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Time-varying exchange rate pass-through: experiences of some industrial countries

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Abstract

This paper estimates exchange rate pass-through of six major industrial countries using a time-varying parameter with stochastic volatility model. Exchange rate pass-through is divided into impacts of exchange rate fluctuations to import prices (first-stage pass-through) and those of import price movements to consumer prices (second-stage pass-through). The paper finds that both stages of pass-through have declined over time for all the sample countries. The decline in second-stage pass-through is associated with the emergence of the low and stable inflation environment as well as a rise in import penetration, while the relationship to the inflation environment is weak for first-stage pass-through.

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Keywords: exchange rate pass-through, impacts of commodity prices, time-varying parameter, stochastic volatility, Markov Chain Monte Carlo.

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Contents

Time-Varying Exchange Rate Pass-Through: Experiences of Some Industrial Countries[*]

Toshitaka Sekine[†]

March, 2006

1 Introduction

In recent years, inflation in a number of industrial and emerging market countries has remained surprisingly stable in the face of wide swings in exchange rates. This development has drawn attention to the issue of exchange rate pass-through to domestic prices and to whether and why it has declined.

A decline in exchange rate pass-through can have important macroeconomic implications. First, it might imply a change in the sensitivity to external shocks such as exchange rate fluctuation, but possibly also to shocks to prices of commodities or other traded goods. The modest impact on inflation of the recent sharp rise in oil prices has attracted much attention (see BIS (2005), Chapter II). Second, exchange rate pass-through to import prices affects expenditure switching in the domestic market by changing the relative prices of imported and domestically produced goods. This raises the question of whether a decline in the exchange rate pass-through has weakened a channel through which current account imbalances can be adjusted.

A growing body of research conducted at central banks as well as in academia has documented the decline in pass-through of exchange rate movements into domestic prices. However, most of the research is done by split sample estimations or rolling regressions: these estimation techniques are based on the assumption that underlying parameters did not alter within the estimated sample periods, and thus they do not necessarily provide precise timing of parameter shifts. It is often the case with rolling regressions that the timing of parameter shifts crucially depends on the size of windows. As a rare exception in the recent literature, Amstad and Fischer (2005) focus on the time-varying nature of pass-through without relying on the above standard estimation techniques, but their approach—an application of event-study procedures used in empirical finance—is quite different from the one taken in this paper. Kim (1990) is methodologically closer to our approach, but his estimation covers only up until the mid-1980s and yields mixed results.[1]

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[1]Another example can be found in Mumtaz et al (2005), who apply a time-varying coefficient model to UK

A contribution of this paper is to estimate the development of pass-through coefficients (and volatilities of inflation) for six major industrial countries (the United States, Japan, Germany, the United Kingdom, France and Italy) by explicitly taking into account their time-varying natures. We will apply a time-varying parameter (with stochastic volatility) model for this purpose. By doing this, we might be able to shed more light on when these coefficients declined and how they were related to other developments and factors in the economy. Moreover, estimation for six major industrial countries enables us to see whether there have been common features across these countries.

The paper finds that (i) pass-through has declined over time in all major industrial countries; (ii) these declines have taken place gradually compared with those estimated by rolling regressions, and in most cases did not show the parameter shift envisaged by split sample estimations; and (iii) the decline in pass-through to consumer prices was related to the emergence of the low and stable inflation environment as well as the rise in import penetration, while the relationship to the inflation environment is weak for pass-through to import prices.

The rest of the paper is organised as follows: Section 2 lays out an analytical framework by discussing specifications and introducing a time-varying parameter cum stochastic volatility model. Section 3 documents estimation results of time-varying pass-through. Section 4 examines the relationship between pass-through and other variables. Section 5 concludes the paper, which is followed by the Appendix elaborating more details of the employed algorithm.

2 Analytical Framework

2.1 First- and second-stage pass-through

Exchange rate pass-through is usually defined as the responsiveness of domestic prices—including consumer prices, producer prices, import prices and sometimes the prices set by domestic exporters—to exchange rate movements. One standard way to estimate exchange rate pass-through is as the coefficient obtained from regressing changes in price indexes on movements in nominal effective exchange rates.

Simple as it may sound, however, a number of specification issues have arisen in the literature. These include:

- *Multivariate models*: Some researchers (McCarthy (2000); Adolfson (2004); de Walque and Wouters (2004)) measure exchange rate pass-through as the responsiveness to an unexpected movement in the exchange rate (a shock: ie the exchange rate movement which a model cannot predict) by estimating multivariate models such as a VAR or a simultaneous equations model. This may differ from the results of regression coefficients estimated by single equations that assume any movement in the exchange rate is exogenous.
- *Cointegration relationships*: Adolfson (2004) and Heath et al (2004) show that, for some small open economies at least,[2] there exists a long-run cointegration relationship among import prices, exchange rates and foreign prices, which corresponds to the PPP relationship. Based on these findings, their favourite specifications take the form of an error

import prices.

[2]Their estimation covers Australia, Canada, New Zealand, Sweden and the United Kingdom.

correction formula. This implies that pass-through to import prices is complete in the long run.

However, whether the PPP holds empirically has been a long-standing contentious issue among researchers. This is especially so for large industrial countries, where strategic considerations might prevent firms from simply passing through exchange rate fluctuations. In fact, various Johansen tests at preliminary investigations fail to find any meaningful cointegration relationships among the relevant variables in this paper.

- *Asymmetry and non-linearity*: Herzberg et al (2003) try to capture asymmetric and/or non-linear response of pass-through for UK import prices by using various specifications (a threshold model, a spline model and a quadratic logistic STAR model). A non-linear model like a regime-switching model would detect structural breaks in the pass-through coefficients. However, as briefly surveyed by Marazzi et al (2005), there is no clear support in general for either asymmetries or non-linearities.

In this paper, we choose a simple specification (single equation analysis; no cointegration relationship; a symmetric linear model). This does not necessarily preclude the possibility of extending the analysis to the above directions in the future. However, as a first step in explicitly incorporating the time-varying nature of pass-through across major industrial countries, we think it worthwhile to keep the specification as simple as possible so that it broadly corresponds to a number of existing studies such as Campa and Goldberg (2004); Marazzi et al (2005); Otani et al (2003, 2005); Gagnon and Ihrig (2004).

Estimation of exchange rate pass-through in this paper is regarded as an atheoretical exercise. The standard specifications, which this paper is based on, are typically derived from a partial equilibrium setup. They miss some of the structural elements of a more general equilibrium framework. For instance, the specifications lack explicit treatment of expectation as well as the conduct of monetary policy. However, exchange rate pass-through thus calculated can provide some insight into the likely underlying structural factors. For example, if pass-through changed at the time of a policy regime shift, it is likely that a change in monetary policy regime altered the pass-through relationship.

In order to gauge time-varying impacts of exchange rate fluctuations on domestic prices, we divide pass-through into two stages. One is the effect of exchange rate movements on import prices ("first-stage" pass-through) and the other is the effect of import price movements on consumer prices ("second-stage" pass-through). The distinction between first- and second-stage pass-through reflects developments in the literature. In the academic literature of international economics or industrial organisation, pass-through has often been calculated based on import prices or the prices set by domestic exporters, while pass-through to consumer prices has more recently come to the attention of researchers, especially those at central banks. The distinction allows for different pricing behaviour along a distribution chain. The pricing behaviour of foreign exporters or domestic importers is thought to affect first-stage pass-through, and that of domestic distributors is thought to be relevant for second-stage pass-through. The difference in these pricing behaviours may lead to different development in each stage of pass-through.

First-stage pass-through is measured by the following reduced form specification.

$$\Delta p_t^m = \alpha_{0t} \Delta p_{t-1}^m + \alpha_{1t}(L) \Delta e_t + \alpha_{2t}(L) \Delta p_t^* + \alpha_{3t}(L) \Delta p_t^{com} + \alpha_{4t}(L) \tilde{y}_t + \alpha_{5t} + \epsilon_t, \qquad (1)$$

where p_t^m is import prices at the period t, e_t is the effective exchange rate (expressed in terms of units of domestic currency per unit of foreign currency), p_t^* is foreign prices (in foreign currency), p_t^{com} is commodity prices (converted in own currency with an assumption of complete pass-through for commodities) and $\tilde{y}_t$ is output gap. All the variables except for $\tilde{y}_t$, are in logarithm, and Δ denotes a first difference operator. $\alpha_{it}(L) = \alpha_{it}(1 + L + L^2 + ...)$ is a lag polynomial where L is a lag operator—we include contemporaneous and one-quarter lag variables for Δe_t, Δp_t^*, Δp_t^{com} and $\tilde{y}_t$, given that pass-through tends to occur rapidly (Marazzi et al, 2005).[3]

As discussed above, the specification is standard in the literature except for the points discussed in the paragraphs below. It can be derived from a first-order condition of a foreign monopolistic exporter's profit maximisation in a static partial equilibrium model:

$$P_t^m = \mu_t C_t^* E_t,$$

where P^m is the import price, C_t^* is the marginal cost of the foreign exporter, E_t is the exchange rate, and μ_t is the markup, which is equal to $\eta/(\eta - 1)$ (where η is the positive price elasticity of demand). p_t^* in equation (1) corresponds to the marginal cost of the foreign exporter. Given the difficulty in obtaining appropriate data, most existing studies construct this as a weighted average of trade partners' CPI (Marazzi et al, 2005) or unit labour costs (Campa and Goldberg, 2004). In the empirical work below, we will follow this conventional practice by calculating a weighted average of trade partners' CPI.

We introduce time-varying nature in two aspects. One is that all the coefficients are assumed to be time-varying as denoted by time subscripts on coefficients. More specifically, we incorporate this by allowing permanent shifts in parameters: $\alpha_{i,t+1} = \alpha_{it} + u_{it}$, where u_{it} is an error term and assumed to follow an i.i.d. normal distribution, $u_{it} \sim N(0, H^{-1})$. The other time variance is the volatility of an error term ϵ_t. We assume that an unobserved log-volatility h_t can vary from time to time such that: $h_{t+1} = h_t + \eta_t$, where η_t is an i.i.d. normal error term, $\eta_t \sim N(0, \sigma_\eta^2)$. By doing this, we can see whether or not an inflation process becomes more stable even conditional on developments of explanatory variables.

The specification, in principle, captures both gradual shifts and sudden changes in state variables α_{it} and h_{it}. Variance H^{-1} (or precision H) and σ_η^2 are key parameters that determine how smoothly these state variables change over time. If they are large, state variables might change abruptly. If they are small, state variables tend to change gradually. At the limit, if they are as small as zero, the stochastic process degenerates to $\alpha_{i,t+1} = \alpha_{it}$ and $h_{t+1} = h_t$, which imply time-invariant coefficients and volatility.

In a Bayesian framework, these key parameters are obtained as a combination of prior belief and information from sample data. We assume a fairly diffuse prior for H^{-1} in order to let the data determine time variation of each parameter. More specifically, we use a Wishart prior of which diagonal elements are set to 0.001 (see Appendix for other priors). This implies that the variance of a change in each parameter is marginally distributed with the mean of 10^{-3} and the variance of 2×10^{-6}. As we will see below, these priors yield considerably smoother developments of pass-through coefficients compared with those obtained by rolling regressions. However, preliminary investigations suggest that putting larger values for these priors does not substantially alter estimation results.

We are primarily interested in the following coefficients. First, *long-run* exchange rate pass-

[3] We find that estimation results do not alter much even if we take two lags for each variable.

through is calculated as $\alpha_{1t}(1)/(1-\alpha_{0t})$.[4] Second, a long-run impact of commodity price fluctuation is calculated as $\alpha_{3t}(1)/(1-\alpha_{0t})$. Developments of this coefficient would reveal whether or not the recent years observe not only a decline in exchange rate pass-through, but also one in the impact of commodity prices. Finally, the long-run inflation rate is calculated as $\alpha_{5t}/(1-\alpha_{0t})$, which reveals inflation rate to converge in the long run, if there is no additional movement in exchange rate, foreign prices and commodity prices, $\Delta e = \Delta p^* = \Delta p^{com} = 0$, and output gap is closed to zero, $\tilde{y} = 0$. Although it does not accord with conventional measures of core inflation such as excluding-food-and-energy and trimmed-mean, we call this "core" inflation rate hereafter, following the terminology of Cogley and Sargent (2005).[5] It would be interesting to see whether the decline in core CPI inflation rate observed by Cogley and Sargent (2005) holds for import prices as well as consumer prices of other countries.

Second-stage pass-through is measured by a backward-looking Phillips curve.

$$\Delta p_t = \alpha'_{0t}\Delta p_{t-1} + \alpha'_{1t}(L)\Delta p^m_t + \alpha'_{2t}\tilde{y}_{t-1} + \alpha'_{3t} + \epsilon'_t. \tag{2}$$

where p_t is consumer prices (excluding food and energy) in logarithm. We include upto two-quarter lags for $\alpha'_{1t}(L)$. However, for the United Kingdom and France, a contemporaneous term is excluded as there is a sign of overfit. One-period lag is taken for output gap, as it tends to lead inflation rate in most countries (Higo and Nakada, 1999). Similar to equation (1), we allow for time variance of parameters $\alpha'_{i,t+1} = \alpha'_{it} + u'_{it}$ and volatility $h'_{t+1} = h'_t + \eta'_t$. The second-stage pass-through (ie impacts of import prices on consumer prices) is captured by $\alpha'_{1t}(1)/(1-\alpha'_{0t})$ and core consumer inflation rate is measured by $\alpha'_{3t}/(1-\alpha'_{0t})$. On top of these coefficients, we are also interested in $\alpha'_{2t}/(1-\alpha'_{0t})$ to see whether or not the effects of output gap have diminished.

2.2 Time-varying parameter cum stochastic volatility model

The above pass-through equations can be put in the following state space form:

$$\begin{aligned} y_t &= Z_t\alpha_t + \epsilon_t, & (3)\\ \alpha_{t+1} &= \alpha_t + u_t,\ u_t \sim N(0, H^{-1}), & (4)\\ \epsilon_t &= \gamma \exp\left(\frac{h_t}{2}\right)\varepsilon_t,\ \varepsilon_t \sim N(0,1), & (5)\\ h_{t+1} &= h_t + \eta_t,\ \eta_t \sim N(0, \sigma^2_\eta), & (6) \end{aligned}$$

and the initial values of state variables are

$$\begin{aligned} \alpha_0 &= 0 \text{ and } u_0 \sim N(0, H_0^{-1}), & (7)\\ h_0 &= 0 \text{ and } \eta_0 \sim N(0, \sigma^2_{\eta 0}). & (8) \end{aligned}$$

[4] In the literature, the term "long-run" pass-through has two different meanings. One is a long-run stationary relationship captured by cointegrating vectors. The other is the cumulative effect of a change in the exchange rate until its effect has died out. The former implies the latter, but not vice versa. This paper uses the latter meaning of long-run pass-through.

[5] However, it shares the idea of gauging expected inflation excluding a certain type of "noise" (Mankikar and Paisley, 2002)—which in this case refers to any movements in exchange rate, foreign prices and commodity prices and deviation of output gap from zero.

Equations (3), (4) and (7) correspond to a time-varying parameter model. In equation (3), y_t is an observable dependent variable, which corresponds to Δp_t^m in (1) and Δp_t in (2). Z_t is a vector of explanatory variables ($\Delta p_{t-1}^m, \Delta e_t, \Delta p_t^*, ...$ in (1) and $\Delta p_{t-1}, \Delta p_t^m, ...$ in (2)), and α_t is a vector of corresponding coefficients ($\alpha_{0t}, \alpha_{1t}, ...$ and $\alpha_{0t}', \alpha_{1t}', ...$). In equation (4), as described above, α_t evolves as an AR(1) process with a unit root coefficient. The time-varying parameter model can be estimated by a Gibbs sampling scheme of a Bayesian Markov Chain Monte Carlo (MCMC), which exploits an efficient Gaussian simulation smoother developed by de Jong and Shephard (1995).

Equations (5), (6) and (8) correspond to a stochastic volatility model. A general formula is expressed as (Omori et al, 2004):

$$\epsilon_t = \gamma \exp\left(\frac{h_t}{2}\right)\varepsilon_t, \text{ for } t = 1, \ldots, T \tag{9}$$

$$h_{t+1} = \mu + \phi(h_t - \mu) + \eta_t, \text{ for } t = 0, \ldots, T \tag{10}$$

and

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim N(0, \Sigma) \text{ and } \Sigma = \begin{pmatrix} 1 & \rho\sigma_\eta \\ \rho\sigma_\eta & \sigma_\eta^2 \end{pmatrix}.$$

Our simplified model assumes $\mu = 0$, $\phi = 1$ and $\rho = 0$.[6] Kim et al (1998) demonstrate that the stochastic volatility model can be estimated by a Bayesian MCMC framework as an extension of the above Gibbs sampling scheme.

A posterior density of an entire model (3)-(8) can be obtained from some priors by combining two Gibbs sampling schemes: one for a time-varying parameter model and the other for a stochastic volatility model. The Appendix will discuss more details of the algorithm and priors used.

2.3 Data

The data used are those of major industrial countries (the United States, Japan, Germany, the United Kingdom, France and Italy). Sample coverage are 1974 Q1-2004 Q4. All the data are more or less conventional in the literature and do not require explanation in the main text. See Table 1 for definition of the variables and data sources.

3 Estimation Results

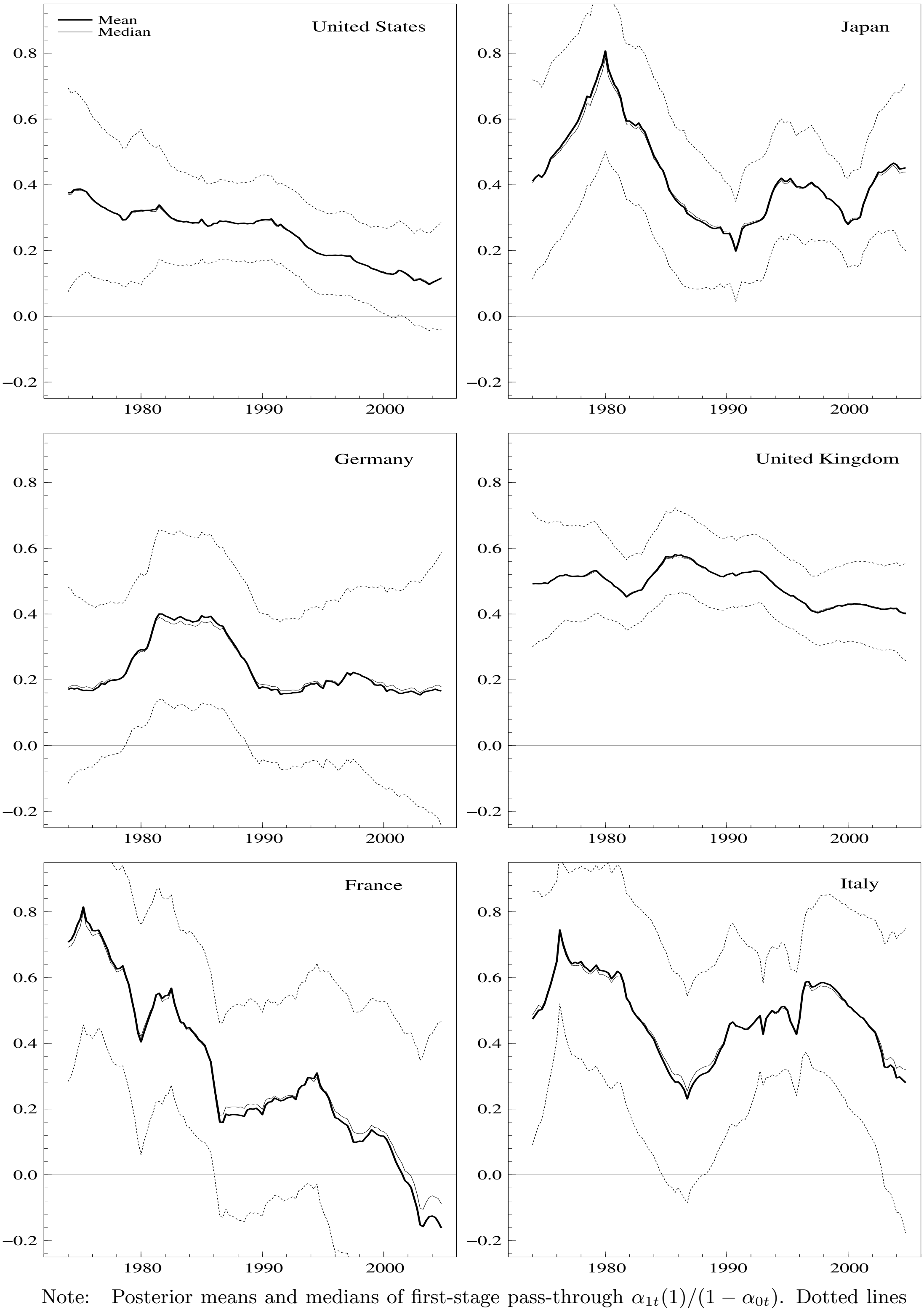
In order to have posterior results for the state space form, we run the above Gibbs sampler for 21,000 replications, with 1,000 burn-in replications discarded and 20,000 replications retained.

3.1 First-stage pass-through

For first-stage pass-through, posteriors support the view that most of the parameters, including volatility, are indeed time-varying. Table 2 contains posterior means and standard deviations

[6] It is customary for a stochastic volatility model to assume $\gamma = 1$, as γ is not identifiable when $\mu \neq 0$ (Kim et al, 1998). However, this is not the case for equation (6).

Figure 1: First-stage pass-through



Note: Posterior means and medians of first-stage pass-through $\alpha_{1t}(1)/(1-\alpha_{0t})$. Dotted lines indicate posterior interquartile ranges.

Table 1: Data list

	Definition	Source
p_t^m	Log of import price index (JP, DE, UK) or import unit value index (US, FR, IT). For French data, the series are spliced at 1999M1, when a break is observed.	BIS/DBS, US-BEA, OECD
p_t	Log of consumer price index excluding food and energy. For Japan, only fresh food is excluded.	BIS/DBS, OECD
p_t^{com}	Log of raw material price index (in US$) compiled by Hamburg Institute of International Economics. Spot exchange rates are used for conversion to own currency.	BIS/DBS
e_t	Log of nominal effective exchange rate (25-country basis).	BIS/DBS
p_t^*	Log of foreign prices obtained by $e_t - e_t' + p_t$ where e_t' is CPI-based real effective exchange rate.	BIS/DBS
$\tilde{y}_t$	Output gap calculated by the HP filter on real GDP (the band-with is 1600).	BIS/DBS
$(m-y)_t$	Log of import penetration ratio (i.e., the share of imports of goods and services in real GDP).	BIS/DBS

Notes:

1. BIS/DBS stands for the BIS Data Bank Service, which collects various national data.
2. US-BEA stands for the US Bureau of Economic Analysis.

of precision H and σ_η^{-2} for first-stage pass-through equation (1). Comparison between means and standard deviations suggests that in most cases, precision associated with state equations of exchange rate pass-through (Δe_t and Δe_{t-1}), impacts of commodity prices (Δp_t^{com} and Δp_{t-1}^{com}), core inflation (a constant term) and volatility (h_t) is statistically significantly different from zero. As discussed above, these findings indicate that stochastic components of state equations (4) and (6) cannot be ignored and corresponding parameters are changing over time.

Figure 1 plots estimated long-run first-stage pass-through, $\alpha_{1t}(1)/(1-\alpha_{0t})$. Together with its posterior mean, the figure shows the posterior median and interquartile range. Standard deviation of $\alpha_{1t}(1)/(1-\alpha_{0t})$ tends to be huge (and is not shown) because of the outliers in the posterior densities resulting from division by $1-\alpha_{0t}$, which sometimes takes on a value close to zero.

First-stage pass-through has declined over time. For instance, in the case of the United States (the top left panel of Figure 1), it decreased to 0.1 in the recent periods from 0.4 in the 1970s. The relatively sharp fall is observed from the 1990s. For other countries, pass-through has also declined, but magnitude and timing of the decline differ. Relatively large declines in pass-through are observed in Japan and France, which are followed by Italy, while changes in pass-through in Germany and the United Kingdom are of a similar order to the United States. Compared with the United States, for Japan, Germany and Italy, declines are rather concentrated before the 1990s. In the United Kingdom, the decline in pass-through somewhat accelerated after 1990.

Our time-varying parameter model suggests that the decline in pass-through took place gradually compared with those obtained by rolling regressions (Figure 2). We believe in the

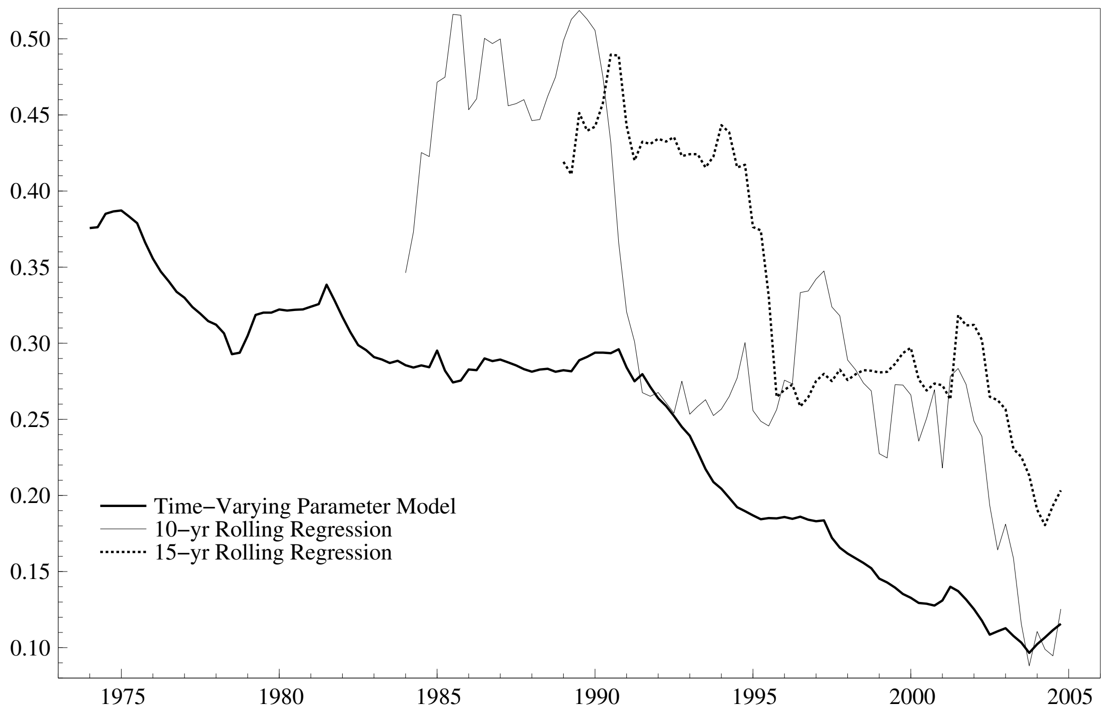
Table 2: Posterior precision of first-stage pass-through

	United States	Japan	Germany
Δp^m_{t-1}	2,904 (2,244)*	3,989 (2,579)*	3,604 (2,423)*
Δe_t	4,497 (2,607)**	3,978 (2,586)*	3,313 (2,228)*
Δe_{t-1}	4,806 (2,691)**	4,170 (2,564)*	3,705 (2,329)*
Δp^{com}_t	6,220 (3,138)**	5,605 (2,952)**	6,949 (3,371)**
Δp^{com}_{t-1}	6,132 (3,065)**	5,225 (2,920)**	6,896 (3,405)**
Δp^*_t	2,211 (1,788)	1,342 (1,334)	1,699 (1,483)
Δp^*_{t-1}	2,755 (2,162)	1,702 (1,662)	2,697 (2,045)*
$\tilde{y}_t$	2,820 (2,131)*	2,684 (2,277)	3,011 (2,192)*
$\tilde{y}_{t-1}$	2,745 (2,150)	2,815 (1,977)*	2,765 (2,018)*
const.	24,456 (6,059)***	17,392 (5,084)***	25,218 (6,082)***
h_t	6.8 (3.1)**	6.7 (3.1)**	6.9 (3.0)**
	United Kingdom	**France**	**Italy**
Δp^m_{t-1}	3,038 (2,207)*	3,832 (2,445)*	4,255 (2,603)*
Δe_t	4,935 (2,870)**	3,142 (2,293)*	2,941 (2,182)*
Δe_{t-1}	4,879 (2,886)**	3,176 (2,193)*	3,197 (2,345)*
Δp^{com}_t	6,637 (3,145)**	5,827 (3,083)**	6,087 (3,195)**
Δp^{com}_{t-1}	6,448 (3,237)**	5,904 (3,039)**	5,522 (3,078)**
Δp^*_t	2,026 (1,859)	1,969 (1,838)	2,381 (2,069)
Δp^*_{t-1}	1,606 (1,489)	2,075 (2,000)	1,979 (1,944)
$\tilde{y}_t$	2,625 (2,053)	2,406 (1,923)	2,629 (2,103)
$\tilde{y}_{t-1}$	2,912 (2,219)*	2,711 (2,153)	2,730 (2,139)
const.	23,294 (5,805)***	17,058 (5,179)***	17,455 (5,160)***
h_t	6.9 (3.1)**	7.1 (3.2)**	7.1 (3.2)**

Notes:

1. Figures are posterior means of precision for state equations of coefficients on corresponding variables (diagonal elements of H in equation (A.14) and H_η in equation (A.15)).
2. Figures in parentheses are posterior standard deviations. "***", "**" and "*" denote statistical significance at the 1%, 5% and 10% levels, respectively.

Figure 2: Comparison with rolling regressions (United States)



Note: Time-varying parameter model is posterior means of United States first-stage pass-through (same as top left panel of Figure 1).

gradual changes in pass-through for at least three reasons: First, rolling regressions tend to yield abrupt changes depending on whether or not a specific sample is in the window. For instance, a sharp drop in the early 1990s detected by the 10-year window rolling regression is due to exclusion of a sample in the early 1980s from the window, as the same sharp drop is found in the middle of the 1990s by the 15-year window. Because of this dependence on the size of windows, rolling regression does not provide precise timings in the changes in parameter. Second, the time-varying parameter model tends to yield gradual changes in pass-through because of smoothing (equations (A.8) to (A.13) in the Appendix), as often seen in the difference between one-side (Kalman) filtered series and smoothed series. However, as pointed out by Sims (2001), it is smoothed series that give a more precise estimate of actual time variation. Finally, as mentioned above, our robustness check of the larger priors in time-variation does not alter smoothness.

Figure 3 decomposes long-run pass-through coefficients into the sum of exchange rate coefficients, $\alpha_{1t}(1)$, and autoregressive coefficients, α_{0t}. Evidence of lower pass-through is more mixed if we focus on exchange rate coefficients (Figure 3, lower panels). For instance, a decline is not evident for the United Kingdom and Italy. For these countries, a change in autoregressive coefficient accounts for the above observed decline in long-run pass-through (Figure 3, upper panels). For the United States and Japan, changes in both $\alpha_{1t}(1)$ and α_{0t} lead to the lower long-run pass-through.

Turning to other coefficients, first, impacts of commodity price fluctuation, $\alpha_{3t}(1)/(1-\alpha_{0t})$, have become smaller (Figure 4, upper panels). For most of the countries, impacts declined relatively sharply in the 1970s when energy conservation was enhanced after the first oil crisis. From the 1980s, the pace of decline became modest for the United States, Germany and the United Kingdom. For Japan, impacts of commodity prices rose around the middle of the 1980s. The timing coincides with a sharp fall in import prices as well as rapid appreciation after the Plaza Accord. Given that the first-stage pass-through decreased constantly during this period, there might be an identification problem.

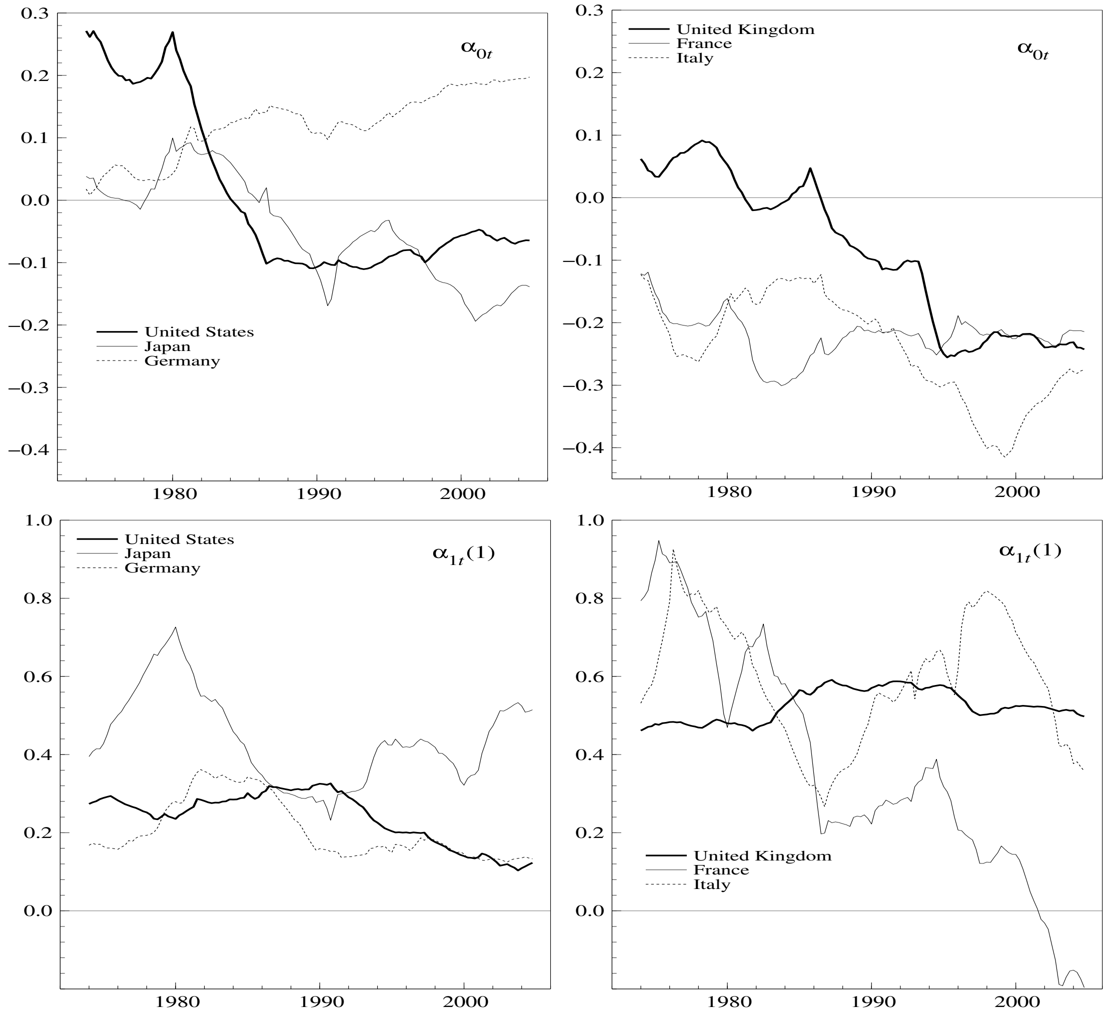
Core import prices inflation rates, in contrast, do not show a unique pattern across countries (Figure 4, middle panels). For the United States and Germany, annualised core inflation rates, $4\alpha_{5t}/(1-\alpha_{0t})$, were about 10% around the time of the second oil crisis of the early 1980s and show modest deceleration thereafter. However, for Japan and the United Kingdom, there is no declining trend observed. France and Italy seem to reveal weak upward trends.

Volatilities have declined for all countries (Figure 4, lower panels). The relatively high volatilities are observed either in the 1970s (the United Kingdom, France, Italy) or in the early 1980s (the United States, Germany, Japan) corresponding to the first and second oil crises. After that, modest declines are observed toward the end of the sample period.

In short, the above estimation confirms that not only long-run exchange rate pass-through, but also impacts of commodity prices fluctuation have become smaller. This may imply that import prices of industrial countries have become more resilient to external shocks of foreign exchange rates and commodity prices. At the same time, a decline in volatility seems to reflect the fact that in the past decade there was no major shock comparable to those associated with the two oil crises.

Comparison of the estimated pass-through coefficients with those of the existing studies provides the following observations. First, the estimated sizes of pass-through coefficients are not

Figure 3: First-stage pass-through: breakdown to α_{0t} and $\alpha_{1t}(1)$



Note: Posterior means of autoregressive coefficient α_{0t} (upper panels) and sum of exchange rate coefficients $\alpha_{1t}(1)$ (lower panels).

Figure 4: First-stage pass-through: other coefficients

Note: Posterior means of long-run impact of commodity price changes $\alpha_{3t}(1)/(1-\alpha_{0t})$ (upper panels); annualised core import price inflation $4\alpha_{5t}/(1-\alpha_{0t})$ (middle panels); and its stochastic volatility $\exp(h_t/2)$ (lower panels).

far from those obtained from in-depth individual country studies of the United States and Japan (Marazzi et al (2005), Otani et al (2003, 2005)). Campa and Goldberg (2004) find relatively large declines in pass-through for Japan, France and Italy, which are largely consistent with our observations. Second, the timing of the declines is, however, different from studies using rolling regressions. These include Marazzi et al (2005) who find that a large decline emerged after 1997, which we do not observe. Third, the fact that the decline in pass-through took place gradually over time does not accord with split sample estimations, which assume a one-time structural break in parameter.

In some countries, the decline in pass-through accelerated after 1990, when China and other previously socialist countries were integrated into the global economy. A typical example is the United States, where a decline took place mainly after 1990. In this respect, it is not surprising to see that Kim (1990) does not find strong evidence on change in pass-through for its import price before the mid-1980s.

3.2 Second-stage pass-through

For second-stage pass-through, posterior precision again clearly supports the view that pass-through has changed from time to time (Table 3). As before, the comparison between means and standard deviations suggests that in all cases, precision associated with state equations of import prices (Δp_t^m, Δp_{t-1}^m and Δp_{t-2}^m) is statistically significantly different from zero. That is, the stochastic components of state equation (4) cannot be ignored, and the corresponding parameters have changed over time. Table 3 also indicates that coefficients on core inflation (a constant term) and volatility (h_t') are also time-varying. Meanwhile, the evidence of time variance is weaker for coefficients on the output gap and there is no support for time variance of autoregressive coefficients.

Long-run second-stage pass-through, $\alpha_{1t}'(1)/(1-\alpha_{0t}')$, has declined for all of the sample countries (Figure 5). For the United States, Japan and the United Kingdom, pass-through has fallen from more than 0.1 in early years to between zero and 0.03 in recent years. Declines are more modest for Germany, France and Italy, where pass-though was as small as 0.05 even in early periods.[7] The timing of declines differs. For the United States, pass-through dropped relatively rapidly after 1980, while a decline is rather concentrated before 1980 for Japan. The United Kingdom and Italy experienced a step-wise fall in pass-through around the mid-1980s and around 1980, respectively. For Germany and France, pass-through declined almost constantly over the sample periods.

These declines in long-run pass-through mainly correspond to those in the sum of import price coefficients, $\alpha_{1t}'(1)$: declines in direct impacts of import prices to consumer prices are clear for all countries, as indicated by the lower panel of Figure 6. Changes in autoregressive coefficients, α_{0t}', also contribute by reducing $1/(1-\alpha_{0t}')$, although, consistent with the insignificance of corresponding precision, these changes are less visible compared with declines in $\alpha_{1t}'(1)$ (Figure 6, upper panel).

As for other coefficients, first, the evidence is mixed for those on the output gap, $\alpha_{2t}'/(1-\alpha_{0t}')$. These coefficients have become smaller for some countries like Japan, the United Kingdom and

[7] Like those in the mid-1990s for the United Kingdom, the recent negative pass-through coefficients for France are not statistically significantly different from zero.

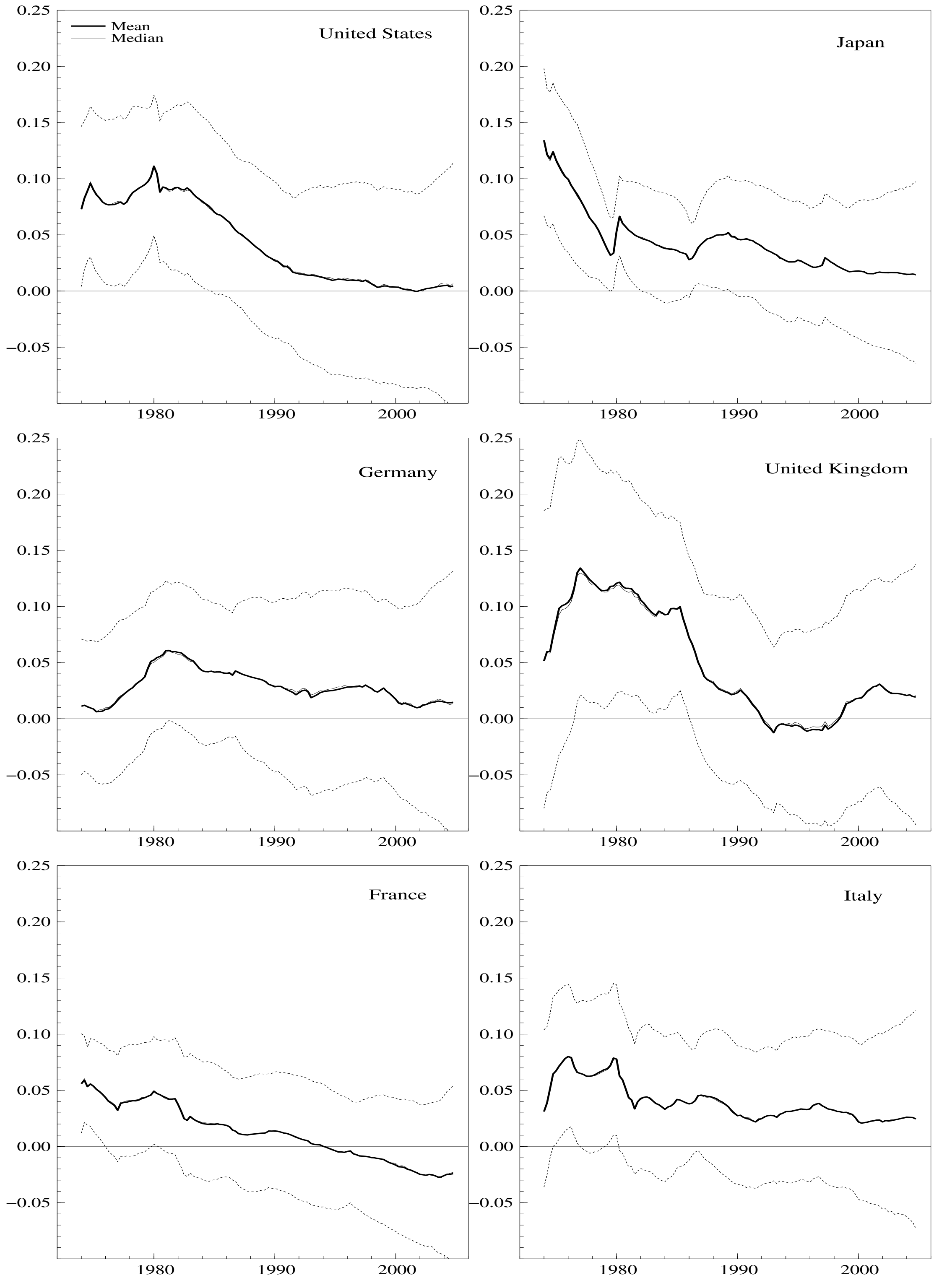
Table 3: Posterior precision of second-stage pass-through

	United States	Japan	Germany
Δp_{t-1}	1,863 (1,860)	1,830 (1,745)	1,651 (1,558)
Δp_t^m	4,447 (2,628)**	5,672 (2,907)**	4,514 (2,581)**
Δp_{t-1}^m	4,470 (2,535)**	6,006 (3,031)**	4,552 (2,556)**
Δp_{t-2}^m	4,447 (2,625)**	6,411 (3,155)**	4,724 (2,722)**
$\tilde{y}_{t-1}$	2,886 (2,103)*	2,434 (1,973)	3,205 (2,194)*
const.	52,298 (9,241)***	43,596 (8,313)***	48,957 (8,793)***
h_t'	6.6 (3.0)**	5.9 (2.7)**	6.8 (3.1)**
	United Kingdom	France	Italy
Δp_{t-1}	2,004 (1,826)	1,787 (1,658)	2,076 (1,756)
Δp_t^m			5,021 (2,760)**
Δp_{t-1}^m	3,160 (2,072)*	4,820 (2,601)**	5,434 (2,933)**
Δp_{t-2}^m	3,677 (2,429)*	5,089 (2,769)**	4,970 (2,734)**
$\tilde{y}_{t-1}$	2,007 (1,842)	2,596 (1,988)*	2,003 (1,770)
const.	34,390 (6,975)***	51,619 (9,181)***	41,697 (7,973)***
h_t'	6.0 (2.8)**	6.7 (3.0)**	7.0 (3.0)***

Notes:

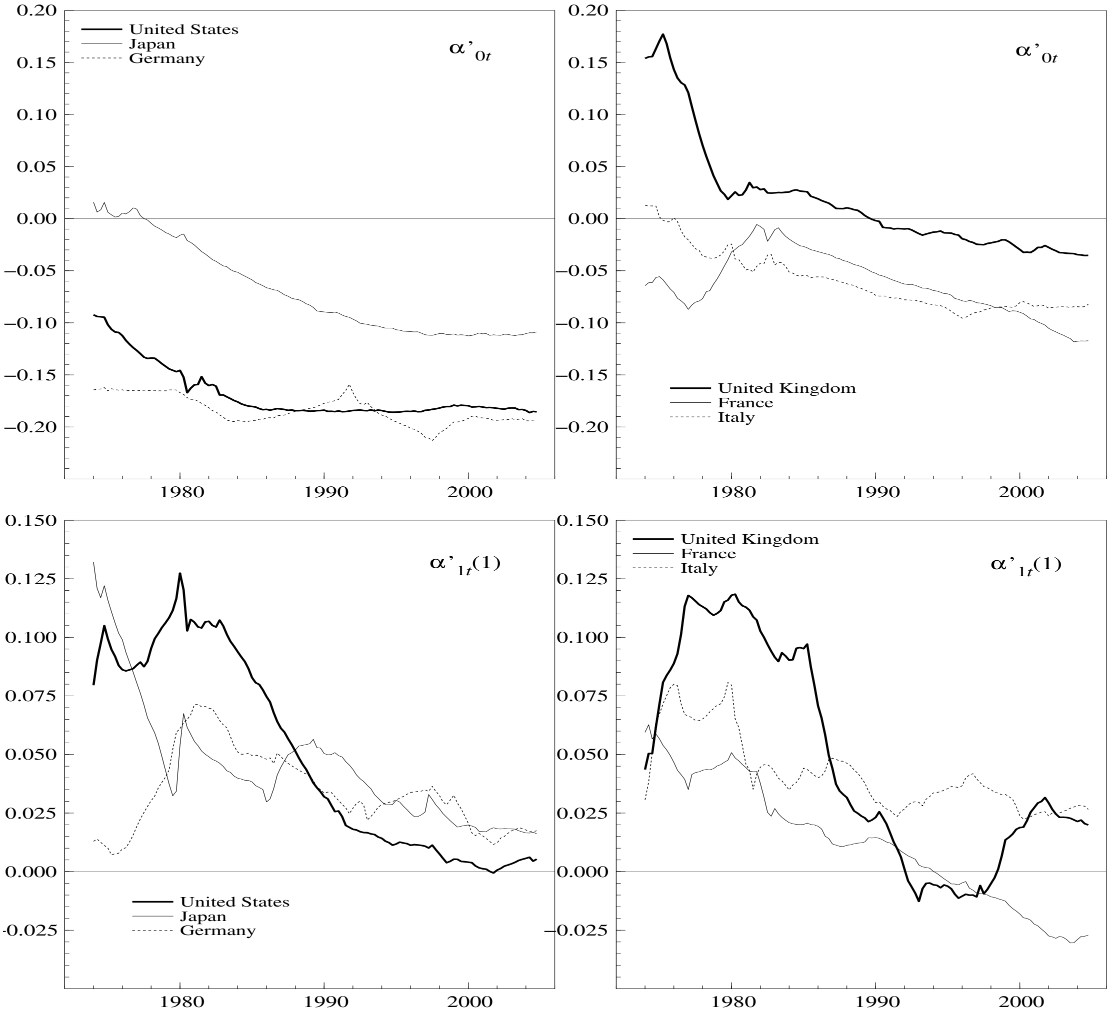
1. Figures are posterior means of precision for state equations of coefficients on corresponding variables (diagonal elements of H in equation (A.14) and H_η in equation (A.15)).
2. Figures in parentheses are posterior standard deviations. "***", "**" and "*" denote statistical significance at the 1%, 5% and 10% levels, respectively.

Figure 5: Second-stage pass-through



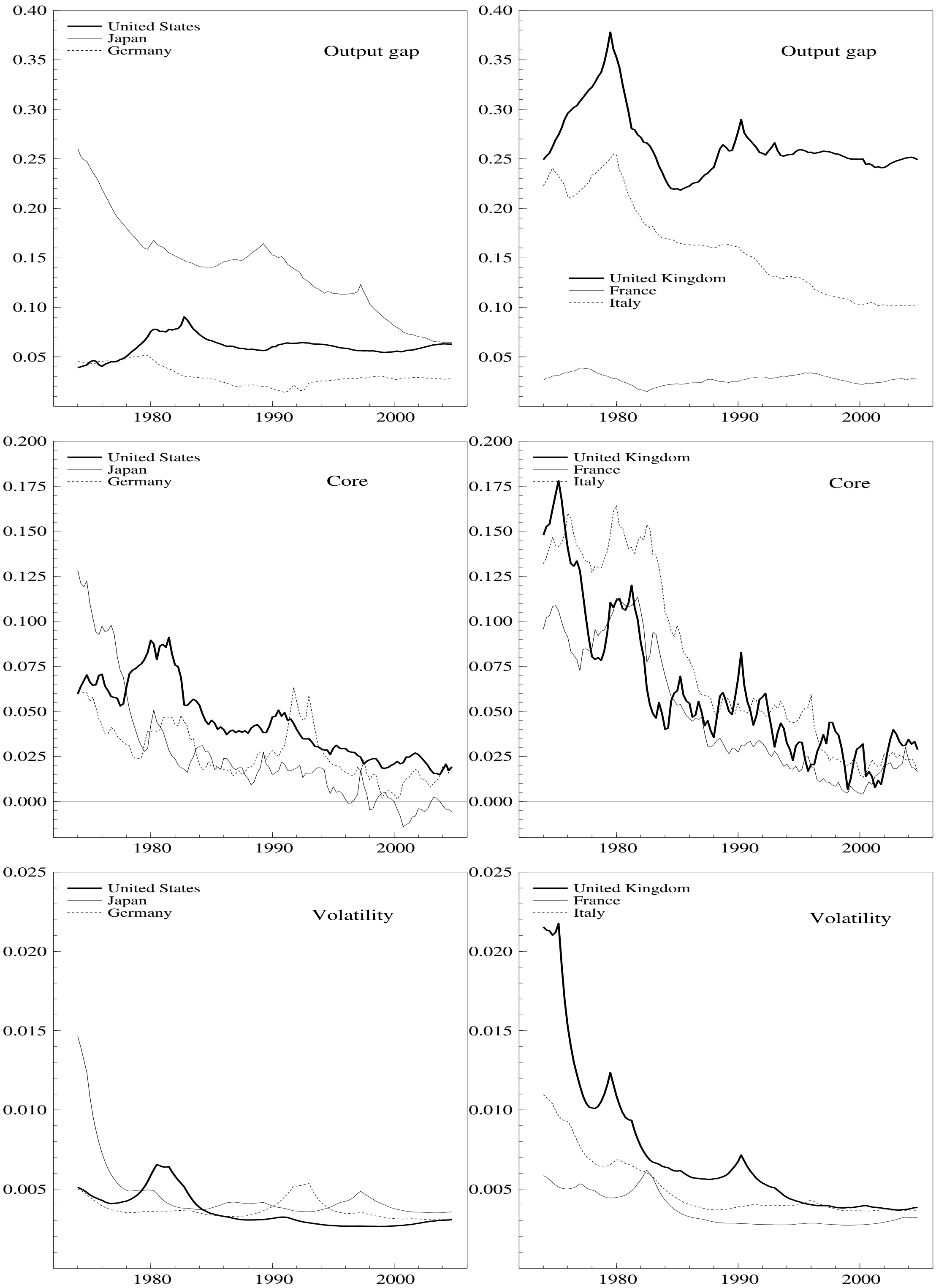
Note: Posterior means and medians of second-stage pass-through $\alpha'_{1t}(1)/(1-\alpha'_{0t})$. Dotted lines indicate posterior interquartile ranges.

Figure 6: Second-stage pass-through: decomposition to α'_{0t} and $\alpha'_{1t}(1)$



Note: Posterior means of autoregressive coefficient α'_{0t} (upper panels) and sum of import price coefficients $\alpha'_{1t}(1)$ (lower panels).

Figure 7: Second-stage pass-through: other coefficients



Note: Posterior means of long-run impact of output gap $\alpha'_{2t}/(1-\alpha'_{0t})$ (upper panels); annualised core import price inflation $4\alpha'_{3t}/(1-\alpha'_{0t})$ (middle panels); and its stochastic volatility $\exp(h'_t/2)$ (lower panels).

Italy (Figure 7, upper panel), while the decline is not obvious for the others. Admittedly, these results need to be taken with caveats, as the output gap is calculated by the conventional HP filter and the specification lacks rigorous treatment of expectation. Further investigation of changes in the slope of the Phillips curve is certainly warranted, but is beyond the scope of the current paper.

Next, core consumer inflation rates have declined for all countries (Figure 7, middle panels). In the United States, the annualised core consumer inflation, $4\alpha'_{3t}/(1-\alpha'_{0t})$, reached nearly 10% in the early 1980s soon after Chairman Volcker took office. Similarly, most other countries experienced high inflation in the 1970s and/or the early 1980s. Core inflation declined thereafter, and reached about 2-3% levels in recent years, except in Japan, whose core inflation rate became slightly negative, reflecting deflation in the country.

Volatilities have also declined for most countries (Figure 7, lower panels). Similar to volatilities of import prices inflation, declines are concentrated in the earlier periods. After that, declines became quite modest. The countries that experienced high inflation in the 1970s/1980s tended to have high volatility at that time. In contrast, Germany, where the core inflation rate was modest even in the early years, continued to have low volatility except a small spike after the unification in the 1990s.

In sum, consumer prices have become less responsive to movement in import prices in major industrial countries. At the same time, the level and volatility of consumer prices inflation have declined. For some countries, autoregressive coefficients and the impacts of output gap have also fallen, even though the evidence is not so decisive.

The estimated sizes of pass-through coefficients are largely consistent with those of existing studies. For instance, Gagnon and Ihrig (2004) also find that second-stage pass-through became almost zero or statistically insignificantly different from zero for most of the countries examined in this paper.

In most of the countries, the fact that the decline in second-stage pass-through took place gradually over time invalidates split sample estimations, as observed in the case of first-stage pass-through. Important exceptions are the United Kingdom and Italy, where second-stage pass-though shifted down around the mid-1980s and around 1980, respectively. These structural breaks in pass-through appear to coincide with regime changes in monetary and exchange rate policy such as the United Kingdom's introduction of de facto fixed exchange rate policy after abandonment of the M3 target policy in 1985 and Italy's participation in the fixed Exchange Rate Mechanism (ERM) in 1979.

4 Relationship with Other Developments

This section intends to give a broad idea of how changes in pass-through coefficients are related to other developments. First, the section examines the relationships with other time-varying coefficients such as autoregressive terms, constant terms, commodity prices, output gap and volatilities. Lower pass-through might be associated with a lower level of inflation, smaller volatility and smaller persistence, because a low and stable inflation environment would induce firms to lower pass-through (Taylor, 2000). Monetary policy that credibly pursues a policy aimed at keeping inflation low and stable may, by anchoring inflation expectations, increase

Table 4: Relationship with other developments

	Relationship with numerator of first-stage pass-through: $\alpha_{1t}(1)$					
	AR (Δp_t^m)	Core (Δp_t^m)	Vol (Δp_t^m)	Commodity	Vol (Δe_t)	Penetration
	α_{0t}	α_{5t}	h_t	$\alpha_{3t}(1)$	h_t^e	$(m-y)_t$
US	0.04 (0.05)	7.93 (1.26)***	22.00 (2.56)***	1.33 (0.09)***	-7.37 (0.93)***	-0.41 (0.02)***
JP	0.73 (0.11)***	2.92 (4.40)	14.76 (3.63)***	-1.23 (0.10)***	-3.33 (1.38)**	-0.47 (0.08)***
DE	-0.39 (0.14)***	6.63 (0.93)***	30.27 (1.39)***	1.24 (0.13)***	22.33 (1.52)***	-0.83 (0.08)***
UK	-0.12 (0.03)***	1.66 (1.29)	-17.05 (3.65)***	0.04 (0.14)	5.81 (0.41)***	-0.22 (0.03)***
FR	0.76 (0.91)	-22.79 (2.08)***	38.77 (6.86)***	1.87 (0.34)***	17.36 (2.30)***	-0.54 (0.15)***
IT	-0.96 (0.16)***	-6.40 (1.98)***	4.74 (2.74)*	-1.12 (0.17)***	-12.75 (1.57)***	1.97 (0.10)***
All	-0.07 (0.08)	-1.67 (22.89)	15.35 (68.32)	0.34 (0.30)	3.63 (33.05)	-0.08 (0.18)
	Relationship with numerator of second-stage pass-through: $\alpha'_{1t}(1)$					
	AR (Δp_t)	Core (Δp_t)	Vol (Δp_t)	Output gap	Vol (Δe_t)	Penetration
	α'_{0t}	α'_{3t}	h'_t	α'_{2t}	h_t^e	$(m-y)_t$
US	1.30 (0.09)***	5.99 (0.59)***	21.14 (3.24)***	-1.29 (0.25)***	-4.68 (0.31)***	-0.13 (0.02)***
JP	0.24 (0.02)***	1.00 (0.22)***	1.69 (3.39)	0.19 (0.07)***	0.42 (0.25)*	0.07 (0.02)***
DE	1.14 (0.09)***	2.97 (0.47)***	10.50 (2.31)***	0.78 (0.06)***	2.66 (0.19)***	-0.09 (0.01)***
UK	0.84 (0.08)***	4.98 (0.37)***	9.63 (0.70)***	0.58 (0.13)***	-1.40 (0.90)	-0.16 (0.04)***
FR	0.13 (0.10)	1.71 (0.11)***	12.07 (0.46)***	0.40 (0.30)	2.79 (0.18)***	-0.04 (0.02)**
IT	0.29 (0.03)***	0.42 (0.05)***	3.62 (0.45)***	0.15 (0.01)***	0.29 (0.09)***	-0.04 (0.01)***
All	0.66 (0.04)***	2.80 (0.85)***	9.68 (8.65)	0.15 (0.10)	0.04 (1.33)	-0.06 (0.00)***

Notes:

1. Coefficients obtained by dynamic OLS (DOLS) regressions of first- and second-stage pass-through on respective variables. "All" is calculated as a random coefficient model of the corresponding DOLS specifications.
2. Figures in parentheses are posterior standard deviations. "***", "**" and "*" denote statistical significance at the 1%, 5% and 10% levels, respectively.

the readiness of firms to absorb exchange rate fluctuation in their profit margins. The low inflation expectation ascribed to changes in monetary policy is supposed to be captured by the core inflation rate in this paper, as these effects are not controlled in pass-through equations (1) and (2). Furthermore, the lower first-stage pass-through might reflect the smaller impact of commodity prices, because the lower share of raw materials in imports accounts for a decline in pass-through as well as a reduction in the direct impact of high commodity prices, as raw materials tend to exhibit a high exchange rate pass-through to import prices.

In addition, this section examines how changes in pass-through coefficients are related to exchange rate volatility, h_t^e, and the import penetration ratio $(m-y)_t$, where import penetration is defined as the share of imports of goods and services in real GDP. Firms tend to invoice their transactions by currencies with relatively low exchange rate variability in order to stabilise their revenues (Devereux et al, 2004). In this case, the lower (higher) volatility of the exchange rate is supposed to make more foreign exporters choose local (producer) currency pricing, and hence decrease (increase) pass-through. The relationship with import penetration might go in both directions. On the one hand, the higher import penetration and the consequent greater competition may bring about an increase in pass-through by turning firms into price takers, and in the limit make pass-through complete (Dornbusch, 1987). On the other hand, greater competition and a commensurate reduction in the market power of dominant firms may reduce pass-through (Bacchetta and van Wincoop, 2005).

Table 4 shows coefficients obtained by the regressing the numerator of pass-through coefficients on respective variables. These simple regressions are intended to reveal correlations between pass-through coefficients and each variable.[8] Because of non-stationarity—by construction, these variables have unit roots as embedded by equations (4) and (6)—the dynamic OLS (DOLS) of Stock and Watson (1993) is used by augmenting three-quarter leads and lags of differenced explanatory variables. For all countries, the coefficients are obtained as a random coefficient model of Swamy (1970): ie a matrix-weighted average of DOLS coefficients of individual countries, with weights inversely proportional to their covariance matrices. Exchange rate volatilities are calculated by fitting a stochastic volatility model (equations (5) and (6)) to changes in effective exchange rate, Δe_t, using the Gibbs sampler of corresponding steps.

For first-stage pass-through, the relationships with coefficients on the level, Core (Δp^m), autoregressive terms, AR (Δp^m), and volatility, Vol (Δp^m), of import prices are mixed (Table 4, upper panel). For instance, for France and Italy, negative coefficients are found on the level of import prices inflation rate. For Japan and the United Kingdom, these coefficients are not significant. Similarly, coefficients on autoregressive terms have wrong sign (Germany, the United Kingdom, and Italy) or are insignificant (the United States and France). A coefficient on volatility is incorrectly signed for the United Kingdom.

In contrast, for second-stage pass-through, the relationships with the level, Core (Δp), autoregressive terms, AR (Δp), and volatility, Vol (Δp), of consumer prices are more evident (Table 4, lower panel). For most of the cases, coefficients on these variables are positive and significant. The finding is consistent with a number of cross-country studies (Gagnon and Ihrig

[8] In a simple linear regression model of $y = a + bx$, if y and x follow a bivariate normal distribution, $\rho = b\sigma_x/\sigma_y$, where ρ is a correlation coefficient, and σ_x and σ_y are standard deviations of x and y respectively. If a regression coefficient b is zero, then a correlation coefficient ρ becomes zero. As noted immediately below, additional complications arise due to the presence of a unit-root, but the regressions in this section is conducted in this spirit.

(2004), Frankel et al (2005), Choudhri and Hakura (2001) and Goldfajn and Werlang (2000)), which found that countries with lower and/or less volatile CPI inflation rates tend to have a lower pass-through to CPI.

Turning to the relationships with other variables, for first-stage pass-through, the signs of coefficients on commodity prices are mixed. This might suggest that shifts in the composition of imported goods cannot account for all of the decline in pass-through. For second-stage pass-through, coefficients on output gap tend to be positive. Perhaps, not surprisingly, countries that fail to find significantly positive coefficients (the United States and France) have no flattening Phillips curve with this measure of the output gap.

The relationships with exchange rate volatility are again mixed. This might be due to the fact that we do not distinguish between permanent and transitory shocks of exchange rates—in order to do that, we need to endogeneise exchange rate movements, as discussed above. It might also be the case that the underlying relationship between pass-through and exchange rate volatility is non-linear, and we fail to detect it by fitting linear regressions.

Finally, the coefficients on the import penetration ratio are found to be negative in most cases. This supports the view that more competitive pressures have reduced pass-through. To the extent that a rise in the import penetration ratio reflects a change in import composition—switching from raw materials to manufactured products is supposed to raise the import penetration ratio, as the latter have larger value added compared with the former—the negative coefficients might also capture that effect.

5 Conclusion

In this paper, we estimate the exchange rate pass-through of major industrial countries (the United States, Japan, Germany, the United Kingdom, France and Italy) as a time-varying parameter with stochastic volatility model. The exchange rate pass-through is divided into impacts of exchange rate fluctuations to import prices (first-stage pass-through) and those of import price movements to consumer prices (second-stage pass-through).

The main findings can be summarised as follows:

- Both the first- and second-stage pass-throughs have declined over time for all of the countries examined. These changes in pass-through are statistically significant and economically non-negligible. For instance, in the case of the United States, when the two pass-throughs are combined together, the long-run responsiveness of consumer prices to 10% exchange rate fluctuation has declined from 0.4 percentage points to almost nil.
- The decline in pass-through took place gradually compared with those estimated by rolling regressions. Except for second-stage pass-through in the United Kingdom and Italy, changes in pass-through did not show a structural break in parameters in a way assumed by split sample estimation.
- The lower second-stage pass-through is associated with the lower and the more stable consumer prices inflation environment. The evidence is mixed for first-stage pass-through given weak relationships with the level and volatility of import prices. For most countries, a decline in both stages of pass-through is related to a rise in import penetration.

As a future step of research, it is important to further examine what accounts for observed declines in pass-through. Given that the decline in second-stage pass-through was associated with the emergence of the low and stable inflation environment, one may argue that a change in monetary policy regime is a factor that accounts for its decline. For instance, the timing of a decline in second-stage pass-through in the United States broadly coincides with a change in the Fed's monetary policy towards interest rate setting that is more reactive to expected inflation Clarida et al (2000). Second-stage pass-through shifted down at the time of adoption of a de facto fixed exchange rate regime (United Kingdom) and participation in the ERM (Italy). At the same time, given that a decline in both stages of pass-through is related to a rise in import penetration for most countries, one may also argue that the effect of globalisation on the competition and constestability of the market have played some role.

In addition, it is important to tackle econometric issues such as endogeneity, cointegration relationships, asymmetry and non-linearity. As stated above, the time-varying coefficient model generally suggests gradual changes in pass-through coefficients rather than structural break-type parameter shifts—this might be because the underlying structural change took place gradually over time (possibly in the case of globalisation) or because it took some time for structural change to be materialised in behavioural changes (e.g. the credibility of the new policy regime might have been gained gradually, even though the shift in the policy regime took place overnight). In order to further confirm this, it would be very interesting to embed regime shifts in pass-through coefficients by estimating a non-linear threshold model, and examine whether or not the regime-switching model is outperformed by our time-varying parameter model in terms of Bayes factors (Koop and Potter, 2001).

Appendix: Bayesian MCMC Algorithm for a State Space Form

This appendix elaborates on the algorithm used. All the codes are written in Ox (Doornik, 2001).

Equations (3) and (4) of the time-varying parameter model can be put in a more general state space form (Koop (2003), Chapter 8).

$$\begin{aligned} y_t &= X_t\beta + Z_t\alpha_t + G_t v_t, \text{ for } t = 1, \dots, T \qquad (A.1) \\ \alpha_{t+1} &= T_t\alpha_t + J_t v_t, \text{ for } t = 0, \dots, T \qquad (A.2) \end{aligned}$$

and $\alpha_0 = 0$. v_t is i.i.d. $N(0, \sigma^2 I_{p+1})$. α_t is $p \times 1$ vector containing p state equations. To obtain (3), (4) and (7) from this general model, there is no exogenous variable X_t and $T_t = I_p$. With $\sigma^2 = 1$, v_t is defined as

$$v_t = \begin{pmatrix} \epsilon_t \\ u_t \end{pmatrix}.$$

G_t is a $(p+1)$ row vector given by[9]

$$G_t = (\gamma \exp(h_t/2), 0, \dots, 0),$$

J_t is a $p \times (p+1)$ matrix and A is a implicitly determined $p \times p$ matrix such that

$$J_t = \begin{bmatrix} 0_p & A \end{bmatrix}, \text{ where } H^{-1} = AA'.$$

Similarly, J_0 (a $p \times (p+1)$ matrix) and A_0 (a $p \times p$ matrix) are given by

$$J_0 = \begin{bmatrix} 0_p & A_0 \end{bmatrix}, \text{ where } H_0^{-1} = A_0 A_0'.$$

Once this mapping to a general state space form is established, we can apply a standard technique of estimating a state space form by a Bayesian MCMC, which exploits an efficient Gaussian simulation smoother developed by de Jong and Shephard (1995).

Equation (9) of the stochastic volatility model can be expressed as

$$\epsilon_t^* = \log \epsilon_t^2 = c + h_t + \zeta_t,$$

where $c = \log \gamma^2$. $\zeta_t = \log \varepsilon_t^2$ follows a log χ_1^2 density.

$$f(\zeta_t) = \frac{1}{\sqrt{2}} \exp\left\{\frac{\zeta_t - \exp(\zeta_t)}{2}\right\}.$$

An idea of Kim et al (1998) is approximating $f(\zeta_t)$ by a mixture of K normal density,

$$g(\zeta_t) = \sum_{i=1}^{K} q_i f_N(\zeta_t | m_i, \omega_i^2),$$

[9]Conditional on h_t, ϵ_t follows a normal distribution $N(0, \gamma \exp(h_t/2))$. Instead of stochastic volatility, if we assume constant variance such that $\epsilon_t \sim N(0, h^{-1})$, G_t becomes $(h^{-1/2}, 0, \dots, 0)$.

where $f_N(\zeta_t|m_i, \omega_i^2)$ denotes the density function of a normal distribution with mean m_i and variance ω_i^2 (m_i, ω_i^2 and q_i are determined by Table 4 of their paper on the basis of $K =$ 7 components). Equivalently, this mixture density can be written in terms of a component indicator $s_t \in 1, 2, \ldots, K$ such that

$$(\zeta_t|s_t = i) \sim N(m_i, \omega_i^2), \text{ where } \Pr(s_t = i) = q_i.$$

In this setup, since ζ_t is expressed as a normal distribution, the model becomes a variant of Gaussian state space form expressed by (A.1) and (A.2). This enables us to apply the MCMC method used for estimating a time-varying parameter model above.

In sum, the following algorithm can be used to obtain a posterior density of an entire model (3)-(8): $p(\alpha_1, ..., \alpha_t, H, H_0, h_0, ..., h_t, \sigma_\eta^2, \sigma_{\eta 0}^2, c|y)$.

1. Initialise H, H_0, h, s, σ_η^2, $\sigma_{\eta 0}^2$ and c.
2. Sample α from $\alpha|y, H, H_0, h$.
3. Sample H and H_0 from $H|\alpha$ and $H_0|\alpha$, respectively.
4. Sample s from $s|\epsilon^*, h$.
5. Sample h from $h|\epsilon^*, s, \sigma_\eta^2, \sigma_{\eta 0}^2, c$.
6. Sample σ_η^2 and $\sigma_{\eta 0}^2$ from $\sigma_\eta^2|h$ and $\sigma_{\eta 0}^2|h$, respectively.
7. Sample c from $c|\epsilon^*, s, h$.
8. Go to 2.

This is essentially a combination of two Gibbs sampling schemes: one for a time-varying parameter model (Steps 2-3) and the other for a stochastic volatility model (Steps 4-7).

Some details of the main steps of the algorithm are as below:

Step 2: sampling from $p(\alpha_1, ..., \alpha_t|y, H, H_0, h)$

We use the simulation smoother developed by de Jong and Shephard (1995). Set $a_0 = 0$ and $P_1 = J_0 J_0'$. First, run the Kalman filtering as

$$\begin{aligned} e_t &= y_t - X_t\beta - Z_t a_t, && \text{(A.3)} \\ D_t &= Z_t P_t Z_t' + G_t G_t', && \text{(A.4)} \\ K_t &= (T_t P_t Z_t' + J_t G_t') D_t^{-1}, && \text{(A.5)} \\ a_{t+1} &= T_t a_t + K_t e_t, && \text{(A.6)} \\ P_{t+1} &= T_t P_t (T_t - K_t Z_t)' + J_t (J_t - K_t G_t)'. && \text{(A.7)} \end{aligned}$$

Then, conduct the smoothing in reverse time order ($t = T, T-1, ..., 1$) by setting $r_T = 0$ and $U_T = 0$.

$$C_t = F_t(I - G_t' D_t^{-1} G_t - [J_t - K_t G_t]' U_t [J_t - K_t G_t]) F_t', \quad \text{(A.8)}$$

$$\begin{aligned}
\xi_t &\sim N(0,\sigma^2 C_t), && \text{(A.9)}\\
V_t &= F_t(G_t' D_t^{-1} Z_t + [J_t - K_t G_t]' U_t [T_t - K_t Z_t]), && \text{(A.10)}\\
r_{t-1} &= Z_t' D_t^{-1} e_t + (T_t - K_t Z_t)' r_t - V_t' C_t^{-1} \xi_t, && \text{(A.11)}\\
U_{t-1} &= Z_t' D_t^{-1} Z_t + (T_t - K_t Z_t)' U_t (T_t - K_t Z_t) + V_t' C_t^{-1} V_t, && \text{(A.12)}\\
u_t &= F_t(G_t' D_t^{-1} e_t' + [J_t - K_t G_t]' r_t) + \xi_t. && \text{(A.13)}
\end{aligned}$$

For u_0, assume $G_0 = 0$ for the last equation. This implies

$$u_0 = F_0 J_0' r_0 + \xi_0,$$

where $\xi_0 \sim N(0, \sigma^2 C_0)$ and $C_0 = F_0(I + J_0' U_0 J_0) F_0'$.

It is proved that this algorithm yields u_t as a random draw from $p(u|y, \beta, h, H)$ where $u_t = F_t v_t$, and u_0 as a random draw from $p(u_0|y, \beta, h, H_0)$ where $u_0 = F_0 v_0$. Then, we can have α_t from equation (2) by using $\alpha_0 = 0$, $u_t = J_t v_t$ (by setting $F_t = J_t$) and $u_0 = J_0 v_0$ (also by setting $F_0 = J_0$).

Step 3: sampling from $p(H|y, \alpha)$ and $p(H_0|y, \alpha)$

Step 3.1: H

If we use a Wishart prior as

$$p(H) = f_W(H|\underline{\nu}_H, \underline{H}),$$

then the posterior is

$$H|\alpha_1, \ldots, \alpha_T \sim W(\overline{\nu}_H, \overline{H}), \tag{A.14}$$

where

$$\overline{\nu}_H = (T-1) + \underline{\nu}_H \text{ and } \overline{H} = \left[\underline{H}^{-1} + \sum_{t=1}^{T-1} (\alpha_{t+1} - \alpha_t)(\alpha_{t+1} - \alpha_t)' \right]^{-1}.$$

We set priors $\underline{\nu}_H = 1$ and $\underline{H}^{-1} = 0.001 \times I_p$.

Step 3.2: H_0

If we assume that the elements of u_0 are independent from one another, like

$$H_0^{-1} = \begin{bmatrix} \lambda_{00}^{-1} & 0 & 0 & . & 0 \\ 0 & \lambda_{01}^{-1} & 0 & . & 0 \\ 0 & 0 & \lambda_{02}^{-1} & . & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & . & \lambda_{0p}^{-1} \end{bmatrix},$$

then we can use a Gamma prior in place of the above Wishart prior for H^{-1} such that

$$p(\lambda_{0i}) = f_G(\lambda_{0i}|\underline{s}_{\lambda 0i}^{-2}, \underline{\nu}_{\lambda 0i}).$$

The corresponding posterior becomes

$$\lambda_{0i}|\alpha_1 \sim G(\overline{s}_{\lambda 0i}^{-2}, \overline{\nu}_{\lambda 0i}),$$

for $i = 0, \ldots, p$, where

$$\overline{\nu}_{\lambda 0i} = 1 + \underline{\nu}_{\lambda 0i} \text{ and } \overline{s}^2_{\lambda 0i} = \frac{\alpha_{i1}^2 + \underline{\nu}_{\lambda 0i}\underline{s}^2_{\lambda 0i}}{\overline{\nu}_{\lambda 0i}}.$$

Prior $\underline{\nu}_{\lambda 0i}$ is set to 1. In order to incorporate uncertainty of initial values, $\underline{s}^2_{\lambda 0i}$ is set as large as 10.

Step 4: sampling from $p(s|\epsilon^*, h, c)$

s_t can be sampled from the probability mass function

$$\Pr(s_t = i|\epsilon_t^*, h_t, c) \propto q_i f_N(\epsilon_t^*|c + h_t + m_i - 1.2704, \omega_i^2).$$

That is, for each time t, we sample κ_t from a uniform distribution and select i which satisfies

$$\sum_{j=1}^{i} \Pr(s_t = j|\epsilon_t^*, h_t) > \kappa_t > \sum_{j=1}^{i-1} \Pr(s_t = j|\epsilon_t^*, h_t),$$

where $\Pr(s_t = 0|\epsilon_t^*, h_t) = 0$. We will use the corresponding m_i and ω_i in the following step.

Step 5: sampling from $p(h|\epsilon^*, s, \sigma_\eta^2, \sigma_{\eta 0}^2, c)$

In terms of a general state space form of (A.1) and (A.2), we put $y_t = \epsilon_t^*$, $X_t\beta = c + m_i - 1.2704$, $Z_t = 1$, $\alpha_t = h_t$, $G_t = (\omega_i, 0)$, $T_t = 1$ and $J_t = (0, \sigma_\eta)$ to have a stochastic volatility part of the model (equations (5) and (6)) with an approximated mixed density. With this setup, we simply run a simulation smoother described in Step 2 above.

Step 6: sampling from $p(\sigma_\eta^2|\epsilon^*, h)$ and $p(\sigma_{\eta 0}^2|\epsilon^*, h)$

Once we define precision $H_\eta = \sigma_\eta^{-2}$ and $H_{\eta 0} = \sigma_{\eta 0}^{-2}$, the step becomes essentially same as Step 3.

Step 6.1: H_η

Instead of a Wishart prior, we use a conjugate Gamma prior as this is a univariate process.

$$p(H_\eta) = f_G(H_\eta|\underline{s}_\sigma^{-2}, \underline{\nu}_\sigma).$$

The corresponding posterior becomes

$$H_\eta|h \sim G(\overline{s}_\sigma^{-2}, \overline{\nu}_\sigma), \tag{A.15}$$

where

$$\overline{\nu}_\sigma = (T-1) + \underline{\nu}_\sigma \text{ and } \overline{s}^2_\sigma = \frac{\sum_{t=1}^{T-1}(h_{t+1} - h_t)^2 + \underline{\nu}_\sigma \underline{s}^2_\sigma}{\overline{\nu}_\sigma}.$$

We set priors as $\underline{\nu}_\sigma = 1$ and $\underline{s}^2_\sigma = 1$.

Step 6.2: $H_{\eta 0}$

Similar to the above, we use a Gamma prior for $H_{\eta 0}$

$$p(H_{\eta 0}) = f_G(H_{\eta 0}|\underline{s}_{\sigma_0}^{-2}, \underline{\nu}_{\sigma_0}).$$

The corresponding posterior becomes

$$H_{\eta 0}|h \sim G(\overline{s}_{\sigma_0}^{-2}, \overline{\nu}_{\sigma_0}),$$

where

$$\overline{\nu}_{\sigma_0} = 1 + \underline{\nu}_{\sigma_0} \text{ and } \overline{s}_{\sigma_0}^2 = \frac{h_1^2 + \underline{\nu}_{\sigma_0}\underline{s}_{\sigma_0}^2}{\overline{\nu}_{\sigma_0}}.$$

We set priors as $\underline{\nu}_{\sigma_0} = 1$ and $\underline{s}_{\sigma_0}^2 = 100$. The large value of $\underline{s}_{\sigma_0}^2$ corresponds to uncertainty of initial values.

Step 7: sampling from $p(c|\epsilon^*, s, h)$

We can use a normal prior

$$p(c|\epsilon^*, s, h) = f_N(\underline{c}, \underline{V}).$$

Then, the posterior is

$$c|\epsilon^*, s, h \sim N(\overline{c}, \overline{V}),$$

where

$$\overline{V} = \left(\underline{V}^{-1} + T\right)^{-1} \text{ and } \overline{c} = \overline{V}\left(\underline{V}^{-1}\underline{c} + \sum_{t=1}^{T}(\epsilon_t^* - h_t - m_i + 1.2704)\right).$$

Used priors are $\underline{c} = 1$ and $\underline{V} = 1$.

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