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# Hard to Shift, Easy to Reshape: Central Bank Reserve Demand and Frictions*

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**Abstract**

How far can central banks shrink their balance sheets? The answer sets the limits to quantitative tightening (QT), and depends critically on the demand for reserves and whether the drivers widely thought to have increased it over the past decade actually do so. We assess the impact of three such drivers: post-crisis liquidity regulation, monetization frictions, and fragmented interbank markets. Building on the canonical Poole (1968) model, we show that these drivers tend to reshape, rather than horizontally shift, reserve demand. Liquidity regulation, such as the Liquidity Coverage Ratio (LCR), does not raise reserve demand when banks can substitute reserves with other high-quality liquid assets (HQLA). Over some regions of the curve, the LCR even reduces demand. Frictions in monetizing non-reserve HQLA into reserves change both the slope of the reserve demand curve and the satiation point when demand flattens. With fragmented interbank markets, the mapping from aggregate reserve supply to the interbank rate becomes non-unique. In this case, the effective reserve demand curve also shifts outward on impact of negative supply shocks, and the more so, the greater the initial level of supply. These findings have direct implications for balance sheet normalization, especially for central banks operating floor or ample reserves frameworks near the satiation point of the reserve demand curve.

**Keywords:** Reserve demand, balance sheet normalization, Liquidity Coverage Ratio, interbank market fragmentation, monetary policy implementation

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# 1 Introduction

A key question for many advanced economy central banks is currently how far they can shrink their balance sheets. It came to the forefront of the policy debate in 2022 when these central banks began quantitative tightening (QT), following nearly 15 years of expansions in the aftermath of the Great Financial Crisis (GFC) and the Covid epidemic (Figure 1). The debate has intensified again recently with the new Federal Reserve chairman, Kevin Warsh, establishing a task force to review the benefits and risks of the Fed's current balance sheet regime (Warsh (2026)). The answer to the question hinges on whether reserve demand has shifted outward compared to pre-expansion levels. Since there is empirical evidence that the demand for reserves has shifted outwards[1], several influential voices, including Miran (2026), Logan (2026b), and Duffie (2026), have argued that an effective approach to balance sheet reduction is to first shift the reserve demand curve inwards and then reduce supply.

This raises a deeper question: what determines the demand for reserves? And why has demand seemingly shifted in some economies such as the United States, the United Kingdom, or the euro area, whereas Korea has managed to operate with zero excess reserves above reserve requirements for the last 20 years (Figure 1)? Several reasons are at the heart of the current debate about why reserve demand may have increased.[2]

One set of arguments relates to liquidity risk and post-GFC liquidity regulation. Liquidity regulation compels banks to hold buffers of high-quality liquid assets (HQLA), with central bank reserves the pre-eminent one. The introduction of the Basel III Liquidity Coverage Ratio (LCR) is therefore widely assumed to have shifted the demand curve outward (e.g. Williams (2025), Logan (2023), Schnabel (2023), and Bailey (2024)). Monetization frictions may play an additional role. Reserves and other HQLA such as government bonds are not perfect substitutes for insuring against liquidity stress: the latter must first be converted into reserves, that is, monetized, before they can be used to meet outflows. The LCR therefore specifies expectations that banks can demonstrate their ability to monetize non-reserve HQLA.[3] But liquidity stress tests emphasize this even more since monetization is especially difficult in stress. Such stress tests are, for instance, mandated in the United States, and are hence seen by US banks as a key driver of their high reserve demand (Nelson et al. (2026)).[4] Moreover, monetization frictions may also have intensified given the size

[1]See e.g. Lopez-Salido and Vissing-Jorgensen (2025), Afonso et al. (2022), and Åberg et al. (2021).

[2]Anderson et al. (2026) review some of the most discussed drivers and how addressing them could reduce the demand for reserves in the United States.

[3]See §30, Basel Committee on Banking Supervision (2017).

[4]Internal liquidity stress tests (or ISLTs) are mandated under Regulation YY in the United States (Board of Governors of the Federal Reserve System (2019)). Part of the stress test is to demonstrate that non-reserve HQLA can be monetized. Initial supervisory practice was to focus on monetizing non-reserve HQLA in private markets. Since private markets may not have the capacity for monetizing a large volume of

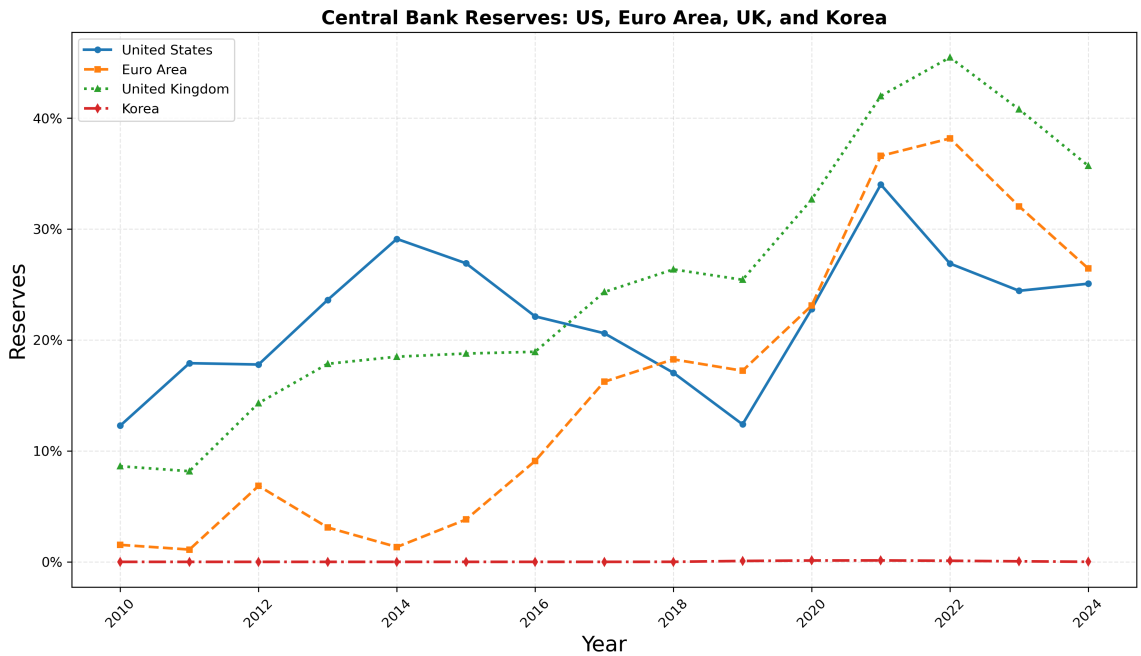


Figure 1: Central Bank Reserve Above Reserve Requirements in Different Countries

*Notes:* Reserves above reserve requirements are normalized by bank credit to the private non-financial sector. Source: Cavallino et al. (2025).

and speed of deposit runs observed in March 2023 (e.g. Hauser (2023) and Saporta (2024)). And a longstanding discussion in this context is also the role of stigma in hindering banks accessing the discount window.

Another set of arguments relates to the role of fragmented money markets and the payment flow volatility. In particular, evidence shows that banks have relied on their central bank accounts to manage payment flows rather than accessing money markets to manage imbalances (e.g. Markets Committee (2019), Thedeen (2025), and Asberg Sommar et al. (2026)). Hence, money markets may no longer effectively redistribute reserves across the banking system, possibly raising banks' precautionary demand (e.g. Schnabel (2023), Borio (2023), and Åberg et al. (2021)). And during a prolonged period of QT effects may be self-reinforcing: large excess reserve supply atrophies the interbank market, increasing the demand for reserves and further undermining interbank market functioning (e.g. Nelson (2025)). Another ratcheting-up effect could have also played a role, in that large central bank reserve supply via asset purchases increases flighty deposits (Acharya and Rajan (2022)), which in turn increase payment flow volatility and potentially the need for greater

government bonds to meet unexpected high outflows of a large GSIB in stress, this effectively required banks to hold reserves. Recent FAQs have clarified that banks can rely on Fed facilities or the Federal Home Loan Banks for monetization (Board of Governors of the Federal Reserve System (2024)), which we also assume in our model.

precautionary reserve buffers. High payment flow volatility more generally, including how this relates to the design of the payment system, has also been argued as another structural driver of the demand for reserves (Logan (2026a) and Duffie (2026)).

Despite the pressing policy debate, there is limited theoretical understanding of how the various factors highlighted in the debate affect the demand for reserves. This paper provides an analytical framework for studying these mechanisms.

To address these questions, we build on the canonical Poole (1968) model. In this baseline model, banks choose between holding reserves on their balance sheet and trading reserves in the interbank market before idiosyncratic late day payment shocks are realized. Banks are subject to a minimum reserve requirement. If payment shocks leave their reserve balance below (above) this requirement, they borrow the shortfall from the discount window (deposit the excess at the deposit facility). This delivers the familiar S-shaped reserve demand curve.

We use the baseline model to illustrate two features of the current policy debate that are well understood. First, an increase in the reserve requirement raises banks' reserve demand one-for-one at every interbank rate, i.e. it shifts the curve horizontally. This is independent of whether the reserve requirement is imposed by the central bank, or by regulation and supervision to address banks' liquidity risks. Second, stigma makes reserve shortfalls more costly thereby increasing the insurance value of reserves while leaving unchanged the opportunity cost of holding reserves rather than lending in the interbank market. Hence, the upper bound of the reserve demand curve increases and the slope changes (see e.g. Whitesell (2006)).

We expand the model to make several contributions to the literature.

Our first contribution is to study the effects of the LCR and to show that it does not raise reserve demand. In contrast to the literature, we capture the institutional design that the LCR is not a hard requirement on reserve holdings but on end-of-day HQLA holdings, i.e. reserves plus other eligible HQLA such as government bonds. We expand the baseline model and allow banks to optimize their portfolios over reserves, government bonds and higher-yielding assets and then study the effects of introducing an LCR. Since banks can satisfy the LCR with either reserves or other HQLA, the requirement does not mechanically change the marginal tradeoff that pins down reserve demand. Below a threshold-level of reserves, the reserve requirement (which can be zero) is the binding constraint. In that region, banks can adjust the composition of HQLA while keeping the LCR slack and maintaining their unconstrained optimal holdings of risky assets. Reserve demand is therefore unchanged. Above the threshold, the LCR becomes the tighter effective constraint than the reserve requirement. To avoid this, banks optimally reduce their holdings of non-HQLA assets. The opportunity cost of holding additional reserves therefore includes not only the forgone interbank return, but also the return forgone on higher-yielding non-HQLA assets. This portfolio tradeoff lowers reserve demand over that region, reshapes the demand curve, and moves the satiation point inwards. But it does not shift the reserve demand curve uniformly.

Our second contribution is to analyze monetization frictions, which do not shift but reshape the reserve demand curve. As discussed, reserves and other HQLA are not perfect substitutes for liquidity-risk management because non-reserve HQLA must be monetized into reserves before they can settle payment flows. As in liquidity stress tests, we capture this by assuming that banks may be unable to monetize government bonds into reserves in private markets in the future. To model this, we extend the baseline model to an infinite horizon. In this case, monetization frictions make reserves valuable not only as insurance against today's payment shocks, but also against future ones when reserves are needed and government bonds cannot be monetized. This asymmetry makes reserves special relative to other HQLA and adds a continuation-value term to the marginal value of reserves. This additional value, however, does not translate into a simple outward shift of reserve demand. Instead, monetization frictions change the shape of the curve, including its slope and the satiation point. When combined with the LCR, reserve specialness does not overturn our earlier result on liquidity regulation. The LCR continues to operate through the portfolio tradeoff and can lower reserve demand relative to the no-LCR benchmark.

Our third contribution is to study the effects of fragmented interbank markets, which leads to uncertain reserve demand for a given level of reserve supply. We first model fragmentation by assuming that a fixed fraction of banks are inactive in money markets and rely solely on their central bank accounts to manage payment flows. Inactive banks become de facto "autonomous factors" in the parlance of central banks.[5] Hence, their payment flows become effectively reserve-supply shocks for active banks. We show that this gives rise to an equilibrium in the money market that can be interpreted as resulting from a shift in the reserve demand curve in line with the payment shock. Standard reserve demand curves trace out the equilibrium interest rate for a given supply of reserves by the central bank. In our case, the aggregate reserve supply remains fixed and the payment shocks change the quantity of reserves that active banks have to intermediate, hence the effective reserve curve shifts. And the shift gets larger, the larger the share of inactive banks because each active bank then has to absorb a greater share of the payment shock of the inactive banks. Since payment flows are random, fragmentation breaks the one-to-one mapping between aggregate reserve supply by the central bank and the interbank equilibrium interest rate.

We also show that fragmentation increases endogenously with supply. This is the case if banks endogenously decide to be active or inactive because of e.g. different participation costs. Reasons for such different participation costs could be operational, because banks have

[5] Autonomous factors change the aggregate supply of reserves independent of central bank monetary policy operations. Autonomous factors include bank notes in circulation, FX reserves or, if the treasury banks with the central bank, the treasury account.

different IT systems or abilities to maintain staff experience,[6] or risk management frameworks that differ and may hinder some banks from trading.[7] We show that with endogenous participation decisions, the share of inactive banks rises as balance sheets increase. This in turn increases the uncertainty around the reserve demand curve far into the region where the reserve demand curve would normally be anchored at the deposit facility rate.

The final contribution of the paper is to show that the effective reserve demand curve shifts outwards on impact of negative supply shocks, and the more so, the greater the initial level of supply. The reason is that with endogenous fragmentation, the share of active/inactive banks does not adjust in response to shocks. Hence, active banks must absorb not only the random payment shocks but also the change in the supply, which is equivalent to an outward (inward) shift in the reserve demand curve if there is a negative (positive) supply shock. And the implied shift in the reserve demand curve is larger, the smaller the endogenously determined fraction of active banks, i.e. the larger the initial level of supply.

An interesting feature of the model is that the endogenous outward shift in the demand curve is not fixed but depends on the unexpected change in the supply of reserves. Fragmentation in general implies uncertainty about the reserve demand curve because of random payment flows of inactive banks. If these were absent, the reserve demand curve would be the same as in the baseline model even with fragmentation. But this is not so, if there are unexpected changes in the supply of reserves. In this case, the decision to be inactive was based on the supply of reserves before the shock, so that the shock can only be absorbed by the then active banks. This, in turn, gives rise to an outward shift in the reserve demand curve for a negative supply shock, which can be substantial if there are only few active market participants.

The implications of our findings for central banks can differ depending on their operational framework. In particular, it can matter whether the central bank implements monetary policy in the steep part of the reserve demand curve, as in a traditional corridor system, or in the flat part, traditionally labeled a floor system.[8] Several central banks operate such floor systems currently, with different characteristics and labels. For instance, the US Fed implements an ample reserves framework, which operates on the flat part of the reserve demand curve close to the satiation point.[9]

[6]These factors have been shown to impact the money market ecosystem over periods of large balance sheet expansions (e.g. Markets Committee (2019))

[7]Afonso et al. (2020a) find that money market desks typically require approvals to exceed internal limits, which may slow banks' response to changing market conditions, especially in repo markets that trade early in the morning.

[8]The standard classification of operating frameworks into corridor, floor and ceiling is too coarse to capture the richness of modern frameworks. Cavallino et al. (2025) provide a new classification based on the size of excess reserves and opportunity costs.

[9]The Bank of England, the ECB, the Reserve Bank of Australia, or the Bank of Canada implement a so-called demand driven framework. Such a system allows banks to demand as many reserves as they want,

With respect to the ongoing policy debate, several conclusions for policy makers stand out from our paper.

First, the LCR or any other liquidity requirement that can be fulfilled by non-reserve HQLA has no major impact on the reserve demand curve. It does not shift the reserve demand curve. It may marginally matter for central banks operating close to the satiation point as this moves inward, allowing them to operate with fewer reserves, if anything.

Second, requirements or supervisory actions that force banks to hold a fixed amount of reserves are nothing else than a reserve requirement in another form. Hence, reducing them reduces the demand for reserves. If for prudential reasons, holding reserves as part of liquidity buffers is considered important, this should be made explicit and communicated to the central bank. If not, such requirements or supervisory actions act as implicit, unobservable reserve requirements, shifting reserve demand and possibly hampering the implementation of monetary policy.

Third, with monetization frictions, the reserve demand curve becomes more elastic and the satiation point moves outward. These effects are therefore especially relevant for central banks operating on the flat part close to the satiation point such as with abundant reserves. In this case, reducing monetization frictions, for instance by improving the depth and resilience of repo and government bond markets, would lower the satiation point and allow central banks to operate with fewer reserves.

Fourth, fragmentation breaks the one-to-one mapping between aggregate reserve supply and the equilibrium interest rate and can lead to an endogenous shift in the reserve demand curve. Policy makers should therefore reduce the risk of fragmentation to ensure that banks do not drop out and that all banks are willing and do trade in money markets to ensure a smooth redistribution of reserves.

Fifth, the endogenous shifts in the reserve demand curve depend on shocks to reserve supply. And the risk that these undermine monetary policy implementation is particularly pronounced during balance sheet normalization after long periods when money markets traded at the deposit rate. At this point, the share of active banks is lowest, implying the largest outward shift in the reserve demand curve for a given negative supply shock. But this may put central banks that normalize balance sheets while operating a floor system in a bind if they want to reduce fragmentation. To incentivize participation in the interbank market, gains from trading in money markets would need to increase, which in turn requires the reduction of supply. But this could undermine monetary policy implementation as the reserve demand curve shifts outward in an unobserved fashion if there are negative supply shocks and banks do not reconsider their participation decisions.

typically once a week. Our model has nothing to say how this demand is determined. But our insights are applicable once supply is determined. And experience so far shows that these systems also operate on the flat part of the demand curve close to the satiation point.

Finally, our results allow for a radical interpretation. While we do not take a stance whether this is optimal or not, none of the factors discussed in this paper prevent central banks from moving back to pre-GFC operational frameworks.[10] Indeed, the Central Bank of Korea has successfully implemented monetary policy over the last 20 years with a very low quantity of reserves and high opportunity costs between money market rates and the deposit rate at the central bank. As discussed, the LCR does not shift the reserve demand curve and thereby has no implications for corridor systems. If money markets are deep, monetization frictions are very low and without material effect on reserve demand. And once opportunity costs are high enough, fragmentation disappears. This transition can also be managed if announced well in advance. Banks then have time to build up their trading activities so that they can adjust to the new environment without the risk of shifts in reserve demand because of supply shocks. Such a transition was for instance successfully undertaken when Norway moved away from a floor system in 2011, significantly reducing reserve supply (see Norges Bank (2014)).

This paper connects to a large literature on the demand for reserves in the tradition of Poole (1968), in which banks hold reserves to insure against payment shocks that would otherwise force costly recourse to the central bank.[11] We add to this work by analyzing liquidity regulation, including monetization frictions, and interbank fragmentation and assessing how these affect the reserve demand curve.[12]

Our contribution to understanding the effects of liquidity regulation is to fully capture the institutional set-up of the LCR that allows banks to optimize across their pool of all HQLA. Recent work that mainly focuses on estimating the demand for reserves, such as Afonso et al. (2022) or Lagos and Navarro (2025), captures liquidity regulation in reduced form via increased reserve requirements. As such, the outward shift in demand is essentially assumed rather than derived. Similarly, d'Avernas et al. (2025) assume binding intraday constraints, motivated by internal liquidity stress tests. In contrast, Bech and Keister (2017), assess the effects of the LCR in a Poole-type model, where they allow banks to meet the LCR with either reserves or government bonds. However, they take bond holdings as exogenous, so that reserves are effectively the only margin for closing an LCR shortfall and the constraint behaves in some states of the world like a reserve requirement. Instead, we allow banks to optimize across their whole portfolio, including reserve, non-reserve HQLA and other investments. We also capture the key institutional feature that the LCR is an end-of-day requirement. As banks can now freely optimize across reserves and government bonds, the

[10]For various arguments on the optimal size of the central bank balance sheet, see, for example, Greenwood et al. (2016), Goodhart (2017), Borio (2023), and Williams (2025).

[11]For an overview, see Bindseil (2014).

[12]A separate strand, which we do not address, concerns leverage-ratio requirements; see, e.g., Kim et al. (2020), d'Avernas and Vandeweyer (2024), Andersen et al. (2019), and Duffie and Krishnamurthy (2016).

LCR leaves reserve demand mainly unchanged, unless the LCR becomes the tighter effective constraint than the reserve requirement, which shifts the satiation point inward. In contrast to other papers in the literature, we also take account of monetization frictions. Explicitly modeling these monetization frictions is revealing as they are captured in internal liquidity stress tests.

Our analysis on fragmentation connects to the search-and-matching literature on interbank markets, starting with Afonso and Lagos (2015a) and Afonso and Lagos (2015b). In this literature, all banks are potentially active but bilateral trade is subject to search frictions so that a bank may fail to find a counterparty. Search and matching frictions as well as central bank access may also differ for different counterparties (e.g. Lagos and Navarro (2025)). We take a different route and assess the effects if the participation status is fixed for the trading day to match the empirical observations of banks becoming fully inactive after long periods of large balance sheets.[13] As random payment flows to inactive banks now determine how much active banks need to absorb, this breaks the one-to-one mapping from aggregate reserve supply to the interbank rate. Moreover, when participation is endogenous, fragmentation increases with reserve supply, effectively shifting reserve demand outward if there are negative supply shocks. This is an important difference from Bianchi and Bigio (2022). In their work, the random distribution of reserves via payment shocks also breaks the one-to-one mapping from supply to the interbank rate. And higher supply also endogenously affects this equilibrium schedule as it reduces money market tightness in their paper. But in their work, the location of the reserve demand curve does not change (given a payment shock).[14] It is only reshaped, becoming steeper in the steep part of the curve and the satiation point shifting inwards.

Our paper has also implications for the empirical literature. For one, our model provides an interesting lens on the mid-September 2019 events in US money markets (Afonso et al. (2020a)). At that point, money market rates unexpectedly surged around large payments to the US treasury. The model would suggest that a large share of inactive banks played a key role. While the treasury payments were expected, banks seemingly did not update their participation decision, so that the reserve demand curve governing the equilibrium in money markets shifted outward in an unexpected fashion. Fragmentation more generally challenges estimations of demand curves based on aggregate data, as e.g. Hamilton (1997) and Lopez-Salido and Vissing-Jorgensen (2025). But even if more granular data are used as e.g. in Afonso et al. (2022), Lagos and Navarro (2025), and Anbil et al. (2026), estimated demand curves will be imprecise, unless fragmentation and the resulting reserve supply shocks from

[13] Åberg et al. (2021) discuss fragmentation in a manner close to ours. These authors do not, however, provide a theoretical model.

[14] Technically, in Bianchi and Bigio (2022) there are also multiple reserve demand curves over the course of the various rounds of search and matching.

inactive banks are taken into account.

The remainder of the paper is organized as follows. Section 2 presents the baseline model and derives the Poole benchmark with mean-variance preferences. Section 3 introduces the LCR alongside non-HQLA assets and characterizes reserve demand under post-crisis liquidity regulation. Section 4 extends the model to an infinite-horizon environment and studies monetization frictions: non-reserve HQLA cannot always be converted into reserves, while reserves can always be converted into non-reserve HQLA. It shows that the superior liquidity of reserves moves the demand curve slightly to the right but still does not make the LCR behave like an additional reserve requirement. Section 5 analyzes interbank market fragmentation. Finally, Section 6 concludes.

# 2 Baseline Model

This section develops the paper's baseline bank reserve demand problem. We solve for the aggregate reserve demand curve for the banking sector. The setup is a version of Poole (1968) with a local CRRA mean–variance approximation where banks care about the variance in addition to the expected value of end-of-day portfolio returns.

## 2.1 Environment

Consider a single-day economy with a continuum of ex-ante identical banks of total measure one. The timeline within the day is as follows.

- **Start of day.** Banks begin the day with the balance sheet size $W$. Each bank chooses its target balance sheet composition: reserve holdings $R$, risk-free government bond holdings $G$, and the residual interbank position $W - R - G$, which is borrowed or lent in a competitive interbank market at rate $i$. Government bonds earn the same return as the interbank position by arbitrage. We work with variables normalized by start-of-day balance sheet size $W$: $r = R/W$ is the reserve share, $g = G/W$ is the government bond share, and $1 - r - g$ is the interbank position share.

- **Intraday.** During the day, banks trade in the secured interbank market to accommodate ordinary payment flows and adjust their balance sheet toward the target. After the interbank market closes, however, each bank faces an idiosyncratic late day payment shock $z = W\zeta$, where $\zeta$ is the shock as a share of balance sheet size, drawn independently across banks from distribution $F$ with density $f$ and mean zero. A positive realization is a reserve inflow and a negative realization is a reserve outflow. After the shock, a bank enters the end-of-day period with reserve balance $W(r + \zeta)$.

- **End of day.** Banks must hold reserves of at least $\bar{X} = \bar{x}W$, where $\bar{x}$ is the required reserve share measured relative to start-of-day balance sheet size. If reserves are insufficient, banks borrow the shortfall from the discount window at rate $i^{DW}$. Reserve balances earn interest at rate $i^{IORB}$. The offsetting claim or liability generated by the payment shock earns or pays the rate $i^{IORB}$.

## 2.2 Bank Problem

Throughout, $W$ denotes the bank's start-of-day scale variable; we refer to it interchangeably as wealth or balance sheet size. The bank chooses portfolio shares to maximize expected utility over end-of-day wealth $\tilde{W}$,

$$\max_{r,g} \mathbb{E}\big[u(\widetilde{W}(r,\zeta))\big].$$

Banks have CRRA preferences, with coefficient of relative risk aversion $\gamma \geq 0$. End-of-day wealth can be written as

$$\widetilde{W}(r,\zeta) = W\left[1 + i + y(r,\zeta)\right], \qquad y(r,\zeta) \equiv \eta r - \chi b(r,\zeta).$$

where discount window borrowing as a share of $W$ is

$$b(r,\zeta) \equiv (\bar{x} - r - \zeta)^{+},$$

and

$$\eta \equiv i^{IORB} - i, \qquad \chi \equiv i^{DW} - i^{IORB}.$$

Here $\eta$ captures the interest foregone from holding reserves rather than lending in the interbank market, while $\chi$ captures the cost of discount window borrowing relative to the rate on reserves.

Approximating expected CRRA utility over end-of-day wealth to second order in the portfolio return component $y(r,\zeta)$, and dropping constants that do not affect portfolio choice, the bank's objective becomes

$$\max_{r} \mathbb{E}[y(r,\zeta)] - \frac{\gamma}{2}\mathrm{Var}(y(r,\zeta)), \qquad \gamma > 0. \tag{1}$$

Appendix A derives the first-order condition for the bank's problem. The baseline reserve demand curve is characterized in the following proposition.

**Proposition 2.1** (Baseline Reserve Demand)**.** *The bank's reserve demand is characterized* *by*

$$0 = (i^{IORB} - i) + (i^{DW} - i^{IORB})F(\bar{x} - r) \\ + \gamma\chi^2 m(\bar{x} - r)\big(1 - F(\bar{x} - r)\big), \tag{2}$$

*where*

$$m(k) \equiv \int_{-\infty}^{k} (k - \zeta) f(\zeta)\, d\zeta$$

*denotes expected discount window borrowing at shortfall threshold $k$.*

The bank chooses reserve holdings such that the marginal value of reserves is zero. The first line of equation (2) reflects the expected return tradeoff: holding more reserves reduces expected discount window borrowing after payment shocks, but also forgoes the return from lending in the interbank market. The second line captures the effect of reserves on the volatility of end-of-day wealth. This variance margin becomes more important as risk aversion $\gamma$ increases.

Government bond holdings do not directly affect the banks' end-of-day wealth because they earn the same rate as the interbank positions. Banks' government bond holdings $g$ are thus pinned down residually after the market clearing of interbank positions that is in net-zero supply.

Because the optimal reserve share does not depend on bank size, every bank chooses the same reserve share $r(i)$ at interbank rate $i$. Aggregate reserve demand as a share of total banking-sector balance sheet size coincides with the individual bank's reserve demand:

$$\frac{\int r(i) W_j\, dj}{\int W_j\, dj} = r(i),$$

where $W_j$ denotes bank $j$'s start-of-day balance sheet size.

Let $S$ denote aggregate reserve supply chosen by the central bank, and define reserve supply as a share of total banking-sector balance sheet size by $s \equiv \frac{S}{\int W_j\, dj}$. An equilibrium is an interbank rate $i^*$ such that aggregate reserve demand equals aggregate reserve supply:

$$r(i^*) = s.$$

## 2.3 The Poole Benchmark

From Proposition 2.1, setting $\gamma = 0$ so that banks care only about expected end-of-day wealth yields the classic Poole reserve demand condition:

$$0 = (i^{IORB} - i) + (i^{DW} - i^{IORB})F(\bar{x} - r). \tag{3}$$

When choosing reserve holdings, banks trade off the opportunity cost of holding reserves rather than lending in the interbank market, $i - i^{IORB}$, against the expected benefit of avoiding discount window borrowing. The latter equals the gain $i^{DW} - i^{IORB}$, weighted by the probability $F(\bar{x} - r)$ that payment shocks push the bank into discount window borrowing at the margin.

The resulting reserve demand curve is

$$r(i; \bar{x}, i^{DW}, i^{IORB}) = \bar{x} - F^{-1}\left(\frac{i - i^{IORB}}{i^{DW} - i^{IORB}}\right). \tag{4}$$

Figure 2 plots the Poole benchmark reserve demand curve.[15] As in Poole (1968), the curve is S-shaped. When reserves are scarce, the marginal unit of reserves mainly substitutes for discount window borrowing, so its value is pinned near $i^{DW}$. When reserves are abundant, the marginal unit is deposited at the central bank, so its value is pinned near $i^{IORB}$. The curve is steepest in the intermediate range, where the probability of discount window borrowing is most sensitive to reserve balances.

Several well-known corollaries result from the standard Poole model.

**Corollary 2.2** (Reserve Requirements)**.** *For any change $h$ in the required reserve share, the Poole reserve demand curve in equation (4) satisfies*

$$r(i; \bar{x} + h, i^{DW}, i^{IORB}) = r(i; \bar{x}, i^{DW}, i^{IORB}) + h.$$

The corollary shows that a change in reserve requirements alters the effective discount window shortfall threshold, shifts the reserve demand curve horizontally one-for-one, without changing its slope or curvature. The higher-reserve-requirement curve in Figure 2 illustrates this implication: raising $\bar{x}$ moves the reserve demand curve to the right without changing its shape.

In the context of discussing liquidity regulation, the corollary is important insofar as it clarifies that any liquidity regulation that explicitly requires banks to hold *reserves* is economically equivalent to a reserve requirement. This implication is distinct from that of liquidity regulation that can be satisfied with either reserves or non-reserve HQLA, even when the two assets are imperfect substitutes. We study such liquidity regulation in the main part of the paper.

**Corollary 2.3** (Discount Window Stigma)**.** *Suppose discount window borrowing carries a*

[15] Although equation (4) writes reserve demand as $r(i)$, the figures plot the inverse representation, with reserve balances on the horizontal axis and the interbank rate on the vertical axis. We refer to this plotted object as the reserve demand curve.

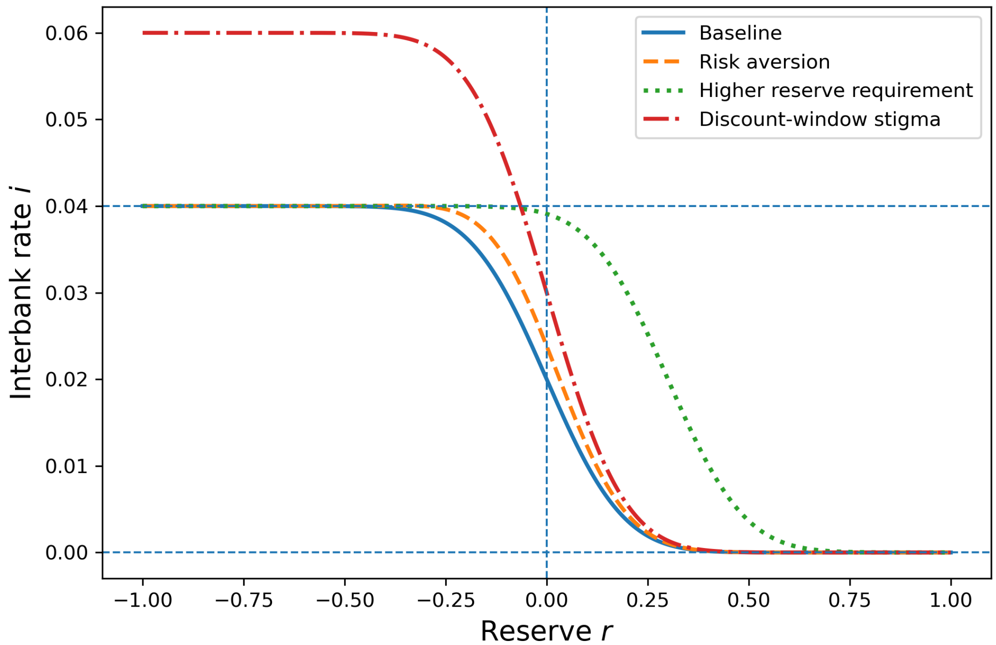


Figure 2: Reserve Demand Curve (Baseline)

*Notes:* The figure plots the reserve demand curve implied by equation (2). The baseline curve sets $\gamma = 0$, $i^{DW} = 4\%$, $i^{IORB} = 0\%$, $\bar{x} = 0$, and $\zeta \sim N(0, 0.15^2)$. The risk-aversion curve changes only $\gamma$ from 0 to 80. The higher-reserve-requirement curve changes only $\bar{x}$ from 0 to 0.3, illustrating the horizontal shift in Corollary 2.2. The discount window-stigma curve raises the effective discount window rate from 4% to 6%, holding the other baseline parameters fixed, as in Corollary 2.3.

*stigma cost $\kappa \geq 0$ per dollar borrowed. Then the Poole reserve demand curve becomes*

$$r(i; \bar{x}, i^{DW} + \kappa, i^{IORB}) = \bar{x} - F^{-1}\left(\frac{i - i^{IORB}}{i^{DW} + \kappa - i^{IORB}}\right).$$

It has long been understood (e.g. Whitesell, 2006) that discount window stigma is equivalent to an increase in the effective discount window rate from $i^{DW}$ to $i^{DW} + \kappa$. By making reserve shortfalls more costly, stigma raises the marginal insurance value of reserves while leaving unchanged the opportunity cost of holding reserves rather than lending in the interbank market. The discount window-stigma curve in Figure 2 illustrates this upward reshaping of inverse reserve demand. The curve is higher especially in the scarce-reserve region, where shortfalls are more likely, and its slope and curvature change because of the extra insurance value.

### 2.4 Risk Aversion and the Shape of Reserve Demand

**Corollary 2.4** (Risk Aversion). *The effect of reserve holdings on the variance of end-of-day wealth is weakly negative:*

$$\frac{d}{dr}\operatorname{Var}\big(y(r,\zeta)\big) \le 0.$$

*Let $i(r;\gamma)$ denote the inverse reserve demand curve implied by equation (2). Then*

$$\frac{\partial i(r;\gamma)}{\partial \gamma} \ge 0.$$

The corollary shows that risk aversion adds a variance margin to the classical tradeoff. Holding more reserves lowers banks' exposure to costly discount window borrowing induced by late day payment shocks and therefore weakly reduces the volatility of end-of-day wealth. The risk-aversion curve in Figure 2 illustrates the resulting reshaping: risk aversion raises banks' willingness to bear the opportunity cost of holding reserves, with the size of this effect varying with the extent to which additional reserves reduce payment-shock risk. Risk aversion therefore changes the slope and the satiation region rather than shifting the curve horizontally.

## 3 Liquidity Regulation

In this section, we incorporate liquidity regulation and assess its effects on reserve demand. Before we do so in Section 3.2, we expand the asset menu to include non-HQLA risky assets, such as corporate bonds in Section 3.1.

### 3.1 Risky Higher-Yielding Assets

In this section, we expand the baseline model by allowing banks to invest in risky, illiquid assets. For simplicity, we label them as corporate bonds. Let $C$ denote corporate bond holdings and $c = C/W$ their balance sheet share. Corporate bonds earn excess risky return over the interbank rate, $i + \tilde{\epsilon}_c$, where $E[\tilde{\epsilon}_c] = \epsilon_c > 0$ and $Var(\tilde{\epsilon}_c) = \sigma_c^2 > 0$. Corporate bond return shocks $\tilde{\epsilon}_c$ are independent of the late day payment shocks. Throughout Section 3, we assume $\gamma > 0$, so that the risky-asset portfolio problem has a finite interior solution. We also assume that banks can lever up in the money market from outside investors.

Relative to the baseline problem, corporate bonds add the excess-return term $c\tilde{\epsilon}_c$ to the bank's portfolio return. The bank's problem becomes

$$\max_{r,c} \mathbb{E}[y(r,\zeta) + c\tilde{\epsilon}_c] - \frac{\gamma}{2}\operatorname{Var}(y(r,\zeta) + c\tilde{\epsilon}_c). \tag{5}$$

**Proposition 3.1** (Portfolio Choice ). *Suppose $\gamma > 0$ and corporate bond return shocks are independent of payment shocks and banks can lever up in the money market from outside investors. Then access to risky higher-yielding assets does not change banks' reserve demand: the optimal reserve holdings are still characterized by equation (2). The unconstrained optimal corporate bond share is*

$$c^{int} = \frac{\epsilon_c}{\gamma \sigma_c^2}. \tag{6}$$

The proof is in Appendix B.

This result follows from two assumptions. First, from the independence between corporate bond return shocks and payment shocks. The contributions of corporate bond holdings and reserve holdings to the mean and the variance of end-of-day wealth are additively separable. As a result, the marginal tradeoff underlying reserve demand, and thus the reserve demand curve itself, is unchanged. The optimal corporate bond position is pinned down by its Sharpe ratio and banks' risk aversion. Second, from the assumption that banks can lever up by borrowing from outsiders in the money market. This ensures that the balance sheet size does not constrain banks' optimal portfolio choice. The proof in Appendix B shows that $g^* = 1 - c^* - r^*$ where $c^*$ is pinned down by proposition 3.1 and $r^* = r(i^*) = s$ given by equation 2.2. Hence, as supply increases, $g^*$ can become negative, i.e. banks lever up. One way to interpret this is to assume that reserves are supplied via asset purchases. And once they are bought from NBFIs, reserve holdings on banks' balance sheets mechanically increase alongside NBFI deposits on the liability side (see e.g. Acharya and Rajan, 2022). If we were to drop this assumption and introduce constraints that limit banks' balance sheet size, such as the leverage ratio, the money market rate would drop below the deposit rate, as banks would need to earn a premium on reserve holdings to compensate for less corporate bond holdings as shown in e.g., Kim et al. (2020), d'Avernas and Vandeweyer (2024), Andersen et al. (2019), and Duffie and Krishnamurthy (2016).

## 3.2 LCR

We now introduce liquidity regulation to capture the key features of the LCR in a stylized fashion. In addition to the reserve requirement, banks must satisfy an end-of-day LCR requirement $\bar{L} = \bar{\ell} W$, where $\bar{\ell}$ is the required HQLA share measured relative to start-of-day balance sheet size.[16] If banks borrow from the interbank market or from outsiders during the day $(g < 0)$ , we assume that this raises required HQLA one-for-one in line with the idea that

[16] We assume that reserves held for reserve requirements can be drawn down in stress and are therefore eligible as level 1 assets under the Basel III rules (Basel Committee on Banking Supervision (2017)). This is the case in many emerging markets with larger reserve requirements such as Brazil, Thailand, or South Africa. In several advanced economies, reserve requirements are currently zero, such as in the United States or the United Kingdom (see Markets Committee (2026)).

these are flighty interbank or NBFI deposits. Hence, in contrast to reserve requirements, banks cannot rely on borrowing from private markets to satisfy an LCR shortfall. HQLA are defined as reserves and government bonds held outright at the end of the day. Hence, the end-of-day LCR constraint reduces to

$$1 - c + \zeta + b(r, c, \zeta) \geq \bar{\ell}.$$

where $b(r, c, \zeta)$ denotes discount window borrowing after both liquidity regulations are imposed. Banks' discount window borrowing is the minimum amount that makes both the reserve requirement and the LCR hold. Define

$$k_1(r) := \bar{x} - r, \qquad k_2(c) := \bar{\ell} - 1 + c, \qquad k(r, c) := \max\{k_1(r), k_2(c)\}.$$

Then

$$b(r, c, \zeta) = (k(r, c) - \zeta)^+.$$

Here $k_1(r)$ is the shortfall threshold implied by the reserve requirement, $k_2(c)$ is the shortfall threshold implied by the LCR, and $k(r, c)$ is the tighter of the two. The key distinction is that the reserve requirement applies directly to reserve holdings $r$, whereas the LCR is a requirement on the HQLA share, which is equivalent to a restriction on the non-HQLA share $c$. We therefore cannot interpret the LCR simply as another reserve requirement.

The bank's problem therefore remains the same as in equation (5), except for a different formula for end-of-day discount window borrowing, $b(r, c, \zeta)$. Because risk-free government bond positions are determined residually by interbank market clearing, the bank's problem reduces to a choice over $(r, c)$:

$$\max_{r,\ c} \quad \mathcal{U}^{LCR}(r, c) := u\big(r, k(r, c)\big) + v(c). \tag{7}$$

where

$$\begin{aligned} u(r, k) &:= \eta r - \chi \mathbb{E}[(k - \zeta)^+] - \frac{\gamma}{2} \operatorname{Var}\Big(\chi (k - \zeta)^+\Big), \\ v(c) &:= \epsilon_c c - \frac{\gamma}{2} \sigma_c^2 c^2. \end{aligned}$$

The solution to equation (7) characterizes optimal reserve demand and corporate bond demand under the LCR.

The effective shortfall threshold $k(r, c)$ depends on which liquidity regulation is tighter at the chosen balance sheet composition. This makes the bank's problem piecewise in $(r, c)$. The reserve requirement is tighter than the LCR regulation and disciplines discount window borrowing when

$$k_1(r) \geq k_2(c) \quad \Longleftrightarrow \quad c \leq \bar{c}(r) \equiv 1 - \bar{\ell} + \bar{x} - r. \tag{8}$$

Equation (8) defines the key cutoff in the bank's balance sheet composition. For any given reserve position $r$, $\bar{c}(r)$ is the maximum corporate bond share consistent with keeping the reserve requirement, rather than the LCR, as the operative liquidity constraint. Let $r_0$ denote the level of reserve holdings at which the LCR constraint is just slack when the bank's corporate bond position is at its unconstrained optimum, i.e.

$$r_0 = 1 - \bar{\ell} + \bar{x} - c^{int}.$$

This cutoff separates the region in which the LCR is irrelevant from the region in which it becomes potentially binding. To the left of $r_0$, the bank can increase reserve holdings by substituting reserves for non-reserve HQLA without making the LCR binding. To the right of $r_0$, keeping $c = c^{int}$ while increasing reserves would make the LCR tighter than the reserve requirement, so the bank must either accept the LCR as the effective constraint or reduce corporate bond holdings. This tradeoff gives rise to the following proposition.

**Proposition 3.2** (Reserve Demand under the LCR)**.** *Let $r^{LCR}$ and $c^{LCR}$ denote reserve demand and corporate bond demand under the LCR. Let $r_1 > r_0$ be the reserve share at which the Crowding-out region reaches the floor, $i = i^{IORB}$.*

1. ***Slack-LCR region.*** *For $r \leq r_0$, the LCR is slack at the unconstrained corporate bond allocation. Then*

$$r^{LCR}(i) = r(i),\ c^{LCR}(r) = c^{int}.$$

2. ***Crowding-out region.*** *For $r_0 < r < r_1$, banks choose their balance sheet composition such that reserve requirement and LCR are equally binding. The corporate bond holding is*

$$c^{LCR}(r) = 1 - \bar{\ell} + \bar{x} - r.$$

*Reserve demand is then characterized by*

$$\begin{aligned} &\eta + \chi F(\bar{x} - r) + \gamma\chi^2 m(\bar{x} - r)\big(1 - F(\bar{x} - r)\big) \\ &\quad = \epsilon_c - \gamma\sigma_c^2\big(1 - \bar{\ell} + \bar{x} - r\big). \end{aligned} \tag{9}$$

*Define $r_1$ as the solution to (9) when $i = i^{IORB}$.*

3. ***Floor Region.*** *For $r \geq r_1$, reserve demand is flat at the floor, $i = i^{IORB}$. In that region, the corporate bond allocation $c^F$ satisfies*

$$\begin{aligned} 0 = & -\chi F(\bar{\ell} - 1 + c) \\ & - \gamma\chi^2 m(\bar{\ell} - 1 + c)\big(1 - F(\bar{\ell} - 1 + c)\big) + \epsilon_c - \gamma\sigma_c^2 c. \end{aligned} \tag{10}$$

The proposition yields a three-region reserve demand curve under the liquidity coverage ratio (LCR) regulations. It shows that the LCR changes reserve demand only once reserve holdings are high enough for the balance sheet cutoff $r_0$ to become relevant. The proof is in Appendix C. In particular, Appendix C.1 shows that, as long as $i > i^{IORB}$, the bank's optimum cannot lie in the strictly LCR-dominant region and therefore remains on the boundary $c = \bar{c}(r)$ in the Crowding-out region.

In the *Slack-LCR region*, the unconstrained corporate bond allocation remains feasible and the LCR is slack. Discount window borrowing is governed by the reserve requirement, so the LCR does not enter the bank's marginal condition. Reserve demand therefore coincides with the baseline, and corporate bonds remain at their unconstrained optimum.

In the *Crowding-out region*, the relevant marginal condition changes. Banks can no longer increase reserves while keeping both a slack LCR and the unconstrained corporate bond allocation. When $i > i^{IORB}$, a strictly LCR-binding allocation cannot be optimal: additional reserves provide no extra current liquidity insurance in that region, yet still earn less than interbank lending. The bank therefore remains on the boundary $c = \bar{c}(r)$, where additional reserves require a one-for-one reduction in corporate bond holdings. Reserve demand is pinned down by equating the marginal liquidity value of reserves to the forgone marginal value of the displaced non-HQLA asset. As a result, reserve demand under the LCR lies below the baseline.

In the *Floor Region*, the interbank rate has reached the floor, $i = i^{IORB}$, so banks are willing to hold additional reserves and reserve demand becomes flat. The LCR is now the operative constraint on discount window borrowing. Corporate bond holdings are determined not only by their risk-adjusted return, but also by the tighter liquidity constraint.

Figure 3 illustrates the reserve demand curve under the LCR. The LCR changes both the slope and the satiation point of the reserve demand curve. Reserve demand coincides with the baseline as long as banks can substitute reserves for non-reserve HQLA without making the LCR tighter than the reserve requirement. Once reserve holdings exceed $r_0$, however, this is no longer possible if banks wish to keep the reserve requirement operative. Additional reserves must then be financed by reducing corporate bond holdings, so the marginal liquidity-insurance benefit of reserves is partly offset by the forgone return on corporate bonds. As a result, reserve demand lies below the baseline. Once the interbank

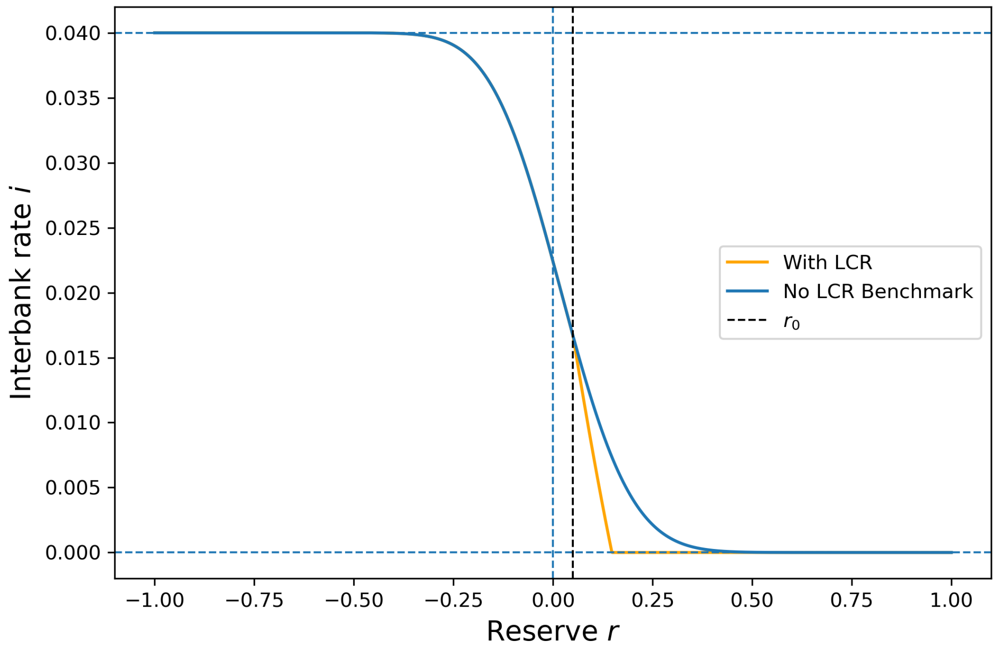


Figure 3: Reserve Demand Curve (LCR)

*Notes:* The figure plots the reserve demand curve under the LCR implied by Proposition 3.2. The no-LCR benchmark uses the baseline reserve demand curve with the same risk aversion, $\gamma = 50$. The vertical line marks $r_0$, the reserve level at which the LCR begins to affect reserve demand. The calibration sets $\bar{x} = 0$, $\bar{\ell} = 0.25$, $i^{DW} = 4\%$, $i^{IORB} = 0\%$, $\zeta \sim N(0, 0.15^2)$, $\epsilon_c = 0.0525$, and $\sigma_c = \sqrt{0.0525/(50 \times 0.7)} \simeq 0.0387$.

rate reaches the floor, reserve demand becomes flat.

The proposition shows that the LCR does not operate like an additional reserve requirement that shifts the reserve demand curve horizontally. When the LCR is slack at the unconstrained optimal corporate bond holding, reserve demand is unchanged. Once higher reserve holdings force banks to give up non-HQLA assets, reserve demand falls below the baseline because additional reserves displace spread-earning corporate bonds. Only after the interbank rate reaches the floor does the LCR become the operative constraint. Thus, the LCR lowers rather than raises reserve demand, but only over a specific region.

Introducing a realistic portfolio choice problem across different types of HQLA is an important contribution of this paper. This differs from approaches that interpret the LCR as an additional reserve requirement shifting the entire reserve demand curve horizontally, as in, for example, Afonso et al. (2022) and Afonso et al. (2020b). Our results also differ from Bech and Keister (2017). In their setting, bond holdings are exogenous. As a result, whether the LCR or the reserve requirement is the operative constraint is largely determined outside the bank's portfolio problem, and reserves are effectively the only margin through which banks can adjust a shortfall of the LCR.

### 3.2.1 Corporate Bond Demand under the LCR

**Corollary 3.3** (Corporate Bond Demand under the LCR)**.** *The corporate bond demand under the LCR is*

$$c^{LCR}(r) = \begin{cases} c^{int}, & \text{if } r \le r_0, \\ 1 - \bar{\ell} + \bar{x} - r, & \text{if } r_0 < r < r_1, \\ c^F, & \text{if } r \ge r_1. \end{cases}$$

The corollary reveals a three-region structure parallel to reserve demand. In the *Slack-LCR region*, the unconstrained corporate bond allocation remains feasible with slack LCR, so the LCR does not distort banks' portfolio choice and corporate bond demand coincides with the benchmark. In the *Crowding-out region*, banks optimally remain on the boundary $\bar{c}(r)$ where each additional unit of reserves must be matched by a one-for-one reduction in corporate bond holdings. In the *Floor Region*, the LCR becomes the operative liquidity constraint, and the optimal corporate bond position is pinned down by the tradeoff between risk-adjusted return and tighter liquidity. Corporate bond demand is therefore flat in reserve supply.

Corollary 3.3 also has an immediate asset-pricing implication. If $\epsilon_c$ is determined endogenously by banking-sector demand rather than taken as exogenous, the LCR creates a non-HQLA spread in corporate bond yields. That spread is zero while the LCR is slack, rises with reserve supply in the Crowding-out region as reserves displace corporate bonds, and becomes constant in the Floor Region.

Now consider how a rise in banks' risk aversion changes corporate bond demand. The next corollary compares a pre-shock economy with risk aversion $\gamma_1$ to a post-shock economy with higher risk aversion $\gamma_2$.

**Corollary 3.4** (Risk Aversion and Corporate Bond Demand under the LCR)**.** *Suppose $0 < \gamma_1 < \gamma_2$. Let $c^{LCR}(r;\gamma_i)$, $r_0^i$, $r_1^i$, and $c_i^F$ denote the objects under the LCR associated with risk aversion $\gamma_i$, for $i \in \{1,2\}$. Define*

$$\Delta^0 := c^{int}(\gamma_1) - c^{int}(\gamma_2) = \frac{\epsilon_c}{\gamma_1\sigma_c^2} - \frac{\epsilon_c}{\gamma_2\sigma_c^2} > 0,$$

*and*

$$\Delta^{LCR}(r) := c^{LCR}(r;\gamma_1) - c^{LCR}(r;\gamma_2).$$

*Then $\Delta^0$, the decline in corporate bond demand absent the LCR, is constant in reserve supply. Under the LCR, by contrast, $\Delta^{LCR}(r)$ varies with reserve supply as follows:*

1. *For $r \le r_0^1$, both economies lie in the Slack-LCR region, and $\Delta^{LCR}(r) = \Delta^0$.*

*2. For $r_0^1 < r < r_0^2$, the pre-shock economy is in the Crowding-out region while the post-shock economy remains in the Slack-LCR region. In this case, $\Delta^{LCR}(r)$ is strictly below $\Delta^0$ and decreases with $r$.*

*3. For $r_0^2 \leq r \leq r_1^1$, both economies lie in the Crowding-out region, and $\Delta^{LCR}(r) = 0$.*

*4. For $r_1^1 < r < r_1^2$, the pre-shock economy is in the Floor Region while the post-shock economy remains in the Crowding-out region. In this case, $\Delta^{LCR}(r) = c_1^F - (1 - \bar{\ell} + \bar{x} - r)$, which increases with $r$ from $0$ to $c_1^F - c_2^F$.*

*5. For $r \geq r_1^2$, both economies lie in the Floor Region, and $\Delta^{LCR}(r) = c_1^F - c_2^F$, which is independent of $r$ and strictly smaller than $\Delta^0$.*

*Hence,*

$$\Delta^{LCR}(r) \leq \Delta^0 \qquad \text{for all } r,$$

*with equality if and only if $r \leq r_0^1$.*

The LCR weakens, and can locally shut down, the transmission from bank risk appetite to non-HQLA risky assets demand. Absent the LCR, a rise in risk aversion shifts bank's corporate bond demand inward by the same amount at every level of reserve supply. Under the LCR, that comparative static becomes state dependent because higher risk aversion lowers the unconstrained optimal corporate bond position, which in turn relaxes the LCR, and moves the cutoff $r_0$. When the economy before the rise in risk aversion lies in the Crowding-out region but remains in the Slack-LCR region afterwards, the direct effect of higher risk aversion is partly offset by the relaxation of the LCR. When both economies lie in the Crowding-out region, corporate bond holdings are pinned down by the boundary $c = 1 - \bar{\ell} + \bar{x} - r$, so the effect of risk aversion on bond demand disappears altogether. Only when the two economies occupy different regions again does the gap reopen.

The pricing implication is particularly striking. If $\epsilon_c$ is determined endogenously by banking-sector demand, a deterioration in banks' risk appetite need not raise corporate bond yields at all. When the economy before and after the rise in risk aversion lies in the Crowding-out region, banks' demand for non-HQLA assets is pinned down by reserve supply rather than by risk appetite, so lower risk tolerance does not translate into lower corporate bond demand or a wider non-HQLA spread. More broadly, the sensitivity of non-HQLA prices to changes in banks' risk appetite depends on where reserve supply lies relative to the reserve demand curve.

Figure 4 illustrates Corollary 3.3 and Corollary 3.4. It shows that the LCR leaves corporate bond demand unchanged at low levels of reserve supply, compresses it once reserve supply exceeds $r_0$, and sharply weakens the effect of lower risk appetite on demand for non-HQLA assets. Most strikingly, over the range in which both the pre-shock and post-shock

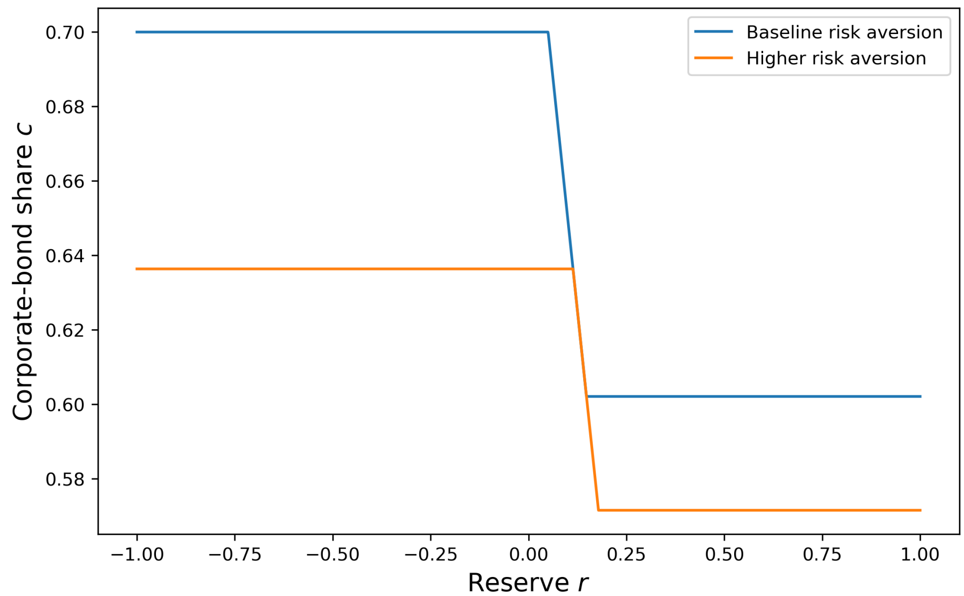


Figure 4: Corporate bond demand under the LCR

*Notes:* The figure plots corporate bond demand under the LCR implied by Corollary 3.3 and Corollary 3.4. The baseline-risk-aversion curve sets $\gamma = 50$; the higher-risk-aversion curve sets $\gamma = 55$, holding all other parameters fixed. The calibration sets $\bar{x} = 0$, $\bar{\ell} = 0.25$, $i^{DW} = 4\%$, $i^{IORB} = 0\%$, $\zeta \sim N(0, 0.15^2)$, $\epsilon_c = 0.0525$, and $\sigma_c = \sqrt{0.0525/(50 \times 0.7)} \simeq 0.0387$.

economies lie in the Crowding-out region, the two corporate bond demand schedules coincide: higher risk aversion does not reduce corporate bond demand and therefore need not move corporate bond prices.

# 4 Monetization Frictions and Reserve Specialness

This section studies monetization frictions as a source of reserve specialness. Reserves and other HQLA are not perfect substitutes for liquidity-risk management because non-reserve HQLA must first be monetized into reserves before they can settle payment flows. In stress, government bonds may be difficult to monetize into reserves, whereas reserves can still be converted into government bonds or lent out. The friction therefore matters when the bank needs to rebuild reserves after an outflow, but not when an inflow leaves the bank with excess reserves.

The within-day environment in each period is the same as in Section 2: banks face the same payment shock $\zeta$, the same reserve requirement share $\bar{x}$, and the same discount window borrowing $b(r, \zeta) = (\bar{x} - r - \zeta)^+$. The bank's problem now also becomes infinite horizon. This extension is necessary because the friction we study affects banks' ability to rebalance their carried-over portfolio and therefore can only be meaningfully analyzed in an intertemporal setting. In each period, banks derive utility from that day's end-of-day wealth in excess of the risk-free return; this flow payoff is consumed within the period, while the underlying asset positions are carried into the next period. [17] To keep the dynamic problem tractable, we set $\gamma = 0$ and abstract from ordinary intraday payment flows. Banks discount future flow payoffs by $\beta \in (0, 1)$.

## 4.1 Monetization Frictions

In this subsection, we abstract from higher-yielding risky assets and LCR regulation so the analysis isolates the effect of monetization frictions on reserve demand. Starting from portfolio shares $(r, g)$, a bank hit by shock $\zeta$ carries into the next day reserves $W(r + \zeta)$ and government bonds $Wg$, while its next-period balance sheet scale is $W(1 + \zeta)$. The payment shock has no standalone payoff; as in Section 2, it matters only because it changes reserve balances and can force costly discount window borrowing. It is useful to write the inherited reserve share after shock $\zeta$ as

$$\tau(r, \zeta) = \frac{r + \zeta}{1 + \zeta}.$$

On the next day, with probability $P$, the bank cannot monetize government bonds into reserves during the intraday period, although it can still purchase government bonds by running down reserves. With probability $1 - P$, it can adjust its portfolio freely.

**Banks' Maximization Problems.** Because the problem is homogeneous in wealth, we work with value per dollar of current wealth. There are three value objects. The function $V(r, g)$ is the value of entering the day with portfolio shares $(r, g)$. The scalar $V^o$ is the flexible continuation value when the bank can freely choose its portfolio. The function $V^-(\tau)$ is the constrained continuation value when the bank inherits reserve share $\tau$ and cannot increase reserves by monetizing government bonds. If the bank is not hit by the monetization friction, it chooses its portfolio during the normal intraday environment:

$$V^o = \max_{0 \le r' \le 1} V(r', 1 - r').$$

If the monetization friction hits, the bank can reduce reserves by converting reserves into government bonds, but it cannot increase reserves by monetizing government bonds. Given inherited reserve share $\tau(r, \zeta)$, the constrained value function is

$$V^-(\tau) = \max_{0 \le r' \le \tau} V(r', 1 - r').$$

[17] This change in time horizon is otherwise innocuous. The settings in Section 2 and Section 3 can also be embedded in an infinite-horizon framework; when banks can reoptimize their portfolio freely at the start of each day, continuation values depend only on end-of-day wealth rather than on portfolio composition. Those problems therefore collapse to the same one-day formulation studied above.

The constrained state only rules out choices with $r' > \tau$. This is due to monetization frictions: reserves can be run down, but government bonds cannot always be turned into additional reserves when needed.

The Bellman equation is

$$V(r, 1-r) = \mathbb{E}_\zeta \big[\eta r - \chi b(r,\zeta) + \beta(1+\zeta)\left((1-P)V^o + PV^-(\tau(r,\zeta))\right)\big]. \tag{11}$$

The monetization friction, however, only bites when the bank suffers a negative payment shock and the inherited reserve share falls below the stationary reserve share. When a bank was hit by a positive payment shock in the last period, and it faces monetization frictions today, it can and will run down excess reserves to the stationary level by buying government bonds or lending. The monetization friction therefore does not bind after positive payment shocks. The Bellman equation therefore can be rewritten as:

$$V(r, 1-r) = \mathbb{E}_\zeta \left[\eta r - \chi b(r,\zeta) + \beta(1+\zeta)V^o + \beta P(1+\zeta)\mathbf{1}_{\{\zeta<0\}}\left(V^-(\tau(r,\zeta)) - V^o\right)\right]. \tag{12}$$

To simplify notation, from this point onward we state the marginal-value and reserve-demand expressions in the limit as $\beta \uparrow 1$.

**Proposition 4.1** (Reserve Demand with Monetization Frictions)**.** *If $Pq < 1$, the reserve demand curve under monetization frictions is*

$$i(r;P) = i^{IORB} + (1-Pq)(i^{DW} - i^{IORB})\sum_{n=0}^{\infty} P^n\left((T^-)^n F(\bar{x} - \cdot)\right)(r). \tag{13}$$

*where $T^-$ is the negative-shock transition operator and $q$ is the probability of a negative late day payment shock:*

$$T^-[h](r) = \int_{-1}^{0} h(\tau(r,\zeta)) f(\zeta)\, d\zeta, \qquad q = T^-[1] = F(0).$$

*When $P = 0$, this collapses to the Poole benchmark:*

$$i(r;0) = i^{IORB} + \chi F(\bar{x} - r).$$

The proof is in Appendix D. The cost of discount window usage is scaled by $1 - Pq$, rather than $1-P$, and future shortfall probabilities are propagated using the negative-shock operator $T^-$. This is because monetization frictions matter only when the late day payment shock is negative and inherited reserves fall below the stationary target.

Figure 5 shows how the friction of monetizing non-reserve HQLA into reserves raises

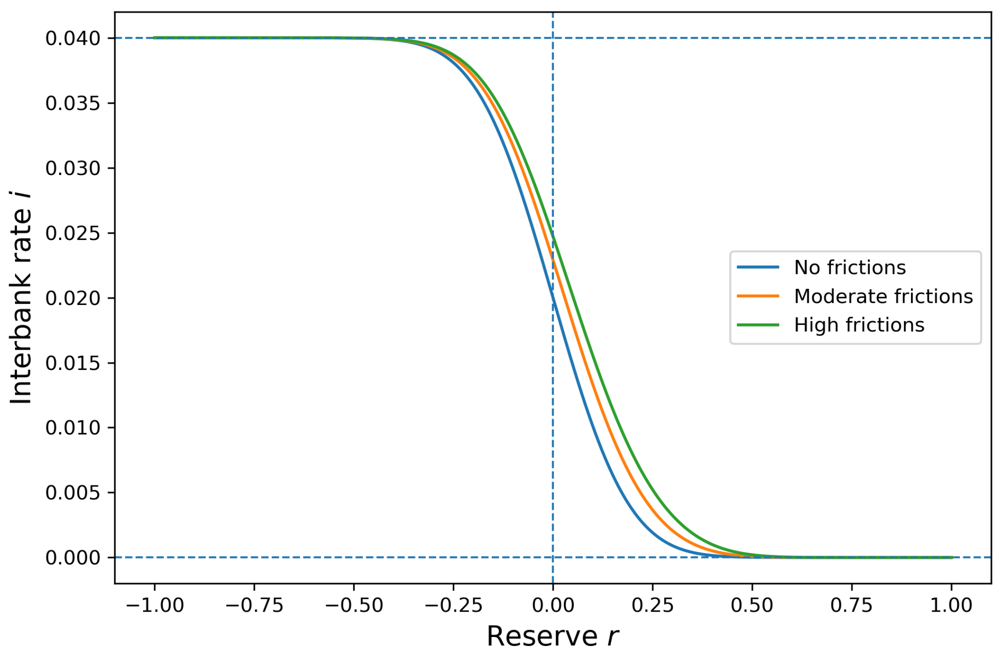


Figure 5: Reserve Demand Curve (Monetization Frictions)

*Notes:* The figure plots the reserve demand curve under one-sided monetization frictions implied by Proposition 4.1. The no-, moderate-, and high-frictions curves correspond to disruption probabilities $P = 0$, $P = 0.5$, and $P = 0.75$, respectively. The simulation evaluates the negative-shock operator $T^-$ by Monte Carlo with 15,000 draws, and sets $i^{DW} = 4\%$, $i^{IORB} = 0\%$, $\bar{x} = 0$, and $\zeta \sim N(0, 0.15^2)$.

banks' demand for reserves ex ante. As monetization frictions increase (i.e., as $P$ rises), the whole reserve demand curve weakly moves outward relative to the Poole benchmark: for a given reserve position, banks require a higher interbank rate, or equivalently demand more reserves at a given rate. Monetization frictions raise the precautionary value of reserves after negative payment shocks, but they do not create an offsetting cost after positive shocks because excess reserves can still be run down. The outward movement is most pronounced near the satiation point, where the current-period marginal liquidity benefit of reserves is small but the future insurance value becomes relatively more important.

The results imply that when non-reserve HQLA can be monetized more easily in stress, banks choose to hold fewer reserves because the insurance value of reserves declines, especially when reserve supply lies on the right-hand, lower-rate portion of the demand curve. The survey evidence in Nelson et al. (2026) provides some evidence on this mechanism. In the survey, bank treasurers report that after the Federal Reserve's Regulation YY FAQ issued on August 13, 2024, banks could point to the Federal Home Loan Bank advances, and the Standing Repo Facility as monetization channels for HQLA in liquidity stress tests; see Board of Governors of the Federal Reserve System (2024). They view this change as an important reason why banks have reduced the share of assets invested in reserve balances. In the language of our model, the FAQ lowers effective monetization frictions by expanding the set of states in which non-reserve HQLA can credibly be turned into reserves when needed; that reduces the need to hold reserves for precautionary purposes and therefore lowers reserve demand.

Appendix E studies a different market disruption in which banks may be unable to rebalance between reserves and government bonds in either direction. Because the disruption is two-sided, reserves not only provide a future insurance benefit after outflows but also entail a potential opportunity cost after inflows, when banks may be stuck with excess reserves remunerated at the lower deposit rate. As shown in Figure E.1, the reserve demand curve therefore rotates: demand rises on the right-hand side of the curve but can fall on the left-hand side. This contrasts with monetization frictions, which weakly increase reserve demand along the entire curve.

## 4.2 Monetization frictions and the LCR

This subsection studies how liquidity coverage ratio (LCR) regulation affects reserve demand when reserves are special in the monetization sense. To do so, we reintroduce higher-yielding risky non-HQLA assets and the LCR regulation.

Although banks are risk-neutral, we model the risky asset, interpreted as corporate bonds, with a risk-adjusted payoff to avoid corner solutions. Let $C$ denote corporate bond holdings, and let $c = C/W$ be the corporate bond share of the balance sheet. Corporate bonds generate the flow payoff

$$\sqsubseteq(C; W) = W \left[ \epsilon_c \frac{C}{W} - \frac{\gamma \sigma_c^2}{2} \left( \frac{C}{W} \right)^2 \right].$$

Equivalently, the return per unit of wealth is a function of share $c$,

$$v(c) = \epsilon_c c - \frac{\gamma \sigma_c^2}{2} c^2.$$

Here $\epsilon_c$ is the expected excess payoff from corporate bonds, $\sigma_c^2$ measures the riskiness of the investment, and $\gamma$ captures the reduced-form risk penalty associated with corporate bond exposure. The unconstrained optimal corporate bond share is therefore $c^{int} = \epsilon_c/(\gamma\sigma_c^2)$.

The analysis proceeds in two steps. We first show that, absent the LCR, higher-yielding corporate bonds remain separable from reserve demand, even in the dynamic environment with monetization frictions. We then introduce the LCR and study how it alters reserve demand.

### 4.2.1 Corporate Bonds Without the LCR

Because the problem is homogeneous in wealth, we again work with value per dollar of current wealth. The function $V(r, g, c)$ is the value of entering the day with portfolio shares $(r, g, c)$. The scalar $V^o$ is the flexible continuation value when the bank can freely choose its portfolio. The function $V^-(\tau)$ is the constrained continuation value when the bank inherits reserve share $\tau$ and faces monetization frictions. Thus,

$$V^o = \max_{\substack{r' \geq 0,\ c' \geq 0 \\ r'+c' \leq 1}} V(r', 1 - r' - c', c'), \tag{14}$$

$$V^-(\tau) = \max_{\substack{0 \leq r' \leq \tau,\ c' \geq 0 \\ r'+c' \leq 1}} V(r', 1 - r' - c', c'). \tag{15}$$

where $\tau(r, \zeta) = (r + \zeta)/(1 + \zeta)$ is the inherited reserve share after the payment shock $\zeta$. The Bellman equation is

$$V(r, 1 - r - c, c) = \mathbb{E}_\zeta \left[\pi(r, c, \zeta) + (1 + \zeta) \left((1 - P)V^o + PV^-(\tau(r, \zeta))\right)\right]. \tag{16}$$

The flow payoff $\pi(r, c, \zeta)$ is given by

$$\pi(r, c, \zeta) = \eta r - \chi b(r, \zeta) + v(c).$$

where $b(r, \zeta)$ is discount window borrowing. Without the LCR, the reserve requirement is the only liquidity constraint:

$$b(r, \zeta) = (\bar{x} - r - \zeta)^+.$$

**Proposition 4.2** (Portfolio Choices and Monetization Frictions)**.** *Access to higher-yielding corporate bonds does not change reserve demand, even when reserves are special because of monetization frictions. The optimal corporate bond share is*

$$c = c^{int} = \frac{\epsilon_c}{\gamma \sigma_c^2},$$

*and reserve demand remains characterized by the monetization-friction reserve demand curve in Proposition 4.1.*

The result extends the separability logic in Proposition 3.1 to the dynamic monetization environment. Corporate bonds affect the flow payoff through $v(c)$, but they do not enter discount window borrowing $b(r, \zeta)$. Because corporate bond holdings can always be adjusted by trading government bonds, their margin is pinned down by $v'(c) = 0$. Reserve demand is therefore governed by exactly the same liquidity wedge as in Section 4. Appendix D.1 gives the formal derivation.

### 4.2.2 The effects of the LCR

We now also impose the LCR on banks. The continuation values and Bellman equation are the same as equation (14) and equation (15) and equation (16); but with a different current flow payoff. As in Section 3, the LCR changes discount window borrowing to

$$b(r, c, \zeta) = \max\{0, \bar{x} - r - \zeta, \bar{\ell} - 1 + c - \zeta\} = (k(r, c) - \zeta)^+,$$

where

$$k_1(r) = \bar{x} - r, \qquad k_2(c) = \bar{\ell} - 1 + c, \qquad k(r, c) = \max\{k_1(r), k_2(c)\}.$$

For each reserve share $r$, let $\bar{c}(r)$ denote the maximum corporate bond share for which the reserve requirement remains weakly tighter than the LCR:

$$\bar{c}(r) \equiv 1 - \bar{\ell} + \bar{x} - r.$$

The current per-wealth flow payoff is

$$\pi(r, c, \zeta) = \eta r - \chi b(r, c, \zeta) + v(c).$$

The key difference from the previous subsection is that corporate bonds now enter discount window borrowing through $k_2(c)$. The corporate bond and reserve margins are therefore no longer separable, so the LCR can affect reserve demand.

**Proposition 4.3** (Reserve Demand under the LCR with Monetization Frictions)**.** *Let $c^*(r)$ denote the optimal corporate bond share chosen in the constrained states with carried-over reserve share $r$, and let $c^L$ solve the optimal corporate bond share when the LCR is binding:*

$$v'(c^L) = \chi F(\bar{\ell} - 1 + c^L).$$

*The corporate bond policy is static and has three regions:*

$$c^*(r) = \begin{cases} c^{int}, & \text{if } c^{int} \le \bar{c}(r), \\ \bar{c}(r), & \text{if } c^L \le \bar{c}(r) < c^{int}, \\ c^L, & \text{if } c^L > \bar{c}(r). \end{cases}$$

*With $T^-$ and $q$ as in Proposition 4.1, define the marginal benefit of reserves as:*

$$S^*(r) = \begin{cases} F(\bar{x} - r), & \text{if } c^{int} \leq \bar{c}(r), \\ F(\bar{x} - r) - \dfrac{v'(\bar{c}(r))}{\chi}, & \text{if } c^L \leq \bar{c}(r) < c^{int}, \\ 0, & \text{if } c^L > \bar{c}(r), \end{cases}$$

*where the second line subtracts the marginal value of the corporate bonds crowded out along the LCR boundary, and the third line is zero because the LCR regulation is binding. The marginal value of reserves is then the sum of future discounted state-dependent payoff:*

$$H^*(r; P) = \sum_{n=0}^{\infty} P^n \left( (T^-)^n S^* \right)(r).$$

*The reserve demand curve is*

$$i(r; P) = i^{IORB} + (1 - Pq)\chi H^*(r; P).$$

*The floor is reached when $H^*(r; P) = 0$, in which case $i(r; P) = i^{IORB}$.*

Because corporate bonds can be reoptimized every period, $c^*(r)$ is pinned down by the current LCR tradeoff. Reserve demand is forward-looking for the same reason as in Proposition 4.1: with monetization frictions, the marginal value of reserves includes not only the current marginal payoff $S^*(r)$, but also the expected marginal value of reserves in future constrained states reached after negative payment shocks.

The reserve demand curve coincides with Proposition 4.1 in the first region. When $c^{int} \leq \bar{c}(r)$, the LCR is slack, the bank chooses $c^{int}$, and future negative shocks only move the bank further into the slack-LCR region. Along this path, $S^*(r) = F(\bar{x} - r)$, so the reserve demand curve is exactly the monetization-friction curve in Proposition 4.1.

Once reserve holdings are high enough that the LCR would bind at unconstrained corporate bond holdings, $r > r_0$, the future marginal payoff of additional reserves is weakly lower. In the boundary region, the marginal payoff is reduced by $v'(\bar{c}(r))/\chi$ because an additional unit of reserves crowds out corporate bonds one-for-one. In the strict LCR-dominant region, the marginal payoff vanishes because reserves do not reduce current discount window borrowing. This lower marginal payoff function gives a reserve demand curve that lies weakly below that of Proposition 4.1.

Figure 6 shows that reserve specialness still does not make the LCR behave like an additional reserve requirement. The dashed curve is the no-LCR monetization-friction benchmark; the solid curves impose the LCR. The two vertical lines, $r_0$ and $r_1$, mark the reserve

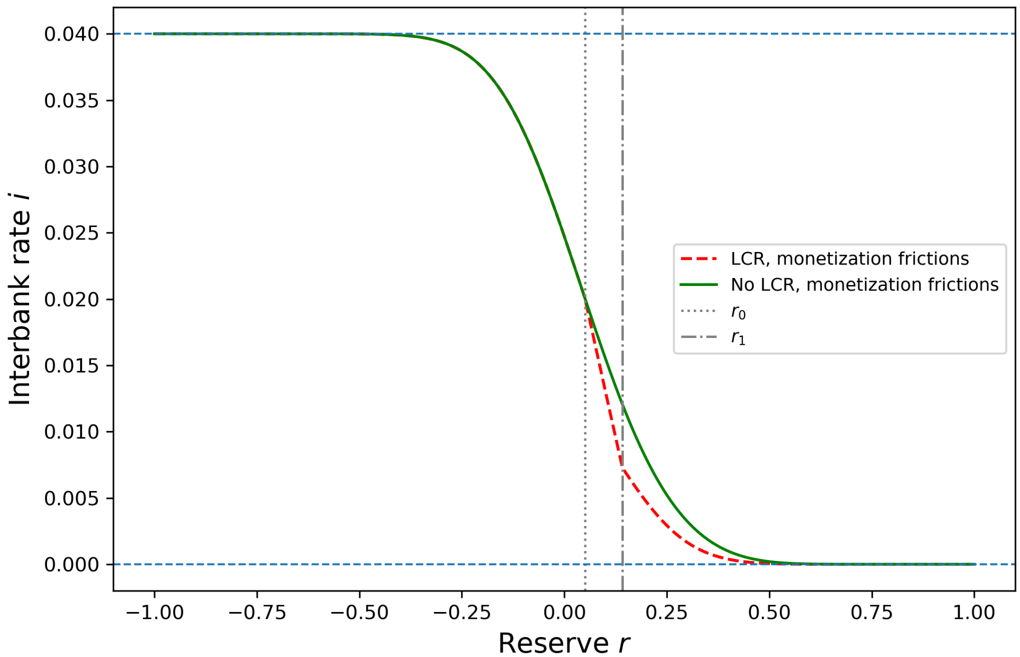


Figure 6: Reserve Demand Curve (Monetization Frictions with the LCR)

*Notes:* The figure plots the reserve demand curve under one-sided monetization frictions with and without the LCR, as implied by Proposition 4.3. Both curves set $P = 0.75$. The vertical lines mark $r_0$ and $r_1$, defined by $\bar{c}(r_0) = c^{int}$ and $\bar{c}(r_1) = c^L$, respectively; $r_0$ is where the LCR begins to bind weakly, and $r_1$ is where the LCR becomes strictly binding. The calibration sets $\gamma = 50$, $i^{DW} = 4\%$, $i^{IORB} = 0\%$, $\bar{x} = 0$, $\bar{\ell} = 0.25$, $\epsilon_c = 0.0525$, $\sigma_c = \sqrt{0.0525/(50 \times 0.7)} \simeq 0.0387$, and $\zeta \sim N(0, 0.15^2)$. The simulation evaluates the negative-shock operator $T^-$ by Monte Carlo with 15,000 draws.

levels at which the LCR changes the marginal value of reserves. To the left of $r_0$, the LCR is slack not only today but also along the relevant future paths after negative payment shocks. The marginal value of reserves is therefore the same as in the no-LCR monetization benchmark, and the curves coincide. Between $r_0$ and $r_1$, additional reserves crowd out higher-yielding corporate bonds. This lowers the marginal payoff from reserves not only in the current state, but also in some future states reached after outflows. The LCR reserve demand curve therefore lies below the no-LCR curve and its slope changes. To the right of $r_1$, the reserve requirement is slack, so an additional unit of reserves has no current insurance value. Reserves can still have a positive forward-looking marginal value because subsequent outflows may move the bank back into states in which the reserve requirement binds. As reserve holdings rise further, those future binding states become increasingly unlikely, and the curve approaches the floor. The figure therefore makes clear that even when reserves are special, the LCR does not act like an additional reserve requirement. It reshapes reserve demand by changing both the assets crowded out by additional reserves and the future states in which reserves provide liquidity insurance.

# 5 Fragmented Interbank Market

This section studies how market fragmentation changes the mapping from aggregate reserve supply to the market-clearing interbank rate. This captures the well-documented reserve-market atrophy in the wake of prolonged periods of abundant reserves when some banks become inactive in money markets ( Asberg Sommar et al. (2026)). Inactive banks never participate in interbank markets and only rely on their reserve holding or central bank facilities to manage all payment flows. Their payment flows then become reserve supply shocks for the active segment of the market. We first characterize the rate implications of a fixed inactive segment to then endogenize fragmentation by allowing banks to choose whether to be active in the interbank market. Finally, we study the impact of aggregate reserve supply shock when markets are fragmented. Throughout the section, banks solve the one-period problem from Section 2, with $\gamma = 0$.

## 5.1 Exogenous Market Fragmentation

Fragmentation generates reserve-supply disturbances endogenously, even when the central bank keeps aggregate reserve supply fixed. The reason is that when an inactive bank receives an ordinary payment flow, it absorbs the flow on its reserve account and vice versa. Thus, whenever the aggregate payment flow absorbed by inactive banks is non-zero, active banks face an effective reserve-supply shock. The interbank rate then fluctuates endogenously because only active banks clear the residual reserve market.

To be specific, we assume that there is a fraction $\psi$ of inactive banks, so that only a fraction $1-\psi$ trades in money markets. Let $\Omega_t$ denote the aggregate ordinary payment flow absorbed by inactive banks at time $t$, measured as a share of total banking-sector balance sheet size. Throughout the section the aggregate supply of reserves $s$ is kept constant.[18]

Market clearing then requires active banks to hold the residual reserve share

$$\rho(\Omega_t; s, \psi) \equiv s - \frac{\Omega_t}{1-\psi}. \tag{17}$$

A positive $\Omega_t$ means that inactive banks receive a net reserve inflow, leaving fewer reserves for active banks; a negative $\Omega_t$ has the opposite effect. Fragmentation therefore changes the residual reserve share that active banks must absorb, while leaving their inverse reserve-demand schedule $\mathcal{I}(r)$ unchanged from the baseline implied by equation (3).

[18] We assume granular banks so that $\Omega_t$ is non-degenerate; with a literal continuum of atomless banks and independent idiosyncratic $\zeta'$, the law of large numbers would imply $\Omega_t = 0$ almost surely.

**Proposition 5.1** (Market fragmentation and Endogenous Interbank Rate Volatility)**.** *Ordinary payment flows absorbed by inactive banks act as an effective reserve-supply shock to active banks. The equilibrium interbank rate is*

$$i(\Omega_t; s, \psi) \equiv \mathcal{I}(\rho(\Omega_t; s, \psi)) = i^{IORB} + (i^{DW} - i^{IORB})F(\bar{x} - \rho(\Omega_t; s, \psi))\,. \tag{18}$$

*Around* $\Omega_t = 0$,

$$\frac{di(\Omega_t; s, \psi)}{d\Omega_t} = \frac{(i^{DW} - i^{IORB})f(\bar{x} - s)}{1 - \psi}. \tag{19}$$

*Hence, if* $\Omega_t$ *is non-degenerate, fragmentation creates interbank-rate volatility even when aggregate reserve supply* $s$ *is fixed.*

The proposition shows that fragmentation can turn payment-flow imbalances into reserve-supply risk for the active part of the banking system. If $\Omega_t = 0$, active banks hold the aggregate reserve supply $s$ and the interbank rate coincides with the baseline rate, $\mathcal{I}(s)$. If inactive banks in aggregate receive a net reserve inflow, active banks must operate with fewer reserves and the interbank rate rises along the usual Poole reserve demand curve.

The proposition has important implications.

First, fragmentation horizontally displaces the aggregate supply-to-rate mapping. The implied effective reserve supply per active bank is $s - \Omega_t/(1 - \psi)$. Consequently, when the interbank rate is plotted against aggregate reserve supply, a positive $\Omega_t$ appears as a rightward shift of the aggregate reserve demand curve by $\Omega_t/(1 - \psi)$, even though active banks' structural reserve demand remains unchanged.

Second, payment flows of inactive banks are random. Hence, there is no one-to-one mapping between aggregate reserve supply by the central bank and the equilibrium money market rate. Figure 7 illustrates this. For a given aggregate reserve supply $s$, the shaded regions show the range of interbank rates generated by realizations of the inactive-flow imbalance $\Omega_t$.

## 5.2 Endogenous Market Fragmentation

In this section, we endogenize the banks' decisions to be active or inactive to capture the empirical evidence that after large balance sheet expansions, several money market participants become inactive (e.g. Asberg Sommar et al. (2026) ). We model this by assuming bank specific participation costs. These could reflect, for instance, different IT infrastructures or abilities to maintain staff experience, which have been identified in impacting the money market ecosystem over periods of large balance sheet expansions (Markets Committee (2019)). It could also reflect different risk management frameworks that may hinder some

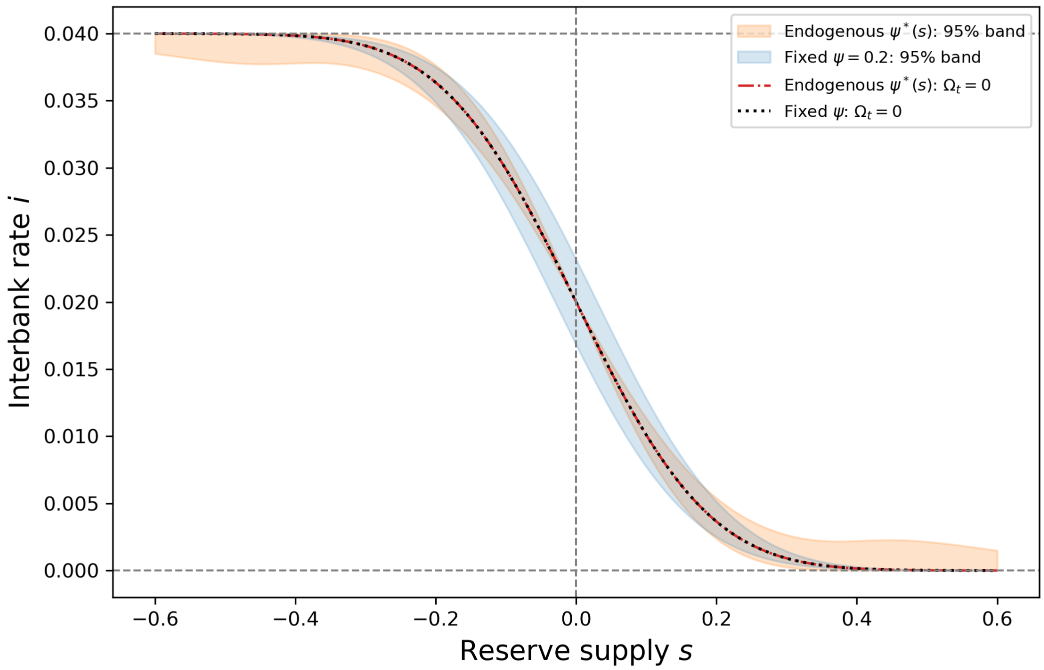


Figure 7: The Effects of Market Fragmentation

*Notes:* The figure plots the interbank rate as a function of aggregate reserve supply $s$ under reserve-supply shocks generated by inactive payment flows. The blue band corresponds to the case in which the inactive fraction is fixed at $\psi = 0.2$, as in Proposition 5.1. The orange band corresponds to the case in which inactive fraction $\psi^*(s)$ is endogenously determined following Proposition 5.2. Both bands are obtained by evaluating the interbank rate equation at the 2.5th and 97.5th percentiles of $\Omega_t$; in the endogenous case these percentiles are those of $H_{\psi^*(s)}$. The $\Omega_t = 0$ curves coincide with the Poole benchmark. The calibration sets $i^{DW} = 4\%$, $i^{IORB} = 0\%$, $\bar{x} = 0$, and intraday payment flows with standard deviation 0.15. The inactive-flow shock follows the granular-bank calibration $\text{Var}_\psi(\Omega_t) = \psi(0.15)^2/N_\Omega$ with $N_\Omega = 30$ effective equal-sized banks. Participation costs follow a lognormal distribution with log standard deviation 2.00, truncated above and calibrated so that $\psi^*(\bar{x}) = 0$. Plotted rates are bounded by the standing-facility corridor $[i^{IORB}, i^{DW}]$.

banks from trading. Afonso et al. (2020a) for instance find that money market desks typically require approvals to exceed internal limits, which may slow banks' response to rapidly changing market conditions, especially in repo markets that trade early in the morning.

To be specific, we assume that banks decide at the beginning of each day (before shocks are realized) whether to be active or inactive. If they are active, bank $j$ pays cost $\kappa_j W_j$ . The cost $\kappa_j$ is i.i.d. from c.d.f. $\Phi_\kappa$ with density $\phi_\kappa$.

To assess the bank's participation decision, consider first the expected gain conditional on the interbank rate $i$ and the reserve share $\bar{r}$. This is

$$\begin{aligned}
\Delta(i,\bar{r}) &= \max_r \left[\eta(i)r - \chi m(\bar{x}-r)\right] - \left[\eta(i)\bar{r} - \chi\tilde{m}(\bar{x}-\bar{r})\right] \\
&= \underbrace{\chi[\tilde{m}(\bar{x}-\bar{r}) - m(\bar{x}-\bar{r})]}_{A(\bar{r})} + \underbrace{\left[\max_r \bar{y}(r;i) - \bar{y}(\bar{r};i)\right]}_{O(i,\bar{r})},
\end{aligned}$$

where $\bar{y}(r;i) \equiv \mathbb{E}[y(r,\zeta)] = \eta(i)r - \chi m(\bar{x}-r)$, and $\eta(i) \equiv i^{IORB} - i$. The term $\tilde{m}(k) \equiv \mathbb{E}[(k-\zeta-\zeta')^+]$ is the inactive bank's expected discount window borrowing under the effective late day payment shock $\tilde{\zeta} = \zeta + \zeta'$.

This gain has two components. The first, $A(\bar{r})$, is the value of accommodating ordinary payment flows via the interbank market rather than relying on reserve holdings at the central bank and risk that payment flows trigger discount window borrowing. The second, $O(i,\bar{r})$, is the option value of adjusting toward the optimal portfolio and reserve shares upon the latest interbank market conditions.

Banks overall expected gains from participating depend on the expected distribution of market-clearing rates, which in turn depends on the expected share of inactive banks. Given a conjectured inactive fraction $\psi$, let $H_\psi$ denote the distribution of $\Omega$. The expected gain from being active for a bank with individual reserve holdings $\bar{r} = s$ in an economy with aggregate reserve supply $s$ is:

$$\begin{aligned}
\mathcal{B}(s,\psi) &= \mathbb{E}_{\Omega\sim H_\psi}\left[\Delta(i(\Omega;s,\psi),\bar{r}=s)\right] \\
&= A(s) + O(s,\psi),
\end{aligned} \tag{20}$$

where

$$O(s,\psi) = \mathbb{E}_{\Omega\sim H_\psi}\left[\bar{y}(\rho(\Omega;s,\psi); i(\Omega;s,\psi)) - \bar{y}(s; i(\Omega;s,\psi))\right].$$

For a given aggregate reserve supply $s$, we focus on the symmetric case in which each bank enters the day with a reserve share $s$. Equilibrium requires the inactive fraction to equal the measure of banks whose participation costs exceed the expected value of being active:

$$\psi^*(s) = 1 - \Phi_\kappa(\mathcal{B}(s,\psi^*(s))). \tag{21}$$

Appendix F.1.2 provides sufficient conditions for the existence and uniqueness of the share of inactive banks $\psi^*(s)$ given a reserve supply, $s$. We also show that the equilibrium inactive fraction $\psi^*(s)$ increases with reserve supply, in particular $d\psi^*(s)/ds > 0$ for $s > \bar{x}$ (and given symmetry $d\psi^*(s)/ds < 0$ for $s < \bar{x}$.) assuming symmetric unimodal payment-shock

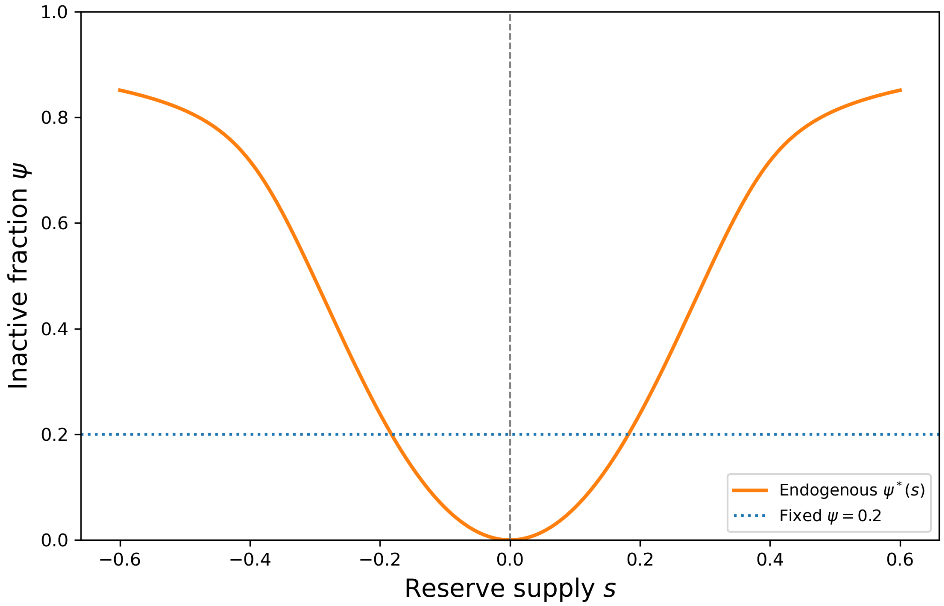


Figure 8: Endogenous Inactive Fraction

*Notes:* The figure plots the equilibrium inactive fraction $\psi^*(s)$ implied by the equation (21). The dotted horizontal line shows the fixed-fragmentation benchmark, $\psi = 0.2$, and the vertical dashed line marks the reserve requirement, $s = \bar{x} = 0$. The calibration is the same as in Figure 7.

distributions.

Figure 8 plots the inactive fraction, $\psi^*(s)$, in equilibrium as reserve supply varies. It reflects the two components of the participation benefit. Starting from zero reserves, the insurance component $A(s)$ falls with reserve supply because discount window shortfalls become less likely. This pushes banks toward inactivity. The option value of adjusting to prevailing market conditions $O(s, \psi)$ is most important near the kink of the reserve demand curve. Holding $\psi$ fixed, the local approximation implies that $O_s(s, \psi)$ is proportional to $-f'(\bar{x} - s)$, so its variation is governed by the curvature of the reserve demand curve. For symmetric unimodal payment shocks, this option value is high around the kink and small in the flat abundant reserve region. As a result, $\psi^*(s)$ is relatively insensitive to reserve supply near the kink, where the increased option value partly offsets the decline in insurance value.

The equilibrium participation decision gives rise to the following proposition.

**Proposition 5.2** (Endogenous Fragmentation and Interbank Rate)**.** *Suppose a unique participation equilibrium exists and has inactive fraction $\psi^*(s)$. Then the interbank rate is*

$$i_t = \mathcal{I}\left(s - \frac{\Omega_t}{1 - \psi^*(s)}\right), \qquad \Omega_t \sim H_{\psi^*(s)}.$$

*Under the granular-bank calibration, the local standard deviation of the interbank rate is*

$$\sigma_i(s) \simeq \chi f(\bar{x} - s) \frac{\sqrt{\psi^*(s)}}{1 - \psi^*(s)} \frac{\sigma'}{\sqrt{N_\Omega}}. \tag{22}$$

*where $\sigma'$ is the standard deviation of the ordinary payment flows and $N_\Omega$ is the effective number of banks in the market.*

The derivation is in Appendix F.1.2.

Proposition 5.2 shares the same fragmented interbank market as Proposition 5.1, but with the fragmentation endogenously determined by banks' participation decisions. In addition, the proposition shows that the volatility of the interbank market rate depends on reserve supply. Given its impact on fragmentation, it changes the distribution of payment flows locked outside the interbank market. Moreover, it changes the size of the active market, so a given imbalance has a larger effect on the portfolio of the banks who participate in redistributing reserves when the active market is thin. And independently of fragmentation, reserve supply changes the local price sensitivity of the market rate, captured by $f(\bar{x} - s)$.

Figure 7 plots the interbank rate as a function of aggregate reserve supply $s$ under fixed and endogenous fragmentation. This highlights two issues. First, when supply equals reserve requirements, i.e. when $s = \bar{x}$ , there is no fragmentation and the demand reserve curve is exactly as in the baseline model. Second, endogenous fragmentation increases the uncertainty around the reserve demand curve compared to the fixed fragmentation point, especially around the satiation point. But uncertainty around the reserve demand curve remains large well into the region where the reserve demand curve would anchored at the deposit facility rate in the baseline model, or even with fixed fragmentation. This also implies that when reserves are abundant, the endogenous band is wider than the fixed fragmentation benchmark because higher reserve supply induces greater market fragmentation. The endogenous error band does not, however, expand monotonically with reserve supply, because the rise in $\psi^*(s)$ is partially offset by the decline in $f(\bar{x} - s)$ as the reserve demand curve becomes flatter.

## 5.3 Aggregate Reserve Supply Shocks in Fragmented Markets

We now introduce an unexpected aggregate reserve supply shock during the day, after banks have made their participation decisions. To be precise, banks choose whether to participate in the interbank market at the pre-shock reserve supply $s_0$, pinning down the inactive fraction $\psi^*(s_0)$. After these decisions are made, an unexpected shock moves aggregate reserve supply to $s_1$.

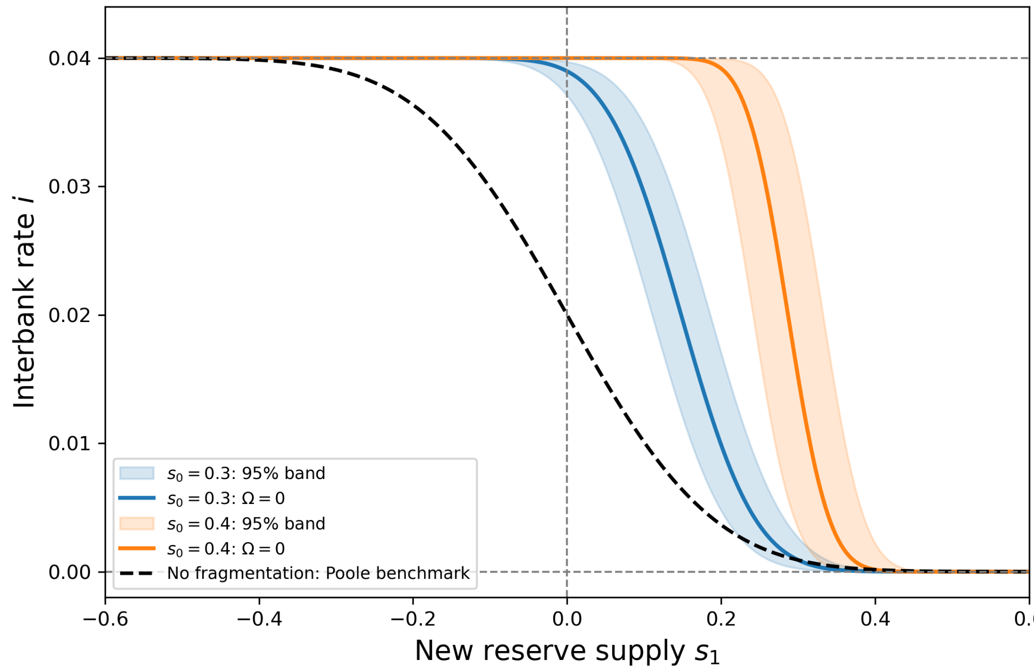


Figure 9: Aggregate Reserve Supply Shocks in Fragmented Markets

*Notes:* The figure plots the interbank rate on impact as a function of the new reserve supply $s_1$ after participation decisions have been made at pre-shock reserve supply $s_0 = 0.3$ or $s_0 = 0.4$. The dashed black line is the no-fragmentation Poole benchmark, $\mathcal{I}(s_1)$, which is the rate that would prevail if all banks participated in the interbank market after the shock. For each $s_0$, the solid line sets $\Omega = 0$, and the shaded band evaluates Proposition 5.3 at the 2.5th and 97.5th percentiles of $H_{\psi^*(s_0)}$. The inactive fraction $\psi^*(s_0)$ and the inactive-flow distribution use the same participation-cost and granular-bank calibration as Figure 7. The calibration gives $\psi^*(0.3) = 0.494$ and $\psi^*(0.4) = 0.717$. Plotted rates are bounded by the standing-facility corridor $[i^{IORB}, i^{DW}]$.

**Proposition 5.3** (Interbank Rates and Reserve Supply Shocks in Fragmented Markets)**.** *Suppose an unexpected shock moves aggregate reserve supply from $s_0$ to $s_1$. The interbank rate on impact is*

$$i = \mathcal{I}\left(s_0 - \frac{\Omega}{1-\psi^*(s_0)} + \frac{s_1 - s_0}{1-\psi^*(s_0)}\right), \qquad \Omega \sim H_{\psi^*(s_0)}.$$

The derivation is in Appendix F.2.

Proposition 5.3 implies that the reserve demand curve endogenously shifts outward as supply increases. This is illustrated in Figure 9. In this case, active banks have to absorb not only the random imbalance of payment flows generated by inactive banks as in Proposition 5.2 but also the reserve supply shock $s_1 - s_0$. The reserve demand curve therefore shifts in line with $s_1 - s_0$, scaled by the fraction of active banks, $1 - \psi^*(s_0)$, i.e. it shifts in line with $\frac{s_1 - s_0}{1-\psi^*(s_0)}$. And the higher the initial reserve supply, $s_0$, the lower the endogenously determined share of active banks, $1 - \psi^*(s_0)$, and the greater the outwards.

An interesting feature of the model is that the endogenous outward shift in the demand curve is not fixed but depends on the unexpected change in the supply of reserves. If there are no unexpected changes in the supply after banks' participation decisions are made, the reserve demand curve is given by Proposition 5.1. But if there are unexpected reductions in reserve supply, the outward shift in the reserve demand curve can be substantial if there are only few active market participants. And this risk is particularly large during balance sheet normalization after long periods where money markets traded at the deposit rate.

# 6 Conclusion

This paper develops a framework for studying how liquidity regulation, monetization frictions and money market fragmentation affect the demand for central bank reserves. These factors have often been identified as structurally increasing the demand for reserves. However, frictions generally do not produce a simple outward shift of the curve but reshape the curve by changing its slope, curvature, and the satiation point. Fragmentation, though, is different. It breaks the one-to-one mapping from aggregate reserve supply to the interbank rate and can give rise to endogenous shifts in the reserves demand curve. As discussed in the introduction, this has important implications for the discussion about balance sheet normalization.

Several important questions remain open. On the empirical side, a key issue is to identify how relevant our various mechanisms are in practice as policy implications differ. More generally, it would be interesting to test the paper's results, which is difficult. Distinguishing a reshaping from a horizontal shift requires identifying variation in slope and satiation point separately from changes in reserve supply—a challenge compounded by the fact that central banks operating near the flat part of the curve provide little information about the steep portion.

On the theoretical side, it would be very interesting to analyze how the mechanisms studied here play out in a demand-driven floor given its prevalent use by several major central banks. They operate close to the satiation point, suggesting that the results of the paper are highly relevant. But banks' individual decisions to draw reserves are endogenous to the same portfolio tradeoffs modeled here, creating a feedback between the demand curve and the quantity of reserves in circulation that our framework does not capture. More generally, the welfare implications of operating at different points on the reserve demand curve, and the optimal design of standing facilities and repo markets and the interaction with prudential considerations underpinning banks' reserve demand remain important areas for future work.

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# Appendix: Additional Results and Proofs

# Appendix A. Proposition 2.1 and Corollary 2.4

This appendix derives Proposition 2.1 and Corollary 2.4 in balance sheet shares.

**Truncated moments.** Define

$$m(k) \equiv \int_{-\infty}^{k} (k-\zeta) f(\zeta)\, d\zeta,$$
$$m_2(k) \equiv \int_{-\infty}^{k} (k-\zeta)^2 f(\zeta)\, d\zeta.$$

Then $m(\bar{x}-r) = \mathbb{E}[b(r,\zeta)]$ is the expected discount window borrowing.

**Mean.** The relevant return payoff is

$$y(r,\zeta) = \eta r - \chi b(r,\zeta).$$

Because $\mathbb{E}[\zeta] = 0$,

$$\mathbb{E}[y(r,\zeta)] = \eta r - \chi m(\bar{x}-r).$$

By Leibniz's rule,

$$\frac{d}{dr} m(\bar{x}-r) = -F(\bar{x}-r).$$

Therefore

$$\frac{d}{dr}\mathbb{E}[y(r,\zeta)] = \eta + \chi F(\bar{x}-r). \tag{A.1}$$

**Variance.** Let

$$Y(r,\zeta) = -\chi b(r,\zeta).$$

Since $\eta r$ is deterministic, $\mathrm{Var}(y(r,\zeta)) = \mathrm{Var}(Y(r,\zeta))$. Using the truncated moments defined above,

$$\mathbb{E}[Y(r,\zeta)] = -\chi m(\bar{x}-r),$$

and

$$\mathbb{E}[Y(r,\zeta)^2] = \chi^2 m_2(\bar{x}-r).$$

Hence

$$\mathrm{Var}(Y(r,\zeta)) = \chi^2 m_2(\bar{x}-r) - \chi^2 m(\bar{x}-r)^2.$$

Moreover,
$$\frac{d}{dr}m(\bar{x}-r) = -F(\bar{x}-r), \qquad \frac{d}{dr}m_2(\bar{x}-r) = -2m(\bar{x}-r).$$
Therefore
$$\begin{aligned}
\frac{d}{dr}\operatorname{Var}\big(y(r,\zeta)\big) &= \chi^2\frac{d}{dr}m_2(\bar{x}-r) - 2\chi^2 m(\bar{x}-r)\frac{d}{dr}m(\bar{x}-r) \\
&= -2\chi^2 m(\bar{x}-r) + 2\chi^2 m(\bar{x}-r)F(\bar{x}-r) \\
&= -2\chi^2 m(\bar{x}-r)\big(1-F(\bar{x}-r)\big). \qquad \text{(A.2)}
\end{aligned}$$

Substituting equation (A.1) and equation (A.2) into the first-order condition for equation (1) yields equation (2), which proves Proposition 2.1.

**Sign of the Variance Derivative.** Since $m(\bar{x}-r) \geq 0$ and $1-F(\bar{x}-r) \geq 0$, equation (A.2) implies
$$\frac{d}{dr}\operatorname{Var}\big(y(r,\zeta)\big) = -2\chi^2 m(\bar{x}-r)\big(1-F(\bar{x}-r)\big) \leq 0.$$
Thus, reserve holdings weakly reduce the variance of end-of-day returns. The inequality is strict whenever expected discount window borrowing is positive and the no-shortfall event has positive probability.

**Risk Aversion and the Inverse Demand Curve.** Solving equation (2) for the interbank rate gives
$$i(r;\gamma) = i^{IORB} + \chi F(\bar{x}-r) + \gamma\chi^2 m(\bar{x}-r)\big(1-F(\bar{x}-r)\big).$$
Differentiating with respect to $\gamma$ yields
$$\frac{\partial i(r;\gamma)}{\partial \gamma} = \chi^2 m(\bar{x}-r)\big(1-F(\bar{x}-r)\big) \geq 0,$$
since $\chi > 0$, $m(\bar{x}-r) \geq 0$, and $1-F(\bar{x}-r) \geq 0$. The effect is strict whenever expected discount window borrowing is positive and the no-shortfall event has positive probability. This proves Corollary 2.4.

# Appendix B. Proposition 3.1

Assume $\gamma > 0$. The return payoff with corporate bonds is

$$y(r,\zeta) + c\tilde{\epsilon}_c, \qquad y(r,\zeta) = \eta r - \chi b(r,\zeta).$$

Because corporate bond return shocks are independent of payment shocks,

$$\mathbb{E}[y(r,\zeta) + c\tilde{\epsilon}_c] = \mathbb{E}[y(r,\zeta)] + \epsilon_c c,$$

and

$$\mathrm{Var}(y(r,\zeta) + c\tilde{\epsilon}_c) = \mathrm{Var}(y(r,\zeta)) + \sigma_c^2 c^2.$$

The mean-variance objective therefore separates into a reserve block and a corporate bond block:

$$\begin{aligned}\mathbb{E}[y(r,\zeta) + c\tilde{\epsilon}_c] - \frac{\gamma}{2}\mathrm{Var}(y(r,\zeta) + c\tilde{\epsilon}_c) &= \Big(\mathbb{E}[y(r,\zeta)] - \frac{\gamma}{2}\mathrm{Var}(y(r,\zeta))\Big) \\ &\quad + \Big(\epsilon_c c - \frac{\gamma}{2}\sigma_c^2 c^2\Big).\end{aligned}$$

Government bond holdings do not appear because they earn the same risk-free rate as the residual interbank position.

The reserve block is exactly the baseline problem studied in Section 2. Hence the optimal reserve choice is still characterized by equation (2). The corporate bond block is strictly concave because $\gamma > 0$ and solves

$$\max_{c \geq 0} \ \epsilon_c c - \frac{\gamma}{2}\sigma_c^2 c^2.$$

Its first-order condition,

$$\epsilon_c - \gamma\sigma_c^2 c = 0,$$

implies the unique interior optimum

$$c^* = c^{int} = \frac{\epsilon_c}{\gamma\sigma_c^2}.$$

Finally, interbank market clearing pins down government bond holdings residually:

$$g^* = 1 - r^* - c^*.$$

This proves Proposition 3.1.

# Appendix C. Proposition 3.2

The Slack-LCR region follows immediately from Proposition 3.1. When $\bar{c}(r) \geq c^{int}$, the unconstrained corporate bond position remains feasible while the reserve requirement is tighter than the LCR, so reserve demand coincides with the benchmark and $c^{LCR}(r) = c^{int}$. It remains to prove the characterizations for the Crowding-out region and the Floor Region.

## C.1 Crowding-out Region

Consider reserve holdings $r$ such that $r_0 < r < r_1$, so that $\bar{c}(r) < c^{int}$ and $i > i^{IORB}$.

**Step 1: the optimum cannot be strictly LCR-dominant.** Suppose $k_2(c) > k_1(r)$, so that $k(r,c) = k_2(c)$ is independent of $r$. Then the discount window term, the variance term, and $v(c)$ do not vary with $r$. The only remaining $r$-dependence in

$$\mathcal{U}^{LCR}(r,c) = u(r, k(r,c)) + v(c)$$

comes from the carry term $(i^{IORB} - i)r$. Therefore,

$$\frac{\partial}{\partial r}\mathcal{U}^{LCR}(r,c) = i^{IORB} - i < 0$$

whenever $i > i^{IORB}$. Hence lowering $r$ strictly raises utility as long as the LCR is strictly tighter than the reserve requirement, so an optimum cannot lie in the strictly LCR-dominant region.

**Step 2: the optimum lies on the boundary $c = \bar{c}(r)$.** Step 1 implies that, in the Crowding-out region, the optimum must satisfy $k(r,c) = k_1(r) = \bar{x} - r$. Accordingly, the bank's problem becomes

$$\begin{aligned} \max_{r,c} \quad & \mathcal{U}^{LCR}(r,c) := u\big(r, k_1(r)\big) + v(c) \\ \text{s.t.} \quad & c \leq \bar{c}(r) = 1 - \bar{\ell} + \bar{x} - r. \end{aligned} \tag{C.1}$$

Because this region is defined by $\bar{c}(r) \leq c^{int}$, we have

$$v'(c) = \epsilon_c - \gamma\sigma_c^2 c \geq 0 \qquad \text{for all } c \leq \bar{c}(r).$$

Hence the objective is weakly increasing in $c$ over the feasible set, so the optimal corporate bond holding is attained at the upper bound:

$$c^{LCR}(r) = \bar{c}(r) = 1 - \bar{\ell} + \bar{x} - r.$$

**Step 3: reserve demand in the Crowding-out region.** Substituting the boundary choice into equation (C.1) yields the reduced one-dimensional problem

$$\max_{r} \left\{ u(r, \bar{x} - r) + v\big(1 - \bar{\ell} + \bar{x} - r\big) \right\}.$$

The interior first-order condition is

$$\frac{d}{dr} u(r, \bar{x} - r) = \frac{d}{dc} v(c)\Big|_{c = 1 - \bar{\ell} + \bar{x} - r}. \tag{C.2}$$

Along the boundary $k_1(r) = \bar{x} - r$, the left-hand side is exactly the marginal value of reserves in the baseline problem:

$$\frac{d}{dr} u(r, \bar{x} - r) = \eta + \chi F(\bar{x} - r) + \gamma \chi^2 m(\bar{x} - r)\big(1 - F(\bar{x} - r)\big).$$

The right-hand side is $v'(c) = \epsilon_c - \gamma \sigma_c^2 c$. Substituting $c = 1 - \bar{\ell} + \bar{x} - r$ into equation (C.2) gives equation (9). This proves the Crowding-out region characterization in Proposition 3.2.

## C.2 The Floor Region

In the Floor Region, the interbank rate has reached the floor, so $i = i^{IORB}$. The carry cost of reserves therefore vanishes, and the argument used in the Crowding-out region to rule out LCR-dominant allocations no longer applies.

**Step 1: a strictly reserve-binding allocation cannot be optimal.** Suppose first that the bank chooses an allocation with the reserve requirement strictly tighter than the LCR, so that $k_1(r) > k_2(c)$. Then for a given $r$ the discount window term is independent of $c$. Since $i = i^{IORB}$, the bank's objective on this side reduces to choosing $c$ only through

$$\max_{0 \le c \le \bar{c}(r)} v(c) = \max_{0 \le c \le \bar{c}(r)} \left\{ \epsilon_c c - \frac{\gamma}{2} \sigma_c^2 c^2 \right\}.$$

Because $v(\cdot)$ is strictly concave with unconstrained maximizer $c^{int} = \epsilon_c / (\gamma \sigma_c^2)$, and the Floor Region is reached only after $\bar{c}(r) < c^{int}$, the best allocation among all reserve-tight choices is attained at the boundary $c = \bar{c}(r)$. Hence a strictly reserve-binding allocation cannot be optimal.

**Step 2: corporate bond choice on the weakly LCR-binding side.** The boundary argument above implies that the optimum must lie in the region $k_2(c) \geq k_1(r)$ where the LCR is weakly tighter than the reserve requirement. In that region, $k(r, c) = k_2(c) = \bar{\ell} - 1 + c$. Conditional on $c$, this threshold does not depend on $r$ and $i = i^{IORB}$, so the objective is flat in $r$. This is why reserve demand is flat at the floor.

Denote $K = \bar{\ell} - 1 + c$ and $Y(K) = -\chi(K - \zeta)^+$. Since $dK/dc = 1$,

$$\frac{d}{dK} m(K) = F(K), \qquad \frac{d}{dK} m_2(K) = 2m(K).$$

Thus

$$\frac{d}{dK} \operatorname{Var}(Y(K)) = 2\chi^2 m(K)(1 - F(K)).$$

Combining the derivative of the expected shortfall term with the derivative of $v(c)$ gives

$$\frac{\partial \mathcal{U}^{LCR}(r, c)}{\partial c} = -\chi F(K) - \gamma\chi^2 m(K)(1 - F(K)) + \epsilon_c - \gamma\sigma_c^2 c.$$

Substituting $K = \bar{\ell} - 1 + c$ yields (10). Let $c^F$ denote the corresponding solution.

# Appendix D. Monetization Frictions and Reserve Specialness

This appendix derives the reserve demand schedule under monetization frictions and records the local approximation used for simulation. Consistent with the convention in Section 4, the derivation below states the marginal-value expressions in the limit as $\beta \uparrow 1$.

**Step 1: Local binding logic.** Start from the primitive normalized Bellman equation in equation (11). Around a stationary equilibrium in which the flexible target reserve share is $r^o$,

$$\tau(r^o, \zeta) - r^o = \frac{(1 - r^o)\zeta}{1 + \zeta},$$

positive shocks imply $\tau(r^o, \zeta) \geq r^o$ whenever $r^o < 1$, so the flexible target share is feasible and $V^-(\tau(r^o, \zeta)) = V^o$. Negative shocks imply $\tau(r^o, \zeta) < r^o$, so returning to the flexible target requires increasing reserves by monetizing government bonds. Under the local interior case in which the constraint binds after negative shocks, the relevant continuation states are exactly the states $\zeta < 0$. Substituting this local optimality logic into equation (11) gives the reduced Bellman equation in equation (12).

**Step 2: Marginal-value recursion.** Define the reserve-versus-government-bond margin

$$\delta(r,g) = V_r(r,g) - V_g(r,g).$$

Differentiating equation (12) with respect to $r$, holding total portfolio shares fixed, gives

$$\delta(r,g) = (i^{IORB} - i) + \chi F(\bar{x} - r) + P \int_{-1}^{0} \delta^{-}(\tau(r,\zeta)) f(\zeta)\, d\zeta.$$

The factor $(1+\zeta)$ multiplying the continuation value cancels the chain-rule derivative $\partial\tau(r,\zeta)/\partial r = 1/(1+\zeta)$. When the monetization constraint binds,

$$\delta^{-}(s) = V_r(s, 1-s) - V_g(s, 1-s),$$

so the liquidity wedge depends only on the inherited reserve share. Hence $\delta^{-}(\tau(r,\zeta)) = \delta(\tau(r,\zeta))$.

Define

$$T^{-}[h](r) = \int_{-1}^{0} h(\tau(r,\zeta)) f(\zeta)\, d\zeta, \qquad q = T^{-}[1] = F(0).$$

The recursion is

$$\delta(r) = (i^{IORB} - i) + \chi F(\bar{x} - r) + PT^{-}\delta(r).$$

Iterating forward gives

$$\delta = \sum_{n=0}^{\infty} P^n (T^{-})^n \left[(i^{IORB} - i) + \chi F(\bar{x} - r)\right].$$

Because $(T^{-})^n 1 = q^n$, for $Pq < 1$,

$$\delta(r) = \frac{i^{IORB} - i}{1 - Pq} + \chi H_{\bar{x}}(r;P), \qquad H_{\bar{x}}(r;P) = \sum_{n=0}^{\infty} P^n \left((T^{-})^n F(\bar{x} - \cdot)\right)(r).$$

The flexible first-order condition $\delta(r) = 0$ yields equation (13).

**Step 3: Local small-shock approximation.** Using the same first-order approximation as in Appendix E,

$$\tau(r,\zeta) = \frac{r+\zeta}{1+\zeta} \approx r + \alpha(r)\zeta, \qquad \alpha(r) = 1 - r,$$

the negative-shock operator becomes

$$T^-[h](r) \approx \int_{-1}^{0} h(r+\alpha(r)\zeta) f(\zeta)\, d\zeta.$$

If shocks are local and Gaussian, $\zeta \sim N(0,\sigma^2)$, then

$$F(\bar{x}-r) = \Phi\left(\frac{\bar{x}-r}{\sigma}\right).$$

For $n \geq 1$,

$$\left((T^-)^n F(\bar{x}-\cdot)\right)(r) \approx \mathbb{E}\left[\Phi\left(\frac{\bar{x}-r-\alpha(r)\sum_{j=1}^n Z_j}{\sigma}\right)\prod_{j=1}^{n} \mathbf{1}_{\{Z_j<0\}}\right],$$

where the $Z_j$ are independent $N(0,\sigma^2)$ shocks. Equivalently, with $Z^- \sim Z \mid Z<0$,

$$\left((T^-)^n F(\bar{x}-\cdot)\right)(r) \approx q^n \mathbb{E}\left[\Phi\left(\frac{\bar{x}-r-\alpha(r)\sum_{j=1}^n Z_j^-}{\sigma}\right)\right].$$

For Gaussian shocks, $Z^- = -|N(0,\sigma^2)|$.

## D.1 Monetization Frictions and the LCR

This appendix proves the propositions in Section 4.2 from the marginal conditions implied by equation (16). We avoid restating the Bellman problem and use the continuation-value notation introduced in the main text.

### D.1.1 Corporate bonds without the LCR

Without the LCR, discount window borrowing is $b(r,\zeta) = (\bar{x}-r-\zeta)^+$. Differentiating equation (16) with respect to reserves, relative to government bonds, gives the reserve margin

$$\delta(r,g,c) = (i^{IORB}-i) + \chi F(\bar{x}-r) + P\int_{-1}^{0} \delta^-(\tau(r,\zeta)) f(\zeta)\, d\zeta. \tag{D.1}$$

The integral is over negative payment shocks because, as in Section 4, the monetization constraint binds only when inherited reserves fall below the target.

The corporate bond margin is

$$V_c(r,g,c) - V_g(r,g,c) = v'(c) = \epsilon_c - \gamma\sigma_c^2 c. \tag{D.2}$$

Thus, if the interior corporate bond allocation is feasible, the corporate bond first-order condition is

$$v'(c) = 0 \quad \Longrightarrow \quad c = c^{int} = \frac{\epsilon_c}{\gamma\sigma_c^2}.$$

Because $c$ does not enter $b(r, \zeta)$, the reserve margin in equation (D.1) is independent of corporate bond holdings. Corporate bonds remain separable, and the reserve demand equation is exactly the monetization-friction reserve demand equation in Proposition 4.1. This proves Proposition 4.2.

### D.1.2 Adding the LCR

With the LCR, discount window borrowing is $b(r, c, \zeta) = (k(r, c) - \zeta)^+$, where

$$k_1(r) = \bar{x} - r, \qquad k_2(c) = \bar{\ell} - 1 + c, \qquad k(r, c) = \max\{k_1(r), k_2(c)\}.$$

Away from the kink $k_1(r) = k_2(c)$, the reserve margin is

$$\delta(r, g, c) = (i^{IORB} - i) + \chi \mathbf{1}_{\{k_1(r) \geq k_2(c)\}} F(k_1(r)) + P \int_{-1}^{0} \delta^-(\tau(r, \zeta)) f(\zeta)\, d\zeta. \tag{D.3}$$

The corporate bond margin is

$$V_c(r, g, c) - V_g(r, g, c) = v'(c) - \chi \mathbf{1}_{\{k_2(c) \geq k_1(r)\}} F(k_2(c)). \tag{D.4}$$

These two explicit margins are the source of the interaction between the LCR and monetization frictions.

Let $c^*(r)$ denote the corporate bond share chosen after reserve share $r$ is inherited, and recall that

$$\bar{c}(r) = 1 - \bar{\ell} + \bar{x} - r$$

is the LCR boundary. If $\bar{c}(r) \geq c^{int}$, the unconstrained corporate bond allocation is feasible and $c^*(r) = c^{int}$. If $\bar{c}(r) < c^{int}$, the left derivative at the boundary is positive, $v'(\bar{c}(r)) > 0$. The allocation then either remains at the kink or moves into the strict LCR-dominant region, where the corporate bond choice solves

$$v'(c^*) = \chi F(\bar{\ell} - 1 + c^*).$$

Denote this solution by $c^L$. Because the strict LCR-dominant objective is concave, the kink is optimal exactly when the strict-LCR optimum lies weakly to the left of the boundary. Equivalently,

$$c^L \leq \bar{c}(r) \quad \Longleftrightarrow \quad v'(\bar{c}(r)) - \chi F(\bar{x} - r) \leq 0,$$

where the equality $F(\bar{\ell}-1+\bar{c}(r)) = F(\bar{x}-r)$ uses the definition of $\bar{c}(r)$. Thus $c^*(r) = \bar{c}(r)$ when $c^L \leq \bar{c}(r)$, while $c^*(r) = c^L$ when $c^L > \bar{c}(r)$. This proves the static corporate bond policy in Proposition 4.3.

It remains to derive reserve demand. The current normalized marginal payoff of reserves is

$$S^*(r) = \begin{cases} F(\bar{x}-r), & \text{if } c^*(r) = c^{int}, \\ F(\bar{x}-r) - \dfrac{v'(\bar{c}(r))}{\chi}, & \text{if } c^*(r) = \bar{c}(r), \\ 0, & \text{if } c^*(r) = c^L. \end{cases}$$

The first line applies when the LCR is slack and corporate bonds are locally independent of reserves. The second applies on the LCR boundary: an additional unit of reserves reduces discount window borrowing but also reduces corporate bond holdings one-for-one, generating the net term $\chi F(\bar{x}-r) - v'(\bar{c}(r))$. The third applies in the strict LCR-dominant region, where reserves do not reduce current discount window borrowing because the LCR shortfall is strictly larger than the reserve shortfall.

Iterating the marginal value of reserves over future constrained states gives

$$H^*(r;P) = \sum_{n=0}^{\infty} P^n \left((T^-)^n S^*\right)(r).$$

The reduced reserve shadow value satisfies

$$\delta(r) = \frac{i^{IORB} - i}{1-Pq} + \chi H^*(r;P).$$

At an interior reserve choice, $\delta(r) = 0$, so

$$i(r;P) = i^{IORB} + (1-Pq)\chi H^*(r;P).$$

If $H^*(r;P) = 0$, then $i(r;P) = i^{IORB}$ and reserve demand is at the floor.

# Appendix E. Rebalancing Disruptions

Using the environment introduced in Section 4, this appendix studies a different market disruption in which banks may be unable to rebalance between reserves and government bonds in either direction. We abstract from higher-yielding risky assets and LCR regulation, so the analysis isolates the rebalancing friction between reserves and government bonds. Specifically, with probability $P$, the bank cannot rebalance its inherited portfolio; with probability $1-P$, it can adjust its portfolio freely.

**Banks' Maximization Problems.** Because the problem is homogeneous in wealth, we again work with value per dollar of current wealth. There are two value functions in addition to the flexible value $V^o$. The function $V^n(r,g)$ is the value of entering the day with portfolio shares $(r,g)$ when the bank can rebalance its portfolio. The function $V^m(r,g)$ is the value of entering the day with the same portfolio shares but the bank faces a rebalancing disruption. In the flexible state, the bank chooses its portfolio freely:

$$\begin{aligned} V^o &\equiv \max_{0\le r'\le 1} V^n(r', 1-r') \\ V^n(r,g) &= \mathbb{E}_\zeta\big[\eta r - \chi b(r,\zeta) + (1+\zeta)\left((1-P)V^o + PV^m\left(\tau(r,\zeta), \frac{g}{1+\zeta}\right)\right)\big]. \end{aligned} \tag{E.1}$$

In the disrupted state, ordinary payment flows $\zeta' W$ cannot be accommodated through intraday rebalancing and therefore effectively become additional late day payment shocks. Let $\tilde{\zeta} = \zeta + \zeta'$ have c.d.f. $\tilde{F}$ and density $\tilde{f}$. Then

$$V^m(r,g) = \mathbb{E}_{\tilde{\zeta}}\big[\eta r - \chi b(r,\tilde{\zeta}) + (1+\tilde{\zeta})\left((1-P)V^o + PV^m\left(\tau(r,\tilde{\zeta}), \frac{g}{1+\tilde{\zeta}}\right)\right)\big]. \tag{E.2}$$

The derivation below shows that the first order conditions with respect to reserves are

$$\begin{aligned} \delta^n(r,g) &\equiv V^n_r(r,g) - V^n_g(r,g) = (i^{IORB} - i) + \chi F(\bar{x} - r) + P\int_{-1}^{\infty} \delta^m\left(\tau(r,\zeta), \frac{g}{1+\zeta}\right) f(\zeta)\, d\zeta, \\ \delta^m(r,g) &\equiv V^m_r(r,g) - V^m_g(r,g) = (i^{IORB} - i) + \chi \tilde{F}(\bar{x} - r) + P\int_{-1}^{\infty} \delta^m\left(\tau(r,\tilde{\zeta}), \frac{g}{1+\tilde{\zeta}}\right) \tilde{f}(\tilde{\zeta})\, d\tilde{\zeta}. \end{aligned} \tag{E.3}$$

The marginal benefit of reserves is the sum of three components. The first two terms are the same as in the Poole benchmark and capture the direct benefit of holding reserves for today's utility. It is the liquidity insurance benefit of avoiding the costly discount window against the opportunity cost of holding reserves rather than lending in the interbank market. The last term captures the expected future marginal value of reserves when banks are unable to rebalance their portfolios between government bonds and reserves.

**Proposition E.1** (Reserve Demand with Rebalancing Frictions)**.** *In a stationary equilibrium with the same interbank rate $i$ in the long run, reserve demand under rebalancing frictions is characterized by*

$$i(r;P) = i^{IORB} + (1-P)(i^{DW} - i^{IORB})\left[F(\bar{x} - r) + \sum_{n=1}^{\infty} P^n \left(T\tilde{T}^{n-1}\tilde{F}(\bar{x} - \cdot)\right)(r)\right]. \tag{E.4}$$

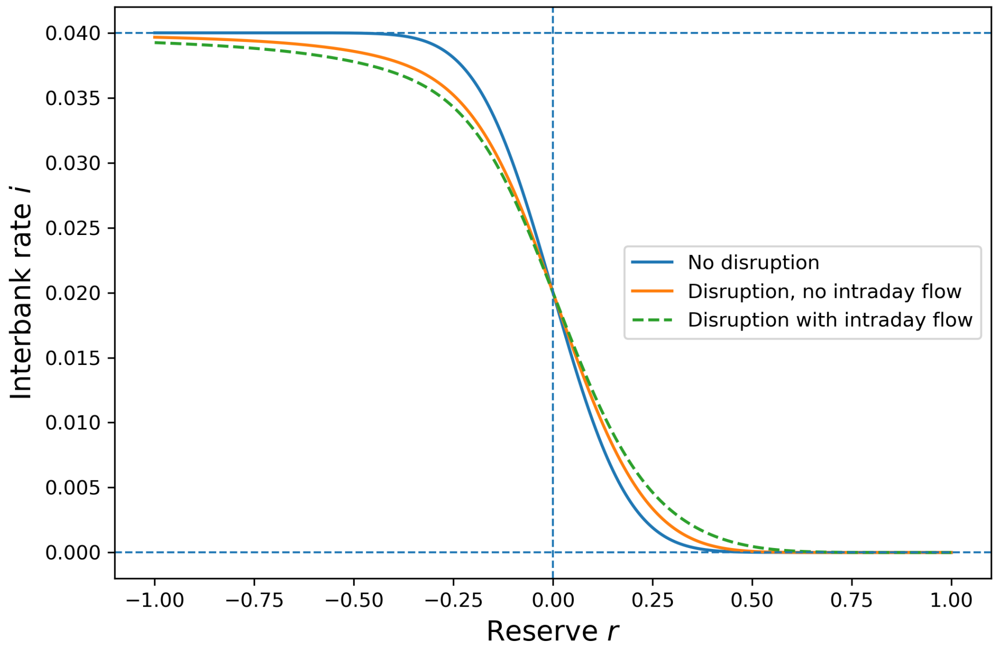


Figure E.1: Reserve Demand Curve (Rebalancing Disruptions)

*Notes:* The figure plots the reserve demand curve under rebalancing disruptions implied by Proposition E.1. The no-disruption curve sets $P = 0$. The disruption-without-intraday-flow curve sets $P = 0.5$ and shuts down the ordinary intraday payment-flow shock, $\zeta' = 0$. The disruption-with-intraday-flow curve sets $P = 0.5$ and adds $\zeta' \sim N(0, 0.15^2)$ independently of the late day payment shock $\zeta$, so that $\tilde{\zeta} = \zeta + \zeta' \sim N(0, 2 \times 0.15^2)$ in disrupted states. The simulation uses the local small-shock approximation and sets $i^{DW} = 4\%$, $i^{IORB} = 0\%$, $\bar{x} = 0$, and $\zeta \sim N(0, 0.15^2)$.

*where $T$ and $\tilde{T}$ are the share-transition operators*

$$T[h](r) = \int_{-1}^{\infty} h(\tau(r,\zeta)) f(\zeta)\, d\zeta, \qquad \tilde{T}[h](r) = \int_{-1}^{\infty} h(\tau(r,\tilde{\zeta}))\tilde{f}(\tilde{\zeta})\, d\tilde{\zeta}.$$

*If the additional ordinary payment-flow shock is absent, $\zeta' = 0$, then $\tilde{\zeta} = \zeta$, $\tilde{F} = F$, and $\tilde{T} = T$. In this case, reserve demand simplifies to*

$$i(r;P) = i^{IORB} + (1-P)(i^{DW} - i^{IORB}) \sum_{n=0}^{\infty} P^n \left(T^n F(\bar{x} - \cdot)\right)(r). \tag{E.5}$$

The derivation below obtains equation (E.4) from equation (E.3) and reports the local Gaussian approximation in equation (E.6), which is used in the simulation.

When there are no rebalancing disruptions, $P = 0$, the reserve demand curve becomes

$$i(r;0) = i^{IORB} + (i^{DW} - i^{IORB})F(\bar{x} - r),$$

which is exactly the same as the Poole (1968) benchmark in Section 2. The extension to an infinite horizon is innocuous when there are no rebalancing disruptions, because continuation values depend only on end-of-day wealth, not on portfolio composition.

Figure E.1 illustrates the reserve demand curve under rebalancing disruptions when payment shocks are normally distributed. Relative to the Poole benchmark, the curve rotates: reserve demand is higher (lower) on the right (left)-hand side of the curve. The reason is that reserves now matter not only for current-period liquidity insurance, but also for their effect on future portfolio positions when banks may be unable to rebalance. This future value has two sides. An additional unit of reserves protects the bank against future reserve shortfalls, but it can also leave the bank carrying costly excess reserves in states in which it cannot rebalance. When current reserve holdings are relatively high, the shortfall-insurance channel dominates, so rebalancing disruptions raise reserve demand. When current reserve holdings are low, the excess-reserve cost is more likely to dominate, so rebalancing disruptions reduce reserve demand. Rebalancing disruptions therefore rotate the reserve demand curve rather than simply shifting it uniformly.

The dashed curve adds the case in which rebalancing disruptions also prevent banks from accommodating ordinary payment flows. Then the intraday payment flow effectively becomes an additional late day payment shock. The increasing payment volatility magnifies the rotation in the reserve demand curve: both the shortfall-insurance motive and the cost of being stuck with excess reserves become stronger.

## E.1 Derivation

This subsection first derives the recursion for the marginal value of reserves and then solves for the reserve demand schedule under two-sided rebalancing disruptions in shares.

**Step 1: Deriving the recursion for $\delta$.** The normal-state and disrupted-state Bellman equations are given in equation (E.1) and equation (E.2). Define

$$\delta^n(r,g) = V_r^n(r,g) - V_g^n(r,g), \qquad \delta^m(r,g) = V_r^m(r,g) - V_g^m(r,g).$$

Differentiating with respect to $r$ and $g$, holding the other state variable fixed, gives

$$\delta^n(r,g) = (i^{IORB} - i) + \chi F(\bar{x} - r) + P \int_{-1}^{\infty} \delta^m \left(\tau(r,\zeta), \frac{g}{1+\zeta}\right) f(\zeta)\, d\zeta,$$
$$\delta^m(r,g) = (i^{IORB} - i) + \chi \tilde{F}(\bar{x} - r) + P \int_{-1}^{\infty} \delta^m \left(\tau(r,\tilde{\zeta}), \frac{g}{1+\tilde{\zeta}}\right) \tilde{f}(\tilde{\zeta})\, d\tilde{\zeta}.$$

The factor $(1+s)$ multiplying continuation value cancels the chain-rule derivative $\partial\tau(r,s)/\partial r = 1/(1+s)$. This proves equation (E.3).

**Step 2: Operator solution.** On the simplex $g = 1 - r$, define

$$T[h](r) = \int_{-1}^{\infty} h(\tau(r,\zeta)) f(\zeta)\, d\zeta, \qquad \tilde{T}[h](r) = \int_{-1}^{\infty} h(\tau(r,\tilde{\zeta})) \tilde{f}(\tilde{\zeta})\, d\tilde{\zeta}.$$

Let

$$A^n(r) = (i^{IORB} - i) + \chi F(\bar{x} - r), \qquad A^m(r) = (i^{IORB} - i) + \chi \tilde{F}(\bar{x} - r).$$

Then

$$\delta^n = A^n + PT\delta^m, \qquad \delta^m = A^m + P\tilde{T}\delta^m.$$

For $P \in [0,1)$,

$$\delta^m = \sum_{j=0}^{\infty} P^j \tilde{T}^j A^m.$$

Substituting this into the normal-state equation and collecting the constant terms yields

$$\delta^n(r) = \frac{i^{IORB} - i}{1-P} + \chi \left[ F(\bar{x} - r) + \sum_{n=1}^{\infty} P^n \left( T\tilde{T}^{n-1}\tilde{F}(\bar{x} - \cdot) \right)(r) \right].$$

In the flexible state, the bank chooses $r$ so that $\delta^n(r) = 0$, which gives equation (E.4).

**Step 3: Local Gaussian approximation.** For fixed $r$,

$$\tau(r,s) = \frac{r+s}{1+s} = r + (1-r)s + O(s^2).$$

For the stationary reserve demand schedule, evaluate this slope at the reserve share $r$ itself and write $\alpha(r) = 1 - r$. Then $\tau(r,s) \approx r + \alpha(r)s$. If $\zeta \sim N(0,\sigma^2)$ and $\tilde{\zeta} \sim N(0,\tilde{\sigma}^2)$, then

$$\left( T\tilde{T}^{n-1}\tilde{F}(\bar{x} - \cdot) \right)(r) \approx \Phi\left( \frac{\bar{x} - r}{\sqrt{\tilde{\sigma}^2 + \alpha(r)^2\sigma^2 + \alpha(r)^2(n-1)\tilde{\sigma}^2}} \right),$$

Substituting this approximation into equation (E.4) gives

$$\begin{aligned} i(r;P) \approx i^{IORB} + (1-P)(i^{DW} - i^{IORB}) \Bigg[ \Phi\left(\frac{\bar{x} - r}{\sigma}\right) \\ + \sum_{n=1}^{\infty} P^n \Phi\left( \frac{\bar{x} - r}{\sqrt{\tilde{\sigma}^2 + (1-r)^2\sigma^2 + (1-r)^2(n-1)\tilde{\sigma}^2}} \right) \Bigg]. \end{aligned} \qquad \text{(E.6)}$$

# Appendix F. Fragmented Interbank Market

This appendix derives Proposition 5.1 and the endogenous-participation results in Section 5.2.

## F.1 Constant Reserve Supply

### F.1.1 Fixed Market Fragmentation

If active status is permanent, active banks do not attach continuation value to the risk of becoming inactive. Their current marginal payoff from reserves is

$$(i^{IORB} - i_t) + (i^{DW} - i^{IORB})F(\bar{x} - r).$$

The active-bank first-order condition therefore gives

$$i_t = \mathcal{I}(r).$$

Inactive banks absorb the aggregate ordinary payment-flow imbalance $\Omega_t$, so reserve-market clearing requires active banks to hold

$$r = r_t^A = \rho(\Omega_t; s, \psi) = s - \frac{\Omega_t}{1-\psi}.$$

Substituting this market-clearing condition into the active-bank inverse demand curve gives equation (18). Differentiating with respect to $\Omega_t$ yields

$$\frac{di(\Omega_t; s, \psi)}{d\Omega_t} = \mathcal{I}'(\rho(\Omega_t; s, \psi))\frac{\partial \rho(\Omega_t; s, \psi)}{\partial \Omega_t} = \left[-(i^{DW} - i^{IORB})f(\bar{x} - \rho(\Omega_t; s, \psi))\right]\left[-\frac{1}{1-\psi}\right].$$

Evaluating at $\Omega_t = 0$, so that $\rho(0; s, \psi) = s$, gives equation (19).

### F.1.2 Endogenous Participation

This subsection first derives the participation gain and the fixed point for the inactive fraction, then signs the reserve-supply comparative static, and finally derives the endogenous rate and volatility expressions.

**Participation gain.** With $\gamma = 0$, the active bank's expected excess return at interbank rate $i$ is

$$\bar{y}(r; i) = \eta(i)r - \chi m(\bar{x} - r), \qquad \eta(i) = i^{IORB} - i.$$

Since $d\,m(\bar{x}-r)/dr = -F(\bar{x}-r)$, the first-order condition is

$$0 = \bar{y}_r(r;i) = \eta(i) + \chi F(\bar{x}-r),$$

or $i = \mathcal{I}(r)$. An inactive bank keeps inherited reserve share $\bar{r}$ and does not accommodate ordinary payment flows. Its expected excess return is

$$\bar{y}^I(i,\bar{r}) = \eta(i)\bar{r} - \chi\tilde{m}(\bar{x}-\bar{r}), \qquad \tilde{m}(k) = \mathbb{E}[(k-\zeta-\zeta')^+].$$

The realized participation gain is therefore

$$\Delta(i,\bar{r}) = \max_r \bar{y}(r;i) - \bar{y}^I(i,\bar{r}) = \chi[\tilde{m}(\bar{x}-\bar{r}) - m(\bar{x}-\bar{r})] + \left[\max_r \bar{y}(r;i) - \bar{y}(\bar{r};i)\right],$$

which gives equation (20) after imposing $\bar{r} = s$ and taking expectations over the inactive-flow imbalance.

**Participation fixed point.** For a given aggregate reserve supply $s$, we focus on the symmetric case in which each bank enters the day with reserve share $s$. Define the inactive-fraction map

$$T_s(\psi) = 1 - \Phi_\kappa(\mathcal{B}(s,\psi)).$$

An equilibrium inactive fraction is a fixed point of $T_s$. Existence follows from a standard self-map argument. Suppose $\mathcal{B}(s,\psi)$ is continuous in $\psi$ and there exists a compact domain $\Psi = [0,\bar{\psi}]$, with $\bar{\psi} < 1$, such that $T_s(\Psi) \subseteq \Psi$. Then $T_s$ is a continuous self-map on a compact convex set, so Brouwer's fixed-point theorem gives a fixed point. Equivalently, in this one-dimensional setting, $G_s(\psi) \equiv T_s(\psi) - \psi$ satisfies $G_s(0) \geq 0$ and $G_s(\bar{\psi}) \leq 0$, so the intermediate value theorem gives a zero of $G_s$.

For uniqueness, define

$$H_s(\psi) = \psi + \Phi_\kappa(\mathcal{B}(s,\psi)) - 1.$$

Any participation equilibrium is a zero of $H_s$. If

$$\mathcal{B}_\psi(s,\psi) \geq 0 \qquad \text{on } \Psi,$$

then

$$H_s'(\psi) = 1 + \phi_\kappa(\mathcal{B}(s,\psi))\mathcal{B}_\psi(s,\psi) \geq 1,$$

so $H_s$ is strictly increasing and can cross zero at most once. The economic content of this condition is that a larger inactive sector does not reduce the value of active participation. In this environment, more inactivity generates more residual payment-flow imbalances, raising the value of being able to adjust reserve holdings after market conditions are realized.

**Reserve-supply comparative statics.** The same condition also pins down the sign of the denominator in the comparative static. If the equilibrium is differentiable in $s$, differentiating $H_s(\psi^*(s)) = 0$ gives

$$\frac{d\psi^*(s)}{ds} = -\frac{\phi_\kappa(\mathcal{B}(s,\psi^*(s)))\mathcal{B}_s(s,\psi^*(s))}{D(s)}, \qquad D(s) \equiv 1 + \phi_\kappa(\mathcal{B}(s,\psi^*(s)))\mathcal{B}_\psi(s,\psi^*(s)).$$

Under $\mathcal{B}_\psi(s,\psi) \geq 0$, $D(s) \geq 1 > 0$. If the participation-cost density is positive at the cutoff, $\phi_\kappa(\mathcal{B}(s,\psi^*(s))) > 0$, then the sign of $d\psi^*(s)/ds$ is the opposite of the sign of $\mathcal{B}_s(s,\psi^*(s))$.

The same differentiation also gives the change in the equilibrium participation gain itself. Along the fixed point,

$$\frac{d}{ds}\mathcal{B}(s,\psi^*(s)) = \mathcal{B}_s(s,\psi^*(s)) + \mathcal{B}_\psi(s,\psi^*(s))\frac{d\psi^*(s)}{ds} = \frac{\mathcal{B}_s(s,\psi^*(s))}{D(s)}.$$

Hence, under the uniqueness condition above, the total derivative of the equilibrium participation gain has the same sign as the partial derivative $\mathcal{B}_s(s,\psi^*(s))$.

**Sign of the participation-gain derivative.** Since

$$\mathcal{B}(s,\psi) = A(s) + O(s,\psi),$$

we have

$$\mathcal{B}_s(s,\psi) = A'(s) + O_s(s,\psi).$$

Moreover,

$$A'(s) = \chi\left[F(\bar{x}-s) - \tilde{F}(\bar{x}-s)\right],$$

because $d\,m(\bar{x}-s)/ds = -F(\bar{x}-s)$ and $d\,\tilde{m}(\bar{x}-s)/ds = -\tilde{F}(\bar{x}-s)$.

It remains to sign the two terms. Write

$$k = \bar{x} - s, \qquad \rho(\Omega; s, \psi) = s - \frac{\Omega}{1-\psi}.$$

Holding $\psi$ fixed, use $i(\Omega; s,\psi) = \mathcal{I}(\rho(\Omega; s,\psi))$. The envelope condition $\bar{y}_r(\rho; i(\Omega; s,\psi)) = 0$ gives the exact derivative of the updating option value:

$$O_s(s,\psi) = \chi\mathbb{E}_{\Omega\sim H_\psi}\left[F\left(k + \frac{\Omega}{1-\psi}\right) - F(k) - \frac{\Omega}{1-\psi}f\left(k + \frac{\Omega}{1-\psi}\right)\right].$$

To see this, differentiate

$$O(s,\psi) = \mathbb{E}_{\Omega \sim H_\psi}\left[\bar{y}\left(s - \frac{\Omega}{1-\psi}; \mathcal{I}\left(s - \frac{\Omega}{1-\psi}\right)\right) - \bar{y}\left(s; \mathcal{I}\left(s - \frac{\Omega}{1-\psi}\right)\right)\right].$$

Since $\bar{y}_i(r;i) = -r$, $\mathcal{I}'(r) = -\chi f(\bar{x} - r)$, and

$$\bar{y}_r\left(s; \mathcal{I}\left(s - \frac{\Omega}{1-\psi}\right)\right) = \chi\left[F(k) - F\left(k + \frac{\Omega}{1-\psi}\right)\right],$$

the displayed expression for $O_s(s,\psi)$ follows.

The insurance component is weakly decreasing in the abundant-reserve region under a standard single-crossing implication of symmetric additional payment-flow risk:

$$\tilde{F}(k) \geq F(k) \quad \text{for } k \leq 0, \qquad \tilde{F}(k) \leq F(k) \quad \text{for } k \geq 0.$$

This condition says that the effective inactive-bank shock $\tilde{\zeta} = \zeta + \zeta'$ is more dispersed than the late day shock $\zeta$ around the same center. It is satisfied, for example, by the Gaussian calibration used in the figures, where $\zeta$ and $\zeta'$ are independent, mean-zero normal shocks. Therefore $A'(s) \leq 0$ for $s \geq \bar{x}$ and $A'(s) \geq 0$ for $s \leq \bar{x}$, with strict inequality away from the kink whenever the additional ordinary payment-flow shock is non-degenerate.

For small inactive-flow imbalances, the corresponding local approximation to the updating option value is

$$O(s,\psi) \simeq \frac{1}{2}\chi f(\bar{x} - s)\frac{\mathrm{Var}_\psi(\Omega)}{(1-\psi)^2}. \tag{F.1}$$

Differentiating equation (F.1) with respect to $s$ gives

$$O_s(s,\psi) \simeq -\frac{1}{2}\chi f'(\bar{x} - s)\frac{\mathrm{Var}_\psi(\Omega)}{(1-\psi)^2}.$$

Thus, if $f$ is symmetric and unimodal around zero, so that $f'(k) > 0$ for $k < 0$ and $f'(k) < 0$ for $k > 0$, then $O_s(s,\psi) < 0$ for $s > \bar{x}$ and $O_s(s,\psi) > 0$ for $s < \bar{x}$ under the local approximation. The exact derivative above has the same negative sign whenever the residual imbalances are small enough that $k + \Omega/(1-\psi) \leq 0$ almost surely in the abundant-reserve region and $f$ is increasing on the relevant left-tail interval.

Combining the two components,

$$\mathcal{B}_s(s,\psi) = \chi\left[F(\bar{x} - s) - \tilde{F}(\bar{x} - s)\right] + O_s(s,\psi).$$

Under the conditions just stated, $\mathcal{B}_s(s,\psi) < 0$ for $s > \bar{x}$ in the abundant-reserve region.

Evaluated at the equilibrium $\psi^*(s)$, this implies

$$\frac{d}{ds}\mathcal{B}(s,\psi^*(s)) < 0, \qquad \frac{d\psi^*(s)}{ds} > 0 \quad \text{if} \quad \phi_\kappa(\mathcal{B}(s,\psi^*(s))) > 0.$$

Thus, reserve abundance lowers the value of active participation and increases the equilibrium inactive fraction. By symmetry, the inequalities reverse on the scarce-reserve side under the analogous right-tail conditions.

**Local uniqueness condition.** The same local approximation verifies the monotonicity condition used for uniqueness. Under the granular-bank calibration $\mathrm{Var}_\psi(\Omega) = \psi(\sigma')^2/N_\Omega$, equation (F.1) becomes

$$O(s,\psi) \simeq C(s)\frac{\psi}{(1-\psi)^2}, \qquad C(s) \equiv \frac{1}{2}\chi f(\bar{x}-s)\frac{(\sigma')^2}{N_\Omega}.$$

Therefore

$$O_\psi(s,\psi) \simeq C(s)\frac{1+\psi}{(1-\psi)^3}.$$

Because $f$ is the p.d.f. of the late day payment-shock distribution, $f(\bar{x}-s) \geq 0$; for a normal distribution it is strictly positive for every $s$. Hence $O_\psi(s,\psi) \geq 0$, and since $A(s)$ does not depend on $\psi$, $\mathcal{B}_\psi(s,\psi) = O_\psi(s,\psi) \geq 0$ under the approximation. The local approximation therefore delivers uniqueness and $D(s) > 0$ whenever the existence self-map condition above is satisfied.

**Endogenous rate and volatility.** The rate equation in Proposition 5.2 follows directly from residual market clearing. Given the equilibrium inactive fraction $\psi^*(s)$, active banks must hold

$$r_t^A = \rho(\Omega_t; s, \psi^*(s)) = s - \frac{\Omega_t}{1-\psi^*(s)}.$$

The active-bank first-order condition then gives

$$i(\Omega_t; s, \psi^*(s)) = \mathcal{I}(r_t^A) = \mathcal{I}\left(s - \frac{\Omega_t}{1-\psi^*(s)}\right).$$

Quantiles follow by applying this monotone transformation to the quantiles of $H_{\psi^*(s)}$. Differentiating with respect to $\Omega_t$ and evaluating at $\Omega_t = 0$ gives

$$\frac{\partial i(\Omega_t; s, \psi^*(s))}{\partial \Omega_t} = \mathcal{I}'(s)\left(-\frac{1}{1-\psi^*(s)}\right) = \frac{\chi f(\bar{x}-s)}{1-\psi^*(s)}.$$

Combining the local derivative of the rate with the granular-bank variance formula above gives equation (22) in Proposition 5.2.

## F.2 Aggregate Reserve-Supply Shocks after Participation

This subsection derives Proposition 5.3. At the beginning of the day, banks choose participation taking aggregate reserve supply $s_0$ as given. The resulting inactive fraction is $\psi^*(s_0)$. After participation decisions are fixed, aggregate reserve supply unexpectedly changes to $s_1$.

Let $\Omega$ denote the aggregate payment-flow imbalance of inactive banks, so that the inactive sector holds reserves equal to

$$\psi^*(s_0)s_0 + \Omega.$$

Because inactive banks do not trade after the shock, active banks must hold the remaining reserves. Their aggregate reserve holdings are therefore

$$s_1 - \psi^*(s_0)s_0 - \Omega.$$

Dividing by the active mass $1 - \psi^*(s_0)$ gives the reserve share held by active banks:

$$\begin{aligned} r^A &= \frac{s_1 - \psi^*(s_0)s_0 - \Omega}{1 - \psi^*(s_0)} \\ &= s_0 + \frac{s_1 - s_0}{1 - \psi^*(s_0)} - \frac{\Omega}{1 - \psi^*(s_0)}. \end{aligned} \tag{F.2}$$

The active-bank first-order condition is the same intraday reserve demand condition used in Section 5.2, so the market-clearing interbank rate is

$$i = \mathcal{I}(r^A) = \mathcal{I}\left(s_0 + \frac{s_1 - s_0}{1 - \psi^*(s_0)} - \frac{\Omega}{1 - \psi^*(s_0)}\right).$$

Since the inactive-flow imbalance is distributed according to the participation equilibrium at $s_0$, $\Omega \sim H_{\psi^*(s_0)}$. This proves Proposition 5.3.