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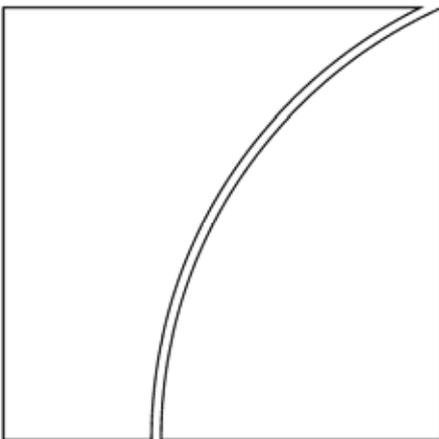
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# Capital Regulation and Banks' Financial Decisions\*

Haibin Zhu<sup>†</sup>

## Abstract

This paper develops a stochastic dynamic model to examine the impact of capital regulation on banks' financial decisions. In equilibrium, lending decisions, capital buffer and the probability of bank failure are endogenously determined. Compared to a flat-rate capital rule, a risk-sensitive capital standard causes the capital requirement to be much higher for small (and riskier) banks and much lower for large (and less risky) banks. Nevertheless, changes in actual capital holdings are less pronounced due to the offsetting effect of capital buffers. Moreover, the non-binding capital constraint in equilibrium implies that banks adopt an active portfolio strategy and hence the counter-cyclical movement of risk-based capital requirements does not necessarily lead to a reinforcement of the credit cycle. In fact, the results from the calibrated model show that the impact on cyclical lending behavior differs substantially across banks. Lastly, the analysis suggests that the adoption of a more risk-sensitive capital regime can be welfare-improving from a regulator's perspective, in that it causes less distortion in loan decisions and achieves a better balance between safety and efficiency.

**JEL Classification Numbers:** G21, G28

**Keywords:** Capital Requirement; Economic Capital; Regulatory Capital; Actual Capital Holding; Procyclicality Effect; Dynamic Programming; Prudential Regulation

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# 1 Introduction

Banking is one of the most regulated industries in the world. Among various regulatory measures, the regulation of bank capital is crucial due to the important role it plays in banks' soundness and risk-taking behavior, and its influence on the competitiveness of banks.<sup>1</sup> In practice, a key aspect of capital regulation is the calculation of minimum regulatory capital, which is typically based on the credit risk of bank assets. Traditionally, the risk matrix was very simple in the sense that the risk weights were practically the same for bank loans of different risk. More recently, more risk-sensitive capital standards have been introduced, mainly because the low-sensitivity had arguably led to severe market distortions as banks swapped low-risk assets against high-risk ones with more favorable risk weighting (regulatory arbitrage).

The adoption of more risk-sensitive capital standards has generated the discussion on its implications for individual banks (a micro perspective) and for the stability of the banking system (a macro view). Two major concerns arise from these studies. First, it tends to cause substantial changes in the level of bank capital. The study of Edwin and Wilde (2001), which is based on a hypothetical bank portfolio in the U.S. in 1990, suggests that the minimum capital requirement could drop from 8% to 6.8% with the introduction of the risk-based capital rule. Similarly, a study conducted by U.S. federal regulators (OCC, Federal Reserve Board, FDIC and OTS, 2004) shows that the new capital rule will cause the minimum required risk-based capital (MRC), the sum of expected and unexpected losses, to drop on average by 12.5%. Across entities, the changes in MRC vary substantially within a range of  $[-50\%, 70\%]$ , with a median reduction of 24%. These results raise concerns that the introduction of the new capital standard may cause abrupt and undesirable changes to the banking industry. Second, the greater risk-sensitivity in the new capital regime may cause additional volatility in economic activity, sometimes cited as the procyclicality problem.<sup>2</sup> In particular, capital requirements can increase as the economy falls into recession and fall as the economy enters an expansion. The increase in capital requirements during the downswing may result in a credit crunch and thus worsen already adverse economic conditions. The empirical relevance of this concern was later confirmed in several countries including the United States (Catarineu-Rabell et al., 2005; Jordan et al., 2003; Gordy and Howells, 2006; Kashyap and Stein, 2004), Spain (Corcostegui et al., 2002) and Mexico (Segoviano and Lowe, 2002).

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<sup>1</sup>See Berger et al. (1995) and Santos (2001) for surveys on the motivations behind capital regulation.

<sup>2</sup>See Borio et al. (2001); Jokivuolle and Kauko (2001) for related discussion and Allen and Saunders (2004) for a literature survey.

However, there are several major caveats in these studies, which suggest that the above conclusions are debatable. First, these studies focus exclusively on the change in regulatory capital. Nevertheless, in order to understand the impact of regulatory measures, it is more important to examine the change in banks' actual capital holdings.<sup>3</sup> A well-known fact is that most banks tend to hold a significant amount of capital above the regulatory requirement in practice, either for efficiency reasons or because the capital cushion is established as a precaution against contingencies (adverse events or regulatory penalties, see Barrios and Blanco, 2003; Elizalde and Repullo, 2006). Taking into account the role of capital buffers may change the above conclusion. Indeed, some researchers (Estrella, 2004b; Heid, 2005; Koopman et al., 2005; Peura and Jokivuolle, 2004) suggest that the existence of capital buffers can potentially mitigate the volatility in total capital. By contrast, the empirical evidence in Germany (Stolz and Wedow, 2005) and Spain (Ayuso et al., 2004) shows that capital buffers are also anti-cyclical, therefore it is not clear whether actual capital holdings are more volatile than regulatory capital, or the vice versa.<sup>4</sup>

Second, these studies adopt a passive portfolio assumption, e.g. banks hold the same portfolio under the two capital regimes or hold portfolios as specified by the minimum capital requirements. This is questionable, given that an important objective of the revision of capital rules is to avoid excessive risk-taking that may arise under the old (less risk-sensitive) regime. Hence, it is more interesting to investigate how banks adjust their risk-taking behavior in their loan extension process. In other words, an active portfolio strategy is more appropriate for the purpose of this study.

Moreover, some of these impact studies are based on the industry information at a particular point at time. Therefore, the results fail to recognize the dynamic aspect of regulatory implications. The dynamic perspective is crucial in the sense that the underlying credit risk of bank assets can change substantially over the business cycle. As a result, the impacts on banks' capital holdings (both regulatory and actual capital) and portfolio composition tend to be different at different phases of the business cycle.

This paper seeks to fill these gaps and provide new insights on these important issues. I propose a stochastic dynamic model in which a bank's objective is to maximize shareholder value by making loans. A larger loan book yields lower return, but is associated with lower

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<sup>3</sup>This paper uses three distinct definitions of bank capital. The *economic capital* refers to banks' capital holdings in the absence of capital regulation, the *regulatory capital* refers to the minimum capital requirements based on specific capital rules, and the *actual capital* (or total capital) refers to banks' equilibrium capital holdings in an economy with capital regulation. Accordingly, capital buffer is defined as the difference between actual and regulatory capital.

<sup>4</sup>Cross-country studies, such as Bikker and Metzmakers (2004) and Kim and Lee (2006), find mixed results on the impact of capital regulation on the dynamics of actual capital holdings.

risk due to benefits from portfolio diversification. Each bank has to choose its optimal capital structure, facing the tradeoff between one-period investment gains and future franchise value. In equilibrium, the bank's leverage ratio, lending strategy and failure probability are jointly determined. An important contribution of this model is that it allows for heterogeneity in the banking industry. Interestingly, the comparative static analysis shows that the adoption of the a more risk-sensitive capital regime has substantially different implications on different banks depending on their size and risk profiles. I consider three economic environments: the baseline economy in which capital regulation is absent, a flat-rate capital regime in which a uniform capital-asset ratio requirement is imposed, and a risk-based capital regime in which the minimum capital requirement is determined by the credit risk of bank assets. The model is calibrated to match aspects of the U.S. economy in 1980-2002 and the equilibrium outcomes under the three economic environments are compared with each other.

The numerical results show that banks' financial decisions depend crucially on the state of the business cycle, banks' initial equity holdings and the form of capital standards. Comparing equilibrium outcomes under different capital regimes sheds new insights on the issues of interest. On the one hand, the results support the view that the adoption of a risk-sensitive capital rule causes significant changes in minimum capital requirements, and the impact is substantially different across banks depending on the risk profile of their loan portfolios. However, they also suggest that changes in actual capital holdings can be much smaller than changes in minimum capital requirements due to the offsetting effect of capital buffers. On the other hand, the results find little support to the idea that capital regulation amplifies procyclicality in bank lending. The volatility of bank credit (and economic output) over the business cycle is smaller under both capital regimes than the volatility under the baseline economy (without capital regulation). Even when comparing between the two capital regimes, higher volatility of capital requirements under the risk-based capital rule does not necessarily lead to amplified swings in banks' lending activity, because the non-binding capital constraint implies that the loan portfolios chosen by banks in equilibrium are often different from those implied by regulatory capital constraints. In fact, for smaller banks the swing in bank lending is less pronounced under the risk-based capital rule than under the flat-rate capital regime.

The model also provides a framework to compare the optimality of the two capital regimes from a regulator's perspective, assuming that the regulator has to choose between maximizing franchise value and minimizing the negative externality caused by bank failures. Illustrative examples suggest that the risk-based capital rule can outperform the flat-rate capital requirement, in that expected franchise value is higher for the same level of bank safety. The results

hold both at the micro level (comparison based on individual banks) and at the macro level (comparison based on the banking system as a whole). The intuition behind this observation is: under the flat-rate capital requirement, the failure to reward sound investments causes a big distortion in limiting the scale of lending for those banks that have access to high-return, low-risk investment opportunities. From a dynamic perspective, such distortion implies that banks have to make a big sacrifice of future franchise value in order to reduce the default risk in the current period. By contrast, the future franchise value at risk is much smaller under the risk-based capital regulation because banks can invest on high-return, low risk projects without incurring high cost of holding capital.

The remainder of the paper is organized as follows. Section 2 outlines the model setup and section 3 discusses the qualitative results. The model is then calibrated to the U.S. economy, allowing us to examine the equilibrium financial decisions under different capital regimes. Section 5 compares between the two capital regimes from a regulator’s perspective. The final section concludes.

## 2 The model

The benchmark model follows a modified version of the model developed by Cooley and Quadrini (2001) for a generic firm. I treat a bank as a special type of firm, whose objective is to maximize the expected discounted value of dividends,  $E_0\{\sum_{t=0}^{\infty} \beta^t d_t\}$ , where  $d_t$  is the dividend payment at time  $t$ ,  $\beta$  the discount factor of bank shareholders, and  $E$  the expectation operator. Accordingly, the assumptions on the asset and liability of the *firm*’s balance sheet will be modified to reflect distinctive features of the banking sector. The modified framework bears considerable similarity to the one developed by van den Heuvel (2002).

### 2.1 Loans

In each period, the bank tries to maximize shareholder value by making loans. To maintain analytical tractability, this model abstracts from the determination of lending rates by working directly with a return function of  $Y_t = (z_t + \epsilon_t + \mu_t)F(L_t)$ , where  $Y_t$  is the revenue from making loans (adjusted by cost of defaults),  $L_t$  is the size of the loan portfolio, and  $F(\cdot)$  satisfies  $F(0) = 0$ ,  $F' > 0$  and  $F'' < 0$ .<sup>5</sup> The concavity of  $F(\cdot)$  function implies decreasing returns to scale for bank investments, which is standard in the literature and has been sup-

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<sup>5</sup>This partial equilibrium assumption enables me to focus exclusively on banks’ investment decisions. As a tradeoff, the financing strategies of borrowers and the impact of banks’ lending decisions on the real economy, which would be important in a general equilibrium setting, are ignored in this paper.

ported by empirical findings in Berger et al. (2005), Carter and McNulty (2005) and Cole et al. (2004), among others.

The other three variables,  $z_t$ ,  $\epsilon_t$  and  $\mu_t$ , jointly determine the profitability of bank lending. In particular,  $z_t$  represents the state of the economy that determines the level of investment yields and follows a first-order Markov process (Assumption 1);  $\epsilon_t$  is a bank-specific shock that determines the risk of bank assets (Assumption 2); and  $\mu_t$  measures the benefit from monitoring as it is explained below (Assumption 3).

**Assumption 1** *The state of the economy,  $z$ , takes values in a finite set  $Z = \{z_1, z_2, \dots, z_N\}$  and follows a first-order Markov process with transition probabilities  $P(z_{t+1}|z_t)$ .*

**Assumption 2** *The bank-specific shock  $\epsilon$  is independently and identically distributed and has a normal density function  $N(0, \sigma_\epsilon^2(L))$ . The volatility is inversely related to the size of bank assets, i.e.  $\frac{d\sigma_\epsilon}{dL} < 0$ .*

At the beginning of each period, each bank observes  $z_t$  but not  $\epsilon_t$ . In other words, each bank knows the level of expected investment yields in the current period, and is aware that the underlying risk depends on its lending strategy. The above two assumptions suggest that, when a bank chooses the size of its loan portfolio, it faces the tradeoff between return and risk. The lower risk of larger portfolios, as assumed in this model, can be justified on the ground of greater scope for diversification as proposed in finance theory. For instance, a report by the Group of Ten (2001) suggests that consolidation of banks within the United States has led to reductions in risk due to geographic diversification.

Finally, there exists another bank-specific term,  $\mu$ , which represents the contribution of bank-specific efforts in improving its investment returns. Such effort can vary across banks due to differences in managerial skills, market power and monitoring incentives. By paying a monitoring cost of  $\theta(\mu)$  per unit of loan, the bank is able to improve the average investment return by  $\mu$ .<sup>6</sup> This unit cost increases in  $\mu$  at a growing rate, implying that the room for return enhancement tends to become smaller, i.e. a diminishing benefit from monitoring.

**Assumption 3** *The function  $\theta(\mu)$  satisfies  $\theta(0) = 0$ ,  $\theta'(\cdot) > 0$  and  $\theta''(\cdot) > 0$ .*

The model abstracts from the determination of lending rates and assumes instead a continuous distribution of net revenues.<sup>7</sup> The continuity assumption can be justified as follows. Suppose that a bank can make many individual loans, with a fixed lending rate

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<sup>6</sup>In the literature, the monitoring effort could have two effects. One is to induce entrepreneurs to choose high-productivity projects, and the other to reduce the business risk. Since the revenue function refers to default-adjusted returns, both effects tend to increase the expected revenue.

<sup>7</sup>The same assumption is adopted by Acharya (2003).

for each loan. Since each loan can go bad with positive probability, the effective average return of the whole portfolio tends to be distributed over a large set of real numbers, with the maximum being the highest possible return if no default occurs and the minimum the worst scenario if all loans default.

## 2.2 Deposits and equity

A bank begins each period with equity capital equal to  $e_t$ . The bank finances its investment by equity capital and bank deposits, the latter being a debt instrument with a fixed return of  $r_t^d$ .

Bank deposits are not risk-free, in the sense that the bank may default if its loan investment does not generate enough profits. Therefore a risk premium is charged to compensate for the default risk. This paper abstracts from the supply of bank deposits from the household sector by assuming that the elasticity of the deposit supply is infinite and:

**Assumption 4** *Depositors are risk-neutral and their reservation return is the risk-free rate,  $r_t^f$ .*

The assumption of perfect access to non-reservable deposits is also adopted in van den Heuvel (2002).<sup>8</sup> The advantage of this assumption is that it allows me to focus on the bank capital channel through which the change in capital regimes affects banks' lending behavior. On the other hand, it implies that the bank lending channel does not exist in this model, since a necessary condition for the existence of an effective bank lending channel – shocks to reservable deposits should affect the bank's loan supply (Bernanke and Blinder, 1988; Kashyap and Stein, 1994) – is absent from this model.

However, this paper departs from the assumption in van den Heuvel (2002) that bank deposits are fully insured (with zero premium) and by extension the deposit rate equals the risk-free rate. Instead, the bank will need to pay a risk premium in line with its likelihood of default. Assuming that depositors are risk-neutral and the expected payoff on bank deposits is  $E(r)$ , the supply function is:

$$D_t^s = \begin{cases} 0 & \text{if } E(r) < r_t^f \\ [0, \infty] & \text{if } E(r) = r_t^f \\ \infty & \text{if } E(r) > r_t^f \end{cases} \quad (1)$$

At the end of each period, banks receive revenues on their asset portfolios and pay back deposits. A solvent bank chooses its dividend payment ( $d_t$ ) or equivalently, its initial equity

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<sup>8</sup>By assuming a perfect access to deposits, this paper also abstracts from the vulnerability issue related to the bank run phenomenon, as examined in Diamond and Dybvig (1983).

holding ( $e_{t+1}$ ) for the next period. Importantly, the dividend is allowed to be negative in this model, i.e., the bank is able to issue new equity if necessary. Nevertheless, issuing new equity is costly. This is consistent with the statement made by practitioners, and can be justified with respect to transaction costs and differential taxation levels for equity. Specifically, the cost of raising new equity is assumed to be linear following Cooley and Quadrini (2001).<sup>9</sup>

**Assumption 5** *The bank pays an issuance cost of  $\Lambda(-d_t) = -\lambda d_t$  to raise new equity in the amount of  $-d_t$  ( $d_t < 0$ ).*

Finally, this model makes the following assumption on the exit of a bank.

**Assumption 6** *A bank defaults if its net asset value drops below zero. A bank cannot re-enter the market once it exits, i.e. its franchise value is zero thereafter.*

Here the bank's net asset value is defined as the end-of-period total revenue minus the bank's debt liability, i.e. the principal and interest payments on bank deposits. If the bank's net asset value drops below zero, its equity capital is negative and it claims bankruptcy. In other words, the renegotiation of bank debt is not allowed in this paper and a default automatically leads to permanent exit.

## 2.3 Capital regulation

Regulation is introduced in the form of a minimum capital requirement imposed by the regulatory authority. Each bank has to meet this requirement at all times. The minimum level of bank capital typically depends on the size of the bank's loan book, and under a risk-based capital regime, related to the risk profile of bank assets.

**Assumption 7** *The bank has to meet the minimum capital requirement,  $e_{min}$ , which depends on the characteristics of bank assets.*

Notice that the capital regulation, combined with the equity issuance cost assumption, causes deviations from the Modigliani-Miller framework. Without these assumptions, whether bank loans are financed with deposits or equity is irrelevant. As explained later, the probability of default and the subsequent losses increase banks' willingness to hold more capital than required.

Owing to the simplified maturity structure of bank assets and liabilities in this paper, the minimum capital requirement is effectively operative only at the moment that the bank

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<sup>9</sup>The cost of raising new equity is assumed to be constant over time. In practice, it is often more expensive for banks to raise fresh capital in economic downturns. Numerical studies show that introducing time-varying cost of equity issuance does not change the results qualitatively.

decides on the size of its loan portfolio.<sup>10</sup> Throughout the paper, I assume that the minimum capital requirement cannot be violated, or equivalently, the penalty cost is infinite and the bank's franchise value goes to zero if it fails to meet the minimum capital requirement.<sup>11</sup>

This paper examines two alternative forms of capital requirements. The first form imposes a flat capital-asset ratio requirement ( $\kappa$ ) of 8% on all banks. That is, the level of regulatory capital depends on the size of bank assets but not their riskiness. By contrast, under the alternative form, the minimum regulatory capital is calculated on the basis of the risk profile of bank assets. In particular, it is determined in a way that the probability of default for the bank does not exceed a threshold level (0.1%). That is, the minimum level of regulatory capital ( $e_{min}$ ) is determined by:<sup>12</sup>

$$(z + \epsilon_{0.001} + \mu)F(L) - \theta(\mu)L = (L - e_{min})r^d$$

where  $\epsilon_{0.001}$  is the 0.1% critical value of the normally distributed variable  $\epsilon$  with mean zero and standard deviation of  $\sigma_\epsilon$ .<sup>13</sup> The above equation implies that, if a bank holds the required level of capital, there is 0.1% probability that the bank's revenue will be insufficient to finance the bank's deposit liability, i.e. the bank will become insolvent. Because the default risk is very small, the deposit rate should be very close to the risk-free rate. Therefore, the above equation can be written as

$$e_{min} \approx -\frac{(z + \mu)F(L) - (r^f + \theta(\mu))L}{r^f} - \frac{\epsilon_{0.001}F(L)}{r^f} \quad (2)$$

In Equation 2, the minimum capital requirement consists of two components. The first component refers to the expected loss and the second to the unexpected loss of the bank's portfolio. The expected loss is jointly determined by the state of the economy and the bank's choice of monitoring effort.

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<sup>10</sup>Given that a bank's portfolio is fixed during each period, meeting the minimum capital requirement at the beginning of a period (when the loan is extended) implies that the requirement is not violated throughout the period. Moreover, it is not meaningful to discuss capital charges after the realization of bank-specific shocks, particularly under the risk-based capital regime, because the uncertainty in asset returns is eliminated.

<sup>11</sup>In a model with richer maturity structure of bank assets and liabilities the capital requirement might be violated subject to a non-infinitely harsh penalty.

<sup>12</sup>Here the calculation is based on the credit VaR (value-at-risk) for the bank portfolio as a whole rather than on the basis of individual loans. Gordy (2003) and Kupiec (2004) prove that, when the default risk is driven by a single common factor and the bank portfolio is fully diversified, the two definitions yield the same level of regulatory capital.

<sup>13</sup>Implicitly, I assume that banks have an incentive to disclose accurate information on their capital adequacy, monitoring effort and loan risk. Estrella (2004a) suggests that it can be achieved via the combination of voluntary disclosure, direct supervision and financial market discipline, which is beyond the scope of this paper.

## 2.4 Timing

At the beginning of each period, there exist a continuum of banks each with equity equal to  $e_{i,t}$ , which is carried over from last period. The state of the economy,  $z_t$ , is known since the end of last period, while the bank-specific shock  $\epsilon_{i,t}$  is realized at the end of the current period. That is, at the moment of making lending decisions, each bank knows  $z_t$  but does not know  $\epsilon_{i,t}$ . Since banks' decisions are independent of each other, in the remaining part the analysis will focus on the optimization problem of an individual bank and the subscript  $i$ , which represents bank-specific variables, will be dropped.

The bank then decides on the volume of loans it will supply ( $L_t$ ) and the amount of monitoring cost it will pay ( $\theta_t$ ). To finance the loan portfolio the bank's liability comprises equity holding ( $e_t$ ) and deposits ( $L_t - e_t$ ), the latter paying an equilibrium deposit rate of  $r_t^d$ . The bank's financial structure has to meet the regulatory capital requirement, if one exists.

At the end of each period, the bank-specific shock  $\epsilon_t$  is realized and depositors are paid back. The bank either continues its operation (if its net asset value is positive) or files for bankruptcy (if its net asset value drops below zero). If a bankruptcy occurs, all depositors receive an equal amount of payment by splitting residual assets on the bank's balance sheet.

Lastly, at the end of each period the state of the economy in the next period,  $z_{t+1}$ , is revealed. Depending on the amount of net profits and the future state of the economy, a solvent bank chooses whether to distribute dividend to its shareholders or to issue new equity ( $d_t$ ). This choice is reflected on the bank's initial equity holding for the next period,  $e_{t+1}$ .

## 3 The solution to the model

The bank's problem can be solved using dynamic programming techniques and the backward induction method. At each period, a bank makes three important decisions: choosing the size of its loan portfolio ( $L_t$ ) and the monitoring effort ( $\theta_t$ ) at the beginning of the period and deciding on the dividend policy ( $d_t$ ) at the end of the period.

I first examine the solvency condition for each bank at the end of each period. For given initial equity ( $e_t$ ), bank lending ( $L_t$ ), deposit rate ( $r_t^d$ ) and monitoring effort ( $\theta_t$ ), the end-of-period net asset value of the bank is

$$\pi(L_t, \mu_t, \epsilon_t, r_t^d) = (z_t + \epsilon_t + \mu_t)F(L_t) - \theta(\mu_t)L_t - r_t^d(L_t - e_t) \quad (3)$$

There are two possible outcomes. If the bank's net asset value is positive, i.e.  $\pi_t > 0$ , depositors receive full payment. Otherwise the bank files for bankruptcy and depositors have to accept a smaller payment prorated by the amount of residual assets on the bank's balance

sheet, which equals  $(z_t + \epsilon_t + \mu_t)F(L_t) - \theta(\mu_t)L_t$ .

Accordingly, the critical value of the bank-specific shock,  $\bar{\epsilon}$ , is defined by:

$$\pi(L_t, \mu_t, \bar{\epsilon}_t, r_t^d) = 0. \quad (4)$$

For  $\epsilon \geq \bar{\epsilon}$ , depositors receive full payment, otherwise the bank defaults. Since depositors are risk-neutral, in equilibrium their expected payoff should equal the return on risk-free investments (see Equation 1). That is,

$$\int_{\bar{\epsilon}_t}^{\infty} r_t^d(L_t - e_t)\phi(\epsilon)d\epsilon + \int_{-\infty}^{\bar{\epsilon}_t} [(z_t + \epsilon_t + \mu_t)F(L_t) - \theta(\mu_t)L_t] \phi(\epsilon_t)d\epsilon = r_t^f(L_t - e_t) \quad (5)$$

where  $\phi(\epsilon)$  is the p.d.f. function of the shock variable  $\epsilon$ . Combining Equations (4) and (5),  $\bar{\epsilon}_t$  and  $r_t^d$  can be jointly determined conditional on the choice of  $L_t$  and  $\mu_t$ .

$$\bar{\epsilon}_t = \bar{\epsilon}(L_t, e_t, \mu_t) \quad (6)$$

$$r_t^d = r^d(L_t, e_t, \mu_t) \quad (7)$$

**Lemma 1**  $\bar{\epsilon}$  increases with bank asset ( $L$ ) and decreases with bank equity ( $e$ ).

Proof: based on Equations (4) and (5), it can be easily derived that

$$\begin{aligned} \frac{\partial \bar{\epsilon}}{\partial L} &= \frac{1}{1 - \Phi(\bar{\epsilon})} \cdot \left\{ \frac{\theta(\mu) + r_f}{F(L)} \cdot \left(1 - \frac{F'(L)L}{F(L)}\right) + \frac{F'(L)e}{F(L)^2} \right\} > 0 \\ \frac{\partial \bar{\epsilon}}{\partial e} &= -\frac{r^f}{F(L) \cdot (1 - \Phi(\bar{\epsilon}))} < 0 \end{aligned}$$

Lemma (1) is quite intuitive. Notice that  $\bar{\epsilon}$  is the threshold value above which a bank can generate positive net profits, hence a higher  $\bar{\epsilon}$  indicates lower profitability and a higher probability of bank failure. Everything else constant, an increase in bank assets tends to lower bank profits and increase bank fragility for two reasons. For one, it lowers the marginal return of bank assets because of the concavity of the revenue function. Moreover, it increases the leverage ratio, pushes upwards the default boundary ( $\bar{\epsilon}$ ) and, consequently, raises the probability of default ( $\Phi(\bar{\epsilon})$ ). Similarly, lower bank equity increases the probability of bank failure because of higher leverage.

Once  $\bar{\epsilon}_t$  is determined, the net asset value of the bank can be rewritten as

$$\pi(L_t, \mu_t) = \begin{cases} (\epsilon_t - \bar{\epsilon}_t)F(L_t) & \text{if } \epsilon_t \geq \bar{\epsilon}_t \\ 0 & \text{if } \epsilon_t < \bar{\epsilon}_t \end{cases} \quad (8)$$

Define  $V(e, z)$  as the value function for a bank with an initial equity of  $e$  and the initial

state of the economy  $z$ . The bank's optimization problem can be written as follows:

$$V(e_t, z_t) = \max_{(L_t, \mu_t)} \sum_{z_{t+1}} \int_{\bar{\epsilon}_t}^{\infty} W(\pi(L_t, \mu_t, \epsilon_t, e_t), z_{t+1}) \cdot P(z_{t+1}|z_t) \phi(\epsilon) d\epsilon \quad (9)$$

subject to Equations (6), (7), (8) and

$$L_t \geq e_t \quad (10)$$

$$e_t \geq e_{min}(L) \quad (11)$$

$$W(\pi(L_t, \mu_t), z_{t+1}) = \max_{e_{t+1}} \{d(\pi_t, e_{t+1}) + \beta V(e_{t+1}, z_{t+1})\} \quad (12)$$

subject to

$$d(\pi(L_t, \mu_t), e_{t+1}) = \begin{cases} \pi(L_t, \mu_t) - e_{t+1} & \text{if } \pi_t \geq e_{t+1} > 0 \\ [\pi(L_t, \mu_t) - e_{t+1}] (1 + \lambda) & \text{if } 0 \leq \pi_t < e_{t+1} \end{cases} \quad (13)$$

$$e_{t+1} = 0 \quad \text{if } \pi_t < 0 \quad (14)$$

By solving the above dynamic problem, the bank maximizes the expected value for its shareholders. The uncertainty in future franchise value comes from both the uncertainty about the future state of the economy and the riskiness of bank assets. Franchise value drops to zero when  $\epsilon < \bar{\epsilon}(L_t, \mu_t)$ , because in that case the bank loses all its equity capital and is forced to exit the market. Equation (11) is the regulatory capital constraint (if applicable). The bank's equity holding must be higher than the minimum requirement, which equals  $\kappa \cdot L$  under the flat-rate capital requirement and is endogenously determined under the risk-based capital requirement (Equation 2). The embedded dynamic programming problem (Equations 12-14) is the dividend policy to be chosen by the bank at the end of each period, which is equivalent to the choice of  $e_{t+1}$ . If the bank chooses to issue new equity,  $d_t$  is negative and the bank pays an additional cost of  $\lambda$  for each unit of fresh capital.

The solution to this problem has the following properties.

**Proposition 1** *There exists a unique function  $V^*(e, z)$  that solves the dynamic programming problem (9). The optimal value function is strictly concave in  $e$ .*

Proof: see Appendix A.

**Proposition 2** *(optimal policy functions)*

1. *The optimal monitoring effort,  $\mu^*$ , is a function of  $L$  and is independent of  $z$  and  $\epsilon$ . It satisfies  $F(L) = \theta'(\mu^*)L$ ;*

2. *The optimal dividend policy follows a cutoff rule, i.e. there exists a pair  $(\pi_1, \pi_2)$  with  $\pi_1 \leq \pi_2$ , for which the bank issues new equity ( $d < 0$ ) if  $\pi < \pi_1$ , retains all profits ( $d = 0$ ) if  $\pi \in [\pi_1, \pi_2]$  and distributes dividend ( $d = \pi - \pi_2$ ) if  $\pi > \pi_2$ .*

Proof: see Appendix B.

Proposition 2.1 suggests that the level of monitoring effort is chosen to maximize the expected net profit in the current period and is independent of the uncertainty in asset returns and the bank's future franchise value. This is because, by assumption, the monitoring effect affects the bank's revenue as an additive component,  $\mu_t F(L_t) - \theta(\mu_t)L_t$ . As a result, the optimal monitoring effort that maximizes this component also generates a current-period profit outcome that strictly dominates any other solution, and by extension maximizes the bank's franchise value. This result is very useful, as it reduces the number of free parameters by one in the value function iteration procedure and simplifies the algorithm significantly.

The strict concavity of  $V^*$  implies that the dividend policy assumes a simple form as specified in Proposition 2.2. Suppose that the feasible values of  $e$  are restricted to a set  $[e_{min}, e_{max}]$ . In the embedded dynamic programming problem (13), the marginal cost of bank equity is 1 if the bank does not issue new equity or  $1 + \lambda$  if the bank issues new equity. At the same time, the marginal benefit of bank equity, irrespective of whether new equity is issued or not, is downward sloping and equals  $V'(e_{t+1}|z_t)$ . This tradeoff is illustrated in Figure 1. Suppose that  $e^1$  (or  $e^2$ ) is the critical value of  $e_{t+1}$  at which the marginal benefit equals the marginal cost if new bank equity is issued (or if no new equity is issued), the bank's dividend policy must follow the cutoff-rule as stated above with  $\pi_1 = e^1$  and  $\pi_2 = e^2$ . The reason is as follows. If the bank's net asset value is smaller than  $e^1$ , the bank has the incentive to raise new equity to the target level  $e^1$ , because the increase in the bank's franchise value exceeds the equity issuance cost. By contrast, when  $\pi > e^2$ , the bank chooses to distribute dividend in the amount of  $\pi - e^2$ , at which point the marginal decrease in bank value equals the marginal benefit of dividend payment. When the net asset value is in the middle range, the bank prefers to reinvest all profits because it increases the shareholders' value by more than paying them out as dividend; nevertheless, no extra equity will be raised because the cost is relatively too high.

Notice that the two critical values,  $\pi_1$  and  $\pi_2$ , change with the state of the business cycle but are independent of bank characteristics. In other words, the "optimal" range of equity holding is the same for all banks. Of course, the choice of equity holding within this range depends largely on the realization of bank-specific shocks, suggesting that the equilibrium size distribution of bank equity exhibits substantial cross-sectional differences (this issue will be re-examined later).

The optimal cutoff rule implies that, when a bank chooses its lending strategy ( $L$ ), it faces the following tradeoffs.<sup>14</sup> First, without considering the possibility of bank failure, banks tend to choose an optimal loan volume that maximizes their current period expected profit. Importantly, this optimal loan volume is the same for all banks because they have the same investment opportunities.<sup>15</sup> In this sense, initial equity size does not play a role as all banks tend to choose the same portfolio size, which implies that smaller banks would choose higher leverage ratios. Second, the possibility that a bank may go under would affect its lending decision in an important way. As Lemma 1 suggests, the probability of bank failure increases in portfolio size and decreases in banks' initial equity holdings. Therefore, in making portfolio decisions, small banks would pay more attention to expected losses in franchise value (due to bank failure) and choose a lower leverage ratio than the one driven by the efficiency concern alone (as discussed above). Lastly, the existence of equity issuance cost adds another layer of complexity in banks' lending decisions, because the size of bank asset has an ambiguous effect on the magnitude of this cost.<sup>16</sup> As a result, a closed-form solution is ruled out in the model. The remaining part of the paper employs numerical examples to illustrate the properties of the solution and the equilibrium relationship among bank size, leverage, profits and the probability of failure.

## 4 Calibration

### 4.1 Parameterization

The model parameters are calibrated based on the U.S. data in 1981-2002. Table 1 summarizes the input parameters used in the numerical analysis.

- The risk-free rate is fixed at 3 percent, close to the average real risk-free rate (short-term Treasury rate adjusted by inflation) in the United States in the period 1981-2002 (which equals 3.07%).
- The concave return function  $F(\cdot)$  is specified as  $F(L) = L^\alpha$  with  $\alpha = 0.985$ . This returns to scale parameter is close to the one adopted by Cooley and Quadrini (2001).

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<sup>14</sup>The first-order condition is too complicated so I do not state it explicitly here.

<sup>15</sup>Based on Equation 3 and Proposition 2.1, expected profit in the current period equals  $E(\pi|z) = zL^\alpha + \frac{L^{2\alpha-1}}{4c_0} - r^f(L - e)$ . It is straightforward to show that there is a uniform  $L$  that maximizes the expected profit for all banks.

<sup>16</sup>Based on Equation (8), the two critical net asset values can be transformed into two critical values for bank-specific shocks. Specifically, a bank chooses to issue new equity when the bank-specific shock is greater than  $\bar{\epsilon}$  but lower than  $\epsilon_1 = \bar{\epsilon} + \frac{e^1}{F(L)}$ . The change in loan volume ( $L$ ) has an ambiguous effect on the probability of new equity issuance,  $\Phi(\epsilon_1) - \Phi(\bar{\epsilon})$ , and the expected cost of equity issuance.

The parameter must be strictly less than one, because otherwise the optimal size of the bank goes to infinity. In addition, it is chosen close to one to avoid the large discrepancy in investment returns between large and small banks.

- The state of the economy. For simplicity, I assume that  $z$  only takes two possible states: high (H) or low (L). The probability of staying at the same state in the next period is set to be 0.800, which implies that the same state of economy lasts for four years on average. The bank-specific shock  $\epsilon$  is assumed to be normally distributed with mean zero and standard deviation  $\sigma_\epsilon$ , which will be discussed later.

The choice of two possible states of the economy,  $z_H$  and  $z_L$ , attempts to match the loan premium observed in the data. I rank the annual loan premium (defined as loan rates minus short-term Treasury rates) during the period 1981-2002, and divide the sample into two subgroups. The average loan premium in the good eleven years is 3.88% and that in the bad eleven years is 2.12%. Hence I set the two shocks as  $z_H = 1.07$  and  $z_L = 1.052$  (i.e. excess returns of 4% and 2.2%), respectively.

- The determination of  $\sigma$ , the return volatility, is not straightforward. Following conventional finance theory, I first assume a particular functional form for the relationship between the volatility and the size of bank assets:  $\sigma(L) = \frac{a_1}{L+a_2}$ . The two constant terms  $a_1$  and  $a_2$  are then chosen so that the Sharpe ratios, defined as the bank's excess return on assets (over the risk-free rate) divided by its standard deviation, are within a reasonable range in equilibrium. Rosen (2005) documents that the median Sharpe ratio for U.S. banks during the period 1977-2003 is about 0.90. For this study,  $a_1 = 1.05$  and  $a_2 = 15$  so that, in equilibrium, the Sharpe ratios vary between 0.40 and 1.40 with a median of 0.92.
- The cost of monitoring effort is specified as a quadratic form  $\theta(\mu) = c_0\mu^2$ , where  $c_0 = 6.5$ . I choose this particular value to offset the decreasing returns to scale effect due to the concavity of the return function, so that in equilibrium the average loan premium matches the target level. Nevertheless, the monitoring cost parameter is not crucial in this study, and varying this parameter has only a minor impact on the results.
- The intertemporal discount factor equals 0.95. This is very close to the one adopted by Cooley and Quadrini (2001) and is also consistent with the business cycle literature.
- Following Cooley and Quadrini (2001), the cost of issuing new equity, or the new share's premium, is set to be 30%. The magnitude of the equity issuance cost is also

in line with the size of the tax burden (40%) discussed in van den Heuvel (2002), an alternative way to introduce the differentiation between equity and debt financing.

- Under the flat-rate capital requirement, the minimum capital-asset ratio equals 8%.

The computational procedure of the dynamic programming problem is described in Appendix C. The results presented are based on the discretization using a grid with 240 points for bank equity ( $e$ ), net asset value ( $\pi$ ) and loan volume ( $L$ ).

## 4.2 A baseline model without capital regulation

The baseline model examines the banks' financial decisions in the absence of regulation, i.e. when the capital requirement is zero. This provides a benchmark for judging the impact of different forms of capital regulation. The characteristics of banks' equilibrium behavior are shown in Figures 2 and 3.

Figure 2 shows the optimal dividend policy adopted by banks at the end of each period. All banks adopt a uniform cut-off rule as described in Proposition 2.2 (top panels). That is, irrespective of banks' initial equity in the current period, their initial equity in the next period will be distributed within the same "optimal" range. However, the size distribution within this range is different across banks, because it depends on the realization of end-of-period net asset values. The middle and bottom panels of Figure 2 illustrate the probability distributions of end-of-period net asset values and next-period initial equities for three hypothetical banks – a large bank ( $e_0 = 2$ ), a small bank ( $e_0 = 1$ ) and a mid-sized bank ( $e_0 = 1.5$ ). The results suggest that large banks are very likely to remain the business leaders in the next period. This is not surprising, because strong capitalization implies that these banks can choose to maximize investment profits without worrying too much about the possibility of failure. By contrast, the growth capacity of small banks is limited due to the concern that a high-leveraged capital structure, which is desirable from the perspective of profit maximization, tends to increase the probability of bank failure substantially.

Figure 3 summarizes the characteristics of equilibrium solutions in the baseline economy, including banks' lending decisions, monitoring efforts, and default probabilities. In each panel, the solid lines refer to equilibrium outcomes in the good state of the economy and the dashed lines represent equilibrium outcomes in the bad state. There are several important observations.

First of all, panel (a) of Figure 3 indicates that the bank's franchise value is higher in the good state and it increases with bank size.

Second, panels (b) and (c) plot the bank's lending decisions. Generally, bank lending increases during the expansion period and falls during the downturn period. This procyclical lending behavior is a result of maximizing profits when the average return is high. The optimal leverage ratio (the inverse of equity/asset ratio) can be as high as in the range of 15-35 in the good state and as low as 6-10 in the bad state.

Moreover, the optimal leverage ratio differs across banks. As shown in the figure, large and very small banks choose low leverage in equilibrium, while mid-sized banks choose high leverage.<sup>17</sup> This cross-sectional difference is driven by the tradeoff between one-period gains (an efficiency concern) and future franchise value (a safety concern) as discussed in Section 3. On the one hand, there is an optimal scale of bank lending that maximizes the expected profit of each bank in the current period due to the decreasing returns to scale assumption (see footnote 15). Other things equal, each bank has the incentive to choose this optimal lending scale, implying that the optimal leverage ratio would be higher for smaller banks. On the other hand, high leverage increases the probability of bank failure and reduces the bank's value in the future. The equilibrium lending decision depends on the relative importance of these two effects. The numerical example suggests, for very small banks, focusing solely on the efficiency leads to very high leverage and high default risk. The future franchise value cost is so high that these banks will choose a prudent lending strategy (i.e. the safety concern dominates). As bank equity increases, the stronger capital position implies that banks can maximize current period gains without increasing the probability of default substantially. Hence the optimal leverage ratio increases to reach the balance between the marginal benefit from investment gains and the marginal cost of bank failure. However, at certain point (when the capital position is strong enough) the efficiency aspect dominates the safety concern and the optimal leverage ratio decreases with bank size. The combined effects explain the "U-shape" optimal equity/asset ratios as observed in the graph.

Third, panels (d) and (e) show the risk profile of bank investments and banks' monitoring efforts. Smaller banks are associated with smaller portfolio investments, which have high return but high risk as well. In order to stay in the business, smaller banks are willing to devote a higher monitoring effort (per unit of asset) to reducing the probability of failure. In addition, when the economic conditions are favorable, high productivity induces banks to lower their monitoring efforts because the marginal benefit of inducing higher returns

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<sup>17</sup>Figure 2 suggests that the optimal bank size is approximately between 1 and 2 for all banks. Although bank size here does not map directly into the observed data, it is reasonable in this paper to interpret banks of size 2 as the largest ones in the industry, banks of size 1 as the small and established banks, and banks of size in between as mid-sized banks. Banks with a size smaller than 1 represent the smallest ones that are still at their infancy stage.

through this channel is smaller.

Lastly, panel (f) shows the probability of bank failure during the two stages of the business cycle. Overall, the predicted bank failure rates are within a reasonable range, with a peak level of 1-2% and a low level of 0.01-0.04%. This is in line with the observations before a mandatory capital requirement was applied in 1998 (Figure 4). In addition, the probability of default is very small for all banks, therefore the difference between default-adjusted deposit rates and the risk-free rate is negligible (recall Assumption 4). However, the failure probability differs substantially across banks. In line with the difference in lending decisions, an inverse “U-shape” curve is observed across banks of different sizes. The largest and smallest banks have lower leverage and their default probabilities are very low. By contrast, mid-range banks engage in aggressive lending and by consequence they incur a high risk of default.

Interestingly, the probability of bank failure is much higher in the upturn of the business cycle. This supports the view that business risks build up during the expansion period.<sup>18</sup> As the former Federal Reserve Chairman Alan Greenspan noted, “the worst loans are made at the top of the business cycle” (Greenspan, 2001). Borio and Lowe (2001) and Borio et al. (2001) document the same pattern that systemic risk tends to build up in the booming period. Notice that, however, in this paper the higher systemic risk during upswings is not caused by underestimation of the business risk. Instead, the forward-looking banks are fully aware of the risk involved: they are willing to increase their risk exposure in the pursuit of higher returns. In other words, the one-period expected gain from risk-taking exceeds the franchise value at risk.

### 4.3 A flat-rate capital requirement

The rest of the paper introduces capital regulation and examines its impact on equilibrium outcomes, including banks’ lending behavior, optimal leverage ratios, probability of failure, etc. The capital requirement is assumed to be exogenously imposed for regulatory reasons that are not explored in this model. This subsection focuses on the effect of a flat-rate capital standard, which requires a minimum capital-asset ratio of 8%.

Figure 5 shows the characteristics of equilibrium outcomes under this flat (risk-insensitive) capital rule. Many qualitative aspects of the results in the baseline economy remain un-

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<sup>18</sup>In this paper, it is hard to distinguish between the “build-up” and the “materialization” of business risks because all bank loans mature in one period. The simplified assumption on maturity structure misses the realistic situation that there exists a lag for risks to materialize. Therefore, the readers should not link directly the realization of bank failures with the state of the business cycle when fitting model predictions with the data. Moreover, for general discussion on the impact of the timing assumption, see Crockett (2000).

changed, as have been proved in Section 3. For instance, the bank’s franchise value is concave and is strictly increasing in bank size and the state of the economy. In equilibrium, all banks choose a cutoff dividend rule as described in Proposition 2.2. Furthermore, numerical results also exhibit similarities in that the initial bank equity is important in determining the bank’s lending behavior. Large banks tend to lend more and spend less resources on monitoring (per unit of asset). The optimal equity/asset ratio is “U-shape” distributed across banks, and the probability of default has an inverse “U-shape” distribution.

However, the imposition of the minimum capital requirement does have important effects on the bank’s financial decisions. The counter-cyclical nature of the economic capital, defined as the capital holding under the unregulated economy (see Figure 3), implies that the flat minimum capital requirement is more likely to be binding in the good state. The binding capital constraint forces banks to lend less aggressively during the expansion period. As a result, the probability of bank failure is much lower than in the baseline economy. By contrast, during downturns the bank’s financial decisions are practically unaffected.<sup>19</sup>

The cross-sectional difference in economic capital under the baseline economy suggests that the effectiveness of the capital requirement is different across banks. In good times, the capital constraint is binding for most banks except for the smallest ones. Accordingly, most banks have to reduce the scale of investment to abide by the regulatory constraint. In bad times, banks are willing to hold capital above the minimum regulatory requirement, and the level of capital buffer, defined as actual capital holding minus the minimum capital requirement, reflects the difference between economic capital and regulatory capital. Consistent with the baseline results, large and small banks hold larger capital buffers than the others.

Importantly, contrary to the baseline results, the probability of bank failure is not always higher in the good state than in the bad state. Except for very small banks, banks’ failure rates are lower in the good state, suggesting that the capital requirement plays a powerful disciplinary role in containing excessive risk-taking during economic upswings. By contrast, it has little impact on banks during the period when banks hold strong capital positions and are less fragile.<sup>20</sup>

The prediction of cross-sectional differences in actual capital holdings is partially sup-

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<sup>19</sup>The bank’s equilibrium financial decisions in bad times differ from the results in the unregulated economy, even if the capital requirement is not binding. This is because banks are forward-looking and the possibility of a binding capital constraint in the future affects their financial decisions in the current period.

<sup>20</sup>The U.S. experience suggests that the flat-rate capital requirement has helped to mitigate the severity of the worst scenario, in that the average default rates in bad years dropped significantly after the introduction of capital regulation (Figure 4). First, I divide the sample into two sub-periods: 1971-1988 and 1991-2002. For each sub-period, I then calculate the average of the half high bank failure rates and that of the half low failure rates. During the first eighteen years, the two averages were 0.67% and 0.04%, respectively. In the twelve years after capital regulation was introduced, the two averages were 0.43% and 0.04%, respectively.

ported by empirical evidence. Figure 4 shows that, in the U.S. banking industry, the smallest banks typically hold the highest capital/asset ratio. This is consistent with the calibration results. What deviates from the model prediction is, however, that the data show a negative relationship between capital ratios and bank size, i.e. the largest banks hold the least amount of capital buffers. This could be attributed to a number of factors that are absent in this model. For instance, the largest banks have better access to the inter-bank market and may, arguably, be considered by market participants to be safer due to the “too-big-to-fail” argument. Hence, they may be able to raise new equity at a lower cost. These privileges entice the largest banks into extra lending; hence their capital ratios could be lower than what the model predicts.

Lastly, the calibration results permit on examination of the procyclicality question. In this paper, I distinguish between the procyclical movement of bank credit and the procyclicality effect of capital regulation. In particular, the latter refers to *excessive* volatility of bank credit as a result of introduction of capital regulation, compared to the cyclic movement under the un-regulated economy. Such distinction is important, because a cyclical movement of bank credit itself could be a manifestation of an optimal allocation of economic resources over the business cycle. By contrast, excessive volatility of bank credit, which in the context of this model is equivalent to an amplified cycle of economic output, could cause potential harm to the real economy and should be avoided.

I quantify the magnitude of credit cycle for each bank by calculating the ratio of economic outputs (investment yields minus the monitoring cost) in good times vs. in bad times. Furthermore, under capital regulation, the cycle is further broken down into two components, one attributed to the cyclical movements in capital charges and the other driven by variation in capital buffers. The comparison of credit cycles between the baseline economy and the one with capital regulation sheds light on the procyclicality effect of capital requirements.

Figure 6 plots the changes in economic outputs over the business cycle, first assuming that the minimum capital requirement is binding for each bank (left panels) and then showing the *actual* outcomes in equilibrium (right panels). Intuitively, the first result isolates the cyclic movement in bank credit caused by capital regulation, and the difference between the two results shows the cyclic effect driven by capital buffer decisions. There are two major observations. First, under the flat-rate capital requirement (the dashed lines), *actual* economic outputs co-move with the business cycle and this cyclical movement is mainly driven by the anti-cyclical movement in capital buffers.<sup>21</sup> By contrast, the capital constraint has only a

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<sup>21</sup>This anti-cyclical movement in capital buffers is consistent with the findings in Germany (Stolz and Wedow, 2005) and Spain (Ayuso et al., 2004). Jokipii and Milne (2006) further point out that the fluctuation of capital buffers over the business cycle can vary across country and with the type and size of banks.

small impact because it merely reflects the difference in the economy-wide shock. Second, whereas the impact of the flat-rate capital constraint is uniform for all banks, distinction in capital buffer decisions implies that the actual cyclic movement of lending activity is bank-specific. Those banks that hold low capital buffers in both states (initial equity  $\approx 1$ ) have the smallest credit cycle in the banking industry.

Importantly, the results suggest that the concern about the procyclicality effect is misplaced under the flat-rate capital requirement, in that the introduction of capital regulation actually causes less volatility in economic activities than in the baseline economy (the solid lines in Figure 6). The mitigation of credit cycles is mainly because capital regulation constrains banks' excessively risky lending during upswings.

#### 4.4 A risk-based capital requirement

This subsection examines the impact of a risk-based capital rule, which is defined in a way that the maximum failure probability for each bank does not exceed a pre-specified threshold value (see Section 2.3). A risk-based capital rule implies that the level of regulatory capital varies not only with the state of the economy but also with the risk profile of bank assets, the latter of which is endogenously determined. Specifically, the new rule rewards those banks with high-quality assets by lowering their capital requirements and penalizes banks with low-quality assets by doing the opposite.

Figure 7 summarizes the characteristics of equilibrium outcomes under the new capital regime. Overall, there is little qualitative difference between the results under a flat rule and a risk-based rule, in terms of the functional form of the optimal dividend policy and the relationship between bank size on the one hand, the loan volume, return on assets, asset volatility and capital structure on the other.

It is worth mentioning that the new capital regime appears to be quite efficient in improving the safety of individual banks in that the probability of bank failure is successfully contained below the target level. The disciplinary role of the risk-sensitive capital regime is particularly significant in the case of small banks, which are forced by higher capital charges to reduce the scale of bank lending substantially in good times. In other words, the build up of excessive risk-taking by those banks is substantially mitigated.

The simulation results permit on re-examination of the impact of the transition from a flat-rate to a risk-based capital regime on the level of bank capital. In addressing this issue one needs to distinguish between the impact on regulatory minimum requirements and on actual bank capital holdings. The impact on the first is very significant. The dotted and dash-dotted lines in the top two panels of Figure 8, and the solid line in the bottom panel,

are indicative of this impact. The risk-sensitive rule favors large (and less risky) banks at the cost of small (and riskier) banks. In particular, it causes the average minimum capital requirement for large banks to be approximately 50% lower than under the flat-rate capital regime, and that for small banks to be 50-60% higher. These findings are roughly in line with previous studies.

By contrast, the impact of the regulatory rule on actual bank capital holdings is much less pronounced. The solid and dashed lines in the top two panels of Figure 8, and the dashed line in the bottom panel, illustrates this impact. Overall, the change in total capital is much smaller than the change in regulatory capital, because of the offsetting effect by endogenous adjustments in banks' capital buffers. Moreover, the impact is less uneven across banks, in that large banks will hold 15-20% less capital and small banks will hold approximately 20% more capital under the risk-sensitive rule than under the flat-rate one.

The uneven impact across banks is not necessarily undesirable. To some extent, it reflects an important motivation for adopting a risk-sensitive capital regime: to level the playing field. The flat-rate capital requirement does not impose sufficiently high capital requirements on high-risk loans, but at the same time penalize high-quality investments. For instance, mid-sized and large banks are forced to forgo high-return, low-risk lending opportunities in good times because of binding capital constraints. By contrast, the risk-based capital regime successfully restores market efficiency by linking capital requirements with asset quality. As the simulation results show, under the risk-based capital regime, the more favorable capital regulation for larger (and safer) banks makes their lending portfolios more similar to those under the baseline economy. The stricter capital requirement for smaller (and riskier) banks forces them to cut loan extension substantially.

Moreover, the simulation results also provide insights on the the procyclicality effect under the risk-sensitive rule, as illustrated in Figure 6. Swings in economic outputs over the business cycle reflect swings in banks' capital structure and in the extension of bank credit. There are three main messages. First, under the risk-sensitive capital rule, the cyclical movement in economic activities is mainly driven by the counter-cyclicality of capital charges, and the variation in capital buffers has hardly any impact.<sup>22</sup> This is in sharp contrast with the result under the flat-rate capital requirement, in which the lending cycle is primarily caused by the

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<sup>22</sup>The large swing in minimum capital requirements is partly driven by the short horizon of the portfolio risk assessment. For simplicity reasons, this model assumes that bank assets mature in one period (or one year), therefore the risk profile of bank portfolio is a point-at-time measure. This is consistent with the prevalent practice of using 1-year default probability in computing risk-based capital requirements. If risk assessments over a longer time horizon are adopted (eg "through-the-cycle" ratings), the variation in regulatory capital over time tends to be smoother. This requires a richer maturity structure of bank assets in a dynamic setting, which is beyond the scope of this paper.

counter-cyclical movement in capital buffers. To some extent, this difference implies that the risk-based capital rule achieves the goal of aligning regulatory capital with actual capital that a bank is willing to hold. Second, it is unclear whether the risk-sensitive capital rule would amplify or mitigate the credit cycle in the aggregate economy. The simulations show that the cycle for very small banks is less pronounced under the risk-sensitive capital rule, whereas it becomes more volatile for large banks. The aggregate effect, therefore, depends on the size distribution of the banking industry. For instance, the simulation results in Figure 6 suggest that the aggregate credit cycle is less pronounced under the risk-based capital standard than under the flat-rate capital rule if the majority of assets in the banking industry are held by very small banks, and vice versa if large and medium banks play a dominant role. Lastly, compared with the results under the baseline economy, the introduction of risk-based capital requirements does not lead to an amplified credit cycle, in that the volatility of bank loans is much smaller for small banks (for which the loan activity is constrained in good times). Even for large banks, for which the capital rule is not binding, the cyclical movement of lending activity mimics the lending cycle in the baseline economy and does not cause extra volatility into the economy.

## 5 The tradeoff between efficiency and safety and the impact of capital regimes

The previous section discusses how the change in capital rules affects banks' equilibrium financial decisions. Nevertheless, there is no indication whether this change is desirable. For instance, lower leverage and higher bank safety are not always appealing if they imply big sacrifice in lending efficiency and growth potential. The analysis in this section can shed some light on this important issue.<sup>23</sup>

Although there is no consensus view on the objective function of a bank regulator, researchers tend to agree that the introduction of capital regulation can be justified on the basis of improving the safety and soundness of the banking sector and to minimize negative externality associated with bank failure. Following the mainstream of the literature (see Kashyap and Stein, 2004; Hellmann et al., 2000; Dell'Ariccia and Marquez, 2006, for example), I assume that there are two major components in a regulator's objective function: franchise value and the probability of bank failure, which stand for the the profitability (or

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<sup>23</sup>The following discussion focuses exclusively on two capital regimes examined in this paper. Implicitly, I assume that a regulator only cares about franchise value and bank fragility but nothing else, and he is fully aware of banks' responses to various capital rules. It does not intend to address the issue on optimal design of capital regulation.

efficiency) and safety of the banking sector, respectively. For obvious reasons, the regulator favors higher franchise value and lower bank fragility.

There are two ways for a regulator to examine the welfare implications of capital regulation, a *micro* perspective that draws conclusion from the analysis based on individual banks, and a *macro* perspective that focuses on the banking system as a whole. This is in line with the two distinct views regarding the objective of prudential regulation, known as the micro- vs. macro-prudential perspectives.<sup>24</sup> The first view emphasizes the safety of *individual* financial institutions, with an implicit assumption that the safety of individual banks is a sufficient and necessary condition for the soundness of the banking system. By contrast, the macro perspective analyzes the financial system as a whole. Therefore, systemically more important banks receive more attention from the regulator than small banks that contribute less to the systemic risk.

Figure 9 illustrates the impact of the two capital regimes on the tradeoff between efficiency and safety from a micro-perspective, using a very small bank (initial equity  $e_0 = 0.5$ ), a small bank ( $e_0 = 1$ ) and a large bank ( $e_0 = 2$ ) as examples. The horizontal axis represents the expected probability of bank failure and the vertical line represents expected franchise value, using the fact that the good and bad states are equally likely to occur in the long run. The exercise is implemented by studying the flat-rate capital regime with different minimum capital ratio requirements ( $\kappa = 2\%, 4\%, \dots, 14\%$ ) and the risk-based capital regime with different threshold default probabilities (based on the credit VaR definition as specified in Equation 2).

The results are very interesting. When the capital standard is very loose, the equilibrium outcome converges to the result under the baseline economy. And the tightening in capital standards (higher uniform capital ratios or lower threshold default probabilities) moves the equilibrium outcome to the lower-left corner. That is, it reduces the probability of bank failure at the cost of banks' franchise value. This is not surprising, since the regulator puts more emphasis on bank safety relative to efficiency. The more interesting finding is that, the risk-based capital rule yields higher franchise value for the same degree of bank safety. This implies that the risk-based capital regulation is more appealing from a regulator's perspective. At a first glance, this result is not quite intuitive. For instance, comparing the flat-rate capital requirement with  $\kappa = 8\%$  and the risk-based capital requirement with the threshold default probability at  $0.1\%$ , a small bank actually faces a more restrictive capital requirement under the second regime and is forced to choose lower leverage and a lower

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<sup>24</sup>See Crockett (2000) and Borio (2003) for further discussion. Nevertheless, the distinction between the two strands of views is not always a clear-cut.

probability of failure in the current period. However, the bank has a higher franchise value under the risk-sensitive regime than under the flat-rate regime, despite the current-period losses.

The answer to this seeming puzzle lies in the dynamic benefit from the risk-based capital regulation. As we have mentioned earlier, a potential problem with the flat-rate capital rule is that it is not capable of discriminating between sound and risky loans, therefore it tends to hamper efficiency as when it reduces bank fragility. This is most obvious in the case of large banks (the bottom panel), which are forced to forego sound investment opportunities in order to achieve a very low probability of failure. This conservative lending strategy is no longer appealing in that efficiency losses are too high. From a dynamic perspective, it implies that all banks pay a very high future franchise value cost under the flat-rate capital regime. This cost tends to overshadow any potential one-period expected gains (as in the small bank example above) and cause expected franchise value to be significantly lower than under a risk-based capital regime.

In addition, Figure 10 illustrates the comparison from a macro-perspective, based on a hypothetical banking sector that consists of 5 large banks ( $e_0 = 2$ ) and 50 small banks ( $e_0 = 1$ ).<sup>25</sup> Bank-specific shocks are assumed to be independent across banks. The horizontal axis represents the fragility of the whole banking sector, defined as the probability of more than 5% of total bank assets default in the current period. In this example, the threshold is equivalent to the failure of one of the 5 large banks, or the failure of 4 small banks simultaneously. In this simplified banking sector, the comparison result between the two capital regimes is in line with the micro-based results. The risk-based capital requirement achieves a better balance between lending efficiency and safety, in that (expected) aggregate franchise value is higher for any given level of banking stability.

Clearly, this is only an illustrative example and the results should not be interpreted as general and conclusive. There are many complicated issues related to a practical implementation of this method, which are beyond the scope of this paper. For instance, the results crucially depend on the size distribution of the banking sector and the inter-dependence of asset returns across banks, both of which are not explored in this paper.

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<sup>25</sup>In equilibrium, the concentration ratio of the top 5 banks is about 30-40%, which is in line with the survey results in the U.S. and other industrial countries (see BIS, 2004, page 131).

## 6 Concluding remarks

This paper develops a dynamic equilibrium model and examines the impact of the transition from a flat-rate and a risk-sensitive capital regulation on banks' financial decisions. Several important policy implications arise. First, in gauging the impact, it is important to take into consideration the potential influence on banks' capital buffers and portfolio decisions. I show that the change in actual capital holdings is much smaller than the change in regulatory capital. Second, the impact of the capital regime differs substantially across banks. The risk-sensitive capital standard leads to a more favorable capital requirement for large (and in this model also less risky) banks. Hence, the cyclical movement in lending behavior is more remarkable for these banks. By contrast, small (and riskier) banks are subject to a stricter capital regulation under the new regime. They have to cut loan extension substantially in good times, when risk is most likely to build up in the absence of regulatory restrictions. As a result, the lending cycle for these banks is actually less volatile under the risk-sensitive regime. Lastly, the analysis suggests that the transition might be welfare-improving from a regulator's perspective, in that the risk-based capital regime finds a better balance between safety and efficiency and causes less distortion in loan decisions.

The analysis provides a starting point to examine issues on the design of bank capital regulation and the interaction between the banking sector and the real economy. It is by no means a completed task. One caveat of this model is that the asset side of the bank's balance sheet is not explicitly modeled, implying that the important feedback effect from the bank's lending behavior to the real economy is missing in the current analysis. An extension of the model into a general equilibrium framework will make it possible to address this issue. Furthermore, there is no active role of monetary policy in this paper in that the risk-free rate is constant over time. An extension of this model could be used to discuss the impact of monetary policy on the channel through which capital regulation affects the bank's financial decisions. These issues could be potentially very interesting in the research along this line.

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# Appendix

## A Proof of Propositions 1

1. The dynamic program problem is equivalent to:

$$V(e_t, z_t) = \max_{(L_t, \mu_t, e_{t+1})} \sum_{z_{t+1}} \left\{ \int_{\bar{e}}^{\infty} [d(\pi_t, e_{t+1}) + \beta V(e_{t+1}, z_{t+1})] \phi(\epsilon) d\epsilon \cdot P(z_{t+1}|z_t) \right\} \quad (\text{A.1})$$

subject to Equations (6), (7), (8), (10), (11) and (13).

It suffices to prove the existence of a unique solution  $V^*$  by showing that the  $V(\cdot)$  function satisfies the two Blackwell's sufficient conditions and therefore is a contraction mapping. To check the two conditions:

- (*Monotonicity*) Take  $V_1 \geq V_2$ , then:

$$\begin{aligned} T(V_1) &= \max E [d(L^*, \mu^*, e^*) + \beta V_1(e^*, z_{t+1})] \\ &\geq \max E [d(L^*, \mu^*, e^*) + \beta V_2(e^*, z_{t+1})] \\ &= T(V_2) \end{aligned}$$

- (*Discounting*) For any positive real number  $c > 0$ ,

$$\begin{aligned} T(V + c) &= E\{d(L^*, \mu^*, e^*) + \beta [V(e^*, z_{t+1}) + c]\} \\ &= E\{d(L^*, \mu^*, e^*) + \beta V(e^*, z_{t+1})\} + c \\ &= T(V) + c \end{aligned}$$

2. The concavity of  $V^*(e)$  function can be proved based on Theorem 4.8 (page 81) in Stokey et al. (1989). For simplicity, I only prove the case in which  $z$  is constant but the proof can be easily extended to the case in which  $z$  is random.

First, it is straightforward that  $d(\pi_t, e_{t+1})$  function is continuous and bounded if we can restrict feasible values of  $e$  within the range  $[e_{min}, e_{max}]$ , where  $e_{min} = 0$  and  $e_{max}$  is sufficiently large so that banks' equity position will never exceed the boundary.

Second,  $d(\cdot)$  function is concave. That is, for any  $(\pi_1, e'_1)$ ,  $(\pi_2, e'_2)$  and all  $h \in [0, 1]$ ,

$$d(\pi_3, e'_3) \geq h d(\pi_1, e'_1) + (1 - h) d(\pi_2, e'_2)$$

where  $\pi_3 \equiv h\pi_1 + (1 - h)\pi_2$  and  $e'_3 \equiv h e'_1 + (1 - h)e'_2$ . This is because

- if  $\pi_1 \geq e'_1$  and  $\pi_2 \geq e'_2$ , then  $\pi_3 \geq e'_3$  and  $d(\pi_3, e'_3) = h d(\pi_1, e'_1) + (1 - h) d(\pi_2, e'_2)$ .
- similarly, if  $\pi_1 < e'_1$  and  $\pi_2 < e'_2$ , then  $\pi_3 < e'_3$  and  $d(\pi_3, e'_3) = h d(\pi_1, e'_1) + (1 - h) d(\pi_2, e'_2)$ .

- if  $\pi_1 \geq e'_1$ ,  $\pi_2 < e'_2$ , and  $\pi_3 \geq e'_3$ , then

$$\begin{aligned} d(\pi_3, e'_3) &= h(\pi_1 - e'_1) + (1 - h)(\pi_2 - e'_2) \\ &> h(\pi_1 - e'_1) + (1 - h)(1 + \lambda)(\pi_2 - e'_2) \\ &= hd(\pi_1, e'_1) + (1 - h)d(\pi_2, e'_2) \end{aligned}$$

- if  $\pi_1 \geq e'_1$ ,  $\pi_2 < e'_2$ , and  $\pi_3 < e'_3$ , then

$$\begin{aligned} d(\pi_3, e'_3) &= h(1 + \lambda)(\pi_1 - e'_1) + (1 - h)(1 + \lambda)(\pi_2 - e'_2) \\ &> h(\pi_1 - e'_1) + (1 - h)(1 + \lambda)(\pi_2 - e'_2) \\ &= hd(\pi_1, e'_1) + (1 - h)d(\pi_2, e'_2) \end{aligned}$$

Finally, the feasible set of  $e'$  is convex if we restrict the feasible value of  $e$  within  $[e_{min}, e_{max}]$ .

Based on the above three properties,  $V^*(e)$  function is strictly concave in  $e$ .

## B Proof of Proposition 2

To determine the optimal monitoring effort in the current period, it is worth noticing that future bank value,  $V(e_{t+1}, z_{t+1})$ , is independent of  $\mu_t$ . Therefore, the optimal  $\mu_t$  should maximize

$$\int_{\bar{\epsilon}}^{\tilde{\epsilon}} (\pi_t - e_{t+1})(1 + \lambda)\phi(\epsilon)d\epsilon + \int_{\tilde{\epsilon}}^{\infty} (\pi_t - e_{t+1})\phi(\epsilon)d\epsilon \quad (\text{B.1})$$

where  $\tilde{\epsilon}$  is the critical value below which new equity needs to be raised. It is determined by  $(\tilde{\epsilon} - \bar{\epsilon})F(L) = e_{t+1}$ .

The first-order condition of the maximization problem can be rearranged as:

$$\frac{d\tilde{\epsilon}}{d\mu} \cdot [e_{t+1}(1 + \lambda)\phi(\tilde{\epsilon}) - (1 + \lambda)F(L)(\Phi(\tilde{\epsilon}) - \Phi(\bar{\epsilon})) - F(L)(1 - \Phi(\tilde{\epsilon}))] = 0$$

Since the bank chooses  $\mu_t$  at the beginning of period  $t$ , at which time the choice of  $e_{t+1}$  is still uncertain, the above condition is satisfied if and only if  $\frac{d\tilde{\epsilon}}{d\mu} = 0$ . From Equations (4) and (5), it is straightforward to derive that  $\frac{d\tilde{\epsilon}}{d\mu} = \frac{\theta'(\mu)L - F(L)}{(1 - \Phi(\bar{\epsilon}))F(L)}$ . Therefore

$$\left. \frac{d\tilde{\epsilon}}{d\mu} \right|_{\mu=\mu^*} = 0 \quad \Rightarrow \quad F(L) = \theta'(\mu^*)L. \quad (\text{B.2})$$

In addition, given the strict concavity of  $V^*$ , the dividend policy assumes a cutoff rule for a given initial bank equity. Since this is quite straightforward, I leave out the strict proof here but the intuitions are described in the text.

## C Computational procedure

The computational procedure for the dynamic programming is based on value function iteration as introduced in Judd (1998). The value function is solved based on the discretization of the state variable  $e_t$ , the quasi-state variable  $\pi_t$  and control variables  $L_t$  and  $e_{t+1}$ . The main features of the numerical algorithm are:

1. Set the minimum and maximum values of bank equity ( $e$ ) and end-of-period net profit ( $\pi$ );
2. The range of bank lending ( $L$ ) is set within  $[e, e/\kappa]$  under the flat-rate capital requirement, or within  $[e, L_{max}]$  under the risk-based capital regime ( $L_{max}$  is calculated from Equation (2) by setting  $e_{min} = e$ ).
3. For each pair of  $(e, L)$ , calculate the corresponding  $\bar{\epsilon}$  and  $\mu$  based on Equations (6), (7) and Proposition 2.1.
4. For each pair of  $(e, L)$ , calculate the probability distribution of end-of-period net asset values.
5. Guess initial values of  $V(e)$  function for  $z = z_H$  and  $z = z_L$ .
6. Solve for the  $W(\pi, z)$  function and the policy function  $e'$  for given  $\pi$ ,  $e$  and  $z$ ;
7. Based on the  $W$  function, solve for the  $V(e, z)$  function and the policy function  $L$  for given  $e$  and  $z$ ; The found values are the new guesses of the  $V$  function. The procedure is then restarted from step 6 until converged.

Table 1: **Model parameters for the calibration**

Risk-free rate	$r^f$	1.030
Return to scale parameter	$\alpha$	0.985
Economic state	$z_H$	1.070
	$z_L$	1.052
Transition matrix	H $\rightarrow$ H	0.800
	L $\rightarrow$ L	0.800
Standard deviation of the shock	$\sigma_\epsilon(L)$	$\frac{1.05}{L+15}$
Intertemporal discount rate	$\beta$	0.950
Monitoring cost parameter	$c_0$	6.500
New equity premium	$\lambda$	0.300
Minimum capital requirement	$\kappa$	0.080

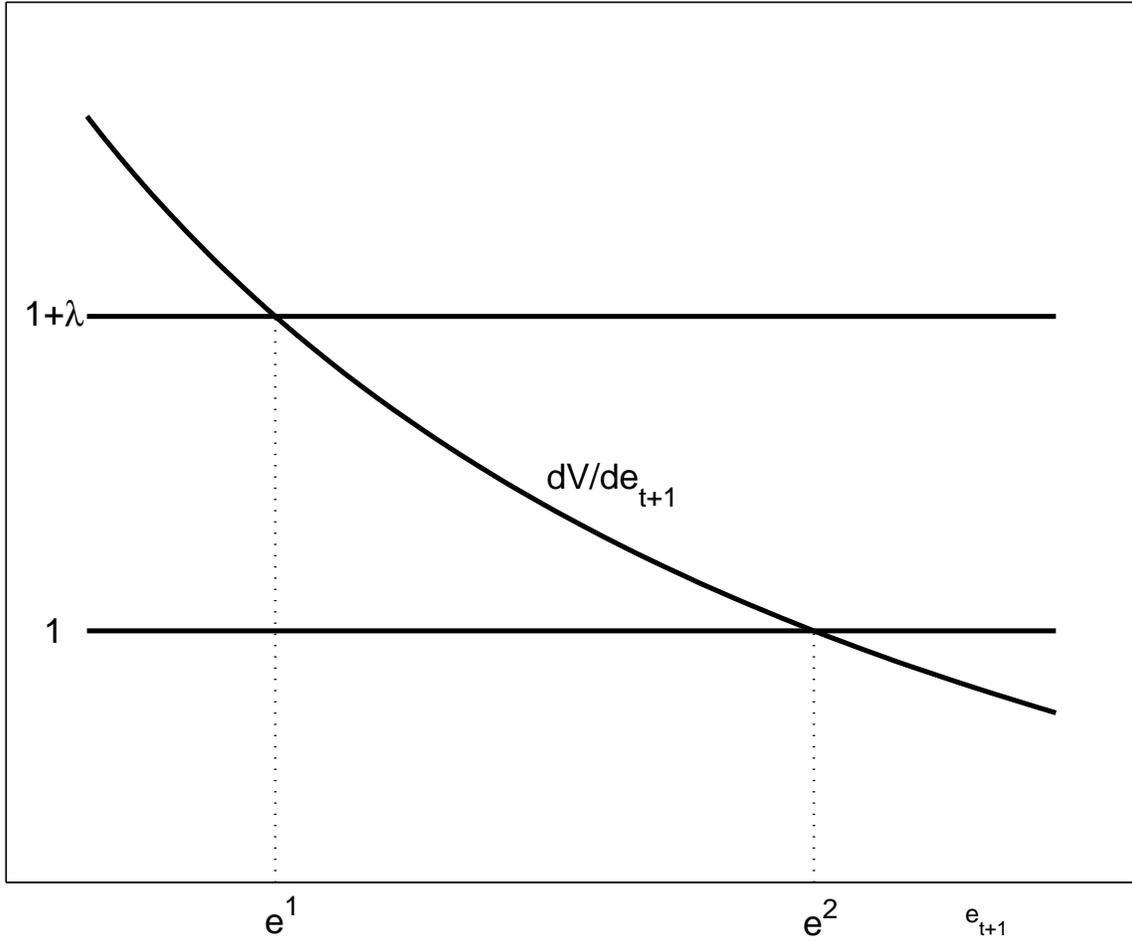


Figure 1: The determination of the bank's dividend policy

*Note:* The two horizontal lines represent the marginal cost of bank equity when new equity is (the upper line) or is not issued (the lower line). The downward-sloping curve represents the marginal benefit of bank equity.

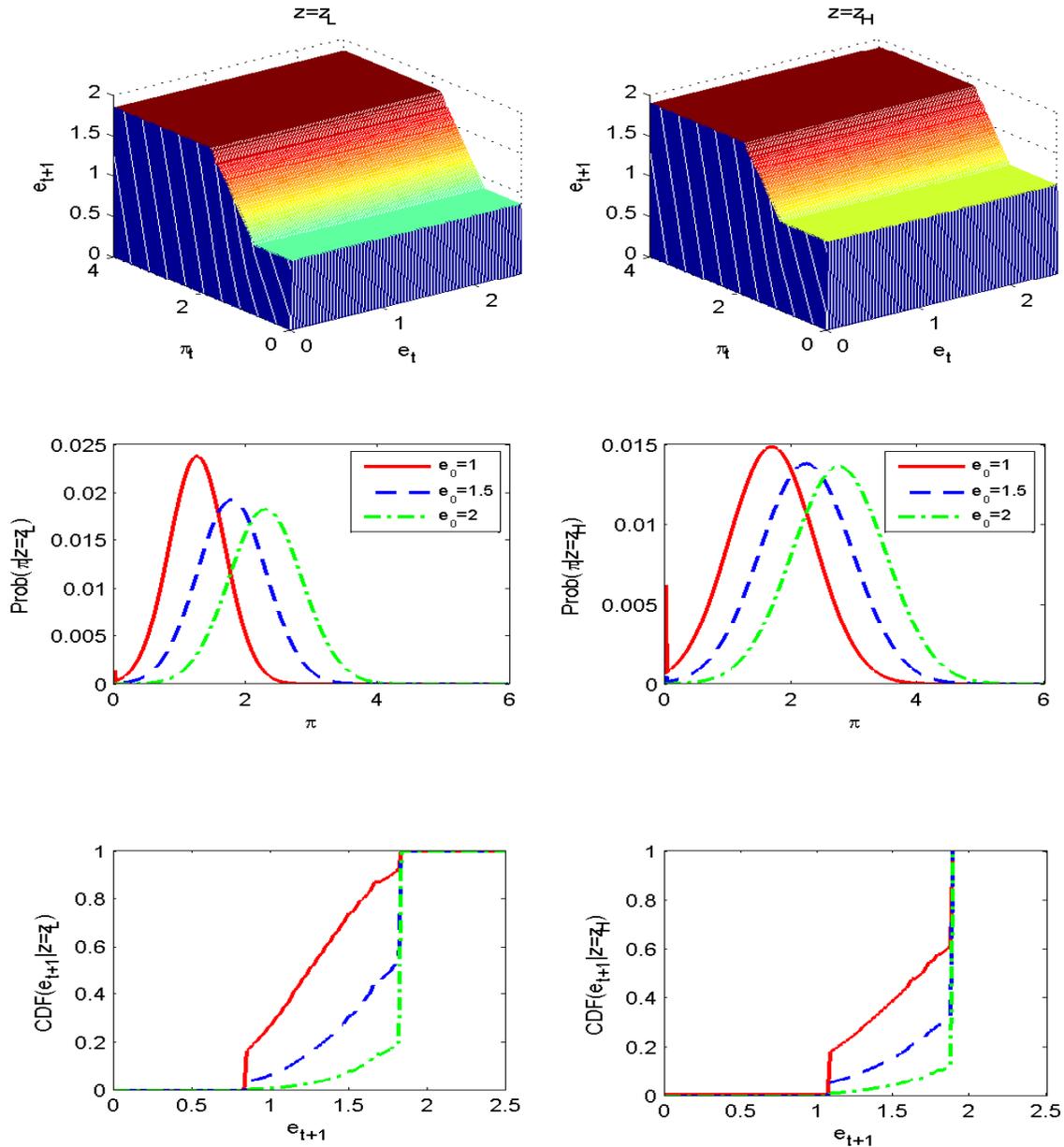


Figure 2: Optimal policy function under the baseline economy without capital regulation

*Note:* The top panels show the optimal dividend policy under the baseline economy, i.e. the determination of next period's equity ( $e_{t+1}$ ) for given initial equity ( $e_t$ ) and end-of-period net asset value ( $\pi_t$ ). The middle panels illustrate the probability distribution of end-of-period net asset values for a small bank (initial equity = 1), a mid-sized bank ( $e_0 = 1.5$ ) and a large bank ( $e_0 = 2$ ). Notice that the most left observations refer to  $Prob(\pi_t \leq 0)$ , i.e. the probability of bank failure. The bottom panels illustrate the cumulative distribution functions of the three banks' equity positions at the beginning of next period.

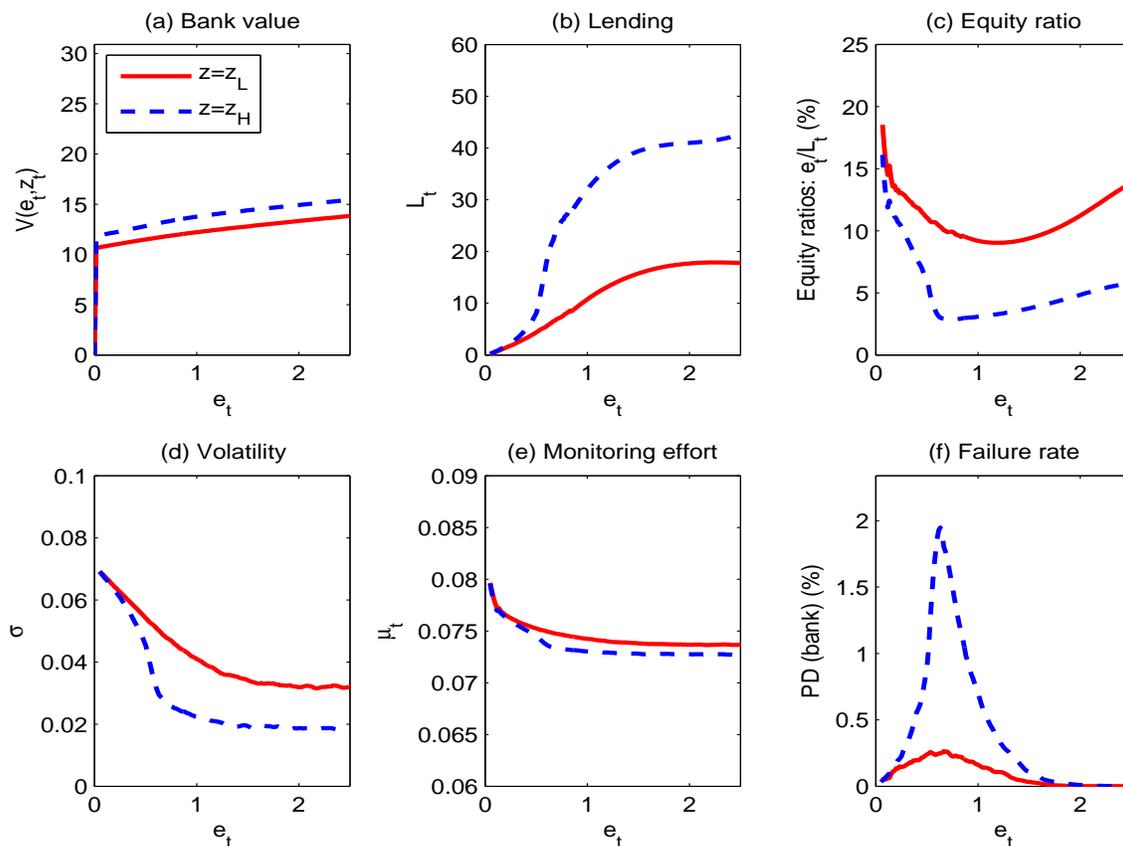


Figure 3: Equilibrium outcomes under the baseline economy

*Note:* The figure plots the equilibrium outcomes under the baseline economy (without capital regulation), including banks' franchise values ( $V$ ), the volume of loan portfolio ( $L$ ), equity/asset ratios, asset return volatility ( $\sigma$ ), monitoring efforts ( $\mu$ ) and the probability of bank failure.

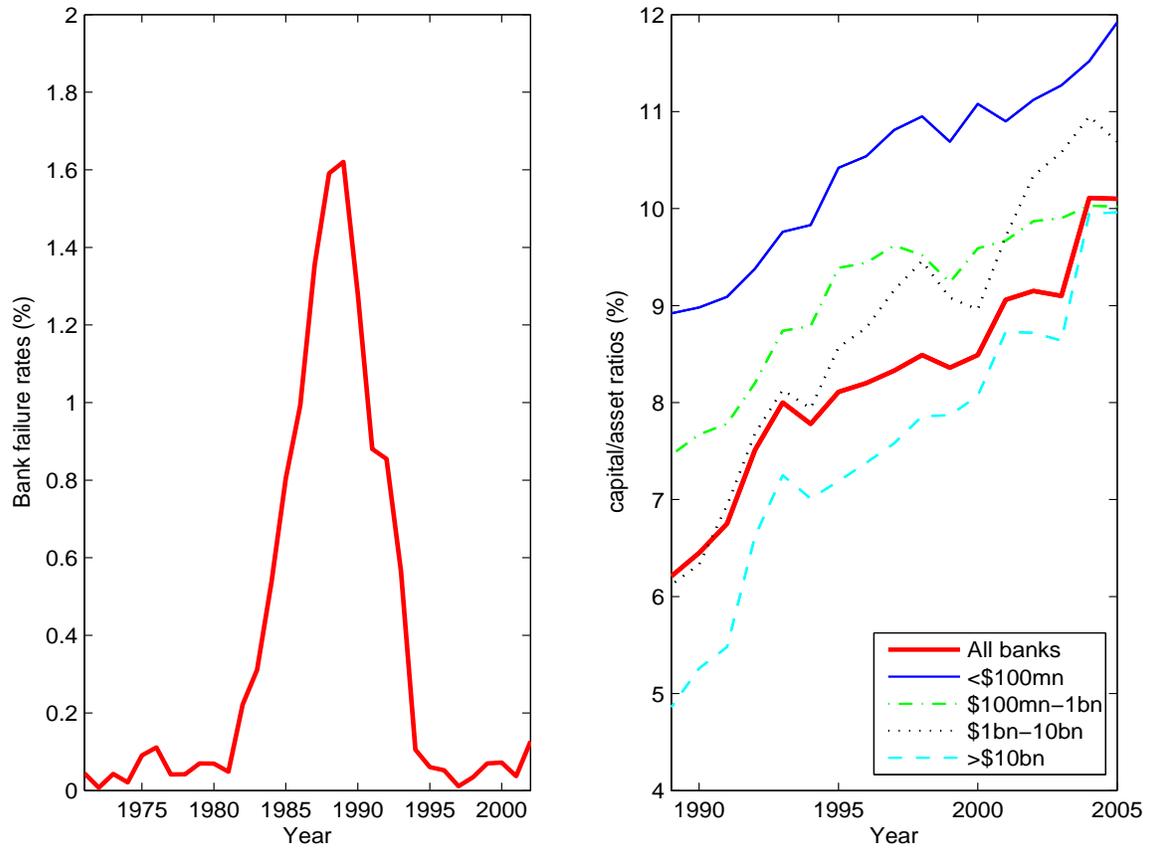


Figure 4: The banking sector in the United States

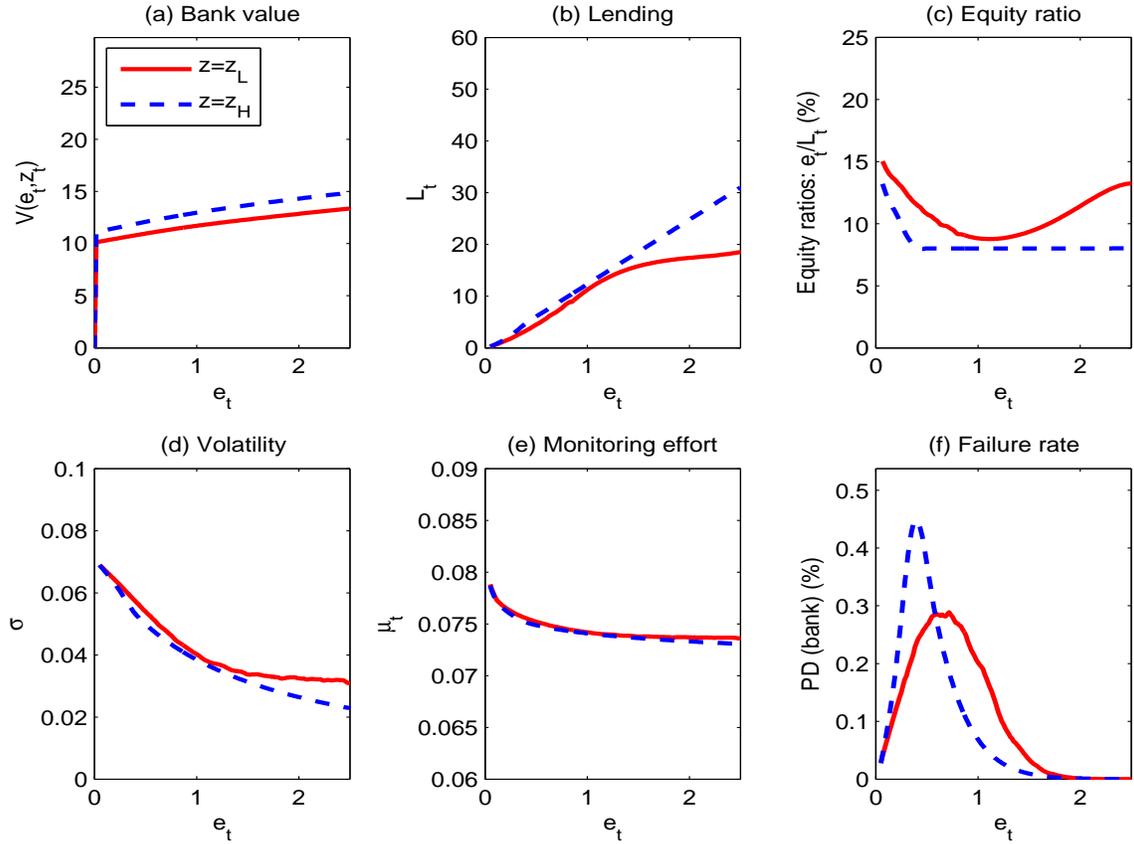


Figure 5: Equilibrium outcomes under a flat-rate capital requirement

*Note:* The figure plots the equilibrium outcomes under a flat-rate capital regime, which imposes a minimum capital/asset ratio of 8% on all banks. Definitions of variables follow Figure 3.

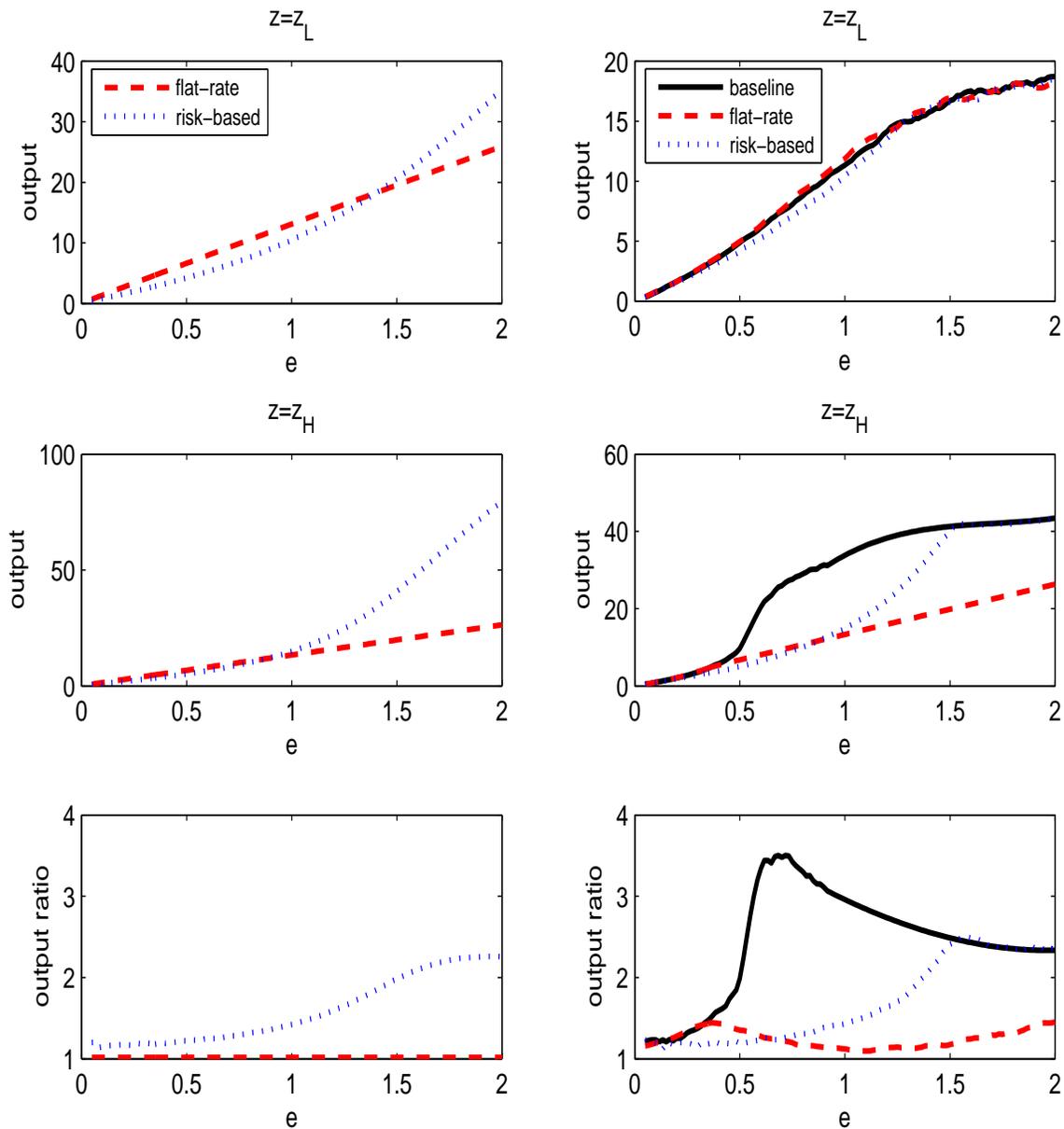


Figure 6: Lending cycles under two capital regimes

*Note:* The figure plots the level of lending revenues for each bank under two distinctive capital regimes, i.e. a uniform capital/asset ratio of 8% (dashed lines) or a risk-based capital standard (dotted lines). The left panels refer to the results if banks choose portfolios as indicated by the minimum capital requirements, and the right panels refer to actual outcomes in equilibrium (equilibrium outputs under the baseline economy are also shown in dark lines for comparison purpose). The bottom panels plot the ratio of outputs in the good state vs. in the bad state, a measure for the magnitude of lending cycles.

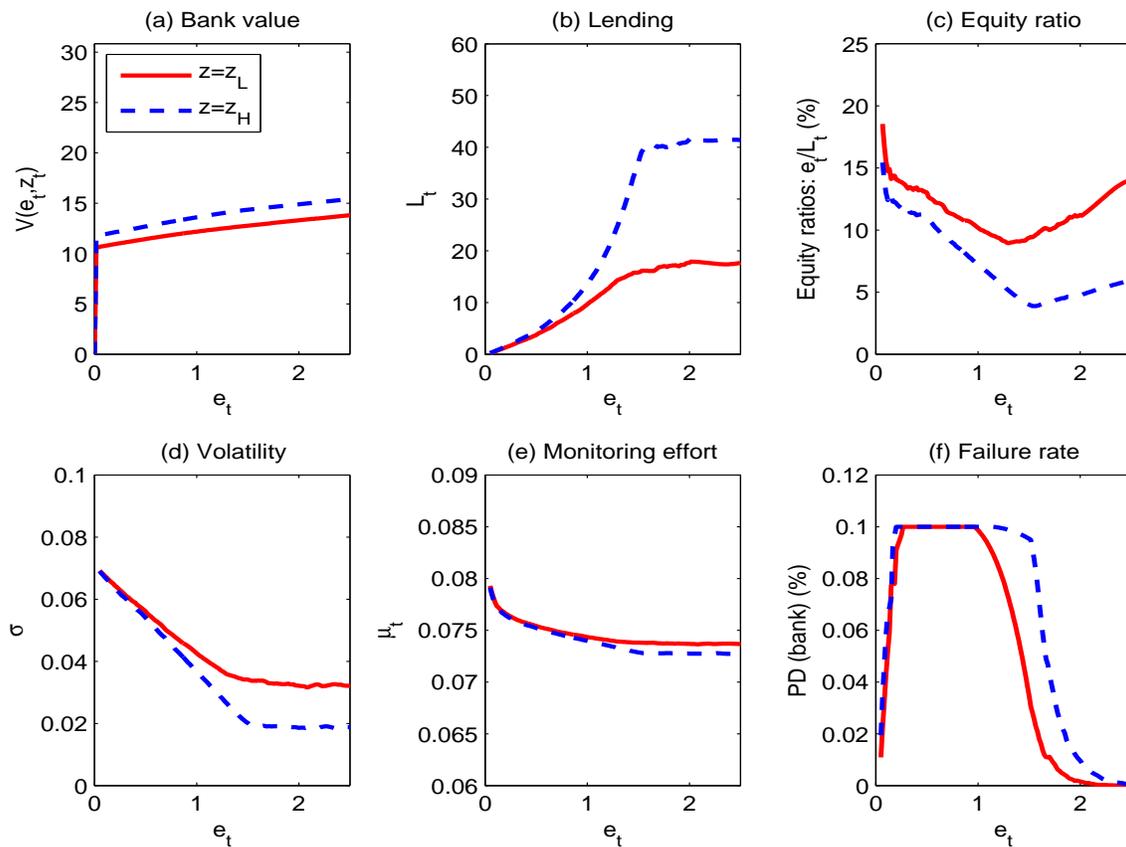


Figure 7: Equilibrium outcomes under a risk-based capital requirement

*Note:* The figure plots the equilibrium outcomes under a risk-based capital rule, in which the minimum capital requirement is determined by the risk profile of bank assets (Equation 2). Definitions of variables follow Figure 3.

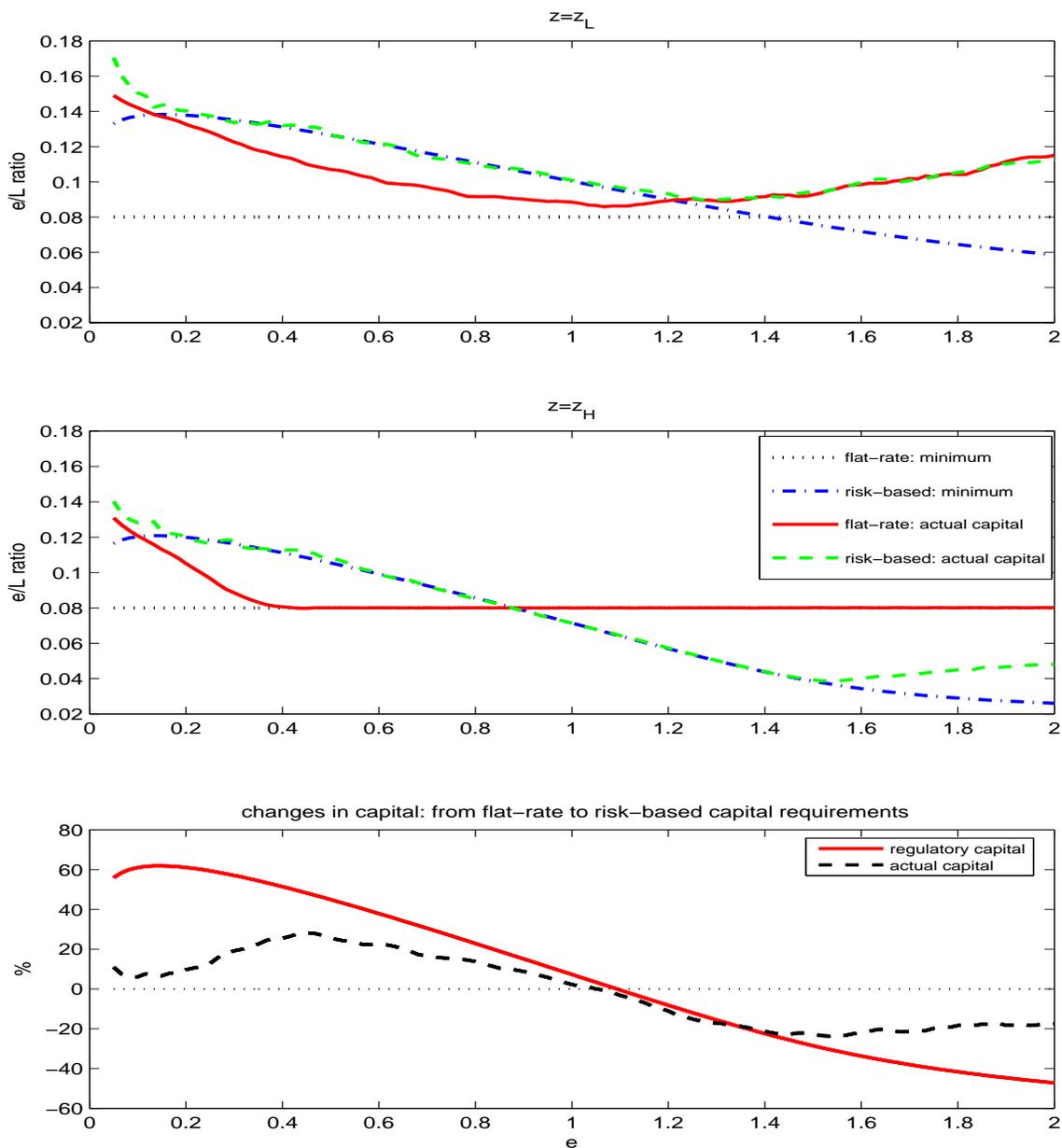


Figure 8: Comparison of capital holdings under two capital regimes

*Note:* In the top two panels, the dotted and dash-dotted lines plot the minimum capital requirements under two distinctive capital regimes, i.e. a uniform capital/asset ratio of 8% or a risk-based capital standard. The solid and dashed lines plot banks' *actual* capital holdings under the two capital regimes. The bottom panel shows the average changes in regulatory capital (the solid line) and actual capital (the dashed line) if the flat-rate capital rule is replaced by the risk-based capital requirement.

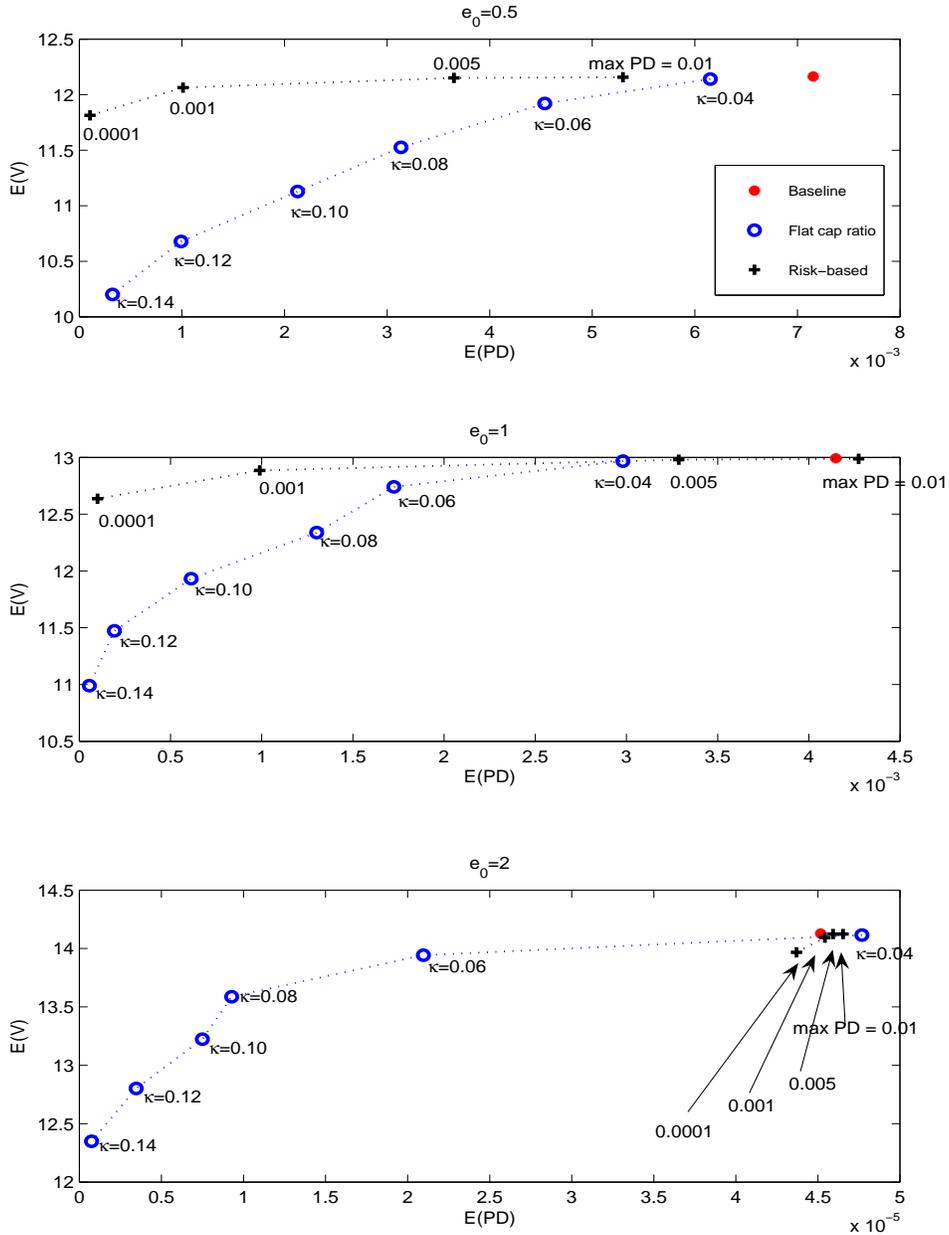


Figure 9: Comparison between the two capital regimes: a micro-perspective

*Note:* This figure plots expected franchise value and expected probability of failure for a very small bank (initial equity of 0.5, the top panel), a small bank (initial equity of 1, the middle panel) and a large bank (initial equity of 2, the bottom panel), with the good and bad states equally likely to occur in the long run. The circles refer to equilibrium outcomes under flat-rate capital requirements, with minimum capital ratios varying from 4% to 14%. The “+” symbols refer to equilibrium outcomes under the credit VaR-based capital regulation, with threshold default rates varying from 1% to 0.01%.

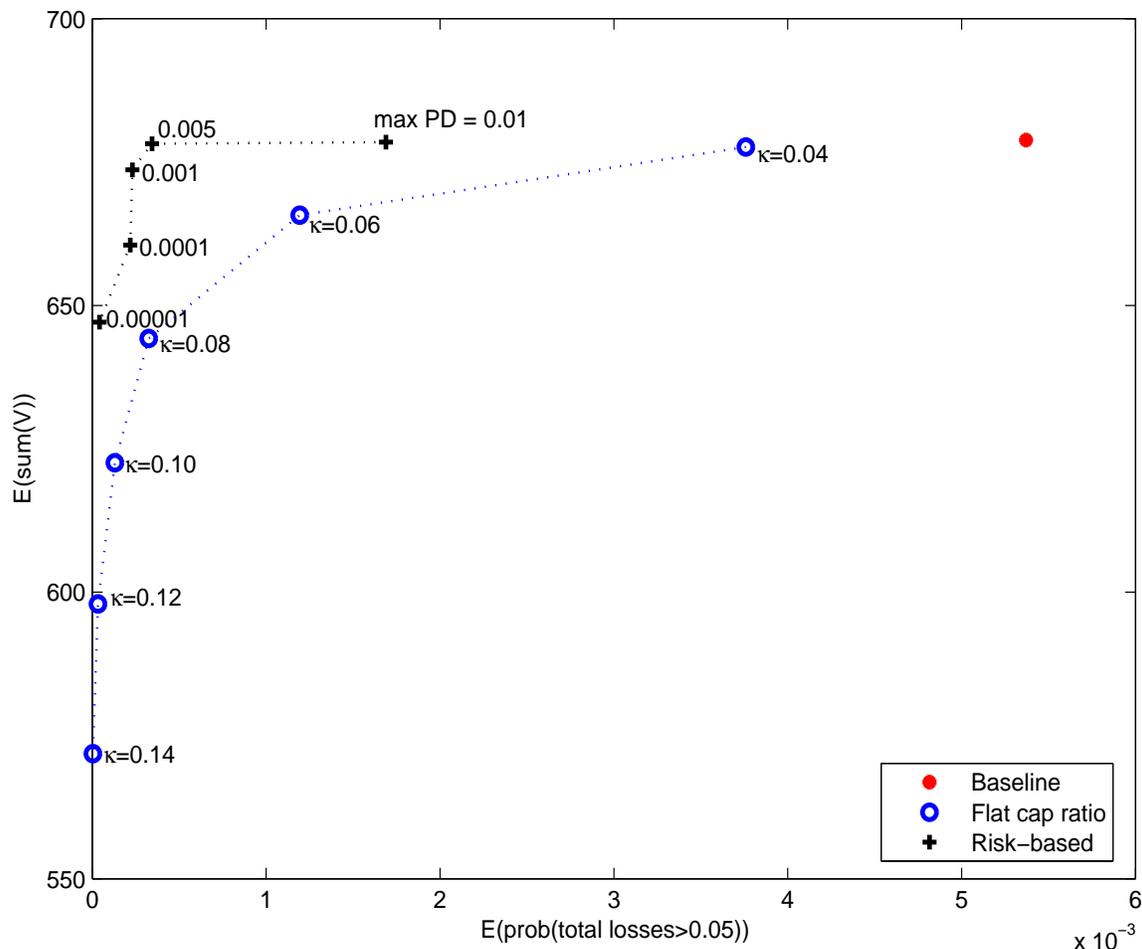


Figure 10: Comparison between the two capital regimes: a macro-perspective

*Note:* This figure plots expected franchise value and expected probability of failure for a group of banks that consist of 5 large banks (initial equity of 2) and 50 small banks (initial equity of 1), with the good and bad states equally likely to occur in the long run. The horizontal axis represents the probability that more than 5% of total bank assets default in one period and the vertical axis represents total franchise value. The circles refer to equilibrium outcomes under flat-rate capital requirements, with minimum capital ratios varying from 4% to 14%. The “+” symbols refer to equilibrium outcomes under the credit VaR-based capital regulation, with threshold default rates varying from 1% to 0.001%.