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Inflation and the Joint Bond-FX Spanning Puzzle*

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Abstract

We generalize the yield spanning condition in the bond literature to non-linear models and to exchange rates. In standard macro-finance models, no variable should predict yield or exchange rate changes once standard yield curve factors are controlled for. We provide novel evidence that this spanning condition is violated, with inflation as a common unspanned predictor of both bond and exchange rate returns. Investors' incomplete information about the Federal Reserve's monetary policy rule emerges as the key driver of this result. We find high inflation to be followed by unexpected monetary policy tightening, which leads to dollar appreciation and low bond returns. We explain these findings by a simple model that departs from full information rational expectations.

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1. Introduction

In standard affine term structure models, the shape of the yield curve encapsulates all information relevant for predicting yield changes and bond returns. Because information in macro variables is already embedded in yields, macro factors should not predict bond returns after linearly controlling for yield curve factors such as level, slope and curvature.

We theoretically generalize the linear spanning result in two dimensions. First, we show that linear spanning also holds in non-linear models, such as the habit model. Hence it represents a broader conceptual property of macro-finance models than previously thought. Second, we show that linear spanning extends to exchange rates. In particular, no variable should predict exchange rate changes or currency excess returns after linearly controlling for home and foreign yield curve factors.

Our key empirical contribution is to show that this spanning condition is violated in the data. In particular, we find that U.S. inflation is an unspanned predictor of both U.S. bond and dollar returns. High inflation is associated with low subsequent bond returns but high dollar returns, which also holds after controlling for the information already embedded in the current shapes of U.S. and foreign yield curves. Broadly speaking, both our theoretical and empirical results imply that the spanning puzzle in the bond literature is actually a *joint* spanning puzzle for bonds *and* currencies.

We also explore the mechanisms through which inflation predicts bond and dollar returns. Using survey data, we find that high inflation anticipates forecast errors concerning long term interest rates. Moreover, high inflation is associated with future monetary policy shocks as inferred from high-frequency reactions to monetary policy news. This is especially the case for shocks to the path of expected future interest rates derived by Gürkaynak et al. (2004). Finally, we find that high inflation predicts future revisions in economic agents' perceived Taylor rule coefficient on inflation. We interpret these results as indications that high inflation triggers unexpected future monetary policy tightening. This policy contraction, in turn, depresses bond returns and strengthens the dollar.

Why do economic agents seemingly misunderstand the relationship between inflation and future monetary policy? Why is inflation unspanned by information contained in the yield curve?

To make inroads in (partially) answering these questions, we present a simple model that offers intuitive insights. In the model, agents observe the interest rate set by the central bank but are unaware of the actual targeted inflation metric. Monetary policy is also subject to random shocks. As a result, agents adopt a simple sticky expectations rule when forecasting interest rates. While our setting is different, this forecasting rule is identical to that in Coibion and Gorodnichenko (2015) and Mankiw and Reis (2002). Our setup implies that current inflation is a noisy proxy of the inflation metric targeted by the Fed. However, these two are positively correlated. Moreover, current inflation is not taken into account by the agents applying a simple sticky expectations rule to forecast interest rates.

Our simple model is consistent with the patterns observed in the data. It implies that high current inflation today predicts future monetary policy tightening. This tightening then leads to low bond returns and U.S. dollar appreciation. At the same time, the model implies that inflation is not fully spanned by information in the yield curves.

Related literature. Duffee (2011) and Joslin et al. (2014) argue that in standard affine term structure models, yield factors fully linearly span macroeconomic variables. Although this result should hold conceptually, they provide evidence that measures of inflation and real activity are unspanned predictors of bond returns. The papers proceed to propose affine models with unspanned variables. Here a macro variable is unspanned if its effect on future interest rates and bond risk premia exactly offset each other. However, a key caveat with such unspanned models is that they effectively require knife-edge restrictions on model parameters. While these restrictions cause a failure of the standard invertibility condition, minor deviations from model parameters again imply full spanning.

Failures of spanning are implicit in the literature on forecasting the business cycle (Stock and Watson, 2003). A large body of literature has also explicitly looked at such violations. Papers explicitly finding that macroeconomic variables contain useful unspanned information include Cieslak and Povala (2015), Coroneo et al. (2016), Bianchi et al. (2021), and Moench and Siavash (2022). Sihvonen (2024) argues that past bond returns hold yet additional useful information on top of both current yields and macro variables. Crump and Gospodinov (2025) find that the incremental predictive ability of trend inflation survives in a bootstrap that imposes only weak restrictions about the true data generating process. Finally, Kroencke et al. (2021) and Boehm and Kroner (2024) find that the Fed affects asset prices through channels unspanned by the yield curve.

Observed spanning failures can be due to measurement error in yields (Cieslak and Povala, 2015; Bauer and Hamilton, 2018). However, most of the literature views the spanning puzzle real and not arising from mechanical issues, including Cieslak (2018) and Bauer and Rudebusch (2020). Relatedly, Sihvonen (2024) finds that yield measurement error is too small to explain why past bond returns are unspanned by current yields.

While our empirical results for U.S. bonds are related to those in previous papers (e.g Duffee, 2011), our paper is the first to relate yield curve spanning and exchange rates. Filippou and Taylor (2017) address the relationship between macro variables, including inflation, and FX return predictability, but do not discuss spanning or the mechanisms behind the results. Whereas our paper focuses on the predictive power of the inflation level, Dahlquist and Hasseltoft (2020) demonstrate that changes in inflation can also predict currency returns. Ang and Chen (2010) and Lustig et al. (2019) study yield curve based predictors of FX rates and Sarno et al. (2012) and Feunou et al. (2024) build joint models relating yield curves and currency dynamics. However, these papers also do not study the issue of spanning.

Dahlquist and Pénasse (2022) use the Kalman filter to extract a hidden factor that predicts FX returns but is unrelated to interest rate differentials. They argue that this hidden factor drives most of real exchange rate volatility. However, they do not explore the relationship between inflation and the hidden factor. Moreover, while the hidden factor is unrelated to interest rate differentials, it can still be partly spanned by yield curve factors such as slope and curvature.

Our paper also contributes to a growing literature studying the effects of expectational errors on asset prices (e.g. Gourinchas and Tornell, 2004; Bordalo et al., 2018). Schmeling et al. (2022) find that expectational errors explain most of the excess returns on money market instruments. Some papers (e.g Cieslak, 2018; Sihvonen, 2024) have proposed that deviations from rational expectations can explain violations from full spanning. However, the mechanism we explore in this paper is novel.

Since we analyze the relationship between inflation and exchange rates, our paper is also loosely related to the vast literature on testing purchasing power parity (PPP) (Engel, 2000; Taylor and Taylor, 2002; Sarno and Taylor, 2002). PPP would in principle predict that high inflation is associated with currency depreciation, which is opposite to what we find. However, this literature has focused on longer horizons than those applied in this paper and still found relatively weak evidence in favor of PPP.

A large literature has studied the contemporaneous relationship between monetary policy shocks and forecast revisions concerning macroeconomic variables (see e.g. Nakamura and Steinsson, 2018; Bauer and Swanson, 2023). Karnaukh and Vokata (2022) also find that forecast revisions concerning GDP growth can predict future monetary policy shocks. They argue that this finding is consistent with noisy information models (see e.g. Mankiw and Reis, 2002). Note that such a model cannot explain our key results since our predictor is the level rather than change in inflation and because the noisy information model would still imply full spanning.

Finally, our paper is related to a literature on non-linear macro-finance models, the canonical example being the habit model (Wachter, 2006; Verdelhan, 2010). Similarly, while standard term structure models are affine, shadow rate models used to impose the zero lower bound constraint on interest rates are generally non-linear in model state variables (see e.g. Christensen and Rudebusch, 2016). Here our results show that all of these models have a linear representation in yield curve factors.

2. Generalizing the spanning hypothesis

In this section, we generalize linear spanning. First, we consider the case of bonds and show that linear spanning also holds in non-linear models. Second, we turn to currencies and show that the linear spanning hypothesis also extends to currency returns, that is, spot rate changes and currency excess returns. In the Appendix, we provide further numerical robustness checks that the below results hold accurately in standard models.

2.1. Bond returns

The Model. We consider a general Markovian model for yields. Let the model state variable be $x_t \in \mathbb{R}^{m \times 1}$ and the zero-coupon yield of an n-maturity bond be

$$y_t^n = g_n(x_t), \tag{1}$$

for some function g_n. This expression is very general, though g_n might not have a simple analytical form.[1] The underlying model could be an asset pricing model such as the habit model of Campbell and Cochrane (1999), a DSGE model like that in Rudebusch and Swanson (2012) or a non-linear term structure model as in Christensen and Rudebusch (2015).

The corresponding one period bond excess return is $rx_{t+1}^n = -(n-1)y_{t+1}^{n-1} + ny_t^n - y_t^1$. The state variable by definition contains all information about future yield changes. Expected excess returns are then also a function of x_t

$$\mathbb{E}_t[rx_{t+1}^n] = \Pi_n(x_t). \tag{2}$$

We call this latter statement the risk premium equation. Note that equivalently we could formulate the results for expected yield changes instead of expected returns.

Exact non-linear spanning. Before moving to the approximation, we first consider an exact non-linear version of the spanning argument. Pick any m yields stacked into a vector $\tilde{y}_t \in \mathbb{R}^{m \times 1}$. Moreover, define $\tilde{g}$ as

$$\tilde{y}_t = \tilde{g}(x_t),$$

where this function simply collects the relevant elements using g_n. Assuming the inverse exists, we can solve

$$x_t = \tilde{g}^{-1}(\tilde{y}_t).$$

[1] Note that any Markov(h) process can be written as Markov(1). Here relevant lags of state variables are included as state variables.

Controlling for $\tilde{g}^{-1}(\tilde{y}_t)$ no variable should explain variation in the factors. Then consider expected bond returns $\mathbb{E}_t[rx^n_{t+1}]$. We have

$$\mathbb{E}_t[rx^n_{t+1}] = \Pi_n(x_t) = \Pi_n(\tilde{g}^{-1}(\tilde{y}_t)).$$

Now, no other variable should predict excess returns controlling for $\Pi_n(\tilde{g}^{-1}(\tilde{y}_t))$.

Approximate linear spanning. We begin by defining a linear approximation to the yield and risk premium equations.

Definition 1: Affine approximation in yield factors. The p-variable affine approximations to the yield Equation (1) and the risk premium Equation (2) are given by $y^n_t \approx A_n + B'_n T_t$ and $\mathbb{E}_t[rx^n_{t+1}] \approx C_n + D'_n T_t$, where T_t consists of the first p principal component scores of yields. Moreover A_n, B_n, C_n and D_n are obtained by (population) least squares.

Note that here the model state variables are instead replaced by yield curve factors. However, generally one should use more yield curve factors than model state variables, $p > m$, as discussed below. The following proposition helps to understand the approximation:

Proposition 1. *Consider the r:th order local approximation to yield and risk premium Equations (1) and (2). Now the $p = \binom{m+r}{m} - 1$ variable affine approximation in yield factors results in the same equations.*

Proof: see the Appendix.

The above proposition can also be stated as follows. Assume we apply a local approximation to the non-linear model. Then if we use as many yield curve factors in an affine approximation as there are terms in the local approximation, we obtain the same yield and risk premium equations. For example, consider a non-linear model with one state variable. Then applying one yield curve factor would result in the same equations than applying a first order local approximation. Similarly, applying two yield curve factors would correspond to a 2nd order local approximation. In this sense, applying more yield curve factors than model state variables can be seen to account for non-linearities.[2] It is straightforward to show that the Proposition also holds when the affine approximation is defined using p yields or forward rates instead of yield principal components.[3]

Local approximations are widely employed to solve DSGE models. For example, Rudebusch and Swanson (2012) study term premia in a model solved using a 3rd order approximation. The above proposition implies that the affine approximation (with $r = 3$) holds perfectly in the local solution applied in the paper. Asset pricing models with long run risk are often solved using the log-linearization of Campbell and Shiller (1988), which implies an affine solution for yields (Bansal and Yaron, 2004). The affine approximation (with $r = 1$) then holds perfectly.[4] The similarity between affine models and linearized long-run risk models is also discussed by Creal and Wu (2020).

One practical implication of the proposition concerns the measurement of risk premia. In standard affine models, variations in risk premia can be captured using linear regressions of bond excess returns on yield factors (e.g. Adrian et al., 2013). The proposition implies that this approach is valid for non-linear models but generally using more yield factors than state variables.

The issue of invertibility is clearer in the affine than in the original non-linear model. As long as the p-variable affine approximation exists, it is invertible. Technically, this is because the principal component scores are orthogonal.

The following proposition provides a linear counterpart to the argument that no variable should predict bond returns controlling for sufficiently many yields:

Proposition 2. *Consider the p-variable affine approximation of the model. No variable can*

[2]The analogue to a local approximation is, however, not perfect. Generally, our approximation is applied directly to a non-linear model and not to a local approximation. For example in the case of one state variable, a two factor approximation does not exactly correspond to a 2nd order local approximation. This is because our approximation attempts to provide the best two factor approximation to the general non-linear problem, which generally entails also accounting for some of the effects of third and fourth order terms and so on. Because of this and the fact that some state variables can be quantitatively unimportant for yield changes, the affine approximation does not necessarily require a large amount of factors to hold accurately.

[3]Assuming these are not perfectly correlated.

[4]However, assume we apply our procedure to a local approximation and use as many yield factors as terms in the local approximation. Now, while our equations capture the same predictive content, the yield curve factors are rotated versions of the terms in the local approximation. Assuming we only observe the yield curve factors, we generally cannot perfectly reverse-engineer the original state variables.

predict returns after linearly controlling for yield factors T_t.

This follows directly from the risk premium equation being affine in T_t. Alternatively, instead of yield factors, we could simply control for p yields.

2.2. FX returns

We now show that linear spanning holds also for FX returns. Assume two countries. On top of the home country, described above, there is also a foreign country. Foreign country state variables are given by $x_t^* \in \mathbb{R}^{m^* \times 1}$ and yields by $y_t^{*n} = g_n^*(x_t^*)$.

This setting is again very general. Note that by separately specifying x_t and x_t^* we allow for some unspanning between the two yield curves, that is, the home yield factors generally do not capture all variation in the foreign yield curve. The setting also allows for different ways to model dependencies between the two countries. For example, x_t and x_t^* might share common state variables on top of independent country specific state variables. Alternatively all of the state variables might be separate but correlated between the two countries.[5]

Let the log exchange rate between the two countries be given by s_t, defined as the foreign currency price of one U.S. dollar, that is, higher values represent a U.S. dollar appreciation. Similarly to before we could equivalently formulate the results for FX rate changes or excess returns but choose to use the latter. This U.S. dollar excess return is given by

$$rx_{t+1}^{FX} = s_{t+1} - s_t + y_t^1 - y_t^{*1}.$$

We could formulate the results also by defining returns as $s_{t+1} - f_t^1$, where f_t^1 is the one period FX forward rate. In standard macro-finance models, the covered interest rate parity (CIP) holds so these two definitions give identical returns. However, such equivalence is not necessary for our results.

We define the FX risk premium as

[5] We can also have $x_t = x_t^*$, although in this case the home and foreign yield curves embed the same information. However, we do not include variables, which do not affect home yields, to x_t and vice versa for x_t^*.

$$\mathbb{E}_t[rx_{t+1}^{FX}] \equiv \Omega_t(x_t, x_t^*).$$

The following definition generalizes the affine approximation to FX rates

Definition 2: Two Country Affine Approximation in Yield Factors. The (p, p^*)-variable affine approximations to the yield and risk premium equations are given by $y_t^n \approx A_n + B_n' T_t$, $y_t^{*n} \approx A_n^* + B_n^{*'} T_t^*$, $\mathbb{E}_t[rx_{t+1}^n] \approx C_n + D_n' T_t$ and $\mathbb{E}_t[rx_{t+1}^{FX}] \approx H + F T_t + F^* T_t^*$. T_t and T_t^* consists of the first p and p^* principal component scores of home and foreign yields respectively. Moreover A_n, B_n, A_n^*, B_n^*, C_n, D_n, H, F and F^* are obtained via (population) least squares.

Note that here we allow for different approximation orders p and p^* for the home and foreign equations. This is especially because the number of home and foreign state variables m and m^* can be different. Given this definition, we are now ready to generalize our key results to include FX rates:

Proposition 3. *Assume the FX risk premium equation is separable in home and foreign state variables $\Omega_t(x_t, x_t^*) = f(x_t) - f^*(x_t^*)$. This holds e.g. when the market is complete, the FX risk premium is exactly affine in state variables or $x_t = x_t^*$. The $p = \binom{m+r}{m} - 1$ and $p^* = \binom{m^*+r}{m^*} - 1$ variable affine approximation in yield factors results in the same model equations than an r:th order local approximation. Given the affine approximation, no variable predicts bond or FX returns once linearly controlling for home and foreign yield factors.*

Relative to Propositions 1 and 2, Proposition 3 includes an additional separability condition. Such separability holds in standard models such as the long-run risk model of Bansal and Shaliastovich (2012) and the habit model of Verdelhan (2010). These models assume complete markets. Bansal and Shaliastovich (2012) also consider a linear economy. Moreover, while the habit model of Verdelhan (2010) is generally non-linear, here the FX risk premium is linear in state variables. In two country DSGE models with incomplete markets such as Benigno and Thoenissen (2008), home state variables such as home total factor productivity affect also the foreign country and vice versa. Hence separability holds since effectively $x_t = x_t^*$. These models also tend to be solved using log-linearization so that separability holds because of the solution technique.[6]

We have not found a realistic example of a model, where separability would not hold. However, generally in non-linear incomplete market models with some unspanning between home and foreign yield curves, the FX risk premium would also depend on interaction terms between home and foreign yield curve factors. We empirically argue that accounting for such interaction terms does not alter our key results, though.

Here we have, for simplicity, analyzed a two country setting. However, in the empirical part we mainly look at the average returns of the U.S. dollar against five major currencies. It is straightforward to extend our results to such average dollar returns. In particular, expected average dollar returns depend only on US yield curve factors and average foreign yield curve factors. However, the weights on the foreign factors can generally be different and must be estimated separately. As before, it is straightforward to show that Proposition 2 also holds when the affine approximation is instead defined using p home and p^* foreign yields or forward rates.

3. Data

We now describe the key data sources and variable definitions. We focus on six advanced economies: Canada, Germany, Sweden, Switzerland, UK and the U.S. For these countries, we were able to obtain long histories of zero coupon government yield curves.[7]

Asset prices. Our data on spot and forward exchange rates is from Datastream and are available from 1983 onwards. Here the five exchange rates are quoted against the U.S. dollar. For Germany, we initially apply the Deutschmark but switch to euro once it becomes available in January 1999. However, we have yield curve data going back to 1983 only for Germany, UK and US. Hence, for Canada, Switzerland and Sweden we start the exchange rate series later when the relevant yield curve data becomes available.

[6] This type of log-linearization also results in a zero risk premium. However, as mentioned our results also hold for exchange rate changes.

[7] Out of the key advanced economy currency areas, we unfortunately had to exclude Australia due to some gaps in yield curve data.

Our zero coupon yield curve data comes from various sources. Our US zero coupon yield curve is from Liu and Wu (2021), who construct the curve using a novel non-parametric method. This curve is also used to compute the excess returns on U.S. 10 year zero coupon bonds.[8] The UK yield curve data is from the Bank of England and built using a spline-based method. The Swiss curve is from the Swiss National Bank and the Swedish curve is from the BIS database. Finally, the German government curve is from Deutsche Bundesbank. The German, Swedish and Swiss curves are constructed using the method of Svensson (1994). There is some unevenness in the availability of yield curve data. The data for the U.S. curve is from March 1973 to December 2023; Canadian data begin later in January 1986, Swiss data in January 1987 and the Swedish curve data in December 1992, while the data for Germany and the UK start in December 1983.[9] These starting dates are summarized in Table A.2 in the Appendix.

Inflation. We compute the annual U.S. inflation rate as the annual log change in the corresponding seasonally unadjusted CPI index, obtained from FRED.[10] U.S. CPI has not been revised backwards but contains a two to three week publication lag. To account for this lag, we follow Cieslak and Povala (2015) and lag inflation rate by one month. That is at the end of each month, we form return predictions using the CPI in the past month. We focus on U.S. inflation due to the lack of availability of long histories of real time CPI indices for the other countries. In particular, the CPI indices for the other countries generally contain some revisions.

Additional data. We combine the aforementioned FX, yield curve and macro data with other data from a range of sources. Annual survey forecasts concerning yields on 10 year Treasury bonds are taken from Consensus Economics. These are available from January 1990. Our

[8]The maturity of 10 years is chosen to match the maturity used in the interest rate survey data. As robustness, we show results using 5 year bonds in the Appendix.

[9]The curve data for Germany and UK in principle go back to the 1970s. However, since we use these data only as controls in the FX regressions and the FX data begins in 1983, we only rely on the yield curve data from that date onwards.

[10]We use annual inflation instead of trend inflation since the persistence in trend inflation can give rise to econometric issues (Crump and Gospodinov, 2025). We use headline rather than core inflation as we want to include sharp shifts to which agents may react sluggishly. Headline inflation can also lead core inflation as evidenced vividly during the pandemic (Ball et al., 2022). From a macro-finance angle headline inflation also more accurately represents changes in the total costs for a representative agent.

monetary policy shock data are updated versions of the data constructed by Gürkaynak et al. (2004) and Nakamura and Steinsson (2018). These data begin in February 1995. [11] Finally, the estimated Taylor rule coefficients are from Lombardi et al. (2025) and are available from March 1990.

4. The spanning hypothesis: an empirical evaluation

We now provide evidence that the spanning conditions derived in Section 2 are violated in the data. In particular we find inflation to be an unspanned predictor of both bond and currency excess returns. Our baseline empirical specification is given by the predictive regression:

$$Dep_{t+1} = a + b\pi_t + \epsilon_{t+1},$$

with π_t denoting the annual U.S. CPI inflation rate that is available in month t[12] and Dep representing the main dependent variables, that is, U.S. bond and dollar excess returns as well as the corresponding yield and U.S. dollar exchange rate changes.

The spanning regressions have the same form but also control for vectors of yield factors:

$$Dep^B_{t+1} = a + b\pi_t + c'yc_t + \epsilon_{t+1};$$

$$Dep^{FX}_{t+1} = a + b\pi_t + c'yc_t + c^{*\prime}\bar{yc}^*_t + \epsilon_{t+1}.$$

Here Dep^B_{t+1} represent bond-related dependent variables (bond excess returns and yield changes), and Dep^{FX}_{t+1} stands for FX-related variables (changes in the dollar exchange rate and dollar excess returns vis-a-vis a broad set of other currencies). The FX spanning regressions also control for foreign yield curve factors $\bar{yc}^*_t$, while the U.S. bond regressions only control for U.S. factors yc_t. The linear spanning hypothesis, reflected by Propositions 2 and 3, implies that in both cases we should have $b = 0$. As yield factors we consider the first 3 or 5 principal

[11] Updated data are available here: https://www.acostamiguel.com/research.html.

[12] As mentioned, this is the inflation rate known in the previous month $t-1$, that is, $\log CPI_{t-1} - \log CPI_{t-12-1}$.

components of yields or the 10 annual forward rates.

Similarly to e.g. Lustig et al. (2019), we focus on monthly excess returns. As robustness checks, we also consider quarterly and annual horizons. These modify the regressions in a straightforward way.

We focus on explaining average dollar excess returns against a basket of currencies. Here the relevant foreign control variables $\bar{yc}_t^*$ are then averages of yield curve factors across the foreign countries. However, our theoretical results imply that the weights on the factors might be different, e.g. yield curve factors in some foreign country might be particularly important for dollar returns. Hence, the weights are obtained by first performing the regressions country by country using bilateral exchange rates. These weights are then used to construct the average foreign factors used in the main FX regressions. In the Appendix, we also show the country specific FX regressions, which also provide direct support that full spanning is violated.

The key empirical results are given in Table 1. We apply Newey and West (1987) standard errors with a conservative choice of 12 lags. Statistical significance would be stronger if we instead used 5 lags implied by the standard lag selection formula.[13] In the Appendix, we also show that the key results are robust to using the bootstrap procedure of Bauer and Hamilton (2018) as well as the Crump and Gospodinov (2025) bootstrap. For brevity, we do not report the constant terms but show key descriptive statistics such as means of variables in the Appendix.

As seen from Panel A, high inflation predicts low bond excess returns and high dollar excess returns. In particular, a 1 percentage point (pp) higher inflation implies a 12bps lower bond excess return and 16bps higher excess return on the U.S. dollar. Relatedly, 1 pp higher inflation is also associated with 1bps lower 10 year yield and 16bps dollar appreciation. Essentially all of the higher dollar excess returns is explained by dollar appreciation rather than interest rate differentials. The yield change may appear small but note that bond returns roughly equal duration times yield change.[14]

[13]For example Duffee (2011) applies Newey and West (1987) standard errors with 4 lags.

[14]The modified duration of a zero coupon 10 year bond is $\frac{10}{1+y}$ and the return up to first order is $\frac{10}{1+y}\Delta y$. An additional but weaker mechanism is that high inflation is associated with high short rates, which lowers excess returns.

Table 1: Testing the spanning hypothesis: baseline results

	Panel A: No YC Controls			
	(1)	(2)	(3)	(4)
	rx^{10Y}	rx^{FX}	Δy^{10Y}	Δs
π	-12.25***	15.78**	0.909*	15.67**
	(-2.73)	(2.00)	(1.73)	(2.03)
N	608	480	608	480
R^2 (in %)	1.22	1.07	0.58	1.07
	Panel B: With YC Controls			
	(1)	(2)	(3)	(4)
	rx^{10Y}	rx^{FX}	Δy^{10Y}	Δs
π	-14.46**	27.04***	1.682**	25.27***
	(-2.29)	(3.15)	(2.36)	(3.07)
N	608	480	608	480
R^2 (in %)	2.88	4.87	2.11	3.92

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Panel A shows the results when predicting monthly U.S. bond and dollar excess returns as well as yield and exchange rate changes with the latest annual U.S. CPI inflation rate. Panel B shows the coefficients on inflation when predicting the same variables using inflation but also controlling for the first three principal components of yields. The t-statistics are based on Newey and West (1987) standard errors with 12 lags. The sample for bond return and yield regressions is between March 1973 and December 2023. The dollar exchange rate is defined against a basket of currencies with varying start dates.

Panel B of Table 1 shows the results when we also control for the yield curve factors. Here we can see that the predictive power of inflation remains strong after adding these controls. In the FX specifications, statistical significance of inflation is actually increased. Overall, while Panel A shows that inflation is a common predictor of bond and dollar excess returns, Panel B shows it is an unspanned predictor. This provides direct evidence that the spanning hypothesis detailed in the previous section is violated for both U.S. bond and dollar returns as well as yield and dollar exchange rate changes.

The coefficients on the yield curve factors are not shown for brevity but the full results are in the Appendix. However, with some exceptions the yield curve factors are insignificant.

Table 2: Baseline results for a quarterly horizon.

	Panel A: No YC Controls			
	(1)	(2)	(3)	(4)
	rx^{10Y}	rx^{FX}	Δy^{10Y}	Δs
π	-33.87***	43.68*	2.422	43.59*
	(-2.60)	(1.87)	(1.57)	(1.90)
N	606	478	606	478
R^2 (in %)	2.88	2.50	1.23	2.58
	Panel B: YC Controls			
	(1)	(2)	(3)	(4)
	rx^{10Y}	rx^{FX}	Δy^{10Y}	Δs
π	-41.31**	68.63***	4.732**	65.23***
	(-2.18)	(2.72)	(2.23)	(2.90)
N	606	478	606	478
R^2 (in%)	7.63	10.50	6.04	7.94

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Panel A shows the results when predicting quarterly U.S. bond and dollar excess returns as well as yield and exchange rate changes with the latest annual U.S. CPI inflation rate. Panel B shows the coefficients on inflation when predicting the same variables using inflation but also controlling for the first three principal components of yields. The t-statistics are based on Newey and West (1987) standard errors with 12 lags. The sample for bond return and yield regressions is between March 1973 and December 2023. The dollar exchange rate is defined against a basket of currencies with varying start dates.

Table 2 shows that the results are similar when applying a quarterly horizon instead of monthly. However, the estimated coefficient values grow somewhat with the horizon. In particular, here a 1 pp higher inflation implies a 34bps drop in bond and a 44bps increase in dollar excess return, as well as a 2.4bps fall in the 10 year yield and a 48bps dollar appreciation. The Appendix further shows results for an annual horizon.

Table 3 shows results similar to Table 1, Panel B but controlling for the 10 annual forward rates instead of principal components.[15] The results are similar to before. The Appendix also shows results when controlling for 5 principal components of yields instead of 3 and performs several robustness checks for the key results.

Table 3: Forward rates as controls

	YC Controls			
	(1)	(2)	(3)	(4)
	rx^{10Y}	rx^{FX}	Δy^{10Y}	Δs
π	-17.78***	17.96***	2.09***	17.58***
	(-2.73)	(2.79)	(2.84)	(2.75)
N	608	480	608	480
R^2 (in %)	6.47	12.19	6.02	10.99

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The Table shows the results when predicting monthly U.S. bond and dollar excess returns as well as yield and exchange rate changes with the latest annual U.S. CPI inflation rate. Instead of yield principal components, we here control for all annual forward rates. The t-statistics are based on Newey and West (1987) standard errors with 12 lags. The sample for bond return and yield regressions is between March 1973 and December 2023. The dollar exchange rate is defined against a basket of currencies with varying start dates.

Our baseline results for U.S. bonds are consistent with Duffee (2011), Joslin et al. (2014) and Cieslak and Povala (2015). However, our results for exchange rate returns and changes are novel.

Note that the result that dollar bond yields and the dollar exchange rate move in the same direction is well consistent with standard models. For example, in models assuming that the uncovered interest rate parity holds, as in the classical model of Dornbusch (1976), an interest rate shock leads to exchange rate appreciation. However, in such classical models dollar and bond returns would be unpredictable. More recent macro-finance models can generate time variation in expected returns and hence predictability. However, as explained

[15] The first forward rate is the annual yield, the second the forward rate for a one year period starting after one year and so on.

in the theoretical section, they imply that this predictability should vanish once controlling for yield curve factors.

5. Understanding the mechanisms

Which economic mechanisms might underlie the predictability patterns documented in the previous section? In this section, we conduct a series of empirical tests demonstrating that our key predictability results stem from investors' incomplete information about the Fed's monetary policy reaction function – a deviation from full-information rational expectations (FIRE). In particular, we show that inflation also predicts future monetary policy shocks, expectational errors concerning bond yields as well as changes in the perceived monetary policy stance of the Fed.

We first document that inflation also predicts monetary policy shocks – a stark violation of the unpredictability condition posited by the FIRE assumption. The shocks considered are shocks to the monetary policy target and to the expected future path of the policy rate in the spirit of Gürkaynak et al. (2004). Moreover, we use the shocks constructed by Nakamura and Steinsson (2018) (NS). The NS shocks represent shocks to the level factor of short- to medium-term interest rates from euro-dollar futures, that is, besides capturing target changes they also entail a forward-looking (path) component.

The results are given by Table 4.[16] Panel A shows that inflation is a significant predictor of path and NS shocks, but not of target shocks. Panel B shows that the predictive power survives after controlling for yield curve factors measured as annual forward rates.[17] Note that under full information rational expectations (FIRE), monetary policy shocks should be unpredictable.

We also construct annual expectational errors on U.S. 10 year yields using yield forecasts from Consensus Economics.[18] Table 5, Panel A shows that a rise in inflation predicts such yield forecast errors. The results become statistically significant once we also control for yield

[16] Because the data is unequally spaced we here only correct for heteroskedasticity similarly to Nakamura and Steinsson (2018). This is also conservative since, if anything, the residuals have some negative autocorrelation.

[17] However, we also found predictability in months without Fed meetings. These months can also contain revisions of beliefs about future policy.

[18] The expectational error is given by $y^{10Y}_{t+12} - \mathbb{E}^S_t[y^{10Y}_{t+12}]$, where $\mathbb{E}^S_t[y^{10Y}_{t+12}]$ is the survey forecast.

Table 4: Predicting U.S. monetary policy shocks

	Panel A: No YC Controls		
	(1)	(2)	(3)
	Target	Path	NS
π	0.0584	1.149***	0.306**
	(0.47)	(2.30)	(2.39)
N	231	231	231
R^2 (in %)	0.06	3.46	2.30
	Panel B: YC Controls		
	(1)	(2)	(3)
	Target	Path	NS
π	0.148	1.14**	0.358**
	(0.89)	(2.22)	(2.37)
N	231	231	231
R^2 (in %)	5.53	11.15	11.99

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: Panel A shows the results from predicting three types of U.S. monetary policy shocks over a monthly horizon using the annual U.S. CPI inflation rate. The shocks considered are the target and path shocks of Gürkaynak et al. (2004) as well as Nakamura and Steinsson (2018) shocks. Panel B shows the coefficients on inflation when also controlling for the 10 annual forward rates. The sample is from February 1995 to December 2023. The standard errors correct for heteroskedasticity.

Table 5: Predicting interest rate forecast errors and changes in the Taylor rule coefficient on inflation

Panel A: Long rate forecast errors		
	(1)	(2)
	No YC Controls	YC Controls
π	13.51	12.74**
	(1.53)	(2.47)
N	395	395
$R^2(\%)$	5.04	42.09

Panel B: Changes in the Taylor coefficient on inflation		
	(1)	(2)
	No YC Controls	YC Controls
π	0.168	0.219**
	(1.01)	(2.09)
N	405	405
$R^2(\%)$	2.49	18.73

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: Panel A shows the results from predicting forecast errors for 10 year yields with the latest available CPI inflation rates. The annual yield forecasts are from Consensus Economics. Panel B shows the results from predicting monthly changes in the perceived Taylor rule coefficient on inflation using the latest available CPI inflation rates. The estimated Taylor rule coefficients on inflation are from Lombardi et al. (2025). Both panels also show the results when controlling for the 10 annual forward rates. The t-statistics are based on Newey and West (1987) standard errors with 12 lags. The sample is from January 1990 to December 2023 for interest rate forecast errors and from March 1990 to December 2023 for changes in Taylor coefficients.

curve factors. An increase in inflation is associated with investors underpredicting future yields. The economic magnitude of the errors is relatively large.[19] Again note that under FIRE, like monetary policy shocks, forecast errors should be unpredictable.

Similarly to Bauer et al. (2024), Lombardi et al. (2025) use panel data of survey forecasts to estimate professional forecasters' perceived Taylor rule coefficients on inflation. Table 5, Panel B shows that high inflation today also tends to be followed by upward revisions of estimated Taylor rule coefficients, although these results are again significant only when

[19]From Table A.5 in the Appendix we can see that the coefficient on inflation when predicting annual yield changes is 9.3 without yield factor controls. This is slightly smaller than the coefficient when predicting forecast errors. Unexpected component of the yield change is hence actually larger than the yield change. Because bond returns roughly equal duration times yield change such yield changes give rise to relatively large bond returns.

controlling for yield factors. Inflation seems to be associated with future updates on investors' perceived hawkishness of the Fed in countering inflation.

Our main interpretation of the above results is that high inflation leads investors to revise upwards their beliefs about the path of short rates in the future. This updating likely relates to investors not fully grasping the central bank reaction function in real-time. As they update their beliefs about the Fed's resolve to rein in on inflation, and the central bank proceeds with tightening, the dollar appreciates and excess returns in the bond market fall.

Why not a risk-based story? As mentioned, monetary policy shocks and expectational errors should be unpredictable under FIRE. Here we provide further evidence that our results are due to expectational errors rather than standard risk-based mechanisms (for a review, see Cieslak and Pflueger, 2023). We use survey data to form subjective expectations of annual excess dollar and bond returns.[20] The subjectively expected excess return on the U.S. dollar is given by

$$\mathbb{E}_t^S[s_{t+12}] - s_t + y_t^1 - y_t^{1*},$$

where $\mathbb{E}_t^S[s_{t+12}]$ denotes the survey expectation of the dollar exchange rate after one year. The subjectively expected bond excess returns use a duration approximation (see e.g. Bacchetta et al., 2009):

$$\approx \frac{D_{10Y} y_t^{10Y} - (D_{10Y} - D_{12})\mathbb{E}_t^S[y_{t+12}^{10Y}]}{D_{12}} - y_t^{12}.$$

Here $\mathbb{E}_t^S[y_{t+12}^{10Y}]$ is the survey forecast of the 10 year yield after one year. Moreover, D_n is the duration of a bond with maturity n.

As shown in Table 6, regressing subjectively expected FX returns on inflation gives a slope coefficient of -11 (column (1)). This is opposite in sign to the predictability coefficient and hence strengthens the error narrative.

[20] We could also apply a term structure model such as Kim and Wright (2011) to decompose long rates to short rate expectations and term premia. However, these models impose FIRE a priori and do not generally measure subjective risk premia.

Table 6: Explaining subjective risk premia

	(1) $\mathbb{E}^S[rx^{FX}]$	(2) $\mathbb{E}^S[rx^{10Y}]$	(3) $\Delta\mathbb{E}^S[rx^{10Y}]$
π	-10.9***	-18.3	-1.24
	(-1.54)	(-2.70)	(-1.44)
N	407	407	406
R^2 (in %)	7.71	0.8	0.8

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The Table shows the results when explaining the subjectively expected annual excess bond and FX returns using current U.S. annual inflation. It also shows the results when predicting monthly changes in the subjectively expected bond return using inflation. The subjectively expected bond and FX excess returns are based on yield and FX forecasts from Concensus Economics. The t-statistics are based on Newey and West (1987) standard errors with 12 lags. The sample is from January 1990 to December 2023.

Regressing subjectively expected bond excess returns on inflation gives a slope coefficient of -18 (column (2)). One percentage point higher inflation is associated with 18bps lower subjectively expected annual excess bond return. This hence has the same sign than the bond predictability coefficient. In particular, predicting annual bond excess returns with inflation results in a slope coefficient of -133 as reported in the Appendix. However, we can decompose this predictability coefficient to a risk premium effect of -18 and error effect of $-133 - (-18) = -115$. Expectational errors explain $-115/(-133) \approx 86\%$ of bond return predictability!

We also regress changes in subjectively expected bond returns on inflation (column (3)). The resulting slope coefficient is -1. Hence high inflation is associated with subsequent declines in subjective bond risk premia. This gives further evidence that long-term yields increase due to a change in the path of short rates, not an increase in (subjectively perceived) term premia.

6. A stylized model

Overall, our empirical results suggest that high inflation is associated with future unexpected monetary policy tightening, especially revisions in beliefs about the path of short rates. This leads to increases in long-term yields, low bond returns, and exchange rate appreciation.

We do not take a strong stance on the specific model that explains the results. However, we provide a stylized example of a model that could. We assume that the Fed sets the short rate according to the following simple rule

$$y_t^1 = \phi \bar{\pi}_t + v_t.$$

Here $\bar{\pi}_t$ is long run inflation and v_t is a random monetary policy shock, $v_t \sim N(0, \sigma_v^2)$. We ignore constant terms for simplicity as they do not affect predictability. We assume $\bar{\pi}_t$ follows AR(1) with persistence λ_l and residual variance σ_l^2 but is unobserved by the agents. More generally, $\bar{\pi}_t$ could represent an unobserved macroeconomic state. Agents short rate expectations follow a sticky expectations process

$$\mathbb{E}_t^S[y_{t+1}^1] = (1-k)\lambda_l \mathbb{E}_{t-1}^S[y_t^1] + k\lambda_l y_t^1,$$

where $\mathbb{E}_t^S$ is the subjective expectations operator. This rule emerges as a solution to a filtration problem with unknown $\bar{\pi}_t$. Here k is given by the standard formulas for the Kalman filter. However, it could also represent a simple behavioral rule.

Note that the full information rational forecast of the next period short rate would be $\lambda_l \phi \bar{\pi}_t$. Here the key deviation from FIRE is omitting information related to $\bar{\pi}_t$. This setting is different from the standard sticky expectations model in which the forecast under FIRE is λr_t (Coibion and Gorodnichenko, 2015)[21]. In the standard sticky expectations model the key deviation from FIRE is instead slow reaction to new information.

We assume that the agents are risk neutral. Long-term interest rates are hence given by:

$$y_t^n = \mathbb{E}_t^S \frac{1}{n} \sum_{i=0}^{n-1} y_{t+i}^1.$$

This is essentially the expectations hypothesis under subjective beliefs.[22] The exchange rate can be obtained by solving the standard UIP equation forward:

[21] Here we also deviate from Granziera and Sihvonen (2024), who use a sticky expectations model to explain bond and currency dynamics.

[22] For derivations of the long rate and exchange rate equations, see Granziera and Sihvonen (2024). The solution for the exchange rate assumes a constant long-run exchange rate, which can be normalized to zero.

$$s_t^n = \mathbb{E}_t^S \sum_{i=0}^{\infty} y_{t+i}^1.$$

Here the foreign interest rate is set to a constant. However, the foreign interest rate can also be ignored if we assume that agents have rational expectations about them so they do not affect predictability.[23]

High long-run inflation $\bar{\pi}_t$ implies higher than expected future short rates. This implies low bond returns and dollar appreciation. On the other hand, current inflation π_t is positively correlated with long-run inflation $\bar{\pi}_t$. Hence high π_t predicts low bond returns and high currency returns. Moreover, here π_t is largely unspanned by information in the yield curves. Note that since r_t and $\mathbb{E}_t^S[r_{t+1}]$ are observed, they are instead fully spanned by the yield curve.

As a simple example we consider an exponential moving average process

$$\bar{\pi}_t = \omega\bar{\pi}_{t-1} + (1-\omega)\pi_t^m,$$

and assume monthly inflation π_t^m follows an AR(1)-process

$$\pi_t^m = \lambda\pi_{t-1}^m + \epsilon_t,$$

with $\epsilon_t \sim N(0, \sigma_e^2)$. $\bar{\pi}_t$ specified by such a moving average process is often called trend inflation. The solution to the filtration problem follows given an AR(1)-approximation for $\bar{\pi}_t$.[24] But again note that the sticky expectations process might also represent a simple behavioral rule.

To calibrate the model, we first estimate the persistence parameter λ^m and residual volatility σ_e directly from inflation data. We set $\phi = 1.5$, a standard Taylor rule value for

[23] We also need to assume that foreign rates do not contain additional information about $\bar{\pi}_t$ or if they do, agents do not use that information. As mentioned before we focus on U.S. inflation but remain agnostic whether similar results hold for foreign inflation rates. However, we have replicated many of our results using available foreign CPI indices. Here we found evidence that foreign inflation also predicts exchange rates and foreign bond returns yet this predictability seems weaker than our benchmark results. However, the predictability results using foreign inflation are less reliable as many of the indices are not available in real time. In particular this predictability might potentially be due to backward data revisions.

[24] We solve this using simulation. Strictly speaking $\bar{\pi}_t$ follows an AR(2) but we find an AR(1) to be very accurate giving an R^2 of 98%.

Table 7: Calibrated values for the theoretical model

Parameter	Value	Source
λ^m	0.49	estimated
σ_e	0.0028	estimated
ϕ	1.5	standard
σ_v	$0.67\sigma_l$	target: $k \approx 0.5$ (Granziera and Sihvonen, 2024)
ω	0.97	target: baseline regression slopes

Table 8: Key coefficients implied by the model and those in the data

Coefficient	Model	Data
regression slope 10Y bond return	-10.8	-12.3
regression slope FX return	15.2	15.8
regression slope 10Y bond return, YC controls	-6.1	-14.5
regression slope FX return, YC controls	8.6	27

Notes: The Table shows the predictability coefficients for bond and dollar excess returns obtained by simulating the calibrated model as well as those measured from the data. It also shows the coefficients when controlling for yield curve factors.

inflation. σ_v is set to target a sticky expectations coefficient $k \approx 0.5$, which is reasonable given the estimates in Granziera and Sihvonen (2024). Finally, ξ is set to target the baseline predictability regression slopes. The calibrated values are given in Table 7.

Table 8 shows the predictability coefficients obtained by simulating the calibrated model as well as those measured from the data. The baseline predictability coefficients from the model are close to their empirical counterparts.

The model somewhat understates the predictability coefficients obtained by also controlling for yield factors. However, these regressions are less comparable. This is because the simple theoretical model features only two yield factors while in the empirical specifications we control for three factors. The results in the theoretical model can be sensitive to the number of factors, e.g. the coefficients are larger if we only control for one yield factor. Second, the model abstracts away from the effects of foreign yield factors on exchange rates.

The model serves as a useful tool to demonstrate a plausible mechanism explaining the key results in this paper. However, it abstracts away from one effect. In the empirical part we found some evidence that high inflation is followed by increases in the perceived Taylor rule coefficient on inflation. Relaxing the assumption that ϕ is known might create an interesting additional mechanism contributing to the empirical results. However, this would complicate the model solution considerably and this extension is hence left to future work.

The above model also generates some exchange rate disconnect in the sense of Meese and Rogoff (1983). In particular the contemporaneous correlation between exchange rate changes and inflation changes is lower than if inflation would be fully incorporated into interest rate forecasts. However, generally the FX spanning puzzle is different from the exchange rate disconnect puzzle. In particular the FX spanning puzzle discussed in this paper refers to the existence of macro factors, which can forecast future exchange rate changes but are unspanned by the yield curve. On the other hand, the exchange rate disconnect puzzle is about the low contemporaneous correlation between exchange rate changes and macroeconomic variables.

Relatedly, Gourinchas et al. (2022) propose a model with some segmentation between bond and FX markets. This weakens the contemporaneous correlation between bond and currency returns relative to standard models. However, full spanning still holds so such a model is inconsistent with the results documented in this paper. Also broadly speaking, while market segmentation can be useful for explaining some features of FX dynamics, our key finding that inflation is an unspanned predictor of both bond and currency returns, points to information frictions rather than market segmentation as a key underlying driver.

Our findings are somewhat more closely related to but still clearly distinct from the FX-bond disconnect discussed by Chernov and Creal (2023). They find that contemporaneus regressions of FX returns on the returns of different maturity bonds yield an R^2 clearly below 100%. That is FX returns cannot be replicated with bonds. On the other hand, the above simple model predicts a perfect fit when using at least two bond returns. Loosely speaking their findings imply that FX rates contain some information that is not priced in bond returns. On the other hand, in our setting there is a common factor missing from both FX and bond pricing. However, it would be interesting to extend our model to include a factor that is reflected only in FX pricing.

7. Conclusion

We argue that the spanning puzzle documented in the bond literature is, in fact, a *joint* bond-FX spanning puzzle. In standard macro-finance models, no variable should predict bond or exchange rate returns — nor changes in yields or exchange rates — after linearly controlling for yield curve factors. This spanning condition is violated in the data. In particular U.S. inflation is not only an unspanned predictor of bond returns but also predicts dollar appreciation.

To explore the underlying drivers of these patterns, we find that high inflation also predicts monetary policy shocks, unexpected increases in long-term interest rates as well as increases in the perceived Taylor rule coefficient on inflation. The most plausible interpretation is that investors grapple with understanding the central bank's reaction function. As investors update their beliefs about the Fed's resolve to rein in on inflation, the dollar appreciates and excess returns in the bond market fall. We argue that the results are consistent with a simple model in which agents are unaware of the inflation metric targeted by the Fed.

References

Adrian, T., Crump, R., and Moench, E. (2013). Pricing the term structure with linear regressions. *Journal of Financial Economics*, 110(1):110–138.

Ang, A. and Chen, J. (2010). Yield curve predictors of foreign exchange returns. Working paper.

Bacchetta, P., Mertens, E., and Van Wincoop, E. (2009). Predictability in financial markets: What do survey expectations tell us? *Journal of International Money and Finance*, 28(3):406–426.

Bai, J. and Perron, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica*, pages 47–78.

Ball, L., Leigh, D., and Mishra, P. (2022). Understanding US inflation during the COVID era. *NBER wp. 30613*.

Bansal, R. and Shaliastovich, I. (2012). A long-run risks explanation of predictability puzzles in bond and currency markets. *Review of Financial Studies*, 26(1):1–33.

Bansal, R. and Yaron, A. (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance*, 59(4):1481–1509.

Bauer, M. and Hamilton, J. (2018). Robust bond risk premia. *Review of Financial Studies*, 31(2):399–448.

Bauer, M., Pflueger, C., and Sunderam, A. (2024). Perceptions about monetary policy. *Quarterly Journal of Economics*, 139(4):2227–2278.

Bauer, M. and Rudebusch, G. (2020). Interest rates under falling stars. *American Economic Review*, 110(5):1316–1354.

Bauer, M. and Swanson, E. (2023). An alternative explanation for the "Fed information effect". *American Economic Review*, 113(3):664–700.

Benigno, G. and Thoenissen, C. (2008). Consumption and real exchange rates with incomplete markets and non-traded goods. *Journal of International Money and Finance*, 27(6):926–948.

Bianchi, D., Büchner, M., and Tamoni, A. (2021). Bond risk premiums with machine learning. *Review of Financial Studies*, 34(2):1046–1089.

Boehm, C. and Kroner, T. (2024). Monetary policy without moving interest rates: The Fed non-yield shock. NBER wp. 32636.

Bordalo, P., Gennaioli, N., and Shleifer, A. (2018). Diagnostic expectations and credit cycles. *The Journal of Finance*, 73(1):199–227.

Campbell, J. and Cochrane, J. (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 107(2):205–251.

Campbell, J. and Shiller, R. (1988). The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies*, 1(3):195–228.

Chernov, M. and Creal, D. (2023). International yield curves and currency puzzles. *The Journal of Finance*, 78(1):209–245.

Christensen, J. and Rudebusch, G. (2015). Estimating shadow-rate term structure models with near-zero yields. *Journal of Financial Econometrics*, 13(2):226–259.

Christensen, J. and Rudebusch, G. (2016). Modeling yields at the zero lower bound: Are shadow rates the solution? In *Dynamic Factor Models*, volume 35, pages 75–125.

Cieslak, A. (2018). Short-rate expectations and unexpected returns in treasury bonds. *Review of Financial Studies*, 31(9):3265–3306.

Cieslak, A. and Pflueger, C. (2023). Inflation and asset returns. *Annual Review of Financial Economics*, 15(1):433–448.

Cieslak, A. and Povala, P. (2015). Expected returns in treasury bonds. *Review of Financial Studies*, 28(10):2859–2901.

Cochrane, J. and Piazzesi, M. (2005). Bond risk premia. *American Economic Review*, 95(1):138–160.

Coibion, O. and Gorodnichenko, Y. (2015). Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review*, 105(8).

Coroneo, L., Giannone, D., and Modugno, M. (2016). Unspanned macroeconomic factors in the yield curve. *Journal of Business & Economic Statistics*, 34(3):472–485.

Creal, D. and Wu, C. (2020). Bond risk premia in consumption-based models. *Quantitative Economics*, 11(4):1461–1484.

Crump, R. and Gospodinov, N. (2025). Deconstructing the yield curve. *Review of Financial Studies*, 38(2):381–421.

Crump, R. K. and Gospodinov, N. (2022). On the factor structure of bond returns. *Econometrica*, 90(1):295–314.

Dahlquist, M. and Hasseltoft, H. (2020). Economic momentum and currency returns. *Journal of Financial Economics*, 136(1):152–167.

Dahlquist, M. and Pénasse, J. (2022). The missing risk premium in exchange rates. *Journal of Financial Economics*, 143(2):697–715.

Dornbusch, R. (1976). Expectations and exchange rate dynamics. *Journal of Political Economy*, 84(6):1161–1176.

Driscoll, J. C. and Kraay, A. C. (1998). Consistent covariance matrix estimation with spatially dependent panel data. *Review of Economics and Statistics*, 80:549–560.

Duffee, G. (2011). Information in (and not in) the term structure. *Review of Financial Studies*, 24(9):2895–2934.

Engel, C. (2000). Long-run PPP may not hold after all. *Journal of International Economics*, 51(2):243–273.

Fama, E. and Bliss, R. (1987). The information in long-maturity forward rates. *American Economic Review*, 77:680–692.

Feunou, B., Fontaine, J.-S., and Krohn, I. (2024). Twin stars: Neutral rates and currency risk premia. Working paper.

Filippou, I. and Taylor, M. (2017). Common macro factors and currency premia. *Journal of Financial and Quantitative Analysis*, 52(4):1731–1763.

Gourinchas, P.-O., Ray, W., and Vayanos, D. (2022). A preferred-habitat model of term premia, exchange rates, and monetary policy spillovers. NBER wp. 29875.

Gourinchas, P.-O. and Tornell, A. (2004). Exchange rate puzzles and distorted beliefs. *Journal of International Economics*, 64(2):303–333.

Granziera, E. and Sihvonen, M. (2024). Bonds, currencies and expectational errors. *Journal of Economic Dynamics and Control*, 158.

Gürkaynak, R., Sack, B. P., and Swanson, E. (2004). Do actions speak louder than words? The response of asset prices to monetary policy actions and statements. *International Journal of Central Banking*, page 55–93.

Joslin, S., Priebsch, M., and Singleton, K. (2014). Risk premiums in dynamic term structure models with unspanned macro risks. *Journal of Finance*, 69(3):1197–1233.

Karnaukh, N. and Vokata, P. (2022). Growth forecasts and news about monetary policy. *Journal of Financial Economics*, 146(1):55–70.

Kim, D. and Wright, J. (2011). An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates. Fed Board Discussion Paper.

Kroencke, T., Schmeling, M., and Schrimpf, A. (2021). The FOMC risk shift. *Journal of Monetary Economics*, 120:21–39.

Liu, Y. and Wu, J. (2021). Reconstructing the yield curve. *Journal of Financial Economics*, 142(3):1395–1425.

Lombardi, M., Schrimpf, A., and Sonja, Z. (2025). Decoding beliefs about the central bank reaction function. Unpublished manuscript.

Longstaff, F. (1989). A nonlinear general equilibrium model of the term structure of interest rates. *Journal of Financial Economics*, 23(2):195–224.

Lustig, H., Stathopoulos, A., and Verdelhan, A. (2019). The term structure of currency carry trade risk premia. *American Economic Review*, 109(12).

Mankiw, G. and Reis, R. (2002). Sticky information versus sticky prices: a proposal to replace the new keynesian phillips curve. *Quarterly Journal of Economics*, 117(4):1295–1328.

Meese, R. and Rogoff, K. (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of International Economics*, 14(1-2):3–24.

Moench, E. and Siavash, S. (2022). What moves treasury yields? *Journal of Financial Economics*, 146(3):1016–1043.

Nakamura, E. and Steinsson, J. (2018). High-frequency identification of monetary non-neutrality: the information effect. *Quarterly Journal of Economics*, 133(3):1283–1330.

Newey, W. and West, K. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55:703–708.

Pohl, W., Schmedders, K., and Wilms, O. (2018). Higher order effects in asset pricing models with long-run risks. *Journal of Finance*, 73(3):1061–1111.

Rudebusch, G. and Swanson, E. (2012). The bond premium in a DSGE model with long-run real and nominal risks. *American Economic Journal: Macroeconomics*, 4(1):105–143.

Sarno, L., Schneider, P., and Wagner, C. (2012). Properties of foreign exchange risk premiums. *Journal of Financial Economics*, 105(2):279–310.

Sarno, L. and Taylor, M. (2002). Purchasing power parity and the real exchange rate. *IMF staff papers*, 49(1):65–105.

Schmeling, M., Schrimpf, A., and Steffensen, S. (2022). Monetary policy expectation errors. *Journal of Financial Economics*, 146(3):841–858.

Sihvonen, M. (2024). Yield curve momentum. *Review of Finance*, 28(3):805–830.

Stock, J. and Watson, M. (2003). Forecasting output and inflation: The role of asset prices. *Journal of Economic Literature*, 41(3):788–829.

Svensson, L. (1994). Estimating and interpreting forward interest rates: Sweden 1992-1994. NBER wp 4871.

Taylor, A. and Taylor, M. (2002). The purchasing power parity debate. *Journal of Economic Perspectives*, 18(4):135–158.

Verdelhan, A. (2010). A habit-based explanation of the exchange risk premium. *Journal of Finance*, 65(1):123–145.

Wachter, J. (2006). A consumption-based model of the term structure of interest rates. *Journal of Financial Economics*, 79:365–399.

Appendix
Inflation and the Joint Bond-FX Spanning Puzzle

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A. Proofs and Numerical Accuracy Checks

A1. Proof of Proposition 1

The r:th order local approximation to the yield equation is

$$y_t^n \approx g_n(\bar{x}) + \sum_{|\alpha| \leq r} \frac{1}{\alpha!} \frac{D^\alpha g(\bar{x})}{\alpha!} (x_t - \bar{x})^\alpha$$

This expression is linear in $(x_t - \bar{x})^\alpha$.

To derive the number of elements in the expansion, by stars and bars there are $\binom{k+m-1}{m-1}$ ways to choose non-negative integers $\alpha_1, \ldots, \alpha_m$ such that $\alpha_1 + \ldots + \alpha_m = k$. This is also the number of unique elements of order k in the Taylor approximation. Then by the Christmas stocking identity, total number of unique terms is

$$\sum_{k=1}^{r} \binom{k+m-1}{m-1} = \binom{r+m}{m} - 1$$

We can write this as $y_t^n = \hat{A}_n + \hat{B}_n X_t$. The new vector state variable in the affine version X_t consists of all the powers of the vector state variable x_t up to order r. For example consider a model with a single state variable x_{t1} and $r = 2$ i.e. a second order approximation. Now the new state variable in the affine version is $X_t = [x_{t1}, x_{t1}^2]'$. Similarly, with two state variables x_{t1} and x_{t2} we have $X_t = [x_{t1}, x_{t2}, x_{t1}^2, x_{t2}^2, x_{t1}x_{t2}]'$.

To now see that the linear approximation results in the same yield equation, note that the principal component scores are linear combinations of yields. Hence for the p scores T_t we can also write

$$T_t = \bar{A} + \bar{B} X_t$$

Here $\bar{B}$ is a square matrix and the scores are uncorrelated. Hence $\bar{B} Var(X_t) \bar{B}'$ is diagonal with each diagonal entry strictly positive. Hence $\bar{B}$ is full rank and therefore invertible. Now we can solve

$$X_t = \bar{B}^{-1}(T_t - \bar{A}).$$

Therefore

$$y_t^n = \hat{A}_n + \hat{B}_n \bar{B}^{-1}(T_t - \bar{A}) = \hat{A}_n - \hat{B}_n \bar{B}^{-1}\bar{A} + \hat{B}_n \bar{B}^{-1} T_t$$

Hence each yield is affine in T_t. The fact that the linear approximation of the risk premium equation also corresponds that resulting from the local approximation follows similarly.

A2. Proof of Proposition 3

We first show that complete markets imply separability. Note that under complete markets:

$$s_{t+1} - s_t = \xi_{t+1} - \xi_{t+1}^*$$

Here ξ_{t+1} and ξ_{t+1}^* are the home and foreign log stochastic discount factors. Then

$$\mathbb{E}_t[rx_{t+1}^{FX}] = \mathbb{E}_t[\xi_{t+1}^*] + y_t^1 - \mathbb{E}_t[\xi_{t+1}] - y_t^{*1} = f(x_t) - f^*(x_t^*)$$

Here the last step uses the fact that $\mathbb{E}_t[\xi_{t+1}^*]$ depends on foreign state variables and $\mathbb{E}_t[\xi_{t+1}]$ on home state variables.

Next we show that the local approximation and the affine approximation gives rise to the same equations. Given separability, the r:th order Taylor approximation gives

$$\mathbb{E}_t[rx_{t+1}^{FX}] \approx \bar{C} + \bar{D}X_t + \bar{D}^* X_t^*$$

where X_t is as in the proof of Proposition 1 and X_t^* collects the powers of foreign state variables analoguously. Similarly to Proposition 1 we can solve

$$T_t = \bar{A} + \bar{B}X_t \tag{A.1}$$

$$T_t^* = \bar{A}^* + \bar{B}^* X_t^* \tag{A.2}$$

and we can solve

$$X_t = \bar{B}^{-1}(T_t - \bar{A})$$

$$X_t^* = \bar{B}^{*-1}(T_t^* - \bar{A}^*)$$

The rest follows by plugging these to Equations A.1 and A.2.

In case x_t and x_t^* contain some of the same variables, F or F^* can include zeros and regressing the FX risk premium on X_t and X_t^* can be subject to a multicollinearity issue. Here we assume one either performs principal component analysis on X_t and X_t^* to reduce the dimension or drops multicollinear variables and sets their coefficient to zero.

A3. Numerical accuracy

The key theoretical results are based on higher order local approximations. In this section we provide additional evidence that the results hold very accurately in standard models. As mentioned before, the results hold exactly in models solved using linearization (e.g. Bansal and Yaron, 2004) or higher order local approximation (e.g. Rudebusch and Swanson, 2012).

The habit model. The first model we consider is the model of Wachter (2006), which is an extended version of the external habit model of Campbell and Cochrane (1999) applied to the term structure of interest rates. Here real yields take the form[25]:

$$y_t^n = -\frac{1}{n}\ln(F_n(z_t))$$

[25] Results are similar when using nominal yields. In this model expected inflation affects nominal yields but not excess returns

$$F_n(z_t) = \mathbb{E}_t[\exp\left(\ln\delta - \gamma g - \gamma(1-\phi)(\bar{z} - z_t) - \gamma(\lambda(z_t)+1)\sigma_c\epsilon_{t+1}\right) F_{n-1}(z_{t+1})]$$

The boundary condition is $F_0(z_t) = 1$. The log surplus consumption z_t describes the consumption of a representative agent relative to a habit formed by past consumption. z_t evolves as

$$z_{t+1} = (1-\phi)\bar{z} + \phi z_t + \lambda(z_t)(\Delta c_{t+1} - \mathbb{E}[\Delta c_{t+1}])$$

$$\Delta c_{t+1} = g + v_{t+1}, \qquad v_{t+1} \sim i.i.d.N(0, \sigma_v^2)$$

$$\lambda(z_t) = \frac{1}{\bar{Z}}\sqrt{1 - 2(z_t - \bar{z})} - 1, \qquad \bar{Z} = \sigma_v\sqrt{\frac{\gamma}{1 - \phi - \frac{b}{\gamma}}}$$

Expected excess bond returns hence depend on a single state variable: surplus consumption z_t. This state variable captures time-variation in expected returns.

Verdelhan (2010) analyzes the implications of the same model for currency returns. Now assume that in addition to the home country there is also a symmetric foreign country. Verdelhan (2010) shows that the expected currency return is given by

$$\mathbb{E}_t[rx_{t+1}^{FX}] = \frac{\gamma\sigma_v^2}{\bar{Z}}(z_t^* - z_t)$$

Here z_t^* is the log foreign habit.

We solve the model using the global solution method, based on numerical integration, and benchmark calibration described in Wachter (2006). We then study the predictability of quarterly excess returns using model simulated yields and excess returns.[26] We predict the true conditional expectation of excess bond return using a linear regression on yield principal

[26] We focus on quarterly returns since the benchmark calibration of Wachter (2006) is for the quarterly horizon. Note that while our main results are for the monthly horizon, we strongly reject full spanning also for the quarterly horizon, see Table 2.

Table A.1: Approximation error in the habit and non-linear models

	Error		
	Habit Model	Non-linear model	
Mat. (y)	1 pc	1 pc	2 pc:s
1	0.02 %	59 %	0 %
2	0.02 %	74 %	0 %
3	0.02 %	69 %	0 %
4	0.02 %	70 %	0 %
5	0.02 %	68 %	0 %
6	0.02 %	62 %	0 %
7	0.02 %	61 %	0 %
8	0.02 %	56 %	0 %
9	0.02 %	56 %	0 %
10	0.03 %	56 %	0 %

Notes: table shows the approximation error when predicting quarterly excess returns of different maturity bonds in the habit model using a linear regression with the first principal component of yields. It also shows the error when predicting monthly excess bond returns in a simple non-linear model using a linear regression with one or two principal component of yields.

components. Approximation error is measured by the mean squared prediction error scaled by the variance of true expected excess bond return.

The results are given in Table A.1. We can see that linear spanning holds very accurately in the habit model when using one yield curve factor. That is the risk premium equation is actually close to linear and there is no need to account for non-linearities using more factors.

We also repeated the exercise for FX returns but now predicting the returns with a home and foreign yield factor. Here the approximation error is merely 0.0002%. However note that in the habit model, unlike the bond risk premium, the FX risk premium is actually exactly linear in the state variables. Hence the approximation error only represents error due to approximating the state variables with single yield factors. There can also be some numerical error in the global solution procedure.

As mentioned asset pricing models with long-run risks are often solved using log-linearization (Bansal and Yaron, 2004). Here linear spanning holds perfectly when using as many yield factors as model state variables. We also experimented the accuracy of the procedure by solving the long-run risk model of Bansal and Shaliastovich (2012) using the global solution procedure described by Pohl et al. (2018). We found that the log-linearization is very accurate for the purposes of analyzing spanning. This was true also when using more non-standard parameter values.

Empirical non-linear model. As explained above the standard asset pricing models tend to be close to linear. We now evaluate linear spanning using a simple term structure model with greater non-linearity, which is loosely motivated by Longstaff (1989). We assume that the short rate y_t^1 follows an AR(1)-process and estimate it using monthly U.S. data. We then assume the following affine yield specification:

$$y_t^n = A_n + B_n^1 y_t^1 + B_n^2 \sqrt{y_t^1}.$$

Here the coefficients A_n, B_n^1 and B_n^2 are solved from U.S. data using least squares. The approximation error for predicting monthly returns using one and two principal components is given in Table A.1.

This model has only one state variable: y_t^1. Still due to non-linearity using one yield factor to predict returns results in large approximation errors. On the other using two factors captures all of predictability and results in zero errors. This is because this model is affine in y_t^1 and $\sqrt{y_t^1}$. That is, while the model is a non-linear model with one state variable, it is also a two factor affine model.

To analyze the accuracy for FX rates we could for example assume that interest rate parity holds for the returns of long maturity bonds (Lustig et al., 2019). However, then the results FX rate returns are as for bond returns and are omitted.

Table A.2: Data overview

Parameter	Value
Canada	January 1986
Germany	December 1983
Switzerland	January 1987
Sweden	December 1992
UK	December 1983

Notes: The Table summarizes the starting dates for the FX and foreign yield curve data.

B. Additional empirical results and discussion

In this section we provide additional empirical results and robustness checks to the results presented in the main text.

B1. Data: starting dates and descriptive statistics

Table A.2 summarizes the starting dates for currency and foreign yield curve data. For Canada, Switzerland and Sweden the starting date is constrained by the availability of yield curve data.

Table A.3 shows descriptive statistics for key variables such as bond and currency excess returns. During our sample the dollar depreciated slightly.

Table A.3: Descriptive statistics

	rx^{10Y}	rx^{FX}	Δy^{10Y}	Δs	π
mean	0.22	-0.02	-0.01	-0.03	3.89
stdev	3.16	2.37	0.25	2.35	2.85
skew	0.09	0.16	-0.04	0.18	1.31
kurt	4.88	3.72	7.35	3.83	4.44

Notes: The Table shows the means, standard deviations as well as skewness and kurtosis values for key variables. The variables are monthly excess bond and dollar returns, change in the 10 year US interest rate and the dollar exchange rate as well as annual US inflation. All variables are expressed as percentages. The sample for bond return and yield regressions is between March 1973 and December 2023. The dollar exchange rate is defined against a basket of currencies with varying start dates.

B2. Spanning regressions: full results for Table 1

In our spanning regressions, coefficients on the yield curve factors are not shown for brevity. Table A.4 replicates Table 1, Panel B but also shows the coefficients on the yield curve factors. For bond returns the slope factor is significant, consistent with Fama and Bliss (1987). The other factors are insignificant. For currency returns, only the foreign level and curvature factors are significant.

B3. Annual horizon

The main text provides predictability results for monthly and quarterly horizons. Table A.5 shows similar results for an annual horizon.

B4. Bauer-Hamilton bootstrap

Bauer and Hamilton (2018), hereafter BH, argue that the standard approach to testing the spanning hypothesis is subject to small sample distortions. They instead propose a parametric bootstrap procedure to evaluate significance in spanning regressions. In this section we implement this bootstrap. The setting also implicitly accounts for the effects of measurement error in yields.

We consider the cases of three and five principal components of yields. In both cases, we estimate a VAR(1)-model for the yield curve factors. We estimate an AR(2)-process for inflation. Here the second lag is still strongly significant. The (demeaned) yields are assumed to be given by:

$$y_t^n = \beta_n' T_t + e_{tn} \tag{B.3}$$

Here T_t is a vector of the yield factors and e_{tn} is a yield specific measurement error, $e_{tn} \sim N(0, \sigma_y^2)$. The loading vector β_n is estimated using least squares.

Using the estimated processes for yield factors and inflation, we simulate 10,000 draws, each of length 1,109 months. We start the simulation from the first empirical observation for the yield curve factors and inflation. The simulated residuals are drawn from the estimated joint

Table A.4: Testing the spanning hypothesis: showing the coefficients on yield curve factors

	(1) rx^{10Y}	(2) rx^{FX}	(3) Δy^{10Y}	(4) Δs
π	-14.46**	27.04***	1.682**	25.27***
	(-2.29)	(3.15)	(2.36)	(3.07)
pc^1	0.0067	0.0060	-0.0010*	0.0011
	(1.45)	(0.86)	(-1.92)	(0.19)
pc^2	-0.047**	0.023	0.0011	-0.0016
	(-2.01)	(0.86)	(0.42)	(-0.07)
pc^3	-0.057	-0.061	0.010	-0.098
	(-0.63)	(-0.97)	(0.95)	(-1.54)
$\bar{pc}^1$		0.78**		0.78
		(2.01)		(1.46)
$\bar{pc}^2$		1.97		1.30
		(1.68)		(0.52)
$\bar{pc}^3$		3.58***		3.37***
		(2.94)		(3.06)
cons	0.79***	-0.67**	-0.07**	-0.69***
	(2.77)	(-2.52)	(-2.19)	(-2.65)
N	608	480	608	480
R^2	2.88	4.87	2.11	3.92

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The Table shows the results when predicting monthly U.S. bond and dollar excess returns as well as yield and exchange rate changes with the latest annual U.S. CPI inflation rate. The regressions also control for the first three principal components of U.S. yields. The exchange rate related regressions also control for weighted averages of foreign yields. The weights are obtained by regressing returns and FX-rate changes on yield curve factors country by country. The t-statistics are based on Newey and West (1987) standard errors with 12 lags. The sample for bond return and yield regressions is between March 1973 and December 2023. The dollar exchange rate is defined against a basket of currencies with varying start dates.

Table A.5: Baseline results for an annual horizon.

	Panel A: No YC Controls			
	(1)	(2)	(3)	(4)
	rx^{10Y}	rx^{FX}	Δy^{10Y}	Δs
π	-133.1***	30.62	9.323	37.23
	(-2.68)	(0.54)	(1.58)	(0.71)
N	597	469	597	469
R^2	10.65	0.24	4.64	0.45
	Panel B: YC Controls			
	(1)	(2)	(3)	(4)
	rx^{10Y}	rx^{FX}	Δy^{10Y}	Δs
π	-174.0***	126.13**	18.59***	106.36*
	(-2.76)	(2.20)	(2.69)	(1.91)
N	597	469	597	469
R^2	26.13	11.30	20.33	8.54

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Panel A shows the results when predicting annual U.S. bond and dollar excess returns as well as yield and exchange rate changes with the latest annual U.S. CPI inflation rate. Panel B shows the coefficients on inflation when predicting the same variables using inflation but also controlling for the first three principal components of yields. The t-statistics are based on Newey and West (1987) standard errors with 12 lags. The sample for bond return and yield regressions is between March 1973 and December 2023. The dollar exchange rate is defined against a basket of currencies with varying start dates.

Table A.6: Bauer-Hamilton bootstrap p-values

	Coefficient	Bootstrap p-value	Yield fitting error
3 pc:s	-14.46	0.054	5.7bps
5 pc:s	-15.31	0.048	3.0bps

Notes: The Table shows the two sided p-values (and coefficient estimates) for the coefficients on inflation in the bond excess return spanning regressions controlling for 3 and 5 principal components of yields. The p-values are solved using the bootstrap procedure described by Bauer and Hamilton (2018). The Table also shows the yield fitting errors of the models with 3 and 5 lags.

empirical distribution of the yield factor and inflation residuals. The first 500 observations are disregarded. Hence we have 10,000 draws for the yield factors and inflation, each matching the length of 609 months of the original data. The yields are then solved using Equation B.3 and by simulating draws for e_{tn} from the normal distribution. These simulated values can then be used to construct distributions for the sample statistics. Here we apply Newey and West (1987) t-statistics as in the main text, but use the simulated distribution to find the corresponding p-value.

Note that full spanning holds for the simulated data as inflation and yield factors are specified separately. However, measurement error generates non-zero estimated values on the coefficient of inflation in spanning regressions.

The results are shown in Table A.6. The yield fitting error obtained by pooling across maturities is 5.7bps when using three yield factors and 3.0bps using five yield factors. The simulated two-sided p-value for the coefficient on inflation in the bond spanning regression in Table 1 is 0.054. On the other hand, for the regression controlling 5 pc:s shown in Table A.11, the p-value for the coefficient on inflation is 0.048. Overall, the bootstrap suggests somewhat weaker but still fairly strong statistical significance.

We leave extending the results to FX predictability regressions to future work. However, note that the corresponding spanning regressions have very high statistical significance with Newey and West (1987) t-statistics above 3. Hence bootstrap procedures are unlikely to change the key conclusions.

B5. Crump-Gospodinov bootstrap

Crump and Gospodinov (2025), hereafter CH, propose a non-parametric bootstrap that is particularly designed to address the high degree of cross-sectional and time-series dependence in yields. Their bootstrap also does not impose restrictions concerning dependencies between macro factors and yields. Here they are critical of the use of the BH bootstrap by Bauer and Rudebusch (2020). The authors apply the bootstrap to test whether the equilibrium real interest rate is an unspanned predictor of bond returns. Crump and Gospodinov (2025) argue that the BH bootstrap is invalid since it cannot accommodate predictors that are themselves a function of yields.

This criticism does not appear to carry to our setting and Crump and Gospodinov (2025) argue that the predictive ability of trend inflation is robust. However, as a robustness check, we also implement the CH bootstrap. We simulate primitive objects called difference returns that represent differences in the returns of adjacent maturity bonds. These have both smaller cross-sectional and time-series dependence than yields. The simulation also includes long maturity forward rates and inflation that are whitened using a VAR(1) model.

Crump and Gospodinov (2025) apply quarterly and annual returns, while we focus on monthly returns. This implies that we need to carry more maturities and increases the bootstrap block size. The bootstrap seems to converge relatively slowly. We therefore focus on five-year bonds as in Table A.10, which lowers the block size and amount of data. However, to ensure converge we use 100 000 bootstrap simulations, 10 times more than applied by Crump and Gospodinov (2025). We apply Newey and West (1987) standard errors with 12 lags as in the main text, but solve for p-values using the simulated distribution.

The results are given in A.7. Overall, the CG-bootstrap suggests slightly lower significance than asymptotic Newey and West (1987) p-values or the BH bootstrap. However, Crump and Gospodinov (2025) note that their method tends to give weaker significance compared to alternatives as it accomodates all of the possible sources of uncertainty, including the high persistence of inflation and the joint dynamics of inflation and yields. The p-values are below the 10% threshold, which is the focus of Crump and Gospodinov (2025).

Table A.7: Crump-Gospodinov bootstrap p-values

	Coefficient	Bootstrap p-value
3 pc:s	-9.52	0.087
5 pc:s	-9.66	0.077

Notes: The Table shows the two- sided p-values (and coefficient estimates) for the coefficients on inflation in the bond excess return spanning regressions controlling for 3 and 5 principal components of yields. The p-values are solved using the bootstrap procedure described by Crump and Gospodinov (2025). The Table also shows the yield fitting errors of the models with 3 and 5 lags.

B6. Discussion: on the factor structure of interest rates

Crump and Gospodinov (2022) note that characterizing the true factor structure of yields is more difficult than commonly appreciated. This is both due to the cross-sectional dependencies of different maturity yields as well as the time-series persistence of especially short maturity yields. They also caution against relying too much on commonly used goodness-of-fit metrics.

While three principal components of yields are sufficient for providing a good fit to yield levels, we have showed that our results are robust to using 5 principal components as well as all the 10 annual forward rates. Here forward rates have much lower cross-sectional dependence than yields. Moreover, in the previous subsection we saw that our results are robust to the bootstrap procedure of Crump and Gospodinov (2025). The bootstrap is based on simulating difference returns that have both weak cross-sectional and time-series dependence.

Overall, Crump and Gospodinov (2022) argue that it is often better to characterize the factor structure using bond returns rather than yield levels. While this view has merit from the angle of statistically characterizing the factor structure, it is important to note that this is not the approach implied by standard macro-finance models. In particular, standard macro-finance models imply that yield factors should span inflation rates. However, they do not generally imply that bond return factors span inflation rates. Theoretically, macro-finance models imply that state variables capture bond return predictability, while the bond return factors are associated with changes in state variables.

Further work is needed on explaining why yield levels seem to be well characterized by relatively few factors. Our theoretical results imply that the (approximate) dimension of the

Table A.8: FX return predictability for individual countries

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: rx^{FX} No YC Controls						
	CA	CH	G	SE	UK	Panel
π	7.76	10.54	16.74*	31.19***	14.50	15.44**
	(0.99)	(1.38)	(1.83)	(2.83)	(1.43)	(1.97)
N	455	431	298	359	481	2024
R^2	0.33	0.31	1.13	2.60	0.62	0.8
Panel B: rx^{FX} YC Controls						
	(1)	(2)	(3)	(4)	(5)	(6)
	CA	CH	G	SE	UK	Panel
π	14.67	18.83**	17.50*	40.91***	30.43**	20.32**
	(1.52)	(2.04)	(1.70)	(3.92)	(2.30)	(2.09)
N	455	431	298	359	481	2024
R^2	2.78	2.42	2.27	5.58	3.32	2.06

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: Panel A shows the results when predicting monthly FX excess returns separately for each currency (against U.S. dollar) with the latest annual U.S. CPI inflation rate. Panel B shows the coefficients on inflation when predicting the same variables using inflation but also controlling for the first three principal components of yields. The t-statistics are based on Newey and West (1987) standard errors with 12 lags for individual regressions and Driscoll and Kraay (1998) standard errors with 12 lags for the panel regression. See Table A.2 for the starting dates of the data.

yield curve is increasing in a) the number of relevant state variables, b) degree of non-linearities. The dimension might be small due to a small number of relevant state variables and/or yields being approximately linear in state variables. It could also be that yields are approximately non-linear only with respect to one state variable.

B7. Individual currencies

The main results for exchange rate predictability are based on average U.S. dollar returns. Table A.8 shows the key monthly predictability results separately for each currency against the U.S. dollar. The panel regression slopes are similar to those obtained using average dollar returns. All of the individual country regressions also have a positive sign though not all of the slopes are statistically significant.

For some countries, especially Canada, the level factor tends to be quite correlated with the US level factor. This can potentially give rise to a multicollinearity issue even though the level factors do not appear significant predictors of returns. As a robustness check, for each country we extracted five principal components from the foreign and U.S. level, slope and curvature factors. That is here the dimension of the control vector is reduced from 6 to 5. We then performed the panel regression controlling for these 5 factors. Here the coefficient on inflation is 22.51 with a t-statistic of 2.51.[27] Hence inflation remains a significant predictor. The results are similar when we instead drop the foreign level factors from the panel regression.

Multicollinearity does not appear an issue in our main empirical specification applying average dollar returns and weighted averages of foreign yield factors. However, the weights are obtained from individual country regressions in which the foreign level factor can be quite correlated with U.S. factors, which might bias the used weights. Hence as a robustness check we instead use equal weights across countries. Here the coefficient on inflation is 26.1 with a t-statistic of 2.85. The coefficient again remains strongly significant. The average foreign level factors are somewhat less correlated with US factors.

As a final robustness check, we performed the main FX regression omitting the foreign level factors. Here we still use country level regressions to determine the weights on the foreign slope and curvature factors. The coefficient on inflation is 28.4 with a t-statistic of 3.30 and inflation thus remains clearly significant.

B8. Longer exchange rate data

Our baseline FX regressions, e.g. those reported in Table 1, are constrained by the availability of yield curve data. However, using data from Barclays, we can extend the results without yield curve controls, in particular those reported in Table 1, Panel A. In particular, we have spot and forward exchange rate data for each country going back to January 1976.

Table A.9 shows the results when predicting average U.S. dollar excess returns and exchange rate changes using U.S. inflation in this longer sample. We can see that, compared to Table 1, Panel A, the coefficient estimates are somewhat lower yet statistical significance is stronger.

[27] Again the t-statistics in these robustness exercises are computed using Newey and West (1987) standard errors with 12 lags.

Table A.9: FX predictability, longer data

	(1) rx^{FX}	(2) Δs
π	10.32***	8.50**
	(2.71)	(2.05)
N	574	574
R^2	1.32	0.85

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The Table shows the results when predicting monthly U.S. dollar and exchange rate changes with the latest annual U.S. CPI inflation rate. The t-statistics are based on Newey and West (1987) standard errors with 12 lags. The sample is from January 1976 to December 2023.

B9. 5 year maturity bonds

Our baseline results are based on zero coupon bonds with a maturity of 10 years. As a robustness check, Table A.10 shows the results when predicting excess returns on bonds with a maturity of 5 years. The results are clearly significant though the coefficient on inflation is smaller. This is natural since interest rate changes have larger effects on bonds with higher maturity and hence longer duration.

B10. More yield factors and interactions

The main spanning regressions control for 3 yield curve principal components, the so called level, slope and curvature factors. These factors capture the bulk of variation in yields. However, some papers have argued that the fourth and fifth principal components of yields contain additional predictive information even though they do not add much to explaining yields contemporaneously (e.g. Cochrane and Piazzesi, 2005).

Table A.11 shows the results when instead controlling for 5 yield curve principal components. Here the coefficients on inflation are very similar to before.

In the theory part, we explained that our FX spanning results assume that the FX risk premium is separable in home and foreign state variables. As mentioned before, we have not found a realistic example of a model in which this would not hold. However, if separability

Table A.10: Predicting monthly excess returns of 5 year bonds

	(1) No YC Controls	(2) YC Controls
π	-6.29**	-9.72*
	(-2.16)	(-2.42)
N	608	608
$R^2(\%)$	0.98	3.22

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: The Table shows the results when predicting monthly excess returns on U.S. bonds with 5 year maturity with the latest annual U.S. CPI inflation rate. It also shows the coefficient on inflation when controlling for the first three principal components of yields. The t-statistics are based on Newey and West (1987) standard errors with 12 lags. The sample is from March 1973 to December 2023.

fails, we should generally account for interactions between home and foreign yield curve factors. The table also shows the FX spanning regression when also controlling for simple interactions formed by multiplying each U.S. principal component by its average foreign counterpart. Again the results are very similar to the baseline results.

B11. Structural breaks

Are the key mechanisms identified in this paper stronger or weaker in some particular time periods? Here we test for structural breaks in the benchmark predictability regressions for monthly bond and FX returns, see Table 1. In particular we implement the test of Bai and Perron (1998) modified with heteroskedasticity and autocorrelation robust standard errors. We test the null of no breakpoints in the slope coefficient against the alternative of at least one breakpoint.

The test gives no indication of breakpoints either for bond or FX returns. Hence while the estimated coefficients can be different in subperiods, the variation does not appear to represent statistically significant breaks.

Table A.11: Additional yield curve factors and interactions

	(1) rx^{10Y} 5 pc:s	(2) rx^{FX} 5 pc:s	(3) rx^{FX} int
π	-15.31**	15.78**	32.68***
	(-2.33)	(1.98)	(2.67)
N	608	480	480
R^2	4.91	7.05	5.80

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The Table shows the monthly predictability results when controlling for 5 principal components instead of 3. It also shows the FX predictability results when controlling for interactions between home and average foreign yield curve factors. The t-statistics are based on Newey and West (1987) standard errors with 12 lags. The sample for bond return and yield regressions is between March 1973 and December 2023. The dollar exchange rate is defined against a basket of currencies with varying start dates.

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