# BIS Working Papers
No 1288

# The capital puzzle

by Eduardo Amaral

Monetary and Economic Department

September 2025

JEL classification: E43, E52, E58

Keywords: monetary policy, New Keynesian Model, natural interest rate

BIS Working Papers are written by members of the Monetary and Economic Department of the Bank for International Settlements, and from time to time by other economists, and are published by the Bank. The papers are on subjects of topical interest and are technical in character. The views expressed in them are those of their authors and not necessarily the views of the BIS.

This publication is available on the BIS website (www.bis.org).



ISSN 1020-0959 (print)
ISSN 1682-7678 (online)

# The Capital Puzzle

Eduardo G. C. Amaral*[a,b]

[a]Banco Central do Brasil (BCB): eduardo.amaral@bcb.gov.br

[b]Bank for International Settlements (BIS): eduardo.amaral@bis.org

August 27, 2025

**Abstract**

Can a central bank tighten monetary policy and real interest rates fall under monetary dominance? Introducing endogenous capital into the New Keynesian model allows real interest rates to move in any direction at the impact of a positive persistent monetary policy shock. This raises concerns that the real interest rate channel is only observational — not structural — in these models. This paper demonstrates that the puzzle goes beyond capital. It emerges when the elasticity of an endogenous state variable to a persistent shock is high enough to sink inflation expectations, inducing the endogenous (or systematic) component of the monetary policy rule to sufficiently offset its exogenous component. The channel is indeed structural, but conventional definitions of the natural interest rate (r-star) and real interest rate gap can be misleading, particularly following events that significantly disrupt investment, such as pandemics, financial crises or trade wars. As an alternative sign-consistent gauge of the monetary policy stance, I propose the real interest rate gap that neutralizes the effect of shocks on endogenous state variables. From 1965Q1 to 2023Q3, it was often a better predictor of future inflation and helped telling the history of monetary policy in the United States.

**Keywords:** Monetary Policy, New Keynesian Model, Natural Interest Rate

**JEL Classification:** E43; E52; E58.

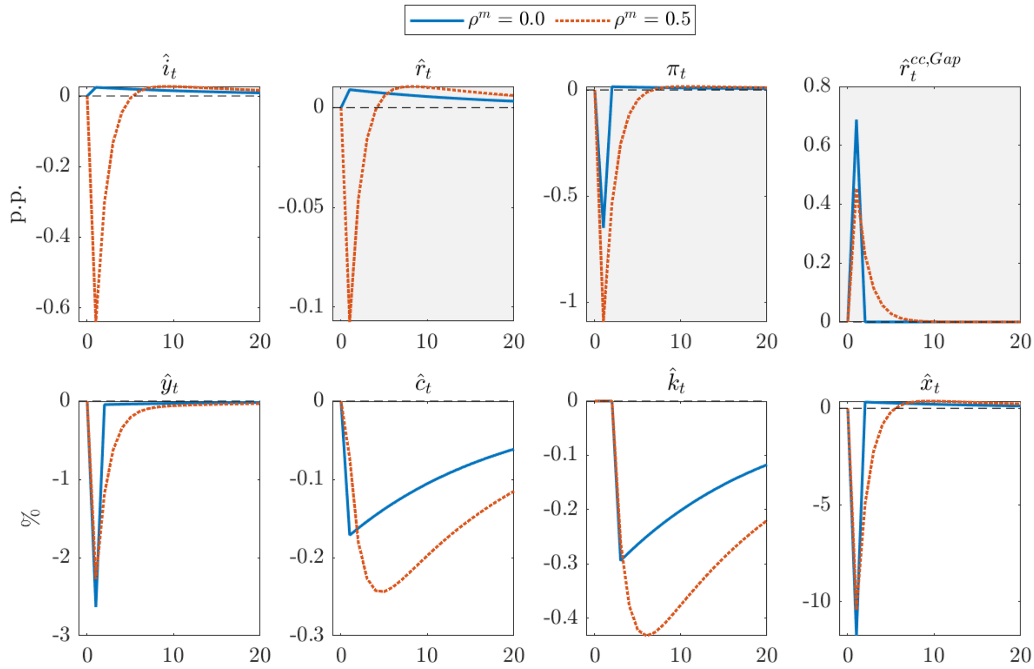
*The views in this paper are those of the author and do not necessarily reflect those of the BCB or the BIS. A previous version of this paper was released as Amaral (2024). I thank Carlos Viana de Carvalho, Eduardo Loyo, Ricardo Reis, André Minella, Diogo Guillén, Jon Frost, Ilhyock Shim, and two anonymous referees for their valuable comments, as well as Frank Smets, Raf Wouters and Johannes Pfeifer for providing helpful codes. I gratefully acknowledge the Economics Department of the London School of Economics for its hospitality during the 2018/2019 academic year. Finally, I thank the Banco Central do Brasil, CAPES, and the Economics Department of PUC-Rio for their support. The author declares there are no conflicts of interest that could have appeared to influence the work reported in this paper. All remaining errors are the author's sole responsibility.

# 1 Introduction

Can a central bank tighten monetary policy and real interest rates fall under monetary dominance? In a recent paper, Rupert and Šustek (2019) challenged the existence of a real interest rate channel of monetary policy transmission in textbook New Keynesian models, e.g., Woodford (2003*a*) and Galí (2015). They showed that introducing endogenous capital into such models allows the real interest rate to move in any direction after a positive persistent monetary policy shock, $\xi_t^m$.[1] Figure 1 displays the effect of that shock under two different specifications for its persistence coefficient, $\rho^m$. The real interest rate rises immediately after a transitory shock ($\rho^m = 0.0$) but falls when the latter is just mildly persistent ($\rho^m = 0.5$), while inflation falls and investment slumps in both cases.

Rupert and Šustek (2019) argue that these puzzling impulse response functions would prove that the real interest rate channel is only observational — not structural — in New Keynesian models. A channel that is not robust to the Lucas (1976) critique raises serious concerns regarding the reliability of these models for policy recommendations. For example, the interpretation of the mechanisms behind their impulse response functions becomes debatable. It would also be quite problematic to assume the real interest rate channel for the identification of vector autoregression (VAR) models, whether through sign restrictions, as in the method proposed by Uhlig (2005); through sequentially ordering nominal and real rates, as in a Cholesky decomposition; or by selecting real rates instead of nominal ones as part of a reduced model's endogenous variables. The importance of this identification problem is straightforward, yet significant, to the extent that it has recently been evoked by Holden (2024) as one of the reasons to motivate a radical shift in central banking from nominal to real interest rate rules.

[1] Woodford (2003*a*, sec. 5.3.3) calls the lack of any effect of variations in private spending on the economy's productive capacity one of the more obvious omissions in the baseline New Keynesian model. He observes that although there are calibrations for which introducing endogenous capital results in similar dynamics for output and inflation after a monetary policy shock, the mechanisms within each model that generate these results are not the same.




Note: hat variables are deviations from the zero-inflation-target steady state. $\hat{i}_t$ denotes the nominal interest rate; $\hat{r}_t$ the real interest rate; $\pi_t$ inflation; $\hat{r}_t^{Gap,cc}$ the real interest rate gap with constant capital; $\hat{y}_t$ output; $\hat{c}_t$ consumption; $\hat{k}_t$ capital at the beginning of period; and $\hat{x}_t$ investment.

Figure 1: Impulse response function to a positive monetary policy shock in a canonical New Keynesian model augmented with endogenous capital

The dynamics in Figure 1 resembles gloomy scenarios such as a pandemic, financial crisis or trade war — episodes where investment collapses, and the central bank risks falling behind.[2] Although investment slumps of this magnitude are uncommon, they do occur, particularly during recessions, as exemplified in Figure 2. It plots the investment history of the United States, from 1965Q1 to 2019Q4, against smoothed monetary policy shocks recovered from the estimated Smets and Wouters (2024)'s model, where shaded areas are recessions identified by the business cycle dating committee of the National Bureau of Economic Research (NBER).[3] In that period, quarterly investment dropped on average 1.3% during expansions

[2]Given the model's linearization, sign-reversed impulse response functions can be derived in the case of a monetary easing, where a sudden boost in investment, e.g., due to a technological discovery, may result in rising demand and rising inflation expectations combined with rising policy real interest rates, which is a sign-switch of the same mechanism discussed in this paper.

[3]It is a stylized fact that investment drops by more during the recessions identified by the NBER. Under their definition, a recession involves a significant widespread decline in economic activity which lasts more than a few months. Depth, diffusion, and duration are treated as somewhat interchangeable criteria. Extreme conditions under one criterion may partially offset weaker indications under another (NBER, 2025).

and 2.9% during recessions, but in at least two episodes, in the early 1980s and during the Great Financial Crisis of 2008, investment plunged by more than 8%. More recently, a similar downfall was observed at the outbreak of the Covid-19 pandemic. Just as investment has been negatively correlated with monetary policy shocks in the last decades, quarters with a contractionary monetary policy stance have been observed even during recessions. That is not surprising since an identified monetary policy shock is not always a voluntary or well-informed action of a central bank. It may be a delayed response, a misinterpretation of the monetary policy stance, or even some binding feature not incorporated into the model, such as the effective lower bound. With hindsight, some of these shocks are called monetary policy mistakes in Economic History books.

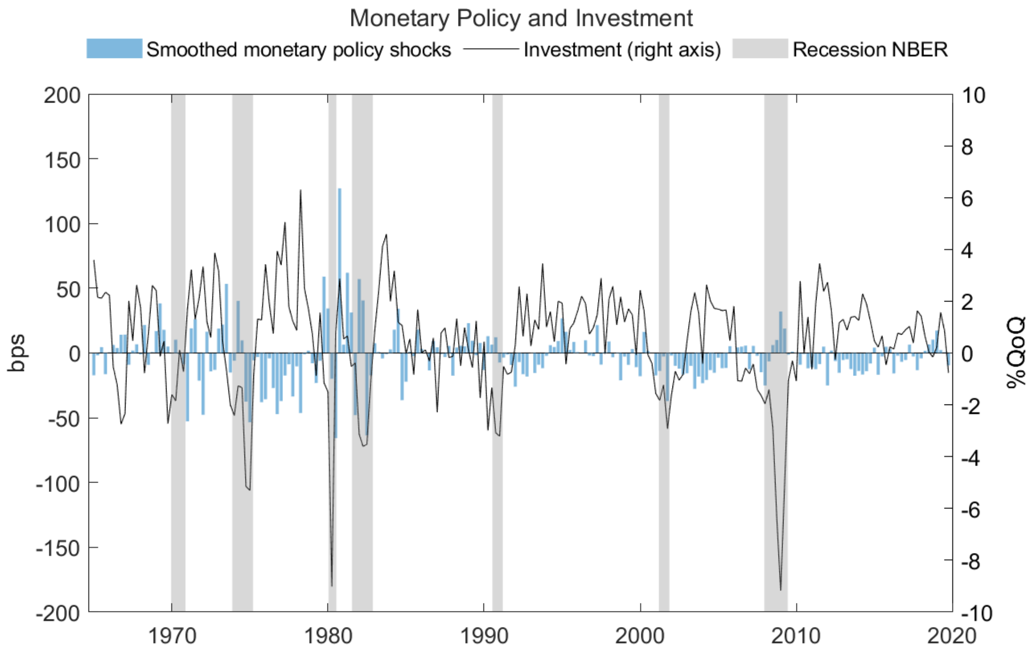



Figure 2: Estimated monetary policy shocks from Smets and Wouters (2024) and observed investment in the United States from 1965Q1 to 2019Q4

Despite the fact that in general equilibrium all variables are determined simultaneously, every model needs a story to tell, and the common view on the transmission of monetary policy in textbook New Keynesian models hinges on the real interest rate channel, which Ireland (2010) describes as follows:

> "A monetary tightening in the form of a shock to the Taylor rule that increases the short-term nominal interest rate translates into an increase in the real interest rate as well when nominal prices move sluggishly due to costly or staggered

price setting. This rise in the real interest rate then causes households to cut back on their spending, as summarized by the IS curve. Finally, through the Phillips curve, the decline in output puts downward pressure on inflation, which adjusts only gradually after the shock."

Galí (2015, p. 5) similarly highlights the real interest rate channel when discussing the short-run non-neutrality of monetary policy in this class of models:

> "As a consequence of the presence of nominal rigidities, changes in short-term nominal interest rates (whether chosen directly by the central bank or induced by changes in the money supply) are not matched by one-for-one changes in expected inflation, thus leading to variations in real interest rates. The latter bring about changes in consumption and investment and, as a result, in output and employment, because firms find it optimal to adjust the quantity of goods supplied to the new level of demand. In the long run, however, all prices and wages adjust, and the economy reverts back to its natural equilibrium."

Schematically, a positive monetary policy shock ($\epsilon_t^m$) raises the nominal interest rate ($i_t$) and, *due to price stickiness*, the real rate ($r_t$). This reduces consumption ($c_t$), output ($y_t$), and ultimately inflation ($\pi_t$).

$$\uparrow \epsilon_t^m \quad \Rightarrow \quad \uparrow i_t \quad \Rightarrow \quad \underbrace{\uparrow r_t}_{\text{if prices are sticky}} \quad \Rightarrow \quad \downarrow c_t \quad \Rightarrow \quad \downarrow y_t \quad \Rightarrow \quad \downarrow \pi_t$$

However, Rupert and Šustek (2019) propose a different story, which they argue is more consistent with the actual mechanics of the model. The transmission does not operate through a real interest rate channel. First, equilibrium inflation is approximately determined as in a flexible-price model.[4] Second, output is pinned down by the New Keynesian Phillips curve, interpreted here to mean that, given the expected inflation trajectory, firms that cannot

[4] In Chapter 2 of Galí (2015), a canonical real business cycle (RBC) model is augmented with a fixed-intercept interest rate rule to pin down inflation and, thus, a trajectory for the price level. Current inflation, as a deviation from its steady-state value, is determined by the expected path of real interest rate deviations from the steady state, as long as the Taylor principle is obeyed. It is important to note that the steady-state value of the real interest rate and the intercept of the monetary policy rule coincide, assuming a zero-inflation target. Chapter 1 of Woodford (2003*a*) shows the same idea in a partial-equilibrium monetary model where the sequence of real interest rates is exogenous.

adjust prices will change output. Finally, the real rate only reflects the feasibility of maintaining smooth consumption when income changes, whereas equivalence with the real interest rate channel depends on the persistence of the monetary policy shock. The canonical model, with fixed capital, is simply a limiting case where capital adjustment costs are infinite. According to this view, monetary transmission should work as follows:

$$\uparrow \epsilon_t^m \quad \Rightarrow \quad \uparrow i_t \quad \Rightarrow \quad \downarrow \pi_t \quad \Rightarrow \quad \underbrace{\downarrow y_t}_{\text{if prices are sticky}} \quad \Rightarrow \quad \downarrow c_t \quad \Rightarrow \quad \underbrace{?r_t}_{\substack{\text{depends on the presence}\\ \text{of capital and calibration}}}$$

In this paper, I show that this puzzle stems from a problem of definition. In the canonical New Keynesian model, monetary policy stance is measured by the real interest rate gap (RIRG), $r_t^{Gap}$, which compares the actual real interest rate of a sticky economy, $r_t$, with the one of an identical counterpart except for the absence of nominal rigidities, that is, the natural real interest rate, $r_t^n$. The gap is represented here by equation (1), where hat-variables are deviations from the non-stochastic steady state. If there is only a history of monetary policy shocks in the economy, $r_t^n$ should not move, implying that $r_t^{Gap}$ is equal to $r_t$.

$$\hat{r}_t^{Gap} = \hat{r}_t - \hat{r}_t^n \tag{1}$$

In the most frequently used definition of $r_t^n$, nominal rigidities are assumed to not exist neither in the past, present, nor future of the model. This definition has been ubiquitously employed in dynamic stochastic general equilibrium (DSGE) models, such as Smets and Wouters (2003, 2007, 2024). I call this definition, the *state-variant definition*, because for any given period state-variables may differ in value for $r_t$ and $r_t^n$. Since the canonical New Keynesian model has no endogenous state variables, such as capital, the difference in states is not an issue, and $r_t^{Gap}$ is strictly connected to the notion of equilibrium determination in that economy. In this case, $r_t^{Gap} = 0$ implies that output is at its potential level and inflation is at the central bank's target. This represents a short-run equilibrium, which also holds in the long run in the absence of additional exogenous shocks, where a single persistent exogenous shock is just a sequence of exogenous shocks.

Table 1 summarizes the algebra of how the capital puzzle emerges under the state-variant definition. The expressions for $\hat{r}_t$, $\hat{r}_t^n$ and $\hat{r}_t^{Gap}$ are translated into a state-space representation, where $\{A, B, C, D\}$ are matrices of coefficients, $\{\hat{s}_t\}$ is a matrix with a given sequence of actual and expected endogenous state variables as deviations from the steady state, $\epsilon_t$ is a vector of exogenous shocks, and $\xi_t$ is a vector of persistent exogenous shocks that follow AR(1) processes. When there is not a single state variable, such as in the canonical New Keynesian model, the presence or not of persistence in any shock does not affect the matrix coefficients of the shock, $B$ and $D$, which reflect the reaction of $\hat{r}_t$ and $\hat{r}_t^n$, respectively, in the period of the shock. Nevertheless, when one adds at least a single state variable to the model (e.g., capital), then this remains the case only if there is no persistence in the shock processes. In the case of persistence, the matrix coefficients of the shocks are now related to the expected trajectory of the state variables, $\frac{\hat{s}_t}{\hat{s}_{t-1}}$, a function of their elasticities. In this case, if a state variable is allowed to suffer a sudden large change, the sign of the shock coefficient of $\hat{r}_t^{Gap}$ will depend on the persistence of the shock and on the expected trajectory of the state variables. This explains why Rupert and Šustek (2019) find that the combination of a just slightly persistent monetary shock ($\rho^m = 0.1$) with low capital adjustment costs (high elasticity) induces the real interest rate to deviate from its conventional trajectory, declining at the impact of a contractionary monetary shock.

| Endogenous State Variable | Shock Persistence | $\hat{r}_t$ | $\hat{r}_t^n$ | $\hat{r}_t^{Gap}$ |
|---|---|---|---|---|
| no | no | $\hat{r}_t = B\epsilon_t$ | $\hat{r}_t^n = D\epsilon_t$ | $(B-D)\,\epsilon_t$ |
| no | yes | $\hat{r}_t = B\xi_t$ | $\hat{r}_t^n = D\xi_t$ | $(B-D)\,\xi_t$ |
| yes | no | $\hat{r}_t = A\hat{s}_{t-1} + B\epsilon_t$ | $\hat{r}_t^n = C\hat{s}_{t-1}^n + D\epsilon_t$ | $A\hat{s}_{t-1} - C\hat{s}_{t-1}^n + (B-D)\,\epsilon_t$ |
| | | $A = \frac{\mathbb{E}_t \hat{r}_{t+1}}{\hat{s}_t}$ | $C = \frac{\mathbb{E}_t \hat{r}_{t+1}^n}{\hat{s}_t^n}$ | $\mathbb{E}_t \hat{r}_{t+1} \frac{\hat{s}_t}{\hat{s}_{t-1}} - \mathbb{E}_t \hat{r}_{t+1}^n \frac{\hat{s}_t^n}{\hat{s}_{t-1}^n} + (B-D)\,\epsilon_t$ |
| yes | yes | $\hat{r}_t = A\hat{s}_{t-1} + B\xi_t$ | $\hat{r}_t^n = C\hat{s}_{t-1}^n + D\xi_t$ | $\left(\mathbb{E}_t \hat{r}_{t+1} - B\rho_m\xi_t\right) \frac{\hat{s}_t}{\hat{s}_{t-1}} - \left(\mathbb{E}_t \hat{r}_{t+1}^n - D\rho_m\xi_t\right) \frac{\hat{s}_t^n}{\hat{s}_{t-1}^n} + (B-D)\,\xi_t$ |
| | | $A = \frac{\mathbb{E}_t \hat{r}_{t+1} - B\rho_m\xi_t}{\hat{s}_t}$ | $C = \frac{\mathbb{E}_t \hat{r}_{t+1}^n - D\rho_m\xi_t}{\hat{s}_t^n}$ | $\mathbb{E}_t \hat{r}_{t+1} \frac{\hat{s}_t}{\hat{s}_{t-1}} - \mathbb{E}_t \hat{r}_{t+1}^n \frac{\hat{s}_t^n}{\hat{s}_{t-1}^n} + \left(B - B\rho_m \frac{\hat{s}_{t-1}}{\hat{s}_t} - D + D\rho_m \frac{\hat{s}_{t-1}^n}{\hat{s}_t^n}\right)\xi_t$ |

Table 1: State-variant definition

Woodford (2003*a*, Ch. 5 sec 3.4) points out that the state-variant definition of the RIRG turns out to be odd once one introduces capital in the model. That is so because, out of the steady state, the stock of capital of the sticky economy will probably differ most of the times from the stock of the counterpart economy without nominal rigidities. He suggests defining the natural rate of this economy at any period $t$ as the one that would result from the lack of nominal rigidities now and in the future, given the same exogenous and predetermined state variables of the actual economy.[5] I call this the *state-consistent definition*, $\hat{r}_t^{n,cons}$. Thus, for the RIRG to represent the current monetary policy stance, it must control for the effect of shocks on state variables, $\hat{r}_t^{Gap,cons}$. A natural rate that matters for measuring the monetary

[5] Woodford (2003*a*, Ch. 5 sec 3.4)'s refers only to price flexibility in the context of the simple model he is analyzing.

policy stance should be one that assumes that state variables are at the same levels as for the real rate, so that they are comparable. However, Table 2 shows that the state-consistent definition also allows for the presence of the capital puzzle as the matrix coefficients of the shocks are still related to the trajectory of the state variables, just like in the state-variant definition.

| Endogenous State Variable | Shock Persistence | $\hat{r}_t$ | $\hat{r}_t^{n,cons}$ | $\hat{r}_t^{Gap,cons}$ |
|---|---|---|---|---|
| no | no | $\hat{r}_t = B\epsilon_t$ | $\hat{r}_t^n = D\epsilon_t$ | $(B-D)\,\epsilon_t$ |
| no | yes | $\hat{r}_t = B\xi_t$ | $\hat{r}_t^n = D\xi_t$ | $(B-D)\,\xi_t$ |
| yes | no | $\hat{r}_t = A\hat{s}_{t-1} + B\epsilon_t$ | $\hat{r}_t^n = C\hat{s}_{t-1} + D\epsilon_t$ | $(A-C)\,\hat{s}_{t-1} + (B-D)\,\epsilon_t$ |
| | | $A = \frac{\mathbb{E}_t \hat{r}_{t+1}}{\hat{s}_t}$ | $C = \frac{\mathbb{E}_t \hat{r}_{t+1}^n}{\hat{s}_t}$ | $\mathbb{E}_t \hat{r}_{t+1} \frac{\hat{s}_t}{\hat{s}_{t-1}} - \mathbb{E}_t \hat{r}_{t+1}^n \frac{\hat{s}_t}{\hat{s}_{t-1}} + (B-D)\,\epsilon_t$ |
| yes | yes | $\hat{r}_t = A\hat{s}_{t-1} + B\xi_t$ | $\hat{r}_t^n = C\hat{s}_{t-1} + D\xi_t$ | $\left(\mathbb{E}_t \hat{r}_{t+1} - B\rho_m\xi_t\right)\frac{\hat{s}_t}{\hat{s}_{t-1}} - \left(\mathbb{E}_t \hat{r}_{t+1}^n - D\rho_m\xi_t\right)\frac{\hat{s}_t}{\hat{s}_{t-1}} + (B-D)\,\xi_t$ |
| | | $A = \frac{\mathbb{E}_t \hat{r}_{t+1} - B\rho_m\xi_t}{\hat{s}_t}$ | $C = \frac{\mathbb{E}_t \hat{r}_{t+1}^n - D\rho_m\xi_t}{\hat{s}_t}$ | $\left(\mathbb{E}_t \hat{r}_{t+1} - \mathbb{E}_t \hat{r}_{t+1}^n\right)\frac{\hat{s}_t}{\hat{s}_{t-1}} + \left(B - B\rho_m\frac{\hat{s}_{t-1}}{\hat{s}_t} - D + D\rho_m\frac{\hat{s}_{t-1}}{\hat{s}_t}\right)\xi_t$ |

Table 2: State-consistent definition

The finding that both the state-variant and the state-consistent definition can result in a sign inconsistent with the real interest rate channel awakens the interest in other ways of measuring the monetary policy stance. In this paper, I propose a *state-invariant definition* for the RIRG. In that, I incorporate the important caveat that state variables must be comparable through a different approach. I decompose the natural real interest rate into an exogenous component, $\hat{r}_t^{n,cc}$, that corresponds to a counterfactual that keeps the endogenous state variables constant – in this case the capital stock – and a remaining component, $\eta_{nk}\hat{k}_t^n + \boldsymbol{\eta_{nkk}}\boldsymbol{\varepsilon}_t$, that captures the state variable and the shocks, where $\eta_{nk}$ is a constant, $\boldsymbol{\eta_{nkk}}$ is a vector of constants, $\hat{k}_t^n$ is the log-deviation from the steady state of the capital stock in a counterfactual economy without nominal rigidities, and $\boldsymbol{\varepsilon}_t$ is a vector of shocks.[6]

$$\hat{r}_t^n = \underbrace{\hat{r}_t^{n,cc}}_{=\,0 \text{ if only shocks are monetary}} + \eta_{nk}\hat{k}_t^n + \underbrace{\boldsymbol{\eta_{nkk}}\boldsymbol{\varepsilon}_t}_{=\,0 \text{ if only shocks are monetary}} \tag{2}$$

Next, I decompose $\hat{r}_t$ as the sum of a counterfactual where capital is fixed, $\hat{r}_t^{cc}$, and a residual, $\left(\eta_k\hat{k}_t + \eta_{kk}\epsilon_t^m\right)$, where $\eta_k$ and $\eta_{kk}$ are constants, and define the state-invariant RIRG, $\hat{r}_t^{Gap,cc}$, as the gap of real interest rates that controls for the endogenous state variables.

$$\hat{r}_t = \hat{r}_t^{cc} + \left(\eta_k\hat{k}_t + \eta_{kk}\epsilon_t^m\right) \tag{3}$$

$$\hat{r}_t^{Gap,cc} = \hat{r}_t^{cc} - \hat{r}_t^{n,cc} \tag{4}$$

[6]Note that the remaining component does not depend on the monetary policy shock due to the lack of nominal rigidities in the counterfactual economy.

Figure 1 shows that, by comparing $\hat{r}_t$ and $\hat{r}_t^{Gap,cc}$, the latter is the RIRG measure whose sign is consistent with the New Keynesian theory in the shock period. Note that the decomposition proposed here isolates in coefficients $\eta_k$ and $\eta_{kk}$ any deep parameter sensitive to the speed of the adjustment of the state variable. The algebra is summarized in Table 3.

| Endogenous State Variable | Shock Persistence | $\hat{r}_t^{cc}$ | $\hat{r}_t^{n,cc}$ | $\hat{r}_t^{Gap,cc}$ |
|---|---|---|---|---|
| no | no | $\hat{r}_t^{cc} = B\epsilon_t$ | $\hat{r}_t^{n,cc} = D\epsilon_t$ | $(B-D)\,\epsilon_t$ |
| no | yes | $\hat{r}_t^{cc} = B\xi_t$ | $\hat{r}_t^{n,cc} = D\xi_t$ | $(B-D)\,\xi_t$ |

Table 3: State-invariant definition

Thus, I propose the following explanation for the New Keynesian capital puzzle: a monetary tightening in the form of a shock to the Taylor rule increases the short-term nominal interest rate (policy rule), causing an increase in the real interest rate when nominal prices move sluggishly (Fisher equation). This rise in the real interest rate causes households to cut back on their spending (IS). If investment sinks too much, households cut consumption even further (IS with capital). The large decline in output puts significant downward pressure on inflation (Phillips Curve), amplifying the endogenous negative response of the policy rule. This results in the nominal interest rate reversing its sign and dropping when its endogenous response is numerically larger than the original monetary policy shock, ultimately causing a drop in the real interest rate because prices are sticky. Then, onward, this flow continues with shrinking and oscillating amplitude until convergence to the in-period equilibrium, as induced by monetary dominance.

$$\uparrow \epsilon_t^m \;\Rightarrow\; \uparrow i_t \;\Rightarrow\; \underbrace{\uparrow r_t}_{\text{if prices are sticky}} \;\Rightarrow\; \underbrace{\downarrow\downarrow c_t}_{\text{if capital sinks}} \;\Rightarrow\; \downarrow\downarrow y_t \;\Rightarrow\; \downarrow\downarrow \pi_t \;\Rightarrow\; \downarrow i_t \;\Rightarrow\; \downarrow r_t$$

This explanation for the capital puzzle is backed by a simple arithmetic. Inflation is determined in a New Keynesian model through two equations: the Fisher relation ($\hat{i}_t = \hat{r}_t + \mathbb{E}_t \hat{\pi}_{t+1}$) and the interest rate rule ($\hat{i}_t = \widehat{\text{endogenous}}_t + \widehat{\text{exogenous}}_t$). By combining both equations and eliminating $\hat{i}_t$, I obtain that the sign of $\hat{r}_t$ will be opposite to that of the monetary policy shock *if and only if* the endogenous response of the monetary policy rule discounted of the expectation for next-period inflation has the opposite sign to the shock and is larger in absolute value.

$$\hat{r}_t = \left(\widehat{\text{endogenous}}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) + \widehat{\text{exogenous}}_t \tag{5}$$

A simple illustration exercise is to pick a policy rule of the form $\hat{i}_t = 1.5\mathbb{E}_t\hat{\pi}_{t+1} + \xi_t^m$ and impose a +50 bps shock. It will generate the capital puzzle if $\mathbb{E}_t\hat{\pi}_{t+1}$ is lower than -100 bps.

From an empirical perspective, this paper also shows that the identification problem of the real interest rate channel of monetary policy can be largely mitigated in the relevant parameter range by adding interest-rate smoothing to the Taylor rule — a feature as prevalent as capital in medium-scale New Keynesian DSGE models, e.g., Smets and Wouters (2003, 2007, 2024). Smoothing interest rates narrows down the numerical difference between the RIRGs measured under the state-invariant and state-variant definitions, while it also makes more likely that $\hat{r}_t^{Gap}$ reflects the sign of the monetary policy shock in impulse response functions such as the one in Figure 1.

I adopt the following modeling strategy to disentangle the observational equivalence of the real interest rate channel from its structural validity. First, I solve the textbook New Keynesian model without and with capital as well as decompose the effect of shocks on $r_t^{Gap}$ as in Equations (2) to (4). This decomposition exposes that the state-invariant component retains the sign predicted by the New Keynesian theory. The sign of the state-variant component, by its turn, will depend on the interaction between the structure of the model and the monetary policy rule. If the state-variant component happens to have the opposite sign of the state-invariant one and it is larger in magnitude than the latter, a positive monetary policy shock can lead to an immediate decrease of the real interest rate. In this case, yes, a central bank can tighten and real interest rates fall, but that reflects the dynamics of the endogenous state variables of that economy and not the first-round effect of monetary policy. This is true for both short and long-term real rates.

Second, I add smoothing in the Taylor rule to show that the latter can mitigate the identification problem. The finding that the real interest rate channel of monetary policy is reestablished with a common ingredient of medium-scale New Keynesian models alleviates concerns regarding that channel's proper identification in policy-oriented DSGE or VAR models. The latter are mostly immune to the problem because lagging terms are ubiquitous in their specification. I check the robustness of my results by exploring different combinations of interest-rate smoothing and also capital adjustment costs, since making capital adjustment sluggish is warranted to prevent excessive output fluctuations after a monetary policy shock.

Finally, I demonstrate how previous results extend to a medium-scale DSGE model —

more closely resembling those employed by contemporary central banks — and explore the potential of the state-invariant RIRG as a predictor of future inflation and as an instrument that helps telling the history of monetary policy. I find that from 1965Q1 to 2023Q3 in the United States, it often surpassed in performance the state-variant and state-consistent versions.

The next sections of this paper are structured as follows. Section 2 presents the related literature. Section 3 describes, solves, and analyzes the New Keynesian model before and after introducing endogenous capital. Section 4 introduces interest-rate smoothing. Section 5 takes the proposed solutions to a medium-scale model. Section 6 explores forecasting properties. Section 7 interprets the monetary policy stance history of the United States. Finally, Section 8 concludes.

# 2 Related literature

In the literature about gauging the monetary policy stance through interest rate gaps, Wicksell (1898, 1907) builds a whole monetary theory over the gap between the money rate and the natural interest rate in a mostly frictionless environment. More recently, Woodford (2003*a*) makes the theoretical case for the use of RIRGs in monetary policy; Neiss and Nelson (2003) and Mésonnier (2011) present evidence in favor of their empirical relevance; and Barsky, Justiniano and Melosi (2014) discuss their usefulness.

In central banking practice, the use of RIRGs has been pervasive (e.g., Waller (2024); Schnabel (2024)), leading to several estimation methods being proposed. Laubach and Williams (2003) estimate the gap in a semi-structural model, defining the natural rate as a trend that depends on output growth. Cour-Thimann, Pilegaard and Stracca (2006) specify the natural rate as the unobservable component of the real interest rate which has neither contemporaneous nor lagged correlations with the output gap. Lubik, Matthes et al. (2015) employ a time-varying parameter VAR model without assuming economic relationships between key macroeconomic variables, in a largely agnostic approach. What all proposed methods in the literature have in common are large confidence bands around their estimates for the natural rate.

Despite the widespread use of the RIRG as a measure of the monetary policy stance, suspicion about the real interest rate channel of New Keynesian models is considerably older than Rupert and Šustek (2019). The early work of Kimball (1995) on the derivation of real business cycle models with sticky prices and a quantity equation with exogenous shocks to the money supply dedicates a whole section to discussing the likelihood of that channel. He concludes that, even when investment adjustment costs are introduced, parameter values perceived by him as "plausible" would imply that the real interest rate increases in response to a monetary expansion.[7] The "implausible" scenario would occur if either adjustment costs were "too high" or convergence back to the long-run equilibrium after a monetary policy shock was "too fast", not unlike what Rupert and Šustek (2019) find.

Nonetheless, two distinctions exist between Kimball (1995)'s Neo-Monetarist model and most New Keynesian models that followed. First, the New Keynesian literature has followed the real-world trend of adopting nominal interest rate rules with an endogenous response to inflation. These last rules, especially when augmented with smoothing, put in sharp relief the speed of convergence back to a long-run equilibrium. Second, the parameterization he deems as "plausible" — an investment adjustment cost elasticity of 0.2 and a (labor-constant) elasticity of intertemporal substitution (EIS) for consumption of 0.2 — does not match modern estimations, which find higher values for the EIS.[8] The lack of realism in modeling assumptions is also a subject of criticism in Brault and Khan (2022), who modify Rupert and Šustek (2019)'s work by including frictions on changes in the flow of investments, as Christiano, Eichenbaum and Evans (2005) do, rather than on capital adjustment. They find that the real interest rate moves in the same direction as the monetary policy shock when the model contains empirically realistic frictions. This suggests that, at least in contemporary (medium-scale) New Keynesian models, the real interest rate channel should be observed.

Concerning interest-rate smoothing, its introduction into the canonical model is empirically motivated. Significant levels of it are found in the response function of the Federal Reserve by Clarida, Galí and Gertler (1999), for both the pre-Volcker (1960Q1-1979Q2) and the Volcker-Greenspan (1979Q3-1996Q4) eras. The same are found by Coibion and Gorodnichenko (2012), whose results employing both hard and narrative real-time data favor that source of purposeful policy inertia over simply serially correlated monetary policy shocks, where

[7] The model is linearized and, therefore, I assume a symmetrical response in the case of a monetary contraction.

[8] Using Bayesian methods, Smets and Wouters (2003) estimate the EIS to be 0.74 for the Euro Area, and Smets and Wouters (2007) estimate it to be 0.68-0.72 for the U.S. All values are posterior modes.

the latter could be arbitrary or motivated by the inherited persistence from underlying data generating processes of omitted variables to which the Federal Reserve also responds. For their part, Smets and Wouters (2007) estimate a medium-scale New Keynesian DSGE model using Bayesian methods for the United States from 1966 to 2004 and find large coefficients for interest-rate smoothing (above 0.7) as well as small coefficients for monetary policy shock persistence (below 0.3). Smets and Wouters (2024) update the sample until 2019 with a more detailed fiscal sector and allow inflation to be only partially backed by fiscal policy, obtaining mostly similar results. All these papers suggest the empirical presence of smoothing through the estimation of either single or multiple equation models, that is, by imposing only a little or a lot of informational restriction on the estimation. However, contrasting results can still be found depending on the estimation strategy as Rudebusch (2006) and Carrillo, Fève and Matheron (2018) demonstrate, favoring the modeling of serially correlated monetary policy shocks employed by Rupert and Šustek (2019).

Smoothing interest rates is also theoretically justified. Sack and Wieland (2000) and Woodford (2003*b*) show that smoothing policy interest rates may be optimal from a welfare perspective. This is a concern already presented in Brainard (1967), for which the existence of uncertainty on the effects of a certain policy recommends moving in its direction in small steps. Taking a related perspective, Goodfriend (1987) rationalizes smoothing in terms of a central bank's preference to maintain "orderly money markets" by minimizing unexpected asset price movements that otherwise could raise the risk of bankruptcies and banking crises.

Thus, although smoothing is a policy choice, high levels of it are generally optimal, and low levels are empirically rare, which warrants the case of this paper. Listening directly to central banks themselves, Amaral et al. (2025) asked several of them to rank their motivations for adjusting the monetary policy rate in small steps and found evidence that central banks do purposefully smooth the policy rate to some extent. Both approaches to monetary policy are not mutually exclusive, though, since a data-driven stance and contingency on new information do not preclude forward guidance and policy inertia. This paper shows that different combinations of these two features are usually enough to restore the identification of the real interest rate channel when using the conventional state-variant RIRG.

# 3 New Keynesian model before and after capital

In this section, I propose and solve a New Keynesian toy model: first the canonical version, followed by the model augmented with a generic endogenous state variable, and then finally with endogenous capital. This exposition strategy facilitates the comparison across the different versions of the model. The reader can find in Appendix A a glossary of all symbols used.

## 3.1 Canonical closed economy (with omitted or fixed capital)

Consider a closed economy without fiscal policy, where a one-period risk-free nominal bond is available in zero net supply and the central bank adopts a fixed-intercept Taylor rule. I expand here on the simplified presentation made by Rupert and Šustek (2019) of the canonical New Keynesian model of Galí (2015), with minor notational changes, whose full derivation is available in Appendix B.1.

The simple linearized model reduces to three equilibrium conditions: an Euler/IS equation (6), a Phillips Curve (7), and a Taylor rule (8). They pin down three variables: real output, $y_t$; nominal interest rate, $i_t$; and inflation, $\pi_t$. Over-lined variables represent their non-stochastic steady-state values ($\overline{\pi} = 0$, $\overline{y} = 1$) and hat variables are deviations from that same steady state, such that $\hat{y}_t \equiv \frac{y_t - \overline{y}}{\overline{y}}$ and $\hat{i}_t \equiv i_t - \overline{i}$. At this stage of the model, the relevant parameters are the subjective discount factor, $\beta$; the inverse of the elasticity of labor supply, $\eta$; the fraction of producers not adjusting prices at any given period, $\theta$; and the Taylor-rule coefficient that gauges the central bank's reaction to current inflation, $\nu$. There is also an exogenous monetary policy shock variable, $\xi_t^m$.

$$\left.\begin{aligned} -\hat{y}_t &= -\mathbb{E}_t \hat{y}_{t+1} + \hat{i}_t - \mathbb{E}_t \pi_{t+1} && (6) \\ \pi_t &= \Omega \hat{y}_t + \beta \mathbb{E}_t \pi_{t+1} && (7) \\ \hat{i}_t &= \nu \pi_t + \xi_t^m && (8) \end{aligned}\right\}$$

where

$$\Omega \equiv \frac{(1+\eta)(1-\theta)(1-\theta\beta)}{\theta} > 0 \tag{9}$$

Notice that when prices are fully flexible, $\theta \to 0$, then $\Omega \to \infty$, whereas when prices are fixed, $\theta \to 1$, then $\Omega \to 0$.

I can proceed further by substituting the policy rule (8) into (6) so that I reduce the system to only two equations:

$$\left.\begin{aligned} -\hat{y}_t &= -\mathbb{E}_t \hat{y}_{t+1} + \nu\pi_t + \xi^m_t - \mathbb{E}_t \pi_{t+1} \\ \pi_t &= \Omega\hat{y}_t + \beta\mathbb{E}_t\pi_{t+1} \end{aligned}\right\} \qquad \begin{aligned}(10)\\(11)\end{aligned}$$

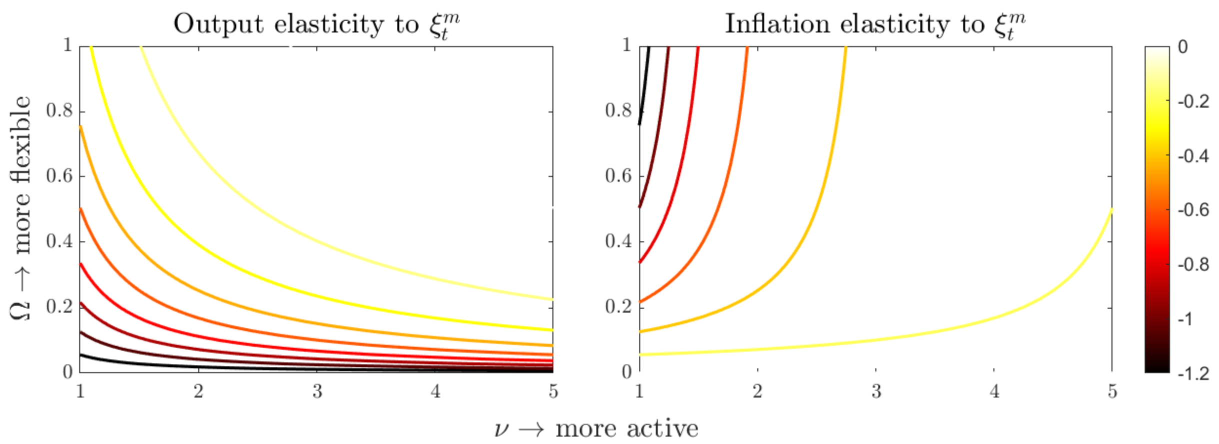
I assume the monetary policy shock follows an AR(1) process given by $\xi^m_t = \rho^m \xi^m_{t-1} + \epsilon^m_t$, where $\rho^m \in [0,1)$ and $\epsilon^m_t$ is i.i.d. N(0,1). Solving the model with the method of undetermined coefficients — also known as guess-and-verify — by conjecturing $\hat{y}_t = a\xi^m_t$ and $\pi_t = b\xi^m_t$, where $a$ and $b$ are the coefficients I want to obtain, and discarding explosive paths for output and inflation leads to

$$a = -\frac{1-\beta\rho^m}{(1-\rho^m)(1-\beta\rho^m) + \Omega(\nu-\rho^m)} < 0 \tag{12}$$

$$b = -\frac{1}{(1-\rho^m)(1-\beta\rho^m)\Omega^{-1} + (\nu-\rho^m)} < 0 \tag{13}$$

where both coefficients imply that a positive monetary policy shock always reduces inflation and output in the canonical New Keynesian model.

Figure 3 plots coefficients $a$ and $b$ for different values of $\nu$ and $\Omega$, under the calibration of Rupert and Šustek (2019)[9]. As expected, flexible prices reduce output elasticity to zero at the same time that inflation elasticity is at its maximum. Moreover, a more active monetary policy reduces both elasticities.




Note: Darker colors imply higher (absolute) elasticity values.

Figure 3: Output and inflation elasticity to a monetary policy shock

[9]The following calibration includes deep parameters, as well as parameters that will later be incorporated into the model presented in this paper: $\beta = 0.99$, $\eta = 1$, $\varepsilon = 0.83$, $\theta = 0.7$, $\nu = 1.5$, $\rho^m = 0.5$, $\alpha = 0.3$, $\delta = 0.025$.

The real interest rate deviation from steady state can be derived from the Fisher identity, $\hat{r}_t = \hat{i}_t - \mathbb{E}_t \pi_{t+1}$. Substituting my solution, I have

$$\hat{r}_t = \underbrace{\left(1 - \underbrace{\frac{1}{1 + \frac{1-\rho^m}{\nu-\rho^m}\frac{1-\beta\rho^m}{\Omega}}}_{\in[0,1]}\right)}_{\geq 0} \xi_t^m \tag{14}$$

whose coefficient on $\xi_t^m$, which is never negative, implies that the real interest rate always increases/decreases right after a positive/negative monetary policy shock. This is observationally and structurally consistent with monetary policy being transmitted through a real interest rate channel.

## 3.2 Model with endogenous state variable

Next, I expand the reduced canonical New Keynesian model of equations (10) and (11) with a generic endogenous state variable in the Phillips curve, $\tilde{e}_{t-1}$, whose data generating process is elastic to $\xi_t^m$ by a factor of $\omega$. To simplify, I assume this endogenous state variable is purely explained by the history of exogenous monetary policy shocks, as in equation (17).[10] I replace hat variables with tilde variables in this session to make the comparison with the canonical model clearer.

$$\left.\begin{aligned} -\tilde{y}_t &= -\mathbb{E}_t \tilde{y}_{t+1} + \nu\tilde{\pi}_t + \xi_t^m - \mathbb{E}_t \tilde{\pi}_{t+1} \quad (15)\\ \tilde{\pi}_t &= \Omega\tilde{y}_t + \beta\mathbb{E}_t\tilde{\pi}_{t+1} + \tilde{e}_{t-1} \quad (16)\\ \tilde{e}_t &= \omega\xi_t^m \quad (17)\end{aligned}\right\}$$

After conjecturing $\tilde{y}_t = \tilde{a}\xi_t^m$ and $\tilde{\pi}_t = \tilde{b}\xi_t^m$, where $\tilde{a}$ and $\tilde{b}$ are the coefficients I want to obtain, and discarding explosive paths for output and inflation ($\rho^m \in (0,1)$) leads to

$$\tilde{a} = a\left(1 + \omega\underbrace{\frac{(\nu - \rho^m)}{\rho^m(1-\beta\rho^m)}}_{>0}\right)_{\text{sign depends on } \omega} \underbrace{?}\ 0 \tag{18}$$

$$\tilde{b} = b\left(1 - \omega\underbrace{\frac{(1 - \rho^m)}{\Omega\rho^m}}_{>0}\right)_{\text{sign depends on } \omega} \underbrace{?}\ 0 \tag{19}$$

[10]Assuming $\tilde{e}_{t-1}$ is purely an exogenous shock which is correlated to $\epsilon_t^m$ can generate similar impulse response functions, but its interpretation as a puzzle is less obvious.

Now, despite $a < 0$ and $b < 0$, the signs of $\tilde{a}$ and $\tilde{b}$ will depend on the sign and size of $\omega$. Moreover, $\tilde{a}$ and $\tilde{b}$ can diverge in sign, allowing for an output expansion amid a persistent disinflation, such as when the income effect of the monetary shock outweighs its intertemporal substitution effect (see Section 4.3.3). Substituting the solution into the Fisher relation, I find

$$\tilde{r}_t = \underbrace{\hat{r}_t}_{\text{same sign as } \xi_t^m} + \underbrace{\omega \underbrace{\frac{\left(1-\rho^m\right)}{\rho^m \Omega \left(1 + \frac{1-\rho^m}{\nu-\rho^m} \frac{1-\beta\rho^m}{\Omega}\right)}}_{>0}}_{\text{sign of } \omega} \xi_t^m \tag{20}$$

This is equivalent to decomposition (3), but with respect to this model. It shows that, upon the introduction of an endogenous state variable, the RIRG can go in any direction in the period of a positive persistent monetary shock. Whether up or down will depend on the sign and size of the elasticity of the state variables to the shock. In that sense, the capital puzzle goes beyond capital.

## 3.3 Model with endogenous capital

This section incorporates endogenous capital and shows that the real interest rate channel is reestablished once one uses the state-invariant RIRG definition. The full derivation of this expansion of the canonical model is available in Appendix B.2.

### 3.3.1 What does change in the model?

Following Rupert and Šustek (2019), I assume an economy-wide rental market for capital, so firms adjust holdings each period. Thus, capital is not firm-specific.[11] Moreover, they assume that whenever households change their stock of capital, there is a quadratic adjustment cost, $-\frac{\kappa}{2}\left(k_{t+1} - k_t\right)^2$, where $k_t$ is the stock of capital inherited from the previous period and $\kappa \geq 0$ is a parameter that governs the size of the adjustment cost in terms of foregone real income. The production function now incorporates capital and labor proportionate to constant returns to scale, where $\alpha$ is the Cobb-Douglas coefficient of capital and $r_t^k$ is the latter's rent.

[11] Altig et al. (2011) estimate a New Keynesian DSGE model for the U.S. and find that this modeling choice for introducing endogenous capital results in firms enduring long spells before readjusting prices, up to 9 quarters on average. They show that firm-specific capital can align that spell more with empirical evidence from micro data to, say, once a year.

Comparing with the three-equation canonical model of Section 3.1, the reduced system has now five equations (21 to 25). There is a new Euler equation for the capital asset (22) and a more complex resource constraint (24), where $\delta \in (0,1)$ is the capital's depreciation rate. Moreover, $q_t$ is the price of capital in terms of current consumption, or Tobin's q, such that $q_t \equiv 1+\kappa(k_{t+1}-k_t)$. Finally, $G_{t+1} \equiv \frac{q_{t+1}}{q_t}$ is the capital gain, so $\hat{g}_t = \hat{q}_t - \hat{q}_{t-1} = \overline{\kappa}\left(\hat{k}_{t+1}-\hat{k}_t\right) - \overline{\kappa}\left(\hat{k}_t - \hat{k}_{t-1}\right)$, where $\overline{\kappa} = \kappa\overline{k}$. For any variable $X$, $\hat{X} \equiv \frac{X_t - \overline{X}}{\overline{X}}$, with the exception of $\hat{i}_t \equiv i_t - \overline{i}$, $\hat{r}_t \equiv r_t - \overline{r}$, and $\hat{r}_t^k \equiv r_t^k - \overline{r}^k$. The systems reduces to

$$\left.\begin{aligned}
-\hat{c}_t &= -\mathbb{E}_t\,\hat{c}_{t+1} + \hat{i}_t - \mathbb{E}_t\,\pi_{t+1} &(21)\\
-\hat{c}_t &= -\mathbb{E}_t\,\hat{c}_{t+1} + \mathbb{E}_t\,\hat{g}_{t+1} + \overline{r}^k\,\mathbb{E}_t\left(\hat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\hat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\hat{k}_{t+1}\right) &(22)\\
\pi_t &= \Psi\left(\frac{\eta+\alpha}{1-\alpha}\hat{y}_t - \alpha\frac{1+\eta}{1-\alpha}\hat{k}_t + \hat{c}_t\right) + \beta\,\mathbb{E}_t\,\pi_{t+1} &(23)\\
\hat{y}_t &= \frac{\overline{c}}{\overline{y}}\hat{c}_t + \frac{\overline{k}}{\overline{y}}\hat{k}_{t+1} - (1-\delta)\frac{\overline{k}}{\overline{y}}\hat{k}_t &(24)\\
\hat{i}_t &= \nu\pi_t + \xi_t^m &(25)
\end{aligned}\right\}$$

where $\Psi \equiv \overline{\chi}\frac{(1-\theta)(1-\theta\beta)}{\theta}$ and $\overline{\chi}$ is the real marginal cost at the steady state, such that when prices are flexible, $\Psi \to \infty$.

### 3.3.2 The real interest rate gap

In the New Keynesian theory, the gauge of monetary policy stance is the real interest rate gap, $\hat{r}^{Gap}$. The conventional way of calculating the gap is by taking the difference between the actual real interest rate, $\hat{r}_t$, and the real interest rate of a counterfactual economy without nominal rigidities, $\hat{r}_t^n$, as in (1). This approach does not take into account the fluctuation of state variables, such as capital, which, I showed in Table 1, is the reason why Rupert and Šustek (2019) find that the real interest rate channel of monetary policy transmission is only observational and not structural in New Keynesian models.

Next, I decompose the effect of the shock into a purely exogenous component that emerges from the counterfactual with fixed state variables, and one which depends on the existence of the state variable, (27). After that, I can calculate a state-consistent RIRG, like in Woodford (2003*a*), but whose sign reflects the actual monetary policy stance. I call this the state-invariant RIRG. This last measure is consistent with the New Keynesian theory and shows that the transmission of monetary policy is truly through a real interest rate channel, although deviations of endogenous state variables from their long-run values blur that same mechanism.

A steady-state economy inflicted with a positive monetary policy shock will always exhibit a positive state-invariant RIRG in the period of the shock.

I start by recalling some definitions introduced in this paper, (1) to (4). $r_t^{cc}$ is the real interest rate of a counterfactual economy in which capital is kept fixed at its long-run (steady-state) value, $r_t^{n,cc}$ is the same counterfactual but with no nominal rigidities, $\hat{r}_t^{Gap,cc}$ is the gap between them, while $\eta_{nk}$, $\eta_{nkk}$, $\eta_k$, and $\eta_{kk}$ are constants.

$$\hat{r}_t^{Gap} = \hat{r}_t - \hat{r}_t^n \tag{1}$$

$$\hat{r}_t^n = \hat{r}_t^{n,cc} + \left(\eta_{nk}\hat{k}_t^n + \eta_{nkk}\epsilon_t^m\right) \tag{2}$$

$$\hat{r}_t = \hat{r}_t^{cc} + \left(\eta_k\hat{k}_t + \eta_{kk}\epsilon_t^m\right) \tag{3}$$

$$\hat{r}_t^{Gap,cc} = \hat{r}_t^{cc} - \hat{r}_t^{n,cc} \tag{4}$$

I can now proceed with the decomposition. I substitute (3) and (2) into (4), and then (1) into its result to obtain (26).

$$\hat{r}_t^{Gap,cc} = \left(\hat{r}_t - \eta_k\hat{k}_t - \eta_{kk}\epsilon_t^m\right) - \left(\hat{r}_t^n - \eta_{nk}\hat{k}_t^n - \eta_{nkk}\epsilon_t^m\right)$$

$$\hat{r}_t^{Gap,cc} = \hat{r}_t^{Gap} - \eta_k\hat{k}_t + \eta_{nk}\hat{k}_t^n - \eta_{kk}\epsilon_t^m + \eta_{nkk}\epsilon_t^m$$

$$\hat{r}_t^{Gap} = \underbrace{\hat{r}_t^{Gap,cc}}_{\text{state-invariant effect}} + \underbrace{\eta_k\hat{k}_t - \eta_{nk}\hat{k}_t^n + \eta_{kk}\epsilon_t^m - \eta_{nkk}\epsilon_t^m}_{\text{state-variant effect}} \tag{26}$$

As the only shock of the model is monetary, $\hat{r}_t^{Gap,cc} = \hat{r}_t^{cc}$, $\eta_{nkk} = 0$, and $\hat{k}_t^n = 0$.

$$\hat{r}_t^{Gap} = \underbrace{\hat{r}_t^{cc}}_{\text{state-invariant effect}} + \underbrace{\eta_k\hat{k}_t + \eta_{kk}\epsilon_t^m}_{\text{state-variant effect}} \tag{27}$$

Returning to the model equations, after fixing capital, I can get rid of (22). Assuming the same policy rule as before, I have:

$$\left.\begin{aligned} -\hat{c}_t^{cc} &= -\mathbb{E}_t\,\hat{c}_{t+1}^{cc} + \nu\pi_t^{cc} + \xi_t^m - \mathbb{E}_t\,\pi_{t+1}^{cc} & (28)\\ \pi_t^{cc} &= \Psi\left(\frac{\eta+\alpha}{1-\alpha}\hat{y}_t^{cc} + \hat{c}_t^{cc}\right) + \beta\mathbb{E}_t\,\pi_{t+1}^{cc} & (29)\\ \hat{y}_t^{cc} &= \frac{\overline{c}}{\overline{y}}\hat{c}_t^{cc} & (30) \end{aligned}\right\}$$

I substitute (30) in (29) to eliminate $\hat{y}_t$. Using the undetermined coefficients methods, I solve next.

$$\hat{c}_t^{cc} = a\xi_t^m \quad \Rightarrow \quad \mathbb{E}_t\,\hat{c}_{t+1}^{cc} = a\mathbb{E}_t\,\xi_{t+1}^m \quad \Rightarrow \quad \hat{c}_t^{cc} = a\epsilon_t^m \quad \Rightarrow \quad \mathbb{E}_t\,\hat{c}_{t+1}^{cc} = a\rho^m\epsilon_t^m$$

$$\hat{\pi}_t^{cc} = b\xi_t^m \quad \Rightarrow \quad \mathbb{E}_t \hat{\pi}_{t+1}^{cc} = b\mathbb{E}_t \xi_{t+1}^m \quad \Rightarrow \quad \hat{\pi}_t^{cc} = b\epsilon_t^m \quad \Rightarrow \quad \mathbb{E}_t \hat{\pi}_{t+1}^{cc} = b\rho^m \epsilon_t^m$$

$$\left.\begin{aligned} -\hat{c}_t^{cc} &= -\mathbb{E}_t \hat{c}_{t+1}^{cc} + \nu\pi_t^{cc} + \xi_t^m - \mathbb{E}_t \pi_{t+1}^{cc} \\ \pi_t^{cc} &= \Psi\left(\frac{\eta+\alpha}{1-\alpha}\frac{\overline{c}}{\overline{y}} + 1\right)\hat{c}_t^{cc} + \beta\mathbb{E}_t \pi_{t+1}^{cc} \end{aligned}\right\}$$

$$\left.\begin{aligned} -a\epsilon_t^m &= -a\rho^m \epsilon_t^m + \nu b\epsilon_t^m + \epsilon_t^m - b\rho^m \epsilon_t^m \\ b\epsilon_t^m &= \Psi\left(\frac{\eta+\alpha}{1-\alpha}\frac{\overline{c}}{\overline{y}} + 1\right)a\epsilon_t^m + \beta b\rho^m \epsilon_t^m \end{aligned}\right\}$$

$$a = \frac{1}{\rho^m - 1}\left(\frac{\left(\nu-\rho^m\right)\Theta}{1-\left(\nu-\rho^m\right)\Theta} + 1\right) \tag{31}$$

$$b = \frac{\Theta}{1-\left(\nu-\rho^m\right)\Theta} \tag{32}$$

where $\Theta \equiv \frac{\Psi\left(\frac{\eta+\alpha}{1-\alpha}\frac{\overline{c}}{\overline{y}}+1\right)}{(1-\beta\rho^m)(\rho^m-1)} < 0$.

Since $\Theta < 0$ and $\left(\nu-\rho^m\right) > 0$, $-1 < \frac{(\nu-\rho^m)\Theta}{1-(\nu-\rho^m)\Theta} < 0$, so that $a < 0$, which means that consumption always decreases with respect to the state-invariant effect of the shock. Moreover, the numerator of $b$ is negative and the denominator is positive, so that $b < 0$, which means that inflation also falls as a consequence of the state-invariant effect of the shock. $\hat{r}_t^{cc}$ and $\hat{r}_t^{n,cc}$ can now be analytically solved, and therefore, also $\hat{r}_t^{Gap,cc}$.

$$\hat{r}_t^{cc} = i_t^{cc} - \mathbb{E}_t \pi_{t+1}^{cc} = \nu\pi_t^{cc} + \epsilon_t^m - \mathbb{E}_t \pi_{t+1}^{cc}$$

$$\hat{r}_t^{cc} = \nu b\epsilon_t^m + \epsilon_t^m - b\rho^m \epsilon_t^m = \left(\nu b + 1 - b\rho^m\right)\epsilon_t^m$$

$$\hat{r}_t^{cc} = \left(1 + (\nu-\rho^m)\frac{\Theta}{1-\left(\nu-\rho^m\right)\Theta}\right)\epsilon_t^m$$

$$\hat{r}_t^{cc} = \underbrace{\left(1 + \frac{1}{\underbrace{\frac{1}{(\nu-\rho^m)\Theta} - 1}_{>\text{-}1}}\right)}_{\geq 0}\epsilon_t^m$$

A positive monetary policy shock will always result in a non-negative state-invariant effect, $\hat{r}_t^{cc} \geq 0$ and therefore $\hat{r}_t^{Gap,cc} \geq 0$, but the sign of the state-variant effect will depend on the interaction between the structure of the model economy and the policy rule. Therefore, $\hat{r}_t^{Gap}$ will be negative if the state-variant effect is negative and has a higher magnitude than the state-invariant effect. Tables (4) to (7) sweep the sign of the impulse response functions of $\hat{r}_t^{Gap}$ and $\hat{r}_t^{Gap,cc}$ right after a positive monetary policy shock for different values of $\kappa$

ranging from 0 to 0.5. Notice that $\hat{r}_t^{Gap}$ turns negative the more persistent is the shock, but higher $\kappa$ delays the sign switch to larger $\rho^m$. What capital adjustment costs do in the model is to introduce a friction to its state variable (i.e. capital), reducing its sensitivity to exogenous shocks overall, and therefore attenuating the magnitude of the state-variant component of the RIRG. Overall, the RIRG truly consistent with the monetary policy stance, and with the real interest rate transmission channel, is the state-invariant one.

Table 4: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.0$

| | $\rho^m = 0$ | $\rho^m = 0.1$ | $\rho^m = 0.2$ | $\rho^m = 0.3$ | $\rho^m = 0.4$ | $\rho^m = 0.5$ | $\rho^m = 0.6$ | $\rho^m = 0.7$ | $\rho^m = 0.8$ | $\rho^m = 0.9$ | $\rho^m = 0.95$ | $\rho^m = 0.99$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r^{Gap}$ | + | - | - | - | - | - | - | - | - | - | - | - |
| $r^{cc,Gap}$ | + | + | + | + | + | + | + | + | + | + | + | + |

Note: + indicates that the specific real interest rate gap increases right after a positive monetary policy shock; - indicates that it decreases.

Table 5: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.1$

| | $\rho^m = 0$ | $\rho^m = 0.1$ | $\rho^m = 0.2$ | $\rho^m = 0.3$ | $\rho^m = 0.4$ | $\rho^m = 0.5$ | $\rho^m = 0.6$ | $\rho^m = 0.7$ | $\rho^m = 0.8$ | $\rho^m = 0.9$ | $\rho^m = 0.95$ | $\rho^m = 0.99$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r^{Gap}$ | + | + | + | + | + | + | + | + | - | - | - | - |
| $r^{cc,Gap}$ | + | + | + | + | + | + | + | + | + | + | + | + |

Note: + indicates that the specific real interest rate gap increases right after a positive monetary policy shock; - indicates that it decreases.

Table 6: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.2$

| | $\rho^m = 0$ | $\rho^m = 0.1$ | $\rho^m = 0.2$ | $\rho^m = 0.3$ | $\rho^m = 0.4$ | $\rho^m = 0.5$ | $\rho^m = 0.6$ | $\rho^m = 0.7$ | $\rho^m = 0.8$ | $\rho^m = 0.9$ | $\rho^m = 0.95$ | $\rho^m = 0.99$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r^{Gap}$ | + | + | + | + | + | + | + | + | + | - | - | - |
| $r^{cc,Gap}$ | + | + | + | + | + | + | + | + | + | + | + | + |

Note: + indicates that the specific real interest rate gap increases right after a positive monetary policy shock; - indicates that it decreases.

Table 7: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.5$

| | $\rho^m = 0$ | $\rho^m = 0.1$ | $\rho^m = 0.2$ | $\rho^m = 0.3$ | $\rho^m = 0.4$ | $\rho^m = 0.5$ | $\rho^m = 0.6$ | $\rho^m = 0.7$ | $\rho^m = 0.8$ | $\rho^m = 0.9$ | $\rho^m = 0.95$ | $\rho^m = 0.99$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r^{Gap}$ | + | + | + | + | + | + | + | + | + | + | - | - |
| $r^{cc,Gap}$ | + | + | + | + | + | + | + | + | + | + | + | + |

Note: + indicates that the specific real interest rate gap increases right after a positive monetary policy shock; - indicates that it decreases.

## 3.4 The long-term real interest rate

In a weaker version of the real interest rate channel, monetary policy affects the economy predominantly through the long-term real interest rate, $\hat{r}_t^l$, instead of through shorter-term real rates. $\hat{r}_t^l$ can be obtained by iterating forward the log-linearized Euler equation of bonds (21) and imposing stationarity, which results in $\hat{r}_t^l$ being the sum of all one-period real rates from today until infinity — the expectations hypothesis of the yield curve.

$$-\hat{c}_t = -\mathbb{E}_t\,\hat{c}_{t+1} + \hat{r}_t \Rightarrow -\hat{c}_t = -\mathbb{E}_t\,\hat{c}_{t+2} + \hat{r}_t + \hat{r}_{t+1} \Rightarrow \dots \Rightarrow -\hat{c}_t = -\underbrace{\mathbb{E}_t\,\hat{c}_{t+\infty}}_{0} + \underbrace{\sum_{j=0}^{\infty} \hat{r}_{t+j}}_{\equiv \hat{r}_t^l} \Rightarrow -\hat{c}_t = \hat{r}_t^l$$

Therefore, $\hat{r}_t^l$ will have the opposite sign of $\hat{c}_t$ whenever the economy is out of the steady state. I showed in section 3.3.2 that consumption always decreases with respect to the state-invariant effect of a positive monetary policy shock, what can now be extrapolated to $\hat{r}_t^l$ always increases with respect to the state-invariant effect of a positive monetary policy shock. Thus, even the weaker version of the real interest rate channel is also structural. It is true that $\hat{r}_t^l$ may fall after a positive monetary policy shock, e.g., if the persistence of the shock is large, like showed in Rupert and Šustek (2019), but that is a case in which the state-variant effect – whose sign and size depend on the state elasticities to the shock and on the strength of the endogenous component in the monetary policy rule – dominates. Section 4.3.3 shows an example in which this happens as the income effect of the monetary policy shock outweighs its intertemporal substitution effect, pushing $\hat{c}_t$ in the same direction of the shock.

# 4 Interest-rate smoothing

This section shows that adopting interest-rate smoothing in the Taylor Rule easily delivers impulse-response functions for the state-variant RIRG, $r_t^{Gap}$, with the sign consistent with the real interest rate channel – at least within the empirically relevant parameter range. This is possible as smoothing introduces a new endogenous state variable whose state-variant effect offsets the state-variant effect of the capital dynamics identified in (27). This finding significantly mitigates the identification problem from an empirical perspective and provides new insight into its mechanics.

## 4.1 Smoothing the policy rule

I substitute the Taylor rule (25) with one that includes interest-rate smoothing (33), whose persistence is governed by $\rho^i \in [0,1)$.

$$i_t = \rho^i i_{t-1} + \left(1-\rho^i\right)(i + \nu\pi_t) + \xi_t^m \Big\} \tag{33}$$

The reduced 4-equation system with the new policy rule becomes:

$$\left.\begin{aligned}
-\hat{c}_t &= -\mathbb{E}_t\,\hat{c}_{t+1} + \rho^i \hat{i}_{t-1} + \left(1-\rho^i\right)\nu\pi_t - \mathbb{E}_t\,\pi_{t+1} + \xi_t^m & (34)\\
-\hat{c}_t &= -\mathbb{E}_t\,\hat{c}_{t+1} + \mathbb{E}_t\,\hat{g}_{t+1} + \overline{r}^k\,\mathbb{E}_t\left(\hat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\hat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\hat{k}_{t+1}\right) & (35)\\
\pi_t &= \Psi\left(\frac{\eta+\alpha}{1-\alpha}\hat{y}_t - \alpha\frac{1+\eta}{1-\alpha}\hat{k}_t + \hat{c}_t\right) + \beta\,\mathbb{E}_t\,\pi_{t+1} & (36)\\
\hat{y}_t &= \frac{\overline{c}}{\overline{y}}\hat{c}_t + \frac{\overline{k}}{\overline{y}}\hat{k}_{t+1} - (1-\delta)\frac{\overline{k}}{\overline{y}}\hat{k}_t & (37)
\end{aligned}\right\}$$

To determine whether the negative response of real interest rates to a positive monetary policy shock persists as an identification problem, I sweep the combinations of parameter values for $\rho^m \in [0:0.1:0.9, 0.95, 0.99]$ and $\rho^i \in [0:0.1:0.9, 0.95, 0.99]$. Table 8 displays the sign of the reaction of the real interest rate right after the shock for all combinations under $\delta = 0.025$ and $\kappa = 0.0$. Tables 9 and 10 increase $\kappa$ to 0.1 and 0.5, respectively. As one can see, under the hypothesis of no adjustment costs, $\rho^i$ must be at least 0.95 to guarantee a positive response under all values of $\rho^m$. However, even a small adjustment cost, like $\kappa = 0.1$, is enough to largely increase the parameter range consistent with a real interest rate channel of monetary policy transmission.

Table 8: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.0$

| | $\rho^i = 0$ | $\rho^i = 0.1$ | $\rho^i = 0.2$ | $\rho^i = 0.3$ | $\rho^i = 0.4$ | $\rho^i = 0.5$ | $\rho^i = 0.6$ | $\rho^i = 0.7$ | $\rho^i = 0.8$ | $\rho^i = 0.9$ | $\rho^i = 0.95$ | $\rho^i = 0.99$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho^m = 0$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.1$ | - | - | - | - | - | - | - | + | + | + | + | + |
| $\rho^m = 0.2$ | - | - | - | - | - | - | - | - | - | + | + | + |
| $\rho^m = 0.3$ | - | - | - | - | - | - | - | - | - | + | + | + |
| $\rho^m = 0.4$ | - | - | - | - | - | - | - | - | - | - | + | + |
| $\rho^m = 0.5$ | - | - | - | - | - | - | - | - | - | - | + | + |
| $\rho^m = 0.6$ | - | - | - | - | - | - | - | - | - | - | + | + |
| $\rho^m = 0.7$ | - | - | - | - | - | - | - | - | - | - | + | + |
| $\rho^m = 0.8$ | - | - | - | - | - | - | - | - | - | + | + | + |
| $\rho^m = 0.9$ | - | - | - | - | - | - | - | - | - | + | + | + |
| $\rho^m = 0.95$ | - | - | - | - | - | - | - | - | + | + | + | + |
| $\rho^m = 0.99$ | - | - | - | - | - | + | + | + | + | + | + | + |

Note: + indicates that the real interest rate increases right after a positive monetary policy shock; - indicates that it decreases.

Table 9: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.1$

| | $\rho^i = 0$ | $\rho^i = 0.1$ | $\rho^i = 0.2$ | $\rho^i = 0.3$ | $\rho^i = 0.4$ | $\rho^i = 0.5$ | $\rho^i = 0.6$ | $\rho^i = 0.7$ | $\rho^i = 0.8$ | $\rho^i = 0.9$ | $\rho^i = 0.95$ | $\rho^i = 0.99$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho^m = 0$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.1$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.2$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.3$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.4$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.5$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.6$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.7$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.8$ | - | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.9$ | - | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.95$ | - | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.99$ | - | + | + | + | + | + | + | + | + | + | + | + |

Note: + indicates that the real interest rate increases right after a positive monetary policy shock; - indicates that it decreases.

Table 10: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.5$

| | $\rho^i = 0$ | $\rho^i = 0.1$ | $\rho^i = 0.2$ | $\rho^i = 0.3$ | $\rho^i = 0.4$ | $\rho^i = 0.5$ | $\rho^i = 0.6$ | $\rho^i = 0.7$ | $\rho^i = 0.8$ | $\rho^i = 0.9$ | $\rho^i$ =0.95 | $\rho^i$ =0.99 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho^m = 0$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.1$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.2$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.3$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.4$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.5$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.6$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.7$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.8$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.9$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m$ =0.95 | - | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m$ =0.99 | - | + | + | + | + | + | + | + | + | + | + | + |

Note: + indicates that the real interest rate increases right after a positive monetary policy shock; - indicates that it decreases.

So, how restricting is the direction-switching behavior of the real interest rate in response to a monetary policy shock for the estimation of VARs and DSGEs? We have seen that, in the presence of interest-rate smoothing, an empirically validated (Coibion and Gorodnichenko, 2012), theoretically desirable (Woodford (2003*b*), Sack and Wieland (2000)), and prevalent feature of medium-scale DSGE models (Smets and Wouters (2003) estimates $\rho^i = 0.956$ for the Euro Area; Smets and Wouters (2007) estimates $\rho^i = 0.75 - 0.84$ for the United States), a plausibly small adjustment cost is enough to reestablish the sign consistency with the real interest rate channel. Thus, observational equivalence is obtained in medium-scale New Keynesian models through a combination of realistic endogenous state variable rigidities.

## 4.2 The mechanics of smoothing

Now, using the method of undetermined coefficients, I explicitly derive the solution for the real interest rate and compare it to the case without smoothing.

Originally, there are three state variables ($\hat{k}_t$, $\xi^m_{t-1}$, $\hat{i}_{t-1}$) and one shock ($\epsilon^m_t$).[12] To reduce the number of coefficients I have to solve for, this representation can be simplified to just three state variables ($\hat{k}_t$, $\xi^m_t$, $\hat{i}_{t-1}$) using the monetary policy shock process equation. For

[12]In Dynare code, there is an additional state variable, $k_{t-1}$, that is used just for plotting capital at the beginning of the period.

the four jump variables, I assume $\hat{c}_t = a_0\hat{k}_t + a_1\xi_t^m + a_2\hat{i}_{t-1}$; $\pi_t = b_0\hat{k}_t + b_1\xi_t^m + b_2\hat{i}_{t-1}$; $\hat{y}_t = d_0\hat{k}_t + d_1\xi_t^m + d_2\hat{i}_{t-1}$; $\hat{k}_{t+1} = f_0\hat{k}_t + f_1\xi_t^m + f_2\hat{i}_{t-1}$. The set of coefficients to be determined for the solution of the full system is $\{a_0, a_1, a_2, b_0, b_1, b_2, d_0, d_1, d_2, f_0, f_1, f_2\}$.

With the log-linearized Fisher relation, $\hat{r}_t = \hat{i}_t - \mathbb{E}_t\pi_{t+1}$, and the Euler equation (34) I can write:

$$
\begin{aligned}
\hat{r}_t &= \mathbb{E}_t\,\hat{c}_{t+1} - \hat{c}_t \\
&= \underbrace{\left(a_0 f_0 - a_0 + a_2\left(1-\rho^i\right)\nu b_0\right)\hat{k}_t + \left(a_0 f_2 - a_2 + a_2\rho^i + a_2\left(1-\rho^i\right)\nu b_2\right)\hat{i}_{t-1}}_{=\,0 \text{ at the shock}} \\
&\quad + \left(\underbrace{\underbrace{\rho^m a_1 - a_1}_{\text{ex-smoothing}} + \underbrace{a_2\left(1-\rho^i\right)\nu b_1 + a_2}_{\text{smoothing}}}_{\text{indirect effect of capital}} \qquad \underbrace{+a_0 f_1}_{\text{direct effect of capital}}\right)\xi_t^m
\end{aligned}
$$

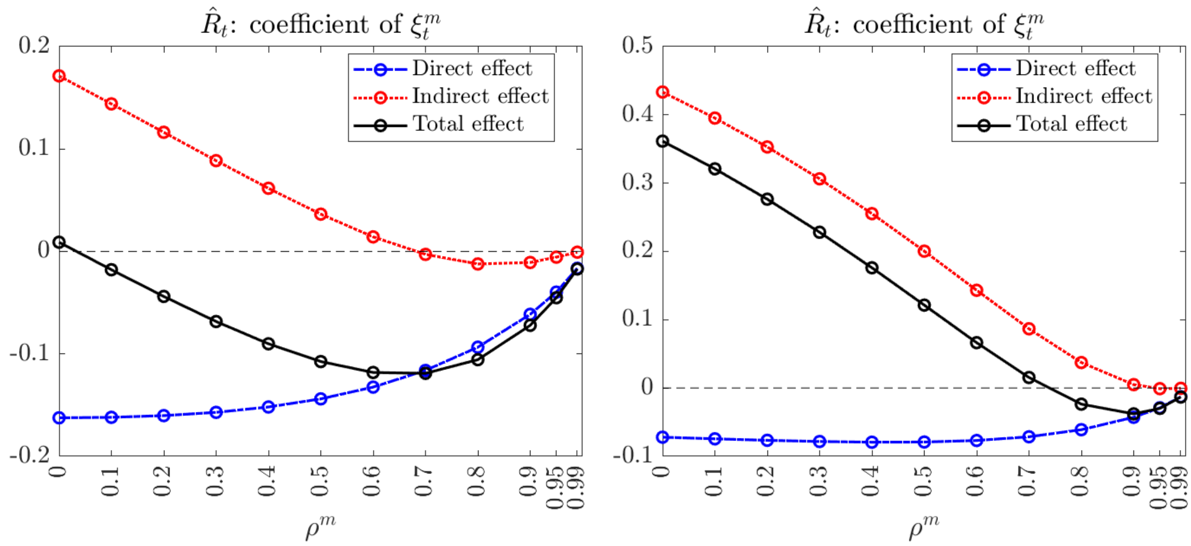
When I remove interest-rate smoothing, that is, when $\rho^i = 0$, $a_2 = 0$, $b_2 = 0$, $d_2 = 0$, and $f_2 = 0$, the model is the same as the one portrayed in Rupert and Šustek (2019).

The decision rules (first-order solutions) from Dynare (Adjemian et al. (2024)) yield numerical coefficients, each representing a partial derivative with respect to a state variable or a shock (e.g., $a_0 \equiv \frac{\partial \hat{c}_t}{\partial \hat{k}_t}$). With that in mind, I can decompose the immediate effect of the shock on the real interest rate into a direct effect of capital and an indirect one. The direct effect is analytically the same as in Rupert and Šustek (2019) since it depends only on the existence of endogenous capital in the model. The indirect effect, on the other hand, can be decomposed into two components: ex-smoothing and smoothing. The ex-smoothing component is the full indirect effect in Rupert and Šustek (2019), whereas the smoothing component appears in my model whenever $\rho^i > 0$. Although the direct effect of capital is always negative, the indirect effect can switch signs depending on how much consumption smoothing is allowed. For that, the shock's persistence, the policy rate's smoothing, and capital adjustment costs are key.[13] I call the total effect the sum of the direct and indirect effects.

[13]The depreciation rate of capital, $\delta$, is also important because it sets the $\overline{k} = \frac{\overline{K}}{\overline{Y}}$, but I prefer to keep it fixed to simplify the analysis.

$$\frac{\partial \hat{r}_t}{\partial \xi_t^m} = \underbrace{\underbrace{\left(\rho^m - 1\right)\frac{\partial \hat{c}_t}{\partial \xi_t^m}}_{\text{ex-smoothing}} + \underbrace{\left(1 + \left(1 - \rho^i\right)\nu\frac{\partial \hat{\pi}_t}{\partial \xi_t^m}\right)\frac{\partial \hat{c}_t}{\partial i_{t-1}}}_{\text{smoothing}}}_{\text{indirect effect of capital}} \quad \underbrace{+\frac{\partial \hat{c}_t}{\partial \hat{k}_t}\frac{\partial \hat{k}_{t+1}}{\partial \xi_t^m}}_{\text{direct effect of capital} < 0}$$

Under the benchmark calibration, with no adjustment costs and no interest-rate smoothing, the direct effect of capital on the real interest rate from a monetary policy shock is negative for all possible values of $\rho^m$, while the indirect effect is mostly positive. The absolute indirect effect is larger than the direct one only at the lowest range of $\rho^m$, as seen in Figure 4a. Note that for considerably persistent monetary policy shocks ($\rho^m > 0.7$), the indirect effect can be negative, which implies $\frac{\partial \hat{c}_t}{\partial \xi_t^m} > 0$, an atypical situation in which the prospect of a long spell of deflation motivates a consumption increase in the present due to the income effect.[14] Meanwhile, in Figure 4b, I show that raising $\kappa$ to 0.1 increases both components of the total effect, amplifying the range with the sign observationally consistent with the real interest rate channel.




(a) $\kappa = 0$ (b) $\kappa = 0.1$

Figure 4: Decomposition of the effect of capital on $\hat{r}_t$ from a monetary policy shock when $\rho^i = 0$

In Figure 5, I introduce interest-rate smoothing by setting $\rho^i = 0.5$, with no capital adjustment costs. In that case, the total effect curve becomes flatter near the zero axis. Raising

[14] This case is explored in more detail in Section 4.3.3.

the adjustment cost to $\kappa = 0.1$, as in Figure 6, is enough to turn the total effect curve positive for all possible values of $\rho^m$ even with just a little interest-rate smoothing ($\rho^i = 0.1$).

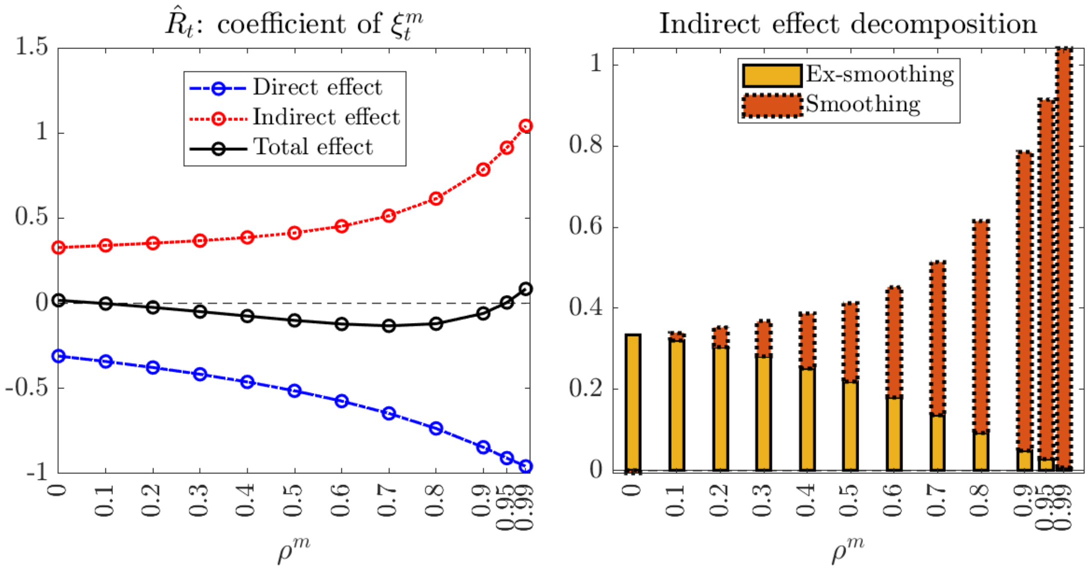



Figure 5: Decomposition of the effect of capital on $\hat{r}_t$ from a monetary policy shock when $\rho^i = 0.5$ and $\kappa = 0$

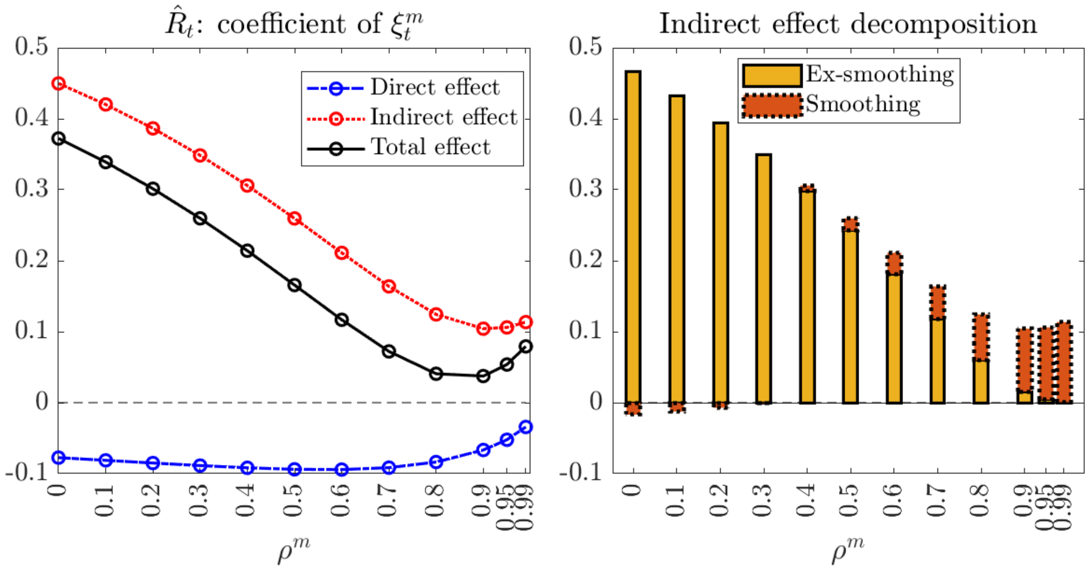



Figure 6: Decomposition of the effect of capital on $\hat{r}_t$ from a monetary policy shock when $\rho^i = 0.1$ and $\kappa = 0.1$

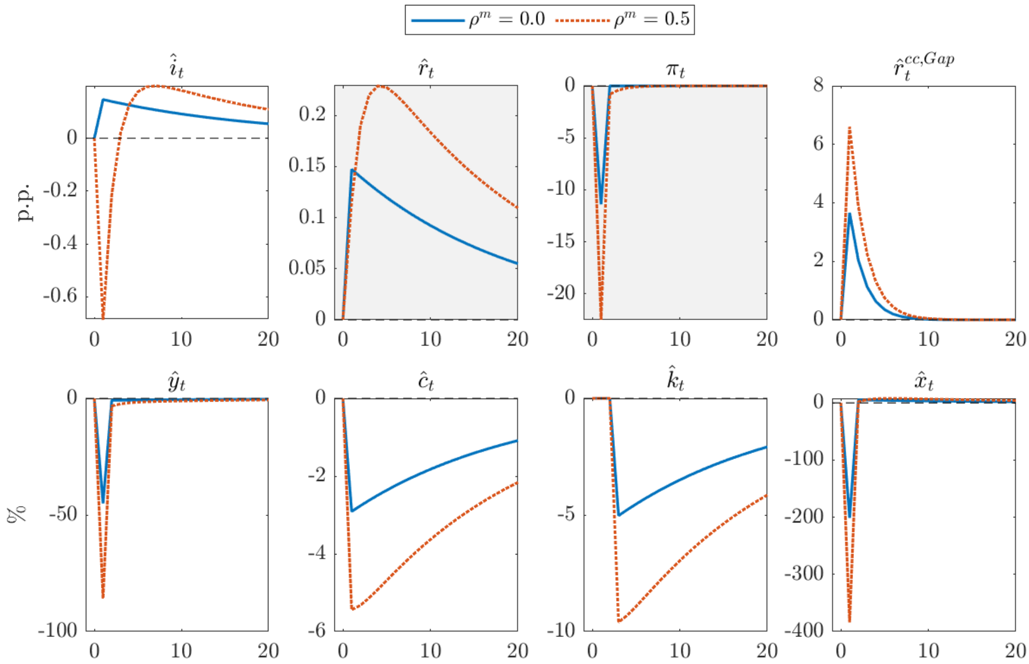
## 4.3 Different solutions for the capital puzzle

In this section, I explore different solutions for the capital puzzle while maintaining the conventional state-variant definition of the RIRG. For that, I plot impulse response functions of the New Keynesian model augmented with endogenous capital, adjustment costs, and interest-rate smoothing. I calibrate the standard deviation of the monetary policy shock to 1 p.p. The graphs display percentage deviations from steady-state values, except for interest rates, which are measured in p.p. deviations from steady-state values. As expected, output, consumption, and inflation respond negatively in the event of a contractionary monetary policy shock, except for the atypical case in which the income effect dominates the intertemporal substitution of consumption effect. In this last case, output and inflation fall, but consumption expands. The capital stock always decreases, but with a lag due to my timing convention. The nominal interest rate may react either positively or negatively, as the sign depends on inflation expectations and actual inflation, both of which may decrease significantly in the presence of persistence of the monetary policy shock, a well-documented pattern (Galí (2015) and Woodford (2003*a*, sec. 4.2.4)). Most importantly, the state-invariant RIRG, $r_t^{Gap,cc}$, is always positive right after a positive monetary policy shock.[15]

#### 4.3.1 Fixing with very high interest-rate smoothing

Figure 7 shows that high interest-rate smoothing ($\rho^i = 0.95$) restores the observational equivalence with the real interest rate channel of monetary policy transmission. However, without capital adjustment costs, the model predicts excessive output fluctuations.

[15]Interest-rate smoothing introduces a new state variable, which is lagged interest rate. Fixing that state would violate the determinacy condition of the model, therefore I refrain from fixing it in the calculation of $r_t^{Gap,cc}$ for the graphs of this section.

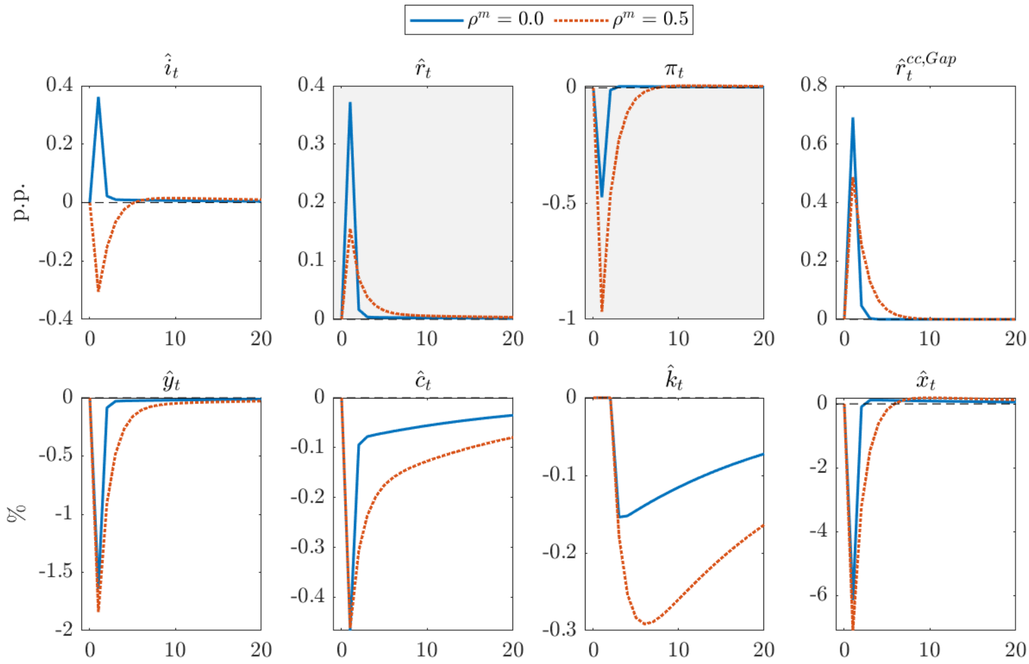



Note: hat variables are deviations from the zero-inflation-target steady state. $\hat{i}_t$ denotes the nominal interest rate; $\hat{r}_t$ the real interest rate; $\pi_t$ inflation; $\hat{r}_t^{Gap,cc}$ the real interest rate gap with constant capital; $\hat{y}_t$ output; $\hat{c}_t$ consumption; $\hat{k}_t$ capital at the beginning of period; and $\hat{x}_t$ investment.

Figure 7: Impulse response function to a one-standard-deviation monetary policy shock under $\rho^i = 0.95$ and $\kappa = 0$

#### 4.3.2 Fixing with very low interest-rate smoothing and small adjustment cost

Figure 8 shows that simply combining a very low level of smoothing ($\rho^i = 0.1$) with a small adjustment cost ($\kappa = 0.1$) resolves the identification problem. The adjustment cost still prevents output from overreacting right after the shock. Moreover, the negative association between changes in inflation and changes in the real interest rate does not depend on inflation expectations, differing from what is observed for changes in the nominal interest rate, whose sign depends on the persistence of the monetary policy shock.




Note: hat variables are deviations from the zero-inflation-target steady state. $\hat{i}_t$ denotes the nominal interest rate; $\hat{r}_t$ the real interest rate; $\pi_t$ inflation; $\hat{r}_t^{Gap,cc}$ the real interest rate gap with constant capital; $\hat{y}_t$ output; $\hat{c}_t$ consumption; $\hat{k}_t$ capital at the beginning of period; and $\hat{x}_t$ investment.

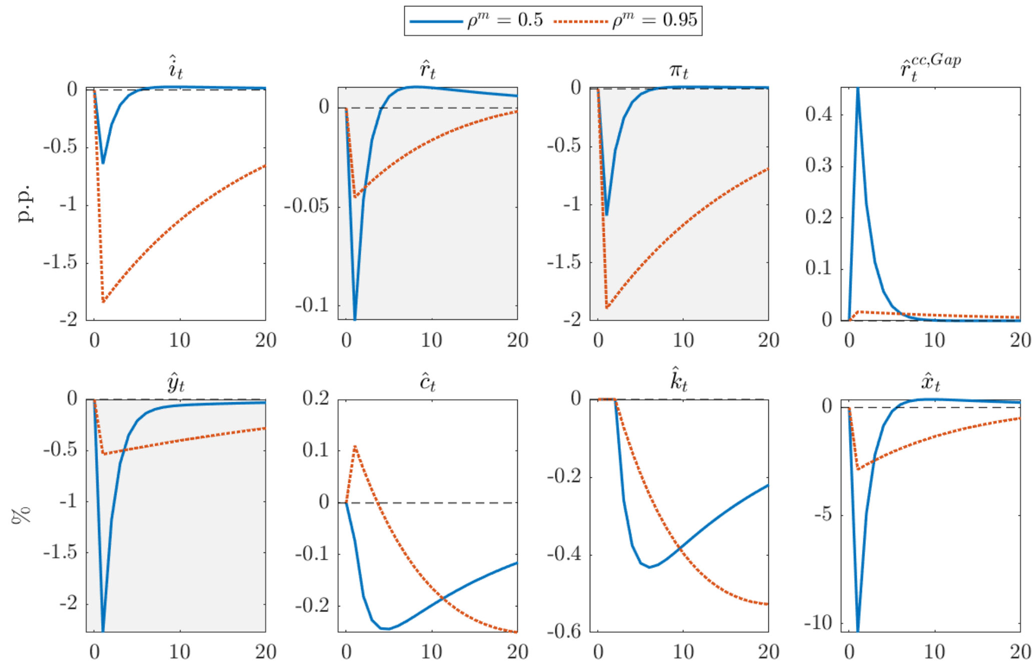
Figure 8: Impulse response function to a one-standard-deviation monetary policy shock under $\rho^i = 0.1$ and $\kappa = 0.1$

### 4.3.3 An atypical situation: when the income effect dominates

Figure 4a showed that for considerably persistent monetary policy shocks, say $\rho^m > 0.7$, the indirect effect of capital can be negative, which implies $\frac{\partial \hat{c}_t}{\partial \xi_t^m} > 0$. This atypical outcome in representative agent New Keynesian (RANK) models arises because the expectation of a prolonged period of deflation encourages households to increase their consumption in the present as they need to save less today to smooth their future consumption. This counterintuitive reaction to a monetary policy shock occurs because, in the absence of either capital or investment adjustment costs, investment becomes significantly more responsive (elastic) to the shock than consumption and, therefore, than output as well. In this scenario, the income effect of the monetary policy shock outweighs the intertemporal substitution effect, leading to the observed increase in current consumption.[16] As can be seen in Figure 9, by

[16]Kaplan, Moll and Violante (2018) comment that, for reasonable parameterizations, monetary policy in RANK models works almost exclusively through intertemporal substitution, while in heterogeneous agent models, indirect effects, such as the ones arising from changes in labor income, play a larger role.

comparing the impulse responses to a contractionary monetary policy shock for $\rho^m = 0.5$ and $\rho^m = 0.95$, the inconsistency with the real interest rate channel is present in this atypical case, but can also be solved with interest-rate smoothing, as shown in Figures 5 and 6. Here, the interpretation of $\hat{r}_t^{Gap,cc}$ is more challenging and raises a caveat. Although its sign at the impact of the monetary policy shock is consistent with theory, the total size of the tightening is smaller when $\rho^m$ is higher and the income effect dominates.




Note: hat variables are deviations from the zero-inflation-target steady state. Nominal interest rate ($\hat{i}_t$), real interest rate ($\hat{r}_t$), inflation ($\pi_t$), real interest rate gap with constant capital ($\hat{r}_t^{Gap,cc}$), output ($\hat{y}_t$), consumption ($\hat{c}_t$), capital at the beginning of period ($\hat{k}_t$), and investment ($\hat{x}_t$).

Figure 9: Impulse response function to a one-standard-deviation monetary policy shock under $\rho^m = 0.5$ and $\rho^m = 0.95$

## 5 Can Smets and Wouters (2007) do it?

Moving from textbooks to real-world central banking practice, how likely is it that the real interest rate channel identification problem will appear? In normal times, quite unlikely. Medium-scale New Keynesian models often include some additional ingredients that smooth consumption and investment, complementing interest-rate smoothing in the Taylor rule. Consumption habits, sticky wages, and investment adjustment costs all favor smooth changes in endogenous state variables. A more complete rule specification, which responds to output change, the output gap, and inflation expectations, may also support gradualism in monetary policy. In that sense, there are plenty of endogenous state variable frictions which, at the end of the day, guarantee observational equivalence with the real interest rate channel. Most clearly, Smets and Wouters (2007) note that investment adjustment costs induce hump-shaped dynamics that appear in real data. Shutting them off and re-estimating the model significantly deteriorates its marginal likelihood to the extent that these costs are the most empirically important real friction in the model. In this re-estimation, the need for heightened persistence is distributed over the remaining frictions: more persistent exogenous shocks, increased nominal rigidities, and more interest-rate smoothing.

### 5.1 Impulse response functions

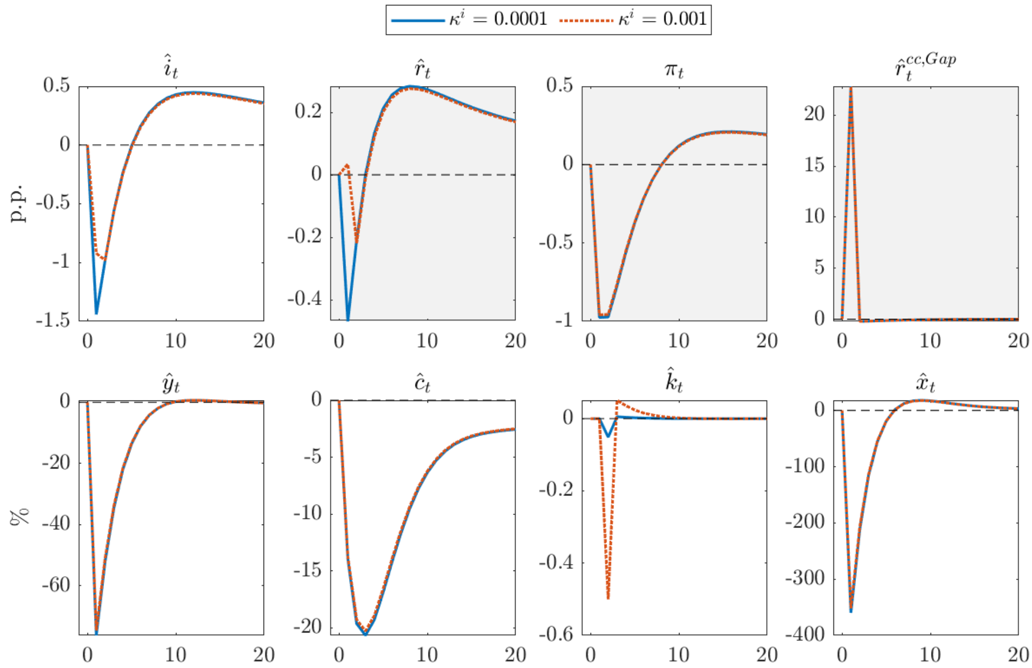
Using Smets and Wouters (2007) calibrated at the posterior mode, I replicate the real interest rate channel identification problem by slashing the investment adjustment cost parameter ($\kappa^i$) from 5.4882 to as low as 0.0001 and setting $\rho^m = \rho^i = 0$ (Figure 10).[17] Since $\kappa^i$ is a structural parameter, not a policy choice like $\rho^m$ and $\rho^i$, the negative association between changes in the real interest rate and changes in inflation is arguably much more robust than that between the latter and changes in the nominal interest rate, as it is immune to the Lucas (1976) critique. In this simulation, investment falls by more than 300%, likely exceeding the limits of the log-linear approximation of the model around a nonstochastic steady state. It is remarkable that the profound drops in output and consumption are not reflected in the real interest rate, but are clearly noticeable from the sign and the amplitude of $\hat{r}_t^{Gap,cc}$, which reaches more than 20 p.p.[18] This indicates that $\hat{r}_t^{Gap,cc}$ is not only sign-consistent with the stance of monetary policy, but it also may be a better gauge of its intensity.

In Figure 11, I change the policy parameter $\rho^m$, keeping $\rho^i = 0$, and apply the same monetary policy shock. Immediately after the shock, it is possible to make the nominal interest rate to move in either the same or the opposite direction as inflation. Nonetheless, $\hat{r}_t^{Gap,cc}$

[17] At that calibration, the sign switching requires higher values of $\kappa^i$. I thank Johannes Pfeifer for providing Dynare codes for the replication of Smets and Wouters (2007).

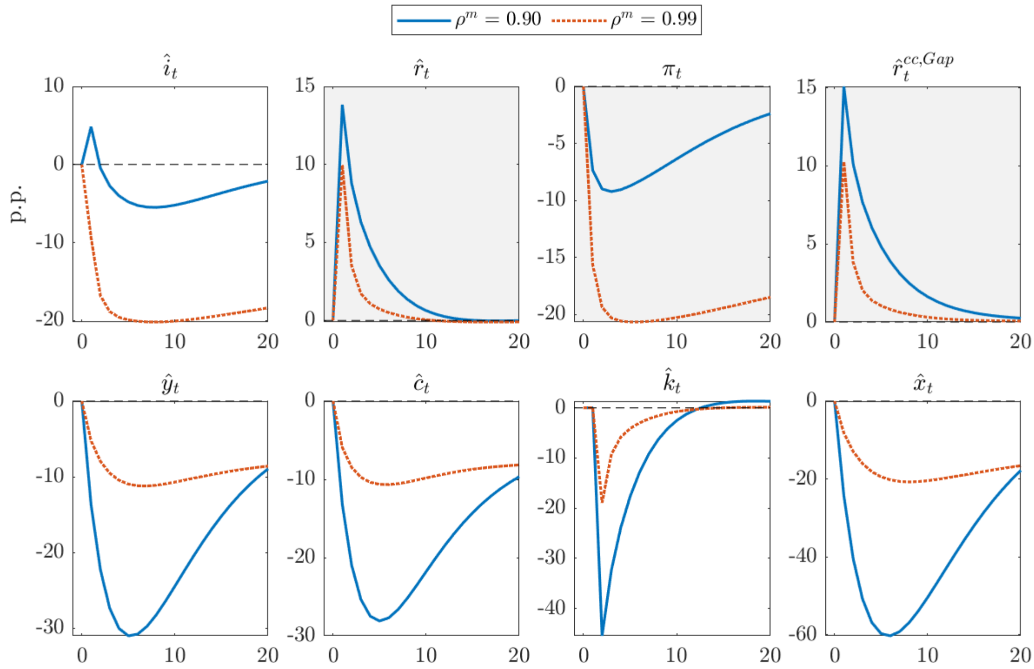
[18] $\hat{r}_t^{Gap,cc}$ is calculated by fixing capital stock, capital services, capital utilization and investment. The removed equations are the "Investment Euler Equation" (SW Equation 3), the "Definition of capital services" (SW Equation 6), the "Definition of degree of capital utilization" (SW Equation 7), and the "Law of motion for capital" (SW Equation 8). In the remaining equations, references to capital stock, capital services, degree of capital utilization or investment are fixed at their steady-state levels.

is always positive on the impact. Note that $\hat{r}_t^{Gap,cc}$ can turn negative in a future period after the shock, which did not happen in the canonical model augmented with capital (Section 4.3). This happens in this medium-scale model due to the effect of the other state variables that exist in it and, for convenience, are still allowed to vary in our calculation of $\hat{r}_t^{Gap,cc}$.




Note: hat variables are deviations from the zero-inflation-target steady state. $\hat{i}_t$ denotes the nominal interest rate; $\hat{r}_t$ the real interest rate; $\pi_t$ inflation; $\hat{r}_t^{Gap,cc}$ the real interest rate gap with constant capital; $\hat{y}_t$ output; $\hat{c}_t$ consumption; $\hat{k}_t$ capital at the beginning of period; and $\hat{x}_t$ investment.

Figure 10: Smets and Wouters (2007)'s impulse response function to a one-standard-deviation monetary policy shock under $\kappa^i = 0.0001$ and $\kappa^i = 0.001$. Both calibrations assume $\rho^m = 0$ and $\rho^i = 0$




Note: hat variables are deviations from the zero-inflation-target steady state. $\hat{i}_t$ denotes the nominal interest rate; $\hat{r}_t$ the real interest rate; $\pi_t$ inflation; $\hat{r}_t^{Gap,cc}$ the real interest rate gap with constant capital; $\hat{y}_t$ output; $\hat{c}_t$ consumption; $\hat{k}_t$ capital at the beginning of period; and $\hat{x}_t$ investment.

Figure 11: Smets and Wouters (2007)'s impulse response function to a one-standard-deviation monetary policy shock under $\rho^m = 0.90$ and $\rho^m = 0.99$. Both calibrations assume $\rho^i = 0$

## 5.2 Interest-rate smoothing

To show how interest-rate smoothing also helps with the identification problem in the Smets and Wouters (2007) model, I raise the investment adjustment cost parameter from 0.001 to 0.005 in Table 11 and sweep for different values of $\rho^m$ and $\rho^i$. Note that increasing interest-rate smoothing makes that channel more likely, as in the textbook model. Finally, in Table 12, I double the investment adjustment costs parameter and reestablish the real interest rate channel.

Table 11: Smets and Wouters (2007)'s parameter sweep with $\kappa^i = 0.005$

| | $\rho^i = 0$ | $\rho^i = 0.1$ | $\rho^i = 0.2$ | $\rho^i = 0.3$ | $\rho^i = 0.4$ | $\rho^i = 0.5$ | $\rho^i = 0.6$ | $\rho^i = 0.7$ | $\rho^i = 0.8$ | $\rho^i = 0.9$ | $\rho^i = 0.95$ | $\rho^i = 0.99$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho^m = 0$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.1$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.2$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.3$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.4$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.5$ | + | + | + | - | - | - | - | - | - | + | + | + |
| $\rho^m = 0.6$ | - | - | - | - | - | - | - | - | - | + | + | + |
| $\rho^m = 0.7$ | - | - | - | - | - | - | - | - | - | + | + | + |
| $\rho^m = 0.8$ | - | - | - | - | - | - | - | - | - | + | + | + |
| $\rho^m = 0.9$ | - | - | - | - | - | - | + | + | + | + | + | + |
| $\rho^m = 0.95$ | - | - | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.99$ | + | + | + | + | + | + | + | + | + | + | + | + |

Note: + indicates that the real interest rate increases right after a positive monetary policy shock; - indicates that it decreases.

Table 12: Smets and Wouters (2007)'s parameter sweep with $\kappa^i = 0.01$

| | $\rho^i = 0$ | $\rho^i = 0.1$ | $\rho^i = 0.2$ | $\rho^i = 0.3$ | $\rho^i = 0.4$ | $\rho^i = 0.5$ | $\rho^i = 0.6$ | $\rho^i = 0.7$ | $\rho^i = 0.8$ | $\rho^i = 0.9$ | $\rho^i = 0.95$ | $\rho^i = 0.99$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho^m = 0$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.1$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.2$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.3$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.4$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.5$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.6$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.7$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.8$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.9$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.95$ | + | + | + | + | + | + | + | + | + | + | + | + |
| $\rho^m = 0.99$ | + | + | + | + | + | + | + | + | + | + | + | + |

Note: + indicates that the real interest rate increases right after a positive monetary policy shock; - indicates that it decreases.

# 6 Forecasting inflation with the real interest rate gap

Neiss and Nelson (2003) find that the state-variant RIRG contains informational content that helps forecasting inflation, although the very own real interest rate is still a better, and more readily available, indicator. Next, I conduct a similar exercise with the models presented in Smets and Wouters (2007) and Smets and Wouters (2024). By scrutinizing the 9 regression specifications in Table 13, I find that the state-invariant gap frequently outperforms other measures of the RIRG.

| Model | Specification |
|---|---|
| 1 | Includes only lagged inflation as the predictor. |
| 2 | Adds the state-variant real interest rate gap $\left(\hat{r}^{\text{Gap}}\right)$. |
| 3 | Adds the state-consistent real interest rate gap $\left(\hat{r}^{\text{Gap,cons}}\right)$. |
| 4 | Adds the state-invariant real interest rate gap $\left(\hat{r}^{\text{Gap,cc}}\right)$. |
| 5 | Adds the real interest rate $(\hat{r})$. |
| 6 | Combines lagged inflation, $\left(\hat{r}^{\text{Gap}}\right)$, and $(\hat{r})$. |
| 7 | Combines lagged inflation, $\left(\hat{r}^{\text{Gap,cons}}\right)$, and $(\hat{r})$. |
| 8 | Combines lagged inflation, $\left(\hat{r}^{\text{Gap,cc}}\right)$, and $(\hat{r})$. |
| 9 | Combines lagged inflation, $\left(\hat{r}^{\text{Gap}}\right)$, $\left(\hat{r}^{\text{Gap,cons}}\right)$, $\left(\hat{r}^{\text{Gap,cc}}\right)$, and $(\hat{r})$. |

Table 13: Regression specifications

## 6.1 The importance of investment adjustment costs

To test how sizable investment adjustment costs affect the puzzle, I simulate the Smets and Wouters (2007)'s model with all its different types of shocks activated[19]. Starting from the non-stochastic steady state and with the model calibrated at the posterior mode estimated by the authors,[20] I simulate 250,000 periods and drop the first 50,000 periods. I find that the state-invariant gap has similar properties to the state-variant gap, also helping with forecast-

[19]There are seven orthogonal structural shocks: TFP, risk premium, investment-specific technology, price markup, wage markup, exogenous spending, and monetary policy. The two markup shocks are not present in the version of the model without nominal rigidities. Steady-state distortion due to monopolistic competition is not undone, which results that real disturbances affect the natural rate of interest and a counterfactual efficient rate differently.

[20]Estimation relies on seven macroeconomic quarterly time series for the United States: log difference of real GDP, real consumption, real investment, real wage, log hours worked, log difference of the GDP deflator, and the federal funds rate. The effective sample ranges from 1966Q1 to 2004Q4, where previous data starting in 1957Q1 is used as a training sample.

ing $\Delta_4\pi_{t+4}$, annual inflation 4-period ahead (Tables 14 and 15), but with the clear advantage of indicating the right sign for the monetary policy stance.

When investment adjustment costs are realistic or sizable, such as at the mode of the estimated parameters' posterior distribution, $\hat{r}_t^{Gap,cc}$ is as effective a predictor as $\hat{r}_t^{Gap}$ and a better predictor than $\hat{r}_t^{Gap,cons}$.[21] This is evidenced by lower and similar adjusted R-squared statistics and root mean square errors achieved in Model 2 and Model 4. However, when investment adjustment costs are negligible, such as by setting $\kappa^i = 0.0001$ while all other parameters remain unchanged, the more volatile endogenous state variables make $\hat{r}_t^{Gap}$ a generally better predictor than $\hat{r}_t^{Gap,cc}$. This can be explained by the fact that the first carries information about the state variables, which become more relevant for explaining the dynamics of the model in this case. Nevertheless, only $\hat{r}_t^{Gap,cc}$ is sign-consistent with actual monetary policy stance. Overall, $\hat{r}_t$ is the variable that individually improves forecasting performance the most, but only when adjustment costs are sizable. Combining all variables in the regression (Model 9) achieves the highest adjusted R-squared statistic and the lowest root mean square error across all forecasting set-ups, including those presented in Tables OA1 to OA6 in the Online Appendix. These last tables display the same forecasting exercise for $\pi_{t+1}$, $\Delta_4\pi_{t+1}$, and $\pi_{t+4}$.

The forecasting capability of the state-invariant gap can also be assessed by plotting the root mean square error of inflation regressions on lags of the three RIRGs, separately, across different horizons, while including lagged inflation as a predictor. Figure 12 plots the statistic for horizons from 1 to 10 periods ahead. As expected, the forecasting error grows as the horizon is pushed forward and it is higher if capital is more volatile. When investment adjustment costs are realistic, the state-invariant gap is as good a predictor as the state-variant gap, albeit, in longer horizons, the state-consistent gap outperforms the others. When adjustment costs are negligible, the state-variant gap is a slightly better predictor than the other two gaps.

[21]The state-consistent natural interest rate has the same policy function as the state-variant one, but the state variables are replaced by sticky-price ones.

Table 14: Forecasting $\Delta_4\pi_{t+4}$ under realistic investment adjustment costs

Forecasting 4-period ahead annual inflation

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-4}$ | 0.705*** | 0.700*** | 0.708*** | 0.703*** | 0.714*** | 0.708*** | 0.711*** | 0.714*** | 0.700*** |
| | (0.00158) | (0.00158) | (0.00179) | (0.00157) | (0.00153) | (0.00152) | (0.00152) | (0.00158) | (0.00175) |
| $\hat{r}^{Gap}_{t-4}$ | | -0.0860*** | | | | -0.103*** | | 0.0940*** | 0.147*** |
| | | (0.00159) | | | | (0.00154) | | (0.0161) | (0.0163) |
| $\hat{r}^{Gap,cons}_{t-4}$ | | | -0.00410*** | | | | | | 0.0217*** |
| | | | (0.00121) | | | | | | (0.00119) |
| $\hat{r}^{Gap,cc}_{t-4}$ | | | | -0.0928*** | | | -0.103*** | -0.196*** | -0.252*** |
| | | | | (0.00157) | | | (0.00152) | (0.0159) | (0.0162) |
| $\hat{r}_{t-4}$ | | | | | -0.809*** | -0.850*** | -0.835*** | -0.821*** | -0.825*** |
| | | | | | (0.00681) | (0.00676) | (0.00674) | (0.00716) | (0.00715) |
| Constant | 0.00331 | 0.00324 | 0.00342 | 0.00325 | 0.00785** | 0.00801** | 0.00794** | 0.00788** | 0.00733** |
| | (0.00327) | (0.00325) | (0.00328) | (0.00325) | (0.00317) | (0.00313) | (0.00313) | (0.00313) | (0.00313) |
| Observations | 199,993 | 199,993 | 199,993 | 199,993 | 199,993 | 199,993 | 199,993 | 199,993 | 199,993 |
| Adjusted R-squared | 0.498 | 0.505 | 0.498 | 0.506 | 0.531 | 0.541 | 0.541 | 0.541 | 0.542 |
| Root Mean Square Error | 1.465 | 1.454 | 1.465 | 1.452 | 1.415 | 1.400 | 1.399 | 1.399 | 1.398 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 15: Forecasting $\Delta_4\pi_{t+4}$ under negligible investment adjustment costs

Forecasting 4-period ahead annual inflation

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-4}$ | 0.690*** | 0.699*** | 0.691*** | 0.678*** | 0.684*** | 0.692*** | 0.674*** | 0.683*** | 0.684*** |
| | (0.00162) | (0.00156) | (0.00162) | (0.00158) | (0.00160) | (0.00154) | (0.00157) | (0.00152) | (0.00152) |
| $\hat{r}_{t-4}^{Gap}$ | | 3.865*** | | | | 3.688*** | | 3.553*** | 3.619*** |
| | | (0.0302) | | | | (0.0300) | | (0.0295) | (0.0296) |
| $\hat{r}_{t-4}^{Gap,cons}$ | | | -0.00400*** | | | | | | -0.00956*** |
| | | | (0.000486) | | | | | | (0.000462) |
| $\hat{r}_{t-4}^{Gap,cc}$ | | | | -0.168*** | | | -0.449*** | -0.420*** | -0.408*** |
| | | | | (0.00166) | | | (0.00487) | (0.00470) | (0.00473) |
| $\hat{r}_{t-4}$ | | | | | 0.126*** | 0.108*** | -0.303*** | -0.293*** | -0.278*** |
| | | | | | (0.00171) | (0.00165) | (0.00495) | (0.00478) | (0.00483) |
| Constant | 0.00331 | 0.00189 | 0.00342 | 0.00328 | 0.00255 | 0.00131 | 0.00504 | 0.00369 | 0.00385 |
| | (0.00351) | (0.00338) | (0.00351) | (0.00342) | (0.00346) | (0.00334) | (0.00339) | (0.00328) | (0.00327) |
| Observations | 199,993 | 199,993 | 199,993 | 199,993 | 199,993 | 199,993 | 199,993 | 199,993 | 199,993 |
| Adjusted R-squared | 0.477 | 0.516 | 0.477 | 0.502 | 0.491 | 0.526 | 0.511 | 0.544 | 0.545 |
| Root Mean Square Error | 1.570 | 1.509 | 1.570 | 1.531 | 1.549 | 1.494 | 1.517 | 1.465 | 1.463 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

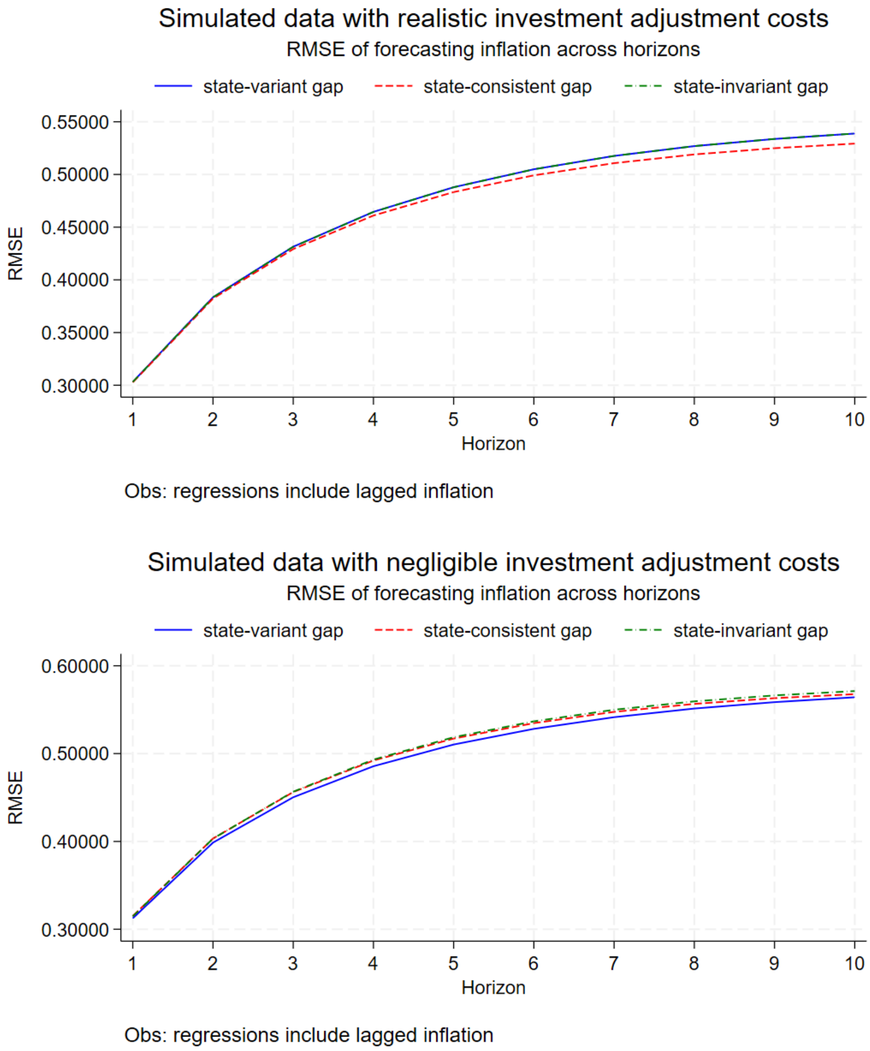



Figure 12: Root mean square error of simulating Smets and Wouters (2007)'s model with random data for different horizons

## 6.2 Forecasting with historical data

The historical performance of the state-invariant gap is evaluated through a *quasi-final* estimation of the model, in the typology proposed by Orphanides and Norden (2002), where parameters are constant and estimated with the full sample, but unobservable states are recovered with data up to the period of the observation by a Kalman filtering process.[22]

[22] Orphanides and Norden (2002) differentiate four types of estimation: real-time, quasi-real, quasi-final, and final. Real-time estimation uses all data available up until the period of the observation but that are still subject to further revision; quasi-real uses the same data but parameters are time-varying as they are re-estimated with data available up until the period of each observation; quasi-final estimates parameters with the full final sample, but states are recovered using data available up until the period of the observation

Figure 13 displays observed inflation for the United States from 1966Q1 to 2004Q4 as well as filtered natural real interest rates and filtered RIRGs for the same period generated by the Smets and Wouters (2007)'s model, after feeding it with the same historical data the authors used for estimating its parameters. Note that the state-invariant gap is considerably more negative than the state-variant one until the beginning of the 1980s. From then onward, both measures become numerically closer.

Forecasting observed inflation using the state-invariant gap demonstrates superior performance compared to the state-variant or state-consistent gaps. This is true for both short and long-term forecasts, or from 1 to 10 periods ahead, in the sample that ranges from 1966Q1 to 2004Q4 (Table 16 and top panel of Figure 14). Here, the state-invariant gap performs better than even the filtered real interest rate. To investigate the stability of these results, the same exercise is conducted for two subsamples with the parameters reestimated: the "Great Inflation" from 1966Q1 to 1979Q2, when Paul Volcker was nominated chairman of the Board of Governors of the Federal Reserve System,[23] and the "Great Moderation" from 1984Q1 to 2004Q4. Overall, the state-invariant RIRG tends to perform better or nearly as good up to four quarters ahead in both subsamples (Tables 17 to 18 in addition to middle and bottom panels of Figure 14). Aligned with theory, forecasting errors are smaller during the "Great Moderation", when inflation was relatively low and stable, compared to during the "Great Inflation", when it was high and quite volatile.

(filtering); final estimates uses the full final sample to estimate parameters and recover the unobserved states (smoothing). A real-time analysis of the forecasting properties of the state-invariant gap is left for future research.

[23] Paul A. Volcker was nominated by President Jimmy Carter on 25 July 1979 (Carter, 1979).

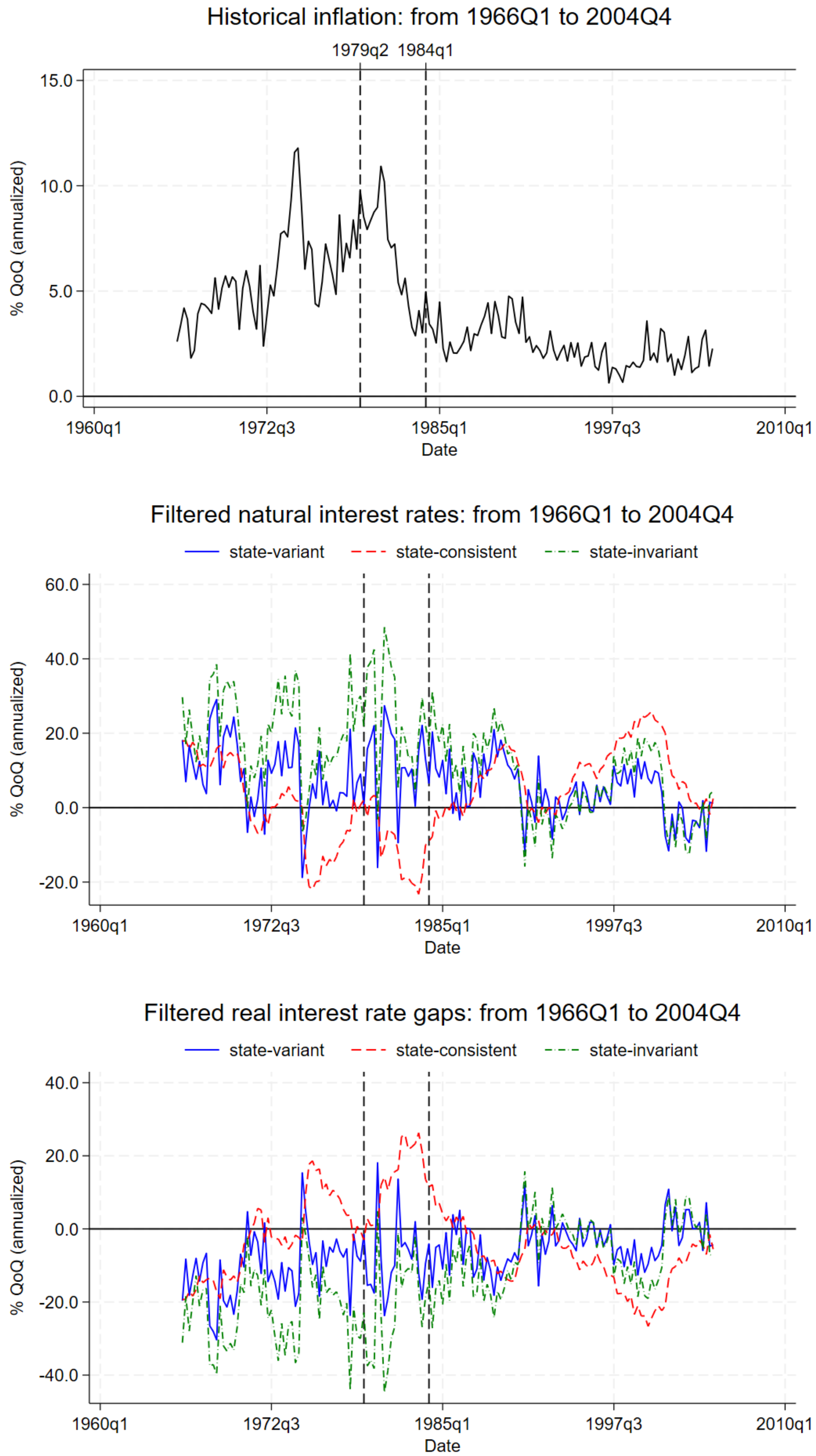



Figure 13: Observed and filtered data obtained from simulating Smets and Wouters (2007)'s model with historical data

Table 16: Forecasting $\Delta_4\pi_{t+4}$ from 1966Q1 to 2004Q4

Forecasting 4-period ahead annual inflation: from from 1966Q1 to 2004Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-4}$ | 0.855*** | 0.830*** | 0.961*** | 0.723*** | 0.862*** | 0.836*** | 0.721*** | 0.590*** | 0.620*** |
| | (0.0414) | (0.0390) | (0.0482) | (0.0399) | (0.0404) | (0.0373) | (0.0372) | (0.0437) | (0.0543) |
| $\hat{r}_{t-4}^{Gap}$ | | -0.208*** | | | | -0.224*** | | 0.440*** | 0.425*** |
| | | (0.0425) | | | | (0.0409) | | (0.0890) | (0.0905) |
| $\hat{r}_{t-4}^{Gap,cons}$ | | | -0.146*** | | | | | | -0.0296 |
| | | | (0.0379) | | | | | | (0.0318) |
| $\hat{r}_{t-4}^{Gap,cc}$ | | | | -0.210*** | | | -0.229*** | -0.525*** | -0.507*** |
| | | | | (0.0285) | | | (0.0269) | (0.0649) | (0.0675) |
| $\hat{r}_{t-4}$ | | | | | -0.447*** | -0.522*** | -0.590*** | -0.629*** | -0.606*** |
| | | | | | (0.146) | (0.135) | (0.122) | (0.114) | (0.116) |
| Constant | 0.115 | -0.242** | -0.101 | -0.534*** | 0.114 | -0.272** | -0.592*** | -0.747*** | -0.764*** |
| | (0.100) | (0.119) | (0.111) | (0.123) | (0.0978) | (0.114) | (0.116) | (0.112) | (0.114) |
| Observations | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 |
| Adjusted R-squared | 0.733 | 0.768 | 0.755 | 0.802 | 0.747 | 0.787 | 0.827 | 0.850 | 0.850 |
| Root Mean Square Error | 1.194 | 1.114 | 1.144 | 1.029 | 1.163 | 1.067 | 0.960 | 0.894 | 0.894 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 17: Forecasting $\Delta_4\pi_{t+4}$ from 1966Q1 to 1979Q2

Forecasting 4-period ahead annual inflation: from from 1966Q1 to 1979Q2

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-4}$ | 0.626*** | 0.800*** | 0.700*** | 0.770*** | 0.640*** | 0.811*** | 0.768*** | 0.419*** | 0.428*** |
| | (0.0944) | (0.0974) | (0.123) | (0.0843) | (0.0962) | (0.0978) | (0.0844) | (0.102) | (0.112) |
| $\hat{r}_{t-4}^{Gap}$ | | -0.445*** | | | | -0.519*** | | 2.662*** | 2.678*** |
| | | (0.122) | | | | (0.141) | | (0.558) | (0.567) |
| $\hat{r}_{t-4}^{Gap,cons}$ | | | -0.102 | | | | | | -0.0202 |
| | | | (0.109) | | | | | | (0.0911) |
| $\hat{r}_{t-4}^{Gap,cc}$ | | | | -0.505*** | | | -0.547*** | -2.800*** | -2.809*** |
| | | | | (0.104) | | | (0.113) | (0.481) | (0.488) |
| $\hat{r}_{t-4}$ | | | | | 0.387 | -0.525 | -0.416 | 0.951** | 0.922* |
| | | | | | (0.480) | (0.496) | (0.433) | (0.461) | (0.483) |
| Constant | 1.245*** | 0.113 | 1.052*** | -0.331 | 1.379*** | -0.257 | -0.604 | -0.386 | -0.420 |
| | (0.286) | (0.403) | (0.352) | (0.404) | (0.332) | (0.534) | (0.494) | (0.415) | (0.448) |
| Observations | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 |
| Adjusted R-squared | 0.448 | 0.553 | 0.447 | 0.614 | 0.444 | 0.554 | 0.614 | 0.731 | 0.726 |
| Root Mean Square Error | 1.419 | 1.276 | 1.421 | 1.186 | 1.424 | 1.275 | 1.187 | 0.990 | 1 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 18: Forecasting $\Delta_4 \pi_{t+4}$ from 1984Q1 to 2004Q4

Forecasting 4-period ahead annual inflation: from from 1984Q1 to 2004Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4 \hat{\pi}_{t-4}$ | 0.721*** | 0.739*** | 0.737*** | 0.701*** | 0.676*** | 0.726*** | 0.809*** | 0.833*** | 0.786*** |
| | (0.0654) | (0.0629) | (0.0775) | (0.0590) | (0.0812) | (0.0802) | (0.0762) | (0.0751) | (0.0813) |
| $\hat{r}_{t-4}^{Gap}$ | | -0.106*** | | | | -0.104*** | | 0.142** | 0.134** |
| | | (0.0363) | | | | (0.0377) | | (0.0625) | (0.0622) |
| $\hat{r}_{t-4}^{Gap,cons}$ | | | -0.00797 | | | | | | 0.0274 |
| | | | (0.0203) | | | | | | (0.0186) |
| $\hat{r}_{t-4}^{Gap,cc}$ | | | | -0.101*** | | | -0.137*** | -0.232*** | -0.242*** |
| | | | | (0.0224) | | | (0.0275) | (0.0498) | (0.0499) |
| $\hat{r}_{t-4}$ | | | | | 0.122 | 0.0353 | -0.312** | -0.495*** | -0.518*** |
| | | | | | (0.130) | (0.129) | (0.144) | (0.162) | (0.161) |
| Constant | -0.135** | -0.276*** | -0.149** | -0.461*** | -0.169** | -0.283*** | -0.492*** | -0.561*** | -0.550*** |
| | (0.0569) | (0.0729) | (0.0677) | (0.0887) | (0.0673) | (0.0769) | (0.0879) | (0.0910) | (0.0906) |
| Observations | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 |
| Adjusted R-squared | 0.592 | 0.627 | 0.588 | 0.670 | 0.592 | 0.622 | 0.684 | 0.700 | 0.704 |
| Root Mean Square Error | 0.510 | 0.488 | 0.513 | 0.459 | 0.510 | 0.491 | 0.449 | 0.438 | 0.434 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

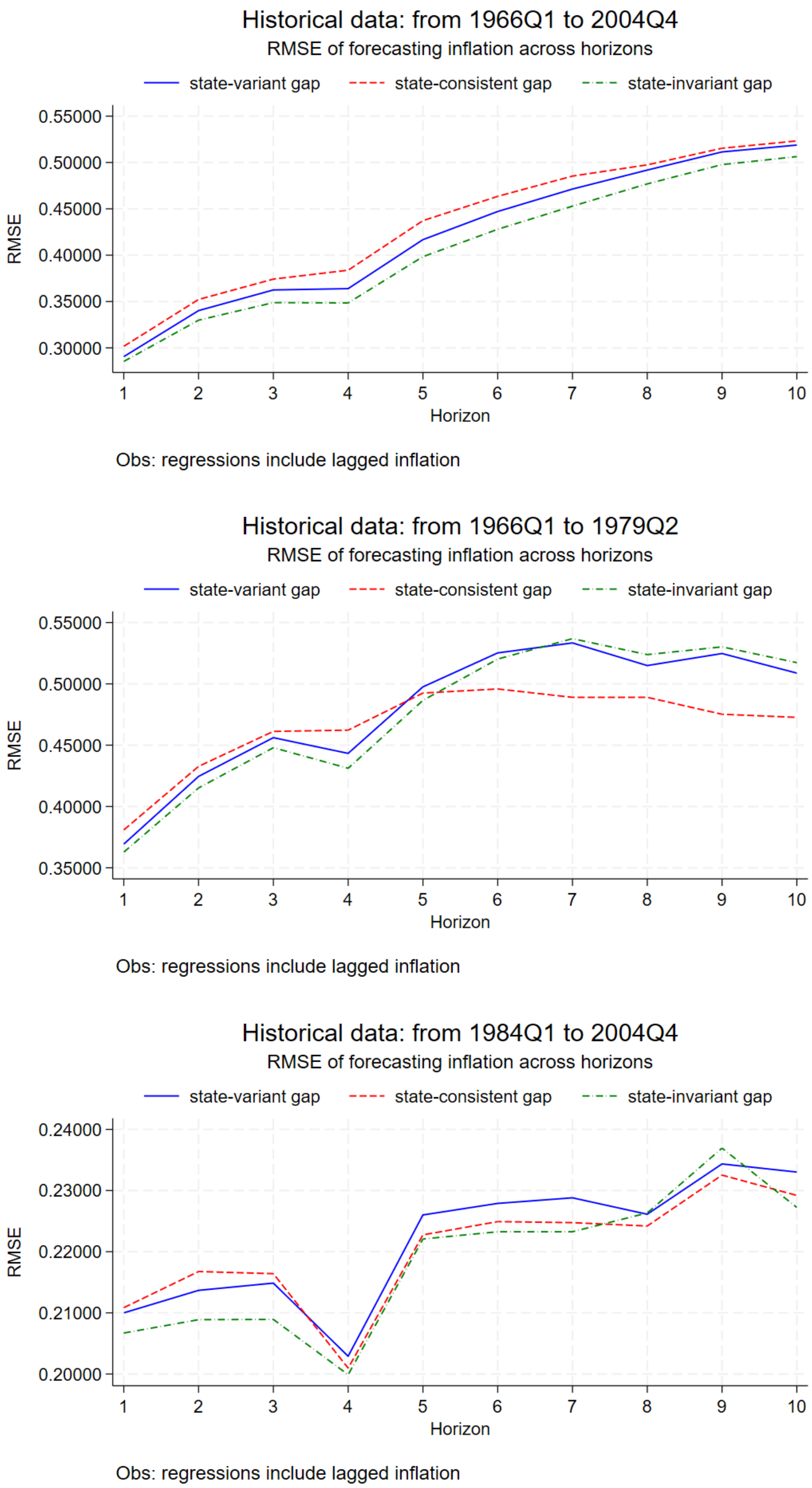



Figure 14: Root mean square error of simulating Smets and Wouters (2007)'s model with historical United States data for different horizons

I extend now the analysis to a more recent period by employing Smets and Wouters

(2024)'s model, which extends Smets and Wouters (2007) in two ways.[24] First, it allows for intermediate monetary/fiscal policy regimes, which means that inflation may be only partially backed by fiscal policy or even not backed at all. Second, it tackles the zero lower bound challenge of the periods after the Global Financial Crisis (GFC) by incorporating an additional monetary policy shock to accommodate for changes in the FED's forward guidance policy. The estimation sample ranges from 1965Q1 to 2019Q4 and it augments the list of data series with the 1-year short-term interest rate and four fiscal variables: (1) the market value of US government debt, (2) the total government primary balance, (3) social security transfers and (4) government spending. Calibrating the parameters of the model at the posterior mode, the validity of the state-invariant gap as a relevant measure of monetary policy stance is confirmed.

Concerning the forecasting capability of the three different RIRGs, the state-invariant definition performs similarly to the state-variant on average in the whole sample for quarterly inflation in most horizons (Figure 15). Moreover, considering annual inflation four quarters ahead, the state-invariant is the one that performs best as can be seen in Table 19, which also shows that forecasting benefits from simultaneously incorporating the three different measures of the RIRG. Table 20 summarises the RMSEs from all regression specifications and shows the p-values from the respective Diebold-Mariano equal forecast accuracy tests (Diebold and Mariano, 1995).[25] While for 1-quarter-ahead quarterly inflation it is not possible to discard that the autoregressive model forecasts as accurate as the models expanded with RIRGs, for 1-quarter-ahead annual inflation, 4-quarter-ahead quarterly inflation and 4-quarter-ahead annual inflation, the models expanded with RIRGs improve forecasting accuracy at the 5% level and most specifications even at the 1% level.

In subsamples, rank positions change depending on the period.[26] Forecasting beyond the estimation sample period, during the COVID-19 pandemic age (2020Q1-2023Q3), when investment plunged and then had a large recovery, and when forecasting errors were especially large, the state-invariant gap was a better predictor of future inflation for all horizons up to 5

[24] I thank Frank Smets and Raf Wouters for kindly sharing the codes for both papers.

[25] Results should not be interpreted as comparisons between models (Diebold, 2015).

[26] Re-estimating the model in subsamples, during the Great Inflation (1965Q1-1979Q2), the state-consistent and state-variant perform better in short horizons, while the state-invariant performs best starting from 7 quarters ahead. However, during the Great Moderation (1984Q1-2004Q4), there is not significant differences in root mean square errors for most forecasting horizons.

quarters ahead. Moreover, when inflation is only monetary-led or only fiscal-led, the state-invariant measure is usually the best predictor (Online Appendix, tables OA25 to OA30). Overall, there is not much robustness on ranking orders, but the result that combining the three definitions for the RIRG improves forecasting is confirmed most times.

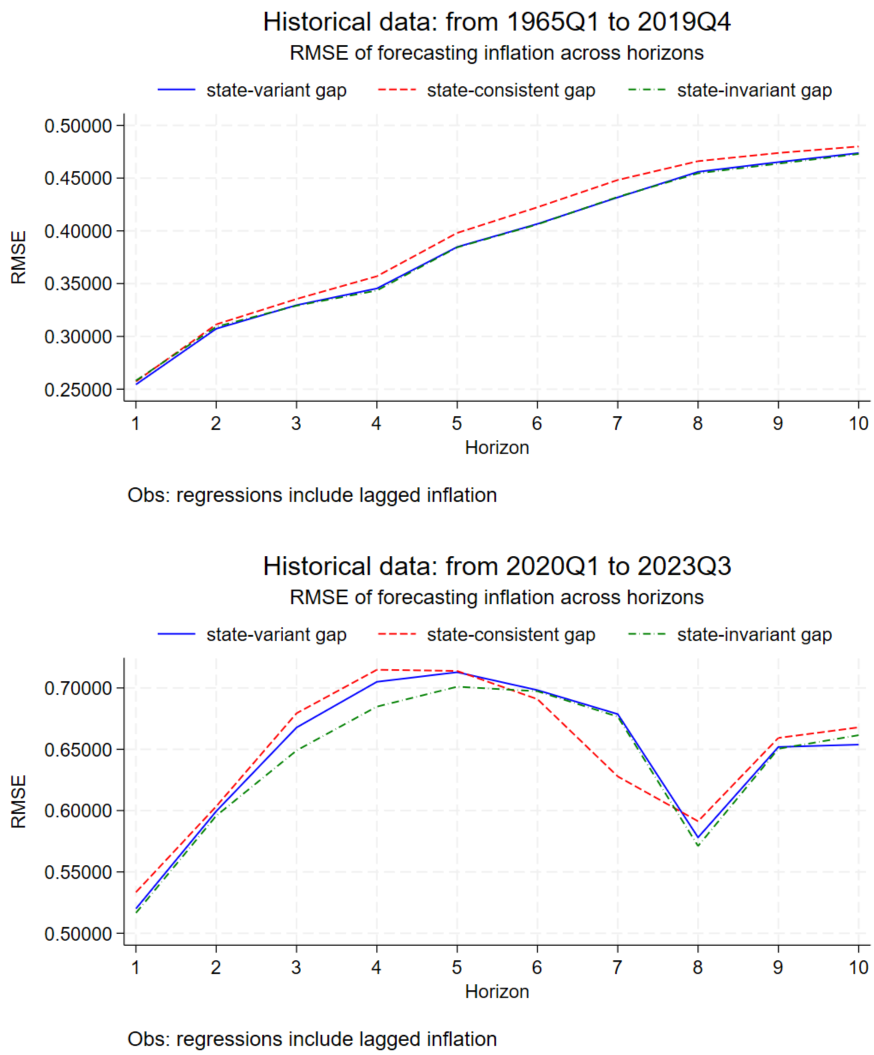



Figure 15: Root mean square error of simulating Smets and Wouters (2024)'s model with historical United States data for different horizons

Table 19: Forecasting $\Delta_4\pi_{t+4}$ from 1965Q1 to 2019Q4

Forecasting 4-period ahead annual inflation: from from 1965Q1 to 2019Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-4}$ | 0.869*** | 0.818*** | 0.866*** | 0.822*** | 0.886*** | 0.833*** | 0.841*** | 0.857*** | 0.839*** |
| | (0.0341) | (0.0349) | (0.0343) | (0.0326) | (0.0356) | (0.0366) | (0.0338) | (0.0354) | (0.0390) |
| $\hat{r}^{Gap}_{t-4}$ | | -0.393*** | | | | -0.383*** | | 0.238 | 0.0871 |
| | | (0.0921) | | | | (0.0922) | | (0.163) | (0.214) |
| $\hat{r}^{Gap,cons}_{t-4}$ | | | -0.0600 | | | | | | -0.0940 |
| | | | (0.0702) | | | | | | (0.0860) |
| $\hat{r}^{Gap,cc}_{t-4}$ | | | | -0.350*** | | | -0.354*** | -0.486*** | -0.416*** |
| | | | | (0.0586) | | | (0.0582) | (0.107) | (0.125) |
| $\hat{r}_{t-4}$ | | | | | -0.199 | -0.159 | -0.225** | -0.260** | -0.223* |
| | | | | | (0.121) | (0.118) | (0.112) | (0.115) | (0.120) |
| Constant | 0.179** | 0.158* | 0.170* | 0.169** | 0.154* | 0.138 | 0.140* | 0.144* | 0.128 |
| | (0.0893) | (0.0861) | (0.0901) | (0.0829) | (0.0903) | (0.0871) | (0.0836) | (0.0834) | (0.0847) |
| Observations | 216 | 216 | 216 | 216 | 216 | 216 | 216 | 216 | 216 |
| Adjusted R-squared | 0.751 | 0.769 | 0.750 | 0.785 | 0.752 | 0.770 | 0.788 | 0.789 | 0.790 |
| Root Mean Square Error | 1.107 | 1.065 | 1.107 | 1.027 | 1.102 | 1.063 | 1.020 | 1.017 | 1.016 |

Standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 20: Forecasting performance from 1965Q1 to 2019Q4

| | RMSE | | | | Diebold-Mariano test p-value | | | |
|---|---|---|---|---|---|---|---|---|
| | $\hat{\pi}_{t+1}$ | $\Delta_4\hat{\pi}_{t+1}$ | $\hat{\pi}_{t+4}$ | $\Delta_4\hat{\pi}_{t+4}$ | $\hat{\pi}_{t+1}$ | $\Delta_4\hat{\pi}_{t+1}$ | $\hat{\pi}_{t+4}$ | $\Delta_4\hat{\pi}_{t+4}$ |
| Model 1 | 0.260 | 0.372 | 0.356 | 1.107 | | | | |
| Model 2 | 0.979 | 0.974 | 0.951 | 0.962 | 0.424 | 0.000 | 0.012 | 0.000 |
| Model 3 | 0.990 | 1.002 | 1.001 | 0.992 | 0.338 | 0.000 | 0.001 | 0.000 |
| Model 4 | 0.994 | 0.941 | 0.924 | 0.936 | 0.247 | 0.000 | 0.014 | 0.000 |
| Model 5 | 0.995 | 0.997 | 1.002 | 1.002 | 0.732 | 0.000 | 0.001 | 0.000 |
| Model 6 | 0.973 | 0.973 | 0.953 | 0.964 | 0.552 | 0.000 | 0.012 | 0.000 |
| Model 7 | 0.990 | 0.936 | 0.925 | 0.936 | 0.347 | 0.000 | 0.016 | 0.000 |
| Model 8 | 0.970 | 0.931 | 0.926 | 0.936 | 0.756 | 0.000 | 0.013 | 0.000 |
| Model 9 | 0.973 | 0.933 | 0.922 | 0.918 | 0.789 | 0.000 | 0.015 | 0.000 |

Note: Model specifications are available in Table 13. The entries in the first row show the RMSE of the benchmark autoregressive forecast, while other entries show the ratio of the RMSE of forecasts based on that row's regression specification to the RMSE of the benchmark forecast. Therefore, a ratio below unity is an improvement in forecast accuracy. Each p-value corresponds to the two-sided distributions of the equal-forecast accuracy test of Diebold and Mariano (1995), where the null hypothesis is that both accuracies are equal. The maximum lag order of the test is calculated from the Schwert criterion as a function of the sample size. The loss criterion used is the mean-squared error. The kernel used in calculating the long-run variance is Bartlett.

# 7 The monetary policy stance under a pre-pandemic lens

The Covid-19 pandemic outbreak in March 2020 posed significant challenges for interpreting DSGE models. First, the lockdown and sanitary measures acted as both a negative demand shock (forced savings) and a negative supply shock (labor hoarding). Second, to alleviate the consequences of the economic slowdown, governments implemented income transfer policies on an unprecedented scale. Third, the intensity and persistence of the shocks, as well as their one-off nature, affected the seasonal adjustment processes adopted in the time series used to evaluate economic models. Given the extent of these challenges, there is yet to be a consensual adaptation of DSGE models to address them. In light of this, I use the

Smets and Wouters (2024)'s model calibrated at the posterior mode of the estimation made with data prior to the pandemic, that is, from 1965Q1 to 2019Q4, and filter the unobserved state variables from 2020Q1 onward. By doing so, it is possible to recover the RIRGs and determine how much of them is explained by the time-cumulative contribution of monetary policy shocks[27] under the lens of what was known about the structure of the economy before the pandemic (Figure 16). While this approach is subject to criticism regarding the uniqueness of the pandemic event, it serves as the minimum benchmark a central bank would have access to in assessing the monetary policy stance. It also shows that in addition to being a more robust signal of the latter, and of its enhanced forecasting capability (see Section 6), the state-invariant RIRG also serves as a narrative instrument.

From a historical perspective, Figure 16 tells us that monetary policy was considerably loose in the late 1960s and throughout the 1970s, the Great Inflation period. That stance was abruptly reversed in 1979 with the escalation of interest rate hikes promoted by Paul Volcker at the head of the Federal Reserve. Monetary policy crossed again the neutrality line in the late 1980s to become moderately loose and remained largely so until the second half of the 1990s, when it shifted in favor of even looser policy. This long period with a loose monetary stance that started at the late 1980s and ended with the GFC is the so-called Great Moderation period. From late 2007 onward, the sub-prime financial crisis that ignited the GFC resulted in an involuntary contractionary shift of monetary policy as the real natural rate became negative and the Federal Reserve was constrained by the effective lower bound of the policy rate. Unconventional monetary policy measures may have reduced the contractionary intensity of the monetary policy stance, but they were not enough to force the latter consistently into stimulative territory. The overall normalization of monetary policy in the late 2010s maintained a moderately contractionary stance. This is the situation faced by the US economy when the Covid-19 shock hit in March 2020.

As was the case during the onset of the sub-prime crisis, the immediate consequence of the pandemic shock was an intense and involuntary contraction of the monetary policy stance of up to near 30 p.p. on an annual basis, where a supposed delayed reaction by the FED and the effective lower bound itself are only accountable for around 6 p.p., or 20% of

[27]The model includes two exogenous monetary policy shocks: one affecting the policy rate, which directly influences short-term interest rates, and another affecting the 1-year zero-coupon yield, which captures expectations of future monetary policy.

that tightening, in this decomposition.[28] The FED's reaction by aggressively cutting rates in March by a total of 150 bps, to close to 0%, helped to loosen the monetary policy stance, but most of the latter's swing stemmed from the pandemic shock itself. As a consequence, Figure 17 shows that quarterly inflation was the lowest in the sample that starts in 1965Q1, while investment plunged almost 8.0% on a quarterly basis.[29] In 2020Q3, the RIRG got into stimulative territory, even though the effective lower bound of the monetary policy rate continued contributing to a tighter monetary policy stance until 2021Q1, quarter in which a rebound of the RIRG coincided with reinforced lockdown measures due to the emergence of new strains of coronavirus.[30] It was only from 2021Q2 onward that the US economy consistently navigated a loose monetary policy stance, which lasted until 2023Q2, when the RIRG reached neutrality. The loose monetary policy stance period overlaps with the period in which inflation was above the 2% target (Figure 17), while investment remained subdued.

[28] As the effective lower bound is not explicitly incorporated into the model, it is also not incorporated into agents expectations. This results into "unexpected" contractionary monetary shocks whenever the constraint is binding.

[29] Nominal fixed private investment is deflated by GDP deflator, following the dataset choice of Smets and Wouters (2024).

[30] US CDC (Centers for Disease Control and Prevention) released data in a Morbidity and Mortality Weekly Report (MMWR) on the emerging and more transmissible COVID-19 B.1.1.7 / "Alpha" variant, and recommended "universal and increased compliance with mitigation strategies, like social distancing and masking, and higher vaccination coverage to protect the public" (CDC, 2025).

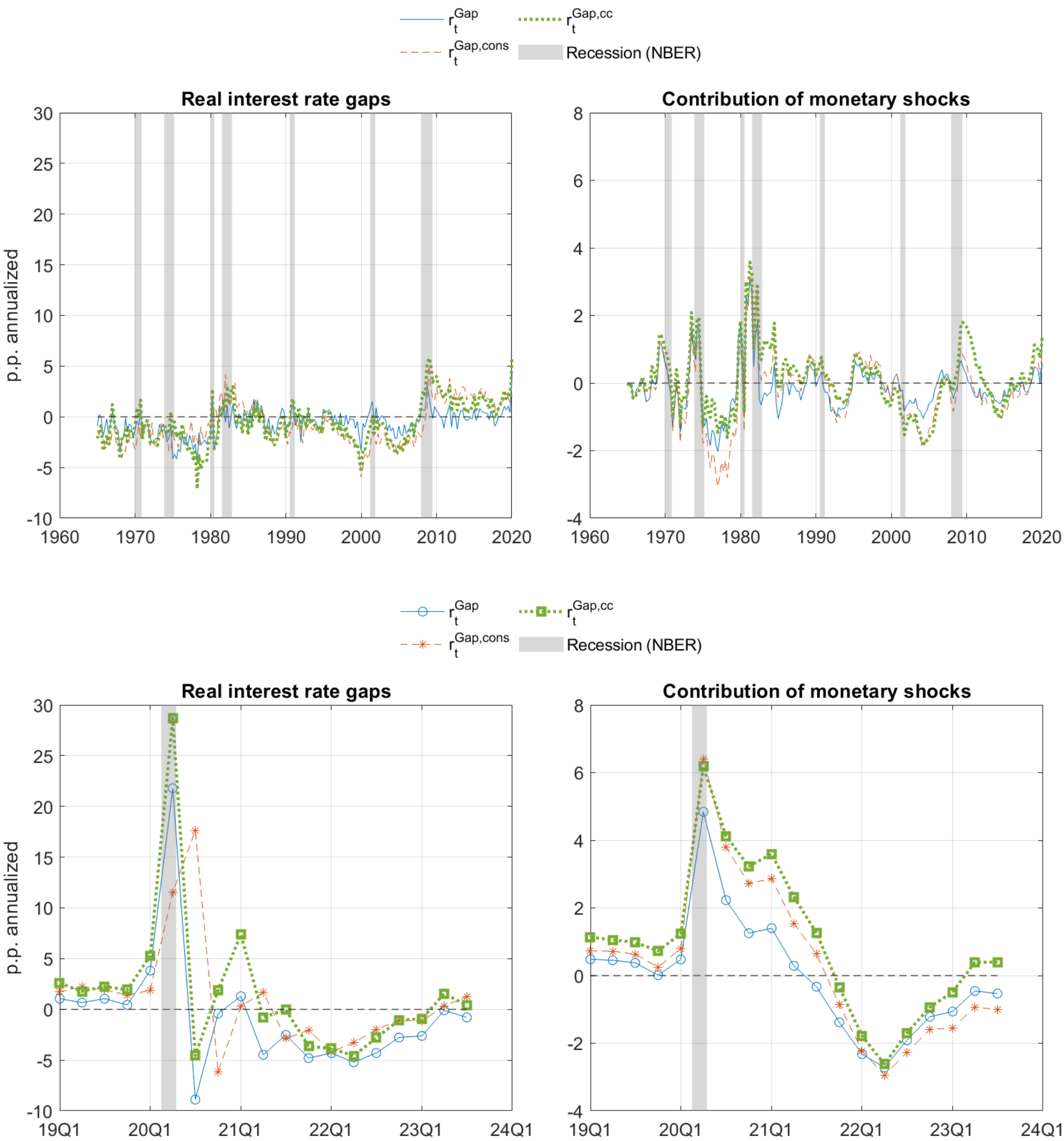



Figure 16: Historical real interest rate gaps recovered from Smets and Wouters (2024)'s model


Figure 17: Historical inflation and investment in the United States

# 8 Conclusion

Yes, a central bank can tighten monetary policy and real interest rates fall under monetary dominance. The puzzle arises when the elasticity to a persistent shock of an endogenous state variable (e.g., capital) is so high that inflation expectations overreact, causing the endogenous – or systematic – component of the monetary policy rule to sufficiently offset its exogenous one. This finding has important implications for the design and implementation of monetary policy, particularly in periods when investment is significantly impacted, such as a pandemic, a financial crisis or even a trade war.

This paper demonstrates that while the real interest rate channel is structural in New Keynesian models, the identification problem revealed by Rupert and Šustek (2019) raises concerns for the use of the real interest rate gap as a gauge of the monetary policy stance. Simply comparing the actual real interest rate to a counterfactual without nominal rigidities — the state-variant definition — as ubiquitously done in the literature (e.g., Neiss and Nelson (2003)) or assuring state variables are the same as in Woodford (2003*a*) — the state-consistent definition — can result in the wrong perception that monetary policy does not operate through the real interest rate channel. This is so as both measures fail to isolate the influence of dynamics in endogenous state variables, such as capital, which can amplify or offset the effect of a monetary policy shock depending on their elasticity to it and the persistence of the shock. The appropriate measure of the real interest rate gap, which accurately reflects the sign of the monetary policy stance and called in this paper the state-invariant definition, compares the counterfactual ex-ante real interest rate as if state variables were invariant to its counterpart without nominal rigidities.

Furthermore, including empirically validated interest-rate smoothing into the Taylor rule can mitigate the identification problem, a feature as prevalent in medium-scale New Keynesian models as capital itself. The sign of changes in the real interest rate right after a positive monetary policy shock is positive under realistic parameters, thereby reestablishing the observational equivalence with the real interest rate channel of monetary policy transmission and weakening the empirical relevance of the New Keynesian capital puzzle. This finding implies that VAR models can be reliably identified by imposing a same-sign restriction on the real interest rate's response to a monetary policy shock. It is also acceptable to sequentially order nominal and real rates, as done in Cholesky decompositions. Moreover, using real rates instead of nominal ones still — at least partially — captures monetary policy shocks. Finally, the interpretation of impulse responses from canonical DSGE models via the real interest rate channel remains consistent, provided the monetary policy rule incorporates a sufficient degree of smoothing.

Ultimately, the state-invariant real interest rate gap effectively assesses the monetary policy stance, gauges its intensity, improves inflation forecasting, and serves as a narrative tool. All these features have been observed for actual historical data in the United States from 1965Q1 to 2023Q3. Alternatively, the state-variant real interest rate gap may be a good approximation if one incorporates a somewhat realistic combination of endogenous state variable rigidities, for example, acknowledging that at least some smoothing is the norm in central banking and that capital adjustment costs are never negligible in the real world. Overall, the negative association between changes in inflation and changes in either short- or long-term real interest rates in New Keynesian models is more robust than that between changes in inflation and changes in the nominal interest rate, which is more likely to be ambiguous.

## References


**Adjemian, Stéphane, Michel Juillard, Fréderic Karamé, Willi Mutschler, Johannes Pfeifer, Marco Ratto, Normann Rion, and Sébastien Villemot.** 2024. "Dynare: Reference Manual, Version 6." CEPREMAP Dynare Working Papers 80.

**Altig, David, Lawrence J Christiano, Martin Eichenbaum, and Jesper Lindé.** 2011. "Firm-specific capital, nominal rigidities and the business cycle." *Review of Economic Dynamics*, 14(2): 225–247.

**Amaral, Eduardo.** 2024. "Smoothing the New-Keynesian Capital Puzzle." *BCB Working Paper Series*, 606.

**Amaral, Eduardo, Torsten Ehlers, Ilhyock Shim, and Alexandre Tombini.** 2025. "Monetary policy decision-making and communication under high uncertainty: insights from central banks in the Americas and other economies." *Manuscript.*

**Barsky, Robert, Alejandro Justiniano, and Leonardo Melosi.** 2014. "The natural rate of interest and its usefulness for monetary policy." *American Economic Review*, 104(5): 37–43.

**Brainard, William C.** 1967. "Uncertainty and the Effectiveness of Policy." *The American Economic Review*, 57(2): 411–425.

**Brault, Joshua, and Hashmat Khan.** 2022. "The Real Interest Rate Channel Is Structural in Contemporary New-Keynesian Models: A Note." *Journal of Money, Credit and Banking*, 54(5): 1551–1563.

**Carrillo, Julio, Patrick Fève, and Julien Matheron.** 2018. "Monetary policy inertia or persistent shocks: a DSGE analysis." *Eighth issue (March 2007) of the International Journal of Central Banking.*

**Carter, Jimmy.** 1979. "Federal Reserve System Nomination of Paul A. Volcker To Be Chairman of the Board of Governors." Online by Gerhard Peters and John T. Woolley, The American Presidency Project.

**CDC.** 2025. "CDC Museum COVID-19 Timeline." Accessed: 2025-05-08.

**Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans.** 2005. "Nominal rigidities and the dynamic effects of a shock to monetary policy." *Journal of political Economy*, 113(1): 1–45.

**Clarida, Richard, Jordi Galí, and Mark Gertler.** 1999. "The science of monetary policy: a new Keynesian perspective." *Journal of Economic Literature*, 37(4): 1661–1707.

**Coibion, Olivier, and Yuriy Gorodnichenko.** 2012. "Why are target interest rate changes so persistent?" *American Economic Journal: Macroeconomics*, 4(4): 126–162.

**Cour-Thimann, Philippine, Rasmus Pilegaard, and Livio Stracca.** 2006. "The output gap and the real interest rate gap in the euro area, 1960–2003." *Journal of Policy Modeling*, 28(7): 775–790.

**Diebold, Francis X.** 2015. "Comparing predictive accuracy, twenty years later: A personal perspective on the use and abuse of Diebold–Mariano tests." *Journal of Business & Economic Statistics*, 33(1): 1–1.

**Diebold, Francis X, and Roberto S Mariano.** 1995. "Comparing Predictive Accuracy." *Journal of Business & Economic Statistics*, 13(3).

**Galí, Jordi.** 2015. *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications.* . 2$^{nd}$ ed., Princeton, USA:Princeton University Press.

**Goodfriend, Marvin.** 1987. "Interest rate smoothing and price level trend-stationarity." *Journal of Monetary Economics*, 19(3): 335–348.

**Holden, Tom D.** 2024. "Robust Real Rate Rules." *Econometrica*, 92(5): 1521–1551.

**Ireland, Peter N.** 2010. “Monetary transmission mechanism.” In *Monetary economics.* 216–223. Springer.

**Kaplan, Greg, Benjamin Moll, and Giovanni L Violante.** 2018. “Monetary policy according to HANK.” *American Economic Review,* 108(3): 697–743.

**Kimball, Miles.** 1995. “The Quantitative Analytics of the Basic Neomonetarist Model.” *Journal of Money, Credit and Banking,* 27(4): 1241–1277.

**Laubach, Thomas, and John C Williams.** 2003. “Measuring the natural rate of interest.” *Review of Economics and Statistics,* 85(4): 1063–1070.

**Lubik, Thomas A, Christian Matthes, et al.** 2015. “Calculating the natural rate of interest: A comparison of two alternative approaches.” *Richmond Fed Economic Brief,* , (Oct): 1–6.

**Lucas, Robert.** 1976. “Econometric policy evaluation: A critique.” Vol. 1 of *Carnegie-Rochester conference series on public policy,* 19–46, North-Holland.

**Mésonnier, Jean-Stéphane.** 2011. “The forecasting power of real interest rate gaps: an assessment for the Euro area.” *Applied Economics,* 43(2): 153–172.

**NBER.** 2025. “Business Cycle Dating.” *https://www.nber.org/research/business-cycle-dating*, accessed on 2025-05-12.

**Neiss, Katharine S, and Edward Nelson.** 2003. “The real-interest-rate gap as an inflation indicator.” *Macroeconomic dynamics,* 7(2): 239–262.

**Orphanides, Athanasios, and Simon van Norden.** 2002. “The unreliability of output-gap estimates in real time.” *Review of economics and statistics,* 84(4): 569–583.

**Rudebusch, Glenn D.** 2006. “Monetary Policy Inertia: Fact or Fiction?” *International Journal of Central Banking.*

**Rupert, Peter, and Roman Šustek.** 2019. “On the mechanics of New-Keynesian models.” *Journal of Monetary Economics,* 102: 53–69.

**Sack, Brian, and Volker Wieland.** 2000. “Interest-rate smoothing and optimal monetary policy: a review of recent empirical evidence.” *Journal of Economics and Business,* 52(1-2): 205–228.

**Schnabel, Isabel.** 2024. "R (ising) star?" *SUERF Policy Note*, 345(Apr).

**Smets, Frank, and Rafael Wouters.** 2003. "An estimated dynamic stochastic general equilibrium model of the Euro Area." *Journal of the European Economic Association*, 1(5): 1123–1175.

**Smets, Frank, and Rafael Wouters.** 2007. "Shocks and frictions in US business cycles: A Bayesian DSGE approach." *American Economic Review*, 97(3): 586–606.

**Smets, Frank, and Rafael Wouters.** 2024. "Fiscal Backing, Inflation and US Business Cycles." CEPR Press 19791, Paris & London.

**Uhlig, Harald.** 2005. "What are the effects of monetary policy on output? Results from an agnostic identification procedure." *Journal of Monetary Economics*, 52(2): 381–419.

**Waller, Christopher J.** 2024. "Some Thoughts on r*: Why Did It Fall and Will It Rise?" Speech at the Reykjavik Economic Conference, Reykjavik, Iceland.

**Wicksell, Knut.** 1898. *Interest and Prices: A Study of the Causes Regulating the Value of Money.* Originally published as Geldzins und Güterpreise in 1898. Translated to English in 1936 for the Royal Economic Society. Republished by Read Books Ltd. in 2013.

**Wicksell, Knut.** 1907. "The influence of the rate of interest on prices." *The Economic Journal*, 17(66): 213–220.

**Woodford, Michael.** 2003*a*. *Interest and Prices: Foundations of a Theory of Monetary Policy.* . $1^{st}$ ed., Princeton, USA:Princeton University Press.

**Woodford, Michael.** 2003*b*. "Optimal interest-rate smoothing." *The Review of Economic Studies*, 70(4): 861–886.

# A Glossary of symbols

Table 21: Model variables

| Symbol | Definition/Description |
|---|---|
| *Original Variables (Levels)* | |
| $c_t$ | Consumption |
| $l_t$ | Labor |
| $y_t$ | Output |
| $w_t$ | Real wage |
| $\chi_t$ | Real marginal cost |
| $i_t$ | Nominal interest rate |
| $r_t$ | Real interest rate |
| $\pi_t$ | Inflation |
| $k_t$ | Capital stock (beginning of period) |
| $r_t^k$ | Rental rate of capital |
| $x_t$ | Investment |
| $q_t$ | Tobin's q ($q_t \equiv 1 + \kappa(k_{t+1} - k_t)$) |
| $G_{t+1}$ | Capital gain ($G_{t+1} \equiv q_{t+1}/q_t$) |
| *Log-Deviations from Steady State (Hat or Tilde Variables)* | |
| $\hat{c}_t$ | $\log(c_t/\overline{c})$ |
| $\hat{y}_t, \tilde{y}_t$ | $\log(y_t/\overline{y})$ |
| $\hat{k}_t$ | $\log(k_t/\overline{k})$ |
| $\hat{x}_t$ | $\log(x_t/\overline{x})$ |
| $\hat{r}_t, \tilde{r}_t$ | $r_t - \overline{r}$ (real interest rate deviation) |
| $\hat{i}_t$ | $i_t - \overline{i}$ (nominal interest rate deviation) |
| $\hat{r}_t^k$ | $r_t^k - \overline{r}^k$ (rental rate deviation) |
| $\hat{\pi}_t, \tilde{\pi}_t$ | $\pi_t - \overline{\pi}$ (inflation deviation, $\overline{\pi} = 0$) |

Table 22: Model variables (part 2)

| Symbol | Definition/Description |
|---|---|
| *Shocks and Policy Variables* | |
| $\xi_t^m$ | Monetary policy shock (AR(1) process) |
| $\epsilon_t^m$ | Innovation to $\xi_t^m$ (i.i.d. normal) |
| $\rho^m$ | Persistence of $\xi_t^m$ |
| *Other Variables* | |
| $\hat{r}_t^n$ | Natural real interest rate (flex-price counterfactual) |
| $\hat{r}_t^{n,cons}$ | Natural real interest rate (flex-price counterfactual with consistent states) |
| $\hat{r}_t^{n,cc}$ | Natural real interest rate with capital fixed (flex-price counterfactual) |
| $\hat{r}_t^{cc}$ | Real interest rate with capital fixed |
| $\hat{r}_t^{Gap}$ | State-variant real interest rate gap: $\hat{r}_t - \hat{r}_t^n$ |
| $\hat{r}_t^{Gap,cons}$ | State-consistent real interest rate gap: $\hat{r}_t - \hat{r}_t^{n,cons}$ |
| $\hat{r}_t^{Gap,cc}$ | State-invariant real interest rate gap: $\hat{r}_t^{cc} - \hat{r}_t^{ncc}$ |
| $\Delta_4\hat{\pi}_t$ | Annual inflation (4-period price level difference) |

Table 23: Model deep parameters

| Symbol | Name/Definition |
|---|---|
| $\beta$ | Subjective discount factor |
| $\eta$ | Inverse elasticity of labor supply |
| $\varepsilon$ | Elasticity of substitution between intermediate goods |
| $\theta$ | Fraction of firms not adjusting prices (Calvo parameter) |
| $\nu$ | Taylor rule coefficient on inflation |
| $\rho^m$ | Persistence of monetary policy shock |
| $\alpha$ | Capital share in production (Cobb-Douglas) |
| $\delta$ | Depreciation rate of capital |
| $\kappa$ | Capital adjustment cost parameter |
| $\rho^i$ | Interest-rate smoothing coefficient in Taylor rule |
| $\omega$ | Elasticity of the endogenous state variable to the monetary policy shock |
| *Smets-Wouters Model Additions* | |
| $\kappa^I$ | Investment adjustment cost parameter |

Table 24: Model derived parameters

| Symbol | Definition/Description |
|---|---|
| $\Omega$ | Slope of the Phillips curve in the canonical model: $\Omega \equiv \frac{(1+\eta)(1-\theta)(1-\theta\beta)}{\theta}$ |
| $\Psi$ | Slope of the Phillips curve with capital: $\Psi \equiv \overline{\chi}\frac{(1-\theta)(1-\theta\beta)}{\theta}$ |
| $\Theta$ | Auxiliary coefficient: $\Theta \equiv \frac{\Psi\left(\frac{\eta+\alpha}{1-\alpha}\frac{\overline{C}}{\overline{Y}}+1\right)}{(1-\beta\rho^m)(\rho^m-1)}$ |
| $\overline{\kappa}$ | Effective capital adjustment cost: $\overline{\kappa} \equiv \kappa\overline{K}$ |

Table 25: Additional coefficients

| Symbol | Definition/Description |
|---|---|
| $a, b$ | Canonical model decision rule coefficients for output and inflation |
| $\tilde{a}, \tilde{b}$ | Augmented model decision rule coefficients for output and inflation |
| $a_0, a_1, a_2$ | Consumption decision rule coefficients |
| $b_0, b_1, b_2$ | Inflation decision rule coefficients |
| $d_0, d_1, d_2$ | Output decision rule coefficients |
| $f_0, f_1, f_2$ | Capital decision rule coefficients |
| $\eta_{nk}, \boldsymbol{\eta_{nkk}}$ | Coefficients linking natural real rate to capital and shocks: $\hat{r}_t^n = \hat{r}_t^{ncc} + \eta_{nk}\hat{k}_t^n + \boldsymbol{\eta_{nkk}}\epsilon_t$ |
| $\eta_k, \eta_{kk}$ | Coefficients linking actual real rate to capital and shocks: $\hat{r}_t = \hat{r}_t^{cc} + \eta_k\hat{k}_t + \eta_{kk}\epsilon_t^m$ |

# B Model derivation

## B.1 Canonical closed economy (with omitted or fixed capital)

Consider a closed economy without fiscal policy, where a one-period risk-free nominal bond is available in zero net supply and the central bank adopts a fixed-intercept Taylor rule. I expand here on the simplified presentation made by Rupert and Šustek (2019) of the canonical New Keynesian model of Galí (2015), with minor notational changes.

The simple model starts with seven variables: real consumption, $c_t$; labor $l_t$; real output, $y_t$; real wage, $w_t$; real marginal cost, $\chi_t$; nominal interest rate, $i_t$; and inflation, $\pi_t$. Overlined variables represent their steady-state values. There are six parameters: the subjective discount factor, $\beta$; the inverse of the elasticity of labor supply, $\eta$; the elasticity of substitution between intermediate goods, $\varepsilon$; the fraction of producers not adjusting prices at any given period, $\theta$; the intercept of the Taylor rule, $i$; and the Taylor-rule coefficient that gauges the central bank's reaction to current inflation, $\nu$. There is also an exogenous monetary policy shock variable, $\xi_t^m$.

Assuming a per-period utility function of the form

$$u_t = \log(c_t) - \frac{l_t^{1+\eta}}{1+\eta} \tag{B.1}$$

and an intermediate goods aggregator like

$$y_t = \left[\int y(j)^{\varepsilon}\,\mathrm{d}j\right]^{\frac{1}{\varepsilon}} \tag{B.2}$$

the equilibrium conditions of that economy are given by the Euler equation (B.3) in conjunction with equations (B.4) to (B.9), namely the first-order conditions (FOC) of labor, the production function, the real marginal cost, the New Keynesian Phillips curve under Calvo pricing already linearized around a zero steady-state inflation, a Taylor rule, and the market-clearing condition.

$$\left.\begin{aligned}
\frac{1}{c_t} &= \beta \mathbb{E}_t\left(\frac{1}{c_{t+1}}\frac{1+i_t}{1+\pi_{t+1}}\right) && \text{(B.3)}\\
\frac{w_t}{c_t} &= l_t^{\eta} && \text{(B.4)}\\
y_t &= l_t && \text{(B.5)}\\
\chi_t &= w_t && \text{(B.6)}\\
\pi_t = \frac{(1-\theta)(1-\theta\beta)}{\theta}&\left(\mathrm{mc}_t - \overline{\mathrm{mc}}\right) + \beta \mathbb{E}_t \pi_{t+1} && \text{(B.7)}\\
i_t &= i + \nu\pi_t + \xi_t^m && \text{(B.8)}\\
y_t &= c_t && \text{(B.9)}
\end{aligned}\right\}$$

As usual, the equilibrium conditions above can be simplified to a three-equation system linearized around a nonstochastic steady state ($\overline{\pi} = 0, \overline{y} = 1$). This is possible by first linearizing (B.3) and substituting the market-clearing condition (B.9) into it, then eliminating (B.9), (B.5), and (B.6) through the substitution of their respective expressions for $c_t$, $l_t$, and $w_t$ into (B.4), which is later linearized such that $\hat{y}_t \equiv \frac{y_t - \overline{y}}{\overline{y}}$. Finally, the Taylor Rule (B.8) is rewritten as deviations of the interest rate from its steady-state value such that $\hat{i}_t = i_t - i$.

$$\left.\begin{aligned}
-\hat{y} &= -\mathbb{E}_t \hat{y}_{t+1} + \hat{i}_t - \mathbb{E}_t \pi_{t+1} && \text{(B.10)}\\
\pi_t &= \Omega \hat{y}_t + \beta \mathbb{E}_t \pi_{t+1} && \text{(B.11)}\\
\hat{i}_t &= \nu\pi_t + \xi_t^m && \text{(B.12)}
\end{aligned}\right\}$$

where

$$\Omega \equiv \frac{(1+\eta)(1-\theta)(1-\theta\beta)}{\theta} > 0 \tag{B.13}$$

## B.2 Model with endogenous capital

I build on the model of Rupert and Šustek (2019), which assumes there is an economy-wide rental market of capital so that firms can rent capital in every period. In that sense, capital is not firm-specific. Moreover, they assume that whenever households change their stock of capital, there is a quadratic adjustment cost, $-\frac{\kappa}{2}(k_{t+1}-k_t)^2$, where $k_t$ is the stock of capital inherited from the previous period and $\kappa \geq 0$ is a parameter that governs the size of the adjustment cost in terms of foregone real income.

Resuming from the canonical model of Appendix B.1, there is a new Euler equation for the capital asset (B.14), where $\delta \in (0,1)$ is a depreciation rate, and $q_t$ is the price of capital in terms of current consumption, Tobin's q, such that $q_t \equiv 1+\kappa(k_{t+1}-k_t)$. The production function (B.5) is replaced by (B.15), which incorporates capital and labor proportionate to constant returns to scale, where $\alpha$ is the Cobb-Douglas coefficient of capital. Equation (B.16) is the condition for the optimal mix of capital and labor in production, which comes from the FOC of the firm. The marginal cost (B.6) is updated to include the rent on capital, $r_t^k$, (B.17). The resource constraint (B.9) must now account for the investment flow so markets can clear (B.18).

$$\left.\begin{aligned} \frac{1}{c_t} &= \beta\mathbb{E}_t\left(\frac{1}{c_{t+1}}\left(\frac{r_{t+1}^k-\delta}{q_t}+\frac{q_{t+1}}{q_t}\right)\right) && \text{(B.14)}\\ y_t &= k_t^{\alpha} l_t^{1-\alpha} && \text{(B.15)}\\ \frac{w_t}{r_t^k} &= \frac{1-\alpha}{\alpha}\left(\frac{k_t}{l_t}\right) && \text{(B.16)}\\ \chi_t &= \left(\frac{r_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} && \text{(B.17)}\\ y_t &= c_t+k_{t+1}-(1-\delta)k_t+\frac{\kappa}{2}(k_{t+1}-k_t)^2 && \text{(B.18)} \end{aligned}\right\}$$

After substituting equation (B.16) into (B.17) by eliminating $r_t^k$, and substituting equation (B.4) into (B.15) so as to eliminate $l_t$, the model is log-linearized around the zero-inflation non-stochastic steady state (Appendix C). For any variable $X$, $\hat{X} \equiv \frac{X_t-\overline{X}}{\overline{X}}$, with the exception of $\hat{i}_t \equiv i_t-\overline{i}$, $\hat{r}_t \equiv r_t-\overline{r}$, and $\hat{r}_t^k \equiv r_t^k-\overline{r}^k$. After that, it is possible to eliminate $\hat{r}_t^k$, $\hat{\chi}_t$, $\hat{w}_t$, and $\hat{l}_t$ to obtain the following reduced system with five equations, including the policy rule specification (B.23).

$$\left.\begin{aligned}
-\hat{c}_t &= -\mathbb{E}_t\,\hat{c}_{t+1} + \hat{i}_t - \mathbb{E}_t\,\pi_{t+1} &\quad \text{(B.19)}\\
-\hat{c}_t &= -\mathbb{E}_t\,\hat{c}_{t+1} + \mathbb{E}_t\,\hat{g}_{t+1} + \overline{r}^k\,\mathbb{E}_t\left(\hat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\hat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\hat{k}_{t+1}\right) &\quad \text{(B.20)}\\
\pi_t &= \Psi\left(\frac{\eta+\alpha}{1-\alpha}\hat{y}_t - \alpha\frac{1+\eta}{1-\alpha}\hat{k}_t + \hat{c}_t\right) + \beta\,\mathbb{E}_t\,\pi_{t+1} &\quad \text{(B.21)}\\
\hat{y}_t &= \frac{\overline{c}}{\overline{y}}\hat{c}_t + \frac{\overline{k}}{\overline{y}}\hat{k}_{t+1} - (1-\delta)\frac{\overline{k}}{\overline{y}}\hat{k}_t &\quad \text{(B.22)}\\
\hat{i}_t &= \nu\pi_t + \xi_t^m &\quad \text{(B.23)}
\end{aligned}\right\}$$

where $\Psi \equiv \overline{\chi}\frac{(1-\theta)(1-\theta\beta)}{\theta}$, such that when prices are flexible, $\Psi \to \infty$. Moreover, $G_{t+1} \equiv \frac{q_{t+1}}{q_t}$ is the capital gain, so $\hat{g}_t = \hat{q}_t - \hat{q}_{t-1} = \overline{\kappa}\left(\hat{k}_{t+1} - \hat{k}_t\right) - \overline{\kappa}\left(\hat{k}_t - \hat{k}_{t-1}\right)$, where $\overline{\kappa} = \kappa\overline{k}$.

# C Steady state

In this section, I derive the nonstochastic steady state of the canonical New Keynesian model augmented with endogenous capital.[31] From their definitions, capital gain and Tobin's q equal 1 at the steady state.

$$\overline{G} = 1 \tag{C.1}$$

$$\overline{Q} = 1 \tag{C.2}$$

I pick a zero-inflation steady state. The real rate of return on the implicit risk-free bond of the model is the one obtained from the Fisher equation. Through the no-arbitrage condition, I combine the two Euler equations of the model, (B.3) and (B.14), to get a relation between the two real rates.

$$\overline{\pi} = 0 \tag{C.3}$$

$$\overline{i} = \overline{r} + \overline{\pi} \quad \Rightarrow \quad \overline{i} = \overline{r} \tag{C.4}$$

$$\left(1+\overline{i}\right) = \left(1+\overline{\pi}\right)\left(1 + \frac{\overline{r}^k - \delta}{\overline{Q}}\right) \quad \Rightarrow \quad \overline{i} = \overline{r}^k - \delta \quad \Rightarrow \quad \overline{r} = \overline{r}^k - \delta \tag{C.5}$$

From the capital Euler equation (B.14) evaluated at the steady state I can isolate $\overline{r}^k$ as a function of the deep parameters:

$$\frac{1}{\overline{C}} = \beta\left(\frac{1}{\overline{C}}\left(\frac{\overline{r}^k - \delta}{\overline{Q}} + \frac{\overline{Q}}{\overline{Q}}\right)\right) \quad \Rightarrow \quad \overline{r}^k = \frac{1}{\beta} + \delta - 1 \tag{C.6}$$

[31] The steady state is the same with and without interest-rate smoothing.

Substituting $\overline{r}^k$ into the FOC of capital (B.16) at the steady state I get $\frac{\overline{K}}{\overline{L}}$:

$$\overline{r}^k = \alpha \overline{K}^{\alpha-1} \overline{L}^{1-\alpha} \quad \Rightarrow \quad \frac{\overline{K}}{\overline{L}} = \left( \frac{\alpha}{\frac{1}{\beta} - 1 + \delta} \right)^{\frac{1}{1-\alpha}} \tag{C.7}$$

The production function (B.15) at the steady state can be rewritten so as to put in evidence $\frac{\overline{K}}{\overline{L}}$ on the right side of the expression:

$$\overline{Y} = \left( \frac{\overline{K}}{\overline{L}} \right)^{\alpha} \overline{L} \tag{C.8}$$

Investment at the steady state only compensates for depreciated capital.

$$\overline{I} = \delta \overline{K} \tag{C.9}$$

Now, the aggregate resource constraint (B.18) may be rewritten as auxiliary ratios by substituting the just derived expressions for $\overline{Y}$ and $\overline{I}$ and then dividing by $\overline{L}$.

$$\overline{Y} = \overline{C} + \overline{I} \quad \Rightarrow \quad \left( \frac{\overline{K}}{\overline{L}} \right)^{\alpha} = \frac{\overline{C}}{\overline{L}} + \frac{\delta \overline{K}}{\overline{L}} \tag{C.10}$$

Combining the condition for the optimal mix of capital and labor in production (B.16) and the intratemporal condition at the steady state (B.4), I can write an expression for $\frac{\overline{C}}{\overline{L}}$:

$$\overline{W} = (1-\alpha) \overline{K}^{\alpha} \overline{L}^{-\alpha} \quad \text{and} \quad \frac{\overline{W}}{\overline{C}} = \overline{L}^{\eta} \quad \Rightarrow \quad \frac{\overline{C}}{\overline{L}} = (1-\alpha) \left( \frac{\overline{K}}{\overline{L}} \right)^{\alpha} \overline{L}^{-(\eta+1)} \tag{C.11}$$

By substituting previously derived expressions for $\frac{\overline{K}}{\overline{L}}$ and $\frac{\overline{C}}{\overline{L}}$ into the aggregate resource constraint, I obtain $\overline{L}$ as a function of the deep parameters of the model.

$$\overline{L} = \left( \left( \frac{1}{1-\alpha} \right) \left( \frac{\delta \alpha}{\frac{1}{\beta} - 1 + \delta} \right) \right)^{\frac{-1}{\eta+1}} \tag{C.12}$$

I get the remaining steady-state variables as functions of the parameters by recursively substituting (C.12) into my previously derived expressions: (C.7) $\Rightarrow$ $\overline{K}$, (C.8) $\Rightarrow$ $\overline{Y}$, (C.9) $\Rightarrow$ $\overline{I}$, and (C.11) $\Rightarrow$ $\overline{C}$ and $\overline{W}$. Finally, I obtain $\overline{\chi}$ by substituting $\overline{r}^k$ and $\overline{W}$ into (B.17) evaluated at the steady state.

# Online Appendix

## OA1 Forecasting other measures of inflation

This section extends the forecasting exercise of section 6 to other measures of inflation: $\pi_{t+1}$, $\Delta_4\pi_{t+1}$, and $\pi_{t+4}$.

Results of tables OA1 to OA6 are similar to those previously found in the paper in the sense that when investment adjustment costs are realistic or sizable, both $\hat{r}_t^{Gap}$ and $\hat{r}_t^{Gap,cc}$ are equally as good predictors, $\hat{r}_t$ is the individual best predictor, and adding all three variables as predictors achieves the highest adjusted R-squared statistic and the lowest root mean square error. When investment adjustment costs are negligible, endogenous state variables become more volatile and, in this case, more informative to the extent that $\hat{r}_t^{Gap}$ is usually the best predictor. Yet, combining all three variables still turns out as the best regression specification for forecasting.

Tables OA7 to OA15 extend the same exercise to actual historical data, with Smets and Wouters (2007)'s model parameters re-estimated for each sample, and I find that the forecasting properties of the state-invariant RIRG remain valid. Tables OA16 to OA30 present a robustness check obtained by extending the sample and employing Smets and Wouters (2024)'s model.

### OA1.1 Canonical NK model expanded with endogenous capital

Table OA1: Forecasting $\pi_{t+1}$ under realistic investment adjustment costs

Forecasting 1-period ahead inflation

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-1}$ | 0.845*** | 0.842*** | 0.833*** | 0.842*** | 0.841*** | 0.838*** | 0.838*** | 0.841*** | 0.825*** |
| | (0.00120) | (0.00121) | (0.00127) | (0.00121) | (0.00121) | (0.00122) | (0.00122) | (0.00127) | (0.00134) |
| $\hat{r}_{t-1}^{Gap}$ | | -0.00525*** | | | | -0.00608*** | | 0.0305*** | 0.0573*** |
| | | (0.000334) | | | | (0.000336) | | (0.00349) | (0.00354) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | 0.00655*** | | | | | | 0.00931*** |
| | | | (0.000235) | | | | | | (0.000246) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.00577*** | | | -0.00630*** | -0.0362*** | -0.0648*** |
| | | | | (0.000330) | | | (0.000331) | (0.00343) | (0.00350) |
| $\hat{r}_{t-1}$ | | | | | -0.0286*** | -0.0317*** | -0.0308*** | -0.0255*** | -0.0304*** |
| | | | | | (0.00147) | (0.00148) | (0.00148) | (0.00159) | (0.00159) |
| Constant | 0.000433 | 0.000433 | 0.000242 | 0.000434 | 0.000606 | 0.000625 | 0.000621 | 0.000597 | 0.000364 |
| | (0.000678) | (0.000678) | (0.000677) | (0.000678) | (0.000678) | (0.000677) | (0.000677) | (0.000677) | (0.000675) |
| Observations | 199,999 | 199,999 | 199,999 | 199,999 | 199,999 | 199,999 | 199,999 | 199,999 | 199,999 |
| Adjusted R-squared | 0.713 | 0.714 | 0.714 | 0.714 | 0.714 | 0.714 | 0.714 | 0.714 | 0.716 |
| Root Mean Square Error | 0.303 | 0.303 | 0.303 | 0.303 | 0.303 | 0.303 | 0.303 | 0.303 | 0.302 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA2: Forecasting $\Delta_4 \pi_{t+1}$ under realistic investment adjustment costs

Forecasting 1-period ahead annual inflation

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4 \hat{\pi}_{t-1}$ | 0.968*** | 0.965*** | 0.993*** | 0.967*** | 0.972*** | 0.969*** | 0.971*** | 0.970*** | 0.987*** |
| | (0.000562) | (0.000556) | (0.000625) | (0.000554) | (0.000530) | (0.000520) | (0.000519) | (0.000542) | (0.000592) |
| $\hat{r}_{t-1}^{Gap}$ | | -0.0407*** | | | | -0.0488*** | | -0.0313*** | -0.0974*** |
| | | (0.000560) | | | | (0.000526) | | (0.00551) | (0.00554) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | -0.0360*** | | | | | | -0.0272*** |
| | | | (0.000423) | | | | | | (0.000405) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.0432*** | | | -0.0481*** | -0.0173*** | 0.0531*** |
| | | | | (0.000554) | | | (0.000519) | (0.00544) | (0.00549) |
| $\hat{r}_{t-1}$ | | | | | -0.376*** | -0.395*** | -0.388*** | -0.393*** | -0.388*** |
| | | | | | (0.00236) | (0.00232) | (0.00231) | (0.00245) | (0.00242) |
| Constant | 0.000365 | 0.000333 | 0.00132 | 0.000338 | 0.00248** | 0.00255** | 0.00251** | 0.00253** | 0.00322*** |
| | (0.00116) | (0.00115) | (0.00114) | (0.00114) | (0.00109) | (0.00107) | (0.00107) | (0.00107) | (0.00106) |
| Observations | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 |
| Adjusted R-squared | 0.937 | 0.938 | 0.939 | 0.939 | 0.944 | 0.946 | 0.946 | 0.946 | 0.947 |
| Root Mean Square Error | 0.520 | 0.513 | 0.511 | 0.512 | 0.489 | 0.479 | 0.479 | 0.479 | 0.474 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA3: Forecasting $\pi_{t+4}$ under realistic investment adjustment costs

| | Forecasting 4-period ahead inflation | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-4}$ | 0.572*** | 0.571*** | 0.537*** | 0.571*** | 0.566*** | 0.564*** | 0.564*** | 0.572*** | 0.529*** |
| | (0.00183) | (0.00185) | (0.00193) | (0.00185) | (0.00185) | (0.00187) | (0.00187) | (0.00195) | (0.00203) |
| $\hat{r}_{t-4}^{Gap}$ | | -0.00262*** | | | | -0.00401*** | | 0.0752*** | 0.147*** |
| | | (0.000512) | | | | (0.000515) | | (0.00534) | (0.00539) |
| $\hat{r}_{t-4}^{Gap,cons}$ | | | 0.0197*** | | | | | | 0.0249*** |
| | | | (0.000358) | | | | | | (0.000374) |
| $\hat{r}_{t-4}^{Gap,cc}$ | | | | -0.00375*** | | | -0.00466*** | -0.0784*** | -0.155*** |
| | | | | (0.000506) | | | (0.000507) | (0.00526) | (0.00532) |
| $\hat{r}_{t-4}$ | | | | | -0.0510*** | -0.0530*** | -0.0525*** | -0.0395*** | -0.0527*** |
| | | | | | (0.00225) | (0.00227) | (0.00226) | (0.00244) | (0.00242) |
| Constant | 0.00119 | 0.00119 | 0.000613 | 0.00119 | 0.00150 | 0.00151 | 0.00151 | 0.00145 | 0.000828 |
| | (0.00104) | (0.00104) | (0.00103) | (0.00104) | (0.00104) | (0.00104) | (0.00104) | (0.00104) | (0.00103) |
| Observations | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 |
| Adjusted R-squared | 0.328 | 0.328 | 0.338 | 0.328 | 0.329 | 0.329 | 0.329 | 0.330 | 0.345 |
| Root Mean Square Error | 0.465 | 0.465 | 0.461 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.459 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA4: Forecasting $\pi_{t+1}$ under negligible investment adjustment costs

Forecasting 1-period ahead inflation

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-1}$ | 0.848*** | 0.844*** | 0.849*** | 0.841*** | 0.844*** | 0.841*** | 0.839*** | 0.836*** | 0.836*** |
| | (0.00118) | (0.00118) | (0.00119) | (0.00122) | (0.00120) | (0.00119) | (0.00124) | (0.00123) | (0.00123) |
| $\hat{r}_{t-1}^{Gap}$ | | 0.363*** | | | | 0.356*** | | 0.352*** | 0.350*** |
| | | (0.00626) | | | | (0.00628) | | (0.00628) | (0.00632) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | 0.000949*** | | | | | | 0.000310*** |
| | | | (9.76e-05) | | | | | | (9.87e-05) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.00839*** | | | -0.0188*** | -0.0166*** | -0.0169*** |
| | | | | (0.000351) | | | (0.00104) | (0.00103) | (0.00104) |
| $\hat{r}_{t-1}$ | | | | | 0.00665*** | 0.00510*** | -0.0110*** | -0.0105*** | -0.0110*** |
| | | | | | (0.000352) | (0.000351) | (0.00104) | (0.00103) | (0.00104) |
| Constant | 0.000403 | 0.000290 | 0.000374 | 0.000414 | 0.000370 | 0.000267 | 0.000483 | 0.000368 | 0.000361 |
| | (0.000705) | (0.000699) | (0.000705) | (0.000704) | (0.000704) | (0.000698) | (0.000704) | (0.000698) | (0.000698) |
| Observations | 199,999 | 199,999 | 199,999 | 199,999 | 199,999 | 199,999 | 199,999 | 199,999 | 199,999 |
| Adjusted R-squared | 0.719 | 0.724 | 0.719 | 0.720 | 0.720 | 0.724 | 0.720 | 0.725 | 0.725 |
| Root Mean Square Error | 0.315 | 0.313 | 0.315 | 0.315 | 0.315 | 0.312 | 0.315 | 0.312 | 0.312 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA5: Forecasting $\Delta_4\pi_{t+1}$ under negligible investment adjustment costs

Forecasting 1-period ahead annual inflation

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-1}$ | 0.967*** | 0.970*** | 0.968*** | 0.961*** | 0.964*** | 0.967*** | 0.959*** | 0.961*** | 0.962*** |
| | (0.000571) | (0.000553) | (0.000562) | (0.000548) | (0.000561) | (0.000545) | (0.000531) | (0.000517) | (0.000506) |
| $\hat{r}_{t-1}^{Gap}$ | | 1.231*** | | | | 1.153*** | | 1.075*** | 1.176*** |
| | | (0.0107) | | | | (0.0106) | | (0.0101) | (0.00989) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | -0.0134*** | | | | | | -0.0148*** |
| | | | (0.000169) | | | | | | (0.000154) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.0785*** | | | -0.253*** | -0.245*** | -0.226*** |
| | | | | (0.000574) | | | (0.00165) | (0.00160) | (0.00158) |
| $\hat{r}_{t-1}$ | | | | | 0.0535*** | 0.0478*** | -0.189*** | -0.186*** | -0.162*** |
| | | | | | (0.000599) | (0.000585) | (0.00168) | (0.00163) | (0.00161) |
| Constant | 0.000358 | -9.20e-05 | 0.000743 | 0.000344 | 3.83e-05 | -0.000349 | 0.00144 | 0.00103 | 0.00128 |
| | (0.00124) | (0.00120) | (0.00122) | (0.00118) | (0.00121) | (0.00118) | (0.00115) | (0.00112) | (0.00109) |
| Observations | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 |
| Adjusted R-squared | 0.935 | 0.939 | 0.937 | 0.940 | 0.937 | 0.941 | 0.944 | 0.947 | 0.949 |
| Root Mean Square Error | 0.554 | 0.537 | 0.545 | 0.530 | 0.543 | 0.528 | 0.514 | 0.500 | 0.489 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA6: Forecasting $\pi_{t+4}$ under negligible investment adjustment costs

Forecasting 4-period ahead inflation

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-4}$ | 0.559*** | 0.550*** | 0.562*** | 0.556*** | 0.556*** | 0.549*** | 0.559*** | 0.553*** | 0.555*** |
| | (0.00185) | (0.00183) | (0.00185) | (0.00191) | (0.00188) | (0.00186) | (0.00194) | (0.00191) | (0.00191) |
| $\hat{r}^{Gap}_{t-4}$ | | 0.774*** | | | | 0.772*** | | 0.775*** | 0.753*** |
| | | (0.00973) | | | | (0.00976) | | (0.00977) | (0.00981) |
| $\hat{r}^{Gap,cons}_{t-4}$ | | | 0.00459*** | | | | | | 0.00320*** |
| | | | (0.000152) | | | | | | (0.000153) |
| $\hat{r}^{Gap,cc}_{t-4}$ | | | | -0.00312*** | | | 0.00888*** | 0.0136*** | 0.0101*** |
| | | | | (0.000550) | | | (0.00163) | (0.00160) | (0.00161) |
| $\hat{r}_{t-4}$ | | | | | 0.00441*** | 0.00104* | 0.0128*** | 0.0139*** | 0.00908*** |
| | | | | | (0.000552) | (0.000545) | (0.00163) | (0.00161) | (0.00162) |
| Constant | 0.00117 | 0.000925 | 0.00103 | 0.00117 | 0.00115 | 0.000920 | 0.00109 | 0.000837 | 0.000774 |
| | (0.00110) | (0.00109) | (0.00110) | (0.00110) | (0.00110) | (0.00109) | (0.00110) | (0.00109) | (0.00108) |
| Observations | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 | 199,996 |
| Adjusted R-squared | 0.313 | 0.334 | 0.316 | 0.313 | 0.313 | 0.334 | 0.313 | 0.334 | 0.335 |
| Root Mean Square Error | 0.493 | 0.486 | 0.492 | 0.493 | 0.493 | 0.486 | 0.493 | 0.486 | 0.485 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

### OA1.2 Smets and Wouters (2007)'s model

Table OA7: Forecasting $\pi_{t+1}$ from 1966Q1 to 2004Q4

Forecasting 1-period ahead inflation: from 1966Q1 to 2004Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-1}$ | 0.873*** | 0.848*** | 0.859*** | 0.776*** | 0.874*** | 0.849*** | 0.777*** | 0.736*** | 0.689*** |
| | (0.0394) | (0.0386) | (0.0433) | (0.0435) | (0.0394) | (0.0388) | (0.0440) | (0.0586) | (0.0639) |
| $\hat{r}_{t-1}^{Gap}$ | | -0.0398*** | | | | -0.0390*** | | 0.0321 | 0.0364 |
| | | (0.0112) | | | | (0.0113) | | (0.0302) | (0.0301) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | 0.00717 | | | | | | 0.0158* |
| | | | (0.00907) | | | | | | (0.00894) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.0355*** | | | -0.0354*** | -0.0580** | -0.0645*** |
| | | | | (0.00822) | | | (0.00845) | (0.0229) | (0.0230) |
| $\hat{r}_{t-1}$ | | | | | 0.0355 | 0.0179 | 0.00242 | -0.00422 | -0.0198 |
| | | | | | (0.0370) | (0.0361) | (0.0360) | (0.0365) | (0.0373) |
| Constant | 0.0235 | -0.0412 | 0.0327 | -0.0810** | 0.0236 | -0.0399 | -0.0806** | -0.0949** | -0.0869** |
| | (0.0252) | (0.0304) | (0.0278) | (0.0340) | (0.0252) | (0.0306) | (0.0345) | (0.0371) | (0.0371) |
| Observations | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 |
| Adjusted R-squared | 0.760 | 0.777 | 0.759 | 0.785 | 0.760 | 0.776 | 0.783 | 0.783 | 0.786 |
| Root Mean Square Error | 0.301 | 0.291 | 0.302 | 0.285 | 0.301 | 0.291 | 0.286 | 0.286 | 0.284 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA8: Forecasting $\Delta_4\pi_{t+1}$ from 1966Q1 to 2004Q4

Forecasting 1-period ahead annual inflation: from 1966Q1 to 2004Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-1}$ | 0.984*** | 0.975*** | 1.028*** | 0.944*** | 0.986*** | 0.977*** | 0.943*** | 0.911*** | 0.947*** |
| | (0.0140) | (0.0136) | (0.0158) | (0.0145) | (0.0140) | (0.0133) | (0.0141) | (0.0172) | (0.0207) |
| $\hat{r}^{Gap}_{t-1}$ | | -0.0606*** | | | | -0.0653*** | | 0.107*** | 0.0894*** |
| | | (0.0148) | | | | (0.0147) | | (0.0344) | (0.0341) |
| $\hat{r}^{Gap,cons}_{t-1}$ | | | -0.0622*** | | | | | | -0.0355*** |
| | | | (0.0125) | | | | | | (0.0120) |
| $\hat{r}^{Gap,cc}_{t-1}$ | | | | -0.0601*** | | | -0.0652*** | -0.138*** | -0.117*** |
| | | | | (0.0103) | | | (0.0102) | (0.0253) | (0.0256) |
| $\hat{r}_{t-1}$ | | | | | -0.0938* | -0.120** | -0.140*** | -0.147*** | -0.122*** |
| | | | | | (0.0494) | (0.0469) | (0.0445) | (0.0433) | (0.0431) |
| Constant | 0.0151 | -0.0838** | -0.0745** | -0.162*** | 0.0129 | -0.0942** | -0.181*** | -0.220*** | -0.238*** |
| | (0.0341) | (0.0405) | (0.0364) | (0.0434) | (0.0338) | (0.0400) | (0.0426) | (0.0433) | (0.0427) |
| Observations | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 |
| Adjusted R-squared | 0.969 | 0.972 | 0.973 | 0.975 | 0.970 | 0.973 | 0.976 | 0.977 | 0.979 |
| Root Mean Square Error | 0.405 | 0.386 | 0.376 | 0.367 | 0.401 | 0.379 | 0.357 | 0.347 | 0.339 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA9: Forecasting $\pi_{t+4}$ from 1966Q1 to 2004Q4

Forecasting 4-period ahead inflation: from from 1966Q1 to 2004Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-4}$ | 0.781*** | 0.749*** | 0.800*** | 0.633*** | 0.777*** | 0.741*** | 0.608*** | 0.453*** | 0.406*** |
| | (0.0499) | (0.0480) | (0.0551) | (0.0521) | (0.0491) | (0.0467) | (0.0501) | (0.0646) | (0.0709) |
| $\hat{r}_{t-4}^{Gap}$ | | -0.0589*** | | | | -0.0639*** | | 0.122*** | 0.127*** |
| | | (0.0139) | | | | (0.0136) | | (0.0338) | (0.0338) |
| $\hat{r}_{t-4}^{Gap,cons}$ | | | -0.00908 | | | | | | 0.0157 |
| | | | (0.0115) | | | | | | (0.0101) |
| $\hat{r}_{t-4}^{Gap,cc}$ | | | | -0.0572*** | | | -0.0647*** | -0.150*** | -0.157*** |
| | | | | (0.00992) | | | (0.00964) | (0.0254) | (0.0256) |
| $\hat{r}_{t-4}$ | | | | | -0.116** | -0.141*** | -0.171*** | -0.197*** | -0.213*** |
| | | | | | (0.0473) | (0.0447) | (0.0425) | (0.0415) | (0.0426) |
| Constant | 0.0434 | -0.0574 | 0.0314 | -0.132*** | 0.0452 | -0.0638* | -0.153*** | -0.206*** | -0.197*** |
| | (0.0321) | (0.0387) | (0.0355) | (0.0422) | (0.0315) | (0.0376) | (0.0405) | (0.0417) | (0.0418) |
| Observations | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 |
| Adjusted R-squared | 0.612 | 0.650 | 0.611 | 0.679 | 0.624 | 0.669 | 0.708 | 0.729 | 0.732 |
| Root Mean Square Error | 0.383 | 0.364 | 0.384 | 0.348 | 0.377 | 0.354 | 0.332 | 0.320 | 0.318 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA10: Forecasting $\pi_{t+1}$ from 1966Q1 to 1979Q2

Forecasting 1-period ahead inflation: from 1966Q1 to 1979Q2

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-1}$ | 0.738*** | 0.787*** | 0.716*** | 0.754*** | 0.761*** | 0.784*** | 0.765*** | 0.494*** | 0.504*** |
| | (0.0966) | (0.0974) | (0.103) | (0.0927) | (0.0944) | (0.0969) | (0.0928) | (0.125) | (0.122) |
| $\hat{r}_{t-1}^{Gap}$ | | -0.0620* | | | | -0.0391 | | 0.496*** | 0.373** |
| | | (0.0322) | | | | (0.0369) | | (0.166) | (0.174) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | 0.0160 | | | | | | 0.0510* |
| | | | (0.0242) | | | | | | (0.0270) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.0706** | | | -0.0545* | -0.489*** | -0.404** |
| | | | | (0.0296) | | | (0.0325) | (0.148) | (0.151) |
| $\hat{r}_{t-1}$ | | | | | 0.251** | 0.176 | 0.156 | 0.355** | 0.389*** |
| | | | | | (0.123) | (0.142) | (0.133) | (0.141) | (0.138) |
| Constant | 0.210** | 0.0761 | 0.228** | 0.0214 | 0.301*** | 0.190 | 0.121 | 0.105 | 0.136 |
| | (0.0830) | (0.107) | (0.0877) | (0.112) | (0.0923) | (0.140) | (0.140) | (0.131) | (0.128) |
| Observations | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 |
| Adjusted R-squared | 0.520 | 0.544 | 0.515 | 0.560 | 0.548 | 0.549 | 0.563 | 0.623 | 0.642 |
| Root Mean Square Error | 0.379 | 0.369 | 0.381 | 0.363 | 0.368 | 0.367 | 0.361 | 0.336 | 0.327 |

Standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table OA11: Forecasting $\Delta_4\pi_{t+1}$ from 1966Q1 to 1979Q2

Forecasting 1-period ahead annual inflation: from 1966Q1 to 1979Q2

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-1}$ | 0.947*** | 1.011*** | 1.030*** | 0.991*** | 0.953*** | 1.011*** | 0.991*** | 0.900*** | 0.947*** |
| | (0.0370) | (0.0367) | (0.0433) | (0.0326) | (0.0365) | (0.0376) | (0.0329) | (0.0449) | (0.0480) |
| $\hat{r}_{t-1}^{Gap}$ | | -0.170*** | | | | -0.173*** | | 0.576*** | 0.604*** |
| | | (0.0436) | | | | (0.0517) | | (0.207) | (0.200) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | -0.116*** | | | | | | -0.0822** |
| | | | (0.0368) | | | | | | (0.0369) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.176*** | | | -0.175*** | -0.650*** | -0.653*** |
| | | | | (0.0374) | | | (0.0416) | (0.175) | (0.169) |
| $\hat{r}_{t-1}$ | | | | | 0.301* | -0.0199 | 0.0129 | 0.300 | 0.159 |
| | | | | | (0.171) | (0.183) | (0.164) | (0.185) | (0.189) |
| Constant | 0.245** | -0.193 | 0.0118 | -0.310** | 0.357*** | -0.208 | -0.301 | -0.205 | -0.361* |
| | (0.118) | (0.154) | (0.132) | (0.154) | (0.132) | (0.208) | (0.194) | (0.185) | (0.192) |
| Observations | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 |
| Adjusted R-squared | 0.925 | 0.941 | 0.936 | 0.947 | 0.928 | 0.940 | 0.946 | 0.952 | 0.956 |
| Root Mean Square Error | 0.523 | 0.464 | 0.484 | 0.441 | 0.513 | 0.469 | 0.446 | 0.418 | 0.402 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA12: Forecasting $\pi_{t+4}$ from 1966Q1 to 1979Q2

Forecasting 4-period ahead inflation: from from 1966Q1 to 1979Q2

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-4}$ | 0.531*** | 0.610*** | 0.496*** | 0.574*** | 0.533*** | 0.606*** | 0.557*** | 0.239 | 0.258* |
| | (0.111) | (0.113) | (0.120) | (0.105) | (0.114) | (0.113) | (0.106) | (0.149) | (0.148) |
| $\hat{r}_{t-4}^{Gap}$ | | -0.0878** | | | | -0.112** | | 0.674*** | 0.530** |
| | | (0.0389) | | | | (0.0445) | | (0.236) | (0.257) |
| $\hat{r}_{t-4}^{Gap,cons}$ | | | 0.0227 | | | | | | 0.0452 |
| | | | (0.0294) | | | | | | (0.0335) |
| $\hat{r}_{t-4}^{Gap,cc}$ | | | | -0.103*** | | | -0.121*** | -0.720*** | -0.612*** |
| | | | | (0.0358) | | | (0.0390) | (0.213) | (0.226) |
| $\hat{r}_{t-4}$ | | | | | 0.0204 | -0.189 | -0.175 | 0.117 | 0.131 |
| | | | | | (0.157) | (0.171) | (0.159) | (0.180) | (0.179) |
| Constant | 0.390*** | 0.196 | 0.417*** | 0.108 | 0.397*** | 0.0783 | 0.00106 | -0.0455 | -0.0118 |
| | (0.0912) | (0.123) | (0.0979) | (0.130) | (0.107) | (0.162) | (0.162) | (0.152) | (0.153) |
| Observations | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 |
| Adjusted R-squared | 0.291 | 0.343 | 0.286 | 0.379 | 0.278 | 0.346 | 0.381 | 0.459 | 0.468 |
| Root Mean Square Error | 0.460 | 0.443 | 0.462 | 0.431 | 0.465 | 0.442 | 0.430 | 0.402 | 0.399 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA13: Forecasting $\pi_{t+1}$ from 1984Q1 to 2004Q4

Forecasting 1-period ahead inflation: from 1984Q1 to 2004Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-1}$ | 0.529*** | 0.542*** | 0.496*** | 0.506*** | 0.436*** | 0.453*** | 0.445*** | 0.453*** | 0.413*** |
| | (0.0934) | (0.0927) | (0.0955) | (0.0915) | (0.0945) | (0.0953) | (0.0950) | (0.0960) | (0.0953) |
| $\hat{r}_{t-1}^{Gap}$ | | -0.0270* | | | | -0.0190 | | -0.0215 | -0.0283 |
| | | (0.0160) | | | | (0.0157) | | (0.0283) | (0.0278) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | 0.0129 | | | | | | 0.0207** |
| | | | (0.00877) | | | | | | (0.00919) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.0242** | | | -0.0113 | 0.00230 | -0.00597 |
| | | | | (0.0103) | | | (0.0120) | (0.0216) | (0.0214) |
| $\hat{r}_{t-1}$ | | | | | 0.126*** | 0.116*** | 0.100** | 0.120** | 0.0967* |
| | | | | | (0.0423) | (0.0429) | (0.0503) | (0.0567) | (0.0562) |
| Constant | -0.0350 | -0.0711** | -0.00571 | -0.113*** | -0.0595** | -0.0830*** | -0.0911** | -0.0796* | -0.0641 |
| | (0.0241) | (0.0320) | (0.0311) | (0.0407) | (0.0244) | (0.0311) | (0.0416) | (0.0443) | (0.0438) |
| Observations | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 |
| Adjusted R-squared | 0.272 | 0.288 | 0.283 | 0.310 | 0.336 | 0.340 | 0.335 | 0.331 | 0.364 |
| Root Mean Square Error | 0.212 | 0.210 | 0.211 | 0.207 | 0.203 | 0.202 | 0.203 | 0.204 | 0.199 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA14: Forecasting $\Delta_4\pi_{t+1}$ from 1984Q1 to 2004Q4

Forecasting 1-period ahead annual inflation: from 1984Q1 to 2004Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-1}$ | 0.953*** | 0.958*** | 0.964*** | 0.943*** | 0.947*** | 0.964*** | 0.986*** | 0.989*** | 0.982*** |
| | (0.0307) | (0.0299) | (0.0337) | (0.0292) | (0.0371) | (0.0367) | (0.0358) | (0.0359) | (0.0390) |
| $\hat{r}_{t-1}^{Gap}$ | | -0.0401** | | | | -0.0412** | | 0.0319 | 0.0296 |
| | | (0.0166) | | | | (0.0172) | | (0.0290) | (0.0296) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | -0.00757 | | | | | | 0.00476 |
| | | | (0.00999) | | | | | | (0.0103) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.0339*** | | | -0.0488*** | -0.0695*** | -0.0703*** |
| | | | | (0.0106) | | | (0.0129) | (0.0228) | (0.0230) |
| $\hat{r}_{t-1}$ | | | | | 0.0147 | -0.0146 | -0.121* | -0.156** | -0.155** |
| | | | | | (0.0536) | (0.0535) | (0.0612) | (0.0688) | (0.0692) |
| Constant | -0.0229 | -0.0766** | -0.0388 | -0.133*** | -0.0265 | -0.0745** | -0.152*** | -0.168*** | -0.164*** |
| | (0.0258) | (0.0335) | (0.0333) | (0.0423) | (0.0290) | (0.0346) | (0.0426) | (0.0450) | (0.0460) |
| Observations | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 |
| Adjusted R-squared | 0.921 | 0.925 | 0.920 | 0.929 | 0.920 | 0.924 | 0.931 | 0.931 | 0.931 |
| Root Mean Square Error | 0.225 | 0.219 | 0.225 | 0.213 | 0.226 | 0.220 | 0.209 | 0.209 | 0.210 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA15: Forecasting $\pi_{t+4}$ from 1984Q1 to 2004Q4

Forecasting 4-period ahead inflation: from from 1984Q1 to 2004Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-4}$ | 0.583*** | 0.585*** | 0.546*** | 0.565*** | 0.550*** | 0.553*** | 0.563*** | 0.553*** | 0.512*** |
| | (0.0877) | (0.0883) | (0.0916) | (0.0876) | (0.0955) | (0.0968) | (0.0956) | (0.0949) | (0.0947) |
| $\hat{r}^{Gap}_{t-4}$ | | -0.00687 | | | | -0.00469 | | 0.0450 | 0.0445 |
| | | (0.0151) | | | | (0.0153) | | (0.0282) | (0.0276) |
| $\hat{r}^{Gap,cons}_{t-4}$ | | | 0.00944 | | | | | | 0.0167** |
| | | | (0.00707) | | | | | | (0.00783) |
| $\hat{r}^{Gap,cc}_{t-4}$ | | | | -0.0160 | | | -0.0158 | -0.0449** | -0.0572** |
| | | | | (0.00979) | | | (0.0116) | (0.0216) | (0.0219) |
| $\hat{r}_{t-4}$ | | | | | 0.0399 | 0.0375 | 0.00188 | -0.0447 | -0.0906 |
| | | | | | (0.0451) | (0.0461) | (0.0529) | (0.0599) | (0.0624) |
| Constant | -0.0364 | -0.0456 | -0.0182 | -0.0887** | -0.0467* | -0.0524 | -0.0885** | -0.111*** | -0.108** |
| | (0.0227) | (0.0304) | (0.0263) | (0.0391) | (0.0255) | (0.0316) | (0.0398) | (0.0420) | (0.0411) |
| Observations | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 |
| Adjusted R-squared | 0.342 | 0.336 | 0.348 | 0.355 | 0.340 | 0.333 | 0.347 | 0.360 | 0.387 |
| Root Mean Square Error | 0.202 | 0.203 | 0.201 | 0.200 | 0.202 | 0.203 | 0.201 | 0.199 | 0.195 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

### OA1.3 Smets and Wouters (2024)'s model

Table OA16: Forecasting $\pi_{t+1}$ from 1966Q1 to 2004Q4

Forecasting 1-period ahead inflation: from 1965Q1 to 2019Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-1}$ | 0.897*** | 0.860*** | 0.909*** | 0.876*** | 0.885*** | 0.846*** | 0.865*** | 0.847*** | 0.844*** |
| | (0.0302) | (0.0317) | (0.0305) | (0.0318) | (0.0309) | (0.0324) | (0.0323) | (0.0323) | (0.0360) |
| $\hat{r}_{t-1}^{Gap}$ | | -0.0706*** | | | | -0.0728*** | | -0.121*** | -0.126** |
| | | (0.0220) | | | | (0.0219) | | (0.0391) | (0.0506) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | 0.0378** | | | | | | -0.00359 |
| | | | (0.0165) | | | | | | (0.0214) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.0294* | | | -0.0283* | 0.0390 | 0.0413 |
| | | | | (0.0150) | | | (0.0150) | (0.0262) | (0.0298) |
| $\hat{r}_{t-1}$ | | | | | 0.0482* | 0.0529* | 0.0460* | 0.0591** | 0.0603** |
| | | | | | (0.0278) | (0.0273) | (0.0277) | (0.0275) | (0.0284) |
| Constant | 0.0349* | 0.0317 | 0.0386* | 0.0361* | 0.0394* | 0.0365* | 0.0404** | 0.0333* | 0.0327 |
| | (0.0204) | (0.0200) | (0.0203) | (0.0203) | (0.0205) | (0.0201) | (0.0204) | (0.0201) | (0.0205) |
| Observations | 219 | 219 | 219 | 219 | 219 | 219 | 219 | 219 | 219 |
| Adjusted R-squared | 0.801 | 0.809 | 0.805 | 0.804 | 0.803 | 0.812 | 0.805 | 0.813 | 0.812 |
| Root Mean Square Error | 0.260 | 0.254 | 0.257 | 0.258 | 0.259 | 0.253 | 0.257 | 0.252 | 0.253 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA17: Forecasting $\Delta_4\pi_{t+1}$ from 1966Q1 to 2004Q4

Forecasting 1-period ahead annual inflation: from 1965Q1 to 2019Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-1}$ | 0.987*** | 0.972*** | 0.986*** | 0.972*** | 0.992*** | 0.977*** | 0.978*** | 0.985*** | 0.984*** |
| | (0.0114) | (0.0119) | (0.0115) | (0.0111) | (0.0120) | (0.0125) | (0.0115) | (0.0121) | (0.0133) |
| $\hat{r}_{t-1}^{Gap}$ | | -0.112*** | | | | -0.109*** | | 0.103* | 0.0998 |
| | | (0.0312) | | | | (0.0313) | | (0.0556) | (0.0728) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | -0.0117 | | | | | | -0.00208 |
| | | | (0.0236) | | | | | | (0.0292) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.107*** | | | -0.108*** | -0.165*** | -0.164*** |
| | | | | (0.0198) | | | (0.0198) | (0.0365) | (0.0426) |
| $\hat{r}_{t-1}$ | | | | | -0.0604 | -0.0492 | -0.0690* | -0.0842** | -0.0833** |
| | | | | | (0.0409) | (0.0400) | (0.0384) | (0.0391) | (0.0408) |
| Constant | 0.0162 | 0.0114 | 0.0145 | 0.0154 | 0.00822 | 0.00505 | 0.00628 | 0.00826 | 0.00793 |
| | (0.0297) | (0.0290) | (0.0300) | (0.0280) | (0.0301) | (0.0294) | (0.0283) | (0.0281) | (0.0286) |
| Observations | 219 | 219 | 219 | 219 | 219 | 219 | 219 | 219 | 219 |
| Adjusted R-squared | 0.972 | 0.973 | 0.971 | 0.975 | 0.972 | 0.973 | 0.975 | 0.975 | 0.975 |
| Root Mean Square Error | 0.372 | 0.362 | 0.373 | 0.350 | 0.371 | 0.362 | 0.349 | 0.347 | 0.347 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA18: Forecasting $\pi_{t+4}$ from 1966Q1 to 2004Q4

Forecasting 4-period ahead inflation: from from 1965Q1 to 2019Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-4}$ | 0.796*** | 0.738*** | 0.801*** | 0.739*** | 0.803*** | 0.743*** | 0.747*** | 0.742*** | 0.725*** |
| | (0.0417) | (0.0432) | (0.0424) | (0.0424) | (0.0427) | (0.0443) | (0.0433) | (0.0441) | (0.0491) |
| $\hat{r}_{t-4}^{Gap}$ | | -0.116*** | | | | -0.115*** | | -0.0341 | -0.0688 |
| | | (0.0300) | | | | (0.0301) | | (0.0534) | (0.0692) |
| $\hat{r}_{t-4}^{Gap,cons}$ | | | 0.0128 | | | | | | -0.0232 |
| | | | (0.0229) | | | | | | (0.0294) |
| $\hat{r}_{t-4}^{Gap,cc}$ | | | | -0.0839*** | | | -0.0846*** | -0.0657* | -0.0507 |
| | | | | (0.0201) | | | (0.0201) | (0.0359) | (0.0406) |
| $\hat{r}_{t-4}$ | | | | | -0.0284 | -0.0206 | -0.0346 | -0.0309 | -0.0233 |
| | | | | | (0.0384) | (0.0373) | (0.0371) | (0.0376) | (0.0388) |
| Constant | 0.0689** | 0.0623** | 0.0704** | 0.0707** | 0.0664** | 0.0605** | 0.0676** | 0.0656** | 0.0614** |
| | (0.0283) | (0.0275) | (0.0285) | (0.0273) | (0.0286) | (0.0277) | (0.0275) | (0.0277) | (0.0283) |
| Observations | 216 | 216 | 216 | 216 | 216 | 216 | 216 | 216 | 216 |
| Adjusted R-squared | 0.629 | 0.651 | 0.628 | 0.655 | 0.628 | 0.650 | 0.655 | 0.654 | 0.653 |
| Root Mean Square Error | 0.356 | 0.345 | 0.357 | 0.343 | 0.357 | 0.346 | 0.344 | 0.344 | 0.344 |

Standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table OA19: Forecasting $\pi_{t+1}$ from 1966Q1 to 1979Q2

Forecasting 1-period ahead inflation: from 1965Q1 to 1979Q2

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-1}$ | 0.785*** | 0.845*** | 0.786*** | 0.807*** | 0.802*** | 0.831*** | 0.731*** | 0.630*** | 0.636*** |
| | (0.0862) | (0.0925) | (0.0848) | (0.105) | (0.0822) | (0.0897) | (0.102) | (0.0960) | (0.0971) |
| $\hat{r}_{t-1}^{Gap}$ | | -0.0639 | | | | -0.0331 | | -0.247*** | -0.233*** |
| | | (0.0392) | | | | (0.0405) | | (0.0670) | (0.0712) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | 0.0970 | | | | | | 0.0333 |
| | | | (0.0584) | | | | | | (0.0538) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.0108 | | | 0.0375 | 0.205*** | 0.195*** |
| | | | | (0.0294) | | | (0.0325) | (0.0540) | (0.0566) |
| $\hat{r}_{t-1}$ | | | | | 0.351** | 0.310** | 0.446*** | 0.562*** | 0.551*** |
| | | | | | (0.134) | (0.144) | (0.157) | (0.144) | (0.146) |
| Constant | 0.212** | 0.185** | 0.205** | 0.176 | 0.276*** | 0.255*** | 0.418*** | 0.888*** | 0.857*** |
| | (0.0861) | (0.0864) | (0.0848) | (0.130) | (0.0855) | (0.0897) | (0.149) | (0.185) | (0.193) |
| Observations | 57 | 57 | 57 | 57 | 57 | 57 | 57 | 57 | 57 |
| Adjusted R-squared | 0.594 | 0.606 | 0.607 | 0.588 | 0.633 | 0.631 | 0.636 | 0.706 | 0.702 |
| Root Mean Square Error | 0.363 | 0.358 | 0.358 | 0.366 | 0.345 | 0.346 | 0.344 | 0.309 | 0.311 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA20: Forecasting $\Delta_4\pi_{t+1}$ from 1966Q1 to 1979Q2

Forecasting 1-period ahead annual inflation: from 1965Q1 to 1979Q2

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-1}$ | 0.961*** | 1.004*** | 0.965*** | 1.036*** | 0.967*** | 0.996*** | 1.008*** | 0.979*** | 0.994*** |
| | (0.0341) | (0.0379) | (0.0352) | (0.0466) | (0.0328) | (0.0375) | (0.0508) | (0.0565) | (0.0577) |
| $\hat{r}^{Gap}_{t-1}$ | | -0.133** | | | | -0.0943 | | -0.135 | -0.162 |
| | | (0.0581) | | | | (0.0612) | | (0.115) | (0.116) |
| $\hat{r}^{Gap,cons}_{t-1}$ | | | -0.0441 | | | | | | -0.106 |
| | | | (0.0875) | | | | | | (0.0869) |
| $\hat{r}^{Gap,cc}_{t-1}$ | | | | -0.108** | | | -0.0617 | 0.0450 | 0.0482 |
| | | | | (0.0470) | | | (0.0581) | (0.108) | (0.107) |
| $\hat{r}_{t-1}$ | | | | | 0.469** | 0.355* | 0.318 | 0.417 | 0.406 |
| | | | | | (0.193) | (0.204) | (0.240) | (0.253) | (0.252) |
| Constant | 0.229* | 0.138 | 0.219* | -0.187 | 0.313** | 0.228 | 0.0478 | 0.385 | 0.355 |
| | (0.128) | (0.130) | (0.130) | (0.220) | (0.128) | (0.138) | (0.281) | (0.401) | (0.400) |
| Observations | 57 | 57 | 57 | 57 | 57 | 57 | 57 | 57 | 57 |
| Adjusted R-squared | 0.934 | 0.939 | 0.933 | 0.939 | 0.939 | 0.941 | 0.940 | 0.940 | 0.941 |
| Root Mean Square Error | 0.519 | 0.500 | 0.523 | 0.500 | 0.497 | 0.491 | 0.497 | 0.495 | 0.493 |

Standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table OA21: Forecasting $\pi_{t+4}$ from 1966Q1 to 1979Q2

Forecasting 4-period ahead inflation: from from 1965Q1 to 1979Q2

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-4}$ | 0.523*** | 0.619*** | 0.522*** | 0.525*** | 0.527*** | 0.620*** | 0.513*** | 0.407*** | 0.390*** |
| | (0.113) | (0.125) | (0.114) | (0.140) | (0.115) | (0.127) | (0.146) | (0.133) | (0.132) |
| $\hat{r}^{Gap}_{t-4}$ | | -0.0885* | | | | -0.0921 | | -0.364*** | -0.427*** |
| | | (0.0527) | | | | (0.0561) | | (0.0981) | (0.107) |
| $\hat{r}^{Gap,cons}_{t-4}$ | | | 0.0244 | | | | | | -0.110 |
| | | | (0.0800) | | | | | | (0.0788) |
| $\hat{r}^{Gap,cc}_{t-4}$ | | | | -0.00115 | | | 0.00730 | 0.250*** | 0.293*** |
| | | | | (0.0384) | | | (0.0450) | (0.0767) | (0.0821) |
| $\hat{r}_{t-4}$ | | | | | 0.0620 | -0.0385 | 0.0800 | 0.279 | 0.332 |
| | | | | | (0.185) | (0.192) | (0.217) | (0.201) | (0.203) |
| Constant | 0.491*** | 0.450*** | 0.489*** | 0.487*** | 0.502*** | 0.441*** | 0.530** | 1.205*** | 1.339*** |
| | (0.110) | (0.111) | (0.112) | (0.170) | (0.116) | (0.120) | (0.207) | (0.259) | (0.274) |
| Observations | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 |
| Adjusted R-squared | 0.277 | 0.302 | 0.264 | 0.263 | 0.265 | 0.288 | 0.250 | 0.403 | 0.414 |
| Root Mean Square Error | 0.465 | 0.457 | 0.469 | 0.470 | 0.469 | 0.462 | 0.474 | 0.423 | 0.419 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA22: Forecasting $\pi_{t+1}$ from 1984Q1 to 2004Q4

Forecasting 1-period ahead inflation: from 1984Q1 to 2004Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-1}$ | 0.766*** | 0.760*** | 0.751*** | 0.770*** | 0.747*** | 0.733*** | 0.743*** | 0.727*** | 0.731*** |
| | (0.0711) | (0.0712) | (0.0718) | (0.0718) | (0.0750) | (0.0755) | (0.0751) | (0.0771) | (0.0779) |
| $\hat{r}_{t-1}^{Gap}$ | | -0.0471 | | | | -0.0571 | | -0.0960 | -0.0624 |
| | | (0.0422) | | | | (0.0432) | | (0.108) | (0.133) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | 0.0327 | | | | | | 0.0152 |
| | | | (0.0258) | | | | | | (0.0343) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.0189 | | | -0.0444 | 0.0415 | 0.0158 |
| | | | | (0.0369) | | | (0.0423) | (0.106) | (0.121) |
| $\hat{r}_{t-1}$ | | | | | 0.0251 | 0.0342 | 0.0437 | 0.0229 | 0.0222 |
| | | | | | (0.0312) | (0.0319) | (0.0360) | (0.0430) | (0.0432) |
| Constant | 0.0244 | 0.0264 | 0.0426* | 0.0254 | 0.0245 | 0.0270 | 0.0271 | 0.0263 | 0.0348 |
| | (0.0173) | (0.0174) | (0.0224) | (0.0175) | (0.0174) | (0.0174) | (0.0175) | (0.0176) | (0.0260) |
| Observations | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 |
| Adjusted R-squared | 0.581 | 0.582 | 0.584 | 0.577 | 0.579 | 0.583 | 0.580 | 0.579 | 0.574 |
| Root Mean Square Error | 0.144 | 0.144 | 0.143 | 0.144 | 0.144 | 0.143 | 0.144 | 0.144 | 0.145 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA23: Forecasting $\Delta_4\pi_{t+1}$ from 1984Q1 to 2004Q4

Forecasting 1-period ahead annual inflation: from 1984Q1 to 2004Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-1}$ | 0.960*** | 0.973*** | 0.964*** | 1.002*** | 0.975*** | 0.980*** | 0.994*** | 0.991*** | 0.996*** |
| | (0.0287) | (0.0263) | (0.0303) | (0.0275) | (0.0322) | (0.0294) | (0.0293) | (0.0303) | (0.0301) |
| $\hat{r}^{Gap}_{t-1}$ | | -0.242*** | | | | -0.238*** | | -0.0448 | -0.199 |
| | | (0.0562) | | | | (0.0570) | | (0.143) | (0.169) |
| $\hat{r}^{Gap,cons}_{t-1}$ | | | -0.0169 | | | | | | -0.0742* |
| | | | (0.0392) | | | | | | (0.0443) |
| $\hat{r}^{Gap,cc}_{t-1}$ | | | | -0.230*** | | | -0.250*** | -0.209 | -0.0941 |
| | | | | (0.0508) | | | (0.0560) | (0.142) | (0.156) |
| $\hat{r}_{t-1}$ | | | | | -0.0476 | -0.0209 | 0.0413 | 0.0318 | 0.0326 |
| | | | | | (0.0484) | (0.0446) | (0.0479) | (0.0569) | (0.0563) |
| Constant | 0.0124 | 0.0137 | 0.00205 | 0.0120 | 0.00963 | 0.0124 | 0.0144 | 0.0141 | -0.0300 |
| | (0.0260) | (0.0236) | (0.0354) | (0.0233) | (0.0261) | (0.0238) | (0.0235) | (0.0237) | (0.0353) |
| Observations | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 |
| Adjusted R-squared | 0.931 | 0.943 | 0.930 | 0.944 | 0.931 | 0.942 | 0.944 | 0.943 | 0.944 |
| Root Mean Square Error | 0.210 | 0.190 | 0.211 | 0.188 | 0.210 | 0.191 | 0.189 | 0.190 | 0.188 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA24: Forecasting $\pi_{t+4}$ from 1984Q1 to 2004Q4

Forecasting 4-period ahead inflation: from from 1984Q1 to 2004Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-4}$ | 0.548*** | 0.538*** | 0.517*** | 0.559*** | 0.555*** | 0.535*** | 0.550*** | 0.520*** | 0.527*** |
| | (0.0894) | (0.0893) | (0.0916) | (0.0905) | (0.0988) | (0.0994) | (0.0990) | (0.102) | (0.102) |
| $\hat{r}^{Gap}_{t-4}$ | | -0.0720 | | | | -0.0726 | | -0.172 | -0.0801 |
| | | (0.0541) | | | | (0.0553) | | (0.140) | (0.172) |
| $\hat{r}^{Gap,cons}_{t-4}$ | | | 0.0410 | | | | | | 0.0360 |
| | | | (0.0290) | | | | | | (0.0388) |
| $\hat{r}^{Gap,cc}_{t-4}$ | | | | -0.0422 | | | -0.0479 | 0.105 | 0.0301 |
| | | | | (0.0478) | | | (0.0536) | (0.135) | (0.157) |
| $\hat{r}_{t-4}$ | | | | | -0.00758 | 0.00294 | 0.0121 | -0.0257 | -0.0281 |
| | | | | | (0.0450) | (0.0455) | (0.0501) | (0.0587) | (0.0588) |
| Constant | 0.0443* | 0.0483** | 0.0654** | 0.0478** | 0.0445* | 0.0482** | 0.0478** | 0.0461** | 0.0658** |
| | (0.0224) | (0.0225) | (0.0268) | (0.0228) | (0.0226) | (0.0226) | (0.0229) | (0.0229) | (0.0312) |
| Observations | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 |
| Adjusted R-squared | 0.306 | 0.312 | 0.314 | 0.304 | 0.297 | 0.303 | 0.295 | 0.300 | 0.299 |
| Root Mean Square Error | 0.185 | 0.184 | 0.184 | 0.185 | 0.186 | 0.185 | 0.186 | 0.186 | 0.186 |

Standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table OA25: Forecasting $\pi_{t+1}$ from 1965Q1 to 2019Q4 (only monetary-led inflation)

Forecasting 1-period ahead inflation: from 2005Q1 to 2019Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-1}$ | 0.468*** | 0.281** | 0.399*** | 0.149 | 0.436*** | 0.284** | 0.135 | 0.0723 | 0.0705 |
| | (0.114) | (0.125) | (0.136) | (0.137) | (0.114) | (0.125) | (0.135) | (0.142) | (0.164) |
| $\hat{r}^{Gap}_{t-1}$ | | -0.265*** | | | | -0.235** | | 0.283 | 0.281 |
| | | (0.0901) | | | | (0.0945) | | (0.213) | (0.225) |
| $\hat{r}^{Gap,cons}_{t-1}$ | | | -0.0390 | | | | | | -0.00113 |
| | | | (0.0410) | | | | | | (0.0514) |
| $\hat{r}^{Gap,cc}_{t-1}$ | | | | -0.301*** | | | -0.289*** | -0.524*** | -0.523** |
| | | | | (0.0837) | | | (0.0828) | (0.195) | (0.201) |
| $\hat{r}_{t-1}$ | | | | | 0.151* | 0.0877 | 0.128 | 0.185** | 0.183 |
| | | | | | (0.0837) | (0.0841) | (0.0769) | (0.0876) | (0.122) |
| Constant | -0.0316 | 0.0426 | -0.0111 | 0.0757* | 0.0795 | 0.0987 | 0.165** | 0.212*** | 0.211** |
| | (0.0291) | (0.0372) | (0.0363) | (0.0400) | (0.0678) | (0.0654) | (0.0667) | (0.0751) | (0.0813) |
| Observations | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| Adjusted R-squared | 0.211 | 0.303 | 0.210 | 0.345 | 0.241 | 0.304 | 0.365 | 0.373 | 0.362 |
| Root Mean Square Error | 0.222 | 0.209 | 0.222 | 0.202 | 0.218 | 0.208 | 0.199 | 0.198 | 0.200 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA26: Forecasting $\Delta_4\pi_{t+1}$ from 1965Q1 to 2019Q4 (only monetary-led inflation)

Forecasting 1-period ahead annual inflation: from 2005Q1 to 2019Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4\hat{\pi}_{t-1}$ | 0.911*** | 0.855*** | 0.906*** | 0.799*** | 0.940*** | 0.885*** | 0.810*** | 0.777*** | 0.775*** |
| | (0.0490) | (0.0594) | (0.0648) | (0.0571) | (0.0577) | (0.0658) | (0.0684) | (0.0672) | (0.0737) |
| $\hat{r}_{t-1}^{Gap}$ | | -0.210 | | | | -0.216 | | 0.636** | 0.631** |
| | | (0.130) | | | | (0.130) | | (0.268) | (0.291) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | -0.00780 | | | | | | -0.00342 |
| | | | (0.0591) | | | | | | (0.0636) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | -0.343*** | | | -0.336*** | -0.843*** | -0.839*** |
| | | | | (0.105) | | | (0.109) | (0.238) | (0.252) |
| $\hat{r}_{t-1}$ | | | | | -0.122 | -0.131 | -0.0371 | 0.119 | 0.114 |
| | | | | | (0.128) | (0.127) | (0.123) | (0.135) | (0.160) |
| Constant | -0.0374 | 0.0198 | -0.0334 | 0.0848 | -0.124 | -0.0715 | 0.0559 | 0.174 | 0.173 |
| | (0.0378) | (0.0513) | (0.0484) | (0.0514) | (0.0987) | (0.102) | (0.109) | (0.116) | (0.120) |
| Observations | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| Adjusted R-squared | 0.854 | 0.858 | 0.851 | 0.874 | 0.853 | 0.858 | 0.872 | 0.882 | 0.880 |
| Root Mean Square Error | 0.288 | 0.284 | 0.290 | 0.266 | 0.288 | 0.283 | 0.269 | 0.258 | 0.260 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA27: Forecasting $\pi_{t+4}$ from 1965Q1 to 2019Q4 (only monetary-led inflation)

Forecasting 4-period ahead inflation: from from 2005Q1 to 2019Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-4}$ | 0.324*** | 0.130 | 0.340** | -0.0382 | 0.327*** | 0.126 | -0.0352 | -0.0859 | -0.0627 |
| | (0.120) | (0.133) | (0.147) | (0.143) | (0.122) | (0.133) | (0.144) | (0.153) | (0.172) |
| $\hat{r}_{t-4}^{Gap}$ | | -0.259*** | | | | -0.283*** | | 0.227 | 0.253 |
| | | (0.0928) | | | | (0.0964) | | (0.224) | (0.241) |
| $\hat{r}_{t-4}^{Gap,cons}$ | | | 0.00801 | | | | | | 0.0156 |
| | | | (0.0432) | | | | | | (0.0517) |
| $\hat{r}_{t-4}^{Gap,cc}$ | | | | -0.316*** | | | -0.317*** | -0.502** | -0.521** |
| | | | | (0.0819) | | | (0.0826) | (0.200) | (0.212) |
| $\hat{r}_{t-4}$ | | | | | -0.0131 | -0.0837 | -0.0256 | 0.0237 | 0.0486 |
| | | | | | (0.0913) | (0.0890) | (0.0820) | (0.0953) | (0.127) |
| Constant | -0.0435 | 0.0202 | -0.0472 | 0.0531 | -0.0535 | -0.0379 | 0.0338 | 0.0725 | 0.0840 |
| | (0.0308) | (0.0370) | (0.0370) | (0.0374) | (0.0765) | (0.0721) | (0.0724) | (0.0818) | (0.0909) |
| Observations | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| Adjusted R-squared | 0.0972 | 0.192 | 0.0819 | 0.271 | 0.0817 | 0.190 | 0.260 | 0.260 | 0.248 |
| Root Mean Square Error | 0.237 | 0.225 | 0.239 | 0.213 | 0.239 | 0.225 | 0.215 | 0.215 | 0.217 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA28: Forecasting $\pi_{t+1}$ from 1965Q1 to 2019Q4 (only fiscal-led inflation)

Forecasting 1-period ahead inflation: from 2020Q1 to 2021Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-1}$ | 0.621 | 1.073* | 0.684 | 1.084* | -1.105 | -0.593 | -0.570 | -0.581 | -1.821 |
| | (0.355) | (0.483) | (0.378) | (0.475) | (0.777) | (0.887) | (0.898) | (1.054) | (1.459) |
| $\hat{r}_{t-1}^{Gap}$ | | 0.118 | | | | 0.0807 | | 0.0432 | -1.306 |
| | | (0.0904) | | | | (0.0724) | | (0.785) | (1.376) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | -0.0420 | | | | | | -0.156 |
| | | | (0.0561) | | | | | | (0.134) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | 0.122 | | | 0.0817 | 0.0382 | 1.167 |
| | | | | (0.0897) | | | (0.0732) | (0.794) | (1.226) |
| $\hat{r}_{t-1}$ | | | | | -2.228* | -1.968 | -1.937 | -1.953 | -2.681 |
| | | | | | (0.942) | (0.949) | (0.956) | (1.140) | (1.246) |
| Constant | 0.282 | 0.0634 | 0.328 | 0.0139 | -2.127* | -1.995 | -1.991 | -1.993 | -2.577 |
| | (0.239) | (0.282) | (0.256) | (0.298) | (1.034) | (1.016) | (1.017) | (1.174) | (1.218) |
| Observations | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| Adjusted R-squared | 0.227 | 0.307 | 0.166 | 0.324 | 0.562 | 0.583 | 0.583 | 0.444 | 0.503 |
| Root Mean Square Error | 0.615 | 0.583 | 0.639 | 0.576 | 0.463 | 0.452 | 0.452 | 0.522 | 0.493 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA29: Forecasting $\Delta_4 \pi_{t+1}$ from 1965Q1 to 2019Q4 (only fiscal-led inflation)

Forecasting 1-period ahead annual inflation: from 2020Q1 to 2021Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\Delta_4 \hat{\pi}_{t-1}$ | 1.174*** | 1.185*** | 1.173*** | 1.193*** | 0.826** | 0.731** | 0.741** | 0.816* | 0.725 |
| | (0.198) | (0.231) | (0.218) | (0.231) | (0.273) | (0.248) | (0.239) | (0.280) | (0.290) |
| $\hat{r}_{t-1}^{Gap}$ | | 0.0135 | | | | 0.139 | | -0.715 | -1.858 |
| | | (0.102) | | | | (0.0890) | | (1.059) | (1.521) |
| $\hat{r}_{t-1}^{Gap,cons}$ | | | -0.00310 | | | | | | -0.131 |
| | | | (0.0766) | | | | | | (0.127) |
| $\hat{r}_{t-1}^{Gap,cc}$ | | | | 0.0235 | | | 0.144 | 0.850 | 1.860 |
| | | | | (0.103) | | | (0.0858) | (1.049) | (1.424) |
| $\hat{r}_{t-1}$ | | | | | -1.297 | -2.063* | -2.043* | -1.769 | -1.598 |
| | | | | | (0.783) | (0.846) | (0.805) | (0.956) | (0.959) |
| Constant | 0.512 | 0.498 | 0.517 | 0.480 | -1.062 | -2.134 | -2.167 | -2.076 | -1.876 |
| | (0.294) | (0.338) | (0.345) | (0.351) | (0.985) | (1.106) | (1.070) | (1.159) | (1.162) |
| Observations | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| Adjusted R-squared | 0.830 | 0.796 | 0.796 | 0.798 | 0.868 | 0.898 | 0.903 | 0.888 | 0.891 |
| Root Mean Square Error | 0.812 | 0.888 | 0.889 | 0.885 | 0.715 | 0.630 | 0.612 | 0.659 | 0.651 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table OA30: Forecasting $\pi_{t+4}$ from 1965Q1 to 2019Q4 (only fiscal-led inflation)

Forecasting 4-period ahead inflation: from from 2020Q1 to 2021Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| VARIABLES | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
| $\hat{\pi}_{t-4}$ | -0.0212 | 1.315 | -0.0725 | 1.765 | -1.375* | -1.090 | -1.104 | -3.971 | -4.008 |
| | (0.758) | (1.898) | (0.962) | (2.045) | (0.581) | (1.435) | (1.652) | (4.571) | (7.658) |
| $\hat{r}_{t-4}^{Gap}$ | | 0.163 | | | | 0.0308 | | 1.710 | 1.761 |
| | | (0.211) | | | | (0.139) | | (2.512) | (7.850) |
| $\hat{r}_{t-4}^{Gap,cons}$ | | | 0.00873 | | | | | | 0.00224 |
| | | | (0.0830) | | | | | | (0.319) |
| $\hat{r}_{t-4}^{Gap,cc}$ | | | | 0.215 | | | 0.0286 | -1.939 | -1.991 |
| | | | | (0.229) | | | (0.161) | (2.895) | (8.244) |
| $\hat{r}_{t-4}$ | | | | | -1.955** | -1.909** | -1.906** | -2.707 | -2.714 |
| | | | | | (0.547) | (0.642) | (0.668) | (1.377) | (2.007) |
| Constant | 0.453 | 0.421 | 0.434 | 0.359 | -1.267* | -1.233 | -1.237 | -1.416 | -1.414 |
| | (0.280) | (0.293) | (0.356) | (0.300) | (0.509) | (0.586) | (0.591) | (0.687) | (0.863) |
| Observations | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| Adjusted R-squared | -0.167 | -0.250 | -0.397 | -0.189 | 0.606 | 0.513 | 0.511 | 0.435 | 0.153 |
| Root Mean Square Error | 0.756 | 0.783 | 0.827 | 0.763 | 0.440 | 0.488 | 0.490 | 0.526 | 0.644 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

## Previous volumes in this series

| | | |
|---|---|---|
| 1287<br>September 2025 | Banks' regulatory risk tolerance | Mikael Juselius, Aurea Ponte Marques and Nikola Tarashev |
| 1286<br>September 2025 | How Do Quantitative Easing and Tightening Affect Firms? | Egemen Eren, Denis Gorea and Daojing Zhai |
| 1285<br>August 2025 | R* in East Asia: business, financial cycles, and spillovers | Pierre L. Siklos, Dora Xia and Hongyi Chen |
| 1284<br>August 2025 | What is needed for convergence? The role of finance and capital | Bryan Hardy and Can Sever |
| 1283<br>August 2025 | Comparing search and intermediation frictions across markets | Gabor Pinter, Semih Üslü and Jean-Charles Wijnandts |
| 1282<br>August 2025 | Market whiplash after the 2025 tariff shock: an event-targeted VAR approach | Gabor Pinter, Frank Smets and Semih Üslü |
| 1281<br>July 2025 | Integrating balance sheet policy into monetary policy conditions | Benoit Mojon, Phurichai Rungcharoenkitkul and Dora Xia |
| 1280<br>July 2025 | CBDC and banks: disintermediating fast and slow | Rhys Bidder, Timothy Jackson and Matthias Rottner |
| 1279<br>July 2025 | Central bank and media sentiment on central bank digital currency: an international perspective | Boris Hofmann, Xiaorui Tang and Feng Zhu |
| 1278<br>July 2025 | Soybean yield prediction in Argentina using climate data | Emiliano Basco, Diego Elías, Maximiliano Gómez Aguirre and Luciana Pastore |
| 1277<br>July 2025 | Firm-level CO2 emissions and production networks: evidence from administrative data in Chile | Pablo Acevedo, Elías Albagli, Gonzalo García-Trujillo and María Antonia Yung |
| 1276<br>July 2025 | Economic activity, inflation, and monetary policy after extreme weather events: ENSO and its economic Impact in the Peruvian economy | John Aguirre, Alan Ledesma, Fernando Perez and Youel Rojas |
| 1275<br>July 2025 | Decoding climate-related risks in sovereign bond pricing: a global perspective | Sofia Anyfantaki, Marianna Blix Grimaldi, Carlos Madeira, Simona Malovana and Georgios Papadopoulos |

All volumes are available on our website www.bis.org.