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by Nikola Tarashev and Haibin Zhu

Monetary and Economic Department

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Keywords: Correlated defaults, value at risk, multiple common factors, granularity, estimation error, tail dependence, bank capital.

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## Abstract

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# 1 Introduction

Assessments of portfolio credit risk, which are often based on analytical models, have attracted much attention in recent years. One reason is that participants in the increasingly popular market for structured finance products rely heavily on estimates of the interdependence of credit risk across various exposures.[1] Such estimates are also of principal interest to financial supervisors who, in enforcing new standards in the banking and insurance industries, have to ensure that regulatory capital is closely aligned with credit risk.

This paper investigates the well-known Asymptotic Single-Risk Factor (ASRF) model of portfolio credit risk. The popularity of this model stems from its implication that the contribution of each exposure to the credit value-at-risk (VaR) of the portfolio – defined as the maximum default loss that can be incurred with a given probability over a given horizon – is independent of the characteristics of the other exposures. This implication – which has been derived rigorously in Gordy (2003) and is known as *portfolio invariance* of marginal credit VaR contributions – underpins the internal-ratings-based (IRB) approach of the Basel II framework (BCBS, 2005). The implication has been interpreted as alleviating the data requirements and computational burden on users of the model. Indeed, portfolio invariance implies that credit VaR can be calculated solely on the basis of exposure-specific parameters, including individual probability of default (PD), loss-given-default (LGD) and dependence on the common factor.

Its popularity notwithstanding, the "portfolio invariance" implication of the ASRF model hinges on two strong assumptions that have been criticized as sources of *misspecification errors*. Namely, the model assumes that the systematic component of credit risk is governed by a single common factor and that the portfolio is so finely grained that all idiosyncratic risks are diversified away. Violations of the "single-factor" and "perfect granularity" assumptions would translate directly into erroneous assessments of portfolio credit risk.

Moreover, an application of the ASRF model may be quite challenging, even if this model is well specified. An important reason is that the portfolio invariance implication does *not* exempt a user of the model from adopting a global approach. In particular, estimates of exposure-specific dependence on the common factor, which are required for a calibration of the ASRF model, hinge on information about the correlation structure in the portfolio

[1] Examples of structured finance products are collateralized debt obligations (CDOs), nth-to-default credit default swaps (CDSs) and CDS indices.

or about the common factor itself.[2] When such portfolio- or market-wide information is imperfect, the user will implement a *flawed calibration* of the model, which will be another source of errors in measured portfolio credit risk.

A contribution of this paper is to develop a *unified* method for quantifying the importance of model misspecification and calibration errors in assessments of portfolio credit risk. In order to implement this method, we rely on a large data set that comprises Moody's KMV estimates of PDs and pairwise asset return correlations for nearly $11,000$ non-financial corporates worldwide. We treat these estimates as the actual credit risk parameters of hypothetical portfolios that match the industrial-sector concentration of typical portfolios of US wholesale banks. For each such portfolio, we derive the "true" probability distribution of default losses and then condense this distribution into unexpected losses, which are defined as a credit VaR net of expected losses. This summary statistic is equivalent to a "target" capital measure necessary to cover default losses with a desired probability.[3] The target capital measure can be compared directly to a "shortcut" capital measure, which is based on the ASRF model and a rule-of-thumb calibration of exposure-specific dependence on a single common factor.

We decompose the difference between the target and shortcut capital measures into four non-overlapping and exhaustive components. Two of these components relate to the sources of misspecification of the ASRF model discussed above and are attributed to a "multi-factor" effect and a "granularity" effect, respectively. The other two components, which we derive after transforming the correlation structure to be consistent with the ASRF model, relate to errors in the calibration of the inter-dependence of credit risk across exposures.[4] Specifically, the calibration errors we consider arise either from an overall bias in the measured correlations of firms' asset returns – which gives rise to what we dub the "correlation level" effect – or from noise in the measured dispersion of these correlations across pairs of firms – the "correlation dispersion" effect.

Another contribution of this paper is that it provides two additional perspectives on flaws in the calibration of the ASRF model. First, we calculate deviations from a desired capital

[2] The IRB capital formula of Basel II avoids the necessity of a global approach by postulating that firm-specific dependence on the single common factor is determined fully by the level of the corresponding PD.

[3] In this paper, we use the terms "assessment of credit risk" and "capital measure" interchangeably. Importantly, our capital measures do not correspond to "regulatory capital", which reflects considerations of bank supervisors, or to "economic capital", which reflects additional strategic and business objectives of financial firms.

[4] In order to sharpen the analysis, we do not analyse the implications of errors in the estimates of PDs and LGDs.

buffer that arise not from the adoption of rule-of-thumb parameter values but from plausible small-sample errors in asset return correlation estimates. Second, motivated by the analysis in Gordy (2000) and Frey and McNeil (2003), we examine the importance of errors in the calibrated tail dependence among individual asset return distributions. The impact of such errors on capital measures is similar to but separate from the impact of errors in estimated asset return correlations.

Our main conclusion is that errors in the practical implementation, as opposed to the specification, of the ASRF model are the main sources of potential miscalculations of the credit risk in large portfolios. Specifically, the misspecification-driven multi-factor effect leads a user of the model to underpredict target capital buffers by only 1%. This is because a single-factor approximation, if chosen carefully, fits well the correlation structure of asset returns in our data. Similarly, the granularity effect causes calculated capital to underpredict the target level by 5%. By contrast, assessments of portfolio credit risk are considerably more sensitive to possible miscalibrations of the single-factor model. The correlation dispersion effect, for example, leads to capital measures that are 12% higher than the target level. In turn, the correlation level effect makes calculated capital over(under)predict the target measure by roughly 8% for each percentage point over(under)estimation of the average correlation coefficient. Furthermore, plausible small-sample errors in correlation estimates – arising when users of the model have 5 to 10 years of monthly asset returns data – translate into capital measures that may deviate from the target level by 30 to 45%.[5] Finally, data on asset returns suggests that the empirical tail dependence among the underlying distributions is at odds with the conventional multi-normality assumption. Concretely, a capital measure that incorporates this assumption underestimates the target level by 22 to 86%.

Among the four effects on capital calculations, only the granularity effect is sensitive to the number of exposures in the portfolio. When this number decreases, the portfolio maintains a larger portion of idiosyncratic risks and, thus, we are not surprised to find that the granularity effect leads to a 19% underestimation of target capital in typical small portfolios.

In comparison to articles in the related literature, this paper covers a wider range of errors in assessments of portfolio credit risk. Most of these articles have focused exclusively

[5] Of course, it is reasonable to expect that plausible small-sample estimation errors would have a significant impact on capital measures based on any model of portfolio credit risk, irrespective of whether the model accommodates a single or multiple factors. We do not quantify such an impact, however, because doing so would be a digression from the main focus of our analysis.

on misspecifications of the ASRF model and have proposed ways to partially correct for them without impairing the model's tractability. Empirical analyses of violations of the perfect granularity assumption include Martin and Wilde (2002), Vasicek (2002), Emmer and Tasche (2003), and Gordy and Luetkebohmert (2006). For their part, Pykhtin (2004), Duellmann (2006), Garcia Cespedes et al. (2006) and Duellmann and Masschelein (2006a) have analyzed implications of the common factor assumption under different degrees of portfolio concentration in a limited number of industrial sectors. In addition, Heitfield et al. (2006) and Duellman et al. (2006) have examined both granularity and sector concentration issues in the context of US and European bank portfolios, respectively.[6] A small branch of the related literature, which includes Loeffler (2003) and Morinaga and Shiina (2005), has analyzed calibration issues and has derived that noise in model parameters can have a significant impact on assessments of portfolio credit risk.

The remainder of this paper is organized as follows. Section 2 outlines the ASRF model and the empirical methodology applied to it. Section 3 describes the data and Section 4 reports the empirical results. Finally, Section 5 concludes.

# 2 Methodology

In this section, we first outline the ASRF model. Then, we discuss how violations of its key assumptions or flawed calibration of its parameters can affect assessments of portfolio credit risk. Finally, we develop an empirical methodology for quantifying and comparing alternative sources of error in such assessments.

## 2.1 The ASRF model[7]

The ASRF model of portfolio credit risk – introduced by Vasicek (1991) – postulates that an obligor defaults when the value of its assets falls below some threshold. In addition, the model assumes that asset values are driven by a single common factor:

$$V_{iT} = \rho_i \cdot M_T + \sqrt{1-\rho_i^2} \cdot Z_{iT} \tag{1}$$

[6]The recent working paper by the Basel Committee on Bank Supervision (BCBS, 2006) provides an extensive review of these articles.

[7]This section provides an intuitive discussion of the ASRF model. For a rigorous and detailed study of this model, see Gordy (2003). In addition, Frey and McNeil (2003) examine the calibration of the ASRF model under the so-called Bernoulli mixture representation.

where: $V_{iT}$ is the value of assets of obligor $i$ at time $T$; $M_T$ and $Z_{iT}$ denote the common and idiosyncratic factors, respectively; and $\rho_i \in [-1, 1]$ is the obligor-specific loading on the common factor. The common and idiosyncratic factors are independent of each other and scaled to random variables with mean 0 and variance 1.[8] Thus, the asset return correlation between borrowers $i$ and $j$ is given by $\rho_i \rho_j$.

The ASRF model delivers a closed-form approximation to the probability distribution of default losses on a portfolio of $N$ exposures. The accuracy of the approximation increases when the number of exposures grows, $N \to \infty$, and the largest exposure weight shrinks, $\sup_i (w_i) \to 0$. In these limits, as the portfolio becomes perfectly granular, the probability distribution of default losses can be derived as follows. First, let the indicator $\mathcal{I}_{iT}$ equal 1 if obligor $i$ is in default at time $T$ and 0 otherwise. Conditional on the value of the common factor, the expectation of the indicator equals

$$\begin{aligned} E(\mathcal{I}_{iT}|M_T) &= \Pr(V_{iT} < \mathcal{F}^{-1}(PD_{iT})|M_T) \\ &= \Pr(\rho_i \cdot M_T + \sqrt{1-\rho_i^2} \cdot Z_{iT} < \mathcal{F}^{-1}(PD_{iT})|M_T) \\ &= \mathcal{H}\left(\frac{\mathcal{F}^{-1}(PD_{iT}) - \rho_i M_T}{\sqrt{1-\rho_i^2}}\right) \end{aligned}$$

where: $PD_{iT}$ is the unconditional probability that obligor $i$ is in default at time $T$; the cumulative distribution function (CDF) of $Z_{iT}$ is denoted by $\mathcal{H}(\cdot)$; and the CDF of $V_{iT}$ is $\mathcal{F}(\cdot)$, implying that the default threshold equals $\mathcal{F}^{-1}(PD_{iT})$.

Second, under perfect granularity, the Law of Large Numbers implies that the conditional total loss on the portfolio, $TL|M$, is deterministic for any value of the common factor $M$:

$$\begin{aligned} TL|M &= \sum_i w_i \cdot E\left(LGD_i\right) \cdot E(\mathcal{I}_i|M) \\ &= \sum_i w_i \cdot E\left(LGD_i\right) \cdot \mathcal{H}\left(\frac{\mathcal{F}^{-1}(PD_i) - \rho_i M}{\sqrt{1-\rho_i^2}}\right) \end{aligned} \qquad (2)$$

where time subscripts have been suppressed. In addition, the loss-given-default of obligor $i$, $LGD_i$, is assumed to be independent of both the common and idiosyncratic factors.[9]

[8] The ASRF model can accommodate distributions with infinite second moments. Nonetheless, we abstract from this generalization in order to streamline the analysis.

[9] The ASRF model does allow for inter-dependence between asset returns and the LGD random variable. Such inter-dependence leads to another dimension in the study of portfolio credit risk, which is explored by Kupiec (2007). We abstract from this additional dimension in order to focus on the correlation of default events.

Finally, the conditional total loss $TL|M$ is a decreasing function of the common factor $M$ and, consequently, the unconditional distribution of $TL$ can be derived directly on the basis of equation (2) and the CDF of the common factor, $\mathcal{G}(\cdot)$. Denoting by $TL_{1-\alpha}$ the $(1-\alpha)^{th}$ percentile in the distribution of total losses, i.e. $\Pr(TL < TL_{1-a}) = 1-\alpha$, it follows that:

$$\begin{aligned} TL_{1-\alpha} &= \sum_i w_i \cdot E(LGD_i) \cdot \mathcal{H}\left(\frac{\mathcal{F}^{-1}(PD_i) - \rho_i \mathcal{G}^{-1}(\alpha)}{\sqrt{1-\rho_i^2}}\right) \\ &= TL|M_\alpha \end{aligned} \tag{3}$$

where $M_\alpha$ is the $\alpha^{th}$ percentile in the distribution of the common factor. The magnitude $TL_{1-\alpha}$ is also known as the credit VaR at the $(1-\alpha)$ confidence level.

Thus, in order to cover unexpected (i.e. total minus expected) losses with probability $(1-\alpha)$, the capital buffer for the entire portfolio should be set to:

$$\begin{aligned} \kappa &= TL_{1-\alpha} - \sum_i w_i \cdot E(LGD_i) \cdot PD_i \\ &= \sum_i w_i \cdot E(LGD_i) \cdot [\mathcal{H}\left(\frac{\mathcal{F}^{-1}(PD_i) - \rho_i \mathcal{G}^{-1}(\alpha)}{\sqrt{1-\rho_i^2}}\right) - PD_i] \\ &\equiv \sum_i w_i \cdot \kappa_i \end{aligned} \tag{4}$$

As implied by this equation, the capital buffer for the portfolio can be set on the basis of exposure-specific parameters. These parameters reflect the weight of a given exposure in the portfolio, the exposure's LGD and PD as well as the underlying dependence on the common factor. The flip side of this implication is that each exposure-specific portion of the capital buffer is independent of the rest of the portfolio and, thus, is *portfolio invariant*.

In practice, an implementation of the ASRF model requires that one specify the distribution of the common and idiosyncratic factors of asset returns. It is standard to assume normal distributions and rewrite equation (4) as:

$$\kappa = \sum_i w_i \cdot E(LGD_i) \cdot [\Phi\left(\frac{\Phi^{-1}(PD_i) - \rho_i \Phi^{-1}(\alpha)}{\sqrt{1-\rho_i^2}}\right) - PD_i] \tag{5}$$

where $\Phi(\cdot)$ is the CDF of a standard normal variable. Notice that equation (5) underpins the regulatory capital formula in the IRB approach of Basel II, in which *LGD* is set to be 45% and $\alpha$ is chosen as 0.1%.

## 2.2 Impact of model misspecification

The portfolio invariance implication of the ASRF model hinges on two key assumptions, i.e. that the portfolio is of perfect granularity and that there is a single common factor. In the light of this, we examine at a conceptual level how violations of either of these assumptions – which give rise to "granularity" and "multi-factor" effects – affect capital measures. For the illustrative examples in this section, we use the ASRF formula in equation (5) and set $\alpha = 0.1\%$.

### 2.2.1 Granularity effect

The granularity effect arises empirically either because of a limited number of exposures or because of exposure concentration in a small number of borrowers. In either of these cases, idiosyncratic risk is not fully diversified away. Therefore, the existence of a granularity effect implies that capital measures based on the ASRF model would be insufficient to cover unexpected losses.

The top-left panel in Figure 1 provides an illustrative example of the granularity effect. In this example, the desired capital level for a homogenous portfolio is computed as a function of the number of exposures (solid line).[10] In addition, the figure also plots the capital measure implied by the ASRF model (dotted line), which differs from the target one only in that it assumes an infinite number of exposures. The difference between the dotted and solid lines equals the magnitude of the granularity effect. As expected, the granularity effect is always negative and decreases when the number of exposures increases.

Gordy and Luetkebohmert (2006) derive a closed-form "granularity adjustment", which should approximate the *negative* of the granularity effect. When the portfolio is homogeneous, the approximation is linear in the reciprocal of the number of exposures, which is largely in line with the properties of the granularity effect plotted in Figure 1.[11]

### 2.2.2 Multi-factor effect

The impact of various macroeconomic and industry-specific conditions on portfolio credit risk may be best accounted for by generalizing equation (1) in order to incorporate *multiple* (potentially unobservable) common factors. Multiple common factors affect the likelihood

[10] The calculation of the desired capital level uses a Gaussian copula (see Appendix A).

[11] Further comparison between the granularity effect and the granularity adjustment of Gordy and Luetkebohmert (2006) is reported in Section 4.1 below.

of default clustering – i.e. the likelihood of a large number of defaults occurring over a given horizon – which influences the tails of the probability distribution of credit losses. In line with our empirical results (reported in Section 4), our conceptual analysis treats a fattening of these tails as implying unambiguously a higher level of the desired capital buffer.[12]

The existence of multiple common factors of credit risk violates the single-factor assumption of the ASRF model, leading to what we call a multi-factor effect in capital measures. Depending on the characteristics of the credit portfolio, the multi-factor effect could be either negative – i.e. implying that the ASRF model underestimates the desired capital – or positive. To illustrate the two possibilities, we generalize equation (1):

$$V_i = \rho_{1,i} \cdot M_1 + \rho_{2,i} \cdot M_2 + \sqrt{1 - \rho_{1,i}^2 - \rho_{2,i}^2} \cdot Z_i \tag{6}$$

assuming that $M_1$, $M_2$ and $Z$ are mutually independent standard normal variables.

In our first example, we consider a portfolio in which all exposures have equal weights, have the same PD and are divided into two groups according to their dependence on the common factors. For exposures in the first group, $0 < \rho_{1,i} = \rho < 1$ and $\rho_{2,i} = 0$; while $\rho_{1,j} = 0$ and $0 < \rho_{2,j} = \rho < 1$ for exposures in the second group. Thus, the common factors are group specific and underpin positive and homogeneous within-group pairwise correlations and zero across-group correlations. The solid line in the top-right panel of Figure 1 plots the desired capital measure for such a portfolio as a function of the relative weight of exposures in group 1.[13] This measure is lowest when the portfolio is most diversified between the two groups of exposures and, thus, the probability of large default losses is minimized. In addition, the dashed line in the figure plots an alternative capital measure, which is based on the ASRF model and is underpinned by a single-factor structure of the asset return correlations. This structure, which allows for providing the ASRF model with as much information about the true correlations as possible (see Appendix C), turns out to match extremely well the true *average* asset return correlation but to approximate only roughly the dispersion of correlation coefficients in the cross section of exposures.

The difference between the dashed and solid lines equals the multi-factor effect. This effect is *negative* because the single-factor assumption of the ASRF model ignores the fact

[12]Note that, depending on the confidence level of the targeted credit VaR, a fattening of the tails of the probability distribution of credit losses may either raise or lower the desired capital buffer. Ceteris paribus, fatter tails of the loss distribution translate into a higher desired level of the capital buffer if the value of $\alpha$ in equation (4) is sufficiently close to zero.

[13]The desired capital buffer is calculated on the basis of Monte Carlo simulations (see Appendix B) for a portfolio consisting of 1000 exposures.

that the common factors are two independent sources of default clustering, which leads to an underestimation of the desired capital. The underestimation is largest when the two groups enter the portfolio with equal weights, in which case the role of multiple factors is greatest.

It is possible, however, to construct another example, in which imposing an erroneous single-factor structure on portfolio credit risk distorts the interaction between asset return correlations and individual PDs in a way that leads to a *positive* multi-factor effect. Consider a portfolio comprising two groups of exposures, with the exposures in the first group being individually riskier but less correlated among themselves than the exposures in the second group. In terms of equation (6), this can be formalized by postulating that firms with high PDs feature $0 < \rho_{1,i} = \rho < 1$ and $\rho_{2,i} = 0$, whereas firms with low PDs feature $0 < \rho_{1,j} = \rho_{2,j} = \rho < 1$. A single-factor approximation to this correlation structure would match the average correlation coefficient but would also imply too high a correlation among riskier exposures. This could raise the probability of default clustering, suggesting a capital buffer that is larger than desired.[14]

## 2.3 Impact of calibration errors

Errors in the calibration of the ASRF model will affect assessments of portfolio credit risk even if this model is well specified. In this paper, we focus on errors in the calibration of the inter-dependence of credit risk across exposures, which can be driven by noise in the adopted values of asset return correlations or by a flawed assumption regarding the distribution of asset returns. When analyzing the consequences of such errors, we maintain our earlier practice and treat fattening of the tails of the loss distribution as implying unambiguously a higher 99.9% credit VaR and, thus, a higher desired level of the capital buffer. In this way, we sharpen the conceptual analysis and keep it in line with our empirical findings.

We study two general types of errors in calibrated asset return correlations: errors in the average correlation coefficient and errors in the dispersion of correlation coefficients across exposure pairs. Each error type is trivially independent of the granularity effect. In addition, extracting the multi-factor effect on the basis of the single-factor approximation described in Section 2.2.2 allows us to separate this effect from errors in the average correlation. By contrast, errors in the calibrated dispersion of asset return correlations could arise either as a result of imposing a single-factor structure on the correlation matrix in the presence of

[14] A similar result emerges when the dispersion of correlation coefficients is distorted and the true correlation structure is driven by a single common factor. In order to avoid overburdening of the exposition, we provide a graphical illustration only in the single-factor context (see Section 2.3 below).

multiple factors (recall Section 2.2.2) or as a result of noise in the estimated factor loadings when there is a single common factor. In this section, we are concerned with the second case, as it is consistent with a correct specification of the ASRF model and refers only to calibration errors.

The two types of errors in calibrated asset return correlations have various potential sources. One possibility is that a user of the ASRF model is data constrained and, hence, relies on rule-of-thumb values, which may simply be correlation estimates for popular credit indices. Such estimates will lead to a discrepancy between desired and calculated capital to the extent that the underlying indices are not representative of the user's own portfolio. Alternatively, a user of the model may have insufficient data on the assets of the obligors in the portfolio, which would lead to small-sample estimation errors in asset return correlations. Indeed, limitations on the availability of data points are likely to be important in practice because: (i) asset value estimates are typically available at low (i.e. monthly or quarterly) frequencies and (ii) supervisory texts require that financial institutions possess only five years of relevant data.[15]

A positive error in the average level of asset return correlations leads to a capital measure that is higher than the desired one (Figure 1, bottom-left panel). This result reflects the intuition that inflating asset return correlations increases the likelihood of default clustering, which fattens the tails of the loss distribution. In the remainder of this paper, the implication of such errors is dubbed the "correlation level" effect.

In turn, the effect of noise in the estimated dispersion of correlation coefficients can be seen in the following example. Suppose that all firms in one portfolio have homogeneous PDs and exhibit homogeneous pairwise asset return correlations. Suppose further that a second portfolio is characterized by the same PDs and average asset return correlation but includes a group of firms that are more likely to default together. The second portfolio, in which pairwise correlations exhibit dispersion, is more likely to experience several simultaneous defaults and, thus, has a loss distribution with fatter tails. Consequently, between the two portfolios, the second one requires higher capital in order to attain solvency with the same probability. This is portrayed by the upward slope of the solid line in the bottom-right panel of Figure 1 and is a particular instance of what we dub the "correlation dispersion" effect, which arises in the context of a single common factor.

[15]Data limitations are likely to be important irrespective of how a user of the model estimates asset return correlations. Such estimates may rely on balance sheet information and stock market data. Alternatively, as derived in Tarashev and Zhu (2006), asset return correlations can be extracted from the CDS market.

This result can be strengthened (dashed line in the same panel) but also weakened or even reversed if PDs vary across firms. To see why, consider the previous example but suppose that the strongly correlated firms in the second portfolio are the ones that have the lowest individual PDs. In other words, the firms that are likely to generate multiple defaults are less likely to default. As a result, greater dispersion of asset return correlations may lower the probability of default clustering in the second portfolio to an extent that depresses the desired capital level below that for the first portfolio. This is illustrated by the negative slope of the dotted line in the bottom-right panel of Figure 1.

Even if asset return correlations were known with certainty, a flawed calibration of the *marginal* distributions of asset returns would still drive errors in the calibrated interdependence of credit risk across exposures. Although the ASRF model imposes quite weak restrictions on asset return distributions, it is common practice to adopt distributions whose main advantages stem not from realistic features but from operational convenience. In particular, the consensus view in the literature is that asset returns have fatter tails than those imposed by the conventional normality assumption. To the extent that this fatness of the tails reflects the distribution of the common factor, the probability of default clustering and, thus, the desired capital level would be higher than those derived under normality (Hull and White, 2004; Tarashev and Zhu, 2006). We study this issue by considering Student-$t$ distributions for both the common and idiosyncratic factors of asset returns.

## 2.4 Evaluating various sources of error

An important contribution of this paper is to present a unified empirical method for quantifying the impact of several sources of error in model-based assessment of portfolio credit risk. In particular, we focus on the difference between target capital measures and shortcut ones, the latter of which are based on the ASRF model and possible erroneous calibration of its parameters. We dissect this difference into four non-overlapping and exhaustive components, attributing them to the multi-factor, granularity, correlation level and correlation dispersion effects. In order to probe further the likely magnitude of the last two effects, we derive plausible small-sample errors that could affect direct estimates of asset return correlations. Finally, we also examine the implications of erroneous assumptions regarding the marginal distribution of asset returns.

The basic empirical method consists of two general steps. In the first step, we construct a hypothetical portfolio that is either "large" – consisting of 1,000 equal exposures – or "small" – consisting of 200 equal exposures.[16] The sectoral composition of such a portfolio is designed to be in line with the typical loan portfolio of large wholesale banks in the United States.[17] Given the constraints of such a composition, the portfolio is drawn at random from our sample of firms. Since each simulated portfolio is subject to sampling noise, we examine 3,000 different draws for both large and small portfolios.

For a portfolio constructed in the first step, the second step calculates five alternative capital measures, which differ in the underlying assumptions regarding the inter-dependence of credit risk across exposures. Each of these alternatives employs the same set of PD values and assumes that asset returns are normally distributed. In addition, each alternative is based on the assumption that LGD is a random variable, which has a symmetric triangular distribution that is identical across exposures, peaks at 50% and has a continuous support on the interval $[0\%, 100\%]$.[18]

Each measure differs from a previous one owing to a single assumption:

1. The *target capital* measure incorporates data on asset return correlations, which are treated as representing the "truth". Using these correlations, we conduct Monte Carlo simulations to construct the "true" probability distribution of default losses at the one-year horizon. The implied 99.9% credit VaR minus expected losses equals target capital (see Appendix B for further detail).

2. The second capital measure differs from target capital only owing to a restriction on the number of common factors governing asset returns. In particular, we adopt a correlation matrix that fits the original one as closely as possible under the constraint that correlation coefficients should be consistent with the presence of a single common factor (see Appendix C). The fitted single-factor correlation matrix is used to derive the one-year probability distribution of joint defaults on the basis of the so-called

[16]The distinction between what we dub large and small portfolios does not reflect the size of the aggregate exposure but rather different degrees of diversification across individual exposures. Importantly, the degree of diversification in the large hypothetical portfolios studied in this paper matches the diversification in the large real-world portfolios studied by Heitfield et al. (2006) (see Section 3.2). The small hypothetical portfolios exhibit similar properties.

[17]Such a portfolio does not incorporate consumer loans and, thus, may not be representative of all aspects of credit risk.

[18]The LGD specification warrants an explanation. The independence between the incidence of defaults and LGDs implies that, in the absence of simulation noise, only the mean of the LGD distribution enters (as a multiplicative factor) capital measures. However, the entire LGD distribution affects measures obtained from Monte Carlo simulations. Importantly, assuming a continuous distribution for LGDs smooths the derived probability distribution of joint defaults, which improves the robustness of simulation-based capital measures.

Gaussian copula method (see Appendix A). This distribution is then mapped into a probability distribution of default losses and, finally, into a capital measure.

3. The third capital measure differs from the second one only in that it assumes that all idiosyncratic risk is diversified away. This assumption allows us to use the fitted single-factor correlation matrix underpinning measure 2 in the ASRF formula (equation 5).

4. The fourth capital measure differs from the third one only in that it is based on the assumption that loading coefficients on the single common factor are the same across exposures. The resulting common correlation coefficient, which is set equal to the average of the pairwise correlations underpinning measures 2 and 3, is used as an input to the ASRF formula (equation 5).

5. Finally, the *shortcut* capital measure differs from the fourth one only in that it incorporates alternative, rule-of-thumb, values for the common correlation coefficient.

The three intermediate measures lead to a straightforward dissection of the difference between target and shortcut capital.[19] Specifically, the difference between measures 5 and 1 is the sum of the following four components: (i) the difference between measures 2 and 1, which equals the multi-factor effect; (ii) the difference between measures 3 and 2, which equals the granularity effect; (iii) the difference between measures 4 and 3, which equals the correlation dispersion effect; and (iv) the difference between measures 5 and 4, which equals the correlation level effect.

The specific ordering and choice of the three intermediate capital measures is a result of the following reasoning. As far as measure 2 is concerned, its position is fixed by the necessity to extract the multi-factor effect first. The reason for this is twofold. First, deriving a capital measure that assumes an infinite number of exposures but allows for multiple factors (i.e. calculating measure 2 after the extraction of the granularity effect) is subject to approximation errors (see Pykhtin, 2004, for example). Second, it is possible to isolate calibration errors (via measures 3 and 4) only after the extraction of the multi-factor effect (via measure 2) has modified the original correlation matrix to render it consistent with the ASRF model. Likewise, an application of this model for the extraction of calibration

[19]Importantly, the method also applies to alternative definitions of *target* and *short-cut* capital, so long as the *true* correlation structure and short-cut correlation estimates chosen by the user are clearly defined.

errors requires the assumption of infinite granularity, which fixes the position of measure 3.[20] Finally, modifying measure 4 by preserving the cross-sectional distribution of the single-factor correlation coefficients but changing their average level would reverse the order in which the correlation level and dispersion effects are extracted. An important problem with this procedure is that it would allow only for an imperfect estimate of the correlation level effect because this estimate would be influenced by changes in the structure of the single-factor correlation matrix.

### 2.4.1 Two extensions

In an attempt to delve further into the impact of *plausible* calibration errors on capital measures, we conduct two additional exercises. Each exercise focuses on a specific type of errors in the calibrated inter-dependence of defaults and incorporates the assumption that the true PDs are identical across exposures. This assumption insulates capital measures from the impact of interaction between heterogeneous PDs and errors in the calibrated inter-dependence of defaults.

In the first exercise, we derive the extent to which plausible limitations on the size of available data can affect assessments of portfolio credit risk by affecting the estimates of asset return correlations. Specifically, we draw time series of asset returns from a joint distribution characterized by constant pairwise correlations equal to the correlation underpinning measure 4. Using the sample correlation matrix of the simulated series and a typical value for the probability of default, we obtain an "estimated" capital measure on the basis of the ASRF formula in equation 5.[21] The difference between this measure and the desired capital, which employs the *exact* correlation structure, are driven by small-sample noise in the estimates of the overall level and dispersion of asset return correlations.

In our second exercise, we examine how measure 4 would change if the common and idiosyncratic factors of asset returns are in fact driven by Student-$t$ distributions. The results of this exercise reveal how flawed calibration of the tail dependence among exposures' asset returns – which is separate from flaws in correlation values – affects capital calculations. In order to carry out the exercise, we use the general ASRF formula in equation (4) and make two technical adjustments to the empirical setup. The first adjustment corrects for the

[20] Despite this observation, we have also experimented with imposing the infinite granularity assumption only after we calculate the correlation dispersion and level effects (i.e. deriving measure 3 after measures 4 and 5). This modifies the meaning of these two effects but alters negligibly their magnitudes, as well as the magnitude of the granularity effect.

[21] This measure abstracts from the granularity and multi-factor effects.

fact that the variance of a Student-$t$ variable is larger than unity.[22] The second adjustment addresses the fact that the generalized CDF of asset returns, $\mathcal{F}(\cdot)$, does not exist in closed form. In concrete terms, we calculate the default threshold $\mathcal{F}^{-1}(PD_i)$ on the basis of 10 million Monte Carlo simulations.

# 3 Data description

This section describes the two major blocks of data that we rely on: (i) credit risk parameter estimates provided by Moody's KMV and (ii) the sectoral distribution of exposures in typical portfolios of US wholesale banks.

## 3.1 Credit risk parameters

Our sample includes the universe of firms covered in July 2006 by both the expected default frequency (EDF$^{\text{TM}}$) model and the global correlation (GCorr$^{\text{TM}}$) model of Moody's KMV. These two models deliver, respectively, estimates of 1-year physical PDs and physical asset return correlation coefficients for publicly traded companies. We abstract from financial firms – whose capital structure makes their PDs notoriously difficult to estimate – and work with 10,891 companies.

The sample covers firms with diverse characteristics. Specifically, 5,709 of the firms are headquartered in the United States, 4,383 in Western Europe, and the remaining 799 in the rest of the world. The distribution of the 10,891 firms across industrial sectors is reported in the last column in Table 1, with the largest share of firms (10.4%) coming from the business service sector. Importantly, only 1,434 (or 13.2%) of the firms have a rating from either S&P or Moody's, which matches the stylized fact that the majority of bank exposures are unrated.

There are several reasons why EDFs and GCorr correlations are natural data for our exercise. First, both measures are derived within the same framework, which builds on the model of Merton (1974) and is in the spirit of the ASRF model (see Das and Ishii, 2001; Crosbie and Bohn, 2003; Crosbie, 2005, for detail). Second, in line with their role in this paper, Moody's KMV EDFs have been widely used as proxies for actual default probabilities (see Berndt et al., 2005; Longstaff et al., 2005, for example). Third, the GCorr correlations have a multi-factor structure, which is crucial for our study of the multi-factor effect. In

[22] Specifically, a Student-$t$ variable with $r > 2$ degrees of freedom has a variance of $\frac{r}{r-2}$.

particular, this model incorporates 120 common factors, including 2 global economic factors, 5 regional economic factors, 7 sector factors, 61 industry-specific factors and 45 country-specific factors.

Table 2 and Figure 2 report summary statistics of the Moody's KMV 1-year PD and asset return correlation estimates. PDs have a long right tail and, thus, their median (0.39%) is much lower than their mean (2.67%). In addition, the favorable credit conditions in July 2006 result in 1,217 firms (i.e. about 11.2% of the total) having the lowest EDF score (0.02%) allowed by the Moody's KMV empirical methodology. At the same time, the upper bound on the Moody's KMV PD estimates (20%) is attained by 643 firms. For their part, GCorr correlations are limited between 0 and 65%. Clustered mainly between 5% and 25%, these correlations average 9.24%.[23]

## 3.2 Characteristics of hypothetical portfolios

The portfolios we simulate match the sectoral distribution of the typical portfolio of US wholesale banks. Specifically, to construct a large portfolio (1000 exposures), we apply the 40 non-financial sector weights reported by Heitfield et al. (2006) (see Table 1). For a small portfolio (200 exposures), we rescale the 10 largest sectoral weights so that they sum up to unity and set all other weights to zero. Within each sector, we draw firms at random.[24] All firms in a portfolio receive equal weights (up to a rounding error) and, thus, there is a one-to-one correspondence between the number of firms in a sector and this sector's weight in the portfolio.

The sector and name concentration indices, reported at the bottom of Table 1, provide justification for our design of large and small portfolios. Calculated as the sum of squared sectoral (name) weights, the sector (name) concentration index of the large portfolios studied in this paper equals 0.0432 (0.001). This belongs to the corresponding interval $[0.03, 0.045]$ ($[0.000, 0.003]$) reported in Heitfield et al. (2006) for large portfolios of US banks. For small portfolios, the analogous indices and intervals are 0.1135 (0.005) and $[0.035, 0.213]$ ($[0.001, 0.008]$), respectively.

[23] The GCorr correlation estimates are quite in line with correlation estimates reported in other studies. For instance, Lopez (2004) documents an average asset correlation of 12.5% for a large number of US firms and Duellman et al. (2006) estimate a median asset return correlation of 10.1% for European firms.

[24] Within each industry sector, we draw randomly *with* replacement. If the same firm is drawn twice, the corresponding pairwise correlation is set equal to the average correlation for the sector. Drawing randomly *without* replacement does not affect materially the results.

# 4 Empirical results

We implement the empirical methodology described in Section 2.4 in order to quantify the impact of various sources of error in ASRF-based assessments of portfolio credit risk. Before reporting our findings, it is useful to highlight several aspects of the methodology.

First, as far as calibration of the model is concerned, the analysis in this paper focuses exclusively on errors in the values of parameters that relate to the inter-dependence of credit risk across exposures. Considering the impact of noise in PD and LGD estimates would make it extremely difficult to isolate the correlation level and dispersion effects we focus on. This is because noise in PDs and LGDs would interact with noise in correlation inputs in a highly non-linear fashion.

Second, we make the stylized assumption that portfolios consist of equally weighted exposures. Considering disparate exposure sizes would require considering an additional dimension of portfolio characteristics, as it will no longer be the case that the granularity of a larger portfolio is necessarily finer. In addition, lower granularity that results from higher concentration in a small number of borrowers would also have a bearing on the number of common factors affecting the portfolio and on the overall correlation of risk. This would make it impossible to isolate the granularity effect from the other three effects we consider.

Finally, our analysis treats the correlation matrix provided by Moody's KMV as revealing the "true" correlation of asset returns. Of course, this matrix is itself an estimate that is subject to errors. Nevertheless, the Moody's KMV correlation matrix provides a reasonable benchmark to work from. In addition, we have verified that results regarding the *relative* importance of alternative sources of error depend only marginally on the accuracy of the GCorr estimates, even though the *absolute* impact of alternative sources of error does change with the benchmark correlation level.

## 4.1 Various errors in shortcut capital measures

To study various sources of error in assessments of portfolio credit risk, we start by calculating the five capital measures listed in Section 2.4 for 3000 large and as many small hypothetical portfolios. Even though they have the same sectoral composition by construction, the simulated portfolios differ from each other with respect to the individual constituent exposures and, thus, with respect to the underlying risk parameters. For example, as reported in Table 3, the average PDs in large portfolios have a mean of 2.42% but, owing to sampling variance, vary between 1.79% and 3.12%. In comparison, the average asset return correlation remains more stable, ranging between 9.14% and 10.73% across large portfolios. In addition, in accordance with an assumption of the Basel II IRB approach, firms with higher PDs tend to be less correlated with the rest of the portfolio.[25] This is illustrated succinctly by the last line in each panel of Table 3, which reports negative correlations between individual PDs and the corresponding loading on a single common factor.[26]

Table 4 reports summary statistics of the target and shortcut capital measures (i.e. the two extremes described in Section 2.4). For large portfolios, the table reveals that target capital averages 3.31% (per unit of aggregate exposure) across the 3000 simulated portfolios. The corresponding shortcut level (based on a rule-of-thumb asset return correlation of 12%) is 81 basis points higher.

Decomposing the difference between target and shortcut capital for large portfolios reveals that errors caused by model misspecification play a minor role. In qualitative terms, the multi-factor effect can be of either sign (fourth column in Table 4) but is more likely to be negative (fourth and fifth columns in Table 4). In the light of the discussion in Section 2.2.2, imposing a single-factor framework is more likely to lead to too low a capital buffer because such a framework ignores the existence of multiple sources of default clustering. In quantitative terms, however, the multi-factor effect entails an average discrepancy that amounts to less than 1% of the average target capital level.[27] This is because the single-factor approximation fits closely the raw correlation matrix. Indeed, our single-factor approximation matches almost perfectly the level of average correlations (with a maximum discrepancy across simulated portfolios of less than 4 basis points) and explains on average 76% of the variability of pairwise correlations in the cross section of exposures.[28,29]

Similarly, the granularity effect is with the expected negative sign but, for large portfolios, leads to a small deviation from target capital. With an average of $-16$ basis points (or 5% of target capital), this deviation is higher than that induced by the multi-factor effect but is

[25] The negative relationship between PDs and correlations (or loading coefficients in a single-factor setting) is likely to be a general phenomenon. See, for example, Lopez (2004), Arora et al. (2005) and Dev (2006) who find that global factors often play bigger roles for firms of better credit quality.

[26] This calculation is conducted under the single-factor approximation of the correlation matrix

[27] A similar result is obtained by Duellmann and Masschelein (2006b) who rely on Pykhtin (2004) to approximate the multi-factor effect in loan portfolios of German banks.

[28] The goodness-of-fit measure for the one-factor approximation is described in Appendix C. Across the 3000 simulations of large portfolios, this measure ranges between 67% and 85%. For small portfolios, the range is 63% to 86%.

[29] Principal component analysis confirms this result. Specifically, the portion of the total variance of asset returns explained by the first principal component is at least 10 times larger than the portion explained by the second principal component.

still small.

Furthermore, the granularity effect we calculate is approximated extremely well by (the negative of) the closed-form granularity adjustment of Gordy and Luetkebohmert (2006), which averages −17 basis points for large portfolios in our sample. In addition, the correlation between the granularity effect and the granularity adjustment across large simulated portfolios is 66%.

By contrast, erroneous calibration of the ASRF model leads to much greater deviations from the target capital. For large portfolios, the correlation dispersion effect raises the capital measure by 39 basis points, which amounts to roughly 12% of the target level.[30] The sign of the effect reflects the regularity that exposures with higher PDs tend to be less correlated with the rest of the portfolio. The shortcut capital measure ignores this regularity and, in line with the intuition provided in Section 2.3, overestimates the target capital.

The correlation level effect has a similarly important implication. Specifically, this effect reveals that raising the average correlation coefficient from 9.78% (the one observed in the data) to a rule-of-thumb value of 12% leads to an 18% overestimation of the target capital level. The sign of the deviation is not surprising in the light of the discussion in Section 2.3. Importantly, the shortcut measure drops (rises) by roughly 8% with each percentage point decrease (increase) in the homogeneous correlation coefficient. Thus, using a rule-of-thumb correlation of 6% leads to a 32% *under*-estimation of the target level.[31]

Turning to small portfolios, the decomposition results are qualitatively the same, with the notable exception of the granularity effect. In these portfolios, a much smaller portion of the idiosyncratic risk is diversified away and the granularity effect equals −73 basis points, which is a 19% underestimation of the target capital. This underestimation is approximated well by the Gordy and Luetkebohmert (2006) granularity adjustment, (the negative of) which averages −86 basis points for small portfolios and whose correlation with the corresponding granularity effect is 89%.

## 4.2 Regression analysis of calibration errors

Given the dominant role of correlation level and dispersion effects as determinants of model-based assessments of portfolio credit risk, we investigate the sources of these two effects via a regression analysis. The regressions – run on the cross section of simulated portfolios – are

[30] On the basis of a hypothetical portfolio of US firms, Hanson et al. (2006) also demonstrate the importance of accounting for cross-sectional heterogeneity in credit risk parameters.

[31] The rule-of-thumb asset return correlations reported in the literature range between 5 and 25%.

simple linear models of *calibration-driven capital discrepancy*, which is defined as shortcut capital (based on a correlation of 12%) minus target capital net of the multi-factor and granularity effects.

We consider two blocks of explanatory variables. The first block comprises the average level and the dispersion of the asset return correlation coefficients underlying each simulated portfolio. These variables are natural drivers of the correlation level and dispersion effects and would explain the two effects completely if assessments of portfolio credit risk did not depend on the interaction of asset return correlations with PDs. In order to account for such interaction, we include a second block of explanatory variables, which comprises average PDs and the cross-sectional correlations between PDs and single-factor loading coefficients. One would recall that the PDs underlying target capital are identical to those underlying the shortcut measure. Thus, the regression coefficient of the first variable in the second block reflects how a general rise in single-name credit risk interacts with the different average correlations and correlation structures behind the two capital measures. Finally, the coefficient of the last explanatory variable captures the component of the correlation dispersion effect that is driven by a systematic relationship between individual firms' riskiness and their dependence on the common factor.[32]

The regression results, reported in Table 5, reveal that the correlation level and dispersion variables have strong explanatory power. Depending on the portfolio size, these variables explain one-third or more of the variation in calibration-driven capital discrepancies across simulated portfolios and enter the regressions with statistically significant coefficients of the expected signs. First, the positive impact of a higher average correlation on target capital translates into a negative impact on capital discrepancy because the correlation underpinning the shortcut measure stays constant across simulated portfolios. Second, the correlation dispersion variable enters with a positive coefficient because – given that the empirical relationship between PDs and asset return correlations is negative and that the shortcut capital measure abstracts from correlation dispersion – shortcut capital overpredicts the target level by more when correlation dispersion is greater. In order to visualize this phenomenon refer back to the dotted line in the bottom-right panel of Figure 1. In this plot, shortcut capital appears at zero correlation dispersion ($c = 0$) and a rise in correlation dispersion (i.e. a rise in $c$) translates into a downward movement of target capital along the

[32]The correlation dispersion variable is calculated as the standard deviation of the loading coefficients under the single-factor approximation of the correlation matrix. The correlation between these loading coefficients and the associated PDs delivers the last explanatory variable.

dotted line.

The second part of the analysis reveals that the interaction between correlation coefficients and PDs is the main driver of the correlation level and dispersion effects. In particular, adding the second block of explanatory variables to the regression raises the goodness of fit measures (adjusted $R^2$) by 50 percentage points (to 89%) for large portfolios and by 53 percentage points (to 86%) for small portfolios. In addition, the positive statistically significant coefficient of average PDs indicates that, although an increase in this variable raises both the target and shortcut capital measures, the effect is stronger under the higher (homogeneous) asset return correlation underpinning the latter measure. Finally, the statistically significant coefficient of the correlation between PDs and loading factors is with the expected negative sign. This is because target capital tends to increase in the correlation between PDs and loadings on the single factor – as illustrated by an upward movements across the lines in the bottom-right panel of Figure 1 – whereas the shortcut measure abstracts from this correlation.

Reassuringly, the regression results are extremely robust across portfolio sizes. The robustness can be seen in that the values of the goodness-of-fit measures and the coefficient estimates obtained in the context of large portfolios match almost exactly their small-portfolio counterparts. Moreover, this match extends to the t-statistics of the estimates. In a further test of the robustness of the regression results, we pool observations across the two portfolio sizes and observe that all estimates change only marginally, leaving the message of the regression analysis intact.[33]

## 4.3 Estimation errors

The above results show that capital measures based on shortcut input estimates can deviate substantially from target. In practice, shortcut measures are likely to be adopted by less sophisticated users of the ASRF model who face constraints in terms of data and analytical capacity. By contrast, larger and more sophisticated users are likely to construct their own estimates of asset return correlations on the basis of in-house data. This section demonstrates that, for realistic sizes of such data, small-sample estimation errors in the correlation parameters are still likely to lead to large flaws in assessments of portfolio credit risk.

In order to quantify plausible estimation errors, we consider a portfolio whose "true"

[33] Background checks reveal that the regression residuals can be attributed to a large extent to interactions among PDs and asset return correlations that are non-linear and difficult to pin down.

credit risk parameters match those of the "typical" portfolio in our data set. For this portfolio, we impose the simplifying assumption of homogeneous PDs (1%), LGDs (50%) and pairwise asset return correlations (9.78%) and consider different numbers of underlying exposures (see Table 6). Abstracting from issues related to granularity and multiple factors, this assumption allows us to use the ASRF model and calculate that the desired capital buffer, dubbed "benchmark", equals 3.31% for each portfolio size. Referring to Table 4, this is seen as the typical (i.e. average) target capital buffer studied in Section 4.1.[34]

Then, we place ourselves in the shoes of a model user who does not know the exact asset return correlations but estimates them from available data.[35] Specifically, we endow the user with 60, 120 or 300 months of asset returns data – drawn from the "true" underlying distribution – and calculate the sample correlation matrix. In order to quantify a plausible range of errors in the estimate of the correlation matrix, we repeat this exercise 1000 times. As reported in panels A and B of Table 6, the sample correlations contain estimation error that remains substantial even for 300 months (or 25 years) of data.

Panel C of the table reveals how estimation errors in correlation coefficients translate into deviation from the desired benchmark capital buffer. First, these deviations are affected little by the number of exposures in the portfolio. Second, at standard confidence levels, the deviations decrease in the size of the available time series of asset returns but remain substantial even if this size is assumed to be unrealistically large. Specifically, if a portfolio comprises 1000 exposures and a user has 120 months of data, estimated capital buffers can deviate from the benchmark level by as much as 30% with a 95% probability. The size of the deviation decreases to 18% on the basis of 300 months of data. Third, estimated capital buffers exhibit a positive bias relative to the benchmark level, i.e. their average level is invariably higher than 3.31%. This is because the true correlation structure is assumed to be homogeneous, while small-sample errors introduce dispersion in estimated correlation coefficients. By the intuition presented in Section 2.3, this dispersion raises the implied capital buffer in the presence of homogenous PDs.

[34]Given that we abstract from model misspecification in this subsection, the benchmark capital measure is conceptually equivalent to what we earlier called target capital.

[35]In order to focus on issues in the estimation of the inter-dependence of credit risk across exposures, we assume that the user knows the true PD and LGD.

## 4.4 Alternative asset return distributions

There is general consensus in the literature that the marginal distributions of asset returns have tails that are fatter than the tails of the convenient normal distribution. Importantly, an erroneous normality assumption tends to bias capital buffers downwards to the extent that the empirical distribution of asset returns is driven by fat tails in the distribution of the common factor.[36] Such a distribution of the common factor implies great tail dependence across exposures, which leads to a large probability of default clustering.

In order to quantify the impact of alternative asset return distributions on capital measures, we consider a homogeneous portfolio in which all PDs equal 1%, all LGDs equal 50% and all asset return correlations equal 9.78% (the same as in Section 4.3). Given these risk parameters, we follow the literature on the pricing of portfolio credit risk (see Hull and White, 2004; Kalemanova et al., 2007) and consider the case in which both the common and idiosyncratic factors of asset returns have the same Student-$t$ distribution. Experimenting with different distributional specifications, we do see that fatter tails of the distribution of the common factor (i.e. fewer degrees of freedom) translate into larger deviations from a capital buffer derived under the normality assumption (Table 7, left panel).

In order to examine which distributional specification is supported by the data, we rely on time series of asset returns estimated by Moody's KMV. For each of the 10,891 firms in our sample, we use the available 59 months of estimated returns (from September 2001 to July 2006) to calculate the sample kurtosis, which is the standard measure of tail fatness. The mean and median of this statistic across firms equal 7.96 and 5.28, respectively. Then, on the basis of 10,000 Monte Carlo simulations, we derive the distributions of the estimators of these mean and median when: (i) the data size matches the size of the Moody's KMV data on asset return estimates and (ii) both the common and idiosyncratic factors of asset returns follow the same Student-$t$ distribution. As revealed by the confidence intervals for these estimators (Table 7, right panel), the Moody's KMV data support only the "double-$t$" specification with 3 degrees of freedom for each factor.

If the asset returns are indeed driven by such marginal distributions – which would be in line with findings in Kalemanova et al. (2007) – then a normality assumption will lead to tremendous underpredictions of the desired capital buffer. Specifically, we find that a double-$t$ specification with 3 degrees of freedom implies a capital buffer of 6.17% per unit

[36] For existing theoretical and empirical analysis of the treatment of tail dependencies by credit risk models, see for example Gordy (2000), Lucas et al. (2002) and Frey and McNeil (2003).

of aggregate exposure. This buffer is 86% higher than the capital buffer calculated under a normality assumption.

Even though this result may be undermined by probable errors in the Moody's KMV estimates of asset returns, alternative double-$t$ specifications studied in the related literature also lead to capital measures that are significantly higher than those implied by a normal distribution. Indeed, given that the available time series of Moody's KMV asset return estimates are short, plausible systemic errors in these estimates across firms might affect substantially the cross-sectional mean and median of the sample kurtosis. This casts doubt on the validity of the double-$t$ specification with 3 degrees of freedom and prompts us to consider alternative specifications, with 4 degrees of freedom (which is recommended by Hull and White, 2004) and 5 degrees of freedom (which is reportedly a market standard). As revealed in Table 7, these alternatives imply capital measures that are, respectively, 39 and 22% higher than the measure incorporating normal distributions.

# 5 Concluding remarks

This paper quantified the relative importance of alternative sources of error in model-based assessments of portfolio credit risk. We found that a misspecification of the popular ASRF model is likely to have a limited impact on such assessments, especially for large well-diversified portfolios. By contrast, erroneous calibration of this model – driven by flaws in popular rule-of-thumb values of asset return correlation, plausible small-sample estimation errors, or a wrong assumption regarding the marginal distributions of asset returns – has the potential to affect substantially measures of portfolio credit risk.

These results reveal a challenging task for risk managers. For one, it is reasonable to anticipate that calibration errors could also have a substantial impact on measures of portfolio credit risk implied by alternative models, irrespective of the number of common factors being incorporated. Moreover, our analysis has abstracted from several additional sources of error in credit risk parameter estimates. In particular, we assumed that PDs and LGDs are free of estimation noise, which is likely to be sizeable in practice and to interact with noise in correlation estimates in generating errors in measured portfolio credit risk. In addition, there may be time variation in credit risk parameters that relate to the likelihood and severity of default losses as well as to the correlation of the occurrence of such losses across exposures. Such time variation, which could be due either to cyclical developments or to structural changes in credit markets, would impair the useful content of the available data and, thus, would make it even more difficult to measure portfolio credit risk.

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# Appendix

## A Gaussian copula

The Gaussian copula is an efficient algorithm for measuring portfolio credit risk when a portfolio consists of a finite number of exposures, the correlation matrix is driven by a factor-loading structure and underlying distributions are normal. The efficiency of the algorithm stems from the fact that, conditional on the realization of the common factor(s), defaults occurrences are independent across exposures. This allows for a closed-form solution for the conditional probability of joint defaults. The corresponding unconditional probability is then derived by integrating over the probability distribution of the common factor(s). For further detail, see Gibson (2004).

## B Monte Carlo simulations

Monte Carlo simulations deliver the target capital level. This method can be applied to any portfolio comprising $N$ equally weighted exposures, provided that the exposure-specific probabilities of default, $PD_i$, the distribution of $LGD_i$ and the correlation matrix of asset returns, $R$, are known.

Given that $LGD_i$ is assumed to be independent of the factors underlying $PD_i$, and that the distribution of $LGD_i$ is identical across exposures, the simulation of portfolio credit losses can be divided in two parts. The first part calculates the probability distribution of joint defaults. Given this distribution, the second part incorporates the LGD distribution to derive the probability distribution of portfolio losses.

Specifically, we estimate the probability of joint defaults as follows:

1. Using the vector $\{PD_i\}_{i=1}^{N}$ and the assumption that asset returns are distributed as standard normal variables, we obtain an $N \times 1$ vector of default thresholds.

2. We draw an $N \times 1$ vector from $N$ standard normal variables whose correlation matrix is $R$. The number of entries in this vector that are smaller than the corresponding default threshold is the number of simulated defaults for the particular draw.

3. We repeat the previous step 500,000 times to derive the probability distribution of the number of defaults, $\Pr(nd = k)$, where $nd$ refers to the number of defaults and $k = 0, 1, \cdots, N$.

Then, we estimate the probability distribution of portfolio credit losses as follows:

1. For each possible number of defaults, $k$, we draw LGDs for the defaulted exposures 1,000 times and calculate the conditional loss distribution, $\Pr(TL|nd = k)$.

2. We repeat the above exercise for each $k = 0, 1, \cdots, N$, and calculate the unconditional probability distribution of portfolio credit losses. Specifically, $\Pr(TL) = \sum_k \Pr(TL|nd = k) \cdot \Pr(nd = k)$.

Finally we set the capital measure to equal $TL_{1-\alpha} - \sum_{i=1}^{N} E(PD_i) \cdot E(LGD_i)$, where $\alpha = 0.1\%$.

# C Fitting a single-factor correlation structure

A single-factor approximation of an empirical correlation matrix is obtained as follows. Denote the empirical correlation matrix by $\Sigma$ and its elements $\sigma_{ij}$, for $i, j \in \{1, \cdots, N\}$. The single-factor loading structure $\rho \equiv [\rho_1, ...\rho_N]$ that minimizes the discrepancies between the elements of $\Sigma$ and their fitted counterparts are given by:

$$\min_{\rho} \sum_{i=1,\cdots,N-1} \sum_{j>i} (\sigma_{ij} - \rho_i \rho_j)^2$$

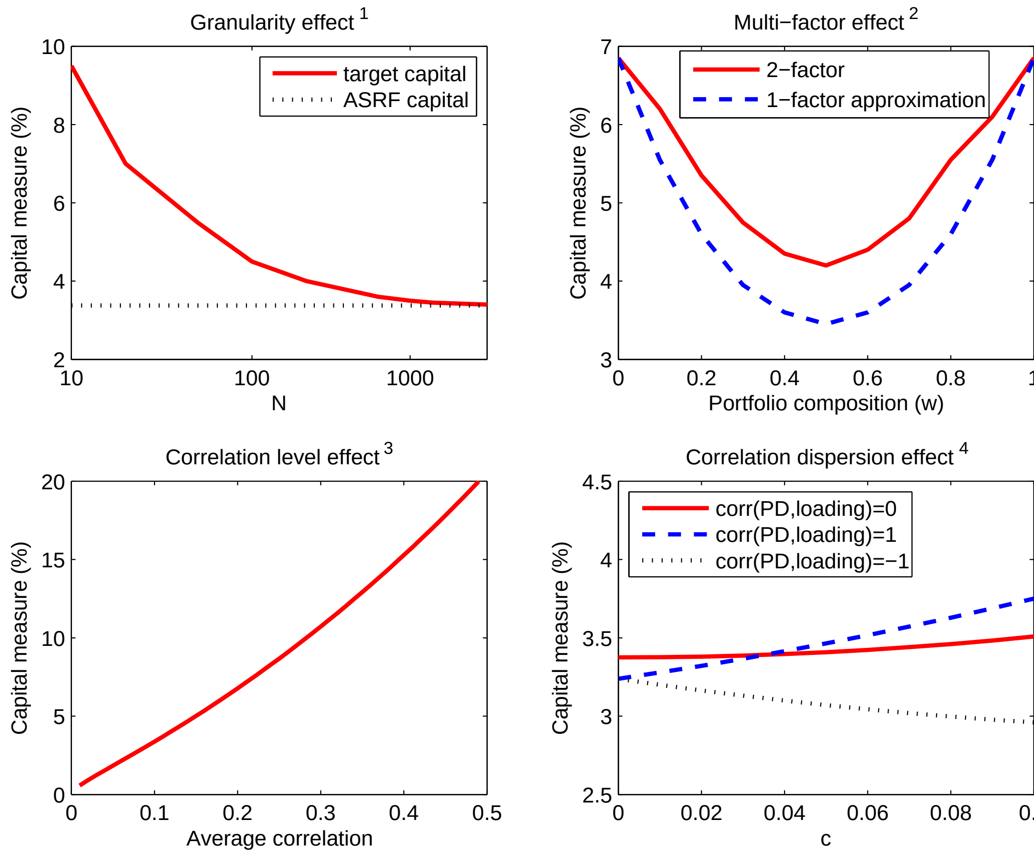
Andersen et al. (2003) propose an efficient algorithm to solve this minimization problem. The fitted correlation matrix $\hat{\Sigma}$ has elements $\rho_i \rho_j$.

We also construct a measure that reflects the "explanatory power" of the single-factor approximation:

$$\text{Goodness-of-fit measure} \equiv 1 - \frac{var(\epsilon)}{var(\sigma)}$$

where $\sigma$ is a vector of all pairwise correlation coefficients $\sigma_{ij}$ $(i, j = 1, \cdots, N, i < j)$ and $\epsilon$ is a vector of the errors $\sigma_{ij} - \rho_i \rho_j$ $(i, j = 1, \cdots, N, i < j)$. This measure reflects the degree to which the cross-sectional variation in pairwise correlations can be explained by common factor loadings in a single-factor framework.

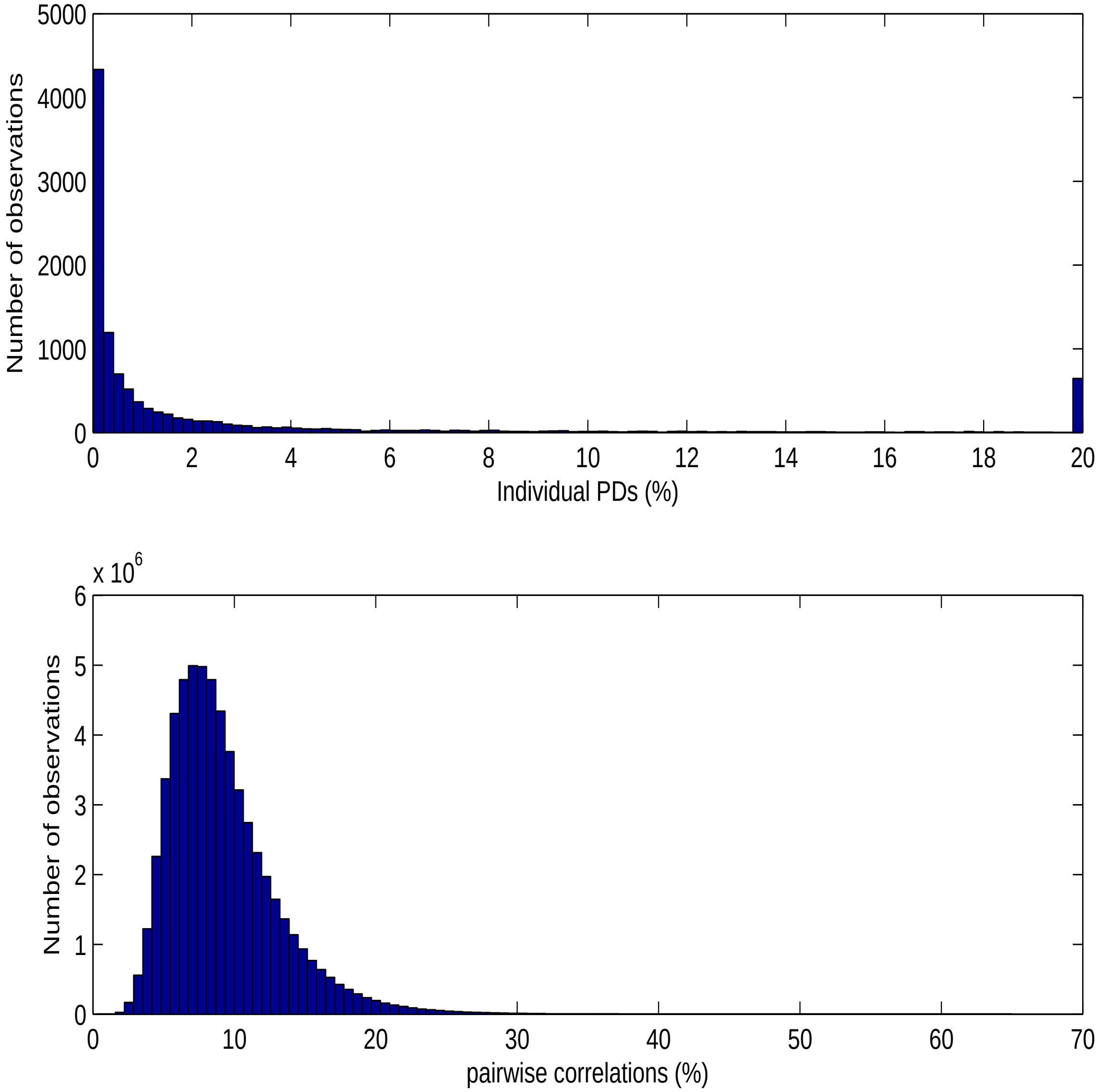
Figure 1: Four sources of error in capital measures




*Note*: Capital measures, in per cent and per unit of aggregate exposure, on the vertical axes. For each panel (unless noted otherwise), PD = 1% and LGD = 50% are the same across exposures.

[1] The solid line plots target capital for a portfolio in which all pairwise asset return correlations equal 10%. The number of exposures in the portfolio ($N$) varies across the horizontal axis. The dotted line plots the corresponding capital estimate when $N = \infty$. [2] The portfolio consists of 1000 exposures that are divided into two groups, with $w$ denoting the weight of the first group. Within each group, the asset return correlation equals 20% for all exposure pairs. Inter-group correlations are zero. The solid line plots the target capital level, which incorporates the two common factors in the simulated data. The dashed line plots the capital calculated under a one-common-factor approximation of the correlation structure (see Appendix C). [3] Capital measures for different levels of homogeneous pairwise asset return correlation. [4] The solid line plots capital measures under the assumption that PD = 1%, there is a single common factor and the loadings on this factor are distributed uniformly in the cross section between $\sqrt{0.1} - c$ and $\sqrt{0.1} + c$. For the other two lines, PDs are distributed uniformly in the cross section between 0.5% and 1.5% and are perfectly positively (dashed line) or negatively (dotted line) correlated with the common-factor loadings.

Figure 2: Distribution of individual PDs and pairwise correlations




*Note*: The parameter estimates are reported by Moody's KMV in July 2006 and relate to 10,891 non-financial firms.

Table 1: **Sectoral composition of simulated portfolios**

| | Large portfolio | | Small portfolio | | *Memo:* |
|---|---|---|---|---|---|
| Sector | Number of names | Exposure weight (%) | Number of names | Exposure weight (%) | number of firms in the sample |
| Aerospace & Defense | 31 | 3.1 | | | 105 |
| Agriculture | 9 | 0.9 | | | 56 |
| Air Transportation | 4 | 0.4 | | | 83 |
| Apparel, Footwear, & Textiles | 17 | 1.7 | | | 357 |
| Automotive | 51 | 5.1 | 19 | 9.5 | 198 |
| Broadcast Media | 43 | 4.3 | 16 | 8.0 | 191 |
| Business Services | 23 | 2.3 | | | 1132 |
| Chemicals | 42 | 4.2 | 15 | 7.5 | 940 |
| Computer Equipment | 10 | 1.0 | | | 746 |
| Construction | 43 | 4.3 | 16 | 8.0 | 277 |
| Electric, Gas, & Sanitary | 107 | 10.7 | 39 | 19.5 | 335 |
| Electronics & Electrical | 18 | 1.8 | | | 693 |
| Entertainment & leisure | 33 | 3.3 | | | 294 |
| Fabricated Metals | 17 | 1.7 | | | 146 |
| Food, Beverages, & Tobacco | 63 | 6.3 | 23 | 11.5 | 490 |
| General Retail | 31 | 3.1 | | | 133 |
| Glass & Stone | 6 | 0.6 | | | 149 |
| Health care | 38 | 3.8 | 14 | 7.0 | 178 |
| Legal & Other Services | 16 | 1.6 | | | 452 |
| Lodging | 22 | 2.2 | | | 70 |
| Machinery & Equipment | 36 | 3.6 | 13 | 6.5 | 645 |
| Medical Equipment | 10 | 1.0 | | | 334 |
| Mining | 6 | 0.6 | | | 486 |
| Miscellaneous Manufacturing | 18 | 1.8 | | | 130 |
| Non-defense Trans. & Parts | 2 | 0.2 | | | 53 |
| Oil Refining & Delivery | 24 | 2.4 | | | 100 |
| Oil & Gas Exploration | 55 | 5.5 | 20 | 10.0 | 458 |
| Other Trans. Services | 22 | 2.2 | | | 108 |
| Paper & Forestry | 23 | 2.3 | | | 172 |
| Personal Services | 7 | 0.7 | | | 31 |
| Primary Metals | 11 | 1.1 | | | 188 |
| Printing & Publishing | 28 | 2.8 | | | 186 |
| Repair Services & Rental | 13 | 1.3 | | | 37 |
| Restaurants | 9 | 0.9 | | | 141 |
| Rubber & Plastics | 18 | 1.8 | | | 120 |
| Semiconductors | 2 | 0.2 | | | 177 |
| Telecommunications | 69 | 6.9 | 25 | 12.5 | 177 |
| Trucking & Warehousing | 5 | 0.5 | | | 68 |
| Water Transportation | 5 | 0.5 | | | 104 |
| Wood, Furniture, & Fixtures | 13 | 1.3 | | | 151 |
| Total | 1000 | 100 | 200 | 100 | 10891 |
| Name concentration index | 0.0010 | | 0.0050 | | |
| Sector concentration index | 0.0432 | | 0.1135 | | |

*Note*: A concentration index is defined as the sum of squared weights, which are set either at the firm or the sector level.

Table 2: **Summary statistics (in percent)**

| | mean | std. dev. | skewness | median | minimum | maximum |
|---|---|---|---|---|---|---|
| 1-year PDs | 2.67 | 5.28 | 2.49 | 0.39 | 0.02 | 20.00 |
| Pairwise correlations | 9.24 | 3.86 | 1.87 | 8.45 | 0.29 | 65.00 |

*Note*: The sample includes 10,891 non-financial firms.

Table 3: **Characteristics of simulated loan portfolios (in percent)**

| A. Large portfolios (1000 firms) | | | | | |
|---|---|---|---|---|---|
| | mean | std. dev. | median | minimum | maximum |
| average PD | 2.42 | 0.19 | 2.42 | 1.79 | 3.12 |
| std. dev. of individual PDs | 5.16 | 0.26 | 5.16 | 4.25 | 6.14 |
| median PD | 0.26 | 0.03 | 0.26 | 0.18 | 0.36 |
| average correlation | 9.78 | 0.22 | 9.77 | 9.14 | 10.73 |
| std. dev. of loadings | 9.33 | 0.31 | 9.32 | 8.33 | 10.47 |
| corr (PD, loadings) | -20.0 | 2.04 | -20.1 | -26.7 | -12.8 |
| **B. Small portfolios (200 firms)** | | | | | |
| | mean | std. dev. | median | minimum | maximum |
| average PD | 2.28 | 0.36 | 2.26 | 1.24 | 3.68 |
| std. dev. of individual PDs | 5.05 | 0.53 | 5.06 | 3.01 | 6.89 |
| median PD | 0.24 | 0.05 | 0.23 | 0.11 | 0.55 |
| average correlation | 10.49 | 0.44 | 10.48 | 8.99 | 12.00 |
| std. dev. of loadings | 10.54 | 0.70 | 10.55 | 7.80 | 12.79 |
| corr (PD, loadings) | -19.8 | 4.59 | -20.2 | -31.8 | -1.2 |

*Note*: The results are based on 3,000 simulated portfolios and are carried out in two steps. First, portfolio-specific characteristics specified by row headings are calculated for each simulated portfolio. Second, summary statistics specified by column headings are calculated for each of the portfolio specific characteristics obtained in the first step. "Loadings" are estimated under a one-common-factor approximation of the correlation structure and refer to the firm-specific loadings of asset returns on the single common factor.

Table 4: **Capital measures and four sources of errors (in percent)**

| A. Large portfolios (1000 firms) | | | | | |
|---|---|---|---|---|---|
| | mean | std. dev. | median | 95% interval | 50% interval |
| **Target capital** [1] | **3.31** | **0.18** | **3.30** | **[2.97, 3.67]** | **[3.19, 3.42]** |
| *Deviation from target due to:* [2] | | | | | |
| Multi-factor effect [3] | -0.03 | 0.02 | -0.03 | [-0.08, 0.01] | [-0.05, -0.02] |
| Granularity effect [4] | -0.16 | 0.004 | -0.15 | [-0.16, -0.15] | [-0.16, -0.15] |
| Correlation dispersion effect [5] | 0.39 | 0.05 | 0.39 | [0.30, 0.48] | [0.36, 0.42] |
| Correlation level effect [6] | 0.61 | 0.06 | 0.62 | [0.49, 0.73] | [0.57, 0.66] |
| **Shortcut capital (corr=12%)** | **4.12** | **0.20** | **4.12** | **[3.75, 4.51]** | **[3.99, 4.25]** |
| *Memo: correlation level effect if:* | | | | | |
| corr=6% | -1.06 | 0.08 | -1.06 | [-1.23, -0.92] | [-1.11, -1.01] |
| corr=18% | 2.24 | 0.10 | 2.24 | [2.05, 2.43] | [2.17, 2.30] |
| corr=24% | 3.86 | 0.14 | 3.86 | [3.59, 4.14] | [3.77, 3.95] |

| B. Small portfolios (200 firms) | | | | | |
|---|---|---|---|---|---|
| | mean | std. dev. | median | 95% interval | 50% interval |
| **Target capital** [1] | **3.86** | **0.32** | **3.85** | **[3.25, 4.55]** | **[3.64, 4.07]** |
| *Deviation from target due to:* [2] | | | | | |
| Multi-factor effect [3] | -0.04 | 0.04 | -0.04 | [-0.12, 0.04] | [-0.07, -0.01] |
| Granularity effect [4] | -0.73 | 0.03 | -0.73 | [-0.79, -0.67] | [-0.75, -0.71] |
| Correlation dispersion effect [5] | 0.42 | 0.12 | 0.42 | [0.19, 0.65] | [0.34, 0.50] |
| Correlation level effect [6] | 0.40 | 0.13 | 0.40 | [0.16, 0.67] | [0.32, 0.48] |
| **Shortcut capital (corr=12%)** | **3.91** | **0.38** | **3.90** | **[3.17, 4.70]** | **[3.64, 4.16]** |
| *Memo: correlation level effect if:* | | | | | |
| corr=6% | -1.19 | 0.13 | -1.19 | [-1.46, -0.95] | [-1.28, -1.10] |
| corr=18% | 1.95 | 0.21 | 1.94 | [1.57, 2.38] | [1.81, 2.08] |
| corr=24% | 3.50 | 0.30 | 3.49 | [2.94, 4.12] | [3.30, 3.70] |

*Note*: Summary statistics for the simulated portfolios underpinning Table 3. The column entitled "95% interval" reports the 2.5th and 97.5th percentiles of the statistics specified in the particular row heading. The column entitled "50% interval" reports the corresponding 25th and 75th percentiles. In all calculations, $LGD_i$ follows a symmetric triangular distribution between 0 and 1.

[1] Based on Moody's KMV estimates of PDs and asset return correlations and a Monte Carlo procedure for calculating the probability distribution of default losses. [2] Four sources of deviation from the target capital level; a negative sign implies underestimation. The sum of the target capital level and the four deviations equals the shortcut capital level. Each deviation is based on the assumptions underlying previous deviations plus one additional assumption. [3] For the multi-factor effect, the correlation matrix underpinning the target capital level is approximated under the assumption that there is a single common factor. [4] For the granularity effect, there is the additional assumption that the number of firms is infinite. [5] For the correlation dispersion effect, the additional assumption is that the loadings on the single common factor are the same across exposures. [6] For the correlation level effect, the additional assumption imposes a different, shortcut, level on the constant pairwise correlation.

Table 5: **Explaining calibration-driven capital discrepancies**

| | Regression 1 | | | Regression 2 | | |
|---|---|---|---|---|---|---|
| | Large portfolio | Small portfolio | Pooled sample | Large portfolio | Small portfolio | Pooled sample |
| constant | 3.01 | 3.00 | 3.29 | 2.58 | 2.40 | 2.46 |
| | (65.6) | (53.0) | (111.6) | (129.6) | (84.0) | (159.8) |
| average corr | -0.24 | -0.23 | -0.24 | -0.28 | -0.26 | -0.27 |
| | (34.8) | (31.5) | (48.8) | (95.6) | (74.4) | (114.5) |
| std dev of loading coefficients | 0.037 | 0.022 | 0.010 | 0.031 | 0.029 | 0.028 |
| | (7.5) | (4.8) | (3.1) | (14.6) | (13.9) | (19.9) |
| average PD | | | | 0.21 | 0.19 | 0.19 |
| | | | | (86.2) | (61.6) | (94.2) |
| corr(PD, loading coefficient) | | | | -0.018 | -0.019 | -0.019 |
| | | | | (80.5) | (79.5) | (113.9) |
| adjusted $R^2$ | 0.39 | 0.33 | 0.56 | 0.89 | 0.86 | 0.91 |

*Note*: t-statistics in parentheses. The regression is based on 3,000 simulations for each portfolio size. The dependent variable equals shortcut capital (based on asset return correlation of 12%) minus target capital, net of the granularity and multi-factor effects (see Table 4). Loading coefficients are estimated under a one-common-factor approximation of the correlation structure of asset returns. The last regressor equals the Kendall rank correlation between PDs and loading coefficients.

Table 6: **Impact of estimation errors (in per cent)**

| | N=100 | N=200 | N=500 | N=1000 |
|---|---|---|---|---|
| **A. Sample average of pairwise correlations** | | | | |
| T=60 | 9.72 [6.5, 13.3] | 9.67 [6.4, 13.3] | 9.64 [6.6, 13.0] | 9.63 [6.9, 12.7] |
| T=120 | 9.77 [7.4, 12.4] | 9.77 [7.6, 12.1] | 9.76 [7.6, 12.1] | 9.72 [7.7, 12.0] |
| T=300 | 9.75 [8.3, 11.3] | 9.79 [8.3, 11.3] | 9.74 [8.4, 11.2] | 9.77 [8.4, 11.25] |
| **B. Sample standard deviation of loading coefficients** | | | | |
| | N=100 | N=200 | N=500 | N=1000 |
| T=60 | 12.11 [10.3, 14.1] | 11.84 [10.5, 13.2] | 11.65 [10.8, 12.5] | 11.58 [10.8, 12.2] |
| T=120 | 8.50 [7.4, 9.8] | 8.34 [7.5, 9.1] | 8.17 [7.6, 8.8] | 8.13 [7.7, 8.6] |
| T=300 | 5.35 [4.6, 6.2] | 5.24 [4.7, 5.7] | 5.16 [4.8, 5.5] | 5.12 [4.9, 5.4] |
| **C. Estimated capital, based on one-factor loading structure** | | | | |
| | N=100 | N=200 | N=500 | N=1000 |
| T=60 | 3.87 [2.8, 5.1] | 3.83 [2.8, 5.0] | 3.80 [2.8, 4.9] | 3.79 [2.9, 4.8] |
| T=120 | 3.59 [2.9, 4.4] | 3.58 [2.9, 4.4] | 3.56 [2.9, 4.3] | 3.54 [2.9, 4.3] |
| T=300 | 3.41 [3.0, 3.9] | 3.42 [3.0, 3.9] | 3.40 [3.0, 3.9] | 3.41 [3.0, 3.9] |
| *benchmark* | 3.31 | 3.31 | 3.31 | 3.31 |

*Note*: Results are based on 1000 simulations of the asset returns of N firms over T months, with the true pairwise correlation, PD and LGD fixed at 9.78%, 1% and 50%, respectively. Each cell contains the mean and the 95% confidence interval of: the average of sample pairwise correlations (panel A); the standard deviation of the loading coefficients in a one-factor approximation (panel B); and the implied capital buffer (panel C).

Table 7: **Alternative distributional assumptions**

| Degrees of freedom[1] | Capital measure[2] (in per cent) | Mean kurtosis[3] | | Median kurtosis[3] | |
|---|---|---|---|---|---|
| | | mean | 95% interval | mean | 95% interval |
| (3, 3) | 6.17 | 7.83 | [7.30, 8.27] | 5.54 | [5.11, 6.54] |
| (4, 4) | 4.59 | 5.72 | [5.47, 5.92] | 4.43 | [4.25, 4.59] |
| (5, 5) | 4.03 | 4.79 | [4.64, 4.93] | 3.96 | [3.85, 4.06] |
| (6, 6) | 3.81 | 4.31 | [4.19, 4.40] | 3.70 | [3.63, 3.78] |
| (7, 7) | 3.67 | 4.02 | [3.93, 4.10] | 3.55 | [3.49, 3.60] |
| (8, 8) | 3.59 | 3.83 | [3.76, 3.89] | 3.44 | [3.39, 3.49] |
| (9, 9) | 3.53 | 3.70 | [3.64, 3.75] | 3.36 | [3.32, 3.40] |
| (10, 10) | 3.49 | 3.60 | [3.55, 3.65] | 3.30 | [3.26, 3.34] |
| ($\infty$, $\infty$) | 3.31 | 3.00 | [2.99, 3.01] | 2.89 | [2.88, 2.90] |

[1] The degrees of freedom of the Student-$t$ distribution of asset returns' common and idiosyncratic factors, respectively. The last row refers to a Gaussian specification. [2] Obtained by applying the general ASRF formula (equation 4) to a homogeneous portfolio, in which PD=1% and LGD=50% for each exposure and asset return correlations equal 9.78% for each pair of exposures. The default boundary for such exposures is calculated on the basis of 10 million simulations. [3] The means and the 95% confidence intervals (based on 10,000 Monte Carlo simulations) of the estimators of the cross-sectional mean and median of the kurtosis when: (i) the marginal Student-$t$ distributions of the common and idiosyncratic factors have the degrees of freedom specified in the row heading and asset return correlations equal 9.78%; (ii) the available data comprise time series of 59 returns for 10,891 firms.