y_t - q_t]^2 + O\left(\|\xi_t\|^3\right) \tag{B-15}$$

for:

$$\delta \equiv \psi\chi\frac{\overline{\alpha}}{1-\overline{\alpha}}$$

where δ measures the effects of oil shocks on labour.

B.2.2 The price dispersion measure

The price dispersion measure is given by

$$\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} dz$$

Since a proportion $1-\theta$ of intermediate firms set prices optimally, whereas the other θ set the price last period, this price dispersion measure can be written as:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*(z)}{P_t}\right)^{-\varepsilon} + \theta\int_0^1 \left(\frac{P_{t-1}(z)}{P_t}\right)^{-\varepsilon} dz$$

Dividing and multiplying by $(P_{t-1})^{-\varepsilon}$ the last term of the RHS:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*(z)}{P_t}\right)^{-\varepsilon} + \theta\int_0^1 \left(\frac{P_{t-1}(z)}{P_{t-1}}\right)^{-\varepsilon}\left(\frac{P_{t-1}}{P_t}\right)^{-\varepsilon} dz$$

Since $P_t^*(z)/P_t = N_t/D_t$ and $P_t/P_{t-1} = \Pi_t$, using equation (2.8) in the text and the definition for the dispersion measure lagged on period, this can be expressed as

$$\Delta_t = (1-\theta)\left(\frac{1-\theta(\Pi_t)^{\varepsilon-1}}{1-\theta}\right)^{\varepsilon/(\varepsilon-1)} + \theta\Delta_{t-1}(\Pi_t)^{\varepsilon} \tag{B-16}$$

which is a recursive representation of Δ_t as a function of Δ_{t-1} and Π_t.

Benigno and Woodford (2005) showed that a second order approximation of the price dispersion depends solely on second order terms on inflation. Then, the second order approximation of equation (B-16) is:

$$\widehat{\Delta}_t = \theta\widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\pi_t^2 + O\left(\|\xi_t\|^3\right) \tag{B-17}$$

which is equation $(B-iii)$ in the previous section. Moreover, we can use equation $(B-17)$ to write the infinite sum:

$$\begin{aligned}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_t &= \theta\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\frac{\pi_t^2}{2} + O\left(\|\xi_t\|^3\right)\\ (1-\beta\theta)\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_t &= \theta\widehat{\Delta}_{t_o-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\frac{\pi_t^2}{2} + O\left(\|\xi_t\|^3\right)\end{aligned}$$

Dividing by $(1-\beta\theta)$ and using the definition of κ :

$$\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_t = \frac{\theta}{1-\beta\theta}\widehat{\Delta}_{t_o-1} + \frac{1}{2}\frac{\varepsilon}{\kappa}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\frac{\pi_t^2}{2} + O\left(\|\xi_t\|^3\right) \tag{B-18}$$

The discounted infinite sum of $\widehat{\Delta}_t$ is equal to the sum of two terms, on the initial price dispersion and the discounted infinite sum of π_t^2.

B.2.3 The second order approximation of the Phillips Curve

The second order expansion for equations $(B-6)$, $(B-7)$ and $(B-8)$ are:

$$\pi_t = \frac{(1-\theta)}{\theta}(n_t - d_t) - \frac{1}{2}\frac{(\varepsilon-1)}{1-\theta}(\pi_t)^2 + O\left(\|\xi_t\|^3\right) \tag{B-19}$$

$$n_t = (1-\theta\beta)\left(a_t + \frac{1}{2}a_t^2\right) + \theta\beta\left(E_t b_{t+1} + \frac{1}{2}E_t b_{t+1}^2\right) - \frac{1}{2}n_t^2 + O\left(\|\xi_t\|^3\right) \tag{B-20}$$

$$d_t = (1-\theta\beta)\left(c_t + \frac{1}{2}c_t^2\right) + \theta\beta\left(E_t e_{t+1} + \frac{1}{2}E_t e_{t+1}^2\right) - \frac{1}{2}d_t^2 + O\left(\|\xi_t\|^3\right) \tag{B-21}$$

Where we have defined the auxiliary variables a_t,b_{t+1},c_t and e_{t+1} as:

$$a_t \equiv (1-\sigma)\, y_t + mc_t \qquad b_{t+1} \equiv \varepsilon \pi_{t+1} + n_{t+1}$$
$$c_t \equiv (1-\sigma)\, y_t \qquad e_{t+1} \equiv (\varepsilon - 1)\, \pi_{t+1} + d_{t+1}$$

Subtract equations $(B-20)$ and $(B-21)$, and using the fact that $X^2 - Y^2 = (X-Y)(X+Y)$, for any two variables X and Y :

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)(a_t - c_t) + \frac{1}{2}(1-\theta\beta)(a_t - c_t)(a_t + c_t) \\ &\quad + \theta\beta E_t (b_{t+1} - e_{t+1}) + \frac{1}{2}\theta\beta E_t (b_{t+1} - e_{t+1})(b_{t+1} + e_{t+1}) \\ &\quad - \frac{1}{2}(n_t - d_t)(n_t + d_t) + O\left(\|\xi_t\|^3\right) \end{aligned} \tag{B-22}$$

Plugging in the values of a_t, b_{t+1}, c_t and e_{t+1} into equation $(B-22)$, we obtain: $(B-23)$

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)\, mc_t + \frac{1}{2}(1-\theta\beta)\, mc_t \left(2(1-\sigma)\, y_t + mc_t\right) \\ &\quad + \theta\beta E_t (\pi_{t+1} + n_{t+1} - d_{t+1}) \\ &\quad + \frac{1}{2}\theta\beta E_t (\pi_{t+1} + n_{t+1} - d_{t+1})\left((2\varepsilon - 1)\, \pi_{t+1} + n_{t+1} + d_{t+1}\right) \\ &\quad - \frac{1}{2}(n_t - d_t)(n_t + d_t) + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{B-23}$$

Taking forward one period equation $(B-19)$, we can solve for $n_{t+1} - d_{t+1}$:

$$n_{t+1} - d_{t+1} = \frac{\theta}{1-\theta}\pi_{t+1} + \frac{1}{2}\frac{\theta}{1-\theta}\frac{(\varepsilon-1)}{1-\theta}(\pi_{t+1})^2 + O\left(\|\xi_t\|^3\right) \tag{B-24}$$

replace equation $(B-24)$ in $(B-23)$ and make use of the auxiliary variable $z_t = (n_t + d_t)/(1-\theta\beta)$

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)\, mc_t + \frac{1}{2}(1-\theta\beta)\, mc_t \left(2(1-\sigma)\, y_t + mc_t\right) \\ &\quad + \frac{\theta}{1-\theta}\beta \left[E_t \pi_{t+1} + \left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right) E_t \pi_{t+1}^2 + (1-\theta\beta)\, E_t \pi_{t+1} z_{t+1}\right] \\ &\quad - \frac{1}{2}\frac{\theta}{1-\theta}(1-\theta\beta)\, \pi_t z_t + O\left(\|\xi_t\|^3\right) \end{aligned} \tag{B-25}$$

Notice that we use only the linear part of equation $(B-24)$ when we replace $n_{t+1} - d_{t+1}$ in the quadratic terms because we are interested in capturing the terms only up to second order of accuracy. Similarly, we make use of the linear part of equation $(B-19)$ to replace $(n_t - d_t) = \frac{\theta}{1-\theta}\pi_t$ in the right hand side of equation $(B-25)$. Replace equation $(B-25)$ in $(B-19)$:

$$\begin{aligned} \pi_t &= \kappa mc_t + \frac{1}{2}\kappa mc_t \left(2(1-\sigma)\, y_t + mc_t\right) \\ &\quad + \beta \left[E_t \pi_{t+1} + \left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right) E_t \pi_{t+1}^2 + (1-\theta\beta)\, E_t \pi_{t+1} z_{t+1}\right] \\ &\quad - \frac{1}{2}(1-\theta\beta)\, \pi_t z_t - \frac{1}{2}\frac{(\varepsilon-1)}{1-\theta}(\pi_t)^2 + O\left(\|\xi_t\|^3\right) \end{aligned} \tag{B-26}$$

for

$$\kappa \equiv \frac{(1-\theta)}{\theta}(1-\theta\beta)$$

where z_t has the following linear expansion:

$$z_t = 2(1-\sigma)y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon-1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\|\xi_t\|^3\right) \tag{B-27}$$

Define the following auxiliary variable:

$$v_t = \pi_t + \frac{1}{2}\left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)\pi_t^2 + \frac{1}{2}(1-\theta\beta)\pi_t z_t \tag{B-28}$$

Using the definition for v_t, equation $(B-26)$ can be expressed as:

$$v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t\left(2(1-\sigma)y_t + mc_t\right) + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\|\xi_t\|^3\right) \tag{B-29}$$

which is equation $(B-iv)$ in the previous section.

Moreover, the linear part of equation (B-29) is:

$$\pi_t = \kappa mc_t + \beta E_t(\pi_{t+1}) + O\left(\|\xi_t\|^2\right)$$

which is the standard New Keynesian Phillips curve, inflation depends linearly on the real marginal costs and expected inflation.

Replace the equation for the marginal costs (B-14) in the second order expansion of the Phillips curve (B-29)

$$\begin{aligned} v_t &= \kappa_y y_t + \kappa_q q_t + \kappa\chi v\widehat{\Delta}_t + \frac{1}{2}\varepsilon\pi_t^2 + \\ &\quad + \frac{1}{2}\kappa\left[c_{yy}y_t^2 + 2c_{yq}y_t q_t + c_{qq}q_t^2\right] + \beta E_t v_{t+1} + O\left(\|\xi_t\|^3\right) \end{aligned} \tag{B-30}$$

where the coefficients of the linear part are given by

$$\begin{aligned} \kappa_y &= \kappa\chi(\sigma+v) \\ \kappa_q &= \kappa(1-\chi) \end{aligned}$$

and those of the quadratic part are:

$$\begin{aligned} c_{yy} &= \chi(\sigma+v)\left[2(1-\sigma)+\chi(\sigma+v)\right] + (1-\psi)\frac{\chi^2(1-\chi)(\sigma+v)^2}{1-\overline{\alpha}} \\ c_{yq} &= (1-\chi)\left[2(1-\sigma)+\chi(\sigma+v)\right] - (1-\psi)\frac{\chi^2(1-\chi)(\sigma+v)}{1-\overline{\alpha}} \\ c_{qq} &= (1-\chi)^2 + (1-\psi)\frac{\chi^2(1-\chi)}{1-\overline{\alpha}} \end{aligned}$$

Equation B-30 is a recursive second order representation of the Phillips curve. However, we need to express the price dispersion in terms of inflation in order to have a Phillips curve only as a function of output, inflation and the oil shock. Equation B-30 can also be expressed as the discounted infinite sum:

$$v_{t_o} = \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left\{ \kappa_y y_t + \kappa_q q_t + \kappa \chi v \widehat{\Delta}_t + \frac{1}{2} \varepsilon \pi_t^2 + \frac{1}{2} \kappa \left[c_{yy} y_t^2 + 2 c_{yq} y_t q_t + c_{qq} q_t^2 \right] \right\} + \left(\|\xi_t\|^3 \right)$$

after making use of equation B-18, the discounted infinite sum of $\widehat{\Delta}_t$, v_{t_o} becomes

$$v_{t_o} = \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left\{ \kappa_y y_t + \kappa_q q_t + \frac{1}{2} \varepsilon \left(1 + \chi v\right) \pi_t^2 + \frac{1}{2} \kappa \left[c_{yy} y_t^2 + 2 c_{yq} y_t q_t + c_{qq} q_t^2 \right] \right\} + \frac{\chi v \theta}{1 - \beta \theta} \widehat{\Delta}_{t_o - 1} + \left(\|\xi_t\|^3 \right) \tag{B-31}$$

This is the Phillips curve expressed as a infinite sum of output, inflation and oil shock.

B.3 A second-order approximation to utility

The expected discounted value of the utility of the representative household

$$U_{t_o} = E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left[u\left(C_t\right) - v\left(L_t\right) \right] \tag{B-32}$$

The first term can be approximated as:

$$u\left(C_t\right) = \overline{C} \overline{u}_c \left\{ c_t + \frac{1}{2} \left(1 - \sigma\right) c_t^2 \right\} + t.i.p. + O\left(\|\xi_t\|^3 \right) \tag{B-33}$$

Similarly, the second term:

$$v\left(L_t\right) = \overline{L} \overline{v}_L \left\{ l_t + \frac{1}{2} \left(1 + v\right) l_t^2 \right\} + t.i.p. + O\left(\|\xi_t\|^3 \right) \tag{B-34}$$

Replace the equation for labour in equilibrium in B-34:

$$v\left(L_t\right) = \overline{L} \overline{v}_L \left\{ v_y y_t + \frac{1}{2} v_{yy} y_t^2 + v_{yq} y_t q_t + v_\Delta \widehat{\Delta}_t \right\} + t.i.p. + O\left(\|\xi_t\|^3 \right) \tag{B-35}$$

where:

$$\begin{aligned}
v_y &= 1 - \delta\left(v + \sigma\right) \\
v_{yy} &= \left(1 + v\right)\left(1 - \delta\left(v + \sigma\right)\right)^2 + \frac{1}{2} \frac{1 - \psi}{1 - \overline{\alpha}} \chi^2 \delta \left(\sigma + v\right)^2 \\
v_{yq} &= \left(1 + v\right) \delta \left(1 - \delta\left(v + \sigma\right)\right) - \frac{1}{2} \frac{1 - \psi}{1 - \overline{\alpha}} \chi^2 \delta^2 \left(\sigma + v\right) \\
v_\Delta &= \frac{\chi}{1 - \overline{\alpha}}
\end{aligned}$$

We make use of the following relation:

$$\overline{L} \overline{v}_L = \left(1 - \Phi\right)\left(1 - \overline{\alpha}\right) \overline{Y} \overline{u}_c \tag{B-36}$$

where $\Phi = 1 - \frac{1-\tau}{\varepsilon/(\varepsilon-1)}$ is the steady state distortion from monopolistic competition. Replace the previous relation, equation B-33 and B-35 in B-32, and make use of the clearing market condition: $C_t = Y_t$

$$U_{t_o} = \overline{Y}\overline{u}_c \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left(u_y y_t + \frac{1}{2} u_{yy} y_t^2 + u_{yq} y_t q_t + u_\Delta \widehat{\Delta}_t \right) + t.i.p. + O\left(\|\xi_t\|^3 \right) \qquad \text{(B-37)}$$

where

$$\begin{aligned} u_y &= 1 - (1-\Phi)(1-\overline{\alpha}) v_y = \Phi_L \\ u_{yy} &= 1 - \sigma - (1-\Phi)(1-\overline{\alpha}) v_{yy} = 1 - \sigma - (1-\Phi_L) v_{yy} / (1 - \delta(v+\sigma)) \\ u_{yq} &= -(1-\Phi)(1-\overline{\alpha}) v_{yq} = -(1-\Phi_L) v_{yq} / (1 - \delta(v+\sigma)) \\ u_\Delta &= -(1-\Phi)(1-\overline{\alpha}) v_\Delta = -(1-\Phi)\chi \end{aligned}$$

where we make use of the following change of variable:

$$\Phi_L = 1 - (1-\Phi)(1-\overline{\alpha})(1-\delta(v+\sigma)) \qquad \text{(B-38)}$$

where Φ_L is a wedge between consumption and labor in the utility function in steady state.

Replace the present discounted value of the price distortion (B-18) in B-37:

$$U_{t_o} = \overline{Y}\overline{u}_c E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left(u_y y_t + \frac{1}{2} u_{yy} y_t^2 + u_{yq} y_t q_t + \frac{1}{2} u_\pi \pi_t^2 \right) + t.i.p. + O\left(\|q_t\|^3 \right) \qquad \text{(B-39)}$$

where

$$u_\pi = \frac{\varepsilon}{\kappa} u_\Delta = -(1-\Phi)\chi \frac{\varepsilon}{\kappa}$$

Use equation B-31, the second order approximation of the Phillips curve, to solve for the expected level of output:

$$\begin{aligned} \sum_{t=t_o}^{\infty} \beta^{t-t_o} y_t &= -\frac{1}{\kappa_y} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left\{ \kappa_q q_t + \frac{1}{2}\varepsilon(1+\chi v)\pi_t^2 + \frac{1}{2}\kappa\left[c_{yy} y_t^2 + 2c_{yq} y_t q_t + c_{qq} q_t^2\right] \right\} \\ &\quad + \frac{1}{\kappa_y}\left(v_{t_o} - \chi v (1-\theta) \widehat{\Delta}_{t_o-1} \right) + \left(\|\xi_t\|^3 \right) \end{aligned} \qquad \text{(B-40)}$$

Replace equation B-40 in B-39 to express it as function of only second order terms:

$$U_{t_o} = -\Omega E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left(\frac{1}{2}\lambda_y (y_t - y_t^*)^2 + \frac{1}{2}\lambda_\pi \pi_t^2 \right) + T_{t_o} + t.i.p. + O\left(\|q_t\|^3 \right) \qquad \text{(B-41)}$$

which is equation B-36 in the text, where:

$$\begin{aligned} \lambda_y &= \Phi_L \frac{\kappa}{\kappa_y} c_{yy} - u_{yy} \\ \lambda_\pi &= \Phi_L \frac{\varepsilon(1+\chi v)}{\kappa_y} - u_\pi \\ y_t^* &= -\frac{\Phi_L \frac{\kappa}{\kappa_y} c_{yq} - u_{yq}}{\Phi_L \frac{\kappa}{\kappa_y} c_{yy} - u_{yy}} q_t \end{aligned}$$

additionally we have that $\Omega = \overline{Y}\overline{u}_c$ and $T_{t_o} = \overline{Y}\overline{u}_c \frac{\Phi_L}{\kappa_y} v_{t_o}$

Make use of the following auxiliary variables:

$$\begin{aligned}
\omega_1 &= (1-\sigma)\,\Phi_L + \chi\,(\sigma+v) \\
\omega_2 &= \chi\,(\sigma+v)\left[\frac{1-\chi}{1-\overline{\alpha}} + (1-\Phi_L)\,\frac{\sigma\psi\overline{\alpha}}{1-\sigma\psi\overline{\alpha}}\right] \\
\omega_3 &= \Phi_L \sigma\overline{\alpha}
\end{aligned}$$

then, λ_y, λ_π and y_t^* can be written as a function of ω_1, ω_2 and ω_3

$$\begin{aligned}
\lambda_y &= \omega_1 + (1-\psi)\,\omega_2 \\
\lambda_\pi &= \frac{\varepsilon}{\kappa_y\,(1-\sigma\psi\overline{\alpha})}\left[\omega_1 + (1-\psi)\,\omega_3\right] \\
y_t^* &= -\frac{1-\chi}{\chi\,(\sigma+v)}\left[\frac{\omega_1 - (1-\psi)\,\frac{\chi}{1-\chi}\omega_2}{\omega_1 + (1-\psi)\,\omega_2}\right] q_t
\end{aligned}$$

using the definitions for χ, y_t^* can be expressed as:

$$y_t^* = -\left(\frac{1+\psi v}{\sigma+v}\right)\left(\frac{\overline{\alpha}}{1-\overline{\alpha}+\eta}\right) \tag{B-42}$$

where

$$\eta \equiv \frac{(1-\psi)\,(1-\overline{\alpha})\,\omega_2}{(1-\chi)\,\omega_1 - (1-\psi)\,\chi\omega_2}$$

Denote α^*, the efficient share in steady state of oil in the marginal costs, where

$$\alpha^* = \frac{\overline{\alpha}}{1+\eta}$$

then y_t^* is

$$y_t^* = -\left(\frac{1+\psi v}{\sigma+v}\right)\left(\frac{\alpha^*}{1-\alpha^*}\right) q_t \tag{B-43}$$

Note from the definition for η that when $\psi = 1$, then $\eta = 0$, $\alpha^* = \overline{\alpha} = \alpha$ and $y_t^* = y_t^n$. For a Cobb-Douglas production function the efficient level of output equals the natural level. Also, when $\psi < 1$, then $\eta > 0$, $\alpha^* < \overline{\alpha}$ and $|y_t^*| < |y_t^n|$. For the elasticity of substitution between inputs lower than one, the efficient level fluctuates less to oil shocks than the natural level. Also note that even when Φ_L is equal to zero, which summarises the effect of monopolistic distortions on the wedge between the marginal rate of substitution and the marginal product of labour, η is still different than zero for $\psi \neq 1$. This indicates that the efficient level of output still diverges from the natural level even if we eliminate the effects of monopolistic distortions.

In the same way, the natural rate of output can be expressed as:

$$y_t^n = -\left(\frac{1+\psi v}{\sigma+v}\right)\left(\frac{\overline{\alpha}}{1-\overline{\alpha}}\right) q_t \tag{B-44}$$

Similarly, we can simplify $\lambda = \lambda_y / \lambda_\pi$ as:

$$\lambda = \frac{\lambda_y}{\lambda_\pi} = \frac{\kappa_y \left(1 - \sigma\psi\overline{\alpha}\right)}{\varepsilon}\gamma$$

where we use the auxiliary variable:

$$\gamma \equiv \left[\frac{\omega_1 + (1-\psi)\,\omega_2}{\omega_1 + (1-\psi)\,\omega_3}\right]$$

Note that when $\psi = 1$, then $\gamma = 1$ and when $\psi < 1$, then $\gamma = 1$ since $\omega_2 > \omega_3$.

C Appendix: Optimal monetary policy

C.1 Optimal response to oil shocks

The policy problem consists in choosing x_t and π_t to maximise the following Lagrangian:

$$\mathcal{L} = -E_{t_o}\left\{\sum_{t=t_o}^{\infty}\beta^{t-t_o}\left[\frac{1}{2}\lambda x_t^2 + \frac{1}{2}\pi_t^2 - \varphi_t\left(\pi_t - \kappa_y\widehat{y}_t - \beta E_t\pi_{t+1} - u_t\right)\right] + \varphi_{t_o-1}\left(\pi_{t_o} - \pi_{t_o}^*\right)\right\}$$

where $\beta^{t-t_o}\varphi_t$ is the Lagrange multiplier associated with the constraint at time t

The first order conditions with respect to π_t and y_t are respectively

$$\pi_t = \varphi_{t-1} - \varphi_t \tag{C-1}$$

$$\lambda x_t = \kappa_y\varphi_t \tag{C-2}$$

and for the initial condition:

$$\pi_{t_o} = \pi_{t_o}^*$$

where $\pi_{t_o}^*$ is the initial value of inflation which is consistent with the policy problem in a "timeless perspective".

Replace conditions C-1 and C-2 in the Phillips Curve:

$$\beta E_t\varphi_{t+1} - \left[(1+\beta)\,\lambda + \kappa_y^2\right]\varphi_t + \lambda\varphi_{t-1} = \lambda u_t \tag{C-3}$$

this difference equation has the following solution[9] :

$$\varphi_t = \tau_\varphi\varphi_{t-1} - \tau_\varphi\sum\nolimits_{j=0}^{\infty}\beta^j\tau_\varphi^j E_t u_{t+j} \tag{C-4}$$

where τ_φ is the characteristic root, lower than one, of C-3, and it is equal to

$$\tau_\varphi = Z - \sqrt{Z^2 - \frac{1}{\beta}}$$

[9] See Woodford (2003), pp. 488-490 for details on the derivation.

for $Z = \left((1+\beta) + \frac{\kappa_y^2}{\lambda}\right)/(2\beta)$. Since the oil price follows an AR(1) process of the form:

$$q_t = \rho q_{t-1} + \xi_t$$

and the mark-up shock is: $u_t = \varpi q_t$, then u_t follows the following process:

$$u_t = \rho u_{t-1} + \varpi \xi_t \tag{C-5}$$

Solution to the optimal problem: Taking into account C-5, equation C-4 can be expressed as:

$$\varphi_t = \tau_\varphi \varphi_{t-1} - \phi q_t \tag{C-6}$$

where:

$$\phi = \frac{\tau_\varphi}{1 - \beta \tau_\varphi \rho} \varpi$$

Initial condition: Iterate backward equation (C-6) and evaluate it at $t_o - 1$, this is the timeless solution to the initial condition φ_{t_o-1} :

$$\varphi_{t_o-1} = -\phi \Sigma_{k=0}^{\infty} (\tau_\varphi)^k q_{t_o-1-k} \tag{C-7}$$

which is a weighted sum of all the past realisations of oil prices.

Equations (C-1), (C-2), (C-6) and (C-7) are the conditions for the optimal unconstrained plan presented in proposition 3.5. **Impulse responses** An innovation of ξ_t to the real oil price affects the current level and the expected future path of the Lagrange multiplier by an amount:

$$E_t \varphi_{t+j} - E_{t-1} \varphi_{t+j} = -\frac{\rho^{j+1} - (\tau_\varphi)^{j+1}}{\rho - \tau_\varphi} \phi \xi_t$$

for each $j \geq 0$. Given this impulse response for the multiplier. (C-1) and (C-2) can be used to derive the corresponding impulse responses for inflation and output gap:

$$\begin{aligned} E_t \pi_{t+j} - E_{t-1} \pi_{t+j} &= \left[\frac{\rho^{j+1} - (\tau_\varphi)^{j+1}}{\rho - \tau_\varphi} - \frac{\rho^j - (\tau_\varphi)^j}{\rho - \tau_\varphi} \right] \phi \xi_t \\ E_t y_{t+j} - E_{t-1} y_{t+j} &= -\frac{\kappa_y}{\lambda} \frac{\rho^{j+1} - (\tau_\varphi)^{j+1}}{\rho - \tau_\varphi} \phi \xi_t \end{aligned}$$

which are expressions that appear in the main text.

C.2 The optimal non-inertial plan

We want to find a solution for the paths of inflation and output gap such that the behaviour of endogenous variables is function only on the current state. That is:

$$\pi_t = \overline{\pi} + f_\pi u_t \tag{C-8}$$

$$x_t = \overline{x} + f_x u_t \tag{C-9}$$

$$\varphi_t = \overline{\varphi} + f_\varphi u_t \tag{C-10}$$

where the coefficients $\overline{\pi}, \overline{y}, \overline{\varphi}, f_\pi, f_x$ and f_φ are to be determined

Replace (C-8), (C-9) and (C-10) in the Lagrangian and take unconditional expected value:

$$
\begin{aligned}
-E\left(L_{t_o}\right) &\equiv E\left\{E_{t_o}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\left[\begin{array}{c}\frac{1}{2}\lambda\left(\overline{x}+f_x u_t\right)^2+\frac{1}{2}\left(\overline{\pi}+f_\pi u_t\right)^2 \\ -\left(\overline{\varphi}+f_\varphi u_t\right)\left(\begin{array}{c}(1-\beta)\,\overline{\pi}-\kappa_y\overline{x} \\ +(1-\beta\rho)\,f_\pi u_t-u_t-\kappa_y f_x u_t\end{array}\right)\end{array}\right]\right\} \\
&\quad +E\left(\left(\overline{\varphi}+f_\varphi u_{t_o-1}\right)\left[\overline{\pi}+f_\pi u_{t_o}\right]\right) \qquad \text{(C-11)}
\end{aligned}
$$

suppressing the terms that are independent of policy and using the law of motion for u_t, this can be simplified as:

$$
\begin{aligned}
-E\left(L_{t_o}\right) &\equiv \frac{1}{2\,(1-\beta)}\left(\lambda\overline{x}^2+\overline{\pi}^2\right)-\frac{1}{2\,(1-\beta)}\overline{\varphi}\left((1-\beta)\,\overline{\pi}-\kappa_y\overline{x}\right) \\
&\quad +\frac{1}{2}\frac{\sigma_u^2}{1-\beta}\left(\lambda {f_x}^2+{f_\pi}^2\right)-\frac{1}{2}\frac{\sigma_u^2}{1-\beta}f_\varphi\left((1-\beta\rho)\,f_\pi-1-\kappa_y f_x\right) \\
&\quad +\rho\sigma_u^2 f_\varphi f_\pi
\end{aligned}
$$

the problem is then to find $\overline{\pi}, \overline{y}, \overline{\varphi}, f_\pi, f_x$ and f_φ such, that they maximise the previous expression. These coefficients are:

$$
\begin{aligned}
\overline{\pi} &= \overline{x}=\overline{\varphi}=0 \\
f_\pi &= \frac{\lambda(1-\rho)}{\lambda\,(1-\beta\rho)\,(1-\rho)+\kappa_y^2} \\
f_x &= -\frac{\kappa_y}{\lambda\,(1-\beta\rho)\,(1-\rho)+\kappa_y^2} \\
f_\varphi &= \frac{\lambda}{\lambda\,(1-\beta\rho)\,(1-\rho)+\kappa_y^2}
\end{aligned}
$$

which is the solution to the optimal non-inertial plan given in proposition 3.6.