BANK FOR INTERNATIONAL SETTLEMENTS

BIS Working Papers

No 228

Inflation risk premia in the term structure of interest rates

by Peter Hördahl and Oreste Tristani

Monetary and Economic Department

May 2007

BIS Working Papers are written by members of the Monetary and Economic Department of the Bank for International Settlements, and from time to time by other economists, and are published by the Bank. The views expressed in them are those of their authors and not necessarily the views of the BIS.

Copies of publications are available from:

Bank for International Settlements
Press & Communications
CH-4002 Basel, Switzerland

E-mail: publications@bis.org

Fax: +41 61 280 9100 and +41 61 280 8100

This publication is available on the BIS website (www.bis.org).

ISSN 1020-0959 (print)

ISSN 1682-7678 (online)

Inflation risk premia in the term structure of interest rates*

Peter Hördahl**
Bank for International Settlements

Oreste Tristani***
European Central Bank

April 2007

Abstract

This paper estimates the size and dynamics of inflation risk premia in the euro area, based on a joint model of macroeconomic and term structure dynamics. Information from both nominal and index-linked yields is used in the empirical analysis. Our results indicate that term premia in the euro area yield curve reflect predominantly *real* risks, i.e. risks which affect the returns on both nominal and index-linked bonds. On average, inflation risk premia were negligible during the EMU period but occasionally subject to statistically significant fluctuations in 2004−2006. Movements in the raw break-even rate appear to have mostly reflected such variations in inflation risk premia, while long-term inflation expectations have remained remarkably anchored from 1999 to date.

JEL classification numbers: E43, E44

Keywords: Term structure of interest rates, inflation risk premia, central bank credibility.

*The opinions expressed are personal and should not be attributed to the Bank for International Settlements or the European Central Bank. We wish to thank Claudio Borio, Andrea Buraschi, Don Kim, Frank Packer, Adrien Verdelhan, David Vestin and seminar participants at the 2005 ECB-BIS workshop on "Macro-finance modelling of the term structure of interest rates", the CEF 2006 and EEA 2006 conferences for useful comments and suggestions.

** Bank for International Settlements, Centralbahnplatz 2, CH-4002, Basel, Switzerland.
E-mail: peter.hoerdahl@bis.org.

*** European Central Bank, DG Research, Kaiserstrasse 29, D-60311, Frankfurt am Main, Germany.
E-mail: oreste.tristani@ecb.int.

1 Introduction

Central banks often interpret the difference between nominal and inflation-linked yields as a measure of expected inflation over the life of the bond, or "break-even inflation rate". Expected inflation over the distant future can, in turn, be viewed as a measure of credibility of the central bank's inflation objective. If the objective is well known because of a public announcement, and if it is credible, it should be reflected in inflation expectations over horizons far into the future. In other words, any current inflationary shocks should be viewed as temporary and long-run inflation expectations should remain anchored at the level consistent with the announced objective.

The break-even inflation rate is, however, a noisy measure of expected inflation, because it includes an inflation risk premium component (and, possibly, differential liquidity premia). Long-term nominal yields could in fact be decomposed into a real yield, average inflation expectations and an inflation risk premium.[1] The main objective of this paper is to estimate the size of the inflation risk premium in euro area yields and to analyze its relationship to inflation, output and the nominal interest rate. If inflation premia were non-negligible, break-even inflation rates would no longer represent a correct measure of expected future inflation. Variations in break-even rates could simply reflect changes in inflation risk premia over time.

The presence of inflation risk premia also complicates the interpretation of raw break-even inflation rates as measures of credibility. While possibly representing *per se* a reason for concern, a large inflation risk premium would not be directly related to the credibility of the inflation objective.

More specifically, an increase in the inflation risk premium could be due either to a higher level of inflation risk, or to an increase in investors' aversion to bearing that risk – i.e. the "market price of risk". In the first case, the higher inflation risk could reflect an increase in the uncertainty of the overall macroeconomic environment, which may render a price stability objective more difficult to attain over the years, but need not be related to the credibility of the central bank. In the second case, which is also the one considered explicitly in this paper, variations in the prices of inflation risk may be due to the particular features of investors' portfolios, namely their exposure to the cyclical position of the economy or the inflation level. Once again, provided inflation expectations

[1] To simplify the discussion, we are here disregarding a convexity term.

remained anchored, it is not immediately clear that an increase in the price of risk should be interpreted as a signal of lower central bank credibility.

In order to estimate inflation risk premia and understand their macroeconomic determinants, a necessary condition is a joint model of macroeconomic and term structure dynamics. Only within a macroeconomic model can notions such as "inflation objective" be defined formally. Only if bond prices are built on a macroeconomic framework can one discuss the impact on yields of inflationary shocks of various sources. Finally, a macro model should also provide a more realistic description of inflation dynamics, compared to a reduced-form model. For these reasons, we adopt the framework developed in Hördahl, Tristani and Vestin (2006), which in turns builds on Ang and Piazzesi (2003). More specifically, we price yields based on the dynamics of the short rate obtained from the solution of a linear forward-looking macro model and using an essentially affine stochastic discount factor (see Duffie and Kan, 1996; Dai and Singleton, 2000; Duffee, 2002).[2]

Compared to the alternative of relying on a microfounded model, our modelling strategy has the advantage of imposing milder theoretical constraints on risk premia (while remaining highly tractable). It is only through a modelling framework capable of generating large premia that we can test whether premia were actually large or small in the EMU sample. With respect to smaller models which can be solved nonlinearly, our approach has the advantage of being independent of special assumptions imposed for analytical tractability, and of relying on a well-established monetary policy transmission mechanism. The drawback is obviously that we are unable to draw a link from the prices of risk to individuals' preferences.

In order to disentangle the inflation risk premium from the total "nominal premium," which includes a real premium to compensate for uncertainty associated with fluctuations in real interest rates, it is also important to enrich the information set available in the estimation. Our aim is really to identify the two theoretical components of a variable, the term premium, which is itself unobservable and is the result of a filtering process. If we relied only on information from the nominal term structure, we would run the risk of reaching conclusions that are difficult to validate. For this reason, we believe that the information provided by index-linked bonds is crucially important in our analysis.

Finally, we present all our results on estimated term and inflation risk premia with

[2] Other recent papers that jointly model macroeconomic and nominal term structure dynamics include Dewachter and Lyrio (2004) and Rudebusch and Wu (2004).

confidence intervals, to emphasize that all these notions are obviously measured with uncertainty within our model. This allows us to make probabilistic statements as to the statistical relevance of premia.

Focusing on the 10-year maturity, our main result is that, on average, the inflation risk premium on euro area nominal yields was insignificantly different from zero over the EMU sample. Fluctuations around the average have also been relatively small, but statistically significant in 2001 and 2002 and, occasionally, in the 2004−2006 period. However, we can be less confident in our estimates over the 2001−2002 period, when the index-linked bond market was relatively thin. In those years, our estimates are likely to be affected by variations in liquidity premia.

All in all, our results suggest that fluctuations in the raw break-even rate have mostly reflected variations in the inflation risk premium: adjusting for such premium, long-term inflation expectations appear to have remained well anchored from 1999 to date. From this standpoint, monitoring the time variation in inflation risk premia is important to understand correctly the information contained in break-even rates.

Ceteris paribus, inflation risk premia appear to be lower when policy interest rates are relatively high. This appears to suggest that investors feel less worried about inflation risk when policy is tightened. Moreover, inflation risk premia seem to rise when the output gap widens, suggesting that investors become more concerned about inflationary pressures as economic activity picks up.

Our paper is organized as follows. The next section contrasts our methodology to estimate the euro area inflation risk premium to other approaches, both theoretical and empirical, that have been used in the literature. Our results from the estimation of zero-coupon real rates derived from index-linked bond yields are presented in Section 3, where we also present some descriptive statistics on our full dataset of real and nominal bonds and macroeconomic variables. Section 4 outlines our model, its implications for the inflation risk premium and our econometric methodology. Our empirical results are presented in Section 5, where we show our parameter estimates and their implications for term premia and inflation risk premia. In this section, we also relate premia to their macroeconomic determinants and calculate risk-adjusted break-even inflation rates. Section 6 concludes the paper.

2 What should one expect regarding inflation risk premia?

It goes without saying that we are not the first to analyze the inflation risk premium in nominal bonds. However, there is little agreement in the theoretical and empirical literature on the size and even the sign of the premium. The raw evidence available from index-linked bonds points to a positive difference between nominal and real yields, and the nominal yield curve also appears to be steeper than the real yield curve (e.g. Roll, 2004). In order to make inferences with regard to the inflation risk premium, however, one needs to take a stance on inflation expectations over the life of the bond. Since the latter are also unobservable, a theoretical framework is necessary to answer the question in the title of this section.

From a theoretical viewpoint, it is clear at least from Fischer (1975) that there is no reason to expect the inflation risk premium to be positive. The sign of the premium depends entirely on the covariance between real returns on nominal bonds and the stochastic discount factor. In simple microfounded models, the log stochastic discount factor is proportional to consumption growth and the inflation risk premium will be positive when consumption growth and inflation are negatively correlated. In US data, where the sample correlation coefficient between consumption growth and inflation is -0.15 in the $1960-1997$ period, one should therefore expect the inflation risk premium to be positive on average. In more general setups, however, this simple intuition is lost. The stochastic discount factor will depend on the marginal utility of consumption, which need not be proportional to consumption growth. Nevertheless, in the approximate solution of a calibrated model with habit persistence and nominal rigidities, Hördahl, Tristani and Vestin (2007) argue that the average inflation risk premium in the US is probably positive, but small.

A number of recent empirical studies also suggest that the inflation risk premium in the US nominal term structure should be positive and non-negligible in economic terms. Buraschi and Jiltsov (2005) use a monetary version of a real business cycle model to characterize and estimate the inflation risk premium, and find an average premium of 15 basis points at the 1-month horizon and 70 basis points at the 10-year horizon. Based on an essentially affine term structure model with regime switching, Ang, Bekaert and Wei (2006) also find positive inflation risk premia in the US, ranging from almost zero to 200 basis points over the $1952-2004$ sample for 5-year bonds. Kim and Wright (2005) report that the US 10-year instantaneous forward inflation premium typically fluctuates within a 50-100 basis point range, based on an affine model supplemented with inflation data and survey expectations.

However, not all of these papers incorporate information from inflation-indexed bonds. Barr and Campbell (1997) use this information, but set risk premia to zero by assumption. Remolona, Wickens and Gong (1998) estimate an affine model on UK data and find a relatively smooth 2-year inflation risk premium of around one percentage point before 1990 and around 0.7 percentage points thereafter. Based on an essentially affine setup which incorporates index-linked UK yields, Risa (2001) also finds a positive inflation risk premium, but on average this is downward sloping in maturity: it is equal to 2.2 percentage points for a theoretical instantaneous bond and it falls to 1.7 percentage points for a 20-year bond. The short term inflation risk premium is also much more volatile than the long term premium. An even starker difference characterizes the results in Evans (2003), where the UK term structure is modelled using a regime switching setup which incorporates information from index-linked bonds. Evans (2003) also finds a downward sloping inflation risk premium, but this is large and negative for most maturities, reaching -1.8 percentage points or even -3.5 percentage points at the 10-year horizon depending on the prevailing state.

All in all, there appear to be no robust results on the sign, size, maturity structure and volatility of inflation risk premia. The different results in the literature could partly be due to differences in samples or country.

3 Data

Our main objective is to extract long-term inflation expectations and premia from the term structure of euro area interest rates. In order to achieve this goal, however, we face a number of data limitations.

More specifically, we need to deal with two main difficulties: first, the possibility that the creation of the single European currency, the euro, induced a structural break in economic relationships; second, the unavailability of accurate bond price data for most European countries before the mid- or even late nineties. Both considerations recommend starting our estimation period in January 1999, based on euro area data, which leaves us with 88 data points at a monthly frequency (from January 1999 to April 2006).

Another difficulty concerns index-linked bonds, of which only very few were available during the earliest part of the sample. We therefore use solely data on nominal yields and macroeconomic variables for the model estimation between January and September 1999, and treat real yields as unobservables; as of October 1999, the dataset is extended to include real yields from index-linked bonds.

3.1 Index-linked zeros

For our analysis, we first derive zero-coupon equivalent rates from index-linked prices and coupons. Specifically, we rely on data for index-linked bonds issued by the French Treasury (obtained from Bloomberg).[3] In this process, as is typically the case in the literature, we abstract from tax and liquidity issues. Concerning liquidity, our index-linked sample starts one year after the introduction of such bonds by the French Treasury, during which time liquidity was at its lowest and one might have expected initial mispricings to be particularly pronounced. Nevertheless, monthly turnover figures shown in Figure 1a suggest that liquidity in the French index-linked market initially remained limited for a couple of years. It is therefore possible that liquidity issues may have had an impact on index-linked bond prices during this period, although it is not obvious how to measure the size of such influences. We return to this issue in the discussion of the empirical results.

We assume that index-linked bonds are truly risk-free, i.e. we dismiss the inflation risk borne by investors because of the indexation lag (the fact that there exists a lag between the publication of the inflation index and the indexation of the bond). In principle, we could use the methodology of Kandel, Ofer and Sarig (1996) and Evans (1998) to account for this lag. However, Evans (1998) estimates the indexation-lag premium to be quite small, notably around 1.5 basis points, in the UK, where the indexation lag is 8 months. Since the lag is of only 3 months in the euro area, we believe that any estimate of the indexation-lag premium would be well within the range of any measurement error.

[3] While index-linked government bonds are available for other countries in the euro area, we rely exclusively on those issued by the French Treasury. The main reason for this is that the French segment of the market is the largest in the euro area: at present the amount of outstanding French index-linked bonds is around 94 bn. EUR. The market segments of Italy and Greece are somewhat smaller at 56 and 46 bn. EUR outstanding value. Another important reason why we rely only on French bonds is to avoid mixing bonds with different credit ratings. French bonds have the highest credit rating possible (AAA by S&P), while bonds issued by Italy and Greece are lower rated (AA- and A respectively). German government bonds are also AAA-rated, but Germany issued its first index-linked bond (9 bn. EUR) only in March 2006.

Finally, we face the constraint that only bonds indexed to the French CPI, rather than the euro area HICP, were available up to late 2001, when the French Treasury began issuing bonds linked to the euro area HICP. While the difference between euro area HICP and French CPI is typically on the order of a few basis points, it is persistent over time, so that yields on HICP-linked bonds consistently tended to be below those of CPI-linked bonds. For the estimation, we use a mixed series: the HICP-linked bond as of October 2002 and the CPI-based series prior to this.[4] However, since the variable of interest for monetary policy, and hence affecting the short-term rate, is the euro area HICP, we adjust the CPI-linked zero-coupon rates downwards by an amount equal to the *average* difference between CPI and HICP-linked yields at each maturity.

In order to construct zero-coupon equivalents for index-linked yields, we follow the spline method in McCulloch and Kochin (2000). The methodology is designed to work with yield data that are only available for few maturities. It is based on a discount function of the form

$$\delta(m) = \exp\left[-\sum_{j=1}^{n} a_j \psi_j(m)\right],$$

where m is the time to maturity and n is the number of maturities available from the data, while the $\psi_j(m)$'s are splines defined by

$$\psi_j(m) = \theta_j(m) - \frac{\theta_j''(m_n)}{\theta_{n+1}''(m_n)}\theta_{n+1}(m), \quad j = 1, ..., n,$$

and the functions $\theta_j(m)$ are given by

$$\begin{aligned} \theta_1(m) &= m \\ \theta_2(m) &= m^2 \\ \theta_j(m) &= \max(0, m - m_{j-2})^3, \quad j = 3, ..., n+1. \end{aligned}$$

The resulting real zero-coupon yields for the 3-year, 5-year and 10-year maturities are shown in Figure 1b. The real zeros are relatively high in 2000 and 2001, when growth was also relatively high, and lower in more recent years. As mentioned above, it is also possible that liquidity considerations may have affected the level of real rates in the French

[4] While HICP-linked bonds were introduced in 2001, sufficient data to allow estimation of zero-coupon real yields is available only as of October 2002.

index-linked market during the earliest part of the sample.

3.2 Nominal yields and macro data

For nominal rates, we use zero-coupon yields derived from German government bonds, which for large parts of the maturity spectrum are seen as benchmarks for euro area nominal yields; Figure 2 displays the yields for the 3-month, 3-year and 10-year maturities. This data originates from the Bundesbank and is provided by the BIS. We can use the real and nominal zero-coupon rates to construct constant-maturity break-even inflation rates, namely the straightforward difference between nominal and real yields. Figure 3 shows zero-coupon break-even inflation rates for 3-, 5- and 10-year maturities. Since 1999, break-even rates have varied within a relatively close range. At the 10-year maturity, in particular, they have mostly oscillated between 1.5 and 2.5 percent.

As for macro variables, our approach requires time series of euro area inflation and the output gap. Inflation is defined as the monthly log-change in the HICP for the euro area.[5] For output, we use log-industrial production. Following Clarida, Galí and Gertler (1998), our output gap series is defined in terms of deviations of industrial production from a quadratic trend, and is calculated in "real time", i.e. estimated at each point in time using only information available up to that point.

In order to specify our model of section 4, we analyze whether long-term real rates appear to include information which is significantly different from that contained in nominal rates. For this purpose, we look separately at the principal components of nominal yields, of nominal and real yields, and of all yields plus our macro variables. We can obviously carry out this analysis only for the period over which our zero-coupon real yields are available, namely, starting in October 1999.

Over this sample, three principal components are necessary to capture 99% of the variance of nominal yields. As soon as we add the real yields, however, four principal components are needed. When we include macro variables, four principal components continue to capture 99% of the variance of all variables, but the fourth becomes much more important: it explains 4% of the variance of the variables, compared to 1% explained

[5] Month-on-month inflation is a relatively volatile series. For practical estimation purposes, the use of year-on-year inflation as in Hördahl, Tristani and Vestin (2006) would therefore be an attractive alternative. However, we need to specify both the nominal and the real pricing kernels in order to price nominal and real bonds, which requires using the one-period (i.e. one-month) rate of inflation (see the Appendix).

in the case without macroeconomic variables. This suggests that it is important to include four different risk factors in the model.

4 Model

We rely on a structural economic model, which is specified directly at the aggregate level. The model includes just two equations which describe the evolution of inflation, π_t, and the output gap, x_t. Since we are going to estimate the model at the monthly frequency, the two equations are specified with a relatively elaborate lead and lag structure:

$$\widehat{\pi}_t = \mu_\pi \frac{1}{12} \sum_{i=1}^{12} E_t \left[\widehat{\pi}_{t+i}\right] + (1-\mu_\pi) \sum_{i=1}^{2} \delta_{\pi i} \widehat{\pi}_{t-i} + \delta_x \widehat{x}_t + \varepsilon_t^\pi \tag{1}$$

$$\widehat{x}_t = \frac{1}{12} \mu_x \sum_{i=1}^{12} E_t \left[\widehat{x}_{t+i}\right] + (1-\mu_x) \sum_{i=1}^{2} \zeta_{xi} \widehat{x}_{t-i} - \zeta_r \left(\widehat{r}_t - E_t \left[\widehat{\pi}_{t+1}\right]\right) + \varepsilon_t^x \tag{2}$$

where r_t is the one-month nominal interest rate, inflation is defined as the monthly change in the log-price level, and hats denote deviations from the mean. The specification of the model is similar to that in Hördahl, Tristani and Vestin (2006), and is motivated by the literature on the so-called "new Keynesian" Phillips curve (e.g. Galí and Gertler, 1999) and on estimation of consumption-Euler equations (e.g. Fuhrer, 2000). Both equations include a forward-looking term capturing expectations over the next year of inflation and output, respectively. The lags in the backward-looking components of the two equations are motivated empirically. In the estimation, we impose $\mu_\pi + (1-\mu_\pi) \sum_i \delta_{\pi i} = 1$, a version of the natural rate hypothesis.

The simple representation of the economy in equations (1) and (2) incorporates explicitly some standard channels of transmission of inflationary shocks and of monetary policy. Inflation can be due to demand shocks ε_t^x, which increase output above potential and create excess demand, and to cost-push shocks ε_t^π, which have a direct impact on prices. In turn, monetary policy can affect inflation via stimuli or restrictions of aggregate demand, i.e. modifying the real interest rate $\widehat{r}_t - E_t \left[\widehat{\pi}_{t+1}\right]$, or influencing expectations.

To solve for the rational expectations equilibrium, we need an assumption on how monetary policy is conducted. We focus on private agents' perceptions of the monetary policy rule followed by the central banks, which is supposedly to set the nominal short rate according to

$$\widehat{r}_t = (1-\rho)\left\{\beta\left(\frac{1}{12}E_t\left[\sum_{i=0}^{11}\widehat{\pi}_{t+i}\right]-\widehat{\pi}_t^*\right)+\gamma\widehat{x}_t\right\}+\rho\widehat{r}_{t-1}+\eta_t \tag{3}$$

where $\widehat{\pi}_t^*$ is the perceived inflation target and η_t is a "monetary policy shock".

This is consistent with the formulation in Clarida, Galí and Gertler (2000). The first two terms represent a forward-looking Taylor (1993) rule, where the rate responds to deviations of expected inflation from the inflation target. The second part of the rule is motivated by interest rate smoothing concerns, i.e. the desire to avoid producing large volatility in nominal interest rates. We also allow for a time-varying, rather than constant, inflation target π_t^*. We adopt this formulation in order to allow for some evolution in the behavior of monetary policy over time, or at least in the way monetary policy was perceived by markets.

Finally, we need to specify the processes followed by the stochastic variables of the model, i.e. the perceived inflation target and the three structural shocks. We assume that our three macro shocks are serially uncorrelated and normally distributed with constant variance. The only factor that we allow to be serially correlated is the unobservable inflation target, which will follow an AR(1) process

$$\widehat{\pi}_t^* = \phi_{\pi^*}\widehat{\pi}_{t-1}^* + u_{\pi^*,t} \tag{4}$$

where $u_{\pi^*,t}$ is a normal disturbance with constant variance uncorrelated with the other structural shocks.[6]

In order to solve the model we write it in the general form

$$\begin{bmatrix} \mathbf{X}_{1,t+1} \\ E_t\mathbf{X}_{2,t+1} \end{bmatrix} = \mathbf{H}\begin{bmatrix} \mathbf{X}_{1,t} \\ \mathbf{X}_{2,t} \end{bmatrix} + \mathbf{K}\widehat{r}_t + \begin{bmatrix} \Sigma\xi_{1,t+1} \\ \mathbf{0} \end{bmatrix}, \tag{5}$$

where $\mathbf{X}_{1,t} = [x_{t-1}, x_{t-2}, x_{t-3}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_t^*, \eta_t, \varepsilon_t^\pi, \varepsilon_t^x, r_{t-1}]'$ is the vector of predetermined variables, $\mathbf{X}_{2t} = [E_t x_{t+11}, ..., E_t x_{t+1}, x_t, E_t\pi_{t+11}, ..., E_t\pi_{t+1}, \pi_t]'$ includes the variables which are not predetermined, $\widehat{r}_t$ is the policy instrument and ξ_1 is a vector of independent, normally distributed shocks. The short-term rate can be written in the feedback

[6] To ensure stationarity of the inflation target process, we impose an upper limit of 0.99 on the ϕ_{π^*} parameter during the estimation process. This restriction is binding.

form

$$\widehat{r}_t = -\mathbf{F}\begin{bmatrix} \mathbf{X}_{1,t} \\ \mathbf{X}_{2,t} \end{bmatrix}. \tag{6}$$

The solution of the (5)−(6) model can be obtained numerically following standard methods. We choose the methodology described in Söderlind (1999), which is based on the Schur decomposition. The result are two matrices $\mathbf{M}$ and $\mathbf{C}$ such that $\mathbf{X}_{1,t} = \mathbf{M}\mathbf{X}_{1,t-1} + \Sigma\xi_{1,t}$ and $\mathbf{X}_{2,t} = \mathbf{C}\mathbf{X}_{1,t}$.[7] Consequently, the equilibrium short-term interest rate will be equal to $\widehat{r}_t = \mathbf{\Delta}'\mathbf{X}_{1,t}$, where $\mathbf{\Delta}' \equiv -(\mathbf{F}_1+\mathbf{F}_2\mathbf{C})$ and $\mathbf{F}_1$ and $\mathbf{F}_2$ are partitions of $\mathbf{F}$ conformable with $\mathbf{X}_{1,t}$ and $\mathbf{X}_{2,t}$. Focusing on the short-term (policy) interest rate, the solution can be written as

$$\begin{aligned} \widehat{r}_t &= \mathbf{\Delta}'\mathbf{X}_{1,t} \\ \mathbf{X}_{1,t} &= \mathbf{M}\mathbf{X}_{1,t-1} + \Sigma\xi_{1,t}. \end{aligned} \tag{7}$$

4.1 Building the term structure

The system (7) expresses the short-term interest rate as a linear function of the vector $\mathbf{X}_1$, which in turn follows a first-order Gaussian VAR. This is the basic model setup in the affine term structure literature. However, in our case, both the short-rate equation and the law of motion of vector $\mathbf{X}_1$ have been obtained endogenously, as functions of the parameters of the macroeconomic model. This contrasts with the standard affine setup based on unobservable variables, where both the short rate equation and the law of motion of the state variables are postulated exogenously.

To derive the term structure, we only need to impose the assumption of absence of arbitrage opportunities, which guarantees the existence of a risk-neutral measure, and to specify a process for the stochastic discount factor. Following the essentially affine formulation (see Duffee, 2002; Dai and Singleton, 2002), an important element of the stochastic discount factor will be the market prices of risk λ_t, which will be affine in the vector $\mathbf{X}_{1t}$, i.e. $\lambda_t = \widetilde{\lambda}_0 + \widetilde{\lambda}_1\mathbf{X}_{1t}$. Note that $\mathbf{X}_{1t}$ includes the four stochastic factors of the system, i.e. the inflation target and the three white noise shocks. These shocks will induce risk premia, but in the essentially affine formulation the premia will also depend

[7] The presence of non-predetermined variables in the model implies that there may be multiple solutions for some parameter values. We constrain the system to be determinate in the iterative process of maximizing the likelihood function.

on the level of the other states. Since our $\mathbf{X}_{1t}$ includes 11 variables – the four stochastic factors plus three lags of the output gap and inflation and one lag of the short-term rate – the maximum number of non-zero elements in the $\widetilde{\lambda}_1$ matrix is 4×11.

Estimation of 44 parameters just for the state-dependent prices of risk is prohibitive. We therefore impose some restrictions on the λ_1 matrix. More specifically, rather than allowing the market prices of risk to be independently influenced by the lags of the macroeconomic variables, we assume that such lag dependence is induced by the current levels of those macro variables. For example, we assume that the inflation lags will potentially affect the prices of risk only through their effect on current inflation, output, or the nominal interest rate. This assumption implies that we can rewrite the market prices of risk as linear functions of only $\widehat{x}_t$, $\widehat{r}_t$, $\widehat{\pi}_t$ and $\widehat{\pi}^*_t$. Since each of these variables can be written as a linear combination of the vector of predetermined variables using the model's solution, this assumption is equivalent to imposing cross-restrictions on the elements of the $\widetilde{\lambda}_1$ matrix.

More precisely, we first define a new vector $\mathbf{Z}_t$ which is a transformation of the original state vector $\mathbf{X}_{1t}$, such that $\mathbf{Z}_{1t} \equiv [x_{t-1}, x_{t-2}, x_{t-3}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi^*_t, r_t, \pi_t, x_t, r_{t-1}]'$, and then rewrite the solution equation for the short-term interest rate as a function of $\mathbf{Z}_t$, $r_t = \overline{\mathbf{\Delta}}'\mathbf{Z}_t$. The $\mathbf{Z}_t$ vector can obviously be expressed as a linear combination of the predetermined variables using the solution $\mathbf{X}_{2,t} = \mathbf{C}\mathbf{X}_{1,t}$, so that $\mathbf{Z}_t = \hat{\mathbf{D}}\mathbf{X}_{1,t}$ for a suitably defined matrix $\hat{\mathbf{D}}$. The (nominal) pricing kernel m_{t+1} is defined as $m_{t+1} = \exp(-r_t)\,\psi_{t+1}/\psi_t$, where ψ_{t+1} is the Radon-Nikodym derivative assumed to follow the log-normal process $\psi_{t+1} = \psi_t \exp\left(-\frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\xi_{1,t+1}\right)$. Finally, market prices of risk are assumed to be affine in the transformed state vector $\mathbf{Z}_t$

$$\lambda_t = \lambda_0 + \lambda_1 \mathbf{Z}_t, \tag{8}$$

where only the 4 elements in λ_0 and the 4×4 sub-matrix in λ_1 corresponding to contemporaneous values are allowed to be non-zero. Since $\mathbf{Z}_t = \hat{\mathbf{D}}\mathbf{X}_{1,t}$, $\lambda_1\mathbf{Z}_t = \lambda_1\hat{\mathbf{D}}\mathbf{X}_{1,t}$ and λ_1 will induce restrictions on $\widetilde{\lambda}_1$ such that $\widetilde{\lambda}_1\hat{\mathbf{D}}^{-1} = \lambda_1$.

In the Appendix we show that the reduced form (7) of our macroeconomic model, coupled with the aforementioned assumptions on the pricing kernel, implies that the continuously compounded yield y^n_t on a zero coupon nominal bond with maturity n is given by

$$y_t^n = A_n + B_n' \mathbf{Z}_t, \tag{9}$$

where the A_n and B_n' matrices can be derived using recursive relations. Stacking all yields in a vector $\mathbf{Y}_t$, we write the above equations jointly as $\mathbf{Y}_t = \mathbf{A} + \mathbf{B}'\mathbf{Z}_t$ or, equivalently, $\mathbf{Y}_t = \mathbf{A}_n + \tilde{\mathbf{B}}_n' \mathbf{X}_{1,t}$, where $\tilde{\mathbf{B}}_n' \equiv \mathbf{B}_n' \hat{\mathbf{D}}$. Similarly, for real bonds y_t^{*n} we obtain

$$y_t^{*n} = A_n^* + B_n'^* \mathbf{Z}_t, \tag{10}$$

4.2 The inflation risk premium

It is instructive to first look at the inflation risk premium which characterizes the short-term rate. Given the nominal and real short rates, r_t and r_t^* respectively, the Appendix shows that the former can be written as

$$r_t = r_t^* + E_t\left[\pi_{t+1}\right] + prem_{\pi,t} + \tfrac{1}{2}\mathbf{C}_\pi \Sigma \Sigma' \mathbf{C}_\pi' \tag{11}$$

where

$$\begin{aligned}
r_t^* &= \mathbf{C}_\pi \Sigma \left(\lambda_0 - \tfrac{1}{2}\Sigma' \mathbf{C}_\pi'\right) + \left(\overline{\mathbf{\Delta}}' - \mathbf{C}_\pi \left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma \lambda_1\right)\right) \mathbf{Z}_t \\
E_t\left[\pi_{t+1}\right] &= \mathbf{C}_\pi \mathbf{M} \hat{\mathbf{D}}^{-1} \mathbf{Z}_t \\
prem_{\pi,t}^1 &= -\mathbf{C}_\pi \Sigma \lambda_0 - \mathbf{C}_\pi \Sigma \lambda_1 \mathbf{Z}_t
\end{aligned}$$

We define $prem_{\pi,t}$ as the inflation risk premium to distinguish it from the convexity term $\frac{1}{2}\mathbf{C}_\pi \Sigma \Sigma' \mathbf{C}_\pi'$, which would affect the short-term rate even if the prices of risk were zero.

The inflation risk premium is related to the full standard deviation of inflation, the term $\mathbf{C}_\pi \Sigma$, irrespective of the actual shock that determines it. For given prices of risk, the inflation risk premium will be higher, the higher the variance of the shocks, and the higher their impact on inflation.

For bonds of other maturities, a more complex expression holds (see the Appendix). As a result, the break-even inflation rate (BE) can be written as

$$BE_{\pi,t}^n = const + \mathbf{C}_\pi \hat{\mathbf{D}}^{-1} \frac{\sum_{i=1}^n \widehat{\mathbf{M}}^i}{n} \mathbf{Z}_t \tag{12}$$

where *const* is a constant component and $\widehat{\mathbf{M}} \equiv \hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right)$ captures the risk-adjustment in the law of motion of the transformed state vector $\mathbf{Z}_t$.

The inflation risk premium is equal to the break-even inflation rate net of expected average inflation over the maturity of the bond, i.e. $\overline{\pi}_{t+n} = \frac{1}{n}\sum_{i=1}^{n} \pi_{t+i}$,

$$prem_{\pi,t}^{ytm,n} = const + \mathbf{C}_\pi \left(\hat{\mathbf{D}}^{-1} \frac{\sum_{i=1}^{n} \widehat{\mathbf{M}}^i}{n} - \frac{\sum_{i=1}^{n} \mathbf{M}^i}{n} \hat{\mathbf{D}}^{-1} \right) \mathbf{Z}_t \tag{13}$$

Equation (13) emphasizes that the inflation risk premium arises because of the difference between the historical and risk-adjusted laws of motions of the state vector $\mathbf{Z}_t$. Appendix A.3 shows that when the market prices of risk are not state dependent, i.e. when $\lambda_1 = 0$, the inflation risk premium becomes constant over time.

Depending on the prices of risk, the matrix $\widehat{\mathbf{M}}$ could have eigenvalues outside the unit circle even if $\mathbf{M}$ does not. If its eigenvalues are within the unit circle, inflation risk premia on long term yields will be bounded from above. Long-term premia will also be more sensitive to changes in the states $\mathbf{Z}_t$ than premia on short-term bonds, because $\sum_{i=1}^{n} \widehat{\mathbf{M}}^i$ tends to increase as n increases. If, instead, the risk-adjusted law of motion is non-stationary, i.e. if some of the eigenvalues of $\widehat{\mathbf{M}}$ are outside the unit circle, then the sum in equation (12) is not bounded and inflation risk premia can play an even larger role on long-term yields.

4.3 Maximum likelihood estimation

In order to estimate the model, we need to distinguish first between observable and unobservable variables in the $\mathbf{X}_{1t}$ vector. We adopt the approach which is common in the finance literature and which involves inverting the relationship between yields and unobservable factors (Chen and Scott, 1993). We also use the common approach of assuming that some of the yields are imperfectly measured to prevent stochastic singularity. More precisely, we use yields on 1-, 3-, and 6-month, as well as 1-, 3-, and 10-year nominal zero-coupon bonds and on 3-, 5-, 7- and 10-year real bonds. We assume that all bonds are imperfectly observable, with the exception of nominal bonds at the 3-month and 10-year maturities.

To deal with the lack of data on real yields before October 1999, we simply treat such yields as unobservable variables. Since these are not state variables, their unobservability has no impact on the likelihood. They are included in the measurement equation as of October 1999 through their impact on the measurement errors. The likelihood function can therefore be written as

$$
\begin{aligned}
\pounds(\theta) = &-(T-1)\left(\ln|J| + \frac{n_p}{2}\ln(2\pi) + \frac{1}{2}\ln\left|\mathbf{\Sigma\Sigma}'\right| + \frac{n_m}{2}\ln(2\pi) + \frac{1}{2}\sum_{i=1}^{n_m}\ln\sigma_{m,i}^2\right) \\
&-\frac{1}{2}\sum_{t=2}^{T}\left(\mathbf{X}_{1,t}^u - \mathbf{M}^u\mathbf{X}_{1,t-1}^u\right)'\left(\mathbf{\Sigma\Sigma}'\right)^{-1}\left(\mathbf{X}_{1,t}^u - \mathbf{M}^u\mathbf{X}_{1,t-1}^u\right) - \frac{1}{2}\sum_{t=2}^{T}\sum_{i=1}^{n_m}\frac{\left(u_{t,i}^m\right)^2}{\sigma_{m,i}^2} \\
&-(T-t_r)\left(\frac{n_r}{2}\ln(2\pi) + \frac{1}{2}\sum_{i=1}^{n_r}\ln\sigma_{r,i}^2\right) - \frac{1}{2}\sum_{t=t_r}^{T}\sum_{i=1}^{n_r}\frac{\left(u_{t,i}^r\right)^2}{\sigma_{r,i}^2}
\end{aligned}
$$

where $\mathbf{X}_{1,t}^u$ are the unobservable variables included in the $\mathbf{X}_{1,t}$ vector, u_t^m are the measurement error shocks, $\mathbf{J}$ is a Jacobian matrix defined in the Appendix, $\mathbf{\Sigma\Sigma}'$ is the variance-covariance matrix of the four macroeconomic shocks, σ_m and σ_r are the standard deviations of the nominal and the real measurement error shocks respectively, T is the sample size, t_r is the observation from which index-linked yields are available, n_m and n_r are the numbers of measurement errors in nominal and real bonds, respectively, and n_p is the number of variables measured without error. To reduce the number of estimated parameters, we assume all nominal (real) measurement errors are characterized by the same standard deviation.

The problem of maximizing the likelihood is nontrivial, given the large size of the parameter space. We employ the method of simulated annealing, introduced to the econometric literature by Goffe, Ferrier and Rogers (1994). The method is developed with an aim towards applications where there may be a large number of local optima. One disadvantage of the simulated annealing method is that it does not provide us with an estimate of the first and second derivatives, evaluated at the maximum, of the likelihood function with respect to the parameter vector, i.e. $\partial\ln(\pounds(\theta))/\partial\theta'$ and $\partial^2\ln(\pounds(\theta))/\partial\theta'\partial\theta$. To deal with this problem, we follow Anderson *et al.* (1996) and rely on analytical results to calculate the Jacobian $\partial\ln(\pounds(\theta))/\partial\theta'$ to obtain the outer product derivative estimate of the variance covariance matrix (as in Hördahl, Tristani and Vestin, 2006).

Our results, however, also show some signs of residual autocorrelation, especially in the measurement errors of index-linked yields. For this reason, our inference is based on HAC standard errors of Newey and West, which also require the computation of an estimate of the variance-covariance matrix based on the Hessian. In this paper, the Hessian matrix of the likelihood function with respect to the parameters is also computed analytically.

5 The term structure of inflation risk premia in the euro area

5.1 Parameter estimates and impulse responses

An advantage of our approach is that the parameters which affect the historical dynamics of the state vector can be interpreted economically. Table 1 reports parameter estimates based on our preferred specification with HAC standard errors.[8] Most parameters are estimated quite precisely, but there are exceptions concerning, most notably, the inflation response coefficient in the Taylor rule, β, and the elasticity of the output gap to the real interest rate, ζ_r. The imprecision in the estimate of β is likely to reflect the small variation in inflation over our sample period, while a small ζ_r is a frequent occurrence in estimates of the output gap equation (see e.g. Jondeau and Le Bihan (2001) for Germany; Fuhrer and Rudebusch (2004) for the United States). While these in principle are important parameters, they do not appear to affect significantly our risk premia estimates. Small perturbations of either β or ζ_r cause minor changes in the inflation risk premia generated by our model.[9]

Concerning our point estimates of macro parameters, these are broadly in line with previous results in the literature. The policy rule is characterized by a high degree of interest rate smoothing, a mild response to inflation deviations from the objective, and a non-negligible response to the output gap. The degree of forward-lookingness of the output gap equation is relatively small, while it is high for the inflation equation. The latter result is significantly different from available estimates based on German data over a longer sample (see e.g. Jondeau and Le Bihan (2001) or Hördahl, Tristani and Vestin (2006)). Rather than signalling structural diversities, however, the difference could simply be due to the different definition of inflation, which here is constructed based on monthly, rather than year-on-year, log-price changes. Intuitively, monthly inflation is much less

[8] In the estimation process, we successively set to zero entries of the λ_1 matrix when this restriction was accepted in a likelihood-ratio test.

[9] More precisely, a 10% increase in either β or ζ_r causes the 10-year inflation risk premium to shift only by a few basis points, almost never pushing it outside its 95% confidence bands.

persistent than year-on-year inflation, thus the lesser role of backward-looking elements.

The estimated standard deviations of fundamental shocks are relatively low, suggesting that the model is capable of accounting endogenously for a large part of macroeconomic dynamics. The standard deviation of measurement errors is broadly consistent with the results of affine models without macroeconomic variables. Filtered measurement error series, however, tend to display some serial correlation for real yields, which occasionally – notably at the very end of the sample period – translates into persistent mispricings.

Figures 4 and 5 show impulse response functions of the macroeconomic variables, the 10-year break-even inflation rate, expected inflation and inflation risk premium to a monetary policy shock and an output gap shock, respectively. We report only responses to these shocks, because they turn out to be the most important drivers of long-term inflation risk premia.

A 20 basis points surprise increase in the policy interest rate has a strong impact on the output gap over time, while current and expected inflation fall only mildly. The response of the 10-year break-even inflation rate reflects mostly the dynamics of the inflation risk premium. The surprise interest rate hike appears to be associated with reduced concern about inflationary risks among investors, so that the inflation risk premium falls on impact and then slowly returns to the baseline after 3 years.

An increase in the output gap by 0.7 percent is met by a progressive increase in policy rates over time, with a peak increase of 20 basis points after 1.5 years. The policy tightening is sufficient to keep both inflation and long-run inflation expectations firmly anchored. Nevertheless, the output gap shock generates some movement in the break-even inflation rate, which increases on impact by 4 basis points for the 10-year maturity, and then falls by up to 5 basis points after 2 years. Once again, the results show that the dynamics of the break-even inflation rate reflect those of the inflation risk premium. This increases when the shock occurs, presumably alongside concerns for inflationary risks, and then falls as the policy response unfolds.

5.2 Yield premia and inflation risk premia

Our estimates of the term structure of average yield premia and inflation risk premia are reported with 95% confidence bands in Figures 6 and 7. The average yield premia reflect the average slope of the yield curve over the period. The interesting part of Figure 6 is the confidence interval, showing that yield premia are significantly different from zero over the whole maturity spectrum. This result contrasts sharply with the evidence on average inflation risk premia in Figure 7, which are much smaller and insignificantly different from zero at all maturities. This result strongly suggests that term premia in the nominal yield curve are mostly a reflection of real sources of risk, i.e. risks which affect both the nominal and index-linked yield curves.

The conditional risk premia results shown in Figures 8 and 9, which focus on the 10-year maturity, are consistent with the average results in Figures 6 and 7. The term premium on 10-year nominal yields is large, time-varying, and strongly significantly different from zero over the whole sample period. In contrast, the inflation risk premium tends to hover around zero and is only occasionally statistically significant. More specifically, the 10-year inflation premium is insignificant until mid-2000; turns significantly negative thereafter and hovers around -40 basis points until end-2002; increases to positive territory again and remains borderline insignificant until the end of our sample (April 2006).

The time variation in inflation risk premia is not necessarily highly correlated across maturities. Figure 10 shows our results for the inflation premium on 3-year bonds, which should reflect more closely risks at business cycle frequencies. This inflation risk premium is significantly positive between the second half of 1999 and the beginning of 2000, while becoming insignificantly different from zero thereafter.

We analyze more closely the relationship of these dynamics with those of macro variables in the next subsection, where it becomes clear that a direct association between the level of actual inflation and the level of inflation risk premia would be misleading. Instead, it turns out that the short-term policy rate and the output gap are important in determining inflation risk premia, in line with the impulse-response results discussed above.

From a more general viewpoint, the finding of a (temporarily) negative inflation risk premium is not inconsistent with the theoretical results reviewed in Section 2. Nevertheless, it may have a more intuitive explanation in terms of liquidity premia. Indeed, one can observe that the risk premium is negative in the period of relatively low liquidity in the index-linked bond market (see figure 1a). *Ceteris paribus*, a higher liquidity premium on real yields would tend to increase their levels, thus to reduce the break-even inflation rate. Hence, for given inflation expectations, differential liquidity conditions could partly explain the *estimated* negative inflation risk premium in 2001−2002. The latter would be lower than the "true" inflation risk premium, because it would include a negative liquidity premium component (namely the difference between the liquidity premia on nominal and real bonds).

5.3 The inflation risk premium and the macroeconomy

In order to make sense of the evolution of the estimated 10-year inflation risk premium, we decompose the time-varying part of this premium into its determinants in Figure 11. As shown in the Appendix, the inflation risk premium is affine in the state variables $\mathbf{Z}_t$, and it is therefore straightforward to obtain a decomposition of the time-varying component of the inflation premium in terms of inflation, output gap, inflation target and the short-term policy rate.

The most striking feature of the results illustrated in this figure is that the premium is insensitive to the evolution of inflation, and very mildly affected by changes in the perceived inflation objective. This result is partly a reflection of the stability of the filtered inflation objective over our estimation sample (see the next section). The consequence is that the most important determinants of variations in investors' attitudes towards risk are variations in the level of short-term interest rates and in the cycle – as captured by the dynamics of the output gap.

Ceteris paribus, investors become more willing to take on inflation risk, i.e. they require a lower inflation risk premium, when short-term interest rates are high. As argued above, this may reflect their stronger confidence in the absence of future upside inflation surprises when policy is tightened. At the same time, inflation risk premia are higher when the output gap is positive, and vice versa. Once again, a positive output gap could be associated with perceptions of a higher risk of inflation surprises on the upside.

Figure 12 compares these results to those related to real risk premia, i.e. premia implicit in the return on index-linked bonds. The 10-year real premium is also insensitive to the evolution of inflation, and little sensitive to cyclical developments – as captured by output gap fluctuations. The real risk premium appears instead mostly correlated with movements in the perceived inflation objective of the ECB. Small increases in the perceived objective tend to be accompanied by increasing fears of their potential effect on the overall macroeconomic performance. Consequently, the estimated 10-year real risk premium increases during the mid-2000 to mid-2002 period, when the perceived target is filtered to be slightly higher than average, and falls in 2005−2006 alongside the reduction in the perceived target.

Finally, Figure 13 reports results from the decomposition of the time-varying part of the estimated 10-year total yield premium in nominal bonds, i.e. the sum of both real and inflation risk premia. Not surprisingly, given the larger absolute value of real risk premia, fluctuations in the total premium are associated with the same variables as those in the real risk premium.

5.4 Premium-adjusted break-even inflation rates

An alternative way to account for the impact of inflation risk premia is via the calculation of premium-adjusted break-even inflation rates, which provide a model-consistent measure of inflation expectations over the life of the bonds. This simply strips out the estimated inflation risk premium component from the standard break-even inflation rate. Figure 14 reports raw and adjusted 10-year break-even inflation rates in the euro area.

The raw break-even rate displays some non-negligible variability, dropping down to 1.5% in 2001 and increasing thereafter up to levels slightly above 2%. To the extent that these fluctuations were directly interpreted as measures of inflation expectations, they could provide reasons for concern. Average inflation expectations over a 10-year horizon either at 1.5% or above 2% could be taken as signals of imperfect credibility of the ECB's price stability objective.

The premium-adjusted break-even inflation rate, however, gives a different message. Fluctuations in the raw break-even rate are interpreted by the model as mostly due to the dynamics of the inflation premium. Average inflation expectations are, on the other hand, quite stable over time. Moreover, taking into account the confidence bands, the estimated risk-adjusted break-even inflation rate has remained at levels consistent with the ECB objective of inflation below but close to 2%. Information from long-horizon survey forecasts are broadly in line with our estimates of 10-year inflation expectations. Alongside the adjusted and unadjusted break-even rates, Figure 14 displays expected euro area inflation 10 years ahead, as reported by Consensus Economics twice per year.[10] The

[10] Consensus Economics began publishing long-horizon inflation forecasts for the euro area only as of 2003. Prior to this, we use a weighted average of survey results for Germany, France, Italy, Spain and the Netherlands. The weights used are proportional to the euro area HICP weights for these countries.

adjusted break-even rates are closer to the survey forecasts in the second half of the sample, while they tend to exceed the survey data more in the early part of the sample. As already discussed, liquidity considerations may have affected the level of index-linked bond yields in particular at the beginning of our estimation period. The results for the early part of the sample should therefore be interpreted with caution.

The estimated adjusted break-even inflation rates are in line with our estimates of the perceived inflation objective π_t^*, displayed in Figure 15. The objective displays very minor variations over the 1999−2006 period, which seems intuitively appealing given the mild inflation fluctuations observed over the same period of time.[11]

6 Conclusions

The difference between nominal and inflation-linked bond yields, the break-even inflation rate, is often used as an indicator of market expectations of future inflation. However, the break-even inflation rate is a noisy measure of expected inflation, because it can include an inflation risk premium component.

This paper uses information from both nominal and index-linked yields to estimate the size and dynamics of inflation risk premia in the euro area. This is done by adopting the macro-finance term structure framework developed in Hördahl, Tristani and Vestin (2006), in which yields are derived from the dynamics of the short rate obtained from the solution of a linear macro model, combined with an essentially affine stochastic discount factor. Apart from delivering estimates of the inflation risk premium, this approach has the advantage that it also makes it possible to analyze its relationship with macroeconomic variables.

The main result of our analysis is that, on average, the inflation risk premium on long-term euro area nominal yields was insignificantly different from zero over the 1999−2006 sample. Fluctuations around the average have also been relatively small, but statistically significant in 2001 and 2002 and, occasionally, in the 2004−2006 period. As a result, the raw break-even inflation rate has often provided inaccurate information on inflation expectations. More specifically, our results suggest that fluctuations in the raw break-

[11] A caveat in this regard is that the available euro data spans a very short period. There are, of course, no guarantees that the relatively limited fluctuations in inflation seen during this period will continue to characterise the euro area economy indefinitely.

even rate have mostly reflected variations in the inflation risk premium, while long-term inflation expectations have always remained remarkably anchored from 1999 to date. Our results suggest that a regular monitoring of inflation risk premia is important to understand correctly the information contained in break-even inflation rates.

A Appendix

A.1 Pricing real and nominal bonds

The solution of the macro model is of the form

$$\mathbf{X}_{1,t+1} = \mathbf{M}\mathbf{X}_{1,t} + \Sigma\xi_{1,t+1},$$
$$\mathbf{X}_{2,t+1} = \mathbf{C}\mathbf{X}_{1,t+1},$$

where the state vector $\mathbf{X}_{1,t} = [x_{t-1}, x_{t-2}, x_{t-3}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_t^*, \eta_t, \varepsilon_t^\pi, \varepsilon_t^x, r_{t-1}]'$ contains the predetermined variables, $\mathbf{X}_{2,t}$ includes the non-predetermined variables, $\mathbf{X}_{2t} = [E_t x_{t+11}, ..., E_t x_{t+1}, x_t, E_t \pi_{t+11}, ..., E_t \pi_{t+1}, \pi_t]'$. The nominal short-term interest rate can be written as

$$\begin{aligned} r_t &= -\left(\mathbf{F}_1 + \mathbf{F}_2\mathbf{C}\right)\mathbf{X}_{1,t} \\ &\equiv \mathbf{\Delta}'\mathbf{X}_{1,t}, \end{aligned}$$

with $\mathbf{F}_1$ and $\mathbf{F}_2$ partitions of $\mathbf{F}$ conformable with $\mathbf{X}_{1,t}$ and $\mathbf{X}_{2,t}$.

Alternatively, we can write this in terms of the transformed state vector $\mathbf{Z}_t$, defined $\mathbf{Z}_t = \hat{\mathbf{D}}\mathbf{X}_{1,t}$ so that $\mathbf{Z}_{1t} \equiv [x_{t-1}, x_{t-2}, x_{t-3}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_t^*, r_t, \pi_t, x_t, r_{t-1}]'$ for a suitably defined matrix $\hat{\mathbf{D}}$, in which case the short rate is given by

$$r_t = \overline{\mathbf{\Delta}}'\mathbf{Z}_t.$$

From the macro model solution, we also know that

$$\begin{aligned} \pi_{t+1} &= \mathbf{C}_\pi\mathbf{M}\mathbf{X}_{1,t} + \mathbf{C}_\pi\Sigma\xi_{1,t+1} \\ &= \mathbf{C}_\pi\mathbf{M}\hat{\mathbf{D}}^{-1}\mathbf{Z}_t + \mathbf{C}_\pi\Sigma\xi_{1,t+1}, \end{aligned}$$

where $\mathbf{C}_\pi$ is the relevant row of $\mathbf{C}$.

Now assume that the real pricing kernel is m_{t+1}^*, so that the following fundamental asset pricing relation holds

$$E_t\left[m_{t+1}^*\left(1 + R_{t+1}^*\right)\right] = 1,$$

where R_{t+1}^* denotes the real return on some asset.

If we now want to price an n-period nominal bond, p_t^n, we get

$$\frac{p_t^n}{q_t} = E_t\left[m_{t+1}^*\frac{p_{t+1}^{n-1}}{q_{t+1}}\right],$$

where q_t is the price level in the economy. In terms of inflation rates, $\pi_{t+1} \equiv \ln q_{t+1} - \ln q_t$, we obtain

$$p_t^n = E_t\left[m_{t+1}^*\frac{p_{t+1}^{n-1}}{\exp\left(\pi_{t+1}\right)}\right].$$

Notice that this is equivalent to postulating a nominal pricing kernel $m_{t+1} \equiv m_{t+1}^*/\exp\left(\pi_{t+1}\right)$, such that

$$p_t^n = E_t\left[m_{t+1}p_{t+1}^{n-1}\right].$$

We now define the nominal pricing kernel as $m_{t+1} = \exp\left(-r_t\right)\frac{\psi_{t+1}}{\psi_t}$, where ψ_{t+1} is the

Radon-Nikodym derivative ψ_{t+1} assumed to follow the log-normal process

$$\psi_{t+1} = \psi_t \exp\left(-\frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\xi_{1,t+1}\right),$$

and where λ_t is the vector of market prices of risk associated with the underlying sources of uncertainty $\xi_{1,t+1}$ in the economy. We also assume that the market prices of risk are affine in the transformed state vector $\mathbf{Z}_t$,

$$\lambda_t = \lambda_0 + \lambda_1 \mathbf{Z}_t.$$

Postulating that nominal bond prices will be exponential-affine functions of the state variables, we obtain

$$p_t^n = \exp\left(\bar{A}_n + \bar{B}_n'\mathbf{Z}_t\right),$$

where $\bar{A}_n$ and $\bar{B}_n'$ are recursive parameters that depend on the maturity n in the following way:

$$\begin{aligned}
\bar{A}_{n+1} &= \bar{A}_n - \bar{B}_n'\hat{\mathbf{D}}\Sigma\lambda_0 + \tfrac{1}{2}\bar{B}_n'\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_n, \\
\bar{B}_{n+1}' &= \bar{B}_n'\hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) - \overline{\mathbf{\Delta}}'.
\end{aligned}$$

Nominal bond yields are then given by

$$\begin{aligned}
y_t^n &= -\frac{\ln\left(p_t^n\right)}{n} \\
&= -\frac{\bar{A}_n}{n} - \frac{\bar{B}_n'}{n}\mathbf{Z}_t \\
&\equiv A_n + B_n'\mathbf{Z}_t.
\end{aligned}$$

A.1.1 Real bonds

The definition of the pricing kernel implies

$$r_t = -\ln m_{t+1} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\xi_{1,t+1},$$

which translates into a real pricing kernel

$$m_{t+1}^* = \exp\left(-r_t + \pi_{t+1}\right)\frac{\psi_{t+1}}{\psi_t},$$

or

$$m_{t+1}^* = \exp\left(-\overline{\mathbf{\Delta}}'\mathbf{Z}_t + \mathbf{C}_\pi\mathbf{M}\mathbf{X}_{1,t} + \mathbf{C}_\pi\Sigma\xi_{1,t+1} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\xi_{1,t+1}\right).$$

We postulate again that real bond prices will be exponential-affine functions of the state variables,

$$p_t^{n*} = \exp\left(\bar{A}_n^* + \bar{B}_n^{*\prime}\mathbf{Z}_t\right),$$

where $\bar{A}_n^*$ and $\bar{B}_n^*$ are parameters that depend on the maturity n, and which can be identified using

$$
\begin{aligned}
p_t^{n+1*} &= E_t\left[m_{t+1}^* p_{t+1}^{n*}\right] \\
&= \exp\left(\bar{A}_n^* - \overline{\boldsymbol{\Delta}}'\mathbf{Z}_t + \mathbf{C}_\pi \mathbf{M}\mathbf{X}_{1,t} + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\mathbf{M}\mathbf{X}_{1,t} - \frac{1}{2}\lambda_t'\lambda_t\right) \\
&\times E_t\left[\exp\left(\left(\mathbf{C}_\pi\Sigma - \lambda_t' + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\Sigma\right)\xi_{1,t+1}\right)\right],
\end{aligned}
$$

where we used

$$
\mathbf{Z}_{t+1} = \hat{\mathbf{D}}\mathbf{M}\mathbf{X}_{1,t} + \hat{\mathbf{D}}\Sigma\xi_{1,t+1}.
$$

Noting that

$$
E_t\left[\exp\left(\left(\mathbf{C}_\pi\Sigma + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\Sigma - \lambda_t'\right)\xi_{1,t+1}\right)\right] = \exp\left(\tfrac{1}{2}\left(\left(\mathbf{C}_\pi + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\Sigma - \lambda_t'\right)\left(\left(\mathbf{C}_\pi + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\Sigma - \lambda_t'\right)'\right),
$$

and rearranging terms, we obtain

$$
\begin{aligned}
p_t^{n+1*} = \exp\Big(&\bar{A}_n^* + \tfrac{1}{2}\left(\mathbf{C}_\pi + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\Sigma\Sigma'\left(\mathbf{C}_\pi + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)' - \left(\mathbf{C}_\pi + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\Sigma\lambda_0 \\
&+ \left(\left(\mathbf{C}_\pi + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) - \overline{\boldsymbol{\Delta}}'\right)\mathbf{Z}_t\Big).
\end{aligned}
$$

We can therefore identify $\bar{A}_n^*$ and $\bar{B}_n^*$ recursively as

$$
\begin{aligned}
\bar{A}_{n+1}^* &= \bar{A}_n^* + \tfrac{1}{2}\left(\mathbf{C}_\pi + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\Sigma\Sigma'\left(\mathbf{C}_\pi + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)' - \left(\mathbf{C}_\pi + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\Sigma\lambda_0, \\
\bar{B}_{n+1}^{*\prime} &= \left(\mathbf{C}_\pi + \bar{B}_n^{*\prime}\hat{\mathbf{D}}\right)\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) - \overline{\boldsymbol{\Delta}}'.
\end{aligned}
$$

For a 1-month real bond, in particular, we obtain

$$
\begin{aligned}
p_t^{1*} &= E_t\left[m_{t+1}\right] \\
&= \exp\left(\left(-\overline{\boldsymbol{\Delta}}' + \mathbf{C}_\pi\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right)\right)\mathbf{Z}_t - \mathbf{C}_\pi\Sigma\left(\lambda_0 - \tfrac{1}{2}\Sigma'\mathbf{C}_\pi'\right)\right),
\end{aligned}
$$

which can be used to start the recursion. Note that the short-term real rate is

$$
r_t^* = \mathbf{C}_\pi\Sigma\left(\lambda_0 - \tfrac{1}{2}\Sigma'\mathbf{C}_\pi'\right) + \left(\overline{\boldsymbol{\Delta}}' - \mathbf{C}_\pi\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right)\right)\mathbf{Z}_t.
$$

A.2 Short-rate spread

The effect of the inflation risk premium is to drive a wedge between riskless real yields and ex-ante real yields, namely nominal yields net of expected inflation. For the short-term rate, in particular, we can write

$$
r_t = r_t^* + E_t\left[\pi_{t+1}\right] + prem_{\pi,t} + \tfrac{1}{2}\mathbf{C}_\pi\Sigma\Sigma'\mathbf{C}_\pi',
$$

where

$$\begin{aligned} r_t^* &= \mathbf{C}_\pi \Sigma \left(\lambda_0 - \tfrac{1}{2}\Sigma' \mathbf{C}_\pi'\right) + \left(\overline{\mathbf{\Delta}}' - \mathbf{C}_\pi \left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right)\right) \mathbf{Z}_t \\ E_t\left[\pi_{t+1}\right] &= \mathbf{C}_\pi \mathbf{M}\hat{\mathbf{D}}^{-1} \mathbf{Z}_t \\ prem_{\pi,t} &= -\mathbf{C}_\pi \Sigma \left(\lambda_0 + \lambda_1 \mathbf{Z}_t\right). \end{aligned}$$

Note that the discrepancy between ex-ante real and risk-free rates is not only due to inflation risk, but also includes a convexity term $\frac{1}{2}\mathbf{C}_\pi \Sigma\Sigma' \mathbf{C}_\pi'$. We define as inflation risk premium the component of the difference which would vanish if market prices of risk were zero.

A.3 Derivation of inflation risk premium and break-even inflation rates

For all maturities, recall that the continuously compounded yield is, for nominal and real bonds, respectively

$$\begin{aligned} y_{t,n} &= -\frac{\bar{A}_n}{n} - \frac{\bar{B}_n'}{n} \mathbf{Z}_t \\ y_{t,n}^* &= -\frac{\bar{A}_n^*}{n} - \frac{\bar{B}_n^{*\prime}}{n} \mathbf{Z}_t. \end{aligned}$$

The yield spread is therefore simply

$$y_{t,n} - y_{t,n}^* = -\frac{1}{n}\left(\bar{A}_n - \bar{A}_n^*\right) - \frac{1}{n}\left(\bar{B}_n' - \bar{B}_n^{*\prime}\right) \mathbf{Z}_t,$$

where

$$\begin{aligned} \bar{A}_{n+1} - \bar{A}_{n+1}^* &= \bar{A}_n - \bar{A}_n^* - \left(\bar{B}_n' - \bar{B}_n^{*\prime}\right) \hat{\mathbf{D}} \Sigma \lambda_0 + \mathbf{C}_\pi \Sigma \lambda_0 - \tfrac{1}{2}\mathbf{C}_\pi \Sigma\Sigma' \mathbf{C}_\pi' \\ &\quad - \mathbf{C}_\pi \Sigma\Sigma' \hat{\mathbf{D}}' \bar{B}_n^* + \tfrac{1}{2}\left(\bar{B}_n' \hat{\mathbf{D}} \Sigma\Sigma' \hat{\mathbf{D}}' \bar{B}_n - \bar{B}_n^{*\prime} \hat{\mathbf{D}} \Sigma\Sigma' \hat{\mathbf{D}}' \bar{B}_n^*\right) \\ \bar{B}_{n+1}' - \bar{B}_{n+1}^{*\prime} &= \left(\bar{B}_n' - \bar{B}_n^{*\prime}\right) \hat{\mathbf{D}} \left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) - \mathbf{C}_\pi \left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right). \end{aligned}$$

Note that the nominal bond equation can be solved explicitly as

$$\begin{aligned} \bar{A}_n &= \bar{A}_1 + \sum_{i=1}^{n-1} \left(\tfrac{1}{2}\overline{\mathbf{B}}_i' \hat{\mathbf{D}} \Sigma\Sigma' \hat{\mathbf{D}}' \overline{\mathbf{B}}_i - \overline{\mathbf{B}}_i' \hat{\mathbf{D}} \Sigma \lambda_0\right), \\ \overline{B}_n' &= -\overline{\mathbf{\Delta}}' \sum_{i=0}^{n-1} \left[\hat{\mathbf{D}} \left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right)\right]^i. \end{aligned}$$

Similar, for the real bond $\bar{A}_n^*$ we obtain

$$\begin{aligned} \bar{A}_n^* &= n\mathbf{C}_\pi \Sigma \left(\tfrac{1}{2}\Sigma' \mathbf{C}_\pi' - \lambda_0\right) + \sum_{i=1}^{n-1} \left(\bar{B}_i^{*\prime} \hat{\mathbf{D}} \Sigma\Sigma' \mathbf{C}_\pi' + \tfrac{1}{2}\bar{B}_i^{*\prime} \hat{\mathbf{D}} \Sigma\Sigma' \hat{\mathbf{D}}' \bar{B}_i^* - \bar{B}_i^{*\prime} \hat{\mathbf{D}} \Sigma \lambda_0\right) \\ \bar{B}_n^{*\prime} &= \left(\mathbf{C}_\pi \left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) - \overline{\mathbf{\Delta}}'\right) \sum_{i=0}^{n-1} \left[\hat{\mathbf{D}} \left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right)\right]^i. \end{aligned}$$

Note that the law of motion of the transformed state vector can be written as $\mathbf{Z}_{t+1} = \hat{\mathbf{D}}\mathbf{M}\hat{\mathbf{D}}^{-1}\mathbf{Z}_t + \hat{\mathbf{D}}\Sigma\xi_{1,t+1}$, so that the term $\hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right)$ represents the expected change in $\mathbf{Z}_t$ under Q. We can then define a new matrix $\widehat{\mathbf{M}} = \hat{\mathbf{D}}\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right)$. Note also that the sum $\sum_{i=0}^{n-1}\widehat{\mathbf{M}}^i$ can be solved out as $\sum_{i=0}^{n-1}\widehat{\mathbf{M}}^i = \left(\mathbf{I} - \widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I} - \widehat{\mathbf{M}}^n\right)$ for bonds of finite maturity.[12] Note that we could equivalently write $\sum_{i=0}^{n-1}\widehat{\mathbf{M}}^i = \left(\mathbf{I} - \widehat{\mathbf{M}}^n\right)\left(\mathbf{I} - \widehat{\mathbf{M}}\right)^{-1}$.

For the state dependent component of bond prices, it follows that

$$\begin{aligned}
\overline{B}_n' &= -\overline{\boldsymbol{\Delta}}'\left(\mathbf{I} - \widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I} - \widehat{\mathbf{M}}^n\right) \\
\bar{B}_n^{*\prime} &= \left(\mathbf{C}_\pi\hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}} - \overline{\boldsymbol{\Delta}}'\right)\left(\mathbf{I} - \widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I} - \widehat{\mathbf{M}}^n\right),
\end{aligned}$$

and

$$\bar{B}_n^{*\prime} - \overline{B}_n' = \mathbf{C}_\pi\hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}}\left(\mathbf{I} - \widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I} - \widehat{\mathbf{M}}^n\right).$$

Note also that

$$E_t\left[\pi_{t+n}\right] = \mathbf{C}_\pi\mathbf{M}^n\hat{\mathbf{D}}^{-1}\mathbf{Z}_t,$$

and that expected average inflation up to $t+n$, $\overline{\pi}_{t+n}$ is

$$\begin{aligned}
E_t\overline{\pi}_{t+n} &= \frac{1}{n}\sum_{i=1}^{n} E_t\pi_{t+i} \\
&= \mathbf{C}_\pi\frac{\sum_{i=1}^{n}\mathbf{M}^i}{n}\hat{\mathbf{D}}^{-1}\mathbf{Z}_t,
\end{aligned}$$

or, writing this out explicitly,

$$E_t\overline{\pi}_{t+n} = \frac{1}{n}\mathbf{C}_\pi\left(\mathbf{I} - \mathbf{M}^n\right)\left(\mathbf{I} - \mathbf{M}\right)^{-1}\mathbf{M}\hat{\mathbf{D}}^{-1}\mathbf{Z}_t.$$

We are now ready to define the break even inflation rate as

$$\begin{aligned}
y_{t,n} - y_{t,n}^* &= \frac{1}{n}\left(\bar{A}_n^* - \bar{A}_n\right) + \frac{1}{n}\left(\bar{B}_n^{*\prime} - \bar{B}_n'\right)\mathbf{Z}_t \\
&= \frac{1}{n}\left(\bar{A}_n^* - \bar{A}_n\right) + \frac{1}{n}\mathbf{C}_\pi\hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}}\left(\mathbf{I} - \widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I} - \widehat{\mathbf{M}}^n\right)\mathbf{Z}_t,
\end{aligned}$$

where $\bar{A}_n^*$ and $\bar{A}_n$ are defined above.

The inflation risk premium can then be defined as

$$y_{t,n} - y_{t,n}^* - E_t\overline{\pi}_{t+n} = \frac{1}{n}\left(\bar{A}_n^* - \bar{A}_n\right) + \frac{1}{n}\left(\bar{B}_n^{*\prime} - \bar{B}_n'\right)\mathbf{Z}_t - E_t\overline{\pi}_{t+n},$$

whose state-dependent component can be written explicitly as

$$\frac{1}{n}\left(\bar{B}_n^{*\prime} - \bar{B}_n'\right)\mathbf{Z}_t - E_t\overline{\pi}_{t+n} = \frac{1}{n}\mathbf{C}_\pi\left[\hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}}\left(\mathbf{I} - \widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I} - \widehat{\mathbf{M}}^n\right) - \mathbf{M}\left(\mathbf{I} - \mathbf{M}^n\right)\left(\mathbf{I} - \mathbf{M}\right)^{-1}\hat{\mathbf{D}}^{-1}\right]\mathbf{Z}_t.$$

[12] For bonds of infinite maturity, the sum will only be defined if all eigenvalues of $\widehat{\mathbf{M}}$ are inside the unit circle. This is not necessarily true, even if the eigenvalues of $\mathbf{M}$ are within the unit circle by construction.

Note that the time-varying component of the inflation risk premium is zero *at all maturities* when the λ_1 prices of risk are zero. To see this, note that for $\lambda_1 = 0$ we obtain $\widehat{\mathbf{M}} = \hat{\mathbf{D}}\mathbf{M}\hat{\mathbf{D}}^{-1}$, so that $\left(\hat{\mathbf{D}}\mathbf{M}\hat{\mathbf{D}}^{-1}\right)^n = \hat{\mathbf{D}}\mathbf{M}^n\hat{\mathbf{D}}^{-1}$, and

$$\begin{aligned}
\frac{1}{n}\left(\bar{B}_n^{*\prime} - \bar{B}_n^{\prime}\right)\mathbf{Z}_t - E_t\overline{\pi}_{t+n} &= \frac{1}{n}\mathbf{C}_\pi\mathbf{M}\left[(\mathbf{I}-\mathbf{M})^{-1}\hat{\mathbf{D}}^{-1}\left(\mathbf{I}-\hat{\mathbf{D}}\mathbf{M}^n\hat{\mathbf{D}}^{-1}\right) - (\mathbf{I}-\mathbf{M}^n)(\mathbf{I}-\mathbf{M})^{-1}\hat{\mathbf{D}}^{-1}\right]\mathbf{Z}_t \\
&= \frac{1}{n}\mathbf{C}_\pi\mathbf{M}\left[(\mathbf{I}-\mathbf{M})^{-1}(\mathbf{I}-\mathbf{M}^n) - (\mathbf{I}-\mathbf{M}^n)(\mathbf{I}-\mathbf{M})^{-1}\right]\hat{\mathbf{D}}^{-1}\mathbf{Z}_t \\
&= \frac{1}{n}\mathbf{C}_\pi\mathbf{M}\left[\sum_{i=0}^{n-1}\mathbf{M}^i - \sum_{i=0}^{n-1}\mathbf{M}^i\right]\hat{\mathbf{D}}^{-1}\mathbf{Z}_t \\
&= 0.
\end{aligned}$$

A.4 Holding period returns

We define the one-period expected holding period return on an n-bond as

$$e_{n,t}^* = E_t\left[\ln p_{t+1}^{n-1*} - \ln p_t^{n*}\right].$$

Using the bond equations, we know that

$$p_{t+1}^{n-1*} = \exp\left(\bar{A}_{n-1}^* + \bar{B}_{n-1}^{*\prime}\mathbf{Z}_{t+1}\right),$$

and

$$\begin{aligned}
e_{n,t}^* = &-\tfrac{1}{2}\left(\mathbf{C}_\pi + \bar{B}_{n-1}^{*\prime}\hat{\mathbf{D}}\right)\Sigma\Sigma'\left(\mathbf{C}_\pi + \bar{B}_{n-1}^{*\prime}\hat{\mathbf{D}}\right)' + \left(\mathbf{C}_\pi + \bar{B}_{n-1}^{*\prime}\hat{\mathbf{D}}\right)\Sigma\lambda_0 \\
&+ \left(\bar{B}_{n-1}^{*\prime}\hat{\mathbf{D}}\Sigma\lambda_1 - \mathbf{C}_\pi\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right) + \overline{\boldsymbol{\Delta}}'\right)\mathbf{Z}_t,
\end{aligned}$$

which in case of the 1-period bond collapses to

$$e_{1,t}^* = -\tfrac{1}{2}\mathbf{C}_\pi\Sigma\Sigma'\mathbf{C}_\pi' + \mathbf{C}_\pi\Sigma\lambda_0 + \left(\overline{\boldsymbol{\Delta}}' - \mathbf{C}_\pi\left(\mathbf{M}\hat{\mathbf{D}}^{-1} - \Sigma\lambda_1\right)\right)\mathbf{Z}_t,$$

i.e. the short-term real rate.

The excess real holding period return is therefore

$$e_{n,t}^* - e_{1,t}^* = -\tfrac{1}{2}\bar{B}_{n-1}^{*\prime}\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_{n-1}^* + \bar{B}_{n-1}^{*\prime}\hat{\mathbf{D}}\Sigma\left(\lambda_0 - \Sigma'\mathbf{C}_\pi'\right) + \bar{B}_{n-1}^{*\prime}\hat{\mathbf{D}}\Sigma\lambda_1\mathbf{Z}_t.$$

Similarly, for the nominal term structure we obtain

$$\begin{aligned}
e_{n,t} &= -\tfrac{1}{2}\bar{B}_{n-1}'\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_{n-1} + \bar{B}_{n-1}'\hat{\mathbf{D}}\Sigma\lambda_0 + \left(\bar{B}_{n-1}'\hat{\mathbf{D}}\Sigma\lambda_1 + \overline{\boldsymbol{\Delta}}'\right)\mathbf{Z}_t \\
e_{n,t} - e_{1,t} &= \bar{B}_{n-1}'\hat{\mathbf{D}}\Sigma\left(\lambda_0 - \tfrac{1}{2}\Sigma'\hat{\mathbf{D}}'\bar{B}_{n-1}\right) + \bar{B}_{n-1}'\hat{\mathbf{D}}\Sigma\lambda_1\mathbf{Z}_t,
\end{aligned}$$

so that the nominal-real spread net of expected inflation is

$$\begin{aligned}
e_{n,t} - e_{n,t}^* - E_t\left[\pi_{t+1}\right] = &-\tfrac{1}{2}\left(\bar{B}_{n-1}'\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_{n-1} - \bar{B}_{n-1}^{*\prime}\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_{n-1}^*\right) \\
&+ \mathbf{C}_\pi\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_{n-1}^* + \tfrac{1}{2}\mathbf{C}_\pi\Sigma\Sigma'\mathbf{C}_\pi' \\
&+ \left(\left(\bar{B}_{n-1}' - \bar{B}_{n-1}^{*\prime}\right)\hat{\mathbf{D}} - \mathbf{C}_\pi\right)\left(\Sigma\lambda_0 + \Sigma\lambda_1\mathbf{Z}_t\right).
\end{aligned}$$

We can rewrite this using the solutions for $\bar{B}'_{n-1}$ and $\bar{B}^{*\prime}_{n-1}$ to obtain

$$
\begin{aligned}
& e_{n,t} - e^*_{n,t} - E_t\left[\pi_{t+1}\right] \\
&= -\mathbf{C}_\pi \hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}}\left(\mathbf{I}-\widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I}-\widehat{\mathbf{M}}^{n-1}\right)\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\left(\mathbf{I}-\left(\widehat{\mathbf{M}}'\right)^{n-1}\right)\left(\mathbf{I}-\widehat{\mathbf{M}}'\right)^{-1}\left(\overline{\boldsymbol{\Delta}}-\tfrac{1}{2}\widehat{\mathbf{M}}'\left(\hat{\mathbf{D}}'\right)^{-1}\mathbf{C}'_\pi\right) \\
&+ \mathbf{C}_\pi\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}^*_{n-1} + \tfrac{1}{2}\mathbf{C}_\pi\Sigma\Sigma'\mathbf{C}'_\pi \\
&- \left(\mathbf{C}_\pi \hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}}\left(\mathbf{I}-\widehat{\mathbf{M}}\right)^{-1}\left(\mathbf{I}-\widehat{\mathbf{M}}^{n-1}\right)\hat{\mathbf{D}}+\mathbf{C}_\pi\right)\left(\Sigma\lambda_0+\Sigma\lambda_1\mathbf{Z}_t\right).
\end{aligned}
$$

A.5 Forward premia

Real 1-period forward rates are defined as

$$
\begin{aligned}
f^*_{n,t} &= \ln p^{n*}_t - \ln p^{n+1*}_t \\
&= \mathbf{C}_\pi\Sigma\lambda_0 - \tfrac{1}{2}\mathbf{C}_\pi\Sigma\Sigma'\mathbf{C}'_\pi - \bar{B}^{*\prime}_n\hat{\mathbf{D}}\Sigma\left(\Sigma'\mathbf{C}'_\pi-\lambda_0\right) - \tfrac{1}{2}\bar{B}^{*\prime}_n\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}^*_n \\
&+ \left(\bar{B}^{*\prime}_n\left(\mathbf{I}-\widehat{\mathbf{M}}\right) - \mathbf{C}_\pi\hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}} + \overline{\boldsymbol{\Delta}}'\right)\mathbf{Z}_t.
\end{aligned}
$$

Note that

$$
E_t r^*_{t+n} = \mathbf{C}_\pi\Sigma\left(\lambda_0 - \tfrac{1}{2}\Sigma'\mathbf{C}'_\pi\right) + \left(\overline{\boldsymbol{\Delta}}' - \mathbf{C}_\pi\hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}}\right)\hat{\mathbf{D}}\mathbf{M}^{n-1}\hat{\mathbf{D}}^{-1}\mathbf{Z}_t,
$$

so that the real forward premium is

$$
\begin{aligned}
f^*_{n,t} - E_t r^*_{t+n} &= -\bar{B}^{*\prime}_n\hat{\mathbf{D}}\Sigma\left(\Sigma'\mathbf{C}'_\pi-\lambda_0\right) - \tfrac{1}{2}\bar{B}^{*\prime}_n\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}^*_n \\
&+ \left(\bar{B}^{*\prime}_n - \bar{B}^{*\prime}_n\widehat{\mathbf{M}} + \left(\overline{\boldsymbol{\Delta}}' - \mathbf{C}_\pi\hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}}\right)\left(\mathbf{I}-\hat{\mathbf{D}}\mathbf{M}^{n-1}\hat{\mathbf{D}}^{-1}\right)\right)\mathbf{Z}_t.
\end{aligned}
$$

The nominal-real forward spread is then given by

$$
\begin{aligned}
f_{n,t} - f^*_{n,t} &= \left(\bar{B}'_n - \bar{B}^{*\prime}_n\right)\hat{\mathbf{D}}\Sigma\lambda_0 - \tfrac{1}{2}\bar{B}'_n\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}_n + \tfrac{1}{2}\bar{B}^{*\prime}_n\hat{\mathbf{D}}\Sigma\Sigma'\hat{\mathbf{D}}'\bar{B}^*_n - \mathbf{C}_\pi\Sigma\lambda_0 + \tfrac{1}{2}\mathbf{C}_\pi\Sigma\Sigma'\mathbf{C}'_\pi \\
&+ \bar{B}^{*\prime}_n\hat{\mathbf{D}}\Sigma\Sigma'\mathbf{C}'_\pi + \left(\bar{B}'_n - \bar{B}^{*\prime}_n - \left(\bar{B}'_n - \bar{B}^{*\prime}_n\right)\widehat{\mathbf{M}} + \mathbf{C}_\pi\hat{\mathbf{D}}^{-1}\widehat{\mathbf{M}}\right)\mathbf{Z}_t.
\end{aligned}
$$

References

[1] Anderson, E. W., L. P. Hansen, E. R. McGrattan and T. J. Sargent (1996), "Mechanics of forming and estimating dynamic linear economies," *Handbook of Computational Economics*, Volume 1, Amsterdam: North-Holland, pp. 171-252.

[2] Ang, A., G. Bekaert and M. Wei (2006), "The term structure of real rates and expected inflation," mimeo, Columbia University.

[3] Ang, A. and M. Piazzesi (2003), "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables," *Journal of Monetary Economics,* 50, 745-787.

[4] Barr, D.G. and J.Y. Campbell (1997), "Inflation, real interest rates, and the bond market: a study of U.K. nominal and index-linked government bond prices," *Journal of Monetary Economics* 39, 361-383.

[5] Buraschi, A. and A. Jiltsov (2005), "Inflation risk premia and the expectations hypothesis", *Journal of Financial Economics*, 75, 429-490.

[6] Clarida, R., J. Galí and M. Gertler (1998), "Monetary policy rules in practice: Some international evidence," *European Economic Review* 42, 1033-1067.

[7] Clarida, R., J. Galí and M. Gertler (2000), "Monetary policy rules and macroeconomic stability: Evidence and some theory," *Quarterly Journal of Economics,* 115, 147-180.

[8] Dai, Q. and K.J. Singleton (2000), "Specification analysis of affine term structure models," *Journal of Finance*, 55, 1943-1978.

[9] Dai, Q. and K. Singleton (2002), "Expectation puzzles, time-varying risk premia, and affine models of the term structure," *Journal of Financial Economics*, 63, 415-441.

[10] Dewachter, H. and M. Lyrio (2004), "Macro factors and the term structure of interest rates," *Journal of Money, Credit, and Banking*, 38, 119-140.

[11] Duffee, G.R. (2002), "Term premia and interest rate forecasts in affine models," *Journal of Finance*, 57, 405-43.

[12] Duffie, D. and R. Kan (1996), "A yield-factor model of interest rates," *Mathematical Finance*, 6, 379-406.

[13] Evans, M.D.D. (1998): "Real rates, expected inflation, and inflation risk premia," *Journal of Finance,* 53, 187-218.

[14] Evans, M.D.D. (2003): "Real risk, inflation risk, and the term structure," *Economic Journal*, 113, 345-89.

[15] Fischer, S. (1975), "The demand for index bonds," *Journal of Political Economy*, 83, 509-34.

[16] Fuhrer, J.C. (2000), "Habit formation in consumption and its implications for monetary policy models," *American Economic Review*, 90, 367-390.

[17] Fuhrer, J.C. and G. Rudebusch (2004), "Estimating the Euler equation for output," *Journal of Monetary Economics*, 51, 1133-53.

[18] Galí, J. and M. Gertler (1999), "Inflation dynamics: a structural econometric analysis," *Journal of Monetary Economics,* 44, 195-222.

[19] Goffe, L.G., G.D. Ferrier and J. Rogers (1994), "Global optimization of statistical functions with simulated annealing," *Journal of Econometrics*, 60, 65-99.

[20] Hördahl, P., O. Tristani and D. Vestin (2006), "A joint econometric model of macroeconomic and term structure dynamics," *Journal of Econometrics*, 131, 405-444.

[21] Hördahl, P., O. Tristani and D. Vestin (2007), "The yield curve and macroeconomic dynamics," mimeo, European Central Bank.

[22] Jondeau, E. and H. Le Bihan (2001), "Testing for a forward-looking Phillips curve. Additional evidence from European and US data," Notes d'Études et de Recherche 86, Banque de France.

[23] Kandel, S., A.R. Ofer, and O. Sarig, (1996), "Real interest rates and inflation: An ex-ante empirical analysis," *Journal of Finance*, 51, 205-225.

[24] Kim, D.H. and J.H. Wright, (2005), "An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates," FEDS working paper # 2005-33, Federal Reserve Board.

[25] McCulloch, J.H. and L.A. Kochin (2000), "The inflation premium implicit in the US real and nominal term structures of interest rates," Ohio State University Working Paper # 98-12.

[26] Remolona, E. M., M. R. Wickens and F. F. Gong (1998), "What was the market's view of U.K. Monetary policy? Estimating inflation risk and expected inflation with indexed bonds," Federal Reserve Bank of New York Staff Reports 57.

[27] Risa, Stefano (2001), "Nominal and inflation indexed yields: separating expected inflation and inflation risk premia," mimeo, Columbia University.

[28] Roll, R. (2004): "Empirical TIPS," *Financial Analyst Journal,* 60, 31-53.

[29] Rudebusch, G. and T. Wu (2004), "A macro-finance model of the term structure, monetary policy, and the economy," Federal Reserve Bank of San Francisco Working Paper 03-17.

[30] Söderlind, P. (1999), "Solution and estimation of RE macromodels with optimal policy," *European Economic Review,* 36, 975-1011.

[31] Svensson, L.E.O. (1994), "Estimating and interpreting forward interest rates: Sweden 1992-94," IMF Working paper WP/94/114.

[32] Taylor, J. B. (1993), "Discretion vs. policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy,* 39, 195-214.

Table 1: Parameter estimates
(Sample period: Jan. 1999 − Apr. 2006)

Parameter	Point estimate	Standard error
ρ	0.955	0.027
β	1.000	1.135
γ	0.777	0.268
μ_π	0.708	0.090
δ_x	0.036	0.010
μ_x	0.146	0.071
ζ_r	0.064	0.055
ϕ_{π^*}	0.99	−
$\sigma_{\pi^*} \times 10^2$	0.014	0.019
$\sigma_\eta \times 10^2$	0.016	0.002
$\sigma_\pi \times 10^2$	0.276	0.025
$\sigma_x \times 10^2$	0.059	0.005
$\sigma_m \times 10^2$	0.012	0.001
$\sigma_r \times 10^2$	0.012	0.001
$\lambda_{0,1}$	−0.094	0.073
$\lambda_{0,2}$	−0.773	0.186
$\lambda_{0,4}$	−0.974	0.209

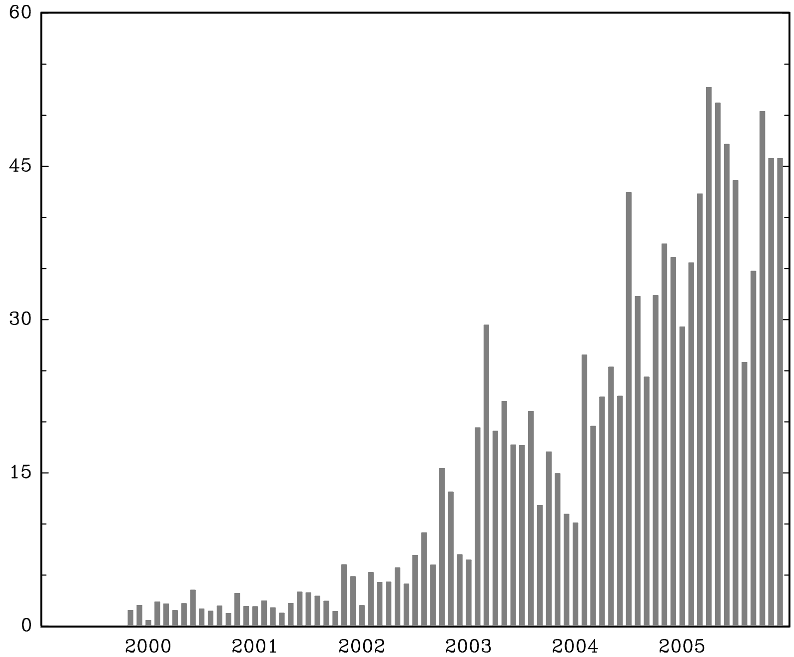
$\lambda_1 \times 10^{-2}$

	π^*	r	π	x
π^*	0.703	0	0	0.142
	(1.070)	(−)	(−)	(0.048)
r	−2.889	−1.520	0.454	2.096
	(3.389)	(1.707)	(0.412)	(1.158)
π	0	0	0	0
	(−)	(−)	(−)	(−)
x	−2.942	0	0	0
	(3.504)	(−)	(−)	(−)

Standard errors in parentheses

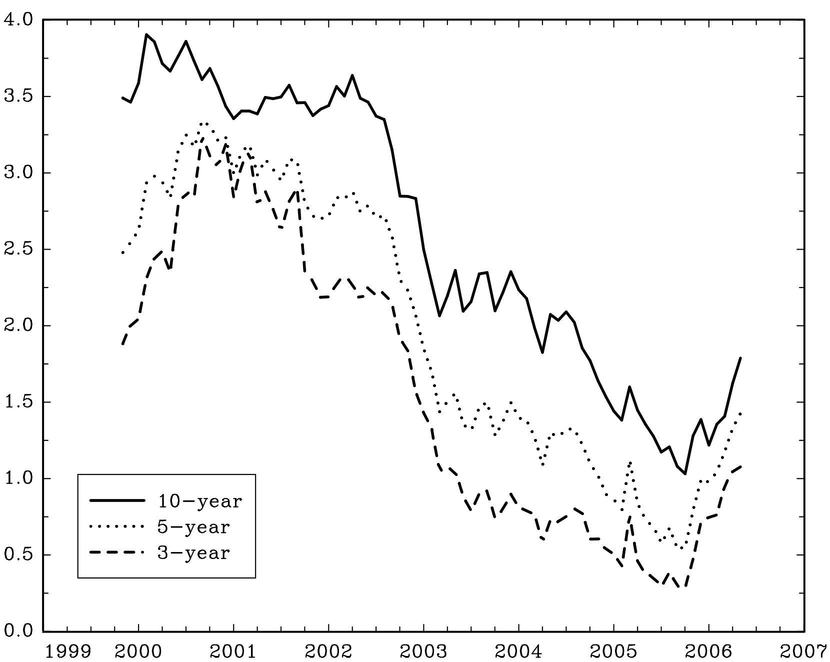
The standard errors are based on a Newey-West (12 lags) HAC variance-covariance matrix calculated using analytical expressions for the Jacobian and Hessian matrices of the likelihood function. The estimates of the lag coefficients for inflation and output are not reported.

Figure 1a: Turnover in the French index-linked bond market



In EUR billion per month. Source: Agence France Tresor.

Figure 1b: Euro area real zero-coupon yields

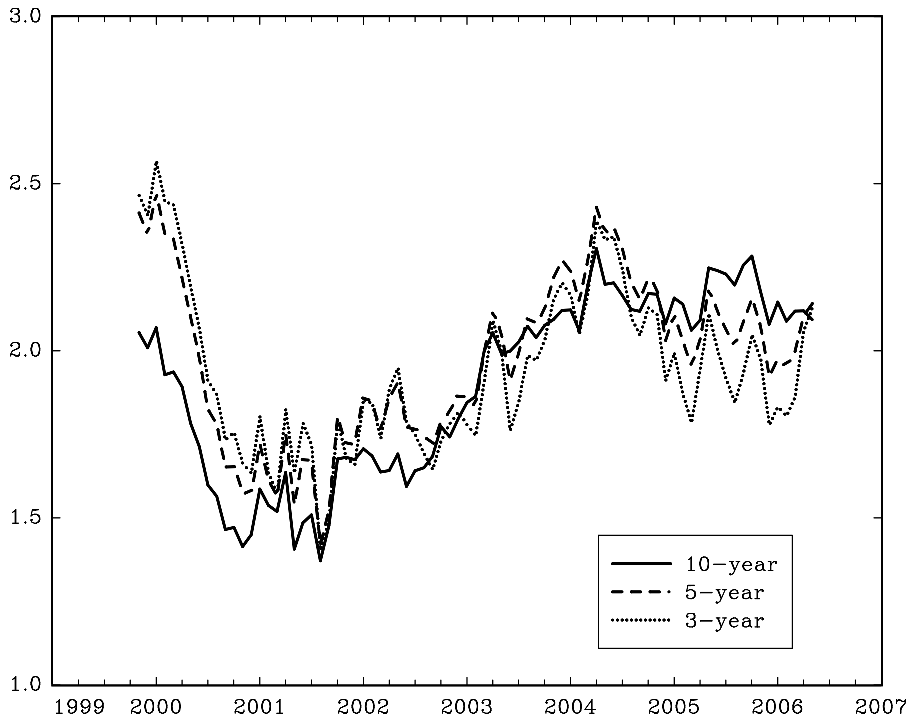


Based on the spline method in McCulloch and Kochin (2000) applied to prices of index-linked bonds issued by the French Treasury. Sample period: October 1999 to April 2006 (percent per year).

Figure 2: Euro area nominal zero-coupon yields

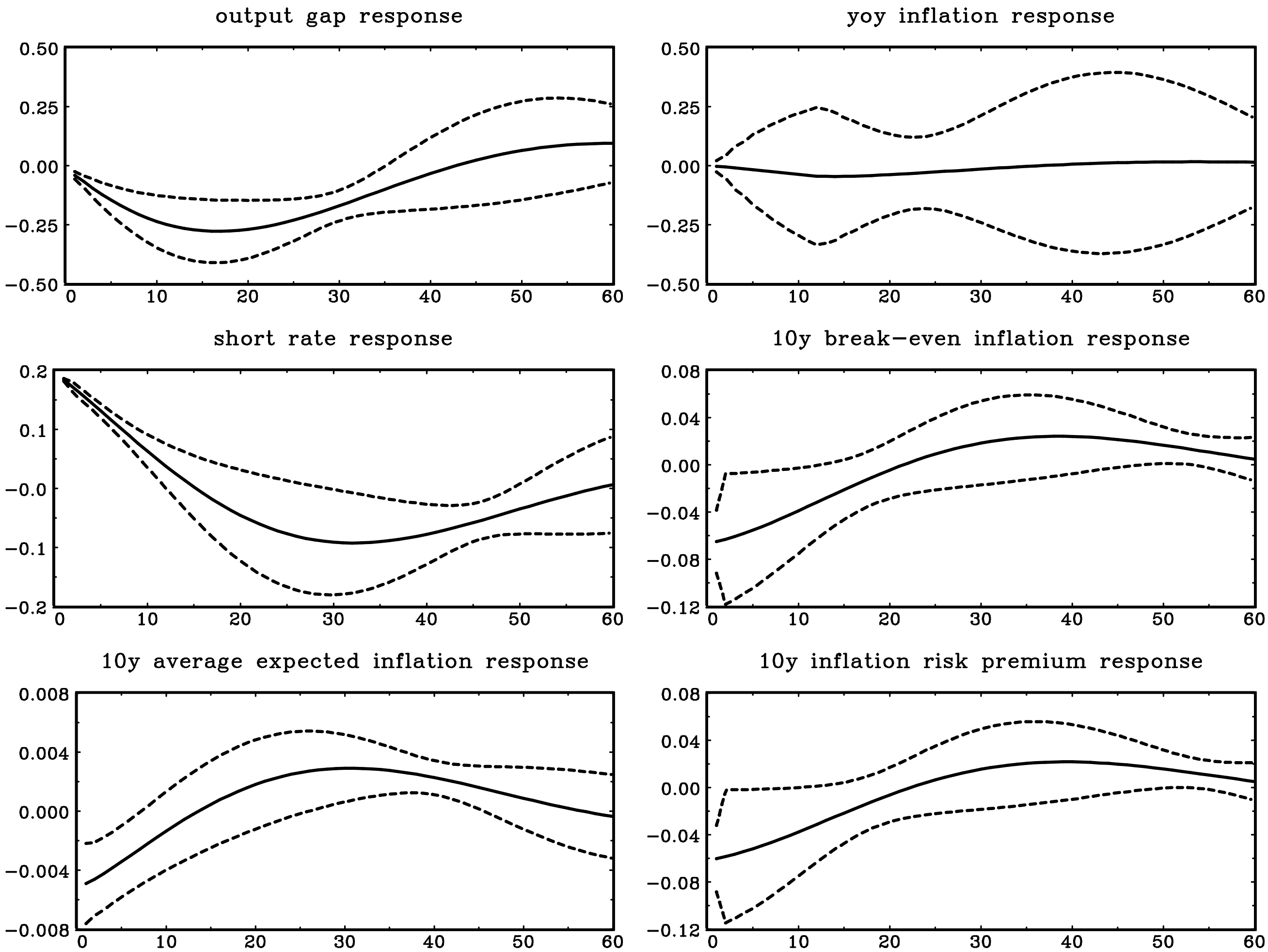
Based on the interpolation method by Svensson (1994) applied to German government bonds, as reported by the Bundesbank. Sample period: January 1999 to April 2006 (percent per year).

Figure 3: Euro area zero-coupon break-even inflation rates



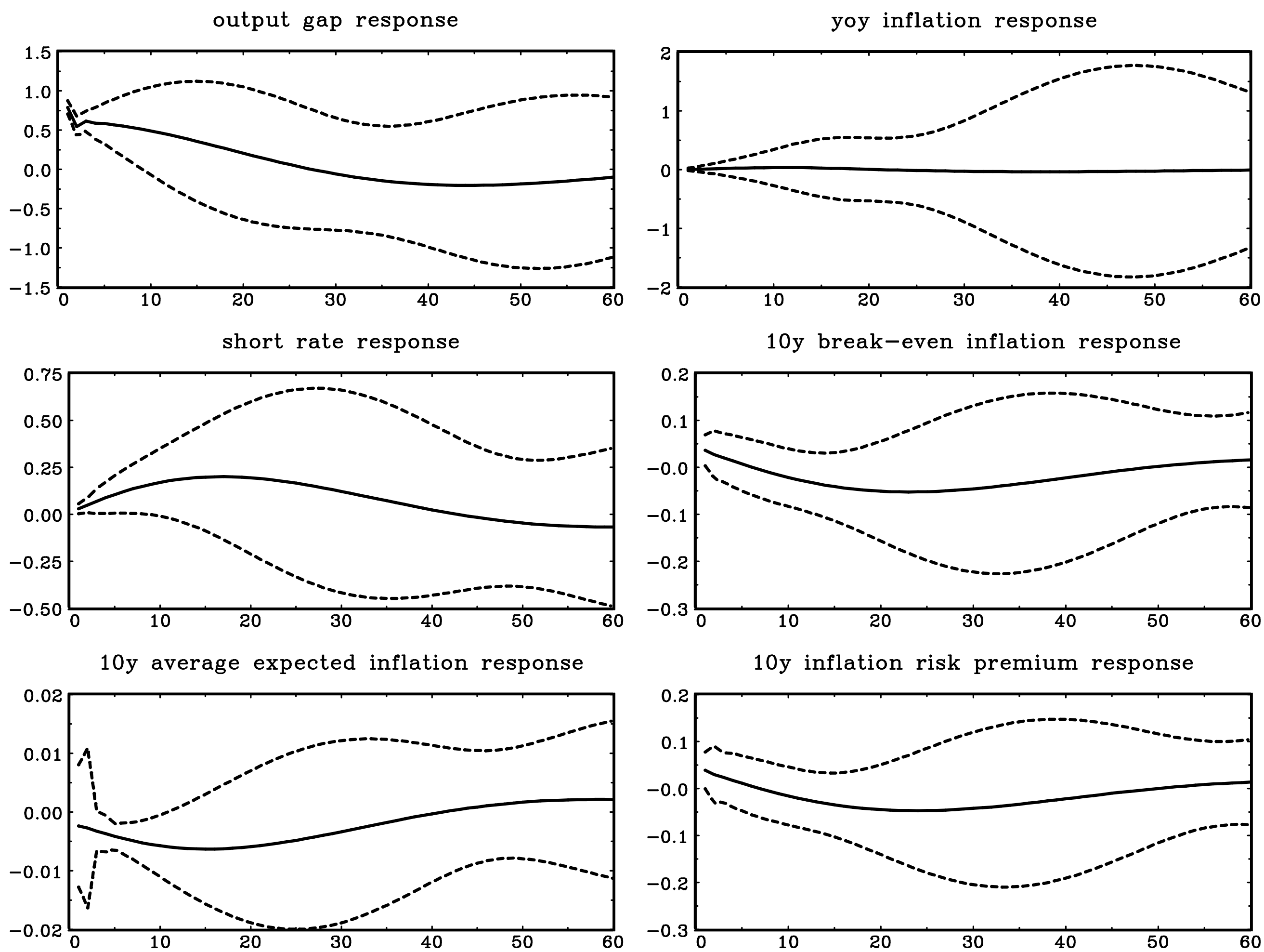
Difference between model-implied nominal and real zero-coupon yields of the same maturity. Sample period: October 1999 to April 2006 (percent per year).

Figure 4: Impulse responses to a monetary policy shock



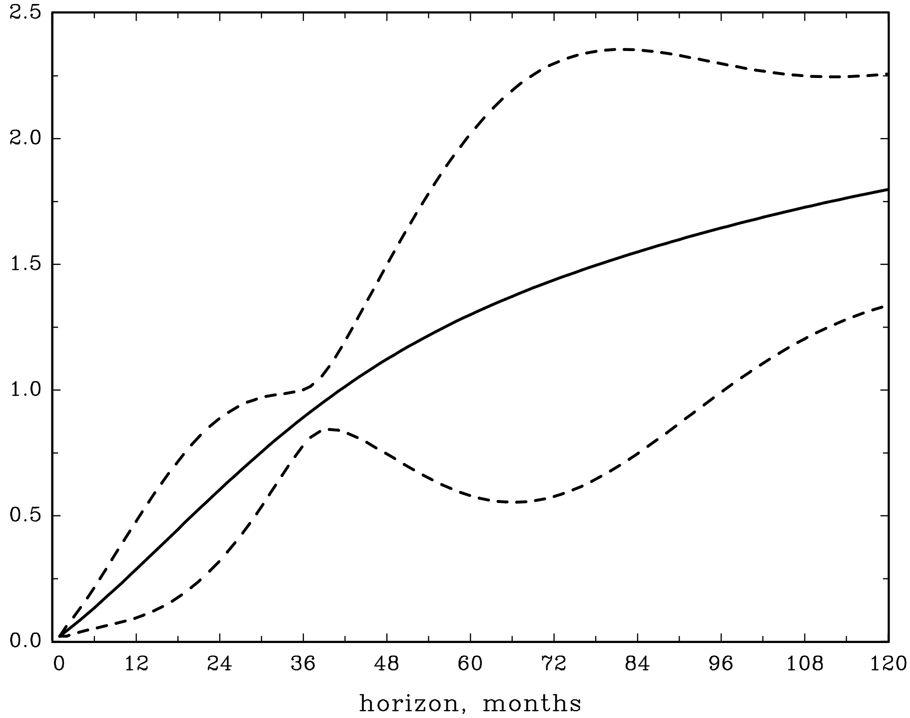
All responses are expressed in annual percentage terms (except the output gap). Dashed lines are 95% confidence bands based on a Newey-West HAC variance-covariance matrix calculated using analytical expressions for the Jacobian and Hessian matrices of the likelihood function. The short-term interest rate was shocked by one standard deviation (around 19 bps.).

Figure 5: Impulse responses to an output shock



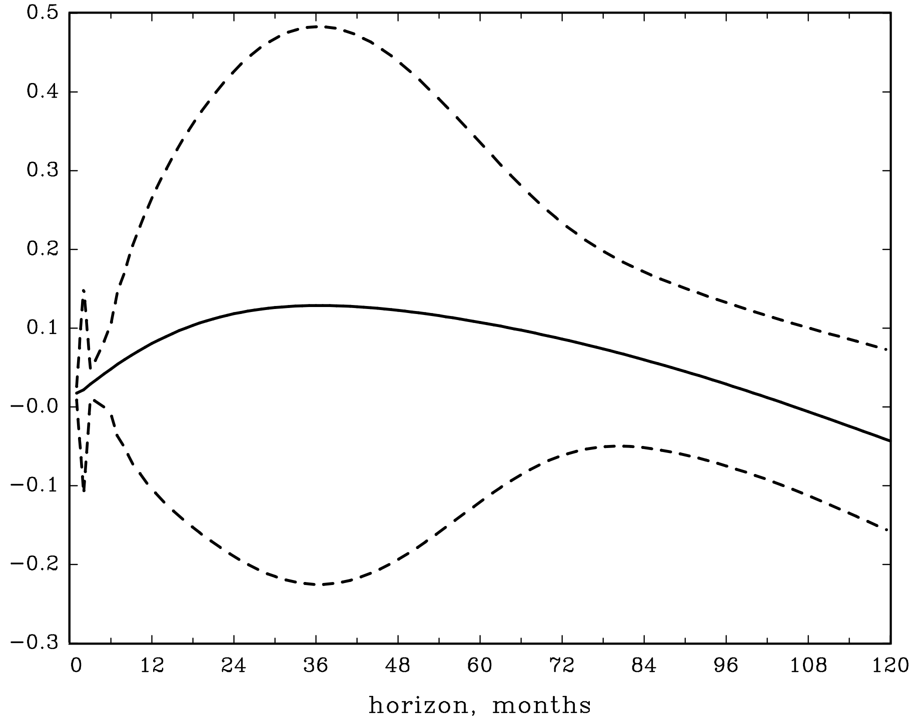
All responses are expressed in annual percentage terms (except the output gap). Dashed lines are 95% confidence bands based on a Newey-West HAC variance-covariance matrix calculated using analytical expressions for the Jacobian and Hessian matrices of the likelihood function. The output gap was shocked by one standard deviation (around 0.7 per cent).

Figure 6: Term structure of average total yield premia



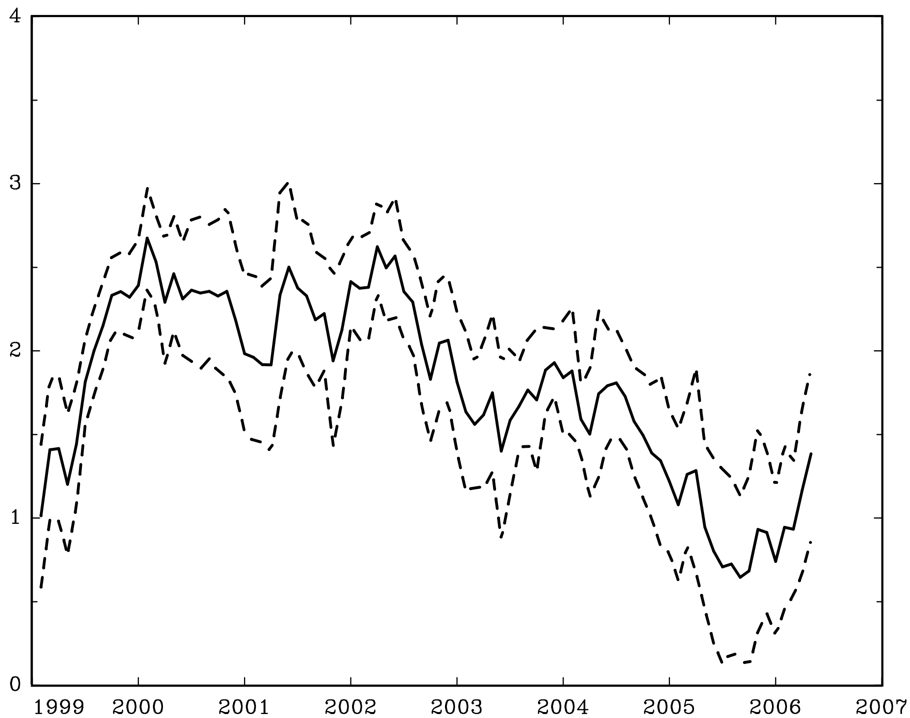
Expressed in percent per year. Dashed lines are 95% confidence bands based on a Newey-West HAC variance-covariance matrix calculated using analytical expressions for the Jacobian and Hessian matrices of the likelihood function.

Figure 7: Term structure of average inflation risk premia



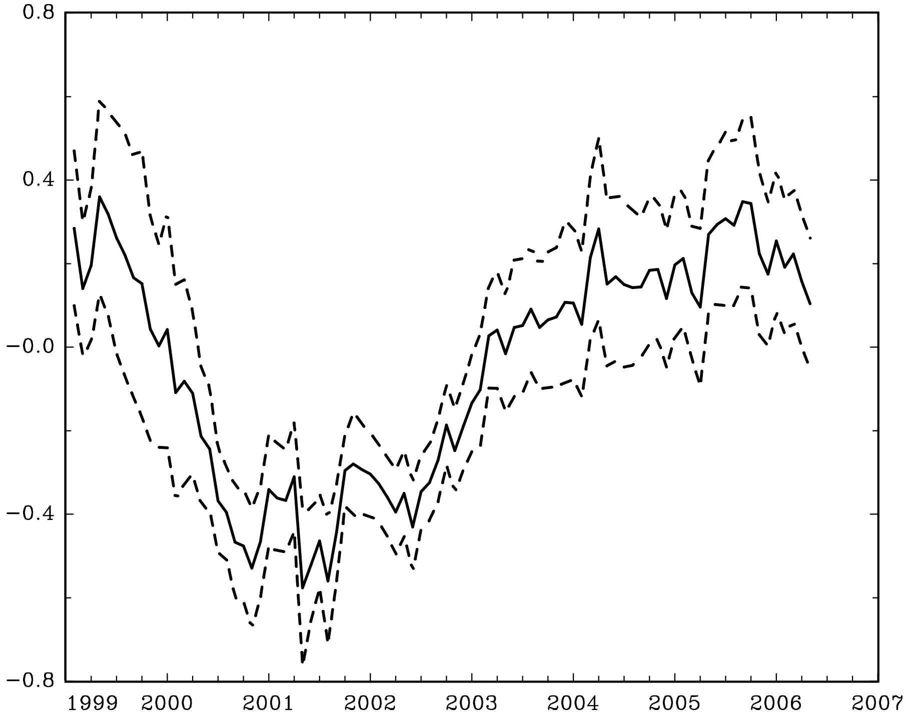
Expressed in percent per year. Dashed lines are 95% confidence bands based on a Newey-West HAC variance-covariance matrix calculated using analytical expressions for the Jacobian and Hessian matrices of the likelihood function.

Figure 8: Estimated total 10-year yield premium



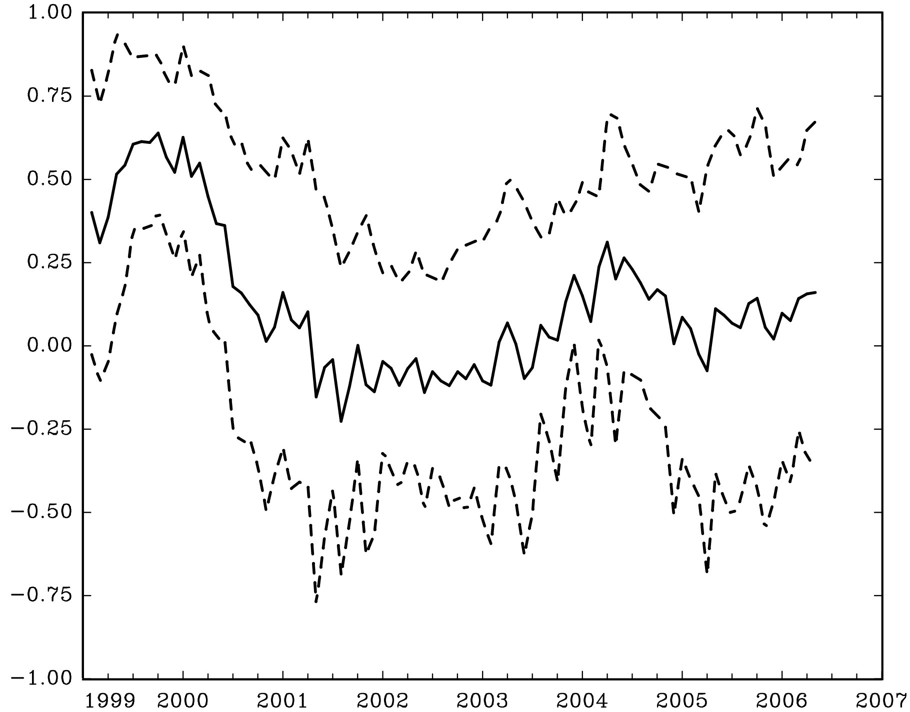
Dashed lines are 95% confidence bands based on a Newey-West HAC variance-covariance matrix calculated using analytical expressions for the Jacobian and Hessian matrices of the likelihood function. Sample period: January 1999 to April 2006; expressed in percent per year.

Figure 9: Estimated 10-year inflation risk premium



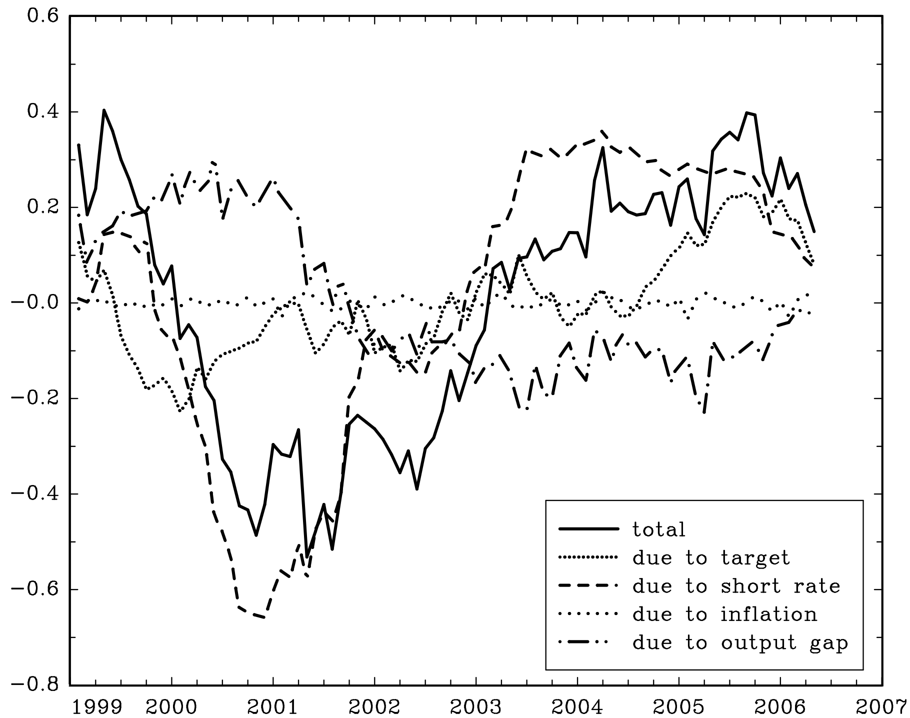
Dashed lines are 95% confidence bands based on a Newey-West HAC variance-covariance matrix calculated using analytical expressions for the Jacobian and Hessian matrices of the likelihood function. Sample period: January 1999 to April 2006; expressed in percent per year.

Figure 10: Estimated 3-year inflation risk premium



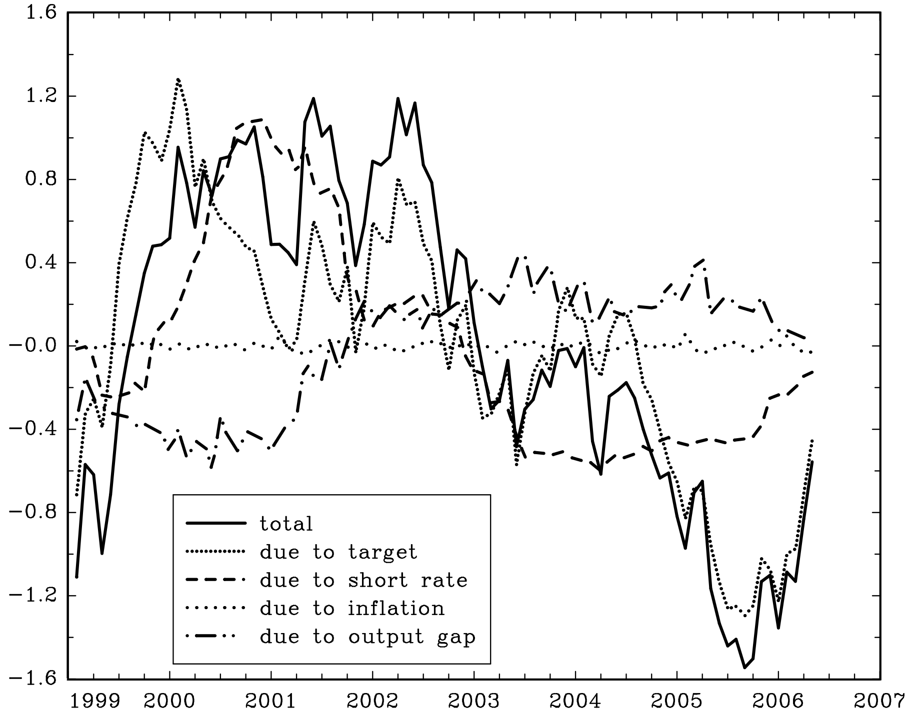
Dashed lines are 95% confidence bands based on a Newey-West HAC variance-covariance matrix calculated using analytical expressions for the Jacobian and Hessian matrices of the likelihood function. Sample period: January 1999 to April 2006; expressed in percent per year.

Figure 11: Estimated 10-year inflation risk premium and its components



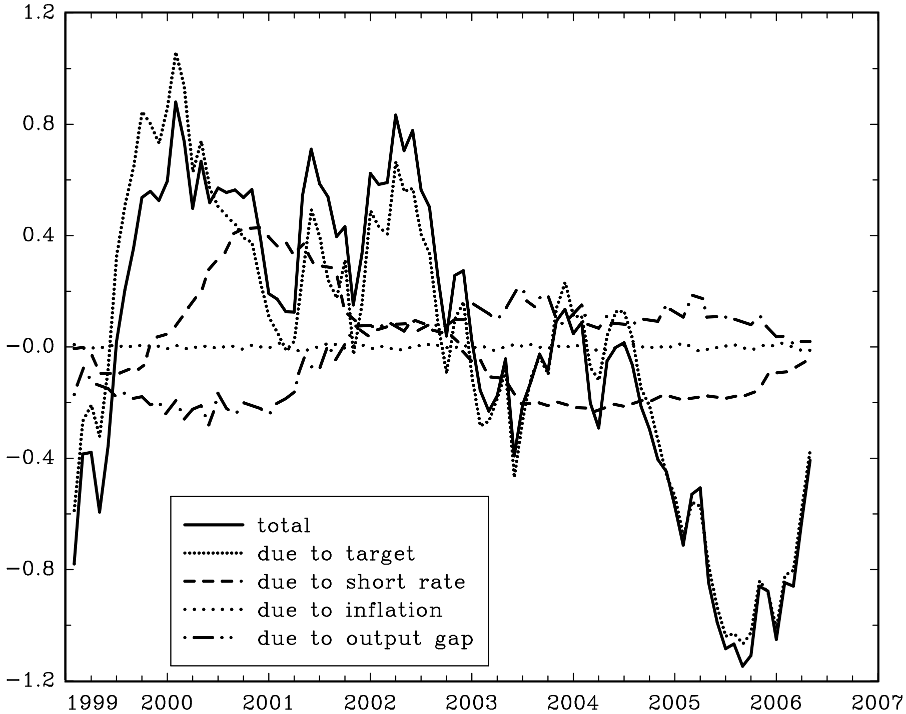
The solid line is the estimated (de-meaned) 10-year inflation risk premium during the sample period, expressed in annual percentage terms. The other lines show the contribution to the premium coming from each of the macro factors.

Figure 12: Estimated 10-year real risk premium and its components



The solid line is the estimated (de-meaned) 10-year real risk premium during the sample period, expressed in annual percentage terms. The other lines show the contribution to the premium coming from each of the macro factors.

Figure 13: Estimated total 10-year yield premium and its components

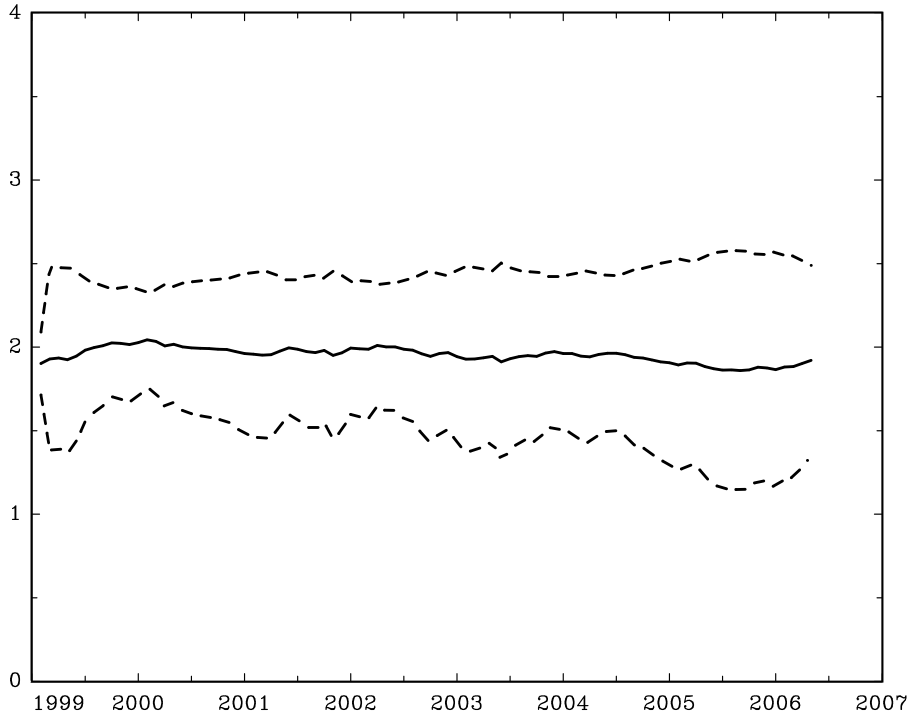


The solid line is the estimated (de-meaned) 10-year total yield premium during the sample period, expressed in annual percentage terms. The other lines show the contribution to the premium coming from each of the macro factors.

Figure 14: 10-year break-even inflation rates and survey inflation forecasts

The solid thin line is the unadjusted model-implied 10-year break-even rate; the solid thick line is the break-even rate adjusted for the inflation risk premium; the dashed lines are 95% confidence bands based on a Newey-West HAC variance-covariance matrix calculated using analytical expressions for the Jacobian and Hessian matrices of the likelihood function. The dots are 10-year ahead euro area inflation forecasts from the biannual long-horizon survey of Consensus Economics. Sample period: January 1999 to April 2006; expressed in percent per year.

Figure 15: Estimated inflation target



Dashed lines are 95% confidence bands based on a Newey-West HAC variance-covariance matrix calculated using analytical expressions for the Jacobian and Hessian matrices of the likelihood function. Sample period: January 1999 to April 2006; expressed in percent per year.