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Currency Crises and the Informational Role of Interest Rates*

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Abstract

In the late 1990s, Morris and Shin proposed a new theoretical framework of financial crises, which generalised traditional models of strategic complementarity and self-fulfilling beliefs by incorporating idiosyncratic uncertainty about the state. The innovative feature of their framework is expressed by its capacity to account for seemingly unwarranted speculative attacks under equilibrium uniqueness and to thus place policy analysis on a firm footing. The macroeconomic implications of the framework have been questioned, however, because it ignores the issue of information aggregation via market prices.

Motivated by such criticism, this paper modifies the Morris-Shin setup by allowing prices to adjust freely to market conditions. It is then shown that all of the appealing characteristics of that setup are preserved even when public information has an endogenously disseminated component. Moreover, the prevailing weak form of strategic complementarity, in conjunction with heterogeneity of private agents' information sets, leads to a less restrictive prerequisite for equilibrium uniqueness. Further, the paper's model delivers new policy implications and suggests a change in the approach of structural currency crisis empirical analysis.

Introduction

In order to explain drastic shifts in financial market behaviour that seem to be unrelated to the underlying fundamentals, economic theory has traditionally made use of a self-fulfilling feature of rational beliefs. This feature leads to equilibrium multiplicity and is the result of a particular instance of strategic complementarity that can be illustrated as follows. When economic agents anticipate a financial crisis, they attempt to shield themselves from its consequences and in the process precipitate it; in contrast, if the crisis is not seen as imminent, agents' resulting behaviour preserves the status quo. Their capacity to account for stylised facts notwithstanding, multiple-equilibria models need to attribute the particular prevailing outcome to economically meaningless variables which act as coordination devices and are frequently referred to as sunspots.

In a series of articles, Stephen Morris and Hyun Song Shin demonstrate that potentially self-fulfilling beliefs might actually have a unique equilibrium realisation. In addition to publicly available information about the economy, agents in the Morris-Shin (henceforth, MS) framework hold diverse perceptions of the world due to their access to private signals. The resulting uncertainty of market participants about their peers' beliefs and actions does not allow for any role of sunspots: the economic model fully explains the timing of seemingly unwarranted financial crises and places related policy analyses on a firm footing.

Its insights notwithstanding, the contribution of the MS framework has been considered to be of limited applicability. One of the reasons for such a verdict is the fact that the framework's main results hinge on a premise that the two types of information (private and public) have independent sources. The premise

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is hard to accept if one believes that publicly observed prices are the outcome of heterogeneous agents' *collective* actions and, as such, aggregate at least some of these agents' private information.

This paper conceptually modifies the MS setup by establishing an *endogenous* link between financial market participants' private and public information. In the resulting economic environment, equilibrium uniqueness is shown to be consistent with dramatic switches in agents' behaviour. Policy analysis can thus be conducted in the spirit of previous work by Morris and Shin while cast in a more realistic context. Moreover, the conditions for equilibrium uniqueness are derived to be less restrictive than in the MS model and are expressed in terms of characteristics of the economy's institutional infrastructure and financial markets. A central authority that actively participates in those markets is seen to be in a position to manage a crisis by influencing the informational content of prices.

The paper's analysis is cast in a context of speculative currency attacks. The model builds on the MS setup, which in turn enriches the informational structure of the second-generation approach to currency crises.

There is certain, albeit inconclusive, evidence that the European exchange rate mechanism (ERM) could have been successfully attacked for some time before the currency crisis eventually erupted in the early 1990s (Eichengreen and Wyplosz (1993)). There is also evidence that the same crisis might have never materialised (Eichengreen (2001), Obstfeld (1996)). The second-generation approach to currency crises reconciles such accounts by focusing on the interdependence of the optimisation problems of two distinct foreign exchange market players: the central authority administering the exchange rate regime and the private sector.[1] Private agents' confidence in the regime is reflected in domestic interest rates and/or capital flows that influence the authority's incentive to abandon the regime. Such strategic complementarity in the players' rational actions is shown to lead to multiple equilibria: for certain fundamentals, the prevailing attack or no-attack equilibrium is chosen by a sunspot.

Using analytical tools of the global games literature,[2] Morris and Shin argue that multiple equilibria within the second-generation approach are a consequence of the overly simplistic assumption that agents' information sets are identical.[3] In the conceptual context of that approach, the assumption is not innocuous: since the authority's decision is based on speculators' *aggregate* behaviour, an attack or a no-attack equilibrium is possible only if there is a sufficient degree of common knowledge that the fundamentals justify such an action. Common knowledge incorporates: knowledge that an event has occurred, knowledge that others know that the event has occurred, knowledge that others know that others know that the event has occurred and so on. If private agents are realistically allowed to have their own "window on to the world", ie to receive private signals, common knowledge might be difficult to obtain and equilibrium uniqueness would emerge.

The MS setup of informational heterogeneity produces a highly non-linear functional relationship between the fundamentals and the incidence of a speculative attack. In this way, the model accounts for sudden crises without having to rely on sunspot variables. By eliminating the role of sunspots, the MS setup makes it possible to rigorously examine policy alternatives for curbing speculative attacks, which are generally viewed as reducing economic welfare.

The appealing features of the MS setup are obtained at the cost of a quite restrictive ad hoc assumption. For example, when the setup is the result of a small perturbation from an environment of informational homogeneity, failure of common knowledge requires that private information be relatively so precise as to render its public counterpart redundant. Foreign exchange transactions, however, produce prices that (partially) aggregate market participants' beliefs.[4] Public signals in the form of prices would thus provide each agent with information about the private signals of others and should be expected to non-trivially

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[1] Obstfeld (1994) and Bensaid and Jeanne (1997), for example, adopt that approach.

[2] Carlsson and van Damme (1993) is a seminal paper in that literature and coined the term "global games". The term stands for a strategic incomplete-information environment. An overview of the evolution of the global games literature is provided in Morris and Shin (2000).

[3] A recent outburst of currency crisis research built on and expanded the original MS framework but preserved its key features: Morris and Shin (1999), Corsetti et al (2000) and Goldstein (2000) are typical examples.

[4] The evolution of interest differentials during the ERM crisis is viewed by Rose and Svensson (1994) and Eichengreen et al (1996) as expressing market participants' belief about the sustainability of the European currency zone.

enrich individual information sets, casting doubt on the realism of the necessary condition for equilibrium uniqueness in the MS setup.[5]

With such observations in mind, I develop an endogenous public signal (EPS) model as a modification of the framework of Morris and Shin (1999): a modification in which market prices play an informational role.[6] Agents in the EPS model communicate their beliefs to the market and learn from it about others' beliefs. In taking opposite sides of foreign exchange transactions, agents settle at an equilibrium price at which there is no incentive to recontract even when new information, revealed by that price, is taken into account. In contrast to the necessary condition for equilibrium uniqueness in the MS setup, the public signal is *deduced* within the model to provide useful information irrespective of private signals' precision.

Such an implication of the EPS model notwithstanding, there is a region of its parameter space on which the prevailing outcome is determined solely by the fundamentals. The result is rooted in agents' capacity to influence the central authority's decisions via different channels, which have mutually offsetting impacts: higher domestic interest rates put pressure on the currency but are also likely to be associated with an increased attractiveness of domestic assets, which leads to a higher *in*flow of official reserves and alleviates that pressure. Thus, when reserve flows are unknown to private agents, the effect of a change in the publicly observable interest rate is quite uncertain, even when beliefs about (predetermined) fundamentals are precise and contain a large common component. The uncertainty is further amplified by the heterogeneity of information sets and, for some parameter values, leads to only one rationally justifiable action of the speculators in each state of the world.

Importantly, the unique equilibrium of the EPS model decouples market behaviour from economic fundamentals. It is thus possible to determine unambiguously how the likelihood of sudden and violent currency attacks depends on, for example, changes in transaction costs or in the authority's commitment to the exchange rate regime. The model further suggests a shift in the focus of structural empirical analysis of currency crises.[7] It is also seen how the incidence of speculative attacks can be influenced by a large market participant who is successful in manipulating the informational content of prices.

The contribution of the EPS model is best understood if compared directly to the MS framework. Hence, in Section 1., I develop a setup which incorporates both models as special cases. Then Sections 2. and 3. analyse them sequentially while treating them as small perturbations from informational homogeneity. The former section states and briefly discusses the main results of the MS setup. The latter section derives the equilibrium with the EPS model, provides the intuition behind the conditions for equilibrium uniqueness and conducts comparative statics. In Section 4., I conclude by summarising the paper's message and then pointing to directions of future research suggested by the analysis.

1. The Model

The model relates to a small open economy which maintains an exchange rate peg and is referred to as the domestic economy. There are only two currencies: domestic and foreign. The foreign price level and interest rate are fixed and purchasing power parity holds.

The economy evolves over two periods. In the first period, the exchange rate, the domestic currency's value of the foreign currency, is fixed and normalised to unity. If the peg is abandoned, the currency is devalued in the second period to a new parity $E > 1$. E is a constant that is publicly known at the beginning of the first period.[8]

I now describe the two types of players: the private agents and the domestic central authority.

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5 Grossman (1989) is the classic reference on the role of prices in a heterogeneous-agent rational-expectations environment. Rey (2000) and Atkeson (2000) point out the issue when commenting on a paper within the MS approach.

6 Dasgupta (2002) endogenises the source of common information in a conceptually different fashion. He does not consider the capacity of prices to instantaneously aggregate agents' idiosyncratic information. Instead, he adopts a dynamic setup of sequential learning in which past aggregate actions of the private sector provide a public signal about the state of the economy.

7 Such analysis is conducted in Jeanne (1997) and Jeanne and Masson (2000).

8 The assumptions that (i) a *re*valuation is not possible and (ii) the (shadow) devaluation rate is a fixed commonly known number are made for expositional simplicity. Relaxing these assumptions would not affect the principal conclusions of the paper.

1.1 Private Agents

The private agents form a continuum and are uniformly distributed on the real unit interval. They act on the foreign exchange market where two assets are traded in the first period: one-period default-free discount bonds denominated in the foreign and domestic currency. The associated interest rates are denoted respectively by i^* and i and the relative price of the two assets is normalised to unity. The assets are traded in the currency in which they are denominated and in the region of the world in which the currency is issued. The domestic authority acts as a counterparty to all foreign exchange transactions.

Agents enter the first period with no liquid wealth and have to borrow if they want to invest. The value of the funds an agent can borrow cannot exceed unity (in either currency).[9] Since borrowing is conducted at nominally riskless interest rates, I assume that each agent possesses wealth that is liquid only in the second period and is sufficient to cover any repayment obligation.[10]

Agents trade bonds in order to maximise the real value of their net expected payoff, which is realised in the second period. Their decisions are based on their rational beliefs about the state of the economy. The state is realised in the first period and is fully described by an exogenous "fundamental" random variable, Θ, and a stochastic shock to the supply of the domestic currency bond, U.[11] In the currency crisis literature, Θ typically represents the domestic economy's relative productivity or employment rate. In turn, U is to be thought of as driven by (not modelled) domestic residents who invest or save at home.

Part of agents' beliefs are formed by interpreting exogenous signals about the state and possess a publicly observed (ie common) and a private (ie idiosyncratic) component. Θ is assumed to be a priori uniformly distributed on the real line.[12] Before agents trade, their perceptions of Θ are sharpened by a public signal, denoted by y, which is the realisation of:

$$Y = \Theta + \sqrt{t}\, Z^y \tag{1}$$

where Z^y is a standard normal variable that is independent of Θ and U and $t > 0$. The exogenous public component of beliefs is completed by the prior:

$$U \sim N\left(\mu_u, \sigma_u^2\right) \tag{2}$$

The idiosyncratic component of agents' beliefs leads to heterogeneous perceptions of the likelihood of a devaluation. That component incorporates each agent j's private signal about θ, which is denoted by x_j and is a realisation of:

$$X_j = \Theta + \sqrt{\varepsilon}\, Z_j \tag{3}$$

where $\varepsilon > 0$ and Z_j is a standard normal variable that is independent of Θ, U, Z^y and Z_k for $k \neq j$. An agent j is identified by his/her private signal and henceforth referred to as an x_j-agent.

Given i, i^*, and E, there exists a level of devaluation probability, $\Pr^d$, at which the return on the foreign currency asset is equal to the expected return on the domestic asset:[13]

$$\Pr^d = \frac{1 - \frac{1+i^*}{1+i}}{1 - \frac{1}{E}} \equiv \iota \in [0, 1] \tag{4}$$

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9 The analysis is not affected by the exact value of the upper limit on borrowing as long as this value is finite.

10 Denote this wealth's second-period real value by W and note that, if interest rates are positive, a maximising agent would not keep any cash. Straightforward algebra shows that any obligation covered for $W \geq (E-1)(1+i^*)$. Note that, for a fixed W, an agent's repayment capability changes with the currency in which his/her bond is denominated. However, modifying agents' borrowing constraint in order to accommodate this consideration would unnecessarily burden the exposition without affecting the model's main thrust.

11 Unless explicitly stated otherwise, random variables are denoted by upper case letters; the corresponding lower case letters are used to denote the variables' specific realisations.

12 The assumption is adopted for algebraic tractability and is inconsequential when the signals about θ are sufficiently more precise than Θ's prior, provided that it has a continuous density.

13 The equation in (4) is obtained after rearranging the uncovered interest parity condition:

$$1 + i^* = (1+i)\left(\left(1 - \Pr^d\right) * 1 + \Pr^d \frac{1}{E}\right)$$

Agents who perceive a devaluation probability equal to $\Pr^d$ play a key role in equilibrium and I denote their private signal by x^p.[14] Given that ι depends only on the relative values of i and i^*, I henceforth refer to it as the interest differential.[15] i and i^* affect the analysis only through ι.

1.2 The Central Authority

In the first period, the central authority noiselessly observes the state of the economy,[16] holds foreign reserves and stands ready to exchange one currency for the other at the preset rate. By intervening (or not) on the foreign exchange market, the authority can influence the cost of borrowing in the domestic currency. The domestic interest rate policy in period one is what distinguishes the MS and EPS frameworks and is described at the beginning of Sections 2. and 3..

In the second period, the authority decides on the exchange rate, at which it again stands ready to exchange one currency for the other. Its decision is modelled in the spirit of the second-generation approach to currency crises and is assumed to depend on developments in the first period. The authority's temptation to abandon the peg increases with the domestic interest rate, a rise in which would typically put pressure on domestic banks, and with the outflow of reserves, which might influence the authority's capacity to service debt obligations. Finally, the authority's tolerance to pressure on the peg increases with θ. The mechanism via which these three factors influence the authority is not explicitly modelled; the relevance of fundamentals, interest rates and reserve flows for currency crises is discussed at length in the second-generation literature.[17]

Denoting the *out*flow of reserves by R, the period two decision of the central authority is formalised by assuming that it devalues if and only if:

$$\tau\iota + R > a(\theta) \tag{5}$$

$\tau > 0$ captures the relative importance of the interest differential and the reserve outflows; $a(\cdot)$ is a continuous function with the following properties:

$$0 < b_1 \leq \frac{da(\theta)}{d\theta} \leq b_2 < \infty \tag{6}$$

where b_1 are b_2 are constants.[18] The left-hand side (LHS) of inequality (5) can be interpreted as an indicator of foreign exchange market conditions. The right-hand side (RHS) sets a threshold value of that indicator, beyond which the currency is devalued.

1.3 Sequence of Events

In the first period, the exchange rate is fixed. The state of the economy is realised and observed only by the central authority. The atomistic agents hold common prior beliefs about the state random variable and receive private signals about its particular realisation. Then agents borrow and invest and, in the process, settle at a domestic interest rate, whose value depends on the authority's intervention in the foreign exchange market.

In the second period, after observing the aggregate investment position of the private sector, the central authority decides whether to preserve or abandon the peg. Finally, private agents make and/or receive payments according to their period one contracts.

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14 x^p turns out to be unique in equilibrium.

15 Note that ι increases monotonically in i. The lowest possible no-arbitrage value of the domestic interest rate, $i = i^*$, implies $\iota = 0$; the highest possible no-arbitrage value of the domestic interest rate, $i = (1 + i^*) E - 1$, implies $\iota = 1$.

16 The authority's beliefs constitute all the information that is relevant for private agents' actions. The assumption that the authority has perfect knowledge about the state is, thus, made without loss of generality.

17 See Obstfeld (1994), Obstfeld and Rogoff (1995) and Bensaid and Jeanne (1997). The latter paper provides references to in-depth analyses of interest rates' impact on the cost of maintaining an exchange rate regime.

18 b_1 assures a minimum level of sensitivity of the central authority's incentives to changes in the fundamental. In turn, b_2 rules out a drastic revision of the authority's incentives that is due solely to the fundamentals. The upper bound on $\frac{da(\theta)}{d\theta}$ implies that the exogenous economic environment can evolve only gradually: if there is a sudden and violent speculative attack, it has to be due to the way market participants interact.

2. The MS Setup

Suppose that the interest rate policy rule of the authority requires that it intervene on the foreign exchange market in order to keep the interest differential fixed. Then, the model outlined above delivers the thrust of the MS setup. In this section, I assume that ι is a constant on the unit interval, state the assumption's equilibrium implications and focus on the intuition behind them. The interested reader is referred to Morris and Shin (1999) for all the underlying proofs.

Recall equation (4) for the definition of an x^p-agent. In the only strategy that is sustainable in equilibrium, this agent is pivotal: all agents with private signals smaller than x^p bet against the domestic currency: they issue the maximum possible amount of domestic debt, convert the obtained currency at the domestic authority's window and invest the proceeds in the foreign asset. According to equation (3) this generates an outflow of reserves equal to $\Phi\left(\frac{x^p-\theta}{\sqrt{\varepsilon}}\right)$.[19] Symmetrically, agents with signals bigger than x^p generate an *in*flow of reserves equal to $1-\Phi\left(\frac{x^p-\theta}{\sqrt{\varepsilon}}\right)$. The net outflow of reserves is then:

$$R = 2\Phi\left(\frac{x^p-\theta}{\sqrt{\varepsilon}}\right) - 1 \in [-1,1] \tag{7}$$

Given a particular ι and a value of the pivotal agent's private signal, x^p, inequality (5) and equation (7) identify a unique critical value of the fundamental, θ^{cr}: the peg is abandoned if and only if $\theta < \theta^{cr}$, where:

$$a\left(\theta^{cr}\right) = \tau\iota + 2\Phi\left(\frac{x^p-\theta^{cr}}{\sqrt{\varepsilon}}\right) - 1 \tag{8}$$

Using equations (1) and (3), equation (4) implies that:

$$\Phi\left(\frac{\theta^{cr} - \left(\frac{t}{t+\varepsilon}x^p + \frac{\varepsilon}{t+\varepsilon}y\right)}{\sqrt{\frac{t\varepsilon}{t+\varepsilon}}}\right) = \iota \tag{9}$$

where the LHS is the devaluation probability, as perceived by an x^p-agent.[20]

For a given public signal, y, and a given interest differential, ι, an equilibrium is defined by a pair $\{x^p, \theta^{cr}\}$ that solves equations (8) and (9) simultaneously. Observe that U, the stochastic shock to the supply of the domestic asset, does not enter the determination of equilibrium: in order to keep ι fixed, the central authority's intervention insulates the private sector's demand for the domestic asset, and thus the outflow of reserves, from the supply shock.

The number of equilibria in the MS setup hinges on the *relative* values of ε and t, the parameters capturing the precision of the exogenous private and public signals. Before I turn to three configurations of ε and t that summarise the main conclusions of the MS setup, I identify the so-called dominance regions on the support of Θ: $(-\infty, \underline{\theta})$ and $(\bar{\theta}, \infty)$, where:

$$\begin{aligned} \underline{\theta} &\equiv a^{-1}(-1+\tau\iota) \\ \bar{\theta} &\equiv a^{-1}(1+\tau\iota) \end{aligned} \tag{10}$$

Equation (8) implies that, if $\theta \in (-\infty, \underline{\theta})$ or $\theta \in (\bar{\theta}, \infty)$, the peg is respectively abandoned or preserved for any aggregate investment position of the private agents.

Case 1. No uncertainty: $\varepsilon = 0$ and/or $t = 0$. The resulting equilibrium is the standard result of the second-generation approach to currency crises.

For $\theta < \underline{\theta}$, all agents borrow domestically and invest abroad in order to shield themselves from the peg's inevitable collapse. For $\theta > \bar{\theta}$, agents' only rational decision is to invest domestically. If $\theta \in (\underline{\theta}, \bar{\theta})$, the strategic complementarity sets in. If all agents decide to invest abroad, the resulting attack on the peg generates a reserve outflow that is too high for the authority to bear and the peg collapses. Conversely,

19 Throughout the paper, Φ (ϕ) stands for the standard normal cumulative distribution function, CDF (probability density function, PDF).

20 From the point of view of an x^p-agent, $\left(\frac{t}{t+\varepsilon}x^p + \frac{\varepsilon}{t+\varepsilon}y\right)$ and $\frac{t\varepsilon}{t+\varepsilon}$ are respectively the posterior mean and variance of Θ.

if agents decide to invest domestically, the peg survives and justifies their decision. For intermediate values of the fundamental, agents' actions are thus independent of its particular realisation and are best thought of as coordinated by a sunspot.

Case 2. Both signals are of an extremely high and comparable precision: consider the limit $\varepsilon \to 0$ and assume that $\lim_{\varepsilon \to 0} \frac{\varepsilon}{t} = \kappa > 0$, where κ is an arbitrary constant.[21] The set of equilibria converges to the one in Case 1.

Recalling the posterior distribution of Θ, the assumption about κ implies that both signals influence non-trivially the formation of agents' beliefs. At any level of its precision, the public signal then plays a dual role: on the one hand, it leads to an improved perception of the fundamental; on the other hand, it anchors the heterogeneous beliefs and establishes common knowledge that they are *similar*. Consequently, when signals' precision increases, there is common knowledge that the fundamental belongs to any fixed-length interval containing the public signal. Due to equations (1) and (3), virtually all agents observe private signals that also belong to the latter interval and, thus, believe that θ belongs to it with high probability; moreover each such agent views as certain that other agents view as certain that θ belongs to that interval, and so on. Specifically, when $y \in (\underline{\theta}, \bar{\theta})$, there is common knowledge that the private sector's investment position matters for the preservation/abandonment of the peg.[22] Thus, conditional on $y \in (\underline{\theta}, \bar{\theta})$, there is common knowledge that the two equilibria of Case 1 are rationally justifiable: a sunspot can lead to either one. Analogous arguments apply when $y > \bar{\theta}$ and $y < \underline{\theta}$.

Case 3. The high precision of private signals renders public signals redundant: consider the limit $\varepsilon \to 0$ and assume that $\lim_{\varepsilon \to 0} \frac{\sqrt{\varepsilon}}{t} = 0$. Equilibrium uniqueness prevails in all states of the world: a single equilibrium pair $\{x^p, \theta^{cr}\}$, where $\theta^{cr} \in (\underline{\theta}, \bar{\theta})$, arises for any y. In other words, an infinitesimal worsening of the state, which pushes θ below θ^{cr}, induces a massive attack: an outflow of reserves that topples the peg.

Even if it is extremely precise in absolute terms, the public signal is of negligible importance in agents' information sets: beliefs are determined predominantly by their idiosyncratic component and are thus *dissimilar*. Let us suppose that $\theta \in (\theta^{cr}, \bar{\theta})$. For $\varepsilon \to 0$, equation (3) implies that virtually all agents believe that $\theta < \bar{\theta}$. Furthermore, if all agents holding such beliefs were to attack, the attack would be justified ex post by a collapse of the peg. However, each agent needs to form a belief not only about θ but also about other agents' beliefs about θ. For the assumed relative precision of public and private information, there are not enough agents who believe that there are enough agents who believe (and so on) that $\theta < \bar{\theta}$ and an attack cannot materialise: ie, there is failure of common knowledge that the private sector's action might influence the future of the peg. As a result, only investment in the domestic asset is rationally justifiable.[23] Symmetrically, for $\theta \in (\underline{\theta}, \theta^{cr})$, only an investment in the foreign asset is justifiable.

Contrasting Case 3 against the other two cases underscores the gist of Morris and Shin's contribution to the currency crisis literature. A massive currency attack that topples the peg but is decoupled from the macroeconomic environment could emerge under equilibrium uniqueness if idiosyncratic signals dominate private agents' beliefs. The limiting condition in Case 3 formalises the meaning of "dominate".

2.1 Aggregate Uncertainty

In the MS setup, failure of common knowledge of the fundamental might be (and trivially so) driven by noise in the private signals that is the same across agents.[24] Sufficiently high aggregate uncertainty (uncertainty that remains even if all agents share their information sets) would thus lead to equilibrium uniqueness by obscuring the ultimate impact of any private sector strategy and, consequently, by preventing agents from rationally justifying different actions for the same perception of the fundamentals.

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21 Since the model's equilibrium is discontinuous at $\varepsilon = 0$, "in the limit $\varepsilon \to 0$" is most usefully thought of as "for a sufficiently small but *strictly positive* ε". Since ε cannot be negative, "$\varepsilon \to 0^+$" is replaced by "$\varepsilon \to 0$" in order to reduce the notational clutter.

22 Refer to Morris and Shin (2000) for a precise definition and rigorous analysis of (levels of) common knowledge.

23 A sunspot variable has no equilibrium role to play.

24 Such noise can be modelled by assuming that Z_j from equation (3) is distributed $N(\zeta, 1)$, where ζ is a random variable that is the same across agents.

A similar result is obtained in the homogeneous-agent second-generation environment when there is sufficiently noisy information about the fundamental.

Unlike the driving force of equilibrium uniqueness in Case 3 of Section 2., aggregate uncertainty is not based on the strategic interaction among agents but relates simply to the quality of their information. Such uncertainty is thus not in a position to decouple, on its own, a speculative attack's intensity from macroeconomic fundamentals.[25] Consequently, if the objective is to capture seemingly unwarranted revisions in the outlook of a currency regime and if the analysis is of a qualitative nature, aggregate uncertainty can be assumed away without loss of generality: this is done in Section 2. and in all analyses within the MS setup that I am aware of.

3. The EPS Model

In this section, I specify the model that delivers the contribution of the paper. The framework is the same as the one described in Section 1. with the qualification that, in the first period, the central authority's interest rate policy rule stipulates no intervention on the foreign exchange market. Thus, market clearing implies that the interest differential would have to adjust in order to induce an aggregate investment position of the private sector that matches the stochastic shock on the domestic asset market. The following equation specifies how that shock translates into the exogenous component of the domestic asset supply, S:

$$S = 2\Phi(U) - 1 \tag{11}$$

The relationship between U and S has two implications. First, the support of S is the interval $[-1, 1]$ and private agents' borrowing constraints allow for market clearing in all states of the world. Second, as illustrated in Section 3.1, equation (11) simplifies the exposition tremendously and allows for algebraic tractability.

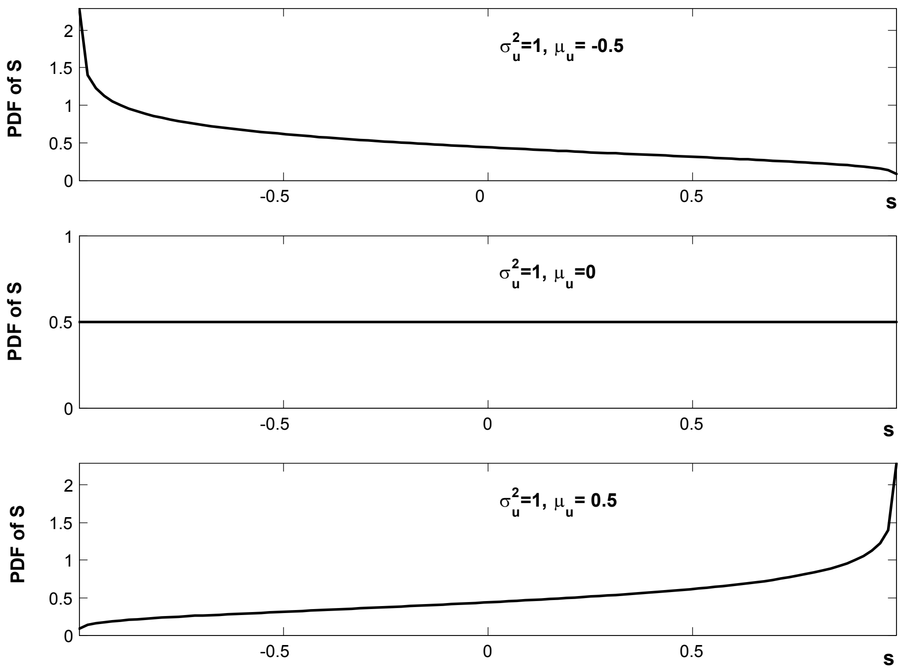

Figure 1: **Possible distributions of the supply shock**

Figure 1 plots the distribution of S for three different values of the shock's mean, μ_u, keeping its variance, σ_u^2, constant. The figure illustrates that the prior information about S becomes more precise as μ_u increases or decreases away from zero: ie μ_u and σ_u^2 act as substitutes in agents' information sets. Changes in μ_u have first-moment implications as well; these are, however, irrelevant in the light of the

25 See Jeanne (1997) for an illustrative example.

analysis' focus on the relative precision of private and public information. Thus, in order to simplify the exposition, the discussion of the EPS model's equilibrium assumes that $\mu_u = 0$. All the conclusions can be revised in a straightforward fashion to incorporate general values of μ_u.[26]

The main difference between the MS and EPS models is that the latter generates an *endo*genous public signal, whereas the former does not. In the light of this difference, I impose there to be no *exo*genous public signal in the EPS model: ie I set $t = \infty$ in equation (1). This assumption allows for a direct parallel between the two setups and underscores the implications of an environment in which the relative precision of private and public information is not imposed in an ad hoc fashion but is an equilibrium outcome.

3.1 Equilibrium in the EPS Model

Agents exhaust their borrowing constraints and, if they issue a bond in one currency, they do so in order to invest the proceeds in a bond denominated in the other currency. An investment strategy can thus be viewed as setting an agent's position in the foreign asset as a function of that agent's private signal.[27]

I assume that the space of equilibrium-implementable strategies consists only of monotone functions of the private signal. Due to the objectives of the paper's analysis, the assumption is not restrictive. As demonstrated in Appendix 1, if a non-monotone function emerges as an equilibrium strategy, certain states of the economy would not fully define agents' aggregate behaviour and it would need to be determined by sunspot variables: the type of variables that the paper attempts to show to be unnecessary for explaining sudden and violent speculative attacks.

Adopting the above-mentioned strategy space, I prove in Appendix 1 that equilibrium-implementable strategies in the EPS framework have to be of the type described in Section 2.: ie decreasing step functions of the private signal. For a given interest differential, the step occurs at the private signal of an agent who perceives the devaluation probability given by equation (4).

For domestic-asset market clearing, x^p needs to satisfy:

$$2\Phi\left(\frac{\theta - x^p}{\sqrt{\varepsilon}}\right) - 1 = s \tag{12}$$

where, on the basis of equation (7), the LHS represents the *net* demand for the domestic asset. Equations (11) and (12) then imply that an equilibrium value of the pivotal agent's private signal is also equal to the realisation of the random variable X^p:

$$X^p \equiv \Theta - \sqrt{\varepsilon} U \tag{13}$$

Since agents' strategy is known to them, x^p is an equilibrium-generated public signal about θ (equivalently, u). Thus, an x_j-agent's information set is fully defined by equations (2), (3), (13) and knowledge of x_j and x^p.

For any $\{\theta, u\}$, an equilibrium in the EPS setup is defined by the vector $\{x^p, \iota, \theta^{cr}\}$ that solves simultaneously equation (12) and the following two equations:

$$a\left(\theta^{cr}\right) = \tau\iota + 2\Phi\left(\frac{x^p - \theta^{cr}}{\sqrt{\varepsilon}}\right) - 1 \tag{14}$$

$$\Phi\left(\frac{\theta^{cr} - x^p}{\sqrt{\varepsilon \frac{\sigma_u^2}{1+\sigma_u^2}}}\right) = \iota \tag{15}$$

Equation (14) is exactly the same as equation (8); equation (15) is the EPS analog of equation (9).[28]

26 A working version of the paper contains the analysis for a general μ_u. That paper is available from the author upon request.

27 As will become clear shortly, the endogenous public signal is a parameter in that function.

28 From the point of view of an x_j-agent, the posterior marginal distributions of Θ and U are as follows:

Θ ~ $N\left(x_j \frac{\sigma_u^2}{1+\sigma_u^2} + x^p \frac{1}{1+\sigma_u^2}, \varepsilon \frac{\sigma_u^2}{1+\sigma_u^2}\right)$

U ~ $N\left(\frac{x_j - x^p}{\sqrt{\varepsilon}} \frac{\sigma_u^2}{1+\sigma_u^2}, \frac{\sigma_u^2}{1+\sigma_u^2}\right)$, where $\frac{x_j - x^p}{\sqrt{\varepsilon}}$ is recognised as the realisation of $(Z_j + U)$.

An equilibrium is to be consistent with the solution of a fixed-point problem, solved by each atomistic agent. Tautologically, the interest differential, at which agents trade, is publicly known to be supported in equilibrium. For such an ι, an agent conjectures an equilibrium investment strategy, ie a value of the pivotal agent's private signal: say, x_1^p. Given ι and x_1^p, equation (14) determines the value of θ^{cr} and thus splits the state space into two regions: the peg survives if and only if $\theta > \theta^{cr}$. Then, using his/her private signal and the signal sent by x_1^p under market clearing, the agent deduces a devaluation probability. If the latter probability is greater than ι, the agent issues a domestic currency bond and invests in the foreign currency. The opposite investment position is optimal if the perceived devaluation probability is lower than ι. Following the same logic, the agent deduces the investment decision of any other agent, receiving any possible private signal. Since the perceived devaluation probability decreases in the private signal,[29] the private sector investment position is summarised by the value of x^p, say, x_2^p, that solves equation (15). For an equilibrium, $x_1^p = x_2^p$. Finally, clearing on the foreign exchange market requires that the prevailing interest differential is such that the resulting x^p satisfies equation (12) for the realised θ and s (equivalently, u).

Note that x^p is determined uniquely by the state, via equation (12) alone. Further, equation (14) defines θ^{cr} as an increasing function of ι. Given a pair $\{\theta, u\}$ and the implied x^p, equilibrium *multiplicity* prevails if at least two different pairs $\{\theta^{cr}, \iota\}$ solve equations (14) and (15).[30] When the latter is true, the prevailing equilibrium value of ι is picked by a sunspot variable, independently of all the random variables in the system.

3.2 Equilibrium Uniqueness in the EPS Model

Paralleling Section 1., I analyse the EPS model under the assumption that private signals are extremely precise, ie that ε is close to zero. Note that, according to equation (13), the variance of the noise in the public signal about θ is equal to $\varepsilon\sigma_u^2$. Thus, recalling equation (3), the *relative* precision of private and public information is *derived* to be equal to σ_u^2, the volatility of the domestic asset's supply shock. Since σ_u^2 does not vary with ε, the EPS equilibrium analysis is conducted in an informational setting that is identical to the one of Case 2 in Section 2..

In the latter case, an increase in the exogenously set precision of private and public signals, attained via a decrease in ε, is the *sole* factor leading to equilibrium multiplicity. For example, the number of equilibria is independent of the value of τ: the parameter influencing the maximum level of official reserve losses that the authority can incur without devaluing. As noted in Section 2., the multiplicity result is due to the following key feature of the informational structure of Case 2: there is (in contrast to Case 3 in the same section) common knowledge that θ belongs to any given interval containing the public signal.[31]

The latter feature emerges within the EPS model as well. However, due to the endogeneity of the information transmission mechanism in that model, parameters unrelated to the exogenous signals' properties but which capture characteristics of the economy's institutional infrastructure and financial markets are *crucial* for the existence of equilibrium multiplicity.[32]

Proposition 1 *There exists $\varepsilon_0 > 0$ such that, for any $\varepsilon \in (0, \varepsilon_0)$ and any x^p, there is exactly one solution to equations (14) and (15) in terms of θ^{cr} and ι if and only if $\{\tau, \sigma_u^2\} \in (0,2) \times \left[\frac{\tau^2}{4-\tau^2}, \infty\right)$.*

The intuition behind the proposition is provided in Section 3.3, whereas the proof is relegated to Appendix 2.

In the EPS context, an abrupt speculative attack is characterised by a drastic increase in the interest differential that is triggered by a small change in the economic environment. The model's capacity to

29 Recall equation (15) and footnote 28.

30 There is always a single x^p associated with any given pair $\{\theta^{cr}, \iota\}$, even under equilibrium multiplicity. This is demonstrated by following Morris and Shin (1999) in its proof of equilibrium uniqueness under a step-function investment strategy.

31 Strictly speaking, the public signal needs to belong to the *interior* of the interval under consideration.

32 At a slightly technical level, the conceptual differences between the EPS model and Case 2 in Section 2. can be understood as follows. Unlike their counterparts, defined by equation (10), the dominance regions in the EPS model change with x^p. Crucially, the interval outside of these regions cannot be considered as given when ε changes: the distance between x^p and at least one of the boundaries of that interval is of the order of $\sqrt{\varepsilon}$. As a result, common knowledge that θ belongs to the EPS counterpart of $(\underline{\theta}, \overline{\theta})$ might fail for all values of x^p even if ε is extremely close to (but not equal to) zero.

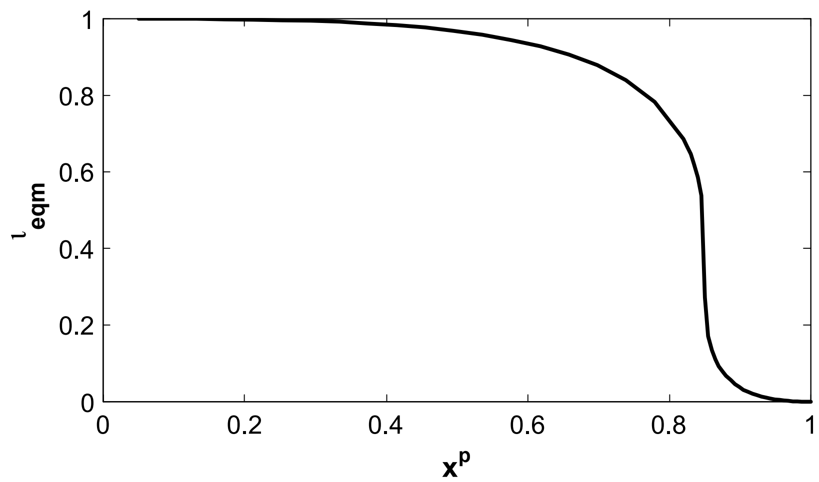

Figure 2: **Equilibrium interest differential as a function of the public signal**

account for such an attack within equilibrium uniqueness is demonstrated in Figure 2, which plots the equilibrium interest differential as a function of the public signal.[33] The result is formalised in Section 3.4, which provides further discussion of the EPS equilibrium implications.

At this stage, it is important to recall equation (13), which implies that:

$$\text{plim}_{\varepsilon \to 0} \left(X^p - \Theta \right) = 0 \tag{16}$$

In other words, when private signals are extremely precise, the graph in Figure 2 may be thought of as depicting the equilibrium interest differential as a function of the *fundamental*.

3.3 Interpreting the EPS Equilibrium Implications

The assumption of no foreign exchange market intervention by the central authority results in two state variables, Θ and U, whereas in the MS setup there is effectively only one, Θ. If a second state variable were to be incorporated in the setup of Section 2., such a variable would add the same noise to each agent's information set. As discussed in Section 2.1, however, the uncertainty generated by such noise cannot influence the qualitative message of the MS setup without hindering its capacity to account for drastic speculative attacks. Thus, the differences in the dimensionality of their respective state spaces does not obscure the comparison of the two models. In this respect, note also that, just as in the setup of Section 2., there is no aggregate uncertainty in the EPS model: knowledge of all the private signals reveals θ and knowledge of θ and x^p reveals the supply shock, u, via equation (12).[34]

Proposition 1 states that, in contrast to Case 2 of Section 2., the EPS model does not produce equilibrium multiplicity simply on the basis of an increase in private signals' precision, ie simply on the basis of a decrease in ε. In order to see why, consider the latter model with one modification: assume that R, which enters the authority's reaction function[35] is a random variable that is *un*related to the private sector's investment position. Consequently, R generates aggregate informational noise and, without loss of generality, its variance can be set to zero if the framework is to be consistent with violent speculative

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33 In order to emphasise the the interest differential's abrupt rise from low levels, I have used $\mu_u < 0$ in Figure 2. More precisely, the model is parameterised as follows: $a(\theta) \equiv \theta$, $\tau = 1$, $\sigma_u^2 = 0.51$, $\mu_u = -0.68$, $\varepsilon = 10^{-10}$.

34 Consequently, as demonstrated by equation (14), an agent needs to explicitly consider the posterior distribution of only one of the two state variables, Θ, when evaluating the likelihood of a devaluation. Of course, in the calculation of that likelihood, the other variable, U, is implicitly taken into account as well.

35 Recall inequality (5).

attacks.[36] The framework's equilibrium condition can then be written as:

$$a\left(\theta^{cr}\right) = \tau\Phi\left(\frac{\theta^{cr} - x^{p}}{\sqrt{\varepsilon\frac{\sigma_u^2}{1+\sigma_u^2}}}\right) + R \tag{17}$$

which is the result of combining equations (5) and (15) by substituting out ι.[37] For each x^p, an equilibrium is fully defined by a value of θ^{cr} solving equation (17).

Observe that the pivotal agent in the modified EPS framework does *not* need to take into account other agents' actions: the private sector participates in the determination of the pressure on the peg only by setting the interest rate, about which there is no uncertainty. Thus, the pivotal agent's problem is isomorphic to the one faced by the representative agent in the second-generation approach to currency crises and, consequently, equilibrium multiplicity emerges for sufficiently low values of ε and/or σ_u^2 and/or for a sufficiently high value of τ.[38] These parameters' impact on the likelihood of equilibrium multiplicity can be understood through the prism of perceived strategic complementarity; mathematically, this complementarity is seen in the fact that both sides of equation (17) increase in θ^{cr}. The bigger the value of τ, the bigger the impact of private sector market behaviour on the authority's incentive to abandon the peg. In turn, the smaller the value of ε and/or σ_u^2, the more precise is private agents' information regarding θ and the more certain they are about the ultimate implications of their actions.

Consider now the counterpart of equation (17) within the original EPS model:[39]

$$a\left(\theta^{cr}\right) = \tau\Phi\left(\frac{\theta^{cr} - x^{p}}{\sqrt{\varepsilon\frac{\sigma_u^2}{1+\sigma_u^2}}}\right) + \left[2\Phi\left(\frac{x^p - \theta^{cr}}{\sqrt{\varepsilon}}\right) - 1\right] \tag{18}$$

The term in square brackets introduces a second strategic link between the private sector and the central authority. A higher interest differential (which constitutes the first link and is equivalent to a higher θ^{cr} in the first RHS term) increases the attractiveness of the domestic asset, which is ceteris paribus associated with a lower outflow of reserves (the term in square brackets).

The two strategic links are mutually offsetting (move in opposite directions when θ^{cr} changes) and ε is the only parameter entering (explicitly) both of them. The impact of τ and σ_u^2 on the likelihood of equilibrium multiplicity is thus the same as in equation (17) and this is what Proposition 1 states. As in equation (17), a drop in ε intensifies the strategic complementarity by increasing the precision of private agents' beliefs. In addition, however, a drop in ε decreases the dispersion of idiosyncratic signals and increases the sensitivity of perceived reserve flows to changes in the interest differential (equivalently, changes in θ^{cr}). This increases the uncertainty of the ultimate impact of a given change in the interest differential and weakens the strategic complementarity in the system. Since private signals' precision and their dispersion are the flip sides of the same coin, the two effects of ε cancel each other.

I conclude this section by noting that, as implied by equations (11) and (12), the equilibrium outflow of official reserves is fully determined by the realisation of the exogenous shock U. The reserve flow, however, possesses two key characteristics due to which it generates strategic interaction among agents and thus plays a crucial role for the type of equilibrium. The reserve flow, on the one hand, enters the authority's reaction function and, on the other hand, matches the aggregate investment decision of informationally heterogeneous agents. The importance of the first characteristic was illustrated in this section by the modified EPS scenario. In turn, if heterogeneity is assumed away in the EPS model, it would fall within the traditional second-generation approach to currency crises.

3.4 Properties of the EPS Model's Unique Equilibrium

In this section, I assume that τ and σ_u^2 belong to the region specified by Proposition 1. The equilibrium interest rate is denoted by ι_{eqm} and its arguments are suppressed in order to reduce the notational burden. The comparative statics results, stated in the section, are derived in Appendix 3.

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36 Recall the discussion in Section 2.1, which applies to the present context as well.

37 To perform the substitution, inequality (5) is transformed into an equation.

38 See, for example, Jeanne (1997).

39 Equation (18) is is the result of substituting ι out of equations (14) and (15).

The equilibrium interest differential decreases with the public signal because the latter increases (on average) with the fundamental, which, via inequality (5), strengthens the authority's capacity to preserve the peg. Specifically:

$$\lim_{\varepsilon \to 0} \iota_{eqm} \begin{cases} = 0 & \text{for } x^p > a^{-1}(1) \\ \in (0,1) & \text{for } x^p \in \left[a^{-1}(\tau - 1), a^{-1}(1)\right] \\ = 1 & \text{for } x^p < a^{-1}(\tau - 1) \end{cases} \tag{19}$$

and, for $x^p \in \left[a^{-1}(\tau - 1), a^{-1}(1)\right]$:

$$\lim_{\varepsilon \to 0} \frac{\partial \iota_{eqm}}{\partial x^p} = \frac{da(\theta)}{d\theta} * G(x^p) < 0 \tag{20}$$

where θ^{cr}_{eqm} is the equilibrium critical value of the fundamental corresponding to ι_{eqm} and $G(\cdot)$ is defined in Appendix 3.

Recall that, as far as common knowledge about the fundamental is concerned, the EPS informational structure is identical to the one of Case 2 in Section 1. Then consider the first line in equation (19) and note that, when $x^p > a^{-1}(1)$, there is common knowledge that $\theta > a^{-1}(1)$. On the basis of inequality (5) and the fact that $R \in [-1, 1]$, it is then commonly known that the equilibrium reserves flow cannot topple the peg *by itself*. Consequently, *an* equilibrium, in which $\iota_{eqm} = 0$, emerges. As long as the assumptions of Proposition 1 are satisfied, this is the *only* equilibrium. An analogous argument applies to the third line in expression (19).

Note that the authority's resistance to speculative attacks might depend strongly on the fundamentals (ie $\frac{da(\theta)}{d\theta}$ might be large) if the commitment to peg preservation is weak. The equality in expression (20) would then stand for an intuitive equilibrium implication: a weaker commitment by the authority is built into agents' expectations and intensifies the public signal's effect on the interest differential.

The incorporation of the interest differential into the authority's reaction function, inequality (5), follows an implicit assumption that a high interest differential hurts the domestic economy. In this respect, the following results reveal the welfare relevance of the precision of *public* information.

For a given τ, there is a value of the public signal $x^p_* \in \left(a^{-1}(\tau - 1), a^{-1}(1)\right)$ such that:[40]

$$\lim_{\sigma_u^2 \searrow \frac{\tau^2}{4-\tau^2}} \left. \frac{\partial \iota_{eqm}}{\partial x^p} \right|_{x^p = x^p_*} = -\infty \tag{21}$$

Further, in the case that $\frac{da(\theta)}{d\theta} = 1$:

$$\frac{\partial \iota_{eqm}}{\partial \sigma_u^2} \begin{cases} > 0 \text{ for } x^p > x^p_* \\ < 0 \text{ for } x^p < x^p_* \end{cases} \tag{22}$$

where $x^p_* = \frac{\tau}{2}$, ie the middle point of the interval on which it is commonly known that the aggregate holdings of the domestic asset can influence the authority's decision, irrespective of the value of the interest differential. Relaxing the assumption $\frac{da(\theta)}{d\theta} = 1$ changes the value of x^p_* but not the qualitative message of inequalities (22).

Equation (21), in conjunction with equation (16), demonstrates formally that the EPS model is in a position to account for sudden and violent speculative attacks: a drastic surge of the interest differential can be triggered by an infinitesimal change in the fundamentals. The potential for a strongly non-linear relationship between x^p and ι_{eqm} is rooted in agents' strategic interaction and in their capacity to synchronise their behaviour by conditioning on the public signal. That capacity increases with the signal's precision, which, in turn, improves as σ_u^2 decreases. If x^p is high and σ_u^2 decreases, agents hold a stronger and stronger common belief that the fundamentals are high. Such a belief sustains a weaker attack, as suggested by the first inequality in expression (22). A symmetric reasoning explains

40 The symbol "$\searrow$" stands for "approaching the limit from above" and, recalling Proposition 1, $\frac{\tau^2}{4-\tau^2}$ stands for the lowest value of σ_u^2 associated with equilibrium uniqueness.

the second inequality. Finally, considering the two inequalities simultaneously, a decrease of the public signal leads to a more drastic transition from low to high values of the interest differential when the precision of public information is higher: this leads to the result in equation (21).

The following two equations illustrate the equilibrium implications of changes in τ:

$$\begin{aligned} \frac{\partial \iota_{eqm}}{\partial \tau} &> 0 \text{ for all } x^p \\ \lim_{\sigma_u^2 \searrow \frac{\tau^2}{4-\tau^2}} \frac{\partial \iota_{eqm}}{\partial \tau} &= +\infty \text{ at } x^p = x_*^p \end{aligned} \tag{23}$$

As implied by equations (4), (14) and (15), a drop in τ would have the same equilibrium impact as the introduction (or an increase) of transaction costs, which would increase pivotal agents' devaluation expectations relative to the interest differential. In the light of such an interpretation of τ, more sand in the wheels of foreign exchange markets would strengthen a pegged currency and, by the second line of expression (23), might even prevent a violent speculative attack from materialising.

4. Conclusion and a Look Ahead

The paper develops a currency crisis model in which beliefs can be self-fulfilling. Further, part of the relevant information in that model is revealed publicly by the equilibrium interest differential, which also constitutes a strategic link between the private sector and the central authority. A second strategic link is introduced by the private sector's aggregate investment position, which is uncertain from individual traders' point of view. A small perturbation of informational homogeneity in the private sector is shown to be in a position to account for abrupt and violent speculative attacks within equilibrium uniqueness. Consequently, policy analysis can be carried out in the paper's EPS model via well-defined comparative statics.

There are two main, mutually related differences between the EPS model and a model in the spirit of Morris and Shin (1998, 1999, 2000). First, public information is disseminated endogenously in the former but in an ad hoc fashion in the latter. Second, the former model derives equilibrium uniqueness under a weaker requirement with respect to the relative precision of private and public information.

In conclusion, I underscore the innovative features of the EPS model by outlining two of its possible extensions. First, the EPS framework suggests scope for managing speculative attacks by influencing prices' informational content. Suppose that, contrary to the assumptions maintained in Section 3., the authority, or any other large player, steps into the domestic asset market in order to (partially) offset or amplify the supply shock. This would alter the variance of the equilibrium quantity demanded (equivalently, supplied) of that asset by the private sector. The random variable driving the latter quantity generates the noise in the endogenously revealed public signal and thus affects the strategic interaction among agents.[41] Consequently, in contrast to its options within the MS setup, the authority is able to influence foreign exchange market developments via purely market means, by sacrificing funds carrying opportunity costs.

The EPS setup thus provides a new point of view for interpreting the frequently observed sterilised interventions on foreign exchange markets, interventions that are deemed futile by analyses based on representative agent models.[42] For example, the first line of expression (22) suggests that, if the fundamentals are sound and public information is rather noisy, increasing its relative precision up to a certain point (via intervention) increases agents' common awareness of the economy's strength and is unambiguously beneficial for the peg.[43] Proposition 1 implies, however, that increasing the latter

41 In the extreme case, where the authority exactly offsets the supply shock, equation (13) becomes $X^p = \Theta$: the private sector net demand for the domestic asset would always be zero and market clearing would perfectly reveal the fundamental.

42 See, for example, Obstfeld and Rogoff (1995).

43 The discussion implicitly assumes that the authority can credibly commit to a particular intervention rule. The incentive compatibility of such a commitment must be verified within the model (a generalisation of the EPS setup) used for analysing the informational impact of interventions on the foreign exchange market.

precision too much would strengthen agents' coordination capacity to an extent that renders the currency regime vulnerable to sunspots.

The second suggested direction of future research is of an empirical nature. A series of speculative attacks, staged on foreign exchange markets in the 1990s, have been described in informal accounts as unwarranted by economic fundamentals. When attempting to determine the type of equilibrium leading to such attacks, structural empirical analysis has relied on the second-generation approach to currency crises.[44] The conclusions of that analysis are potentially biased because, under standard assumptions, its underlying *homogeneous*-agent framework captures sudden and violent attacks *only* within equilibrium multiplicity.

To my knowledge, the EPS model is the first one to (i) account for seemingly unwarranted speculative attacks within both equilibrium multiplicity *and* uniqueness and (ii) allow for endogenous interest rates. The first feature suggests that it is not possible to reach an unambiguous conclusion, regarding the type of a currency attack's underlying equilibrium, by examining only the attack itself. The intertemporal evolution of market devaluation expectations needs to be examined during times of speculative tranquility as well. The second feature of the model allows for extracting these expectations from interest rate data that are available readily and at high frequencies. In sum, the EPS model warrants a shift away from the approach of the above-mentioned structural empirical analysis and provides the foundations for carrying out that shift.[45]

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44 See Jeanne (1997) and Jeanne and Masson (2000), who focus on the attack on the French franc in the autumn of 1992.

45 Tarashev (2003) provides a first attempt at developing this idea.

5. Appendix 1

This appendix analyses the space of equilibrium-implementable strategies in the EPS model. Equations (2) and (3) imply that, before any action has taken place in the first period, an x_j-agent perceives the following prior:

$$\begin{bmatrix} \Theta \\ U \end{bmatrix} \tilde{} N \left(\begin{bmatrix} x_j \\ \mu_u \end{bmatrix}, \begin{bmatrix} \varepsilon & 0 \\ 0 & \sigma_u^2 \end{bmatrix} \right) \quad (24)$$

Let Π be the set of investment strategies that may arise in equilibrium. Strategies within Π are denoted by π: $\pi(x_j) \in [0,1]$ sets the amount of currency that an x_j-agent invests in the foreign asset. For immediate use, divide Π into classes of π's. For any π_a and π_b in the same class, there exists a real λ such that $\pi_a(x_j + \lambda) = \pi_b(x_j)$ for all x_j; conversely, for any real λ and π_a, there exists a strategy π_b in π_a's class such that $\pi_a(x_j + \lambda) = \pi_b(x_j)$ for all x_j.

The net outflow of official reserves is denoted by $R(\theta, \pi)$, where:

$$R(\theta, \pi) = 2 \int \phi \left(\frac{n - \theta}{\sqrt{\varepsilon}} \right) \pi(n)\, dn - 1 \quad (25)$$

is a generalisation of equation (7). In turn, $A(\iota, \pi) \equiv \{\Theta = \theta | R(\theta, \pi) \geq a(\theta) - \tau\iota\}$ denotes the event in which the authority abandons the peg when agents follow the strategy π and the interest differential is ι. Finally, market clearing implies a generalisation of equation (12):

$$-R(\theta, \pi) = 2\Phi(u) - 1 = s \quad (26)$$

Consider equation (25) and observe that, for a non-monotone π, there are values of θ that are not recoverable even if one knows π and $R(\theta, \pi)$. Conversely, if Π contains non-monotone functions, then there exist multiple π's in the same class that produce the same $R(\theta, \pi)$ for a given value of θ. In such cases, the prevailing π is to be set by a sunspot. This motivates the following:

Assumption A.1

Π consists only of monotone functions of the private signal.

Assumption A.2

There is some well-defined prior distribution of Θ, U and $\pi(\cdot)$. $\pi(\cdot)$ is independent of Z_j for all j (recall equation (3)).

Lemma A.1

Any $\pi \in \Pi$ is a decreasing function. Moreover, there exist private-signal values, x_j and x_k, such that $\pi(x_j) = 1$ and $\pi(x_k) = 0$.

Proof

Let us consider an arbitrary equilibrium pair $\{\iota, \pi\}$ and fix $\varepsilon > 0$ at an arbitrary finite value. Assumption A.2, in conjunction with the existence of dominance regions defined in equation (10), implies that, if an agent's private signal is sufficiently small, that agent would choose to invest in the foreign asset irrespective of his/her perception of the measure of agents investing in either asset. A symmetric argument applies to an agent with a sufficiently high private signal. Thus, there exist two values of the private signal, x_j and x_k, such that $x_j < x_k$ and $1 = \pi(x_j) > \pi(x_k) = 0$. Assumption A.1 then implies that π is a monotonically *de*creasing function. **QED**

Lemma A.2

The joint prior distribution of Θ, U and π can be fully defined by equation (24)-(26) and the distribution of a sunspot variable that is independent of Θ, U and Z_j for all j.

Proof

Consider arbitrary θ and u. Lemma A.1 implies that, within each class in Π, there exists *exactly* one π that satisfies equation (26). This has two implications. First, it is possible to conjecture a sunspot variable that sets the equilibrium class of π's and is independent of Θ and U and Z_j for all j. Second, conditional on the class of π's, the joint prior distribution of Θ, U and π is fully determined by equations (24)-(26). **QED**

Denote x_j-agent's information set by Υ_j: it consists of equation (24) and π.

Proposition A.1

Let the sunspot variable from Assumption A.3 set the equilibrium π. *If* $\pi \in \Pi$, *then there exists a real number* x^p *such that* $\pi(x_j) = \begin{cases} 1 & \text{for } x_j < x^p \\ 0 & \text{for } x_j > x^p \end{cases}$

Proof

Let us consider an arbitrary equilibrium pair (ι, π). Consider an x^p-agent from whose point of view equation (4) is satisfied. The existence of such agents is consistent with Lemmas A.1 and A.2. Consider next an x_m-agent, where $x^p < x_m$. Denote by $H(\cdot|\Upsilon_p)$ and $H(\cdot|\Upsilon_m)$ the posterior distributions of Θ when the private signal is x^p, respectively, x_m. Equations (24)-(26) and Lemmas A.1 and A.2 imply that $H(\cdot|\Upsilon_m)$ first-order stochastically dominates $H(\cdot|\Upsilon_p)$. Further, Lemma A.1 implies that $R(\theta, \pi)$ decreases strictly in θ. As a result, $A(\iota, \pi)$ is fully defined by a single cutoff value of the fundamental: for θ below (above) this value, the peg is abandoned (sustained). Thus, an x_m-agent attributes a strictly lower probability to devaluation than an x_p-agent and, thus, invests in the domestic currency asset: $\pi(x_m) = 0$. Analogously, $\pi(x_n) = 1$ for $x_n < x^p$. **QED**

6. Appendix 2

This appendix contains a proof of Proposition 1.

Equations (6) and (14) imply that θ^{cr} is an increasing unbounded function of ι. Thus, as ι increases (decreases), the LHS of equation (15), asymptotes gradually to unity (zero). As a result, an equilibrium pair $\{\iota, \theta^{cr}\}$ exists irrespective of the values of ε, σ_u^2 and τ.

Equations (6) and (18) imply that an equilibrium pair $\{\iota, \theta^{cr}\}$ is unique if and only if it implies the following inequality:

$$\frac{2\sqrt{\frac{\sigma_u^2}{1+\sigma_u^2}}}{\tau} > \frac{\phi\left(\frac{\theta^{cr}-x^p}{\sqrt{\varepsilon\frac{\sigma_u^2}{1+\sigma_u^2}}}\right)}{\phi\left(\frac{\theta^{cr}-x^p}{\sqrt{\varepsilon}}\right)} \tag{27}$$

$O(\sqrt{\varepsilon})$ terms are eliminated in inequality (27), which states that, at an equilibrium, $a(\theta^{cr})$ should increase with θ^{cr} faster than the RHS of equation (18).

Fix an arbitrary x^p. Since $\frac{\sigma_u^2}{1+\sigma_u^2} < 1$, the RHS of inequality (27) is weakly smaller than 1 at all values of θ^{cr} and equals 1 at $\theta^{cr} = x^p$. Further, the LHS is bigger than or equal to 1 only if $\tau \leq 2$ and $\sigma_u^2 \geq \frac{\tau^2}{4-\tau^2}$. This concludes the "if" part of the proposition's statement.

For the "only if" part, observe that equation (18) defines x^p as a continuous *function* of θ^{cr}. (This is derived formally by mimicking the proof of Lemma 3 in Morris and Shin (1999).) Further, $\theta^{cr} = x^p$ is a solution to equation (18) when $x^p = a^{-1}(\tau/2)$. By continuity then, if the LHS of inequality (27) is smaller than 1, there is an interval on the real line, Ω, with the following properties: (i) $a^{-1}(\tau/2) \in \Omega$; (ii) if $x^p \in \Omega$, then there is a value of θ^{cr}, solving equation (18), at which inequality (27) does not hold. This completes the proof of the proposition.

7. Appendix 3

In this appendix, I assume that $\{\tau, \sigma_u^2\} \in (0,2) \times \left[\frac{\tau^2}{4-\tau^2}, \infty\right)$ and prove expressions (19)-(23). Denoting the solution of equation (18) by θ_{eqm}^{cr}, differentiation implies:

$$\lim_{\varepsilon \to 0} \frac{d\left(\theta_{eqm}^{cr} - x^p\right)}{dx^p} < 0 \tag{28}$$

In order to prove expression (19), suppose that x^p belongs to the interior of the interval $\left(a^{-1}(\tau - 1), a^{-1}(1)\right)$ and that $\lim_{\varepsilon \to 0}\left(\frac{\theta_{eqm}^{cr} - x^p}{\sqrt{\varepsilon}}\right) = -\infty$. This would imply that the RHS of equation (18) converges to 1, which then means that $\lim_{\varepsilon \to 0} \theta_{eqm}^{cr} = a^{-1}(1)$. The latter equality, however, contradicts the implicit assumption that $\theta_{eqm}^{cr} < x^p$. A symmetric argument applies to $\lim_{\varepsilon \to 0}\left(\frac{\theta_{eqm}^{cr} - x^p}{\sqrt{\varepsilon}}\right) = +\infty$. This proves the middle line of expression (19).

Suppose now that $x^p > a^{-1}(1)$ and $\theta^{cr} > a^{-1}(1)$. Since $\tau < 2$ and $\theta_{eqm}^{cr} = x^p$ when $x^p = a^{-1}(\tau/2)$, equation (6), inequality (28) and $x^p > a^{-1}(1)$ imply that $x^p > \theta^{cr}$. In turn, $x^p > \theta^{cr}$ implies that the RHS of equation (18) is smaller than unity, contradicting the assumption that $\theta^{cr} > a^{-1}(1)$. Continuity then implies the first line of expression (19). The third line of that expression is proven with a symmetric argument.

Defining $G(x^p) \equiv \left(\tau - \sqrt{\frac{\sigma_u^2}{1+\sigma_u^2}} \frac{2\phi\left(\frac{\theta_{eqm}^{cr} - x^p}{\sqrt{\varepsilon}}\right)}{\phi(\iota_{eqm})}\right)^{-1}$, expression (20) is an immediate implication of equations (14) and (15), inequality (28) and Proposition 1. In turn, equation (21) holds because, according to Proposition 1, there is a value of x^p at which $\lim_{\sigma_u^2 \searrow \frac{\tau^2}{4-\tau^2}} G(x^p) = 0$. Equation (19) then implies that such a value belongs to $\left(a^{-1}(\tau - 1)\ ,\ a^{-1}(1)\right)$.

To prove expression (22), I assume that $\frac{a(\theta)}{d\theta} = 1$. Then, using equation (18), one derives that $\frac{d\theta_{eqm}^{cr}}{dx^p}$, and, consequently, $\frac{d\iota_{eqm}}{dx^p}$, attains its maximum at $x^p = \tau/2$. Using equations (15) and (18), the sign of $\frac{d\iota_{eqm}}{d\sigma_u^2}$ is found to be the same as the sign of $\left(x^p - \theta_{eqm}^{cr}\right)$. The latter result, in conjunction with inequality (28) and the fact that $x^p = \theta_{eqm}^{cr}$ when $x^p = \tau/2$, implies expression (22).

Finally, expression (23) is obtained after totally differentiating equation (18) with respect to θ_{eqm}^{cr} and τ and then using equation (15) and the conditions for equilibrium uniqueness.

References

Atkeson, Andrew (2000): "Discussion of Morris and Shin's 'Rethinking multiple equilibria in macroeconomic modelling'", *NBER Macroeconomics Annual*.

Bensaid, Bernard and Olivier Jeanne (1997): "The instability of fixed exchange rate systems when raising the nominal interest rate is costly", *European Economic Review*, vol 41, pp 1461-78.

Carlsson, Hans and Eric van-Damme (1993): "Global games and equilibrium selection", *Econometrica*, vol 61, pp 989-1018.

Corsetti, Giancarlo, Amil Dasgupta, Stephen Morris and Hyun Song Shin (2000): "Does one Soros make a difference? A theory of currency crises with large and small traders", *www.econ.yale.edu/~smorris*.

Dasgupta, Amil (2002): "Coordination, learning and delay", *mimeo*, Yale University.

Eichengreen, Barry (2001): "The EMS crisis in retrospect", *CEPR Working Paper 2704*.

Eichengreen, Barry and Charles Wyplosz (1993): "The unstable EMS", *Brookings Papers on Economic Activity*, pp 51-143.

Eichengreen, Barry, Andrew Rose and Charles Wyplosz (1996): "Speculative attacks on pegged exchange rates: an empirical exploration with special reference to the European monetary system", in Canzoneri, Ethier and Grilli (eds), *The new transatlantic economy*, pp 191-235.

Goldstein, Itay (2000): "Interdependent banking and currency crises in a model of self-fulfilling beliefs", *manuscript*, University of Tel Aviv.

Grossman, Sanford J (1989): *The Informational role of prices*, MIT Press.

Jeanne, Olivier (1997): "Are currency crises self-fulfilling? A test", *Journal of International Economics*, vol 43, pp 263-86.

Jeanne, Olivier and Paul Masson (2000): "Currency crises, sunspots and Markov-switching regimes", *Journal of International Economics*, vol 50(2), pp 327-50.

Morris, Stephen and Hyun Song Shin (1998): "Unique equilibrium in a model of self-fulfilling currency attacks", *American Economic Review*, vol 88, no 3, pp 587-97.

—— (1999): "A theory of the onset of currency attacks", in Agenor, Miller, Vines and Weber (eds), *The Asian crisis: causes, contagion and consequences*, Cambridge, Cambridge University Press, pp 230-64.

—— (2000): "Global games: theory and applications", forthcoming in the *Volume of the Eighth World Congress of the Econometric Society*.

Obstfeld, Maurice (1994): "The logic of currency crises", *Cahiers Economiques et Monétaires*, vol 43, pp 189-213.

—— (1996): "Are currency crises self-fulfilling?", comment, in Ben Bernanke and Julio Rotemberg (eds), *NBER Macroeconomics Annual*, Cambridge and London, MIT Press, pp 393-403.

Obstfeld, Maurice and Kenneth Rogoff (1995): "The mirage of fixed exchange rates", *NBER Working Paper #5191*.

Rey, Hélène (2000): "Discussion of Morris and Shin's 'Rethinking multiple equilibria in macroeconomic modelling'", *NBER Macroeconomics Annual*.

Rose, Andrew and Lars Svensson (1994): "European exchange rate credibility before the fall", *European Economic Review*, vol 38, pp 1185-216.

Tarashev, Nikola (2003): "Devaluation expectations prior to a speculative attack", in Ph.D. dissertation, Princeton University.

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