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International Risk Sharing and Wealth Allocation with Higher Order Cumulants *

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Abstract

We study international risk sharing across countries differing in size, openness, and productivity distributions, emphasizing fat tails. In a canonical IRBC model, safer economies benefit through asset and terms-of-trade revaluations, while riskier ones smooth consumption at the cost of lower wealth. Calibrated to non-Gaussian shocks, country size and openness, the model predicts welfare gains between 0.03% and 6.9% of permanent consumption (median 6%). Assuming Gaussian shocks reduces gains by about 2 percentage points, while assuming equal country size and no home bias renders them negligible. Clustering economies by openness, size, and higher moments accounts for the cross-country distribution of gains.

Keywords: Asymmetries in Risk, Openness, Country size, Tail Risk, Gains from Risk Sharing, Consumption Smoothing, Terms of Trade, Wealth Transfers.
JEL Classification: F15, F41, G15.

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1 Introduction

The Global Financial Crisis, the euro-area sovereign debt crisis, the early impacts of climate change, and, more recently, the COVID-19 pandemic have all highlighted the global economy's vulnerability to disruptive shocks. Policymakers face an environment characterized by heightened macroeconomic and financial risks, in which both idiosyncratic and global disturbances propagate asymmetrically across borders, deepening cross-country disparities in wealth and welfare. As shown in Table 1, countries differ substantially in the cyclical properties of log per-capita real GDP: across all moments reported in the table, values in the top five percent of the distribution are several times larger than those in the bottom five percent. This evidence motivates a re-examination of the potential benefits of cross-border insurance, placing key dimensions of heterogeneity—including the marked differences in country size and openness also highlighted in the table— at the forefront of the analysis.

This paper studies the welfare gains from sharing production risk across countries that differ in size, openness, and underlying risk, under non-Gaussian productivity shocks. Using a canonical international real business cycle (IRBC) framework, we provide both analytical insight into the mechanisms through which heterogeneous economies benefit from cross-border insurance and a quantitative evaluation of how heterogeneity shapes potential welfare gains.

Our contribution is threefold. First, we map the gains from risk sharing onto the equilibrium adjustment of asset prices, wealth, consumption, and terms of trade, distinguishing between level effects (LE) on unconditional consumption, and smoothing effects (SE) on the variability of the stochastic discount factor. Second, using perturbation methods we derive an analytical decomposition of macroeconomic and financial effects of risk sharing by cumulants (equivalently moments), enabling a detailed assessment of how asymmetries in risk—up to the fourth order—affect welfare differences across countries of unequal size and openness.[1] Finally, we complement our analytical results with numerical

[1] Cumulants are tightly related to moments. In Section 3 we provide a proper definition of cumulants and explain that cumulants are useful in solving our model via perturbation methods.

Table 1: Distribution of size, openness and higher moments of cyclical GDP

	Population size†	**Consumption import share**‡	**Standard deviation**	**Skewness**	**Kurtosis**
Min	0.34	0.02	0.01	-2.59	2.36
5%	0.89	0.03	0.01	-1.57	2.51
50%	18.02	12.65	0.03	-0.44	4.12
95%	267.65	23.58	0.05	0.76	11.37
Max	1433.78	25.00	0.09	1.79	13.25

† In millions. ‡ In percentage points.
Source: Penn World Table (10.0). Cyclical component of GDP per capita at 2011 constant prices; 1960-2019; 178 countries. Data filtered using the Boosted-HP filter (Phillips and Shi, 2020).

simulations based on global methods, calibrating the model to data on size, openness, and output distributions for 55 economies.

Based on our theoretical decomposition, we show analytically and numerically that, all else equal, countries more exposed to extreme events—manifested in larger variance, more negative skewness, or thicker tails in the distribution of income (endogenous to production uncertainty)—derive the greatest benefits in terms of consumption smoothing in the transition from financial autarky to complete markets. Safer countries, instead, benefit from the macroeconomic analog of an insurance premium driven by the equilibrium re-pricing of assets and goods. This market-based implicit wealth transfer tends to improve their terms of trade. Remarkably, we find that safety commands both a welfare and a financial premium: theory predicts a positive relationship between welfare and asset prices under perfect risk sharing. This is in sharp contrast with financial autarky, under which non-traded risk causes this relationship to be uniformly negative.

We characterize how the interactions between elasticities, country size, openness, and output variability crucially weigh on the welfare relevance of macroeconomic insurance. The integration of smaller economies into complete global markets has negligible effects on state-contingent prices (i.e., world asset prices). As a result, smaller economies thus face a larger budget set, since the prices of the states in which they buy or sell insurance do not move against them. For given size, in turn, gains from risk sharing vary with openness, i.e., home bias in demand. While openness tends to reduce a country exposure to own output shocks, it also makes it easier to stabilize utility by substituting domestic with imported goods. We show that the trade elasticity plays a comparably important role: the larger this elasticity, the more efficient the substitution, but also the greater the relative exposure of domestic income to domestic, as opposed to foreign, productivity shocks. Analytically, complementing Martin (2008), we specifically characterize how the welfare gains from sharing output tail risk depend jointly on trade elasticities and risk aversion, both governing the mapping of productivity shocks onto income and consumption risk.

Most crucially, our findings shows that an assessment of the potential gains from risk sharing requires a proper calibration of heterogeneity in economic structures. In our empirical application, the standard IRBC model—calibrated with non-Gaussian output shock estimates, country size (governed by population size), and openness data—predicts welfare gains from sharing productivity risk within a 90% range of 0.03% to 6.9% of permanent consumption, with a median of about 6%. The magnitude of these gains is much higher than suggested by most leading contributions on the subject. For comparison, Coeurdacier et al. (2020) find that financial integration under complete markets yields gains around 0.5% of permanent consumption, up to 1% in some extreme specifications—somewhat comparable to the welfare gains from eliminating the business cycle in Lucas (2003).[2] Yet, departing from log-normality, Martin proposes a striking revision of Lucas' estimates, with gains up to about 14%, "largely attributable to higher cumulants" (Martin, 2008, p. 75).

In spite of its stylized nature, our model allows countries to differ along five dimensions—three moments, size, and openness—all shaping the gains from trade through complex interactions. Indeed, in our evidence-based assessment, the ranking of gains closely aligns with a statistical clustering of countries by size, openness, and moments, cutting across the divide between emerging and advanced economies. A key takeaway from our analysis is that output uncertainty in general, and non-Gaussianity (fat tails)

[2] Lucas (2003) estimates that fully eliminating business-cycle uncertainty would raise permanent consumption by less than 1% (possibly as little as 0.01%).

in particular, play a significant role in generating large gains from risk sharing, provided that economies are modeled allowing for the heterogeneity in size and openness observed in the data.

Methodologically, we adopt a two-pronged approach. For economic insight, Section 4 solves the model analytically using standard higher-order perturbation methods.[3] Under suitable simplifying restrictions, these methods yield transparent, tractable expressions, in line with a large body of macroeconomic and financial research where they are the dominant technique. As a byproduct, we present an algorithm for deriving time-0 relative marginal utility (consumption) under complete markets—naturally interpreted as Negishi weights—for a general class of DSGE models. To facilitate comparison with existing work, in the analytical section we adopt a two-country specification of the model.

For the quantitative analysis, we use global solution methods and conduct two sets of exercises. First, we perform a controlled experiment with generated data to assess the generality of the insights from the analytical solution. Specifically, we re-evaluate the analytical results numerically, relaxing simplifying assumptions but still relying on two-country version of the model. Second, we calibrate the full three-country version of the model to data, examining the integration of each country's financial market with those of two macro regions—advanced economies and emerging market economies. This approach balances the need to capture cross-country heterogeneity with computational tractability, allowing us to quantify both the scale and the composition (level vs. smoothing) of potential welfare gains across borders, as shaped by asymmetries in size, openness, and cumulants.

Literature. Our work draws on three main strands. First is the extension of portfolio and asset-pricing theory beyond mean-variance. Early contributions, such as Samuelson (1970), warned against limiting analysis to first and second moments. Higher moments jointly determine asset values and the degree of hedging they provide.[4] Second, from

[3]See, among others, Holmes (1995), Judd (1998), Schmitt-Grohé and Uribe (2004), Lombardo and Sutherland (2007), Kim et al. (2008), Andreasen et al. (2018), and Lombardo and Uhlig (2018).

[4]Ingersoll (1975) shows that skewness influences efficient portfolio frontiers similarly to second moments. Kraus and Litzenberger (1976) extend the CAPM to include the third moment, improving

a macro-finance perspective, following Rietz (1988), Barro (2006) and Barro and Ursúa (2008) document the importance of non-Gaussian shocks for both equity prices and welfare. A large literature has since estimated the frequency and severity of rare events using macroeconomic data (e.g., Martin, 2013a; Nakamura et al., 2013) or financial data (e.g., Martin, 2008; Backus et al., 2011). Backus et al. (2011) highlight cumulants as both a source of intuition and a practical tool.[5]

Bridging these two strands of literature, our analysis is closely related to Martin's (2008; 2013a; 2013b) integration of cumulants into asset-pricing theory (see also Kyle and Todorov (2022)). We apply this approach to the welfare and asset-price implications of sharing production risk among heterogeneous countries, highlighting the role of trade elasticities and openness in mapping productivity shocks into income risk.

Third, we build on the large literature on cross-border risk sharing (e.g., van Wincoop, 1999; Athanasoulis and van Wincoop, 2000; Lewis and Liu, 2015). In line with recent work (e.g., Gourinchas and Jeanne, 2006; Coeurdacier et al., 2020), we stress that cross-border insurance can reduce precautionary saving and induce reallocation of production factors. We emphasize the role of higher-order cumulants and level effects, and show how the resulting equilibrium allocation of demand and labor shapes the terms of trade. This novel result—linking the market-based transfers from risk sharing to the Keynes-Ohlin debate on the "transfer problem" (Keynes, 1929; Ohlin, 1929)—follows from the fact that risk sharing entails a permanent shift in relative wealth, which can raise the relative price of a country's goods if either consumption is home-biased, or labor supply falls as households become richer, or both.

Our results also speak to the policy debate, where gains from risk sharing are often assessed solely in terms of consumption smoothing (e.g., Viñals, 2015; Constâncio, 2016). Most empirical evidence in policy reports measures consumption volatility and

empirical fit. Harvey and Siddique (2000) and Smith (2007) argue that co-skewness helps explain the cross-section of returns. Bekaert and Engstrom (2017) build a model generating skewness in consumption growth and its observed correlation with option-implied volatility. Extending to the fourth moment, Fang and Lai (1997) find that investors require compensation for higher variance and kurtosis, but accept lower returns for higher skewness.

[5] Guvenen et al. (2018) show that idiosyncratic income fluctuations display non-Gaussian features, with skewness and kurtosis affecting the welfare cost of incomplete insurance.

cross-country consumption correlations.[6] Our analysis shows that smoothing is only one part of total gains. Relatively safe countries benefit from implicit wealth transfers through asset revaluation and terms-of-trade adjustments. Such transfers are fundamental to a full assessment of the benefits and costs of capital-market integration among countries with differing risk profiles and economic structures.[7]

The paper is structured as follows. Section 2 presents the model. Section 3 outlines the solution methods. Section 4 analytically decomposes the level and smoothing effects of risk sharing by cumulants. Section 5 discusses numerical results. In particular, Section 5.2 applies the analysis to 55 economies. Section 6 concludes.

2 Model

As a general framework for our analysis, we specify a model economy consisting of three economies, labeled A, B and C, identical except for their size, degree of openness and the probability distribution of total factor productivity (TFP). Country size (n_A, n_B and n_C) is normalized to sum up to 1. In each period the state of the economy consists of a realization of the stochastic TFP processes and a distribution of financial assets. See Online Appendix A for a compact summary of the equations needed to solve the model.

By setting $n_C = 0$ in the analytical section 4.1 and part of the quantitative section 5, we will be able to compare and contrast our contribution to the large body of work on international risk sharing relying on two-country models. Furthermore, by setting labor share in the production function to zero and eliminating home bias in consumption the model simplifies to the Lucas'(1978) orchard model (Martin, 2013b), which we use in the analytical section. We will use the full model at the end of the paper in an empirical application.

[6]See, e.g., Obstfeld (1994) and the review in Kose et al. (2009). A common approach is to test the consumption risk-sharing condition, which predicts perfect correlation of consumption growth under complete markets.

[7]See Ljungqvist and Sargent (2012), Engel (2016), and Coeurdacier et al. (2020) for related discussions.

2.1 Consumer Problem

The representative agent in country A consumes a bundle of domestic and foreign goods $C_{A,t}$. Domestically she trades in units of capital at price $P_{A,K,t}$, and supplies labor L_t at a wage $w_{A,t}$ and capital $K_{A,t}$ at the rate $r_{A,K,t}$, to firms. Internationally, she trades in Arrow-Debreu securities with residents in countries B and C, at the price $\Lambda_{A,t+1|t}$, that pay one unit of consumption in period $t+1$. We denote by $S_{A,t} = S_{A,B,t} + S_{A,C,t}$ the net-trade of securities with the other two countries. Capital must be purchased one period in advance and is assumed to be constant at the aggregate level. All prices are in units of the consumption basket.

Denoting households' welfare at time 0 as $\mathcal{W}_{A,0}$, the representative country A household solves the following problem:

$$\max_{C_{A,t},S_{A,t+1},K_{A,t+1},L_{A,t}} \mathcal{W}_{A,0} := E_0 \sum_{t=0}^{\infty} \delta^t U\left(C_{A,t}, L_{A,t}\right) \tag{2.1}$$

subject to the individual budget constraint:

$$\begin{aligned} C_{A,t} + E_t\left(\Lambda_{A,t+1|t} S_{A,t+1}\right) + P_{A,K,t} K_{A,t+1} = \\ r_{A,K,t} K_{A,t} + w_{A,t} L_{A,t} + S_{A,t} + P_{A,K,t} K_{A,t}. \end{aligned} \tag{2.2}$$

For computational simplicity we follow Farhi and Gabaix (2016) by assuming that constant relative risk aversion (CRRA) preferences, i.e.

$$U\left(C_{A,t}, L_{A,t}\right) := \frac{C_{A,t}^{1-\rho} - 1}{1-\rho} - \chi \frac{L_{A,t}^{1+\varphi}}{1+\varphi}, \tag{2.3}$$

with $\rho > 0$, the degree of risk aversion, $\varphi > 0$, the inverse Frisch elasticity of labor supply.[8]

[8] Our baseline case adopts CRRA preferences for computational convenience. Codes available on request compare welfare gains under CRRA preferences with gains obtained under Epstein and Zin (1989) preferences. Epstein-Zin (EZ) preferences require additional state variables, strongly affecting the speed of convergence. We opted for CRRA preferences in light of this computational burden given the large number of cases we consider. The main effect of using EZ preferences is to allow for a higher degree of risk aversion and thus larger welfare effects.

Total domestic consumption in each country is a constant elasticity of substitution (CES) function of domestic and foreign goods, e.g. for country A:

$$C_{A,t} = \left(\nu_A^{\frac{1}{\theta}} c_{A,A,t}^{\frac{\theta-1}{\theta}} + (1-\nu_A)^{\frac{1}{\theta}} C_{A,t}^{*\,\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}; \tag{2.4}$$

$$C_{A,t}^{*} = \left(\varsigma_A^{\frac{1}{\theta}} c_{A,B,t}^{\frac{\theta-1}{\theta}} + (1-\varsigma_A)^{\frac{1}{\theta}} c_{A,C,t}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}. \tag{2.5}$$

$C_{A,t}$ is total consumption of goods in country A, $C_{A,t}^*$ is total foreign goods consumed in country A, small letter $c_{A,B,t}$ and $c_{A,C,t}$ denote country B and C goods consumed by country A, $\theta > 0$ is the trade elasticity and $\nu_A \in (0,1)$ is a function of relative size of countries, and the degree of home bias (or equivalently degree of openness), $\lambda_A \in (0,1]$, such that $1-\nu_A = (1-n_A)\lambda_A$ is the import consumption share. Finally, $1-\varsigma_A = \frac{n_C}{1-n_A}$ represents share of country C's goods within country A's foreign goods basket.

The relative prices associated with the above preferences obey the following:

$$1 = \nu_A p_{A,t}^{1-\theta} + (1-\nu_A) {P_{A,t}^{*}}^{1-\theta} \tag{2.6}$$

$${P_{A,t}^{*}}^{1-\theta} = \varsigma_A p_{B,t}^{1-\theta} + (1-\varsigma_A) {p_{C,t}}^{1-\theta}, \tag{2.7}$$

where $p_{i,t}$ is the price of output of country i in terms of country A's consumption basket. Define $Q_{i,j}$ as the real, bilateral exchange rate between country i and j and $ToT_{i,j}$ as terms of trade between country i and j. For example, $ToT_{A,B,t} = \frac{p_{B,t}}{p_{A,t}}$. As the law of one price holds in our economy, we have $p_{B,t} = Q_{A,B,t} p_{B,t}^*$ and $p_{C,t} = Q_{A,C,t} p_{C,t}^*$.

Demands for country specific goods are

$$c_{A,A,t} = \nu_A \left(p_{A,t}\right)^{-\theta} C_{A,t}, \tag{2.8a}$$

$$c_{A,B,t} = \varsigma_A \left(\frac{p_{B,t}}{P_{A,t}^*}\right)^{-\theta} C_{A,t}^*, \; c_{A,C,t} = (1-\varsigma_A)\left(\frac{p_{C,t}}{P_{A,t}^*}\right)^{-\theta} C_{A,t}^*, \tag{2.8b}$$

$$C_{A,t}^* = (1-\nu_A)\left(P_{A,t}^*\right)^{-\theta} C_{A,t}. \tag{2.8c}$$

Total demand for home goods is:

$$Y_t = \left(c_{A,A,t} + \frac{n_B}{n_A} c_{B,A,t} + \frac{n_C}{n_A} c_{C,A,t} \right). \tag{2.9}$$

The first order conditions of the consumer problem are:

$$C_{A,t} : U_C(C_{A,t}, L_{A,t}) - \zeta_{A,t} = 0, \tag{2.10a}$$

$$L_{A,t} : U_L(C_{A,t}, L_{A,t}) + w_{A,t}\zeta_{A,t} = 0, \tag{2.10b}$$

$$S_{A,t+1} : \delta\zeta_{A,t} - \zeta_{A,t-1}\Lambda_{A,t|t-1} = 0, \tag{2.10c}$$

$$K_{A,t+1} : P_{A,K,t} = E_t \delta \frac{\zeta_{A,t+1}}{\zeta_{A,t}} \left(r_{A,k,t+1} + P_{A,K,t+1} \right), \tag{2.10d}$$

where $\zeta_{A,t}$ is the Lagrange multiplier on the budget constraint, so that $\Lambda_{A,t|t-1}$ is the stochastic discount factor (SDF).

2.2 Risk-sharing and the cross-border distribution of wealth

By combining equations (2.10a) and (2.10c) (and the foreign counterparts) and iterating backward, we obtain risk-sharing conditions:

$$\zeta_{B,t} = Q_{A,B,t}\zeta_{A,t}\kappa_{A,B}^{-\rho} \qquad \zeta_{C,t} = Q_{A,C,t}\zeta_{A,t}\kappa_{A,C}^{-\rho}. \tag{2.11}$$

where

$$\kappa_{A,B} := \frac{C_{B,0}}{C_{A,0}} Q_{A,B,0}^{\frac{1}{\rho}}, \qquad \kappa_{A,C} := \frac{C_{C,0}}{C_{A,0}} Q_{A,C,0}^{\frac{1}{\rho}}. \tag{2.12}$$

and $\kappa_{B,C}$ can be obtained by taking the ratio of the above.[9] The variable κ, typically referred to either as "risk-sharing constant" or "Negishi weight", is the ratio of the marginal utilities of consumption of residents in two countries, evaluated in the risk sharing equilibrium at the time in which the complete markets regime starts (the initial "time zero").[10]

[9] When now ambiguous we drop the subscripts of $\kappa_{A,j}$, $j = \{A, B, C\}$, when referring to the generic risk-sharing constant.

[10] In our text, we will use interchangeably "risk-sharing constant" and Negishi weight.

It reflects the initial cross-border distribution of wealth assessed at the equilibrium prices of real and financial assets owned by residents in each country.[11] Complete markets allow agents to optimally smooth fluctuations of consumption around each κ, preventing relative marginal utilities from diverging and opening inefficient wealth gaps.[12] As well known (e.g. Ljungqvist and Sargent, 2012), κ is pinned down by the "time-0" asset allocation conditions. With two weights to be determined, we need two conditions, i.e. we impose zero net foreign asset at time 0:

$$S_{j,0} = E_0 \sum_{t=0}^{\infty} \Lambda_{j,t|0} \left(C_{j,t} - p_{j,t} Y_{j,t} \right) = 0; \;\; j = \{A, B\}. \tag{2.13a}$$

2.3 Firms

Each country produces a differentiated good. In each country, an infinite number of firms operate in perfectly competitive markets using a Cobb-Douglas technology in capital ($K_{A,t}$) and labor ($L_{A,t}$), subject to an exogenous stochastic process for TFP (possibly correlated across countries) $D_{A,t}$. The aggregate production functions is:

$$Y_{A,t} = D_{A,t} K_{A,t}^{\alpha} L_{A,t}^{1-\alpha}. \tag{2.14}$$

Goods markets are competitive. For tractability and to highlight the macroeconomic implications of wealth effects on relative demand and labor supply, throughout the analysis we posit that the aggregate capital stock is constant, and conveniently abstract from labor in the analytical section—we treat labor as an endogenous variable in our quantitative section, as it is potentially important in determining relative price movements and welfare.

In each period the representative firm rents capital at rate $r_{A,K,t}$ and hires workers at the real wage $w_{A,t}$ (in units of the consumption basket) from households to

[11] Negishi (1960) devised an algorithm to find equilibrium allocations under complete markets. Under CRRA, via a simple transformation weights can be referred to relative consumption.

[12] For a definition and discussion of wealth gap, see Corsetti et al. (2010).

solve the following problem:

$$\min_{K_{A,t},L_{A,t}} r_{A,K,t}K_{A,t} + w_{A,t}L_{A,t} \tag{2.15}$$

subject to the production function (2.14). The associated factor demands satisfy the standard conditions: $K_{A,t} = p_{A,t}\alpha\frac{Y_{A,t}}{r_{A,K,t}}$ and $L_{A,t} = p_{A,t}(1-\alpha)\frac{Y_{A,t}}{w_{A,t}}$. For future reference, it is useful to define global income (in country A consumption units) as:

$$I_{w,t} := n_A p_{A,t}Y_{A,t} + n_B p_{B,t}Y_{B,t} + n_C p_{C,t}Y_{C,t}. \tag{2.16}$$

2.4 Output and Productivity

In line with the literature we posit a stochastic process driving TFP specific to each country j ($j = \{A, B, C\}$), assumed to follow an AR(1) process in logs:

$$\ln D_{j,t} = (1-\varphi_j)\ln \bar{D}_j + \varphi_j \ln D_{j,t-1} + \omega\varepsilon_{j,t}; \tag{2.17}$$

where the parameter $\varphi_j \in (0,1)$ measures the persistence of the TFP process, $\varepsilon_{j,t}$ is a serially-uncorrelated exogenous innovation, ω is the perturbation parameter (identical across countries) such that if $\omega = 0$ the model is deterministic, and a bar over variables indicate the deterministic steady state value. The process is analogous for the other two countries. In the global solution of the model we assume that $D_{j,t}$ is generated by mixed-Gaussian distributions (to match the skewness and excess kurtosis observed in the data) as described in the Online Appendix H.[13] To derive our analytical solution, we restrict the probability distribution of $\varepsilon_{j,t}$ as follows:

Assumption 1 (Probability distribution for the analytical solution)**.** *The probability distribution of the innovation $\varepsilon_{j,t}$, $j = \{A, B, C\}$, is characterized as follows:*

[13]Leading papers in the "disaster risk" literature such as Barro (2006); Barro and Jin (2011) and especially Backus et al. (2011), calibrate or estimate Markov processes to capture tail events. According to these contributions, the approach we adopt in this paper—calibrating mixed-Gaussian distributions using observed variance, skewness, and kurtosis—is likely to underestimate fat tails.

Mean	*Cross-Moment*	*Variance*	*Skewness*	*Kurtosis*
$E\left(\varepsilon_{j,t}\right)=0$	$E\left(\varepsilon^r_{A,t}\varepsilon^l_{B,t}\varepsilon^s_{C,t}\right)=\chi^{\dagger}_{r,l,s}$	$E\left(\varepsilon^2_{j,t}\right)=\gamma_j$	$E\left(\varepsilon^3_{j,t}\right)=\phi_j$	$E\left(\varepsilon^4_{j,t}\right)=\eta_j$

$^{\dagger}r,l,s\in\{1,2,3,4\}$

In the perturbation analysis we compute solutions up to the fourth order of accuracy. For analytical transparency and feasibility of our numerical applications, in the main text we restrict $\chi_{r,l,s}=0$. One key takeaway from our analysis is that, even in symmetric economies, gains from risk sharing are generated by asymmetries in the *intensity* of shocks at country level. In the Online Appendix F, we show that, because of these asymmetries, gains are positive and can be substantial even if shocks are perfectly correlated across borders.

Importantly, we model the distribution of the log of TFP, as opposed to its level. For the sake of illustration, note that, setting $\varphi_A=0$, a series expansion of the log of $D_{A,t}$ around $\omega=0$ (see below for a broader discussion) yields

$$E\left(D^{\tau}_{A,t}\right)=1+\frac{\tau^2}{2}\gamma_A+\frac{\tau^3}{6}\phi_A+\frac{\tau^4}{24}\eta_A+\mathcal{O}\left(\omega^5\right), \tag{2.18}$$

where $\tau\in\mathbb{R}$ is an arbitrary constant. This shows that the distribution of the level of TFP is not mean invariant: even with $\tau=1$ the mean of TFP depends on higher moments.[14]

3 Solution Methods

In our analysis, we rely on both higher-order perturbation methods and global solution methods—the former to derive tractable analytical expressions providing economic insight, the latter to corroborate and generalize our analytical insight with numerical analysis, as well as to carry out our quantitative exercises into the assessment of the

[14] We don't adjust the process to rule out this dependence. First, it would be computational cumbersome. Second, and more importantly, there is no empirical reason to believe that the underlying process has a mean-invariant property. Third, it is precisely through the effect on mean consumption and labor that higher moments can have a particularly strong impact on welfare.

potential gains from risk sharing.[15]

Higher-Order Perturbation. We generate an order-decomposition of the variables in the model by applying higher-order series expansions with respect to ω around the riskless equilibrium ($\omega = 0$)—that allows us to characterize the higher-order state-space solution of the model. In our model, the state space solution is a function of exogenous state variables only, hence the series expansion amounts to an order-decomposition of the variables in terms of higher-order terms of the exogenous shocks. Integrating over the probability distribution of these shocks this decomposition will collapse to a decomposition in moments.

The perturbation method we use in our analysis is standard—it is described by Holmes (1995) in general terms, and by Judd (1998) and Lombardo and Uhlig (2018) for DSGE models in particular.[16] Our approach begins with the recognition that, in solving the model, all variables—including endogenous states—must be expressed as functions of a scaling parameter that embeds the stochastic process into the economy. In our notation this scaling parameter is denoted by ω, loading the innovation in the TFP AR(1) process (2.17). So, for example, we write $C_t := C(t;\omega)$, which highlights the dependence of consumption on the scale parameter ω. Then the $m - order$ log-series-expansion of $C(t;\omega)$ around $\omega = 0$ is

$$\ln C\left(t,\omega\right) \approx \sum_{i=0}^{m} = C_t^{(i)} \frac{1}{i!}\omega^i, \text{ where } C_t^{(i)} := \left. \frac{\partial^i \ln C(t;\omega)}{\partial \omega^i} \right|_{\omega=0}. \tag{3.1}$$

For our purposes, it is worth stressing that standard perturbation approaches to the solution of DSGE models are closely related to the derivation of cumulants discussed by Martin (2008, 2013a) and Backus et al. (2011).[17] To wit, following Backus et al.

[15]As is well understood, perturbation methods (series expansions) are an ideal tool for digging deep into analytical solution in terms of cumulants, although not (necessarily) accurate in numerical applications— e.g., with significant departures from the steady state in strongly non-linear systems, they may yield poor quantitative results. We address this issue by comparing analytical results with numerical global solution in Appendix J.

[16]See also Lombardo and Sutherland (2007) and in particular Devereux and Sutherland (2011) in which perturbation methods are used to solve for portfolio choices in open economies.

[17]It is important to note that our solution algorithm is distinct from the "risky-steady-state" approaches discussed in the literature (e.g. Coeurdacier et al., 2011). In the case of the Negishi weights

(2011), given the *moment-generating* function for a random variable x and a real variable ω, $h(\omega, x) = E(e^{\omega x})$, then $f(\omega, x) = \ln h(\omega, x)$ defines the *cumulant-generating* function. Provided that $h(\omega, x)$ is analytical, we can write its series expansion as

$$f(\omega, x) = \sum_{m=1}^{\infty} f_m(x) \frac{\omega^m}{m!}, \tag{3.2}$$

where $f_m := \left. \frac{\partial^m \ln f(\omega, x)}{\partial \omega^m} \right|_{\omega=0}$ and $f_m(x)$ is the $m - order$ cumulant. Cumulants are tightly related to moments: in line with Assumption 1 one can show that:

$$f_{2,j} = \gamma_j,\ f_{3,j} = \phi_j,\ f_{4,j} = \eta_j - 3\gamma_j^2 : j = \{A, B, C\} \tag{3.3}$$

Technical details of our approach are presented in the Online Appendix B, which also offers a practical algorithm specifically designed for numerical solutions of DSGE models of any size, e.g. using Dynare (Juillard, 1996).[18]

Global methods. To solve the model with global methods, we adopt a standard approach (e.g., Judd (1998)). Since for each value of κ allocations and prices can be determined from a static problem, the solution proceeds in two steps. In the first step, we compute policy functions for consumption, labor, and prices—that is, the variables that enter the value functions and the budget constraint. In the second step, which requires evaluating expectations, we solve for the policy functions of the value functions and Arrow securities using a fixed-point iteration. Further details are provided in the Online Appendix C.

4 Analytics of Risk Sharing

For comparison with the literature, throughout the section we will rely on a two-country version of our model. For analytical transparency, we further impose a set of educated

(κ) in the class of models we are considering (i.e. with power utility also of the Epstein and Zin (1989) type) the recursivity of the solution is a natural outcome of the functional form of the model combined with the time-invariance of κ. No ad-hoc deviation from standard perturbation methods is required.

[18] To solve our model, one can also use the Wolfram's Mathematica code, available from the authors.

assumptions (relaxed in Section 6.1).

Assumption 2 (Model in the analytical section)**.** *Two country model ($n_C = 0$), no labor in production ($\alpha = 1$), fixed aggregate capital ($K_t = 1$); identical steady state endowments $\bar{D}_A = \bar{D}_B$, purchasing power parity (PPP) meaning that $\lambda_A = \lambda_B = \lambda = 1$.*

We solve our model up to 4th order approximation, the minimum order that can capture kurtosis. We will use superscripts "*cm*" and "*au*" respectively, to denote variables under complete markets and financial autarky.

4.1 Level vs. smoothing effects (LE vs. SE) of risk sharing

The point of departure of our analysis is the observation that, per equations (2.12), (2.10a) (and the foreign counterparts) and the resource constraint, κ determines the consumption share of a country in global income. Under Assumption 2, this share is

$$\frac{C_{A,t}}{I_{w,t}} = \mu_A := \frac{E_0 \sum_{t=0}^{\infty} \Lambda_{t|0} p_{A,t} Y_{A,t}}{E_0 \sum_{t=0}^{\infty} \Lambda_{t|0} I_{w,t}} = \frac{1}{n_A + n_B \kappa_{A,B}}, \tag{4.1}$$

where $\Lambda_{t|0} = \Lambda_{A,t|0} = \Lambda_{B,t|0}$ and $I_{w,t} = n_A p_{A,t} Y_{A,t} + n_B p_{B,t} Y_{B,t}$, consistent with (2.16). Note that if $\kappa_{A,B} = 1$, then $C_{A,t} = I_{w,t}$.

By using the foreign counterpart of equation (4.1), $\kappa_{A,B}$ can be written as the ratio of the relative present discounted value of the flow of output in each country—assessed at equilibrium prices:[19]

$$\kappa_{A,B} = \frac{E_0 \sum_{t=0}^{\infty} \Lambda_{t|0} p_{B,t} Y_{B,t}}{E_0 \sum_{t=0}^{\infty} \Lambda_{t|0} p_{A,t} Y_{A,t}} = \frac{p_{B,0} Y_{B,0} + P_{B,K,0}}{p_{A,0} Y_{A,0} + P_{A,K,0}}, \tag{4.2}$$

with $P_{j,K,t} = E_t \sum_{i=1}^{\infty} \delta^i \left(\frac{I_{w,t+i}}{I_{w,t}} \right)^{-\rho} p_{j,t+i} Y_{j,t+i}$ where $j = A, B$.[20] Equation (4.2) establishes a tight relationship between relative consumption shares, asset prices and the dynamics of relative output and relative good prices, i.e. the terms of trade. As further

[19] To derive analogous expressions (as (4.1)) for the Foreign country, make use of the fact that $C_{B,t} = \frac{1 - n_A \mu_A}{1 - n_A} I_{w,t}$. (4.2), directly follows taking the ratio of equation (4.1) and its foreign counterpart.

[20] $P_{j,K,t}$ is derived from equations (2.10d) using Assumption 2.

discussed below, through the equilibrium adjustment in asset prices and the terms of trade, transitioning from financial autarky to perfect risk sharing results in an implicit wealth transfer between economies that are asymmetric in risk.[21]

In light of the above, it is apparent that risk sharing arrangements impinge on the equilibrium allocation through two key channels. The first one works through the repricing of a country assets and output at the time when the new financial regime comes into effect (time zero). The equilibrium distribution of wealth and consumption across borders changes when the human and physical asset endowment of a country is re-valued at the new equilibrium prices, reflecting the adjustment in the agents' stochastic discount factor. The second one works via equilibrium consumption smoothing under efficient risk sharing, reflecting the new equilibrium process for consumption and labor. Throughout the paper, we will structure our analysis around these two channels, distinguishing between Level Effects and Smoothing Effects of risk sharing, defined as follows:

Definition 1. Level Effect (LE) of risk sharing is the time-zero equilibrium adjustment in consumption and leisure, captured by κ.

Definition 2. Smoothing Effect (SE) of risk sharing is the equilibrium adjustment in the (mean) response of consumption and leisure to shocks.

The gains from risk sharing stem from the combination of these two effects, which do not need to go in the same direction. To fix ideas, anticipating our results below, consider two countries of equal size with asymmetric income risk. The relatively safe country will be an equilibrium provider of insurance—hence it will benefit mainly from an appreciation of its assets. Its residents will enjoy higher consumption (of goods and leisure) on average. Conversely, the relatively risky country will gain from smoothing its residents' consumption, at the cost of a drop in their average consumption. Most of the applied work on international risk sharing assumes ex-ante symmetric economies, in

[21] Observe that this result is in line with the textbook example in Obstfeld and Rogoff (1996, Ch 5, equation (45) where $n_A = n_B = 1/2$). As shown by these authors, with CRRA preferences and PPP, the share of a country consumption in world output, denoted μ_j, coincide with the portfolio shares of a country j in the world optimal portfolio of claims to countries' current and future output. Thus μ_j captures the optimal exposure to country risk from the vantage point of investors.

which case the LE is irrelevant. But any heterogeneity and asymmetry across borders, in technology, taste or size, translates into relative wealth movements.

For future reference, it is useful to define the counterpart of κ in financial autarky, as the ratio of the multipliers of the budget constraint (see chapter 22 in Azzimonti et al. (Forthcoming)). In financial autarky, this ratio varies period by period in response to shocks. Yet, one can obtain a measure of relative wealth comparable with κ by taking unconditional means. For our purposes, one important feature of such measure is that, holding Assumption 2, unconditionally consumption is identical across borders. Hence, in the rest of the analytical sections, we can interpret log-$\kappa_{A,B}$ under complete markets in terms of cross-border wealth adjustment relative to autarky.[22]

4.2 Decomposition of LE and SE by cumulants

Perturbation methods allow us to offer a tight and insightful expression of the contribution of level and smoothing effects to a country gains from risk sharing by cumulants.[23] In particular, solving our model under Assumption 2 to fourth order of approximation, the gains from risk sharing for country A can be written as:

$$\mathcal{W}^{cm}_{A,0} - \mathcal{W}^{au}_{A,0} = \underbrace{-\left(a_{\kappa,2}\kappa^{(2)}_{A,B} + a_{\kappa,2^2}{\kappa^{(2)}_{A,B}}^2 + a_{\kappa,3}\kappa^{(3)}_{A,B} + a_{\kappa,4}\kappa^{(4)}_{A,B} + a_{\kappa,\Gamma}\kappa^{(2)}_{A,B}\Gamma\right)}_{\text{level effect LE}}$$
$$+ \underbrace{\left(a_{\gamma,A}\gamma_A - a_{\gamma,B}\gamma_B\right) + \left(a_{\phi,B}\phi_B - a_{\phi,A}\phi_A\right) + \left(a_{\eta,A}\eta_A - a_{\eta,B}\eta_B\right)}_{\text{smoothing effect SE}} + \mathcal{O}\left(\omega^5\right), \qquad (4.4)$$

[22] Under Assumption 2, we have seen that $\kappa_{A,B} = \frac{C_{B,0}}{C_{A,0}}$. Under autarky, relative consumption is identical to relative income. From the goods-demand system (equations (2.8)) we can express relative income, and thus relative consumption, as a function of the terms of trade, i.e.

$$\frac{C_{B,t}}{C_{A,t}} = \frac{p_{B,t}Y_{B,t}}{p_{A,t}Y_{A,t}} = \left(\frac{p_{A,t}}{p_{B,t}}\right)^{\theta-1}. \qquad (4.3)$$

where, in our endowment economy, relative prices (the terms of trade) are identical to relative TFP. This implies that the average log-ratio of consumption is equal to the average log-ratio of TFP, which is zero by Assumption 1. Thus the implicit log-$\kappa_{A,B}$ under autarky—defined as average log-consumption-ratio—is zero.

[23] Since the only state variables in our model are the exogenous shocks, cumulants depend only on shocks' moments.

where $\Gamma = \gamma_A + \gamma_B$ is total variance, and $\kappa_{A,B}^{(m)}$ $(m = 2, 3, 4)$ are the $m - order$ terms of the ln-series-expansion of κ around $\omega = 0$:

$$\kappa_{A,B}^{(2)} = b_\gamma (\gamma_A - \gamma_B), \tag{4.5}$$

$$\kappa_{A,B}^{(3)} = b_\phi (\phi_B - \phi_A), \tag{4.6}$$

$$\kappa_{A,B}^{(4)} = -b_{\gamma_A^2}\gamma_A^2 + b_{\gamma_B^2}\gamma_B^2 + b_{\gamma_B\gamma_A}\gamma_B\gamma_A + b_\eta (\eta_A - \eta_B), \tag{4.7}$$

and $a_\bullet$ and $b_\bullet$ as complex convolutions of deep parameters (ρ, θ, δ and n_A). While the $a_\bullet$ and $b_\bullet$ may have either sign, they are nonnegative in our model calibration as well as in most standard calibrations adopted by the literature (see Online Appendix D).[24]

The above expressions detail the analytics of our definitions of LE and SE. The Level Effect of risk sharing includes only terms in $\kappa_{A,B}$, determined at time 0 and time invariant.[25] As opposed to LE, SE reflects the equilibrium response of consumption (of goods and leisure) to shocks over time, and is thus written as a convolution of moments (pinned down by the time-0 expectations of the convolutions of shocks). Notably, LE and SE interact through the last term in the first line of (4.4). Unless, unconditionally consumption is identical across countries (i.e., unless $\kappa_{A,B}^{(2)} = 0$), this interaction term suggests that welfare gains of a country vary with total variance Γ.

The expression (4.4) provides a framework to study the marginal contribution of smoothing and wealth effects to the welfare gains from risk sharing reflecting cross-country asymmetries. Provided $a_\bullet > 0$ and $b_\bullet > 0$, $\kappa_{A,B}$ is increasing in the relative variance $(\gamma_A - \gamma_B)$, relative (negative) skewness $(\phi_B - \phi_A)$ and relative kurtosis $(\eta_A - \eta_B)$ of the productivity distribution in country A. As country A becomes riskier, country B—now safer—gets a compensation for providing insurance.

In general, cumulants and deep parameters interact in complex ways in determining the gains from risk sharing, as discussed in the three subsections below.

[24]In Online Appendix D, it is easy to verify that under symmetry in size and moments $\kappa_{A,B}^{(4)} = 0$.

[25]In our stylized model, under autarky, there are no forward looking equations: consumption and leisure have no "level shift" reflecting precautionary saving. Under complete markets the level shift reflects the time-0 solution of asset allocations.

4.2.1 Risk aversion and trade elasticities

We start by demonstrating that assessing the gains from risk sharing requires a careful mapping of output quantity fluctuations (at second and higher moments), onto income and consumption risk—a mapping that is a complex function of the economy's structure. We start elaborating on this point stating the following proposition, derived under some strong assumptions to ensure transparency in interpretation:

Proposition 1 (Gains from risk sharing)**.** *Define* $\Gamma := \gamma_A + \gamma_B$*,* $\Phi := \phi_A + \phi_B$ *and* $H := \eta_A + \eta_B$*. Under Assumption 2 and countries with equal size (*$n_A = n_B = \frac{1}{2}$*):*

$$\begin{aligned}\mathcal{W}^{cm}_{A,0} - \mathcal{W}^{au}_{A,0} =& \frac{\delta(\theta-1)^2\rho}{8(1-\delta)\theta^2}\Gamma + \\ &+\frac{\delta(\theta-1)^2\rho}{24(1-\delta)\theta^3}\left[-\frac{((\rho+1)+2\theta(\rho-2))}{2}\Phi - (\rho+1)(\theta-1)\phi_A\right] + \\ &+\frac{\delta(\theta-1)^2\rho}{48(1-\delta)\theta^3}\left[\frac{a_H}{8\theta}H + (\theta-1)(\rho-1)(\rho+1)\eta_A + 6\delta\theta(\rho-1)^2\left(\gamma_B\gamma_A + \frac{1}{4}\Gamma^2\right)\right] \\ &+\mathcal{O}\left(\omega^5\right),\end{aligned} \tag{4.8}$$

where $a_H = 3\rho + \rho^2 - 6 + 2\theta(1+\rho^2) + \theta^2(10 - 15\rho + 3\rho^2)$*.*

Proof. The proof follows from direct calculation. □

Proposition 1 offers a decomposition of welfare gains in cumulants, where subsequent rows relate to increasing orders. Under our simplifying assumptions, the second order cumulant contributes positively and symmetrically across countries to the welfare gains. Under our assumptions, the variance does not affect the *relative* gains from risk sharing: equally-sized countries gain the same, irrespective of the ratio γ_A/γ_B. While special, this case illustrates the relevance of the SE and LE decomposition: varying the ratio γ_A/γ_B, in relative terms, any gains in smoothing are exactly offset by losses in wealth level. However, in (4.8), there is no perfect offset between the LE and SE effects of the third and fourth moments—total and country-specific moments can increase or decrease the gains depending on parameter values, especially on trade elasticity θ and and risk aversion ρ. In particular, as in Cole and Obstfeld (1991), if $\theta = 1$ (under PPP) there are no gains from moving from financial autarky to complete markets: terms of trade movements perfectly insure risk emanating from output fluctuations, by ensuring that relative output prices move in proportion to relative quantities so to keep relative income constant.[26]

In general (with $\theta \neq 1$), trade elasticity and risk aversion jointly shape the gains from risk sharing. Intuitively, θ regulates the income exposure of residents in a country to domestic vs. foreign shocks. To show this most clearly, we write the first order approximation of country A's income under Assumption 2 and equal-size countries as follows:

$$p_{A,t}^{(1)\,cm} + Y_{A,t}^{(1)} = (2\theta)^{-1} (\ln D_{B,t} - \ln D_{A,t}) + \ln D_{A,t}. \tag{4.9}$$

If $\theta > 1$, implying that relative prices move less than proportionally to relative output, home income responds more to home productivity shocks than to foreign shocks (in the limit case $\theta \to \infty$ domestic income responds only to domestic TFP); vice-versa if $\theta < 1$. For future purposes, it is worth noting here that home bias reduces the trade elasticity threshold ($\theta > 2 - \frac{1}{\lambda}$) at which home income responds more to home productivity shocks than to foreign ones— see Figure 1 in the Online Appendix E.[27] With some conceptual latitude, we regard home bias as a co-determinant of the "quantity" of domestic income risk.

Risk aversion, ρ, modulates the (consumption) "price" of risk—in the solution of the model, ρ enters only through $\kappa_{A,B}$.[28] Observe that, with log utility ($\rho = 1$) and equal-sized countries, $n_A = n_B = \frac{1}{2}$, there is no LE from risk sharing (zeroing κ's); $\rho = 1$ does make welfare gains independent of cross-border differences in even moments

[26]However, a unit trade elasticity does not grant efficient sharing of productivity risk if demand is home biased, except in the special case of symmetric openness and log utility, see Corsetti and Pesenti (2005).

[27]With home bias ($\lambda \neq 1$), home income can be written as

$$p_{A,t}^{(1)\,cm} + Y_{A,t}^{(1)} = \frac{\ln D_{A,t}((2\theta - 3)\lambda + 2) + \ln D_{B,t}\lambda}{2(\theta - 1)\lambda + 2}. \tag{4.10}$$

[28]To wit, the second order term of the $\kappa_{A,B}$ expansion under PPP is

$$\kappa_{A,B}^{(2)} = \frac{\delta(\theta - 1)}{\theta^2} \left\{ \Gamma \left[n_B - n_A + \theta(1 - 2\rho n_B) \right] + 2\gamma_A \theta(\rho - 1) \right\}. \tag{4.11}$$

(eliminating η_A from (4.8)), but not of differences in odd moments ((4.8) still a function of ϕ_A).

4.2.2 Elasticities and non-Gaussianity

The following corollary details how trade elasticity (θ) and risk aversion (ρ) jointly drive the contribution of deviations from Gaussianity to welfare:

Corollary 1 (Non-Gaussianity and Welfare Gains)**.** *Under Assumption 2 and countries with equal size ($n_A = n_B = \frac{1}{2}$) the contribution of non-Gaussian tails to the welfare gains from risk sharing is regulated by the following coefficients:*

$$\begin{aligned} \phi_A &\Rightarrow -\tfrac{\delta(\theta-1)^3\rho(\rho+1)}{24(1-\delta)\theta^3}; & \Phi &\Rightarrow -\tfrac{\delta(\theta-1)^2\rho(2\theta(\rho-2)+\rho+1)}{48(1-\delta)\theta^3}; \\ \xi_A &\Rightarrow \tfrac{\delta(\theta-1)^3\rho\left(\rho^2-1\right)}{48(1-\delta)\theta^3}; & \Xi &\Rightarrow \tfrac{\delta(\theta-1)^2\rho a_H}{384(1-\delta)\theta^4}, \end{aligned}$$

where $\xi_A := \eta_A - 3\gamma_A^2$ denotes excess kurtosis, $\Xi := \xi_A + \xi_B$ and a_H is defined in Proposition 1.

Proof. Under Gaussianity $\phi_j = \xi_j = 0$ for $j = A, B$. The terms in equation (4.8) associated with individual or aggregate "skewness" (ϕ_j and Φ respectively) measure the contribution of asymmetric tails relative to Gaussianity, without further calculation. In line with definition of cumulants in (3.3), the coefficients for individual excess kurtosis and aggregate excess kurtosis are derived by replacing η_j with $f_{4,j} + 3\gamma_j^2$ in (4.8) and collecting terms. □

It is well known that, unless risk aversion is sufficiently high, risk premia remain moderate (Campbell, 2017). Backus et al. (2011) specifically characterize how premia respond to tail risk, showing how the relevance of third and fourth-order cumulants rise with risk aversion. Our results highlight once more that in open economy risk aversion and trade elasticity are both crucial in determining premia and thus allocations and welfare gains.[29]

[29] To fix ideas, using the parameter values discussed in Section 5 below, i.e. $\theta = 1.5$, $\delta = 0.98$ and

4.2.3 Country size

Gains from risk sharing can vary significantly with the relative size of a country, since size determines the incidence of the demand for and supply of a country assets and goods in the global equilibrium. Specifically, the integration in the world financial markets of a large (i.e., with non-infinitesimal size) country affects state-contingent prices. This means that the price of states of the world in which the large country seeks insurance will rise. The price of the state of the world in which it provides insurance will fall. It follows that, upon joining globally efficient capital markets, the budget set faced by a country will tend to be smaller, the larger its size. By the same token, a country with infinitesimal size will enjoy a "privilege": relative to the pre-insurance status quo, the price of the insurance it seeks in global markets will not be rising when the country joins the world financial market. For a country with a small size n_A, global income is largely determined by country B's income—which in turn implies a low $P_{B,K,t}$—everything else equal $\kappa_{A,B}$ is decreasing in n_A.

That said, country size interacts with all other parameters of the model in shaping the gains from risk sharing. As we show in Section 5, this interaction generates a rich set of results, potentially implying a non-monotonic relationship between size and welfare gains. The following proposition illustrates a case in point, showing that a necessary condition for smaller countries to benefit more from risk sharing—relative to larger ones—depends on the TFP distribution and on intra- and intertemporal elasticities. For clarity, the proposition is derived under cross-country symmetry in all parameters except size.

Assumption 3 (Simplified model with size difference)**.** *Countries have identical TFP probability distributions, i.e.* $\gamma_A = \gamma_B = \frac{\Gamma}{2}$, $\phi_A = \phi_B = \frac{\Phi}{2}$, $\eta_A = \eta_B = \frac{H}{2}$, *but possibly different size, i.e.* $n_A \neq n_B$.

Proposition 2 (Size and Welfare Gains from Risk Sharing)**.** *Under Assumption 3 and*

$\rho = 4$, the coefficients displayed in Corollary 1 amount to $\phi_A \approx -1.51$, $\Phi \approx -3.33$, $\xi_A \approx 2.27$ and $\Xi \approx 1.73$ (for comparison $\Gamma \approx 2.72$). It should be noted that, while the magnitudes of the coefficients are comparable, those of the moments are *typically* not—recall that these are raw moments, i.e. not scaled.

using a fourth-order approximation,

$$\left.\frac{\partial\left(\mathcal{W}^{cm}_{A,0}-\mathcal{W}^{au}_{A,0}\right)}{\partial n_A}\right|_{n_A=n_B}\begin{cases}=0 \quad if & \theta=1\\ <0 \quad iff & \theta\neq 1\,\&\,\Gamma>\frac{1}{2}(\rho-1)\Phi-\frac{1}{24\theta^2}P(\rho,\theta)H,\end{cases} \tag{4.13}$$

where $P(\rho,\theta)=-5+\rho^2+\theta(6+3\rho-\rho^2)+\theta^2(2-9\rho+3\rho^2)$.

Proof. The proof follows from direct calculation (see Appendix L). □

We will comment on this proposition below, comparing it with its analog derived for openness. For now, we note a specific implication of the interaction of size and cumulants, discussed in the Appendix M: if a country is sufficiently small, smoothing and level effects may have the same sign. Infinitesimal-size countries may gain both in smoothing and wealth.[30]

4.2.4 Country Openness

While openness is negatively related, if only weakly, to country size in the data,[31] it has a distinct and significant effect on risk sharing. For any given size, a country with a high degree of home bias in consumption tends to be more exposed to own income risk—yet a strong home bias in consumption constraints the ability of residents to stabilize marginal utility by substituting domestic goods with imports. The extent to which this substitution is efficient, in turn, depends on the trade elasticity. In light of these considerations, the effect of openness on the gains from risk sharing can be expected to have different signs, depending on its interplay with other parameters, and the overall relation need not be monotonic.

Mirroring our discussion of size, with the following proposition we provide insight on the complex effect of openness on risk sharing imposing symmetry in economic

[30]Appendix M studies the limit case of a small-open economy (SOE) with infinitesimal size, assuming that productivity in the rest of the world is constant. In this case the country wealth is predominantly driven by anticipated income flows (equilibrium output prices multiplied by equilibrium quantities) capitalized at constant discount rate, with convexities magnifying the present value of these flows.

[31]See footnote 41.

structure.

Assumption 4 (Simplified model with home bias)**.** *Countries have identical TFP probability distributions, i.e.* $\gamma_A = \gamma_B = \frac{\Gamma}{2}$*,* $\phi_A = \phi_B = \frac{\Phi}{2}$*,* $\eta_A = \eta_B = \frac{H}{2}$ *and equal size, i.e.* $n_A = n_B$*.* $\lambda_A = \lambda_B = \lambda$ *and* λ *is a free parameter measuring the degree of home bias.*

Proposition 3 (Openness and Relative Gains from Risk Sharing)**.** *Under Assumption 4 and using a fourth-order approximation,*

$$\left.\frac{\partial\left(\mathcal{W}_{A,0}^{cm} - \mathcal{W}_{A,0}^{au}\right)}{\partial\lambda}\right|_{\lambda=1} \begin{cases} = 0 \quad if & \theta = 1 \\ > 0 \quad iff & \theta \neq 1 \,\&\, \Gamma > \frac{1}{2}(\rho-1)\Phi + \frac{1}{24\theta^2}Q(\rho,\theta)H, \end{cases} \tag{4.14}$$

where $Q(\rho,\theta) = 4 - \theta(5 + 2\rho + \rho^3) + \theta^2(-2 + 8\rho - 3\rho^2 + \rho^3)$*.*

Proof. The proof follows from direct calculation (see Appendix L). □

Together, Propositions 2 and 3 show that, while (at least locally) openness and size affect welfare gains from risk sharing in similar ways, there are significant differences in their impact. With $\Phi < 0$, provided $\rho > 1$, relative welfare gains are decreasing in size if $P(\rho,\theta) > 0$, while they are increasing in openness if $Q(\rho,\theta) < 0$. Both conditions are satisfied in our calibration in the next section. More in general, the condition $P(\rho,\theta) > 0$ on size is always satisfied for mildly high risk aversion (e.g. $\rho > 3$). In contrast, in the condition $Q(\rho,\theta) < 0$ the interplay of ρ and θ is more involved (see Online Appendix L). Under the conditions of the proposition, the benefits from risk sharing increase with openness when countries have a small exposure to domestic risk (relatively low θ) or price of risk is small (relatively low ρ). We will revisit this result, providing a more general characterization, in Section 5.

4.3 Welfare, asset prices, and the terms of trade

We conclude this section discussing the equilibrium implications of risk sharing for the international prices of assets and goods, both contributing to the adjustment in relative wealth.

Asset prices. The following proposition establishes that welfare gains from risk sharing are positively related to domestic asset prices, although the link is not necessarily tight.

Proposition 4 (Relative welfare and asset prices)**.** *Under 4th order approximation and holding Assumption 2, the difference in welfare in country A and country B under complete market and financial autarky can be written as:*

$$\left(\mathcal{W}^{cm}_{A,0} - \mathcal{W}^{cm}_{B,0}\right) - E_0\left(P^{cm}_{A,K,t} - P^{cm}_{B,K,t}\right) =$$
$$\frac{\delta^2(\theta-1)^2(n_B-n_A)\rho}{8(1-\delta)\theta^4}\left(\theta(\rho-1)(\gamma_A-\gamma_B)-(n_B-n_A)\Gamma(\theta\rho-1)\right)^2+\mathcal{O}(\omega^5), \tag{4.15}$$

and

$$\mathcal{W}^{au}_{A,0} - \mathcal{W}^{au}_{B,0} - \frac{E_0(P^{au}_{A,K,t} - P^{au}_{B,K,t})}{(1-\rho)} = \mathcal{O}(\omega^5). \tag{4.16}$$

Proof. The proof follows from direct calculation. □

Focusing on the case of equally sized country, i.e., $n_A = \frac{1}{2}$, for analytical transparency, note that the right hand side of (4.15) is zero up to fourth order of approximation. Comparing countries with identical size, under perfect risk sharing, the one with higher asset valuation enjoys higher welfare. Remarkably, Proposition 4 establishes that, in autarky, the opposite is true if $\rho > 1$. This result suggests that it may pay to be on the relatively low side of the risk distribution when integrating markets. The level of welfare and wealth are simultaneously higher. This is in sharp contrast with the case of autarky. Under financial autarky, for standard model parameterization with $\rho > 1$, welfare is negatively correlated with financial wealth. To appreciate why, recall that, without trade in financial assets, SDFs and risk-free rates are not equalized across borders (as they are under PPP if risk is efficiently shared). In countries with a higher variability of income, the SDF will also be relatively more variable—implying a lower safe autarky interest rate and larger risk premium. So, on the one hand, a lower safe autarky interest rate will raise the PDV of current and future income flows, boosting asset prices. On the other hand, higher risk premia depress asset prices. When $\rho > 1$ the former dominates, with a positive net effect on asset prices.[32]

[32] We thank an anonymous referee for pointing out that the effect of risk on asset prices works through the intertemporal elasticity of substitution (EIS) which under CRRA is $EIS = 1/\rho$.

Nonetheless, the link between welfare and asset prices is not necessarily tight. As shown by Proposition 4, for $n_A \neq \frac{1}{2}$, cross country differences in welfare relative to asset prices become a function of both the relative and total variance—other moments affect relative welfare and prices symmetrically.[33] For the larger country ($n_B > n_A$), the right-hand side of (4.15) is higher, the higher own output or total variance (γ_B or Γ), or both. Observe that Proposition 4 and equation (4.2) together imply a positive relationship between relative welfare (across countries) and κ. Remarkably, in our quantitative assessment of Section 5 we find that relative welfare, relative asset prices and κ's are all close to proportional under complete markets.

Terms of trade. Differences in cross-border average consumption under perfect risk sharing may be expected to impact the international price of national outputs—i.e., the terms of trade. Although, as explained below, terms of trade cannot respond to risk insurance in the simplified specification used in this section, we find it useful to bring forward a discussion providing insight on a key result of our quantitative exercises below, relying on full model specification detailed in section 2.

As is well understood, under PPP, changes in relative consumption would not directly affect the composition of global demand. Even if a country gets a higher share of the global consumption basket, its relative demand for home and foreign goods remains unchanged. In contrast, with home bias in preferences (a deviation from PPP), a rise in the average relative consumption of a country will also coincide with a rise in the global demand for its domestically-produced goods—contributing to appreciating their relative price. This is a classical argument, resonating with the controversy on the "transfer problem" between Keynes and Ohlin concerning the effects of the reparations imposed on Germany after World War I (Keynes, 1929; Ohlin, 1929). Intuitively, in our context the asset repricing upon entering risk sharing agreement translates into a (market based) "transfer" of purchasing power between countries (e.g., from a riskier to a safer economy).

However, the terms of trade can adjust to risk sharing also under PPP, if supply is endogenous. Under CRRA preferences, for instance, wealth effect impinges on labor

[33] Observe that the additional term in total variance is zero when either $\theta = 1$ or $\rho = 1$.

supply. Because of this, labor will fall in countries experiencing an increase in relative wealth. Even if, under PPP, the relative demand for national goods is unaffected at global level, international relative prices will need to adjust to cross-border changes in labor and thus output supply.

5 Quantitative Analysis

In this section, we use global methods to pursue two goals. First, we revisit quantitatively the analytical results from applying perturbation methods. We do so by solving a two-country version of our model in Section 2 for a sample of artificial economies generated to best explore the range of mechanisms through which risk sharing affects welfare and the global allocation discussed so far. Second, we offer a numerical assessment of gains from risk sharing solving our three-country model in its general form and calibrating size (n_j), import share ($1 - \nu_j$) and higher order moments (γ_j, ϕ_j and η_j), to the countries in our sample.

Throughout our quantitative analysis, we set the frequency of the model to annual (the frequency of the Penn World Table database). The share of labor in production is $1 - \alpha = 0.7$ (in the ballpark of US labor share), the risk aversion parameter is $\rho = 4$ (e.g. Martin, 2008), the inverse Frisch elasticity of labor supply is $\varphi = 1.75$ (see Attanasio et al., 2018), the weight of disutility from working is normalized to $\chi = 1$, the trade elasticity of substitution is $\theta = 1.5$ (in the ballpark of the values used in the international macro literature), and the discount factor is $\delta = 0.98$. Concerning the import share $(1 - \nu_j)$, for transparency in the interpretation of the results from the simulated-data analysis, in the next subsection 5.1 we calibrate $\nu_j = 1-(1-n_j)\lambda_i$ using population data but setting $\lambda_j = 0.3\ \forall j$ (unless differently specified).[34] In the empirical application, instead, we calibrate λ_j using the share of imported consumer goods based on the OECD's Inter-Country Input-Output tables (vintage 2020).

[34]Allowing for an independent degree of openness increases the number of determinants and their interactions, introducing complexities that make the interpretation considerably more involved.

In line with the literature, we posit that output fluctuations are driven by standard autoregressive productivity shocks. In the first exercise, we set the autocorrelation coefficient of the TFP process to 0.5. This value, low relative to empirical estimates, improves the accuracy of discretization, without affecting inference. In the empirical application, we proxy the TFP process by an AR(1) process estimated on detrended real per-head GDP—hence the parameters of the TFP process are specific to each country in the model. The Online Appendix H provides details on the estimation of the AR(1) processes based on mixed-Gaussian distributions. Finally, we report absolute and relative welfare gains in permanent consumption units (PCU).

Three comments are in order. First, we are mindful that the distribution of GDP across countries is affected by shocks of different nature, as documented, e.g., by the SVAR and DSGE literature. We motivate our focus on productivity on the ground that our contribution is primarily theoretical. It is therefore natural for us to conduct our exercises in line with leading contributions that typically assume an exogenous productivity process as the source of macroeconomic risk.[35] Second, we have estimated our productivity process using GDP data. In principle, TFP estimates would have been more directly comparable, but their quality is known to differ significantly across countries, and in some cases to be poor. Finally, we could have calibrated our TFP processes to match the GDP's moments in the model with those observed in the data. However, given the large number of countries in our sample and our global solution technique, computationally, this approach would have been extremely costly relative to the potential benefits.[36] Similar considerations explain why we estimate the process of TFP separately for each country, rather than relying on VARs, given the high computational toll of discretizing the resulting multivariate processes. The Online Appendices H and F discuss the discretization approach and the case of correlated processes, respectively.

[35]The main driver of output risk in the literature is either endowment shocks (e.g. van Wincoop, 1999 or Gourinchas and Rey, 2022) of TFP in production economies, e.g. Coeurdacier et al. (2020). For tractability and transparency, a focus on one single source of shocks allows us to build a clear map onto the observed process for GDP.

[36]In our exercises, model parameters other than TFP and country size are symmetric across countries. Differences in TFP estimates across methods may be expected to have minor effects on the welfare ranking of countries produced by our calibration.

5.1 Numerical analysis under more general assumptions

We start by showing that our results derived using perturbation methods under educated restrictions on parameters carry over to a more general specification solved using global solution methods. In particular, allowing for endogenous labor supply and home bias in consumption, our exercise confirms that gains from risk sharing are higher for countries with smaller size, higher openness, larger variance, more negative skewness and kurtosis of their TFP process, and that the gains are larger with a larger trade elasticity. We also find that gains from risk sharing are tightly related to the level effect and to relative asset prices. Finally, we show that the level effect drives the average equilibrium terms of trade.[37]

To conduct the exercises in this subsection, we construct an ad hoc sample of artificial economies, specifically generated to produce a random sample of country size and cumulants (see the Online Appendix I for the construction of these data). As stressed by Bekaert and Popov (2019) among others, higher moments are strongly correlated in the data. Given a high correlation, the individual contribution of size and moments tends to be blurred in samples including only a relatively small number of countries. Our algorithm generates 477 economies (selected under the criterion of displaying independent higher moments). Each of these differs in size and moments of TFP, allowing us to better distinguish their individual contribution to welfare gains and other variables of interest. In all the analysis in the rest of the section, each of these 477 countries is matched with the same base country, labelled country B. Table 2 summarizes the DGP of the generated sample and of the base country B.

5.1.1 Relative welfare, κ, and the terms of trade as a function of cumulants and country size

To examine the relationship between endogenous variables and exogenous determinants, we estimate regression models using simulated data from our 477 surrogate two-country

[37] Online Appendix J provides an indication that the perturbation-based solution for κ is similar to that obtained with the global methods approach.

Table 2: Quantiles of size and moments of generated data

Quantiles	**Size**	**Standard deviation**	**Skewness**	**Kurtosis**
0%	0.0005	0.0211	-2.9484	3.0157
25%	0.0034	0.0939	-0.7158	9.0786
50%	0.0259	0.1199	0.0571	13.3176
75%	0.1884	0.1357	0.8448	16.8538
100%	0.6643	0.1497	2.8778	19.9787
Base Country B	0.5000	0.1417	0.5992	3.7890

Note: The sample consists of 477 countries, obtained by drawing mixed-Gaussian distributions with mean zero and independent 2nd through 4th moments. Size is drawn randomly from our empirical dataset.

economies. The results are reported in Table 3. The three columns present, respectively, relative welfare gains, LE (i.e., κ relative to its financial-autarky counterpart), and the terms of trade. Consistent with our analytical findings, each of these outcomes depends linearly on the relevant moments, country size, and their first-order interactions.[38] Both the dependent variables and the regressors are demeaned, with regressors expressed as differences between country A's and country B's values.[39]

Looking at the second row of the table, the regression results suggest that, holding cumulants constant and symmetric, the size of country A contributes negatively to its relative welfare gains (second column of the table). Remarkably, when moving from autarky to complete markets, the larger country A is, (a) the larger the wealth transfer to country B, i.e., the larger is $\kappa_{A,B}$ (third column); and (b) the larger the terms of trade deterioration (last column). All these results are consistent with our analysis and propositions in Section 4.

Rows 3 through 5 in the table show the effect cross-border differentials in each

[38]Proposition 1 implies that total moments (i.e., Γ, Φ, and H) vanish in the perturbation solution when computing the difference in welfare gains across countries. We therefore focus on *relative* rather than absolute welfare gains: changing the cumulants of country A's TFP, while holding those of country B fixed, also changes total riskiness.

[39]In all regressions reported in the table, the fit is virtually perfect ($R^2 \approx 1$). By construction, only the listed regressors vary across observations, although the exact non-linear functional form linking them to the dependent variable is unknown. We employ Bayesian linear regression for convenience, as it yields readily available posterior marginal predictions. Similar results obtain using OLS with bootstrapped standard errors.

moment in isolation, for countries with equal size ($n_A = 0.5$). Also in line with the results in Section 4, countries with larger variance and/or kurtosis and more negative skewness in the output distribution, gain more; have higher $\kappa_{A,B}$ and weaker terms of trade. The implicit wealth transfer through $\kappa_{A,B}$ is consistent with its interpretation as compensation for risk. The adjustment in the terms of trade aligns with the change in relative wealth/consumption.

Relative to the analytical section, our quantitative analysis sheds light on the interactions between size and cumulants—shown in rows 6 through 8 of the table. Using the results in these rows, we can indeed compute the critical value n_A^*, at which the marginal effect of the moments changes sign. To do so, we calculate, for each moment,

$$n_A^* = \bar{n}_A - \frac{d}{2f},$$

where—recalling that the data is demeaned—$\bar{n}_A$ is the mean size, d is the coefficient on the moment differential (e.g. for variance, $d \approx 38.5$ in the second column of Table 3), f the coefficient on the interaction term of the same moment differential (for variance, $f \approx -52.0$) and we use the fact that $n_A - (1 - n_A) - (\bar{n}_A - (1 - \bar{n}_A)) = 2(n_A - \bar{n}_A)$, with $\bar{n}_A \approx 0.13$. For relative welfare (second column), the threshold for each moments is, respectively, $n_A^* \approx \{0.5, 0.68, 0.9\}$. Observe that, when countries have equal size, the second moment differential has no effect on the relative welfare gains, consistently with Proposition 1. Furthermore, in line with our analysis in Section 4.2.3, the table results confirm that the composition of the relative welfare gains, i.e. the SE/LE mix, can vary significantly with the size of the country.

5.1.2 Relative welfare, wealth and asset prices

The analytical results derived in Section 4 establishes that, under PPP, there is positive relationship between relative welfare and relative asset prices, tight under financial autarky, and possibly less so under complete markets unless countries are symmetric in size (Proposition 4). By the same token, if PPP holds and countries have equal size, there is a tight relation between κ and relative asset prices (equation 4.2). Together, these two

Table 3: Relative welfare, κ, and the terms of trade as a function of cumulants and country size

	$\frac{PCU_A}{PCU_B}$	$\frac{\kappa_{cm}}{\kappa_{au}}$	$\frac{ToT_{cm}}{ToT_{au}}$
Constant	0.0	0.0	0.0
	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]
$n_A - n_B$	-2.2	1.3	2.1
	[-2.2, -2.2]	[1.2, 1.3]	[2.1, 2.1]
$\gamma_A - \gamma_B$	38.5	5.5	26.3
	[38.4, 38.6]	[5.5, 5.6]	[26.2, 26.4]
$\phi_A - \phi_B$	-29.4	-4.1	-9.4
	[-29.6, -29.2]	[-4.2, -4.0]	[-9.6, -9.2]
$\eta_A - \eta_B$	14.2	0.8	1.4
	[13.6, 14.8]	[0.5, 1.1]	[0.9, 1.9]
$n_A - n_B \times \gamma_A - \gamma_B$	-51.9	28.6	48.4
	[-52.2, -51.6]	[28.5, 28.8]	[48.2, 48.6]
$n_A - n_B \times \phi_A - \phi_B$	26.9	-15.6	-26.6
	[26.3, 27.5]	[-16.0, -15.3]	[-27.1, -26.1]
$n_A - n_B \times \eta_A - \eta_B$	-8.8	5.4	9.4
	[-10.2, -7.3]	[4.7, 6.2]	[8.2, 10.6]
Number of Observations	477.0	477.0	477.0
R^2	1.0	1.0	1.0

Bayesian estimation of a linear model with interaction terms. [95% Highest Posterior Density].

results suggest that κ and relative welfare should also be related.

In accord with our analytical propositions, our numerical results show that asset prices are tightly related to both relative welfare and κ: the correlation between $(\mathcal{W}_A^{cm} - \mathcal{W}_B^{cm})$ and $\ln(P_{A,K,0}/P_{B,K,0})$ is approximately 1.0; equally tight is the correlation between $\ln \kappa_{A,B}$ and $\ln(P_{A,K,0}/P_{B,K,0})$. We also find that the correlation between the welfare differential under complete markets and the gap between κ and the autarky counterpart $(\ln(\kappa_{cm}/\kappa_{au}))$ is as high as -0.99.[40]

Moreover, simulated data analysis lends support to the transfer effect mechanism discussed at the end of Section 4. Namely, we find that $\ln \kappa = \hat{\kappa}$ (in deviation from the autarky counterpart) is strongly correlated with the natural logarithm of the deviation of the terms of trade from the autarky counterpart (approximately 0.99). Moving to perfect risk sharing, richer (safer) countries experience an appreciation of their terms of trade.

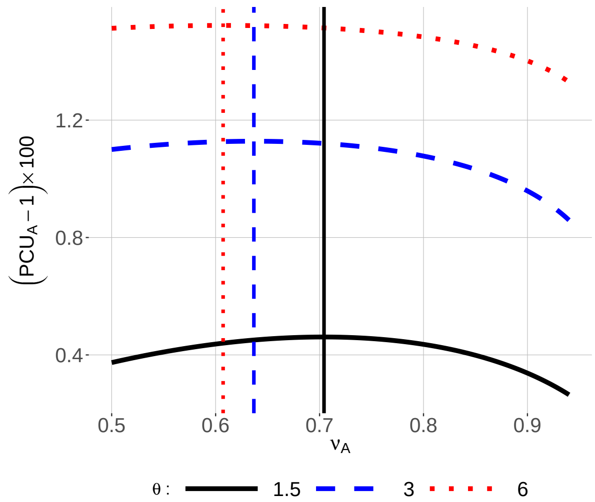
5.1.3 Openness

In Section 4, the Proposition 3 illustrates how openness affects welfare gains around a symmetric benchmark. Building on this proposition we now construct a counterfactual in which all 477 economies face identical TFP shock distributions (same as our base country B), have equal size, but have different home bias—i.e., we only vary the parameter ν_A (keeping $\nu_B = n_B = 0.5$ and varying effectively λ_A). For these economies, Figure 1 plots the (smoothed) gains from risk sharing against their ν_A, for three alternative trade elasticities, $\theta = \{1.5, 3, 6\}$. For all θs, the pattern in the figure is a clear non-monotonic relationship. As discussed above, non-monotonicity can arise because a stronger home bias raises the exposure of a country to own income risk (thus raising the potential benefits from risk sharing), but at the same time reduces the scope for smoothing consumption by substituting domestic with foreign consumption goods (with opposite effects on the potential gains from insurance).

There is nonetheless a broader takeaway from the exercise: the effects of a

[40] In line with Proposition 3, we also find that relative welfare and asset prices remains tightly related, with the right sign, under financial autarky.

Figure 1: Openness and trade elasticity: effects on welfare gains



Note: $\lambda_B = 1$, $n_A = n_B = 0.5$. All Markov processes are identical across cases and identical to the base-country's process. The vertical lines denote the value of ν_A that maximizes the gains.

single parameter on the gains from financial integration are shaped by its interactions with other parameters and the distribution of the fundamental shocks. Observe that the value of ν_A at which welfare gains are at their maximum, marked by vertical lines (for respective values of θ) in Figure 1, is model-dependent—that is, it is sensitive to alternative parameterizations. Similar to home bias, the trade elasticity also regulates the relative "exposure" of a country income and consumption to productivity risk (see Section 4.2.1). In the case illustrated in the figure, a higher trade elasticity—causing a higher exposure to own production risk—implies that the curve reaches a maximum at a lower level of home bias: it takes a lower home bias —which also impinges on the exposure to own production risk—to start seeing a deterioration in the gains from risk sharing. More in general, the effects of home bias on these gains may display different patterns depending on asymmetries in country size and shock distribution.

5.2 An evidence-based assessment of welfare gains

So far, we have explored in analytical and quantitative detail the mechanisms by which cross-country asymmetries in fundamental uncertainty in output shape income risk and welfare gains from risk sharing across countries differing in size and openness. This section brings this material together, with an attempt to gauge potential gains from efficient sharing of income risk across countries through the lens of our (full-fledged) model.

We construct our exercise by solving, with global methods, the three-country version of the model presented in Section 2, with moments and size calibrated using Penn World Table data, and consumption home bias calibrated using OECD's ICIO tables.[41] For the aggregate regions (Advanced Economies (AE), and Emerging Market Plus economies (EME+)) we compute the population-weighted mean consumption share.[42]

For each of the 55 countries in our sample,[43] we assess gains from sharing risk with the rest of the world, distinguishing two foreign-economy blocks: AE and EME+. The population size corresponds to the aggregate population of each block, and the data-generating-process (DGP) of the TFP is calibrated using the aggregate real per-capita GDP of these regions.[44] At the same time, to synthesize country characteristics and explore their importance in driving the gains from risk sharing, we group countries using a statistical (Gaussian-Mixture Model) clustering technique based on size, openness and moments (e.g. see Scrucca et al., 2023).[45] Table 4 provides a concise description of the clusters' determinants and the color coding. The I-green cluster includes countries of modest size, not excessively open, with moderate output variability, hence relatively safe. The II-red cluster includes small, open economies with highly volatile output, hence high income risk. The other two clusters, III-blue and IV-orange, only include 5 large and relatively closed economies, differentiated by riskiness. The two IV-orange economies (IND and CHN) have a relatively pronounced left skewness.

[41]The OECD's ICIO tables are not available for all the countries in our sample. For the few missing we estimate their shares based on the fitted values regressing the available (log) consumption import shares (cshare) on (log) population share (n), which gives $\ln(cshare) = -3.04^{***} - 0.18^{***} \log(n)$, where *** means a a p-value below 1%.

[42]The definition of advanced and emerging market economies are taken from the IMF's Fiscal Monitor Database. EME+ includes some developing economies with moments not too extreme relative to standard emerging market economies. These regions don't include the domestic economy. For example, EME+ trading with China, does not include China; AE trading with the USA does not include the latter, etc.

[43]We excluded 7 countries due to poor convergence of the solution.

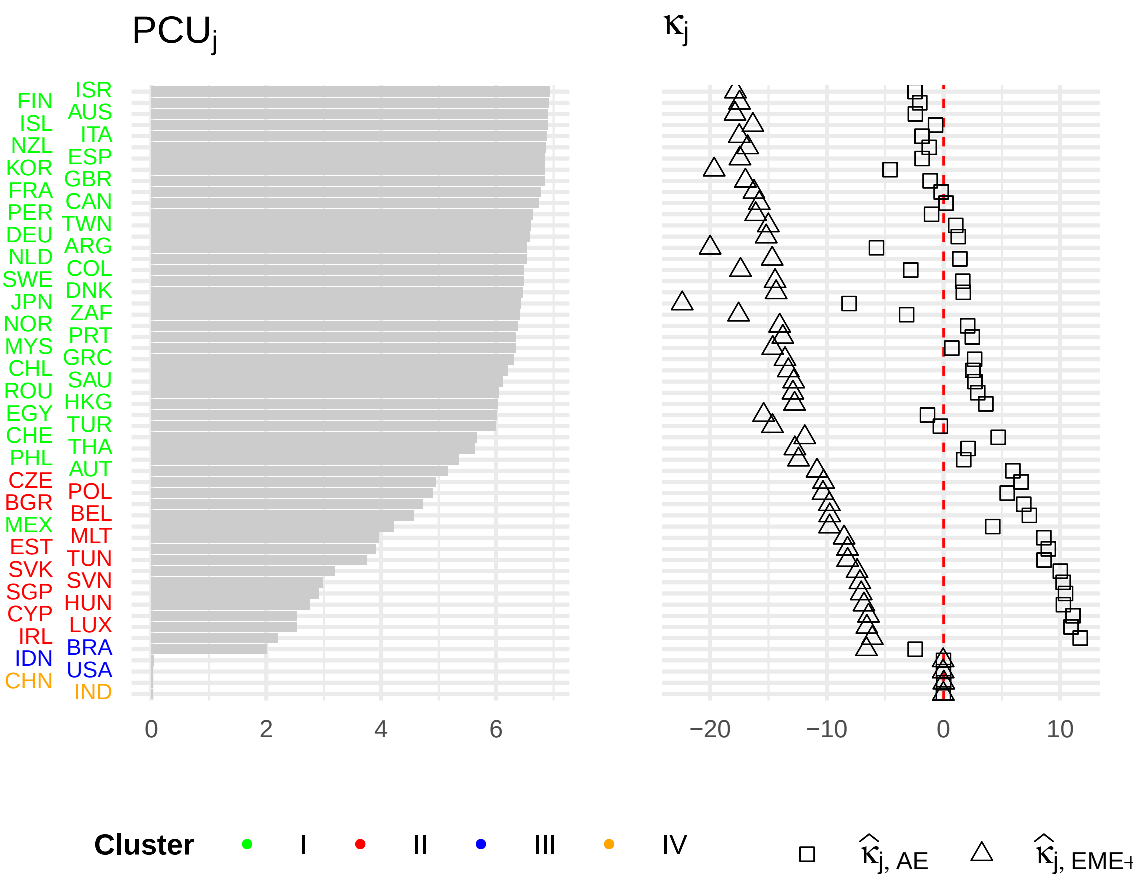
[44]Our approach has its advantages and drawbacks. Aggregating the rest of the world into two regions allows us to demonstrate the heterogeneity of gains and their LE and SE components across individual countries reflecting their cumulants and size. Yet, it may conceal nuances in the distribution of gains, κ_s and consumption shares that a more granular approach would reveal (see Appendix K).

[45]We model the data as a weighted mix of Gaussian distributions. Each country is assigned to the Gaussian component that makes its features most likely. See Online Appendix N for a technical description.

5.2.1 Gains from risk sharing and κ' across countries

Results are shown in Figure 2. The left panel plots welfare gains from perfect risk sharing relative to autarky in permanent consumption units (PCU $\times100$). The right panel plots $\kappa_{j,AE}$ and $\kappa_{j,EME+}$ relative to their counterparts under financial autarky, i.e., the unconditional mean of the consumption in the AE and EME+ blocks relative to country j, adjusted by the real exchange rate.

Figure 2: Distribution of welfare gains and κ_s (in percentages)



Note: $\hat{\kappa}_{j,\cdot}$ denotes the difference between the κ under complete markets and the average of the implied κ under autarky. See Table 4 for a description of the clusters. ISO country codes can be found here: https://www.iso.org/obp/ui/#search/code/.

Consistently with the cross-border heterogeneity in fundamentals and economic structure (moments, size and openness) in the data, there is considerable variation in the potential welfare gains across countries. Total gains vary from a minimum of approximately 0.02% to a maximum of approximately 6.9% of permanent consumption, with a

Table 4: Interpretation of country clusters based on scaled distributional features

Cluster[†]	**Size**	**Openness**	**Standard deviation**	**Skewness**	**Kurtosis**	**Summary**
I-Green	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	≈ 0	Mid-sized economies, mildly closed, modest upside skewness, normal tails.
II-Red	$\downarrow$	$\uparrow\uparrow$	$\uparrow\uparrow$	$\downarrow$	$\uparrow$	Small, open, highly volatile with mild left-tail risk, thick tails.
III-Blue	$\uparrow$	$\downarrow\downarrow$	$\downarrow$	$\downarrow\downarrow$	$\uparrow$	Moderately sized and closed, exposed to strong downside risk and fat tails.
IV-Orange	$\uparrow\uparrow$	$\downarrow\downarrow$	$\downarrow$	$\uparrow\uparrow$	$\downarrow$	Very large and closed, asymmetric with right-tail risks but thin-tailed distribution.

Note: † Clusters obtained via Gaussian Mixture Model (GMM) as discussed in Scrucca et al. (2023) and Online Appendix N. The standardized input features are: population size, openness (consumption import share) and standard deviation, skewness and kurtosis of cyclical GDP. Model selection based on the Bayesian Information Criterion (BIC) together with a sufficiently large number of clusters. We selected the EEI model (diagonal covariance with equal volume and shape), yielding four groups.

median gain of above 6%. Nonetheless, there is structure in this heterogeneity. To start with, for each country, the Level Effect is systematically more favorable relative to the (riskier) EME+ region than relative to the (safer) AE region. As shown in the figure, $\kappa_{j,AE}$'s range between -10.0% and just above 11.0%; $\kappa_{j,EME+}$'s range between just below -20.0% and zero.

More crucially, with the exception of one country, Mexico,[46] the clusters based on countries' characteristics appear to account remarkably well for the welfare ranking and the composition of welfare gains. The smaller, riskier countries in the II-red cluster do derive significant welfare gains, ranging from 2 to 5%, from risk sharing. However, looking at the net κ's on the right panel of the figure, it is apparent that in equilibrium they transfer resources to the safer AEs—they pay a price in terms of own average consumption. The LE thus moderates the gains from risk sharing that accrue in terms of consumption smoothing. Similar considerations apply to the subset of countries in the

[46] It is worth stressing that the probability of belonging to the next most likely group (group II-red) is less than 0.2%. That is, Mexico is not a borderline case.

I-green group with net gains ranging between 5 and 6%. Relative to the II-red group, these countries are less exposed to adverse output variability: the gains of this subset are somewhat higher, in part because the LEs are relatively more favorable. Relative wealth effects are nonetheless stronger in the I-green countries at the top of the distribution, with gains above 6%. These countries tend to benefit from LEs in relation to both the EME+ and the AE region. Finally, reflecting their size and moderate openness, the four largest countries in terms of population (two IV-orange—IND and CHN—two III-blue—USA and IDN) gain little, if at all, from risk sharing, either through SE or LE (note that there is no improvement in average relative consumption with respect to financial autarky). The exception in the III-blue group is Brazil, reflecting a particular and instructive combination of features detailed in the Online Appendix N.[47]

5.2.2 Quantitative importance of non-Gaussianity

We conclude our study with an assessment of the relative importance of tail risk and economic asymmetries as drivers of the potential gains from risk sharing. To do so, we run two exercises. First, we recompute these gains together with κ's under complete markets and the change in κ's from autarky to complete markets, under three calibrations of the model: i) our baseline calibration with Mixed-Gaussian shocks; ii) our baseline calibration but with Gaussian shocks; and iii) PPP and equal country size while keeping Mixed-Gaussian shocks.[48] Tables 5 summarizes our results. To facilitate the comparison with Figure 2, for each variable in the table we report results for the 5th, the 50th and the 95th percentile of the distribution.

Contrasting the columns for the Mixed-Gaussian and Gaussian shocks reveals the relevance of fat tails in output distributions. The first numerical row of Table 2 show that, in permanent consumption units, the median country enjoys a 6% gain—a

[47]Brazil's relatively large population and modest openness, together with somewhat higher riskiness, account for the position of Brazil in the ranking. Note that, relative to the AE group, Brazil's level effect lies markedly to the left—more than other Group III-blue (and IV-orange) members—in line with Group I-green countries. Even so, Brazil's overall profile—especially its size—is more balanced vis-à-vis the EME+ group.

[48]We discretize the shocks using the same technique used for the mixed-Gaussian case, obtaining very similar variances implied by the respective Markov Processes, as discussed in Appendix H.

gain that rises to almost 7 at the top of the distribution under the Mixed Gaussian. Under a Gaussian distribution, in contrast, gains are about two percentage points lower for the median country, and somewhat less dispersed across countries. Strikingly—as shown in the last three columns of the table—once countries' asymmetries in size and openness are removed, the gains from risk sharing become negligible. This is so whether the distribution is assumed to be Mixed-Gaussian (as in the table) or Gaussian. The assessment of the welfare effects of financial integration requires a proper calibration of the heterogeneity in expenditure baskets, country size and risk that characterizes the data.

Moving to the Level Effects, rows 2 and 3 of Table 5 show that both the mixed-Gaussian and Gaussian specifications yield virtually identical distributions of κ's under complete markets. In turn, the last two rows, 4 and 5, indicate that major differences emerge under autarky: for the median country, relative consumption under the Gaussian distribution is almost 1 percentage point lower vis-à-vis AEs and nearly 5 percentage points lower vis-à-vis EMEs+. In other words, moving from autarky to complete markets generally entails a larger adjustment of relative consumption under "fat tails" (Mixed-Gaussian)—except for some EMEs+ in the upper segment—as higher risk typically calls for larger compensations.

Second, we compute Smoothing Effects reporting second and higher moments of (log) consumption and labor under the same specifications of Table 2. Results are shown in Table 6. Recall from our discussion that, because of level effects, countries may experience a deterioration in their consumption smoothing, still draw positive net gains from risk sharing. One notable result from the table is that standard deviations under complete markets are lower than under autarky across most but not all specifications—the exception is for countries in the upper fringe of the distribution. Furthermore, not only variance, but also higher moments matter for welfare—the table highlights that moments need not all move in the direction of improved smoothing in the transition from autarky to complete markets. It may well be possible that smoothing deteriorates in terms of one moment and improve in terms of another. All this illustrates the problem of relying on standard deviations (of consumption) as the only indicators of risk sharing.

Table 5: Percentiles of PCU and κ_s

	Mix.-Gaussian baseline			**Gaussian baseline**			**Mix.-Gaussian PPP and equal size**		
Variable†	5%	50%	95%	5%	50%	95%	5%	50%	95%
PCU	0.03	6.04	6.90	0.46	3.91	4.80	0.01	0.02	0.04
$\kappa_{j,AE}^{cm}$	−7.86	1.66	10.55	−7.85	1.66	10.63	−0.03	0.00	0.03
$\kappa_{j,EME+}^{cm}$	−19.77	−14.34	−6.48	−19.74	−14.33	−6.47	−0.03	0.00	0.03
$\kappa_{j,AE}^{cm-au}$	−3.59	1.64	10.59	−3.45	0.73	7.27	−0.01	0.00	0.02
$\kappa_{j,EME+}^{cm-au}$	−18.40	−13.30	−0.04	−12.17	−8.41	−1.01	−0.01	0.00	0.02

† Percent.

6 Conclusions

In this paper, we have reconsidered the welfare and macroeconomic effects of insuring fundamental output risk among countries differing in size and openness, offering a decomposition of welfare gains into a smoothing and a level effect, by moments (cumulants) of the distribution of output, as a function of trade elasticities and risk aversion. Our analysis brings asymmetries in volatility, skewness and kurtosis—and especially their interactions with economic size and openness of a country—center stage in the assessment of risk sharing.

Our study contributes to the literature an in-depth and transparent analysis of a country exposure to income risk stemming from production uncertainty—providing economic insight on how heterogeneity in size, openness, and higher-order moments (cumulants) shapes the potential gains from cross-border insurance. Most notably, we show that a proper assessment of the relevance of risk requires careful calibration of cross-border heterogeneity in these dimensions. In our quantitative exercise, the potential gains from risk sharing range from a fraction of 1% to more than 7% of permanent consumption. A substantial part of these gains are due to "fat tails". By clustering countries based on openness, size, and higher-order moments, we uncover a ranking of welfare gains that is almost perfectly explained by these features, broadly confirming our theoretical results. Remarkably, the distribution of gains cuts across the traditional divide between advanced

Table 6: Percentiles of (log) consumption and labor moments

	Mix.-Gaussian baseline			**Gaussian baseline**			**Mix.-Gaussian PPP and equal size**		
Moment[†]	5%	50%	95%	5%	50%	95%	5%	50%	95%
				Consumption					
$Stdev_C^{cm-au\ddagger}$	−1.54	−0.60	−0.14	−0.85	−0.41	−0.08	−0.78	−0.28	0.04
$Skew_C^{cm-au}$	−0.17	0.19	0.49	−0.04	−0.01	0.00	−0.22	0.17	0.51
$Kurt_C^{cm-au}$	−1.12	−0.18	0.31	−0.01	0.00	0.00	−1.31	−0.23	0.27
				Labor					
$Stdev_L^{cm-au\ddagger}$	−2.10	−0.75	0.00	−1.48	−0.64	−0.03	−1.09	−0.31	0.14
$Skew_L^{cm-au}$	−0.92	−0.40	0.24	0.01	0.02	0.05	−0.73	−0.22	0.38
$Kurt_L^{cm-au}$	−1.61	−0.26	0.53	−0.01	0.00	0.01	−1.70	−0.12	0.41

[‡] Percentage points. [†] $moment_X^{cm-au}$, with $X = \{C, L\}$, represents the difference between complete markets and autarky of the particular moment of (log) X computed using the Markov process ergodic distribution.

and emerging-market economies.

In economic and policy evaluations, risk-sharing and the benefits of capital market integration are routinely assessed using the volatility and correlation of consumption. Our analysis emphasizes that these indicators offer only a partial view. A key challenge for future research is to develop complementary indicators that capture level effects and relative wealth adjustments, thereby bridging the gap between macroeconomics and the finance and asset pricing literature. This line of inquiry could clarify the extent to which even relatively "safe" countries benefit from cross-border risk sharing, particularly during episodes of heightened tail risk—possibly based on richer models incorporating investment, financial frictions, and government activity, all of which play a central role in the risk-sharing debate. A related challenge is to bring new theoretical and applied work to bear on the scope for risk sharing in the context of rising trade and financial fragmentation, climate-related shocks, as well as heightened geopolitical risks, currently facing the global economy.

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International Risk Sharing and Wealth Allocation with Higher Order Cumulants

Online Appendix

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A Summary of the three-country model

In this section we present the model in a way that lends itself for recursive solution, as implemented in the Fortran codes (see `core_model.f90`). In particular, the solution strategy consists of guessing B's and C's prices in country A's consumption units, recursively solving for the other variables and thus verifying the guess. Hence we guess $p_{B,t}$ and $p_{C,t}$. Using the aggregate price equations obtain

$$p_{A,t} = \left(\frac{1 - (1 - \nu_A)\big(\varsigma_A p_{B,t}^{1-\theta} + (1 - \varsigma_A) p_{C,t}^{1-\theta}\big)}{\nu_A} \right)^{\frac{1}{1-\theta}} \tag{A.1}$$

$$Q_{A,B,t} = \left(\nu_B p_{B,t}^{1-\theta} + (1 - \nu_B) \left(\varsigma_B p_{A,t}^{1-\theta} + (1 - \varsigma_B) p_{C,t}^{1-\theta}\right)\right)^{\frac{1}{1-\theta}} \tag{A.2}$$

$$Q_{A,C,t} = \left(\nu_C p_{C,t}^{1-\theta} + (1 - \nu_C) \left(\varsigma_C p_{A,t}^{1-\theta} + (1 - \varsigma_C) p_{B,t}^{1-\theta}\right)\right)^{\frac{1}{1-\theta}} \tag{A.3}$$

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Using the risk-sharing constants

$$C_{B,t} = Q_{A,B,t}^{-\frac{1}{\rho}} \kappa_{A,B} C_{A,t} \qquad\qquad C_{C,t} = Q_{A,C,t}^{-\frac{1}{\rho}} \kappa_{A,C} C_{A,t}. \tag{A.4}$$

the goods market equilibrium for the three countries are given by

$$n_A p_{A,t} Y_{A,t} = (p_{A,t})^{1-\theta} \left(n_A \nu_A + n_B \varsigma_B (1-\nu_B) Q_{A,B,t}^{\theta-\frac{1}{\rho}} \kappa_{A,B} + n_C \varsigma_C (1-\nu_C) Q_{A,C,t}^{\theta-\frac{1}{\rho}} \kappa_{A,C} \right) C_{A,t} \tag{A.5}$$

$$n_B p_{B,t} Y_{B,t} = (p_{B,t})^{1-\theta} \left(n_B \nu_B Q_{A,B,t}^{\theta-\frac{1}{\rho}} \kappa_{A,B} + n_A \varsigma_A (1-\nu_A) + n_C (1-\varsigma_C)(1-\nu_C) Q_{A,C,t}^{\theta-\frac{1}{\rho}} \kappa_{A,C} \right) C_{A,t} \tag{A.6}$$

$$n_C p_{C,t} Y_{C,t} = (p_{C,t})^{1-\theta} \left(n_C \nu_C Q_{A,C,t}^{\theta-\frac{1}{\rho}} \kappa_{A,C} + n_A (1-\varsigma_A)(1-\nu_A) + n_B (1-\varsigma_B)(1-\nu_B) Q_{A,B,t}^{\theta-\frac{1}{\rho}} \kappa_{A,B} \right) C_{A,t}. \tag{A.7}$$

Summing up these three equations gives

$$C_{A,t} = \mu_{A,t} Y_{W,t} \tag{A.8}$$

where

$$\mu_{A,t} = \left[n_A + n_B Q_{A,B,t}^{1-\frac{1}{\rho}} \kappa_{A,B} + n_C Q_{A,C,t}^{1-\frac{1}{\rho}} \kappa_{A,C} \right]^{-1} \tag{A.9}$$

Note that $\mu_{A,t} > 0$ can be larger or smaller than 1. Likewise,by applying equations (A.4) we have

$$C_{B,t} = \mu_{B,t} Y_{W,t}, \text{ and } C_{C,t} = \mu_{C,t} Y_{W,t} \tag{A.10}$$

where per equations (A.4)

$$\mu_{B,t} = \mu_{A,t} \kappa_{A,B} Q_{A,B,t}^{-\frac{1}{\rho}}, \text{ and } \mu_{C,t} = \mu_{A,t} \kappa_{A,C} Q_{A,C,t}^{-\frac{1}{\rho}}. \tag{A.11}$$

Note also that $n_A \mu_A + n_B \mu_B Q_{A,B,t} + n_C \mu_C Q_{A,C,t} = 1$.

Then use the labor supply equations to generate global output, i.e. from

$$L_{A,t} = \left(C_{A,t}^{-\rho} \frac{1-\alpha}{\chi} p_{A,t} D_{A,t} \right)^{\frac{1}{\varphi+\alpha}} \tag{A.12}$$

$$L_{B,t} = \left({C^*_{B,t}}^{-\rho} \frac{1-\alpha}{\chi} p_{B,t} Q^{-1}_{A,B,t} D_{B,t} \right)^{\frac{1}{\varphi+\alpha}} \tag{A.13}$$

$$L_{C,t} = \left({C^*_{C,t}}^{-\rho} \frac{1-\alpha}{\chi} p_{C,t} Q^{-1}_{A,C,t} D_{C,t} \right)^{\frac{1}{\varphi+\alpha}} \tag{A.14}$$

Or, using the results so far

$$n_A p_{A,t} D_{A,t} L^{1-\alpha}_{A,t} = n_A p_{A,t} D_{A,t} \left(\mu_A^{-\rho} \frac{1-\alpha}{\chi} p_{A,t} D_{A,t} \right)^{\frac{(1-\alpha)}{\varphi+\alpha}} Y_{W,t}^{-\frac{\rho(1-\alpha)}{\varphi+\alpha}} \tag{A.15}$$

$$n_B p_{B,t} D_{B,t} L^{1-\alpha}_{B,t} = n_B p_{B,t} D_{B,t} \left(\mu_B^{-\rho} \frac{1-\alpha}{\chi} p_{B,t} Q^{-1}_{A,B,t} D_{B,t} \right)^{\frac{(1-\alpha)}{\varphi+\alpha}} Y_{W,t}^{-\frac{\rho(1-\alpha)}{\varphi+\alpha}} \tag{A.16}$$

$$n_C p_{C,t} D_{C,t} L^{1-\alpha}_{C,t} = n_C p_{C,t} D_{C,t} \left(\mu_C^{-\rho} \frac{1-\alpha}{\chi} p_{C,t} Q^{-1}_{A,C,t} D_{C,t} \right)^{\frac{(1-\alpha)}{\varphi+\alpha}} Y_{W,t}^{-\frac{\rho(1-\alpha)}{\varphi+\alpha}} \tag{A.17}$$

Adding them up yields

$$Y_{W,t} = \begin{bmatrix} n_A p_{A,t} D_{A,t} \left(\mu_A^{-\rho} \frac{1-\alpha}{\chi} p_{A,t} D_{A,t} \right)^{\frac{(1-\alpha)}{\varphi+\alpha}} + \\ n_B p_{B,t} D_{B,t} \left(\mu_B^{-\rho} \frac{1-\alpha}{\chi} p_{B,t} Q^{-1}_{A,B,t} D_{B,t} \right)^{\frac{(1-\alpha)}{\varphi+\alpha}} + \\ n_C p_{C,t} D_{C,t} \left(\mu_C^{-\rho} \frac{1-\alpha}{\chi} p_{C,t} Q^{-1}_{A,C,t} D_{C,t} \right)^{\frac{(1-\alpha)}{\varphi+\alpha}} \end{bmatrix}^{\frac{\varphi+\alpha}{\rho(1-\alpha)+\varphi+\alpha}} \tag{A.18}$$

Finally use

$$n_B Y_{B,t} = (p_{B,t})^{-\theta} \left(n_B \nu_B Q^{\theta-\frac{1}{\rho}}_{A,B,t} \kappa_{A,B} + n_A \varsigma_A (1-\nu_A) + n_C (1-\varsigma_C)(1-\nu_C) Q^{\theta-\frac{1}{\rho}}_{A,C,t} \kappa_{A,C} \right) C_{A,t} \tag{A.19}$$

$$n_C Y_{C,t} = (p_{C,t})^{-\theta} \left(n_C \nu_C Q^{\theta-\frac{1}{\rho}}_{A,C,t} \kappa_{A,C} + n_A (1-\varsigma_A)(1-\nu_A) + n_B (1-\varsigma_B)(1-\nu_B) Q^{\theta-\frac{1}{\rho}}_{A,B,t} \kappa_{A,B} \right) C_{A,t} \tag{A.20}$$

to verify the guess for $p_{B,t}$ and $p_{C,t}$.

B A useful efficient procedure

We detail a practical step which is particularly useful for numerical solutions of DSGE models of any size, e.g., using Dynare (Juillard, 1996).[1] We then offer higher-order accurate solution of the Negishi weights. For this illustration we focus on the two-country version of the model, *wlog*.

We illustrate the procedure focusing on Negishi weights, which also happen to feature differently than the other variables in the model.[2] We begin by observing that the risk-sharing condition can be solved backward to yield:

$$\ln \zeta_{A,t} - \ln \zeta_{B,t} + \ln Q_{A,B,t} = \ln \zeta_{A,0} - \ln \zeta_{B,0} + \ln Q_{A,B,0} := \rho \ln \kappa_{A,B}. \tag{B.1}$$

where the subscript 0 indicates the time zero in which the risk-sharing agreement is entered for the first time.

The initial distribution of wealth under a full set of period-by-period state contingent (Arrow) securities is pinned down by a condition on the initial distribution of Arrow securities across borders. Take the $m - order$ series-expansion of $\kappa_{A,B}$ around $\omega = 0$

$$\ln \kappa_{A,B}(\omega) \approx \kappa_{A,B}^{(0)} + \kappa_{A,B}^{(1)}\omega + \ldots + \kappa_{A,B}^{(m)} \frac{1}{m!}\omega^m \tag{B.2}$$

where $\kappa_{A,B}^{(m)} := \left.\frac{\partial^m \ln \kappa_{A,B}}{\partial \omega^m}\right|_{\omega=0}$. At each order m, solve for $\kappa_{A,B}^{(m)}$ and proceed recursively, starting from $\kappa_{A,B}^{(0)} = \bar{\kappa}_{A,B}$ and $\kappa_{A,B}^{(1)} = 0$ (as certainty equivalence holds at first order). By way of example, a second order expansion implies that

$$\widetilde{\kappa}_{A,B} := \ln \kappa_{A,B}(\omega) - \ln \kappa_{A,B}^{(0)} \approx \frac{1}{2}\kappa_{A,B}^{(2)} \tag{B.3}$$

[1] We solve our model using a specific code in Wolfram's Mathematica for which we don't need to use this practical suggestion.

[2] For non-separable preferences or recursive preferences, e.g. à la Bansal and Yaron (2004), a similar decomposition can be obtained. Details are available from the authors on request.

where *wlog* we set $\omega = 0$.[3]

Our solution algorithm (whether applied analytically or numerically) proceeds as follows

1. Expand to the order of interest the system of equations constituting the model;
2. Find the RE solution for all variables as a function of $\widetilde{\kappa}_{A,B}$;
3. Use the appropriate series expansion of budget constraint under the condition $S_{A,0} = 0$ to solve for $\widetilde{\kappa}_{A,B}$.

For higher orders of approximation this algorithm can be used recursively starting from lower orders to build the solution for higher orders, i.e. to construct a solution for each of the variables of the model with the same structure as in equation (B.2).

It is worth stressing that this solution technique does not amount to finding the "risky steady state" à la Coeurdacier et al. (2011). Despite some similarities in the two methods, our technique is specific to the derivation of $\kappa_{A,B}$. It relies on the existence of an explicit condition that can be imposed to solve for $\kappa_{A,B}$—a given initial distribution of assets. The intuition is simple. Under complete markets we can solve the model economy (i.e. find the state-space representation of the endogenous dynamic variables) conditionally on the initial, time-invariant distribution of assets. Consider the representation of a DSGE model under a second order perturbation[4]

$$AE_t\widetilde{X}_{t+1}^{(2)} + B\widetilde{X}_t^{(2)} + CE_t\left(\widetilde{X}_{t+1}^{(1)} \otimes \widetilde{X}_{t+1}^{(1)}\right) + D\left(\widetilde{X}_t^{(1)} \otimes \widetilde{X}_t^{(1)}\right) + F\left(\widetilde{X}_t^{(1)} \otimes \varepsilon_t\right) + G\widetilde{\kappa}_{A,B} = 0 \tag{B.4}$$

where X_t is a vector of variables, $\tilde{X}^{(i)} := X^{(i)} - X^{(0)}$, ε_t is an i.i.d. vector of innovations, A, B, C, D, F, G and H (below) are conformable matrices of coefficients. Importantly, $\kappa_{A,B}$ does not appear in the coefficient matrices A, B, C, D, F, G which are only reflecting

[3] The accuracy of the approximation clearly depends on the size of ω. Nevertheless, we can normalize this to 0 and scale appropriately the standard deviation of the underlying shocks, *wlog*.

[4] To simplify the illustration we assume that the model has at most one period ahead expectations and that that the system is represented in "companion form" with all lags subsumed in the vector X_t.

deterministic steady state information. As argued above, the non-linear terms are of lower order (here first order). In this example these terms are solved separately from the system

$$E_t A \widetilde{X}_{t+1}^{(1)} + B \widetilde{X}_t^{(1)} + H \varepsilon_t = 0. \tag{B.5}$$

Notably, $\widetilde{\kappa}_{A,B}^{(1)}$ is missing from the first order, as it is zero under certainty equivalence.

Since $\widetilde{\kappa}_{A,B}^{(2)}$ is time invariant, we can re-write equation (B.4) as

$$A E_t \check{X}_{t+1}^{(2)} + B \check{X}_t^{(2)} + C E_t \left(\widetilde{X}_{t+1}^{(1)} \otimes \widetilde{X}_{t+1}^{(1)} \right) + D \left(\widetilde{X}_t^{(1)} \otimes \widetilde{X}_t^{(1)} \right) + F \left(\widetilde{X}_t^{(1)} \otimes \varepsilon_t \right) = 0 \tag{B.6}$$

where $\check{X}_t^{(2)} = \widetilde{X}_t^{(2)} + (A+B)^{-1} G \widetilde{\kappa}_{A,B}^{(2)}$. Note that under complete markets the system (B.6) does not need to include the households budget constraint, which will instead be used in a second step to solve for $\widetilde{\kappa}_{A,B}^{(2)}$.

Summing up, in the class of models where $\kappa_{A,B}$ enters log linearly,[5] the solution procedure consists naturally of two steps: i) solve for allocations and prices using system (B.6); ii) use the (approximated) budget constraint at time 0 to solve for $\widetilde{\kappa}_{A,B}^{(2)}$. These two steps will then allow to recover $\widetilde{X}_t^{(2)}$. The same procedure holds for any order of approximation.

To avoid misinterpretations of the algorithm, it is important to stress that there is only one (standard) perturbation taking place, i.e. along the "risk" loading parameter ω. The solution of the resulting system of series expansions is recursive with respect to $\kappa_{A,B}$ at each order of approximation – at least in this class of models. The solution could as well be obtained all at once (less efficiently). The key is that we go from the original non-linear model to a system of series expansions of all the "risk-sensitive" variables, including $\kappa_{A,B}$.[6]

[5] This is the case of not only CRRA preferences, but also Epstein-Zin preferences.

[6] A similar recursivity between dynamic allocations and time-invariant financial allocations emerges in long-run portfolio decisions, as discussed by Devereux and Sutherland (2011). There, the "zero order" asymptotic portfolio composition can be solved recursively from the dynamic allocation of the model's variables. This is possible since the "zero order" portfolio shares are time-invariant (like our $\kappa_{A,B}$) and the dynamics of the model can be defined conditionally on these shares. Like for the portfolio case, no ad-hoc assumptions or heuristic techniques are needed. The solution emerges from the mechanical application

The second-order solution of a DSGE model can be written in a second-order VAR form as (e.g. following Dynare notation)

$$\check{X}_t = \check{A}\check{X}_{t-1} + \check{B}\varepsilon_t + \frac{1}{2}\left[C\left(\check{X}_{t-1}\otimes\check{X}_{t-1}\right) + \check{D}\left(\varepsilon_t\otimes\varepsilon_t\right) + 2\check{F}\left(\check{X}_{t-1}\otimes\varepsilon_t\right)\right] + \frac{1}{2}\check{G}\vec{\Sigma^2} \tag{B.7}$$

$\check{A}$, $\check{B}$, $\check{C}$, $\check{D}$, $\check{F}$, $\check{G}$ are conformable matrices, and for any column vectors x and z, $(x\otimes z)$ is the vectorized outer product of these vectors. $\Sigma^2 := E_t\left(\varepsilon_{t+1}\varepsilon_{t+1}'\right)$, and $\vec{\cdot}$ is the vectorization operator.[7]

The key term in equation (B.7) is the last one, which shifts the mean of variables in proportion to the exogenous risk, captured by the variance matrix Σ^2 (also referred to as the stochastic steady state in the literature). Using regular perturbations (see e.g. Lombardo and Uhlig, 2018), none of the matrices in (B.7) depends on exogenous risk. This means that the only place where σ_κ appears is in Σ^2.

The vector $\check{X}_{A,t}$ contains the variable measuring Arrow-Debreu securities. Assume the latter are in position i_{AD}, and that σ_κ occupies position j_{σ_κ} in the vector $\vec{\Sigma^2}$. Then we have that

$$\begin{aligned}\check{X}_{A,t}[i_{AD}] = \check{A}[i_{AD},:]\check{X}_{t-1} + \check{B}[i_{AD},:]\varepsilon_t + \frac{1}{2}\left[\check{C}[i_{AD},:]\left(\check{X}_{t-1}\otimes\check{X}_{t-1}\right)\right.\\ \left. + \check{D}[i_{AD},:]\left(\varepsilon_t\otimes\varepsilon_t\right) + 2\check{F}[i_{AD},:]\left(\check{X}_{t-1}\otimes\varepsilon_t\right)\right] + \frac{1}{2}\check{G}[i_{AD},:]\vec{\Sigma^2}\end{aligned} \tag{B.8}$$

where for a matrix $\check{X}$, $\check{X}[i,j]$ denotes the element in row i and column j, and where $X[i,:]$ denotes the row i of matrix X; for a vector z, $z[j_{\sigma_\kappa}]$ is the $j_{\sigma_\kappa} - th$ element in z. In particular, $\vec{\Sigma^2}[j_{\sigma_\kappa}] = \sigma_\kappa^2$.

Note that if we set $\bar{\kappa}_{A,B} = 1$, we can solve for σ_κ^2 that satisfies some restriction on $\check{X}_{A,t}[i_{AD}]$. In particular we know that under complete markets it must be that $\check{X}_0[i_{AD}] = 0$

of series expansion techniques and solution techniques for dynamic rational expectation models.

[7]To date, Dynare returns only the product $\check{G}\vec{\Sigma^2}$ in the variable "oo_.dr.ghs2". In order to implement our algorithm this product must be factorized in the two components. This can be easily done by modifying Dynare function `dyn_second_order_solver.m` at about line 173, by adding a new variable e.g. `dr.G=LHS\(-RHS);`, where LHS and RHS are variables defined in the function.

(Ljungqvist and Sargent, 2012). One way to implement this condition is to assume that at time 0 and -1 the economy was at the stochastic steady state, i.e. all elements of equation (B.8) are zero except the last one, i.e.[8]

$$\check{X}_0[i_{AD}] = 0 = \frac{1}{2}\check{G}[i_{AD}, :]\vec{\Sigma^2} \tag{B.9}$$

Then we can solve for σ_κ^2 as

$$\sigma_\kappa^2 = -\frac{\check{G}[i_{AD}, j_{\sigma_\kappa}^\perp]\vec{\Sigma^2}[j_{\sigma_\kappa}^\perp]}{\check{G}[i_{AD}, j_{\sigma_\kappa}]} \tag{B.10}$$

where $j_{\sigma_\kappa}^\perp$ denotes all the elements excluding j_{σ_κ}. Now we simply need to swap values, i.e. $\bar{\kappa}_{A,B} \leftarrow \sigma_\kappa^2$ $\sigma_\kappa^2 \leftarrow 1$. With this assignment of values, $\kappa_{A,B}$ is the second-order accurate risk-sharing constant that implements complete markets.

Our proposed algorithm, correctly implements complete markets up to second order accuracy. It should be noted also that our approach does not affect the first-order solution. This solution correctly describes growth rates of variables, since the risk-sharing constant is invariant to time (Ljungqvist and Sargent, 2012).

This approach is reminiscent of the solution algorithm proposed by Devereux and Sutherland (2011) (DS) to solve for portfolio shares up to second order. DS introduce an auxiliary iid shock in the budget constraint of investors as a placeholder for portfolio shares. By knowing the position of this auxiliary shock DS can then use simple linear algebra to derive the shares. Although we solve a different problem, our algorithm shares with DS the idea of using auxiliary iid shocks as placeholders for parameters that would otherwise drop out of the perturbed solution.

[8]Equally easily implementable is any other condition, e.g. $E\check{X}_0[i_{AD}] = 0$.

C Global solution method

We solve the model globally using standard time-iteration methods. With CRRA preferences, finding allocations and prices for given κ does not require solving for expectations. In practice we first solve the model (once) for all possible values of the states. Then we solve for the value function and Arrow securities by iteration. Alternatively these two variables could be found by simulation. With Epstein-Zin preferences (which we consider in codes available on request, but that we omit to discuss for conciseness) the model and the value function must be solved simultaneously. In particular, (a) we iterate on the Bellman equations to determine the policy function relating welfare to the state variables of our model; and (b) we iterate on the budget constraint to determine Arrow securities as a function of the states. In the simple CRRA case, we only have exogenous states (TFP); we considered both possibilities: including κ as a pseudo state variable and solving for κ directly: for each candidate κ we solve the model.[9] We can thus describe the policy functions for welfare and Arrow securities as $\mathcal{W}_{i,t} = \mathcal{W}_i\left(\kappa_{A,B}, \kappa_{A,C}, D_{A,t}, D_{B,t}, D_{C,t}\right)$ and $\mathcal{S}_{i,t} = \mathcal{S}_i\left(\kappa_{A,B}, \kappa_{A,C}, D_{A,t}, D_{B,t}, D_{C,t}\right)$, $i = \{A, B, C\}$. Once we find a solution (i.e. a fixed point such that the residual of the underlying three equations is 10^{-14}), we can determine the optimal κ_s, i.e. $\kappa^{\dagger}_{A,B}$ and $\kappa^{\dagger}_{A,C}$ such that

$$\mathcal{S}_i\left(\kappa^{\dagger}_{A,B}, \kappa^{\dagger}_{A,C}, 1, 1, 1\right) = 0. \tag{C.1}$$

To determine $\kappa^{\dagger}_s$ and evaluate welfare at that value, we approximate the policy functions using cubic splines.

The solution for κ consists of finding Negishi weights that satisfy the symmetric asset allocation at time zero (see main text), the relative share of consumption consistent with a symmetric distribution of Arrow securities across countries at time zero, assuming that the economy is not hit by any shock at that time. The same concept we use under perturbation.[10]

[9]Technically speaking κ is a state variable, since it depends on time-zero moments and allocations. Nevertheless κ is time invariant and could be treated as a deep parameter, that ensures Arrow securities to satisfy the "time zero" asset allocation condition. We thank Felix Kubler for suggesting this approach.

[10]The global solution of our three country model is implemented in Fortran—a useful reference for

D Analytical results

In the text, we have referred to $a_\bullet$ and $b_\bullet$ as complex convolutions of deep parameters (ρ, θ, δ and n_A). Below we show the analytical expression for $a_\bullet$ and $b_\bullet$ for the special case of $n_A = \frac{1}{2}$:

$$a_{\kappa,2} = \frac{1}{4(1-\delta)} \tag{D.1}$$

$$a_{\kappa,2^2} = \frac{\rho}{32(1-\delta)} \tag{D.2}$$

$$a_{\kappa,3} = \frac{1}{12(1-\delta)} \tag{D.3}$$

$$a_{\kappa,4} = \frac{1}{48(1-\delta)} \tag{D.4}$$

$$a_{\kappa,\Gamma} = \frac{\delta(\rho-1)(\theta(\rho-2)-1)}{32(1-\delta)\theta} \tag{D.5}$$

$$a_{\gamma,A} = \frac{\delta(2\theta-2\theta^2+\rho-4\theta\rho+3\theta^2\rho)}{8(1-\delta)\theta^2} \tag{D.6}$$

$$a_{\gamma,B} = \frac{\beta(\rho-\theta^2-2\theta)}{8(1-\delta)\theta^2} \tag{D.7}$$

$$a_{\phi,A} = \frac{\delta(\theta-1)(\rho-1)(-2-\rho+\theta(1+5\rho)-\theta^2(7\rho-4))}{48(1-\delta)\theta^3} \tag{D.8}$$

$$a_{\phi,B} = \frac{\delta(\theta-1)(\rho-1)(-2-\rho-\theta(-1+\rho)-\rho-\theta^2(\rho-4))}{48(1-\delta)\theta^3} \tag{D.9}$$

$$a_{\eta,A} = \frac{\delta(\theta-1)}{384(1-\delta)\theta^4}(\rho(-6+3\rho+\rho^2)-\theta^2(-4+27\rho^2-17\rho^3)- \tag{D.10}$$

$$\theta(8-24\rho+3\rho^2+7\rho^3)-\theta^3(-8+30\rho-39\rho^2+15\rho^3)) \tag{D.11}$$

$$a_{\eta,B} = \frac{\delta(\theta-1)}{384(1-\delta)\theta^4}(\rho(-6+3\rho+\rho^2)-\theta^2(-4+3\rho^2-\rho^3)- \tag{D.12}$$

$$\theta(8-3\rho+\rho^2)-\theta^3(-8+18\rho-9\rho^2+\rho^3)) \tag{D.13}$$

the methods we adopt is Fehr and Kindermann (2018). The codes are available from the authors upon request. Solving the three-country model takes several hours on a standard PC. In recent literature, important advancements in the solution of non-linear DSGE models make use of Machine Learning, e.g. Scheidegger and Bilionis (2019), Maliar et al. (2021) and Azinovic et al. (2022). We make a Python implementation of these techniques to solve a two-country version of our model available online. We leave extensions to more than two countries to future research.

$$b_\gamma = \frac{\delta(\theta-1)(\rho-1)}{\theta} \tag{D.14}$$

$$b_\phi = \frac{\delta(\theta-1)(2-\theta-3\theta\rho-\theta^2(4-9\rho+3\rho^2)}{4\theta^3} \tag{D.15}$$

$$b_{\gamma_A^2} = \frac{\delta(\theta-1)(\rho-1)(-3\beta\theta(1+\theta(\rho-2))(\rho-1)}{2\theta^3} \tag{D.16}$$

$$b_{\gamma_B^2} = -\frac{\delta(\theta-1)(\rho-1)(-3\beta\theta(1+\theta(\rho-2))(\rho-1)}{2\theta^3} \tag{D.17}$$

$$b_{\gamma_B\gamma_A} = 0 \tag{D.18}$$

$$b_\eta = \frac{\delta(\theta-1)(\rho-1)(-2+\theta+3\theta\rho+\theta^2(2-5\rho+\rho^2)}{2\theta^3} \tag{D.19}$$

Table 1: Values of parameters a and b under our calibration and $n_A = 0.5$

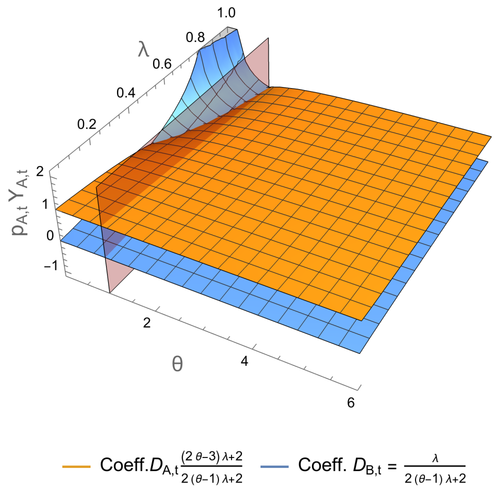
$a_{\kappa,2}$	$a_{\kappa,2^2}$	$a_{\kappa,3}$	$a_{\kappa,4}$	$a_{\kappa,\Gamma}$	$a_{\gamma,A}$	$a_{\gamma,B}$	$a_{\phi,A}$
12.5	6.25	4.17	1.04	12.25	14.97	9.52	12.93

$a_{\phi,B}$	$a_{\eta,A}$	$a_{\eta,B}$	b_γ	b_ϕ	$b_{\gamma_A^2}$	$b_{\gamma_B^2}$	$b_{\gamma_A\gamma_B}$	$b_{\eta,B}$
4.73	6.94	1.22	0.98	1.94	11.52	11.52	0	2.83

E Home bias and trade elasticity

In the main text, we have argued that trade elasticity regulates the relative "exposure" of a country to own and foreign production's shocks. We also argue that relative exposure is affected by the degree of home bias in consumption. We now offer a close-up analysis of the interaction between these two parameters. Figure 1 plots the response coefficient of a country income to domestic and foreign output shocks. The plot shows that home bias can shift the threshold value of θ at which the relative exposure to own vs. foreign shocks falls from above to below 1. With sufficiently strong home bias (e.g., $\lambda \leq 0.3$), nonetheless, a country's income remain relatively more exposed to domestic shocks than to foreign shocks.

Figure 1: Response coefficients of country A's income to domestic and foreign shocks



Note: Coefficients of country A's and country B's TFP in the first-order approximation of country A's income. The vertical plane crosses the θ axis at 1.

F Correlation of TFP across countries and gains from risk sharing

In the main text, for the sake of simplicity, we have assumed that the stochastic processes for TFP are independent across countries. It is well understood that these processes may nonetheless reflect some degree of correlation of the underlying sources of risk. The traditional IRBC literature, e.g. Backus et al. (1992) indeed assumes that the TFP processes are correlated across countries. In this appendix we briefly discuss the implication for risk sharing. We do so analytically focusing on the instructive limit case in which shocks are perfectly correlated, yet they hit countries with different intensities. Without loss of generality, we develop our argument using the the two-country version of the model.

In particular, we assume that

$$D_{B,t} = \zeta D_{A,t};\ \zeta \geq 0. \tag{F.1}$$

Equation (F.1) implies that $corr\left(D_{A,t}, D_{B,t}\right) = 1$. Yet, the intensity of the shocks differs across countries. In a stylized yet compelling way, this case captures one of the core premises of our analysis—strong evidence that shocks, even when global in nature, affect countries in largely asymmetric ways.

To show that perfect correlation of shocks does not rule out gains from risk sharing, we solve the model imposing equation (F.1). Focusing on skewness, up to fourth order of accuracy, we will have:

$$\frac{\partial RGRS}{\partial \phi_A} = \frac{\beta(\zeta-1)^3(\theta-1)^2 n^2 \rho(\theta\rho+\theta+\rho-3)}{2(\beta-1)\theta^3} + \frac{\beta(\zeta-1)^2(\theta-1)^2\rho(\zeta(2\theta\rho-\theta+\rho-2)+(\theta-1)(\rho-2)+3n(-\zeta(2\theta\rho+\rho-3)+2\theta+\rho-3))}{6(\beta-1)\theta^3} \tag{F.2}$$

Similar expressions, albeit more involved, hold for variance and kurtosis. The main takeaway is straightforward: as long as global shocks hit countries with different intensities (or transmit across borders asymmetrically), the gains from risk sharing are not zero, and will generally differ across borders.

We conclude with a comment on the maintained view that the benefits from international insurance are small if shocks are positively correlated across borders or have global nature. From our analysis above, it follows that a higher degree of correlation of GDP across countries may reduce or increase the gains from risk sharing, depending on the underlying heterogeneity of shock intensity across borders. In this sense, the high degree of dispersion in the distribution of moments that we document in the main text is likely to generate gains from insurance dominate the mitigating effect of positive correlations.

G Cumulants' effects

In the text, we have shown that effects of cumulants on both welfare gains and relative asset prices depend on the precise numerical constellation of deep parameters—trade elasticity, risk aversion and size—all having a bearing on relative income risk. Here-after we show that the predictions of the perturbation solution are remarkably close to true properties of the model, including the possibility of a *sign* switch of the effects of cumulants.

Among the model properties, particular relevant are non-linearities that may arise from skewness. For transparency, we illustrate the implications of these non-linearities relying on parameter values that considerably simplify the analysis. In particular we posit $\theta \to \infty$, $n_A = n_B = 0.5$, $\alpha = 1$ and $\beta = 0.98$—so that we are left with one single deep parameter to care about. Moreover, we assume that $D_{B,t} = \bar{D}_B = 1$ and $\varphi_D = 0$—so that the only source of risk comes from country A's *iid* TFP shock—and that $D_{A,0} = \bar{D}_A = 1$. Under these numerical assumptions, the asset price equations reduce to

$$P_{A,K,0} = \frac{\beta}{1-\beta} 0.5^{-\rho} E_0 \left[\left(D_{A,t} + \bar{D}_B \right)^{-\rho} D_{A,t} \right] \tag{G.1a}$$

$$P_{B,K,0} = \frac{\beta}{1-\beta} 0.5^{-\rho} E_0 \left[\left(D_{A,t} + \bar{D}_B \right)^{-\rho} \bar{D}_B \right] \tag{G.1b}$$

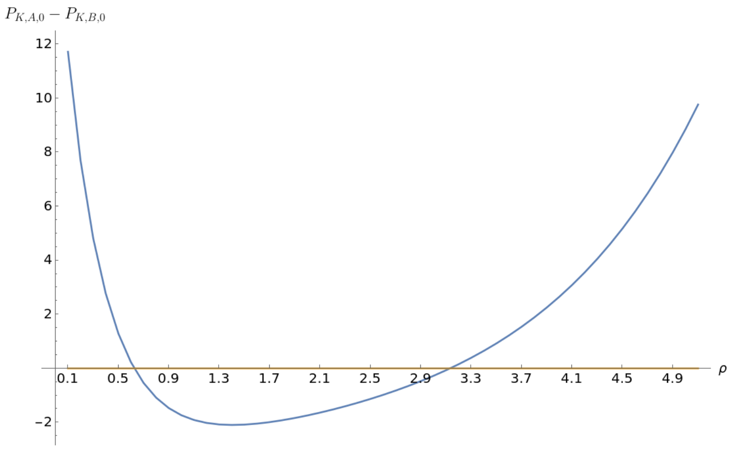
where, by our Proposition 1, we know that (holding PPP) the difference in asset prices maps into differences in welfare.

Next, we posit a skewed normal distribution for $D_{A,t}$,[11] and compute $P_{K,A,0} - P_{K,B,0}$ for different degrees of risk aversion (ρ). Figure 2 shows the slope coefficient of regressing 601 realizations of time-0 asset price differences on the corresponding third moment of $D_{A,t}$ (vertical axis) against different values of ρ (horizontal axis). Remarkably, the price difference (hence the difference in welfare under our assumptions) switches sign twice. In line with the result in this figure, by taking a series expansion of equations (G.1a) and (G.1b) we can pin down an approximation of the values of ρ at which the sign switch occurs: the coefficient on the third moment switches sign twice at $\rho = 0.54$ and $\rho = 2.46$.[12]

[11]The PDF of a variable x with skewed-normal probability distribution is $f(x) = \frac{2}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)\Phi\left(\xi\left(\frac{x-\mu}{\sigma}\right)\right)$, where μ is the location parameter, σ is the scale parameter, ξ is the skewness-controlling parameter, $\phi(\cdot)$ is the PDF of the Gaussian distribution and $\Phi(\cdot)$ is its CDF. We can thus adjust μ and σ to keep mean and variance constant while changing ξ to obtain different degrees of skewness. In each simulation we draw $10K$ values for $D_{A,t}$, 801 values for the skewness parameter (from -20 to 20, *step* $= 0.05$) and 51 values for ρ (from 0.1 to 5.1, *step* $= 0.1$). The implied range of the third un-centered moment $\left(E\left(D^3_{A,t}\right)\right)$ is ± 1.8.

[12]While not necessarily quantitatively precise (given the known limits of perturbation methods), the

Figure 2: Relative asset-price effect of $D_{A,t}$ skewness by varying ρ: sign-switches



So, the model predicts that, under perfect risk sharing, the valuation of the assets in the country with the larger negative skewness is lower for ρ's in the range entertained in many quantitative studies, but can, counter-intuitively, be higher for either very low or sufficiently high values of ρ—often advocated to match equity premia.[13] The interest in this result lies in unveiling how, in the model, the combination of LE and SE shaping the gains from trade in assets may vary with the curvature of the utility function—in our example we focus on one homogeneous goods, hence ruling out income effects from relative output price adjustment. The change is entirely driven by the equilibrium process of the stochastic discount factor.

approximation nonetheless provides an accurate *qualitative* prediction that we can verify using our global solution.

[13]For example, in the open-macro literature, the relative risk-aversion parameter (with CRRA preferences) is often 1 (the log case), or slightly above (e.g. Devereux and Engel, 2007 Obstfeld and Rogoff, 2000). In the equity-premium literature, with CRRA preference, values often need to be larger. See the review by Mehra and Prescott (2003) and Cochrane (2008). Higher values are typically inconsistent with estimates of the intertemporal elasticity of substitution (ρ^{-1} under CRRA), calling for preferences that allow for a separation of the two parameters à la Epstein and Zin (1989). We make available codes for the global solution under these preferences. The main effect is to generate larger gains, as well known in the literature, and thus omit them for reason of space.

H Generating skewed-leptokurtic distributions

We generate skewed and leptokurtic distributions by adopting the mixed-Gaussian distribution approach discussed Farmer and Toda (2017, FT henceforth).[14] In particular we proceed in two steps. First, we calibrate the parameters of a three-elements Gaussian mixture (i.e. weights, means and standard deviations of each element) by minimizing the relative distance between the observed moments and those generated by this distribution. Second, we implement the algorithm suggested by FT. This consists of matching low-order moments of the conditional distribution using relative entropy as the objective function (i.e. the Kullback-Leibler information).[15]

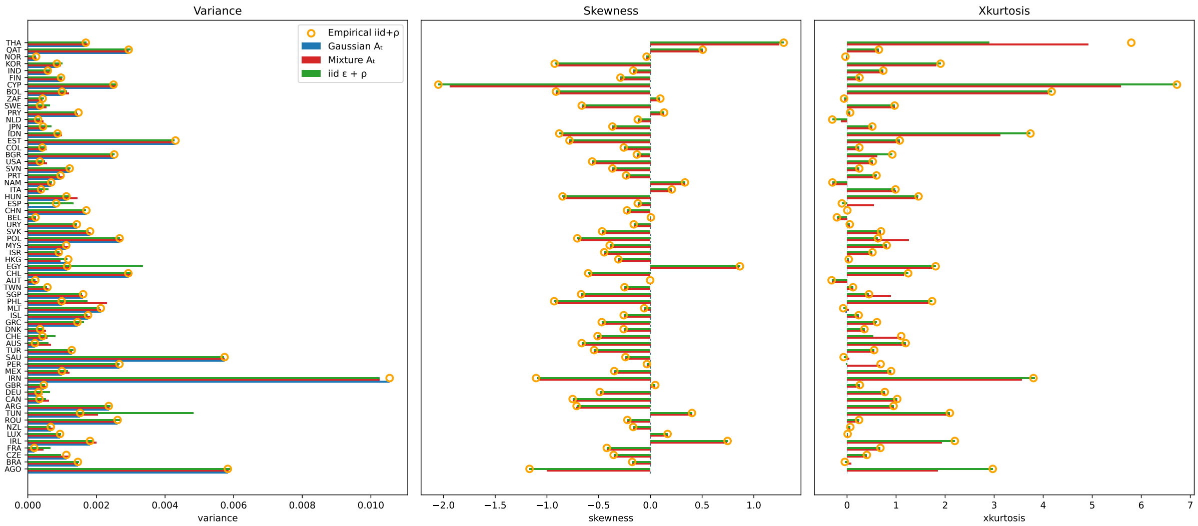
From the PWT version 10.01, we take the distribution of detrended ln real GDP (national accounts measure) as a proxy of country risk (in our model as proxy for the TFP distribution). We remove countries that display very irregular GDP series (these typically consist of countries torn by long conflicts, regime changes, extreme poverty or simply very short series). We further drop countries at the lower and upper 5% of the distribution of kurtosis (of the cyclical component; see below). We take kurtosis as trimming criterion as it is the one with most extreme variation among moments. Our final sample consists of 151 countries with annual series ending in 2019 and starting at variable dates.

On the basis of this data we proceed as follows. First, we ln-detrend real GDP using the "boosted" HP (Hodrick and Prescott, 1997) filter developed by Phillips and Shi (2020). Second, we fit an AR(1) process on detrended component of ln-GDP. Third, we compute the first four moments of the exogenous component of the AR(1) process (the residual). Fourth, we use these moments to calibrate a mixed Gaussian distribution for each country. Fifth, and finally, we discretize the mixed Gaussian using the FT method. Figure 3 shows the alignment between the empirical moments (obtained by simulating the

[14]See also Civale et al. (2017). The empirically relevant case is of leptokurtic distributions (fatter tails than the Gaussian). We thus refer for simplicity to leptokurtosis as shorthand for non-Gaussian kurtosis.

[15]We refer to Farmer and Toda (2017) for details. We implemented their method converting and adapting their Matlab codes into Python.

Figure 3: Moment matching: Gaussian, mixed Gaussian and iid



Note: Empirical moments obtained from the simulation of an estimated AR(1) process for the cyclical component of GDP (trend estimated using the Phillips and Shi (2020) method). "Discretized" moments obtained from the simulation of a discretized Gaussian mixture model (à la Farmer and Toda, 2017) calibrated on empirical moments.

estimated AR process) and those obtained from the ergodic discretized distribution.[16]. In the figure we show four values per panel: i) The empirical moment (circles); ii) The discretized Gaussian version; iii) The discretized mixed-Gaussian version; and iv) The moments obtained by discretizing the iid component of the AR process, and then using the estimated autoregressive coefficients to compute the moment of the full AR variable.

I Generation of an artificial sample of economies

A mixed-Gaussian distribution (with the underlying constraints on its parameters) can map into a limited set of values for its moments. Only within this set it is possible to draw moments that are independent of each other. Along the border of this set, changing one

[16]As discussed by FT, the discretization is not always feasible. In particular, FT decrease the number of moments to be matched for each point on the grid of state variables of the AR(1) process, depending on the success of matching the highest moment targeted. While our fourth step yields negligible residuals for all countries, the fifth step (the FT method) generates heterogeneous results across countries: not all tuples of moments can be matched equally well by a Gaussian mixture. To our knowledge this is an unavoidable limitation of matching more than the first two moments of empirical distributions.

moment requires adjusting other moments, thus generating a correlation among them. Algorithm 1 is designed to draw from the set of admissible moments (second to fourth) using a three-term mixed Gaussian distribution.

Algorithm 1 Construction of the artificial sample of countries

1: Express the TFP process of each country in terms of a three-term mixed Gaussian distribution, e.g. for country A (analogously for country B)

$$\sigma_A \varepsilon_{A,t} \approx p_1 \mathcal{N}(m_1, v_1) + p_2 \mathcal{N}(m_2, v_2) + (1 - p_1 - p_2) \mathcal{N}(m_3, v_3)$$

where $\mathcal{N}(m_i, v_i)$ is the Gaussian PDF with mean m_i and variance v_i and p_i is the weight of the i term in the mixed Gaussian distribution;

2: Express the parameters v_1, v_2 and v_3 in terms of $\gamma_A = E\left(\sigma_A^2 \varepsilon_{A,t}^2\right)$, $\phi_A = E\left(\sigma_A^3 \varepsilon_{A,t}^3\right)$ and $\eta_A = E\left(\sigma_A^4 \varepsilon_{A,t}^4\right)$ and the other parameters of the distributions.

3: Draw random values (from uniform distributions) for p_1, p_2,γ_A, ϕ_A, η_A, m_2 and m_3. Where m_1 is pinned down by the assumption that $E\left(\sigma_A \varepsilon_{A,t}\right) = 0$;

4: Discretize the resulting mixed-Gaussian distributions. Denote by N_D the number of these distributions;

5: Use the N_D distributions to parameterize the TFP process of country A in N_D economies, where sizes are set randomly and country B is the same in all of the N_D economies.

6: Find the solution for all the N_D economies.

To search for the admissible set we drew 100,000 values for the second-to-fourth uncentered moments from uniform distributions. The limits of these distributions were set so that: *standard deviation* $\in (0.001, .15)$, *skewness* $\in (-3, 3)$ and *kurtosis* $\in (3, 20)$. Moreover, we drew the weights of the first two terms of the mixed-Gaussian distribution from the $(0, 1)$ interval (imposing that the sum of all three weights should be 1). Finally the mean of the second and third Gaussian terms were drawn from the $(-1, 1)$ interval; the mean of the first term being set so to obtain a zero-mean for the whole mixed Gaussian distribution.

The distribution of the key parameters of this generated sample is summarized in Table 2.

Figure 4 shows a sub-set of variables of the artificial sample generated by following Algorithm 1. Particularly noteworthy is are the scatter plots relating the three moments of interest. For example, the third and fourth columns of the second-last row

Table 2: Distribution of generated sample

	5%	25%	50%	75%	95%
size	0.0007	0.0034	0.0259	0.1884	0.5646
standard deviation	0.0557	0.0939	0.1199	0.1357	0.1470
skewness	-1.8391	-0.7158	0.0571	0.8448	2.0182
kurtosis	4.8536	9.0786	13.3176	16.8538	19.1461

show the set of admissible values for the fourth uncentered moment mapped against the second and third moments. Only within the set it is possible to pick independent moments.

J Comparison of the perturbation solution with the global solution

In the main text we have argued that the perturbation-based solution of the model delivers qualitatively reliable results. This section offers an example of the gap between the perturbation and global solution for a key variable in our analysis, κ.[17] For this purpose we solve the two-country model calibrated using the 477 ad-hoc constellations of moments and sizes as well as the baseline parametrization discussed in the text. In the perturbation method we use the ergodic moments implied by the discretization of the DGP for home and foreign TFP.

Figure 5 show the value of $\kappa_{A,B}$ (κ for short) obtained using perturbation method (x-axis) against the value obtained using global methods (y-axis). The two methods give mostly consistent results concerning the sign of $\ln \kappa$ (the correlation is 0.97). That said, perturbation methods tend to overstate the magnitude of κ. Although using the ad-hoc sample magnifies the discrepancies (as it include more diverse values for the moments), these result warrant the use of global solution methods.

[17]The global solution is typically more accurate than "asymptotic" (also known as local) approximations. Global methods entail some approximations (e.g. in discretizing the state space). Yet, the error can be more easily quantified and improved using global methods (e.g. by refining the grid) than perturbation methods. See e.g. Judd (1998) for a discussion and comparison of these methods.

Figure 4: Distributions of the key variables in our ad-hoc sample.

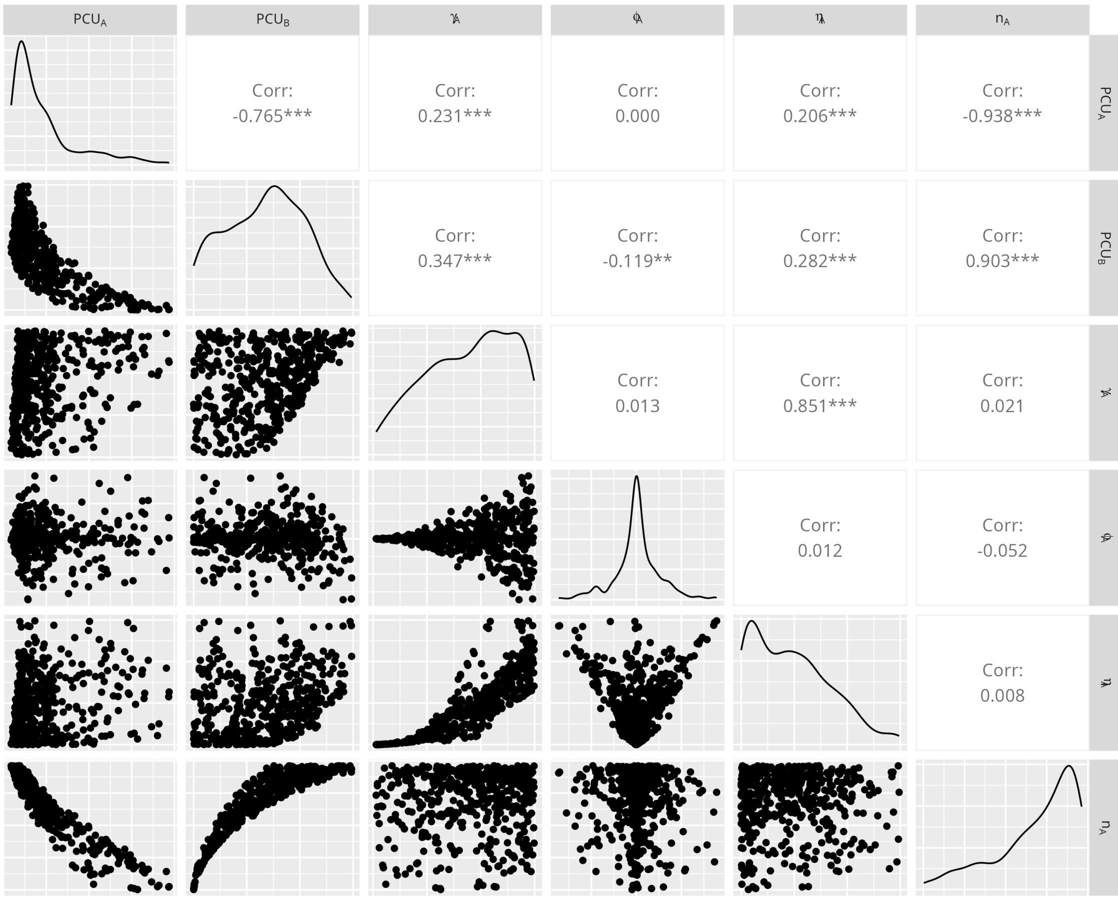
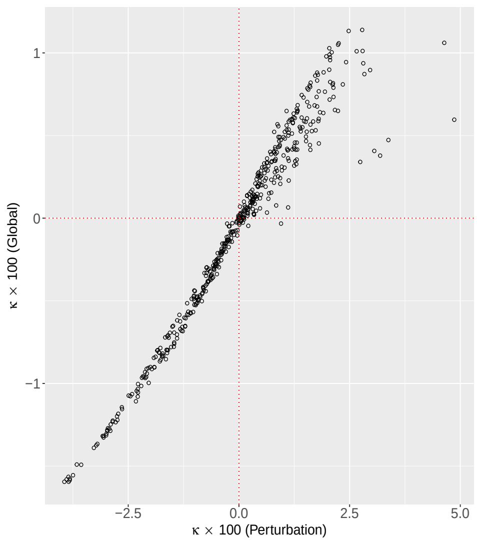


Figure 5: Comparison of solution methods for κ



K Unpacking the RoW into smaller units

To gauge whether subdividing the large regions into smaller units will affect our results, note that by the equilibrium expression for $\mu_{A,0}$

$$\mu_{A,0} = \left[n_A + (1 - n_A) \sum_{i=1}^{N} N^{-1} Q_{A,i,0}^{\frac{\rho-1}{\rho}} \kappa_{A,i} \right]^{-1} \tag{K.1}$$

a country consumption share of total output will not vary as we increase N only if $Q_{A,i,0}^{\frac{\rho-1}{\rho}} \kappa_{A,i}$ remains constant. Consumption smoothing should not be affected by this sub-partition of RoW, as the SDF will still be determined by global income. But the regression results in the main text suggest that both κ and Q (via ToT)[18] vary with size, albeit considerably less than one-to-one. We can thus conclude that breaking up RoW into sub-units does have material implications for country A's consumption share and welfare. Quantifying this effect would require solving our model for a sufficiently large number of countries, at a very large computational cost.[19]

L Proof of Propositions 2 and 3

Some algebra manipulation of the solution of the model yields

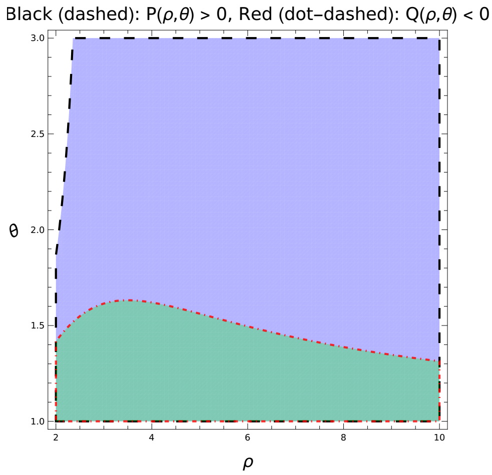
$$\begin{aligned} \frac{\partial \left(\mathcal{W}_{A,0}^{cm} - \mathcal{W}_{A,0}^{au} \right)}{\partial n_A} = \frac{\beta(\theta-1)^2 \rho}{(\beta-1)\theta^2} \cdot \Gamma - \frac{\beta(\theta-1)^2(\rho-1)\rho}{2(\beta-1)\theta^2} \cdot \Phi \\ + \frac{\beta(\theta-1)^2 \rho \left(-5 + \rho^2 + \theta(6 + 3\rho - \rho^2) + \theta^2(2 - 9\rho + 3\rho^2) \right)}{24(\beta-1)\theta^4} \cdot H \end{aligned}$$

Rearranging terms yields the results in Proposition 2.

[18]Note that $ToT_{au,0} = 1$.

[19]Solving our three country model starting from a reasonable guesses of the state-space (e.g. solving the equal-size case starting from the heterogeneous-size solution) takes approximately 19 hours on a Laptop with 4-core Intel(R) Core(TM) i7-4810MQ CPU @ 2.80GHz (3.80 max).

Figure 6: Regions of $P(\rho,\theta) > 0$ and $Q(\rho,\theta) < 0$



By setting $n_A = \frac{1}{2}$, $\nu_A = 1 - n_B\lambda$ and $\nu_B = 1 - n_A\lambda$ we have also that

$$\left.\frac{\partial\left(\mathcal{W}^{cm}_{A,0} - \mathcal{W}^{au}_{A,0}\right)}{\partial\lambda}\right|_{\lambda=1} = \frac{\beta\Gamma(\theta-1)^2}{(1-\beta)\theta^3} - \frac{\beta(\theta-1)^2(\rho-1)}{2(1-\beta)\theta^3}\Phi + \\ - \frac{\beta(\theta-1)^2\left[\theta^2(\rho^3-3\rho^2+8\rho-2)-\theta(\rho^3+2\rho+5)+4\right]}{24(1-\beta)\theta^5}H$$

Provided $\Phi < 0$, Figure 6 shows the range of values for ρ and θ for which the conditions in Proposition 2 and 3, provided $\Phi < 0$, are always satisfied. The effect of size under PPP is robust, provided the degree of risk aversion is sufficiently large. The sign of $Q(\rho,\theta)$ changes for sufficiently large trade elasticity. In that case kurtosis mitigates the effect of openness on the welfare gains (around PPP), and in extreme cases it can also overturn it.

M Smoothing and Level Effects in a SOE

We have argued that smoothing and level effects can have the same sign in a SOE, i.e. when $n_A \to 0$, under the assumption that Country B output is deterministic and constant: $\gamma_B = \phi_B = \eta_B = 0$. Under this assumption, perfect risk sharing implies that the SDF of country A is determined solely by foreign factors, and constant. Now, writing the country A's expected income stream ($E\left(p_{A,t}Y_{A,t}\right)$) as a function of cumulants and taking its derivative with respect to γ_A, ϕ_A and η_A yields:

$$\lim_{n_A\to 0}\frac{\partial E\left(p_{A,t}Y_{A,t}\right)}{\partial\gamma_A}=\frac{1}{2}\left(\frac{\theta-1}{\theta}\right)^2+\mathcal{O}\left(\omega^5\right), \tag{M.1a}$$

$$\lim_{n_A\to 0}\frac{\partial E\left(p_{A,t}Y_{A,t}\right)}{\partial\phi_A}=\frac{1}{6}\left(\frac{\theta-1}{\theta}\right)^3+\mathcal{O}\left(\omega^5\right), \tag{M.1b}$$

$$\lim_{n_A\to 0}\frac{\partial E\left(p_{A,t}Y_{A,t}\right)}{\partial\eta_A}=\frac{1}{24}\left(\frac{\theta-1}{\theta}\right)^4+\mathcal{O}\left(\omega^5\right). \tag{M.1c}$$

In expectations, country A's income is increasing in the variance and kurtosis of the country A's TFP, as well as in its skewness provided $\theta > 1$. As expected income changes with cumulants, so do expected consumption and κ. Specifically:

$$\lim_{n_A\to 0}\frac{\partial\kappa_{A,B}}{\partial\gamma_A}=-\frac{\delta}{4}\frac{(\theta-1)^2}{\theta^4}\left(2\theta^2-\delta\gamma_A(\theta-1)^2\right)+\mathcal{O}\left(\omega^5\right), \tag{M.2a}$$

$$\lim_{n_A\to 0}\frac{\partial\kappa_{A,B}}{\partial\phi_A}=-\frac{\delta}{6}\left(\frac{\theta-1}{\theta}\right)^3+\mathcal{O}\left(\omega^5\right), \tag{M.2b}$$

$$\lim_{n_A\to 0}\frac{\partial\kappa_{A,B}}{\partial\eta_A}=-\frac{\delta}{24}\left(\frac{\theta-1}{\theta}\right)^4+\mathcal{O}\left(\omega^5\right). \tag{M.2c}$$

Given the convexity of the underlying functions, the results above reflect the distribution of the log of TFP we have assumed in our analysis.[20] Observe that, in standard calibrations with $\gamma_A << 1$ and $\theta >> 0$ (including our baseline parameterization in the main text), an increase in country A's variance reduces $\kappa_{A,B}$. When n_A is very small, $\kappa_{A,B}$ becomes predominantly driven by the curvature of the functions that map output into domestic income. We will revisit this result in the next section where, using global methods, we will show that it holds more generally, relaxing the simplifying assumptions underlying the above expressions.

[20]Setting $\tau = \dfrac{\theta-1}{\theta}$ in the main text, clarifies that all the three moments contribute to an increase in the average TFP (even in the case of a one-good economy, with $\tau = 1$ ($\theta \to \infty$).

N Country clustering with GMM

Model–based clustering and interpretation. To gain intuition about the drivers of the cross–country welfare ranking, we use *model–based clustering* via finite Gaussian mixture models, estimated by EM as implemented in the `mclust` package (Scrucca et al., 2023). All features are standardized to mean zero and unit variance prior to fitting so that cluster means are comparable across dimensions. The number of clusters G and the covariance parameterization are compared using the Bayesian Information Criterion (BIC). The global BIC optimum across all parameterizations favors a solution with only two clusters, which is too coarse to capture the observed heterogeneity in welfare gains. Accordingly, for exposition we report results under the EEI specification (diagonal covariance with equal volume and equal shape), which delivers a richer but still BIC–competitive partition.[21]

Under EEI, the implied clusters align closely with the welfare ranking (with Mexico being the main exception), supporting the narrative in the main text. For each cluster, Table 3 reports the average of the standardized features; these moments underpin the descriptive labels used in the discussion. (Posterior membership probabilities—“responsibilities”—are used to gauge assignment certainty but are omitted for brevity.)

Table 3: Standardised cluster means for population size, openness, standard deviation, skewness and kurtosis.

Cluster	**Population size**	**Openness**	**Standard deviation**	**Skewness**	**Kurtosis**
1(II)	−0.328	1.420	0.595	−0.095	0.276
2 (III)	0.666	−1.420	−0.237	−0.452	0.363
3 (I)	−0.203	−0.344	−0.205	0.060	−0.113
4 (IV)	4.940	−1.580	−0.115	0.254	−0.438

[21] EEI implies spherical clusters after axis–wise rescaling; differences arise primarily through the component means. Alternatives such as VVV (unconstrained covariance) yield similar qualitative patterns but fewer, more diffuse groups by BIC.

N.1 Interpreting Per–Country Component Diagnostics (GMM)

We fit a Gaussian mixture model (GMM) with G components to the p-dimensional country feature vector $x_i \in \mathbb{R}^p$ (here $p = 5$: `pop`, `cshare`, `stdev`, `skewness`, `kurtosis`). The fitted parameters are

$$\{(\pi_k, \mu_k, \Sigma_k)\}_{k=1}^{G}, \quad \pi_k > 0, \ \sum_{k=1}^{G} \pi_k = 1, \ \mu_k \in \mathbb{R}^p, \ \Sigma_k \in \mathbb{R}^{p \times p} \ \text{(p.d.)}.$$

Responsibilities and winning component. The posterior membership probabilities ("responsibilities") for country i are

$$r_{ik} \equiv \Pr(Z_i = k \mid x_i) = \frac{\pi_k \, \phi(x_i; \mu_k, \Sigma_k)}{\sum_{h=1}^{G} \pi_h \, \phi(x_i; \mu_h, \Sigma_h)},$$

where $\phi(\cdot; \mu, \Sigma)$ is the multivariate normal density. Define the *winner* and the *runner–up* as

$$\hat{k} \ = \ \arg\max_k r_{ik}, \qquad \hat{j} \ = \ \text{index of the second largest } r_{ik}.$$

Whitened residuals and Mahalanobis distance. Let $\Sigma_k = L_k L_k^\top$ be a Cholesky factorization (with L_k invertible). Define the whitened residuals of x_i relative to component k by

$$z_k \ \equiv \ L_k^{-1}(x_i - \mu_k) \in \mathbb{R}^p,$$

so the squared Mahalanobis distance is

$$d_k^2 \ = \ (x_i - \mu_k)^\top \Sigma_k^{-1} (x_i - \mu_k) \ = \ \|z_k\|_2^2 \ = \ \sum_{\ell=1}^{p} z_{k,\ell}^2.$$

Columns reported in Table 4. For each feature (dimension) $\ell \in \{1, \ldots, p\}$ we report:

1. **`pct_of_mahal_k`**: percentage contribution of feature ℓ to the squared Mahalanobis

distance from the winning component $\hat{k}$,

$$\texttt{pct\_of\_mahal\_k}(\ell) \;=\; 100 \times \frac{z^2_{\hat{k},\ell}}{\sum_{m=1}^{p} z^2_{\hat{k},m}} \;=\; 100 \times \frac{z^2_{\hat{k},\ell}}{d^2_{\hat{k}}}.$$

This sums to approximately 100% over ℓ (up to rounding). Larger values indicate features that matter more for how close/far the country is to its assigned component center.

2. **`tilt_k_vs_j`**: a per–feature tilt showing whether feature ℓ supports the winner $\hat{k}$ against the runner–up $\hat{j}$,

$$\texttt{tilt\_k\_vs\_j}(\ell) \;\equiv\; \frac{1}{2}(z^2_{\hat{j},\ell} - z^2_{\hat{k},\ell}).$$

Positive values mean feature ℓ fits $\hat{k}$ better than $\hat{j}$ (pushes toward the winner); negative values mean the opposite; magnitudes indicate strength.

Decomposition of the log–posterior difference. The log–posterior kernel (up to a constant common to all components) for component k is

$$\log \pi_k \;-\; \frac{1}{2}\log|\Sigma_k| \;-\; \frac{1}{2}d_k^2.$$

Hence the difference between winner $\hat{k}$ and runner–up $\hat{j}$ decomposes as

$$[\log \pi_{\hat{k}} - \log \pi_{\hat{j}}] \;-\; \frac{1}{2}[\log|\Sigma_{\hat{k}}| - \log|\Sigma_{\hat{j}}|] \;+\; \sum_{\ell=1}^{p} \underbrace{\frac{1}{2}(z^2_{\hat{j},\ell} - z^2_{\hat{k},\ell})}_{\texttt{tilt\_k\_vs\_j}(\ell)}.$$

Thus $\sum_\ell \texttt{tilt\_k\_vs\_j}(\ell) \;=\; \frac{1}{2}(d^2_{\hat{j}} - d^2_{\hat{k}})$ captures the *distance* part of the decision; the mixing–weight term $\log \pi$ and the covariance–volume term $\frac{1}{2}\log|\Sigma|$ are global (not feature–specific) contributions.

How to read a row (example). If `pop` has `pct_of_mahal_k` = 61.8 and `tilt_k_vs_j` = +12.5, then population explains about 62% of this country's distance to its assigned component and strongly favors the winner over the runner–up. If `kurtosis` shows 21.6 and -0.41, it is the second most influential feature but slightly favors the runner–up; its effect is outweighed by `pop`. Tiny percentages (e.g., `cshare` = 0.4) indicate features with little leverage in the distance, even if their tilt is positive.

Scaling. All computations use the scaled features (the same inputs used to fit the GMM). If features are standardized (mean zero, unit variance), contributions are directly comparable across dimensions.

Brazil's position in the clusters Table 4 summarizes the fitness of the labelling of Brazil as belonging to the third group along each of the five features. Population is the main factor determining Brazil's membership. Each of the moments would have Brazil tilting towards the first group. Yet, none is strong enough (the probability remains 1 from group III).

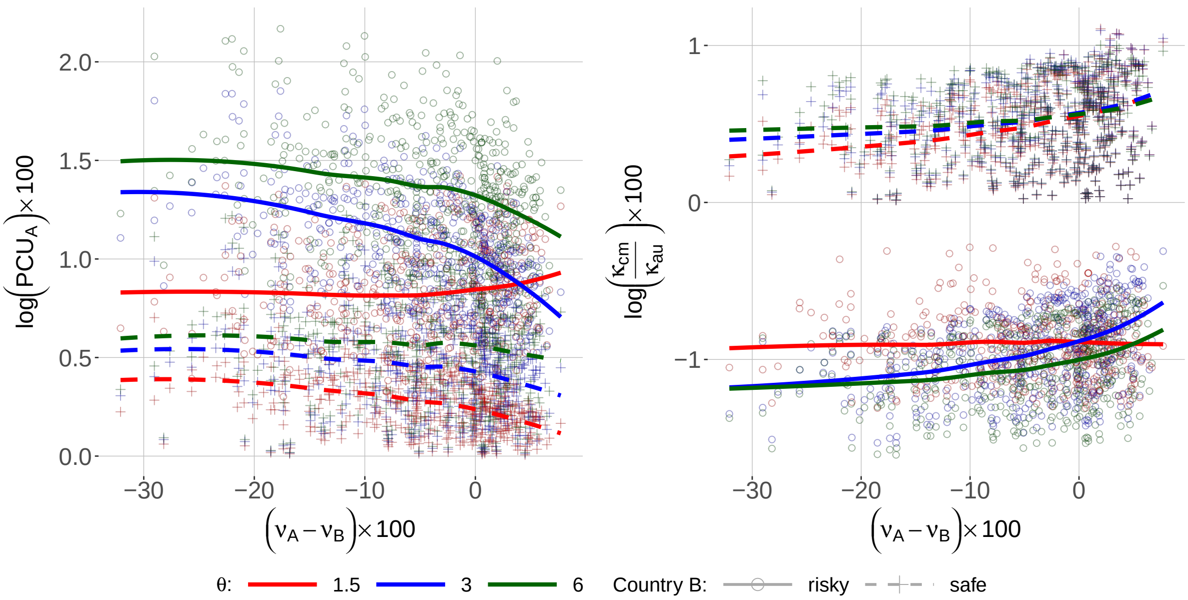
Table 4: Feature-wise contributions for Brazil (BRA). Winner: III (p=1.000); Runner-up: I (p=0.000).

Feature	Perc. of Mahal. (winner)	Tilt	Winner	Run.-up	p(Winner)	p(Runner-up)
pop	61.797	12.545	III	I	1	0
cshare	0.394	2.527	III	I	1	0
stdev	5.684	-0.029	III	I	1	0
skewness	10.533	-0.262	III	I	1	0
kurtosis	21.593	-0.412	III	I	1	0

O Openness and trade elasticity

In the main text we argue that openness and trade elasticity affect the gains from risk sharing—and their composition into LE and SE—by varying the exposure of countries to foreign shocks. To visualize this result, Figure 7 compares welfare gains and κ_s under different relative values of the home-bias parameters $\nu_A - \nu_B$ (along the horizontal axis) and for different values of the trade elasticity θ. We use the generated sample of 477

Figure 7: Openness, welfare gains and LE: the role of trade elasticity



Note: circles (solid fitting line) denote welfare gains for different countries "A" (left panel) and $\kappa_{A,B}$ relative to autarky (right panel) when country B's production is risky. Crosses (dashed fitting line) denote the case of safe country B's production.

countries discussed in the main text and in this Appendix. Since the baseline country in the sample has a particular degree of risk, we also show the case in which country B's production is not stochastic. Recall also that country B's has $\nu_B = 1 - n_A\lambda$, with $\lambda = 0.3$.

Figure 7 shows that the closer is country A, the smaller are its gains from risk sharing. The gains are larger for larger values of the trade elasticity. The gains are inversely correlated with κ, as argued in the main text. These results are qualitatively independent of the fact that B's production is risky. That said, the implicit transfer captured by κ is in absolute larger, the larger is the trade elasticity. So if a country is a recipient of the transfer, it tends to receive a larger transfer for larger θ_s.

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