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A tractable menu cost model with an aggregate markup drift *

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Abstract

This paper extends the menu cost model of Gertler and Leahy (2008) by introducing a drift in the aggregate markup. Assuming that the drift is always negative and not large, consistent with moderate and positive trend inflation, the paper analytically characterizes firms' value function and markup distribution. It derives explicit equations sufficient to close the model in general equilibrium, making the calculation of impulse responses to aggregate shocks as easy as in conventional representative-agent New Keynesian models. In addition, the paper shows two implications of the model. First, the model replicates the empirically observed positive correlation between the inflation rate and the frequency of price changes. Second, the model yields an explicit equation representing the Phillips curve, with additional terms that make the inflation rate more responsive to aggregate shocks.

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1 Introduction

Among models of price stickiness in monetary economics, menu cost models match the evidence from empirical studies of micro prices better than conventional time-dependent models, such as the Calvo model (Calvo (1983)). For example, menu cost models, when combined with a firm-level idiosyncratic shock, often generate a positive correlation between the inflation rate and the frequency of firms' price changes, as found in empirical studies.[1] In pursuit of better understanding of inflation and macroeconomic dynamics, an increasing number of researches have examined the implications of menu cost models.[2]

However, menu cost models are still far less popular than the time-dependent models in macroeconomic policy analysis in general, primarily for technical reasons. The former models feature state dependence in firms' price setting: firms are not exogenously or randomly given a chance to change price; instead, they do so if and only if their current price (markup) is far enough from the reset value. This state dependence, while key for the empirical success of menu cost models, generates non-linearity in firms' policy rule (i.e., the rule that firms follow in choosing whether and how much they change their prices) and a non-trivial price (markup) distribution. Solving the non-linear optimization problem while keeping track of such a distribution is numerically demanding. Moreover, most menu cost models do not yield an explicit equation representing the Phillips curve, a simple relationship between the inflation rate and the real marginal cost for firms, which policy analysis often relies on.[3]

To address the issues, this paper extends the menu cost model of Gertler and Leahy (2008) (GL). The key assumption of this model is that idiosyncratic shocks hit firms only

[1]For the studies covering the recent post-COVID period, see Montag and Villar Vallenas (2025) and Gautier et al. (2025). Earlier studies using the data of high-inflation periods include Nakamura et al. (2018) and Alvarez et al. (2019).

[2]Major studies on general properties of menu cost models include Golosov and Lucas (2007); Midrigan (2011); Alvarez et al. (2016); Alvarez and Lippi (2022).

[3]Many papers in the literature define the Phillips curve equation as the relationship between inflation rate and slacks in the economy. The Phillips curve equation in this paper instead represents the relationship between inflation rate and the real marginal cost for firms, corresponding to the "primitive formulation" according to Gagliardone et al. (2025). We can convert the latter equation into the former by substituting an additional relationship between the real marginal cost and a variable representing slacks.

occasionally and follow a uniform distribution with a wide support (Assumption 1 in Section 3.1). Further assuming zero trend inflation and adopting a first-order perturbation, GL derive a Phillips curve equation that looks identical to that in the Calvo model except for the coefficient value on the real marginal cost. This paper instead analyzes nonlinear dynamics of firms' value function and markup distribution by employing two assumptions. First, firms' aggregate markup always decreases relative to the three markup values that characterize firms' policy rule, unless they are hit by an idiosyncratic shock (Assumption 2). The monotonically decreasing markup, which we call a negative drift in the markup, is consistent with positive trend inflation and represents the steady erosion of firms' markup due to the monotonic increase in the aggregate nominal marginal cost. Second, the drift is so small that the firms that have just adjusted their prices are going to keep the prices for a long time, unless they are hit by an idiosyncratic shock (Assumption 3). Using the two assumptions, this paper derives explicit equations sufficient to close the model in general equilibrium. These equations make the calculation of impulse responses to aggregate shocks as easy as in conventional representative-agent New Keynesian DSGE models. This paper then shows two implications of the model. First, the model replicates the empirically observed positive correlation between the inflation rate and the frequency of price changes (see Sections 3.4 and 4.3), which is absent in the case of zero trend inflation analyzed by GL. Second, adopting a first-order perturbation, this paper derives a simple equation representing the Phillips curve (Section 3.3), with terms absent in the case of zero trend inflation. The additional terms, proportional to levels and changes in the real marginal cost, make the inflation rate more responsive to both aggregate productivity shocks and monetary policy shocks.

Behind these results is a combination of two mechanisms: one is related to the asymmetry of the policy rule, defined at the bottom of Section 3.1; the other originates from the asymmetry of the markup distribution, defined at the bottom of Section 3.2.2. Both are tightly linked to the perpetual negative drift in the aggregate markup posed by Assumption 2. Specifically, the future expectation of the negative drift generates asymmetry of the policy rule, because the firms on the verge of increasing their prices have effectively shorter expectation horizon than those on the verge of decreasing their prices. This asymmetry results in the significant response of the gap between the upper and lower markup thresholds to aggregate shocks, contributing to flexibility in the aggregate price index. Meanwhile, the history of the drift that has always been negative generates asymmetry of the markup distribution, because more firms accumulate around the lower markup threshold. Due to the asymmetry, aggregate inflationary shocks expand the number of firms increasing their prices by more than the decline in the number of firms decreasing their prices, again contributing to price flexibility.

The importance of Assumption 2 clarifies the condition under which one should employ our analysis instead of GL. Namely, our analysis becomes relevant when the trend inflation is large enough or aggregate uncertainty is small enough that aggregate markup does not experience a positive drift. Interestingly, even a temporary violation of the assumption changes the dynamics in a non-linear manner: for example, for the case of a temporary large deflationary shock examined in Section 5.2, the contribution of the asymmetric markup distribution to inflation is dampened, making inflation less sensitive to a change in real marginal cost for a while.

The main contribution of this paper to the vast literature of menu cost models is the analytical characterization of a menu cost model which features a significant fluctuation in the frequency of price changes. As mentioned above, most menu cost models with both aggregate and idiosyncratic shocks do not allow one to analytically derive explicit forms of the equations sufficient to close the model in general equilibrium. Two important exceptions are Danziger (1999) and GL. However, under their settings, the frequency of price changes is constant over time, which is inconsistent with empirical evidence during periods of relatively high inflation. This paper instead replicates the empirical correlation without losing the tractability by introducing the two assumptions on the markup drift.

Moreover, few theoretical analysis has shown how the state dependence of firms' price setting affects the Phillips curve equation. The reason is presumably the inability to derive such simple equations from most menu cost models. While GL is again a notable exception, their Phillips curve is identical to that in the Calvo model apart from the difference in the value of a coefficient. On the other hand, the additional terms in our Phillips curve equation, deriving from firms' state-dependent price setting, contribute to higher sensitivity of inflation to aggregate shocks.

Regarding the proposed mechanisms, this paper is closely related to Ball and Mankiw (1994), Karadi and Reiff (2019), Alexandrov (2020), and Bunn et al. (2024). All of these studies discuss at least one of the two mechanisms regarding how trend inflation affects firms' price setting. However, unlike this paper, they do not analytically characterize a fully state-dependent model in general equilibrium: Ball and Mankiw (1994) partly introduce time-dependent price settings for tractability; and Karadi and Reiff (2019), Alexandrov (2020), and Bunn et al. (2024) analyze a state-dependent menu cost models in general equilibrium using numerical techniques for heterogeneous-agent problems.[4] In addition, their primary focus is on the asymmetric response to aggregate shocks when the size of the shocks is large. While we also observe similar asymmetry by examining a large deflationary shock in Section 5.2 and a large inflationary shock in Section 5.3, our main focus is on the case of small aggregate shocks.

This paper complements other studies in the literature that compare menu cost models with the Calvo model. For example, Auclert et al. (2024) show that the impulse responses of a wide range of menu cost models can be well approximated by a response of the Calvo model by choosing an appropriate value for the coefficient on the real marginal cost. The value of this paper lies on the opposite end of the spectrum: it analyzes one special model and shows how the state dependence generates a deviation of the impulse response from the Calvo model.[5]

This paper is also related to another strand of studies, including Gasteiger and Grimaud (2023) and Blanco et al. (2024), which employ non-standard but tractable settings in firms' optimization problem in order to replicate the significant fluctuations in the frequency of price changes. The contribution of our paper compared to theirs is to employ

[4]Alexandrov (2020) also analytically characterizes a partial-equilibrium menu cost model to examine the effect of trend inflation on monetary non-neutrality.

[5]The strong deviation from the Calvo model shown in this paper may not necessarily contradict with the results of Auclert et al. (2024). In fact, they find that the approximation by the Calvo model deteriorates when they adopt an infrequent idiosyncratic shock, as shown in the online appendix D.5.2 of their paper.

more conventional setting following the literature of menu cost models and to show the endogenous fluctuations in the frequency of price changes.

Finally, this paper contributes to the literature of how trend inflation affects the slope of the Phillips curve, which is inversely related to monetary non-neutrality. As shown by Ascari and Ropele (2007), trend inflation in the Calvo model flattens the Phillips curve, thus enhancing monetary non-neutrality. In contrast, as shown in Karadi and Reiff (2019), trend inflation in menu cost models reduces monetary non-neutrality due to the endogenous fluctuation in the frequency of price changes. This paper reinforces the latter claim by explicitly deriving the terms that steepen the slope of the Phillips curve, as far as the aggregate uncertainty is not so large to reverse the effect of positive trend inflation on the aggregate markup.

The rest of the paper is organized as follows. Section 2 introduces the basic building blocks of the model. Section 3 shows the main analytical results. It derives the laws of motion for variables sufficient to determine inflation, and adopts a first-order perturbation to obtain an equation representing the Phillips curve. In addition, it discusses the frequency of price changes and how it relates to the Phillips curve. Section 4 calibrates the model and numerically calculates the impulse responses of endogenous variables to aggregate shocks. Section 5 discusses the importance of our assumptions as well as the effects of their violations. Section 6 concludes.

2 Model

Our model has the same building blocks as simplest New Keynesian models: household, firms, and a central bank. The setup for the household and the central bank is conventional. Firms face both idiosyncratic productivity shocks and aggregate uncertainty and determine whether they pay a menu cost to adjust their prices.

2.1 Household

Time t is discrete. A representative household consumes consumption bundle C_t, which combines a range of goods $C_{i,t}$ indexed by $i \in [0, 1]$ using constant elasticity of substitution

(CES) function $C_t = \left(\int_0^1 C_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$.The household solves the cost minimization problem given prices $P_{i,t}$. The result is the following demand curve for each good

$$C_{i,t} = C_t \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon}, \tag{1}$$

where the price index is naturally defined as

$$P_t = \left(\int_0^1 P_{i,t}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}. \tag{2}$$

In addition to the consumption bundle C_t, the household chooses nominal short-term bond B_t and labor L_t, given the price index P_t, nominal wage W_t, nominal interest rate i_t and lump-sum transfer from the government and firms Π_t. The expected lifetime utility of the household is given by

$$\max\ E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \frac{1}{1+\varphi} L_{t+s}^{1+\varphi}\right], \tag{3}$$

subject to the budget constraint

$$P_t C_t + B_t e^{-i_t} = W_t L_t + B_{t-1} + \Pi_t. \tag{4}$$

The first-order conditions are

$$\beta E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} e^{i_t - \pi_{t+1}}\right] = 1 \tag{5}$$

and

$$\frac{W_t}{P_t} = L_t^{\varphi} C_t^{\sigma}, \tag{6}$$

where $\pi_t \equiv \ln(P_t/P_{t-1})$ is the inflation rate.

2.2 Firms

The setup for firms mostly follows the model of Gertler and Leahy (2008). There are an infinite number of firms with index $i \in [0, 1]$. Each firm i produces output $Y_{i,t}$ using labor

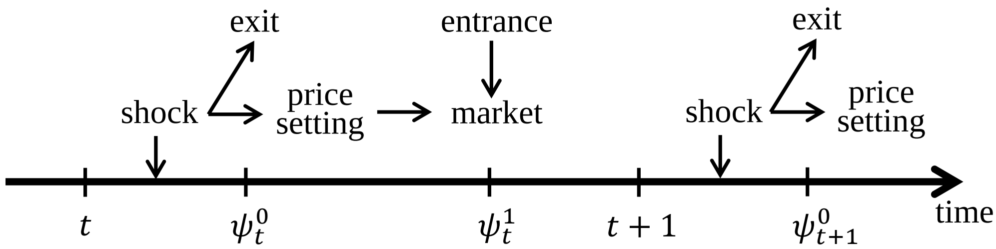

Figure 1: Sequence of events in the model. ψ_t^0 represents the distribution for firms who optimize their prices at time t, while ψ_t^1 represents that of firms selling their products in the market.

input $L_{i,t}$ with a linear production function

$$Y_{i,t} = Z_t e^{z_{i,t}} L_{i,t}, \tag{7}$$

where Z_t is an aggregate productivity while $z_{i,t}$ is an idiosyncratic deviation.

Figure 1 depicts the sequence of events. After entering period t and aggregate productivity Z_t as well as other exogenous variables are updated, a shock may hit each firm with probability $1 - \alpha$. Firms hit by the shock either exit the market with probability $1 - \tau$ or remain in the market with probability τ. The latter firms experience a shift in their idiosyncratic productivity by $\xi_{i,t}$ as

$$z_{i,t} = z_{i,t-1} + \xi_{i,t}, \tag{8}$$

where $\xi_{i,t}$ follows an *i.i.d.* uniform distribution with a support $[-\phi/2, \phi/2]$ and a density $1/\phi$. The purpose of assuming exogenous exits by a probability $(1 - \alpha)(1 - \tau)$ is to make the distribution of $z_{i,t}$ stationary even in the presence of random walk process in Equation (8). Assuming the random walk, in turn, is for analytical tractability as it makes firms' optimization problem essentially independent of the idiosyncratic productivity level $z_{i,t}$.

Firms that do not exit the market choose whether to adjust their prices by paying a nominal menu cost $bW_t e^{(\varepsilon-1)z_{i,t}}$. Following Golosov and Lucas (2007) and Nakamura and Steinsson (2008), firms pay a menu cost in terms of wage to additional labor. Following

Gertler and Leahy (2008), the menu cost depends on firms' idiosyncratic productivity level $z_{i,t}$ with an elasticity $(\varepsilon - 1)$ in order to keep the firm's optimization problem independent with the size of each firm.[6]

The production function (7) implies that the real profit of a firm in each period net of a menu cost is

$$\widetilde{\Pi}_{i,t} = \frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_t}{P_t Z_t e^{z_{i,t}}} Y_{i,t} - b_t \frac{W_t}{P_t} e^{(\varepsilon-1) z_{i,t}}, \tag{9}$$

where b_t is equal to b if $P_{i,t} \neq P_{i,t-1}$ and zero otherwise. By taking into account the demand (1) and using the equations $Y_{i,t} = C_{i,t}$ and $Y_t = C_t$, we rewrite the profit as

$$\widetilde{\Pi}_{i,t} = Z_t e^{-x_t + (\varepsilon-1) z_{i,t}} \left[A_t e^{-\varepsilon x_{i,t}} \left(e^{x_{i,t}} - 1 \right) - b_t \right], \tag{10}$$

where $x_{i,t} \equiv \log(P_{i,t} Z_t e^{z_{i,t}} / W_t)$ is the logarithm of each firm's markup, $x_t \equiv \log(P_t Z_t / W_t)$ is the logarithm of aggregate markup (corresponding to a markup for firms with the price equal to the price index and $z_{i,t} = 0$), and $A_t \equiv (Y_t / Z_t) e^{\varepsilon x_t}$ is a variable representing the scale of firms' profit. The markup for those firms that do not adjust their prices is $\widetilde{x}_{i,t} = \log(P_{i,t-1} Z_t e^{z_{i,t}} / W_t) = x_{i,t-1} + z_{i,t} - z_{i,t-1} - \kappa_t$, where κ_t represents the growth rate of aggregate nominal marginal cost and is defined by

$$\kappa_t \equiv \log \frac{W_t Z_{t-1}}{Z_t W_{t-1}} = \pi_t - x_t + x_{t-1}. \tag{11}$$

The profit in (10) suggests that we can re-scale each firm's value by a factor $Z_t e^{-x_t + (\varepsilon-1) z_{i,t}}$. The value after re-scaling is

$$v_t(\widetilde{x}_{i,t}) \equiv v(\widetilde{x}_{i,t}, \Omega_t) = \max\{\tilde{v}_t(\widetilde{x}_{i,t}), \max_x \tilde{v}_t(x) - b\}, \tag{12}$$

where Ω_t represents a set of all relevant macroeconomic variables in the past as well as rational expectations of future macroeconomic variables as of time t, and the subscript t on the functions $v_t(\cdot)$ and $\tilde{v}_t(\cdot)$ is a shorthand for the dependence on Ω_t. The Bellman

[6] Gertler and Leahy (2008) do not assume that the menu cost is proportional to wage. It is straightforward to apply the main analysis of this paper to the case in which the form of the menu cost exactly follows that assumed in their paper.

equation for the function $\tilde{v}_t(x)$ is

$$\tilde{v}_t(x) = A_t e^{-\varepsilon x}\left(e^x - 1\right) + \alpha\beta E_t e^{\lambda_{t+1}} v_{t+1}\left(x - \kappa_{t+1}\right) \\ + (1-\alpha)\tau\beta E_t e^{\lambda_{t+1}+(\varepsilon-1)\xi_{i,t+1}} v_{t+1}\left(x - \kappa_{t+1} + \xi_{i,t+1}\right), \tag{13}$$

where $\lambda_{t+1} \equiv \log(Z_{t+1}/Z_t) - x_{t+1} + x_t + \log(C_{t+1}^{-\sigma}/C_t^{-\sigma})$ is the sum of the growth rate of the scale factor and the logarithm of stochastic discount factor. The first term in Equation (13) represents the re-scaled flow profit excluding a menu cost. The second term is the expected value for the case in which a shock does not hit the firm in time $t+1$ and thus the markup is only reduced by the drift in markup $(-\kappa_{t+1})$. The third term is the expected value for the case in which a shock hits the firm without letting it exit the market, shifting firm's markup by $(\xi_{i,t+1} - \kappa_{t+1})$. The value function does not depend on each firm's idiosyncratic productivity level $z_{i,t}$ thanks to the assumed form of menu cost $bW_t e^{(\varepsilon-1)z_{i,t}}$. The value function and the associated optimal policy are therefore common across all firms.

New entrants arrive at the market with idiosyncratic productivity level $z_{i,t}$ equal to 0 and markup equal to the reset markup, which we define as the markup after adjusting price in Section 3.1. Both new entrants and survivor firms participate in the market.

2.3 Central bank

The central bank follows a standard Taylor rule

$$i_t = \bar{r} + \bar{\pi} + \phi_\pi(\pi_t - \bar{\pi}) + v_t^m, \tag{14}$$

where $\bar{r}$ is the steady-state real interest rate, $\bar{\pi}$ is the trend inflation rate, $\phi_\pi > 1$ represents the degree to which the central bank stabilizes the inflation, and v_t^m represents a discretionary part of monetary policy.

2.4 Market clearing

The condition for goods market clearing is conventional:

$$C_t = Y_t. \tag{15}$$

As for labor market, in addition to the usual demand for labor to make products, firms need to hire additional workers to change their prices because menu cost is paid as a wage to those workers. Specifically, the labor market clearing condition is

$$L_t = \int_0^1 di L_{i,t} + b \int_{\text{firms paying } b} di \; e^{(\varepsilon-1)z_{i,t}}, \tag{16}$$

where "firms paying b" include both survivor firms changing their prices and new entrants.

3 Analysis

This section presents analytical characterizations of the firms' problem introduced in Section 2.2. Section 3.1 analytically derives firms' policy rule. Section 3.2 analytically derives the law of motion for the price index. The analysis in these two sections, together with the conventional equations described in Section 2 and the exogenous shock process, yields a set of equations sufficient to close the model in general equilibrium. Section 3.3 applies a first-order perturbation to the equations to derive an equation representing the Phillips curve. Finally, Section 3.4 discusses the frequency of price changes and how it relates to the Phillips curve.

3.1 Firms' policy

This section solves the optimization problem for firms defined in Equations (12) and (13). To proceed, we guess that the policy rule is in a form conventional for menu cost models: the value function $\tilde{v}_t(x)$ has a maximum at the reset markup $x_t^* \equiv \operatorname{argmax}_{x\in\mathbb{R}} \tilde{v}_t(x)$ and an inaction region $S_t \equiv [x_t^L, x_t^H]$, which we define by the following conditions: for $\forall x \in S_t$,

$$\tilde{v}_t(x) \geqq \tilde{v}_t(x_t^*) - b, \tag{17}$$

and $x_t^* \in S_t$. x_t^L and x_t^H both satisfy (17) with equality and are distinguished by the sign of the first derivatives: $d\tilde{v}_t(x_t^L)/dx > 0$ and $d\tilde{v}_t(x_t^H)/dx < 0$, assuming they exist. Based on this guess of the policy rule, we construct an explicit form of the value function using Equation (13). Using the value function, we derive the equations that determine the values of the triplet (x_t^*, x_t^L, x_t^H). This section presents an outline of the derivation. Appendix A shows more fundamental aspects of the policy rule, including the verification of the inequality (17).

Based on the guess, we analyze the value function. We first rewrite Equation (12) as $v_t(x) = \tilde{v}_t(x_t^*) - b + (\tilde{v}_t(x) - \tilde{v}_t(x_t^*) + b)\, I\,(x \in S_t)$, where $I\,(\cdot)$ is an indicator function which is equal to 1 if the condition inside the parentheses is satisfied and equal to 0 otherwise. Putting this expression back into Equation (13), substituting the uniform density $p(\xi) = I\,(|\xi| \leqq \phi/2)\,/\phi$ for $\xi_{i,t+1}$, and rearranging it, we obtain

$$\begin{aligned}
\tilde{v}_t(x) = {} & A_t \left(e^{(-\varepsilon+1)x} - e^{-\varepsilon x}\right) + \left(\alpha + (1-\alpha)\tau\varepsilon^{\phi}\right) \beta E_t e^{\lambda_{t+1}} \left(\tilde{v}_{t+1}(x_{t+1}^*) - b\right) \\
& + (1-\alpha)\tau\beta E_t e^{\lambda_{t+1} + (\varepsilon-1)\kappa_{t+1}} e^{(-\varepsilon+1)x} \int_{-\infty}^{\infty} \frac{dx'}{\phi} e^{(\varepsilon-1)x'} \\
& \times I\left(|x' - x + \kappa_{t+1}| \leqq \frac{\phi}{2}\right) I\left(x' \in S_{t+1}\right) \left[\tilde{v}_{t+1}(x') - \tilde{v}_{t+1}(x_{t+1}^*) + b\right] \\
& + \alpha\beta E_t e^{\lambda_{t+1}} \left[\tilde{v}_{t+1}(x - \kappa_{t+1}) - \tilde{v}_{t+1}(x_{t+1}^*) + b\right] I\left(x - \kappa_{t+1} \in S_{t+1}\right), \qquad (18)
\end{aligned}$$

where $\varepsilon^{\phi} \equiv \left(e^{(\varepsilon-1)\phi/2} - e^{-(\varepsilon-1)\phi/2}\right)/\phi(\varepsilon-1)$ is a constant.

In order to simplify the third term in Equation (18), we adopt an assumption corresponding to that used by Gertler and Leahy (2008) (see the inequality (12) in their paper): the support ϕ of the uniform distribution of idiosyncratic shock ξ is wide enough.

Assumption 1 (Wide support of idiosyncratic shock) *The following inequality holds for* $\forall t$ *with probability* 1 [7]*:*

$$\frac{\phi}{2} \geqq \max\left(x_t^H - x_{t-1}^L + \kappa_t, x_{t-1}^H - x_t^L - \kappa_t\right). \qquad (19)$$

[7] Strictly speaking, in order to obtain firms' policy rule, we only need to assume that firms expect that the inequality holds with probability 1 in the future. The reason why we also assume that this inequality has to hold for $\forall t$, including past periods, is to reuse this assumption later in Section 3.2. Similar remarks apply to Assumptions 2 and 3, too.

Intuitively, Assumption 1 makes it possible that even firms on the verge of decreasing their prices at $t-1$ instead increase their prices at t, and vice versa, if they are hit by a large idiosyncratic shock. This assumption, together with the fact that firms' markup x after optimization at t should satisfy $x \in S_t$, ensures that the condition for the second indicator function ($x' \in S_{t+1}$) in Equation (18) is sufficient for the condition for the first indicator function ($|x' - x + \kappa_{t+1}| \leqq \frac{\phi}{2}$).[8] The latter is thus redundant, which in turn imply that the integral is independent of x. We can therefore denote this term as $A_{4,t}e^{(-\varepsilon+1)x}$, where

$$A_{4,t} \equiv (1-\alpha)\tau\beta E_t e^{\lambda_{t+1}+(\varepsilon-1)\kappa_{t+1}} A_{5,t+1}, \tag{20}$$

$$A_{5,t} \equiv \int_{x_t^L}^{x_t^H} \frac{dx}{\phi} e^{(\varepsilon-1)x} \left[\tilde{v}_t(x) - \tilde{v}_t(x_t^*) + b\right]. \tag{21}$$

We next analyze the last term of Equation (18), which represents the case in which no idiosyncratic shock hit the firm at $t+1$. Whether those firms can stay inside the inaction region at $t+1$ depends on the drift in firms' aggregate markup ($-\kappa_{t+1}$) and the dynamics of the thresholds $x_{t+1}^{L,H}$. This consideration motivates another assumption.

Assumption 2 (Monotonic drift) *There exists a positive real number $\epsilon > 0$ such that the following inequalities hold for $\forall t$ with probability 1:*

$$x_t^* + \kappa_t - x_{t-1}^* \geqq \epsilon, \tag{22}$$

$$x_t^L + \kappa_t - x_{t-1}^L \geqq \epsilon, \tag{23}$$

$$x_t^H + \kappa_t - x_{t-1}^H \geqq \epsilon. \tag{24}$$

These inequalities imply perpetual negative drift of firms' aggregate markup relative to the triplet (x_t^*, x_t^L, x_t^H) representing policy rule.[9] This assumption is a sufficient condition

[8] A simple algebra confirms this statement. For example, to confirm $x' - x + \kappa_{t+1} \leqq \phi/2$, we proceed as

$$x' - x + \kappa_{t+1} \leqq x_{t+1}^H - x_t^L + \left(\frac{\phi}{2} - x_{t+1}^H + x_t^L\right) = \frac{\phi}{2},$$

where the inequality is a combination of the following three inequalities: $x' \leqq x_{t+1}^H$, $x \geqq x_t^L$, and $\frac{\phi}{2} \geqq x_{t+1}^H - x_t^L + \kappa_{t+1}$.

[9] The assumption concerns aggregate markup, which should not be confused with each firm's idiosyncratic markup. In other words, even under the assumption, each firm's idiosyncratic markup may fluctuate around zero by a large idiosyncratic shock.

for positive trend inflation: in the steady-state in which $(\bar{x^*}, \bar{x^L}, \bar{x^H})$ are all time-invariant, where the bar above each variable represents its steady-state value, these inequalities imply $\bar{\kappa} = \bar{\pi} \geqq \epsilon > 0$. Outside the steady state, the inequalities imply that, for example, if a firm has a markup exactly equal to the reset markup x^*_{t-1} at period $t-1$ and is not hit by an idiosyncratic shock at period t, the markup of the same firm at t is definitely below the reset markup x^*_t due to the drift $(-\kappa_t)$, unless it changes the price. This assumption is the basis for the main economic mechanisms we highlight in this paper. Clearly, it is a rather restrictive assumption, especially when trend inflation is low or the economy is subject to considerable aggregate uncertainty that may induce positive drift in markup at times. Later in Section 5.2, we revisit the case in which the assumption fails .

Furthermore, in order to clarify the exposition, we temporarily assume that firms perceive no aggregate uncertainty. We later remove this auxiliary assumption when we instead introduce Assumption 3.

Using these assumptions, we solve Equation (18) in a recursive manner to obtain the value function as schematically shown in Figure 2. First, for $x \in (-\infty, x^L_{t+1} + \kappa_{t+1}) \cup (x^H_{t+1} + \kappa_{t+1}, \infty)$, the last term in Equation (18) is equal to zero and $\tilde{v}_t(x)$ is an explicit function only consisting of $e^{(-\varepsilon+1)x}$, $e^{-\varepsilon x}$ and a constant with respect to x.[10] Specifically,

$$\tilde{v}_t(x) = \tilde{v}^{(0)}_t(x) = A^{(0)}_{1,t} e^{(-\varepsilon+1)x} - A^{(0)}_{2,t} e^{-\varepsilon x} + A^{(0)}_{3,t}, \tag{25}$$

where

$$A^{(0)}_{1,t} \equiv A_t + A_{4,t} \tag{26}$$

$$A^{(0)}_{2,t} \equiv A_t \tag{27}$$

$$A^{(0)}_{3,t} \equiv \left(\alpha + (1-\alpha)\tau\varepsilon^{\phi}\right) \beta E_t e^{\lambda_{t+1}} \left(\tilde{v}_{t+1}(x^*_{t+1}) - b\right). \tag{28}$$

[10] Strictly speaking, when we evaluate the third term in Equation (18) above, we assume that $x \in [x^L_t, x^H_t]$ as shown in footnote 8. For $x \in (x^H_t, x^H_{t+1} + \kappa_{t+1}]$, which is outside this range, we need to confirm the inequality $-x' + x - \kappa_{t+1} \leqq \frac{\phi}{2}$. We show this inequality as follows:

$$-x' + x - \kappa_{t+1} \leqq -x^L_{t+1} + \left(x^H_{t+1} + \kappa_{t+1}\right) - \kappa_{t+1} < \frac{\phi}{2},$$

where the second inequality derives from the combination of Assumptions 1 and 2.

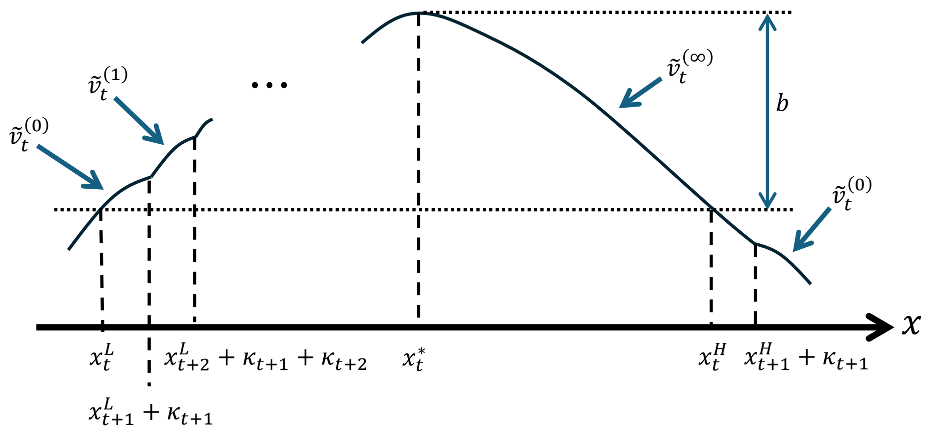

Figure 2: Schematic picture of the value function under the assumption of no aggregate uncertainty.

Because Assumption 2 ensures that $x_t^L < x_{t+1}^L + \kappa_{t+1}$, $\tilde{v}_t(x_t^L) = \tilde{v}_t^{(0)}(x_t^L)$.

Next, we shift the time index to $t+1$ in Equation (25) and substitute the expression into the last term of Equation (18). Under the auxiliary assumption that firms perceive no aggregate uncertainty, we can express $\tilde{v}_t(x)$ in an explicit form similar to Equation (25) for $\forall x \in [x_{t+1}^L + \kappa_{t+1}, x_{t+2}^L + \kappa_{t+1} + \kappa_{t+2})$. Assumption 2 ensures that this domain of x is not empty and covers the region of x strictly larger than x_t^L. By repeating the recursive substitution, the range of x for which we obtain an explicit form of $\tilde{v}_t(x)$ moves to the right in the x axis, eventually covering $\forall x \geqq x_{t+1}^L + \kappa_{t+1}$ until x reaches $x_{t+1}^H + \kappa_{t+1}$. As a result of this recursive calculation, for $\forall x \in [x_{t+1}^L + \kappa_{t+1}, x_{t+1}^H + \kappa_{t+1}]$, by implicitly defining a positive integer n by $x \in [x_t^{(n)}, x_t^{(n+1)})$ where $x_t^{(n)} \equiv x_{t+n}^L + \sum_{s=1}^n \kappa_{t+s}$, we obtain the explicit form of the function $\tilde{v}_t(x)$ as

$$\tilde{v}_t(x) = \tilde{v}_t^{(n)}(x) = A_{1,t}^{(n)} e^{(-\varepsilon+1)x} - A_{2,t}^{(n)} e^{-\varepsilon x} + A_{3,t}^{(n)}, \tag{29}$$

where

$$A_{1,t}^{(n)} \equiv A_t + A_{4,t} + \alpha\beta e^{\lambda_{t+1}+(\varepsilon-1)\kappa_{t+1}} A_{1,t+1}^{(n-1)} \tag{30}$$

$$A_{2,t}^{(n)} \equiv A_t + \alpha\beta e^{\lambda_{t+1}+\varepsilon\kappa_{t+1}} A_{2,t+1}^{(n-1)} \tag{31}$$

$$A_{3,t}^{(n)} \equiv (1-\alpha)\tau\varepsilon^{\phi}\beta e^{\lambda_{t+1}} \left(\tilde{v}_{t+1}(x_{t+1}^*) - b\right) + \alpha\beta e^{\lambda_{t+1}} A_{3,t+1}^{(n-1)}. \tag{32}$$

Because this formula covers the entire range of $x \in [x^L_{t+1} + \kappa_{t+1}, x^H_{t+1} + \kappa_{t+1}]$, we can write $\tilde{v}_t(x^*_t)$ and $\tilde{v}_t(x^H_t)$ in the above form if we choose the right values of n. The piecewise nature of the formula, however, complicates the derivation of x^*_t and x^H_t. In fact, as mentioned at the bottom of Appendix A.5, even the uniqueness of x^*_t is not assured. This motivates us to introduce another assumption, which significantly reduces the complexity of the problem.

Assumption 3 (Slow drift) *Let n^*_t be an integer variable such that the markup of firms adjusting their prices and new entrants at period t shifts out of the inaction region at time $t + n^*_t$ without being hit by an idiosyncratic shock. Then $\alpha^{n^*_t} \ll 1$.*

This assumption allows us to approximate the function $\tilde{v}_t(x)$ for $\forall x \in [x^{(n^*_t - 1)}_t, x^H_{t+1} + \kappa_{t+1}]$ as

$$\tilde{v}_t(x) \approx \tilde{v}^{(\infty)}_t(x) = A_{1,t} e^{(-\varepsilon+1)x} - A_{2,t} e^{-\varepsilon x} + A_{3,t}, \tag{33}$$

where

$$A_{1,t} \equiv A_t + A_{4,t} + \alpha\beta E_t e^{\lambda_{t+1} + (\varepsilon - 1)\kappa_{t+1}} A_{1,t+1} \tag{34}$$

$$A_{2,t} \equiv A_t + \alpha\beta E_t e^{\lambda_{t+1} + \varepsilon\kappa_{t+1}} A_{2,t+1} \tag{35}$$

$$A_{3,t} \equiv (1 - \alpha)\tau\varepsilon^{\phi}\beta E_t e^{\lambda_{t+1}} \left(\tilde{v}_{t+1}(x^*_{t+1}) - b\right) + \alpha\beta E_t e^{\lambda_{t+1}} A_{3,t+1}. \tag{36}$$

Intuitively, if firms expect that the drift $(-\kappa_{t+s})$ in firms' aggregate markup is going to be slow for all $s > 0$, the size of each domain $[x^{(n)}_t, x^{(n+1)}_t)$ for Equation (29) is expected to be small. Firms adjusting their prices and new entrants at period t then expect that it will take a long time n^*_t to exit the inaction region, unless they are hit by an idiosyncratic shock. Because the probability of not being hit by an idiosyncratic shock is α, the approximation of replacing n^*_t by ∞ has an error of the order of $\alpha^{n^*_t}$. We ignore this error by assuming large enough n^*_t. Appendix G.1 explicitly confirms this argument.

Using the approximation based on the assumption, because $x^*_t \in [x^{(n^*_t - 1)}_t, x^{(n^*_t)}_t)$, the function $\tilde{v}_t(x)$ takes a maximum at

$$x_t^* = \log\left(\frac{\varepsilon}{\varepsilon - 1}\right) + \log\left(\frac{A_{2,t}}{A_{1,t}}\right). \tag{37}$$

We implicitly obtain the upper threshold of the inaction region x_t^H by

$$\tilde{v}_t^{(\infty)}(x_t^H) = \tilde{v}_t^{(\infty)}(x_t^*) - b \tag{38}$$

$$\text{and} \quad \left.\frac{d\tilde{v}_t^{(\infty)}}{dx}\right|_{x=x_t^H} < 0. \tag{39}$$

Using Equation (25), we also obtain the lower threshold x_t^L by

$$\tilde{v}_t^{(0)}(x_t^L) = \tilde{v}_t^{(\infty)}(x_t^*) - b \tag{40}$$

$$\text{and} \quad \left.\frac{d\tilde{v}_t^{(0)}}{dx}\right|_{x=x_t^L} > 0. \tag{41}$$

Together with Equation (20) that defines $A_{4,t}$ and Equation (80) in Appendix A.6 that characterizes $A_{5,t}$, Equations (25) and (33) to (41) completely determine the triplet (x_t^*, x_t^L, x_t^H) of firms' policy rule.

It is now appropriate to reconsider the auxiliary assumption introduced above. Namely, in order to derive Equation (29), we assume that firms perceive no aggregate uncertainty. Without this auxiliary assumption, we cannot write the explicit form of the function $\tilde{v}_t(x)$ for the whole range of $x \in [x_{t+1}^L + \kappa_{t+1}, x_{t+1}^H + \kappa_{t+1}]$ as shown in Figure 2. However, under Assumption 3, the only explicit forms of the function necessary to obtain the triplet (x_t^*, x_t^L, x_t^H) are $\tilde{v}_t^{(0)}(x)$ and $\tilde{v}_t^{(\infty)}(x)$ in Equations (25) and (33), both of which are independent of n. In addition, Appendix A.6 shows that the derivation of the law of motion for $A_{5,t}$ does not need this auxiliary assumption. We therefore do not need the auxiliary assumption to obtain the triplet.[11]

We conclude this section by briefly comparing the triplet (x_t^*, x_t^L, x_t^H) with that obtained by Gertler and Leahy (2008)(GL) for the case of zero trend inflation. Equations

[11] Appendix A relies on the auxiliary assumption to discuss various aspects of the value function in more detail, .

(37) and (38) are similar to what they obtain.[12] However, Equation (40) is qualitatively different: the form of our value function around the lower threshold of the inaction region, as depicted in Figure 2, is distinct from the value function around x_t^* and x_t^H, whereas no such distinction is present in the analysis by GL. The distinction is due to the drift in firms' aggregate markup, which is expected to be always negative relative to the triplet (x_t^*, x_t^L, x_t^H) by Assumption 2. Intuitively, this drift makes the behavior of firms with current markup just above x_t^L different from other firms. The former firms rationally expect that, unless being hit by an idiosyncratic shock, they are destined to change their prices soon due to the drift, so they become effectively myopic. On the contrary, other firms need to foresee far future. We term this distinction as "asymmetry of the policy rule". We later discuss that the asymmetry is a crucial element for one of the two mechanisms that significantly affect inflation dynamics as well as the frequency of price changes.[13]

3.2 Markup distribution and the price index

Given the triplet (x_t^*, x_t^L, x_t^H) obtained in the previous section, we derive the law of motion for the price index in this Section.

In general, state-dependent models are starkly different from time-dependent models in the derivation of the price index. For example, the derivation is trivial for the time-dependent Calvo model. The assumption of random selection of firms changing their prices implies that the average price of those not changing their prices corresponds to the price index of the previous period. This in turn implies that the price index at the current period is simply a weighted average of the reset price at the current period and the price index at the previous period, with the weight equal to the frequency of price changes. On the contrary, we cannot resort to a similar simplification in state-dependent models. Instead,

[12] The only minor difference between this paper and GL is their use of second-order perturbation for the value function around x^* and first-order perturbation for the policy functions. In other words, our expressions for x_t^* and x_t^H are identical to those by GL if we use the same perturbation in the limit of zero trend inflation. Section 3.3 and Appendix D.1 explicitly show that a first-order approximation similar to that used by GL yields $A_{4,t} = A_{5,t} = 0$ both in the steady state and in the log-deviation from that.

[13] A related issue of some interest is whether the value of x_t^L in our model converges to that of GL in the limit of zero trend inflation. As shown in Appendix A.7, at least in a steady state, $\tilde{v}^{(0)}(\bar{x^L}) = \tilde{v}^{(\infty)}(\bar{x^L})$ in the limit of zero trend inflation, implying that there is no discontinuity in the value of $\bar{x^L}$ between GL and our model.

the very nature of state dependence and the presence of idiosyncratic shocks necessitate us to keep track of the whole distribution of firms' markup and their idiosyncratic productivity level, making analytical characterization difficult.

The menu cost model in this paper is exceptional in that, despite being fully state-dependent, it allows us to analytically characterize firms' markup distribution. By doing so, we explicitly derive the law of motion for the price index. This section sketches the outline of the derivation, while Appendix B shows the details of intermediate calculations.

3.2.1 Definition of density functions

Before diving into the calculation of the price index, we define the density of firms as a function of markup $x_{i,t}$ and idiosyncratic productivity level $z_{i,t}$ at two different timings within a period, as shown in Figure 1. Namely, $\psi_t^0(x,z)$ represents the density of firms right before they optimize their prices, while $\psi_t^1(x,z)$ represents the density of firms after optimization, thus participating in the market.

The two densities are related to each other through two master equations. On the one hand, the transition from ψ_t^0 to ψ_t^1 follows

$$\begin{aligned}\psi_t^1(x,z) &= (1-\alpha)(1-\tau)\delta\left(x-x_t^*\right)\delta(z) \\ &\quad + \delta\left(x-x_t^*\right)\int_{S_t^c} dx' \psi_t^0\left(x',z\right) + \psi_t^0(x,z) I\left(x\in S_t\right),\end{aligned} \tag{42}$$

where $\delta(\cdot)$ represents Dirac's delta function and $S_t^c \equiv (-\infty, x_t^L)\cup(x_t^H,\infty)$ denotes the region outside the inaction region.[14] In this equation, the first term represents firms newly entering the market. The second term represents the firms that adjust their prices because they are outside the inaction region just before optimizing the price. The third term represents the firms that do not adjust their prices.

[14] Dirac's delta function satisfies the followings: $\delta(x)=0$ for $\forall x\neq 0$; and $\int_{-\infty}^{\infty} dx\ \delta(x)=1$. This paper extensively uses the following basic property of the delta function: for any function f continuous around x_0, $\int_{-\infty}^{\infty} dx\ f(x)\delta(x-x_0)=f(x_0)$.

On the other hand, the master equation governing the transition from ψ_t^1 to ψ_{t+1}^0 is

$$\psi_{t+1}^0(x,z) = \alpha\psi_t^1\left(x+\kappa_{t+1},z\right) + (1-\alpha)\tau\int_{-\infty}^{\infty} d\xi\ p(\xi)\psi_t^1\left(x-\xi+\kappa_{t+1},z-\xi\right), \tag{43}$$

where $p(\xi) = I\left(|\xi| \leqq \phi/2\right)/\phi$ represents the probability density of an idiosyncratic shock. The first term represents firms that are not hit by an idiosyncratic shock and therefore experiences only the drift in markup ($-\kappa_{t+1}$), and the second term represents those being hit by the idiosyncratic shock. The number of firms represented by the two density functions are different: while the number of firms in $\psi_t^1(x,z)$, corresponding to $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dxdz\ \psi_t^1(x,z)$, is 1, the number of firms in $\psi_t^0(x,z)$ is $\{\alpha+(1-\alpha)\tau\} < 1$, because the latter does not take into account exiting firms.

We integrate out idiosyncratic productivity z and focus on the distribution of markup x. Specifically, we define

$$\Psi_t(x) \equiv \int_{-\infty}^{\infty} dz\ e^{(\varepsilon-1)(z-x)}\psi_t^0(x,z). \tag{44}$$

The exponential factor $e^{(\varepsilon-1)(z-x)}$ is to facilitate the calculation of the price index, which also has this factor as shown in Equation (81) in Appendix B. Strictly speaking, this variable does not represent a density function of firms' markup because of this additional factor $e^{(\varepsilon-1)(z-x)}$; we nevertheless call it "markup density" for convenience unless that causes a confusion.[15] By combining the master equations (42) and (43) and integrating out z using Equation (44), we obtain the master equations for $\Psi_t(x)$, shown in Equations (86)-(94) in Appendix B.

3.2.2 Explicit calculations of the density $\Psi_t(x)$ around the inaction region

To calculate the law of motion for price index, we need to explicitly characterize some part of $\Psi_{t+1}(x)$ using the master equations (86) to (94). First, $\Psi_{t+1}(x)$ is flat for $x \in$

[15] To evaluate the frequency and size of price changes in Appendix E, we define another function $\Psi_t^0(x)$,which exactly represents the density of firms as a function of markup x. The function $\Psi_t(x)$ should not be confused with $\Psi_t^0(x)$.

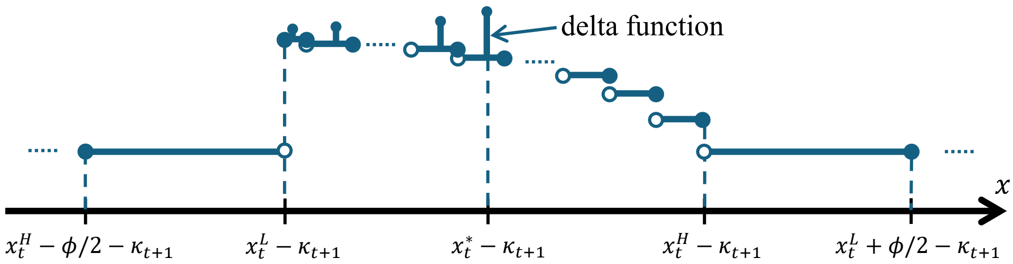

Figure 3: Schematic picture of $\Psi_{t+1}(x)$ around the inaction region.

$[x_t^H - \phi/2 - \kappa_{t+1}, x_t^L - \kappa_{t+1}) \cup (x_t^H - \kappa_{t+1}, x_t^L + \phi/2 - \kappa_{t+1}]$, as depicted in Figure 3:

$$\Psi_{t+1}(x) = \eta \Gamma^1_{t+1}, \tag{45}$$

where $\eta \equiv \frac{(1-\alpha)\tau}{\phi(\alpha+(1-\alpha)\tau)}$ is a constant and Γ^1_t is defined by

$$\Gamma^1_t \equiv \int_{-\infty}^{\infty} dx \ \Psi_t(x). \tag{46}$$

The right-hand side of Equation (45) represents the density of firms being hit by an idiosyncratic shock. Assumptions 1 and 2 ensure that neither of the two flat regions are empty: for example, for the left flat region in Figure 3, $x_t^L - \kappa_{t+1} \geqq x_{t+1}^H - \phi/2 > x_t^H - \phi/2 - \kappa_{t+1}$, where the first inequality follows from Assumption 1 and the second follows from Assumption 2.

Next, we derive an explicit expression of $\Psi_{t+1}(x)$ for x slightly below $x_t^H - \kappa_{t+1}$. The master equations (86)-(94) suggest that for $x \in [x_t^L - \kappa_{t+1}, x_t^H - \kappa_{t+1}]$ excluding the point $x = x_t^* - \kappa_{t+1}$,

$$\Psi_{t+1}(x) = \eta \Gamma^1_{t+1} + \alpha e^{(\varepsilon-1)\kappa_{t+1}} \Psi_t(x + \kappa_{t+1}). \tag{47}$$

The second term on the right-hand side is the contribution of firms which are not hit by idiosyncratic shock at the beginning of time $t+1$ and therefore experience only a drift. This equation suggests that we obtain closed-form expressions for $\Psi_{t+1}(x)$ by gradually lowering x from $x_t^H - \kappa_{t+1}$: for $x \in (x_{t-1}^H - \kappa_{t+1} - \kappa_t, x_t^H - \kappa_{t+1}]$, $\Psi_{t+1}(x) = \eta\left(\Gamma^1_{t+1} + \alpha e^{(\varepsilon-1)\kappa_{t+1}}\Gamma^1_t\right)$; for $x \in (x_{t-2}^H - \kappa_{t+1} - \kappa_t - \kappa_{t-1}, x_{t-1}^H - \kappa_{t+1} - \kappa_t]$, $\Psi_{t+1}(x) =$

$\eta\left(\Gamma^1_{t+1}+\alpha e^{(\varepsilon-1)\kappa_{t+1}}\Gamma^1_t+\alpha^2 e^{(\varepsilon-1)(\kappa_{t+1}+\kappa_t)}\Gamma^1_{t-1}\right)$; and so on. As a result, $\Psi_{t+1}(x)$ increases by a series of step functions, as depicted in Figure 3. Assumption 2 ensures that the regions thus sequentially defined are always of finite length and therefore, after a large enough number of steps, surely reach $x^L_t-\kappa_{t+1}$. The step-wise increase in $\Psi_{t+1}(x)$ eventually saturates by the rate $\alpha^{n^H(x)}$, where $n^H(x)$ represents the number to steps before reaching x from $x^H_t-\kappa_{t+1}$. Assumption 3 then justifies the replacement of the finite summation of $\Psi_{t+1}(x)$ by infinite summation for x close enough to the lower inaction threshold.

Meanwhile, for $x\in[x^L_t-\kappa_{t+1},x^*_t-\kappa_{t+1}]$, another set of terms in a form of delta function contributes to $\Psi_{t+1}(x)$. These terms represent firms changing their prices and new entrants at a past period that have experienced the drift since then without being hit by an idiosyncratic shock. The contribution of these terms, however, also diminishes by the rate $\alpha^{n^*(x)}$, where $n^*(x)$ represents the number of steps before reaching x by lowering x from $x^*_t-\kappa_{t+1}$. Assumption 3 again implies that they become negligible for x around the lower threshold.[16]

These considerations suggest that for $x\in(x^L_t-\kappa_{t+1},x^L_{t+1}]$, $\Psi_{t+1}(x)$ is well approximated by a flat density as

$$\Psi_{t+1}(x)\approx\eta\left(\Gamma^1_{t+1}+\Gamma^5_{t+1}\right), \tag{48}$$

where Γ^5_{t+1} derives from an infinite sum of step functions and is expressed in a recursive form

$$\Gamma^5_{t+1}\equiv\alpha e^{(\varepsilon-1)\kappa_{t+1}}\left(\Gamma^1_t+\Gamma^5_t\right).$$

The first term in the parenthesis on the right-hand side of Equation (48) represents firms being hit by an idiosyncratic shock at the beginning of period $t+1$. On the other hand, the second term represents the contributions of firms that were hit by an idiosyncratic shock

[16] Strictly speaking, delta functions are infinite at a point by definition, so the statement that they become negligible may appear confusing. However, we always evaluate the contribution of the delta functions by taking integrals over some finite region of x, for example, through the calculation of Γ^3_{t+1} and Γ^4_{t+1}. After taking integrals, delta functions become finite, and it makes sense to discuss the magnitude of their contributions.

at a past period $t-\tau$, landed inside the inaction region, and has drifted since then without being hit by another idiosyncratic shock, eventually reaching the region $x \in [x_t^L - \kappa_{t+1}, x_{t+1}^L]$ around the lower threshold.

Importantly, we clearly see the difference between the density around the upper threshold in Equation (45) and the density around the lower threshold in Equation (48). We term the difference as the "asymmetry of the markup distribution". The term Γ_{t+1}^5 reflects the degree of the asymmetry. It derives from the history of negative drift in aggregate markup, following Assumption 2. It is an important constituent of the second mechanism that we highlight in this paper.

3.2.3 Derivation of the law of motion for the price index

We are ready to derive the law of motion for the price index. While the details of the derivation is presented in Appendix (from Appendix B.1 to B.5), we here sketch the outline of the derivation. We first rewrite the price level (2) by the density $\psi_t^1(x,z)$ and expand the expression by using Equation (42). This leads us to two integrals (Γ_t^2 and Γ_t^3 in Appendix B.1), corresponding to the contributions of firms adjusting and not adjusting their prices, respectively. The integrals are evaluated using the master equations (86)-(94) for the density $\Psi_t(x)$ as well as its explicit expressions around the Ss region shown in Equations (45), (47) and (48).

As a result of the calculations, we obtain the following equation:

$$e^{(1-\varepsilon)p_t} = (1-\alpha)Ce^{(1-\varepsilon)p_t^*} + \left(\alpha + f_{1,t} - f_{2,t}\widetilde{\Gamma}_{t-1}\right) e^{(1-\varepsilon)p_{t-1}} \tag{49}$$

$$\widetilde{\Gamma}_t \equiv \frac{\Gamma_{t+1}^5}{\Gamma_{t+1}^1} = \alpha e^{(\varepsilon-1)\pi_t}\left(1+\widetilde{\Gamma}_{t-1}\right) \tag{50}$$

$$f_{1,t} \equiv \frac{(1-\alpha)}{\phi}\tau\left[\Delta_t^H + \Delta_t^L - \frac{e^{(\varepsilon-1)\Delta_t^H} - e^{-(\varepsilon-1)\Delta_t^L}}{\varepsilon-1}\right] \tag{51}$$

$$f_{2,t} \equiv \frac{(1-\alpha)}{\phi}\tau\left[(x_t^L - x_{t-1}^L + \kappa_t) - e^{-(\varepsilon-1)\Delta_t^L}\frac{1-e^{-(\varepsilon-1)(x_t^L - x_{t-1}^L + \kappa_t)}}{\varepsilon-1}\right], \tag{52}$$

where $C \equiv \frac{1-\tau}{1-\tau\varepsilon^{\phi}}$ is a constant and $\Delta_t^H \equiv x_t^H - x_t^*$ and $\Delta_t^L \equiv x_t^* - x_t^L$ are the gaps between the reset markup x_t^* and the inaction thresholds, which we call the upper gap and lower gap, respectively. p_t and p_t^* are defined as the logarithm of the prices P_t and P_t^*, respectively, i.e., $p_t^* \equiv \log P_t^*$ and $p_t \equiv \log P_t$, where $P_t^* \equiv W_t e^{x_t^*}/Z_t$ represents the reset price for firms with idiosyncratic productivity level $z_{i,t} = 0$.

Equation (49) is an intuitive representation of the law of motion for the price index, as is clear by the comparison with the corresponding equation for the time-dependent Calvo model. Specifically, if firms adjust their prices only when an exogenous Calvo fairy visit them with a probability $1 - \alpha'$, Equation (2) implies that the price level would evolve as $e^{(1-\varepsilon)p_t} = (1 - \alpha')e^{(1-\varepsilon)p_t^*} + \alpha' e^{(1-\varepsilon)p_{t-1}}$. In this expression, the price level at t is simply determined by a weighted average of the contribution of $(1 - \alpha')$ firms changing their prices to p_t^* at period t and the contribution of α' firms staying at the same price. Because of the exogeneity of the arrival of a Calvo fairy, the average price of the latter firms is exactly equal to the price level at $t - 1$. For our menu cost model, Equation (49) suggests that the price level at t is expressed by a similar weighted average of the two contributions. In fact, if the variables $f_{1,t}$ and $f_{2,t}\widetilde{\Gamma}_{t-1}$ were equal to 0 and C were equal to 1, Equation (49) would be identical to the law of motion in the Calvo model with the probability of Calvo fairy arrival $(1 - \alpha)$. The latter is essentially the law of motion for price level derived by Gertler and Leahy (2008)(GL) for the case of zero trend inflation.

The important departure of our model from GL is therefore expressed by the terms $f_{1,t}$ and $f_{2,t}\widetilde{\Gamma}_{t-1}$[17]. These terms reflect a different manifestation of the state dependence in price setting. Specifically, $f_{1,t}$ represents how the upper and lower gaps affect the price index. We see this more clearly by expanding the exponential terms in Equation (51) up to a second order: $f_{1,t} \approx (1 - \alpha)\tau(\varepsilon - 1)\left(\left(\Delta_t^L\right)^2 - \left(\Delta_t^H\right)^2\right)/(2\phi)$. This expression suggests that if Δ_t^L decreases or Δ_t^H increases, $f_{1,t}$ becomes smaller, which translates into higher price index p_t in Equation (49) because $\varepsilon > 1$. Intuitively, this term represents the contribution of firms being hit by an idiosyncratic shock at the beginning of current period. These firms change their prices if and only if they land outside the inaction region, the width of which is given by $\left(\Delta_t^H + \Delta_t^L\right)$. If the lower gap Δ_t^L shrinks, after being hit by

[17]The value of the constant C is mostly irrelevant for the dynamics of the model, though it affects the steady-state aggregate markup $\bar{x}$ with respect to the reset markup $\bar{x}^*$.

an idiosyncratic shock, more firms would land below the lower threshold and thus increase their prices. If, on the other hand, the upper gap Δ_t^H widens, an idiosyncratic shock would make less firms decrease their prices.

The term $f_{2,t}\widetilde{\Gamma}_{t-1}$ reflects the contribution of the asymmetry of the markup distribution, as shown in Figure 3 and discussed at the bottom of Section 3.2.2.[18] Specifically, $\widetilde{\Gamma}_{t-1}$ represents the degree of the asymmetry, as is clear from the definition $\widetilde{\Gamma}_{t-1} = \Gamma_t^5/\Gamma_t^1$ and Equation (48). We can translate the degree of the asymmetry into its contribution to the number of firms increasing price by multiplying $\widetilde{\Gamma}_{t-1}$ by $(x_t^L - x_{t-1}^L + \kappa_t)$, because firms with markup $x \in [x_{t-1}^L, x_t^L + \kappa_t]$ at the end of period $t-1$ slide out of the lower inaction threshold at t. The contribution of these firms to the price index is related to Δ_t^L, roughly the size of price changes for these firms. In fact, by first-order expansion of Equation (52) in terms of $(x_t^L - x_{t-1}^L + \kappa_t)$, we see that

$$f_{2,t} \propto (x_t^L - x_{t-1}^L + \kappa_t)(1 - e^{-(\varepsilon-1)\Delta_t^L}), \tag{53}$$

consistent with this interpretation.

While this concludes the analysis of firms' price setting in goods market, we have not discussed another problem that requires non-trivial analysis in general equilibrium: labor demand. Specifically, we need to evaluate the two integrals appearing on the right-hand side of Equation (16). As shown in Appendix B.6 and B.7, evaluating the integrals introduces additional endogenous variables and their laws of motion. Appendix C collects all of 28 equations with 28 endogenous variables to close the model in general equilibrium, apart from exogenous shock variables and their process.[19]

[18] As emphasized in the discussion following Equation (44), especially in footnote 15, strictly speaking, the function $\Psi_t(x)$ does not represent the density of firms as a function of markup x. However, as shown in Appendix E, the shape of genuine density function $\Psi_t^0(x)$ looks qualitatively similar to that of $\Psi_t(x)$ in Figure 3, and the discussion here is valid even with respect to $\Psi_t^0(x)$.

[19] As shown in Equation (128), it is more convenient to rewrite the law of motion for the price index (49) in terms of the markup x_t by using $p_t^* - p_t = x_t^* - x_t$, because the price index p_t and the reset price p_t^* are not stationary in the presence of trend inflation.

3.3 Phillips curve

The set of equations shown in the previous sections allow us to derive an explicit and intuitive equation representing the Phillips curve.

To do this, we adopt standard log-linearizion or linearization of each endogenous variable around the steady-state value, and denote the steady-state value by a bar and the deviation from it by a hat on top of the variable. More specifically, the variables expressed in lower-case letters (such as x_t^* and κ_t) and the two gaps (Δ_t^H and Δ_t^L) are linearized around their steady states —for example, $x_t^* = \bar{x}^* + \hat{x}_t^*$—, while the other variables, expressed in upper-case letters such as A_t and $\tilde{\Gamma}_t$, are log-linearized, i.e., $A_t = \bar{A}e^{\hat{a}_t} \approx \bar{A}\,(1+\hat{a}_t)$.

We also adopt two additional approximations. First, trend inflation rate $\bar{\pi}$ is so small that we ignore any term that represents the interaction of a log-deviation and $\bar{\pi}$: for example, $e^{(\varepsilon-1)\bar{\pi}}\hat{a}_t \approx \hat{a}_t$. Clearly, once this approximation is imposed, we can no longer discuss some of the non-linear effects of trend inflation on actual inflation dynamics; such analysis is to be conducted in the full model without (log-)linearization. Secondly, we assume that the steady-state values of the upper and lower gaps, $\bar{\Delta}^H = \bar{x}^H - \bar{x}^*$ and $\bar{\Delta}^L = \bar{x}^* - \bar{x}^L$, are also small enough to justify a first-order approximation when interacted with a deviation: for example, $e^{\varepsilon\bar{\Delta}^L}\hat{\Delta}_t^L \approx \left(1+\varepsilon\bar{\Delta}^L\right)\hat{\Delta}_t^L$. Note that the hierarchy $\bar{\Delta}^L, \bar{\Delta}^H \gg \bar{\pi}$ presumed in the two approximations is consistent with Assumption 3. The steady-state values of these variables based on the calibration of our baseline model, shown in Section 4.1, are consistent with the hierarchy: $\bar{\Delta}^L = 3.5\times 10^{-2}$, $\bar{\Delta}^H = 2.4\times 10^{-2}$, and $\bar{\pi} = 1.7\times 10^{-3}$ (corresponding to 2 % annual inflation).

While Appendix D shows the details of the derivation, we briefly sketch the outline. First, under the above approximations, $A_{4,t}$ is negligible compared with A_t in Equations (26) and (34). This implies that the upper gap is constant within our approximations:

$$\hat{\Delta}_t^H \approx 0. \tag{54}$$

However, as discussed at the bottom of Section 3.1, the asymmetry of the policy rule implies that firms around the lower threshold behave rather differently. These firms rationally expect that, unless being hit by an idiosyncratic shock, they are destined to change their prices soon due to the negative drift, regardless of the exact size of the shock. This expectation dampens the response of x_t^L to aggregate external shocks. In fact, our approximations yield $\hat{x}_t^L \approx 0$, which in turn implies

$$\hat{\Delta}_t^L \approx \hat{x}_t^*. \tag{55}$$

Using the expressions for the dynamics of the gaps $\hat{\Delta}_t^H$ and $\hat{\Delta}_t^L$, we rewrite Equations (51) and (52) as

$$\hat{f}_{1,t} \approx \frac{(1-\alpha)}{\phi}\tau(\varepsilon-1)\left(-\bar{\Delta}^H\hat{\Delta}_t^H + \bar{\Delta}^L\hat{\Delta}_t^L\right) \approx (1-\alpha)(\varepsilon-1)\zeta_1\hat{x}_t^*, \tag{56}$$

$$\hat{f}_{2,t} \approx (1-\alpha)(\varepsilon-1)\zeta_1\hat{\kappa}_t, \tag{57}$$

where the constant ζ_1 is defined as $\zeta_1 \equiv \tau\bar{\Delta}^L/\phi$. Substituting these expressions into Equation (49) and combining it with Equations (34), (35), and (37), while ignoring $A_{4,t}$, we obtain the Phillips curve equation:

$$\begin{aligned}\hat{\pi}_t \approx \beta E_t\hat{\pi}_{t+1} + \lambda_c\hat{mc}_t + \frac{1-\alpha}{\alpha}\zeta\left(\hat{mc}_t - \alpha\beta E_t\hat{mc}_{t+1}\right) \\ + \zeta\left(\Delta\hat{mc}_t - \alpha\beta E_t\Delta\hat{mc}_{t+1}\right),\end{aligned} \tag{58}$$

where $\hat{mc}_t \equiv -\hat{x}_t$ represents the log-deviation of aggregate real marginal cost from the steady state, and $\Delta\hat{mc}_t \equiv \hat{mc}_t - \hat{mc}_{t-1}$ is a shorthand of the change in real marginal cost from the previous period. The constant $\lambda_c \equiv (1-\alpha)(1-\alpha\beta)/\alpha$ is the same as the coefficient attached to the real marginal cost in a hypothetical Calvo model with the probability of price adjustment equal to $(1-\alpha)$. The constant ζ is defined as $\zeta \equiv \zeta_1/(1-\zeta_1)$.

While the first two terms on the right-hand side is identical to what Gertler and Leahy (2008)(GL) obtain for the case of zero trend inflation, the third and forth terms are new. The third term, proportional to the current and future levels of real marginal cost $(\hat{mc}_t - \alpha\beta E_t\hat{mc}_{t+1})$, primarily derives from $\hat{f}_{1,t}$ and therefore measures how the gaps Δ_t^H and Δ_t^L affect the price index. Because of the asymmetry of the policy rule, the response of $\hat{\Delta}_t^L$ to shocks is approximated by the response of reset markup $\hat{x}_t^*$. The latter by definition can be decomposed into the relative reset price and a negative of the real marginal cost as $\hat{x}_t^* = (\hat{p}_t^* - \hat{p}_t) - \hat{mc}_t$.[20] If we ignore the former term, the negative dependence on the real marginal cost contributes to the narrowing of the lower gap $\hat{\Delta}_t^L$ against an aggregate inflationary shock, effectively enhancing the price flexibility and thus steepening the slope of the Phillips curve. Meanwhile, the forth term, proportional to the current and future changes in the aggregate real marginal cost ($\Delta \hat{mc}_t - \alpha\beta E_t \Delta \hat{mc}_{t+1}$), is related to $\hat{f}_{2,t}$, which arises due to the asymmetry of the markup distribution. Equation (57) shows that $\hat{f}_{2,t}$ is proportional to the growth rate of the nominal marginal cost $\hat{\kappa}_t$, which can be decomposed into the inflation rate and the change in the real marginal cost as $\hat{\kappa}_t = \hat{\pi}_t + \Delta \hat{mc}_t$. If we ignore the former term, the latter term generates an extra contribution to inflation, especially when the real marginal cost abruptly changes from the previous period.

The two terms that we ignore in the discussion in the previous paragraph, i.e., the contribution of relative reset price in $\hat{x}_t^*$ and the contribution of inflation rate in $\hat{\kappa}_t$, offset each other. This statement becomes clear by an explicit calculation of the contributions of $\hat{f}_{1,t}$ and $\hat{f}_{2,t}$ to the price index. Namely, by substituting Equations (56) and (57) into Equation (49) after log-linearization, we obtain

$$
\begin{aligned}
\hat{p}_t - (1-\alpha)\hat{p}_t^* - \alpha\hat{p}_{t-1} &\approx \frac{1}{\varepsilon - 1}\left(\hat{f}_{2,t}\bar{\bar{\Gamma}} - \hat{f}_{1,t}\right) \\
&\approx (1-\alpha)\zeta_1\left[(\hat{p}_t - \hat{p}_{t-1} + \hat{mc}_t - \hat{mc}_{t-1})\frac{\alpha}{1-\alpha} - (\hat{p}_t^* - \hat{p}_t - \hat{mc}_t)\right] \\
&\approx \zeta_1\left[\{\hat{p}_t - (1-\alpha)\hat{p}_t^* - \alpha\hat{p}_{t-1}\} + (1-\alpha)\hat{mc}_t + \alpha\Delta\hat{mc}_t\right]. \qquad (59)
\end{aligned}
$$

The curly bracket on the right-hand side is identical to the left-hand side. It therefore simply scales the contribution of the real marginal costs, and the price index is directly

[20] We here slightly abuse a notation of log-deviation to make the expression more intuitive: because the logarithms of price level p_t and reset price level p_t^* are not stationary due to the trend inflation, strictly speaking, the deviation of these variables with respect to steady-state values does not make sense. However, if we define a new variable representing the logarithm of relative reset price $q_t^* \equiv p_t^* - p_t$, which is stationary, then $\hat{q}_t^*$ is well-defined. We therefore interpret $(\hat{p}_t^* - \hat{p}_t)$ as a shorthand of $\hat{q}_t^*$. A similar comment applies to the expression $\{\hat{p}_t - (1-\alpha)\hat{p}_t^* - \alpha\hat{p}_{t-1}\}$ in Equation (59): This expression should be regarded as $\{\alpha\hat{\pi}_t - (1-\alpha)\hat{q}_t^*\}$.

affected by $\frac{\zeta_1}{1-\zeta_1}\left[(1-\alpha)\hat{mc}_t + \alpha\Delta\hat{mc}_t\right]$, which enters the Phillips curve equation (58).

The negative coefficients attached to the expectation terms inside the parentheses of the new terms in Equation (58) do not imply that future expected positive value (or future expected positive growth) of real marginal cost pushes down current inflation. To see this, we consider the contribution of the third term for the case in which the real marginal cost remains at the steady state at the current period t but is expected to increase at $t+1$: $E_t\hat{mc}_{t+1} > 0$. Equation (58) at $t+1$ suggests that the inflation rate $\hat{\pi}_{t+1}$ is pushed up by the third term by $\frac{1-\alpha}{\alpha}\zeta\hat{mc}_{t+1} > 0$. This contribution also affects the current inflation rate $\hat{\pi}_t$ through the inflation expectation $E_t\hat{\pi}_{t+1}$, resulting in the net effect of marginal cost tomorrow as $\frac{1-\alpha}{\alpha}\zeta\beta(1-\alpha)E_t\hat{mc}_{t+1} > 0$. The additional term is thus always inflationary as long as $E_t\hat{mc}_{t+1} > 0$, and the apparent negative coefficient is to be offset by the contribution through the inflation expectation. Similarly, the forth term is always inflationary as long as $E_t\Delta\hat{mc}_{t+1} > 0$.

We conclude this section by emphasizing that the coefficients attached to the new terms in Equation (58) are not small. In fact, according to the calibration of parameters discussed in Section 4.1, the coefficient on the third term $\frac{1-\alpha}{\alpha}\zeta = 0.027$ is similar to that on the second term $\lambda_c = 0.027$, and the coefficient on the forth term $\zeta = 0.16$ is much larger than that. The size of the coefficients on the additional terms suggest their significant impact on the dynamics of inflation, which we quantitatively investigate in Section 4.

3.4 Frequency of price changes

Finally, we consider the frequency of price changes, the variable of interest in many empirical studies in the literature. We can derive the laws of motion for a few variables to fully characterize the frequency/size of price increases/decreases. While the details of the calculation are presented in Appendix E, the expression for the frequency of price changes, which is the sum of the frequency of price increases and that of price decreases, is

$$fr_t = (1-\alpha)\tau\left[1 - \frac{\Delta_t^H + \Delta_t^L}{\phi}\right] + \alpha\tau\frac{x_t^L - x_{t-1}^L + \kappa_t}{\phi}. \tag{60}$$

Intuitively, the two terms on the right-hand side reflect the same economic mechanisms as the terms $f_{1,t}$ and $f_{2,t}\widetilde{\Gamma}_{t-1}$ in the law of motion of price index (49). The first term on the right-hand side, which is closely related to $f_{1,t}$, represents the contribution of firms being hit by an idiosyncratic shock at period t. These firms change their prices if and only if they are pushed outside the inaction region, the width of which is $\left(\Delta_t^H + \Delta_t^L\right)$. While this term is present even in the case of zero trend inflation analyzed by Gertler and Leahy (2008), it is constant within their perturbation. In contrast, in our model, this term significantly responds to aggregate shocks because the asymmetry of the policy rule induces significant fluctuations in the gap Δ_t^L, as argued in Section 3.3.

The second term, on the other hand, reflects the term $f_{2,t}\widetilde{\Gamma}_{t-1}$ in Equation (49). It represents those firms that were hit by an idiosyncratic shock at a past period and have kept drifting downward since then under Assumption 2 without being hit by another idiosyncratic shock, eventually crossing the lower threshold at period t. The history of the negative drift results in the asymmetry of markup distribution. When an aggregate inflationary shock pushes up real marginal cost at period t, because of the asymmetry, the number of firms raising price expands by more than the decline in the number of firms lowering price, leading to the net increase in the total frequency of price changes.

The dynamics of the frequency of price changes in Equation (60) is thus closely related to the additional terms in the Phillips curve (58). In fact, by applying the same linear approximations introduced in Section 3.3 to Equation (60), we obtain

$$\begin{aligned} \hat{fr}_t &\approx \frac{\tau}{\phi}\left[-(1-\alpha)\hat{\Delta}_t^L + \alpha\hat{\kappa}_t\right] \\ &\approx \frac{\tau}{\phi}\left[\{\hat{p}_t - (1-\alpha)\hat{p}_t^* - \alpha\hat{p}_{t-1}\} + (1-\alpha)\hat{mc}_t + \alpha\Delta\hat{mc}_t\right] \\ &\approx \frac{\tau}{\phi}\frac{1}{1-\zeta_1}\left[(1-\alpha)\hat{mc}_t + \alpha\Delta\hat{mc}_t\right], \end{aligned} \tag{61}$$

where the second approximate equality is by a rearrangement similar to the derivation of Equation (59), and the third approximate equality directly follows from the same equation. Clearly, the frequency of price changes moves in tandem with the additional terms in the Phillips curve (58): both depend on the level and change in the aggregate real marginal cost.

4 Numerical exercise

In the previous sections, we obtain all the explicit equations to close the model in general equilibrium, which are listed in Appendix C. In this section, we use these equations to numerically calculate impulse response of endogenous variables against aggregate shocks. The numerical exercise is as easy as in conventional representative-agent DSGE models, as long as the shock does not cause a violation of any of the assumptions 1-3. Of particular interest is the response of most important macroeconomic variables, such as inflation rate and output, as well as empirically relevant quantities, especially the frequency of price changes.

Section 4.1 calibrates model parameters, explain the solution method and describe the shock process. Section 4.2 shows the results of the simulations for inflation and output. Section 4.3 shows the results for the frequency and size of price changes.

4.1 Calibration, solution method, and the specification of shocks

For parameters not specific to the firms' price setting, we borrow standard values for the United States from the literature. The unit of time period is a month and the discount factor is equal to $\beta = 0.96^{1/12}$. The inverse Frisch elasticity in the baseline model is set at $\varphi = 1$, though some of the exercises below also adopt $\varphi = 0$, which corresponds to the case of inelastic labor supply. We choose the elasticity of substitution between differentiated goods $\varepsilon = 4$ following Nakamura and Steinsson (2008). The coefficient for inflation in the Taylor rule $\phi_\pi = 1.5$. The inflation rate $\bar{\pi}$ in the steady state for the calibration is set to the average monthly inflation rate 0.212% for the 1998-2005 period in the United States, while the rest of the numerical exercise employs $\bar{\pi} = 0.167\%$, corresponding to 2% annual inflation.

For the parameters α, τ, ϕ and b that are important for firms' price setting, we calibrate the values such that the steady-state values of selected endogenous variables in the model match the empirical estimates for the period 1998-2005 in Nakamura and Steinsson (2008) and Zbaracki et al. (2004). Three out of the four calibration conditions are straightforward: the steady-state value of the frequency of price changes $\bar{fr}$ plus exogenous market exit rate $(1-\alpha)(1-\tau)$ is matched to the frequency of price changes excluding sales and including substitution in data, 11.8%; the average absolute size of price changes $(\bar{fr}^+\bar{sz}^+ + \bar{fr}^-\bar{sz}^-)/\bar{fr}$ is equal to 8.5%, where $\bar{fr}^+$ ($\bar{fr}^-$) represents the steady-state value of the frequency of price increases (decreases) and $\bar{sz}^+$ ($\bar{sz}^-$) represents the steady-state value of the absolute size of price increases (decreases); and the size of the menu cost as a share of sales adjusted for the frequency of price changes, $b \times \bar{fr}/\bar{Y}$, is equal to 0.04%.[21] The remaining one condition, which concerns the frequency of price changes $\bar{fr}$ in Equation (60), requires somewhat non-trivial considerations. Specifically, as shown in Bils (2009), when firms discontinue an old model and introduce a new one, some of the price difference between the two models are not justified by a quality difference: it includes a price change as a result of firms' price setting. We therefore cannot regard all of the substitutions in the data of Nakamura and Steinsson (2008) as exogenous exit in our model. Given the difficulty of quantifying this effect in data, we assume that a half of product substitutions is exogenous and the other half reflects price changes. This assumption suggests that we regard a simple average of the frequency of price changes excluding substitutions (9.9% shown in Table 1 of Nakamura and Steinsson (2008)) and that including substitutions (11.8%) as the frequency of price changes in the model $\bar{fr}$. This assumption, albeit rough, is consistent with the share of non-comparable item substitutions among total substitutions (about 0.5) in Appendix Table A1 of Bils and Klenow (2004). Table 1 shows the summary of the calibration.

We solve the model by a standard deterministic simulation, in which the economy is in the steady state for $t \leqq 0$, is hit by an exogenous shock at $t = 1$ which none of the agents expect ex ante, and never experiences a shock again, which the agents correctly anticipate.[22] The method has two major advantages in the context of this paper. First, strictly speaking, many equations obtained in Section 3 are not applicable to the case in which any of the three inequalities in Assumption 2 is violated even for a single period. Stochastic solution

[21] We include only the physical menu costs and exclude other costs associated with price adjustment, such as managerial and customer costs, all of which are measured by Zbaracki et al. (2004). If we included all of these costs, the parameter b would become too large to yield real solutions.

[22] We use Dynare with Matlab for the simulation throughout this paper. For Dynare, see Adjemian et al. (2024).

Table 1: Parameter values for the calibrated baseline model

β	Discount factor (monthly)	$0.96^{1/12}$
φ	Inverse Frisch elasticity of labor supply	1
ε	Elasticity of substitution for goods	4
ϕ_π	Coefficient for inflation in the Taylor rule	1.5
α	Probability that an idiosyncratic shock does not hit a firm	0.86
τ	Probability that a firm stays in the market conditional on an idiosyncratic shock	0.93
ϕ	Support for the uniform distribution of an idiosyncratic shock	0.28
b	Size of a menu cost	0.0038

methods with Gaussian support of aggregate shocks, while most popular in the literature, may by chance fail to comply with this restrictive assumption over repeated simulations to obtain reliable simulated impulse response. In a deterministic simulation, on the other hand, we only need to run a single simulation, which significantly reduces the chance that the assumption is violated, and we easily confirm whether the assumption is strictly satisfied by checking the inequalities over the entire simulation period. The second advantage of the deterministic simulation is that it can incorporate potential deterministic non-linearity of the model. That said, in practice, our experience suggests (not shown in the paper) that when the shocks are not too large, the impulse response obtained by the deterministic simulation is almost identical to that obtained by first-, second-, or third-order stochastic simulation, as far as it adopts pruning for second- and third-order perturbations. As we gradually increase the size of the shocks, we often see that stochastic simulations fail to obtain the solution before the deterministic one fails.

As for the aggregate shocks, we consider a negative productivity shock and an accommodative monetary policy shock. The negative productivity shock ε_t^z affects firms' productivity by a standard AR(1) process: $\log Z_t = \rho^z \log Z_{t-1} - \varepsilon_t^z$, where we choose the coefficient $\rho^z = 0.8$, which roughly corresponds to quarterly persistence 0.5. Similarly, the accommodative monetary policy shock ε_t^m affects the discretionary part of the monetary policy rule, shown in Equation (14), as $v_t^m = \rho^m v_{t-1}^m - \varepsilon_t^m$, where $\rho^m = 0.8$. We choose the sign of these shocks to be inflationary, which many countries experienced in the post-

COVID period. The choice is also motivated by the concern that if we use opposite shocks, Assumption 2 may fail to hold unless the shock is small. We separately discuss the case of a dis-inflationary shock in Section 5.2.

4.2 Inflation and output

We numerically calculate the impulse response of key macroeconomic variables to aggregate shocks in our menu cost model to better understand its characteristics. For this purpose, we consider four models. The first model, which we call the "main" model, is our full menu cost model, with all the equations listed in Appendix C and all the parameters calibrated as in Table 4.1. The second model is a "PC" model, in which Equation (128) in the "main" model, representing the law of motion for price index, is replaced by the Phillips curve (PC) equation (58). The third model, which we call the "PC-ILS" model, is identical to the "PC" model except that the inverse Frisch elasticity φ is set to 0 instead of 1, implying inelastic labor supply (ILS). Finally, the forth model, which we call the "quasi-Calvo" model, is an alternative model which modifies the "PC" model in the following way: (A) the third and forth terms in Equation (58) are ignored; and (B) inelastic labor supply is again assumed as $\varphi = 0$. The name derives from the observation that the Phillips curve equation is identical to that for the Calvo model except for the value of the coefficient λ_c. This model corresponds to what Gertler and Leahy (2008) (GL) derive for the case of zero trend inflation.[23]

The reason why the "quasi-Calvo" model also assumes inelastic labor supply in (B) is because of the non-trivial expression of labor demand on the right-hand side of Equation (16), which we rewrite in a more explicit form in Equations (105) and (108) in Appendix. Since GL did not use similar equations, we do not take into account the labor demand in the "quasi-Calvo" model. The modification (B) serves this purpose by making the labor demand equation (16) irrelevant for the dynamics of the rest of the variable. This in turn

[23]Strictly speaking, GL additionally assumes a structure of islands in which households can supply labor only locally. This introduces real rigidity, which significantly affects the slope of the Phillips curve, as discussed in Woodford (2003). While such effect may be quantitatively important for monetary non-neutrality, we do not introduce such a setting because it is not directly relevant for our discussion.

motivates us to employ the "PC-ILS" model because of the better comparability with the "quasi-Calvo" model than the other models: the "PC-ILS" and "quasi-Calvo" models are different only in the third and forth terms in the Phillips curve (58) .

Figure 4 (a) and (b) show the impulse response of inflation and output, respectively, to a 1% negative productivity shock. While all the models show, as are standard, an increase in inflation and a decrease in output, several observations are noteworthy. First, the response of the "PC" model, despite being derived using non-trivial approximations described at the top of Section 3.3, closely matches with the response of the "main" model. This assures that the Phillips curve equation (58) succinctly summarizes the inflation dynamics of our full menu cost model. Secondly, the comparison between the "PC" model and the "PC-ILS" model suggests that labor supply elasticity amplifies the response to the shock, as is also well established in the literature: for example, see Woodford (2003).

The third observation is that, unlike the "quasi-Calvo" model, the first three models —"main", "PC", and "PC-ILS" models —all exhibit a sharp spike in inflation and a sharp negative spike in output around the onset of the shock. By construction, the third and forth terms on the right-hand side of the Phillips curve equation (58) are the reason for the difference between the "PC-ILS" model and the "quasi-Calvo" model. A natural question is how each of the two new terms impacts the response of inflation. To answer this question, we eliminate the inflation expectation term from the Phillips curve (58) by substituting the inflation rate forward:

$$\hat{\pi}_t \approx \lambda_c \sum_{\tau=0}^{\infty} \beta^{\tau} E_t \hat{m}c_{t+\tau} + \frac{1-\alpha}{\alpha}\zeta \sum_{\tau=0}^{\infty} \beta^{\tau} E_t \left(\hat{m}c_{t+\tau} - \alpha\beta \hat{m}c_{t+\tau+1}\right)$$
$$+ \zeta \sum_{\tau=0}^{\infty} \beta^{\tau} E_t \left(\Delta \hat{m}c_{t+\tau} - \alpha\beta \Delta \hat{m}c_{t+1+\tau}\right).$$

We call the first term on the right-hand side "Calvo term", the second "f_1 term" and the third "f_2 term". Figure 4 (c) shows this decomposition of inflation rate for the "PC-ILS" model in addition to the inflation response for the "quasi-Calvo" model. We see that the contribution of the "Calvo" term is similar to the full response of inflation for the "quasi-Calvo" model. This is consistent with the observation that, as shown in the

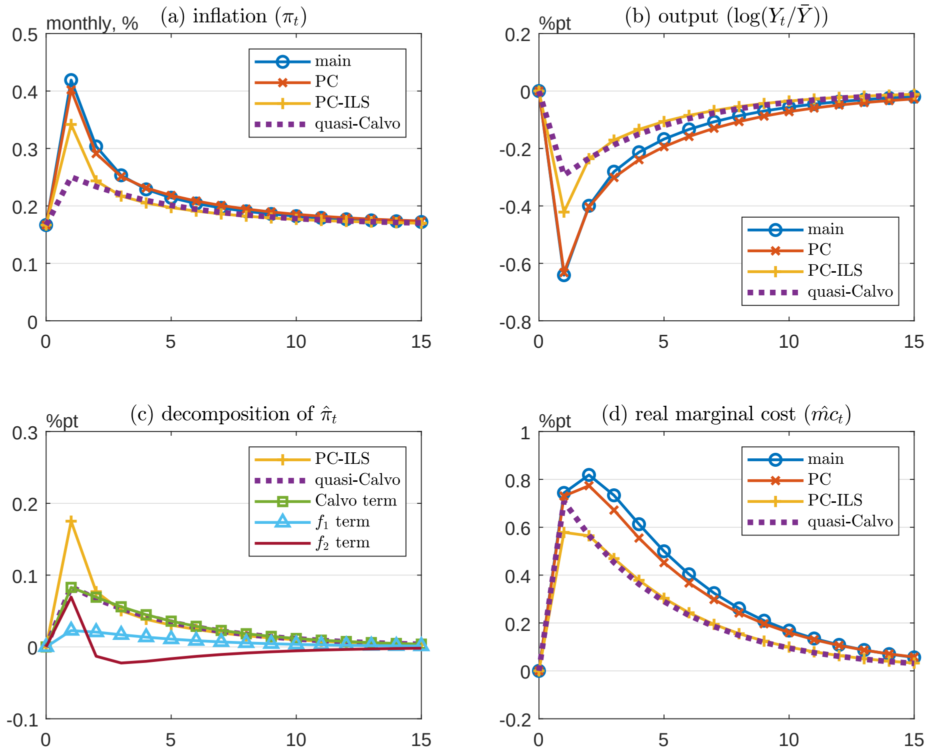

Figure 4: Impulse response of selected endogenous variables to a negative productivity shock of 1%, with AR(1) persistence 0.8. The panel (a) shows the response of monthly inflation rate. The panel (b) shows the response of output expressed as a log-deviation from the steady-state value. The panel (c) shows the contribution of each term in the Phillips curve, defined in the text, to the response of inflation for the "PC-ILS" model, in addition to the response of inflation for the "quasi-Calvo" model. The panel (d) shows the response of (log-) real marginal cost expressed as a deviation from the steady-state values.

panel (d), the dynamics of real marginal cost for the "PC-ILS" model is similar to that for the "quasi-Calvo" model. Meanwhile, the response of inflation for the "PC-ILS" model is significantly affected by the other two terms as well. For the case of AR(1) shock adopted in this exercise, both of the two terms push up the response of inflation at $t = 1$, thereby generating the sharp spike, with the contribution of the "f_2 term" somewhat stronger. For $t \geqq 2$, the net effect of the two terms is much smaller as the two contributions offset each other: while the "f_1 term" keeps pushing up inflation, the contribution of the "f_2 term" turns negative because the change in real marginal cost becomes negative as shown in panel (d).

Figure 5 shows a similar exercise for the case of an accommodative monetary policy shock of 25 basis points. The implications of the panels (a), (c) and (d) are similar to the case of negative productivity shock discussed above: the "PC" model is still a good approximation to the "main" model; labor supply elasticity amplifies the response of inflation to the shock; our menu cost model, unlike the "quasi-Calvo" model, shows a sharp spike in inflation at $t = 1$, which derives from both "f_1 term" and "f_2 term", with the latter effect somewhat stronger; and for $t \geqq 2$, the two contributions offset each other, making the net effect smaller. The implication for the panel (b), which shows the response of output, is rather different from the case of negative productivity shock. Specifically, against an accommodative monetary policy shock, the output response becomes smaller for the model in which the shock generates a larger impact on inflation. This is due to the selection effect, well-known in the literature such as Golosov and Lucas (2007): against a monetary shock, firms are not randomly selected to change price, but they do so because the difference between their price and the reset price is large. This effect lowers the degree of monetary non-neutrality while making the price more flexible, as seen from the comparison between the "quasi-Calvo" model and the "PC-ILS" model.

4.3 Frequency and size of price changes

Figure 6 shows the response of frequency and size of price changes against the same negative productivity shock as in Figure 4. The panel (a) shows that the total frequency ("fr_t")

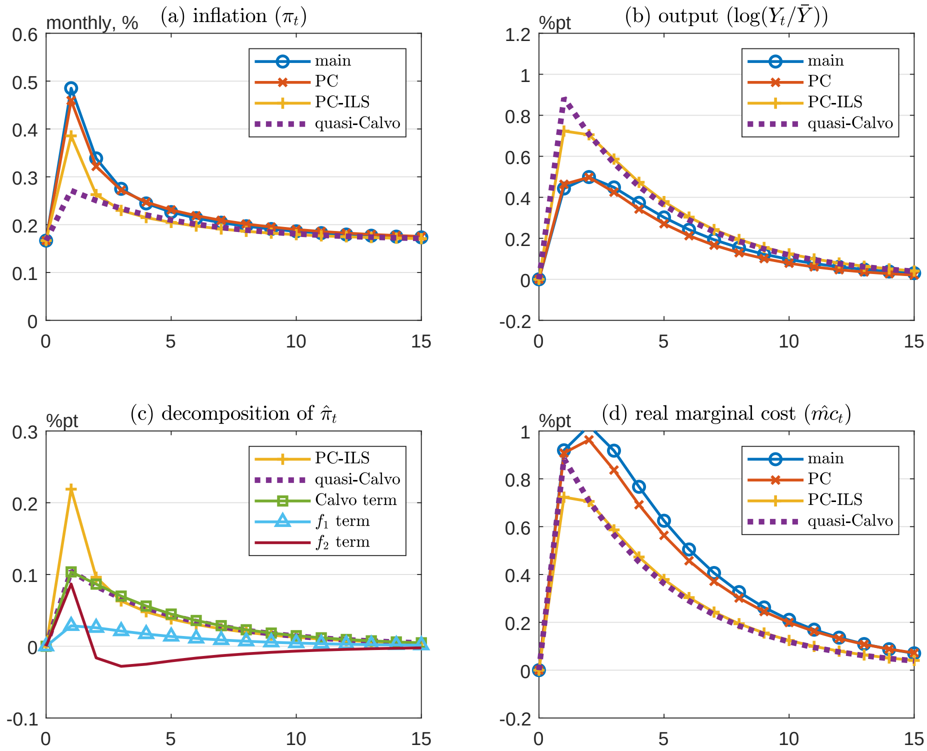

Figure 5: Impulse response of selected endogenous variables to an accommodative monetary policy shock of 25 basis points, with AR(1) persistence 0.8. The panel (a) shows the response of monthly inflation rate. The panel (b) shows the response of output expressed as a log-deviation from the steady-state value. The panel (c) shows the contribution of each term in the Phillips curve, defined in the text, to the response of inflation for the "PC-ILS" model, in addition to the response of inflation for the "quasi-Calvo" model. The panel (d) shows the response of (log-) real marginal cost expressed as a deviation from the steady-state values.

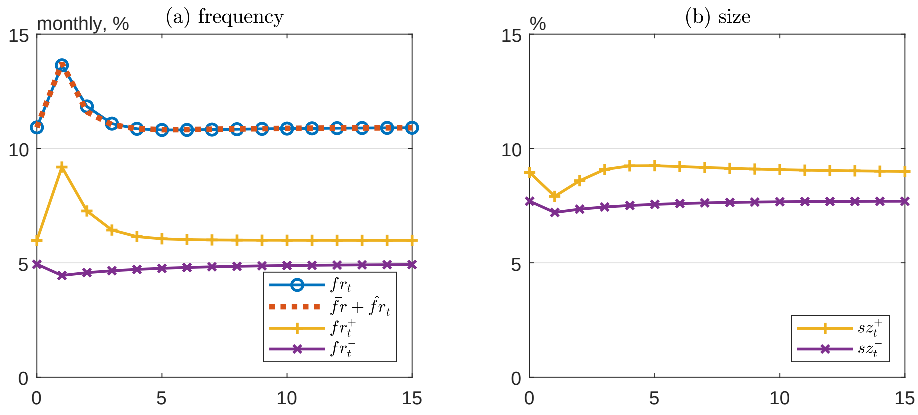

Figure 6: Impulse response of the frequency and size of price changes to a negative productivity shock of 1%, with AR(1) persistence 0.8. The panel (a) shows the response of total frequency (fr_t), frequency of price increases (fr_t^+) and that of price decreases (fr_t^-) as well as the linear approximation of the total frequency ($\bar{fr} + \hat{fr}_t$) according to Equation (61). The panel (b) shows the response of absolute size of price increases (sz_t^+) and that of price decreases (sz_t^-).

significantly responds to the shock. Such a response is absent not only in the Calvo model by assumption, but also in the GL model. Meanwhile, the response of the total frequency is well approximated by the dotted line ("$\bar{fr} + \hat{fr}_t$"), which represents the linear combination of the level and change in the real marginal cost on the right-hand side of Equation (61). This clearly confirms that the underlying mechanisms for the responsiveness of the frequency are the same as the extra terms in the Phillips curve (58), as discussed in Section 3.4.

Underlying the developments of the total frequency are the movements of the frequency of price increases ("fr_t^+") and that of price decreases ("fr_t^-"). While the former responds strongly to the shock, the response of the latter is comparatively muted. This contrast is qualitatively consistent with many empirical observations, including Nakamura and Steinsson (2008), Montag and Villar Vallenas (2025) and Gautier et al. (2025).

The absolute size of price increases and decreases, as shown in the panel (b) of Figure 6, declines after shocks, albeit by much less than the increase in the frequency of price changes. The relatively small responses of the absolute size of price changes are also consistent with the above-mentioned empirical studies. In addition, some studies, such as Karadi and Reiff (2019), find small declines in the absolute size against a relatively large inflationary shock.

We omit the impulse response of the frequency and size of price changes against an accommodative monetary policy shock, because the implications are similar to the case of negative productivity shock in Figure 5.

5 Discussion

5.1 Positive trend inflation and our two mechanisms

As discussed for the Phillips curve in Section 3.3 and for the frequency of price changes in Section 3.4, two mechanisms are behind the qualitative difference of our analysis from that by GL under zero trend inflation. One of the two is associated with the term $f_{1,t}$ and the other is associated with $f_{2,t}\widetilde{\Gamma}_{t-1}$ in Equation (49). Both mechanisms are essentially due to the interaction of state-dependent price setting with Assumption 2. Namely, the first mechanism derives from the future expectation on negative drift in aggregate markup, which generates the asymmetry of the policy rule. The second mechanism, on the other hand, derives from the history of the negative drift, which generates the asymmetry of the markup distribution.

These two mechanisms imply that positive trend inflation is not sufficient for our results. Rather, Assumption 2, which is stronger than assuming positive trend inflation, is necessary. Namely, even in the presence of positive trend inflation, if the aggregate uncertainty often invalidate the assumption and makes the drift in the aggregate markup turn positive, neither the firms' policy rule nor the markup distribution would exhibit similar asymmetry.

A natural question that follows from this argument would be what happens if Assumption 2 is violated. This is the topic of the next section.

5.2 Temporary positive drift

One of the most serious limitations in this paper's analysis is Assumption 2, which implies permanently negative drift in aggregate markup relative to the triplet (x_t^*, x_t^L, x_t^H). This may be a small concern when the aggregate shocks are inflationary as in the exercises in Section 4. In addition, because $\kappa_t = \bar{\pi} + \hat{\pi}_t + \Delta \hat{mc}_t$, we can analyze the cases of dis-inflationary shocks in the same way as long as the shocks are so small that the inequality $|\hat{\pi}_t + \Delta \hat{mc}_t| < \bar{\pi}$ always holds. However, when trend inflation is low enough or dis-inflationary shocks are large enough to invalidate the inequalities in Assumption 2, both the derivation of the triplet (x_t^*, x_t^L, x_t^H) in Section 3.1 and the derivation of the law of motion of price index in Section 3.2 would generally fail.

While it is not possible to analytically characterize inflation dynamics in general without this assumption, there is a case that allows us to perform some analysis: an unexpected once-and-for-all violation of all the inequalities in Assumption 2. Specifically, we consider the following setup. The economy is in the steady state for $t \leqq 0$ and is hit by an unexpected temporary deflationary shock at $t = 1$ such that all the inequalities in Assumption 2 are violated at $t = 1$. For $t \geqq 2$, all the inequalities in Assumption 2 resumes, which firms correctly anticipate as of $t = 1$. This case, though arguably special, serves us to develop intuitions on what happens to the price index and inflation under a temporary deflationary shock.

An advantage of this setting is that the temporary failure of Assumption 2 does not invalidate the derivation of the triplet (x_t^*, x_t^L, x_t^H). Because firms do not expect the shock as of $t = 0$, the triplet at $t = 0$ is the same as their steady-state values. At $t = 1$, because Assumption 2 is only used to deal with the expectation terms in Equation (18), all the arguments in Section 3.1 to characterize the triplet are unaffected. The only part of our analysis that needs revision is the derivation of price index in Section 3.2.

While Appendix F explains the details, we show primary findings. At period $t = 1$, when firms' drift relative to the triplet (x_t^*, x_t^L, x_t^H) becomes positive, the law of motion for the price index in Equation (49) is replaced by

$$e^{(1-\varepsilon)p_1} = (1-\alpha)Ce^{(1-\varepsilon)p_1^*} + \left(\alpha + f_{1,t=1} - f_{2,t=1}^{\dagger}\widetilde{\Gamma}_{t=0}^{\dagger}\right)e^{(1-\varepsilon)p_0}, \tag{62}$$

where

$$\widetilde{\Gamma}_t^{\dagger} \equiv \alpha e^{(\varepsilon-1)\pi_t} \tag{63}$$

$$f_{2,t}^{\dagger} \equiv \frac{(1-\alpha)}{\phi}\tau\left[(x_{t-1}^H - x_t^H - \kappa_t) - e^{(\varepsilon-1)\Delta_t^H}\frac{e^{(\varepsilon-1)(x_{t-1}^H - x_t^H - \kappa_t)} - 1}{\varepsilon - 1}\right]. \tag{64}$$

Most importantly, because $\bar{\tilde{\Gamma}}^{\dagger} \approx \alpha < \frac{\alpha}{1-\alpha} \approx \bar{\tilde{\Gamma}}$, the effect of the term $f_{2,t=1}^{\dagger}$, which is roughly proportional to κ_1, is dampened relative to the effect of $f_{2,t}$ in Equation (49).[24,25] Intuitively, when Assumption 2 is valid, the effect of the drift on the price index derives from those firms that are pushed down through the lower threshold, where firms have accumulated due to the history of the negative drift. In contrast, at $t = 1$, the drift affects the price index via the firms that are pushed up through the upper threshold, where such accumulation has not taken place. This effectively lowers the sensitivity of inflation to a change in the real marginal cost.

Equally interesting are the subsequent periods $t \geqq 2$. At $t = 2$, all the inequalities in Assumption 2 again hold. However, as shown schematically in Figure 9 in Appendix F, the positive drift at $t = 1$ has moved the entire distribution upwards, shifting the accumulated firms as of $t = 0$ away from the lower threshold. In fact, while $f_{2,t}$ again appears in the law of motion for price index as in Equation (49), it is multiplied by a factor significantly less than $\bar{\tilde{\Gamma}}$ at $t = 2$. This implies that the sensitivity of inflation to the recovery in the real marginal cost remains lower. The reduced sensitivity lasts until the accumulated firms as of $t = 0$ entirely return to the region in which firms are pushed out of the lower threshold. In other words, the sensitivity of inflation to the size of shock is dampened not only at the period of positive drift $t = 1$ but also during the recovery period, until the negative drift in markup for $t \geqq 2$ completely offset the effect of the positive drift at $t = 1$.

[24]This argument ignores several other channels for simplicity. First, $\bar{\Delta}^H < \bar{\Delta}^L$, which also tends to lower the response of the price level to the shock in κ_1. On the other hand, the response of $\left(x_t^H - x_{t-1}^H\right)$ to the shock in $f_{2,t}^{\dagger}$ may be more subtle. However, the effect of κ_t tends to quantitatively dominate at the onset of a shock $t = 1$.

[25]To derive Equation (62), we assume that the deflationary shock is not too large that the following inequality is satisfied: $\bar{x}^H - \bar{\kappa} - x_1^H < \kappa_1 \leqq \bar{x}^H - x_1^H$. If the shock is so large that $\kappa_1 \leqq \bar{x}^H - \bar{\kappa} - x_1^H$, then the term corresponding to $\widetilde{\Gamma}_{t=0}^{\dagger}$ becomes somewhat larger, making the sensitivity of inflation to the deflationary shock higher, though still lower than the normal value $\bar{\tilde{\Gamma}} \approx \frac{\alpha}{1-\alpha}$.

These non-linear responses to a deflationary shock clearly demonstrates the importance of Assumption 2. Namely, the assumption is not just a sufficient condition to make the model tractable; it is so deeply connected to the dynamics of our model that even a temporary violation of the assumption immediately results in non-linearity.

5.3 Correction associated with large drift

Another important assumption that we adopt throughout our analysis is Assumption 3. The assumption implies that the drift in aggregate markup is not so large that firms remain inside the inaction region for an extended period after they adjust their prices, unless they are hit by an idiosyncratic shock. This assumption allows us to employ certain approximations both in Section 3.1 and in Section 3.2, making the analysis tractable. The approximations based on this assumption are reasonably good in our calibrated model in Section 4: for example, the period n^* in Assumption 3 in the steady state is about $\bar{\Delta}^L/\bar{\pi} \approx 21.2$, implying $\alpha^{n^*} \approx 0.039$,which is small enough to be ignored compared to other factors of order 1. Appendix G confirms this argument by explicitly calculating the errors associated with the approximation.

Very large shocks may, however, significantly deteriorate the approximations, possibly making some of the corrections important. One such correction is the contribution of the delta functions that we ignore to derive Equation (48). Intuitively, it derives from the firms that change their prices or enter the market at a past period $t_0 = -n^* + 1 < 0$, have drifted since then without being hit by an idiosyncratic shock, and eventually cross the lower threshold at $t = 1$ solely by the drift. For concreteness, suppose that the economy is in the steady state for $t \leqq 0$ and is hit by an unexpected once-and-for-all aggregate inflationary shock at $t = 1$. If $\hat{\kappa}_1 = \kappa_1 - \bar{\pi}$ is small enough to allow only one delta function to cross the lower threshold at $t = 1$, as shown in Appendix G.2, the contribution of the delta function to the price index is approximately

$$(1-\alpha)C\left(1-\tau\frac{\bar{\Delta}^H+\bar{\Delta}^L}{\phi}\right)\alpha^{n^*}\left(\hat{\kappa}_1+\hat{x}_1^*\right). \tag{65}$$

By comparing it to the direct contribution of the term $\hat{f}_{2,t}$, given by

$$\alpha\tau\frac{\bar{\Delta}^L}{\phi}\hat{\kappa}_t, \tag{66}$$

the coefficient for the hat variables in the former expression is about 7.6% of the latter, suggesting that ignoring the former is not a big issue. However, if the inflationary shock at $t = 1$ is large and the ratio of κ_t to $\bar{\kappa}$ is significantly larger than 1, multiple delta functions, deriving from firms that change their prices or enter the market at multiple past periods, may cross the lower threshold at once at $t = 1$. The correction would then be large: for example, if three delta functions cross the border at $t = 1$, this makes the ratio of the contribution to the expression (66) about 27%, which is not negligible.[26] This argument suggests the existence of another non-linearity in our model.

6 Conclusion

This paper shows that introducing a drift in the aggregate markup to the menu model of Gertler and Leahy (2008) gives rise to a qualitatively new behavior, including significant fluctuations in the frequency of price changes and an extra responsiveness of inflation to aggregate shocks. Importantly, while being fully state-dependent, the model is still analytically tractable and yields an intuitive Phillips curve equation. It is thus suitable for policy analysis, such as the economic projection and simulations by central banks, especially when inflationary shocks are of particular concern.

Our main results originate from the interaction of state-dependent price setting with Assumption 2, regarding the perpetual negative drift in the aggregate markup. The assumption is necessarily stronger than just imposing a positive trend inflation. In other words, the inflation dynamics are qualitatively different if and only if the trend inflation is large enough or aggregate uncertainty is small enough that aggregate markup does not experience a positive drift. As far as the drift is negative and not large, the model is well approximated by a linear Phillips curve. Meanwhile, the violation of the assumptions

[26]The contribution is estimated by multiplying the contribution of (65) by a factor $\left(1 + 1/\alpha + 1/\alpha^2\right)$, which is larger than 3 because $\alpha < 1$.

immediately generates a non-linear behavior.

While this paper only analyzes a particular model, the proposed mechanisms, being intuitive and generic, may exist in a wider range of menu cost models. To confirm this is an important research agenda. Another issue of interest is to measure the quantitative importance of the mechanisms in empirical data.

Meanwhile, the model is still so simple that it does not capture a variety of features found in empirical studies, such as temporary sales (see, for example, Midrigan (2011)), seasonality in the frequency of price changes (e.g., Nakamura and Steinsson (2008)), inflation inertia (e.g., Nimark (2008)), and the effect of multi-sector output-input linkages (e.g., Rubbo (2023)). Incorporating these features without losing the tractability would further expand the applicability of the model in policy analysis.

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Appendix

A Details of firms' policy rule

This Appendix presents details regarding firms' policy rule derived in Section 3.1. Specifically, we first show the continuity of the value function $\tilde{v}_t(x)$, which has explicit forms shown in Equations (25) and (29). Next, we discuss the behavior of first derivatives, which are discontinuous at domain boundaries. Based on these basic properties of the value function and an additional assumption 4 introduced below, we prove the inequality (17), which we guess but do not verify in the main text. Finally, we briefly discuss the existence and uniqueness of the the triplet (x_t^*, x_t^L, x_t^H).

Throughout this Appendix, we rely on the auxiliary assumption, introduced temporarily in the main text, that firms perceive no aggregate uncertainty.[27]

In addition, throughout this paper, we use basic properties of the functions of the following form, with which $\tilde{v}_t^{(n)}$ for an arbitrary non-negative integer n conforms:

$$f(x) = A_1 e^{(-\varepsilon+1)x} - A_2 e^{-\varepsilon x} + A_3,$$

where the parameters A_1, A_2 and A_3 are positive and $\varepsilon > 1$. Figure 7 shows the schematic shape of this function: it is continuous and infinitely differentiable; it has a single peak at $x^* = \log\left(\frac{\varepsilon}{\varepsilon-1}\right) + \log\left(\frac{A_2}{A_1}\right)$ and is increasing (decreasing) for $x < x^*$ ($x > x^*$). We tend to focus on the region $x \in [x^L, x^H]$ defined by two values, x^L and x^H, which both satisfy the equation $f(x) = f(x^*) - b$ for some small positive parameter b.

[27] Clearly, without this auxiliary assumption, the value function in Equation (29) does not make sense. As of this writing, we do not know how to construct the value function while fully taking into account the effect of uncertainty. However, this may not be as restrictive as it may look. The properties of the value function investigated in this Appendix, such as the continuity and the inequality (17), are all independent of any particular realization of future macroeconomic variables as long as Assumptions 1 to 3 are respected. It therefore follows that we can average over all realizations of future macroeconomic variables to show the same properties in more general cases if those properties are preserved by taking averages.

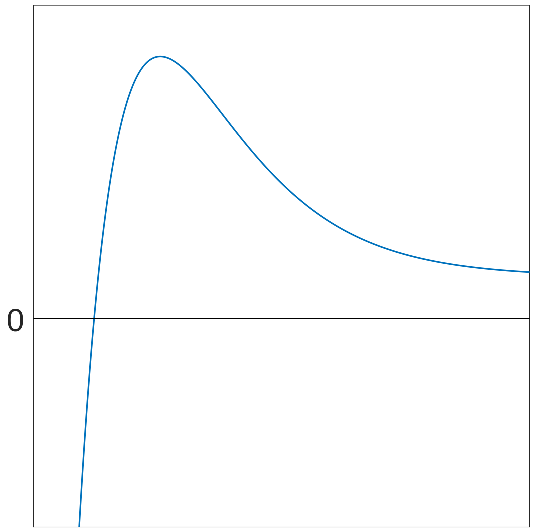

Figure 7: Schematic picture of the function $f(x)$.

A.1 Continuity of the value function

In this section, we establish continuity of the value function, which may not look trivial due to the piece-wise construction of Equation (29).

Because the continuity is obvious in the interior value x for each domain $x \in [x_t^{(n)}, x_t^{(n+1)})$, we only need to examine the continuity at domain boundaries. There are two cases to consider separately: the lower boundaries of each domain at $x_t^{(n)}$ for $n = 1, \ldots, N_t$, where N_t is implicitly defined by $(x_{t+1}^H + \kappa_{t+1}) \in [x_t^{(N_t)}, x_t^{(N_t+1)})$; and the upper boundary of the right-most domain $x_{t+1}^H + \kappa_{t+1}$. We examine the continuity at $x_t^{(n)}$ by evaluating

$$\begin{aligned} &\tilde{v}_t^{(n)}(x_t^{(n)}) - \tilde{v}_t^{(n-1)}(x_t^{(n)}) \\ &\quad = \left(A_{1,t}^{(n)} - A_{1,t}^{(n-1)}\right) e^{(-\varepsilon+1)x_t^{(n)}} - \left(A_{2,t}^{(n)} - A_{2,t}^{(n-1)}\right) e^{-\varepsilon x_t^{(n)}} + \left(A_{3,t}^{(n)} - A_{3,t}^{(n-1)}\right). \quad (67) \end{aligned}$$

In order to evaluate the right-hand side, we use the definitions of $A_{1,t}^{(n)}$, $A_{2,t}^{(n)}$, and $A_{3,t}^{(n)}$ in

Equations (30)-(32). For example,

$$
\begin{aligned}
A_{1,t}^{(n)} - A_{1,t}^{(n-1)} &= \alpha\beta e^{\lambda_{t+1}+(\varepsilon-1)\kappa_{t+1}} \left(A_{1,t+1}^{(n-1)} - A_{1,t+1}^{(n-2)} \right) \\
&= \cdots = (\alpha\beta)^{n-1} e^{\sum_{\tau=1}^{n-1}[\lambda_{t+\tau}+(\varepsilon-1)\kappa_{t+\tau}]} \left(A_{1,t+n-1}^{(1)} - A_{1,t+n-1}^{(0)} \right) \\
&= (\alpha\beta)^{n} e^{\sum_{\tau=1}^{n}[\lambda_{t+\tau}+(\varepsilon-1)\kappa_{t+\tau}]} A_{1,t+n}^{(0)}, \qquad (68)
\end{aligned}
$$

where we also use the definition of $A_{1,t}^{(0)}$ in Equation (26) for the last equality. Similarly,

$$
\begin{aligned}
A_{2,t}^{(n)} - A_{2,t}^{(n-1)} &= (\alpha\beta)^{n} e^{\sum_{\tau=1}^{n}[\lambda_{t+\tau}+\varepsilon\kappa_{t+\tau}]} A_{2,t+n}^{(0)}, \qquad (69) \\
A_{3,t}^{(n)} - A_{3,t}^{(n-1)} &= (\alpha\beta)^{n} e^{\sum_{\tau=1}^{n}\lambda_{t+\tau}} \left[A_{3,t+n}^{(0)} - \tilde{v}_{t+n}(x_{t+n}^{*}) + b \right]. \qquad (70)
\end{aligned}
$$

By substituting Equations (68)-(70) into (67) and using the definition of $x_t^{(n)} \equiv x_{t+n}^L + \sum_{s=1}^{n} \kappa_{t+s}$, we obtain

$$
\begin{aligned}
&\tilde{v}_t^{(n)}(x_t^{(n)}) - \tilde{v}_t^{(n-1)}(x_t^{(n)}) \\
&\quad = (\alpha\beta)^{n} e^{\sum_{\tau=1}^{n}\lambda_{t+\tau}} \left[A_{1,t+n}^{(0)} e^{(-\varepsilon+1)x_{t+n}^L} - A_{2,t+n}^{(0)} e^{-\varepsilon x_{t+n}^L} + A_{3,t+n}^{(0)} - \tilde{v}_{t+n}(x_{t+n}^{*}) + b \right] \\
&\quad = 0, \qquad (71)
\end{aligned}
$$

which establishes the continuity, where the last equality follows from Equation (40).

Next, we examine the continuity at the upper boundary $x_{t+1}^H + \kappa_{t+1}$. Before doing this, we establish the following property of the domains. Namely, for $\forall t$ and $n = 1, \ldots, N$,

$$
x \in [x_t^{(n)}, x_t^{(n+1)}) \Longleftrightarrow (x - \kappa_{t+1}) \in [x_{t+1}^{(n-1)}, x_{t+1}^{(n)}). \qquad (72)
$$

It is straight-forward to verify this property as

$$\begin{aligned} x \in [x_t^{(n)}, x_t^{(n+1)}) &\Longleftrightarrow x_{t+n}^L + \sum_{s=1}^{n} \kappa_{t+s} \leqq x < x_{t+n+1}^L + \sum_{s=1}^{n+1} \kappa_{t+s} \\ &\Longleftrightarrow x_{t+n}^L + \sum_{s=2}^{n} \kappa_{t+s} \leqq x - \kappa_{t+1} < x_{t+n+1}^L + \sum_{s=2}^{n+1} \kappa_{t+s} \\ &\Longleftrightarrow x_{t+1+n-1}^L + \sum_{s'=1}^{n-1} \kappa_{t+s'+1} \leqq x - \kappa_{t+1} < x_{t+1+n}^L + \sum_{s'=1}^{n} \kappa_{t+s'+1} \\ &\Longleftrightarrow x - \kappa_{t+1} \in [x_{t+1}^{(n-1)}, x_{t+1}^{(n)}). \end{aligned}$$

Note that we slightly generalize the notation of $x_t^{(n)}$ by defining $x_t^{(0)} \equiv x_t^L$. For $n = 1$, $\sum_{s'=1}^{n-1} \kappa_{t+s'+1}$ is interpreted as 0.

The continuity of the value function at $x_{t+1}^H + \kappa_{t+1}$ is verified by

$$\begin{aligned} &\tilde{v}_t^{(N_t)}(x_{t+1}^H + \kappa_{t+1}) - \tilde{v}_t^{(0)}(x_{t+1}^H + \kappa_{t+1}) \\ &\quad = \left(A_{1,t}^{(N_t)} - A_{1,t}^{(0)}\right) e^{(-\varepsilon+1)(x_{t+1}^H + \kappa_{t+1})} - \left(A_{2,t}^{(N_t)} - A_{2,t}^{(0)}\right) e^{-\varepsilon(x_{t+1}^H + \kappa_{t+1})} + \left(A_{3,t}^{(N_t)} - A_{3,t}^{(0)}\right) \\ &\quad = \alpha\beta e^{\lambda_{t+1}} \left[A_{1,t+1}^{(N_t-1)} e^{(-\varepsilon+1)x_{t+1}^H} - A_{2,t+1}^{(N_t-1)} e^{-\varepsilon x_{t+1}^H} + A_{3,t+1}^{(N_t-1)} - \tilde{v}_{t+1}(x_{t+1}^*) + b\right], \end{aligned}$$

where the second equality is obtained by using Equations (26)-(28) and (30)-(32). Because of (72), $x_{t+1}^H \in [x_{t+1}^{(N_t-1)}, x_{t+1}^{(N_t)})$. This suggest that

$$\begin{aligned} &\tilde{v}_t^{(N_t)}(x_{t+1}^H + \kappa_{t+1}) - \tilde{v}_t^{(0)}(x_{t+1}^H + \kappa_{t+1}) \\ &\quad = \alpha\beta e^{\lambda_{t+1}} \left[\tilde{v}_{t+1}(x_{t+1}^H) - \tilde{v}_{t+1}(x_{t+1}^*) + b\right] \\ &\quad = 0 \end{aligned} \tag{73}$$

by the definition of x_t^H, which prove the continuity.

A.2 Analysis of first derivative

Due to the piece-wise nature of the value function, the first derivative of the function is discontinuous at domain boundaries. It is therefore useful to analyze the characteristics of the first derivative. Similarly to the analysis of continuity in the previous section, this section considers two different cases for the boundaries: the lower boundaries of each domain at $x_t^{(n)}$ for $n = 1, \ldots, N_t$; and the upper boundary of the right-most domain $x_{t+1}^H + \kappa_{t+1}$.

The calculation closely follows that of the continuity presented in the previous section. Namely, for the lower boundaries,

$$\begin{aligned}
&\tilde{v}_t^{(n)\prime}(x_t^{(n)}) - \tilde{v}_t^{(n-1)\prime}(x_t^{(n)}) \\
&\quad = (-\varepsilon + 1)\left(A_{1,t}^{(n)} - A_{1,t}^{(n-1)}\right) e^{(-\varepsilon+1)x_t^{(n)}} + \varepsilon\left(A_{2,t}^{(n)} - A_{2,t}^{(n-1)}\right) e^{-\varepsilon x_t^{(n)}} \\
&\quad = (\alpha\beta)^n e^{\sum_{\tau=1}^{n} \lambda_{t+\tau}} \left[(-\varepsilon + 1) A_{1,t+n}^{(0)} e^{(-\varepsilon+1)x_{t+n}^L} + \varepsilon A_{2,t+n}^{(0)} e^{-\varepsilon x_{t+n}^L}\right] \\
&\quad = (\alpha\beta)^n e^{\sum_{\tau=1}^{n} \lambda_{t+\tau}} \left[\tilde{v}_{t+n}'(x_{t+n}^L)\right] > 0, \qquad (74)
\end{aligned}$$

where the last inequality is from (41). This implies that in the vicinity of boundaries, the value function is always convex. At the same time, the degree of discontinuity becomes small by a factor $(\alpha\beta)^n$ as n becomes large, and under Assumption 3, for $x \in [x_t^*, x_{t+1}^H + \kappa_{t+1}]$ the discontinuity is negligible.

For the upper boundary at $x_{t+1}^H + \kappa_{t+1}$, the calculation is again similar to the previous section.

$$\begin{aligned}
&\tilde{v}_t^{(0)\prime}(x_{t+1}^H + \kappa_{t+1}) - \tilde{v}_t^{(N_t)\prime}(x_{t+1}^H + \kappa_{t+1}) \\
&\quad = -(-\varepsilon + 1)\left(A_{1,t}^{(N_t)} - A_{1,t}^{(0)}\right) e^{(-\varepsilon+1)(x_{t+1}^H + \kappa_{t+1})} - \varepsilon\left(A_{2,t}^{(N_t)} - A_{2,t}^{(0)}\right) e^{-\varepsilon(x_{t+1}^H + \kappa_{t+1})} \\
&\quad = -\alpha\beta e^{\lambda_{t+1}} \left[(-\varepsilon + 1) A_{1,t+1}^{(N_t-1)} e^{(-\varepsilon+1)x_{t+1}^H} + \varepsilon A_{2,t+1}^{(N_t-1)} e^{-\varepsilon x_{t+1}^H}\right], \\
&\quad = -\alpha\beta e^{\lambda_{t+1}} \left[\tilde{v}_{t+1}'(x_{t+1}^H)\right] > 0, \qquad (75)
\end{aligned}$$

where the last inequality is from (39). Again, in the vicinity of the boundary, the value function is convex. In contrast to the boundaries at $x_t^{(n)}$ for large enough n, however, the discontinuity at $x_{t+1}^H + \kappa_{t+1}$ is not negligible.

A.3 Additional assumption to prove the inequality (17)

We now introduce another assumption. To do this, we expand the domain of the function $\tilde{v}_t^{(0)}(x)$, introduced in Equation (25), to the entire $x \in \mathbb{R}$.

Assumption 4 (Technical assumption for the proof of the inequality (17)) *Let $x_t^{H(0)}$ be a variable that satisfies $\tilde{v}_t^{(0)}(x_t^{H(0)}) = \tilde{v}_t^{(0)}(x_t^L)$ and $\left.\frac{d\tilde{v}_t^{(0)}}{dx}\right|_{x=x_t^{H(0)}} < 0$. Then*

$$x_t^{H(0)} > x_t^*. \tag{76}$$

This assumption does not have a clear economic interpretation but for a technical requirement that we use in the proof of the inequality (17) in Appendix A.4. In our numerical exercise, we confirm that the assumption holds throughout the simulation period.

While we have not succeeded in rigorously deriving the inequality (76) from other assumptions or proving the inequality (17) without the above assumption, we speculate that the inequality (17) is likely to hold in most calibrations of the model as far as Assumption 3 is a good approximation. To see this, we consider two equations: $\tilde{v}_t^{(\infty)}(x_t^H) = \tilde{v}_t^{(0)}(x_t^{H(0)})$ and $\tilde{v}_t^{(\infty)}(x_{t+1}^H + \kappa_{t+1}) = \tilde{v}_t^{(0)}(x_{t+1}^H + \kappa_{t+1})$. The former equation is from the definition of $x_t^{H(0)}$ and Equations (38) and (40), while the latter derives from the continuity shown in Equation (73). Assumption 3 suggests that $x_{t+1}^H + \kappa_{t+1} - x_t^H$, the relative drift in firms' markup, is much smaller than the upper gap $\Delta_t^H = x_t^H - x_t^*$. This roughly means that the two equations hold at x close to x_t^H and thus far from x_t^*, implying $x_t^* < x_t^{H(0)}$.

Somewhat more formally, by expanding the former equation $\tilde{v}_t^{(\infty)}(x_t^H) = \tilde{v}_t^{(0)}(x_t^{H(0)})$ up to a first order around $(x_{t+1}^H + \kappa_{t+1})$, we see that the left-hand side is $\tilde{v}_t^{(\infty)}(x_{t+1}^H + \kappa_{t+1}) + \tilde{v}_t^{(\infty)\prime}(x_{t+1}^H + \kappa_{t+1})(x_t^H - x_{t+1}^H - \kappa_{t+1})$, while the right-hand side is $\tilde{v}_t^{(0)}(x_{t+1}^H + \kappa_{t+1}) + \tilde{v}_t^{(0)\prime}(x_{t+1}^H + \kappa_{t+1})(x_t^{H(0)} - x_{t+1}^H - \kappa_{t+1})$. By using the latter equation $\tilde{v}_t^{(\infty)}(x_{t+1}^H + \kappa_{t+1}) = \tilde{v}_t^{(0)}(x_{t+1}^H + \kappa_{t+1})$, we rewrite it as

$$x_t^H - x_t^{H(0)} \approx \left(\frac{\tilde{v}_t^{(\infty)\prime}}{\tilde{v}_t^{(0)\prime}} - 1 \right) \left(x_{t+1}^H + \kappa_{t+1} - x_t^H \right),$$

where the derivatives are also evaluated at $(x_{t+1}^H + \kappa_{t+1})$. The ratio of the two derivatives is roughly about $1/(1-\alpha\beta)$ according to Equation (75), leading to

$$x_t^H - x_t^{H(0)} \approx \frac{\alpha\beta}{1-\alpha\beta}\left(x_{t+1}^H + \kappa_{t+1} - x_t^H\right).$$

The factor $\frac{\alpha\beta}{1-\alpha\beta}$ is typically not so large: for example, in our calibration presented in Section 4.1, $\alpha\beta/(1-\alpha\beta) \approx 5.6$. This implies that the distance of $x_t^{H(0)}$ from x_t^H is within 6 periods of the relative drift in firms' markup.

We now suppose that the inequality (76) does not hold, therefore $x_t^* \geqq x_t^{H(0)}$. It would then follow that the upper gap $\Delta_t^H = x_t^H - x_t^*$ is narrower than 6 periods of the drift. When trend inflation is not too large, the lower gap Δ_t^L has to be of similar order to the upper gap Δ_t^H, so Δ_t^L should be similarly narrow, in contradiction to Assumption 3. This rough evaluation, albeit not mathematically rigorous, supports the argument that the inequality (76) is likely to hold.

A.4 Proof for the inequality (17)

Under Assumption 3, for $\forall x \in [x_t^*, x_{t+1}^H + \kappa_{t+1}]$, $\tilde{v}_t^{(\infty)}(x)$ is a good approximation for the value function. Because Assumption 2 ensures $x_t^H < x_{t+1}^H + \kappa_{t+1}$, it follows that the inequality (17) holds for the upper half of the inaction region $[x_t^*, x_t^H]$. We therefore need to show the inequality only for the lower half of the inaction region $[x_t^L, x_t^*]$. To do this, we instead show the inequality (17) for $\forall x \in [x_t^L, x_{t+1}^{H(0)}]$, which covers the range of our interest $[x_t^L, x_t^*]$ because of Assumption 4.

Before we dive into the proof, we remember that the function $\tilde{v}_t^{(0)}(x)$, when its domain is expanded to the entire $x \in \mathbb{R}$, satisfies the following inequality

$$x \in [x_t^L, x_t^{H(0)}] \Longrightarrow \tilde{v}_t^{(0)}(x) \geqq \tilde{v}_t^{(0)}(x_t^L) = \tilde{v}_t^{(0)}(x_t^{H(0)}). \tag{77}$$

We are now ready for the proof. For $\forall x \in [x_t^{(0)}, x_t^{(1)})$, where $x_t^{(0)} \equiv x_t^L$, the inequality is trivial from the inequality (77) because $\tilde{v}_t(x) = \tilde{v}_t^{(0)}(x)$ in this range.[28] For the rest of the range, the proof is based on induction. Suppose the inequality holds for arbitrary t

[28]As defined around Equation (29) in the main text, $x_t^{(n)} = x_{t+n}^L + \sum_{s=1}^n \kappa_{t+s}$ for $n = 1, 2, \ldots, N_t$.

and $\forall x \in [x_t^{(n-1)}, x_t^{(n)})$. We evaluate $\tilde{v}_t(x) - \tilde{v}_t^{(0)}(x)$ for $\forall x \in [x_t^{(n)}, x_t^{(n+1)})$ as

$$\begin{aligned}
&\tilde{v}_t(x) - \tilde{v}_t^{(0)}(x) \\
&\quad = \tilde{v}_t^{(n)}(x) - \tilde{v}_t^{(0)}(x) \\
&\quad = \left(A_{1,t}^{(n)} - A_{1,t}^{(0)}\right) e^{(-\varepsilon+1)x} - \left(A_{2,t}^{(n)} - A_{2,t}^{(0)}\right) e^{-\varepsilon x} + \left(A_{3,t}^{(n)} - A_{3,t}^{(0)}\right) \\
&\quad = \alpha\beta e^{\lambda_{t+1}} \left[A_{1,t+1}^{(n-1)} e^{(-\varepsilon+1)(x-\kappa_{t+1})} - A_{2,t+1}^{(n-1)} e^{-\varepsilon(x-\kappa_{t+1})} + A_{3,t+1}^{(n-1)} - \tilde{v}_{t+1}(x_{t+1}^*) + b\right].
\end{aligned}$$

The third equality is based on Equations (26)-(28) and (30)-(32). Using the proposition expressed in (72), we obtain

$$\tilde{v}_t(x) - \tilde{v}_t^{(0)}(x) = \alpha\beta e^{\lambda_{t+1}} \left[\tilde{v}_{t+1}(x-\kappa_{t+1}) - \tilde{v}_{t+1}(x_{t+1}^*) + b\right] \geqq 0,$$

where the induction hypothesis is used for the last inequality. We therefore see that as long as $x \in [x_t^L, x_t^{H(0)}]$, the inequality (77) suggests that

$$\tilde{v}_t(x) \geqq \tilde{v}_t^{(0)}(x) \geqq \tilde{v}_t^{(0)}(x_t^L) = \tilde{v}_t(x_t^*) - b, \tag{78}$$

which concludes the proof.

A.5 Existence and uniqueness of the triplet (x_t^*, x_t^L, x_t^H)

This section discusses the existence and uniqueness of the triplet (x_t^*, x_t^L, x_t^H). First, these properties are obvious for the reset markup x_t^* defined in Equation (37), because $A_{1,t} > 0$ and $A_{2,t} > 0$ ensures that the function $\bar{v}_t^{(\infty)}(x)$ has a unique maximum at x_t^*, as shown schematically in Figure 7.[29] In addition, $\tilde{v}_t^{(\infty)}(x)$ is monotonically decreasing (increasing) in x for x larger (smaller) than x_t^*. This ensures the existence and uniqueness of the upper threshold x_t^H defined by the implicit equation (38) and the negative first derivative (39) as long as $b > 0$ is not too large.

Next, we consider the lower threshold x_t^L. Using x_t^* obtained above, we can uniquely specify the function $\tilde{v}_t^{(0)}(x)$ as in Equation (25) and define x_t^L by the implicit equation (40)

[29] $A_{4,t} > 0$ because of Equations (20) and (21) and the inequality (17). This immediately implies $A_{1,t} > 0$.

and the positive first derivative (41). Given the existence of x_t^L, it must be unique because the shape of $\tilde{v}_t^{(0)}(x)$ is similar to what is depicted in Figure 7.

Finally, we comment on the uniqueness of the triplet for the case without Assumption 3. To consider this case, we remember from Section A.2 above that, as x increases, the first derivative of the value function $\tilde{v}_t(x)$ discontinuously increases at every boundary, making the value function convex in the vicinity of the boundary. This means that there can be multiple values of x_t^*, each belonging to different domain, that give a maximum for the function $\tilde{v}_t(x)$. This possibility makes firms' optimization problem significantly more complicated, which in turn highlights the importance of our Assumption 3.

A.6 Derivation of the equation for $A_{5,t}$

The equations shown in Section 3.1 in the main text are sufficient to characterize firms' policy represented by the triplet (x_t^*, x_t^L, x_t^H), with one exception. Namely, we need to analyze the integral in Equation (21).

For the analysis, we first rewrite Equation (18) using $A_{1,t}^{(0)}$, $A_{2,t}^{(0)}$ and $A_{3,t}^{(0)}$ defined in Equation (25).

$$\begin{aligned}\tilde{v}_t(x) = &A_{1,t}^{(0)} e^{(-\varepsilon+1)x} - A_{2,t}^{(0)} e^{-\varepsilon x} + A_{3,t}^{(0)} \\ &+ \alpha\beta E_t e^{\lambda_{t+1}} \left[\tilde{v}_{t+1}(x-\kappa_{t+1}) - \tilde{v}_{t+1}(x_{t+1}^*) + b\right] I\left(x-\kappa_{t+1} \in S_{t+1}\right). \end{aligned} \tag{79}$$

Using this expression of the value function,

$$\begin{aligned} A_{5,t} &= \int_{x_t^L}^{x_t^H} \frac{dx}{\phi} e^{(\varepsilon-1)x} \left[\tilde{v}_t(x) - \tilde{v}_t(x_t^*) + b\right] \\ &= A_{1,t}^{(0)} \frac{x_t^H - x_t^L}{\phi} - A_{2,t}^{(0)} \frac{-e^{-x_t^H} + e^{-x_t^L}}{\phi} + \left(A_{3,t}^{(0)} - \tilde{v}_t(x_t^*) + b\right) \frac{e^{(\varepsilon-1)x_t^H} - e^{(\varepsilon-1)x_t^L}}{\phi(\varepsilon-1)} \\ &\quad + \alpha\beta E_t e^{\lambda_{t+1}+(\varepsilon-1)\kappa_{t+1}} \int_{x_t^L-\kappa_{t+1}}^{x_t^H-\kappa_{t+1}} \frac{dx'}{\phi}\, e^{(\varepsilon-1)x'} \left[\tilde{v}_{t+1}(x') - \tilde{v}_{t+1}(x_{t+1}^*) + b\right] I\left(x' \in S_{t+1}\right). \end{aligned}$$

The integral in the last term is evaluated as

$$\begin{aligned}
&\int_{x_t^L-\kappa_{t+1}}^{x_t^H-\kappa_{t+1}} \frac{dx'}{\phi}\; e^{(\varepsilon-1)x'} \left[\tilde{v}_{t+1}(x') - \tilde{v}_{t+1}(x^*_{t+1}) + b\right] I\left(x' \in S_{t+1}\right) \\
&\quad = \left(\int_{x_{t+1}^L}^{x_{t+1}^H} - \int_{x_t^H-\kappa_{t+1}}^{x_{t+1}^H}\right) \frac{dx'}{\phi}\; e^{(\varepsilon-1)x'} \left[\tilde{v}_{t+1}(x') - \tilde{v}_{t+1}(x^*_{t+1}) + b\right] \\
&\quad = A_{5,t+1} - \int_{x_t^H-\kappa_{t+1}}^{x_{t+1}^H} \frac{dx'}{\phi}\; e^{(\varepsilon-1)x'} \left[\tilde{v}_{t+1}(x') - \tilde{v}_{t+1}(x^*_{t+1}) + b\right],
\end{aligned}$$

where Assumption 2 is used for the first equality. For the evaluation of the last integral, we remember that due to Assumption 3, $\tilde{v}_t(x)$ is well approximated by Equation (33) for the region $[x_t^H - \kappa_{t+1}, x_{t+1}^H]$ close to the upper threshold.

To sum up, we obtain the equation to determine $A_{5,t}$ as

$$\begin{aligned}
A_{5,t} = {} & A_{1,t}^{(0)} \frac{x_t^H - x_t^L}{\phi} - A_{2,t}^{(0)} \frac{-e^{-x_t^H} + e^{-x_t^L}}{\phi} + \left(A_{3,t}^{(0)} - \tilde{v}_t(x_t^*) + b\right) \frac{e^{(\varepsilon-1)x_t^H} - e^{(\varepsilon-1)x_t^L}}{\phi(\varepsilon-1)} \\
& + \alpha\beta E_t e^{\lambda_{t+1}+(\varepsilon-1)\kappa_{t+1}} \Bigg[A_{5,t+1} \\
& \quad - \Bigg\{ A_{1,t+1} \frac{x_{t+1}^H - x_t^H + \kappa_{t+1}}{\phi} - A_{2,t+1} \frac{-e^{-x_{t+1}^H} + e^{-x_t^H+\kappa_{t+1}}}{\phi} \\
& \quad + \left(A_{3,t+1} - \tilde{v}_{t+1}(x^*_{t+1}) + b\right) \frac{e^{(\varepsilon-1)x_{t+1}^H} - e^{(\varepsilon-1)(x_t^H-\kappa_{t+1})}}{\phi(\varepsilon-1)} \Bigg\} \Bigg].
\end{aligned}$$

By using the definitions of x_t^H and x_t^L in Equations (38) and (40), the above equation is rewritten as

$$\begin{aligned}
A_{5,t} = {} & A_{1,t}^{(0)} \left[\frac{x_t^H - x_t^L}{\phi} - \frac{e^{(\varepsilon-1)(x_t^H-x_t^L)} - 1}{\phi(\varepsilon-1)}\right] - A_{2,t}^{(0)} e^{-x_t^L} \left[\frac{-e^{-x_t^H+x_t^L} + 1}{\phi} - \frac{e^{(\varepsilon-1)(x_t^H-x_t^L)} - 1}{\phi(\varepsilon-1)}\right] \\
& + \alpha\beta E_t e^{\lambda_{t+1}+(\varepsilon-1)\kappa_{t+1}} \Bigg[A_{5,t+1} \\
& \quad - A_{1,t+1} \left\{ \frac{x_{t+1}^H - x_t^H + \kappa_{t+1}}{\phi} - \frac{1 - e^{-(\varepsilon-1)(x_{t+1}^H - x_t^H + \kappa_{t+1})}}{\phi(\varepsilon-1)} \right\} \\
& \quad + A_{2,t+1} e^{-x_{t+1}^H} \left\{ \frac{-1 + e^{x_{t+1}^H - x_t^H + \kappa_{t+1}}}{\phi} - \frac{1 - e^{-(\varepsilon-1)(x_{t+1}^H - x_t^H + \kappa_{t+1})}}{\phi(\varepsilon-1)} \right\} \Bigg]. \qquad (80)
\end{aligned}$$

A.7 The gaps of the inaction region in the limit of zero trend inflation in a steady state

Equations (38) and (40) in the main text suggest that there is an intrinsic asymmetry between lower and upper thresholds, as discussed at the bottom of Section 3.1. This Appendix shows that the asymmetry vanishes in the limit of zero trend inflation at least in a steady state, thereby confirming the intuition that the asymmetry derives from Assumption 2, which is consistent with positive trend inflation.

In a steady state with zero trend inflation, $\lambda_{t+1} = \kappa_{t+1} = 0$. These conditions significantly simplifies Equations (28) and (34)-(36) as

$$
\begin{aligned}
\bar{A}_3^{(0)} &= \left(\alpha + (1-\alpha)\tau\varepsilon^{\phi}\right)\beta\left(\bar{\tilde{v}}(\bar{x}^{*}) - b\right), \\
\bar{A}_1^{(0)} &= (1-\alpha\beta)\bar{A}_1, \\
\bar{A}_2^{(0)} &= (1-\alpha\beta)\bar{A}_2, \\
\bar{A}_3 &= \frac{(1-\alpha)\tau\varepsilon^{\phi}\beta}{1-\alpha\beta}\left(\bar{\tilde{v}}(\bar{x}^{*}) - b\right).
\end{aligned}
$$

Substituting these equations into Equation (40), we obtain

$$
(1-\alpha\beta)\bar{A}_1 e^{(-\varepsilon+1)\bar{x}^{L}} - (1-\alpha\beta)\bar{A}_2 e^{-\varepsilon\bar{x}^{L}} + \left(\alpha + (1-\alpha)\tau\varepsilon^{\phi}\right)\beta\left(\bar{\tilde{v}}(\bar{x}^{*}) - b\right) = \bar{\tilde{v}}(\bar{x}^{*}) - b.
$$

On the other hand, using Equation (33), we rewrite

$$
\bar{\tilde{v}}(\bar{x}^{*}) - b = \bar{A}_1 e^{(-\varepsilon+1)\bar{x}^{*}} - \bar{A}_2 e^{-\varepsilon\bar{x}^{*}} + \frac{(1-\alpha)\tau\varepsilon^{\phi}\beta}{1-\alpha\beta}\left(\bar{\tilde{v}}(\bar{x}^{*}) - b\right) - b.
$$

By removing $\bar{\tilde{v}}(\bar{x}^{*}) - b$ from these two expressions, we obtain

$$
\bar{A}_1 e^{(-\varepsilon+1)\bar{x}^{L}} - \bar{A}_2 e^{-\varepsilon\bar{x}^{L}} = \bar{A}_1 e^{(-\varepsilon+1)\bar{x}^{*}} - \bar{A}_2 e^{-\varepsilon\bar{x}^{*}} - b.
$$

This last expression is rewritten as

$$
\bar{\tilde{v}}^{(\infty)}(\bar{x}^{L}) = \bar{\tilde{v}}^{(\infty)}(\bar{x}^{*}) - b,
$$

implying that $\bar{x}^L$ is determined in a similar way as $\bar{x}^H$ in Equation (38). In other words, there is no asymmetry between $\bar{x}^L$ and $\bar{x}^H$ in the limit of zero trend inflation in a steady state .

B Details of analysis on markup distribution

This Appendix shows intermediate calculations for the derivation of markup distribution and price level as outlined in Section 3.2.

B.1 Definitions of key variables

We start from the price level in Equation (2). Using the definition of firm-level markup $x_{i,t} \equiv \log(P_{i,t} Z_t e^{z_{i,t}} / W_t)$,

$$\begin{aligned}
\log P_t &= \frac{1}{1-\varepsilon} \log \left[\int_0^1 \left(\frac{W_t}{Z_t} e^{x_{i,t} - z_{i,t}} \right)^{1-\varepsilon} di \right] \\
&= \log \frac{W_t}{Z_t} + \frac{1}{1-\varepsilon} \log \left[\int_0^1 e^{(\varepsilon-1)(z_{i,t} - x_{i,t})} di \right].
\end{aligned}$$

By rearranging this equation, we obtain

$$\begin{aligned}
x_t = \log \frac{P_t Z_t}{W_t} &= \frac{1}{1-\varepsilon} \log \int_0^1 e^{(\varepsilon-1)(z_{i,t} - x_{i,t})} di \\
&= \frac{1}{1-\varepsilon} \log \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dz \ e^{(\varepsilon-1)(z-x)} \psi_t^1(x, z). \qquad (81)
\end{aligned}$$

To proceed further, we define a few key variables to rewrite the integral in the right-hand side of Equation (81). Specifically, we define the following variables.

$$\Gamma_t^2 \equiv \int_{-\infty}^{\infty} dz \int_{S_t^c} dx \ e^{(\varepsilon-1)(z - x_t^*)} \psi_t^0(x, z) = \int_{S_t^c} dx \ e^{(\varepsilon-1)(x - x_t^*)} \Psi_t(x) \qquad (82)$$

$$\Gamma_t^3 \equiv \int_{-\infty}^{\infty} dz \int_{S_t} dx \ e^{(\varepsilon-1)(z - x)} \psi_t^0(x, z) = \int_{S_t} dx \ \Psi_t(x) \qquad (83)$$

$$\Gamma_t^4 \equiv \int_{-\infty}^{\infty} dz \int_{S_t} dx \ e^{(\varepsilon-1)(z - x_t^*)} \psi_t^0(x, z) = \int_{S_t} dx \ e^{(\varepsilon-1)(x - x_t^*)} \Psi_t(x) \qquad (84)$$

Γ_t^2 and Γ_t^3 are the contributions of firms adjusting and not adjusting their prices, respectively, to the aggregate markup in Equation (81). In fact, by substituting Equation (42)

Table 2: Notation of each term in the law of motion of $\Psi_{t+1}(x)$.

term	(A)	(B)	(C)	(D)	(E)	(F)
behavior in t	new entrants		changing price		not changing price	
ξ_{t+1} shock	Yes	No	Yes	No	Yes	No

into (81) and combining it with the above definitions, we obtain

$$x_t = \frac{1}{1-\varepsilon}\log\left((1-\alpha)(1-\tau)e^{-(\varepsilon-1)x_t^*} + \Gamma_t^2 + \Gamma_t^3\right). \tag{85}$$

Γ_t^4 does not have a clear economic meaning but is defined for later convenience.

B.2 Master equations for the density $\Psi_t(x)$

We next derive the law of motion for $\Psi_t(x)$ using the law of motion for $\psi_t^0(x,z)$, which is obtained by simply combining Equation (42) with (43).

It is clearer to consider each term separately, because otherwise the equation would become too lengthy to write. As shown in Table 2, we categorize each term appearing in the law of motion for $\Psi_{t+1}(x)$ by (A)-(F). For example, the term (A) represents those firms which newly enter the market at t and are subsequently hit by an idiosyncratic shock at the beginning of $t+1$. Similarly, the term (F) represents those firms which do not change their prices at t and do not experience an idiosyncratic shock at the beginning of $t+1$. We obtain the expression for each of $\Psi_{t+1}^{(A)}(x)$ to $\Psi_{t+1}^{(F)}(x)$ as follows.

(A):

$$\begin{aligned}
\psi_{t+1}^{0(A)}(x,z) &= (1-\alpha)\tau\int_{-\infty}^{\infty} d\xi\ p(\xi)(1-\alpha)(1-\tau)\delta\left(x-\xi+\kappa_{t+1}-x_t^*\right)\delta(z-\xi) \\
&= (1-\alpha)\tau(1-\alpha)(1-\tau)p\left(x+\kappa_{t+1}-x_t^*\right)\delta\left(z-x-\kappa_{t+1}+x_t^*\right) \\
&= \frac{(1-\alpha)\tau}{\phi}(1-\alpha)(1-\tau)\delta\left(z-x-\kappa_{t+1}+x_t^*\right)I\left(|x+\kappa_{t+1}-x_t^*| \leqq \frac{\phi}{2}\right) \\
\Rightarrow \Psi_{t+1}^{(A)}(x) &= \int_{-\infty}^{\infty} dz\ e^{(\varepsilon-1)(z-x)}\psi_{t+1}^{0(A)}(x,z) \\
&= \frac{(1-\alpha)\tau}{\phi}(1-\alpha)(1-\tau)e^{(\varepsilon-1)(\kappa_{t+1}-x_t^*)}I\left(|x+\kappa_{t+1}-x_t^*| \leqq \frac{\phi}{2}\right).
\end{aligned} \tag{86}$$

(B):

$$\begin{aligned}\psi^{0(B)}_{t+1}(x,z) &= \alpha(1-\alpha)(1-\tau)\delta\left(x+\kappa_{t+1}-x^*_t\right)\delta(z)\\ \Rightarrow \Psi^{(B)}_{t+1}(x) &= \int_{-\infty}^{\infty} dz\ e^{(\varepsilon-1)(z-x)}\psi^{0(B)}_{t+1}(x,z)\\ &= \alpha(1-\alpha)(1-\tau)\delta\left(x+\kappa_{t+1}-x^*_t\right)e^{(\varepsilon-1)(\kappa_{t+1}-x^*_t)}. \end{aligned} \tag{87}$$

(C):

$$\begin{aligned}\psi^{0(C)}_{t+1}(x,z) &= (1-\alpha)\tau\int_{-\infty}^{\infty} d\xi\ p(\xi)\delta\left(x-\xi+\kappa_{t+1}-x^*_t\right)\int_{S^c_t} dx'\ \psi^0_t\left(x',z-\xi\right)\\ &= (1-\alpha)\tau p\left(x+\kappa_{t+1}-x^*_t\right)\int_{S^c_t} dx'\ \psi^0_t\left(x',z-x-\kappa_{t+1}+x^*_t\right)\\ &= \frac{(1-\alpha)\tau}{\phi}I\left(\left|x+\kappa_{t+1}-x^*_t\right|\leqq\frac{\phi}{2}\right)\int_{S^c_t} dx'\ \psi^0_t\left(x',z-x-\kappa_{t+1}+x^*_t\right)\\ \Rightarrow \Psi^{(C)}_{t+1}(x) &= \int_{-\infty}^{\infty} dz\ e^{(\varepsilon-1)(z-x)}\psi^{0(C)}_{t+1}(x,z)\\ &= \frac{(1-\alpha)\tau}{\phi}I\left(\left|x+\kappa_{t+1}-x^*_t\right|\leqq\frac{\phi}{2}\right)\int_{S^c_t} dx'\ e^{(\varepsilon-1)(x'+\kappa_{t+1}-x^*_t)}\Psi_t(x')\\ &= \frac{(1-\alpha)\tau}{\phi}I\left(\left|x+\kappa_{t+1}-x^*_t\right|\leqq\frac{\phi}{2}\right)e^{(\varepsilon-1)\kappa_{t+1}}\Gamma^2_t. \end{aligned} \tag{88}$$

(D):

$$\begin{aligned}\psi^{0(D)}_{t+1}(x,z) &= \alpha\delta\left(x+\kappa_{t+1}-x^*_t\right)\int_{S^c_t} dx'\ \psi^0_t\left(x',z\right)\\ \Rightarrow \Psi^{(D)}_{t+1}(x) &= \int_{-\infty}^{\infty} dz\ e^{(\varepsilon-1)(z-x)}\psi^{0(D)}_{t+1}(x,z)\\ &= \alpha\delta\left(x+\kappa_{t+1}-x^*_t\right)\int_{S^c_t} dx'\ e^{(\varepsilon-1)(x'+\kappa_{t+1}-x^*_t)}\Psi_t(x')\\ &= \alpha\delta\left(x+\kappa_{t+1}-x^*_t\right)e^{(\varepsilon-1)\kappa_{t+1}}\Gamma^2_t. \end{aligned} \tag{89}$$

(E):

$$\begin{aligned}
\psi_{t+1}^{0(E)}(x,z) &= (1-\alpha)\tau \int_{-\infty}^{\infty} d\xi \ p(\xi)\psi_t^0(x-\xi+\kappa_{t+1}, z-\xi) I\left(x-\xi+\kappa_{t+1} \in S_t\right) \\
\Rightarrow \Psi_{t+1}^{(E)}(x) &= \int_{-\infty}^{\infty} dz \ e^{(\varepsilon-1)(z-x)} \psi_{t+1}^{0(E)}(x,z) \\
&= \frac{(1-\alpha)\tau}{\phi} e^{(\varepsilon-1)\kappa_{t+1}} \int_{-\infty}^{\infty} dx' \ \Psi_t(x') I\left(\left|x+\kappa_{t+1}-x'\right| \leqq \frac{\phi}{2}\right) I\left(x' \in S_t\right),
\end{aligned} \tag{90}$$

where $x' \equiv x - \xi + \kappa_{t+1}$. Because Assumptions 1 and 2 ensure $\phi/2 > x_t^H - x_t^L$, the multiplication of the two indicator functions in the above equation is non-zero for the following three cases, depending on how the wide support of the uniform shock covers the inaction region.

(E1) : For $x \in [x_t^L - \phi/2 - \kappa_{t+1}, x_t^H - \phi/2 - \kappa_{t+1})$,

$$\Psi_{t+1}^{(E1)}(x) = \frac{(1-\alpha)\tau}{\phi} e^{(\varepsilon-1)\kappa_{t+1}} \int_{x_t^L}^{x+\kappa_{t+1}+\frac{\phi}{2}} dx' \ \Psi_t(x'). \tag{91}$$

(E2) : For $x \in [x_t^H - \phi/2 - \kappa_{t+1}, x_t^L + \phi/2 - \kappa_{t+1}]$,

$$\begin{aligned}
\Psi_{t+1}^{(E2)}(x) &= \frac{(1-\alpha)\tau}{\phi} e^{(\varepsilon-1)\kappa_{t+1}} \int_{S_t} dx' \ \Psi_t(x') \\
&= \frac{(1-\alpha)\tau}{\phi} e^{(\varepsilon-1)\kappa_{t+1}} \Gamma_t^3.
\end{aligned} \tag{92}$$

(E3) : For $x \in (x_t^L + \phi/2 - \kappa_{t+1}, x_t^H + \phi/2 - \kappa_{t+1}]$,

$$\Psi_{t+1}^{(E3)}(x) = \frac{(1-\alpha)\tau}{\phi} e^{(\varepsilon-1)\kappa_{t+1}} \int_{x+\kappa_{t+1}-\frac{\phi}{2}}^{x_t^H} dx' \ \Psi_t(x'). \tag{93}$$

(F):

$$\begin{aligned}
\psi_{t+1}^{0(F)}(x,z) &= \alpha \psi_t^0(x+\kappa_{t+1}, z) I\left(x+\kappa_{t+1} \in S_t\right) \\
\Rightarrow \Psi_{t+1}^{(F)}(x) &= \int_{-\infty}^{\infty} dz \ e^{(\varepsilon-1)(z-x)} \psi_{t+1}^{0(F)}(x,z) \\
&= \alpha e^{(\varepsilon-1)\kappa_{t+1}} \Psi_t(x+\kappa_{t+1}) I\left(x+\kappa_{t+1} \in S_t\right).
\end{aligned} \tag{94}$$

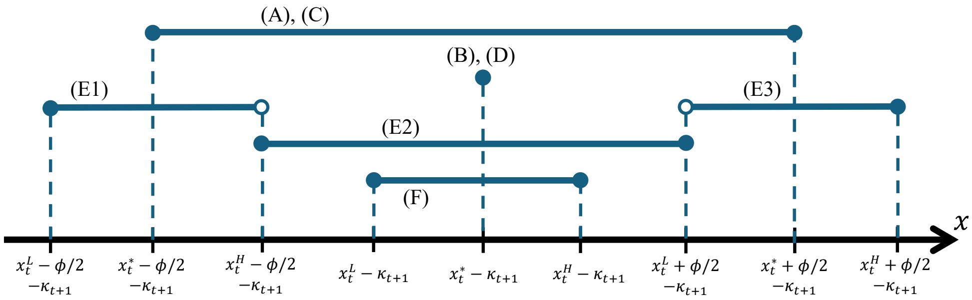

Figure 8: Schematic figure showing the ranges in which each of $\Psi_{t+1}^{(X)}(x)$ is not zero, where $X = A, B, C, D, E1, E2, E3, F$.

We use these master equations to calculate various integrals. For example, to derive the expression for Γ_t^1 defined in Equation (46) in the main text, we integrate each term (A) to (F) above and obtain:

$$\Gamma_{t+1}^1 = (\alpha + (1-\alpha)\tau)\, e^{(\varepsilon-1)\kappa_{t+1}} \left[(1-\alpha)(1-\tau)e^{-(\varepsilon-1)x_t^*} + \Gamma_t^2 + \Gamma_t^3\right]. \tag{95}$$

Finally, it is convenient to summarize the range of each term in $\Psi_{t+1}(x)$ along the x axis. Namely, each term has a different range in which it is not zero, as shown in Figure 8. For example, to derive the expression for $\Psi_{t+1}(x)$ for $x \in [x_t^H - \phi/2 - \kappa_{t+1}, x_t^L - \kappa_{t+1}) \cup (x_t^H - \kappa_{t+1}, x_t^L + \phi/2 - \kappa_{t+1}]$, this figure shows that we only need to consider the terms (A), (C) and (E2). By summing these three terms and using Equation (95), we obtain Equation (45) in the main text. Similarly, we obtain Equation (47) by adding the terms (A), (C), (E2) and (F).

B.3 Calculation of $\int_{-\infty}^{\infty} dx\ e^{(\varepsilon-1)x}\Psi_t(x)$

Next, we show that $\int_{-\infty}^{\infty} dx\ e^{(\varepsilon-1)x}\Psi_t(x)$ is a constant, which we denote by C'. This implies that we only need to calculate Γ_t^4 to obtain Γ_t^2, as Equations (82) and (84) lead to the equality

$$\Gamma_t^2 = e^{-(\varepsilon-1)x_t^*}C' - \Gamma_t^4. \tag{96}$$

Similarly to the evaluation of Γ_t^1, we obtain the integral at $t+1$ by adding up the contribution of each term (A)-(F).

$$\begin{aligned}\int_{-\infty}^{\infty} dx\ e^{(\varepsilon-1)x}\Psi_{t+1}(x) &= \sum_{I=A \text{ to } F} \int_{-\infty}^{\infty} dx\ e^{(\varepsilon-1)x}\Psi_{t+1}^{(I)}(x) \\ &= \left(\alpha + (1-\alpha)\tau\varepsilon^{\phi}\right)\left[(1-\alpha)(1-\tau) + e^{(\varepsilon-1)x_t^*}\left(\Gamma_t^2 + \Gamma_t^4\right)\right] \\ &= \left(\alpha + (1-\alpha)\tau\varepsilon^{\phi}\right)\left[(1-\alpha)(1-\tau) + \int_{-\infty}^{\infty} dx\ e^{(\varepsilon-1)x}\Psi_t(x)\right],\end{aligned} \tag{97}$$

where ε^{ϕ} is a constant defined by

$$\varepsilon^{\phi} \equiv \frac{e^{(\varepsilon-1)\frac{\phi}{2}} - e^{-(\varepsilon-1)\frac{\phi}{2}}}{\phi(\varepsilon-1)}.$$

Equation (97) is a simple first-order recurrence relation for the integral on the left-hand side, and no term or factor on the right-hand side other than the lag of the left-hand-side integral depends on time. It therefore follows that the integral itself is a constant with respect to time t and is equal to

$$\int_{-\infty}^{\infty} dx\ e^{(\varepsilon-1)x}\Psi_t(x) = C' \equiv \frac{\left(\alpha + (1-\alpha)\tau\varepsilon^{\phi}\right)(1-\alpha)(1-\tau)}{1 - \left(\alpha + (1-\alpha)\tau\varepsilon^{\phi}\right)}. \tag{98}$$

It is noteworthy that we can establish the fact that this integral is constant in a more intuitive way. Specifically, by using the definition of $\Psi_t(x)$ in Equation (44),

$$\int_{-\infty}^{\infty} dx\ e^{(\varepsilon-1)x}\Psi_t(x) = \int_{-\infty}^{\infty} dz\ e^{(\varepsilon-1)z} \int_{-\infty}^{\infty} dx\ \psi_t^0(x,z).$$

The integral over x in the right-hand side represents a density function of firms in terms of the productivity level z. Because the shock process z is exogenous, this density function, after reaching a stationary distribution, does not depend on macroeconomic variables and is thus constant over time.[30]

B.4 Evaluation of Γ^3_{t+1}, Γ^4_{t+1}, and Γ^2_{t+1}

Next, we calculate Γ^3_{t+1} and Γ^4_{t+1}, again using the master equations (86)-(94) as well as Equations (45), (47) and (48) in the main text. To evaluate the integral for Γ^3_{t+1}, we proceed as

$$\Gamma^3_{t+1} = \int_{x^L_{t+1}}^{x^H_{t+1}} dx\ \Psi_{t+1}(x) = \left(\int_{x^H_t - \kappa_{t+1}}^{x^H_{t+1}} - \int_{x^L_t - \kappa_{t+1}}^{x^L_{t+1}} + \int_{x^L_t - \kappa_{t+1}}^{x^H_t - \kappa_{t+1}} \right) dx\ \Psi_{t+1}(x). \tag{99}$$

We evaluate this expression by using Equation (45) for the first integral inside the parenthesis, Equation (48) for the second integral, and Equation (47) together with the contribution of delta functions at $x = x^*_t - \kappa_{t+1}$ shown in Equations (87) and (89) for the third integral, respectively. After some rearrangement,

$$\begin{aligned}
\Gamma^3_{t+1} &= \frac{(1-\alpha)\tau}{\alpha + (1-\alpha)\tau} \frac{x^H_{t+1} - x^L_{t+1}}{\phi} \Gamma^1_{t+1} \\
&\quad - \frac{(1-\alpha)\tau}{\alpha + (1-\alpha)\tau} \frac{x^L_{t+1} - x^L_t + \kappa_{t+1}}{\phi} \Gamma^5_{t+1} \\
&\quad + \alpha e^{(\varepsilon-1)\kappa_{t+1}} \left[(1-\alpha)(1-\tau) e^{-(\varepsilon-1)x^*_t} + \Gamma^2_t + \Gamma^3_t \right]. \\
&= \frac{1}{\alpha + (1-\alpha)\tau} \left[\alpha + (1-\alpha)\tau \frac{x^H_{t+1} - x^L_{t+1}}{\phi} \right] \Gamma^1_{t+1} \\
&\quad - \frac{(1-\alpha)\tau}{\alpha + (1-\alpha)\tau} \frac{x^L_{t+1} - x^L_t + \kappa_{t+1}}{\phi} \Gamma^5_{t+1},
\end{aligned} \tag{100}$$

where we use Equation (95) for the second equality.

---

[30]This line of arguments, while standard in the literature of macroeconomic models with heterogeneous agents, uses an assumption that firms' density does not exhibit any fluctuation. This is not a trivial assumption. In other scientific field, for example in statistical mechanics used to study many-body physical systems or chemical systems in finite temperatures, such a fluctuation is fundamentally importance. Nirei and Scheinkman (2024) is an exception that studies such a fluctuation in finite number of firms in a menu cost model. The effect of the fluctuations in firms' number on our menu cost model, while beyond the scope of this paper, is an important future research topic.

Similarly, we obtain the expression for Γ^4_{t+1} as

$$
\begin{aligned}
\Gamma^4_{t+1} &= \int_{x^L_{t+1}}^{x^H_{t+1}} dx\ e^{(\varepsilon-1)(x-x^*_{t+1})}\Psi_{t+1}(x) \\
&= \left(\int_{x^H_t-\kappa_{t+1}}^{x^H_{t+1}} - \int_{x^L_t-\kappa_{t+1}}^{x^L_{t+1}} + \int_{x^L_t-\kappa_{t+1}}^{x^H_t-\kappa_{t+1}} \right) dx\ e^{(\varepsilon-1)(x-x^*_{t+1})}\Psi_{t+1}(x), \\
&= \frac{(1-\alpha)\tau}{\alpha+(1-\alpha)\tau} \frac{e^{(\varepsilon-1)(x^H_{t+1}-x^*_{t+1})} - e^{(\varepsilon-1)(x^L_{t+1}-x^*_{t+1})}}{\phi(\varepsilon-1)} \Gamma^1_{t+1} \\
&\quad - \frac{(1-\alpha)\tau}{\alpha+(1-\alpha)\tau} \frac{e^{(\varepsilon-1)(x^L_{t+1}-x^*_{t+1})} - e^{(\varepsilon-1)(x^L_t-\kappa_{t+1}-x^*_{t+1})}}{\phi(\varepsilon-1)} \Gamma^5_{t+1} \\
&\quad + \alpha e^{(\varepsilon-1)(x^*_t-x^*_{t+1})} \left[(1-\alpha)(1-\tau)e^{-(\varepsilon-1)x^*_t} + \Gamma^2_t + \Gamma^4_t \right]. \\
&= \frac{(1-\alpha)\tau}{\alpha+(1-\alpha)\tau} \frac{e^{(\varepsilon-1)(x^H_{t+1}-x^*_{t+1})} - e^{(\varepsilon-1)(x^L_{t+1}-x^*_{t+1})}}{\phi(\varepsilon-1)} \Gamma^1_{t+1} \\
&\quad - \frac{(1-\alpha)\tau}{\alpha+(1-\alpha)\tau} \frac{e^{(\varepsilon-1)(x^L_{t+1}-x^*_{t+1})} - e^{(\varepsilon-1)(x^L_t-\kappa_{t+1}-x^*_{t+1})}}{\phi(\varepsilon-1)} \Gamma^5_{t+1} \\
&\quad + \alpha e^{-(\varepsilon-1)x^*_{t+1}} \left[(1-\alpha)(1-\tau) + C' \right],
\end{aligned}
$$

where we use Equation (96) for the last equality. Using this expression and Equation (96), we obtain Γ^2_{t+1} as

$$
\begin{aligned}
\Gamma^2_{t+1} &= C' e^{-(\varepsilon-1)x^*_{t+1}} - \Gamma^4_{t+1} \\
&= -\frac{(1-\alpha)\tau}{\alpha+(1-\alpha)\tau} \frac{e^{(\varepsilon-1)(x^H_{t+1}-x^*_{t+1})} - e^{(\varepsilon-1)(x^L_{t+1}-x^*_{t+1})}}{\phi(\varepsilon-1)} \Gamma^1_{t+1} \\
&\quad + \frac{(1-\alpha)\tau}{\alpha+(1-\alpha)\tau} \frac{e^{(\varepsilon-1)(x^L_{t+1}-x^*_{t+1})} - e^{(\varepsilon-1)(x^L_t-\kappa_{t+1}-x^*_{t+1})}}{\phi(\varepsilon-1)} \Gamma^5_{t+1} \\
&\quad + \left[C'(1-\alpha) - \alpha(1-\alpha)(1-\tau) \right] e^{-(\varepsilon-1)x^*_{t+1}}.
\end{aligned} \tag{101}
$$

B.5 Derivation of the law of motion for the price index (49)

By substituting Equations (100) and (101) into (95) and defining Γ_t and $\widetilde{\Gamma}_t$ as $\Gamma_t \equiv e^{(\varepsilon-1)(x^*_t-\kappa_{t+1})}\Gamma^1_{t+1}/(\alpha+(1-\alpha)\tau)$ and $\widetilde{\Gamma}_t \equiv \Gamma^5_{t+1}/\Gamma^1_{t+1}$, we obtain the law of motion for Γ_t

as

$$\begin{aligned}
\Gamma_t &\equiv e^{(\varepsilon-1)(x_t^*-\kappa_{t+1})}\frac{\Gamma_{t+1}^1}{\alpha+(1-\alpha)\tau} \\
&= (1-\alpha)(1-\tau)+e^{(\varepsilon-1)x_t^*}\left(\Gamma_t^2+\Gamma_t^3\right) \\
&= (1-\alpha)\frac{1-\tau}{1-\tau\varepsilon^{\phi}} \\
&\quad + \frac{e^{(\varepsilon-1)x_t^*}\Gamma_t^1}{\alpha+(1-\alpha)\tau}\left[\alpha+\frac{(1-\alpha)\tau}{\phi}\left\{x_t^H-x_t^L-\frac{e^{(\varepsilon-1)(x_t^H-x_t^*)}-e^{(\varepsilon-1)(x_t^L-x_t^*)}}{\varepsilon-1}\right\}\right. \\
&\quad \left. -\frac{(1-\alpha)\tau}{\phi}\left\{x_t^L-x_{t-1}^L+\kappa_t-\frac{e^{(\varepsilon-1)(x_t^L-x_t^*)}-e^{(\varepsilon-1)(x_{t-1}^L-x_t^*+\kappa_t)}}{\varepsilon-1}\right\}\frac{\Gamma_t^5}{\Gamma_t^1}\right] \\
&= (1-\alpha)C+\left(\alpha+f_{1,t}-f_{2,t}\widetilde{\Gamma}_{t-1}\right)e^{(\varepsilon-1)(x_t^*-x_{t-1}^*+\kappa_t)}\Gamma_{t-1}. \qquad (102)
\end{aligned}$$

The auxiliary variables $\widetilde{\Gamma}_t$, $f_{1,t}$ and $f_{2,t}$ and a constant C are defined by

$$\begin{aligned}
\widetilde{\Gamma}_t &\equiv \frac{\Gamma_{t+1}^5}{\Gamma_{t+1}^1}=\alpha e^{(\varepsilon-1)(x_t^*-x_{t-1}^*+\kappa_t)}\frac{\Gamma_{t-1}}{\Gamma_t}\left(1+\widetilde{\Gamma}_{t-1}\right) \qquad (103) \\
f_{1,t} &\equiv \frac{(1-\alpha)}{\phi}\tau\left[\Delta_t^H+\Delta_t^L-\frac{e^{(\varepsilon-1)\Delta_t^H}-e^{-(\varepsilon-1)\Delta_t^L}}{\varepsilon-1}\right] \\
f_{2,t} &\equiv \frac{(1-\alpha)}{\phi}\tau\left[(x_t^L-x_{t-1}^L+\kappa_t)-e^{-(\varepsilon-1)\Delta_t^L}\frac{1-e^{-(\varepsilon-1)(x_t^L-x_{t-1}^L+\kappa_t)}}{\varepsilon-1}\right] \\
C &\equiv \frac{1-\tau}{1-\tau\varepsilon^{\phi}},
\end{aligned}$$

where $\Delta_t^H\equiv x_t^H-x_t^*$ and $\Delta_t^L\equiv x_t^*-x_t^L$.

The variable Γ_t thus defined represents the gap between the aggregate price index and the reset price. By combining the definition of Γ_t with Equations (85) and (95), after rearranging, we obtain

$$\Gamma_t=e^{(\varepsilon-1)(p_t^*-p_t)}, \qquad (104)$$

where the lower case letters on the right-hand side represent the logarithm of prices P_t and P_t^*, i.e., $p_t^*\equiv\log P_t^*$ and $p_t\equiv\log P_t$, respectively, where $P_t^*\equiv W_te^{x_t^*}/Z_t$ represents the reset price for firms with idiosyncratic log-productivity level $z_{i,t}=0$.

By substituting Equation (104) and an equality $x_t^*-x_{t-1}^*+\kappa_t=p_t^*-p_{t-1}^*$ into (102) and (103), we obtain Equation (49):

$$e^{(1-\varepsilon)p_t} = (1-\alpha)Ce^{(1-\varepsilon)p_t^*} + \left(\alpha + f_{1,t} - f_{2,t}\widetilde{\Gamma}_{t-1}\right) e^{(1-\varepsilon)p_{t-1}}$$
$$\widetilde{\Gamma}_t = \alpha e^{(\varepsilon-1)\pi_t}\left(1+\widetilde{\Gamma}_{t-1}\right).$$

B.6 Labor demand for production

This Appendix derives the expression for the first term of the right-hand side in Equation (16). Using Equations (1) and (7) as well as the definitions of the markup $x_{i,t} \equiv \log(P_{i,t}Z_t e^{z_{i,t}}/W_t)$ and aggregate markup $x_t \equiv \log(P_t Z_t/W_t)$,

$$\begin{aligned}\int_0^1 di\ L_{i,t} &= \frac{Y_t}{Z_t}e^{\varepsilon x_t}\int_0^1 di\ e^{(\varepsilon-1)z_{i,t}}e^{-\varepsilon x_{i,t}} \\ &= A_t\int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dx\ e^{(\varepsilon-1)z}e^{-\varepsilon x}\psi_t^1(x,z),\end{aligned}$$

where we also use the definition of $A_t \equiv Y_t e^{\varepsilon x_t}/Z_t$. By substituting Equation (42) into this equation, we obtain

$$\int_0^1 di\ L_{i,t} = A_t\left[(1-\alpha)(1-\tau)e^{-\varepsilon x_t^*} + e^{-x_t^*}\Gamma_t^2 + \Gamma_t^6\right], \tag{105}$$

where Γ_t^2 is defined in Equation (82) and Γ_t^6 is defined as

$$\Gamma_t^6 \equiv \int_{S_t} dx\ e^{-x}\Psi_t(x).$$

For the closed-form expression for Γ_2, we substitute the definitions of $\Gamma_t \equiv e^{(\varepsilon-1)(x_t^*-\kappa_{t+1})}\Gamma_{t+1}^1/(\alpha+(1-\alpha)\tau)$ and $\widetilde{\Gamma}_t \equiv \Gamma_{t+1}^5/\Gamma_{t+1}^1$ into Equation (101) and obtain

$$\begin{aligned}\Gamma_t^2 = -(1-\alpha)\tau e^{(\varepsilon-1)(\kappa_t - x_{t-1}^*)}\Gamma_{t-1}&\left[\frac{e^{(\varepsilon-1)(x_t^H-x_t^*)} - e^{(\varepsilon-1)(x_t^L-x_t^*)}}{\phi(\varepsilon-1)}\right. \\ &\left. - \frac{e^{(\varepsilon-1)(x_t^L-x_t^*)} - e^{(\varepsilon-1)(x_{t-1}^L-x_t^*-\kappa_t)}}{\phi(\varepsilon-1)}\widetilde{\Gamma}_{t-1}\right] \\ &+ \left[C'(1-\alpha) - \alpha(1-\alpha)(1-\tau)\right]e^{-(\varepsilon-1)x_t^*}.\end{aligned} \tag{106}$$

On the other hand, we evaluate Γ_{t+1}^6 in a similar way to Γ_{t+1}^3. Specifically, following the strategy to derive Equation (100),

$$
\begin{aligned}
\Gamma^6_{t+1} &= \left(\int_{x_t^H-\kappa_{t+1}}^{x_{t+1}^H} - \int_{x_t^L-\kappa_{t+1}}^{x_{t+1}^L} + \int_{x_t^L-\kappa_{t+1}}^{x_t^H-\kappa_{t+1}}\right) dx\ e^{-x}\Psi_{t+1}(x) \\
&= \frac{(1-\alpha)\tau}{\alpha+(1-\alpha)\tau}\frac{-e^{-x_{t+1}^H}+e^{-x_{t+1}^L}}{\phi}\Gamma^1_{t+1} - \frac{(1-\alpha)\tau}{\alpha+(1-\alpha)\tau}\frac{-e^{-x_{t+1}^L}+e^{-x_t^L+\kappa_{t+1}}}{\phi}\Gamma^5_{t+1} \\
&\quad + \alpha\Bigg[(1-\alpha)(1-\tau)e^{(\varepsilon-1)(\kappa_{t+1}-x_t^*)}e^{-x_t^*+\kappa_{t+1}} + e^{(\varepsilon-1)\kappa_{t+1}}e^{-x_t^*+\kappa_{t+1}}\Gamma_t^2 \\
&\qquad + e^{(\varepsilon-1)\kappa_{t+1}}\int_{x_t^L-\kappa_{t+1}}^{x_t^H-\kappa_{t+1}} dx\ e^{-x}\Psi_t(x+\kappa_{t+1})\Bigg] \\
&= (1-\alpha)\tau e^{(\varepsilon-1)(\kappa_{t+1}-x_t^*)}\Gamma_t\left[\frac{-e^{-x_{t+1}^H}+e^{-x_{t+1}^L}}{\phi} - \frac{-e^{-x_{t+1}^L}+e^{-x_t^L+\kappa_{t+1}}}{\phi}\widetilde{\Gamma}_t\right] \\
&\quad + \alpha e^{\varepsilon\kappa_{t+1}}\left[(1-\alpha)(1-\tau)e^{-\varepsilon x_t^*} + e^{-x_t^*}\Gamma_t^2 + \Gamma_t^6\right].
\end{aligned} \tag{107}
$$

B.7 Labor demand for menu cost

This Appendix derives the expression for the second term of the right-hand side in Equation (16). In order to evaluate this integral, we realize that the number of firms paying the menu cost b corresponds to the first and second terms on the right-hand side of Equation (42). The rest of the calculation is straight-forward:

$$
\begin{aligned}
&\int_{\text{firms paying } b} di\ e^{(\varepsilon-1)z_{i,t}} \\
&\quad = \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dx\ e^{(\varepsilon-1)z}\left[(1-\alpha)(1-\tau)\delta\left(x-x_t^*\right)\delta(z) + \delta\left(x-x_t^*\right)\int_{S_t^c} dx'\psi_t^0\left(x',z\right)\right] \\
&\quad = (1-\alpha)(1-\tau) + e^{(\varepsilon-1)x_t^*}\Gamma_t^2,
\end{aligned} \tag{108}
$$

where we can use Equation (106) to explicitly obtain Γ_t^2.

C Equations to solve the model in general equilibrium

This Appendix collects all the equations for numerical calculations of our baseline menu cost model for convenience.

Euler equation (5):

$$\beta E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} e^{i_t - \pi_{t+1}} \right] = 1. \tag{109}$$

Labor supply curve (6):

$$\frac{W_t}{P_t} = L_t^{\varphi} C_t^{\sigma}. \tag{110}$$

Aggregate markup:

$$x_t = \log \left(\frac{P_t Z_t}{W_t} \right). \tag{111}$$

A macroeconomic variable determining the scale of firms' profit:

$$A_t = (Y_t / Z_t) e^{\varepsilon x_t}. \tag{112}$$

Growth rate of a factor scaling the value function plus stochastic discount factor:

$$\lambda_{t+1} = \log \left(\frac{Z_{t+1}}{Z_t} \right) - x_{t+1} + x_t + \log \left(\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \right). \tag{113}$$

Growth rate of nominal marginal cost, as defined in Equation (11):

$$\kappa_{t+1} = \pi_{t+1} - x_{t+1} + x_t. \tag{114}$$

Coefficients of the value function around the lower inaction threshold:

$$A_{1,t}^{(0)} = A_t + A_{4,t} \tag{115}$$

$$A_{2,t}^{(0)} = A_t \tag{116}$$

$$A_{3,t}^{(0)} = \left(\alpha + (1-\alpha)\tau\varepsilon^{\phi} \right) \beta E_t e^{\lambda_{t+1}} \Big[A_{1,t+1} e^{(-\varepsilon+1)x_{t+1}^*} - A_{2,t+1} e^{-\varepsilon x_{t+1}^*} + A_{3,t+1} - b \Big]. \tag{117}$$

Coefficients of the value function around the region between the reset markup and the upper inaction threshold:

$$A_{1,t} = A_{1,t}^{(0)} + \alpha\beta E_t e^{\lambda_{t+1}+(\varepsilon-1)\kappa_{t+1}} A_{1,t+1} \tag{118}$$

$$A_{2,t} = A_{2,t}^{(0)} + \alpha\beta E_t e^{\lambda_{t+1}+\varepsilon\kappa_{t+1}} A_{2,t+1} \tag{119}$$

$$A_{3,t} = \frac{(1-\alpha)\tau\varepsilon^{\phi}}{\alpha+(1-\alpha)\tau\varepsilon^{\phi}} A_{3,t}^{(0)} + \alpha\beta E_t e^{\lambda_{t+1}} A_{3,t+1}. \tag{120}$$

Reset markup:

$$x_t^* = \log\left(\frac{\varepsilon}{\varepsilon-1}\right) + \log\left(\frac{A_{2,t}}{A_{1,t}}\right). \tag{121}$$

Upper and lower thresholds defined in Equations (38)-(41), as well as the definitions of the gaps Δ_t^H and Δ_t^L:

$$A_{1,t} e^{(-\varepsilon+1)x_t^H} - A_{2,t} e^{-\varepsilon x_t^H} + A_{3,t} = A_{1,t} e^{(-\varepsilon+1)x_t^*} - A_{2,t} e^{-\varepsilon x_t^*} + A_{3,t} - b \tag{122}$$

$$A_{1,t}^{(0)} e^{(-\varepsilon+1)x_t^L} - A_{2,t}^{(0)} e^{-\varepsilon x_t^L} + A_{3,t}^{(0)} = A_{1,t} e^{(-\varepsilon+1)x_t^*} - A_{2,t} e^{-\varepsilon x_t^*} + A_{3,t} - b \tag{123}$$

$$\Delta_t^H = x_t^H - x_t^* \tag{124}$$

$$\text{s.t.} \quad \Delta_t^H > 0$$

$$\Delta_t^L = x_t^* - x_t^L \tag{125}$$

$$\text{s.t.} \quad \Delta_t^L > \log\left(\frac{A_{2,t}}{A_{1,t}}\right) - \log\left(\frac{A_{2,t}^{(0)}}{A_{1,t}^{(0)}}\right)$$

Other variables to determine policy rule:

$$A_{4,t} = (1-\alpha)\tau\beta E_t e^{\lambda_{t+1}+(\varepsilon-1)\kappa_{t+1}} A_{5,t+1}, \tag{126}$$

$$\begin{aligned} A_{5,t} = A_{1,t}^{(0)} & \left[\frac{\Delta_t^H + \Delta_t^L}{\phi} - \frac{e^{(\varepsilon-1)(\Delta_t^H+\Delta_t^L)} - 1}{\phi(\varepsilon-1)} \right] \\ & - A_{2,t}^{(0)} e^{-x_t^L} \left[\frac{-e^{-(\Delta_t^H+\Delta_t^L)} + 1}{\phi} - \frac{e^{(\varepsilon-1)(\Delta_t^H+\Delta_t^L)} - 1}{\phi(\varepsilon-1)} \right] \\ & + \alpha\beta E_t e^{\lambda_{t+1}+(\varepsilon-1)\kappa_{t+1}} \Bigg[A_{5,t+1} \\ & - A_{1,t+1} \left\{ \frac{x_{t+1}^H - x_t^H + \kappa_{t+1}}{\phi} - \frac{1 - e^{-(\varepsilon-1)(x_{t+1}^H - x_t^H + \kappa_{t+1})}}{\phi(\varepsilon-1)} \right\} \\ & + A_{2,t+1} e^{-x_{t+1}^H} \left\{ \frac{-1 + e^{x_{t+1}^H - x_t^H + \kappa_{t+1}}}{\phi} - \frac{1 - e^{-(\varepsilon-1)(x_{t+1}^H - x_t^H + \kappa_{t+1})}}{\phi(\varepsilon-1)} \right\} \Bigg]. \end{aligned} \tag{127}$$

The law of motion for price stickiness (49), rewritten by using $p_t^* - p_t = x_t^* - x_t$:

$$e^{(\varepsilon-1)(x_t - x_t^*)} = \frac{1 - \left(\alpha + f_{1,t} - f_{2,t}\widetilde{\Gamma}_{t-1}\right) e^{(\varepsilon-1)\pi_t}}{(1-\alpha)C}. \tag{128}$$

Auxiliary variables for the law of motion for price stickiness:

$$\widetilde{\Gamma}_t = \alpha e^{(\varepsilon-1)\pi_t} \left(1 + \widetilde{\Gamma}_{t-1}\right), \tag{129}$$

$$f_{1,t} = \frac{(1-\alpha)}{\phi}\tau \left[\Delta_t^H + \Delta_t^L - \frac{e^{(\varepsilon-1)\Delta_t^H} - e^{-(\varepsilon-1)\Delta_t^L}}{\varepsilon-1} \right] \tag{130}$$

$$f_{2,t} = \frac{(1-\alpha)}{\phi}\tau \left[(x_t^L - x_{t-1}^L + \kappa_t) - e^{-(\varepsilon-1)\Delta_t^L} \frac{1 - e^{-(\varepsilon-1)(x_t^L - x_{t-1}^L + \kappa_t)}}{\varepsilon-1} \right] \tag{131}$$

Taylor rule Equation (14):

$$i_t = \bar{r} + \bar{\pi} + \phi_\pi(\pi_t - \bar{\pi}) + v_t^m. \tag{132}$$

Goods market clearing shown in Equation (15):

$$C_t = Y_t. \tag{133}$$

Labor market clearing of Equation (16), combined with the expressions for labor demand in Equations (105) and (108):

$$
\begin{aligned}
L_t = A_t &\left[(1-\alpha)(1-\tau)e^{-\varepsilon x_t^*} + e^{-x_t^*}\Gamma_t^2 + \Gamma_t^6\right] \\
&+ b\left[(1-\alpha)(1-\tau) + e^{(\varepsilon-1)x_t^*}\Gamma_t^2\right],
\end{aligned} \tag{134}
$$

where auxiliary variables in the above equation are

$$
\begin{aligned}
\Gamma_t^2 = -(1-\alpha)\tau e^{(\varepsilon-1)(\kappa_t - x_{t-1})} &\left[\frac{e^{(\varepsilon-1)\Delta_t^H} - e^{-(\varepsilon-1)\Delta_t^L}}{\phi(\varepsilon-1)}\right. \\
&\left. - \frac{e^{-(\varepsilon-1)\Delta_t^L} - e^{-(\varepsilon-1)(x_t^* - x_{t-1}^* + \kappa_t + \Delta_{t-1}^L)}}{\phi(\varepsilon-1)}\widetilde{\Gamma}_{t-1}\right] \\
&+ \left[C'(1-\alpha) - \alpha(1-\alpha)(1-\tau)\right] e^{-(\varepsilon-1)x_t^*}.
\end{aligned} \tag{135}
$$

and

$$
\begin{aligned}
\Gamma_{t+1}^6 = (1-\alpha)\tau e^{(\varepsilon-1)(\kappa_{t+1}-x_t)} e^{-x_{t+1}^*} &\left[\frac{-e^{-\Delta_{t+1}^H} + e^{\Delta_{t+1}^L}}{\phi} - \frac{-e^{\Delta_{t+1}^L} + e^{x_{t+1}^* - x_t^* + \kappa_{t+1} + \Delta_t^L}}{\phi}\widetilde{\Gamma}_t\right] \\
+ \alpha e^{\varepsilon \kappa_{t+1}} &\left[(1-\alpha)(1-\tau)e^{-\varepsilon x_t^*} + e^{-x_t^*}\Gamma_t^2 + \Gamma_t^6\right].
\end{aligned} \tag{136}
$$

There are therefore 28 equations and 28 unknown endogenous variables: C_t, i_t, π_t, W_t/P_t, L_t, x_t, Y_t, A_t, λ_{t+1}, κ_{t+1}, $A_{1,t}^{(0)}$, $A_{2,t}^{(0)}$, $A_{3,t}^{(0)}$, $A_{1,t}$, $A_{2,t}$, $A_{3,t}$, $A_{4,t}$, $A_{5,t}$, x_t^*, x_t^H, x_t^L, Δ_t^H, Δ_t^L, $f_{1,t}$, $f_{2,t}$, $\widetilde{\Gamma}_t$, $\Gamma_{2,t}$, $\Gamma_{6,t}$. Together with exogenous shock process for Z_t or v_t^m, discussed in Section 4.1, and initial conditions, we can completely characterize the dynamics of the general equilibrium.

D Details of the Phillips curve

This Appendix shows details of the steps to derive the Phillips curve equation (58) from the equations listed in Appendix C. The derivation is organized as follows. Section D.1 shows that, under the approximations introduced at the beginning of Section 3.3, $A_{4,t}$ and $A_{5,t}$ are small both in the steady state and in log-deviation, which allows us to ignore these terms in the following discussions. Sections D.2 and D.3 evaluate upper and lower gaps, $\hat{\Delta}_t^H$ and $\hat{\Delta}_t^L$, respectively, which are important to analyze $\hat{f}_{1,t}$ and $\hat{f}_{2,t}$. Finally, Section D.4 derives Equation (58).

D.1 Evaluation of $A_{5,t}$

We evaluate the steady-state values of the variables in Equation (127) with respect to $\bar{A}_1^{(0)}$. First, we observe that the second and third terms inside the expectation E_t on the right-hand side are small. For the second term, we see that, using $\bar{\kappa} = \bar{\pi}$,

$$-\frac{\bar{A}_1}{\bar{A}_1^{(0)}}\left[\frac{\bar{\pi}}{\phi} - \frac{1-e^{-(\varepsilon-1)\bar{\pi}}}{\phi(\varepsilon-1)}\right] = \frac{1}{1-\alpha\beta e^{(\varepsilon-1)\bar{\pi}}}\left[\frac{\bar{\pi}}{\phi} - \frac{1-e^{-(\varepsilon-1)\bar{\pi}}}{\phi(\varepsilon-1)}\right] = \mathcal{O}\left(\bar{\pi}^2\right),$$

where the first equality derives from Equation (118), and the second is based on the Taylor expansion of the terms inside the square bracket. Similarly, for the third term,

$$\begin{aligned}
\frac{\bar{A}_2}{\bar{A}_1^{(0)}}e^{-\bar{x}^H}\left[\frac{-1+e^{\bar{\pi}}}{\phi} - \frac{1-e^{-(\varepsilon-1)\bar{\pi}}}{\phi(\varepsilon-1)}\right] &= \frac{\varepsilon-1}{\varepsilon}\frac{\bar{A}_1}{\bar{A}_1^{(0)}}e^{-\bar{\Delta}^H}\left[\frac{-1+e^{\bar{\pi}}}{\phi} - \frac{1-e^{-(\varepsilon-1)\bar{\pi}}}{\phi(\varepsilon-1)}\right] \\
&= \frac{\varepsilon-1}{\varepsilon}\frac{1}{1-\alpha\beta e^{(\varepsilon-1)\bar{\pi}}}e^{-\bar{\Delta}^H}\left[\frac{-1+e^{\bar{\pi}}}{\phi} - \frac{1-e^{-(\varepsilon-1)\bar{\pi}}}{\phi(\varepsilon-1)}\right] \\
&= \mathcal{O}\left(\bar{\pi}^2\right),
\end{aligned}$$

where the first equality uses Equations (121) and (124).

Secondly, we examine the first two terms of the right-hand side of Equation (127) in the steady state. Adopting the notation $\bar{\Delta} \equiv \left(\bar{\Delta}^H + \bar{\Delta}^L\right)/2$ for simplicity and using similar equations as above,

$$
\begin{aligned}
&\frac{\bar{A}_1^{(0)}}{\bar{A}_1^{(0)}}\left[\frac{2\bar{\Delta}}{\phi}-\frac{e^{2(\varepsilon-1)\bar{\Delta}}-1}{\phi(\varepsilon-1)}\right]-\frac{\bar{A}_2^{(0)}}{\bar{A}_1^{(0)}}e^{-\bar{x}^L}\left[\frac{-e^{-2\bar{\Delta}}+1}{\phi}-\frac{e^{(\varepsilon-1)2\bar{\Delta}}-1}{\phi(\varepsilon-1)}\right]\\
=&\left[\frac{2\bar{\Delta}}{\phi}-\frac{e^{2(\varepsilon-1)\bar{\Delta}}-1}{\phi(\varepsilon-1)}\right]-\left(1-\alpha\beta e^{\varepsilon\bar{\pi}}\right)\frac{\bar{A}_2}{\bar{A}_1^{(0)}}e^{-\bar{x}^*+\bar{\Delta}^L}\left[\frac{-e^{-2\bar{\Delta}}+1}{\phi}-\frac{e^{2(\varepsilon-1)\bar{\Delta}}-1}{\phi(\varepsilon-1)}\right]\\
=&\left[\frac{2\bar{\Delta}}{\phi}-\frac{e^{2(\varepsilon-1)\bar{\Delta}}-1}{\phi(\varepsilon-1)}\right]-\left(1-\alpha\beta e^{\varepsilon\bar{\pi}}\right)\frac{\bar{A}_1}{\bar{A}_1^{(0)}}\frac{\varepsilon-1}{\varepsilon}e^{\bar{\Delta}^L}\left[\frac{-e^{-2\bar{\Delta}}+1}{\phi}-\frac{e^{2(\varepsilon-1)\bar{\Delta}}-1}{\phi(\varepsilon-1)}\right]\\
=&\left[\frac{2\bar{\Delta}}{\phi}-\frac{e^{2(\varepsilon-1)\bar{\Delta}}-1}{\phi(\varepsilon-1)}\right]-\frac{1-\alpha\beta e^{\varepsilon\bar{\pi}}}{1-\alpha\beta e^{(\varepsilon-1)\bar{\pi}}}\frac{\varepsilon-1}{\varepsilon}e^{\bar{\Delta}^L}\left[\frac{-e^{-2\bar{\Delta}}+1}{\phi}-\frac{e^{(\varepsilon-1)2\bar{\Delta}}-1}{\phi(\varepsilon-1)}\right]\\
=&\mathcal{O}\left(\left(\bar{\Delta}\right)^2\bar{\Delta}^L\right),
\end{aligned}
$$

where we obtain the last expression by the first-order expansion of $\bar{\Delta}$ and $\bar{\Delta}^L$. This evaluation suggests that $\bar{A}_5$ in Equation (127) and hence $\bar{A}_4$ in Equation (126) are much smaller than $\bar{A}_1^{(0)}$, justifying the approximation $\bar{A}_1^{(0)} \approx \bar{A}$ in Equation (115). The steady-state values of these variables based on the calibration of our baseline model are consistent with the evaluations: $\bar{A} = 1.6$ and $\bar{A}_4 = 1.7 \times 10^{-4}$.

Next, we move on to show that the log-deviation of the variable $A_{5,t}$, expressed as $\hat{a}_{5,t}$, is also so small that we can ignore under our approximations introduced at the beginning of Section 3.3. We first observe that the second and third terms inside the expectation E_t on the right-hand side of Equation (127) are small:

$$
\bar{A}_1\left(1+\hat{a}_{1,t+1}\right)\left[\frac{\bar{\pi}+\hat{x}_{t+1}^H-\hat{x}_t^H+\hat{\kappa}_{t+1}}{\phi}-\frac{1-e^{-(\varepsilon-1)\bar{\pi}}\left(1-(\varepsilon-1)\left(\hat{x}_{t+1}^H-\hat{x}_t^H+\hat{\kappa}_{t+1}\right)\right)}{\phi(\varepsilon-1)}\right]\approx 0,
$$

$$
\begin{aligned}
&\bar{A}_2\left(1+\hat{a}_{2,t+1}\right)e^{-\bar{x}^H}\left(1-\hat{x}_{2,t+1}^H\right)\\
&\times\left[\frac{-1+e^{\bar{\pi}}\left(1+\hat{x}_{t+1}^H-\hat{x}_t^H+\hat{\kappa}_{t+1}\right)}{\phi}-\frac{1-e^{-(\varepsilon-1)\bar{\pi}}\left(1-(\varepsilon-1)\left(\hat{x}_{t+1}^H-\hat{x}_t^H+\hat{\kappa}_{t+1}\right)\right)}{\phi(\varepsilon-1)}\right]\approx 0,
\end{aligned}
$$

respectively, where we use the approximation to ignore the interaction terms of log-deviation variables with $\bar{\pi}$.

Secondly, we examine the first two terms of the right-hand side of Equation (127) in their log-linearized form. Using the notation $\hat{\Delta}_t \equiv \left(\hat{\Delta}_t^H + \hat{\Delta}_t^L\right)/2$,

$$
\begin{aligned}
&\bar{A}_1^{(0)}\left(1+\hat{a}_{1,t}^{(0)}\right)\left[2\frac{\bar{\Delta}+\hat{\Delta}_t}{\phi} - \frac{e^{2(\varepsilon-1)\bar{\Delta}}\left(1+2(\varepsilon-1)\hat{\Delta}_t\right)-1}{\phi(\varepsilon-1)}\right] \\
&- \bar{A}_2^{(0)}\left(1+\hat{a}_{2,t}^{(0)}\right)e^{-\bar{x}^L}\left(1-\hat{x}_t^L\right)\left[\frac{-e^{-2\bar{\Delta}}\left(1-2\hat{\Delta}_t\right)+1}{\phi} - \frac{e^{2(\varepsilon-1)\bar{\Delta}}\left(1+2(\varepsilon-1)\hat{\Delta}_t\right)-1}{\phi(\varepsilon-1)}\right] \\
&\approx SS \ + \frac{\bar{A}_1^{(0)}}{\phi}\left[\left(2\bar{\Delta} - \frac{e^{2(\varepsilon-1)\bar{\Delta}}-1}{\varepsilon-1}\right)\hat{a}_{1,t}^{(0)} + 2\left(1-e^{2(\varepsilon-1)\bar{\Delta}}\right)\hat{\Delta}_t\right] \\
&- \frac{\bar{A}_2^{(0)}}{\phi}e^{-\bar{x}^*+\bar{\Delta}^L}\left[\left(1-e^{-2\bar{\Delta}} - \frac{e^{2(\varepsilon-1)\bar{\Delta}}-1}{\varepsilon-1}\right)\left(\hat{a}_{2,t}^{(0)}-\hat{x}_t^L\right) + 2\left(e^{-2\bar{\Delta}} - e^{2(\varepsilon-1)\bar{\Delta}}\right)\hat{\Delta}_t\right] \\
&\approx SS \ - 4\left[\frac{\bar{A}_1^{(0)}}{\phi}(\varepsilon-1) - \frac{\bar{A}_2^{(0)}}{\phi}e^{-\bar{x}^*+\bar{\Delta}^L}\varepsilon\right]\bar{\Delta}\hat{\Delta}_t \\
&\approx SS \ - 4\left[\frac{\bar{A}_1^{(0)}}{\phi}(\varepsilon-1) - \frac{\bar{A}_2^{(0)}}{\phi}\left(\frac{\varepsilon-1}{\varepsilon}\frac{\bar{A}_1}{\bar{A}_2}\right)e^{\bar{\Delta}^L}\varepsilon\right]\bar{\Delta}\hat{\Delta}_t \\
&\approx SS \ - 4(\varepsilon-1)\frac{\bar{A}_1}{\phi}\left[\left(1-\alpha\beta e^{(\varepsilon-1)\bar{\pi}}\right) - \left(1-\alpha\beta e^{\varepsilon\bar{\pi}}\right)e^{\bar{\Delta}^L}\right]\bar{\Delta}\hat{\Delta}_t \\
&\approx SS,
\end{aligned}
$$

where SS denotes steady-state values, and Equations (118), (119) and (121) in the steady state are used. The last (approximate) equality follows because the log-deviation term is $\mathcal{O}\left(\bar{\Delta}^L\bar{\Delta}\hat{\Delta}_t\right)$. To sum up, we can safely ignore $\hat{a}_{5,t}$ and hence $\hat{a}_{4,t}$.

In the following discussion in Appendix D, we therefore ignore $A_{4,t}$ altogether and use the equality $A_{1,t}^{(0)} \approx A_t$ instead of Equation (115).

D.2 Derivation of $\hat{\Delta}_t^H \approx 0$

We next log-linearize Equation (122), replicated below, to obtain $\hat{\Delta}_t^H$:

$$
A_{1,t}e^{(-\varepsilon+1)\left(x_t^*+\Delta_t^H\right)} - A_{2,t}e^{-\varepsilon\left(x_t^*+\Delta_t^H\right)} + A_{3,t} = A_{1,t}e^{(-\varepsilon+1)x_t^*} - A_{2,t}e^{-\varepsilon x_t^*} + A_{3,t} - b.
$$

After removing $A_{3,t}$ from both sides, the right-hand side of the above equation is log-linearized as

$$\begin{aligned}&\bar{A}_1 e^{(-\varepsilon+1)\bar{x}^*}\left(1+\hat{a}_{1,t}-(\varepsilon-1)\,\hat{x}_t^*\right)-\bar{A}_2 e^{-\varepsilon\bar{x}^*}\left(1+\hat{a}_{2,t}-\varepsilon\hat{x}_t^*\right)-b\\ &= SS\ +\bar{A}_2 e^{-\varepsilon\bar{x}^*}\left[\frac{\varepsilon}{\varepsilon-1}\hat{a}_{1,t}-\hat{a}_{2,t}\right],\end{aligned} \tag{137}$$

where Equation (121) in the steady state is used to derived the equality. The left-hand side is similarly log-linearized as

$$\begin{aligned}&\bar{A}_1 e^{(-\varepsilon+1)\left(\bar{x}^*+\bar{\Delta}^H\right)}\left(1+\hat{a}_{1,t}-(\varepsilon-1)\left(\hat{x}_t^*+\hat{\Delta}_t^H\right)\right)-\bar{A}_2 e^{-\varepsilon\left(\bar{x}^*+\bar{\Delta}^H\right)}\left(1+\hat{a}_{2,t}-\varepsilon\left(\hat{x}_t^*+\hat{\Delta}_t^H\right)\right)\\ &= SS\ +\bar{A}_2 e^{-\varepsilon\bar{x}^*}\left[\frac{\varepsilon}{\varepsilon-1}e^{(-\varepsilon+1)\bar{\Delta}^H}\hat{a}_{1,t}-e^{-\varepsilon\bar{\Delta}^H}\hat{a}_{2,t}-\varepsilon\left(e^{(-\varepsilon+1)\bar{\Delta}^H}-e^{-\varepsilon\bar{\Delta}^H}\right)\left(\hat{x}_t^*+\hat{\Delta}_t^H\right)\right]\\ &\approx SS\ +\bar{A}_2 e^{-\varepsilon\bar{x}^*}\left[\frac{\varepsilon}{\varepsilon-1}\hat{a}_{1,t}-\hat{a}_{2,t}-\varepsilon\left(e^{(-\varepsilon+1)\bar{\Delta}^H}-e^{-\varepsilon\bar{\Delta}^H}\right)\hat{\Delta}_t^H\right],\end{aligned}$$

where the first equality derives from Equation (121) in the steady state and the second is obtained from the approximation $e^{\varepsilon\bar{\Delta}^H} \approx \left(1+\varepsilon\bar{\Delta}^H\right)$ as well as the same equation (121) expressed in the log-deviation: $\hat{x}_t^* = \hat{a}_{2,t} - \hat{a}_{1,t}$. By combining the right-hand side with the left-hand side, we obtain the relation $\hat{\Delta}_t^H \approx 0$.

D.3 Derivation of $\hat{\Delta}_t^L \approx \hat{x}_t^*$

Similarly to the previous section D.2, we log-linearize Equation (123), replicated below, to obtain $\hat{\Delta}_t^L$:

$$A_{1,t}^{(0)} e^{(-\varepsilon+1)x_t^L} - A_{2,t}^{(0)} e^{-\varepsilon x_t^L} + A_{3,t}^{(0)} = A_{1,t} e^{(-\varepsilon+1)x_t^*} - A_{2,t} e^{-\varepsilon x_t^*} + A_{3,t} - b.$$

The right-hand side is similar to the case of Equation (122), shown in Equation (137) above:

$$SS\ +\bar{A}_2 e^{-\varepsilon\bar{x}^*}\left[\frac{\varepsilon}{\varepsilon-1}\hat{a}_{1,t}-\hat{a}_{2,t}\right]+\bar{A}_3\hat{a}_{3,t}.$$

On the other hand, the left-hand side is

$$
\begin{aligned}
&\bar{A}_1^{(0)} e^{(-\varepsilon+1)\left(\bar{x}^*-\bar{\Delta}^L\right)} \left(1+\hat{a}_{1,t}^{(0)}-(\varepsilon-1)\,\hat{x}_t^L\right) \\
&-\bar{A}_2^{(0)} e^{-\varepsilon\left(\bar{x}^*-\bar{\Delta}^L\right)} \left(1+\hat{a}_{2,t}^{(0)}-\varepsilon\hat{x}_t^L\right)+\bar{A}_3^{(0)}\left(1+\hat{a}_{3,t}^{(0)}\right) \\
\approx\ & SS\ +(1-\alpha\beta)\bar{A}_2 e^{-\varepsilon\bar{x}^*}\left[\frac{\varepsilon}{\varepsilon-1}e^{(\varepsilon-1)\bar{\Delta}^L}\hat{a}_{1,t}^{(0)}-e^{\varepsilon\bar{\Delta}^L}\hat{a}_{2,t}^{(0)}\right. \\
&\qquad\qquad\left.-\varepsilon\left(e^{(\varepsilon-1)\bar{\Delta}^L}-e^{\varepsilon\bar{\Delta}^L}\right)\hat{x}_t^L\right]+\bar{A}_3^{(0)}\hat{a}_{3,t}^{(0)},
\end{aligned}
$$

where we use Equations (118), (119) and (121) as well as the approximation that the trend inflation is small. Put together,

$$
\begin{aligned}
(1-\alpha\beta)\varepsilon\left(e^{(\varepsilon-1)\bar{\Delta}^L}-e^{\varepsilon\bar{\Delta}^L}\right)\hat{x}_t^L \approx\ & (1-\alpha\beta)\left[\frac{\varepsilon}{\varepsilon-1}e^{(\varepsilon-1)\bar{\Delta}^L}\hat{a}_{1,t}^{(0)}-e^{\varepsilon\bar{\Delta}^L}\hat{a}_{2,t}^{(0)}\right] \\
&-\left[\frac{\varepsilon}{\varepsilon-1}\hat{a}_{1,t}-\hat{a}_{2,t}\right]-\frac{e^{\varepsilon\bar{x}^*}}{\bar{A}_2}\left(\bar{A}_3\hat{a}_{3,t}-\bar{A}_3^{(0)}\hat{a}_{3,t}^{(0)}\right)
\end{aligned}
\tag{138}
$$

We now consider the terms proportional to $\hat{a}_{1,t}^{(0)}$ and $\hat{a}_{1,t}$ in Equation (138). Using Equation (118), these terms are rewritten as

$$
\begin{aligned}
&(1-\alpha\beta)\frac{\varepsilon}{\varepsilon-1}e^{(\varepsilon-1)\bar{\Delta}^L}\hat{a}_{1,t}^{(0)}-\frac{\varepsilon}{\varepsilon-1}\hat{a}_{1,t} \\
&\approx\frac{\varepsilon}{\varepsilon-1}\left[(1-\alpha\beta)\left(e^{(\varepsilon-1)\bar{\Delta}^L}-1\right)\hat{a}_{1,t}^{(0)}-\alpha\beta E_t\left(\hat{\lambda}_{t+1}+(\varepsilon-1)\hat{\kappa}_{t+1}+\hat{a}_{1,t+1}\right)\right] \\
&\approx(1-\alpha\beta)\varepsilon\bar{\Delta}^L\hat{a}_{1,t}^{(0)}-\alpha\beta E_t\left(\frac{\varepsilon}{\varepsilon-1}\hat{\lambda}_{t+1}+\varepsilon\hat{\kappa}_{t+1}+\frac{\varepsilon}{\varepsilon-1}\hat{a}_{1,t+1}\right).
\end{aligned}
$$

Similarly, using Equation (119), the terms proportional to $\hat{a}_{2,t}^{(0)}$ and $\hat{a}_{2,t}$ become

$$
\begin{aligned}
&-(1-\alpha\beta)e^{\varepsilon\bar{\Delta}^L}\hat{a}_{2,t}^{(0)}+\hat{a}_{2,t} \\
&\approx(1-\alpha\beta)\left(-e^{\varepsilon\bar{\Delta}^L}+1\right)\hat{a}_{2,t}^{(0)}+\alpha\beta E_t\left(\hat{\lambda}_{t+1}+\varepsilon\hat{\kappa}_{t+1}+\hat{a}_{2,t+1}\right) \\
&\approx-(1-\alpha\beta)\varepsilon\bar{\Delta}^L\hat{a}_{2,t}^{(0)}+\alpha\beta E_t\left(\hat{\lambda}_{t+1}+\varepsilon\hat{\kappa}_{t+1}+\hat{a}_{2,t+1}\right)
\end{aligned}
$$

By combining these terms and using $\hat{a}_{1,t}^{(0)} \approx \hat{a}_t = \hat{a}_{2,t}^{(0)}$, established in Section D.1, we have

$$\alpha\beta E_t\left(-\frac{1}{\varepsilon-1}\hat{\lambda}_{t+1} - \frac{\varepsilon}{\varepsilon-1}\hat{a}_{1,t+1} + \hat{a}_{2,t+1}\right).$$

Next, we consider remaining terms related to $\hat{a}_{3,t}$ and $\hat{a}_{3,t}^{(0)}$ in Equation (138). To make the exposition clearer, we define $V_t \equiv A_{1,t}e^{(-\varepsilon+1)x_t^*} - A_{2,t}e^{-\varepsilon x_t^*} + A_{3,t} - b$. It then follows that Equation (117) in a log-linearized form becomes

$$\bar{A}_3^{(0)}\hat{a}_{3,t} \approx \left(\alpha + (1-\alpha)\tau\varepsilon^{\phi}\right)\beta\bar{V}E_t\left(\hat{\lambda}_{t+1} + \hat{v}_{t+1}\right).$$

Similarly, Equation (120) in a log-linearized form is

$$\bar{A}_3\hat{a}_{3,t} \approx (1-\alpha)\tau\varepsilon^{\phi}\beta\bar{V}E_t\left(\hat{\lambda}_{t+1} + \hat{v}_{t+1}\right) + \alpha\beta\bar{A}_3E_t\left(\hat{\lambda}_{t+1} + \hat{a}_{3,t+1}\right).$$

The remaining terms in Equation (138) are therefore rearranged as

$$\begin{aligned}
-\frac{e^{\varepsilon\bar{x}^*}}{\bar{A}_2}\left(\bar{A}_3\hat{a}_{3,t} - \bar{A}_3^{(0)}\hat{a}_{3,t}^{(0)}\right) &\approx -\frac{e^{\varepsilon\bar{x}^*}}{\bar{A}_2}\alpha\beta E_t\left[\left(\bar{A}_3 - \bar{V}\right)\hat{\lambda}_{t+1} + \bar{A}_3\hat{a}_{3,t+1} - \bar{V}\hat{v}_{t+1}\right] \\
&\approx -\frac{e^{\varepsilon\bar{x}^*}}{\bar{A}_2}\alpha\beta E_t\left[\left(\bar{A}_3 - \bar{V}\right)\hat{\lambda}_{t+1} - \bar{A}_2e^{-\varepsilon\bar{x}^*}\left(\frac{\varepsilon}{\varepsilon-1}\hat{a}_{1,t+1} - \hat{a}_{2,t+1}\right)\right] \\
&\approx \alpha\beta E_t\left(\frac{\varepsilon}{\varepsilon-1}\hat{a}_{1,t+1} - \hat{a}_{2,t+1}\right) - \frac{e^{\varepsilon\bar{x}^*}}{\bar{A}_2}\alpha\beta E_t\left(\bar{A}_3 - \bar{V}\right)\hat{\lambda}_{t+1},
\end{aligned}$$

where we use the definition of V_t in a log-linearized and rearranged form (Equation (137) apart from $\bar{A}_3\hat{a}_t$) for the second approximate equality.

By putting back all the terms, we rewrite Equation (138) as

$$\begin{aligned}
(1-\alpha\beta)\varepsilon\left(e^{(\varepsilon-1)\bar{\Delta}^L} - e^{\varepsilon\bar{\Delta}^L}\right)\hat{x}_t^L &\approx \alpha\beta E_t\left(-\frac{1}{\varepsilon-1}\hat{\lambda}_{t+1} - \frac{\varepsilon}{\varepsilon-1}\hat{a}_{1,t+1} + \hat{a}_{2,t+1}\right) \\
&\quad + \alpha\beta E_t\left(\frac{\varepsilon}{\varepsilon-1}\hat{a}_{1,t+1} - \hat{a}_{2,t+1}\right) - \frac{e^{\varepsilon\bar{x}^*}}{\bar{A}_2}\alpha\beta E_t\left(\bar{A}_3 - \bar{V}\right)\hat{\lambda}_{t+1} \\
&= \alpha\beta\left(e^{\varepsilon\bar{x}^*}\frac{\bar{V} - \bar{A}_3}{\bar{A}_2} - \frac{1}{\varepsilon-1}\right)E_t\hat{\lambda}_{t+1}. \qquad (139)
\end{aligned}$$

To evaluate the importance of the term on the right-hand side, we combine the definition of V_t with Equation (122) in the steady state as

$$\begin{aligned}
\bar{V} &= \bar{A}_1 e^{(-\varepsilon+1)\left(\bar{x}^*+\bar{\Delta}^H\right)} - \bar{A}_2 e^{-\varepsilon\left(\bar{x}^*+\bar{\Delta}^H\right)} + \bar{A}_3 \\
\Longleftrightarrow e^{\varepsilon\bar{x}^*}\frac{\bar{V}-\bar{A}_3}{\bar{A}_2} - \frac{1}{\varepsilon-1} &= \frac{\bar{A}_1}{\bar{A}_2} e^{\bar{x}^*} e^{(-\varepsilon+1)\bar{\Delta}^H} - e^{-\varepsilon\bar{\Delta}^H} - \frac{1}{\varepsilon-1} \\
&= \frac{\varepsilon}{\varepsilon-1} e^{(-\varepsilon+1)\bar{\Delta}^H} - e^{-\varepsilon\bar{\Delta}^H} - \frac{1}{\varepsilon-1} \\
&\approx 0
\end{aligned}$$

up to the first order in $\bar{\Delta}^H$,where we use Equation (121) in the steady state. Equation (139) therefore reduces to

$$\hat{x}_t^L \approx 0,$$

which immediately implies

$$\hat{\Delta}_t^L \approx \hat{x}_t^*.$$

D.4 Derivation of the Phillips curve equation

We first linearize $f_{1,t}$ and $f_{2,t}$ in Equations (130) and (131). The steady-state values of the two terms are rewritten as

$$\begin{aligned}
\bar{f}_1 &\approx \frac{(1-\alpha)}{\phi}\tau\frac{1}{2}(\varepsilon-1)\left[\left(\bar{\Delta}^L\right)^2 - \left(\bar{\Delta}^H\right)^2\right] \\
\bar{f}_2 &\approx \frac{(1-\alpha)}{\phi}\tau(\varepsilon-1)\bar{\pi}\bar{\Delta}^L.
\end{aligned}$$

These values are clearly much smaller than α in Equation (128), so we ignore them.

The linearized deviations from the steady state of these terms are not negligible, however. After straight-forward calculations, we obtain

$$\begin{aligned}
\hat{f}_{1,t} &\approx \frac{(1-\alpha)}{\phi}\tau(\varepsilon-1)\left(-\bar{\Delta}^H\hat{\Delta}_t^H + \bar{\Delta}^L\hat{\Delta}_t^L\right), \\
\hat{f}_{2,t} &\approx \frac{(1-\alpha)}{\phi}\tau(\varepsilon-1)\bar{\Delta}^L\left(\hat{x}_t^L - \hat{x}_{t-1}^L + \hat{\kappa}_t\right).
\end{aligned}$$

Given the evaluations of $\hat{\Delta}_t^H$, $\hat{\Delta}_t^L$, and $\hat{x}_t^L$ in Sections D.2 and D.3, we simplify the expressions as

$$\hat{f}_{1,t} \approx \frac{(1-\alpha)}{\phi}\tau\left(\varepsilon-1\right)\bar{\Delta}^{L}\hat{x}_{t}^{*} = \left(1-\alpha\right)\left(\varepsilon-1\right)\zeta_{1}\hat{x}_{t}^{*},$$
$$\hat{f}_{2,t} \approx \frac{(1-\alpha)}{\phi}\tau\left(\varepsilon-1\right)\bar{\Delta}^{L}\hat{\kappa}_{t} = \left(1-\alpha\right)\left(\varepsilon-1\right)\zeta_{1}\hat{\kappa}_{t},$$

where we define the constants by $\zeta_1 \equiv \tau\bar{\Delta}^L/\phi$. In addition, Equation (129) in the steady state is approximately written as

$$\bar{\bar{\Gamma}} = \frac{\alpha e^{(\varepsilon-1)\bar{\pi}}}{1-\alpha e^{(\varepsilon-1)\bar{\pi}}} \approx \frac{\alpha}{1-\alpha}.$$

We substitute these expressions and the definition of κ_t in Equation (114) into log-linearized Equation (128). After some rearrangement, we obtain

$$\begin{aligned}
& e^{(\varepsilon-1)(\bar{x}-\bar{x}^*)}\left[1+(\varepsilon-1)(\hat{x}_t-\hat{x}_t^*)\right]. \\
& \approx \frac{1-\left(\alpha+\hat{f}_{1,t}-\hat{f}_{2,t}\bar{\bar{\Gamma}}\right)\left(1+(\varepsilon-1)\hat{\pi}_t\right)}{(1-\alpha)C} \\
\Longleftrightarrow & (\varepsilon-1)(\hat{x}_t-\hat{x}_t^*) \approx -\frac{1}{1-\alpha}\left(\hat{f}_{1,t}-\hat{f}_{2,t}\bar{\bar{\Gamma}}\right)-\frac{\alpha}{1-\alpha}(\varepsilon-1)\hat{\pi}_t \\
\Longleftrightarrow & \hat{x}_t^* \approx \frac{\alpha}{1-\alpha}\hat{\pi}_t+\frac{1}{1-\zeta_1}\hat{x}_t+\frac{\alpha}{1-\alpha}\frac{\zeta_1}{1-\zeta_1}\left(\hat{x}_t-\hat{x}_{t-1}\right).
\end{aligned} \tag{140}$$

On the other hand, Equation (37) representing the optimal policy suggests that

$$\hat{x}_t^* = \hat{a}_{2,t} - \hat{a}_{1,t}, \tag{141}$$

while Equations (34) and (35) suggest that

$$\hat{a}_{1,t} \approx (1-\alpha\beta)\hat{a}_t + \alpha\beta E_t(\hat{\lambda}_{t+1} + (\varepsilon-1)\hat{\kappa}_{t+1} + \hat{a}_{1,t+1}), \tag{142}$$

and

$$\hat{a}_{2,t} \approx (1-\alpha\beta)\hat{a}_t + \alpha\beta E_t(\hat{\lambda}_{t+1} + \varepsilon\hat{\kappa}_{t+1} + \hat{a}_{2,t+1}). \tag{143}$$

By subtracting (142) from (143) and substituting (141) and (114) into it, we obtain

$$\hat{x}_t^* \approx \alpha\beta E_t(\hat{\pi}_{t+1} - \hat{x}_{t+1} + \hat{x}_t + \hat{x}_{t+1}^*). \tag{144}$$

Finally, by combining Equation (140) with (144) and using $\hat{mc}_t \equiv -\hat{x}_t$, we obtain

$$\begin{aligned}\hat{\pi}_t \approx \beta E_t\hat{\pi}_{t+1} + \lambda_c\hat{mc}_t \\ &+ \frac{1-\alpha}{\alpha}\frac{\zeta_1}{1-\zeta_1}\left(\hat{mc}_t - \alpha\beta E_t\hat{mc}_{t+1}\right) \\ &+ \frac{\zeta_1}{1-\zeta_1}\left(\Delta\hat{mc}_t - \alpha\beta E_t\Delta\hat{mc}_{t+1}\right).\end{aligned}$$

By defining $\zeta \equiv \zeta_1/(1-\zeta_1)$,we obtain the Phillips curve in Equation (58).

It is interesting to see, though not discussed in the main text, that we can rewrite the above Phillips curve equation in another form. Namely, it is easy to show that the inflation rate is decomposed into two contributions as $\hat{\pi}_t \approx \hat{\pi}_{1,t} + \hat{\pi}_{2,t}$: The former term is

$$\hat{\pi}_{1,t} \equiv \beta E_t\hat{\pi}_{1,t+1} + \lambda_c\left(1+\zeta\right)\hat{mc}_t,$$

representing a conventional, though steeper, Calvo-like Phillips curve; and the latter term is

$$\hat{\pi}_{2,t} \equiv \zeta\Delta\hat{mc}_t,$$

representing an instantaneous effect of a change in the real marginal cost. The asymmetry in policy rule primarily steepens the the slope of the Phillips curve through the former term $\hat{\pi}_{1,t}$, while the impact of the asymmetry in markup distribution primarily generates the latter term $\hat{\pi}_{2,t}$.

E Derivation of frequency and size of price adjustment

The set of equations used to solve the general equilibrium New Keynesian model in this paper, shown in Appendix C, does not directly use frequency or size of price adjustment. However, empirical studies using micro data of prices typically measure these variables. It is therefore of interest to derive the expression for these variables in our model. Moreover, the calibration of key parameters shown in Section 4.1 uses the expressions for these variables.

E.1 Frequency of price adjustment

To proceed, we first define firms' density function $\Psi_t^0(x)$ as

$$\Psi_t^0(x) \equiv \int_{-\infty}^{\infty} dz\ \psi_t^0(x,z). \tag{145}$$

The frequency of price adjustment, denoted by fr_t, is related to the density $\Psi_t^0(x)$ as

$$fr_t = \int_{S_t^c} dx\ \Psi_t^0(x), \tag{146}$$

where $S_t^c \equiv (-\infty, x_t^L) \cup (x_t^H, \infty)$ is the region outside the inaction region S_t. In addition, we define the frequency of not adjusting price fr_t^c as

$$fr_t^c = \int_{S_t} dx\ \Psi_t^0(x), \tag{147}$$

which is related to fr_t by $fr_t^c = \alpha + (1-\alpha)\tau - fr_t$, excluding the frequency of firm exits $(1-\alpha)(1-\tau)$.

We derive master equations for the density $\Psi_t^0(x)$ from Equations (42) and (43), in a way similar to the master equations for $\Psi_t(x)$ in Appendix B.2. It is convenient to use the same notation (A)-(F) as $\Psi_t(x)$ shown in Table 2.

(A):

$$\Psi_{t+1}^{0(A)}(x) = \frac{(1-\alpha)\tau}{\phi}(1-\alpha)(1-\tau)I\left(|x+\kappa_{t+1}-x_t^*| \leqq \frac{\phi}{2}\right). \tag{148}$$

(B):

$$\Psi_{t+1}^{0(B)}(x) = \alpha(1-\alpha)(1-\tau)\delta\left(x+\kappa_{t+1}-x_t^*\right). \tag{149}$$

(C):

$$\Psi_{t+1}^{0(C)}(x) = \frac{(1-\alpha)\tau}{\phi} I\left(|x+\kappa_{t+1}-x_t^*| \leqq \frac{\phi}{2}\right) \int_{S_t^c} dx' \ \Psi_t^0(x')$$
$$= \frac{(1-\alpha)\tau}{\phi} I\left(|x+\kappa_{t+1}-x_t^*| \leqq \frac{\phi}{2}\right) fr_t. \tag{150}$$

(D):

$$\Psi_{t+1}^{0(D)}(x) = \alpha\delta\left(x+\kappa_{t+1}-x_t^*\right) \int_{S_t^c} dx' \ \Psi_t^0(x')$$
$$= \alpha\delta\left(x+\kappa_{t+1}-x_t^*\right) fr_t. \tag{151}$$

(E1) : For $x \in [x_t^L - \phi/2 - \kappa_{t+1}, x_t^H - \phi/2 - \kappa_{t+1})$,

$$\Psi_{t+1}^{0(E1)}(x) = \frac{(1-\alpha)\tau}{\phi} \int_{x_t^L}^{x+\kappa_{t+1}+\frac{\phi}{2}} dx' \ \Psi_t^0(x'). \tag{152}$$

(E2) : For $x \in [x_t^H - \phi/2 - \kappa_{t+1}, x_t^L + \phi/2 - \kappa_{t+1}]$,

$$\Psi_{t+1}^{0(E2)}(x) = \frac{(1-\alpha)\tau}{\phi} \int_{S_t} dx' \ \Psi_t^0(x')$$
$$= \frac{(1-\alpha)\tau}{\phi} fr_t^c. \tag{153}$$

(E3) : For $x \in (x_t^L + \phi/2 - \kappa_{t+1}, x_t^H + \phi/2 - \kappa_{t+1}]$,

$$\Psi_{t+1}^{0(E3)}(x) = \frac{(1-\alpha)\tau}{\phi} \int_{x+\kappa_{t+1}-\frac{\phi}{2}}^{x_t^H} dx' \ \Psi_t^0(x'). \tag{154}$$

(F):

$$\Psi_{t+1}^{0(F)}(x) = \alpha\Psi_t^0\left(x+\kappa_{t+1}\right) I\left(x+\kappa_{t+1} \in S_t\right). \tag{155}$$

Because each term of $\Psi_t^0(x)$ is non-zero in the exactly same region as the corresponding term of $\Psi_t(x)$ shown in Figure 8, we see that for $x \in [x_t^H - \phi/2 - \kappa_{t+1}, x_t^L - \kappa_{t+1}) \cup (x_t^H -$

$\kappa_{t+1}, x_t^L + \phi/2 - \kappa_{t+1}]$,

$$\begin{aligned}
\Psi_{t+1}^0(x) &= \sum_{I=A,C,E2} \Psi_{t+1}^{0(I)}(x) \\
&= \frac{(1-\alpha)\tau}{\phi}(1-\alpha)(1-\tau) + \frac{(1-\alpha)\tau}{\phi} fr_t + \frac{(1-\alpha)\tau}{\phi} fr_t^c \\
&= \frac{(1-\alpha)\tau}{\phi},
\end{aligned} \tag{156}$$

i.e., the density is constant. For $x \in [x_t^L - \kappa_{t+1}, x_t^H - \kappa_{t+1}]$ excluding at $x = x_t^* - \kappa_{t+1}$,

$$\begin{aligned}
\Psi_{t+1}^0(x) &= \sum_{I=A,C,E2,F} \Psi_{t+1}^{0(I)}(x) \\
&= \frac{(1-\alpha)\tau}{\phi} + \alpha \Psi_t^0\left(x + \kappa_{t+1}\right).
\end{aligned} \tag{157}$$

Following an argument similar to that used to derive Equation (48), we obtain that, under Assumptions 2 and 3, for $\forall x \in [x_t^L - \kappa_{t+1}, x_{t+1}^L]$,

$$\Psi_{t+1}^0(x) = \frac{(1-\alpha)\tau}{\phi}\left(1 + \frac{\alpha}{1-\alpha}\right). \tag{158}$$

These expressions suggest that $\Psi_t^0(x)$ is qualitatively similar to $\Psi_t(x)$: both look like the schematic picture in Figure 3. Using the expressions for $\Psi_t^0(x)$, we explicitly evaluate fr_{t+1}^c in a similar way to Γ_{t+1}^3in Equation (100):

$$\begin{aligned}
fr_{t+1}^c &= \int_{x_{t+1}^L}^{x_{t+1}^H} dx\ \Psi_{t+1}^0(x) \\
&= \left(\int_{x_t^H - \kappa_{t+1}}^{x_{t+1}^H} - \int_{x_t^L - \kappa_{t+1}}^{x_{t+1}^L} + \int_{x_t^L - \kappa_{t+1}}^{x_t^H - \kappa_{t+1}}\right) dx\ \Psi_{t+1}^0(x) \\
&= \frac{(1-\alpha)\tau}{\phi}\left(x_{t+1}^H - x_{t+1}^L\right) - \frac{(1-\alpha)\tau}{\phi}\frac{\alpha}{1-\alpha}\left(x_{t+1}^L - x_t^L + \kappa_{t+1}\right) \\
&\quad + \alpha \int_{x_t^L - \kappa_{t+1}}^{x_t^H - \kappa_{t+1}} dx\ \Psi_t^0\left(x + \kappa_{t+1}\right) \\
&\quad + \alpha(1-\alpha)(1-\tau) + \alpha fr_t \\
&= \alpha + (1-\alpha)\tau\left(\frac{x_{t+1}^H - x_{t+1}^L}{\phi} - \frac{\alpha}{1-\alpha}\frac{x_{t+1}^L - x_t^L + \kappa_{t+1}}{\phi}\right).
\end{aligned} \tag{159}$$

By shifting a time index t by one period backward and using $fr_t = \alpha + (1-\alpha)\,\tau - fr_t^c$, we obtain Equation (60) in the main text:

$$fr_t = (1-\alpha)\tau\left[1 - \frac{\Delta_t^H + \Delta_t^L}{\phi}\right] + \alpha\tau\frac{x_t^L - x_{t-1}^L + \kappa_t}{\phi}. \tag{160}$$

Next, we consider the frequency of price increases and price decreases. The frequency of price increases at period $t+1$, represented by fr_{t+1}^+, corresponds to the number of firms with markup below x_{t+1}^L before price adjustment. We therefore obtain the expression fr_{t+1}^+ by using the master equations (148)-(155) for $\Psi_t^0(x)$ together with Figure 8:

$$\begin{aligned}
fr_{t+1}^+ &= \int_{-\infty}^{x_{t+1}^L} dx\ \Psi_t^0(x) \\
&= \left(\int_{x_t^L-\kappa_{t+1}}^{x_{t+1}^L} + \int_{x_t^H-\frac{\phi}{2}-\kappa_{t+1}}^{x_t^L-\kappa_{t+1}} + \int_{x_t^L-\frac{\phi}{2}-\kappa_{t+1}}^{x_t^H-\frac{\phi}{2}-\kappa_{t+1}}\right) dx\ \Psi_t^0(x) \\
&= \int_{x_t^L-\kappa_{t+1}}^{x_{t+1}^L} dx\ \frac{(1-\alpha)\tau}{\phi}\left(1+\frac{\alpha}{1-\alpha}\right) + \int_{x_t^H-\frac{\phi}{2}-\kappa_{t+1}}^{x_t^L-\kappa_{t+1}} dx\ \frac{(1-\alpha)\tau}{\phi} \\
&\quad + \int_{x_t^*-\frac{\phi}{2}-\kappa_{t+1}}^{x_t^H-\frac{\phi}{2}-\kappa_{t+1}} dx\ \frac{(1-\alpha)\tau}{\phi}\left[(1-\alpha)(1-\tau)+fr_t\right] \\
&\quad + \int_{x_t^L-\frac{\phi}{2}-\kappa_{t+1}}^{x_t^H-\frac{\phi}{2}-\kappa_{t+1}} dx\ \frac{(1-\alpha)\tau}{\phi}\int_{x_t^L}^{x+\frac{\phi}{2}+\kappa_{t+1}} dx'\ \Psi_t^0(x'),
\end{aligned}$$

where the first term in the last expression is from Equation (158), the second term is from (156), the third term is from the sum of (148) and (150), and the forth term is from (152). By explicitly evaluating the integrals in the first, second and third terms and changing the order of integrals in the forth term, we obtain

$$\begin{aligned}
fr_{t+1}^+ = \frac{1}{2}(1-\alpha)\tau - (1-\alpha)\tau\frac{\Delta_{t+1}^L}{\phi} + \alpha\tau\frac{x_{t+1}^L - x_t^L + \kappa_{t+1}}{\phi} \\
+ (1-\alpha)\tau\frac{x_{t+1}^* - x_t^* + \kappa_{t+1}}{\phi} + \frac{(1-\alpha)\tau}{\phi}\gamma_t^1,
\end{aligned} \tag{161}$$

where $\gamma_t^1 \equiv \int_{s_t} dx\ \ (x_t^* - x)\,\Psi_t^0(x)$.

Similarly, the frequency of price decreases fr^-_{t+1} is obtained as

$$\begin{aligned}
fr^-_{t+1} &= \int_{x^H_{t+1}}^{\infty} dx\ \Psi^0_t(x) \\
&= \left(\int_{x^H_{t+1}}^{x^L_t+\frac{\phi}{2}-\kappa_{t+1}} + \int_{x^L_t+\frac{\phi}{2}-\kappa_{t+1}}^{x^H_t+\frac{\phi}{2}-\kappa_{t+1}} \right) dx\ \Psi^0_t(x) \\
&= \int_{x^H_{t+1}}^{x^L_t+\frac{\phi}{2}-\kappa_{t+1}} dx\ \frac{(1-\alpha)\tau}{\phi} \\
&\quad + \int_{x^L_t+\frac{\phi}{2}-\kappa_{t+1}}^{x^*_t+\frac{\phi}{2}-\kappa_{t+1}} dx\ \frac{(1-\alpha)\tau}{\phi}\left[(1-\alpha)(1-\tau)+fr_t\right] \\
&\quad + \int_{x^L_t+\frac{\phi}{2}-\kappa_{t+1}}^{x^H_t+\frac{\phi}{2}-\kappa_{t+1}} dx\ \frac{(1-\alpha)\tau}{\phi} \int_{x+\kappa_{t+1}-\frac{\phi}{2}}^{x^H_t} dx'\ \Psi^0_t(x'),
\end{aligned}$$

where the first term in the last expression is from Equation (156), the second term is from the sum of (148) and (150), and the third term is from (154). By explicitly evaluating the integrals in the first and second terms and changing the order of integrals in the third term, we obtain

$$\begin{aligned}
fr^-_{t+1} &= \frac{1}{2}(1-\alpha)\tau - (1-\alpha)\tau\frac{\Delta^H_{t+1}}{\phi} \\
&\quad - (1-\alpha)\tau\frac{x^*_{t+1}-x^*_t+\kappa_{t+1}}{\phi} - \frac{(1-\alpha)\tau}{\phi}\gamma^1_t.
\end{aligned} \tag{162}$$

Finally, we evaluate γ^1_t in a way similar to the evaluation of Γ^3_t in Equation (100).

$$\begin{aligned}
\gamma^1_{t+1} &= \int_{x^L_{t+1}}^{x^H_{t+1}} dx\ \left(x^*_{t+1}-x\right)\Psi^0_{t+1}(x) \\
&= x^*_{t+1} fr^c_{t+1} - \left(\int_{x^H_t-\kappa_{t+1}}^{x^H_{t+1}} - \int_{x^L_t-\kappa_{t+1}}^{x^L_{t+1}} + \int_{x^L_t-\kappa_{t+1}}^{x^H_t-\kappa_{t+1}} \right) dx\ x\Psi_{t+1}(x) \\
&= x^*_{t+1}\left[\alpha + (1-\alpha)\tau\left(\frac{x^H_{t+1}-x^L_{t+1}}{\phi} - \frac{\alpha}{1-\alpha}\frac{x^L_{t+1}-x^L_t+\kappa_{t+1}}{\phi}\right)\right] \\
&\quad - \int_{x^L_{t+1}}^{x^H_{t+1}} dx\ (1-\alpha)\tau\frac{x}{\phi} + \int_{x^L_t-\kappa_{t+1}}^{x^L_{t+1}} dx\ (1-\alpha)\tau\frac{\alpha}{1-\alpha}\frac{x}{\phi} \\
&\quad - \int_{x^L_t-\kappa_{t+1}}^{x^H_t-\kappa_{t+1}} dx\ \alpha x\Psi^0_t\left(x+\kappa_{t+1}\right) \\
&\quad - \alpha\left[(1-\alpha)(1-\tau)+fr_t\right]\left(x^*_t-\kappa_{t+1}\right).
\end{aligned}$$

After simplifying the expression, we obtain a recursive relation for γ_t^1 as

$$\begin{aligned}\gamma_{t+1}^1 &= \alpha\gamma_t^1 + \alpha\left(x_{t+1}^* - x_t^* + \kappa_{t+1}\right)\\&- (1-\alpha)\tau\frac{\left(\Delta_{t+1}^H\right)^2 - \left(\Delta_{t+1}^L\right)^2}{2\phi}\\&- \alpha\tau\frac{\left(x_{t+1}^L - x_t^L + \kappa_{t+1}\right)\left(x_{t+1}^* - x_t^* + \kappa_{t+1} + \Delta_{t+1}^L + \Delta_t^L\right)}{2\phi}.\end{aligned} \tag{163}$$

E.2 Size of price adjustment

We denote the absolute size of price increases and decreases as sz_t^+ and sz_t^-, respectively. The derivation of these quantities is similar to that of frequency (fr_t^+ and fr_t^-) described above. Namely, for the frequency of price increases,

$$\begin{aligned}fr_{t+1}^+ sz_{t+1}^+ &= \int_{-\infty}^{x_{t+1}^L} dx\ \left(x_{t+1}^* - x\right)\Psi_{t+1}^0(x)\\&= fr_{t+1}^+ x_{t+1}^* - \int_{-\infty}^{x_{t+1}^L} dx\ x\Psi_{t+1}^0(x).\end{aligned} \tag{164}$$

The integral on the right-hand side is evaluated as

$$\begin{aligned}\int_{-\infty}^{x_{t+1}^L} dx\ x\Psi_{t+1}^0(x) &= \left(\int_{x_t^L-\kappa_{t+1}}^{x_{t+1}^L} + \int_{x_t^H-\frac{\phi}{2}-\kappa_{t+1}}^{x_t^L-\kappa_{t+1}} + \int_{x_t^L-\frac{\phi}{2}-\kappa_{t+1}}^{x_t^H-\frac{\phi}{2}-\kappa_{t+1}}\right) dx\ x\Psi_t^0(x)\\&= \int_{x_t^L-\kappa_{t+1}}^{x_{t+1}^L} dx\ x\frac{(1-\alpha)\tau}{\phi}\left(1+\frac{\alpha}{1-\alpha}\right) + \int_{x_t^H-\frac{\phi}{2}-\kappa_{t+1}}^{x_t^L-\kappa_{t+1}} dx\ x\frac{(1-\alpha)\tau}{\phi}\\&+ \int_{x_t^*-\frac{\phi}{2}-\kappa_{t+1}}^{x_t^H-\frac{\phi}{2}-\kappa_{t+1}} dx\ x\frac{(1-\alpha)\tau}{\phi}\left[(1-\alpha)(1-\tau) + fr_t\right]\\&+ \int_{x_t^L-\frac{\phi}{2}-\kappa_{t+1}}^{x_t^H-\frac{\phi}{2}-\kappa_{t+1}} dx\ x\frac{(1-\alpha)\tau}{\phi}\int_{x_t^L}^{x+\frac{\phi}{2}+\kappa_{t+1}} dx'\ \Psi_t^0(x').\end{aligned}$$

After some calculations, it is simplified as

$$\begin{aligned}\int_{-\infty}^{x_{t+1}^L} dx\ x\Psi_{t+1}^0(x) &= \alpha\tau\frac{\left(x_{t+1}^L\right)^2 - \left(x_t^L - \kappa_{t+1}\right)^2}{2\phi} + (1-\alpha)\tau\frac{\left(x_{t+1}^L\right)^2 - \left(x_t^* - \frac{\phi}{2} - \kappa_{t+1}\right)^2}{2\phi}\\&+ (1-\alpha)\tau\frac{\gamma_t^2}{2\phi} - (1-\alpha)\tau\left(\frac{1}{2}+\frac{\kappa_{t+1}}{\phi}\right)\gamma_t^1,\end{aligned}$$

where $\gamma_t^2 \equiv \int_{S_t} dx \ \left[(x_t^*)^2 - x^2\right] \Psi_t^0(x)$. Substituting this expression into Equation (164) and rearranging terms, we obtain

$$\begin{aligned} fr_{t+1}^+ sz_{t+1}^+ = fr_{t+1}^+ x_{t+1}^* - \alpha\tau \frac{\left(x_{t+1}^L\right)^2 - \left(x_t^L - \kappa_{t+1}\right)^2}{2\phi} \\ - (1-\alpha)\tau \frac{\left(x_{t+1}^L\right)^2 - \left(x_t^* - \frac{\phi}{2} - \kappa_{t+1}\right)^2}{2\phi} \\ - (1-\alpha)\tau \frac{\gamma_t^2}{2\phi} + (1-\alpha)\tau \left(\frac{1}{2} + \frac{\kappa_{t+1}}{\phi}\right) \gamma_t^1. \end{aligned} \tag{165}$$

By similar calculations for sz_{t+1}^-, we obtain

$$\begin{aligned} fr_{t+1}^- sz_{t+1}^- = -fr_{t+1}^- x_{t+1}^* + (1-\alpha)\tau \frac{\left(x_t^* + \frac{\phi}{2} - \kappa_{t+1}\right)^2 - \left(x_{t+1}^H\right)^2}{2\phi} \\ - (1-\alpha)\tau \frac{\gamma_t^2}{2\phi} - (1-\alpha)\tau \left(\frac{1}{2} - \frac{\kappa_{t+1}}{\phi}\right) \gamma_t^1. \end{aligned} \tag{166}$$

The average size of price adjustment sz_{t+1} is obtained by taking the sum of the above two expressions as

$$\begin{aligned} fr_{t+1} sz_{t+1} &= fr_{t+1}^+ sz_{t+1}^+ + fr_{t+1}^- sz_{t+1}^- \\ &= \left(fr_{t+1}^+ - fr_{t+1}^-\right) x_{t+1}^* - \alpha\tau \frac{\left(x_{t+1}^L\right)^2 - \left(x_t^L - \kappa_{t+1}\right)^2}{2\phi} \\ &\quad + (1-\alpha)\tau \frac{\left(x_t^* + \frac{\phi}{2} - \kappa_{t+1}\right)^2 + \left(x_t^* - \frac{\phi}{2} - \kappa_{t+1}\right)^2 - \left(x_{t+1}^H\right)^2 - \left(x_{t+1}^L\right)^2}{2\phi} \\ &\quad - (1-\alpha)\tau \frac{\gamma_t^2}{\phi} + (1-\alpha)\tau \frac{2\kappa_{t+1}}{\phi} \gamma_t^1. \end{aligned} \tag{167}$$

Finally, we evaluate γ_t^2 in a similar way as γ_t^1 shown in Equation (163).

$$\begin{aligned}
\gamma_{t+1}^2 &= \int dx \ \left[\left(x_{t+1}^*\right)^2 - x^2\right] \Psi_{t+1}^0(x) \\
&= \left(x_{t+1}^*\right)^2 fr_{t+1}^c - \left(\int_{x_t^H-\kappa_{t+1}}^{x_{t+1}^H} - \int_{x_t^L-\kappa_{t+1}}^{x_{t+1}^L} + \int_{x_t^L-\kappa_{t+1}}^{x_t^H-\kappa_{t+1}}\right) dx \ \Psi_{t+1}(x) \\
&= \left(x_{t+1}^*\right)^2 \left[\alpha + (1-\alpha)\tau \left(\frac{x_{t+1}^H - x_{t+1}^L}{\phi} - \frac{\alpha}{1-\alpha}\frac{x_{t+1}^L - x_t^L + \kappa_{t+1}}{\phi}\right)\right] \\
&\quad - \int_{x_{t+1}^L}^{x_{t+1}^H} dx \ (1-\alpha)\tau \frac{x^2}{\phi} + \int_{x_t^L-\kappa_{t+1}}^{x_{t+1}^L} dx \ (1-\alpha)\tau \frac{\alpha}{1-\alpha}\frac{x^2}{\phi} \\
&\quad - \int_{x_t^L-\kappa_{t+1}}^{x_t^H-\kappa_{t+1}} dx \ \alpha x^2 \Psi_t^0\left(x+\kappa_{t+1}\right) \\
&\quad - \alpha\left[(1-\alpha)(1-\tau) + fr_t\right]\left(x_t^* - \kappa_{t+1}\right)^2 .
\end{aligned}$$

After simplifying the expression, we obtain a recursive relation for γ_t^2 as

$$\begin{aligned}
\gamma_{t+1}^2 &= \alpha\gamma_t^2 - 2\alpha\kappa_{t+1}\gamma_t^1 + \alpha\left[\left(x_{t+1}^*\right)^2 - \left(x_t^* - \kappa_{t+1}\right)^2\right] \\
&\quad - (1-\alpha)\tau x_{t+1}^* \frac{\left(\Delta_{t+1}^H\right)^2 - \left(\Delta_{t+1}^L\right)^2}{\phi} - (1-\alpha)\tau\frac{\left(\Delta_{t+1}^H\right)^3 + \left(\Delta_{t+1}^L\right)^3}{3\phi} \\
&\quad - \alpha\tau\left(x_{t+1}^*\right)^2 \frac{x_{t+1}^L - x_t^L + \kappa_{t+1}}{\phi} + \alpha\tau\frac{\left(x_{t+1}^L\right)^3 - \left(x_t^L - \kappa_{t+1}\right)^3}{3\phi}.
\end{aligned} \tag{168}$$

While the right-hand side of Equation (167) is complex, the largest contribution comes from the third term. In fact, in the limit of zero trend inflation and in a steady state, we can significantly simplifies the expression. In this case, $x_{t+1}^L - x_t^L + \kappa_{t+1} = x_{t+1}^* - x_t^* + \kappa_{t+1} = 0$. Moreover, the result shown in Appendix A.7 suggests that the difference between $\bar{\Delta}^L$ and $\bar{\Delta}^H$ is small and vanishes if we resort to second-order approximation. These considerations imply that all the terms except for the third term are either zero or small. By ignoring the difference between $\bar{\Delta}^L$ and $\bar{\Delta}^H$ and denoting both by $\bar{\Delta}$, the third term in Equation (167) becomes

$$\begin{aligned}
\bar{frsz} &= (1-\alpha)\tau\frac{\left(\bar{x^*} + \frac{\phi}{2}\right)^2 + \left(\bar{x^*} - \frac{\phi}{2}\right)^2 - \left(\bar{x^*} + \bar{\Delta}\right)^2 - \left(\bar{x^*} - \bar{\Delta}\right)^2}{2\phi} \\
&= (1-\alpha)\tau\left(1 - \frac{2\bar{\Delta}}{\phi}\right)\left(\frac{\phi}{4} + \frac{\bar{\Delta}}{2}\right).
\end{aligned}$$

On the other hand, Equation (160) suggests $\bar{fr} = (1-\alpha)\tau\left(1 - 2\bar{\Delta}/\phi\right)$ in the limit of zero trend inflation in a steady state. It therefore follows that

$$\bar{sz} = \frac{\phi}{4} + \frac{\bar{\Delta}}{2},$$

which corresponds to the expression derived by Gertler and Leahy (2008).

F Details of analysis on deflation

This Appendix considers the case of deflation outlined in Section 5.2 in detail.

Given the setting of one-period positive drift proposed in the main text, the only part of our analysis that needs revision is the derivation of price index in Section 3.2. More specifically, we reconsider the evaluation of the density $\Psi_t(x)$ around the inaction region shown in Section 3.2.2 and the calculation of the integrals Γ_t^3 and Γ_t^4 in Appendix B.4.

F.1 Period $t = 1$

First, we consider the period $t = 1$, when all the inequalities in Assumption 2 fail.

At this period, the markup density $\Psi_1(x)$ is still given by Figure 3: the density is flat in the regions $x \in [\bar{x}^H - \phi/2 - \kappa_1, \bar{x}^L - \kappa_1) \cup (\bar{x}^H - \kappa_1, \bar{x}^L + \phi/2 - \kappa_1]$ and increases in the step-wise manner as x decreases from $\left(\bar{x}^H - \kappa_1\right)$ until x reaches $\left(\bar{x}^L - \kappa_1\right)$. This is because the shape of the density function $\Psi_1(x)$ is determined by the past developments up to $t = 0$, when the economy stayed in the steady state. The only effect of the violation of Assumption 2 is that the thresholds x_1^L and x_1^H are set to the left of the boundaries of the flat region $\left(\bar{x}^L - \kappa_1\right)$ and $\left(\bar{x}^H - \kappa_1\right)$, respectively. This implies that the evaluation of the integral for $\Gamma_{t=1}^3$ in Equation (99) should be replaced by

$$\Gamma_1^3 = \int_{x_1^L}^{x_1^H} dx\ \Psi_1(x) = \left(-\int_{x_1^H}^{\bar{x}^H-\kappa_1} + \int_{x_1^L}^{\bar{x}^L-\kappa_1} + \int_{\bar{x}^L-\kappa_1}^{\bar{x}^H-\kappa_1}\right) dx\ \Psi_1(x),$$

where we use $\Psi_{t+1}(x) = \eta\left(\Gamma_{t+1}^1 + \alpha e^{(\varepsilon-1)\kappa_{t+1}}\Gamma_t^1\right)$, shown in the discussion below Equation (46), for the first integral inside the parenthesis, Equation (45) for the second integral, and Equation (47) together with the contribution of delta functions at $x = \bar{x}^* - \kappa_1$ for the third integral, respectively.[31] The evaluation of the integral $\Gamma^4_{(t=)1}$ is similar. By proceeding in an entirely analogous manner to Appendixes B.4 and B.5, we obtain the law of motion for price index (62) in the main text.

F.2 Periods $t \geqq 2$

Next, we consider the period $t = 2$. This time, all the inequalities in Assumption 2 again hold, implying that we can evaluate the integrals $\Gamma^3_{t=2}$ and $\Gamma^4_{t=2}$ by applying a strategy similar to Equation (99). However, the positive drift at $t = 1$ has moved the entire distribution to the right, making the resulting distribution $\Psi_2(x)$ qualitatively different from that shown in Figure 3: instead, as depicted in Figure 9, the accumulated firms as of $t = 0$ is now somewhat away from $x_1^L - \kappa_2$, the lower threshold as of $t = 1$ minus the drift at $t = 2$. Between them lies a region of not-so-large density. More specifically, for $x \in [x_1^L - \kappa_2, x_0^L - \kappa_1 - \kappa_2)$,

$$\begin{aligned}\Psi_2(x) &= \eta\Gamma^1_{t=2} + \alpha e^{(\varepsilon-1)\kappa_2}\Psi_1(x+\kappa_2) \\ &= \eta\left(\Gamma^1_{t=2} + \alpha e^{(\varepsilon-1)\kappa_2}\Gamma^1_{t=1}\right),\end{aligned} \tag{169}$$

where the first equality derive from Equation (47) and the second equality is due to Equation (45), because $(x + \kappa_2) \in [x_1^L, x_0^L - \kappa_1)$ lies in the flat region in Figure 3 for $\Psi_1(x)$. Suppose x_2^L lies inside this range, i.e., $x_2^L \in [x_1^L - \kappa_2, x_0^L - \kappa_1 - \kappa_2)$. Then to evaluate the integral $\Gamma^3_{t=2}$, we use Equation (169) instead of Equation (48) to evaluate the second integral inside the parenthesis on the right-hand side of Equation (99). The evaluation of the integral $\Gamma^4_{t=2}$ is similarly modified. It then follows that the law of motion for price index $t = 2$ becomes

$$e^{(1-\varepsilon)p_2} = (1-\alpha)Ce^{(1-\varepsilon)p_2^*} + \left(\alpha + f_{1,t=2} - f_{2,t=2}\widetilde{\Gamma}^{\dagger}_{t=1}\right)e^{(1-\varepsilon)p_1}, \tag{170}$$

[31] The evaluation of the first integral inside the parenthesis depends on where exactly x_1^H is located. If $x_1^H \in (\bar{x}^H - \bar{\kappa} - \kappa_1, \bar{x}^H - \kappa_1]$, which we here assume for simplicity, then we can evaluate the integral with $\Psi_1(x)$ entirely on the first step; however, if the deflationary shock is so large that $x_1^H \in (\bar{x}^H - 2\bar{\kappa} - \kappa_1, \bar{x}^H - \bar{\kappa} - \kappa_1]$, then we need to further split the integral as $\int_{x_1^H}^{\bar{x}^H - \kappa_1} dx\ \Psi_1(x) = \left(\int_{\bar{x}^H - \bar{\kappa} - \kappa_1}^{\bar{x}^H - \kappa_1} + \int_{x_1^H}^{\bar{x}^H - \bar{\kappa} - \kappa_1}\right) dx\ \Psi_1(x)$, where the two integrals inside the parenthesis correspond to the first and the second step. In the latter case, the sensitivity of inflation to the deflationary shock becomes somewhat larger than that in the former case.

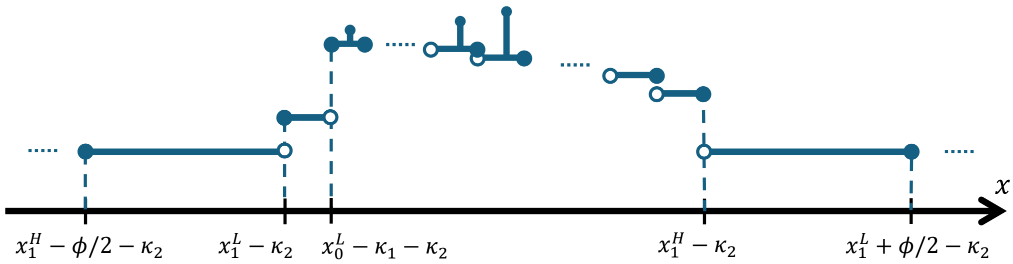

Figure 9: Schematic picture of $\Psi_2(x)$ around the inaction region after a positive drift in firms' markup at $t = 1$.

where $\tilde{\Gamma}^{\dagger}_{t=1}$ is common to period $t = 1$ and is defined in Equation (63). Because $\tilde{\bar{\Gamma}}^{\dagger} \approx \alpha < \frac{\alpha}{1-\alpha} \approx \bar{\bar{\Gamma}}$, this law of motion clearly implies that the sensitivity of inflation to the change in the nominal marginal cost at period $t = 2$ is dampened as in period $t = 1$.

The behavior at $t = 3$ depends on how much the negative drift $(-\kappa_2)$ at period $t = 2$ has offset the positive drift $(-\kappa_1)$. If, as assumed above, $x_2^L \in [x_1^L - \kappa_2, x_0^L - \kappa_1 - \kappa_2)$, then there is still a gap between the accumulated firms and the lower threshold at period $t = 2$ minus the drift at $t = 3$, again implying a reduced sensitivity of inflation to the change in nominal marginal cost. On the other hand, if κ_2 is large enough that $x_2^L \geqq x_0^L - \kappa_1 - \kappa_2$, the behavior at period $t = 3$ is the same as in Section 3.2, and the law of motion of price index is given by Equation (49). In other words, the sensitivity of inflation recovers to the pre-shock level only after the cumulative negative drift in markup for $t \geqq 2$ completely offset the positive drift at $t = 1$.

G Details on the validity of Assumption 3

This Appendix evaluates the accuracy of the approximation based on Assumption 3. Because the assumption is used both for the derivation of firms' policy triplet (x_t^*, x_t^L, x_t^H) in Section 3.1 and for the evaluation of a part of firms' markup distribution in Section 3.2.2, these are treated in Appendix G.1 and G.2, respectively.

G.1 Correction to the reset markup

This section explicitly confirms that the error in the approximation regarding x_t^* based on Assumption 3 is of order $\alpha^{n_t^*}$, thus is not large for our calibrated model in Section 4. More specifically, by defining $x_t^{*(n)} \equiv \log\left(\frac{\varepsilon}{\varepsilon-1}\right) + \log\left(\frac{A_{2,t}^{(n)}}{A_{1,t}^{(n)}}\right)$ for an arbitrary non-negative integer n, we evaluate the difference between x_t^* and $x_t^{*(n_t^*)}$. For this evaluation, we use the first-order perturbation introduced in the derivation of the Phillips curve in Section 3.3. We also adopts the auxiliary assumption of no aggregate uncertainty, introduced in the discussion preceding Equation (25).

We first consider the steady state. Equation (30) implies

$$\bar{A}_1^{(n)} \equiv \bar{A} + \alpha\beta \bar{A}_1^{(n-1)} = \cdots = \frac{1-(\alpha\beta)^{n+1}}{1-\alpha\beta}\bar{A},$$

where we ignore both the trend inflation $\bar{\pi}$ and $A_{4,t}$ following the discussion in Section 3.3. Similarly, Equation (31) implies

$$\bar{A}_2^{(n)} \equiv \bar{A} + \alpha\beta \bar{A}_2^{(n-1)} = \cdots = \frac{1-(\alpha\beta)^{n+1}}{1-\alpha\beta}\bar{A}.$$

Next, we consider the log-linearization of the corresponding variables using the steady-state values above. For example, from Equation (30), we obtain

$$\hat{a}_{1,t}^{(n)} = \frac{1-\alpha\beta}{1-(\alpha\beta)^{n+1}}\hat{a}_t + \alpha\beta\frac{1-(\alpha\beta)^n}{1-(\alpha\beta)^{n+1}}\left[\hat{\lambda}_{t+1} + (\varepsilon-1)\hat{\kappa}_{t+1} + \hat{a}_{1,t+1}^{(n-1)}\right]. \tag{171}$$

Similarly, log-linearizing Equation (31) yields

$$\hat{a}_{2,t}^{(n)} = \frac{1-\alpha\beta}{1-(\alpha\beta)^{n+1}}\hat{a}_t + \alpha\beta\frac{1-(\alpha\beta)^n}{1-(\alpha\beta)^{n+1}}\left[\hat{\lambda}_{t+1} + \varepsilon\hat{\kappa}_{t+1} + \hat{a}_{2,t+1}^{(n-1)}\right]. \tag{172}$$

From Equations (171) and (172) together with the definition of $x_t^{*(n)}$, we obtain

$$\begin{aligned}\hat{x}_t^{*(n)} &= \hat{a}_{2,t}^{(n)} - \hat{a}_{1,t}^{(n)} \\ &= \alpha\beta \frac{1-(\alpha\beta)^n}{1-(\alpha\beta)^{n+1}} \left[\hat{\kappa}_{t+1} + \hat{x}_{t+1}^{*(n-1)}\right] \\ &= \cdots = \sum_{s=1}^{n} (\alpha\beta)^s \frac{1-(\alpha\beta)^{n+1-s}}{1-(\alpha\beta)^{n+1}} \hat{\kappa}_{t+s},\end{aligned}$$

where the third equality follows from the recursive substitution based on the second equality. Similarly, $x_t^* = \lim_{n\to\infty} x_t^{*(n)}$ is rewritten as

$$\begin{aligned}\hat{x}_t^* &= \hat{a}_{2,t} - \hat{a}_{1,t} \\ &= \alpha\beta \left[\hat{\kappa}_{t+1} + \hat{x}_{t+1}^*\right] \\ &= \cdots = \sum_{s=1}^{\infty} (\alpha\beta)^s \hat{\kappa}_{t+s}.\end{aligned}$$

A straight-forward rearrangement of these expressions yield

$$\hat{x}_t^* - \hat{x}_t^{*(n)} = \frac{(\alpha\beta)^{n+1}}{1-(\alpha\beta)^{n+1}} \sum_{s=1}^{n} \left(1-(\alpha\beta)^s\right) \hat{\kappa}_{t+s} + (\alpha\beta)^n \sum_{s=1}^{\infty} (\alpha\beta)^s \hat{\kappa}_{t+n+s}.$$

Clearly, both of the two terms on the right-hand side is of order $(\alpha\beta)^n$, making these terms very small for $n = n_t^*$ large enough by Assumption 3.

G.2 Correction to the markup distribution

Similarly to Appendix G.1, this Appendix explicitly calculates the leading correction associated with the approximation used to obtain the law of motion for price index (49). This correction is associated with the delta functions shown schematically in Figure 3, corresponding to the firms that change their prices or enter the market at a past period, have drifted since then without being hit by an idiosyncratic shock, and eventually cross the lower inaction threshold.[32] As in Section 5.3, t_0 represents the period at which these firms

[32] Another correction is due to the incomplete saturation of the step functions. This correction is however significantly smaller than the correction due to the delta functions because the former is of order $\alpha^{((\bar{\Delta}^L+\bar{\Delta}^H)/\bar{\kappa})}$ while the latter is of order $\alpha^{(\bar{\Delta}^L/\bar{\kappa})}$.

change their prices or enter the market. We then follow their contribution to the density in subsequent periods, which we denote by $\delta^{(t_0)}\Psi_t(x)$, excluding those hit by an idiosyncratic shock.

At the next period $t = t_0 + 1$, the relevant contributions are (B) and (D) in Table 2, so

$$\delta^{(t_0)}\Psi_{t_0+1}(x) = \alpha\left[(1-\alpha)(1-\tau)e^{-(\varepsilon-1)x^*_{t_0}} + \Gamma^2_{t_0}\right]\delta\left(x + \kappa_{t_0+1} - x^*_{t_0}\right)e^{(\varepsilon-1)\kappa_{t_0+1}}.$$

At subsequent periods, we use Equation (47) to keep track of the contribution, and obtain

$$\delta^{(t_0)}\Psi_{t_0+n}(x) = \alpha^n e^{(\varepsilon-1)\sum_{s=1}^{n}\kappa_{t_0+s}}\left[(1-\alpha)(1-\tau)e^{-(\varepsilon-1)x^*_{t_0}} + \Gamma^2_{t_0}\right]\delta\left(x + \sum_{s=1}^{n}\kappa_{t_0+s} - x^*_{t_0}\right).$$

When these firms cross the lower threshold and therefore change their prices at $t = t_0 + n^*$, the following inequality has to be satisfied:

$$x^L_{t_0+n^*-1} - \kappa_{t_0+n^*} \leqq x^*_{t_0} - \sum_{s=1}^{n}\kappa_{t_0+s} < x^L_{t_0+n^*}.$$

It then follows that the evaluation of the integrals $\Gamma^3_{t_0+n^*}$ and $\Gamma^4_{t_0+n^*}$, discussed in Appendix B.4, needs to take into account the term $\delta^{(t_0)}\Psi_{t_0+n^*}(x)$ in this range. For example, the contribution of this density to the former integral, which we denote by $\delta^{(t_0)}\Gamma^3_{t_0+n^*}$, is

$$\delta^{(t_0)}\Gamma^3_{t_0+n^*} = -\alpha^{n^*}e^{(\varepsilon-1)\sum_{s=1}^{n^*}\kappa_{t_0+s}}\left[(1-\alpha)(1-\tau)e^{-(\varepsilon-1)x^*_{t_0}} + \Gamma^2_{t_0}\right].$$

Similarly, the correction of the integral $\Gamma^4_{t_0+n^*}$ becomes

$$\delta^{(t_0)}\Gamma^4_{t_0+n^*} = -\alpha^{n^*}\left[(1-\alpha)(1-\tau)e^{-(\varepsilon-1)x^*_{t_0}} + \Gamma^2_{t_0}\right]e^{(\varepsilon-1)\left(x^*_{t_0} - x^*_{t_0+n^*}\right)}.$$

The rest is analogous to the calculation of price index in Appendix B.5. With the help of log-linear approximation introduced in Section 3.3, we obtain the expression (65) in the main text.

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