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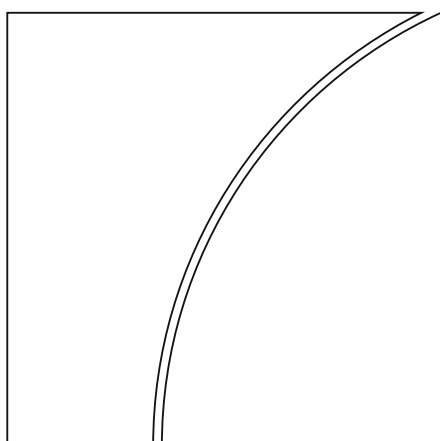
No 1185

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Monetary and Economic Department

May 2024

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Allocative Efficiency and the Productivity Slowdown[*]

Lin Shao[†]   Rongsheng Tang[‡]

April 2024

Abstract

This paper evaluates the contribution of cross-sector allocative efficiency to the productivity slowdown in the US during the 1970s and 2000s. We extend the framework of Oberfield (2013) to derive sufficient statistics for allocative efficiency and decompose aggregate productivity growth in a multi-sector economy with or without input-output linkages. We find approximately two-thirds of the productivity slowdown can be explained by the lack of improvement in allocative efficiency. Furthermore, data shows that increased sector-level volatility is associated with the deterioration of allocative efficiency.

JEL codes: O47; E23.
Keywords: productivity slowdown; allocative efficiency; volatility; adjustment costs.

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1   Introduction

Growth in US real output per worker decreased significantly in the 1970s and the 2000s (see Figure 1). The slowdown in productivity growth is among the most significant macroeconomic developments in the past few decades and has captured the attention of academic researchers and policymakers. This paper evaluates the role of allocative efficiency across sectors in explaining aggregate productivity dynamics during these periods. We show that allocative efficiency across sectors—or more precisely, the lack of improvement in it—is the common factor behind both episodes of slow productivity growth.

Figure 1: Labor productivity in the US
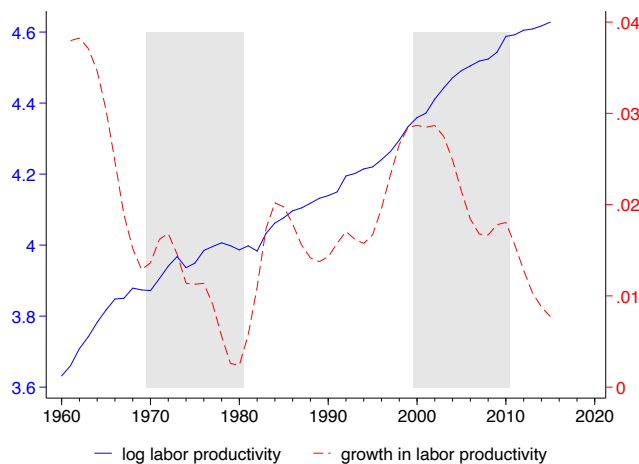
Notes: This figure plots the logarithms and growth of real output per worker in the United States business sector. The growth rate is computed as the hp-filtered log difference in real output per worker. The slow growth during the 1970s and 2000s is also documented using real output per hour (Figure 1 of Vandenbroucke, 2021) and TFP (Figure 1 of Aum et al., 2018). The gray bars indicate the two productivity slowdown episodes.

We begin by documenting two key facts of allocative efficiency across sectors. To do so, we use a multi-sector model, with and without input-output linkages, and sector-level data from the KLEMS database and the World Input-Output Table to measure allocative efficiency. First, we find that the gradual improvement in allocative efficiency is a significant driver of overall productivity growth in the long run. From 1960 to 2007, allocative efficiency gradually improved, contributing to approximately 20 percent of the aggregate labor productivity growth during this period. Second, the periods of productivity slowdown in the 1970s and

2000s stand out as two exceptions to this long-run trend. Allocative efficiency declined during the 1970s and then stagnated in the 2000s, following two decades of continuous improvement.

From a broader perspective, productivity growth is driven by either (i) enhancements in fundamental productivity (i.e., due to technological advancements) or (ii) improved resource allocation across sectors. If there is a stagnation or decline in allocative efficiency, it can lead to slower productivity growth. While data show that the growth rates of observed labor productivity in the 1970s and 2000s decelerated compared to those of the 1960s and 1990s, our analysis attributes approximately two-thirds of these decelerations to either stagnation or decline in allocative efficiency. Consequently, fundamental productivity contributes to only about a third of the observed slowdown in productivity growth.

Next, we investigate the volatility of sectoral productivity as a potential contributor to the lack of improvement in allocative efficiency during the 1970s and 2000s. Previous theoretical and empirical research has highlighted a mechanism linking higher time-series volatility to a decline in allocative efficiency when (non-convex) adjustment costs are present. The option value associated with making an adjustment results in an inaction region in which firms adopt a wait-and-see approach in adjusting their inputs rather than choosing input quantities that would maximize output in every period. This inaction region broadens with higher volatility as the option value of waiting increases, resulting in a widening gap between the actual allocation of resources and the optimal allocation implied by productivity.

We document a significant variation in volatility over time and across sectors, with higher volatility tending to be associated with a deterioration in allocative efficiency relative to the long-term trend. Through an estimated reduced-form model, we find that this increased volatility plays a substantial role in the observed productivity slowdown. In line with this, sectors subjected to more volatile productivity shocks show slower growth in allocative efficiency. Our analysis further reveals that when sectors experience positive productivity shocks, the amount of resources flowing into those sectors falls short of what the optimal allocation would predict. We also observe that periods characterized by high volatility correspond to an increased dispersion of factor utilization rates across sectors, a pattern possibly

driven by the wait-and-see motive.

In summary, our analysis reveals three related findings. First, the gradual improvement in allocative efficiency is a long-run trend, contributing approximately 20 percent of the productivity growth over our sample period. Second, deviations from this trend help give rise to periods of faster- or slower-than-normal productivity growth. And lastly, we emphasize the crucial role of lower volatility in the productivity process for efficient resource allocation, which, in turn, plays a substantial part in driving productivity growth.

Our measure of allocative efficiency follows the tradition of the misallocation literature (Hsieh and Klenow, 2009; Oberfield, 2013; Monge-Naranjo et al., 2019, among others).[1] Our paper also is part of a strand of literature that utilizes sector-level data to examine allocative efficiency between sectors or distortions at the sector level (Basu and Fernald, 2002; Caliendo et al., 2022; Liu, 2019).[2] Our analysis reveals that cross-sector allocative efficiency plays a significant role in explaining aggregate productivity dynamics, in line with findings in Oberfield (2013) and Behrens et al. (2020).

Among the studies exploring the dynamics of productivity in the US during the 1970s and 2000s, a few have examined the role of allocation. Investigating job reallocation across manufacturing industries, Davis and Haltiwanger (2001) propose that oil shocks during the 1970s and 1980s could have created a discrepancy between the actual and desired factor distribution at the industry level. Decker et al. (2020) demonstrate that reallocation across firms significantly decelerated in the 2000s compared to the 1980s and 1990s, suggesting this trend might negatively impact aggregate productivity. Our study diverges from Davis and Haltiwanger (2001) and Decker et al. (2020) in our direct measurement of allocative efficiency, as opposed to analyzing the reallocation rates.

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[1]The existing literature, such as studies like Basu and Fernald (2002) and Baqaee and Farhi (2020), adopts different notions of allocative efficiency that diverge from the one used in the misallocation literature. Hence, results derived using these different definitions are not directly comparable.

[2]Note that we use the terms *sector* and *industry* interchangeably here. There also exists a large body of literature utilizing firm- or establishment-level data to study allocative efficiency. An advantage of sector-level data is their broad availability and the extensive time periods they cover. In the specific context of our research, using sector-level data provides us with the opportunity to examine the early slowdown episode (the productivity slowdown of the 1970s).

A majority of papers studying the two slowdown episodes have sought causes that could explain the decline in fundamental productivity. However, as Bloom et al. (2020) note, without accounting for changes in allocative efficiency, the observed productivity dynamics in the raw data differ from those of fundamental productivity. Consequently, the theories developed to explain these dynamics might be flawed. For instance, labor productivity observed in the raw data shows evidence of a gradual decline from 1960 to 2007. In contrast, our result does not show a long-term secular decline in fundamental productivity. Instead, fundamental productivity growth during 1960–2007 can best be characterized as a relatively stable trend marked by prolonged weak growth that began in the 1970s and extended into the 1980s. Recognizing the timing of the slowdown in fundamental productivity could be useful in identifying its underlying causes.

In the same vein, most policy responses to the productivity slowdown primarily target fundamental productivity. Examples include proposals for fiscal and monetary stimuli to boost aggregate demand (Summers, 2018). While these policies may or may not be good ideas, two issues arise from our results. Firstly, the data suggest the decline in fundamental productivity is not as severe as it seems, indicating the need for more moderate subsidies than what the raw data might suggest. Secondly, existing evidence shows that stimulative policies can negatively impact allocative efficiency (Bai et al., 2016) or even promote the survival of zombie firms (Banerjee and Hofmann, 2018), which could counteract any beneficial effects on fundamental productivity.

Previous studies have explored the impact of productivity volatility on allocative efficiency. For instance, Bloom et al. (2018) examine the impact of volatility shocks on the allocative efficiency (among other things) in a quantitative model, while Asker et al. (2014) focus on differences in allocative efficiency across countries. Our contribution to this body of literature lies in identifying volatility as pivotal to understanding the prolonged productivity slowdowns in the US. The insights provided by Bloom et al. (2018) and Asker et al. (2014) suggest that a decline in allocative efficiency during periods of heightened volatility stems from firms' optimization in the presence of adjustment costs. A significant policy implication is that initiatives aimed at reducing the volatility of the productivity process or

lowering adjustment costs could be the key to mitigating the issue of prolonged productivity slowdown.

The rest of the paper is organized as follows: Section 2 builds the theoretical framework, and Section 3 discusses the data as well as the mapping between the model and the data. In Section 4, we present the main results. Section 5 includes extensions of the model and robustness checks. Section 6 concludes.

2 Measuring allocative efficiency

This section presents the theoretical framework. We first characterize optimal allocation across sectors to solve the planner's problem. Then we derive sufficient statistics to measure allocative efficiency. Finally, we decompose aggregate labor productivity growth in the data into two components.

2.1 Value-added economy

We first consider a multi-sector value-added economy. There are N sectors in the economy ($i = \{1, ..., N\}$). In year t, each sector produces a good $Y_{i,t}$ using capital, labor:

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}},$$

where $A_{i,t}$ is the sectoral productivity. There is one final good Y_t, which is produced by aggregating all sectoral goods, such that

$$Y_t = \prod_{i=1}^{N} Y_{i,t}^{\theta_{i,t}},$$

where $\sum_i \theta_{i,t} = 1$. This final good producer is a stand-in for the preference of price-taking consumers, as in Oberfield (2013).

Planner's problem   The planner's problem is to allocate aggregate capital K_t and labor L_t into the N sectors to maximize the output of final good Y_t:[3]

$$\max\ Y_t = \prod_{i=1}^{N} Y_{i,t}^{\theta_{i,t}}, s.t.\ Y_{i,t} = A_{i,t} K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}},\ \sum_i K_{i,t} = K_t,\ \sum_i L_{i,t} = L_t. \tag{1}$$

The optimal allocation of capital and labor under problem (1) is such that $K_{i,t}^* = \chi_{i,t}^{k*} K_t$ and $L_{i,t}^* = \chi_{i,t}^{l*} L_t$, where $\chi_{i,t}^{k*} = \frac{\theta_{i,t}\alpha_{i,t}}{\sum_i \theta_{i,t}\alpha_{i,t}}$ and $\chi_{i,t}^{l*} = \frac{\theta_{i,t}(1-\alpha_{i.t})}{\sum_i \theta_{i,t}(1-\alpha_{i,t})}$. The optimal distribution, $\chi_{i,t}^{k*}$ and $\chi_{i,t}^{l*}$, reflect the relative importance of sector i's capital and labor in producing the final good.

Allocative efficiency   We define allocative efficiency $\mathbf{E}_t$ as the ratio between output in the data (Y_t) and output under optimal allocation (Y_t^*), such that $\mathbf{E}_t = \frac{Y_t}{Y_t^*}$. We can write $\mathbf{E}_t$ as follows:

$$\mathbf{E}_t = \prod_{i=1}^{N} \left[\left(\frac{\chi_{i,t}^{k}}{\chi_{i,t}^{k*}} \right)^{\alpha_{i,t}} \left(\frac{\chi_{i,t}^{l}}{\chi_{i,t}^{l*}} \right)^{1-\alpha_{i,t}} \right]^{\theta_{i,t}}, \tag{2}$$

where $\chi_{i,t}^{k} = \frac{K_{i,t}}{K_t}$ and $\chi_{i,t}^{l} = \frac{L_{i,t}}{L_t}$ are sector i's capital and labor as a share of aggregate K_t and L_t in the data, respectively. Intuitively, $(\frac{\chi_{i,t}^{k}}{\chi_{i,t}^{k*}})^{\alpha_{i,t}}(\frac{\chi_{i,t}^{l}}{\chi_{i,t}^{l*}})^{1-\alpha_{i,t}}$ measures sector i's allocative efficiency, which represents the deviation of the observed allocation in the data from the optimal allocation. Aggregate allocative efficiency $\mathbf{E}_t$ is the weighted geometric mean of sectoral allocative efficiency with sectoral weights θ_i.[4]

2.2   Input-output economy

Next we consider an economy with input-output linkages. In the input-output economy, each sector $i \in \{1, ..., N\}$ produces good $Q_{i,t}$ using capital, labor, domestic and imported

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[3]The optimal allocation problem takes the economy's total capital and labor inputs as exogenously given. Consequently, the measure of allocative efficiency we derive below is a static measure. It abstracts from the dynamic implications of misallocation.

[4]Details of the model can be found in the Appendix Section B.1.

intermediate goods, such that

$$Q_{i,t} = A_{i,t}(K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}} \left(\prod_{j=1}^{N} d_{ij,t}^{\sigma_{ij,t}}\right)\left(\prod_{j=1}^{N} m_{ij,t}^{\lambda_{ij,t}}\right),$$

where $d_{ij,t}$ is the domestic intermediate good j used by sector i, $m_{ij,t}$ is the imported inter-mediate good j used by sector i, $\sigma_{i,t} = \sum_{j=1}^{N} \sigma_{ij,t}$, and $\lambda_{i,t} = \sum_{j=1}^{N} \lambda_{ij,t}$. There is one final good produced by aggregating over these N sectoral goods:

$$Y_t = \prod_i Y_{i,t}^{\theta_{i,t}},$$

where $\sum_{i=1}^{N} \theta_{i,t} = 1$.

The resource constraint on the sectoral good i, therefore, can be written as

$$Q_{i,t} = Y_{i,t} + \sum_{j=1}^{N} d_{ji,t},$$

and the total expenditure on imported goods is

$$X_t = \sum_{i=1}^{N} \sum_{j=1}^{N} \bar{P}_{j,t} m_{ij,t},$$

where $\bar{P}_{j,t}$ is the price of imported intermediate good j relative to the final good.

Planner's problem   The planner's problem is to allocate aggregate capital K_t, aggregate labor L_t, sectoral output $Q_{i,t}$ and choose imported intermediate good $m_{ij,t}$ such that the

aggregate output net of imports $(Y - X)$ is maximized:

$$\max_{\{K_{i,t}, L_{i,t}, d_{ij,t}, m_{ij,t}\}_{i,j=1}^N} \quad Y_t - X_t = \prod_i Y_{i,t}^{\theta_{i,t}} - \sum_{i=1}^N \sum_{j=1}^N \bar{P}_{j,t} m_{ij,t} \tag{3}$$

$$s.t. \quad Q_{i,t} = A_{i,t} (K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}} \left(\prod_{j=1}^N d_{ij,t}^{\sigma_{ij,t}} \right) \left(\prod_{j=1}^N m_{ij,t}^{\lambda_{ij,t}} \right),$$

$$Q_{i,t} = Y_{i,t} + \sum_{j=1}^N d_{ji,t}, \quad \sum_i K_{i,t} = K_t, \quad \sum_i L_{i,t} = L_t.$$

The optimal allocation of capital, labor and intermediate goods can be characterized with a set of optimal shares $(\chi_{i,t}^{k*}, \chi_{i,t}^{l*}, \gamma_{ij,t}^*, \chi_{i,t}^{y*})$, such that $K_{i,t}^* = \chi_{i,t}^{k*} K_t$, $L_{i,t}^* = \chi_{i,t}^{l*} L_t$, $d_{ij,t}^* = \gamma_{ij,t}^* Q_{j,t}^*$, $Y_{j,t}^* = \chi_{j,t}^{y*} Q_{j,t}^*$, and $m_{ij,t}^* = (\frac{\theta_{i,t}\lambda_{ij,t}}{\chi_{i,t}^{y*}}) \frac{Y_t^*}{\bar{P}_{j,t}}$.[5] The optimal shares can be solved using the following systems of equations:

(i) $\chi_{i,t}^{k*} = \frac{\theta_{i,t}\alpha_{i,t}(1-\sigma_{i,t}-\lambda_{i,t})}{1-\sum_j \gamma_{ji,t}^*} / \sum_s \frac{\theta_{s,t}\alpha_{s,t}(1-\sigma_{s,t}-\lambda_{s,t})}{1-\sum_j \gamma_{js,t}^*}$, $\forall i \in \{1, ..., N\}$.

(ii) $\chi_{i,t}^{l*} = \frac{\theta_{i,t}(1-\alpha_{i,t})(1-\sigma_{i,t}-\lambda_{i,t})}{1-\sum_{j,t} \gamma_{ji,t}^*} / \sum_s \frac{\theta_{s,t}(1-\alpha_{s,t})(1-\sigma_{s,t}-\lambda_{s,t})}{1-\sum_j \gamma_{js,t}^*}$, $\forall i \in \{1, ..., N\}$.

(iii) $\{\chi_{i,t}^{y*}\}_{i=1}^N$ solves the system of equations

$$\frac{1}{\chi_{i,t}^y} = 1 + \frac{1}{\theta_{i,t}} \sum_s \left(\frac{\theta_{s,t}}{\chi_{s,t}^y} \sigma_{si,t} \right), i \in \{1, ..., N\}$$

and

$$\gamma_{ij,t}^* = \frac{\theta_{i,t}\chi_{j,t}^{y*}}{\theta_{j,t}\chi_{i,t}^{y*}} \sigma_{ij,t}.$$

(iv) $\{Q_{i,t}^*\}_{i=1}^N$ solves for the system of equations

$$Q_{i,t} = \chi_{Qi,t} \left(\prod_{s=1}^N Q_{s,t}^{\sigma_{is,t}+\lambda_{i,t}\theta_{s,t}} \right), i \in \{1, ..., N\},$$

where $\chi_{Qi,t} = A_{i,t}[(\chi_{i,t}^{k*}K_t)^{\alpha_{i,t}} (\chi_{i,t}^{l*}L_t)^{1-\alpha_{i,t}}]^{1-\sigma_{i,t}-\lambda_{i,t}} (\prod_{j=1}^N \gamma_{ij,t}^{*\sigma_{ij,t}})[\theta_{i,t} \prod_s (\frac{\chi_{s,t}^{y*}}{\chi_{i,t}^{y*}})^{\theta_{s,t}}]^{\lambda_{i,t}} \prod_{j=1}^N (\frac{\lambda_{ij,t}}{\bar{P}_{j,t}})^{\lambda_{ij,t}}$.

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[5]See Appendix Section B.1 for details of the solution.

Allocative efficiency   Allocative efficiency $\mathbf{E}_t$ is the ratio between the output net of imports in the data and that under optimal allocation, that is, $\mathbf{E}_t = \frac{Y_t - X_t}{Y_t^* - X_t^*}$. It can be written as a product of allocative efficiency of capital and labor E_t^{kl}, domestic intermediate goods E_t^d, imported intermediate goods E_t^m, and goods used for the final good production E_t^y:

$$\mathbf{E}_t = E_t^{kl} \cdot E_t^d \cdot E_t^m \cdot E_t^y, \tag{4}$$

(i) $E_t^{kl} = \prod_{i=1}^N (((\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}})^{\alpha_{i,t}} (\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}})^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}})^{\sum_n \theta_{n,t} C_{ni,t}}$,

(ii) $E_t^d = \prod_{i=1}^N (\prod_{j=1}^N (\frac{\gamma_{ij,t}}{\gamma_{ij,t}^*})^{\sigma_{ij,t}})^{\sum_n \theta_{n,t} C_{ni,t}}$,

(iii) $E_t^m = \frac{1 - \sum_{n=1}^N \frac{\theta_{n,t}\lambda_{n,t}}{\chi_{n,t}^y}}{1 - \sum_{n=1}^N \frac{\theta_{n,t}\lambda_{n,t}}{\chi_{n,t}^{y*}}}$,

(iv) $E_t^y = \prod_{n=1}^N (\frac{\chi_{n,t}^y}{\chi_{n,t}^{y*}})^{\theta_{n,t}} \prod_{i=1}^N (\frac{\prod_s (\frac{\chi_{s,t}^y}{\chi_{i,t}^y})^{\theta_{s,t}}}{\prod_s (\frac{\chi_{s,t}^{y*}}{\chi_{i,t}^{y*}})^{\theta_{s,t}}})^{\lambda_{i,t} \sum_n (\theta_{n,t} C_{ni,t})}$,

where C_t is the $N \times N$ Leontief inverse matrix adjusted for imported intermediate goods, such that $C_t = (I - \Omega_t)^{-1}$ and $\Omega_t(i,j) = \sigma_{ij,t} + \lambda_{i,t}\theta_{j,t}$. Details of the model can be found in Appendix Section B.2.[6]

2.3   Decomposition of aggregate productivity in the data

According to the definition of $\mathbf{E}_t$, the following equation holds: $Y_t = Y_t^* \mathbf{E_t}$. Dividing both sides by aggregate labor inputs yields the decomposition of the labor productivity level:[7]

$$\mathrm{LP}_t = \mathrm{LP}_t^* \mathbf{E_t}. \tag{5}$$

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[6]Input-output linkages alter the measurement of allocative efficiency in two important ways. First, the measure now accounts for the allocative efficiency of intermediate inputs. Second, the weights representing the importance of each sector now take into account input-output effects, represented by the Leontief inverse matrix. For the second point, we provide more discussion in Appendix Section B.4.

[7]Equation (5) shows that allocative efficiency measures the distance between data (LP) and the production possibility frontier (LP*). See discussions in Baqaee and Farhi (2020) for three different notions of allocative efficiency used in the literature.

Taking the log difference gives the decomposition equation for productivity growth:

$$\Delta \log \text{LP}_t \;=\; \Delta \log \text{LP}_t^* + \Delta \log \mathbf{E_t}. \tag{6}$$

Equation (6) shows that it is the changes (growth rates) of allocative efficiency, not the levels, that matter for productivity growth. A useful statistic for our empirical exercise is $\frac{\Delta \log \mathbf{E_t}}{\Delta \log \text{LP}_t}$, which measures the contribution of allocative efficiency to aggregate productivity growth.

A few remarks about the framework are in order before moving on to the empirical exercise. Our model abstracts from examining the impact of *within-sector* allocative efficiency on aggregate productivity growth. Conceptually, within-sector efficiency dynamics will be embedded in sector-level productivity, $A_{i,t}$, which we take as given. Therefore, we interpret our results strictly as an analysis of the impact of between-sector allocative efficiency on aggregate productivity dynamics.

3 Application to the US data

Next, we introduce the datasets used in the empirical analysis and discuss the mapping between data and our model.

3.1 Data description

We use the 2013 version of the KLEMS dataset and the World Input-Output Table (WIOT). These versions of KLEMS and WIOT use the ISIC Rev. 3 classification, thus allowing a straightforward mapping of sectors. We restrict our analysis to 28 private sectors in the economy. Table A.1 in the Appendix lists these sectors. The KLEMS dataset covers the period 1947–2010, while the input-output table covers 1995–2011, thus restricting the analysis with input-output linkages to the period 1995–2010. Below we list all the variables

used in the empirical exercise. We distinguish whether each variable is the nominal value ($) or quantity.

KLEMS (i) sector-level value-added and gross output ($), (ii) sector-level capital and labor compensation, and cost of intermediate goods ($), (iii) sector-level real capital stock and the number of workers (quantity).[8]

WIOT (i) sector i's use of domestic sector j good ($), (ii) sector i's use of foreign sector j good ($), (iii) sector i good used in the final good production ($).

3.2 Mapping between model and data

In this section, we explain the mapping between model and data. Recall that to calculate $\mathbf{E}_t$, we need (i) the allocation of capital, labor and intermediate inputs across sectors in the data $(\chi_{i,t}^k, \chi_{i,t}^l, \chi_{i,t}^y, \gamma_{ij,t})$ and (ii) output elasticities $(\alpha_{i,t}, \sigma_{ij,t}, \lambda_{ij,t}, \theta_{i,t})$, which allows us to solve the optimal allocation.

Cross-sector allocation in the data First, we calculate the data allocation of capital, labor and intermediate inputs: $\chi_{i,t}^k, \chi_{i,t}^l, \chi_{i,t}^y, \gamma_{ij,t}$. Ideally, we would like to use the quantity of the inputs to calculate the allocation across sectors. We are able to do so for capital and labor, such that $\chi_{i,t}^k = \frac{K_{i,t}}{\sum_i K_{i,t}}$ and $\chi_{i,t}^l = \frac{L_{i,t}}{\sum_i L_{i,t}}$, where $K_{i,t}$ is the real capital stock and $L_{i,t}$ is the number of workers in sector i. Due to the lack of a quantity measure of intermediate inputs, $\gamma_{ij,t}$ and $\chi_{i,t}^y$ are computed using expenditure, such that $\gamma_{ij,t} = \frac{\$d_{ij,t}}{\$Q_{j,t}}$ and $\chi_{j,t}^y = \frac{\$Y_{j,t}}{\$Q_{j,t}}$, where $\$d_{ij,t}$ is sector i's use of sector j good, $\$Q_{j,t}$ is sector j's gross output and $\$Y_{j,t}$ is sector j good used in final good production, all nominal values.[9]

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[8]The 2013 version of KLMES reports a capital quantity index for each year. An early vintage of EUK-LEMS (2009 version) contains real capital stock based on 1995 prices, but the data are only available for a shorter time series. We construct real capital stock for all years using the 1995 real capital stock and the quantity index.

[9]Using expenditure as a proxy for quantity will provide an exactly accurate measure of allocative efficiency if the cost of intermediate goods is the same across all sectors. In our model, the measure of allocative efficiency only depends on the distribution of intermediate goods in terms of quantity across sectors. When there is no price variation, the distribution of intermediate goods by expenditure is the same as the distri-

In the baseline result, we do not distinguish between different types of capital and labor inputs. We chose this as the baseline for two reasons. First, the data cover an extended period, thus making it possible to study the earlier decades. Second, real capital stock and total employment are taken from national accounts statistics rather than estimated using survey data. Likely, they suffer less from measurement errors. In Section 5.5, we study the case where capital and labor composition might differ across sectors, and we confirm the robustness of our main results.

Output elasticities To solve the planner's optimal allocation problem, we need to know the output elasticities $(\alpha_{i,t}, \sigma_{ij,t}, \lambda_{ij,t}, \theta_{i,t})$.

Among them, $\theta_{i,t}$ comes from the final-good production function. The final-good production function is a stand-in for the preference of households (consumers) in the economy. They represent the demand system in the data, and their value can vary across years, as in Oberfield (2013) and Bils et al. (2020), capturing changes in the demand system over time. We back out $\theta_{i,t}$ from the data using the expenditure share of each sector's output in the final good consumption. In the value-added economy, $\theta_{i,t} = \frac{P_{i,t}^Y Y_{i,t}}{\Sigma_i P_{i,t}^Y Y_{i,t}}$, where $P_{i,t}^Y Y_{i,t}$ represents the value of sector i's value-added output. In the input-output economy, $\theta_{i,t} = \frac{P_{i,t}^Q(Q_{i,t} - \Sigma_j d_{ji,t})}{\Sigma_i P_{i,t}^Q(Q_{i,t} - \Sigma_j d_{ji,t})}$, where $P_{i,t}^Q(Q_{i,t} - \Sigma_j d_{ji,t})$ represents the value of sector i's gross output used for final consumption.

The output elasticities of capital, labor, and intermediate inputs, $\alpha_{i,t}$, $\sigma_{ij,t}$, and $\lambda_{ij,t}$, are intimately linked to their expenditure shares observed in the data. Nevertheless, these expenditure shares cannot be applied directly to back out output elasticities because they might be *distorted*.

To give an example of why these expenditure shares in the data may be distorted: suppose the data is generated by a price-taking representative firm making optimization decisions subject to a set of *wedges* on their capital and labor inputs, represented by $(\tau_{i,t}^k, \tau_{i,t}^l)$.[10] The

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bution by quantity. If there is variation in the cost of intermediate goods across sectors, this approximation will not capture the effect of price dispersion on allocative efficiency.

[10]The wedges may reflect statutory provisions such as tax code and regulations, discretionary provisions

first order conditions lead to the following equation:

$$\frac{R_t K_{i,t}}{w_t L_{i,t}} = \frac{(1 - \tau_{i,t}^l)\alpha_{i,t}}{(1 - \tau_{i,t}^k)(1 - \alpha_{i,t})},$$

where $R_t K_{i,t}$ and $w_t L_{i,t}$ are the capital and labor expenditure incurred by the firm, respectively. We observe these expenditures in the data, and we can calculate the left-hand side of the above equation. But without any assumptions, $\alpha_{i,t}$ cannot be identified separately from $\tau_{i,t}^k$ and $\tau_{i,t}^l$ on the right-hand side.

This is a well-known issue in the literature, and it is usually addressed by making certain assumptions (see Restuccia and Rogerson, 2017). Hsieh and Klenow (2009) presumed that the US economy is undistorted, and its expenditure shares were used as the benchmark value to study China and India. Oberfield (2013) dealt with this problem by exploiting the panel structure of his dataset and assumed that firms' expenditure shares might be distorted in each year but undistorted on average over time. We will conduct our analysis in two specifications following these approaches.

We first follow the strategy in Oberfield (2013) and assume that, in the long run, wedges are not biased towards one factor so that expenditure shares are on average undistorted. Using the previous example, this means that in any sector i, $\tau_{i,t}^k$ and $\tau_{i,t}^l$ have the same average over time. Hence in the data, the average expenditure shares in each sector over time are equal to the true output elasticities. In practice, we conduct this specification by computing the nominal factor expenditure shares and taking the average in a rolling window centered around the current year. We perform the analysis with a rolling window of 3, 5, 7, and 9 years and find minimal differences in the results; hence, we report results from a rolling window of three years in the main text.[11]

In the second specification, we assume that expenditure shares are undistorted in the

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such as preferential treatment by the government, and market frictions and imperfections (Restuccia and Rogerson, 2017).

[11]Although the assumption is necessary methodologically, one might wonder how it affects our results. To answer this, we ask: what if wedges are biased towards one factor? We find that our main result still stands even if, on average, the capital wedge is much larger than the labor wedge, or vice versa (see Section 5.7 for details).

latter years of our sample. This assumption is based on the empirical evidence that, in the long run, allocative efficiency has improved in the US (Baily et al., 1992; Ziebarth, 2013). More specifically, we use 2010, the last year of our sample, as the benchmark and assume it is undistorted when estimating the elasticities.

We interpret estimates under this specification as allocative efficiency relative to the base-year level. Although they might differ from the actual level of allocative efficiency, they are informative statistics as our focus is on the changes in allocative efficiency over time. In a way, this specification is analogous to Hsieh and Klenow (2009), who assumed that the US economy was undistorted and applied US factor shares to the more distorted economies of China and India. We show that our main results are robust under this specification (for more details, see Section 5.7).

4   Results

This section presents the main results of the paper. We first examine the evolution of allocative efficiency and establish its role in explaining productivity growth, both in the long run and during the slowdown. Then we provide evidence of the relationship between volatility and slow growth in allocative efficiency.

4.1   Contribution of allocative efficiency to productivity slowdown

Allocative efficiency stagnated or declined in the 1970s and 2000s.   We begin with an analysis of allocative efficiency spanning the period 1960–2010. In Figure 2, Panel (a), we observe a gradual improvement in measured allocative efficiency, approaching an optimal value of 1 over this time frame. However, this upward trend is interrupted by two notable exceptions: (i) a decline during the 1970s and (ii) a plateau in the 2000s, following two decades of continuous improvement. The plateau in the 2000s is evident in both the value-added and input-output economies, although it is more pronounced in the former due to its

longer time series. Although the value-added economy displays a higher *level* of measured allocative efficiency (Panel a), the *growth rates* of allocative efficiency—the object of interest when it comes to productivity growth—remain similar across both economies (Panel b).

Figure 2: Evolution of allocative efficiency over time

(a) Allocative efficiency   **(b)** Growth in allocative efficiency
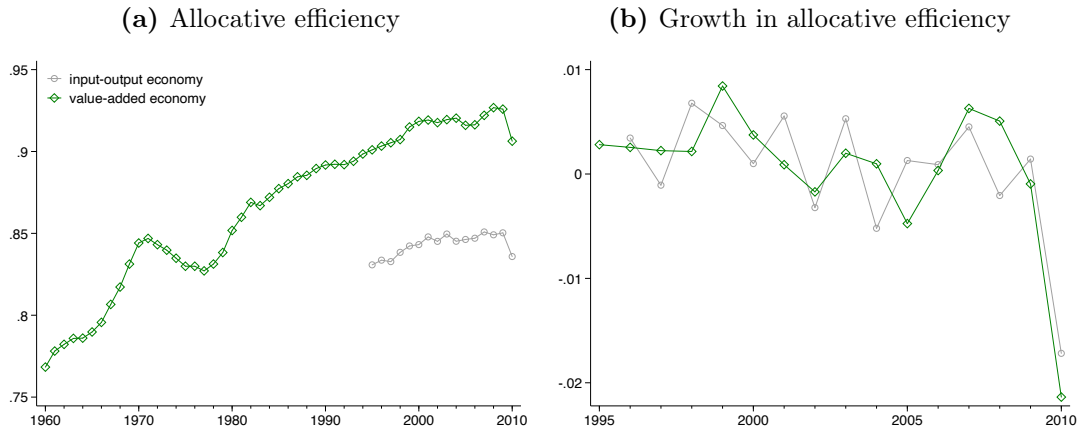
Notes: Panel (a) displays the measured allocative efficiency ($\mathbf{E}_t$) in each year, while Panel (b) shows the annual growth rates ($\Delta \log \mathbf{E}_t$). The gray lines represent the input-output economy, and the green lines represent the value-added economy.

Next, we examine the contribution of improved allocative efficiency to overall productivity growth. Data presented in Table 1 suggests that improved allocative efficiency contributes to roughly 20% of productivity growth from 1960 to 2007, though this contribution fluctuates considerably across different decades (Panel a). Notably, allocative efficiency contributes most significantly to productivity growth during the 1960s, at 32%, while it contributes the least during the 1970s, at -5%, and the second least during the 2000s, at 3%. Panel (b) shows results for the input-output economy from 1995 to 2010, divided into three five-year periods. Allocative efficiency contributes 13% to productivity growth in the latter half of the 1990s, diminishes to 2% in the early 2000s, and drops further to -13% by the end of the decade. Lastly, the value-added economy exhibits a similar trend during this period (Panel c).

Taken together, the data presented in Table 1 and Figure 2 suggest that allocative efficiency, a key driver of US labor productivity growth, either stagnated or declined in the 1970s and 2000s. This likely resulted in a reduced or even negative impact on productivity

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Table 1: Contribution of allocative efficiency to productivity growth $\left(\frac{\Delta \log \mathbf{E_t}}{\Delta \log \mathrm{LP}_t} \right)$

(a) Value-added economy (1960–2007)		(b) Input-output economy (1995–2010)		(c) Value-added economy (1995–2010)	
1960–69	0.32	1995–99	0.13	1995–99	0.13
1970–79	-0.05	2000–04	0.02	2000–04	0.03
1980–89	0.30	2005–10	-0.13	2005–10	-0.10
1990–99	0.13	1995–2010	0.02	1995–2010	0.02
2000–07	0.03				
1960–2007	0.20				

Notes: This table shows the contribution of allocative efficiency to productivity growth $\frac{\Delta \log \mathbf{E_t}}{\Delta \log \mathrm{LP}_t}$. $\Delta \log \mathbf{E_t}$ and $\Delta \log \mathrm{LP}$ are based on the long differences in allocative efficiency and labor productivity between the end and the beginning of each period. They are adjusted to reflect the growth rate over periods of the same length (ten-year window in Panel a and five-year window in Panels b and c). Panels (a) and (c) report results from the value-added economy while Panel (b) shows the results of the input-output economy.

growth during these periods.

Quantifying allocative efficiency's role in explaining productivity slowdown. Following the previous discussion, Table 2 quantifies the contributions of allocative efficiency to the productivity growth *slowdowns* experienced in the 1970s and 2000s. In this context, a slowdown refers to a decrease in the rate of productivity growth when compared to preceding decades. Column (1) of Panel (a) presents the decade-by-decade growth rates of observed labor productivity. These rates are computed as the logarithmic differences between each decade's starting and ending points and are adjusted to reflect growth rates over periods of the same length (10-year windows). Column (4) then calculates changes in the growth rates compared with the previous decades. It shows that productivity growth rates in the 1970s and 2000s (the two slowdown episodes) were 12 and 3 percentage points lower, respectively, than those of the preceding decades.

To quantify the role of allocative efficiency during periods of productivity slowdown, we start by decomposing observed labor productivity (LP_t) into two components: allocative efficiency ($\mathbf{E}_t$) and fundamental labor productivity (LP_t^*) using Equation (5). Subsequently, Columns (2) and (3) display the growth rates of fundamental labor productivity and allocative efficiency, respectively, calculated in a manner similar to Column (1). Lastly, Columns

Table 2: Slowdown in productivity growth and the role of allocative efficiency

Periods	Growth rates by periods (long log-difference)			Changes in growth rates from preceding period		
	(1)	(2)	(3)	(4)	(5)	(6)
	labor productivity			labor productivity		
	data	"fundamental"	$\mathbf{E}_t$	data	"fundamental"	$\mathbf{E}_t$
(a) VA economy (1960–2007)						
1960–69	0.24	0.16	0.08	–	–	–
1970–79	0.13	0.13	-0.01	**-0.12**	**-0.03**	**-0.08**
1980–89	0.15	0.10	0.04	0.02	-0.03	0.05
1990–99	0.19	0.16	0.03	0.05	0.06	-0.02
2000–07	0.16	0.16	0.01	**-0.03**	**-0.01**	**-0.02**
(b) IO economy (1995–2010)						
1995–99	0.11	0.10	0.01	–	–	–
2000–04	0.11	0.11	0.00	0.00	0.02	-0.0
2005–10	0.10	0.11	-0.01	**-0.02**	**0.00**	**-0.02**
(c) VA economy (1995–2010)						
1995–99	0.11	0.10	0.01	–	–	–
2000–04	0.11	0.11	0.00	0.00	0.02	-0.0
2005–10	0.10	0.11	-0.01	**-0.02**	**0.00**	**-0.02**

Notes: Columns (1)–(3) present the growth rates in LP_t (labor productivity, data), LP_t^* (labor productivity, fundamental), and $\mathbf{E}_t$. These growth rates are based on the long differences in labor productivity between the end and the beginning of each period. They are adjusted to reflect the growth rate over periods of the same length (ten-year window in Panel a and five-year window in Panels b and c). Columns (4)–(6) present the changes in these growth rates from the preceding periods. Panels (a) and (c) present results from the value-added economy while Panel (b) presents the results from the input-output economy.

(5) and (6) demonstrate the changes in these growth rates from their respective previous decades.

Column (6) shows that the growth rates of allocative efficiency in the 1970s and 2000s slowed down by 8 and 2 percentage points, respectively, compared to the preceding decades. Concurrently, the growth rates of fundamental productivity fell by 3 and 1 percentage points (Column 5). This suggests that approximately two-thirds (8/12, or 2/3) of the productivity growth slowdown in these periods is attributable to a lack of improvement in allocative efficiency, while the remaining third results from slowdowns in fundamental productivity. Further, focusing on 1995–2010 in Panels (b) and (c), we see that productivity growth decreased by 2 percentage points in the second half of the 2000s, which can be entirely accounted for by the slower growth in allocative efficiency. Taken together, results in Table 2 reveal that the lack of improvement in allocative efficiency plays a quantitatively important

role in explaining the productivity slowdown during the 1970s and 2000s.

So far, we have emphasized the importance of allocative efficiency in understanding the productivity growth slowdown during the 1970s and 2000s. This understanding also provides valuable insights into broader long-term productivity trends. The raw data in Column (1) reveal a deceleration in productivity growth in the long run, with the 1960s standing out due to an exceptionally rapid pace of growth at 24%. However, it is worth noting that this extraordinary growth can be partly attributed to a fast improvement in allocative efficiency. Indeed, we find no evidence of a secular deceleration in the *fundamental* productivity growth. The growth of fundamental productivity can best be described as a relatively stable trend marked by prolonged weak growth, which began in the 1970s and extended into the 1980s. This result highlights the importance of distinguishing between the influences of allocative efficiency and fundamental productivity in understanding long-term productivity trends.

4.2 Decomposing allocative efficiency

Next, we carry out two decomposition exercises to further investigate the dynamics of aggregate allocative efficiency. First, we break down the aggregate allocative efficiency into the respective allocative efficiencies of capital and labor. Following this, we dissect the aggregate measure into sectoral allocative efficiency.

Capital and labor. We decompose $\mathbf{E_t} = E_t^k \cdot E_t^l$ to identify whether capital or labor is the main driver. We define the measures of capital and labor allocative efficiency in the value-added economy as $E_t^k = \prod_{i=1}^{N}(\frac{\chi_{i,t}^{k}}{\chi_{i,t}^{k*}})^{\alpha_{i,t}\theta_{i,t}}$ and $E_t^l = \prod_{i=1}^{N}(\frac{\chi_{i,t}^{l}}{\chi_{i,t}^{l*}})^{(1-\alpha_{i,t})\theta_{i,t}}$, and for the input-output economy, the measures are similarly defined.[12]

Figure 3 reveals that capital allocation is the more important driver across the sample period. In contrast, labor allocation is more efficient than capital allocation in almost all years, but it did not become more efficient over time. Indeed, labor allocation seems so

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[12] $E^{k,t} = \prod_{i=1}^{N}(((\frac{\chi_{i,t}^{k}}{\chi_{i,t}^{k*}})^{\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}})^{\sum_n \theta_{n,t} C_{ni,t}}$ and $E^{l,t} = \prod_{i=1}^{N}(((\frac{\chi_{i,t}^{l}}{\chi_{i,t}^{l*}})^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}})^{\sum_n \theta_{n,t} C_{ni,t}}$.

efficient that it has little room for improvement and thus cannot possibly be a large driver. As with the long-run dynamics, capital allocation plays a more significant role during the two slowdown episodes.[13]

Figure 3: Capital and labor allocation
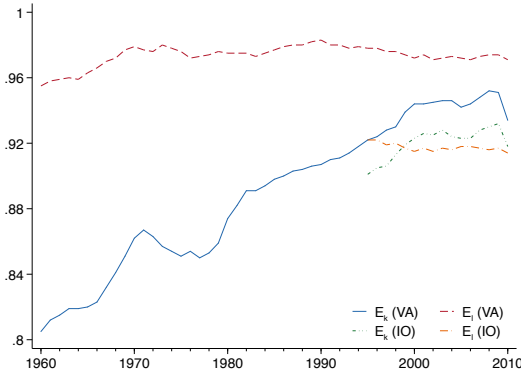
Notes: This figure plots the evolution of capital (E_k) and labor allocation (E_l) over time using measures from the value-added (VA) and the input-output (IO) economy.

Sectoral allocative efficiency. To discern whether a particular sector drives the overall trends, we decompose the aggregate measure into sectoral allocative efficiency, represented as $\mathbf{E}_t = \prod_{i=1}^{N} E_{i,t}^{\theta_{i,t}}$, where $E_{i,t} = (\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}})^{\alpha_{i,t}} (\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}})^{1-\alpha_{i,t}}$ is the allocative efficiency of sector i.[14]

Figure 4, Panel (a) presents the distribution of $E_{i,t}^{\theta_{i,t}}$, where different shades of colors correspond to different percentiles of the distribution. According to its definition, $E_{i,t} = 1$ means that sector i is at the optimal level. Therefore, an optimal allocation would see the cross-sectional distribution of $E_{i,t}$ collapse to a single point ($E_{i,t} = 1$ for all i), and a narrower distribution indicates a more efficient allocation. Notably, the distribution significantly narrows between 1960–1970 and 1980–2000 but widens in the 1970s and stabilizes post-2000, mirroring aggregate allocative efficiency trends. Furthermore, the changes in $E_{i,t}$ observed

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[13]However, given the data do not differentiate between capital returns and profits, caution is necessary when interpreting these results, particularly for policy implications, as we discuss further in Section 5.6.

[14]We discuss results for the value-added economy in the main text as the input-output model yields similar results.

at different percentiles indicates that these aggregate dynamics are not driven by a single sector.

We also examine the allocative efficiency of the manufacturing and service sectors, represented in Figure 4, Panel (b).[15] Both sectors experience a decline in allocative efficiency in the 1970s, with the service sector recovering slightly earlier than manufacturing. Conversely, the 2000s see a more concentrated drop in allocative efficiency of the manufacturing sectors.

Figure 4: Sector-level allocative efficiency

(a) Full distribution

(b) Manufacturing and service sector
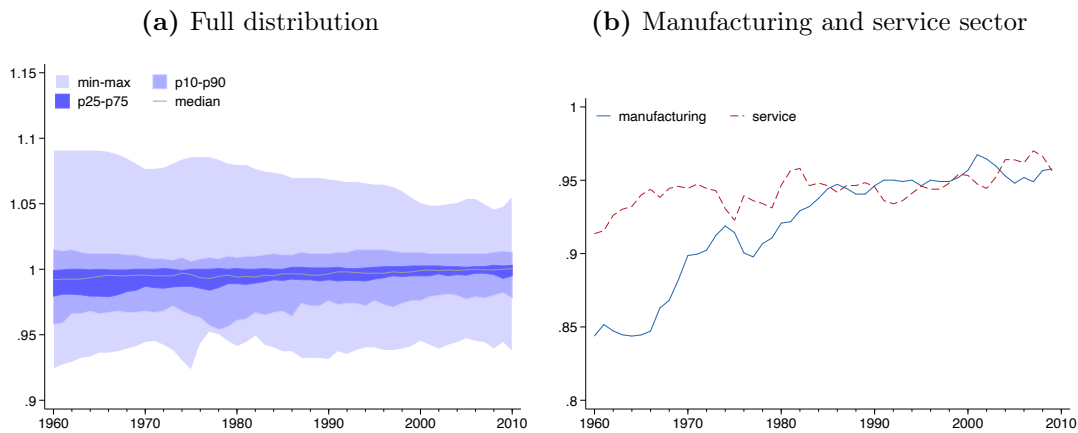
Notes: Panel (a) plots the distribution of $E_{i,t}$ in a model without input-output linkages. The different shades of color represent different percentiles of the $E_{i,t}$ distribution in year t. The grey line represents the median value of $E_{i,t}$. Panel (b) plots the allocative efficiency for the manufacturing and service sectors in both models. The classification of manufacturing and service sectors can be found in footnote 15.

To summarize, our analysis underscores the significant role of allocative efficiency in explaining the deceleration of aggregate productivity growth during the 1970s and 2000s. Moreover, we find that capital is the primary driver of change in allocative efficiency. The sluggish growth in allocative efficiency during productivity slowdown episodes can be attributed to sectors collectively moving away from the optimum. In Appendix Section A.2, we conduct an in-depth analysis of the long-run changes in allocative efficiency. The remainder of Section 4 will delve into an investigation of a potential driver behind the patterns of allocative efficiency during the two slowdown episodes.

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[15] The manufacturing and service sectors are defined as $E_t^m = \prod_{i \in manu} E_{i,t}^{\theta_{i,t}}$ and $E_t^s = \prod_{i \in serv} E_{i,t}^{\theta_{i,t}}$, respectively. See Table A.1 for sector classification details.

4.3 Volatility and allocation during the slowdown episodes

We explore the role of volatility of sector-level productivity in explaining the cross-sector allocative efficiency during the 1970s and 2000s. First, using a variety of measures, we document that the 1970s and 2000s experience an increase in sector-level volatility. Next, we exploit the variations in volatility over time and across sectors to provide evidence linking the increase in volatility to the deterioration in allocative efficiency.

Previous theoretical and empirical studies have demonstrated a mechanism that correlates higher time-series volatility with a decline in allocative efficiency. In the presence of a non-convex adjustment cost, there exists an inaction region in which firms do not adjust their inputs as a response to a productivity shock. With higher volatility, this inaction region expands because the option value of waiting rises, leading to a widening gap between actual capital/labor allocation and the optimal levels suggested by productivity.[16] Bloom et al. (2018), for instance, illustrate this mechanism with a quantitative model. Their findings reveal that after a shock, which increases the volatility of the productivity process, allocative efficiency declines and only gradually returns to pre-shock levels. Similarly, evidence presented by Asker et al. (2014) suggests that the high volatility of firms' productivity processes can partly account for the low allocative efficiency observed in less developed countries.

Productivity slowdown episodes are accompanied by high volatility. As a first pass of the data, we calculate the sectoral growth volatility in employment, real value-added output, and TFP, using the standard deviation of their respective annual growth rates over a rolling 5-year window. As shown in Panel (a) of Figure 5, volatility begins to rise at the onset of the 1970s, remaining high throughout the decade. At the highest point, standard

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[16]The existing literature identifies two impacts of an increase in volatility. The first, known as the *option value effect*, widens the inaction regions and results in a larger disparity between actual and optimal allocations. The second, known as the *volatility effect*, triggers a medium-term surge in aggregate net hiring and investment. Research indicates that in the medium term, the volatility effect tends to be more dominant, leading to an overshoot in aggregate labor and investment. However, it is important to highlight that we do not observe a similar overshoot in allocative efficiency. As shown in Figures 6 and 7 of Bloom et al. (2018), labor misallocation worsens after a shock and gradually recovers back to pre-shock levels, but it doesn't overshoot them. In contrast, aggregate labor and investment both exceed their pre-shock levels 3 to 6 quarters after the shock.

deviations were nearly double those of the 1960s. Volatility then gradually decreases between the 1980s and the early 2000s, after which it starts to ascend again.

Figure 5: Sector-level shocks were more volatile during the 1970s and 2000s

(a) S.D. in sectoral growth rates   **(b)** Dispersion in TFP shocks
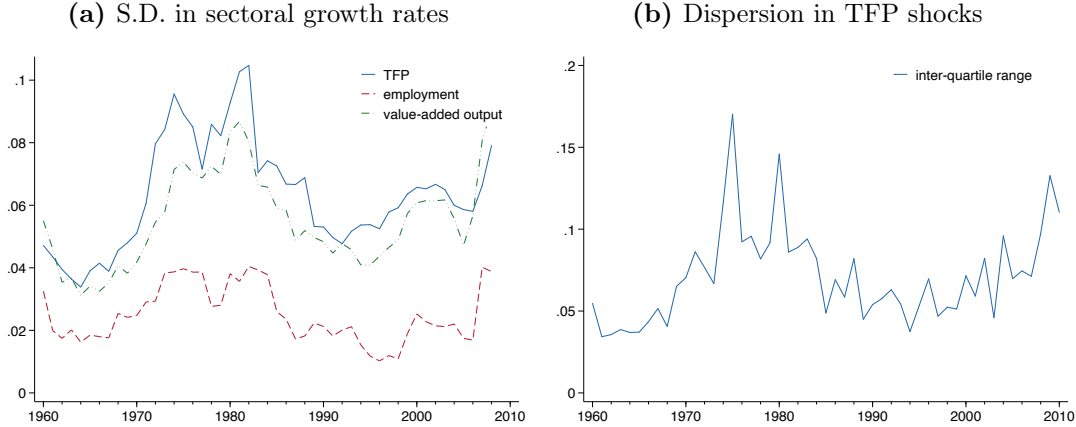
Notes: Panel (a) plots the cross-sectional s.d. of sectoral growth rates in employment, real value-added output, and TFP. Panel (b) plots the cross-sectional dispersion (inter-quartile range) in TFP shocks, computed as the residual terms from regression $\log A_{i,t} = \rho \log A_{i,t-1} + \mu_t + \chi_i + \epsilon_{i,t}$.

Next, we construct a measure of volatility based on dispersion in sector-level TFP. Following Bloom et al. (2018), we first calculate TFP shocks as the residual ($\varepsilon_{i,t}$) from a regression equation for sector-level log TFP, $\log A_{i,t} = \rho \log A_{i,t-1} + \mu_t + \chi_i + \varepsilon_{i,t}$, and use the cross-sectional dispersion of $\varepsilon_{i,t}$ to measure the volatility of sectoral TFP processes.[17] Panel (b) of Figure 5 shows the inter-quartile range of $\varepsilon_{i,t}$ within each year. The 1970s and 2000 are again marked by greater volatility.[18]

Prior to discussing our empirical findings, a few remarks are in order. Decisions related to adjusting inputs are typically made at the firm level. Then how does a firm's behavior reduce *cross-sector* allocative efficiency when sector-level volatility increases? To provide some intuition, consider an environment where a sector consists of many firms, each making decisions about capital and labor inputs in the presence of adjustment costs. A firm's

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[17]This regression controls for the sector- and year-fixed effects, which removes permanent differences in sectoral TFP levels and removes average growth rate differences across years.

[18]Appendix Figure A.3 plots an alternative measure of sector-level volatility, in which we first compute the over-time dispersion of $\varepsilon_{i,t}$ within a rolling window for each sector and then calculate the median value of these dispersions among all sectors. We find a similar dynamic as Figure 5, Panel (b), particularly in the rising volatility during the 1970s and 2000s.

productivity is a multiplicative composite of a sector-level shock, $A_{i,t}$, and a firm-specific shock. As such, an increase in the level of sector productivity $A_{i,t}$ enhances the productivity of all firms in sector i. Likewise, a rise in the volatility of $A_{i,t}$ also increases productivity volatility for all firms in that same sector.[19]

Imagine a situation where $A_{i,t}$—sector i's productivity level—increases relative to other sectors. This change should cause an increase in the optimal level of capital and labor for all firms in sector i. Absent any friction, all firms in this sector would be expected to increase their capital and labor. However, some firms, finding themselves within the inaction region, decide against adjusting their resources, while firms outside of their inaction regions adjust to the new optimal levels of capital and labor. When we aggregate the behavior of all firms in sector i, a gap emerges between the actual and optimal levels of capital and labor. Specifically, sector i as a whole will employ less capital and labor than what the increased productivity level would optimally require. Additionally, increased volatility exacerbates this gap: as volatility rises, more firms fall into the inaction region due to the increased option value and fail to adjust their inputs accordingly. In essence, increased sector-level volatility widens the gap between the actual and optimal resource allocations in response to changes in productivity, leading to reduced cross-sector allocative efficiency.

Relationship between volatility and allocative efficiency. Our next step is to systematically evaluate the relationship between volatility and allocative efficiency by exploiting variations in volatility over time. The results are shown in Table 3. Columns (1)–(3) regress aggregate allocative efficiency $\log(\mathbf{E}_t)$ on the volatility of TFP shocks in year t, $t-1$, and $t-2$, while controlling for $\log(\mathbf{E}_{t-1})$. The inclusion of $t-1$'s and $t-2$'s measure of volatility is motivated by the insight that the history of TFP shocks might have a long-lasting impact on allocation in the presence of adjustment costs. The inclusion of $\log(\mathbf{E}_{t-1})$ aims at controlling for the long-run trend.

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[19]This environment is akin to that of Bloom (2009), where the author investigates scenarios in which input adjustment decisions occur at a lower level (the establishment level) than what is observable in the data (the firm level). He concludes that increases in volatility at the macro, firm, and establishment levels all yield qualitatively comparable effects.

Table 3: Relationship between volatility and allocative efficiency

	(1)	(2)	(3)	(4)	(5)	(6)
Dispersion of TFP shocks in year t	-0.072**	-0.087**	-0.080**	-0.081**	-0.097**	-0.095**
	(0.031)	(0.037)	(0.037)	(0.032)	(0.038)	(0.039)
Dispersion of TFP shocks in year $t-1$		0.003	0.009		0.005	0.007
		(0.043)	(0.046)		(0.041)	(0.046)
Dispersion of TFP shocks in year $t-2$			-0.006			0.002
			(0.027)			(0.030)
Allocative efficiency in year $t-1$	0.973***	0.981***	0.976***			
	(0.013)	(0.012)	(0.012)			
Dependent variables	$\log(\mathbf{E}_t)$	$\log(\mathbf{E}_t)$	$\log(\mathbf{E}_t)$	$\Delta\log(\mathbf{E}_t)$	$\Delta\log(\mathbf{E}_t)$	$\Delta\log(\mathbf{E}_t)$
N	63	62	61	63	62	61
R^2	0.993	0.993	0.993	0.093	0.137	0.128
Observed slowdown in 1970s	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12
Predicted slowdown in 1970s	-0.10	-0.10	-0.10	-0.07	-0.08	-0.08
Observed slowdown in 2000s	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
Predicted slowdown in 2000s	-0.02	-0.03	-0.03	-0.03	-0.03	-0.03

Notes: The top half of the table presents the regression results. Columns (1)–(3) regress logarithms of $\mathbf{E_t}$ on the cross-sectional dispersion in TFP shocks from years $t, t-1, t-2$ while also controlling for the logarithms of $\mathbf{E_{t-1}}$. Columns (4)–(6) regress the log difference in $\mathbf{E_t}$ on the cross-sectional dispersion in TFP shocks of years $t, t-1, t-2$. Robust standard errors are reported and in parentheses. The bottom half of the table presents the predicted productivity slowdown using the estimated models. The four rows correspond to the same two productivity slowdown periods in Table 1: 1970–79 (1970s) and 2000–07 (2000s). The predicted growth rates are computed as $\Delta\log\widehat{\mathrm{LP}}_t = \Delta\log\mathrm{LP}_t^* + \Delta\log\widehat{\mathbf{E}}_t$, where $\Delta\log\mathrm{LP}_t^*$ are taken from previous estimates in Table 2. The observed and actual growth rates are calculated as the long difference between the beginning and end of each period. They are adjusted to reflect the growth rate over periods of the same length (ten-year window). The slowdown in productivity growth (observed and predicted) is calculated as the changes in growth rates from the previous periods.

The results confirm that higher volatility in year t is associated with a significantly less efficient allocation, with an estimated coefficient of -0.072 to -0.087. In Columns (4)–(6), we find very similar results when using the changes in allocative efficiency as the dependent variables. Again, the estimated correlations range from -0.081 to -0.097 and remain highly significant.[20]

These regression results indicate that the impact of increased volatility on productivity

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[20]We note two things about the estimation results. First, R^2 is high in Columns (1)–(3) due to controlling for the lagged value of the dependent variable (allocative efficiency in year $t-1$). Second, the estimated coefficients for TFP dispersion are similar in Columns (1)–(3) and Columns (4)–(6) despite the use of different dependent variables. This similarity exists because the estimated coefficients for the lagged dependent variable in Columns (1)–(3) are close to 1, effectively making the regression models similar in these columns and resulting in similar estimates.

growth can be economically significant. For instance, compared to the 1960s, the inter-quartile range of TFP shocks increased by 0.052 in the 1970s. This increase, according to the estimates in Table 3, translates into a decline in the left-hand-side variable, $\log \mathbf{E}_t$, by a size ranging from 0.004 to 0.005. Furthermore, in the data, the average annual growth rate of labor productivity was approximately 0.015 during the 1970s. This means that if the volatility of TFP shocks in the 1970s had stayed the same as the previous decade, the annual productivity growth rate would have been approximately 30 percent higher.

Next, we quantify to what extent volatility slows down productivity growth. To answer this question, we first obtain the predicted value for $\widehat{\mathbf{E}}_t$ using estimates from Table 3, then calculate predicted growth rates using the following equation: $\Delta \log \widehat{\mathrm{LP}}_t = \Delta \log \mathrm{LP}_t^* + \Delta \log \widehat{\mathbf{E}}_t$.[21] As shown at the bottom of Table 3, model-predicted slowdown during the 1970s ranges somewhere between 8 pp and 10 pp when we apply the full set of regressors (Columns 3 and 6), accounting for more than two-thirds of the observed 12 pp slowdown in the data. Further, the model-predicted slowdown during the 2000s is 3 pp, accounting for the entire observed slowdown in the data. According to Table 2, $\Delta \log \mathrm{LP}_t^*$ slows down by 3 pp during the 1970s and by 1 pp during the 2000s, indicating that the remaining 5–7 pp and 2 pp, respectively, are predicted by the increase in volatility. Based on these findings, volatility plays an even greater role than fundamentals in driving the productivity slowdown.

Evidence at the sector level. As of now, we have established the correlation between volatility and aggregate allocation efficiency. Similarly, allocative efficiency should also decline in sectors facing high volatility. We next provide evidence linking these two phenomena by exploiting differences in volatility across sectors.

Before we delve into the results, it would be helpful to clarify how we measure the changes in sectoral allocative efficiency. It is important to remember that $E_{i,t}$ could be greater or less than 1 with $E_{i,t} = 1$ indicating that resources allocated to this sector are at

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[21] For the regressions in Columns (4)–(6), we first obtain the predicted growth rates $\Delta \log \widehat{\mathbf{E}}_t$, and using the first year of $\mathbf{E_t}$ as a starting point, we compute $\widehat{\mathbf{E}}_t$ recursively for each year. The growth rate under optimal allocation, $\Delta \log \mathrm{LP}_t^*$, is taken from Table 2.

the optimal level. Moreover, a decrease in $E_{i,t}$ does not indicate worsening allocation. For instance, a shift from $E_{i,t} = 1.2$ to $E_{i,t} = 0.9$ actually indicates an improvement in allocation efficiency, as the gap between $E_{i,t}$ and 1, which represents how distant the sector is from optimum, decreases from 0.2 to 0.1. As such, the appropriate measure for sectoral allocation efficiency is the absolute difference between $E_{i,t}$ and 1, which in logarithmic terms can be expressed as $|\log E_{i,t}|$. Accordingly, the change in sectoral allocative efficiency is captured by $|\log E_{i,t}| - |\log E_{i,t-\Delta t}|$, wherein a positive value indicates a deterioration in allocation over the period $[t - \Delta t, t]$.

Table 4: Sector-level relationship between volatility and allocative efficiency

Dependent variable $\|\log E_{i,t}\| - \|\log E_{i,t-\Delta t}\|$	$\Delta t = 2$			$\Delta t = 4$		
Dispersion of TFP shock in $[t - \Delta t, t]$	0.141**	0.168*	0.154*	0.235**	0.253**	0.224*
	(0.0713)	(0.0866)	(0.0925)	(0.101)	(0.124)	(0.134)
Sector FEs	N	Y	Y	N	Y	Y
Year FEs	N	N	Y	N	N	Y
N	1593	1593	1593	1593	1593	1593
R^2	0.021	0.060	0.104	0.029	0.081	0.127
	$\Delta t = 6$			$\Delta t = 8$		
Dispersion of TFP shock in $[t - \Delta t, t]$	0.317***	0.437***	0.396***	0.541***	0.722***	0.671***
	(0.0895)	(0.120)	(0.134)	(0.123)	(0.173)	(0.191)
Sector FEs	N	Y	Y	N	Y	Y
Year FEs	N	N	Y	N	N	Y
N	1593	1593	1593	1593	1593	1593
R^2	0.056	0.139	0.188	0.070	0.181	0.231

Notes: This table regresses changes in allocative efficiency over $[t - \Delta t, t]$ on growth volatility—measured by the dispersion of TFP shocks—at the sector level. The regressions with or without sector- and year-fixed effects are presented in different columns. In addition, we also show the regression results with rolling windows of different lengths, where $\Delta t = 2$, $\Delta t = 4$, $\Delta t = 6$, and $\Delta t = 8$ correspond to 3-year, 5-year, 7-year and 9-year rolling windows, respectively. Robust standard errors are reported in parentheses.

Table 4 presents the correlation estimates between each sector's volatility, measured by the dispersion in productivity shock $\varepsilon_{i,t}$ over a rolling window from $t - \Delta t$ to t, and the associated changes in allocative efficiency within that same window. With a three-year rolling window ($\Delta t = 2$), the estimated coefficient on the dispersion of TFP shocks stands at 0.14, and this value increases somewhat upon controlling for one or both sets of sector- and year-fixed effects. For longer rolling windows ($\Delta t = 2, 4, 6, 8$), a consistently significant

relationship is found between heightened volatility and decreased allocative efficiency. Taken together, results in this table suggest that volatility increases are linked to notable declines in allocative efficiency at the sector level.

Table 5: Direction of movement in $E_{i,t}$ is correlated with the sign of productivity shocks

Dependent variable $\mathbb{I}(E_{i,t} < E_{i,t-\Delta t})$	$\Delta t = 2$	$\Delta t = 4$	$\Delta t = 6$	$\Delta t = 8$
Dummy indicator of positive accumulative TFP shocks	0.204***	0.214***	0.119***	0.116***
	(0.0251)	(0.0248)	(0.0247)	(0.0250)
N	1593	1593	1593	1593
R^2	0.170	0.196	0.205	0.231
Dummy indicator of positive median TFP shocks	0.195***	0.162***	0.0997***	0.0882***
	(0.0250)	(0.0247)	(0.0240)	(0.0238)
N	1593	1593	1593	1593
R^2	0.167	0.179	0.202	0.227
TFP shocks (accumulative)	0.258***	0.234***	0.133***	0.137***
	(0.0765)	(0.0691)	(0.0492)	(0.0408)
N	1593	1593	1593	1593
R^2	0.110	0.137	0.175	0.210
TFP shocks (median)	0.886***	0.923***	0.591*	0.820**
	(0.305)	(0.288)	(0.316)	(0.367)
N	1593	1593	1593	1593
R^2	0.106	0.133	0.172	0.206

Notes: This table examines the correlation between the sign of productivity shocks and the direction of the movement in $E_{i,t}$. The left-hand-side variable is a dummy variable indicating if there is a decline in $E_{i,t}$ over $[t - \Delta t, t]$. On the right-hand side, we include a dummy variable indicating a positive accumulative or median TFP shock and the actual values of the shocks. All regressions include a set of sector- and year-fixed effects. Robust standard errors are reported and in parentheses.

The mechanism highlighted here also has predictions about the direction of $E_{i,t}$'s movement in relation to the sign of the productivity shock. As discussed previously, absent any frictions, a sector receiving a positive productivity shock should witness an increase in capital and labor. However, due to adjustment costs, the actual resource inflow to this sector usually lags behind the optimal level. This likely leads to a decrease in the value of $E_{i,t}$ after a positive productivity shock.

We test this prediction in Table 5. The top panel regresses a binary variable $\mathbb{I}(E_{i,t} <$

$E_{i,t-\Delta t}$), indicating a reduction in $E_{i,t}$, on another binary variable, reflecting whether the sector has seen a positive accumulative TFP shock over the time interval $[t - \Delta t, t]$, with sector- and year-fixed effects as controls.[22] A three-year rolling window yields an estimated correlation of 0.2, which slightly diminishes with longer windows but remains highly significant. This positive and significant estimate is consistent with our prediction that a rise in TFP correlates with a reduction in the value of $E_{i,t}$. The second panel substitutes the accumulative TFP shocks with the median value of the TFP shocks over the time interval $[t - \Delta t, t]$ and returns similar outcomes, despite slightly lower point estimates. This pattern is once again validated when using the actual TFP shock values as dependent variables. As shown in the bottom two panels, a greater positive TFP shock (median or accumulative) is associated with an increased likelihood of a decrease in $E_{i,t}$.

In summary, the two exercises in this section provide insights into the movement of each $E_{i,t}$ as the cross-sectional distribution of $E_{i,t}$ determines aggregate allocative efficiency. The magnitude, as well as the direction of changes in $E_{i,t}$, are closely linked to the underlying TFP shocks.

Evidence from factor utilization. Next, we present additional evidence based on factor utilization rates. Recent studies documented that dispersions of capacity utilization rates are an essential contributor to dispersions of the marginal product of capital and labor (Gorodnichenko et al., 2021). Using a model of investment with adjustment costs, Abel and Eberly (1998) showed that factor utilization rates represent the position in the inaction region. As volatility increases, the inaction region expands, resulting in a broader range of utilization rates in the cross-section. Consequently, the two episodes of productivity slowdown should be accompanied by increased dispersion of utilization rates, which we investigate next.

To do so, we first construct measures of factor utilization rates. For capital, a common approach uses energy consumption to calculate utilization-adjusted capital service (Burnside et al., 1995). According to the estimation in Burnside et al. (1995), the actual capital service used, $\hat{K}_{i,t}$, can be written as a Leontief function of capital stock and energy consumption.

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[22]The accumulative TFP shock is defined as the sum of TFP shocks $\epsilon_{i,t}$ over the period $[t - \Delta t, t]$.

Using the specification in Oberfield (2013), we let $\hat{K}_{i,t} = \min\{E_{i,t}/b_i, u_{i,t}K_{i,t}\}$, where $u_{i,t}$ is the capital utilization rate, $E_{i,t}$ is energy consumption, and b_i represents the energy intensity of capital usage in each sector determined by its fundamental technology. The difference between growth rates of $\hat{K}_{i,t}$ and $K_{i,t}$, therefore, represents changes in capital utilization rates: $\Delta \log u_{i,t}^k = \Delta \log \hat{K}_{i,t} - \Delta \log K_{i,t}$.

For labor utilization rates, we follow the tradition of growth accounting literature and define the rate as the average hours worked per employed person (Basu et al., 2006; Fernald, 2014). This then allows us to compute changes in labor utilization rate $\Delta \log u_{i,t}^l$ for each sector on a yearly basis. The cross-sectional dispersions in $\Delta \log u_{i,t}^k$ and $\Delta \log u_{i,t}^l$ are used to measure the dispersion in factor utilization rates.[23]

Figure 6: Cross-sectional dispersions of factor utilization rates

(a) Standard deviation **(b)** Inter-quartile range
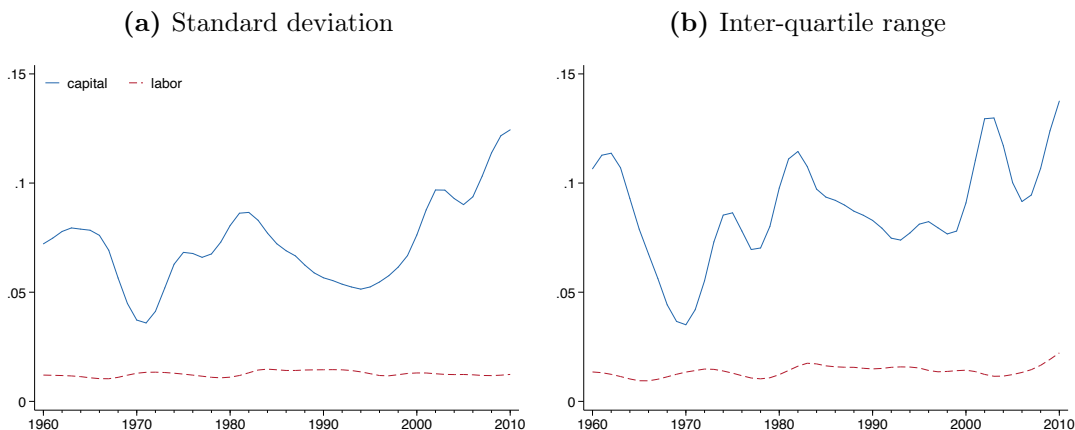
Notes: This table plots the cross-sectional dispersions in capital and labor utilization rates every year. Panels (a) and (b) plot the standard deviation and inter-quartile range, respectively. The time series are HP-filtered with a smoothing parameter of 6.25.

Figure 6 displays the HP-filtered time-series of the standard deviation (Panel a) and inter-quartile range (Panel b) of the utilization rates every year. Labor utilization dispersion is relatively small and stable, whereas capital utilization dispersion varies significantly over time. Notably, during the 1970s and 2000s, the dispersion of capital utilization rates increases significantly compared to the previous decades. Despite being suggestive, this figure provides

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[23]A caveat to this analysis is we can only calculate changes in capital utilization rates but not levels; hence we are unable to quantify to what extent utilization rates contribute to the cross-sectional variations, in $E_{i,t}$, as in Gorodnichenko et al. (2021).

further evidence of the mechanism we highlight.

5 Extensions and robustness checks

This section discusses extensions and robustness checks of the baseline results. We consider (i) an extension of the Cobb-Douglas framework to CES production systems, (ii) an alternative timing of the productivity slowdown, (iii) a closer look at manufacturing industries using more granular data, (iv) post-2010 development in allocative efficiency, (v) heterogeneity in capital and labor composition across sectors, (vi) non-zero profits, and (vii) alternative specifications to estimate output elasticities. To avoid repetition, for each exercise, we provide a summary of the results in the main text and include more details in the Appendix Section A.

5.1 CES production system

Our baseline results use the Cobb-Douglas production system, a valuable benchmark. However, recent papers have shown that the measurement of allocative efficiency depends on the elasticity of substitution between sectors and inputs (see Epifani and Gancia, 2011; Osotimehin and Popov, Forthcoming). This section extends the benchmark value-added economy to a more flexible CES production system.

The final good is a CES aggregation of intermediate goods: $Y = (\sum_i \omega_i Y_i^{1-\frac{1}{\rho}})^{\frac{\rho}{\rho-1}}$. The intermediate good Y_i is produced using capital and labor, $Y_i = A_i(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}$. In these production functions, (ω_i, ν_i) are the CES weights and (ρ, ϵ) are the elasticity of substitution. Similar to before, the planner solves the following optimization problem: $\max Y, s.t \sum_i K_i = K, \sum_i L_i = L$. Appendix Section B.3 details the model solution.

To evaluate our results in the CES framework, we first estimate the elasticity of substi-

tution parameters, ρ and ϵ, using the following specification:

$$\log \frac{p_{i,t}Y_{i,t}}{p_{N,t}Y_{N,t}} = \beta_\rho \log \frac{Y_{i,t}}{Y_{N,t}} + \chi_i + u_{i,t}, \tag{7}$$

$$\log \frac{R_i K_i}{w_i L_i} = \beta_\epsilon \log \frac{K_i}{L_i} + \chi_i + u_{i,t}, \tag{8}$$

where $\beta_\rho = \frac{\rho}{\rho-1}$, $\beta_\epsilon = \frac{\epsilon}{\epsilon-1}$, χ_i represents the sectoral fixed effects and $u_{i,t}$ is the error term.[24] In Equation (7), $p_{i,t}Y_{i,t}$ is the nominal value-added output for sector $i \in \{1,...,N\}$ and $Y_{i,t}$ is the corresponding real value-added output. In Equation (8), $R_i K_i$ and $w_i L_i$ are capital and labor income in nominal terms while K_i and L_i are real capital stock and the number of workers in each sector.[25] After obtaining the elasticity of substitution parameters, the CES weights in the production functions can be calculated using the expenditure shares over a rolling window, as in the Cobb-Douglas case.

The point estimate for the elasticity of substitution between sector goods is $\rho = 0.96$, smaller than but not significantly different from one. The elasticity of substitution ρ measures how easy it is for consumers to substitute across a broad set of goods or services. Not surprisingly, the estimates would vary somewhat across different sectoral classification schemes. Aum et al. (2018) categorized all non-agriculture industries into ten broad sectors. Their estimated elasticity was 0.77 across these sectors. Oberfield and Raval (2021) showed that the estimates of elasticity across two-digit manufacturing sectors centered around one from various specifications. In Herrendorf et al. (2013), the benchmark specification estimated the elasticity between broad sectors—agriculture, service, and manufacturing—to be around 0.9. Atalay (2017) suggested that a value smaller but closer to one best characterizes the demand elasticity from consumers. Both Atalay (2017) and Oberfield and Raval (2021) chose elasticity equal to one in their baseline parametrization. Overall, our estimate is within the range of estimates in the literature.

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[24]Both specifications are derived from the cost-minimization conditions following the approach in Aum et al. (2018). The conditions that generate Equations (7) and (8) are $\log(\frac{P_i Y_i}{P_N Y_N}) = \log(\frac{\omega_i}{\omega_N}) + \frac{\rho-1}{\rho}\log(\frac{Y_i}{Y_N})$ and $\log \frac{R_i K_i}{w_i L_i} = \log \frac{\lambda_i}{1-\lambda_i} + \frac{\epsilon-1}{\epsilon}\log(\frac{K_i}{L_i})$, respectively.

[25]The identification strategy exploits the relationship between changes in input expenditure and changes in input quantity over time. Given the expenditure of inputs, input price and quantity provide essentially the same information. Therefore, the above specification is equivalent to one that regresses expenditure on prices (Atalay, 2017) or regresses quantity on prices (Oberfield and Raval, 2021).

Our estimated elasticity of substitution between capital and labor is $\epsilon = 0.81$, suggesting that capital and labor are gross complements in the sectoral production functions. Several recent papers also estimated the elasticity of substitution between capital and labor at the sectoral/industry level. Among them, Herrendorf et al. (2015) considered three broad sectors, agriculture, manufacturing, and service, and the estimated elasticity of substitution between capital and labor were 1.58, 0.8, and 0.75 in these three sectors, respectively. Alvarez-Cuadrado et al. (2017) found a slightly lower value for the manufacturing (0.78) and service (0.57) sectors. By aggregating up elasticities at the plant level, Oberfield and Raval (2021) obtained an elasticity of 0.72 for manufacturing sectors in 1987 and suggested that it has been trending down since. Aum et al. (2018) combined labor with two types of capital—computer and non-computer—using a nested CES structure and found that the estimated elasticity between computer capital and labor ranged from 1.2 to 1.8.[26]

Given the empirical challenges associated with estimating the elasticity of substitution and the lack of consensus in the literature, applying a range of values for these parameters would be necessary. We set our baseline specification to $\rho = 0.96$ and $\epsilon = 0.81$, which are estimates from our data. We also consider the case of a lower value for the elasticity between sectoral goods, $\rho = 0.77$ (Aum et al., 2018). For the elasticity between capital and labor, we take the estimated value from Oberfield and Raval (2021) (0.72) and from Karabarbounis and Neiman (2014) (1.25), which reflect different views of whether capital and labor are gross substitutes or complements. Therefore we have six combinations of values for ρ and ϵ.

Table 6 presents the changes in the fundamental productivity growth compared to the previous decade.[27] With the baseline parameterization ($\rho = 0.96, \epsilon = 0.81$), productivity growth in the 1970s and 2000s slows down by 4 pp and 1 pp, respectively. We note that the difference between the baseline CES result and the Cobb-Douglas case is small. Further, we

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[26]Researchers have also estimated the elasticity of substitution between capital and labor at the aggregate level. There exists a relatively wide range of estimates (see Chirinko, 2008, for a summary). For example, Karabarbounis and Neiman (2014) estimated that capital and labor are gross substitutes with an estimated elasticity between 1.2 and 1.5 using several cross-country aggregate datasets (see also the estimates in Piketty, 2017). On the other hand, most other papers have found that capital and labor are gross complements, including Antràs (2004), Klump et al. (2007), and León-Ledesma et al. (2010), among others.

[27]In Appendix Section A.4, we provide more details, including the evolution of allocative efficiency under the different values of elasticity over the whole sample period.

Table 6: Changes in fundamental productivity growth from the previous decade

	CD	CES					
		ρ = 0.96			ρ = 0.77		
		ε = 0.81	ε = 0.72	ε = 1.25	ε = 0.81	ε = 0.72	ε = 1.25
		baseline		highest elas.		lowest elas.	
1960–69	–	–		–		–	–
1970–79	-0.03	-0.04	-0.05	-0.02	-0.05	-0.06	-0.04
1980–89	-0.03	-0.02	-0.01	-0.04	-0.02	0.00	-0.03
1990–99	0.06	0.06	0.06	0.05	0.06	0.06	0.05
2000–07	-0.01	-0.01	-0.01	-0.00	-0.02	-0.01	-0.00

Notes: This table presents the change in fundamental productivity growth compared to the previous decade in the CES framework. The results are presented under six combinations of parameter values for ρ and ε, the elasticity of substitution parameters in the production system. The baseline parameterization is ρ = 0.96 and ε = 0.81. As a comparison, we also include the results from the Cobb-Douglas case, taken from the last column of Table 2.

find that higher elasticity is generally associated with a more prominent role for allocative efficiency in explaining the productivity slowdown.[28] For the case with the highest elasticity of substitution ($\rho = 0.96, \epsilon = 1.25$), productivity slowed down by 2 pp during the 1970s. Further, there was no slowdown in productivity during the 2000s. Even for the scenario with the lowest elasticity ($\rho = 0.77, \epsilon = 0.72$), allocation can explain at least half of the productivity slowdown.[29]

Estimating the elasticity of substitution between capital and labor is notoriously difficult. For instance, there can be endogeneity issues regarding estimating Equations (7) and (8). Given the challenges with the estimation, we performed the robustness check under a range of values found in the literature, and it is reassuring that our results are not very sensitive to different elasticities.[30]

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[28]Epifani and Gancia (2011) and Osotimehin and Popov (Forthcoming) show that the elasticity of substitution also matters for the measured *level* of allocative efficiency.

[29]Our exercise abstracts from the complementarity between intermediate inputs. With the availability of better data, we can extend this framework to incorporate this dimension, thereby potentially enhancing the impact of complementarity (see models in Atalay, 2017, and Osotimehin and Popov, Forthcoming).

[30]Complementarities, coupled with uneven growth, can also lead to a decline in fundamental productivity as resources flow into stagnant sectors, i.e., the Baumol's diseases (Duernecker et al., 2017; Aum et al., 2018). Baumol's diseases occur even when resources are allocated optimally across sectors. The degree of complementarity determines the force of Baumol's diseases and the magnitude of the aggregate productivity slowdown. As such, looking back at the decomposition Equation (6), the elasticity of substitution affects both terms on the right-hand side, which makes it a crucial parameter to estimate for the understanding of productivity growth.

5.2 Timing of the slowdown

So far, we have conducted our analysis in decades, focusing on the 1970s and post-2000s as periods of slowdown. However, the timing of the slowdown may not align precisely with these decade-long periods. Indeed, while it is widely recognized in previous papers that the 1970s and 2000s were periods of significant slowdown in productivity, the specific periods studied by these papers vary. For instance, like us, Aum et al. (2018), Decker et al. (2020), and Vandenbroucke (2021) use the decade approach. In comparison, Byrne et al. (2016) argue that the post-2000 slowdown begins in 2005, while the 1970s episode spans 1973–1982.

In this section, we examine the start and the end dates of the slowdown episodes more closely. To do this, we first divide our sample into subperiods based on trends in labor productivity growth. This process generates five subperiods: 1960–1973, 1973–1982, 1982–1995, 1995–2005, and post-2005, all of which show a change in trend growth from the previous one. This division is identical to that used in Byrne et al. (2016). Consistent with Byrne et al. (2016), we identify 1973–1982 and post-2005 as two periods notable for their slower-than-normal labor productivity growth rates.

During the periods 1973–1982 and 2005–2010, the annual growth rate in labor productivity was lower than in preceding periods by 1.88 and 1.08 pp, respectively. Moreover, the growth rate in allocative efficiency was also lower during these periods, by 0.39 and 0.26 percentage points, respectively, compared to preceding periods. As a result, slower growth in allocative efficiency contributed 21% and 24% to the slowdown in labor productivity growth during 1973–1982 and 2005–2010, respectively. Details of the exercise can be found in Appendix A.5.

The contribution of allocative efficiency calculated using this alternative timing is smaller than the baseline results for two reasons. First, with this alternative timing, the productivity growth slowdown becomes more prominent. This is especially true for the post-2005 episode, while the subperiod 1995–2005 displays extraordinarily fast productivity growth. In our benchmark analysis, the 2000s decade reflects a blend of these two vastly different periods.

Second, the slowdown in allocative efficiency does not coincide perfectly with the slowdown in productivity growth. Particularly in the 1970s episode, this timing difference led to underestimating the role of allocative efficiency.

Lastly, our analysis shows that the timing of the slowdowns in labor productivity and allocative efficiency do not match perfectly. Empirically, the timing of the slowdown in economic activities varies slightly based on which variable we consider. Moreover, the timing of the slowdown in allocative efficiency is sensitive to model specifications, as we will illustrate in our robustness-check exercises. A timing difference can lead to either underestimation or overestimation of allocative efficiency's contribution. For these reasons, we retain the decade approach as our baseline specification for its transparency and include a discussion of the alternative timing as the robustness check.

5.3 Manufacturing industries

In this section, we delve deeper into the manufacturing sector, utilizing the NBER-CES Manufacturing Industry Database (1958–2018). This database provides industry-level data for 364 distinct manufacturing industries.[31]

To begin with, we document that the manufacturing sector exhibits similar slowdown dynamics as the whole economy. In particular, based on the changes in growth trends in the manufacturing labor productivity series, we divide the entire sample into four sub-periods: 1960–73, 1973–82, 1982–2005, and post-2005.[32] Additionally, using the NBER-CES database, we can measure allocative efficiency across these manufacturing industries. The

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[31]We use the term manufacturing *industries* to differentiate from the broader manufacturing *sector* that encompasses all these industries.

[32]Note that because of the added noise in the measured $\mathbf{E}_t$, we find it difficult to derive any valuable insights if we continue to analyze the data by decades (as we did in the baseline exercise), as the high-frequency fluctuations obscure the underlying trends. Therefore, we first identify periods of labor productivity slowdown based on changes in its growth trends. As a result, we divide the sample into four subperiods: 1960–73, 1973–82, 1982–2005, and post-2005, and note that 1973–82 and post-2005 are the slowdown episodes. Within each subperiod, we compute the average annual growth rates by running a linear regression of $\log \mathrm{LP}_t$ (or $\log \mathbf{E}_t$) on time. Overall, this method allows us to extract the median-term growth trends without too much interference from the high-frequency fluctuations.

dynamics of $\mathbf{E}_t$ appear noisier than the baseline results, perhaps because the NBER-CES database is constructed using firm surveys and has a much finer industry classification than KLEMS. Nevertheless, despite the noise, the result indicates that 1973–82 and post-2005 have significantly slower growth rates in labor productivity as well as allocative efficiency. During these periods, 45% and 20%, respectively, of the slowdown in labor productivity growth can be attributed to a lack of improvement in allocative efficiency. Appendix Section A.6 contains details of this exercise.

We also investigated the correlation between industry-level volatility and cross-industry allocative efficiency within the manufacturing sector, using the NBER-CES manufacturing database to measure industry-level volatility, as described in Section 4.3. Our findings revealed heightened levels of volatility in the manufacturing sector during the 1970s and from 2000 onwards. This volatility correlated with a decline in allocation efficiency, as detailed in Appendix Section A.7. Overall, these insights corroborate and enhance our baseline findings by applying them to more granular datasets.

5.4   Post-2010 dynamics

Our baseline analysis uses the 2013 version of the KLEMS and WIOT databases, which only include data up to 2010. However, the slowdown in productivity growth extends beyond 2010. In this section, we examine the post-2010 dynamics using a different version of the KLEMS dataset, published by the Luiss Lab of European Economics (LLEE) in 2021. The LLEE KLEMS dataset has a different industry classification and covers a more recent period up until 2015. However, the LLEE KLEMS has several limitations. For instance, several key variables needed to measure allocative efficiency only became available after 1997. Also, several sectors are missing capital stock information altogether. Keeping in mind these caveats, we use this dataset as a robustness check for the post-2010 dynamics. Appendix Section A.8 contains details of the exercise.

Our baseline result shows that the allocative efficiency starts to flatten around 2000 after

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experiencing decades of positive growth. Using the new dataset, we find this flattening trend continues into the post-2010 period. As the growth of allocative efficiency continues to slow, it accounts for almost all the productivity slowdown in 2000–15.

5.5 Heterogeneity in capital and labor

So far, our empirical analysis relies on the 2013 version of KLEMS and WIOT. Although the construction of KLEMS data takes into account different compositions of capital types in each sector, one might wonder if this aggregation would cause mismeasurement.[33] The same concern applies to the measurement of labor.

In this section, we repeat the exercise using alternative input measures. We take capital inputs of different types from the 2009 version of KLEMS, which covers a shorter period (1977–2007). The data reports eight asset types: (i) computing equipment, (ii) communications equipment, (iii) software, (iv) transport equipment, (v) other machinery and equipment, (vi) total non-residential investment, (vii) residential structures, and (viii) other assets. The first three asset types constitute the ICT asset group, and the latter five constitute the non-ICT group. We follow the literature to use labor compensation as a proxy for labor inputs.[34] Details of the exercises can be found in Appendix Section A.9.

When extending the analysis to two types of assets, ICT and non-ICT capital, the result is only available from the end of the 1970s. We find a rapid improvement in allocation during the 1980s and a gradual deterioration since the beginning of the 2000s. The exercise with eight types of assets shows a similar trend. In these cases, allocative efficiency accounts for two-thirds of the productivity slowdown in the two-asset specification and one-third in the eight-asset specification. Moreover, by replacing employment with wage bills as a measure of labor input, the level of allocative efficiency is higher, but the trend remains very similar

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[33]See Jorgenson et al. (2014) for details on how the industry-level capital data are constructed.

[34]These alternative measures, however, come with their own set of challenges. In constructing the returns to capital by type, KLEMS makes assumptions to split the total capital income into returns of different types, which may introduce measurement errors. Regarding labor inputs, wage bills may address the composition problem, but they cannot distinguish the role of price and quantity in measuring allocative efficiency.

to the baseline result. Our results indicate that approximately half of the productivity slowdown in the 1970s and the entire observed slowdown in the 2000s can be attributed to allocative efficiency.

5.6   Positive profits

In KLEMS, capital income equals value-added minus labor income; in other words, the underlying assumption is that sectors make zero pure profits. This data treatment is partly motivated by findings that pure profits as a share of value-added output are close to zero in the US (Rotemberg and Woodford, 1995). However, recent studies show that profits shares have been rising in the US, which implies that the data overestimated capital income. Consequently, this leads to an upward bias in capital-output elasticity estimates and capital weights in measuring aggregate allocative efficiency.

Next, we relax the assumption of zero pure profits to test if such an assumption biases our results. We first split the raw capital compensation into pure profits and actual capital income using the estimated profits to capital income ratio of Barkai (2020). There exists a range of estimates for this ratio over time. We choose two values that are on the upper end of the range: (i) ratio=1/5 in 2010, the last year of our sample, and (ii) ratio=1/2 in 2007, the highest level over the sample period in Barkai (2020). Appendix Section A.10 includes details of the exercise. Here we provide a summary of the main take-aways: we find that under the assumption of positive profits, the changes in allocative efficiency are of a slightly smaller magnitude, which perhaps indicates the presence of bias in the baseline result. However, our main finding is robust under this alternative assumption. Even with the highest profit share, allocative efficiency still explains at least half of the productivity slowdown in the data.[35]

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[35]Note that in this exercise we abstract from the case where profit rates differ across sectors. This abstraction is due to data availability as we cannot distinguish between capital return and pure profits at the sector level.

5.7 Alternative specifications

We use two specifications to obtain output elasticity estimates, and in the main text, we report results based on the first specification and a 3-year rolling window. In this paragraph, we discuss the results of alternative specifications. Details of the exercises can be found in Appendix Sections A.11, A.12, and A.13. Below we summarize the main findings.

First, we consider various lengths of the rolling window within which we calculate the average expenditure shares. We find that the evolution of allocative efficiency with longer rolling windows is slightly smoother than the baseline, but the resulting implications for productivity slowdown are virtually unchanged.

Second, the critical identification assumption in the first specification is that, on average, the wedges are not biased towards any factor. Therefore on average, expenditure shares reflect the actual output elasticities. This assumption is necessary to our methodology, but we would like to check if it drives our results. To do so, we consider alternative cases where the wedge is (significantly) higher for one factor than the other. The finding shows that the level of $\mathbf{E}_t$ can be either higher or lower than the baseline result. However, the trends of $\mathbf{E}_t$ remain very similar to the baseline case. As a result, allocative efficiency still plays a prominent role in explaining productivity slowdown.

Third, we assume that expenditure shares in the later years are undistorted and apply them to the earlier years of the sample. We choose two base years for this exercise: (i) 2010, the last year of our sample, and (ii) 2005, to avoid the impact of the Great Recession. Under this specification, we find that the deterioration in allocation is slightly less prominent than the baseline during the 1970s. However, allocative efficiency remains a quantitatively significant force as it still explains more than half of the productivity slowdown in the data.

Taking stock. To summarize, the theoretical framework we use in this paper relies on relatively strong parametrical assumptions. This section scrutinizes these assumptions to ensure that they do not drive our results. There are two key takeaways. First, while the

measure of allocative efficiency is affected by different assumptions and specifications, we find that the level of these measures is more sensitive than the change (growth rate) of these measures. Second, our main results are robust. Allocative efficiency plays a significant role in driving aggregate productivity growth in all the cases we have considered.[36]

6   Conclusion

This paper quantifies how much of the slowdown in productivity growth can be explained by factor allocation. We apply a tractable decomposition framework to the US economy and show that allocative efficiency explains approximately two-thirds of productivity slowdown in both the 1970s and 2000s. Providing additional evidence on the role of volatility, we find that the slow growth in allocative efficiency in both slowdown episodes is partly due to increased volatility.

These findings demonstrate the critical role of volatility in affecting allocation and aggregate productivity growth. In theory, the worsening allocative efficiency in highly volatile times stems from firms optimizing their production in the presence of adjustment costs. In this case, a departure from the unconstrained optimal allocation might be constrained-efficient. Policies that directly target firms' investment and hiring decisions are not necessarily welfare-enhancing. In contrast, policy initiatives that reduce the volatility firms face or lower their adjustment costs are unambiguously conducive to growth.

We hope that our method may help study other issues related to allocation and growth. One particular direction of work comes to mind. Several countries have experienced fast growth and significant catching up to the frontier in recent decades. The improvement in allocation efficiency has played an essential role in this process (Song et al., 2011; Buera and Shin, 2013). In general, however, there is significant heterogeneity in convergence patterns

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[36]Our methodology in measuring allocative efficiency closely follows Hsieh and Klenow (2009) and Oberfield (2013). Recent papers have raised concerns about the framework's assumptions of constant returns to scale production functions and full pass-through (CES demand system). Ruzic and Ho (2021), for instance, find that allowing for industry-specific markups and returns to scale parameters can alter the conclusions of Hsieh and Klenow (2009).

across countries, even within the group of advanced economies (Cette et al., 2016). On average, developing countries as a whole have not made much progress in closing the income gaps in relation to the US (Johnson and Papageorgiou, 2020). How much of these patterns are explained by variations in allocation efficiency growth across countries? Extending the current framework to a cross-country setting could provide insights into this question.

References

ABEL, A. B. AND J. C. EBERLY (1998): "The Mix and Scale of Factors with Irreversibility and Fixed Costs of Investment," *Carnegie-Rochester Conference Series on Public Policy*, 48, 137–144.

ALVAREZ-CUADRADO, F., N. LONG, AND M. POSCHKE (2017): "Capital-labor Substitution, Structural Change and Growth," *Theoretical Economics*, 12.

ANTRÀS, P. (2004): "Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution," *Contributions to Macroeconomics*, 4.

ASKER, J., A. COLLARD-WEXLER, AND J. D. LOECKER (2014): "Dynamic Inputs and Resource (Mis)allocation," *Journal of Political Economy*, 122, 1013–1063.

ATALAY, E. (2017): "How Important Are Sectoral Shocks?" *American Economic Journal: Macroeconomics*, 9, 254–280.

AUM, S., S. Y. T. LEE, AND Y. SHIN (2018): "Computerizing Industries and Routinizing Jobs: Explaining Trends in Aggregate Productivity," *Journal of Monetary Economics*, 97, 1–21.

BAI, C. E., C. T. HSIEH, AND Z. M. SONG (2016): "The Long Shadow of China's Fiscal Expansion," *Brookings Papers on Economic Activity*, 2016, 129–181.

BAILY, M. N., C. HULTEN, D. CAMPBELL, T. BRESNAHAN, AND R. E. CAVES (1992): "Productivity Dynamics in Manufacturing Plants," *Brookings Papers on Economic Activity. Microeconomics*, 1992, 187.

BANERJEE, R. N. AND B. HOFMANN (2018): "The Rise of Zombie Firms: Causes and Consequences," *BIS Quarterly Review*.

BAQAEE, D. R. AND E. FARHI (2020): "Productivity and Misallocation in General Equilibrium," *Quarterly Journal of Economics*, 135, 105–163.

BARKAI, S. (2020): "Declining Labor and Capital Shares," *Journal of Finance*, 75, 2421–2463.

BASU, S. AND J. G. FERNALD (2002): "Aggregate Productivity and Aggregate Technology," *European Economic Review*, 46, 963–991.

BASU, S., J. G. FERNALD, AND M. S. KIMBALL (2006): "Are Technology Improvements Contractionary?" *American Economic Review*, 96, 1418–1448.

BEHRENS, K., G. MION, Y. MURATA, AND J. SUEDEKUM (2020): "Quantifying the Gap Between Equilibrium and Optimum under Monopolistic Competition," *The Quarterly Journal of Economics*, 135, 2299–2360.

BILS, M., P. J. KLENOW, AND C. RUANE (2020): "Misallocation or Mismeasurement?" Working paper.

BLOOM, N. (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77, 623–685.

BLOOM, N., M. FLOETOTTO, N. JAIMOVICH, I. SAPORTA-EKSTEN, AND S. J. TERRY (2018): "Really Uncertain Business Cycles," *Econometrica*, 86, 1031–1065.

BLOOM, N., C. I. JONES, J. VAN REENEN, AND M. WEBB (2020): "Are Ideas Getting Harder to Find?" *American Economic Review*, 110, 1104–44.

BUERA, F. J. AND Y. SHIN (2013): "Financial Frictions and the Persistence of History: A Quantitative Exploration," *Journal of Political Economy*, 121, 221–272.

BURNSIDE, C., M. EICHENBAUM, AND S. REBELO (1995): "Capital Utilization and Returns to Scale," in *NBER Macroeconomics Annual 1995, Volume 10*, National Bureau of Economic Research, Inc, NBER Chapters, 67–124.

BYRNE, D. M., J. G. FERNALD, AND M. B. REINSDORF (2016): "Does the United States Have a Productivity Slowdown or a Measurement Problem?" *Brookings Papers on Economic Activity*, 2016, 109–182.

CALIENDO, L., F. PARRO, AND A. TSYVINSKI (2022): "Distortions and the Structure of the World Economy," *American Economic Journal: Macroeconomics*, 14, 274–308.

CETTE, G., J. FERNALD, AND B. MOJON (2016): "The Pre-Great Recession Slowdown in Productivity," *European Economic Review*, 88, 3–20.

CHIRINKO, R. S. (2008): "σ: The Long and Short of It," *Journal of Macroeconomics*, 30, 671–686.

DAVIS, S. J. AND J. HALTIWANGER (2001): "Sectoral Job Creation and Destruction Responses to Oil Price Changes," *Journal of Monetary Economics*, 48, 465–512.

DECKER, R. A., J. HALTIWANGER, R. S. JARMIN, AND J. MIRANDA (2020): "Changing Business Dynamism and Productivity: Shocks versus Responsiveness," *American Economic Review*, 110, 3952–3990.

DUERNECKER, G., B. HERRENDORF, AND A. VALENTINYI (2017): "Structural Change within the Service Sector and the Future of Baumol's Disease," CEPR Discussion Papers 12467, C.E.P.R. Discussion Papers.

EPIFANI, P. AND G. GANCIA (2011): "Trade, Markup Heterogeneity and Misallocations," *Journal of International Economics*, 83, 1–13.

FERNALD, J. G. (2014): "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity," *Federal Reserve Bank of San Francisco, Working Paper Series*, 01–28.

GORODNICHENKO, Y., D. REVOLTELLA, J. SVEJNAR, AND C. WEISS (2021): "Resource Misallocation in European Firms: The Role of Constraints, Firm Characteristics and Managerial Decisions," Working Paper.

HERRENDORF, B., C. HERRINGTON, AND A. VALENTINYI (2015): "Sectoral Technology and Structural Transformation," *American Economic Journal: Macroeconomics*, 7, 104–33.

HERRENDORF, B., R. ROGERSON, AND A. VALENTINYI (2013): "Two Perspectives on Preferences and Structural Transformation," *American Economic Review*, 103, 2752–2789.

HSIEH, C.-T. AND P. J. KLENOW (2009): "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 124, 1403–1448.

JOHNSON, P. AND C. PAPAGEORGIOU (2020): "What Remains of Cross-Country Convergence?" *Journal of Economic Literature*, 58, 129–75.

JORGENSON, D. W., M. S. HO, A. PATEL, AND J. D. SAMUELS (2014): "Data Appendix to 'Economic Growth in the Information Age'," Working Paper.

KARABARBOUNIS, L. AND B. NEIMAN (2014): "The Global Decline of the Labor Share," *Quarterly Journal of Economics*, 129, 61–103.

KLUMP, R., P. MCADAM, AND A. WILLMAN (2007): "Factor Substitution and Factor-Augmenting Technical Progress in the United States: A Normalized Supply-Side System Approach," *The Review of Economics and Statistics*, 89, 183–192.

LEÓN-LEDESMA, M. A., P. MCADAM, AND A. WILLMAN (2010): "Identifying the elasticity of substitution with biased technical change," *American Economic Review*, 100, 1330–1357.

LIU, E. (2019): "Industrial Policies in Production Networks," *The Quarterly Journal of Economics*, 134, 1883–1948.

MONGE-NARANJO, A., J. M. SÁNCHEZ, AND R. SANTAEULÀLIA-LLOPIS (2019): "Natural Resources and Global Misallocation," *American Economic Journal: Macroeconomics*, 11, 79–126.

OBERFIELD, E. (2013): "Productivity and Misallocation During a Crisis: Evidence from the Chilean Crisis of 1982," *Review of Economic Dynamics*, 16, 100–119.

OBERFIELD, E. AND D. RAVAL (2021): "Micro Data and Macro Technology," *Econometrica*, 89, 703–732.

OSOTIMEHIN, S. AND L. POPOV (Forthcoming): "Misallocation and Intersectoral linkages," *Review of Economic Dynamics*.

PIKETTY, T. (2017): *Capital in the Twenty-First Century*, Harvard University Press.

RESTUCCIA, D. AND R. ROGERSON (2017): "The Causes and Costs of Misallocation," *Journal of Economic Perspectives*, 31, 151–174.

ROTEMBERG, J. J. AND M. WOODFORD (1995): "Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets," in *Frontiers of Business Cycle Research*, ed. by T. Cooley, Princeton University Press, 243–293.

RUZIC, D. AND S.-J. HO (2021): "Returns to Scale, Productivity Measurement, and Trends in US Manufacturing Misallocation," *The Review of Economics and Statistics*, 1–47.

SONG, Z., K. STORESLETTEN, AND F. ZILIBOTTI (2011): "Growing Like China," *American Economic Review*, 101, 196–233.

SUMMERS, L. H. (2018): "Secular Stagnation and Macroeconomic Policy," *IMF Economic Review*, 66, 226–250.

VANDENBROUCKE, G. (2021): "The Baby Boomers and the Productivity Slowdown," *European Economic Review*, 132, 103609.

ZIEBARTH, N. L. (2013): "Are China and India Backward? Evidence from the 19th Century U.S. Census of Manufactures," *Review of Economic Dynamics*, 16, 86–99.

Appendix

A Empirical appendix

A.1 KLEMS and WIOT sector codes

Table A.1 shows the sector definition in KLEMS and WIOT. Sectors highlighted in red are included in our empirical analysis. For the manufacturing sector definition, we include all subsectors listed under the "D Manufacturing" header. The service sector encompasses all subsectors under "G Wholesale and Retail Trade," "I Transportation, Storage and Communication," "K Real Estate, Renting and Business Activities," as well as sectors H, J, M, and N.

Table A.1: List of sectors in KLEMS and WIOT (2013 version)

AtB Agriculture hunting forestry and fishing
C Mining and quarrying
D Manufacturing
    15t16 Food products, beverages and tobacco
    17t19 Textiles, textile products leather and footwear
    20 Wood and products of wood and cork
    21t22 Pulp paper, paper products, printing and publishing
    23 Coke refined petroleum products and nuclear fuel
    24 Chemicals and chemical products
    25 Rubber and plastics products
    26 Other non-metallic mineral products
    27t28 Basic metals and fabricated metal products
    29 Machinery nec
    30t33 Electrical and optical equipment
    34t35 Transport equipment
    36t37 Manufacturing nec; recycling
E Electricity gas and water supply
F Construction
G Wholesale and retail trade
    50 Wholesale trade and commission trade except of motor vehicles and motorcycles
    51 Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of fuel
    52 Retail trade except of motor vehicles and motorcycles; repair of household goods
H Hotels and restaurants
I Transport and storage and communication
    60t63 Transport and storage
    64 Post and telecommunications
J Financial intermediation
K Real estate, renting and business activities
    70 Real estate activities
    71t74 Renting of m&eq and other business activities
M Education
N Health and social work

A.2 Improvement in allocative efficiency in the long run

This section delves deeper into the improvement in allocative efficiency over the sample period. Figure A.1 plots the sectoral allocative efficiency $E_{i,t}$, averaged over 1960–65 (beginning of our sample) and 2000–05 (highest overall allocative efficiency years). Recall that E_{it} is the measure of how far away the actual allocation is from the optimal allocation, such that

$$E_{i,t} = \left(\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}} \right)^{\alpha_{i,t}} \left(\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}} \right)^{1-\alpha_{i,t}} .$$

If $E_{i,t} > 1$, the actual amount of capital and labor allocated to this sector is above the optimal level. Conversely, if $E_{i,t} < 1$, the actual allocation is below the optimal level. Further, if $\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}} > 1 (< 1)$, sector i is over-(under-)capitalized. Similarly, $\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}} > 1 (< 1)$ means that sector i has abundant (not enough) labor.

Figure A.1: Sector-level allocative efficiency $E_{i,t}$, 1960–65 to 2000–05
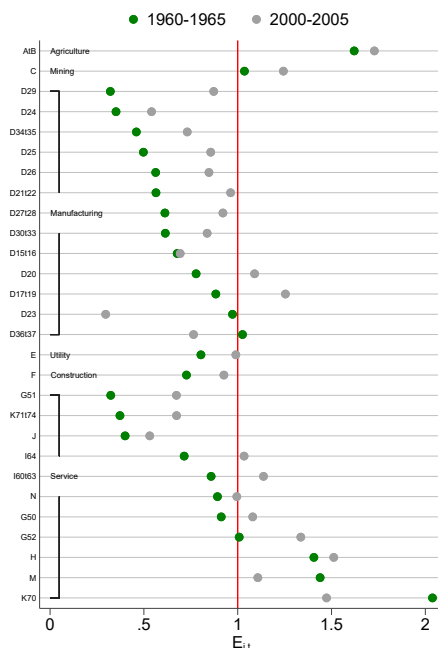
Notes: This figure plots sector-level allocative efficiency measures $E_{i,t}$, averaged over 1960–1965 (green) and 2000–2005 (grey).

Figure A.1 shows that $E_{i,t}$ of manufacturing sectors were almost all below 1 (except for sector D36t37) during 1960–65, indicating that manufacturing sectors needed more capital and labor. In comparison, among service sectors, some had too much capital and labor ($E_{i,t} > 1$) and others did not have enough ($E_{i,t} < 1$). From 1960–65 to 2000–05, $E_{i,t}$ has moved closer to 1 for most sectors in the economy. This means that manufacturing sectors have received more capital and labor ($E_{i,t}$ increased). For the service sectors, this means

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some sectors received more capital and labor while others gave away capital and labor.

Figure A.2: Sector-level capital and labor allocation, 1960–65 to 2000–05

(a) Capital **(b)** Labor
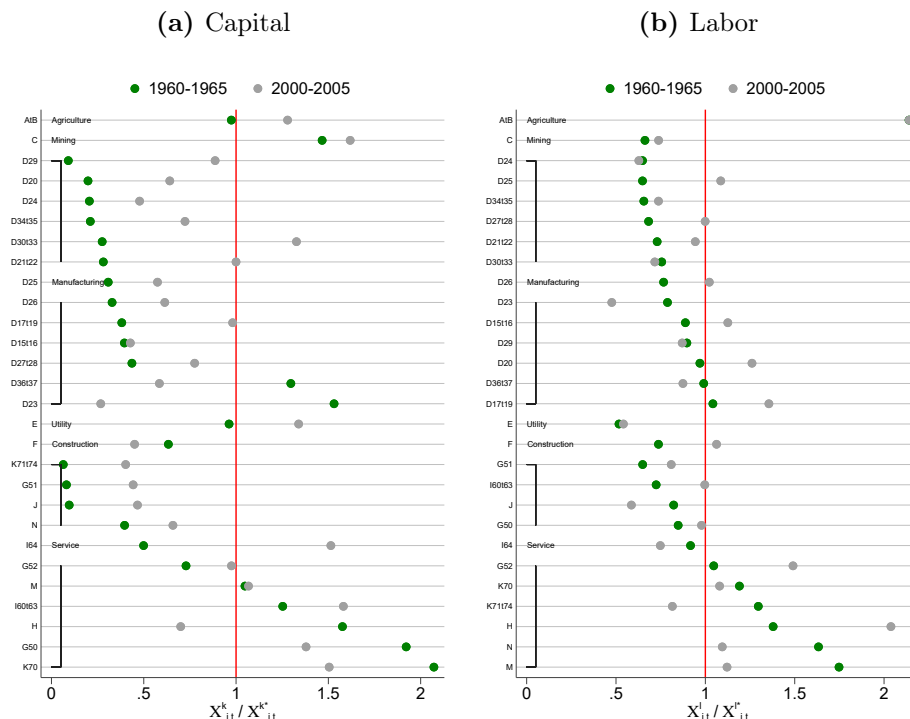
Notes: This figure plots sector-level measures of the capital ($\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}}$, Panel a) and labor ($\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}}$, Panel b) allocative efficiency, averaged over 1960–1965 (green) and 2000–2005 (grey).

Figure A.2 displays the allocation of capital ($\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}}$) and labor ($\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}}$) separately for each sector. Not surprisingly, the manufacturing sector's capital was significantly below the optimal level in the early years. Labor was also scarce in manufacturing sectors, but labor allocation was closer to the optimal level than capital.

Taken together, Figures A.1 and A.2 indicate that manufacturing sectors, particularly their capital allocation, were far below optimal in the early years but improved over time, which contributed to a long-term improvement in allocation efficiency.

A.3 Alternative measure of sector-level volatility

In our paper, we follow Bloom et al. (2018) to construct a measure of sector-level volatility ("microeconomic uncertainty"). To do so, we first run a regression for sector-level log TFP: $\log A_{i,t} = \rho \log A_{i,t-1} + \mu_t + \chi_i + \varepsilon_{i,t}$. Then, the variance of $\varepsilon_{i,t}$, the innovation of the productivity process, is a measure of volatility faced by the sectors.

3

There are two ways to measure the variance of $\varepsilon_{i,t}$. First, it can be calculated as the cross-sector dispersion in ε in any given year. Second, it can be calculated as the dispersion in ε over time for any given sector. In the main text, we use the first approach (see Panel (b) of Figure 5), and, as a robustness check, we apply the second approach in Figure A.3.

Figure A.3: Dispersion of TFP shock in a rolling 7-year window
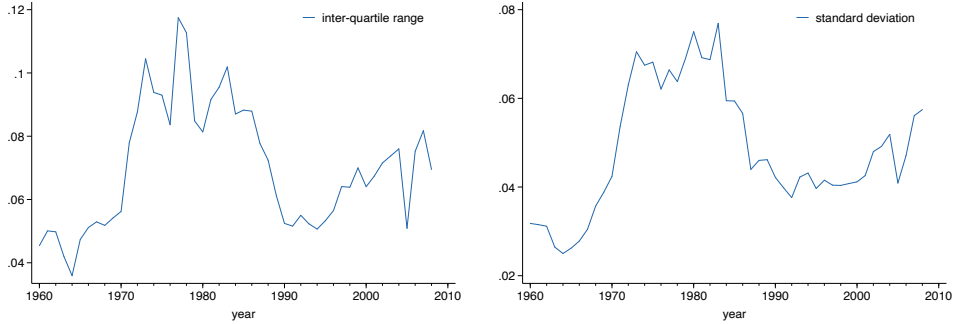
Notes: This figure plots the dispersion of the sector-level TFP shock, calculated as the variance of $\varepsilon_{i,t}$, which is the residual term from regression $\log A_{i,t} = \rho \log A_{i,t-1} + \mu_t + \chi_i + \epsilon_{i,t}$. To plot this figure, we first compute, for each sector, the inter-quartile range or the standard deviation of the $\varepsilon_{i,t}$ within each 7-year rolling window centered around the current year. We then compute the median value of the inter-quartile range (Panel a) or the standard deviation (Panel b) across sectors and plot the time series of the median value.

More formally, in Figure A.3, we first compute, for each sector, the standard deviation or inter-quartile range of the $\varepsilon_{i,t}$ within each 7-year rolling window centered around the current year. We then compute the median value of the inter-quartile range (Panel a) or the standard deviation (Panel b) across sectors and plot the time series of the median value in Figure A.3. The figures exhibit very similar dynamics as Figure 5, Panel (b) in the main text, particularly regarding the rise in volatility during the 1970s and 2000s.

A.4 CES production system

Figure A.4 displays the evolution of $\mathbf{E_t}$ under different values of elasticity for ρ, ϵ. The blue line shows the result under our baseline parametrization where $\rho = 0.96, \epsilon = 0.81$. Two patterns emerge from this figure. First of all, in terms of the level of measured allocative efficiency, this value is in general lower with a higher elasticity of substitution. Among the six combinations of the parameter values, the lowest allocative efficiency occurs with $\rho = 0.96, \epsilon = 1.25$. This pattern is consistent with findings in Epifani and Gancia (2011) and Osotimehin and Popov (Forthcoming). Further, the percent changes in allocative efficiency are also larger for the high-elasticity cases. Secondly, for all six cases, there exists a significant deterioration in allocation during the 1970s and either stagnation or deterioration in allocation during the 2000s.

In Table A.2, we present the contribution of allocative efficiency to productivity growth,

defined by $\frac{\Delta \log \mathbf{E_t}}{\Delta \log \mathrm{LP}_t}$, by decades for the different combinations of (ρ, ϵ). Again, we observe a similar pattern as in Table 1. During the 1970s and 2000s, the contribution of the improvement in allocative efficiency $(\Delta \log \mathbf{E_t})$ to productivity growth $\Delta \log \mathrm{LP}_t$ is considerably smaller than the other decades. In fact, in most specifications, the contribution was negative during these two decades. In Table 6 in the main text, we formally evaluate the slowdown in productivity under these different parameterizations.

Figure A.4: Evolution of $\mathbf{E}_t$ over time under different values of elasticity
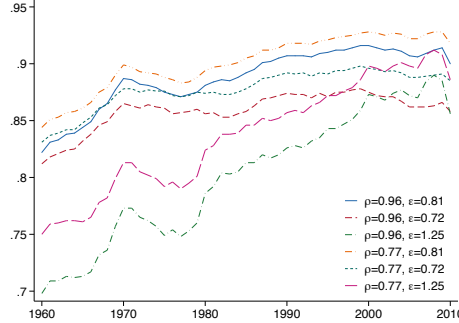
Notes: This figure shows the evolution of allocative efficiency under different values of elasticity of substitution. The baseline parametrization is $\rho = 0.96, \epsilon = 0.81$, which we estimated directly from the data. We also consider a lower value of $\rho = 0.77$ (Aum et al., 2018). For the value of ϵ, we consider two alternative estimates: 1.25 (Karabarbounis and Neiman, 2014) and 0.72 (Oberfield and Raval, 2021).

Table A.2: Contribution of allocative efficiency to productivity growth $\left(\frac{\Delta \log \mathbf{E_t}}{\Delta \log \mathrm{LP}_t} \right)$

	CD	CES					
			$\rho = 0.96$			$\rho = 0.77$	
	$\epsilon = 0.81$	$\epsilon = 0.72$	$\epsilon = 1.25$	$\epsilon = 0.81$	$\epsilon = 0.72$	$\epsilon = 1.25$
	baseline			highest elas.		lowest elas.	
1960–69	0.32	0.27	0.23	0.34	0.22	0.20	0.26
1970–79	-0.05	-0.11	-0.05	-0.13	-0.10	-0.03	-0.12
1980–89	0.30	0.18	0.13	0.29	0.15	0.12	0.23
1990–99	0.13	0.05	0.02	0.20	0.05	0.03	0.17
2000–07	0.03	-0.05	-0.09	0.08	-0.01	-0.05	0.07

Notes: This table shows the contribution of allocative efficiency to productivity growth $\frac{\Delta \log \mathbf{E_t}}{\Delta \log \mathrm{LP}_t}$ in the CES framework. $\Delta \log \mathbf{E_t}$ and $\Delta \log \mathrm{LP}$ are based on the long differences in allocative efficiency and labor productivity between the end and the beginning of each period. They are adjusted to reflect the growth rate over periods of the same length (ten-year window). The results are presented under six combinations of parameter values for ρ and ϵ, the elasticity of substitution parameters in the production system. The baseline parameterization is $\rho = 0.96$ and $\epsilon = 0.81$. As a comparison, we also include the results from the Cobb-Douglas case, taken from the last column of Table 1.

A.5 Timing of the slowdown

In the main text, we examine the data in decades, focusing on the 1970s and post-2000s as periods of slowdown. In this section, we will examine the start and end dates of the slowdown episodes more closely.

To begin with, Panel (a) in Figure A.5 shows the log of labor productivity from 1960 to 2019, divided into five periods: 1960–1973, 1973–1982, 1982–1995, 1995–2005, and post-2005. These periods match the identified slowdown episodes in Byrne et al. (2016) and are consistent with changes in growth trends in our labor productivity time series. The two periods of 1973–1982 and post-2005 are notable for their slower-than-normal labor productivity growth rates. Panel (b) shows the time series of allocative efficiency $\mathbf{E}_t$ divided into the same subperiods as in Panel (a). There, 1973–1982 and post-2005 also experienced slower-than-normal growth in allocative efficiency $\mathbf{E}_t$.

Table A.3 evaluates the contribution of allocative efficiency to productivity slowdowns. Columns (1)–(3) display the average annual growth rates for each subperiod in LP (data), LP* ("fundamental"), and $\mathbf{E}_t$, calculated as the slope of the fitted linear functions shown in the previous figure. Columns (4)–(6) show the *changes* in annual growth rates in percentage points from the preceding periods. During the periods 1973–1982 and 2005–2010, the annual growth rate in labor productivity was lower than in preceding periods by 1.88 and 1.08 percentage points, respectively. Moreover, the growth rate in allocative efficiency was also lower during these periods, by 0.39 and 0.26 percentage points, respectively, compared to preceding periods. As a result, slower growth in allocative efficiency contributed 21% (0.39/1.88) and 24% (0.26/1.08) to the slowdown in labor productivity growth during 1973–1982 and 2005–2010, respectively.

Figure A.5: Productivity slowdown and allocative efficiency, alternative timing
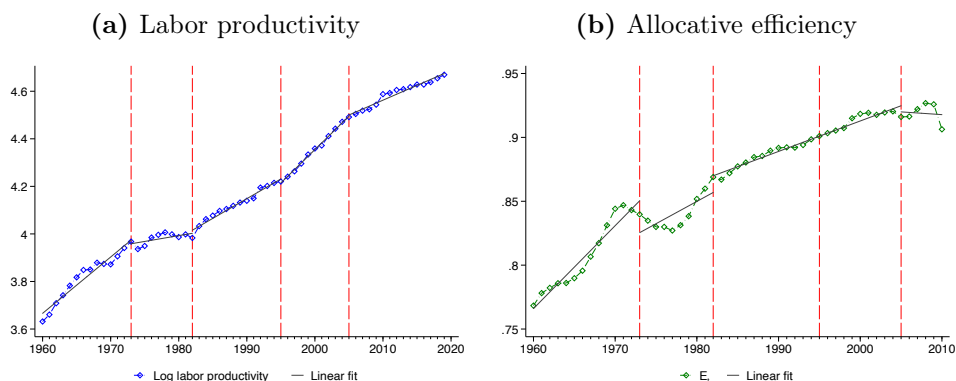
(a) Labor productivity   **(b)** Allocative efficiency

Notes: This figure shows log labor productivity (Panel a) and allocative efficiency measures (Panel b) over time. The time series in both panels are divided by red vertical lines at the years 1973, 1982, 1995, and 2005. The black lines are linear fits of the time series within each of these subperiods.

**Table A.3:** Productivity slowdown and the role of allocative efficiency, alternative timing

Period	Average annual growth rates (percent)			Changes in growth rates from previous period (pp)		
	(1)	(2)	(3)	(4)	(5)	(6)
	labor productivity			labor productivity		
	data	"fundamental"	$\mathbf{E}_t$	data	"fundamental"	$\mathbf{E}_t$
1960–73	2.37	1.57	0.80	–	–	–
1973–82	0.49	0.08	0.41	**-1.88**	**-1.49**	**-0.39**
1982–95	1.66	1.37	0.29	1.17	1.29	-0.13
1995–05	2.81	2.60	0.21	1.15	1.22	-0.08
2005–10	1.73	1.77	-0.05	**-1.08**	**-0.82**	**-0.26**

Notes: Columns (1)–(3) display the annual growth rate (percent) in LP_t (data), LP_t^* ("fundamental"), and $\mathbf{E}_t$, calculated by regressing $\log LP_t$, $\log LP_t^*$ and $\log \mathbf{E}_t$ on time. Columns (4)–(6) show changes in these growth rates (in percentage points) from the previous periods. The periods highlighted in bold indicate a slowdown in observed labor productivity compared to the previous period.

A.6   Manufacturing sector productivity slowdown

In Figure A.6, Panel (a) shows the evolution of log labor productivity divided into four subperiods: 1960–73, 1973–82, 1982–2005, and post-2005. As there is no significant trend break during 1982–2005, we consider this period of rapid productivity growth as a whole. Moreover, Panel (b) shows the dynamics of $\mathbf{E}_t$, which appears to be noisier than the baseline results, perhaps because the NBER-CES dataset is constructed using firm surveys and has a much finer industry classification. Nevertheless, despite the noise, the figure indicates that 1973–82 and post-2005 have significantly slower growth rates in labor productivity as well as allocative efficiency.

Figure A.6: Manufacturing labor productivity and allocative efficiency over time
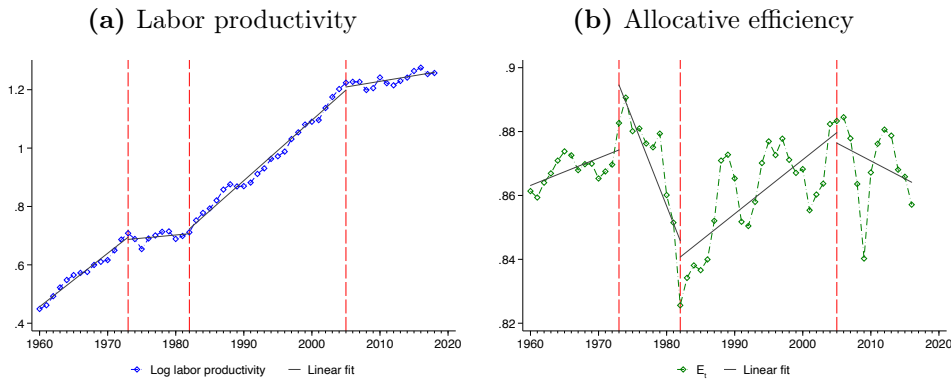
Notes: This figure shows log labor productivity (Panel a) and allocative efficiency measures (Panel b) over time. The time series in both panels are divided by red vertical lines for the years 1973, 1982, and 2005. The black lines are linear fits of the time series within each of these subperiods.

Table A.4 examines the role of allocative efficiency more formally. Columns (1)–(3) show that the average growth rate in observed labor productivity was only 0.22% and 0.39% during

the periods 1973–82 and 2005–16, respectively. At the same time, the annual growth rates in allocative efficiency in these two periods were negative. As shown in Columns (4)–(6), compared with the preceding periods, labor productivity growth rates during 1973–82 and 2005–16 are lower by 1.61 and 1.68 percentage points, respectively, while allocative efficiency growth rates are 0.73 and 0.33 percentage points lower. The lack of improvement in allocative efficiency contributes to 45% (0.73/1.61) and 20% (0.33/1.68) of the slowdown in observed labor productivity growth in these periods.

Table A.4: Manufacturing-sector productivity slowdown: The role of allocative efficiency

	Average annual growth rates (percent)				Changes in growth rates from previous period (pp)		
	(1)	(2)	(3)		(4)	(5)	(6)
	labor productivity				labor productivity		
Period	data	"fundamental"	$\mathbf{E}_t$		data	"fundamental"	$\mathbf{E}_t$
1960–1973	1.83	1.73	0.10		–	–	–
1973–1982	0.22	0.85	-0.63		**-1.61**	**-0.89**	**-0.73**
1982–2005	2.07	1.87	0.20		1.85	1.02	0.83
2005–2016	0.39	0.51	-0.13		**-1.68**	**-1.36**	**-0.33**

Notes: Columns (1)–(3) display the annual growth rate (percent) in LP_t (data), LP_t^* ("fundamental"), and $\mathbf{E}_t$, calculated by regressing $\log LP_t$, $\log LP_t^*$ and $\log \mathbf{E}_t$ on time. Columns (4)–(6) show changes in these growth rates (in percentage points) from the previous periods. The periods highlighted in bold indicate a slowdown in observed labor productivity compared to the previous period.

A.7 Relationship between volatility and allocative efficiency: Evidence from manufacturing industries

In this section, we will study the volatility of industry-level shocks and investigate its relationship with the efficiency of resource allocation using the NBER-CES manufacturing industry database.

We construct measures of industry-level volatility using the NBER-CES manufacturing database and follow the methodology in Section 4.3. Panel (a) of Figure A.7 plots the cross-sectional standard deviation of industry-level TFP, employment, and value-added output growth rates. Panel (b) shows the variance of industry-level TFP shocks unforecasted by an AR(1) model. This figure is analogous to Figure 5 in the main text. Similar to Figure 5, we observe elevated volatility during the 1970s and post-2000.

Table A.5 examines the relationship between the industry-level volatility measures cross-industry allocative efficiency $\mathbf{E}_t$ in the manufacturing sector. More formally, we regress changes in allocative efficiency ($\Delta \log \mathbf{E}_t$) measures on measures of industry-level volatility, measured as the cross-sectional dispersion (IQR) of $\varepsilon_{i,t}$.

The estimated relationship between volatility and changes in allocative efficiency is negative and statistically significant at the 5 percent level when controlling for the volatility of

Figure A.7: Volatility of sector-level shocks, manufacturing only

(a) S.D. in sectoral growth rates

(b) Dispersion in TFP shocks
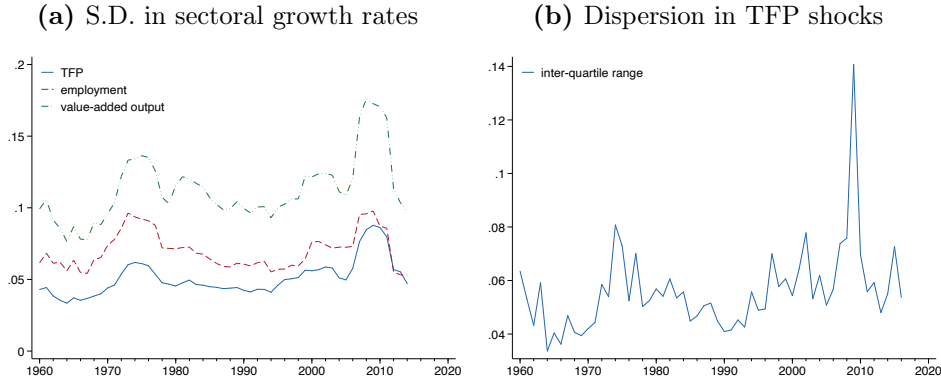
Notes: These figures are based on the NBER-CES manufacturing database. Panel (a) plots the cross-sectional s.d. of sectoral growth rates in employment, real value-added output and TFP. Panel (b) plots the cross-sectional dispersion (inter-quartile range) in TFP shocks, computed as the residual terms from regression $\log A_{i,t} = \rho \log A_{i,t-1} + \mu_t + \chi_i + \epsilon_{i,t}$.

Table A.5: Relationship between allocative efficiency and volatility measures, manufacturing sector only

	(1)	(2)	(3)
year t	-0.190	-0.347**	-0.351**
	(0.161)	(0.156)	(0.158)
year $t-1$		0.305*	0.287
		(0.179)	(0.195)
year $t-2$			
N	58	57	56
R^2	0.046	0.142	0.142

Notes: This table reports results from regressing log difference in $\mathbf{E_t}$ on the volatility of the industry-level shock of $t, t-1, t-2$. Industry-level volatility is measured as the cross-sectional dispersion (IQR) of $\varepsilon_{i,t}$.

$t-1$ and $t-2$. With industry-level volatility, the estimated correlation is around 0.2–0.35. Compared with the estimates of Columns (4)–(6) in Table 3 using the KLEMS data, the estimates found in this table are twice or three times as large.

Overall, the manufacturing sector experienced elevated levels of volatility during the 1970s and again since 2000. Additionally, industry-level volatility is associated with declining allocation efficiency. Using this more granular dataset focusing on the manufacturing sector, the results confirm and expand our baseline findings using the KLEMS dataset.

A.8 Post-2010 dynamics

Our baseline analysis is based on the 2013 version of the KLEMS and WIOT datasets, which only include data up to 2010. However, as shown in Figure A.8, the slowdown in productivity growth extends beyond this point.

Figure A.8: Productivity slowdown and allocative efficiency, post-2010 dynamics
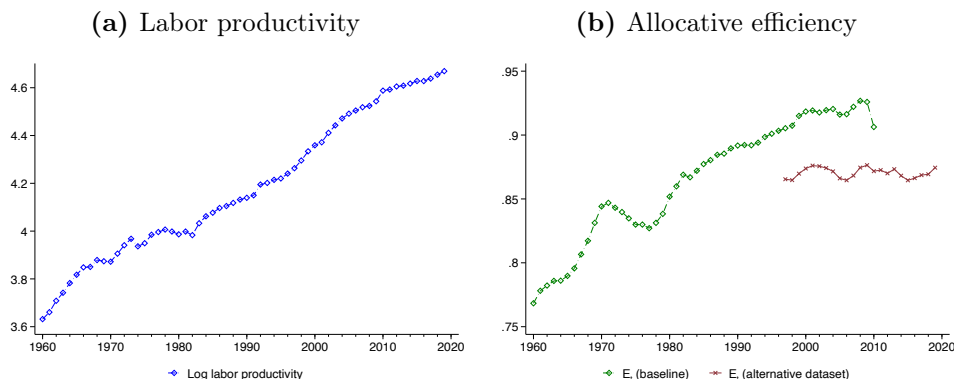
(a) Labor productivity   **(b)** Allocative efficiency

Notes: This figure shows log labor productivity (Panel a) and allocative efficiency measures (Panel b) over time. The green line in Panel (b) is the same as the baseline result while the red line is based on the KLEMS 2021 data.

To study the post-2010 dynamics, we use a different version of the KLEMS database published by the Luiss Lab of European Economics (LLEE) in 2021. The LLEE KLEMS dataset is based on a different industry classification, covering a more recent period extending to 2015. However, it has some limitations. For example, several key variables needed to measure allocative efficiency only became available after 1997. In addition, several sectors are missing capital stock information altogether. Keeping these caveats in mind, we use this dataset as a robustness check for the post-2010 dynamics.

Recall that our baseline result finds that $\mathbf{E}_t$, shown as the green line in Panel (b) of Figure A.8, starts to flatten around 2000 after experiencing decades of positive growth. As demonstrated by the red line calculated using the new dataset, this flattening trend continues into the post-2010 period.

In Table A.6, we extend the baseline results to 2015. The pre-2010 results are based on the KLEMS 2013 data and are identical to Panel (a) of Table 2 in the main text. The post-2010 results are based on the LLEE KLEMS dataset. This table suggests that taking the 2000–15 period as a whole, it exhibits a very similar pattern to the period of 2000–07. Notably, growth rates in allocative efficiency continue to slow, and they can now account for the entirety of the productivity slowdown during this period. Lastly, note that, considering the short length of the time series and the issues of the data sets as discussed, this should be viewed only as suggestive evidence.

Table A.6: Slowdown in productivity growth and the role of allocative efficiency, post-2010 experience

	Growth rates by periods (long log-difference)			Changes in growth rates from preceding period		
	(1)	(2)	(3)	(4)	(5)	(6)
	labor productivity			labor productivity		
Period	data	"fundamental"	$\mathbf{E}_t$	data	"fundamental"	$\mathbf{E}_t$
1960–69	0.24	0.16	0.08	–	–	–
1970–79	0.13	0.13	-0.01	**-0.12**	**-0.03**	**-0.08**
1980–89	0.15	0.10	0.04	0.02	-0.03	0.05
1990–99	0.19	0.16	0.03	0.05	0.06	-0.02
2000–15	0.16	0.17	-0.01	**-0.03**	**0.01**	**-0.03**

Notes: Columns (1)–(3) present the growth rates in LP_t (labor productivity, data), LP_t^* (labor productivity, fundamental), and $\mathbf{E}_t$. These growth rates are based on the long differences in labor productivity between the end and the beginning of each period. They are adjusted to reflect the growth rate over periods of the same length (ten-year window). Columns (4)–(6) present the changes in these growth rates from the preceding periods.

A.9   Alternative measure for capital and labor

Before discussing the results, we first list the asset types provided in the 2009 version of KLEMS. More details can be found in Jorgenson et al. (2014). In this exercise, we consider the most detailed asset classification (eight types) and a broader classification (two types, ICT versus Non-ICT).

- ICT assets

  – Computing equipment

  – Communications equipment

  – Software

- Non-ICT assets

  – Transport equipment

  – Other machinery and equipment

  – Total non-residential investment

  – Residential structures

  – Other assets

We first extend our theoretical framework to include more than one asset type. More

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formally, the planner's optimization problem can be written as

$$\max\ Y_t = \prod_{i=1}^{N} Y_{i,t}^{\theta_{i,t}}, s.t.\ Y_{i,t} = A_{i,t} \prod_{s=1}^{S} K_{i,t}^{s\alpha_{i,t}^s} L_{i,t}^{1-\sum_s \alpha_{i,t}^s},\ \forall s,\ \sum_i K_{i,t}^s = K_t^s,\ \sum_i L_{i,t} = L_t,$$

where $s \in \{1,...,S\}$ represents the different asset types. The optimal allocation of capital and labor is such that $K_{i,t}^{s*} = \chi_{i,t}^{ks*} K_t^s$ and $L_{i,t}^* = \chi_{i,t}^{l*} L_t$, where $\chi_{i,t}^{ks*} = \frac{\theta_{i,t}\alpha_{i,t}^s}{\sum_i \theta_{i,t}\alpha_{i,t}^s}$ and $\chi_{i,t}^{l*} = \frac{\theta_{i,t}(1-\sum_s \alpha_{i,t}^s)}{\sum_i \theta_{i,t}(1-\sum_s \alpha_{i,t}^s)}$. Lastly, the sufficient statistic for allocative efficiency can be written as $\mathbf{E}_t = \prod_{i=1}^{N}\{\prod_s[(\frac{\chi_{i,t}^{ks}}{\chi_{i,t}^{ks*}})^{\alpha_{i,t}^s}](\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}})^{1-\sum_s \alpha_{i,t}^s}\}^{\theta_{i,t}}$.

Figure A.9: Evolution of $\mathbf{E}_t$ over time, alternative measures of inputs
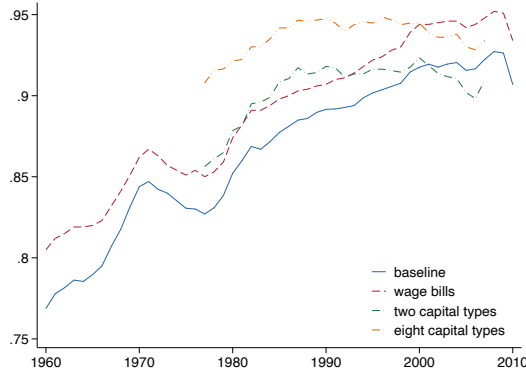
Notes: This figure shows the evolution of allocative efficiency under different measures of capital and labor inputs. The baseline result is the measure with one type of capital and total employment. The other three alternative measures are 1) the use of wage bills (labor compensation) instead to measure labor inputs, 2) the consideration of two types of capital (ICT and non-ICT), and 3) the consideration of eight different types of assets.

Figure A.9 displays the evolution of allocative efficiency under the alternative measures of the capital and labor inputs. The blue line is the benchmark result where we consider only one type of capital and measure labor inputs using employment. Replacing employment with wage bills as measures for labor input, allocative efficiency is slightly higher, but the trend remains very similar to the benchmark result. When considering two types of assets—ICT and non-ICT—the time series only start from the end of the 1970s. The allocative efficiency increases rapidly during the 1980s, stays relatively stable during the 1990s, and starts to decline in the beginning of the 2000s. A similar trend is found when considering eight asset types instead of two, although the level of allocative efficiency is now slightly higher than the two-asset case.

Table A.7 presents the changes in growth rates of observed productivity, fundamental productivity, and allocative efficiency compared to previous decades. The results using wage bills indicate that approximately half of the productivity slowdown in the 1970s and the

Table A.7: Productivity slowdown and the role of allocative efficiency, alternative measures of inputs

Changes in growth rates compared to the preceding periods (pp)

Period	data	two K types fundamental	$\mathbf{E}_t$	eight K types fundamental	$\mathbf{E}_t$	wage bills fundamental	$\mathbf{E}_t$
1960–69	–	–	–	–	–	–	–
1970–79	**-0.12**	–	–	–	–	**-0.06**	**-0.06**
1980–89	0.02	–	–	–	–	-0.02	0.04
1990–99	0.05	0.08	-0.03	0.09	-0.04	0.05	0.00
2000–07	**-0.03**	**-0.01**	**-0.02**	**-0.02**	**-0.01**	**0.00**	**-0.03**

Notes: This table presents the productivity growth rates under the optimal allocation ($\Delta \log \mathrm{LP}^*$) and the changes in these growth rates from the previous decade ("change"). The baseline result is the measure with one type of capital and where labor inputs are measured using employment. The other three alternative measures are 1) the use of wage bills (labor compensation) instead to measure labor inputs, 2) the consideration of two types of capital separately (ICT and non-ICT), and 3) the consideration of eight different types of capital.

entire observed slowdown in the 2000s can be attributed to allocative efficiency. Since the capital by type data is only available after 1977, we can only analyze the later slowdown episode. In these instances, allocative efficiency accounts for two-thirds of the productivity slowdown in the two-capital specification and one-third in the eight-capital specification.

A.10 Non-zero profits

For the profit versus capital income split, we follow Barkai (2020), in which the author estimated the profits and capital income between 1984 and 2014. The results showed that profits as a share of value-added started to increase at the beginning of the 2000s. Over Barkai's whole sample period, the ratio of profit to capital income reached the highest level of 1/2 in 2007. In 2010, the last year of our sample, the ratio of profit to capital income was approximately 1/5. We repeat our exercises with these two alternative values. More formally, we reestimate the output elasticity in the production functions after taking out profits from capital returns. As a result, this adjustment lowers the output elasticity for capital and increases that for labor.

Figure A.10 shows the changes in measured allocative efficiency under these two alternative specifications. Compared to the baseline results where we assume zero profits, the magnitude of the changes in allocation is generally smaller. However, for the periods of interest, there still exists an apparent stagnation or deterioration in allocation.

Table A.8 presents a more formal evaluation of the role of allocation under the assumption of positive profits. When considering a profit to capital income ratio of 1/5, allocative

efficiency accounts for 7/12 of the observed productivity slowdown in the 1970s and 2/3 of the slowdown in the 2000s. With a profit to capital income ratio of 1/2, these ratios slightly decrease to 1/2 and 2/3, respectively. Importantly, our main conclusion remains unchanged: we find that at least half of the productivity slowdown can be attributed to allocative efficiency.

Figure A.10: Evolution of $\mathbf{E}_t$ over time, non-zero profits
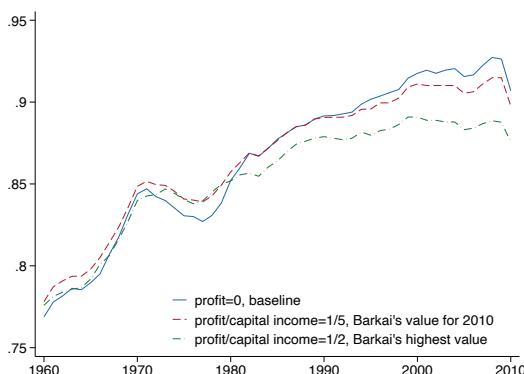
Notes: This figure shows the evolution of allocative efficiency under different profit to capital income ratios. The baseline result is the one with zero profits. The two other values are taken from Barkai (2020): (i) 1/5 for 2010 and (ii) 1/2, the highest point for the period 1984–2014.

Table A.8: Productivity slowdown and the role of allocative efficiency, non-zero profits

Changes in growth rates compared to the preceding periods (pp)

| Period	data	$\frac{\text{profit}}{\text{capital income}} = \frac{1}{5}$		$\frac{\text{profit}}{\text{capital income}} = \frac{1}{2}$	
		fundamental	$\mathbf{E}_t$	fundamental	$\mathbf{E}_t$
1960–69	–	–	–	–	–
1970–79	**-0.12**	**-0.05**	**-0.07**	**-0.06**	**-0.06**
1980–89	0.02	-0.02	0.04	0.00	0.02
1990–99	0.05	0.06	-0.01	0.06	-0.01
2000–07	**-0.03**	**-0.01**	**-0.02**	**-0.01**	**-0.02**

Notes: This table presents the productivity growth rates under the optimal allocation ($\Delta \log \text{LP}^*$) and the changes in these growth rates from the previous decade ("change"). In the figure we plot three specifications with different profit to capital income ratios. The baseline result is the one with zero profits. The two other values are taken from Barkai (2020): (i) 1/5 for 2010, and (ii) 1/2, the highest value during his sample period.

A.11   Length of the rolling window

Our baseline specification relies on the assumption that, although the expenditure shares of the inputs might be distorted each year, they are undistorted on average over time. Consequently, the output elasticities in the production system are equal to the average input expenditure shares. In practice, we consider a rolling window of 3 years centered in the current period—$[t-1, t, t+1]$ for the period t—and assign the average expenditure shares within this rolling window to the output elasticity in year t.

To evaluate the impact of the rolling window length on our analysis, we examine allocative efficiency using rolling windows of 3, 5, 7, and 9 years, as shown in Figure A.11. The results indicate that while a longer rolling window leads to a slightly smaller decline in allocative efficiency, the overall dynamics of measured allocative efficiency remain almost identical across these different specifications.

Table A.9 provides additional support for our findings, showcasing the changes in growth rates of observed labor productivity, fundamental labor productivity, and allocative efficiency compared to previous periods. Importantly, our baseline results remain robust when varying the length of the rolling window. For instance, even with a 9-year rolling window, allocative efficiency explains 7/12 of the observed productivity slowdown in the 1970s and 4/3 of the slowdown in the 2000s.

Figure A.11: Evolution of $\mathbf{E}_t$ over time, alternative rolling windows
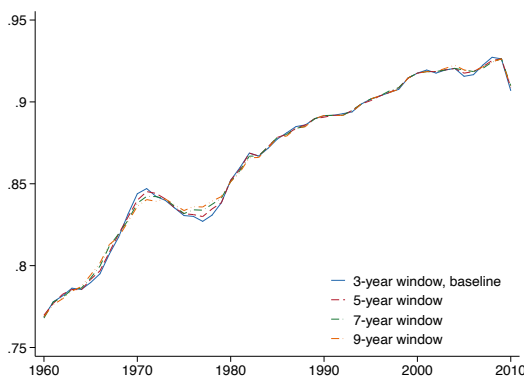
Notes: This figure shows the evolution of allocative efficiency. The baseline result is the line with a three-year rolling window.

A.12   Biased wedges

In the first specification, we follow Oberfield (2013) and assume that, on average, the wedges are not biased towards one factor. In this section, we will test the sensitivity of this assumption. More specifically, we ask: what if, on average, the capital wedge is much higher than

Table A.9: Productivity slowdown and the role of allocative efficiency, rolling windows of different lengths

Changes in growth rates compared to the preceding periods (pp)

Period	data	5-year window fundamental	$\mathbf{E}_t$	7-year window fundamental	$\mathbf{E}_t$	9-year window fundamental	$\mathbf{E}_t$
1960–69	–	–	–	–	–	–	–
1970–79	**-0.12**	**-0.04**	**-0.08**	**-0.04**	**-0.08**	**-0.05**	**-0.07**
1980–89	0.02	-0.03	0.05	-0.02	0.04	-0.02	0.04
1990–99	0.05	0.06	-0.01	0.06	-0.01	0.06	-0.01
2000–07	**-0.03**	**-0.01**	**-0.02**	**-0.01**	**-0.02**	**0.01**	**-0.04**

Notes: This table presents the productivity growth rates under the optimal allocation ($\Delta \log \mathrm{LP}^*$) and the changes in these growth rates from the previous decade ("change") under different lengths of the rolling windows

the labor wedge, or vice versa? We show two cases in which either capital or labor faces larger wedges on average than the other.

In Figure A.12, we show the evolution of $\mathbf{E}_t$ over time under two alternative assumptions. The blue line represents the case in which the capital wedge is higher, $\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 0.5$, for all sectors, the green line represents the other case, in which the labor wedge is higher, $\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 2.0$, and the red line shows the baseline result as a comparison. The figures demonstrate that biased wedges impact the level of allocative efficiency ($\mathbf{E}_t$). A larger capital wedge is associated with a higher $\mathbf{E}_t$, while a larger labor wedge is associated with a lower $\mathbf{E}_t$. However, the trends of $\mathbf{E}_t$ remain similar across all specifications.

Figure A.12: Evolution of $\mathbf{E}_t$ over time, biased wedges
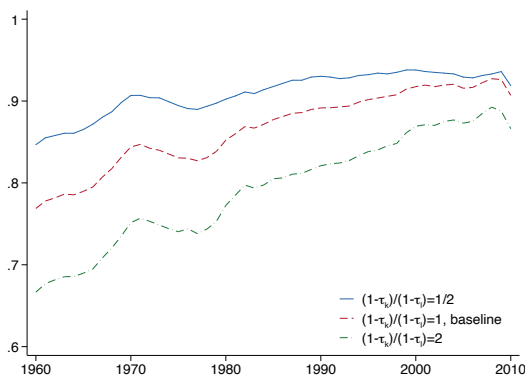
Notes: This figure shows the evolution of allocative efficiency where the wedges are biased on average towards one factor.

The data presented in Table A.10 suggest that when the capital wedge is larger ($\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 0.5$), approximately half of the productivity slowdown witnessed in the 1970s is attributable

to allocative efficiency. This ratio decreases to about one-third for the slowdown experienced in the 2000s. In contrast, when the labor wedge is larger ($\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 2.0$), allocative efficiency accounts for a greater proportion of the productivity slowdown, explaining roughly 5/6 and 2/3 of the observed slowdown during the 1970s and 2000s, respectively. The take-away message from this exercise is that, even if the wedges are significantly biased towards one input (capital or labor), allocative efficiency still accounts for at least 1/3 of the observed productivity slowdown.

Table A.10: Productivity slowdown and the role of allocative efficiency, biased wedges

Changes in growth rates compared to the preceding periods (pp)

Period	data	$\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 0.5$ fundamental	$\mathbf{E}_t$	$\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 2.0$ fundamental	$\mathbf{E}_t$
1960–69	–	–	–	–	–
1970–79	**-0.12**	**-0.06**	**-0.06**	**-0.02**	**-0.10**
1980–89	0.02	-0.02	0.04	-0.03	0.05
1990–99	0.05	0.07	-0.02	0.06	-0.01
2000–07	**-0.03**	**-0.02**	**-0.01**	**-0.01**	**-0.02**

Notes: This table presents the productivity growth rates under the optimal allocation ($\Delta \log LP^*$) and the changes in these growth rates from the previous decade ("change") where we assume that wedges are biased on average towards one factor.

To clarify, our intention is not to suggest that in reality $\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 1/2$ or $\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 2$. The purpose of this exercise is to estimate the impact of biases in wedges on our results. The two quantitative exercises presented assume significant biases in the wedges. For instance, if we assume $\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 1/2$ with $\bar{\tau}_l = 0.2$, it implies $\bar{\tau}_k = 0.6$. In the US data, the capital and labor income shares are approximately 1/3 and 2/3, respectively. Under the assumption of $\bar{\tau}_l = 0.2$ and $\bar{\tau}_k = 0.6$, the true labor and capital income shares would be 4/5 and 1/5, respectively, which significantly deviates from the observed values. By considering these relatively extreme biases, our goal is to provide an upper bound estimation of the potential impact of these assumptions on our results.

A.13 Using expenditure shares of later years

In this section, we consider one more specification. The assumption behind this specification is that resources are more efficiently allocated in the later years than in the earlier years; therefore, the expenditure shares of the later years are more likely to be undistorted and equal to the real output elasticity in the production functions. Therefore, the output elasticity can be backed out using the expenditure shares of the later years.

We choose 2010 (the last year in our sample) and 2005 (the last year unaffected by

the Great Recession) as the base years for computing the expenditure shares.[37] Figure A.13 demonstrates that the long-run evolution in allocative efficiency and the dynamics during the 2000s are similar under this alternative to the baseline specification. There is also a noticeable deceleration in the improvement of allocative efficiency during the 1970s, although it is less severe than the baseline. Finally, results from Table A.11 show that allocative efficiency can still explain half (6/12) of the observed productivity slowdown during the 1970s and essentially all of the observed slowdown during the 2000s.

Figure A.13: Evolution of $\mathbf{E}_t$ over time, using expenditure shares of later years
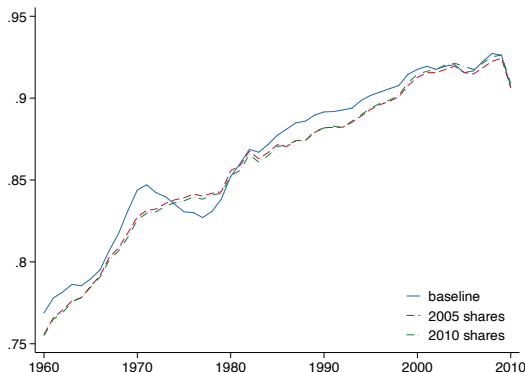
Notes: This figure shows the evolution of allocative efficiency where we assume that the factor expenditure shares are undistorted in later years. We pick two base years for this exercise: (i) 2010, the last year of the sample, and (ii) 2005, to avoid the impact of the Great Recession.

Table A.11: Productivity slowdown and the role of allocative efficiency, using expenditure shares of later years

Changes in growth rates compared to the preceding periods (pp)

| | | 2005 shares | | 2010 shares | |
Period	data	fundamental	$\mathbf{E}_t$	fundamental	$\mathbf{E}_t$
1960–69	–	–	–	–	–
1970–79	**-0.12**	**-0.06**	**-0.06**	**-0.06**	**-0.06**
1980–89	0.02	0.01	0.01	0.01	0.01
1990–99	0.05	0.04	0.01	0.05	0.00
2000–07	**-0.03**	**0.00**	**-0.03**	**0.00**	**-0.03**

Notes: This table presents the productivity growth rates under the optimal allocation ($\Delta \log \mathrm{LP}^*$) and the changes in these growth rates from the previous decade ("change") where we assume that the factor expenditure shares are undistorted in later years. We pick two base years for this exercise: (i) 2010, the last year of the sample, and (ii) 2005, to avoid the impact of the Great Recession.

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[37]2005 is the last unaffected year before the Great Recession (which started in 2007) when estimating output elasticities using a three-year rolling window.

B   Model appendix

B.1   Measuring allocative efficiency in the value-added economy

Solving planner's problem   The solution to the planner's problem requires the equalization of MPK and MPL across sectors, such that,

$$\frac{\partial \log Y}{\partial K_i} = \lambda$$

$$\frac{\partial \log Y}{\partial L_i} = \eta.$$

They can be written as

$$K_i^* = \frac{\theta_i \alpha_i}{\lambda}$$

$$L_i^* = \frac{\theta_i (1 - \alpha_i)}{\eta}.$$

Given the resource constraint, we get

$$K_i^* = \chi_{i,t}^{k*} K$$

$$L_i^* = \chi_{i,t}^{l*} L,$$

where $\chi_{i,t}^{k*} = \frac{\theta_i \alpha_i}{\sum_i \theta_i \alpha_i}$ and $\chi_{i,t}^{l*} = \frac{\theta_i (1 - \alpha_i)}{\sum_i \theta_i (1 - \alpha_i)}$.
Q.E.D.

Deriving allocative efficiency   The final good output under optimal allocation can be written as

$$
\begin{aligned}
Y^* &= \prod_i Y_i^{* \theta_i} \\
&= \prod_i (A_i K_i^{* \alpha_i} L_i^{* 1 - \alpha_i})^{\theta_i} \\
&= \prod_i (A_i (\chi_i^{k*} K)^{\alpha_i} (\chi_i^{l*} L)^{1 - \alpha_i})^{\theta_i}.
\end{aligned}
$$

Similarly, the final output in the data is

$$
\begin{aligned}
Y &= \prod_i Y_i^{\theta_i} \\
&= \prod_i (A_i K_i^{\alpha_i} L_i^{1-\alpha_i})^{\theta_i} \\
&= \prod_i (A_i (\chi_i^k K)^{\alpha_i} (\chi_i^l L)^{1-\alpha_i})^{\theta_i}.
\end{aligned}
$$

As a result,

$$
\mathbf{E}_t = \prod_i \left[\left(\frac{\chi_i^{k*}}{\chi_i^k} \right)^{\alpha_i} \left(\frac{\chi_i^{l*}}{\chi_i^l} \right)^{1-\alpha_i} \right]^{\theta_i}.
$$

B.2 Measuring allocative efficiency in the input-output economy

Solving planner's problem   The planner's problem is

$$
C = \prod_{i=1}^N (Q_i - \sum_{j=1}^N d_{ji})^{\theta_i} - \sum_i \sum_j \bar{P}_j m_{ij}.
$$

The FOCs for K_i, L_i, d_{ij}, m_{ij} are

$$
\begin{aligned}
\frac{\partial C}{\partial K_i} &= \theta_i \frac{Y^*}{Y_i^*} \frac{Q_i^*}{K_i^*} \alpha_i (1 - \sigma_i - \lambda_i) \\
\frac{\partial C}{\partial L_i} &= \theta_i \frac{Y^*}{Y_i^*} \frac{Q_i^*}{K_i^*} (1 - \alpha_i)(1 - \sigma_i - \lambda_i) \\
\frac{\partial C}{\partial d_{ij}} &= \theta_i \frac{Y^*}{Y_i^*} [\frac{Q_i^*}{d_{ij}^*} \sigma_{ij} - I_{\{i=j\}}] + \theta_j \frac{Y^*}{Y_j^*} [\frac{Q_j^*}{d_{jj}^*} \sigma_{jj} I_{\{i=j\}} - 1] \\
\frac{\partial C}{\partial m_{ij}} &= \theta_i \frac{Y^*}{Y_i^*} \frac{Q_i^*}{m_{ij}^*} \lambda_{ij} - \bar{P}_j.
\end{aligned}
$$

The FOC $\frac{\partial C}{\partial d_{ij}} = 0$ implies

$$
d_{ij}^* = \frac{\theta_i Y_j^*}{\theta_j Y_i^*} \sigma_{ij} Q_i^*. \tag{9}
$$

Therefore,

$$Y_j^* = Q_j^* - \sum_{i=1}^{N} d_{ij}^* = Q_j^* - \sum_{i=1}^{N} \frac{\theta_i Y_j^*}{\theta_j Y_i^*} \sigma_{ij} Q_i^*,$$

$$Y_j^* [1 + \frac{1}{\theta_j} \sum_i (\frac{\theta_i Q_i^*}{Y_i^*} \sigma_{ij})] = Q_j^*.$$

Letting $\chi_j^{y*} = \frac{Y_i^*}{Q_i^*}$, $\{\chi_i^{y*}\}_{i=1}^N$ solve the following equations:

$$\frac{1}{\chi_i^{y*}} = 1 + \frac{1}{\theta_i} \sum_s (\frac{\theta_s}{\chi_s^{*y}} \sigma_{si}) \tag{10}$$

or

$$1 - \chi_j^{y*} = \sum_i \sigma_{ij} \frac{\theta_i \chi_j^{y*}}{\theta_j \chi_i^{y*}}.$$

Letting $\gamma_{ij}^* = \frac{\theta_i \chi_j^{y*}}{\theta_j \chi_i^{y*}} \sigma_{ij}$ in Equation (9), then $d_{ij}^* = \gamma_{ij}^* Q_j^*$. The market clearing condition for Q_i^* implies

$$\chi_i^{y*} = 1 - \sum_s \gamma_{si}^*.$$

FOC $\frac{\partial C}{\partial m_{ij}} = 0$ implies

$$m_{ij}^* = \theta_i \frac{Y^*}{Y_i^*} Q_i^* \frac{\lambda_{ij}}{\bar{P}_j} \tag{11}$$

Since

$$Y^* = \prod_i Y_i^{*\theta_i} = \prod_i (\chi_i^* Q_i^*)^{\theta_i},$$

we have

$$m_{ij}^* = \theta_i \prod_s (\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s} \prod_s (Q_s^*)^{\theta_s} \frac{\lambda_{ij}}{\bar{P}_j}. \tag{12}$$

The FOC $\frac{\partial C}{\partial K_i} = 0$ and $\frac{\partial C}{\partial L_i} = 0$ lead to

$$K_i^* = \chi_i^{k*} K \tag{13}$$

$$L_i^* = \chi_i^{l*} L, \tag{14}$$

where

$$\chi_i^{k*} = \frac{\frac{\theta_i \alpha_i (1 - \sigma_i - \lambda_i)}{(1 - \sum_j \gamma_{ji}^*)}}{\sum_s \frac{\theta_s \alpha_s (1 - \sigma_s - \lambda_s)}{(1 - \sum_j \gamma_{js}^*)}}, \chi_i^{l*} = \frac{\frac{\theta_i (1 - \alpha_i)(1 - \sigma_i - \lambda_i)}{(1 - \sum_j \gamma_{ji}^*)}}{\sum_s \frac{\theta_s (1 - \alpha_s)(1 - \sigma_s - \lambda_s)}{(1 - \sum_j \gamma_{js}^*)}}. \tag{15}$$

To fully characterize d_{ij} and m_{ij}, we need to solve for Q_i. Replacing d_{ij} and m_{ij} in the

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production function using $d_{ij}^* = \gamma_{ij}^* Q_j^*$ and (12), we get

$$Q_i^* = A_i(K_i^{*\alpha_i}L_i^{*1-\alpha_i})^{1-\sigma_i-\lambda_i}(\gamma_{i1}^*Q_1^*)^{\sigma_{i1}}\cdots(\gamma_{iN}^*Q_N^*)^{\sigma_{iN}}\prod_{j=1}^{N}\{\theta_i\prod_s(\frac{\chi_s}{\chi_i})^{\theta_s}\prod_s(Q_s^*)^{\theta_s}\frac{\lambda_{ij}}{\bar{P}_j}\}^{\lambda_{ij}}$$

$$= A_i(K_i^{*\alpha_i}L_i^{*1-\alpha_i})^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N}\gamma_{ij}^{\sigma_{ij}})(\prod_{j=1}^{N}Q_j^{*\sigma_{ij}})[\prod_s(Q_s^*)^{\theta_s}]^{\lambda_i}[\theta_i\prod_s(\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i}\prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}$$

$$= A_i[(\chi_i^{k*}K)^{\alpha_i}(\chi_i^{l*}L)^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N}\gamma_{ij}^{\sigma_{ij}})[\theta_i\prod_s(\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i}\prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}(\prod_{s=1}^{N}Q_s^{*\sigma_{is}+\lambda_i\theta_s}). \quad (16)$$

Define

$$\chi_{Qi}^* = A_i[(\chi_i^{k*}K)^{\alpha_i}(\chi_i^{l*}L)^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N}\gamma_{ij}^{\sigma_{ij}})[\theta_i\prod_s(\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i}\prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}. \quad (17)$$

The above equation can be written as

$$Q_i^* = \chi_{Qi}^*(\prod_{s=1}^{N}Q_s^{*\sigma_{is}+\lambda_i\theta_s}). \quad (18)$$

Q.E.D.

Deriving allocative efficiency   Taking the log of Equation (18) gives $\log Q_i^* = \log\chi_{Qi}^* + \sum_{j=1}^{N}[(\sigma_{ij}+\lambda_i\theta_j)\log(Q_j^*)]$. Let $q^* = [\log(Q_1^*),\ldots,\log(Q_N^*)]'_{N\times1}$, Equation (18) can be written as

$$q_{N\times1}^* = b_{N\times1}^* + \Omega_{N\times N}q_{N\times1}^*,$$

where $b^*(i) = \log\chi_{Qi}^*$ and $\Omega(i,j) = \sigma_{ij} + \lambda_i\theta_j$.

Therefore, q can be solved as $q = Cb^*$, where $C_{N\times N} = (I-\Omega)^{-1}$ and $Q_n^* = \prod_{i=1}^{N}(\chi_{Qi}^{*C_{ni}})$.

Rewrite Equation (17) as

$$\chi_{Qi}^* = z_i^* K^{\alpha_i(1-\sigma_i-\lambda_i)}L^{(1-\alpha_i)(1-\sigma_i-\lambda_i)},$$

where $z_i^* = A_i[(\chi_i^{k*})^{\alpha_i}(\chi_i^{l*})^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N}\gamma_{ij}^{\sigma_{ij}})[\theta_i\prod_s(\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i}\prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}$.

Then Q_n^* can be rewritten as

$$Q_n^* = \prod_{i=1}^{N}(\chi_{Qi}^{*C_{ni}}) = \tilde{A}_n^* K^{\tilde{\alpha}_n}L^{\tilde{\beta}_n}, \quad (19)$$

where $\tilde{A}_n^* = \{\prod_{i=1}^N z_i^{*C_{ni}}\}$ and $\tilde{\alpha}_n = \sum_i(\alpha_i(1-\sigma_i-\lambda_i)C_{ni})$, $\tilde{\beta}_n = \sum_i((1-\alpha_i)(1-\sigma_i-\lambda_i)C_{ni})$.

Aggregate output under optimal allocation can be written as a function of aggregate capital K and aggregate labor L:

$$Y^* = \prod_i Y_i^{*\theta_i} = \prod_i (\chi_i^{y*}\tilde{A}_i^* K^{\tilde{\alpha}_i} L^{\tilde{\beta}_i})^{\theta_i} = \bar{A}^* K^{\bar{\alpha}} L^{\bar{\beta}}, \tag{20}$$

where $\bar{A}^* = \prod_{i=1}^N (\chi_i \tilde{A}_i^*)^{\theta_i}$ is the aggregate TFP under optimal allocation and $\bar{\alpha} = \sum_n(\tilde{\alpha}_n \theta_n)$, $\bar{\beta} = \sum_n(\tilde{\beta}_n \theta_n)$.

Replacing Q_s^* in Equation (12) using the expression in (19), we can write the expenditure on imported good j as

$$\bar{P}_j m_{ij}^* = [\prod_s (\chi_s^{y*}\tilde{A}_s^*)^{\theta_s}]\{\frac{\theta_i}{\chi_i^{y*}}K^{\sum_s \theta_s \tilde{\alpha}_s} L^{\sum_s \theta_s \tilde{\beta}_s}\}\lambda_{ij} = (\frac{\theta_i \lambda_{ij}}{\chi_i^{y*}})Y^*.$$

The total expenditure on imported goods is

$$X^* = [\sum_{i=1}^N (\frac{\theta_i \lambda_i}{\chi_i^{y*}})]Y^*.$$

The output net of imported goods is

$$C^* = Y^* - X^* = Y^*[1 - \sum_{i=1}^N (\frac{\theta_i \lambda_i}{\chi_i^{y*}})].$$

Next, we write the data output Y as a function of data allocation (without the stars). The data analog of Equation (16) is

$$Q_i = A_i(K_i^{\alpha_i} L_i^{1-\alpha_i})^{1-\sigma_i-\lambda_i}(\gamma_{i1}Q_1)^{\sigma_{i1}} \cdots (\gamma_{iN}Q_N)^{\sigma_{iN}} \prod_{j=1}^N \{\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s} \prod_s (Q_s)^{\theta_s} \frac{\lambda_{ij}}{\bar{P}_j}\}^{\lambda_{ij}}. \tag{21}$$

Let $\chi_{Qi} = A_i[(\chi_i^k K)^{\alpha_i}(\chi_i^l L)^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^N \gamma_{ij}^{\sigma_{ij}})[\theta_i \prod_s(\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i} \prod_{j=1}^N (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}$. Equation (21) can be written as

$$Q_i = \chi_{Qi}(\prod_{s=1}^N Q_s^{\sigma_{is}+\lambda_i\theta_s}).$$

Let $q = [\log(Q_1), \ldots, \log(Q_N)]'_{N\times 1}$, we can solve q as

$$q_{N\times 1} = b_{N\times 1} + \Omega_{N\times N} q_{N\times 1},$$

where $b(i) = \log\chi_{Qi}$ and $\Omega(i,j) = \sigma_{ij} + \lambda_i\theta_j$.

Therefore, q can be solved as $q = Cb$, where $C_{N\times N} = (I - \Omega)^{-1}$.

Then,

$$Q_n = \prod_{i=1}^{N}(\chi_{Qi}^{C_{ni}}) = \tilde{A}_n K^{\tilde{\alpha}_n} L^{\tilde{\beta}_n},$$

where $\tilde{A}_n = \{\prod_{i=1}^{N} z_i^{C_{ni}}\}$ and $\tilde{\alpha}_n = \sum_i(\alpha_i(1-\sigma_i-\lambda_i)C_{ni})$, $\tilde{\beta}_n = \sum_i((1-\alpha_i)(1-\sigma_i-\lambda_i)C_{ni})$.

We can write the data output as

$$Y = \prod_i Y_i^{\theta_i} = \prod_i(\chi_i^y \tilde{A}_i K^{\tilde{\alpha}_i} L^{\tilde{\beta}_i})^{\theta_i} = \bar{A}K^{\bar{\alpha}}L^{\bar{\beta}}, \tag{22}$$

where $\bar{A} = \prod_{i=1}^{N}(\chi_i^y \tilde{A}_i)^{\theta_i}$ is the aggregate TFP in the data.

In addition, we assume that the expenditure shares of imported intermediate goods are not distorted, such that

$$\bar{P}_j m_{ij} = \lambda_{ij} P_i Q_i = \frac{\lambda_{ij} P_i Y_i}{\chi_i^y} = \frac{\theta_i \lambda_{ij}}{\chi_i^y}Y.$$

Thus,

$$X = [\sum_{i=1}^{N}(\frac{\theta_i \lambda_i}{\chi_i^y})]Y$$

and

$$C = Y - X = (1 - \sum_{i=1}^{N}(\frac{\theta_i \lambda_i}{\chi_i^y}))Y.$$

Now we can compute the allocative efficiency as

$$\mathbf{E} = \frac{C}{C^*} = \frac{[1 - \sum_{n=1}^{N}(\frac{\theta_n \lambda_n}{\chi_n^y})]\prod_{n=1}^{N}(\chi_n^y \tilde{A}_n)^{\theta_n} K^{\bar{\alpha}} L^{\bar{\beta}}}{[1 - \sum_{n=1}^{N}(\frac{\theta_n \lambda_n}{\chi_n^{y*}})]\prod_{n=1}^{N}(\chi_n^{y*} \tilde{A}_n^*)^{\theta_n} K^{\bar{\alpha}} L^{\bar{\beta}}},$$

where

$$\frac{\tilde{A}_n}{\tilde{A}_n^*} = \prod_{i=1}^{N}\{\frac{A_i(\chi_i^{k\alpha_i}\chi_i^{l1-\alpha_i})^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N}\gamma_{ij}^{\sigma_{ij}})[\theta_i \prod_s(\frac{\chi_s^y}{\chi_i^y})^{\theta_s}]^{\lambda_i}\prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}}{A_i(\chi_i^{k*\alpha_i}\chi_i^{l*1-\alpha_i})^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N}\gamma_{ij}^{*\sigma_{ij}})[\theta_i \prod_s(\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s}]^{\lambda_i}\prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}}\}^{C_{ni}}$$

$$= \prod_{i=1}^{N}\{[(\frac{\chi_i^k}{\chi_i^{k*}})^{\alpha_i}(\frac{\chi_i^l}{\chi_i^{l*}})^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}\frac{[\prod_s(\frac{\chi_s^y}{\chi_i^y})^{\theta_s}]^{\lambda_i}}{[\prod_s(\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s}]^{\lambda_i}}\prod_{j=1}^{N}(\frac{\gamma_{ij}}{\gamma_{ij}^*})^{\sigma_{ij}}\}^{C_{ni}}.$$

Rearranged, we get

$$\mathbf{E} = E^{kl}E^d E^m E^y,$$

where

- $E^{kl} = \prod_{i=1}^{N}(((\frac{\chi_i^k}{\chi_i^{k*}})^{\alpha_{i,t}}(\frac{\chi_i^l}{\chi_i^{l*}})^{1-\alpha_i})^{1-\sigma_i-\lambda_i})^{\sum_n \theta_n C_{ni}}$,

- $E^d = \prod_{i=1}^{N}(\prod_{j=1}^{N}(\frac{\gamma_{ij}}{\gamma_{ij}^*})^{\sigma_{ij}})^{\sum_n \theta_n C_{ni}}$,

- $E^m = \frac{1-\sum_{n=1}^{N}\frac{\theta_n \lambda_n}{\chi_n^y}}{1-\sum_{n=1}^{N}\frac{\theta_n \lambda_n}{\chi_n^{y*}}}$,

- $E^y = \prod_{n=1}^{N}(\frac{\chi_n^y}{\chi_n^{y*}})^{\theta_{n,t}} \prod_{i=1}^{N}(\frac{\prod_s(\frac{\chi_s^y}{\chi_i^y})^{\theta_s}}{\prod_s(\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s}})^{\lambda_i \sum_n(\theta_n C_{ni})}$.

In addition, we can show that the value-added aggregate production function that features a constant returns to scale. That is, $\bar{\alpha} + \bar{\beta} = 1$. To show this, we only need to show that $(\tilde{\alpha}_n + \tilde{\beta}_n) = 1$. Then it follows that $\bar{\alpha} + \bar{\beta} = \sum_n((\tilde{\alpha}_n + \tilde{\beta}_n)\theta_n) = \sum_n \theta_n = 1$.

To show that $\tilde{\alpha}_n + \tilde{\beta}_n = 1$, let $B = I - \Omega$. Therefore,

$$\sum_j B(i,j) = \sum_j(1 - (\sigma_i + \lambda_i \theta_j)) = 1 - (\sigma_i + \lambda_i).$$

The first equality is because of the definition of Ω. The second equality holds because $\sum_j \theta_j = 1$. Note that

$$\tilde{\alpha}_n + \tilde{\beta}_n = \sum_i(C_{ni}(1 - \sigma_i - \lambda_i)) = \sum_i \sum_j C(n,i)B(i,j).$$

Since by definition, $BC = CB = I$, $\sum_j \sum_i C(n,i)B(i,j) = 1$ holds for any n. Therefore, $(\tilde{\alpha}_n + \tilde{\beta}_n) = 1$.

B.3   Measuring allocative efficiency in a CES production system

Solving planner's problem   Recall the planner's problem:

$$max \ Y = (\sum_i \omega_i Y_i^{1-\frac{1}{\rho}})^{\frac{\rho}{\rho-1}},$$

$$s.t \ \ \sum_i K_i = K, \sum_i L_i = L, Y_i = A_i(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}.$$

FOC wrt K_i is

$$[K_i] : \omega_i Y_i^{*1-\frac{1}{\rho}} = \frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})\eta_K Y^{*-\frac{1}{\rho}},$$

where the multiplier η_k measures the marginal production for capital in each sector.

Summing up the LHS of the above equation over all sectors gives

$$Y^{*\frac{\rho-1}{\rho}} = \sum \omega_i Y_i^{*1-\frac{1}{\rho}} = \sum \frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})\eta_K Y^{*-\frac{1}{\rho}},$$

$$\frac{Y^*}{\sum \frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})} = \eta_K = \omega_i Y^{*\frac{1}{\rho}} Y_i^{*-\frac{1}{\rho}} \frac{Y_i^*}{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}\nu_i K_i^{*-\frac{1}{\varepsilon}},$$

$$\omega_i\left(\frac{Y_i^*}{Y^*}\right)^{1-\frac{1}{\rho}} = \frac{\frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})}{\sum_i \frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})},$$

where the first equation on the second line follows directly from the first line and the second equation of the second line comes from the FOC w.r.t. K_i. Note that the numerator can be written as

$$\frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}) = \frac{K_i^{*\frac{1}{\varepsilon}-1}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})K_i^*$$

$$= \frac{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}}}K_i^* = \frac{K_i^*}{\frac{\nu_i K_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}}+(1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}}.$$

Let $\alpha_i^* = \frac{\nu_i K_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}}+(1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}$, rewrite the above equation as

$$\frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}) = \frac{K_i^*}{\alpha_i^*}.$$

We next apply a similar approach to labor:

$$\omega_i\left(\frac{Y_i^*}{Y^*}\right)^{1-\frac{1}{\rho}} = \frac{\frac{L_i^{*\frac{1}{\varepsilon}}}{1-\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})}{\sum_i \frac{L_i^{*\frac{1}{\varepsilon}}}{1-\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})}.$$

Again, the numerator of the above equation can be written as

$$\frac{L_i^{*\frac{1}{\varepsilon}}}{1-\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}) = \frac{L_i^*}{\frac{(1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}}+(1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}} = \frac{L_i^*}{1-\alpha_i}.$$

To summarize, so far we have derived three equations for the planner's problem,

$$\alpha_i^* = \frac{\nu_i K_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}, \ \ \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} = \frac{\frac{K_i^*}{\alpha_i}}{\sum_i \frac{K_i^*}{\alpha_i}}, \ \ \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} = \frac{\frac{L_i^*}{1-\alpha_i}}{\sum_i \frac{L_i^*}{1-\alpha_i}}.$$

To further simplify the notation, let $\tilde{K}_i^* = K_i^*/\alpha_i^*$ and $\tilde{L}_i^* = L_i^*/(1-\alpha_i^*)$. Using the last two equations from the line above, we get $\frac{\tilde{K}_i^*}{\tilde{L}_i^*} = \frac{\sum_i \tilde{K}_i^*}{\sum_i \tilde{L}_i^*}$. Further, let $\tilde{K}^* = \sum_i \tilde{K}_i^*$ and $\tilde{L}^* = \sum_i \tilde{L}_i^*$. The last two equations from the line above can be written as

$$K_i^* = \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}}\tilde{K}^*\alpha_i^*; \ \ L_i^* = \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}}\tilde{L}^*(1-\alpha_i^*).$$

Substitute K_i^* and L_i^* in the sector i's production function:

$$Y_i^* = A_i\{\nu_i[\omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}}\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}}\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}}A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}.$$

Therefore,

$$1 = \omega_i(\frac{Y_i^*}{Y^*})^{-\frac{1}{\rho}}\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*},$$

$$1 = \omega_i^{1-\rho}(\frac{Y_i^*}{Y^*})^{-\frac{1-\rho}{\rho}}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{1-\rho},$$

$$1 = \omega_i^{1-\rho}(\frac{Y_i^*}{Y^*})^{\frac{\rho-1}{\rho}}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{1-\rho},$$

$$\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1} = \omega_i(\frac{Y_i^*}{Y^*})^{\frac{\rho-1}{\rho}} = \frac{K_i^*}{\tilde{K}^*\alpha_i^*},$$

$$K_i^* = \alpha_i^*\tilde{K}^*\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1}.$$

Similarly, we apply a similar approach to labor:

$$L_i^* = (1-\alpha_i^*)\tilde{L}^*\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1}.$$

Let $H_i = A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}$, we can rewrite the above equations:

$$K_i^* = \alpha_i^*\tilde{K}^*\omega_i^{\rho}[\frac{H_i}{Y^*}]^{\rho-1}, \ \ L_i^* = (1-\alpha_i^*)\tilde{L}^*\omega_i^{\rho}[\frac{H_i}{Y^*}]^{\rho-1}.$$

Take them back to sectoral production functions:

$$Y_i^* = A_i(\nu_i K_i^* + (1-\nu_i)L_i^*)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= A_i(\nu_i(\alpha_i^* \tilde{K}^* \omega_i^\rho [\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1})^{1-\frac{1}{\varepsilon}}$$

$$+ (1-\nu_i)((1-\alpha_i^*)\tilde{L}^* \omega_i^\rho [\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1})^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \omega_i^\rho [\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1} A_i(\nu_i(\alpha_i^*\tilde{K}^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)((1-\alpha_i^*)\tilde{L}^*\omega_i^\rho)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \omega_i^\rho [\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1} A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \omega_i^\rho Y^{*1-\rho}\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^\rho.$$

Take Y_i^* back to the final good production function:

$$Y^* = (\sum_i \omega_i(\omega_i^\rho Y^{1-\rho}\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^\rho)^{1-\frac{1}{\rho}})^{\frac{\rho}{\rho-1}},$$

$$Y^{*\rho-1} = \sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}.$$

Sum K_i^* and L_i^* up across all sectors:

$$K = \sum_i K_i^* = \sum_i \alpha_i^* \tilde{K}^* \omega_i^\rho [\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1},$$

$$L = \sum_i L_i^* = \sum_i (1-\alpha_i^*)\tilde{L}^* \omega_i^\rho [\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1}.$$

Divide both sides by $\tilde{K}^*$ or $\tilde{L}^*$:

$$\frac{K}{\tilde{K}^*} = \frac{\sum_i \alpha_i^* \omega_i^\rho\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}{\sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}},$$

$$\frac{L}{\tilde{L}^*} = \frac{\sum_i (1-\alpha_i^*) \omega_i^\rho\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}{\sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}.$$

Recall that $\frac{K}{\tilde{K}^*} + \frac{L}{\tilde{L}^*} = 1$. Let $\alpha^* = \frac{K}{\tilde{K}^*}$. Then it follows that $1 - \alpha^* = \frac{L}{\tilde{L}^*}$. Rewrite the above equations with α^*:

$$\alpha^* = \frac{\sum_i \alpha_i^* \omega_i^\rho\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}{\sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}},$$

$$1 = \frac{\sum_i \frac{\alpha_i^*}{\alpha^*} \omega_i^\rho\{A_i\{\nu_i(K\frac{\alpha_i^*}{\alpha^*})^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(L\frac{1-\alpha_i^*}{1-\alpha^*})^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}{\sum_i \omega_i^\rho\{A_i[\nu_i(K\frac{\alpha_i^*}{\alpha^*})^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(L\frac{1-\alpha_i^*}{1-\alpha^*})^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}.$$

Deriving allocative efficiency   We have derived the optimal output as

$$Y^* = \{\sum_i \omega_i^\rho \{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}\}^{\frac{1}{\rho-1}}.$$

Now, replacing $A_i = \dfrac{Y_i}{(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i)^{\frac{\varepsilon}{\varepsilon-1}}}$ in this equation yields[38]

$$Y^* = \{\sum_i \omega_i^\rho \{\frac{Y_i}{(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}}[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}\}^{\frac{1}{\rho-1}}$$

$$= \{\sum_i \omega_i^\rho Y_i^{\rho-1}\{\frac{[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}}{(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}}\}^{\rho-1}\}^{\frac{1}{\rho-1}}.$$

Substitute $Y_i = (\frac{P_i Y_i/\omega_i}{PY})^{\frac{-\rho}{\rho-1}}Y$ in the above equation:

$$Y^* = \{\sum_i \omega_i^\rho (\frac{P_i Y_i/\omega_i}{PY})^\rho Y^{\rho-1}\{\frac{[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}}{(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}}\}^{\rho-1}\}^{\frac{1}{\rho-1}}$$

$$\mathbf{E_t} = \frac{Y^*}{Y} = \{\sum_i (\frac{P_i Y_i}{PY})^\rho\{\frac{\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}}{\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}}}\}^{\frac{\varepsilon}{\varepsilon-1}(\rho-1)}\}^{\frac{1}{\rho-1}}$$

$$= \{\sum_i (\frac{P_i Y_i}{PY})^\rho\{\frac{\nu_i(K\frac{\alpha_i^*}{\alpha^*})^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(L\frac{1-\alpha_i^*}{1-\alpha^*})^{1-\frac{1}{\varepsilon}}}{\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}}}\}^{\frac{\varepsilon}{\varepsilon-1}(\rho-1)}\}^{\frac{1}{\rho-1}},$$

where the last equation is derived by replacing $\tilde{K}^*$ with K/α^*.

B.4   The role of input-output linkages

Recall that incorporating input-output linkages brings about two significant modifications in measuring allocation. Firstly, it accounts for the allocation of intermediate inputs, and secondly, it adjusts the sectoral weights to reflect the relative importance of each sector, taking into account the input-output effects. This section provides a more detailed examination of the second modification—the sectoral weights in the two economies.

First, focusing on the terms that measure capital and labor allocation, Equation (2) and the E_{kl} term in Equation (4), we see that both are the weighted geometric average of sectoral level allocative efficiency but with different sets of sectoral weights: the weights are θ_i in the value-added economy and $(1 - \sigma_i - \lambda_i)\sum_n \theta_n C_{ni}$ in the input-output economy, where C is the Leontief inverse matrix.

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[38]Note that here Y_i, K_i, L_i are the data, whereas Y_i^*, K_i^*, L_i^* are the optimal allocation derived from the planner's problem.

Second, different sectoral weights are used to derive the optimal allocation of capital and labor: in the value added economy, the sectoral weight is again θ_i and in the input-output economy, the weight is $\theta_{i,t}(1 - \sigma_{i,t} - \lambda_{i,t})/(1 - \sum_j \gamma_{ji,t}^*)$.[39]

Do these weights differ between the two economies? The following paragraphs demonstrate that these weights are the same between an undistorted value-added economy and an undistorted input-output economy without *imported* intermediate goods. Furthermore, these weights are equal to the respective sectoral value-added shares in both cases.

(i) θ_i **in the value-added economy** The FOC of the final good producer gives $\theta_i = \frac{P_i Y_i}{Y}$, where $P_i Y_i$ is the value-added output of sector i and Y is the total value-added output (final good price normalized to 1).

(ii) $(1 - \sigma_i - \lambda_i) \sum_n \theta_n C_{nj}$ **in the input-output economy** Note that without international trade, matrix C is just the Leontief inverse matrix. Therefore, $\sum_n \theta_n C_{ni}$ is equal to the undistorted Domar weight, sector i's sales (gross output) to GDP. Multiplying it by sector i's value-added share $(1 - \sigma_i - \lambda_i)$ yields the value-added share of sector i.

(iii) $\frac{\theta_i(1-\sigma_i-\lambda_i)}{1-\sum_j \gamma_{ji}^*}$ **in the input-output economy** We only need to show that $\frac{\theta_i}{1-\sum_j \gamma_{ji}^*}$ is equal to the undistorted Domar weight: $\frac{\theta_i}{1-\sum_j \gamma_{ji}^*} = \frac{\theta_i}{\chi_i^*} = \frac{P_i \theta_i Q_i^*}{P_i Y_i^*} = \frac{P_i Q_i^*}{Y^*}$. The first equality is derived from the resource constraint on good i. The second equality holds because of the definition of χ_i^*, such that $\chi_i^* = Y_i^*/Q_i^*$. The last equality holds because of the optimality condition of the final good production: $\theta_i Y^* = P_i Y_i^*$.

The above discussion presents an equivalence result between the value-added economy and the input-output economy. However, it is worth noting that this equivalence result may not hold in cases where the economy experiences distortions or incorporates imported intermediate goods. We plan to investigate this result in future research.

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[39]To see this, take capital allocation as an example: the optimal shares are equal to $\frac{\theta_{i,t}\alpha_{i,t}}{\sum_i \theta_{i,t}\alpha_{i,t}}$ and $\frac{\theta_{i,t}\alpha_{i,t}(1-\sigma_{i,t}-\lambda_{i,t})/(1-\sum_j \gamma_{ji,t}^*)}{\sum_s [\theta_{s,t}\alpha_{s,t}(1-\sigma_{s,t}-\lambda_{s,t})/(1-\sum_j \gamma_{js,t}^*)]}$ in the value-added and input-output economies, respectively. These optimal shares reflect the relative importance of sectors' capital in producing the final good, which are $\theta_{i,t}\alpha_{i,t}$ and $\theta_{i,t}\alpha_{i,t}(1 - \sigma_{i,t} - \lambda_{i,t})/(1 - \sum_j \gamma_{ji,t}^*)$. In other words, since $\alpha_{i,t}$ represents the relative importance of capital in sector i, the sectoral weights are $\theta_{i,t}$ and $\theta_{i,t}(1 - \sigma_{i,t} - \lambda_{i,t})/(1 - \sum_j \gamma_{ji,t}^*)$, respectively.

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