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The fragility of the Phillips curve: A bumpy ride in the frequency domain

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Abstract

We provide a robustness check of the US Phillips curve in the frequency domain. We design frequency-specific coefficients of correlation (FSCC) and regression (FSCR), based on our frequency-specific data extraction procedure. Being real-valued, signed and normalised, the FSCC is superior to traditional indicators such as coherence and cospectrum. Our FSCC and FSCR estimates suggest that the Phillips tradeoffs vary greatly across frequencies, with frequent sign reversals. They seem to be stable in higher frequencies, but unstable in low and medium frequencies, and they are sensitive to the level and boundaries of frequency aggregation, to the way data are processed prior to analysis (eg detrending) and to the type of variables used. In this sense, the Phillips curves are fragile. The impact of potential cross-frequency model inconsistency on model estimation using conventional time domain methods needs careful scrutiny.

Keywords: Phillips curve, inflation-output tradeoff, filtering, frequency-specific coefficient of correlation, spectral regression.

JEL Classification: C19, E30

1 Introduction[1]

Ever since the publication of Phillips (1958),[2] Lipsey (1960) and Samuelson and Solow (1960), the Phillips curve has been one of the most studied and disputed empirical relationship in economics. Early versions of the Phillips curve postulated that in the long run, the rate of wage inflation depended on the unemployment rate and on lagged rates of price inflation. The relationship held up remarkably well until the 1960s, and it quickly became an essential part of macroeconomic modelling and policy analysis of the time,[3] as well as an important component of the Neoclassical Synthesis championed by Paul Samuelson.

The ascension of monetarism in the 1960s posed a direct challenge to the Neoclassical Synthesis, questioning the long-term validity and structural stability of the Phillips curve. Friedman (1968) proposed the idea of a "natural rate of unemployment", at which the rate of price inflation neither increases nor decreases. The corresponding rate of inflation became known as the non-accelerating inflation rate of unemployment (NAIRU). The concept of NAIRU implies the non-existence of exploitable long-run inflation-unemployment tradeoff for policymakers. In fact, increased perception of inflation persistence and the breakdown of the Phillips tradeoff in the early 1970s cast serious doubt on the theoretical and empirical validity of the Phillips curve.

Lucas (1972b, 1973, 1975) made an extensive analysis and critique of the nature of the empirical inflation-output tradeoff often exploited by policymakers. Lucas (1972b) demonstrated the close relationship between the rational expectations and the natural rate hypotheses. When correctly formulated, the latter has no testable implications for the distributed-lag coefficients in a Phillips curve. Furthermore, under rational expectations, permanent changes in inflation lead to changes in both the policy parameters and the structure of econometric models, and therefore a new policy cannot be evaluated by testing restrictions on those implied by the previous, now irrelevant, policy. This is the celebrated Lucas-Sargent Critique, articulated in Sargent (1971) and Lucas (1975).

One implication of the Critique is that what has not appeared in the data may not be expected, and effects of unannounced policy changes cannot be predicted using the existing model. In general, one cannot test long-run neutrality propositions using reduced-form models. If exogenous variables were stationary, data generated in the rational expectations models do not contain permanent or sustained changes for direct tests of the long run. In fact, the Phillips curve broke down soon after the publication of Gordon (1970).

Friedman's "discovery" of the NAIRU and the Lucas-Sargent critique prompted economists to change the basic specification of the Phillips curve: expectations were incorporated along with surrogates for various supply shocks, such as the oil prices. The so-called NAIRU Phillips curve takes the price

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[2]The first paper describing a negative correlation between price inflation and unemployment was probably written by Irving Fisher (1926), later reprinted in the Journal of Political Economy (1973).

[3]The Phillips long-run inflation-output tradeoff was first incorporated as a wage-price sector in the 1955 version of the Klein-Goldberger model.

acceleration rate as the dependent variable rather than the inflation rate. The new specification worked well and remained stable throughout the 1970s and early 1980s. However, as documented by Stock and Watson (1999) and Atkeson and Ohanian (2001), the NAIRU Phillips curve shifted and became substantially flatter after 1984. As many of the structural characteristics of the US economy have changed since the 1980s,[4] the NAIRU specification itself became unstable.

Decades after Fisher (1926) and Phillips (1958), the empirical validity of the Phillips curve continues to be at the heart of current macroeconomic debate. In the words of Lucas and Sargent (1979), the Phillips curve was an "econometric failure on a grand scale". However, a recent strand of literature seems to have been successful in reviving the fortunes of the Phillips curve, suggesting an important role for the curve in short-term macroeconomic forecasting, and challenging the empirical relevance of the Lucas-Sargent critique. In fact, Blinder (1997) claimed that the reliability of the Phillips curve was the "clean little secret" of modern macroeconomics. Mankiw (2000) claimed that the inflation-unemployment tradeoff remained a basic "inexorable" fact in economics.

King and Watson (1994) distinguished the traditional Keynesian, rational-expectations monetarist and real business-cycle identification schemes and found that the short-term[5] US Phillips curve was remarkably stable in the post-war years, and the curve at every horizon is less stable than it is over time. Fuhrer (1995) argued that agents may not form expectations "rationally", such that the Lucas-Sargent critique loses empirical content. Moreover, "even if agents' expectations react to changes in monetary policy, we cannot know *a priori* whether the historical changes in monetary policy have been large enough to cause empirically significant shifts in expectations in the Phillips curve." In a stationary environment, the Lucas-Sargent critique, just as the Phillips tradeoff, cannot be validated empirically in reduced-form models. Fuhrer concluded that the Phillips curve exhibited remarkable stability over the 1960-93 period, "as an empirical matter, the Lucas critique does not apply to the Phillips curve."

Stock and Watson (1999) investigated the short-term forecasting capability of alternative Phillips curve specifications. Of all methods considered, the Phillips curve produced the most reliable and accurate short-run forecasts of US price inflation.[6] Brayton at al (1999) found that the Phillips curve with capacity utilisation rather than the unemployment rate had been stable and yielded more accurate inflation forecasts. Including an error-correction mechanism involving the markup of prices over trend unit labour costs provides even better results.

Almost all existing time domain regression and correlation analyses of the Phillips curve use *aggregate* time series data, largely disregard any possible frequency-wise variations. Results from these studies are potentially fragile, as correlation and regression coefficients of different signs and magnitudes are aggregated over the whole frequency range. Regressions with filtered data and band spectrum regressions lessen the problem but suffer from the same drawback, as the aggregation proceeds in subjective and arbitrarily pre-specified frequency bands. It is known that aggregate macroeconomic relationships are often frequency dependent. However, frequency domain analyses of macroeconomic

[4]For example, Clarida et al (2000) discussed significant changes in monetary policy in the early 1980s. After 1984, business cycle fluctuations attenuated, and inflation became less volatile and less persistent.

[5]They define "short term" as less than one year.

[6]However, Atkeson and Ohanian (2001) came to the contrary conclusion that the NAIRU Phillips curve had no value for short-term inflation forecasting.

relationships are uncommon, and the empirical validity of the Phillips curve in the frequency domain has not yet been investigated.

In this paper, we design frequency domain time series techniques to examine the validity of the Phillips relations across different frequencies and frequency bands, using available US data. We first apply the Hodrick-Prescott (1980) filter and the Baxter-King (1999) band-pass filter to time series to study the Phillips tradeoffs in conventionally defined frequency bands. Using filtered data, we find that the tradeoff holds only in the medium and higher frequency bands, where the cumulated responses (i.e., "long-run" slopes) in the NAIRU Phillips models are jointly significant and have the correct sign. For some price indices, such as the CPI series, and certain measures of real activity, the Phillips tradeoff turns out to be more solid. The results contradict the conventional view that there is no long-run but a strong short-run Phillips tradeoff.

We then propose a simple procedure to retrieve frequency-specific data from covariance stationary time series, using Fourier and inverse Fourier transforms. Based on this data extraction procedure, we design a *frequency-specific coefficient of correlation* (FSCC), which is superior to traditional frequency domain indicators such as coherence and cospectrum, because it provides a real-valued, normalised and signed measure of the strength of multiple correlations. Unlike coherence, the FSCC delivers the sign of frequency-wise correlations between two or more series. Compared to the cospectrum, the FSCC takes into account both the real and imaginary components. It is standardised and takes values only in the $[-1, 1]$ range, therefore delivering a clear indication of the strength of correlation that is independent of the scale of data. Similarly, we devise a *frequency-specific coefficient of regression* (FSCR), for which conventional asymptotic theory remain valid, since our data retrieval procedure is a linear transformation of original data.

We compute estimates of frequency-specific cospectra, coherences, real coherences, coefficients of correlation and regression for a large number of different Phillips nominal and real variables commonly used in the literature. Our results suggest that for most cases, the Phillips relations vary both in sign and in strength across the whole frequency range, with frequent sign reversals. A small change in the level and boundaries of frequency aggregation may cause the aggregated signs of correlation and regression to reverse. Moreover, the Phillips relations seem to be fairly stable in the high frequency range (less than 6 quarters), but is much less stable in the business cycle frequency range, and quite unstable in the low frequency range (over 32 quarters). Many of the 95% confidence intervals for the Phillips spectral correlation and regression coefficients include zero. Furthermore, the Phillips relations are sensitive to the type of data used and to the way data are processed prior to analysis (i.e., how they are detrended). Summing up, common time and frequency-domain analysis of the Phillips curve can be misleading, and the Phillips relations are fragile in this sense.

In Section 2, we describe a new methodological approach to empirically measuring the Phillips tradeoff in the frequency domain. We propose a simple procedure to retrieve frequency-specific data from covariance stationary time series, using Fourier and inverse Fourier transforms. Based on this procedure, we design a frequency-specific coefficient of correlation that provides a real-valued, normalised and signed measure of the strength of multiple correlations. In Section 3, we present empirical tests of the NAIRU Phillips regression models using conventionally filtered data, and estimates of frequency-specific coefficients of correlation and regression for the Phillips relations.

Section 4 concludes. Technical supplements, tables and figures are collected in Section 5.

2 Frequency domain analysis of the Phillips curve

Spectral analysis of time series is appealing, because covariance stationary processes can be uniquely decomposed into mutually uncorrelated components, each associated with a particular data frequency or frequency band. It has been a time-honoured practice in economic analysis to equate low-frequency characteristics of multiple time series to their average long-run behaviour.[7] While high-frequency seasonal or irregular components are of particular importance in finance,[8] medium-frequency components are of greater interest to business cycle analysts. Differentiating the inflation-unemployment tradeoff across frequency bands helps us to understand the characteristics of different frequency components of the Phillips correlation.[9]

Although we do not always expect to uncover different data generation processes across different frequency bands,[10] we may still want to examine whether one single model fits all frequencies. Engle's (1974) remark was particularly lucid: "... there is little discussion of whether the same model applies to all frequencies. It may be too much to ask of a model that it explain both slow and rapid shifts in the variables, or both seasonal and non-seasonal behaviour. It is at least reasonable to test the hypothesis that the same model applies at various frequencies." Possible frequency-wise variations of any particular relationship imply that pure time domain analysis, which effectively aggregates or averages the relationship across the whole frequency range, may be misleading, by implicitly assuming that the relationship hold over the whole frequency range.

There are two general approaches to frequency domain time series analysis. The first relies on carefully designed filters to extract data corresponding to different frequency bands. Statistical analysis can then be conducted on the basis of filtered data. Filtering methods have a long tradition in economic analysis, being now part of the standard toolkit of modern quantitative analysis of business cycle fluctuations. Examples of this include the linear detrending (LD) filter, the ARIMA filter,[11] the first difference (FD) filter, the Lucas (1980) exponential smoothing filter, the Baxter-King (1999) band-pass (BP) filter, and the Hodrick-Prescott (HP, 1980, 1997) filter, which was popularised by Kydland and Prescott (1982) in their seminal work on a calibrated Real Business Cycle (RBC) model. However, due to finite data length and consequent truncation of data, these filters are only approximations to the ideal filters, and *filter leakage*, *compression* and *exacerbation* are inevitable.[12]

[7]For instance, Summers (1983) used Engle's (1974) band-spectrum regression estimator to examine the Fisher equation as a proposition valid only at low frequencies.

[8]Volatility plays a key role in the analysis of financial time series. See, for example, Engle (2000).

[9]Using a low-pass filter (isolating periodicities over eight years) and the Baxter-King band-pass filter (isolating periodicities between 18 months and eight years), King and Watson (1994) documented a pronounced and stable (over time) negative correlation between inflation and unemployment at business-cycle frequencies, but very unstable lower frequency comovements.

[10]For example, one can impute most of the low-frequency characteristics of macroeconomic time series to a "growth mechanism" and the business-cycle movements to a "cycle mechanism". However, one cannot assume that the two mechanisms are economically or statistically independent. The prevalent use of linear filters to isolate the business cycles frequency in order to study economic fluctuations is subject to this caveat.

[11]See, for example, Blanchard and Fischer (1989), p 9.

[12]See Appendix 5.2 and Baxter and King (1999) for details.

Furthermore, possible variations in multivariate correlations and regressions inside the pre-specified frequency bands will continue to be masked: averaging now takes places in the frequency band rather than across the whole frequency range.

The second approach conducts time series analysis directly in the frequency domain. Early applications in economics focussed on simple spectral indicators, such as gain, phase, spectral and cospectral densities, and coherence. Examples are Granger and Morgerstern (1963), Granger (1966), Granger and Rees (1968) and Granger (1969). Recent applications include Pakko (2000), Estrella (2003), Cogley and Sargent (2001, 2004) and Cogley and Sbordone (2004). More sophisticated spectral regression analysis was pioneered by Hannan (1963). Band spectrum regressions were introduced to economics by Engle (1974, 1978, 1980) and Harvey (1978). These methods allow one to focus on one or more frequency bands of interest, and permit a nonparametric treatment of regression errors. Phillips (1991) demonstrated how spectral regression techniques can be applied to integrated time series to obtain asymptotically median unbiased estimates of cointegrating coefficients. Corbae et al (1994) used the Phillips (1991) procedure to examine the permanent income hypothesis. Diebold et al (1998) propose a measure of divergence between model and data spectra both for the full spectrum and for particular frequency bands to evaluate the goodness-of-fit of calibrated models. Berkowitz (2001) suggested a generalised spectral estimator which can be frequency-specific. Macroeconomic applications of band spectrum regressions include Thoma (1994) and Tan and Ashley (1997, 1998, 1999).

We use HP and BP filters to decompose individual stationary time series into short- (seasonal and irregular), medium- (business-cycle) and long-run components. We run least squares regressions with filtered series and examine the validity of the Phillips tradeoff in different frequency bands. Time domain regressions using filtered data are similar, and in ideal settings, equivalent to band spectrum regressions, but have the drawback of allowing filter leakage, exacerbation and compression effects to pass through into regression analysis.[13] Furthermore, *a priori* definitions of frequency bands are subjective and arbitrary, leaving it open the possibility of averaging different frequency-specific relations in a band.

To demonstrate potential shortcomings of conventional time domain analysis and analysis based on pre-defined frequency bands, and to improve upon the filtering approach, we take a direct approach and focus on the frequency-wise behaviour of the bivariate Phillips correlations and regressions across the entire frequency range. We propose a simple procedure to retrieve frequency-specific data from covariance stationary time series using Fourier and inverse Fourier transforms. The data extraction procedure can be extended to select any specific frequency band. Analysis based on transformed data will not be affected by the filter leakage, exacerbation and compression effects caused by data truncation necessary in common filters.

Based on this data extraction procedure, we design a frequency-specific coefficient of correlation, which is superior to traditional indicators, such as coherence and cospectrum, by providing a real-valued, normalised and signed measure of the strength of multiple correlations. Unlike coherence, the FSCC makes it transparent the sign of frequency-wise correlations between any two series. Compared to cospectrum, the FSCC is standardised and takes values only in the $[-1, 1]$ range, therefore

[13]The mechanics of the Lucas, FD, HP and BP filters, are illustrated in the Appendix 5.2.

delivering a clear indication of the strength of correlation which is independent of the scale of data. We also take advantage of the data extraction procedure and devise a frequency-specific coefficient of regression.

2.1 A frequency-specific data extraction procedure

Consider a time series vector $x = [x_1, x_2, \ldots, x_T]^T$. For $s = 1, \ldots, T$, define the fundamental frequencies as $\omega_s = 2\pi s/T$. The discrete Fourier transform of x at frequency ω_s is

$$w_s x = T^{-1/2} \sum_{t=1}^{T} x_t e^{(t-1)i\omega_s} \tag{1}$$

where

$$w_s = T^{-1/2} \begin{bmatrix} 1 & e^{i\omega_s} & e^{2i\omega_s} & \cdots & e^{(T-1)i\omega_s} \end{bmatrix}$$

Define

$$W = \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_{T-1} \end{bmatrix} \tag{2}$$

Now, W is a unitary matrix such that $W^*W = WW^* = I$, where $*$ indicates the Hermitian conjugate (i.e., transpose of the complex conjugate). Then $\tilde{x} = Wx$ is the vector of discrete Fourier transform of time series x at all fundamental frequencies ω_s, $s = 0, \ldots, T-1$.

Define A_s as a $T \times T$ selection matrix which selects the s-th element or row from any data vector or matrix, respectively. It has 1 as the s,s-th element and zeros elsewhere.[14] The data vector of the discrete Fourier transform of time series x at the s-th frequency is

$$A_s\tilde{x} = A_s W x \tag{3}$$

There are now T data vectors $A_s\tilde{x}$, extracted from the original time series x, each is of length T, and corresponds to a distinct frequency ω_s. All but the s-th elements of the s-th data vector $A_s\tilde{x}$ are zero. We then use inverse Fourier transform to convert the complex data vector $A_s\tilde{x}$ back into the time domain. Write the frequency-ω_s inverse Fourier transform of the time series x as

$$\hat{x}(\omega_s) = L_s x = W^* A_s W x \tag{4}$$

where $L_s \doteq W^* A_s W$ is a linear operator. Using Fourier and inverse Fourier transforms and the selection matrix A_s, from any data vector x, we can extract T time series $\hat{x}(\omega_s) = [x_1(\omega_s), x_2(\omega_s), \ldots, x_T(\omega_s)]^T$, each corresponding to a specific frequency ω_s, where $s = 1, \ldots, T$. Based on these frequency-specific data, we can then design frequency-wise correlation and regression coefficients in a conventional way.

[14] To select a frequency band $[\omega_s, \omega_t]$, one can simply let the s-th to t-th diagonal elements of A all be one.

For any time series, the data extraction procedure retrieves frequency-specific data and transforms them back into the time domain. Given that the procedure is linear in nature, all conventional time domain techniques and the related finite sample and asymptotic apparatus remain valid and can be applied to the extracted time series without any modification. Furthermore, by an appropriate change in the selection matrix A_S, one can also extract band-specific data for any frequency bands. This method is flexible, and it is superior to conventional data filtering methods by avoiding the unavoidable negative effects of filter leakage, compression and exacerbation.

2.2 Correlation analysis

For bivariate[15] stochastic processes $z_t = [\begin{array}{cc} x_t & y_t \end{array}]^T$, which are assumed to be jointly weakly stationary with continuous spectra, we write the corresponding spectral density matrix as

$$f_{zz}(\omega) = \begin{bmatrix} f_{xx}(\omega) & f_{xy}(\omega) \\ f_{yx}(\omega) & f_{yy}(\omega) \end{bmatrix} \tag{5}$$

where the spectral densities $f_{xx}(\omega)$, $f_{yy}(\omega)$ and the cross-spectral density $f_{xy}(\omega)$ determine the relationship between x_t and y_t at frequency ω. In Cartesian form, the cross-spectral density $f_{xy}(\omega)$ can be written as

$$f_{xy}(\omega) = c_{xy}(\omega) - iq_{xy}(\omega) \tag{6}$$

where $c_{xy}(\omega)$ and $q_{xy}(\omega)$ are real-valued functions known as cospectrum (or cospectral density) and quadspectrum (or quadrature spectral density), respectively. The cospectrum $c_{xy}(\omega)$ represents the covariance between coefficients of the in-phase components of two time series, while the quadspectrum $q_{xy}(\omega)$ represents the covariance between coefficients of the out-of-phase components. Cospectrum estimation is equivalent to studying the off-diagonal elements of the variance-covariance matrix between two time series, which are uniquely related to cospectra by Fourier and inverse Fourier transforms.

In polar form,

$$f_{xy}(\omega) = |f_{xy}(\omega)| \exp(i\varphi_{xy}(\omega)) \tag{7}$$

where

$$\varphi_{xy}(\omega) = -\arctan\left(\frac{q_{xy}(\omega)}{c_{xy}(\omega)}\right) \tag{8}$$

The phase $\varphi_{xy}(\omega)$ measures the average phase lead of x_t over y_t, and $\varphi_{xy}(\omega)/\omega$ indicates the extent of time lag. The gain $G_{xy}(\omega)$ is defined as

$$G_{xy}(\omega) = \frac{|f_{xy}(\omega)|}{f_x(\omega)} \tag{9}$$

which is a standardised version of the regression coefficient of y on x at frequency ω. A small $G_{xy}(\omega)$ indicates that x has little effect on y at frequency ω.

[15] All concepts described in Sections 2.2 and 2.3 for bivariate time series can be easily generalised to multivariate stochastic processes, where exogenous variables can be introduced.

Define the complex coherence $ccoh_{xy}(\omega)$ at frequency ω as

$$ccoh_{xy}(\omega) = \frac{f_{xy}(\omega)}{[f_{xx}(\omega) f_{yy}(\omega)]^{1/2}} \tag{10}$$

The complex coherence $ccoh_{xy}$ is the frequency domain analogue of the time domain coefficient of correlation, but since f_{xy} and $ccoh_{xy}$ are complex, it is hard to interpret this indicator in terms of the overall strength of linear correlation between x and y. Real coherence $rcoh_{xy}$, which we define as the real part of $ccoh_{xy}$, is just the cospectrum c_{xy} standardised by the square root of the product of f_{xx} and f_{xy}. It is the coefficient of correlation between coefficients of the in-phase components of two time series x and y. However, a true frequency-specific correlation coefficient needs to account for both the real and complex parts of the complex coherence $ccoh_{xy}$.

One obvious solution is the coefficient of coherence[16] of x_t over y_t at frequency ω, defined as $cohe_{xy}(\omega) = |ccoh_{xy}(\omega)|$. But although it delivers a real number, it fails to reveal the sign of linear correlation at frequency ω. The coherency $coh_{xy}(\omega)$ of x_t over y_t at frequency ω is

$$coh_{xy}(\omega) = cohe_{xy}(\omega)^2 = |ccoh_{xy}(\omega)|^2 \tag{11}$$

Analogous to the coefficient of determination (i.e. R^2) in the time domain, the coherency coh_{xy} is the standardised modulus of cross spectral density. It measures the strength of linear association between two or more variables of interest across frequencies. By Schwarz Inequality, $\forall\omega$, $coh_{xy}(\omega) \in [0, 1]$. At frequencies for which $f_{xx}(\omega) f_{yy}(\omega) = 0$, we define $coh_{xy}(\omega) = 0$. If $coh_{xy}(\omega) = 0$, the two series $x(t)$ and $y(t)$ are completely unrelated at frequency ω. If $coh_{xy}(\omega) = 1$, then one series is an exactly linearly filtered version of the other at frequency ω. In general, coh_{xy} varies with frequency ω, indicating the changing pattern of linear association across frequencies. Regions of high coherence are of particular interest.

Focussing on estimated cospectra $\hat{c}_{xy}(\omega)$, Pakko (2000) found important differences in the low- and higher-frequency components of the output-inflation tradeoff, and gave examples of how cospectra can be used for model evaluation. However, without appropriate standardisation, the magnitude of cospectra estimates depend on the scale of data and they do not provide an appropriate measure of correlative strength. Indeed they may produce a misleading picture of cross-frequency variations in bivariate correlations when relationships are put to comparison.

On the contrary, estimates of the coefficient of coherency $\widehat{coh}_{xy}$ are scale-independent, being normalised to fall in the $[0, 1]$ interval. By definition, it has the disadvantage of taking only non-negative values, therefore failing to differentiate positive and negative correlations between two variables at different frequencies. Aggregating a bivariate relationship across frequencies, when the sign of correlation changes from frequency to frequency, may lead to wrong conclusions as positive and negative correlations cancel each other out and leave an aggregated value of correlation close to zero. Indeed, time domain methods, which aggregate over the entire frequency domain, and methods based on arbitrarily pre-defined frequency bands that aggregate over each band, are not robust when cross-

[16] Extending the conceptual construct of bivariate coherence to multiple time series, we have multiple and partial coherences.

frequency variations are large, and when sign reversals are frequent.

What we need is a frequency-domain analogue of the time-domain coefficient of correlation, corresponding either to a specific frequency ω, or to a frequency band $[\omega_l, \omega_m]$, where $0 \leq \omega_l \leq \omega_m \leq 2\pi$. One natural choice would be the complex coherence $ccoh_{xy}$. But although $ccoh_{xy}$ is signed and normalised, since in general f_{xy} is complex-valued, so is $ccoh_{xy}$. There is no easy way to graphically illustrate, and to interpret, the interplay between the real and complex parts of the complex coherence, even if we are able to represent the indicator in a three-dimensional diagramme. Our solution is to take advantage of the proposed simple frequency-domain data recovery procedure, and we define $\rho(\omega)$, the *frequency-specific coefficient of correlation* (FSCC) at frequency ω, as follows

$$\rho(\omega) = \frac{Cov(\hat{x}(\omega), \hat{y}(\omega))}{\sqrt{Var(\hat{x}(\omega))}\sqrt{Var(\hat{y}(\omega))}} \tag{12}$$

where $\hat{x}(\omega)$ and $\hat{y}(\omega)$ are frequency-specific time series extracted from data vectors x and y, and $Cov(\bullet)$ and $Var(\bullet)$ stand for covariance and variance, respectively. The confidence interval for $\rho(\omega)$ can be computed in the conventional way.

The frequency-specific coefficient of correlation is normalised to take values in the $[-1, 1]$ interval. Unlike cospectral density c_{xy}, the FSCC ρ is free from data scale, hence a true measure of the strength of frequency-ω correlation between x and y. Comparing to coherence coh_{xy}, the FSCC ρ signs the direction of correlation existing in the data. The FSCC estimate is a clear improvement upon cospectrum and coherence estimates, and we use it as the main indicator of strength of bivariate correlation for Phillips relations. When the distribution theories for the cospectral density c_{xy} and the coherence coh_{xy} are complicated, p-values and confidence intervals for the FSCC estimates $\hat{\rho}$'s can be provided in the usual way. In fact, these are often supplied automatically in an econometric or statistical software package.

2.3 Regression analysis

Early applications of spectral regression methods in economics include Engle (1974, 1978), Lucas (1980) and Summers (1983), which rely on low-frequency aspects of macroeconomic time series to test important economic hypotheses concerning the long run. Lucas (1980) designed a filter to single out the very low frequency components of two time series and then studied their bivariate correlations using graphical methods.[17] Engle (1974, 1978) provided good examples of band spectrum regressions and a statistical test of the null hypothesis that a particular data generation process held across different frequency bands. We use HP and BP-filtered data to run *approximate* band spectrum regressions and the corresponding approximate band spectrum causality tests. We then use our data retrieval procedure to run frequency-specific regressions.

[17] Lucas' two-step method is similar to fitting a least squares regression line through filtered time series data. Engle (1978) and McCallum (1984) proved that this method combined with an *ideal* low-pass filter is equivalent to performing a band-spectrum regression with frequencies restricted to a small band around $\omega = 0$, or computing an approximate zero-frequency regression estimate $\hat{\beta}(0)$. This, in a bivariate system, is equal to the sum of coefficients estimated in a distributed-lag regression. Therefore, "long-run relationship" at zero frequency is interpreted as the sum of distributed-lag coefficients, and hypothesis tests about long run are based on testing the sum being equated to a particular value specified by theory.

2.3.1 Regressions with filtered data

Regressions with filtered data are based on the following NAIRU Phillips curve specification

$$\Delta\pi_t = \alpha + A(L)\Delta\pi_{t-1} + B(L)\left(y_t - \bar{y}_t\right) + C(L)x_t + \varepsilon_t \tag{13}$$

or equivalently,

$$\pi_t = \alpha + A'\left(L\right)\pi_{t-1} + B(L)\left(y_t - \bar{y}_t\right) + C(L)x_t + \varepsilon_t \tag{14}$$

where $A'\left(1\right) = 1$. y_t can be the unemployment rate, or growth in real output or in any other measure of real activity, such as the Stock-Watson Experimental Leading Index, and $\bar{y}_t$ is the (possibly time-varying) trend level of y_t.[18] Hence $\left(y_t - \bar{y}_t\right)$ is the unemployment or output gap. In the case of the unemployment rate or real output, $\bar{y}_t$ represents either the NAIRU or the non-accelerating inflation level of output (NAIRO), respectively.[19] The exogenous variables x_t reflect observable supply shocks to the economy.

The current response or short-run slope and the accumulated response or long-run slope of the Phillips curve are, respectively

$$S_{SR} = B_0 \tag{15a}$$

$$S_{LR} = B\left(1\right)/\left[1 - A\left(1\right)\right] \tag{15b}$$

We consider a maximum of six lags for $a\left(L\right)$, $B\left(L\right)$ and $C\left(L\right)$ in Equation (13), therefore a total of 216 specifications are considered for each pair of Phillips variables. Model selection criteria (Akaike Information, Schwarz Bayesian Information and Final Prediction Error Criterion) are used to choose the adequate filtered models, for which bi-directional Granger (1969) and Sims (1972) tests are conducted to examine the empirical validity of the Phillips curves in conventional frequency bands.

2.3.2 Frequency-specific spectral regression

Consider a simple model for two time series $y = \left[y_1, y_2, \ldots, y_T\right]^T$ and $x = \left[x_1, x_2, \ldots, x_T\right]^T$

$$y = \beta x + \varepsilon \tag{16}$$

where $\varepsilon \sim iid\left(\mathbf{0}, \sigma^2 I\right)$ and x is uncorrelated with ε.[20] The periodogram of x and the cross-periodogram between x and y at frequency ω_s are, respectively

$$I_x\left(\omega_s\right) = \left|w_s x\right|^2 \tag{17a}$$

$$I_{xy}\left(\omega_s\right) = \left(w_s x\right)^* \left(w_s y\right) \tag{17b}$$

[18] Contrary to what has often been claimed, the HP and BP filters may not render integrated processes stationary. We use an $I\left(0\right)$ specification here. Lags of $\Delta\pi_t$ are included to reflect expectations of the acceleration rate of inflation.

[19] We use three measures of NAIRU or NAIRO: (1) constant means of the level and growth rate of a real variable; (2) linear trends in the level and growth rate of a real variable; (3) the BLS Real Potential GDP series.

[20] These assumptions are strong for the issues we intend to clarify. However, a relaxation of these assumptions is straightforward and should not change our basic results.

where w_s is defined as in Section 2.1.

The s-th frequency spectral regression is

$$A_s \tilde{y} = \beta_s A_s \tilde{x} + A_s \tilde{\varepsilon} \tag{18}$$

where $\tilde{q} = Wq$, for any q, and W is defined as in Equation (2). The s-th frequency spectral regression coefficient is

$$\begin{aligned} \tilde{\beta}_s &= \left(\tilde{x}^* A_s \tilde{x}\right)^{-1} \tilde{x}^* A_s \tilde{y} \\ &= I_x\left(\omega_s\right)^{-1} I_{xy}\left(\omega_s\right) \end{aligned} \tag{19}$$

Since the unsmoothed periodogram I_x and the cross-periodogram I_{xy} are not consistent estimators of the spectral and cross-spectral densities, and we are interested in frequency-specific regression coefficients that do not involve averaging over a frequency band to obtain consistent estimates of the sums of periodogram and cross-periodogram ordinates, we may instead use smoothed spectral estimates to estimate β_s:[21]

$$\hat{\beta}_s = \hat{f}_x\left(\omega_s\right)^{-1} \hat{f}_{xy}\left(\omega_s\right) \tag{20}$$

In general, the estimator $\hat{\beta}_s$ will be complex-valued. To obtain a real-valued estimate, one can take the real part of $\hat{\beta}_s$, but typically, both the real and complex parts of $\hat{\beta}_s$ matter. Or we may simply use the gain, i.e., the modulus $\left|\hat{\beta}_s\right|$ of $\hat{\beta}_s$, which has the drawback of not allowing us to discern the sign of spectral regressions. We take advantage of the proposed data extraction procedure and run OLS regressions with frequency-specific data. Since the Fourier transform and inverse Fourier transform are linear operations, conventional asymptotic theory continues to apply, and the confidence interval for $\hat{\beta}_s$ can be computed in the usual way (except at the zero frequency). Write the inverse Fourier transform of Equation 18 as

$$L_s y = \beta_s L_s x + L_s \varepsilon$$

where again $L_s \doteq W^* A_s W$. Simple OLS spectral regressions lead to the *frequency-specific coefficient of regression* (FSCR) $\hat{\beta}_s$ corresponding to frequency ω_s

$$\begin{aligned} \tilde{\beta}_s &= \left(x^T L_s^T L_s x\right)^{-1} x^T L_s^T L_s y \\ &= \left(\hat{x}_s^T \hat{x}_s\right)^{-1} \hat{x}_s \hat{y}_s \end{aligned} \tag{21}$$

The great advantage of the data extraction procedure is that it is linear in nature, so that all inferential apparatus in the conventional OLS regression theory can be used as usual.

[21] In a more general distributed lag model

$$y = A(L)\, x + \varepsilon$$

where x and ε are uncorrelated covariance stationary processes,

$$\begin{aligned} f_{xy}(\omega) &= A\left(z^{-1}\right) f_x(\omega) \\ f_y(\omega) &= |A(z)|^2 f_x(\omega) + f_\varepsilon(\omega) \end{aligned}$$

The ratio f_{xy}/f_x is known as Hannan's inefficient estimate of the distributed lag coefficients $A(L)$.

3 Empirical results[22]

Given the large number of variables involved in the analysis, and some similarities in results across certain Phillips relations, in the next two sections, we will be selective and report only the results for pairs of Phillips variables that are of substantial interest or are distinctive.

3.1 Phillips regressions with filtered data

Regression analyses with HP and BP-filtered data are carried out for two separate groups of regressions and the estimation results are summarised in Tables 2-4. We only report results for regressions of nominal variables against gaps (in terms of demeaned values) in output growth or unemployment.

In the first group of regressions, we use acceleration rates of core price indices PCEPILFE and CPILFESL as dependent variables. With PCEPILFE as the regressand, most of the long slopes for HP-filtered trend series and BP-filtered cyclical series have correct signs,[23] while most of the corresponding short slopes have the wrong sign. With CPILFESL as the regressand, almost all long slopes for HP-filtered cyclical series have correct signs. This is not the case with HP-filtered trend series and BP-filtered cyclical series. The validity of the NAIRU Phillips curve is sensitive to whether PCEPILFE or CPILFESL is taken as the regressand, and depends on which filtered series are used.

In the second group of regressions,[24] we use CPIENGSL, CPIUFDSL and POILBRE as control variables and regress filtered series of acceleration rates based on comprehensive price indices (GDPDEF, GDPCTPI, PCECTPI, CPIAUCSL, PPIACO, HCNF and WASCUR) against expectations of price acceleration, current and lagged real variables, and current and lagged controls. The control variables are used to account for the impact on the Phillips tradeoff of exogenous supply shocks, as reflected in food and energy prices. Results from regressions using energy or oil prices as the control variable largely conform to those obtained with filtered core price series. In most cases, signs of long slopes in regressions with HP-filtered trend series and BP-filtered cyclical series are incorrect. For regressions with HP-filtered trend series, most longs slopes have correct signs.

Two key results emerge from the two groups of regressions. First, for almost all filtered series, Phillips regressions based on the NAIRU specification (13) yield insignificant current responses (termed "Short Slope" in Tables 2-4) and highly significant accumulated responses (long slope). This indicates that the Phillips curve might only be valid when one focusses on cumulated responses. Secondly, for both groups, the Durbin-Watson (DW) statistics are very low for models with HP-filtered trend series and BP-filtered cyclical series, suggesting possible model mis-specification. On the contrary, regressions with HP-filtered cyclical series seem to be correctly specified, with a DW test statistic value close to 2. Conflicting results from regressions with BP-filtered and HP-filtered cyclical series suggest that studies of the type are sensitive to the definition of frequency bands, and cast serious doubt on the validity of the Phillips curve.

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[22] Data used are described and the Phillips nominal and real variables defined in Appendix 5.3.

[23] UNRATE is an important exception. In this context, a sign is deemed correct if it is positive for measures of real activity and negative for unemployment rate.

[24] To economise space, tables reporting these results can be obtained from the author's website.

The Phillips tradeoff seems to be valid only in medium and high frequency bands (with HP-filtered cyclical series), in terms of accumulated responses. The tradeoff fails to materialise in the Baxter-King business-cycle frequency band. This might be a reflection of the sensitivity of the Phillips tradeoff to slightly different levels and cutoffs of frequency aggregation, a symptom of fragility of the Phillips curve from the frequency-domain perspective. Given that the cumulated responses often die out within a relatively short time horizon, statistically significant long slopes occur within business-cycle time horizon.

Results from regression with filtered series suggest that the Phillips relations are fragile, in the sense that they are frequency dependent and are sensitive to the level and cutoffs of frequency-band aggregation. The Phillips relations are also fragile in that the validity largely depends on particular variables in use. We find that Friedman's (1968) contention of no long-run Phillips tradeoff may not hold, and King and Watson's (1994) remarkable short-run Phillips correlation may only hold when business-cycle frequency bands are conveniently defined to include the high-frequency component.

3.2 Frequency-specific Phillips correlations and regressions

Frequency-specific estimates of correlation and regression are presented in Figures 2-10, over the $[0, \pi]$ range.[25] The vertical lines indexed by letters L and H indicate the lower $(\pi/16)$ and higher $(\pi/3)$ frequency cutoffs. Given quarterly data frequency, L (or $\pi/16$) corresponds to 32 quarters and H (or $\pi/3$) to 6 quarters. Below L is the low-frequency range that is often used to describe long-run characteristics of economic relations, while the high-frequency range (above H) is useful for studying short-run irregular comovements. Between L and H lies the medium or business-cycle frequency range.

Estimated values of coherence, cospectrum and the real part of coherence (henceforth referred to as *real coherence*) are computed for a total of 44 groups of bivariate $I(0)$ Phillips variables. They are presented in Figures 2-4, collected in Appendix 5.5. The first row of each figure lists coherence estimates between a rate of price inflation (or acceleration), and the growth rate of a real variable, the level (or growth rate) of demeaned and detrended processes of the real variable, and the variable's demeaned and detrended growth rates. Corresponding estimates of cospectra and real coherences are arranged in the second and third rows, respectively.[26] Whenever possible, we also present estimates of real coherence, coherence and cospectrum for bivariate correlations between price inflation (or acceleration) and gaps in real GDP level (and growth), computed with the BLS real potential GDP series.

From Figures 2-4, coherence estimates appear to be large and remarkably stable, the values of which are tightly concentrated in the $[0.95, 1]$ range. Essentially there is no visible cross-frequency variations. On the contrary, estimates of real coherences and cospectra reveal large cross-frequency variations. However, it is difficult to discern variational patterns in these estimates. Many cospectrum estimates show large bivariate correlations in low and business-cycle frequencies, with the strength of correlation

[25] Due to space limit, only a selected number of graphs are presented. The entire group of graphs can be accessed on the author's website.

[26] In the second rows of Figures 2-4, because of scaling, values of all cospectrum estimates should be read after a division by $1,000$.

diminishing and dying down in the high-frequency range. This implies a concentration of correlative power in low and medium frequencies, contradicting high correlation almost constant across the whole spectrum as coherence estimates suggest. This pattern, however, is far from universal. Moreover, estimates at one or two specific frequencies can be very large and often determine the overall sign for the estimate over the entire frequency range. Estimates of real coherences, being normalised and scale-free, reveal even larger cross-frequency variations, with more frequent and quite dramatic sign reversals. Compared to cospectra estimates, real coherence estimates reveal greater volatility at higher (above H) than at low and business-cycle frequencies.

The sign of correlation varies across frequencies, and from one frequency band to another. Changing the boundaries of a frequency band, whether conventionally defined or not, may lead to a change in the sign of average correlation over the band. Essentially, frequency-wise correlations of different signs have been averaged in a simplistic way to deliver information about business cycles or the long run. Large cross-frequency variations in our estimates indicate that conventional results relying on averaging across frequency bands or over the whole range of frequency are not reliable. Even for a narrow band (eg, near-zero frequencies), correlations may vary from being highly positive to very negative, the average of which can be positive, negative, or most likely, close to zero, depending on the exact definition of frequency ranges in use.

Coherence and cospectrum estimates are not good indicators of cross-frequency correlations.[27] Coherence estimates reveals the strength but not the sign of correlation at each frequency. Cospectrum estimates, without standardisation, can be misleading. Since cospectra and real coherences fail to take into account the complex part of bivariate correlations, it is difficult to determine the overall sign of correlation. Our estimates (in both directions) of frequency-specific coefficients of bivariate correlation and regression are superior. They are graphed in Figures 5-10. The first, second and third rows contain estimates of the frequency-specific coefficients of correlation, and of regression coefficients in reverse directions, respectively. The solid lines in the graphs represent the point estimates, while the shaded areas surrounding these depict the 95% confidence intervals.

Figures 5-10 lead us to draw a different conclusion. We present the estimation results in six different groups, depending on the pair of real and nominal variables used in the analysis. The first group pairs real output GDPC96 or real output gap pGDPC96 against different price indices (GDPDEF, GDPCTPI, PCECTPI, qPCEPILFE, qCPILFESL, qCPIAUCSL, qPPIACO and qPPIFCG). The variational patterns can be divided into four subgroups: (1) GDPDEF, GDPCTPI and PCECTPI (see Figure 5), where estimates of correlations and regressions are mostly positive, but the coefficient estimates of reverse regressions (Beta 2) are often insignificant; (2) qPCEPILFE and qCPILFESL (see Figure 6), signs in the low or medium frequencies are reversed, more estimates become insignificant; and (3) qCPIAUCSL, qPPIACO and qPPIFCG; (4) HCMA, HCNF and WASCUR.

In the second group (see Figure 7), PCECC96, the real personal consumption expenditure, is paired

[27] Cospectrum and real coherence estimates can still be informative. In our case, they suggest that it is simplistic to try to determine low or business-cycle Phillips tradeoff by averaging over a pre-defined frequency band. For certain pairs, such as UNRATE or UNEMPLOY and HCNF, positive correlation dominates in both low and business-cycle frequencies. For pairs such as OHMA and PCECTPI, positive correlation dominates the low-frequency range while negative correlation dominates the business-cycle frequencies. Nevertheless, for many other pairs (for instance, RHCMA and PCECTPI), no discernible pattern emerges. There is no uniform and stable business-cycle negative correlation claimed by King and Watson (1994).

with PCECTPI and qPCEPILFE, while real disposable personal income (DPIC96) is paired with qCPIAUCSL and qCPILFESL. One cannot fail to notice that many coefficient estimates are not significantly different from zero. In Group 3, employment variables qCE16OV, qUNRATE and qUNEMPLOY are paired with HCNF (see Figure 8). In Group 4 (see Figure 9), productivity variables are examined as follows: (1) OHMA versus GDPCTPI and PCECTPI; (2) OHNF versus GDPCTPI and PCECTPI; (3) RHCMA versus GDPCTPI and PCECTPI; (4) RHCNF versus GDPCTPI and PCECTPI.

In the fifth group (see Figure 10), analyses with the Stock-Watson Recession (qXRIM), Leading (qXLIM), Coincident (qXCIM), Recession 2 (qXRI2M) and Leading 2 (qXLI2M) indices exhibit different shape patterns: (1) qXLIM and qXRI2M versus GDPCTPI; (2) qXLIM and qXRI2M versus qCPILFESL; (3) qXLIM and qXRI2M versus qPPIACO; (4) qXCIM versus GDPCTPI, PCECTPI and qPCEPILFE; (5) qXCIM versus qCPIAUCSL, qPPIACO and HCNF. In Group 6, real private non-residential fixed investment (PNFIC96) and real gross private domestic investment (GPDIC96) are paired with different price variables, and the shapes can be classified into: (1) PNFIC96 and GPDIC96 versus GDPCTPI; and (2) PNFIC96 versus qCPIAUCSL, qPPIACO and HCNF, and GPDIC96 versus qCPIAUCSL, qPPIACO and HCNF.

From estimates of coefficients of correlation and regression, it is clear that patterns of cross-frequency Phillips relations are diverse and change from (sub)group to (sub)group, and from one pair of Phillips variables to another. Some clear and common characteristics in cross-frequency variations in the Phillips relations emerge. In general, bivariate Phillips correlations and regressions (in both directions) seem to be stable across the high-frequency range, while in low and medium frequencies, cross-frequency variations are relatively large in magnitude, with more frequent sign reversals. Many times, confidence intervals for estimates of correlation and regression estimates include zero for part or all of the frequency range, suggesting that the estimates are not significantly different from 0. Moreover, bivariate Phillips regressions in reverse directions generally do not have similar shapes. In addition, the cross-frequency variational pattern and even the signs of correlation and regression may change, depending on how the variables are processed prior to analysis (eg, demeaning, detrending, or differencing)

Obviously, the Phillips relations may suffer from sign reversals, depending on slightly different definitions of frequency bands of interest. The signs and magnitudes of the Phillips relations are sensitive to the level and cutoffs of frequency-domain aggregation, i.e., the exact frequencies to be included in an analysis. Averaging frequency-dependent correlations across the whole spectrum or within a conventionally defined frequency band may lead to incorrect inference. In this sense, usual time-domain and frequency-domain analyses of the Phillips curve are fragile.

4 Conclusion

In this paper, we use frequency domain techniques to investigate the robustness of the Phillips curve tradeoffs across the entire frequency range. We first use the Hodrick-Prescott and Baxter-King band-pass filters to examine the Phillips tradeoffs in conventionally defined frequency bands. We then propose a frequency-specific data extraction procedure, which enables us to design frequency-specific coefficients of correlation and regression as measures of the strength of "linear" association between time series. We use the coefficient estimates to assess the correlative power and examine cross-frequency variations in the Phillips relations.

Causality tests suggest that long-run Phillips correlations are valid for almost all real and nominal variables we have examined in the paper. Running NAIRU-type Phillips regressions with HP and BP-filtered data, we find that the Phillips tradeoff holds true only in the medium and higher frequency bands, where the cumulated responses ("long" slopes) are jointly significant and have the correct sign. For some price indices, such as CPI series (CPILFESL and CPIAUCSL), and certain measures of real activity, the Phillips tradeoff turns out to be more solid. By using a large number of different nominal and real variables and by focussing on the simple Phillips bivariate relations and the NAIRU specification, our first results contradict the conventional view that there is no long-run but a strong short-run Phillips tradeoff.

Estimates of frequency-specific coefficients of correlation and regression for a large number (44 groups) of Phillips variables cast further doubts on the reliability of empirical work on Phillips curve. For the majority of cases, the Phillips relations vary both in sign and in strength across the frequency range, with frequent sign reversals. Taking a simple average of these correlations across the whole spectrum or in the conventionally defined frequency bands may lead one to draw incorrect inference. A small change in the level and boundaries of frequency aggregation may cause the averaged signs of correlation and regression to reverse.

The Phillips relations seem to be fairly stable in the high frequency range (less than 6 quarters), but much less stable in the business cycle frequency range, and quite unstable in the low frequency range (over 32 quarters). Moreover, many of the 95% confidence intervals for the Phillips spectral correlation and regression coefficients include zero. Furthermore, the Phillips relations are sensitive to the type of data used and to the way how data are processed prior to analysis (i.e., how they are detrended). Summing up, usual time and frequency-domain analyses of the Phillips curve may be misleading, and in this sense, the Phillips relations are fragile.

In fact, there are good economic reasons for the relationship between price inflation and real economic activity to differ across frequencies, i.e., across different time horizons. Cyclical Long-run inflation may react to cyclical output growth, say, differently from how long-run inflation reacts to long-run output growth. Indeed cyclical inflation may react to long-run output growth differently from how it reacts to cyclical output growth. Whether a particular macroeconomic model holds true across different time horizons or frequencies, and how potential cross-frequency model inconsistency may impact on model estimation with aggregate time series using standard time domain methods, needs careful examination.

5 Appendices

5.1 Empirical validity of the Phillips curve: causality tests

The Phillips curve is essentially an empirical phenomenon.[28] Traditionally, the slope of the Phillips curve has been estimated either from a price equation, as in Gordon (1970) and Lucas (1973), or from an aggregate supply equation as in Sargent (1976):

$$\alpha(L)\pi_t = \beta(L)u_t + \varepsilon_t^\pi \tag{22}$$

$$\theta(L)u_t = \phi(L)\pi_t + \varepsilon_t^u \tag{23}$$

where π_t and u_t are the inflation and unemployment rate, respectively, and L is the lag operator. In the price equation (22), ε_t^π is typically a shock to the aggregate demand, while in the supply equation (23), ε_t^u is a supply shock. Since the bivariate correlation is certainly influenced by many other third variables which interact among themselves and with both π_t and u_t, there are serious omitted-variable and endogeneity biases in both equations.

In light of the Lucas-Sargent critique and recent advances in time series analysis, knowledge of the orders of integration of the Phillips variable is key to the validity of a test of the long-run Phillips tradeoff in reduced form. Assuming stationarity for π_t and u_t, Sargent (1971) and Lucas (1972b) gave examples in which it may be impossible to test long-run neutrality using reduced-form models. As Sargent (1971) pointed out, if sustained or permanent changes in the rate of inflation do not materialise in the data during the estimation period, essentially one cannot test long-run propositions that involve such changes.[29] From the policy perspective, if the inflation rate is raised permanently to a higher level, can we permanently raise the real activity to a new steady-state level?[30] Once π_t is known to be integrated, one can define and test long-run neutrality without the full knowledge of the underlying behavioural model.

Rewriting (22) as follows,

$$\pi_t = \beta_0 u_t + \beta'(L)u_{t-1} + \alpha'(L)\,\pi_{t-1} + \varepsilon_t^\pi \tag{24}$$

where $\alpha'(L)$ and $\beta'(L)$ are finite polynomials in L. The short-run or contemporaneous slope S_{SR} of

[28] As Lucas and Sargent (1979) remarked, "the question of whether a particular model is structural is an empirical, not a theoretical one." Yet much efforts have been made to provide theoretical foundations for the Phillips Curve, which include Lucas (1972a), Mankiw (2000), Razin and Yuen (2001) and Woodford (2003).

[29] Fisher and Seater's (1993) neutrality tests and their concept of "long-run derivative" are based on the observation that orders of integration of variables of interest play a key role in non-structural tests of long-run propositions in reduced-form models.

[30] McCallum (1984) distinguishes the Friedman-Phelps accelerationist hypothesis, where unemployment can be kept permanently low only by permanently accelerating the rate of inflation, from the Lucas (1972) natural rate hypothesis, where no monetary policy can keep unemployment permanently low.

the Phillips curve is $S_{SR} = \partial\pi_t/\partial u_t = \beta_0$ and the long-run slope S_{LR} is[31]

$$S_{LR} = \partial\pi/\partial u = \frac{\beta(1)}{\alpha(1)} = \frac{\beta_0 + \beta'(1)}{1 - \alpha'(1)} \tag{25}$$

The dynamic Phillips curve specification (24) embodies both long-run average behaviour and short-term dynamics. It seems to be conventional wisdom that there is no long-run inflation-unemployment tradeoff (i.e., vertical long-run Phillips curve with $S_{LR} = 0$) but a strong negative short-run tradeoff $(S_{SR} < 0)$.[32]

On the basis of (24), and following the example of Sargent (1976), we use both Granger's (1969) and Sims' (1972) causality tests to examine the Phillips correlations. Focussing on the long-run, we assume that there is no instantaneous causality ($\beta_0 = 0$). For jointly covariance stationary processes π_t and u_t, we have a bivariate system

$$\pi_t = \sum_{j=1}^{p} \alpha_j \pi_{t-j} + \sum_{j=1}^{q} \beta_j u_{t-j} + \zeta_t \tag{26}$$

$$u_t = \sum_{j=1}^{m} \theta_j u_{t-j} + \sum_{j=1}^{n} \phi_j \pi_{t-j} + \xi_t \tag{27}$$

where ζ_t and ξ_t can be interpreted as shocks to aggregate demand and supply, respectively. We say that u (π) does *not Granger-cause* π (u), if the null hypothesis of $\beta_j = 0$ $(\phi_j = 0)$, $\forall j$, cannot be rejected. Alternatively, Sims' (1972) causality test is based on the following regression equation

$$u_t = \sum_{j=-m}^{p} \gamma_j \pi_{t-j} + \sum_{j=1}^{q} \theta_j u_{t-j} + \varepsilon_t \tag{28}$$

Here u is said *not* to *Granger-cause* π, if we cannot reject the null that $\gamma_j = 0$, $\forall j < 0$. We apply the Granger and Sims causality tests in both directions of regression, and the bivariate tests are applied to a wide variety of candidate variables, which have been transformed (into gaps) and differenced until their order of integration is reduced to zero ($I(0)$).

Tests of Granger causality constitute a basic test of the long-run Phillips correlation. Variables that cannot Granger-cause each other at least in one direction are essentially unrelated, i.e., long-run tradeoff (or more precisely, cumulated responses) cannot exist in the data. The long-term tradeoff is an empirical phenomenon, which does not depend on short-run dynamics and hence the structural details of the economy. Relatively structure-free tests of the tradeoff in reduced-form models are useful, as long as it is not exploited for policy purposes.

Granger (1969) and Geweke (1982) designed simple measures of feedback and causality in the frequency domain. Granger (1969) showed that for covariance stationary bivariate processes x and y, the cospectrum $f_{xy}(\omega)$ can be decomposed into two components, one depending on causality

[31]It would be more appropriate to call $\partial\pi_t/\partial u_t$ the instantaneous response of π to u, and $\partial\pi/\partial u$ the cumulated response, so that these are nor confused with business-cycle and low-frequency responses. There can be both immediate response in the low-frequency and accumulated response in the business-cycle frequency.

[32]One of Mankiw's (2000) 10 principles of economics is: "Society faces a short-run tradeoff between inflation and unemployment."

running from x to y, the other depending on causality running from y to x. To measure the strength of causality from one series to the other, he focussed on one single component and proposed the concept of causality coherence. Geweke (1982) defined measures of linear feedbacks and dependence based on the regression errors in the system (26) and (27) and provided their counterparts in the frequency domain. Geweke (1986) applied these concepts to study the long-run superneutrality of money.

Given the important implications of non-stationarity for our analysis,[33] we test the order of integration for all variables, using Said and Dickey's (1984) augmented Dickey-Fuller (ADF) test, Phillips' (1987) and Phillips and Perron's (1988) Z_α and Z_t tests. Test results indicate that almost all price indices, with the exception of a few producer price indices, contain unit roots in levels. The null of unit root is rejected at 1% significance level by all three tests for rates of price acceleration. Test results for rates of price change (inflation) are mixed and sometimes inconclusive.[34] For real variables, test results are much more uniform. Except for UNRATE, RHCMA, RHCNF, XLIM, XLI2M, XRIM and XRI2M, which are already $I(0)$ or close to stationarity in levels, all other real variables contain unit roots in levels, while the presence of unit roots in their rates of change and acceleration is rejected at 1% significance level.

We estimate the spectrum for all variables involved. While the levels and rates of change of the nominal variables have the "typical spectral shape" of Granger (1966), their rates of acceleration do not.[35] Levels of the real variables, in general, have the typical spectral shape. For their rates of growth, the power appears to be more evenly distributed, but with more cross-frequency fluctuations. The spectral shapes of different variables largely conform to the unit root test results.

We carry out the Granger and Sims causality tests for variables that are $I(0)$ or differenced enough times to be stationary. In the Granger tests, a maximum of six lags is allowed for both the lagged dependent variable and the regressor, so a total of 36 tests are run in each direction for every pair of Phillips variables. In the Sims tests, a maximum number of six leads is combined with a maximum number of six lags, so a total of 216 tests are run in each direction for every pair of Phillips variables. Model selection criteria (AIC, BIC and FPE) are used to select the adequate model, and the null hypothesis of causality is tested in the adequate model. P-values for both the F and the LM tests of the null of non-causality are obtained.

[33] McCallum (1984) argued that "the association of low-frequency time-series statistics with 'long-run' economic propositions is not generally warranted. Instead, many so-called long-run propositions involve *expectational* relationships which have little or nothing to do with low frequencies *per se*, so that such an association is inappropriate in a fundamental sense." The critique is valid for stationary models in reduced form, and it is equivalent to the Lucas-Sargent critique articulated in the time domain. The key issue is that low-frequency measures are not "designed to reflect the distinction between anticipated and unanticipated fluctuations that is crucial for accurately characterising intervariable relationships in many dynamic models."

The Phillips tradeoff cannot be tested in reduced form in the absence of good knowledge about the underlying structural model, unless the data contain permanent stochastic changes in price level or inflation. Moreover, long-run correlation between inflation and real activity depends on their relative orders of integration. In this context, non-stationarity plays a key role in making inferences about important economic hypotheses.

[34] The order of integration of price indices may change over time, and such changes reflect fundamental changes in economic structure or in the making of economic policies. In a period of price stability since the early 1990s, price indices are likely to be stationary.

[35] Curiously, the only exception seems to be POILBRE, the oil price of UK Brent, light blend 38 API, FOB U.K.

Comparing the Granger and the Sims test results (see Table 1),[36] the rejection rate of the null of non-causality is much lower in the latter case. According to the Granger tests, most pairs of Phillips variables seem to Granger-cause each other, and long-run Phillips correlations are valid for a wide variety of real and nominal variables. The only exception seems to occur with the pair of demeaned RHCMA (productivity gap) growth and GDPCTPI. From Sims tests, however, the majority of causality relations are unidirectional, and some pairs apparently have no causal relationship.

For all causally related pairs of Phillips variables, in most cases, the sum of distributed lag coefficients is negative when measures of real activity are used as the dependent variable, and positive when measures of unemployment are used instead. The observed signs largely conform to prior expectations. However, sign reversals are frequent in Granger regressions, which occur when real variables become the regressand. One explanation might be that the cumulated responses of price inflation or acceleration to a change in real growth are positive, while price inflation or acceleration tend to reduce growth rates in the long run. In Sims regressions, signs for most pairs of variables are consistent. In terms of both tests and signs of cumulated responses, the Granger and Sims regressions disagree in many cases.

Results from the Sims tests reveal distinctive patterns. First, although the vast majority of the Phillips variables appear to be causally related, many causal relations are unidirectional, and there are more cases of no causality; second, signs of sums of distributed leads in the Sims tests suggest the same interpretation as in the Granger tests, whenever the null of non-causality can be rejected; third, for most pairs of Phillips variables, when price indices are all inclusive, causality runs from real to nominal variables, but not the other way round. Once we use core price indices which exclude energy and food prices (CPILFESL and PCEPILFE), or nominal wages (HCNF and WASCUR), causality runs only from nominal to real variables. Controlling for energy and food prices inverts the direction of causality. Test results for regressions with gaps in real output and in real output growth computed using the BLS Real Potential GDP series are broadly in agreement with those obtained from output and unemployment gaps calculated as demeaned or detrended processes. In conclusion, both Granger and Sims causality tests attest to the validity of the Phillips tradeoff.

5.2 Conventional filters in macroeconomic analysis

For the first difference (FD) filter $FD(L) = 1 - L$, the transfer function is

$$\psi_{FD}(\omega) = 1 - e^{-i\omega} \tag{29a}$$

$$= \begin{cases} 2\left|\sin(\omega/2)\right| \exp\left(i(\pi - \omega)/2\right) & \text{if } \omega > 0 \\ 2\left|\sin(\omega/2)\right| \exp\left(-i(\pi + \omega)/2\right) & \text{if } \omega < 0 \end{cases} \tag{29b}$$

[36]Tables with complete causality test results can be obtained from the author's website.

and the gain and squared gain of the FD filter are, respectively

$$G_{FD}(\omega) = 2\left|\sin(\omega/2)\right| \quad (30)$$

$$SG_{FD}(\omega) = 2\sin^2(\omega/2) \quad (31)$$

The FD filter and Hodrick and Prescott's (1980, 1997) HP filter are approximate high-pass filters. Inverting their squared gain functions $SG_{HP}(\omega)$ and $SG_{FD}(\omega)$, we obtain the corresponding inverse squared gain functions $SG^*_{HP}(\omega)$ and $SG^*_{FD}(\omega)$:[37]

$$SG^*_{HP}(\omega) = 1 - SG_{HP}(\omega) \quad (32)$$

$$SG^*_{FD}(\omega) = 1 - SG_{FD}(\omega) \quad (33)$$

The Lucas' (1980) filter takes a two-sided exponentially weighted symmetric moving average of the original time series:

$$y_t = \frac{1-\rho}{1+\rho}\sum_{j=-\infty}^{\infty}\rho^{|j|}x_{t+j} \quad (34)$$

with $0 < \rho < 1$. Its Fourier transform and squared gain are, respectively

$$\psi_L\left(e^{-i\omega}\right) = \frac{(1-\rho)^2}{1+\rho^2-2\rho\cos(\omega)} \quad (35)$$

$$SG_L(\omega) = \left|\psi_L\left(e^{-i\omega}\right)\right|^2 = \frac{(1-\rho)^4}{\left[1+\rho^2-2\rho\cos(\omega)\right]^2} \quad (36)$$

The Lucas filter is an approximate low-pass filter that assigns little weight to the medium and high-frequency components of a time series but retains the power at very low frequencies. As the weighting parameter ρ increases from 0.5 to 0.95, the filter becomes more concentrated around the zero frequency.

The Lucas filter, the *inverse* HP and FD filters are approximate low-pass filters. Using mean squared error criterion, Baxter and King (1999) developed an approximate band-pass (BP) filter, in fact, a combination of approximate high-pass and low-pass filters. Ideal filters can be designed for data of infinite length. In reality, finite data length implies the necessity of truncation of filter weights. This, in turn, leads to filter leakage, exacerbation and compression. Comparing to an ideal filter, approximate filters with cutoffs at $-\bar{\omega}$ and $\bar{\omega}$ allow through a sizeable amount of power from frequencies outside of the range $[-\bar{\omega}, \bar{\omega}]$, the phenomenon being known as *filter leakage*. Inside the range $[-\bar{\omega}, \bar{\omega}]$, an approximate filter may amplify or scale down the power of the input series, leading to "*exacerbation*" and "*compression*" effects (See Figure 1).

The squared gains of the FD, Lucas, HP and BP filters are graphed in Figure 1. The FD filter is a highly inaccurate approximate high-pass filter, with a sizeable amount of leakage appearing at low frequencies, and too great weights assigned to medium- and high-frequency components. As the

[37] The analytical formulae for squared gains of the HP and BP filters are well known. Equations (32) and (33) are not the true squared gains of the *inverse* HP and FD filters. Because the spectral density of the output series is equal to that of the input series multiplied by squared gain, the effect of the *inverse* HP and FD filters can be illustrated by "inverting" their squared gain functions.

smoothing parameter λ increases from 10 to $25,600$, the HP filter increasingly resembles an ideal high-pass filter and the leakage becomes smaller. For the Lucas filter, the picture is typical of an exponential smoothing filter. Besides its elaborate design, the Lucas filter is not any more successful in isolating very low-frequency component of a time series than other symmetric moving average filters with exponential weights. For comparability with the existing literature, we use the HP and BP filters in our analysis.

5.3 Data description

We collect our postwar US macroeconomic data, seasonally adjusted and at quarterly frequency, from the Bureau of Labour Statistics (BLS), US Department of Labor, except for the series of oil prices, which are taken from the CPS. We use a large number of nominal (18) and real (22) variables that have been used in the literature to provide a frequency-domain robustness check of the empirical Phillips relations. While most of the series end in 2001:4 or 2002:1, their starting dates differ greatly, ranging from 1946:1 to 1975:1.[38] We always choose the longest sample period available in each set of bivariate correlation and regression analyses, taking advantage of as much information as data availability allows.

The nominal or price variables are divided into the following six groups:

(1) Gross Domestic Product (GDP) and Gross National Product (GNP) Implicit Price Deflators and the corresponding Chain-type Price Indices (GDPDEF, GDPCTPI, GNPDEF and GNPCTPI);

(2) Personal Consumption Expenditures (PCE) Chain-type Price Index (PCECTPI), PCE Chain-type Price Index Less Food and Energy (PCEPILFE);

(3) Wages and Salaries as Compensation of Employees (WASCUR), Hourly Compensation in the Manufacturing Sector (HCMA), Hourly Compensation in Non-farm Businesses (HCNF);

(4) Consumer Price Index (CPI) for All Urban Consumers, All Items (CPIAUCSL), CPI for All Urban Consumers, All Items excluding Food and Energy (CPILFESL), CPI for All Urban Consumers, Energy (CPIENGSL), CPI for All Urban Consumers, Food (CPIUFDSL);

(5) Producer Price Index (PPI) for all commodities (PPIACO), PPI for finished consumer goods (PPIFCG), PPI for finished goods excluding energy (PPIFLE), PPI for finished goods excluding food and energy (PPILFE);

(6) Oil Prices for UK Brent, light blend 38 API, FOB U.K. (POILBRE).

On the other hand, the (real) variables that we use to measure aggregate real activity, such as real output or unemployment, are divided into the following six groups:

[38] Data ranges for the time series are:
(a) Nominal variables: GDPDEF, GDPCTPI, PCECTPI, HCNF, CPIAUCSL, CPIUFDSL, 1947.1 - 2002.1; GNPDEF, GNPCTPI, 1947.1 - 2001.4; CPIAUCNS, PPIACO, WASCUR, 1946.1 - 2002.1 ; HCMA, 1949.1 - 2002.1; CPILFESL, CPIENGSL, 1957.1 - 2002.1; PPIFCG, 1947.2 - 2002.1; PPIFLE, 1975.1 - 2002.1; PPILFE, 1974.1 - 2002.1; PCEPILFE, 1963.1 - 2002.1; POILBRE, 1957.1 - 2001.2;
(b) Real variables: GDPC96, PCECC96, DPIC96, PNFIC96, GPDIC96, OHNF, RHCNF, 1947.1 - 2002.1; GDPPOT, 1949.1 - 2012.4; OHMA, RHCMA, 1949.1 - 2002.1; GNPC96, 1947.1 - 2001.4; UNRATE, CE16OV, UNEMPLOY, CIVPART, EMRATIO, 1948.1 - 2002.1; PCEC96, 1967.1 - 2002.1; DSPIC96, 1959.1 - 2002.1; XLIM , XCIM, XLI2M, 1959.1 - 2002.1; XRICM, 1959.2 - 2002.1; XRIM, XRI2M, 1961.1 - 2002.1.

(1) Real Gross Domestic and National Product, in chained 1996 dollars (GDPC96 and GNPC96), Real Potential GDP in chained 1996 dollars (GDPPOT);

(2) Real Personal Consumption Expenditures in chained 1996 dollars (PCECC96), Real Disposable Personal Income in chained 1996 dollars (DPIC96);

(3) Civilian Unemployment Rate (UNRATE), Unemployed, 16 years and over (UNEMPLOY), Civilian Employment for 16 years and over (CE16OV), Civilian Participation Rate (CIVPART), Civilian Employment-Population Ratio (EMRATIO);

(4) Real Private Nonresidential Fixed Investment in chained 1996 dollars (PNFIC96), Real Gross Private Domestic Investment (GPDIC96);

(5) Productivity, Output per Hour in Manufacturing (OHMA), Output per Hour in Non-farm Businesses (OHNF), Real Hourly Compensation in Manufacturing (RHCMA), Real Hourly Compensation in Non-farm Businesses (RHCNF);

(6) Stock-Watson NBER Experimental Recession Index (XRIM), Experimental Leading Index (XLIM), Experimental Coincident Index (XCIM), Experimental Coincident Recession Index (XRICM), alternative Non-financial Experimental Leading Index-2 (XLI2M), and alternative Non-financial Experimental Leading Recession Index-2 (XRI2M).

We first transform the data by taking logarithms of all series except for those which are already expressed as percentage rates, such as UNRATE, CIVPART and EMRATIO. Rates of change are then computed either as difference in the log-transformed variables or as percentage change in levels. The rate of acceleration of each variable is calculated as the first difference of its rate of change. For real variables, we define gaps in real activity (in both levels and growth rates) as deviations from a linear trend and as demeaned processes. We take first differences in these gaps as many times as needed to create new variables that are stationary. Using the Real Potential GDP series provided by the BLS, we calculate alternative real output gaps (in both level and growth rate) as the difference between the historical values of Real and Real Potential GDP's.

5.4 Tables

Table 1: **Granger and Sims causality test results: output gaps (GDPC96)**

	Granger tests		Sims tests	
Variable Y	*Levels*	*Growth rates*	*Levels*	*Growth rates*
GDPDEF	0.04124†(1,6)	0.02553♯(2,6)	0.68414†(4,4,1)	0.53736†(4,6,3)
	-0.03622†(5,3)	-0.19934‡(6,3)	0.03728‡(3,1,4)	-0.00669§(1,1,6)
GDPCTPI	0.04222†(1,6)	0.02704†(1,6)	0.76847†(4,4,1)	0.88587†(4,6,1)
	0.10733§(5,1)	-0.06777§(6,1)	0.00111§(1,1,6)	-0.01410§(1,1,6)
PCECTPI	0.04530†(1,6)	0.03579†(1,6)	0.79485†(4,4,1)	0.99663†(4,6,1)
	0.06139§(5,1)	-0.15989§(6,1)	0.00495§(1,1,6)	-0.00084†(1,1,6)
PCEPILFE	0.03692†(2,6)	0.04411†(4,5)	-0.33552§(1,2,5)	-0.11671§(1,5,2)
	-0.42798‡(3,1)	-0.94169†(5,2)	-0.02349‡(2,2,6)	-0.03335†(2,4,6)
CPIAUCSL	0.04367‡(1,6)	0.04802†(1,6)	0.81772†(6,3,1)	0.26787§(1,5,6)
	-0.07460†(5,1)	-0.07812†(6,1)	-0.04063§(1,1,6)	-0.03776§(1,1,6)
CPILFESL	0.05679†(1,5)	0.05705†(3,5)	0.04343†(4,4,1)	0.08334†(4,6,1)
	-0.47721†(3,2)	-0.61019†(6,3)	-0.03872†(2,1,6)	-0.05134†(2,3,6)
PPIACO	0.03584♯(2,6)	0.06177§(1,6)	0.07556♯(1,5,1)	0.10689♯(1,6,1)
	-0.03701‡(5,2)	-0.04347‡(6,1)	0.03401§(4,1,6)	-0.00323§(4,1,6)
HCNF	0.06044†(1,6)	0.02108§(1,6)	0.09335§(1,5,1)	0.04856§(1,6,1)
	-0.07768†(5,1)	-0.11504†(6,1)	-0.05821§(1,1,6)	-0.08918†(4,1,6)
WASCUR	-0.03003†(6,6)	0.00954§(1,6)	-0.00175†(4,1,6)	-0.02152§(1,5,5)
	-0.04623†(3,5)	-0.07228†(6,3)	0.18367†(2,6,6)	0.12913†(2,5,6)

Notes:
(1) Numbers in parentheses indicate lags (in Sims tests, both leads and lags) of the "adequate" model selected using model selection criteria.
(2) †, ‡ and ♯ indicate that the null hypothesis of non-causality can be rejected at 1%, 5% and 10% significance levels, respectively. § indicates that the null cannot be rejected. DW is the Durbin-Watson test statistic.

Table 2: **Phillips regressions with HP-filtered trend series: core price indices**

	PCEPILFE			**CPILFESL**		
Real variable	*Short slope*	*Long slope*	*DW*	*Short slope*	*Long slope*	*DW*
GDPC96 gap	-3.66887§	-0.00010†(3)	0.06	-0.08811§	-0.00269†(8)	0.05
GDPC96 growth gap	0.12381†	0.02248†(1)	0.05	-0.91259§	-0.00732†(4)	0.03
GDPC96	-1.13623§	0.10919†(2)	0.05	-0.49343§	-0.01767†(5)	0.05
PCECC96	-11.75411§	0.08617†(3)	0.07	0.26909§	-0.00690†(4)	0.04
DPIC96	-8.90313§	0.02430†(8)	0.12	-2.76084§	-0.00240†(5)	0.06
UNRATE	8.72337†	0.00162†(3)	0.05	4.96367§	0.00363†(5)	0.05
OHNF	-3.30163§	0.11990†(2)	0.05	-0.40229§	-0.02226†(8)	0.10
RHCNF	-0.04116§	0.00086†(1)	0.04	1.69077§	0.00091†(8)	0.07
XLIM	-0.00524§	0.00034†(2)	0.04	0.00848†	-0.00007†(3)	0.05
PNFIC96	0.12090†	0.00735†(1)	0.04	-1.54864§	0.00582†(4)	0.06
GPDIC96	0.10024†	0.00282†(1)	0.04	-0.22767§	-0.00157†(7)	0.04

Notes:
(1) Regressions use PCEPILFE and CPILFESL as dependent variables, which exclude food and energy prices;
(2) Numbers in parentheses indicate lags of real variables in the "adequate" model selected using model selection criteria;
(3) †, ‡ and ♯ indicate that the null hypothesis of zero coefficient value can be rejected at 1%, 5% and 10% significance levels, respectively. § indicates that the null cannot be rejected. DW is the Durbin-Watson test statistic.

Table 3: **Phillips regressions with HP-filtered cyclical series: core price indices**

	PCEPILFE			CPILFESL		
Real variable	*Short slope*	*Long slope*	*DW*	*Short slope*	*Long slope*	*DW*
GDPC96 gap	0.02969§	0.33551†(2)	2.06	0.07305‡	0.52545†(1)	2.12
GDPC96 growth gap	-0.00812§	0.11720†(2)	2.02	0.06212‡	1.03384†(3)	2.10
GDPC96	0.00296§	-0.36275†(1)	1.97	0.04698§	3.29507†(6)	2.10
PCECC96	-0.07836§	0.47699†(7)	2.03	-0.10134§	2.66675†(8)	2.21
DPIC96	-0.06529§	-0.31779†(3)	2.12	-0.01346§	0.41789†(3)	2.06
UNRATE	-0.04661§	-0.65789†(1)	2.05	-0.32801§	-1.15685†(1)	2.14
OHNF	-0.02530§	-0.98200†(1)	2.00	-0.04301§	-1.08182†(1)	2.01
RHCNF	-0.04521§	0.27194†(1)	2.03	-0.05038§	0.65492†(3)	2.11
XLIM	-0.00035§	-0.00048†(2)	2.13	-0.00086§	0.00129†(7)	2.27
PNFIC96	0.00410§	0.15678†(1)	2.02	0.02325♯	1.22274†(6)	2.18
GPDIC96	0.00101§	0.00217†(1)	2.00	0.01774†	1.14238†(6)	2.09

Notes:
(1) Regressions use PCEPILFE and CPILFESL as dependent variables, which exclude food and energy prices;
(2) Numbers in parentheses indicate lags of real variables in the "adequate" model selected using model selection criteria;
(3) †, ‡ and ♯ indicate that the null hypothesis of zero coefficient value can be rejected at 1%, 5% and 10% significance levels, respectively. § indicates that the null cannot be rejected. DW is the Durbin-Watson test statistic.

Table 4: **Phillips regressions with BP-filtered cyclical series: core price indices**

	PCEPILFE			**CPILFESL**		
Real variable	*Short slope*	*Long slope*	*DW*	*Short slope*	*Long slope*	*DW*
GDPC96 gap	-0.28845§	-0.08849†(3)	0.57	-0.45085§	-0.15832†(3)	0.49
GDPC96 growth gap	-0.27242§	-0.04162†(3)	0.57	-0.45835§	-0.15034†(3)	0.52
GDPC96	-0.55860§	0.10551†(3)	0.59	-0.73174§	-0.39528†(5)	0.54
PCECC96	-0.69032§	0.30340†(4)	0.63	-0.92914§	-0.30676†(5)	0.56
DPIC96	-0.46241§	-0.09988†(3)	0.61	-0.26402§	-0.32918†(3)	0.54
UNRATE	0.61137†	0.15495†(3)	0.54	2.48880†	-0.03633†(8)	0.52
OHNF	-0.51698§	0.41863†(3)	0.60	-0.22246§	0.27032†(1)	0.51
RHCNF	-0.91791§	0.01723†(4)	0.59	0.16413♯	-0.19762†(2)	0.53
XLIM	-0.00056§	0.00027†(1)	0.58	-0.00248§	0.00004†(3)	0.52
PNFIC96	-0.17270§	-0.05086†(3)	0.55	-0.36059§	-0.13020†(3)	0.56
GPDIC96	-0.08587§	0.01551†(3)	0.56	-0.14091§	-0.15731†(8)	0.55

Notes:
(1) Regressions use PCEPILFE and CPILFESL as dependent variables, which exclude food and energy prices;
(2) Numbers in parentheses indicate lags of real variables in the "adequate" model selected using model selection criteria;
(3) †, ‡ and ♯ indicate that the null hypothesis of zero coefficient value can be rejected at 1%, 5% and 10% significance levels, respectively. § indicates that the null cannot be rejected. DW is the Durbin-Watson test statistic.

5.5 Figures

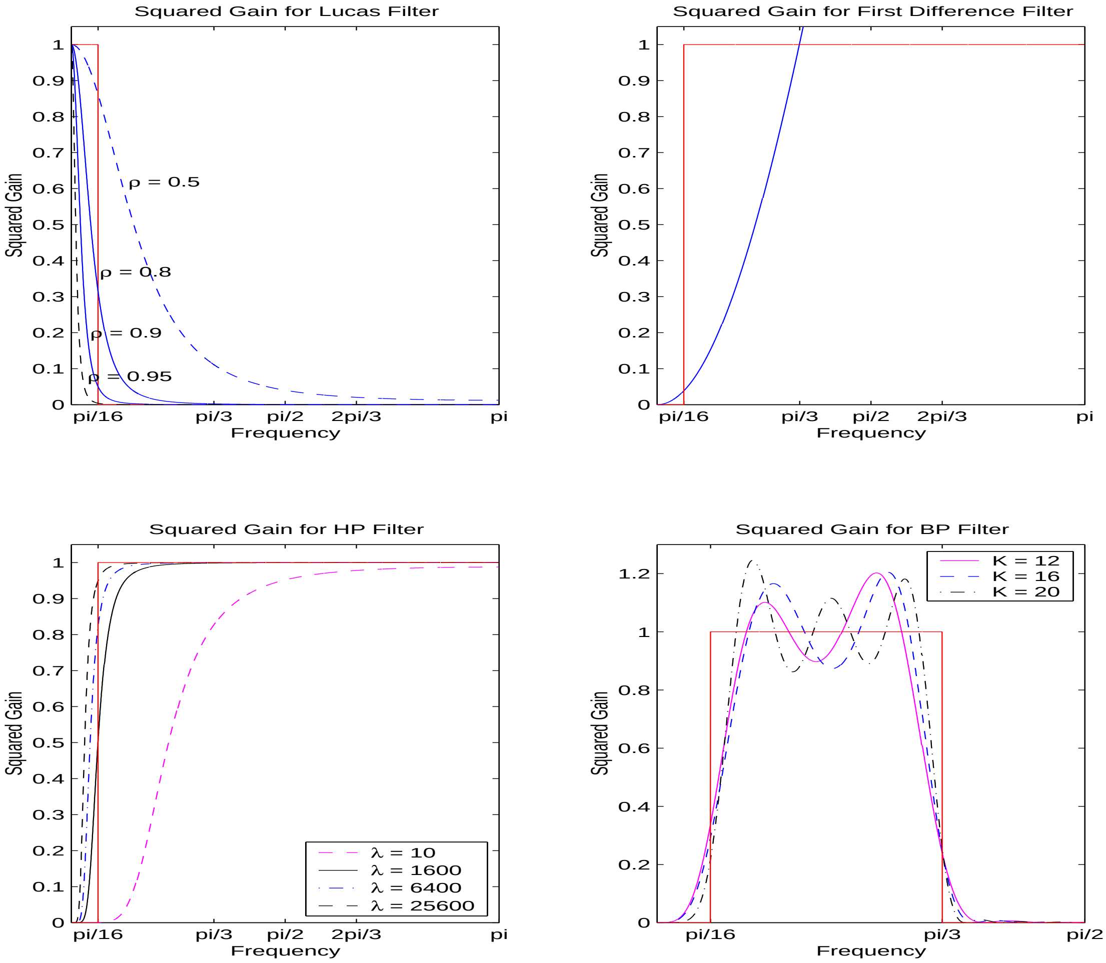

Figure 1: **Squared gains for the Lucas, FD, HP and BP filters**

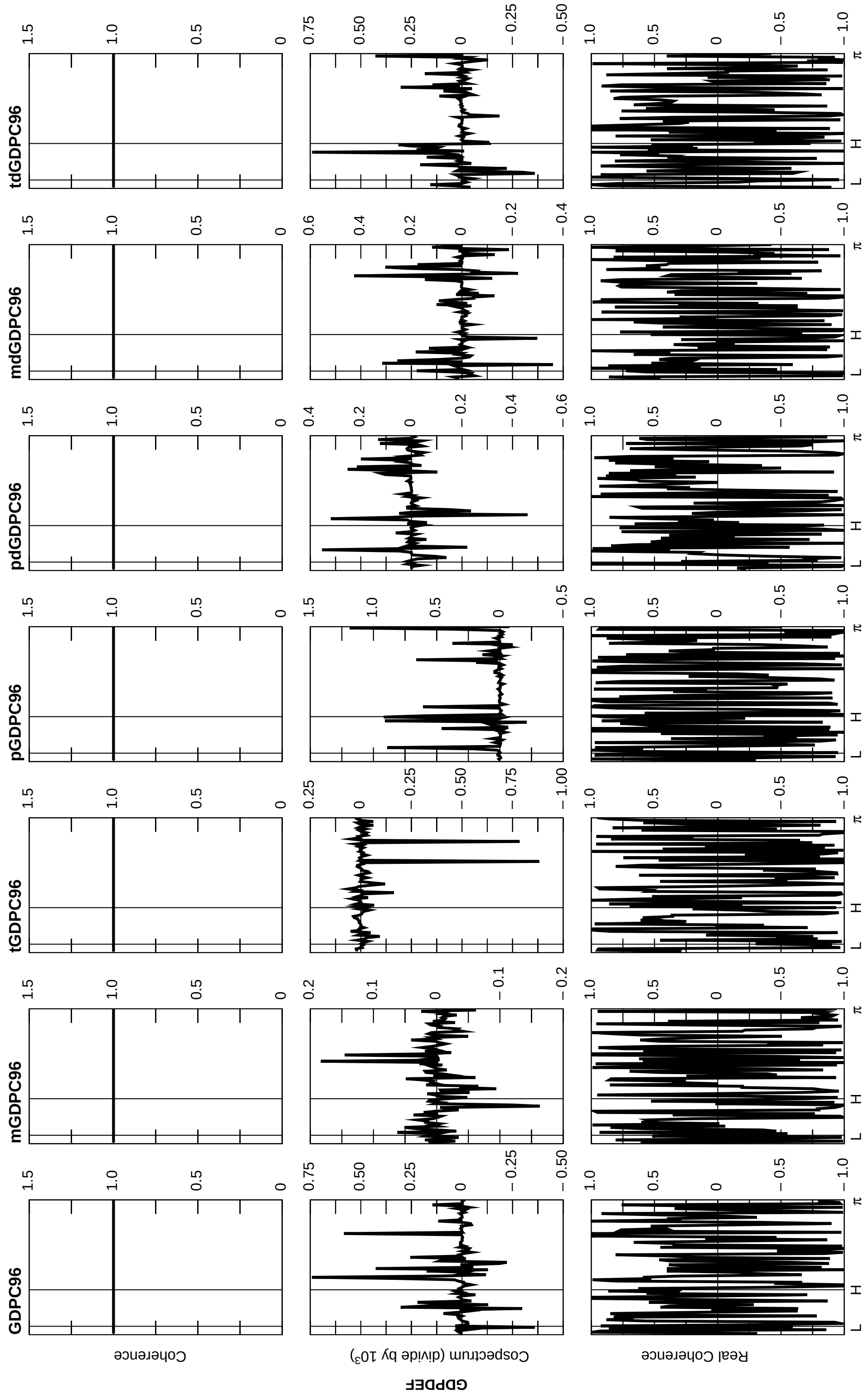

Figure 2: **Spectral indicators of Phillips correlations: GDPC96 versus GDPDEF**

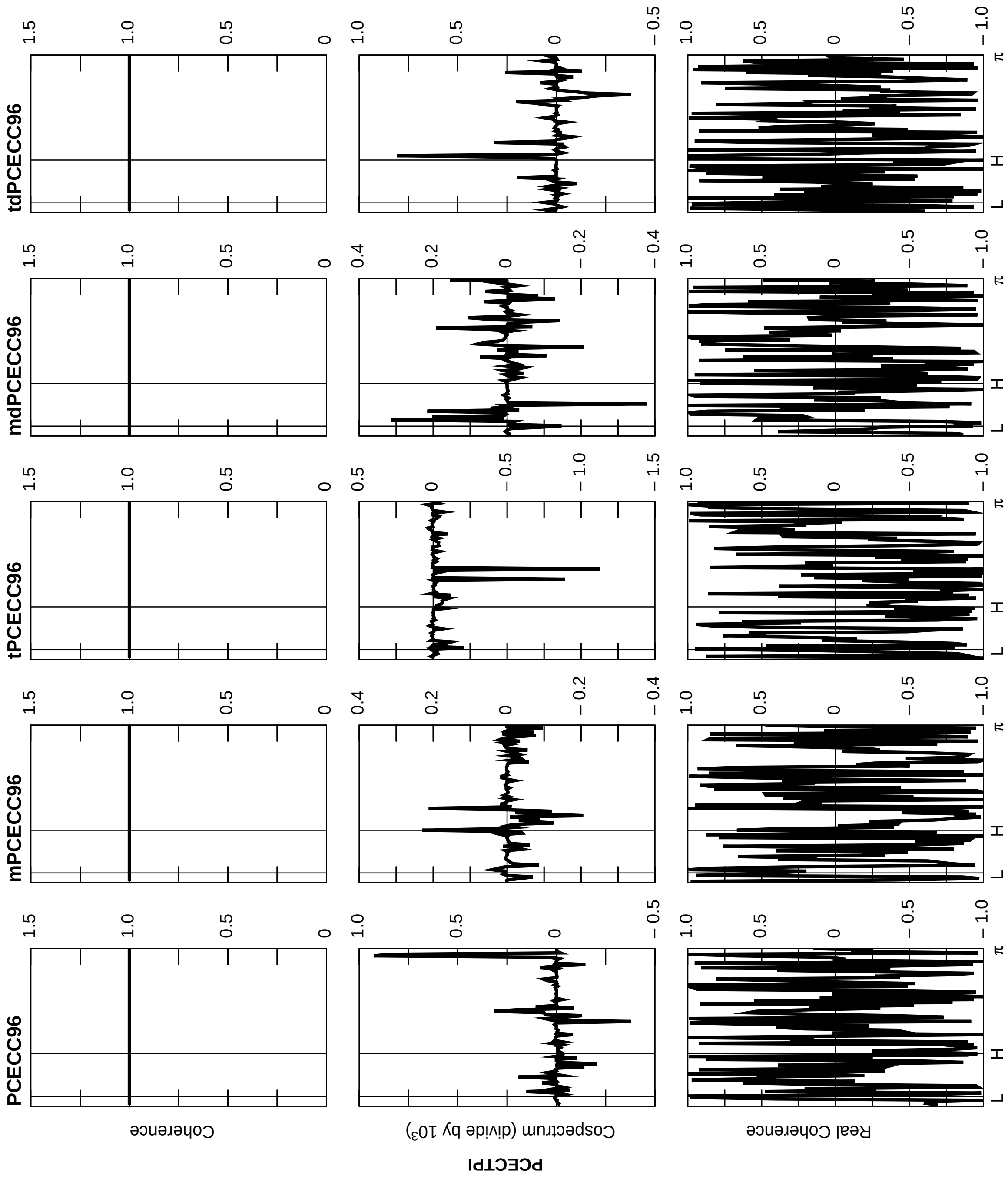

Figure 3: **Spectral indicators of Phillips correlations: PCECC96 versus PCECTPI**

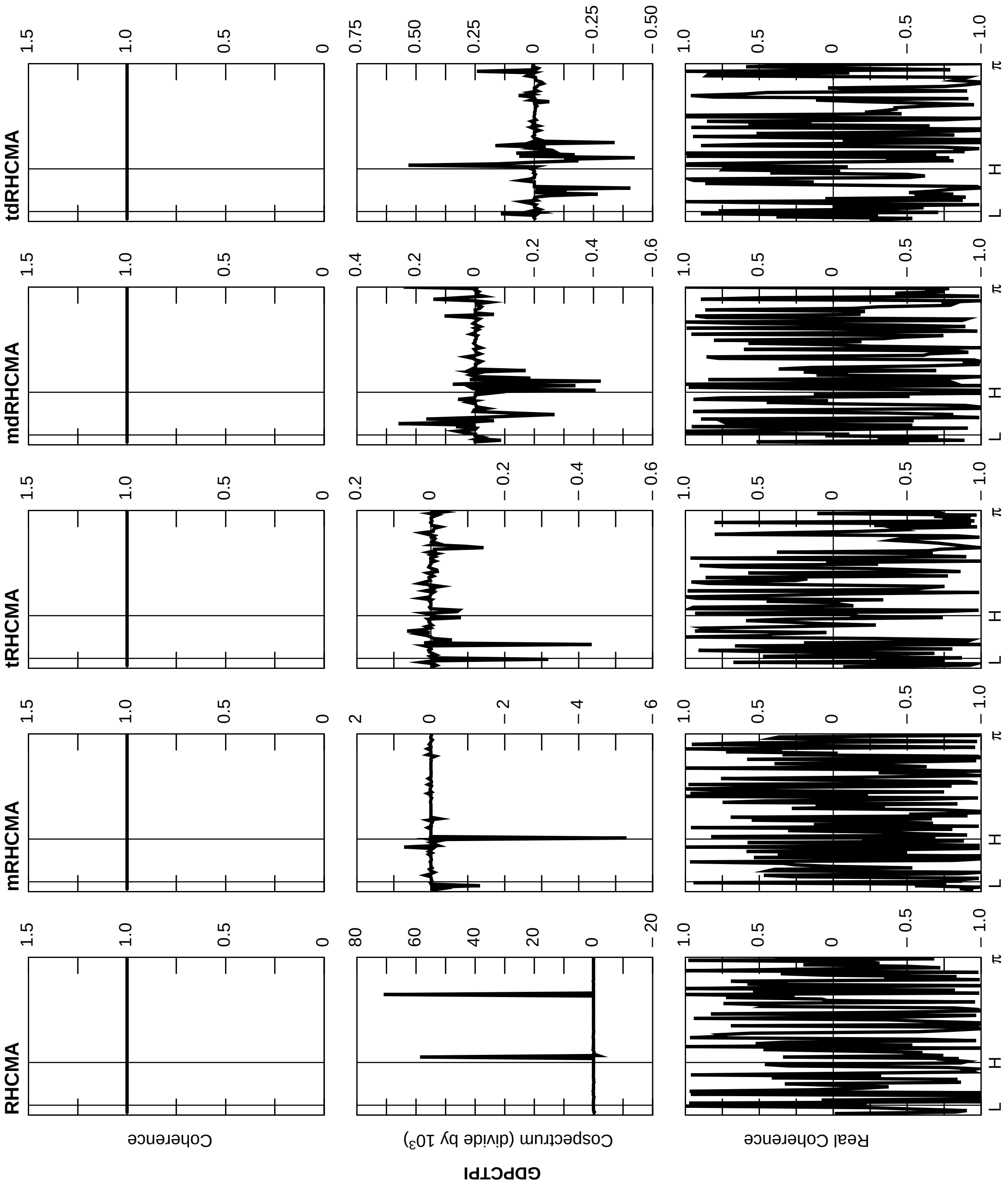

Figure 4: **Spectral indicators of Phillips correlations: RHCMA versus GDPCTPI**

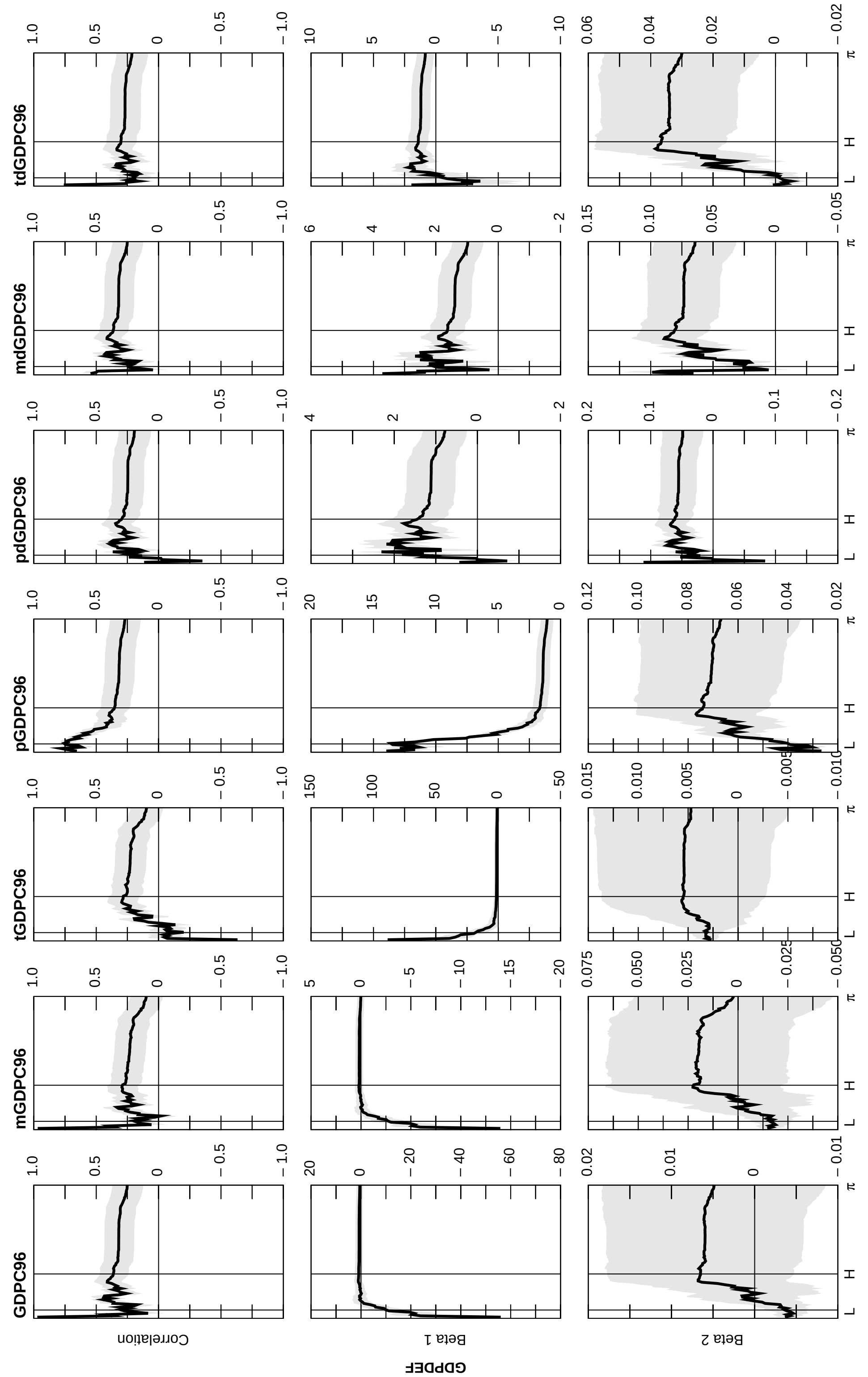

Figure 5: **Spectral indicators of Phillips correlations: GDPC96 versus GDPDEF**

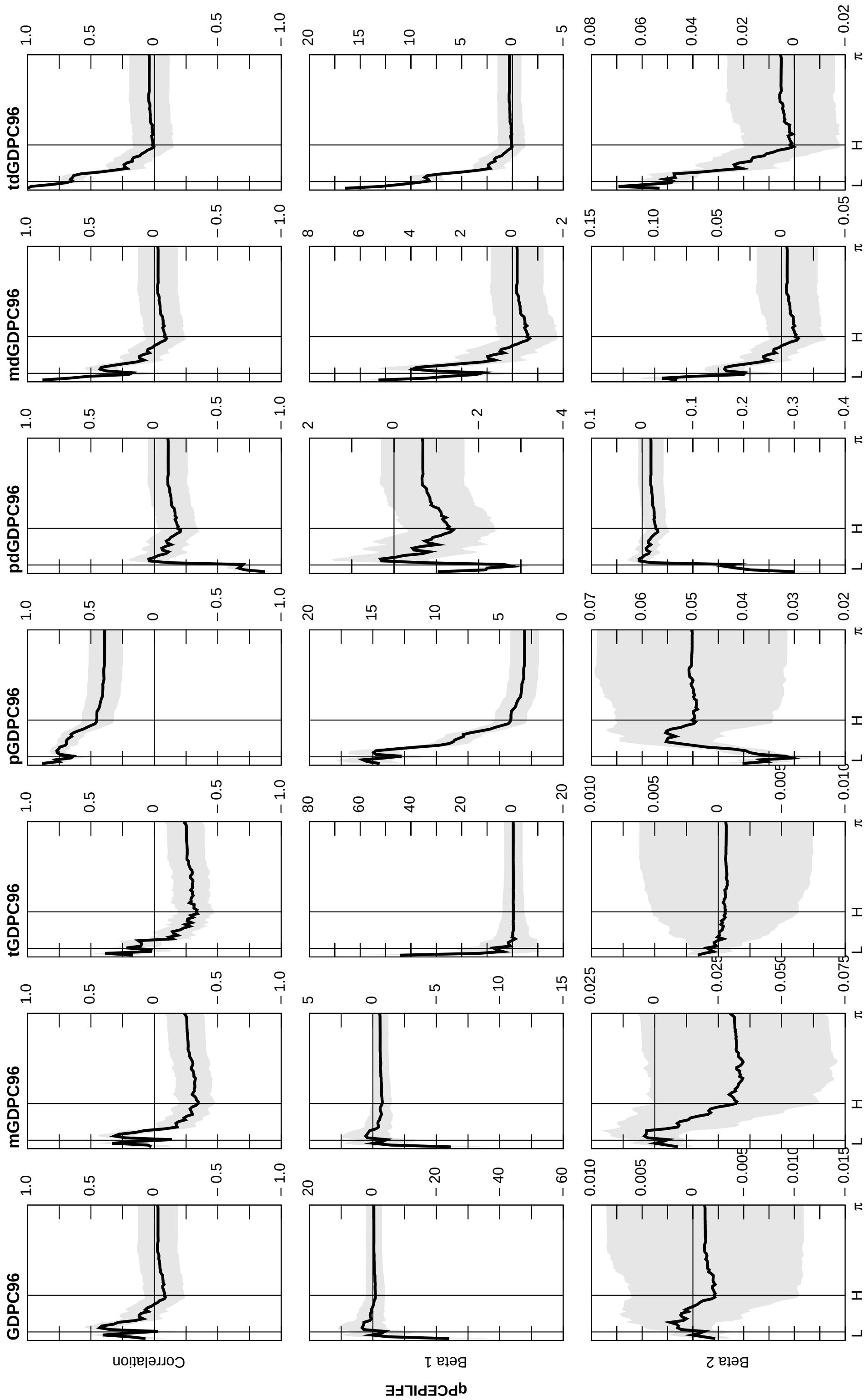

Figure 6: **Spectral indicators of Phillips correlations: GDPC96 versus qPCEPILFE**

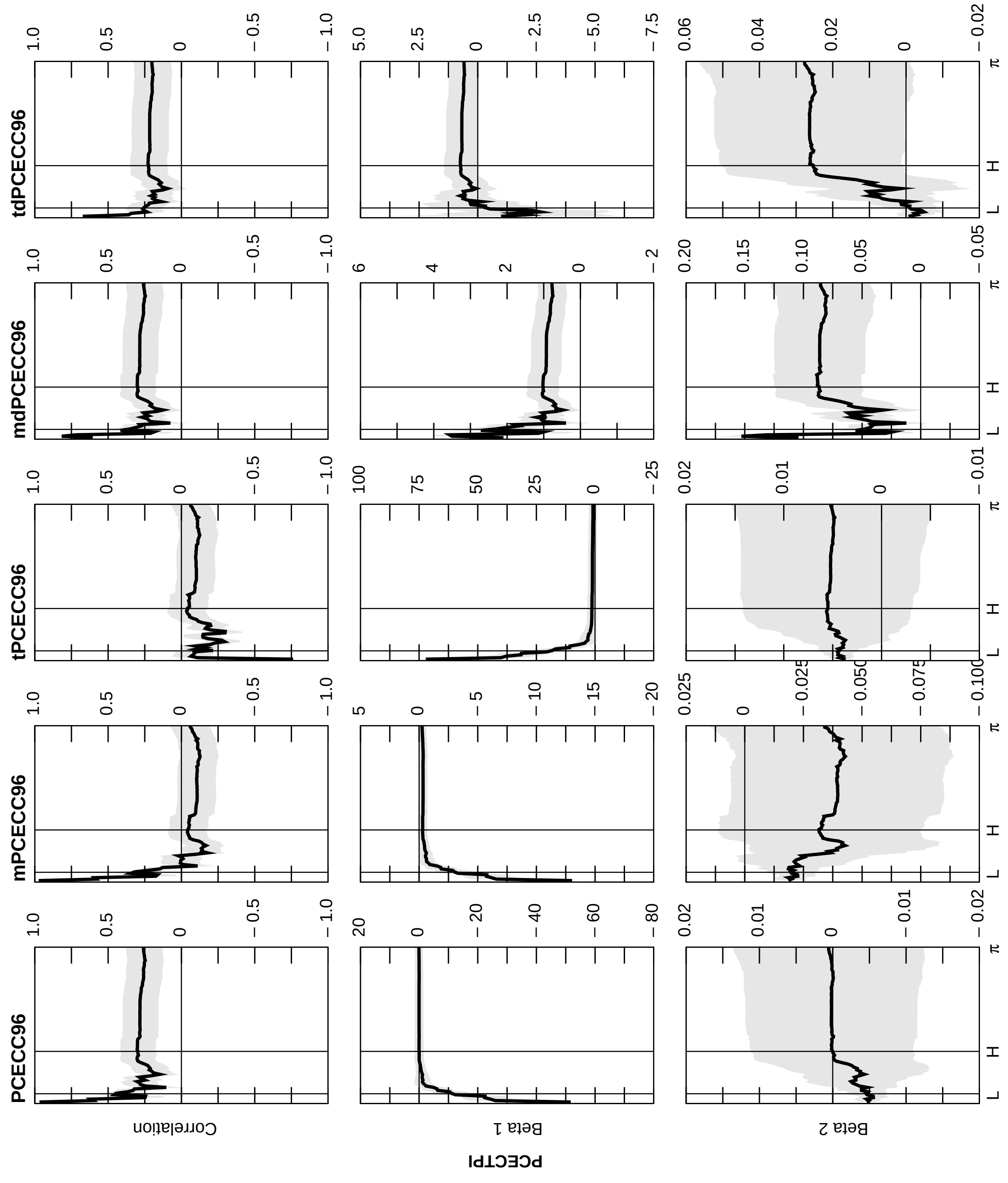

Figure 7: **Spectral indicators of Phillips correlations: PCECC96 versus PCECTPI**

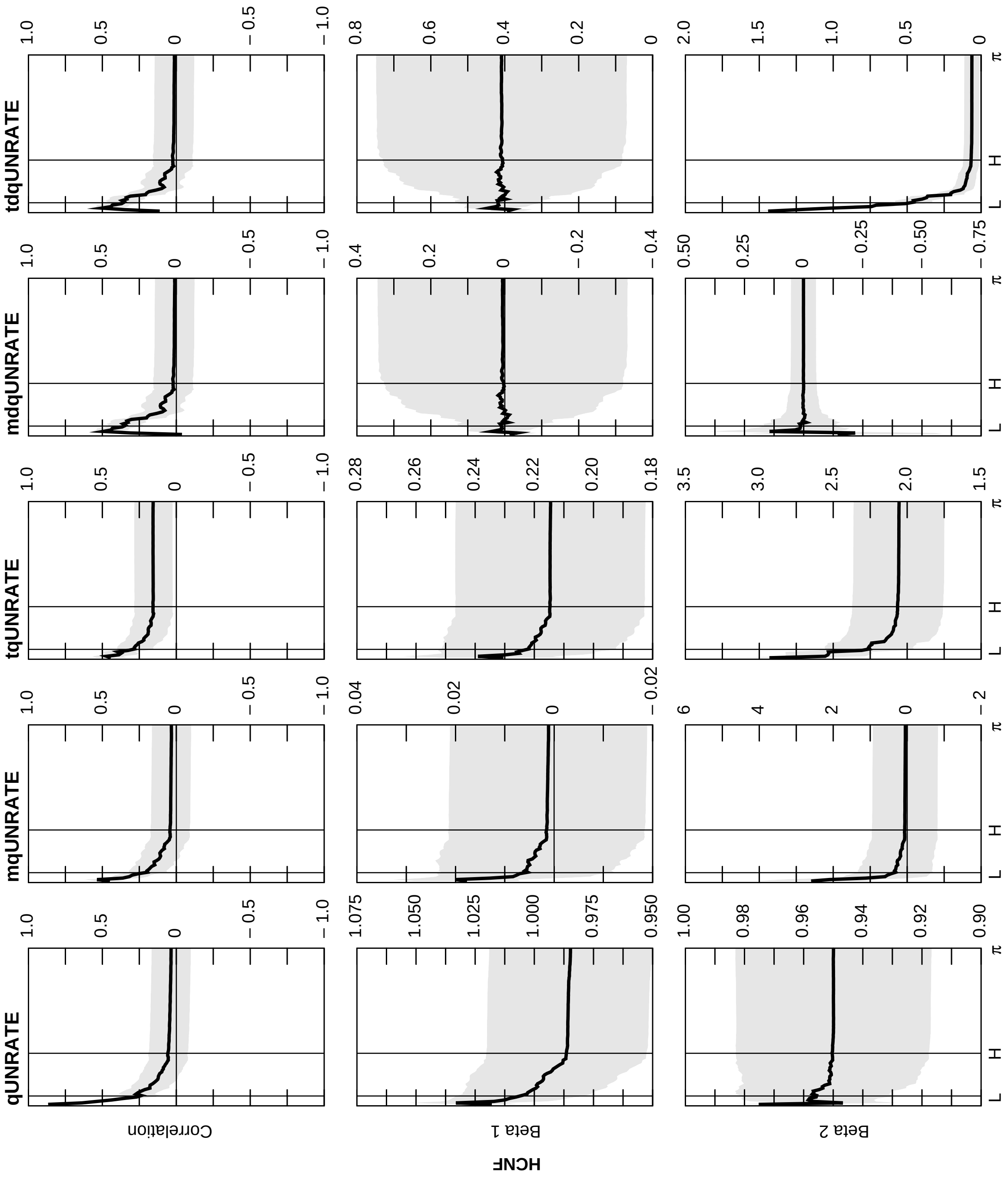

Figure 8: **Spectral indicators of Phillips correlations: qUNRATE versus HCNF**

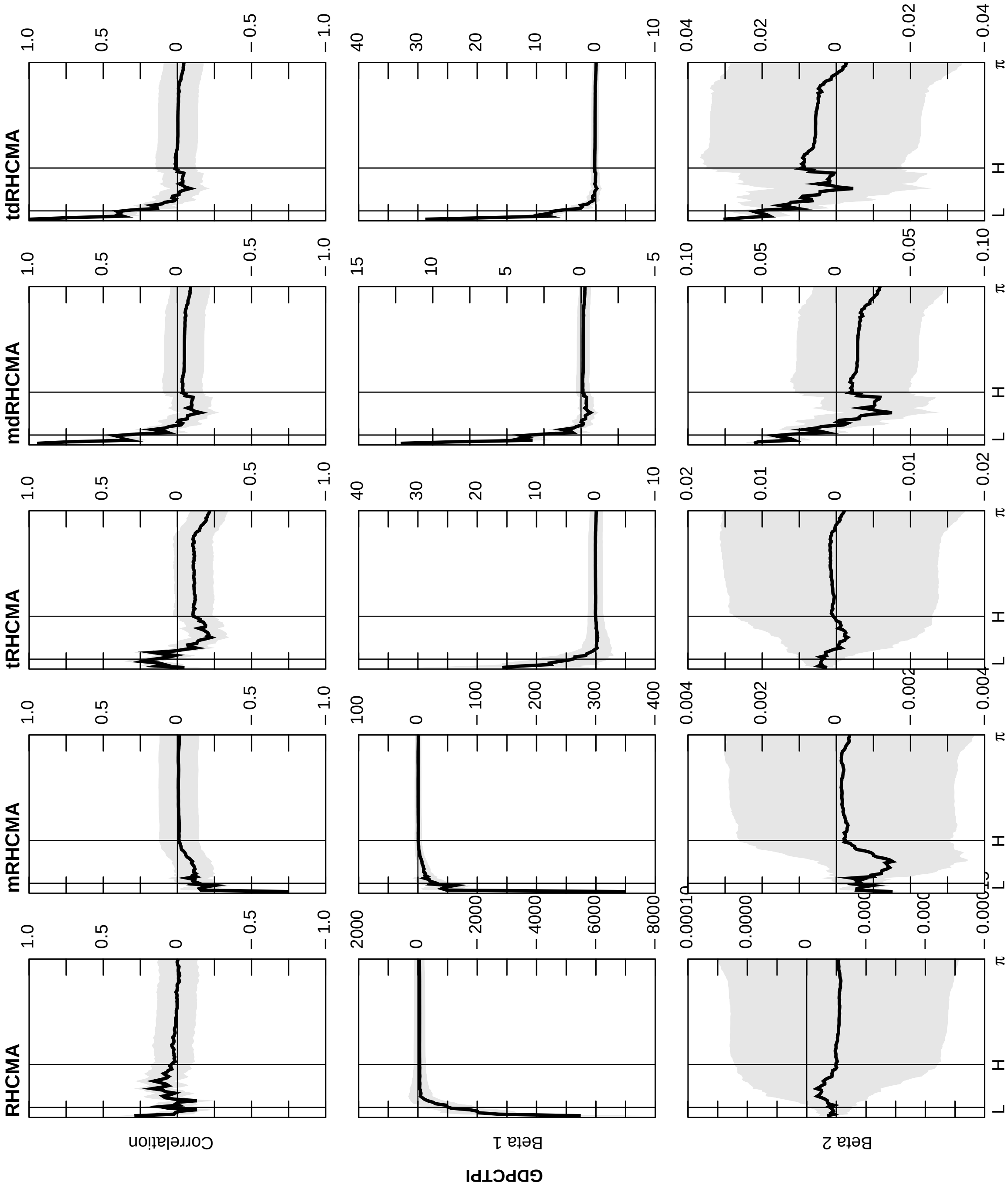

Figure 9: **Spectral indicators of Phillips correlations: RHCMA versus GDPCTPI**

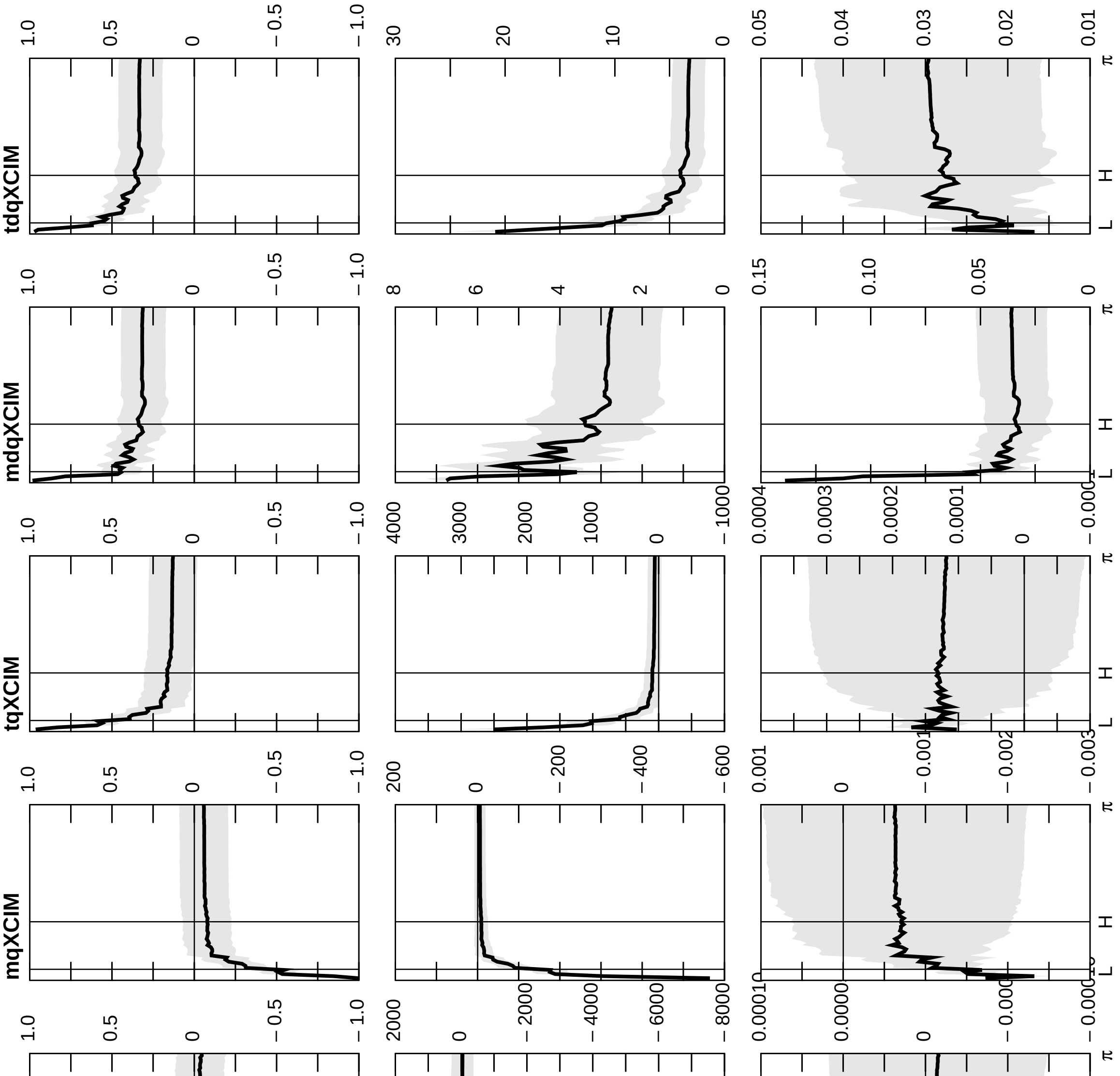

Figure 10: **Spectral indicators of Phillips correlations: qXCIM versus GDPCTPI**

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